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# Using Demand Response to Improve Power System Stability Margins

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## Introduction

Electric power systems with high penetrations of fluctuating renewable generation may operate close to their stability limits. Demand response (DR) can be used to improve power system stability. For example, it is already used to provide frequency control, improving frequency stability. Generation and load levels and production/consumption patterns affect voltage stability margins. Generator re-dispatch takes time because of generator ramp limits. In contrast, flexible loads coordinated to provide DR via low-latency communication systems can respond more quickly. Rather than shifting load in time, we propose to improve voltage stability margins by **shifting load in space**.

## Problem Description

Path 1: A disturbance may cause the operating point to move towards boundary  
Path 2: Choosing a new loading pattern to improve stability margin.  
Path 3: The generators are re-dispatched, system return to the initial operating point.

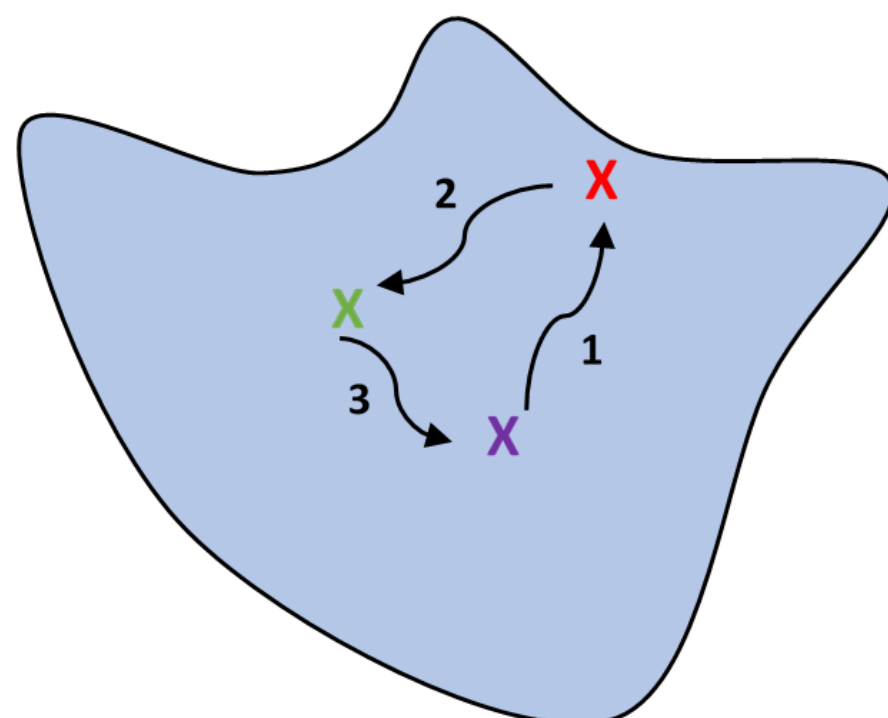


Figure 1: Conceptual illustration of the problem.

In this research, we propose an effective algorithm to computing a loading pattern (Path 2) that maximizes the **smallest singular value (SSV)** of Jacobian matrix, which serves as a measure of voltage stability.

## Methodologies

Because the singular values of the Jacobian matrix  $J$  are the square roots of the eigenvalues matrix  $J^T J$ , an equivalent problem is to maximize the smallest eigenvalue  $\lambda_0$ . To reformulate the original non-convex optimization problem linear **eigenvalue sensitivities** and linearized AC power flow equations are used, so that we can apply iterative linear programming (advanced SSV approach).

