Hard Magnetic Material Selection by Multiple Attribute Decision Making

by

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Author

[Sunny Pinnam]

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Abstract

A novel material selection methodology is introduced in this thesis work, which perceives a material selection problem as a multiple attribute decision making (MADM) phenomena with emphasis on "voice of data" eliminating subjective bias and uncertainty. The methodology was applied to the hard-material selection. To make this methodology more robust, a couple of statistical data analysis techniques, i.e., Hierarchical clustering (HC) and Principal Component Analysis (PCA) have been employed. There are many MADM techniques available to employ for identifying the optimal solution to the given selection problem, but in this methodology, twenty-five techniques, each possessing a unique principle to achieve this purpose were utilized. Each of these MADM tools develops different sets of ranks which will be co-relatable, and a final ranking order is established for further evaluation. Finally, two-dimensional plots, similar to Ashby charts, were developed based on MADM and PCA to provide a visual representation.

Chapter 1: Introduction

Magnetic materials play a prominent role in modern environment-friendly energy conversion technologies [1]. The technologies such as wind turbines, hydro turbines, electric cars, and magnetocaloric refrigeration, etc., have been some of the consequences of this sustainable development mindset. With the onset of scientific and technological advancements, many magnetic materials have been discovered and employed in various sectors of the industry over the past five decades. With such a long catalog of magnets available for the engineer, material selection becomes a crucial part of the product design and conceptualization [2]. Material selection is a process of selecting an optimal material out of the lot, that is best suited for a given design and application. Usually, this process involves many compromises between various material attributes such as mechanical properties, physical properties, cost, machinability, availability, etc., which constitute a list of essential criteria for selecting a material [6]. Material selection has become mainstream in the past couple of decades due to the emphasis on optimal conceptual design requirements.

To address this issue of material selection, many selection methods have been proposed over the years [3-5] that depend on large data banks of materials and their properties to make an optimal decision. The most acceptable modus operandi followed in the industry include the following key steps-

i. Translation: translating the design requirements into material requirements.

ii. Translation: translating the design requirements into material requirements.

iii. Screening: enables to quickly narrow the field of possible materials to a manageable few.

iv. Ranking: narrows the choice further and ranks the choices to identify the optimal material(s) [3].

v. Documentation: obtain supporting information on the shortlisted ranked candidates to further evaluate them on other factors which have not been considered in the previous steps to make a definitive final selection.

Methods that are commonly adopted for materials selection are Ashby's approach (materials' property charts) and Pareto-optimal solution [3 & 7] while the less commonly adopted methods viz. artificial neural network, multiple attribute decision making (MADM) approaches, computational methods are gaining acceptance [5, 7, & 8].

This thesis proposes a novel material selection methodology for magnetic materials which utilizes Multiple Attribute Decision Making (MADM) approach for ranking the material candidates. Moreover, conducts Principal component analysis (PCA) and Hierarchical clustering (HC) on the material data to provide a robust supporting information that can assist the decision-maker in making a final choice on the material for the given design. MADM approach has been applied extensively over the last three decades and is gaining overwhelming acceptance all around engineering spectrum. Moreover, data analysis techniques such as PCA and HC have been utilized to analyze material data to extract important information to influence the final material choice. In this proposed methodology, a robust material selection framework has been developed using the MADM approach and to add on to that, PCA and HC have been integrated to make this methodology more versatile and balanced.

Chapter 2: Literature Review and Background

Materials have been playing a crucial part in the evolution of science and technology. On the other hand, advancements in science and technology have led to the discovery of more than 160000 materials over the history of humankind. Figure 1 shows the discovery of materials over time. When we look around, we find different objects such as cars, buildings, tables, carpets, computers, etc., which can be fundamentally differentiated based on their application, size or appearance, but all of them are made up of materials which is the basic commonality between them. Classification of materials can be done on various parameters such as chemical compositions, physical properties, mechanical properties, etc. Based on chemical composition and atomic structure, materials in the solid state are classified broadly into four categories: Metals, Polymers, Ceramics, and Composites [10].

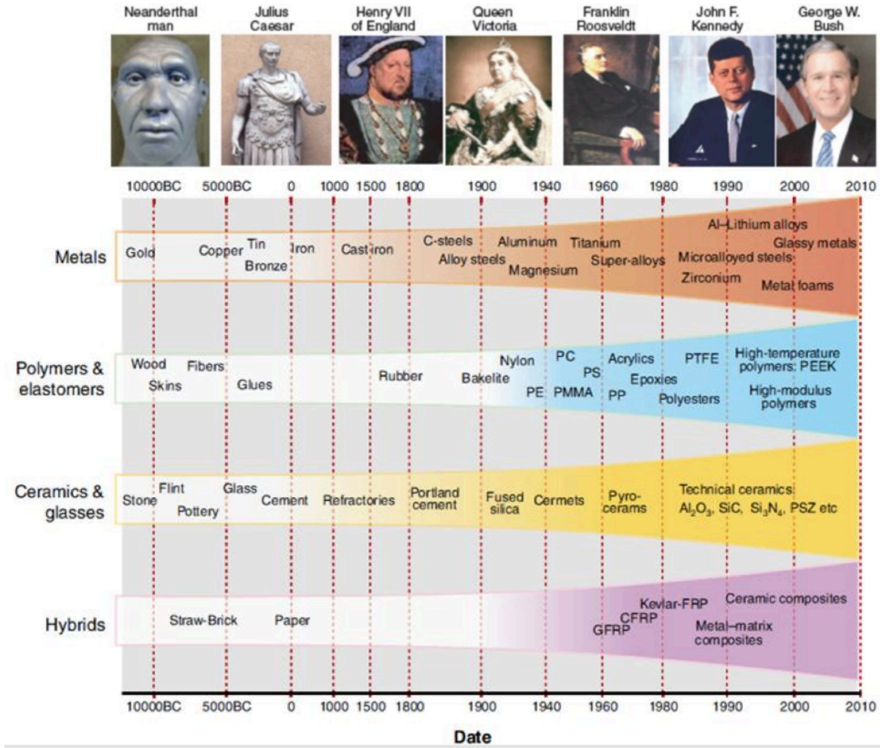


Figure 1: Evolution of materials over human history [10].

Based on properties, materials are broadly categorized as Structural materials and Functional materials [9]. Structural materials are characterized primarily by their mechanical properties and are employed to tasks such as load bearing and load transmission. Functional materials, on the other hand, are known for their unique properties such as magnetism, ferroelectricity, piezoelectricity, etc., and are employed for specific tasks to use their unique properties. There are many other material classification criteria in the world of material science. In recent times, there has been more emphasis on functional materials due to many factors such as sustainable energy generation, transmission, distribution, etc. The next section is dedicated to this topic.

2.1 Functional materials

Functional materials are generally characterized as those materials which possess particular native properties and functions of their own. For example, ferroelectricity, piezoelectricity, magnetism, or energy storage functions [9]. In this general definition, a broad spectrum of materials can be included together with an ample range of material properties and applications. They can be found in all classes of materials: ceramics, metals, polymers, and organic molecules.

Some examples of functional materials are dielectrics, pyroelectrics, piezoelectric, ferroelectrics, ferroelectric relaxors, incipient ferroelectrics, semiconductors, ionic conductors, superconductors, electro-optics, and magnetic materials [9].

It is common to classify functional materials concerning their applications such as Materials for Information Technology, Materials for Electrical Energy Conversion, Materials for Biologic Applications, Materials for Space Technology, among others. The application of functional materials, typified by electroactive materials including piezoelectric, pyroelectrics, and ferroelectrics, for sensing and actuation, spans most if not all industrial sectors [9]. This includes medical diagnostics such as ultrasonic imaging, aerospace such as accelerometers and micro-positioners, automotive such as solid-state piezoelectric fuel injectors, and chemical and process control, which requires the use of thermal, strain and force sensors. Although the discovery of specific properties of functional materials goes back to the nineteenth century, some of the materials that exhibit such properties became useful only during the Second World War and others very recently [9]. The utility of functional materials in these applications reflects their unique properties, such as spontaneous polarization, piezoelectricity, superconductivity, and magnetoresistance. All these properties are directly dependent on the chemical composition, singularities of the crystallographic structure, and manufacture process [9] Functional materials are also of critical importance in materials for energy such as electro- and magnetocaloric materials, for energy storage and solar harvesting functions.

One functional material which is receiving much attention is magnetic materials due to a long list of reasons. In the next sections, a brief account on magnetic materials, their properties, applications, and subsequently the role of material selection in a design based on magnetic functions is presented.

2.1.1 Magnetic materials

Magnetic materials play an essential role in the advancement of industrial and scientific growth. They are invariably used in power generation and transmission, electronic appliances, analog, and digital data storage, medical appliances like magnetic resonance imaging (MRI), magnetic therapy and drug delivery, sensors and actuators, scientific instruments, etc. Functional magnetic materials are a group of materials having relevant and unusual physical properties, which can be influenced by the application of an external perturbation such as an applied magnetic field. These materials are also called the smart magnetic materials of the future.

A significant change of magnetic entropy across a magnetic ordering temperature of a material can have an application in magnetic refrigerators. This functionality of magnetic material has an enormous possibility for use as an alternative cooling technology. The basis of this is the magnetocaloric effect (MCE), defined as the cooling or heating of a magnetic material upon application of a varying magnetic field. This functionality offers the prospect of a compact, highly efficient, less noisy, and environment-friendly alternative to the most commonly used vapor compression- based refrigeration system. The main challenges in the realization of a magnetic refrigerator are the availability of high magnetocaloric materials in large quantities that exhibit large MCE at room temperature in a moderate magnetic field as well as low hysteretic losses. The design of the magnetic field is also crucial as properly designed permanent magnet arrays can enhance the efficiency of these refrigerators to a great extent. The structural and magnetic properties of some of the interesting high magnetocaloric materials, namely TbCo22xFex and La0.67Ca0.33MnO3, will be presented given their usefulness in magnetic cooling at or near room temperature and low-temperature regimes [11]

2.1.2 Basic concepts

Magnetization and Magnetic field

The phenomenon by which materials exert an attractive or repulsive force or influence on other materials is broadly defined as magnetization [12]. This phenomenon can be further explained by understanding terms such as magnetic force or field. Magnetic forces are generated by moving electrically charged particles [12]. Often it is convenient to think of magnetic forces in terms of fields. Imaginary lines of force may be drawn to indicate the direction of the force at positions in the vicinity of the field source.

Magnetic dipoles are found to exist in magnetic materials, which, in some respects, are analogous to electric dipoles. Magnetic dipoles may be thought of as small bar magnets composed of north and south poles instead of positive and negative electric charges [12]. Magnetic dipoles are influenced by magnetic fields like how electric dipoles are affected by electric fields. Within a magnetic field, the force of the field exerts a torque that tends to orient the dipoles with the field.

A familiar example is a way in which a magnetic compass needle lines up with the Earth's magnetic field [12].

