An Optimal Power Flow Approach to Improve Power System Voltage Stability using Demand Response

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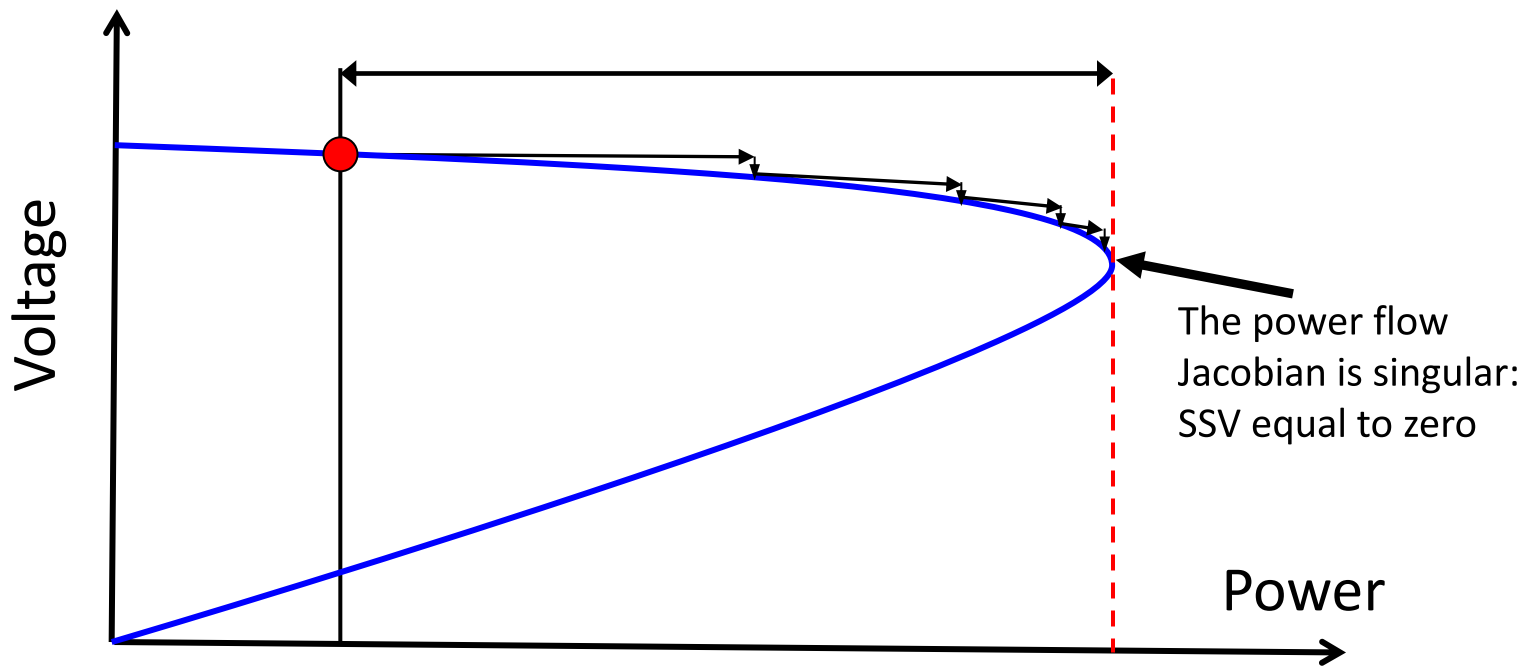
Power System Stability

- Frequency Instability
  - Associated with an imbalance between load and generation
  - Demand response based on *temporal* load shifting [Short et al. 2007; Callaway 2009; Zhang et al. 2013; Mathieu et al. 2013]
- Static Voltage Instability
  - Associated with operation that nears the limits of the network's power transfer capability
  - Demand response based on load *shedding* [Berizzi et al. 1996; Feng et al. 1998; Yu et al. 2016]

Our work: Demand response based on *spatially* shifting load, *without load shedding*, in order to improve voltage stability after a disturbance.

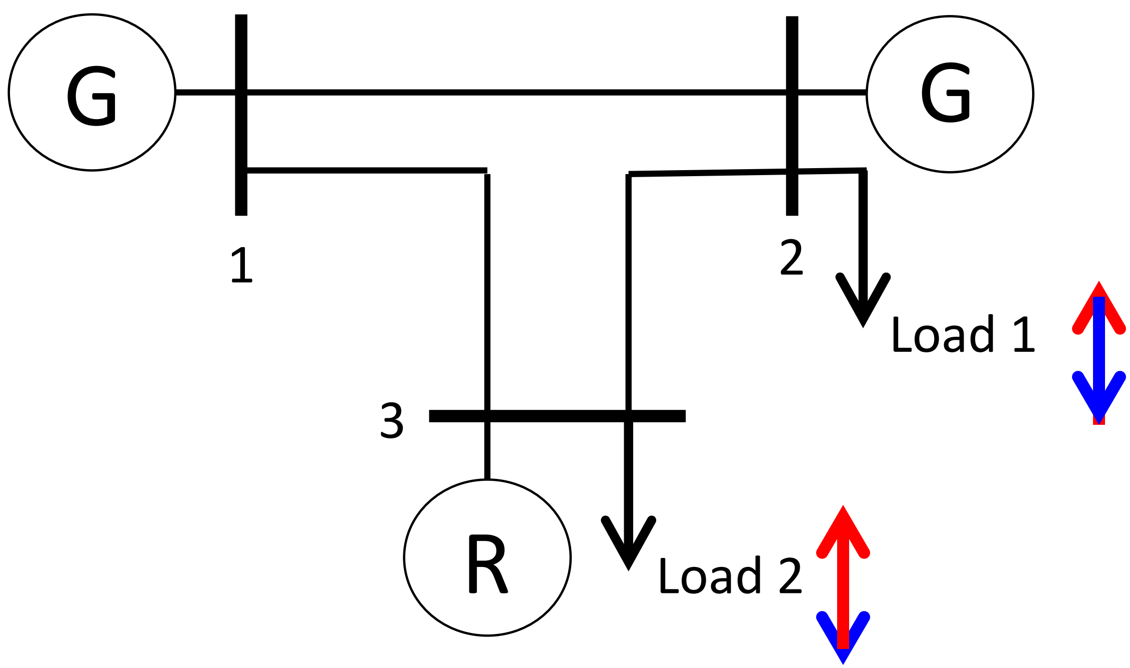
Static Voltage Stability

- Distance to the "nose point" of the PV curve
  - Often computed using continuation methods, which are difficult to embed within an optimization problem
  - The smallest singular value (SSV) of the power flow Jacobian is often used as a voltage stability metric [Tiranuchit and Thomas 1988; Tiranuchit et al. 1988; Lof et al. 1992; Berizzi et al. 1998, Berizzi et al. 2000; Mallada and Tang 2013]



Using Demand Response to Improve Voltage Stability

- Objective: maximize the smallest singular value of the power flow Jacobian via spatial shifting of flexible load
- Constraint: total demand held constant over time to maintain frequency stability