Optimization model:

$$\max_{\substack{\Delta P_g, \Delta Q_g, \Delta P_d, \\ \Delta Q_d, \Delta V, \Delta \theta, \Delta \lambda_0}} \Delta \lambda_0 \quad \text{subject to} \quad (1)$$

$$\Delta \lambda_0 = \sum_i \left[ w_0^T \frac{\partial(J^T J)}{\partial \theta_i} u_0 \right] \Delta \theta_i + \sum_j \left[ w_0^T \frac{\partial(J^T J)}{\partial V_j} u_0 \right] \Delta V_j \quad \forall i \in S_{PV}, S_{PQ}, \forall j \in S_{PQ} \quad (2)$$

$$f_n^P(\Delta \theta, \Delta V) = \Delta P_{g,n} - \Delta P_{d,n} \quad \forall n \in \mathcal{N} \quad (3)$$

$$f_n^Q(\Delta \theta, \Delta V) = \Delta Q_{g,n} - \Delta Q_{d,n} \quad \forall n \in \mathcal{N} \quad (4)$$

$$\sum_{n \in S_{DR}} \Delta P_{d,n} = 0 \quad (5)$$

$$\Delta P_{d,n} \cdot \mu_n = \Delta Q_{d,n} \quad \forall n \in S_{PQ} \quad (6)$$

$$\Delta P_{d,n} = 0 \quad \forall n \in S_{PQ} \setminus S_{DR} \quad (7)$$

$$\Delta P_{g,n} = 0 \quad \forall n \in S_{PV} \quad (8)$$

$$\Delta V_n = 0 \quad \forall n \in S_{PV} \quad (9)$$

$$\Delta V_{ref} = 0, \Delta \theta_{ref} = 0 \quad (10)$$

$$h_{nm}(\Delta \theta, \Delta V) \leq \kappa_{nm} \quad (11)$$

$$h_{mn}(\Delta \theta, \Delta V) \leq \kappa_{mn} \quad (12)$$

$$\underline{P}_{g,ref} \leq \Delta P_{g,ref} + P_{g,ref}^0 \leq \overline{P}_{g,ref} \quad (13)$$

$$\underline{Q}_{g,ref} \leq \Delta Q_{g,ref} + Q_{g,ref}^0 \leq \overline{Q}_{g,ref} \quad (14)$$

$$\underline{Q}_{g,n} \leq \Delta Q_{g,n} + Q_{g,n}^0 \leq \overline{Q}_{g,n} \quad \forall n \in S_{PV} \quad (15)$$

$$\underline{V}_n \leq \Delta V_n + V_n^0 \leq \overline{V}_n - V_n^0 \quad \forall n \in S_{PQ} \quad (16)$$

$$\Delta \lambda_0 \leq \overline{\Delta \lambda_0} \quad (17)$$

To verify the correctness of the advanced SSV method, we benchmark its solution against that of three other approaches:

- Basic SSV:** we compute the SSV for all possible loading patterns and determine the maximum.
- Basic Loading Margin (LM):** we use MATPOEWR to compute the LM for all possible loading patterns and determine the maximum.
- Advanced Loading Margin:** we use an Optimal-Power-Flow-based Direct method to maximize the loading factor subject to both the power flow equations and engineering constraints.

## Results and Discussion

We demonstrate the performance of our advanced SSV approach on the IEEE 9- and 30-bus systems.