2.1.3 Magnetic properties of materials

All the magnetic properties exhibited by materials are primarily dependent on the composition and temperature. However, the metallurgical conditions of the materials also greatly influence these properties. Metallurgical conditions such as the shape, size, and orientation of the grains; the concentration and distribution of various crystal imperfections and the state of lattice regarding impurities, residual stresses, and atomic arrangement in alloys, etc. Based on the metallurgical conditions, magnetic properties are divided into two groups: 1) Structure insensitive properties and 2) Structure sensitive properties. Structure insensitive refers to properties not markedly affected by changes in materials processing (heat treatment or mechanical deformation) or by small changes in composition, including small amounts of certain impurities. Structure-insensitive properties include Saturation magnetization, Curie temperature, and resistivity, etc. In contrast, Structure sensitive properties are largely dependent on the composition of the particular alloy and are not changed substantially in the process of manufacturing a component from the alloy [13]. These properties are further classified into static and dynamic on the basis of their dependence on frequency. Induction, permeability, hysteresis loop, and associated energy loss, coercive force, and magnetic remanence are structure sensitive. Static properties. Eddy-current loss and resonance of spins and domain walls are dynamic properties.

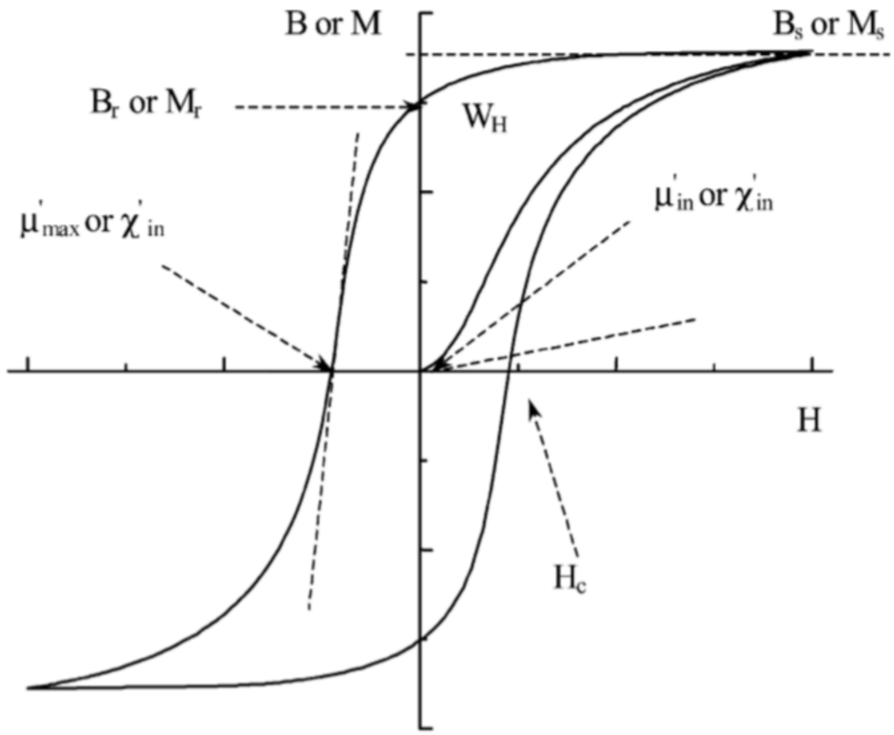


Figure 2: Hysteresis curve [14].

Magnetic properties of the materials on the *BH* plane or flux density *B* versus magnetic field *H*, (or the *MH* plane of magnetization *M* versus magnetic field H). These include coercivity *Hc,* remanence *BR* (*MR*), hysteresis loss *WH*, initial permeability min, maximum differential permeability "m" max (maximum differential susceptibility *in*) and saturation flux density Bs (saturation magnetization *Ms*).

Structure-sensitive properties are those that are drastically affected by impurities. Small amounts of elements such as carbon, oxygen, nitrogen, and sulfur are commonly found in small quantities in magnetic materials. These elements tend to locate at interstitial sites in the crystalline lattice, and consequently, the lattice can be severely strained. As a result, small concentrations of these elements can have significant effects on some of the magnetic properties of the materials.

Permeability, coercivity, hysteresis losses, remanence, and magnetic stability are all considered to be structure sensitive [13]. The structure-sensitive properties are controlled through the processing of the material, including mechanical and thermal treatments. One important realization from magnetic hysteresis is that magnetizing does not happen immediately, nor does demagnetizing. It takes some time for each to take hold, and by then, the opposite may be starting. This is very important in some electronic applications, where a magnetizing coil is wound on an iron core. The core cannot be magnetized or demagnetized instantly; there is always a gradual lag both ways. In technical terms, the hysteresis curve shows a relationship between the induced flux density and magnetizing force.

Coercivity

Coercivity is the parameter which is used to distinguish hard and soft magnetic materials. Traditionally a material with a coercivity of less than 1000 A m^{-1} is considered magnetically 'soft'. A material with a coercivity of greater than 10,000 A m^{-1} is considered magnetically 'hard'. Low coercivities are achieved in nickel alloys such as perm alloy in which the coercivity can be as low as 0.4 A m^{-1} [13].

Permeability

Permeability is the most important parameter for soft magnetic materials since it indicates how much magnetic induction *B* is generated by the material in a given magnetic field strength H. Initial permeabilities of all magnetic materials range from *mr* = 1,000,000 [12] in materials such as amorphous alloys down to as low as *mr* = 1.1 in some of the permanent magnets. It is known that initial permeability and coercivity have, in broad terms, a reciprocal relationship so that materials with high coercivity necessarily have low initial permeability and vice versa [14 & 13].

Saturation magnetization

The highest saturation magnetization available in bulk magnetic materials is $M = 1.95 \times 10^6$ A/m ($Bs = 2.43$ T) which is achieved in an iron-cobalt alloy containing 35% cobalt. The possible values of saturation magnetization then range downward continuously to effectively zero. There has been little progress in improving the range of saturation magnetization of materials for about 100 years [13].

Hysteresis loss

The hysteresis loss is the area enclosed by the hysteresis loop on the B, H plane. It represents the energy expended per unit volume during one cycle of the hysteresis loop. The hysteresis loss increases as the maximum magnetic field reached during the cycle increases. This loss is closely related to the coercivity so that processing of materials to reduce coercivity also reduces the hysteresis loss [13]. Generally, low hysteresis loss is a desirable characteristic of soft magnetic materials [14].

Energy dissipation and power losses

A related property is the power loss which arises when a soft magnetic material is subjected to a time-dependent magnetic field. The hysteresis loss is only one component of the power loss, being the power loss obtained when the field is cycled very slowly under quasi-static conditions. The total power loss depends on the frequency of excitation, the amplitude of magnetic induction, the hysteresis loss, the physical dimensions of the material being magnetized, and the eddy current dissipation. Also, there is usually a discrepancy between the measured power loss and the loss expected from the sum of hysteresis and eddy current losses, and this is usually referred to as the "anomalous loss." The total electrical power loss can be expressed as the sum of these various components. Total power losses can be reduced if the conductivity of the material is reduced [13].

2.1.4 Soft magnets and hard magnets

The magnetic properties of materials are measured from certain defined points and derivatives obtained from the variation of magnetization with the magnetic field, as shown in Figure 2. Magnetic materials are broadly classified into two main groups with either hard or soft magnetic characteristics. Soft magnetic materials can be magnetized by relatively low-strength magnetic fields, and when the applied field is removed, they return to a state of relatively low residual magnetism. Soft magnetic materials typically exhibit coercivities values of approximately 400 A m^(-1)(5 Oe) to as low as 0.16 A m^(-1) (0.002 Oe). Soft magnetic behavior is important in any application involving a change in magnetic induction. Hard magnetic materials retain a large amount of residual magnet- ism after exposure to a magnetic field. These materials typically have coercivities, Hc, of 10 kA/m (125 Oe) to 1 MA/m (12 kOe) [13]. The materials at the high coercivity end of this range are known as permanent magnets. These materials are used principally to supply a magnetic field [13]. Some of the most familiar hard magnets are:

Alnicos

In the development of permanent magnets, the first improvement over steels came about in the early1930s with the discovery of the group of alloys called the Alnico alloys. These alloys are based mainly on the elements nickel, cobalt, and iron with smaller amounts of aluminum, copper, and titanium (Typical weight%: Fe-35, Co-35, Ni-15, Al-7, Cu-4, Ti-4). The alloy composition and processing were developed over the years, and the properties reached a maximum in 1956 with the introduction of anisotropic columnar alnico 9, with an energy product of <~80 kJm^{-3} [15]. These alloys are still used today as they have a high Curie temperature (~850°C), and as a result can operate at higher temperatures as well as having more stable properties around room temperature than some of the more modern alloys.

However, their main disadvantage is that they have low intrinsic coercivity (~50 kAm^{-1}) and as a consequence must be made in the form of horseshoes or long thin cylinders, which cannot be exposed to significant demagnetizing fields. The magnets are either sintered or directionally cast, and then annealed in a magnetic field. This processing route develops an oriented microstructure consisting of rods of strongly magnetic Fe-Co (α') in a matrix of weakly magnetic Ni-Al (α). The coercivity derives from the rod-shaped nature of the α' phase, generating shape anisotropy along with the weak magnetism of a phase pinning the domain walls [15].

Hard ferrites

The next advance in the development of permanent magnets came in the 1950s with the introduction of hard hexagonal ferrites, often referred to as ceramic magnets. These materials are ferrimagnetic and considering the proportion of iron within the material have quite a low remanence (~400 mT). The coercivity of these magnets (~250 kAm^{-1}), however, is far in excess of any previous material. The low remanence means that the maximum energy product is only ~40 kJm^{-3}, which is lower than the alnicos, but due to the high coercivity these, magnets can be made into thinner sections. The magnets could also be exposed to moderate demagnetizing fields and hence could be used for applications such as permanent magnet motors. The magnets are made by a powder metallurgy processing route and there are no problems with oxidation of the powder during processing, as the material is already a stable oxide. The powder processing route ensures that the magnets comprise of very small grains (<1mm), which is essential for generating coercivity in these magnets. During processing the powder is compacted in a magnetic field in order to align the easy direction of magnetization of the particles and hence enhances the remanence and the maximum energy product [15].

Samarium Cobalt type

In 1966 the magnetic properties of the YCo5 phase were discovered. This was the first phase based on a rare-earth (RE) and a transition metal (TM) to be found to have permanent magnetic properties. The combination of RE and TM is ideal as the RE provides the anisotropy to the phase, and the TM provides the high magnetization and Curie temperature. The discovery of SmCo5 soon followed in 1967, and this became the first commercial RE/TM permanent magnetic material, which was polymer bonded and had an energy product of ~40 kJm^{-3}. It was later (1969) found that SmCo5 sintered magnets could be made with energy products of the ~160 kJm^{-3}. These magnets have excess Sm which forms a smoothing grain boundary phase and coercivity is achieved by prevention of the nucleation of reverse domains [15].

