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The Impact of Load Models in an Algorithm for Improving Voltage Stability via Demand Response

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Introduction

This work employs **demand response** to improve voltage stability via **virtual spatial shifting** of loads. The total load is kept constant so that frequency stability is unaffected.

Proper load models are particularly important in stability studies. The load models we use in this work: **ZIP and Induction Machine**

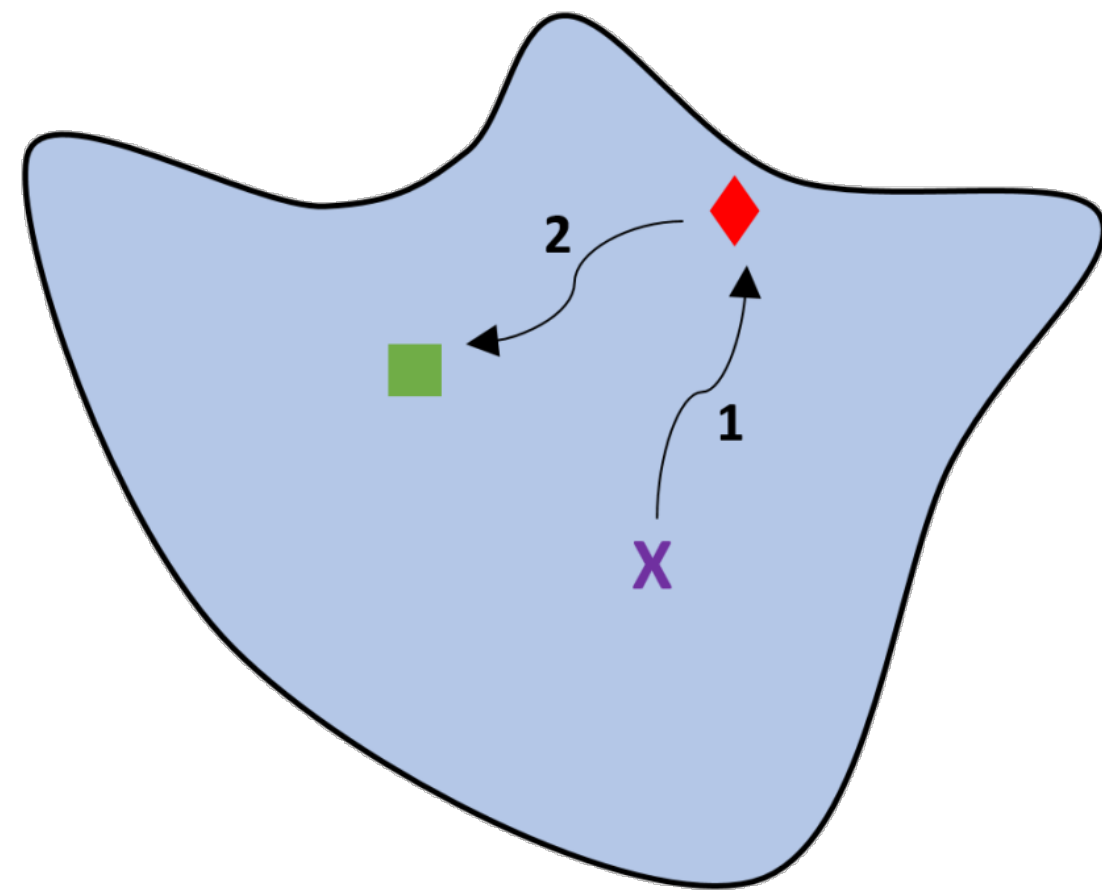


Figure 1: Conceptual description of the problem

Load Models

1. Controllable ZIP Model

$$\mathcal{F}_k^{ZIP}(V_k, \varepsilon_k) = \varepsilon_k P_{d,k}^0 \left[a_{1,k} \left(\frac{V_k}{V_k^0} \right)^2 + a_{2,k} \left(\frac{V_k}{V_k^0} \right) + a_{3,k} \right]$$

$$\mathcal{F}_k^{ZQ}(V_k, \varepsilon_k) = \varepsilon_k Q_{d,k}^0 \left[b_{1,k} \left(\frac{V_k}{V_k^0} \right)^2 + b_{2,k} \left(\frac{V_k}{V_k^0} \right) + b_{3,k} \right]$$