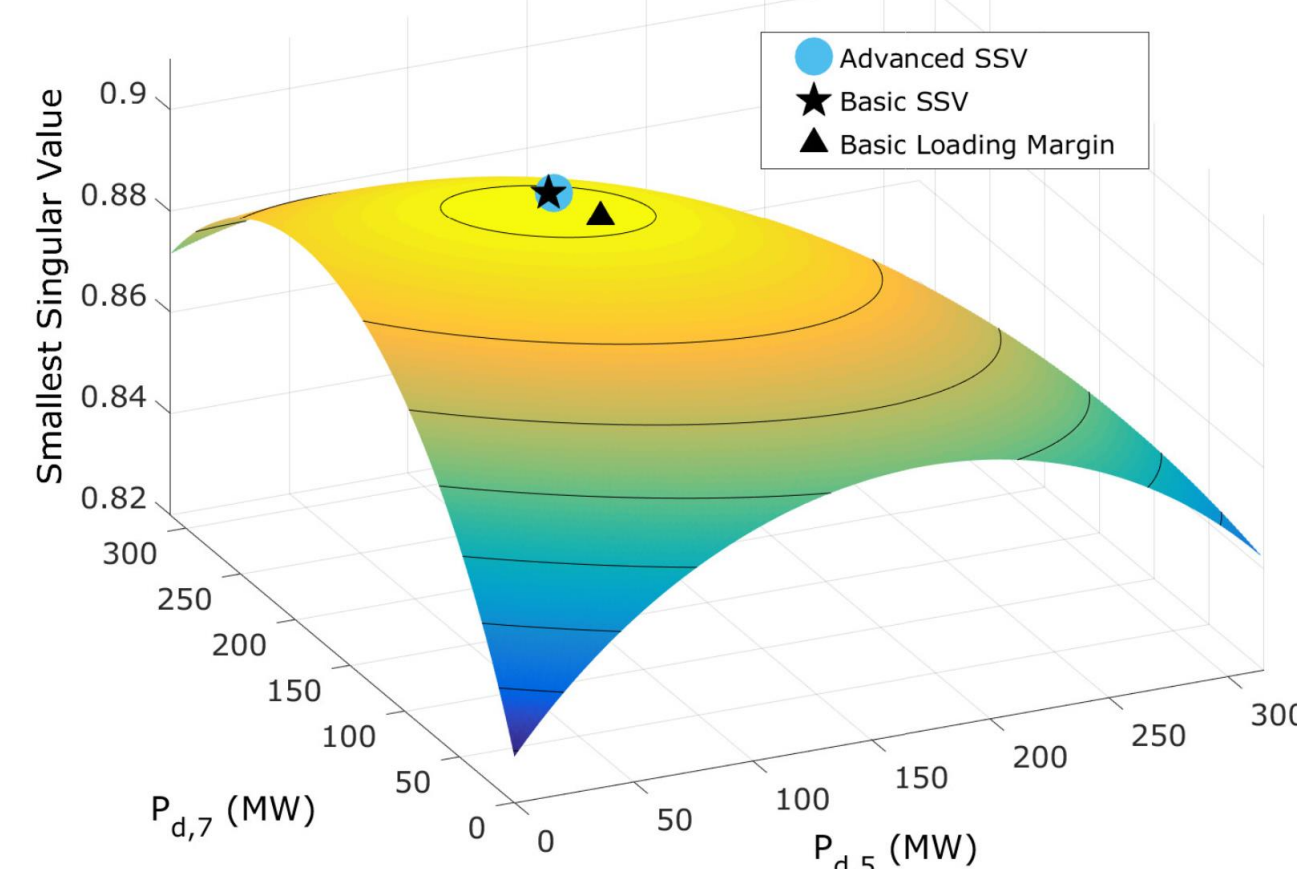


Figure 2: Smallest singular value of the power flow Jacobian for the 9-bus system as a function of bus 5 and 7 loading.