In 1976 the record maximum energy product was increased to 240 kJm^{-3}, with a *Sm2Co17* based alloy. These materials are based on the general composition Sm^2(*Co, Fe, Cu, Zr)*17 and achieve their permanent magnetic properties by careful control of the microstructure. The magnets are produced by powder metallurgy and are solution treated at ~1100°C, where they are single phase. This homogenizing stage is followed by several aging treatments at lowest temperature where a cellular microstructure is formed. The cells are based on the *Sm2Co17* type phase, which is enriched in Fe and, the cell boundaries comprise of a layer of SmCo5 type phase, which is enriched in *Cu*. The intrinsic magnetic properties of the cells and the cell boundaries vary such that the magnetic domain wall energy is greatly reduced within the cell boundary and hence pin the domain walls, leading to permanent magnetic properties. There is still a great deal of interest in these materials as they have the potential for operating at high temperature (~500 ℃), making new applications possible, for example as bearings in gas turbine engines. The main problem with *Sm/Co* based magnets is the expense of the raw materials. Samarium is much less abundant than other light rare-earth elements, such as *La, Ce, Pr* and, *Nd*, which account for over 90% of rare-earth metals in typical rare-earth ores. Cobalt is classified as a strategically important metal and, hence the sales restricted [15].

Neodymium Type

In 1984 the magnetic properties of NdFeB were discovered simultaneously by General Motors in the USA, and Sumitomo Special Metals of Japan. Both groups produced materials based on the magnetic phase Nd2Fe14B but employed different processing routes [15]. Sintered NdFeB based magnets achieve their coercivity by an Nd-rich phase at the grain boundaries which acts to produce liquid phase sintering, smooth the boundaries and hence prevent nucleation of reverse magnetic domains [15].

2.2 Material selection

In recent times, the role of material selection has gained much attention in the industry. Globalization and environmental conservation have put a lot of pressure on the organizations worldwide to strive for leaner and economically efficient product designs. This pursuit has made engineers to emphasis on the role of material selection in a new light. Material selection is an integral part of the product/design development process. It complements the design by proposing an optimal and often compromising solution. To comprehend the role of material selection in the design process, it is necessary to understand the product development process itself. The design or product development is initiated by necessity, as they say, "Necessity is the mother of all inventions." This necessity may arise due to various reasons such as to improve living standards, to enhance comfort, the emergence of newer technology, demand created in the market, to maintain a competitive edge, etc. Once the reasons emerge from developing a design, the requirements for the design are listed. Cascading of the requirements is done on the degree of importance that each requirement holds. For example, an engineering firm has undertaken a task to develop a better design of helmets that gives better protection than the previous model available in the market. So, the list of design requirements may include light weight, ability to withstand impact, long lasting, comfort, availability in different colors to appeal to most customers, etc. Here, requirements are cascaded into primary and secondary requirements and light weight, ability to withstand impact will fall into the primary requirement list, whereas, availability in different colors will fall under secondary requirements. Both the resistance to impact and availability in different colors are the design requirements but, latter holds less importance as compared to the former.

Cascading requirements is followed by setting up the targets for the design which drives the design process until the end. For example, considering the same example of helmet design, the targets or objectives for it may be to develop a low cost and a low weight helmet. The objectives are usually influenced by economics, company standards, and design requirements. Once these preliminaries are addressed, the concept/product development process begins. Many iterations are done in this process to develop a design that complies with design requirements and objectives. The next phase in the process is called embodiment [16], which is concerned with as the name suggests embodying the product in the form of prototype and then next details engineering phase [17]. It is an iterative process where information is exchanged between every phase. Figure 3 shows the basic flow-chart of the product development process.

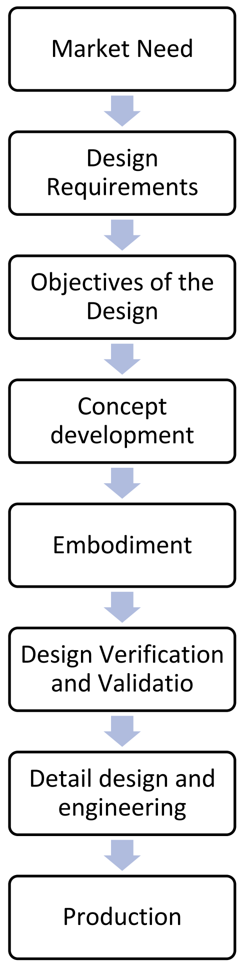


Figure 3: Product development flow-chart.

Material selection forms an inherit a part in all the phases from concept development to detail design and engineering.

2.2.1 Factors affecting material selection

Various factors influence material selection decision. Most of these factors can be clubbed broadly into five categories, and they are:

1. Function
2. Process
3. Geometry
4. Cost
5. Availability

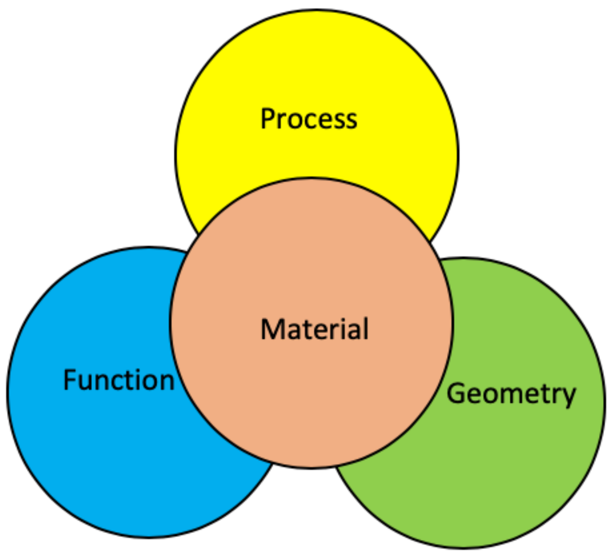


Figure 4: Interaction between function, process, geometry, and material.

The function can be defined as a specific task(s) that should be performed by design. For example, the side mirrors in a vehicle must accomplish a primary task of viewing. In this case, the function of the side mirror is the most crucial influencer for material selection. In the above figure 4, the three spheres overlap each other shows the inter-dependency of the aspect of the design. In general, functions of the product influence the choice of the material, which in turn impacts the process.

“Process” here indicates the manufacturing process which encompasses primary forming process (like forming or casting process), material removal process (machining processes), joining process (welding, etc.) and finishing operations (surface finishing, painting or electroplating) [16]. The process further effects the geometry (shape, size, precision, etc.) of the design. The interactions between these primary factors can have different sequences depending upon the organization and design characteristics.

In some cases, the geometry of the design can have major influence on the material choice, or the process may be the driving force for the material selection process. For example, In the automobile industry, companies often change the design of the vehicles to keep up with the latest trend of the market. Under such volatile conditions, engineers try to accommodate changes in design by the available manufacturing systems. Manufacturing systems are valuable assets to an organization, and any significant changes to it require high investments. The design under such situations is driven by the process [16 & 17].

Cost and availability of the materials are also important factors that have a huge influence on a material selection strategy. Both these factors directly influence each other in a non-proportional manner. If material “A” is available in abundance, the cost of it is usually low. On the other hand, if that same material is scarcely found, then its cost is usually very high [16]. This non-proportional condition is not always true, but in general, it can be reliable.

2.2.2 Material selection strategies

There are many strategies developed to assist engineers for the task of selecting optimal material for a given design. Some of the strategies are mentioned in this section.

Dargie's Method

The initial screening of materials and processes can be a tedious task if performed manually from handbooks and supplier catalogs. This difficulty has prompted the introduction of several computer-based systems for materials and process selection (Dargie, Esawi and Ashby, and Weiss are examples). The system proposed by Dargie et al. (1982) [18] will be briefly described here and the method proposed by Esawi and Ashby in the next section. Dargie's system proposes a part classification code similar to that used in group technology. The first five digits of the MAPS 1 code are related to the elimination of unsuitable manufacturing processes. The first digit is related to the batch size. The second digit characterizes the bulk and depends on the major dimension and whether the part is long, flat, or compact. The third digit characterizes the shape, which is classified based on being prismatic, axisymmetric, cup-shaped, non-axisymmetric, and non-prismatic [18]. The fourth digit is related to tolerance, and the fifth digit is related to surface roughness. The next three digits of the MAPS 1 code are related to the elimination of unsuitable materials. The sixth digit is related to service temperature. The seventh digit is related to the acceptable corrosion rate. The eighth digit characterizes the type of environment to which the part is exposed. The system uses two types of databases for preliminary selection:

- The suitability matrices
- The compatibility matrix

The suitability matrices deal with the suitability of processes and materials for the part under consideration. Each of the code digits has a matrix. The columns of the matrix correspond to the value of the digit, and the rows correspond to the processes and materials in the database [18]. The elements of the matrix are either 0, indicating unsuitability, or 2 indicating suitability. The compatibility matrix expresses the compatibility of the different combinations of processes and materials. The columns of the matrix correspond to the materials, whereas the rows correspond to the processes. The elements of the matrix are either 0 for incompatible combinations, 1 for difficult or unusual combinations, or 2 for combinations used in usual practice [18].

Based on the part code, the program generates a list of candidate combinations of materials and processes to produce it. This list helps the designer to identify possible alternatives early in the design process and to design for ease of manufacture [18].

Esawi and Ashby's method

Another quantitative method of initial screening is proposed by Esawi and Ashby. The method compares the approximate cost of resources of materials, energy, capital, time, and information needed to produce the component using different combinations of materials and manufacturing processes. The method can be used early in the design process and is capable of comparing combinations of materials and processes such as the cost of a polymer component made by injection molding with that of a competing design in aluminum made by die-casting [18].

According to this method, the total cost of a component has three main elements— material cost, tooling cost, and overhead cost. The material cost is a function of the cost per unit weight of the material and the amount of the material needed. Since the cost of tooling (dies, molds, jigs, fixtures, etc.) is normally assigned to a given production run, the tooling cost per component varies as the reciprocal of the number of components produced in that run [18]. The overhead per component varies as the reciprocal of the production rate. Application of this method in initial screening requires a database, such as CES 4, which lists material prices, attributes of different processes, production rates, tool life, and the approximate cost of equipment and tooling. CES 4 software contains records for 112 shaping processes such as vapor deposition, casting, molding, metal forming, machining, and composite forming. The software output can be in the form of graphs giving the variation of cost with the batch size for competing for process/material combinations. Another type of output is the relative cost per unit when a given component is made by different processing routes [18].

Weighted Properties Method

In the weighted properties method, each material requirement, or property, is assigned a certain weight, depending on its importance to the performance of the part in service. Weighted property value is obtained by multiplying the numerical value of the property by the weighting factor (α). The individual weighted property values of each material are then summed to give a comparative materials performance index (γ). Materials with a higher performance index (γ) are considered more suitable for the application [18].

Ashby method

Dr. Michael F. Ashby has proposed a systematic material selection strategy which is easy to understand and utilize. This strategy can be summarized into four steps: Translation, Screening, Ranking, and Documentation [16]. Figure 5 pictorially represents the four stages of Ashby's material selection strategy.