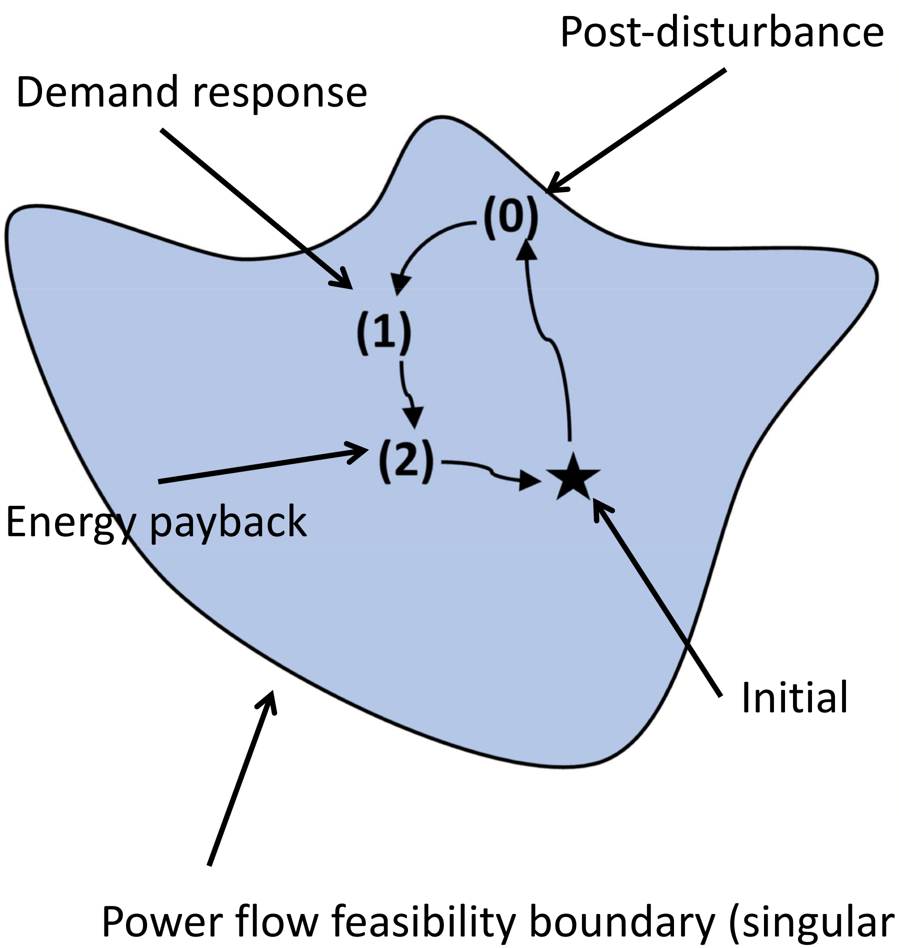


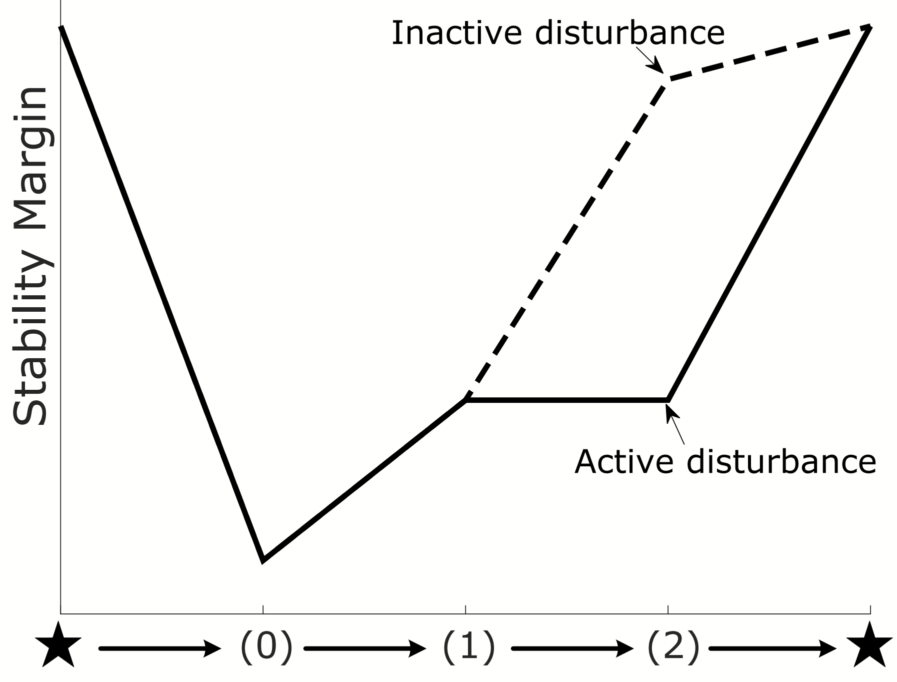
[Yao et al. Submitted 2018]

Demand Response Assumptions

- Loads are faster than generators; DR used for noncritical disturbances
- For occasional use; DR capabilities not purpose-built
- Demand response actions are contractual
  - consumers sign a contract with an aggregator, who dispatches loads within the limits of the contract; loads respond as contracted, or pay a penalty
- Demand responsive loads modeled as virtual batteries at transmission buses
- Loads modeled as “constant real/reactive power and constant power factor” (first half of talk)

Problem Description





Formulation

$$\min_{x(t)} -\alpha\sigma_0(1) + \mathcal{C}(\boldsymbol{P}_{\mathbf{g}}(2))$$ -SSV i

$$\text{s.t.} \quad (\forall t \in \mathcal{T})$$

$$\sigma_0(t) = \sigma_{\min}\{J(x(t))\}$$

$$\sum_{n\in\mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(1) = \sum_{n\in\mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(0)$$

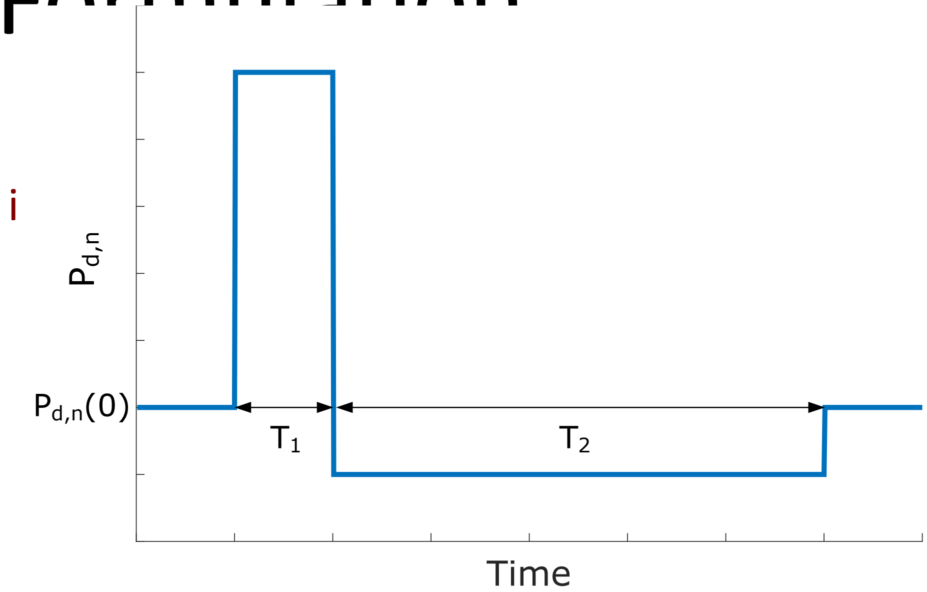
$$T_1 P_{\mathrm{d},n}(1) + T_2 P_{\mathrm{d},n}(2) = (T_1 + T_2) P_{\mathrm{d},n}(0)$$ Energy payback

$$P_{\mathrm{d},n}(t) \cdot \mu_n = Q_{\mathrm{d},n}(t)$$ Constant power load model

$$\sigma_0(2) \geq \sigma_0(1)$$ SSV in Period 2 greater than or equal to that in Period 1

$$\mathcal{F}(x(t)) = 0$$ AC power flow equations (nonlinear, nonconvex)

$$\mathcal{G}(x(t)) \leq 0$$ Generator, voltage, and line limits; etc.



Base case is to choose only DR load real power consumption in Period 1

Period 1: Generator real power production/voltage magnitudes fixed, except slack bus

Period 2: Generators adjust to payback loads

Solution Approaches

- Interior point methods [Kodsi and Canizares 2007]
  - Require the Hessian
- Semidefinite programming [Lavaei and Low 2007; Molzahn and Hiskens 2016]

$$J(t)^T J(t) - \lambda_0(t) I \succeq 0$$
$$\sigma_0(t) = \sqrt{\lambda_0(t)}$$

  - AC power flow equations need to be relaxed; solution of relaxed problem may not be feasible

Solution Approaches

- Iterative nonlinear programming [Avalos, Canizares, and Anjos 2008]
  - Singular value decomposition

$$J(t) = U(t)\Sigma(t)W(t)^T,$$

  - Around a given operating point, the approximate SSV is

$$\widetilde{\sigma}_0(t) = u_0(t)^T J(t) w_0(t),$$

  - Symbolic matrix multiplication complex for large systems; nonlinear programming problem solved in each iteration

Singular Value Sensitivities

[Tiranuchit and Thomas 1988]

- For a general problem

$$\Delta\sigma_i \approx \sum_k u_i^T \frac{\partial A}{\partial \chi_k}\bigg|_{\chi^*} w_i \Delta\chi_k,$$

- For our problem

$$\Delta\sigma_0 \approx \sum_{n\in\{\mathcal{S}_{\mathrm{PV}},\mathcal{S}_{\mathrm{PQ}}\}} \left[u_0^T \frac{\partial J}{\partial \theta_n} w_0\right] \Delta\theta_n + \sum_{n\in\mathcal{S}_{\mathrm{PQ}}} \left[u_0^T \frac{\partial J}{\partial V_n} w_0\right] \Delta V_n,$$

Iterative Linear Programming Algorithm

1. Linearize the objective function and constraints at the operating point; the decision variable is now the *change* in system states.
2. Solve the linear program to obtain the best change in system states; compute the new system states.
3. Solve the AC power flow equation to compute the new operating point.
4. Evaluate the objective function; if it isn't improving go back to step 1; otherwise end.