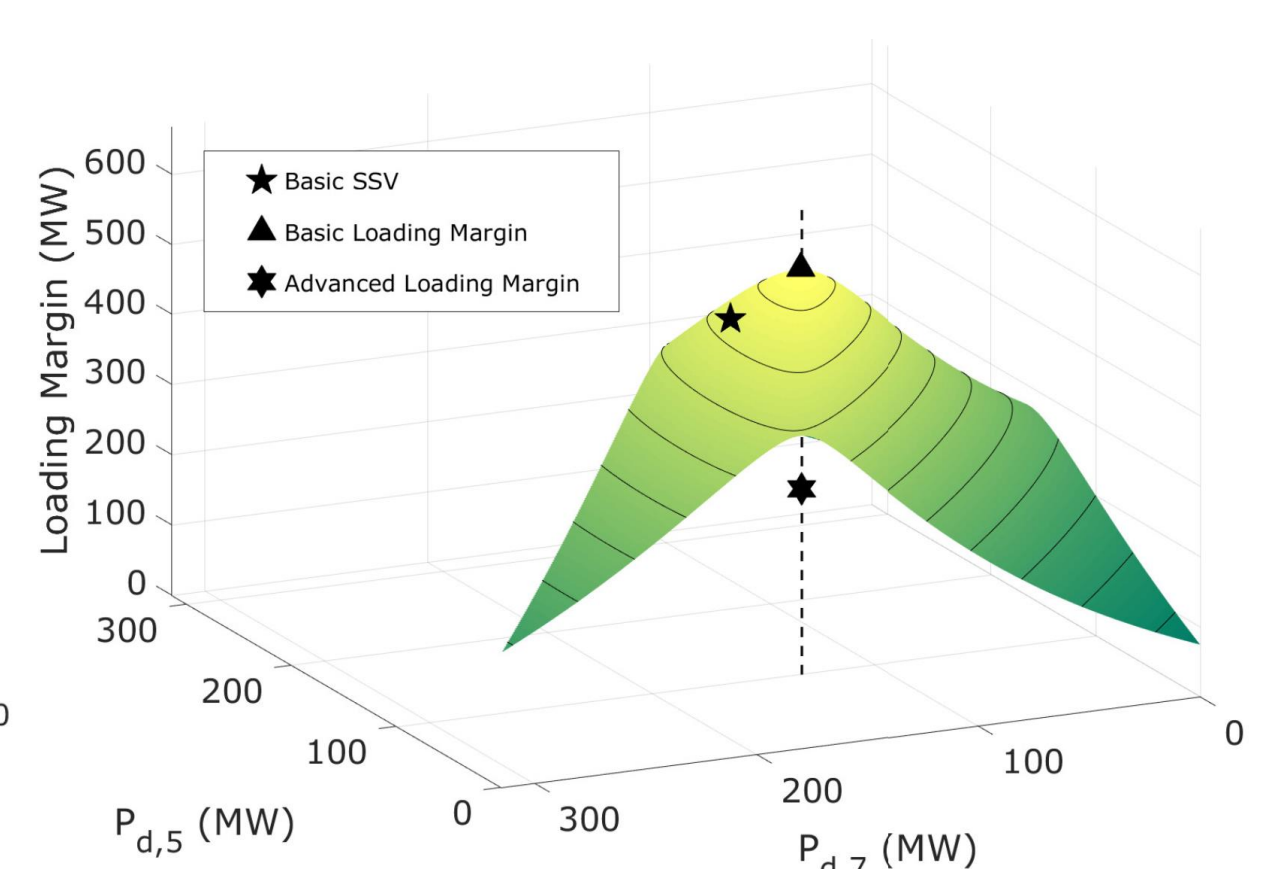


Figure 3: Loading margin for the 9-bus system as a function of bus 5 and 7 loading.

Table 1: Optimal loading patterns for 9-bus system

Approach	$P_{d,5}$	$P_{d,7}$	$P_{d,9}$	SSV	LM
Basic SSV	89	162	64	0.9011	528
Advanced SSV	91	162	61	0.9011	529
Basic Loading Margin	98	134	83	0.9002	571
Advanced Loading Margin	99	133	82	0.9002	264