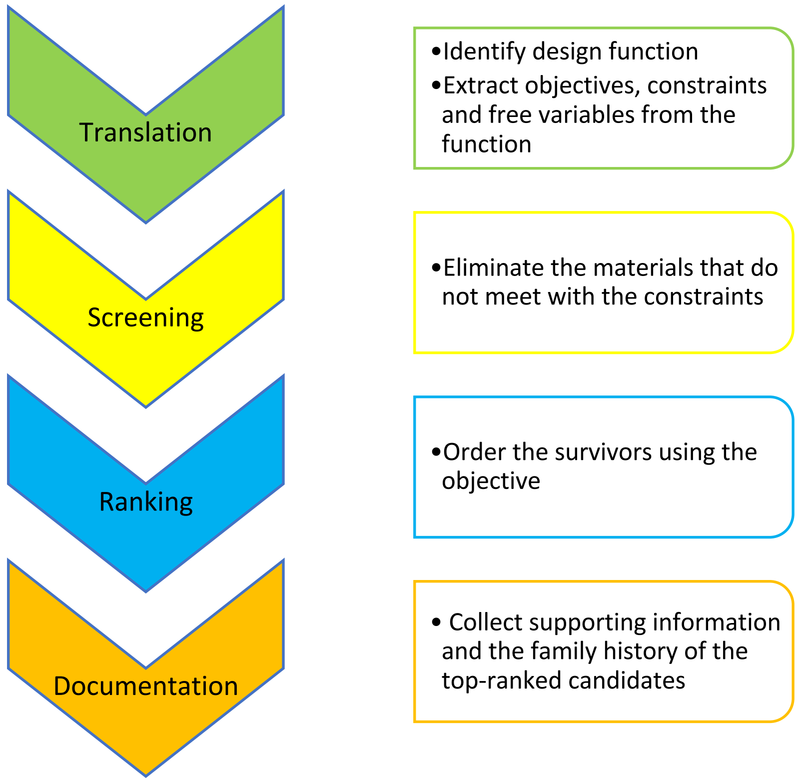


Figure 5: Material selection strategy.

a. Translation

In this step, the design requirements are translated into material requirements. Thus, identifying the objectives of the selection process, deriving constraints, and free variables. For example, consider the same case of helmet design as earlier, the design requirements of helmet such as lightweight, resistance to breaking on impact translates to low density and high impact resistance material properties (attributes) [16]. Once the material attributes are derived, corresponding constraints and independent variables are identified, and the decision-maker sets the objectives for the selection strategy.

b. Screening

Unbiased selection requires considering all the materials available as potential candidates until proven otherwise. Screening is a process of eliminating the undeserving materials that fail to comply with the limits set by material selection criteria (attributes) [16]. The material candidates that satisfy the minimum threshold set by the constraints proceed to the ranking phase.

c. Ranking

Ordering of the screened-out material candidates is done using optimizing criteria. An optimizing criterion is expressed in terms of material indices, which is defined as a property or property-group that maximizes performance for a given design [16]. Material indices provide criteria of excellence that allow ranking of materials by their ability to perform well in the given application [16]. Thus, we can say that a material index is a compromising solution roped between two or more properties. Property charts are plotted using material indices. Dr. Ashby developed property charts based on the concept of optimization criteria. A property chart provides a visual aide for the decision-maker. Figure 6 represents the modulus versus density property chart with density along the x-axis and Young's modulus along the y-axis. In this chart, the material alternatives are ranked using the material index. In some cases, maximizing the criteria of excellence (material index) is required, and in other cases, minimizing these criteria is the need of the hour.

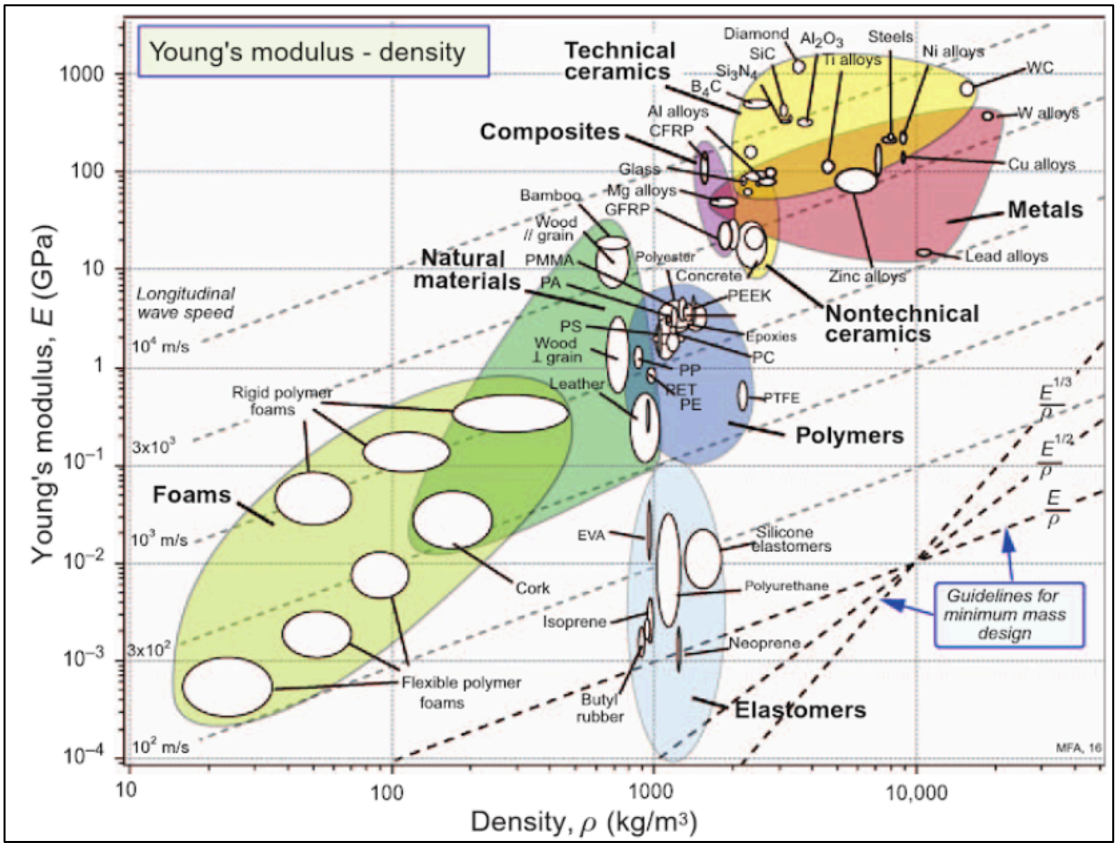


Figure 6: Young’s modulus vs density Ashby chart [16].

d. Documentation

Once the material alternatives are ranked, that does not mean the end of the selection process. The ranked shortlisted material alternatives are further researched to explore other strengths and weaknesses that might play good or bad concerning the design considered. Detail documentation of each shortlisted candidate is prepared, studied and compared with other to evaluate each candidate comprehensively on other aspects which are not considered earlier to select an optimal material for the given design [16].

e. Limitations of Ashby's method

Some of the limitations of the Ashby's method are:

- The number of attributes that can be considered for evaluating material candidates are limited. The complexity of the selection task increases with increase in attributes, and at some point, the application of the method become unviable. The ideal number of attributes can be in the range of three to five.
- Similarly, the number of objectives of the material selection problem must be less to avoid complications in the application of this method. The ideal number of objectives is two.

Multiple criteria decision analysis

Multiple criteria decision analysis (MCDA) has been extensively applied to material selection problem since the past decade. Multi-criteria analysis has spawned from the two key concepts of human behavior: satisficing and bounded rationality [19]. Expressing a decision situation in terms of multiple criteria evaluation of alternatives to determine an optimal solution to the problem is the basic concept behind MCDA. The ability to solve decision problem involving multiple and often conflicting criteria has made this approach extremely popular in the industry for solving a wide range of decision problems. In the wake of globalization and policies mandating sustainable development, material selection has been reckoned as a non-monotonous decision problem involving more than one attribute to consider as a selection criterion for developing an economical, efficient engineering design. Application of MCDA has had impressive results and has been used as a material selection strategy by many engineers.

The next chapter is entirely dedicated to MCDA and more particularly to the sub-branch of it, known as Multiple Attribute Decision Making (MADM).

2.3 Decision making

Decision making is a cognitive process that involves the union of various mental activities such as gathering information from experience, comprehending the problem, assessing the options, setting up the fundamental aspects (criteria) for decision making, acquiring knowledge related to it, etc. Great philosophers like Aristotle, Plato, and Thomas Aquinas, to mention only a few have claimed directly or indirectly that this ability is what distinguishes humans from animals [19]. Decision making is an activity that is performed daily. A simple action of selecting a dress to a complex task of choosing a material for a complex machine involves cognition, but of course with varying intricacy. Decision-making in case of selecting toothpaste is straightforward when the options are minimal. However, the same action becomes a little complicated when the options increase. Yet, it is still elementary as compared to choosing a supplier for a steel manufacturing company or selecting a renewable energy project. These kinds of problems involve not only numerous options (alternatives) but also multiple aspects (criteria) that influence the selection process. The complexity of the decision-making process increases drastically when dealing with engineering, business, welfare, politics, environment, etc., as compared to domestic problems that concern only with few individuals. For example, consider an OEM (original equipment manufacturer) belonging to automobile industry, the decision related to recruitment or layoff will have a significant impact on the economy and lives of many individuals. Similarly, the selection of the wrong design or project will lead to financial chaos for the company.

2.3.1 Decision-making in the industry

In the twenty-first century, decision-making has much more profound implications on the economy, which can be credited to the rise of globalization and humankind's pursuit for sustainable means of energy generation and consumption. Due to these factors, local markets have been exposed to the multi-national companies, and the competition between the local and international organizations to sustain themselves in the market and in the process make profits has increased ten folds. Under such circumstances, decision making has become extremely crucial for the success of an organization.

Nowadays, decision-makers cannot abstain themselves to consider cost as the only criterion while taking a decision, which was the case until the early 1970s. In any industry, a decision problem usually presents itself with a list of possible actions (alternatives) that can accomplish the desired task(s), but the objective(s) of the problem sets a wide range of selection aspects (criteria) that the potential action must satisfy. For example, let us consider a material selection problem for a flywheel. A flywheel is an energy storage device that stores energy in the form of kinetic energy and releases it when there is no external supply. The design requirements of the flywheel determine the material properties. Some of the design requirements are low mass, no catastrophic failure, maximum energy storage capacity, long life, etc. These design requirements will be translated into material properties such as low density, high fatigue strength, high fracture toughness, etc [269].

The materials which excel in these properties and presents as a potential solution to the problem will be at least twenty. Selecting an optimal material from such a list of potentials is an extremely tough task. This pretty much summarizes the nature of decision making in the present scenario. To deal with such problems, decision-makers have been referring to multiple criteria decision making

(MCDM) theory, which embraces the plural nature of decision making (the "plural nature of decision making," means the inherent quality of a decision situation to involve multiple aspects that determine the solution to the decision problem, refer [19])

2.3.2 Multiple criteria decision making (MCDM)

Multiple Criteria Decision Making (MCDM) is a sub-discipline of operations research that deals with decision problems involving multiple decision criteria by establishing a preference model which aides in evaluating each potential alternative along the lines of the criteria [19 & 20]. Multiple criteria decision problems pervade almost all decision situations ranging from common household decisions to complex strategic and policy level decisions in corporations and governments [20]. Before the development of MCDM as a discipline, such problems were traditionally addressed as single-criterion optimization problems, wherein a composite measure that defines the objectives were derived. Moreover, a process to optimize it was formulated, or only one of the objectives was selected as the pivotal decision objective for optimization and solving the problem by requiring an acceptable level of satisfaction in each of the other objectives [19 & 21]. CDM, as a discipline, was founded on two key concepts of human behavior: satisficing and bounded rationality [22]. The two are intertwined because satisficing involves finding solutions that satisfy constraints rather than optimizing the objectives, while bounded rationality involves setting the constraints and then searching for solutions satisfying the constraints, adjusting the constraints, and then continuing the process until a satisfactory solution is found [21]. Therefore, MCDA intuition is closely related to the way humans have always been making decisions [19].