Linear Program

Original Formulation

$$\min_{x(t)} -\alpha\sigma_0(1) + \mathcal{C}(\boldsymbol{P}_{\mathbf{g}}(2))$$

s.t. $(\forall t \in \mathcal{T})$

$$\sigma_0(t) = \sigma_{\min}\{J(x(t))\}$$

$$\sum_{n\in\mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(1) = \sum_{n\in\mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(0)$$

$$T_1 P_{\mathrm{d},n}(1) + T_2 P_{\mathrm{d},n}(2) = (T_1 + T_2) P_{\mathrm{d},n}(0)$$

$$P_{\mathrm{d},n}(t)\cdot\mu_n = Q_{\mathrm{d},n}(t)$$

$$\sigma_0(2) \geq \sigma_0(1)$$

$$\mathcal{F}(x(t)) = 0$$

$$\mathcal{G}(x(t)) \leq 0$$

Linearization about Operating Point

$$\min_{x(t)} -\alpha\Delta\sigma_0(1) + \sum_{n\in\mathcal{S}_{\mathrm{G}}} \left.\frac{\partial\mathcal{C}(\boldsymbol{P}_{\mathbf{g}}(2))}{\partial P_{\mathrm{g},n}(2)}\right|_{\boldsymbol{P}^*_{\mathrm{g}}(2)} \Delta P_{\mathrm{g},n}(2)$$

s.t. $(\forall t \in \mathcal{T})$

$$\Delta\sigma_0(t) = \sum_{n\in\{\mathcal{S}_{\mathrm{PV}},\mathcal{S}_{\mathrm{PQ}}\}} \left[u_0(t)^T \frac{\partial J(t)}{\partial\theta_n} w_0(t)\right] \Delta\theta_n(t) + \sum_{n\in\mathcal{S}_{\mathrm{PQ}}} \left[u_0(t)^T \frac{\partial J(t)}{\partial V_n} w_0(t)\right] \Delta V_n(t)$$

$$\sum_{n\in\mathcal{S}_{\mathrm{DR}}} \Delta P_{\mathrm{d},n}(1) = 0$$

$$T_1 \Delta P_{\mathrm{d},n}(1) + T_2 \Delta P_{\mathrm{d},n}(2) = 0$$

$$\Delta P_{\mathrm{d},n}(t)\cdot\mu_n = \Delta Q_{\mathrm{d},n}(t)$$

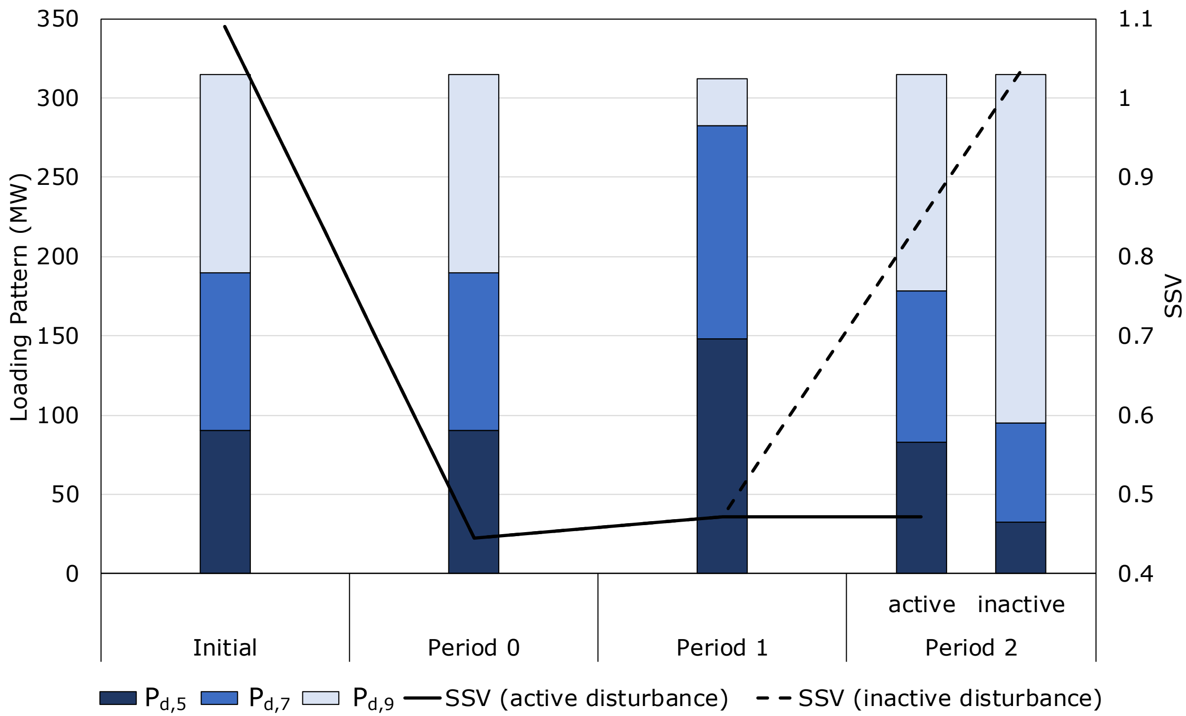
$$\sigma_0^*(2) + \Delta\sigma_0(2) \geq \sigma_0^*(1) + \Delta\sigma_0(1)$$