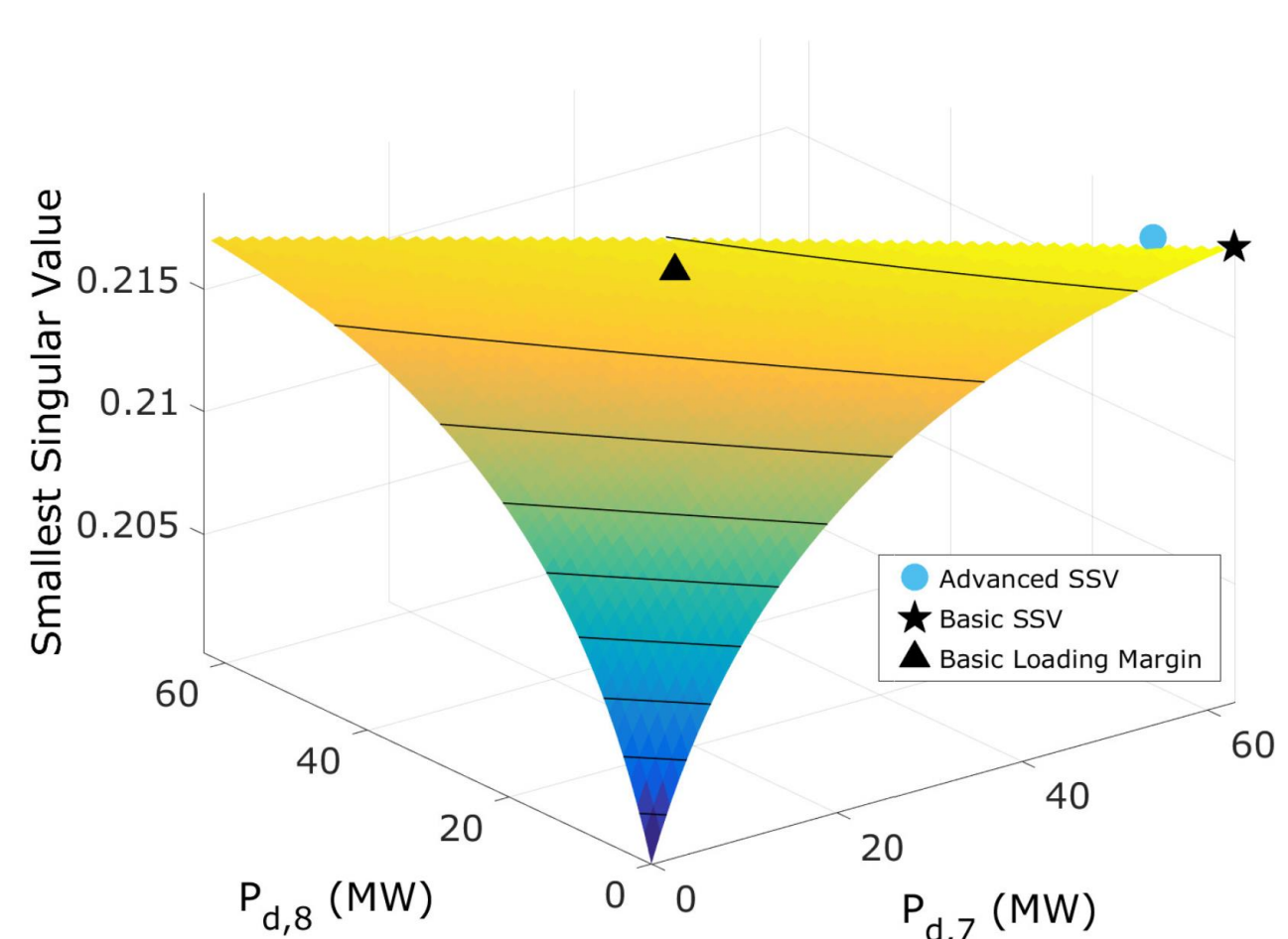


Figure 4: Smallest singular value of the power flow Jacobian for the 9-bus system as a function of bus 7 and 8 loading.