To facilitate systematic research in the field of MCDM, et el Hwang (in 1981) suggested that MCDM problems can be classified into two main categories: multiple attribute decision making (MADM) and multiple objective decision making (MODM), based on the different purposes and different data types. MADM is applied in the evaluation facet, which is usually associated with a limited number of predetermined alternatives and discrete preference ratings. Moreover, MODM is especially suitable for the design/planning facet, which aims to achieve the optimal or aspired goals by considering the various interactions within the given constraints [20]. One of the underlying differences is that MADM deals with discrete alternatives and approach to optimize the set of alternatives, whereas MODM deals with continuous alternatives and is objective oriented (refer [23]).

The next section focusses on the MADM approach, which is the core topic of this thesis work.

2.3.3 Multiple attribute decision making (MADM)

Multiple Attribute Decision Making (MADM) refers to making preference decisions (e.g., evaluation, prioritization, selection) over the available alternatives that are characterized by multiple, usually conflicting, attributes [24]. The problems of MADM are diverse. However, even with the diversity, all the problems that are considered will be processed through a robust framework that works to formulate an optimal solution to the problems. The characteristic components of the MADM framework are:

Alternatives

Alternatives are also referred to as potential action is usually designated as that which constitutes the object of the decision, or that which decision aiding is directed towards. The concept of action does not a priori incorporate any notion of feasibility, or possible implementation. An action is qualified as potential when it is deemed possible to implement it, or simply when it deserves some interest within the decision-aiding process. [19]

A finite number of alternatives, from several to thousands, are screened, prioritized, selected, and ranked. For example, the number of alternative manufacturing plant sites in the United States an automaker can select from may be less than ten, whereas an elite college may consider thousands of applicants for admission each year. The term “alternative” is synonymous with “option,” “policy,” “action,” or “candidate,” among others [24].

Attributes

Attributes express the goals or objectives of the decision problem. They articulate the essential characteristics that each action (alternative) must possess in order to qualify as a potential alternative for a problem. Each decision problem usually has multiple attributes. A decision maker (DM) must generate relevant attributes for each problem set. The number of attributes depends on the nature of the problem. For example, one may use price, gas mileage, safety, warranty period, workmanship, and style to evaluate cars; whereas there may be more than 100 factors to be considered in selecting a site for an auto assembly plant. The term “attributes” may be referred to as “goals” or “criteria.” [24]

Each attribute has different units of measurement. In the car selection problem, gas mileage is expressed in miles per gallon, ride comfort in cubic feet (if measured by passenger space), and selling price in dollars, but safety is expressed in a nonnumerical way [24].

Attribute weights

As established earlier, an MCDM/MADM involves numerous attributes, and not all attributes are likely to be considered equally important for a decision problem. The role of weight serves to express the importance of each attribute relative to the others. Hence, the assignment of weights plays a vital role in the MADM process and may vary from one DM to another. Weights should, of course, reflect the purpose of the evaluation [24].

Many MADM methods require weight information from the DM. A DM may use either an ordinal or a cardinal scale to express his/her preference among attributes. Although it is usually more comfortable for a DM to assign weights by an ordinal scale, most MADM methods require cardinal weights, that is, $W = (W_1 \dots, W_j \dots\dots W_N)$, where W_j is the weight assigned to the jth attribute. Cardinal weights are normalized to sum to 1, that is, $\sum_{j=1}^{N} W_j = 1$ [24].

Weights are determined in three ways [26]:

a. Objective method

The objective method only considers data of the problem to derive attribute weights. It usually utilizes mathematical models to process decision information without considering the decision maker's preferences. Since in the most real problems, the decision maker's expertise and judgment should be taken into account, subjective weighting may be preferable, but when obtaining such reliable subjective weights is severe, the use of objective weights is useful [25]. One of the highly accepted objective weighting measures is Shannon's entropy.

b. Subjective method

In the subjective method, weights of the attributes are solely based on preference information derived from the expert evaluations and can be according to the previous experience, particular constraints of design or designer's preferences. There are many subjective methods available for the DM to obtain weights of the attributes [26]. One of the most prominent subjective methods is AHP (analytic hierarchy process).

c. Combination approach

Sometimes, the weights determined by objective methods are inconsistent with the DM's subjective preferences. Contrariwise, the judgments of the decision-makers usually depend on their knowledge or experience, and the error in weights to some extent is unavoidable. None of the two approaches are perfect, and the integration produces the best compromise, which might be the most appropriate for determining the criteria weights [26]. An example of such a technique is the combinative weighting method [26].

Decision matrix

A MADM problem is concisely expressed in a matrix format, where columns indicate attributes considered in a given problem and rows consists of list competing alternatives. It is called as a decision matrix, which incorporates all the characteristics components of the MADM framework to facilitate an aggregation operation to evaluate and propose an optimal solution(s).

A typical Decision Matrix looks like,

$$
(DM)_{M\times N} = \begin{array}{cc}
\begin{array}{r} \mathit{Weight} \rightarrow \\ \mathit{Attribute} \rightarrow \end{array} & \begin{array}{cccc} W_1 & W_2 \dots & & W_N \\ C_1 & C_2 \dots & & C_N \end{array} \\
\begin{array}{c} \mathit{Alternative} \downarrow \\ A_1 \\ A_2 \\ \\ A_M \end{array} & \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1N} \\ x_{21} & x_{22} & \ddots & \vdots \\ \vdots & \vdots & & \\ x_{M1} & x_{M2} & \cdots & x_{MN} \end{bmatrix}
\end{array}
$$

Where,

"x_{ij}" is the performance rating/evaluation of ith alternative with respect to jth attribute.

Performance ratings

Performance ratings x_{ij} from the decision matrix indicates the performance of alternative 'i' with respect to the attribute 'j'. The performance rating can be objective or subjective in nature depending upon the corresponding attribute.

Figure 7 shows the flow-chart describing the modus operandi of the MADM approach with its main components.

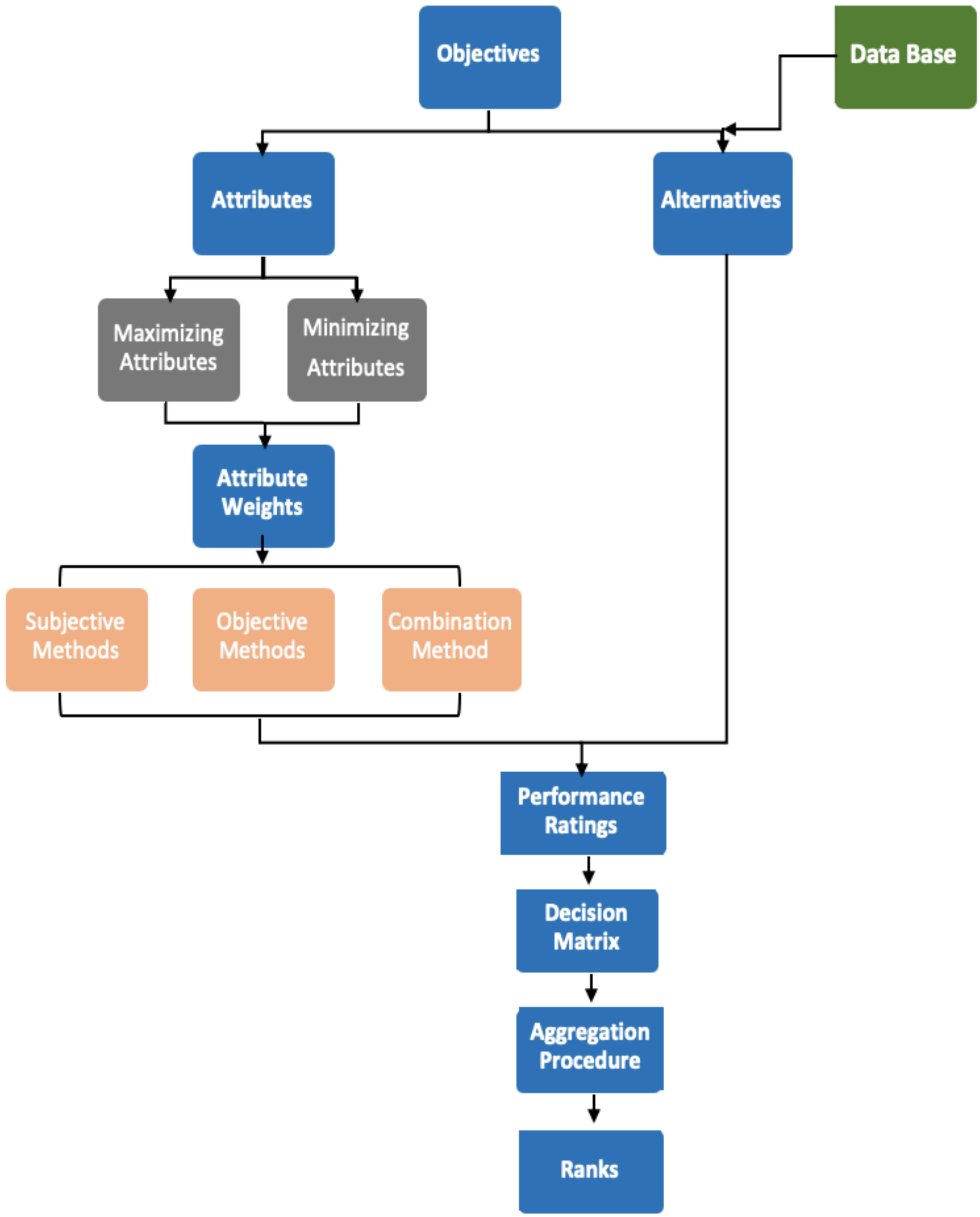


Figure 7: Flow-chart of MADM process.

2.4 MADM methods

2.4.1 SAW (Simple Additive Weighting) method

Simple Additive Weighting (SAW) method is one of the most popular and widely applied multi-attribute decision-making techniques [27], credited to its simplicity and relatively reliable assessment of alternatives. It is a quantitative method based on the weighted average using the arithmetic mean [27,38]. It assumes additive aggregation of decision outcomes, which is controlled by weights expressing the importance of attributes [31]. Its simplicity lies in the application of basic arithmetical operations such as multiplication and summation, to obtain final performance value of each alternative [29 & 31]. This method is called by many names such as weighted linear combination or scoring method [28], weighted sum method (WSM) [27], etc.

2.4.1A Mathematical representation

A multi-attribute decision making problem consisting of "M" alternatives and "N" attributes, an evaluation score can be calculated for each alternative by multiplying the scaled value (score or "utility") given to the alternative of that attribute with the weights of relative importance assigned for each attribute, followed by summing of the products for all criteria/attribute [27, 28 & 31]. Here, weights and scaled value ("utility" or score) can be obtained in both subjective and objective approaches. The assumption that governs this method is the additive utility assumption [27 & 28].