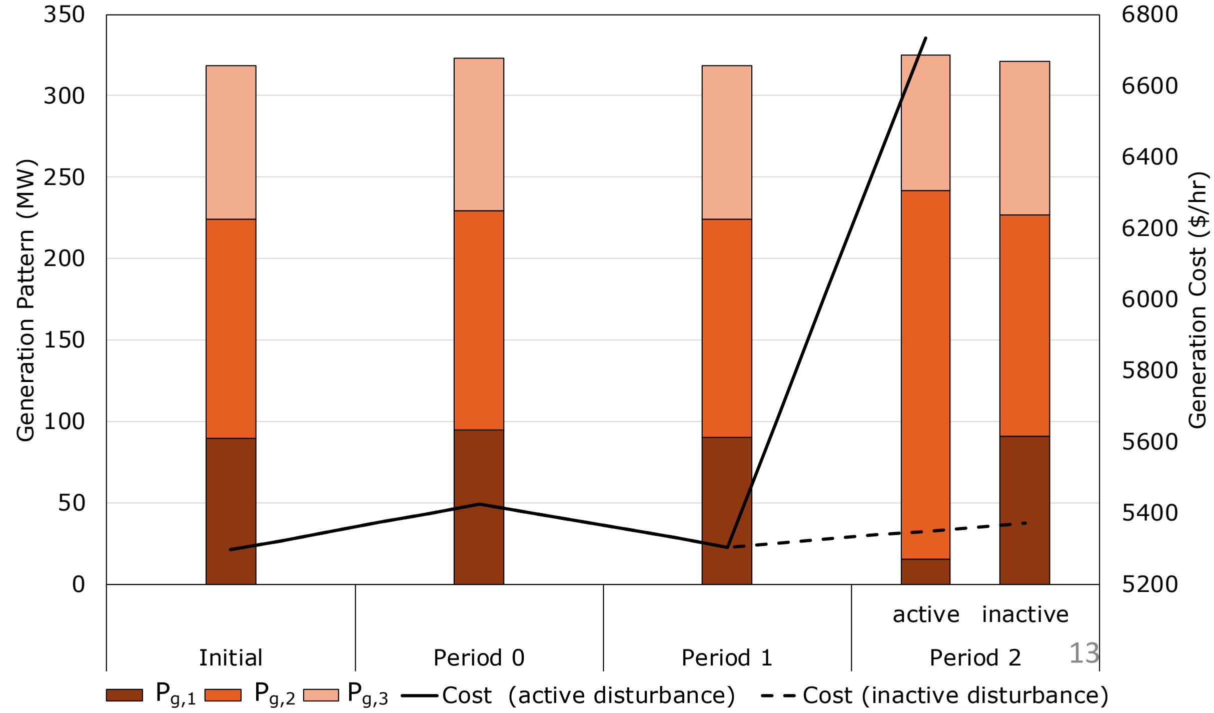
$$f(\Delta x(t)) = 0$$

$$g(\Delta x(t)) \leq 0$$

$$\Delta\sigma_0(t) \leq \overline{\Delta\sigma_0}$$

Results: IEEE 9-Bus System

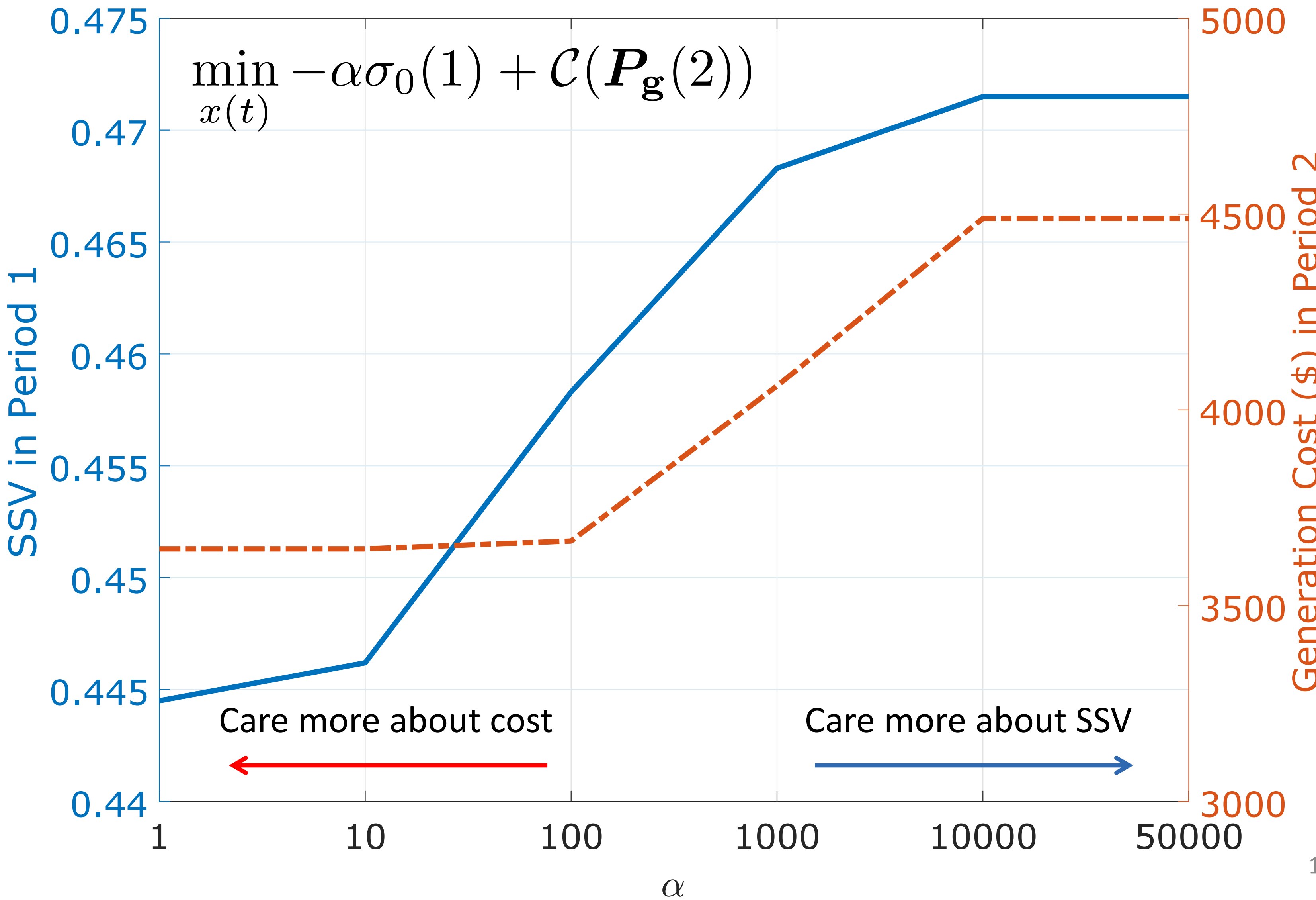




Assumptions

- All loads are 100% demand responsive
- T_1 = 5 min, T_2 is set to the minimum multiple of 5 min to achieve a feasible solution (40 min for active disturbance, 5 min for inactive disturbance)