2. Induction Machine Model

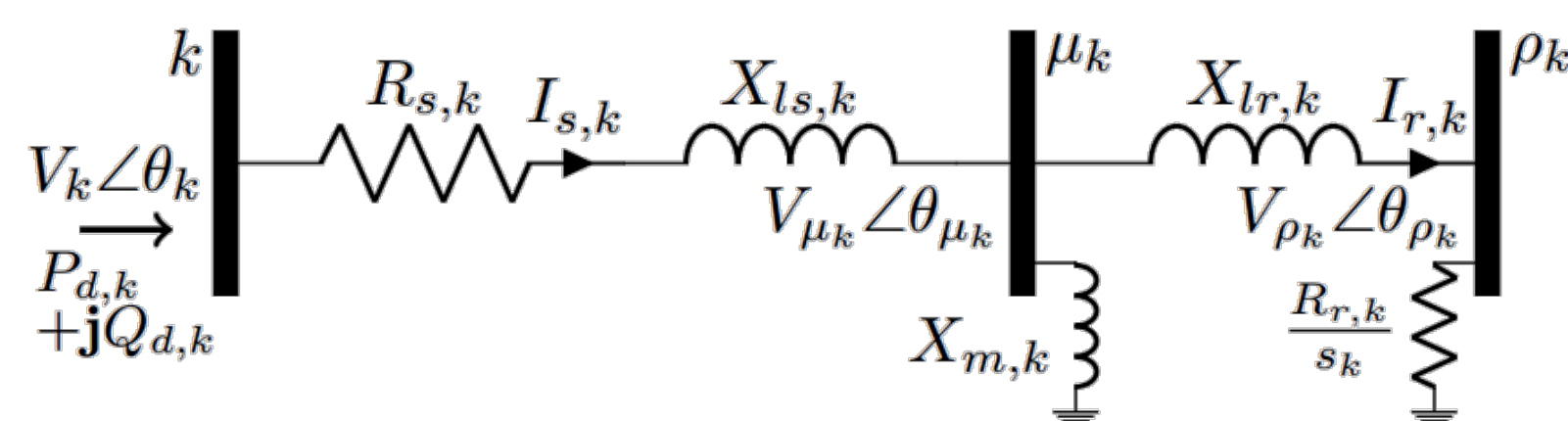


Figure 2: Steady-state equivalent circuit of a IM

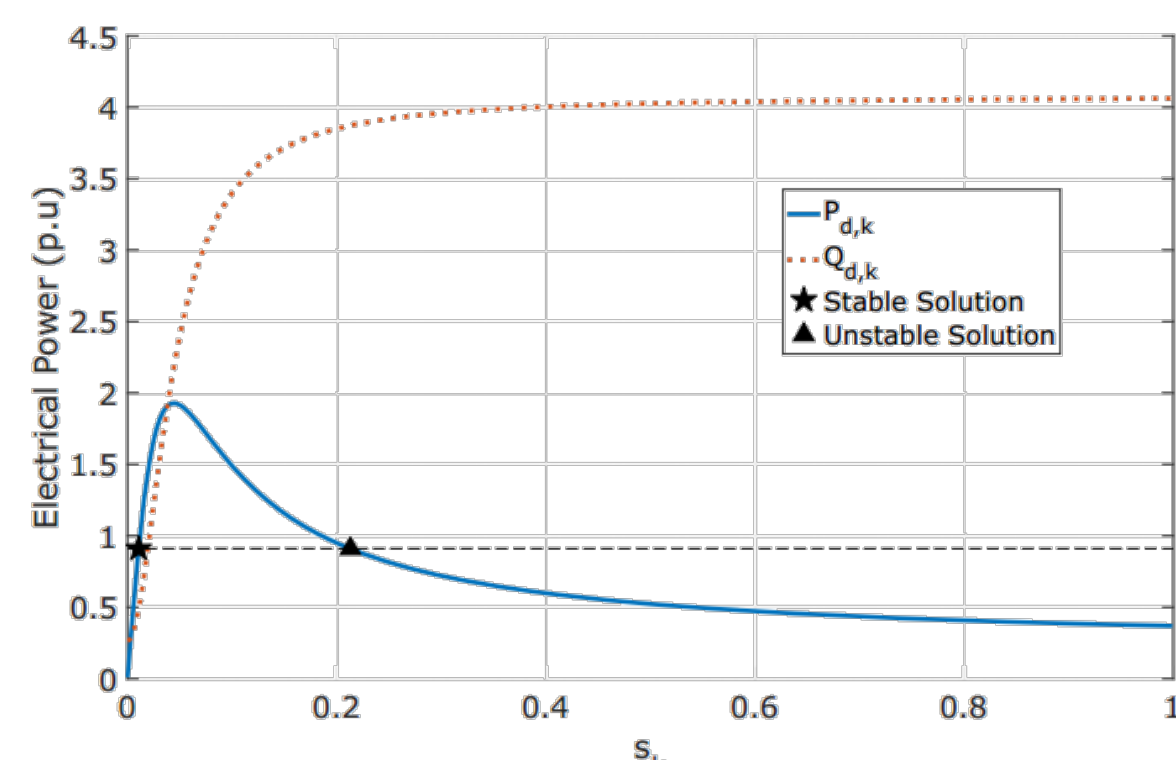


Figure 3: Power as a function of the slip

3. Composite Load Model $S_{d,k} = (1 - \alpha)S_{ZIP,k} + \alpha S_{IM,k}$

Optimization Formulation

1. Jacobian matrix

$$J_{cnv} = \begin{bmatrix} \frac{\partial \mathcal{F}_i^P}{\partial \theta_i} & \frac{\partial \mathcal{F}_i^P}{\partial V_j} \\ \frac{\partial \mathcal{F}_j^Q}{\partial \theta_i} & \frac{\partial \mathcal{F}_j^Q}{\partial V_j} \end{bmatrix} \quad J_{ZIP} = J_{cnv} + \begin{bmatrix} \mathbf{0}_{n-1 \times n-1} & \frac{\partial \mathcal{F}_i^{ZIP}}{\partial V_j} \\ \mathbf{0}_{n_{pq} \times n-1} & \frac{\partial \mathcal{F}_j^{ZIP}}{\partial V_j} \end{bmatrix}$$

$$J_{IM} = \begin{bmatrix} J_{cnv} & \mathbf{0}_{m \times 5n_{dr}} \\ \frac{\partial \mathcal{F}_i^{IP}}{\partial \theta_i} & \frac{\partial \mathcal{F}_i^{IP}}{\partial V_j} & \frac{\partial \mathcal{F}_i^{IP}}{\partial \theta_{\mu,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial V_{\mu,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial \theta_{\rho,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial V_{\rho,k}} & \frac{\partial \mathcal{F}_i^{IP}}{\partial s_k} \\ \frac{\partial \mathcal{F}_j^{IQ}}{\partial \theta_i} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial V_j} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial \theta_{\mu,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial V_{\mu,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial \theta_{\rho,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial V_{\rho,k}} & \frac{\partial \mathcal{F}_j^{IQ}}{\partial s_k} \end{bmatrix}$$

$$J_{com} = (1 - \alpha) [J_{ZIP} \quad \mathbf{0}_{m \times 5n_{dr}}] + \alpha J_{IM}$$

2. Formulation

$$\begin{aligned} \max \quad & \lambda_0 \\ \text{subject to} \quad & J_{com} J_{com}^T - \lambda_0 I \succeq 0 \quad \text{smallest singular value of the power flow Jacobian} \\ & \sum_{i \in \mathcal{S}_{DR}} P_{d,i} = P_{d,total} \quad \text{Total flexible demand is constant} \\ & \mathcal{F}(x) = 0 \quad \text{AC power flow equations, Generator P, V are fixed} \\ & \mathcal{G}(x) \geq 0 \quad \text{Operational limits} \end{aligned}$$

$$x = \{P_g, Q_g, V, \theta, V_\mu, \theta_\mu, V_\rho, \theta_\rho, s, \varepsilon\}$$