It is mathematically represented as:

$$P_i^{(WSM)} = \sum_{j=1}^{N} x_{ij} * w_j \quad \ldots\ldots\ldots\ldots (1)$$

, for *i=1, 2, 3, 4………., M.*

Where,

x_{ij} = score assigned to the ith alternative with respect to the jth attribute.

w_j = importance rating or relative weight of jth attribute,

2.4.1B SAW method in MADM problems

The advantage of the SAW method is that it is a proportional linear transformation of the raw data. It means that the relative order of magnitude of the standardized scores remains equal [28]. In single-dimensional cases, in which all the units are the same (e.g., dollars, feet, seconds), the SAW can be used without difficulty. The complexity with method emerges when it is applied to multi-dimensional decision-making problems [27, 28 & 29]. Then, in combining different dimensions, and consequently, different units, the additive utility assumption is violated [27]. To avoid this, the normalization of each score/utility (x_{ij}) is done to convert dimensional attributes to non-dimensional attributes [42].

Different normalization techniques can be applied in the SAW method. Out of those, Linear Normalization method and Linear Max-Min method are used quite common [for more details refer to N-1] [30].

Another important parameter required for this method is the relative importance rating or weights of attributes [32]. Weights are assigned to attributes both in subjective or objective or a combination of both. One of the most popular objective weight calculating method is Shannon's Entropy method (for more details refer to [290]).

After obtaining the normalized scores and weights, the SAW method can be applied to MADM problems to seek the best alternative.

2.4.1C Applications of SAW

a) Management science

SAW method is used extensively to solve human resource and management related decision-making problems such as:

- Selection of qualified personnel for a business organization for recruitment [28].
- Assessing the teaching staff and to evaluate the best one based on appropriate performance measurement [32].
- The process of determining employee remuneration [34], etc.

b) Communication and information technology

SAW method is seen as a "go to" method to solve the computer network and information technology-related decision-making problems. These problems usually involve multiple attributes in their decision problems and SAW provides a simple and easy approach to identify an optimum solution [33 & 36]. Some of the application are:

- Application SAW method to filter out the best suitable resources for parallel and distributed computing [33].
- Network selection in heterogeneous wireless networks in collaboration with a framework of noncooperative computing game [36].

c) Industrial engineering

The introduction of MCDM approach had an irrevocable change in how engineer approached decision making (DM) problems. There have been many MADM techniques applied to industrial DM problems and SAW in particular found full range application different industrial sectors such as:

Pharmaceutical sector

Supply-chain plays a key role in the sustainability of an industry and in the pharmaceutical industry, its role is more critical. A robust supply-chain will yield good stability and profits to the industry. So, it is critical to conduct a risk assessment before setting up a supply chain. SAW method has found its application in identifying and evaluating local supply-chain associated risks and finally present the ranking of the top risks [for more details refer 29]

Automotive sector

Inventory management is one of the important activities that help to sustain the industry in this fast-paced globalized world, and this is mainly applied to the Automobile industry [37]. Moreover, an effective ordering policy will ensure sustainability and many cost savings to the industry [37]. SAW was applied to provide a quantitative evaluation and ranking of ordering policies on the bases of multiple risk criteria and performance factors [for more information refer to 37].

Food sector

SAW is applied for selecting the best choice of food out of multiple alternatives by numerically analyzing various attributes [38].

Selection of best DNA extraction method for bees from honey, out of multiple extraction alternatives and a quantified attribute data using the SAW method [39].

d) Sensitivity analysis

The SAW method is commonly used in management sciences for not only evaluation or assessment or ranking alternatives, but also as a robust approach for sensitivity analysis to understand the uncertainties present in the data of decision models [30].

These are some of the applications of the SAW method.

2.4.1D Steps of operation involved in Simple Additive Weighting (SAW) [27, 28 & 31]

Step-1: Normalization of Decision Matrix

Normalization is done to convert a dimensional criterion to dimensionless entity [42]. To avoid violating linear additive utility assumption [27 & 29]

For maximizing attributes, i.e., “the larger, the better” kind of criteria such as quality, strength, etc.

Normalization formula is:

$$N_{ij}^{(LN)} = \frac{x_{ij}}{x_j^{max}} \quad \ldots\ldots\ldots\ldots \text{(2)}$$

, for *i=1, 2……, M; & j=1, 2……, N.*

For minimizing attributes, i.e., “the smaller, the better” kind of criteria such as cost, density, etc. Normalization formula is:

$$N_{ij}^{(LN)} = \frac{x_j^{min}}{x_{ij}} \quad \ldots\ldots\ldots\ldots (3)$$

, for *i=1, 2……, M; & j=1, 2……, N.*

Where,

$N_{ij}^{(LN)}$ is the normalized score/value/utility and “LN” abbreviates to “linear normalization”

Step-2: Weighted Normalization of Decision Matrix

It is a crucial step in this method, where relative importance ratings or weights of each criterion is multiplied to the normalized scores. Mathematically, it is represented as:

$$U_{ij}^{(WSM)} = w_j * N_{ij}^{(LN)} \quad \ldots\ldots\ldots\ldots (4)$$

, for *j=1, 2…., N.*

Where,

“U_{ij}” is the weighted normalized utility/score.

Step-3: Calculating the performance index of each alternative

It is done by summation of weighted normalized scores of each alternative:

$$P_i = \sum_{j=1}^{N} U_{ij}^{(WSM)} \quad \ldots\ldots\ldots\ldots (5)$$

Step-4: Ranking

The ranking of the alternatives is done in descending order, where the alternative with the highest performance is ranked as “best” and alternative with the least performance is ranked as “worst.”

2.4.1E Expansion of SAW method

SAW method had been adopted to various MADM problems, modified continuously to respond to different constraints and objectives. It is tough to find decision problems with crisp data all the time. Sometimes, DM problems are associated with fuzzy data, and it poses a challenge to assess the data and apply MADM methods to find the best alternatives. Usually, in a fuzzy MADM problem (MADM problem involving fuzzy data), criteria values and the relative weights are characterized by fuzzy numbers. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1[40]. To deal with such problems SAW is modified to accommodate fuzzy values and is known as Fuzzy SAW (FSAW) method [39]. There are many successful applications for this method in various fields [41].

2.4.2 SMART (Simple Multi-Attribute Rating Technique) method

SMART (Simple Multi-Attribute Rating Technique, [Edwards, 1971, 1977]) provides a simple way to implement the principles of multi-attribute utility theory (MAUT) [48]. SMART requires no judgments of preference or indifference among hypothetical alternatives, as is required with *Logical Decision* and most MAUT methods [43]. This multi-attribute decision-making method is guided on the linear additive principle, and decision-making model where each alternative consists of some attributes that have values and each attribute has weights that describe how important compared to another attribute [44]. The multi-linear function model of this method is mathematically represented as [43, 44 & 45]:

$$P_i = \sum_{j=1}^{N} x_{ij} * w_j \quad \ldots\ldots\ldots\ldots (6)$$

, for $i=1, 2, 3 \ldots\ldots\ldots, M$.

This method is easy to use and also a good trade-off method between modeling error and elicitation error" [46]. Due to these reasons, it is widely used in many fields such as management, supply-chain, military, engineering, etc.

2.4.2A An overview of SMART methodology

The SMART method proposed by W. Edwards (1971, 1977), has been developed from multi-attribute utility theory (MAUT) [46]. The basic idea of multi-attribute utility measurement is that every outcome of an action may have value on several different dimensions. MAUT seeks to measure these values one dimension at a time, followed by aggregation of these values across dimensions through a weighting procedure [47 & 48]. The simplest and most widely used aggregation rule is to take a weighted linear average (Eq-1). This is the general working principle behind SMART.

Edwards proposed a ten-step approach, which includes identifying objectives, shortlisting the alternatives, allotting value to the alternative with respect to attribute, developing relative importance ratings (or weights) for each attribute, normalizing the weights and scores (utilities), calculating a final performance value for each alternative obtained from an aggregation principle [47& 50].

The major advantage of this method is its Value independence, which implies that the extent of preference for one alternative over another on a particular dimension (attribute) is unaffected by relative attainment values (utility) [47]. Edwards argued that while value independence is a strong assumption, quite substantial amounts of deviation from value independence will make little difference to an alternative's and even less difference to the ranking of alternatives [45 & 47].

2.4.2B A SMART approach to MADM problem

A MADM problem can be expressed as $(M \times N)$ matrix consisting of "M" number of alternatives and "N" number of attributes, in which element "x_{ij}" indicates the performance of the alternatives with respect to attributes [27]. Once the decision matrix is constructed, the SMART approach can proceed by the following steps [44, 47 & 50]:

Step-1: Normalization of Decision Matrix

Linear max-min normalization method [42] is employed in SMART technique.

For maximizing attributes,

$$N_{ij}^{LM} = \frac{x_{ij}-x_j^{min}}{x_j^{max}-x_j^{min}} \quad \ldots\ldots\ldots\ldots (7)$$

, for all *i=1, 2……, M; & j=1, 2……., N.*

For minimizing attributes,

$$N_{ij}^{LM} = \frac{x_j^{max}-x_{ij}}{x_j^{max}-x_j^{min}} \quad \ldots\ldots\ldots\ldots (8)$$

, for all *i=1,2……, M; & j=1,2……., N.*

Where,

"N_{ij}^{LM}" is the normalized element, and "LM" indicates the type of normalization.

Step-2: Weighted Normalization of Decision matrix

It is a crucial step in this method, where relative importance ratings or weights of each criterion is multiplied to the normalized scores. Mathematically, it is represented as:

$$U_{ij} = w_j * N_{ij}^{LM} \quad \text{………..} (9)$$

, for *j=1, 2…., N.*

Where,

"U_{ij}" is the weighted normalized utility/score.

Step-3: Performance Value/Score

It is done by summation of weighted normalized scores of each alternative:

$$P_i = \sum_{j=1}^{N} U_{ij} \quad \text{………..} (10)$$

Step-4: Ranking

The ranking of the alternatives is done in descending order, where the alternative with the highest performance is ranked as "best" and alternative with the least performance is ranked as "worst."

2.4.2C SAW and SMART: similarities and differences

Both SMART and SAW methods are similar when it comes to aggregation method employed, i.e., weighted linear average [27 & 44]. However, both the approaches are recognized as different methods due to a simple distinction. SMART is a MADM model, where a detailed layout is presented to solve MADM problem from identifying an objective to cascading the important alternatives, to weighing the attribute to finally ranking the alternatives based on performance index [50]. Whereas, SAW is a ranking technique that deals with a numerical approach to find the optimal solution to a MADM problem [28].