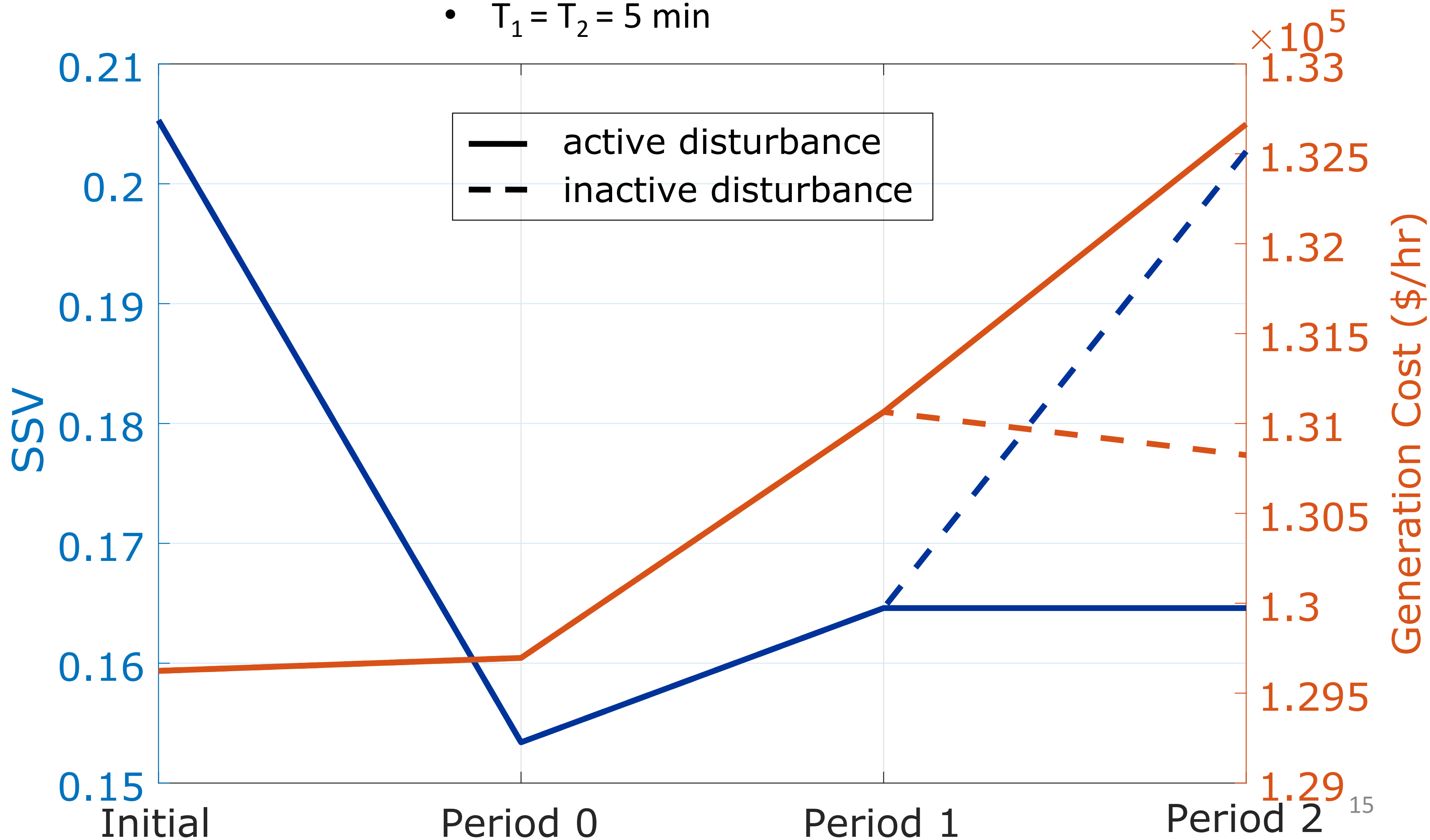
Effect of Cost Function Weighting



Results: IEEE 118-Bus System

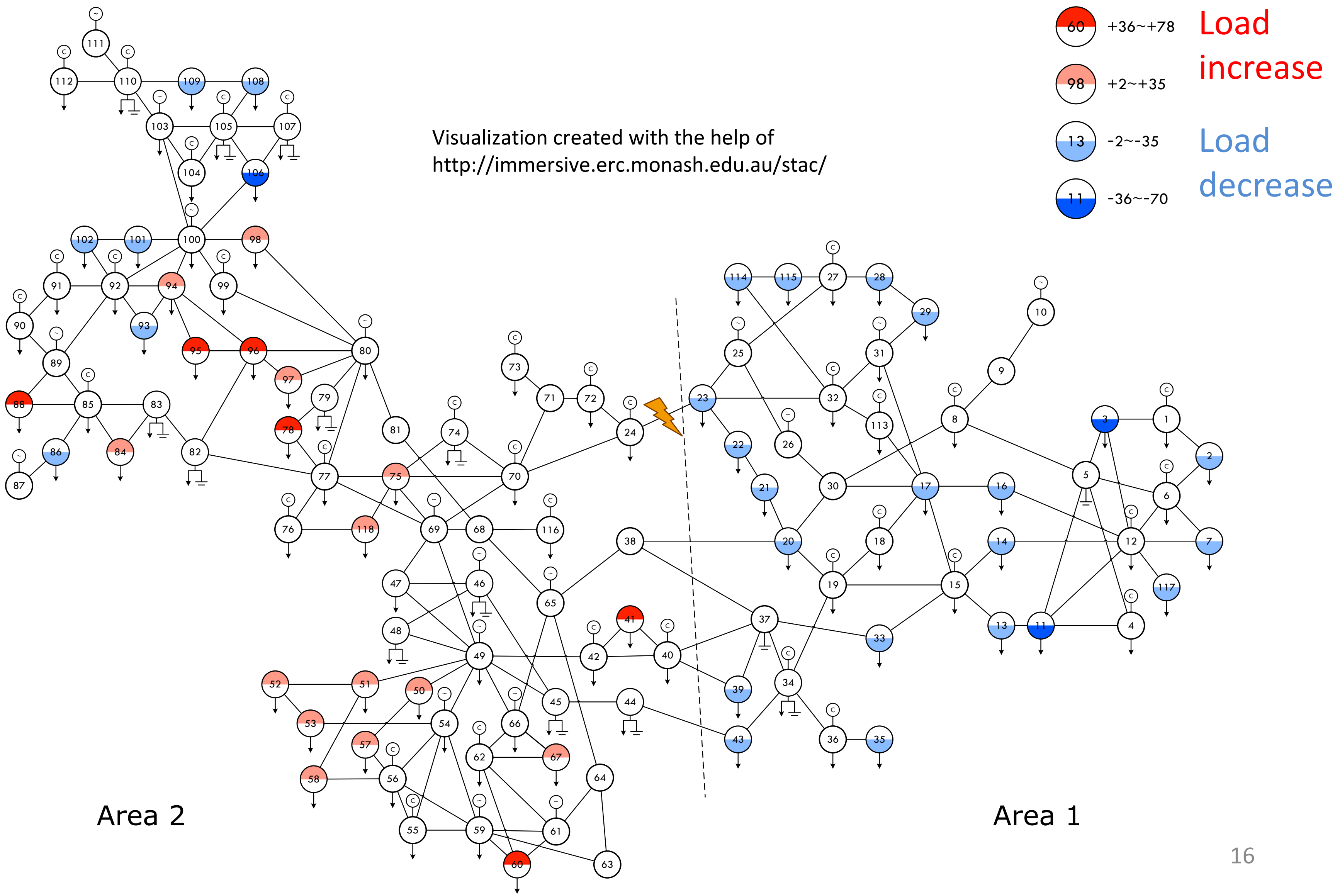
Assumptions

- Loads at PQ buses are 100% demand responsive
- $T_1 = T_2 = 5$ min



Loading changes in IEEE 118-Bus System

Visualization created with the help of
http://immersive.erc.monash.edu.au/stac/



Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$ (slack bus)	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n}, Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
Optimal SSV	0.4715	0.4703	0.4732	0.4569	0.4783	0.4469	0.4885	0.4802
Percent improvement	6.1	5.8	6.5	2.3	7.6	0.5	9.9	8.0
Generation cost ($/hr)	5304.6	5424.5	8270.4	5501.6	8502.6	5424.5	7107.8	5428.1

Base Case

Cost of Period 1 only

Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$ (slack bus)	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n},\ Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
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Generation cost ($/hr)	5304.6	5424.5	8270.4	5501.6	8502.6	5424.5	7107.8	5428.1

$$\sum_{n \in \mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(1) = \sum_{n \in \mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(0)$$

Slack bus manages change in losses

$$\sum_{n \in \mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(1) = \sum_{n \in \mathcal{S}_{\mathrm{DR}}} P_{\mathrm{d},n}(0) + \varepsilon \left(P_{\mathrm{loss}}(0) - P_{\mathrm{loss}}(1) \right)$$

Loads manage change in losses

Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$ (slack bus)	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n}, Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
Optimal SSV	0.4715	0.4703	0.4732	0.4569	0.4783	0.4469	0.4885	0.4802
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Loads vs. Generators

Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$ (slack bus)	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n}, Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
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Generation cost ($/hr)	5304.6	5424.5	8270.4	5501.6	8502.6	5424.5	7107.8	5428.1

Loads vs. Generators with Ramp Rates

Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$ (slack bus)	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n}, Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
Optimal SSV	0.4715	0.4703	0.4732	0.4569	0.4783	0.4469	0.4885	0.4802
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Benefit of Voltage Control

Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n}, Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
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Voltage Control Alone

Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n}, Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
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Just Loads vs. Everything (Optimistic)

Varying the Formulation

Case	1	2	3	4	5	6	7	8
$P_{\mathrm{g,ref}}$ (slack bus)	✓		✓	✓	✓		✓	✓
$P_{\mathrm{g},n} \forall n \in \mathcal{S}_{\mathrm{PV}}$			✓	✓	✓		✓	✓
$V_n \forall n \in \mathcal{S}_{\mathrm{G}}$					✓	✓	✓	✓
$P_{\mathrm{d},n}, Q_{\mathrm{d},n} \forall n \in \mathcal{S}_{\mathrm{DR}}$	✓	✓					✓	✓
1% Ramp Rate				✓				✓
ε	0	1	N/A	N/A	N/A	N/A	0	0
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Generation cost ($/hr)	5304.6	5424.5	8270.4	5501.6	8502.6	5424.5	7107.8	5428.1

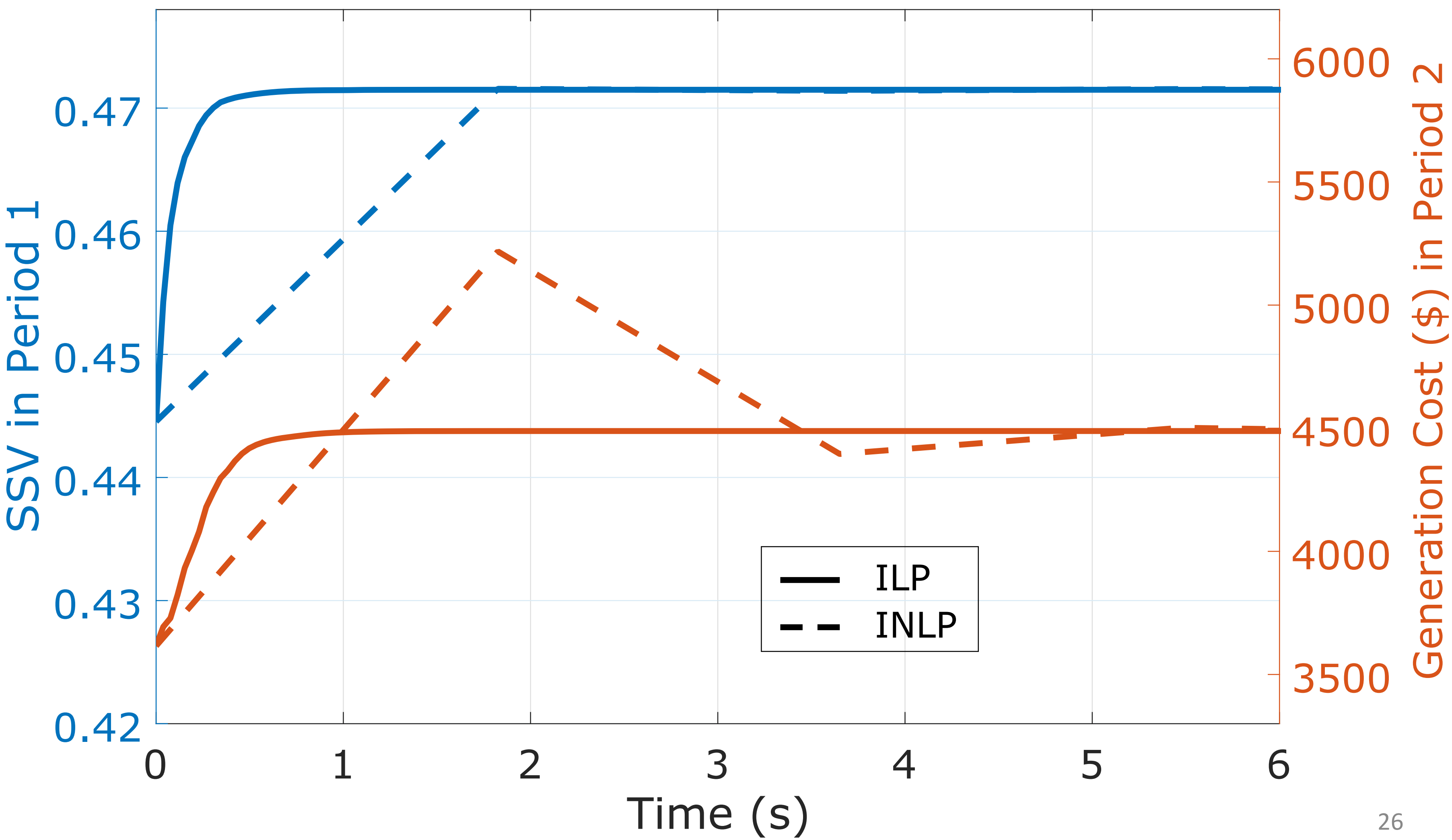
Just Loads vs. Everything (Bounded)