3. Solution approach: Iterative linear programming

Results

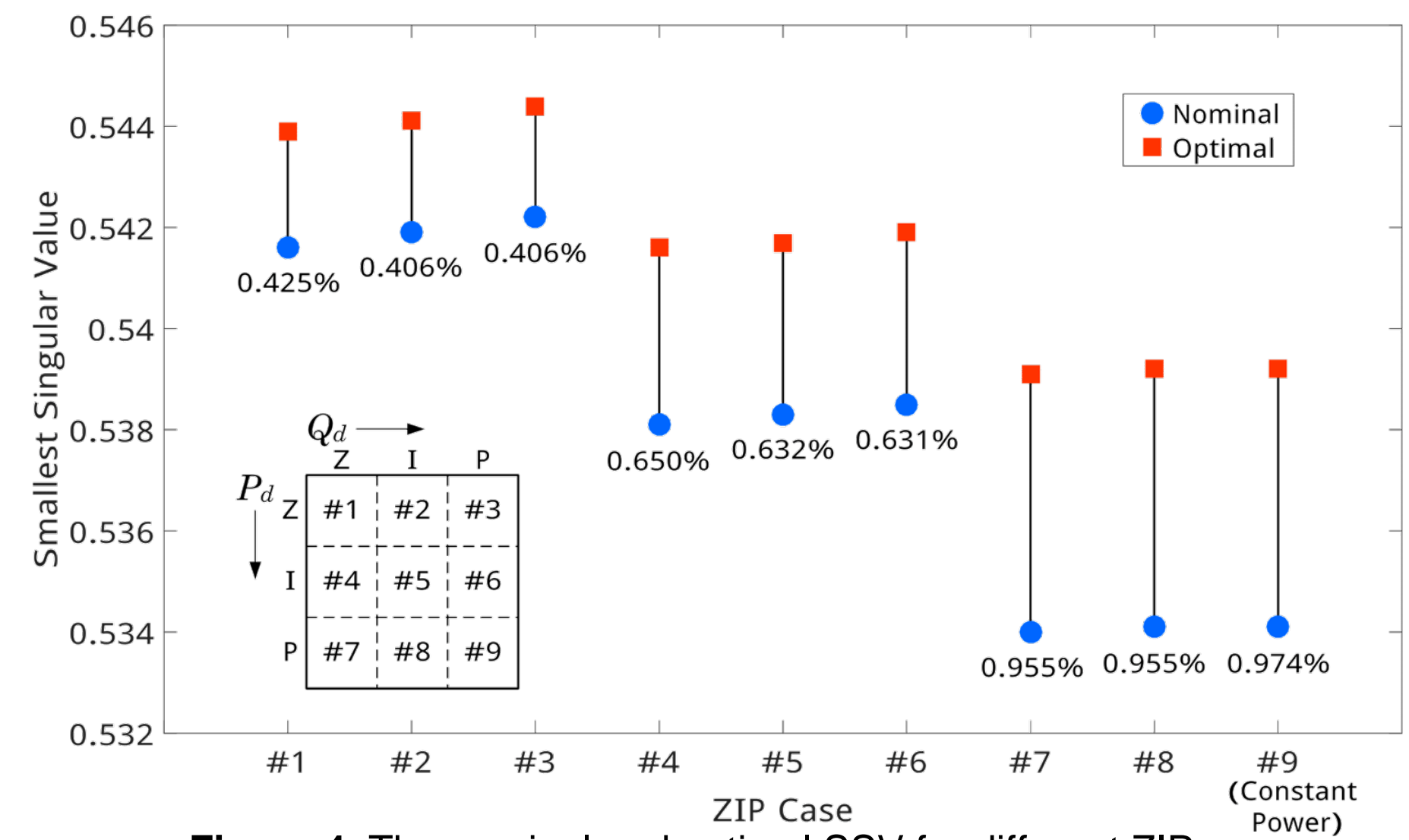


Figure 4: The nominal and optimal