2.4.2D "SMART" application to various MADM problems

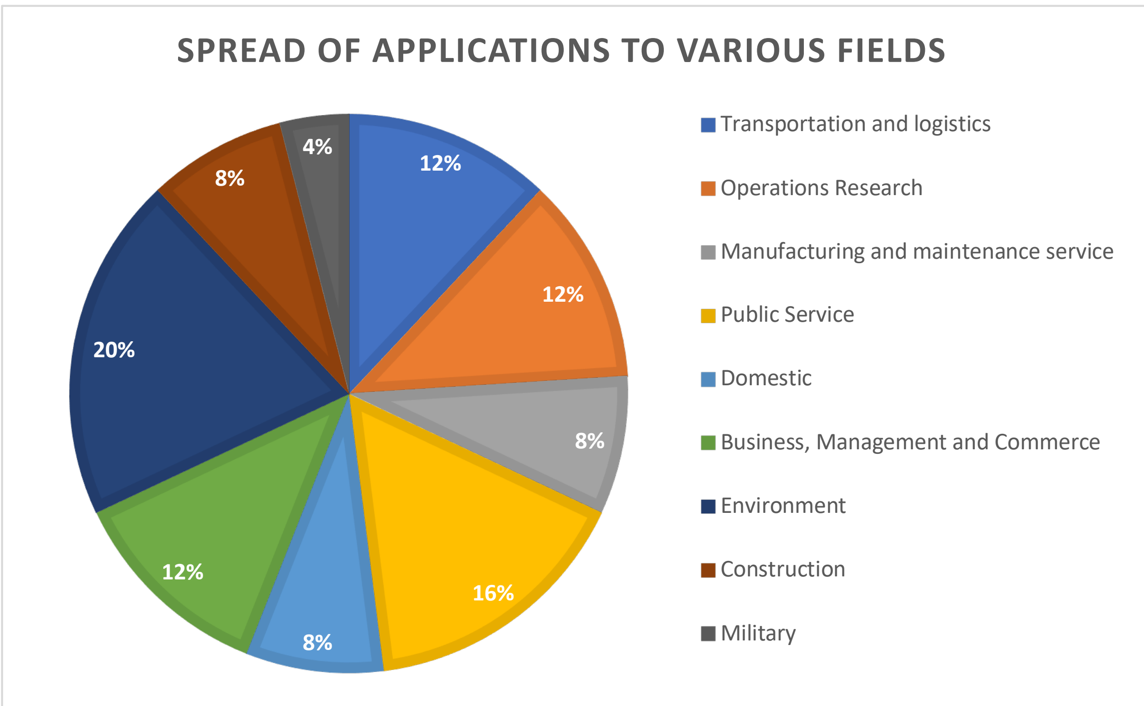


Figure 8: Areas of application of SMART (data is based on my research alone on just published papers on the particular topic, there may be better data presented elsewhere) [43 to 61].

The SMART model has a wide range of application in various fields stretching from Environment to simple domestic cases, as shown in figure 8.

For a detailed insight of application for a specific area, please refer the following references:

- Transportation and Logistics [43,50 & 52].
- Operations Research [44, 48 & 49].
- Manufacturing & Maintenance service [45].
- Public Service [56, 57 & 59].
- Business, Management, and Commerce [58].
- Environment [47, 51, 53, 55 & 60].
- Construction [54 & 61].

2.4.2E Extensions of SMART methods

SMARTS (Simple Multi-attribute Rating Technique using Swings)

SMARTS is a modification over the standard SMART, to ensure that the full range of plausible values is represented while extending weights to select the range of objective measure being weighted as well as its importance and not just limiting its value to single-dimensional utilities [45 & 47].

SMARTER (Simple Multi-attribute Rating Technique Exploiting Ranks)

SMARTER (SMART Exploiting Ranks) uses Barron and Barrett's [1995] rank weights to eliminate the most difficult judgmental step in SMARTS. The relative importance of objectives is based on the order of objectives, using the centroid method, which uses rank order to estimate the set of weights, minimizing the maximum error by identifying the centroid of all possible weights maintaining the rank order of objective importance [45 & 47].

2.4.3 ARAS (Additive Ratio Assessment) method

A typical MADM problem is concerned with the task of ranking a finite number of decision alternatives, each of which is explicitly described in terms of different decision attribute which have to be considered simultaneously. According to the ARAS method, a utility function value determining the complex relative efficiency of a feasible alternative is directly proportional to the relative effect of values and weights of the main criteria considered in a problem [62].

This method was proposed by Edmundas Kazimieras Zavadskas and Zenonas Turskis, both academicians and professors affiliated to Vilnius Gediminas Technical University. The ARAS method is based on quantitative measurements and utility theory [66], where alternatives are assessed on the degree of utility and ranked accordingly in the descending order.

2.4.3A The "Degree of Utility" in ARAS method

Most of the MADM methods have three common steps in their procedure:1) Formation of Decision matrix, 2) Normalization of the elements (scores/values/utilities) of the matrix, 3) Aggregation operation [27]. Moreover, usually, the step that separates one method from another and makes them unique is how the data is further processed to determine the ranking, i.e., Aggregation operation.

The unique concept of "Degree of utility" is demonstrated in the aggregation steps. "Degree of utility" for each alternative is measured as the ratio of optimal function to performance index of each alternative [62 & 68], where the optimal function is the best solution obtained from the list of performance indices of all the alternatives [62, 63 & 64].

Mathematical Representation of "Degree of Utility":

$$K_i = \frac{P_i}{P^*} \quad \ldots\ldots\ldots\ldots (11)$$

, for *i=1, 2 …., M.*

Where,

"K_i" is the Degree of Utility,

"P_i" is the Performance Index of "ith" alternative,

" P^* " is the Optimal function.

2.4.3B The modus operandi of ARAS method [63, 65, 66 & 68.]

Step-1: Normalization of Decision Matrix (DM)

A normalization operation is performed to convert dimensional attributes into dimensionless attributes so that further aggregation can be done without violating linear additive utility assumption [61] [27]. ARAS method employs linear sum-based normalization technique [61].

For Maximizing attributes:

$$N_{ij}^{LS} = \frac{x_{ij}}{\sum_i^m x_{ij}} \quad \ldots\ldots\ldots\ldots (12)$$

, For *i=1, 2……., M; & j=1, 2........., N.*

For Minimizing attributes:

$$N_{ij}^{LS} = \frac{1/x_{ij}}{\sum_i^{m} 1/x_{ij}} \quad \text{………… (13)}$$

, For *i=1, 2……., M; & j=1, 2........., N.*

Where,

"x_{ij}" indicates element from the DM belonging to i^{th} alternative and j^{th} attribute.

"N_{ij}^{LS}" is the normalized element, LS stands for Linear sum-based normalization technique.

Step-2: Weighted Normalization of the DM

This is an important step, where priority (weights/relative importance) and preference (individual criterion preference scores) merges into a meaning full co-relation. It is done by a weighted average method and mathematically represented as:

$$U_{ij} = w_j * N_{ij}^{(LS)} \quad \text{………… (14)}$$

, for *j=1, 2…., N.*

Where,

"w_j" is the relative importance ratings or weight of j^{th} attribute.

"U_{ij}" is the weighted normalized score.

Step-3: Calculating Performance index/Optimal function

The performance index for each alternative is calculated by the linear additive method and represented as:

$$P_i = \sum_{j=1}^{N} U_{ij} \quad \ldots\ldots\ldots\ldots (15)$$

, for *i=1, 2……., M.*

Where,

"P_i" is the performance index of i^{th} alternative.

Step-4: Determining the "ideal Optimal function."

An ideal optimal function/performance index is the best performance score obtained by an alternative, which in this case is the highest score attained.

$$P^* = \max_i P_i \quad \ldots\ldots\ldots\ldots (16)$$

, for *i=1, 2……., M.*

Where,

"P^*" is the ideal optimal function.

Step-5: Computing the "Degree of utility."

$$K_i = \frac{P_i}{P^*} \quad \ldots\ldots\ldots\ldots (17)$$

, for *i=1, 2……., M.*

Where,

"K_i" is the Degree of Utility of "i^{th}" alternative.

Step-6: Ranking of the alternatives

The alternatives are ranked in descending order, where alternatives with the highest degree of utility scores are ranked at the top and those with a low score at the bottom.

2.4.3C Extension of ARAS method to deal with uncertainty

Over the years, Additive Ratio Assessment (ARAS) method has been extended to deal with uncertain and incomplete information by utilizing Fuzzy sets and Grey theory [83].

Grey Additive Ratio Assessment (ARAS-G) Method

Grey theory is one of the methods used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information. The advantage of grey theory over fuzzy sets theory is that grey theory can deal flexibly with the fuzziness situation. Alternative's selection can be viewed as a grey system process. We use grey theory to resolve it. The ratings of criteria are described by linguistic variables that can be expressed in grey numbers [81].

The defining step in this method is the formation of grey decision-making matrix (GDMM. Then, the rest of the procedure is similar to the standard ARAS method [81].

Fuzzy Additive Ratio Assessment Method (ARAS–F)

When dealing with MADM problems, usually there exists information which is incomplete and uncertain, so the decision makers cannot easily express their judgments on the candidates with exact and crisp values. As well as there are many real-life complex problems that need to involve a wide domain of knowledge. These conditions in which decisions are based on obscure and unreliable information or lack of knowledge and personal preferences of the experts can create difficulties in the decision-making process. These difficulties can lead to deceptive and uncertain decisions. Therefore, this led t the introduction of Fuzzy sets. Fuzzy sets generally provide a more adequate description to model real-life decision problems than real numbers and presented to fix these challenges and provided a basis for the development of a variety of fuzzy decision-making models [77].

The general procedure of simple ARAS method is carried out in this method, with the inclusion of fuzzy sets in place of crisp criteria values in the decision matrix (DM) [77, 78 & 79].

2.4.3D Different fields of applications

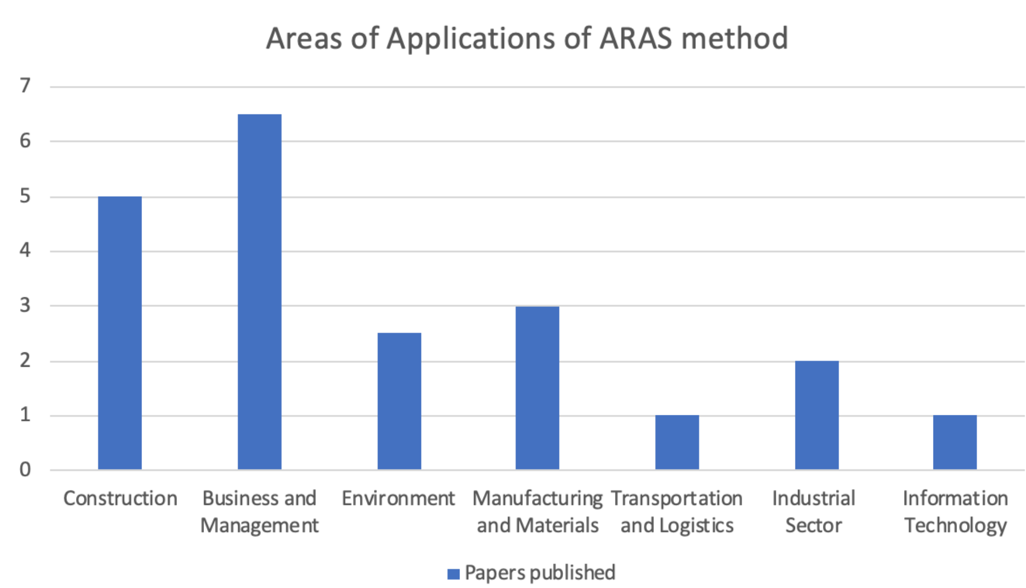


Figure 9: Application of ARAS and its extensions in various fields (data is based on my research).