Cost of Different Strategies

Hourly cost of all Periods (DR, Energy Payback, OPF with SSV constraint) to achieve the same SSV

T_{restored}	Resource	9-bus	118-bus
5 min	DR	5303	129545
	Generation	5360	129905
1 hour	DR	6441	132777
	Generation	6043	132961

Computational Time



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Comparison to Load Shedding

- Formulated/solved optimization problem to minimize load shedding to achieve an SSV at least as good as the one obtained with spatial load shifting [similar to Berizzi et al. 1996]

- For the IEEE 9-bus system the load would need to drop 17% to achieve the same SSV we obtain by spatial load shifting without any net load shedding

Voltage-Dependent Load Models

- If we explicitly model the flexible load's voltage dependence, what will be the effect on the optimal operating point and the smallest singular value improvement?
- Why is this important?
  - Maybe we can get away with simple load models…
  - Insights into which types of systems would stand to benefit more…

[Yao et al. Allerton 2017]

Models

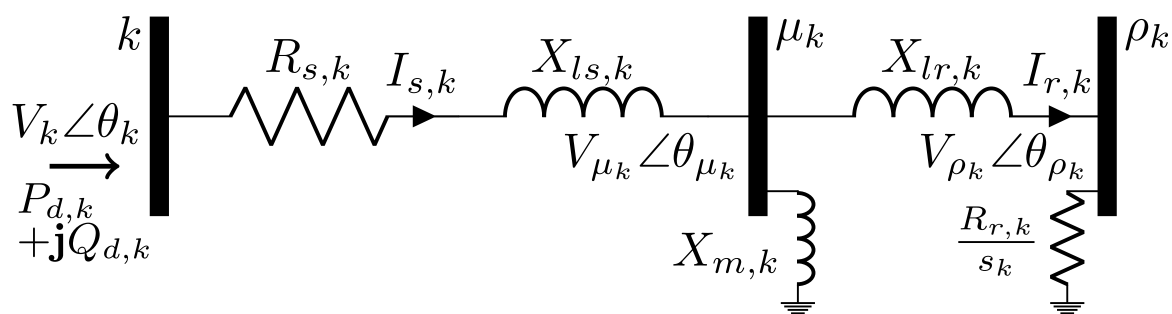
- ZIP model

$$\mathcal{F}_k^{ZP}(V_k, \varepsilon_k) = \varepsilon_k P_{d,k}^0 \left[a_{1,k} \left(\frac{V_k}{V_k^0} \right)^2 + a_{2,k} \left(\frac{V_k}{V_k^0} \right) + a_{3,k} \right]$$

$$\mathcal{F}_k^{ZQ}(V_k, \varepsilon_k) = \varepsilon_k Q_{d,k}^0 \left[b_{1,k} \left(\frac{V_k}{V_k^0} \right)^2 + b_{2,k} \left(\frac{V_k}{V_k^0} \right) + b_{3,k} \right]$$

where ε_k is the ratio between the controlled and nominal demand

- Induction machine model



[Molzahn 2017]

Impact on the Formulation

- Conventional Jacobian

$$J_{\text{cnv}} = \begin{bmatrix} \frac{\partial \mathcal{F}_i^P}{\partial \theta_i} & \frac{\partial \mathcal{F}_i^P}{\partial V_j} \\ \frac{\partial \mathcal{F}_j^Q}{\partial \theta_i} & \frac{\partial \mathcal{F}_j^Q}{\partial V_j} \end{bmatrix}$$

- Jacobian with ZIP models

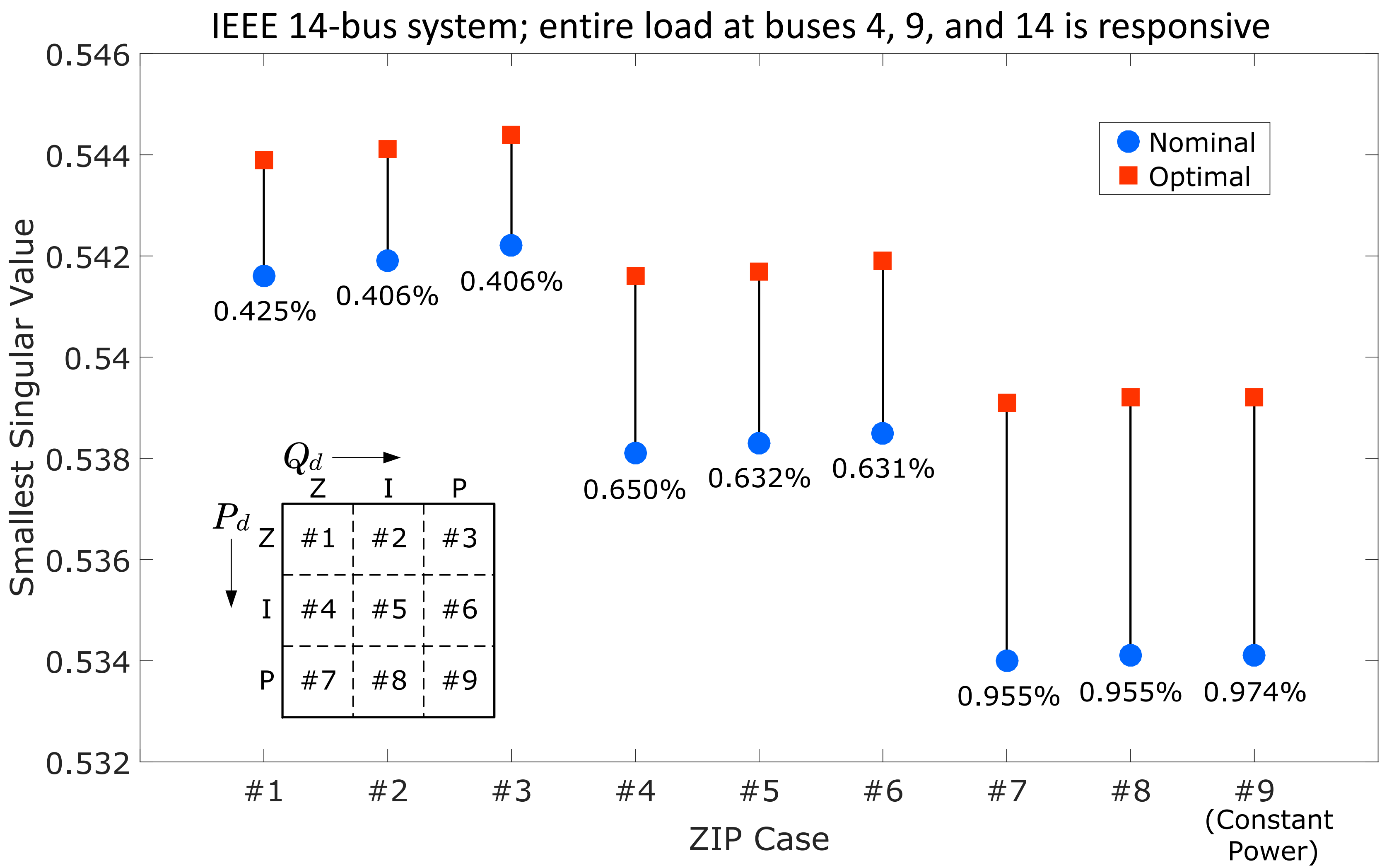
$$J_{\text{ZIP}} = J_{\text{cnv}} + \begin{bmatrix} \mathbf{0}_{n-1\times n-1} & \frac{\partial \mathcal{F}_i^{ZP}}{\partial V_j} \\ \mathbf{0}_{n_{\text{pq}}\times n-1} & \frac{\partial \mathcal{F}_j^{ZQ}}{\partial V_j} \end{bmatrix}$$

- Jacobian with induction machine models

$$J_{\text{IM}} = \begin{bmatrix} J_{\text{cnv}} & \mathbf{0}_{m\times 5n_{\text{dr}}} \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathcal{F}_i^{IP}}{\partial \theta_i} & \frac{\partial \mathcal{F}_i^{IP}}{\partial V_j} & \frac{\partial \mathcal{F}_i^{IP}}{\partial \theta_{\mu,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial V_{\mu,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial \theta_{\rho,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial V_{\rho,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial s_k} \\ \frac{\partial \mathcal{F}_j^{IQ}}{\partial \theta_i} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial V_j} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial \theta_{\mu,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial V_{\mu,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial \theta_{\rho,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial V_{\rho,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial s_k} \end{bmatrix}$$