SSV for different ZIP cases

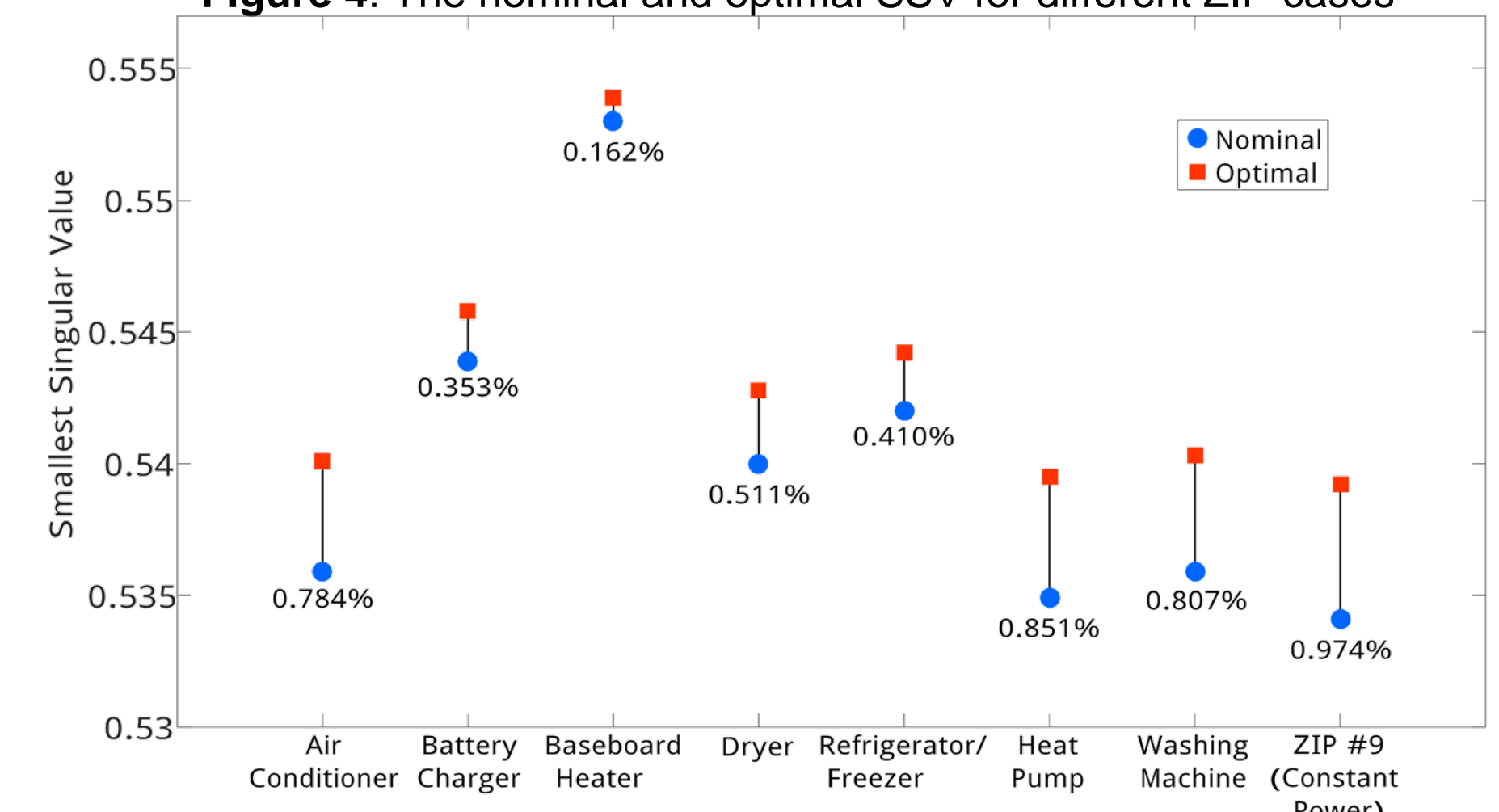


Figure 5: The nominal and optimal SSV for common DR