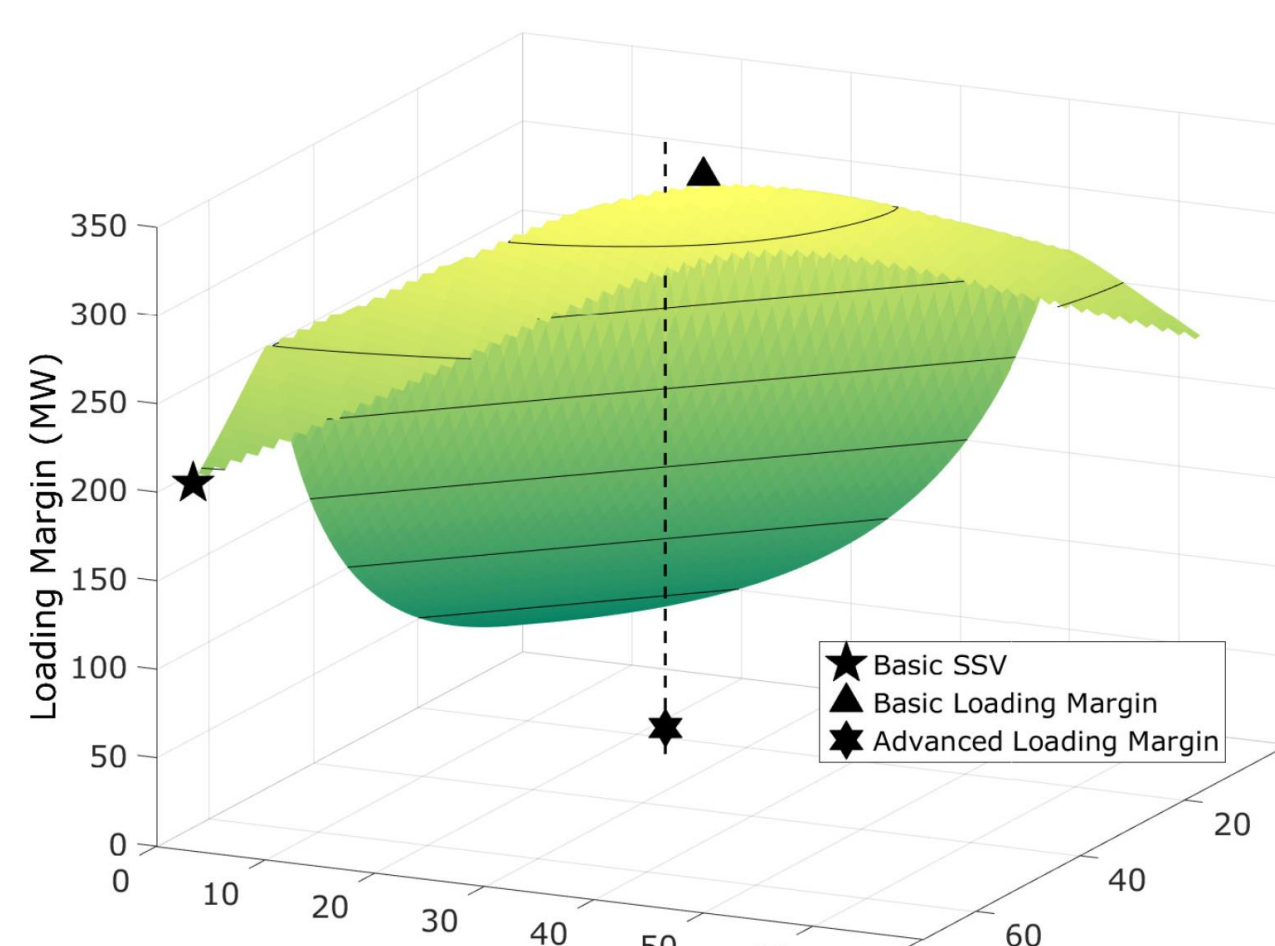


Figure 5: Loading margin for the 9-bus system as a function of bus 7 and 8 loading.

Table 2: Optimal loading patterns for 30-bus system

Approach	$P_{d,7}$	$P_{d,8}$	$P_{d,30}$	SSV	LM
Basic SSV	63	0	0	0.2187	194
Advanced SSV	58	6	0	0.2187	209
Basic Loading Margin	24	28	11	0.2173	323
Advanced Loading Margin	25	25	13	0.2172	15

The Advanced SSV approach produced loading patterns close to the optimum (as determined by the Basic SSV approach).

## Conclusions

We proposed a method for using demand response to improve steady-state voltage stability margins. To solve the optimization problem, we proposed an iterative linear programming algorithm using eigenvalue sensitivities and a linearization of the AC power flow equations.

The test case results show that demand response actions can improve voltage stability margins. We also found that our computationally tractable iterative linear programming method produced loading patterns close to the optimum (as determined by a brute force approach). The results further show that we may obtain significantly different loading patterns when maximizing the smallest singular value of the Jacobian versus maximizing the loading margin. This means that improving one margin may worsen another, and so the system operator should consider the trade-off between different margins.

## Acknowledgements

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