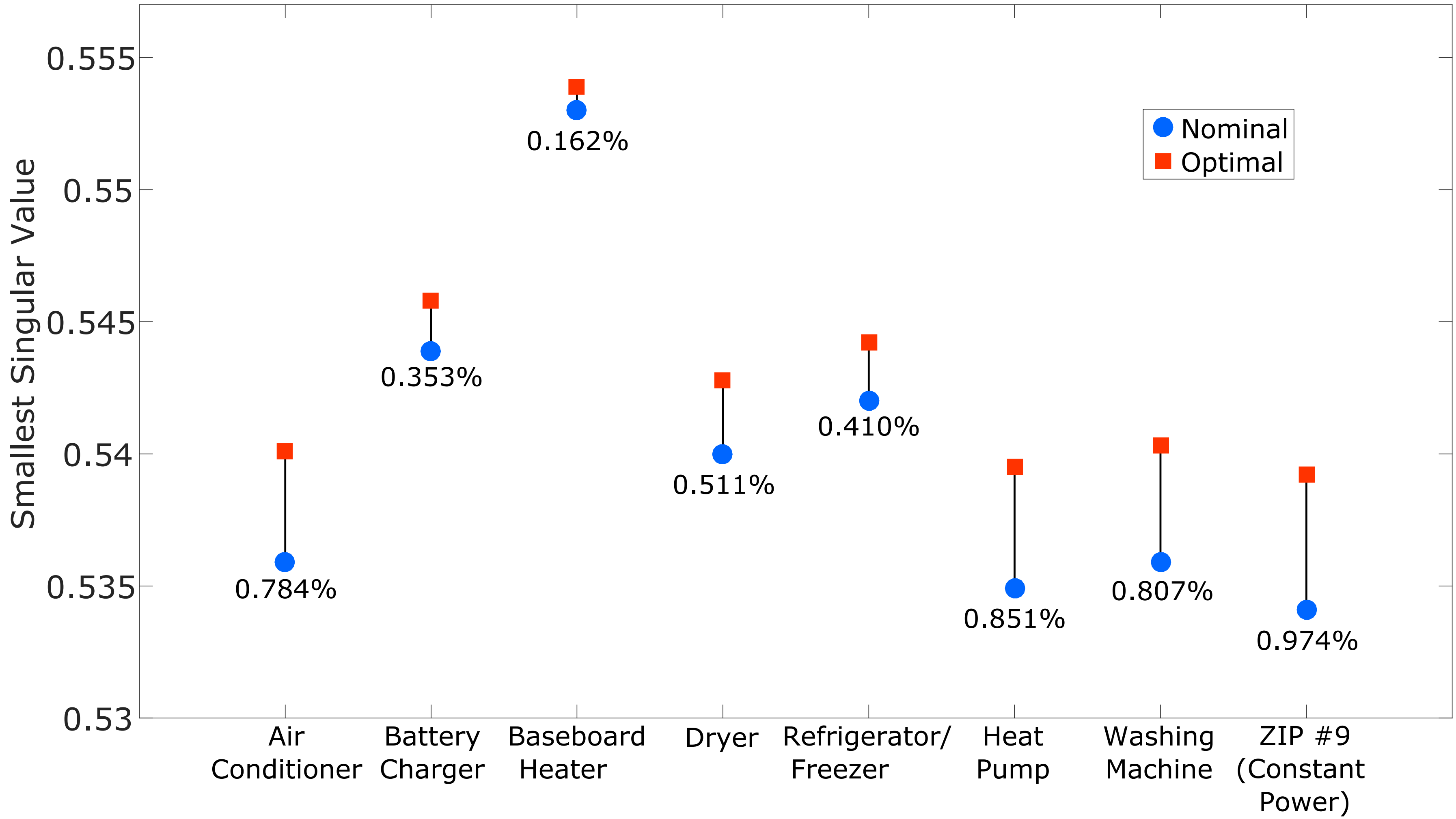
The smallest singular value sensitivity expression now contains partial derivatives with respect to the ZIP states and the induction machine states. $\chi_{\text{ZIP}} = [\theta_i, V_j, \varepsilon_k\ ]^T$ $\chi_{\text{IM}} = [\theta_i, V_j, \theta_{\mu,k}, V_{\mu,k}, \theta_{\rho,k}, V_{\rho,k}, s_k]^T$

Smallest Singular Values for Single-Component ZIP Loads



Smallest Singular Values for Flexible Loads

All flexible load modeled with ZIP parameters corresponding to the load type.



Optimal Loading Patterns

Load Model Jacobian	Nominal Constant Power J_{cnv}		Optimal Constant Power J_{cnv}		Optimal ZIP #3 J_{ZIP}		Optimal IM J_{IM}	
	P_d	Q_d	P_d	Q_d	P_d	Q_d	P_d	Q_d
Bus 4	47.80	32.62	92.20	62.90	92.20	63.99	90.99	48.90
Bus 9	29.50	20.73	0.00	0.00	0.00	0.00	0.62	15.55
Bus 14	14.90	16.39	0.00	0.00	0.00	0.00	0.59	15.79
SSV	0.5341		0.5393		0.5444		2.4533	

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Optimal Loading Patterns

ZIP #9

Load Model Jacobian	Nominal Constant Power J_{cnv}		Optimal Constant Power J_{cnv}		Optimal ZIP #3 J_{ZIP}		Optimal IM J_{IM}	
	P_d	Q_d	P_d	Q_d	P_d	Q_d	P_d	Q_d
Bus 4	47.80	32.62	92.20	62.90	92.20	63.99	90.99	48.90
Bus 9	29.50	20.73	0.00	0.00	0.00	0.00	0.62	15.55
Bus 14	14.90	16.39	0.00	0.00	0.00	0.00	0.59	15.79
SSV	0.5341		0.5393		0.5444		2.4533	

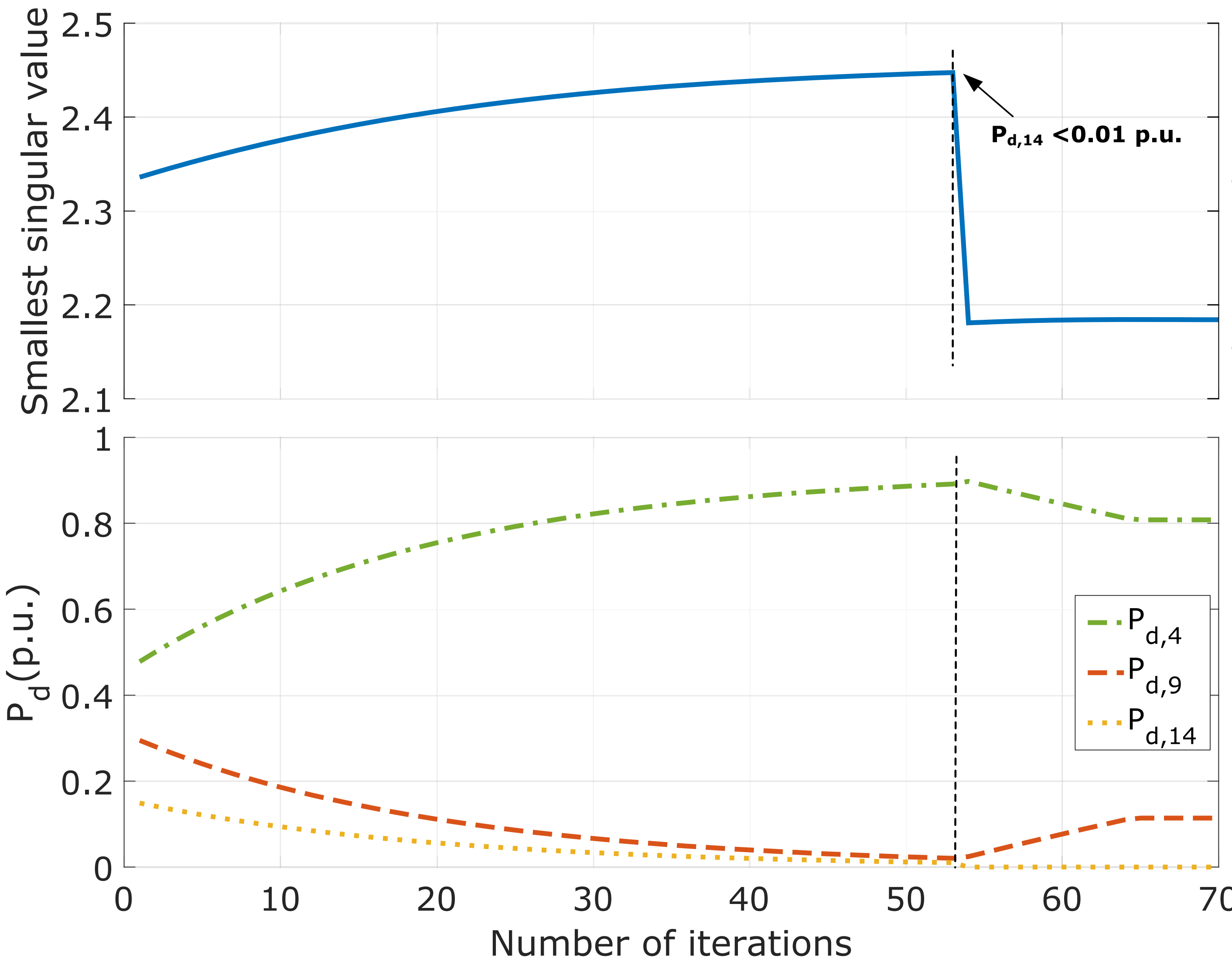
Though the SSVs aren’t comparable across different load models P_d is the same in all optimal ZIP cases!