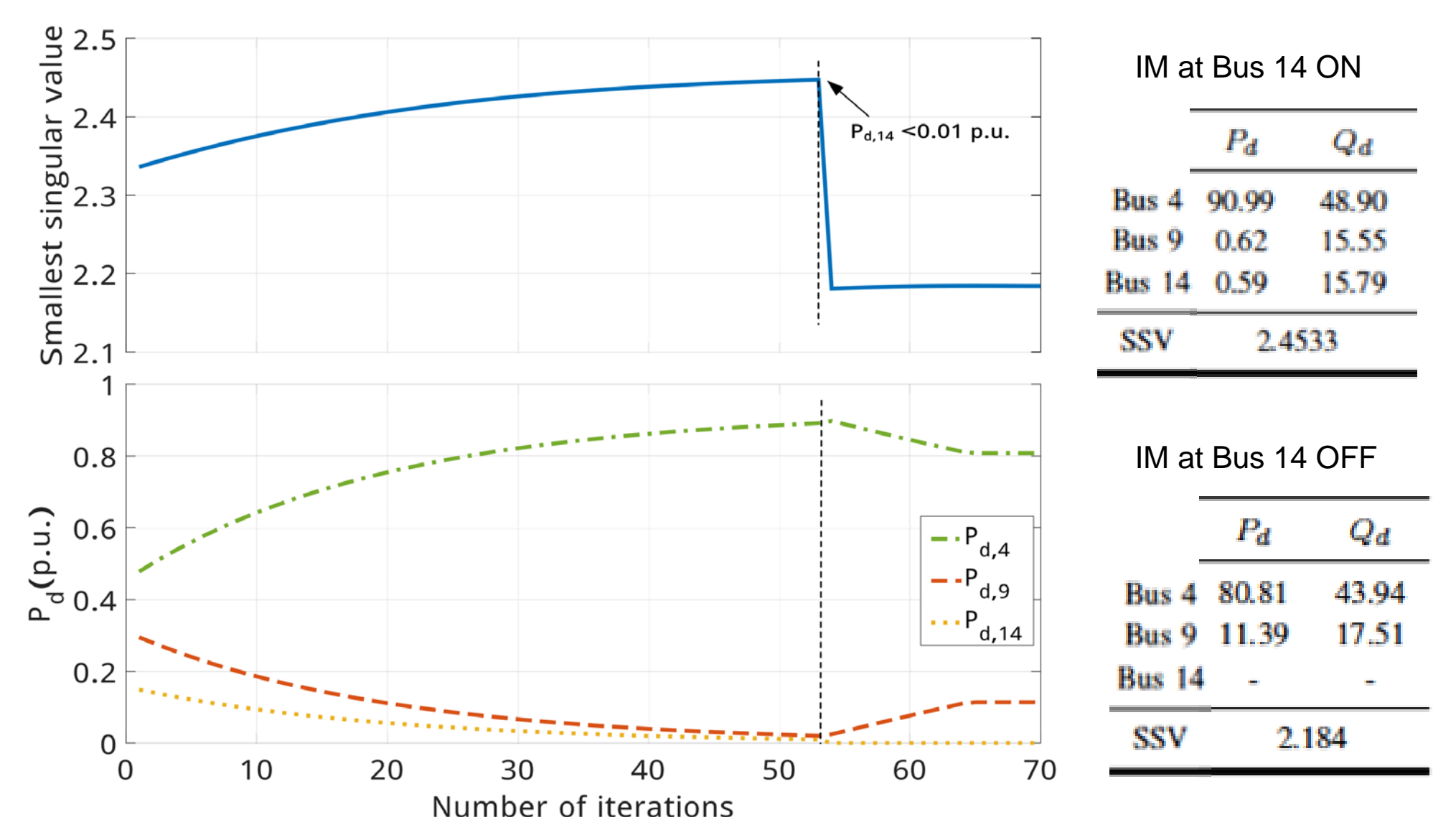


Figure 6: Convergence of SSV and real power demand of the DR if IM at bus 14 is disconnected at low loading

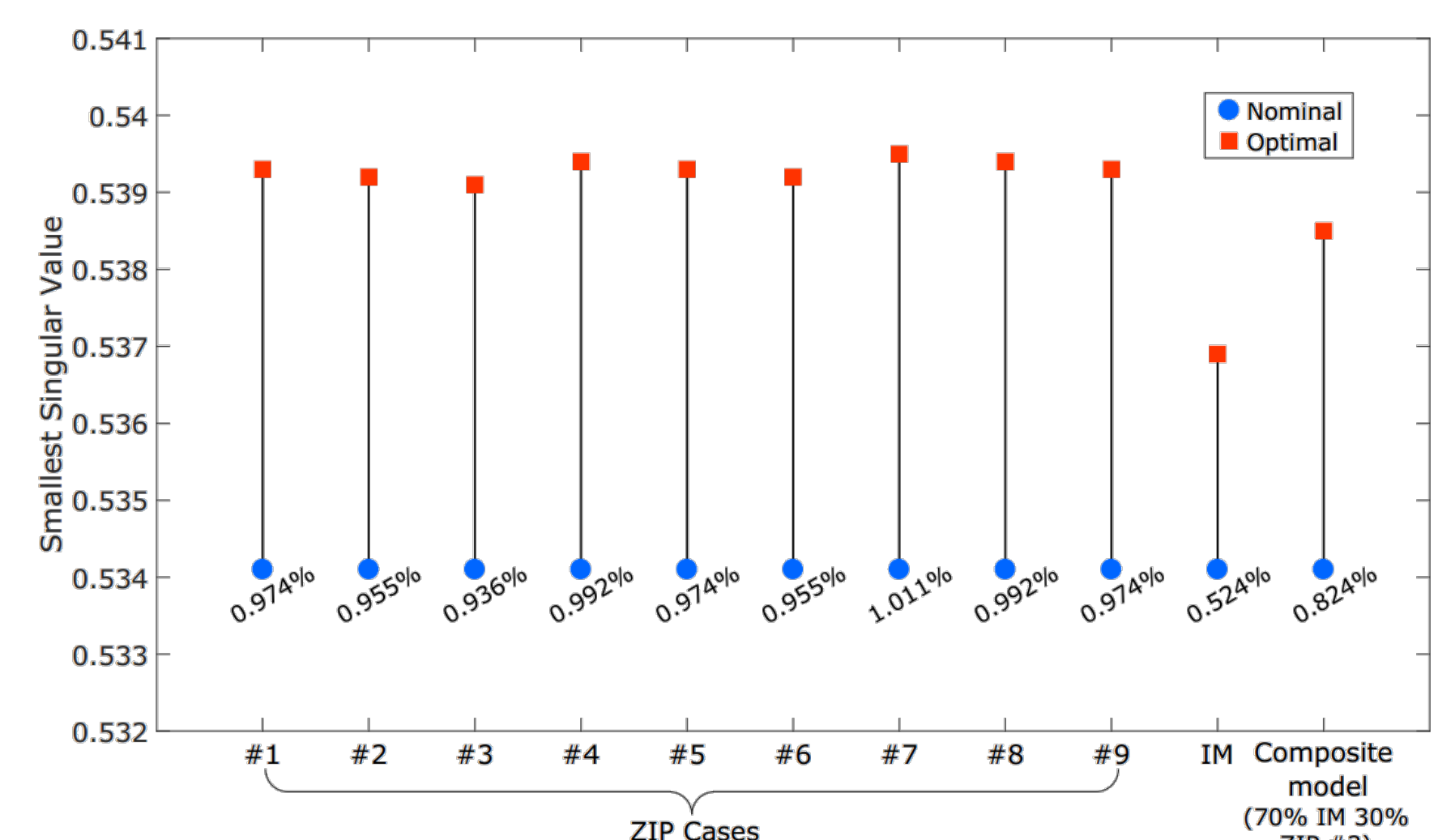


Figure 7: The nominal and optimal SSV of conventional Jacobian matrix

Table 1: Optimal loading patterns and SSV

Load Model	Nominal Constant Power		Constant Power		Optimal for 3 DR buses				IM		IM	
	P_d	Q_d	P_d	Q_d	ZIP #3		ZIP #3		P_d	Q_d	P_d	Q_d
Jacobian	J_{cnv}		J_{cnv}		J_{ZIP}		J_{cnv}		J_{IM}		J_{cnv}	
Bus 4	47.80	32.62	92.20	62.90	92.20	63.99	92.20	63.99	90.99	48.90	71.28	39.94
Bus 9	29.50	20.73	0.00	0.00	0.00	0.00	0.00	0.00	0.62	15.55	20.34	18.50
Bus 14	14.90	16.39	0.00	0.00	0.00	0.00	0.00	0.00	0.59	15.79	0.59	15.73
SSV	0.5341		0.5393		0.5444		0.5391		2.4533		0.5369	

Acknowledgements

This work was supported by NSF Grant EECs-1549670 and the U.S. DOE, Office of Electricity Delivery and Energy Reliability under contract DE-AC02-06CH11357.

[PEN-013]