Optimal Loading Patterns

Load Model Jacobian	Nominal Constant Power J_{cnv}		Optimal Constant Power J_{cnv}		Optimal ZIP #3 J_{ZIP}		Optimal IM J_{IM}	
	P_d	Q_d	P_d	Q_d	P_d	Q_d	P_d	Q_d
Bus 4	47.80	32.62	92.20	62.90	92.20	63.99	90.99	48.90
Bus 9	29.50	20.73	0.00	0.00	0.00	0.00	0.62	15.55
Bus 14	14.90	16.39	0.00	0.00	0.00	0.00	0.59	15.79
SSV	0.5341		0.5393		0.5444		2.4533	

IM constraints keep the load at Bus 9, 14 above zero.

Switching off the Induction Machine at Bus 14



IM at Bus 14 ON

	P_d	Q_d
Bus 4	90.99	48.90
Bus 9	0.62	15.55
Bus 14	0.59	15.79
SSV	2.4533	

IM at Bus 14 OFF

	P_d	Q_d
Bus 4	80.81	43.94
Bus 9	11.39	17.51
Bus 14	-	-
SSV	2.184	

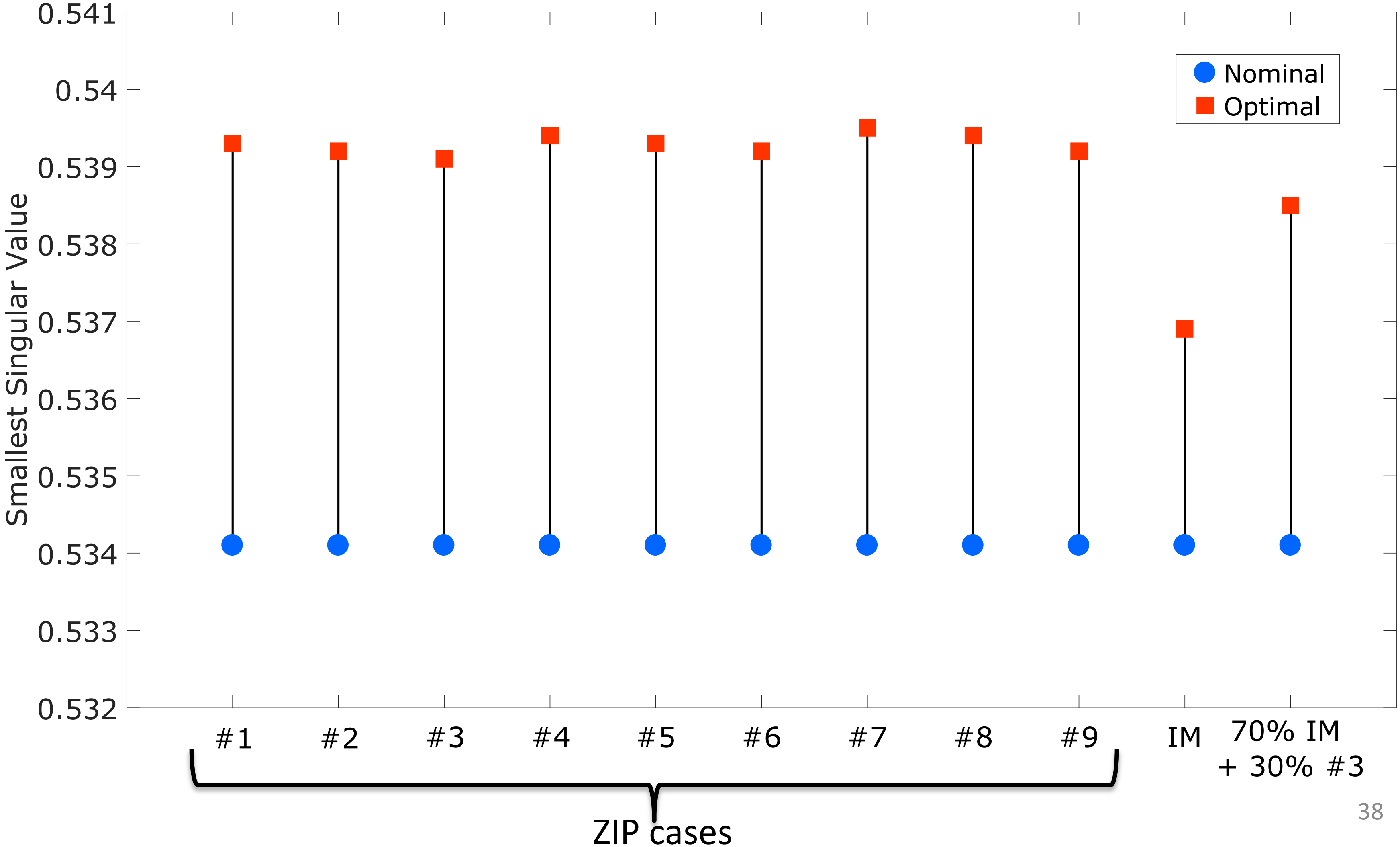
Smallest Singular Value Comparison

Load Model	Nominal	Optimal	Δ	%
Constant Power (ZIP #9)	0.5341	0.5393	0.0052	0.98
ZIP #3	0.5442	0.5444	0.0002	0.04
Induction Machine	2.3360	2.4533	0.1173	5.02
70% IM + 30% ZIP #3	2.2994	2.4078	0.1084	4.71
30% IM + 70% ZIP #3	2.2402	2.3383	0.0981	4.37

It is difficult, if not impossible to compare SSVs across systems with different load models!

Maximizing the SSV of the Conventional Jacobian??

The full Jacobian is still used to solve power flow.



Impact on the Optimal Loading Pattern

Load Model	ZIP #3		ZIP #3		IM		IM	
Jacobian	J_{ZIP}		J_{cnv}		J_{IM}		J_{cnv}	
	P_d	Q_d	P_d	Q_d	P_d	Q_d	P_d	Q_d
Bus 4	92.20	63.99	92.20	63.99	90.99	48.90	71.28	39.94
Bus 9	0.00	0.00	0.00	0.00	0.62	15.55	20.34	18.50
Bus 14	0.00	0.00	0.00	0.00	0.59	15.79	0.59	15.73
SSV	0.5444		0.5391		2.4533		0.5369	

ZIP: Same optimal load pattern

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Impact on the Optimal Loading Pattern

Load Model	ZIP #3		ZIP #3		IM		IM	
Jacobian	J_{ZIP}		J_{cnv}		J_{IM}		J_{cnv}	
	P_d	Q_d	P_d	Q_d	P_d	Q_d	P_d	Q_d
Bus 4	92.20	63.99	92.20	63.99	90.99	48.90	71.28	39.94
Bus 9	0.00	0.00	0.00	0.00	0.62	15.55	20.34	18.50
Bus 14	0.00	0.00	0.00	0.00	0.59	15.79	0.59	15.73
SSV	0.5444		0.5391		2.4533		0.5369	

IM: Different optimal load pattern

Main Findings

- Different load models impact the nominal and optimal smallest singular value
- Different ZIP models give us the same optimal loading pattern (in the cases explored)
- Induction machine models can produce different optimal loading patterns than ZIP models
- How can we measure/compare stability margins when a system undergoes structural changes?

A final note on using the SSV as a stability metric

- Advantages
  - Captures any change in power directions
  - It can (easily) be included in optimization formulations
- Disadvantages
  - Only provides implicit information on the distance to the solvability boundary
  - Does not capture the impact of engineering constraints, which may be encountered first
  - May not be well behaved
  - Its numeric value is system dependent
- Alternatives: loading margin [Greene et al. 1997; Yao et al. PowerTech 2017], distance to closest saddle node bifurcation [Dobson and Lu 1992; Dobson 2003]

Conclusions

- Spatiotemporal load shifting can be used to improve power system static voltage stability after a disturbance; complementing slower generator-based actions
- Ongoing research
  - Maximizing the distance to the closest saddle node bifurcation [Yao, Hiskens, Mathieu, CDC 2018]
  - Maximizing the smallest damping ratio of the generator modes [Koorehdavoudi, Yao, Mathieu, Roy IREP 2017]