Even though this method is relatively new, it has a wide range of applications [63]. Some of its applied fields are illustrated in figure 9, which is based on limited research data, and there is a continuous expansion of this method to other areas even as we speak (For more detail understanding of some of the applications, please refer [62-76, 77-80 & 81-82]).

2.4.3E Advantages of ARAS method

- Comparison of alternatives with the optimal alternative, which is the core principle of ARAS method can be adopted to find ways of improving alternative projects [62].
- Both subjective and objective methods of obtaining relative weights for each alternative can be employed.
- Relative more straightforward method to execute in a MADM problem and easy to understand for the decision maker.

2.4.4 LFA (Loss Function Approach) method

Decision making is a key factor to achieve success in any discipline, especially in a field which requires handling large amounts of information and knowledge such as construction, environment, engineering, and management, etc. [84 The monopolistic approach in dealing with a decision problem has ceased to exist in this highly competitive world that strives on efficiency both operationally and cost associated. Decision making in such an environment can often be an arduous and difficult task to execute [84]. Moreover, continuous research is leading to newer discoveries, which in return, making a decision process more and more complicated with multiple alternatives to analyze. This has led to the foundation of Multiple attribute decision making (MADM) methods. MADM approach is employed in a situation where the decision maker has to choose the best alternative among a finite set, based on multiple and usually conflicting attributes. The MADM problems are classified as scoring methods, compromising methods, and concordance methods. The concordance methods are difficult to understand compared to scoring and compromising methods [85].

Loss Function approach proposed by Vommi *et al.* and Kakollu *et al. [85]* is simple to understand and easy-to-convince method for multiple attribute decision-making problems. This method is based on the philosophy of both scoring and comprising methods and relies on the loss for not choosing the ideal best alternative [85]. It is a relatively new method officially published in 2016 and steadily gaining applications to various fields.

2.4.4A The philosophy behind LFA method

Multiple attribute decision-making problems pose a challenge to the decision maker to select the best alternative among the set of alternatives. Each alternative consists of few attributes based on which the decision maker chooses the best alternative. Some MADM methods such as TOPSIS (The Technique for Order of Preference by Similarity to Ideal Solution) chooses the best alternative based on the Euclidean distances of the alternatives from the ideal best and ideal worst alternatives [TOPSIS-4]. Whereas, VIKOR ('Vlse Kriterijumska Optimizacija Kompromisno Resenje') ranks an alternative foremost on the basis of best compromise among all the attributes by measuring the closeness to the ideal solution [269]. Whereas, this method relies on the principle of measuring the loss caused by each attribute for not being the best concerning the best value available among all the alternatives [85].

To calculate the loss of any alternative, initially, the loss caused by each attribute of that particular alternative has to be calculated. To do that, the attributes of the chosen alternative are to be compared with those of the ideally best alternative (IBA) [85].

2.4.4B Application of LFA method to MADM problem [85]

Step-1: Developing a Decision Matrix

A MADM problem is usually expressed in a matrix format. A decision matrix DM is an (*M* × *N*) matrix in which element. "x_{ij}" indicates the performance of alternative "A_i" when it is evaluated in terms of decision criterion "C_j", (for *i* = 1,2, 3..., *M*, and *j* = 1,2, 3..., *N*) [27].

Step-2: Identifying the Maximizing and Minimizing attributes

It is very important to identify the beneficial and non-beneficial attributes and if possible, segregate them into two groups. So, that further aggregation steps can be applied without any confusion.

Step-3: Computing relative Weights for each alternative:

Weights or relative importance ratings can be obtained in two ways, i.e., subjective and objective. Subjective methods are based on the interaction with experts or decision makers, and the other hand, objective methods are determined by "Voice of Data" [290].

Step-4: Calculating the loss function for Maximizing and Minimizing attributes

For Maximizing/Beneficial attributes:

$$L_{ij}^{B} = \frac{w_j(x_{ij}^{max} - x_{ij})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \quad \ldots\ldots\ldots\ldots (18)$$

, for all $i = 1, 2, 3..., M;$ & j $= 1, 2, 3..., N.$

where,

"L_{ij}^{B}" is the linear loss function of i^{th} alternative for j^{th} attribute and superscript "B" indicates Beneficial,

"w_j" is the relative weight of j^{th} attribute.

For Minimizing/Non-beneficial attributes:

$$L_{ij}^{NB} = \frac{w_j(x_{ij} - x_{ij}^{min})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \quad \text{………… (19)}$$

for all $i = 1, 2, 3..., M$; & $j = 1, 2, 3..., N$.

Where,

"L_{ij}^{NB}" is the linear loss function of i^{th} alternative for j^{th} attribute and superscript "NB" indicates Non-beneficial,

"w_j" is the relative weight of j^{th} attribute.

Step-5: Calculating the Total Loss caused by Beneficial and Non-beneficial attributes

$$L_i^{B*} = \sum_{j=1}^{S} \frac{w_j(x_{ij}^{max} - x_{ij})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \quad \text{………… (20)}$$

, for $j \in S$.

Where,

"L_i^{B*}" is total loss caused by beneficial criteria,

"S" indicates the set of beneficial attributes.

$$L_i^{NB*} = \sum_{j=1}^{H} \frac{w_j(x_{ij} - x_{ij}^{min})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \quad \ldots\ldots\ldots\ldots (21)$$

, for $j \in H$.

"L_i^{NB*}" is totaa l loss caused by beneficial criteria,

"H" indicates the set of non-beneficial attributes.

Step-6: Determining the Total Loss function (L_i) for each alternative

$$L_i = L_i^{B*} + L_i^{NB*} \quad \ldots\ldots\ldots\ldots (22)$$

Step-7: Ranking of the Alternatives

Unlike the methods discussed so far, the ranking of the alternative is done in the ascending order where the alternative with lowest loss function is the best solution, and the alternative with highest loss function is the worst solution to the problem.

2.4.4C LFA method vs TOPSIS method

The popularity of the TOPSIS technique is quite evident by the number of its applications in different areas. In TOPSIS methodology, the best alternative is chosen based on the Euclidean distances of the alternatives from the ideal best and ideal worst alternatives [265]. Irrespective of the nature of attribute, it considers Euclidean distances only. However, in reality, the effect of the attribute need not always be proportional to the straight-line distance. For example, in Taguchi's loss function approach, the losses are taken proportional to the square of the deviation of the quality characteristic from the desired nominal value [85].

In LFA method, initially, the ideal best alternative is obtained from the available alternatives. The ideal best alternative consists of all higher values for the beneficial attributes and all lower values for the non- beneficial values. The loss of choosing each alternative is calculated concerning the ideal best alternative. The alternative with the lowest possible total loss is chosen as the best alternative. The losses can be calculated not only based on quadratic function, but they can be calculated using linear and cubic loss functions also [85].

In case of Quadratic functions: [85]

For Beneficial criteria:

$$L_{ij}^{B} = w_j \left[\frac{(x_{ij}^{max} - x_{ij})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \right]^2 \quad \ldots\ldots\ldots\ldots (23)$$

, for all i = 1, 2, 3..., M; & j = 1, 2, 3..., N.

For Non-beneficial criteria:

$$L_{ij}^{NB} = w_j \left[\frac{(x_{ij} - x_{ij}^{min})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \right]^2 \quad \ldots\ldots\ldots\ldots (24)$$

for all i = 1, 2, 3..., M; & j = 1, 2, 3..., N.

In the case of Cubic functions: [85]

For Beneficial criteria:

$$L_{ij}^{B} = w_j \left[\frac{(x_{ij}^{max} - x_{ij})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \right]^3 \quad \text{………… (25)}$$

for all i = 1,2, 3..., M; & j = 1, 2, 3..., N.,

For Non-beneficial criteria:

$$L_{ij}^{NB} = w_j \left[\frac{(x_{ij} - x_{ij}^{min})}{\left(x_{ij}^{max} - x_{ij}^{min}\right)} \right]^3 \quad \text{………… (26)}$$

for all i = 1, 2, 3..., M; & j = 1, 2, 3..., N.

The ability of this method to extend to the quadratic and cubic criteria function has extended its range to incorporate different criteria models to produce a reasonable selection of alternative.

2.4.5 WP (Weighted Product) method

The weighted product model (or WPM) is similar to the SAW. The main contrast between the two is seen in the aggregation phase, where WPM follows multiplication operation instead of an addition to obtain performance values for the alternatives [86, 87 and 27] It is also referred as Multiplicative Exponent Weighting (MEW) [86] because the assigned scores/values for each alternative are raised to the power equivalent to the relative weight of the corresponding attribute [27].

In general, the over-all performance of each alternative is calculated using the following equation:

$$P_i^{(WPM)} = \prod_j^N x_{ij}{}^{w_j} \quad \ldots\ldots\ldots\ldots (27)$$

where: N is the number of attributes, "x_{ij}" is the normalized value of the i^{th} alternative in terms of the j^{th} criterion, and "w_j" is the weight of importance of the j^{th} criterion.

2.4.5A Solving a MADM problem using the WPM method [27, 86 & 87]

Step-1: Normalization of Decision Matrix

Linear Normalization is employed in this method to obtain dimensionless values. It is mathematically represented by the formula:

For a Maximizing/Beneficial Attribute:

$$N_{ij}^{(LN)} = \frac{x_{ij}}{x_j^{max}} \quad \ldots\ldots\ldots\ldots (28)$$

, for $i=1,2\ldots\ldots, M;$ & $j=1,2\ldots\ldots, N.$

For a Minimizing/Non-beneficial Attribute:

$$N_{ij}^{(LN)} = \frac{x_j^{min}}{x_{ij}} \quad \ldots\ldots\ldots\ldots (29)$$

, for $i=1,2\ldots\ldots, M;$ & $j=1,2\ldots\ldots, N.$

Where,

"$N_{ij}^{(LN)}$"is the normalized value of i^{th}alternative for j^{th} attribute and "LN" indicates linear normalization method.

Step-2: Weighted Normalization of Decision Matrix

It is a crucial step in this method, where relative importance ratings or weights of each criterion is multiplied to the normalized scores. Mathematically, it is represented as:

$$U_{ij}^{WPM)} = \left(N_{ij}^{(LN)}\right)^{w_j} \quad \ldots\ldots\ldots\ldots (30)$$

, for *j=1, 2…., N.*

Where,

U_{ij} is the weighted normalized value of i^{th} alternative with respect to j^{th} attribute.

Step-3: Calculating over-all performance score for each alternative

Multiplying each weighted normalized value (U_{ij}) will give the over-all performance score of each alternative. The mathematical representation is:

$$P_i = \prod_j^N U_{ij}^{(WPM)} \quad \ldots\ldots\ldots\ldots (31)$$

, for all *i=1,2………, M.*

Step-4: Ranking

Alternatives are ranked in descending order, where alternative with highest performance value will be ranked as "best," and similarly the lowest performer will be ranked as "worst."

2.4.5B Utilization of WPM method in various fields

WPM method has penetrated most of the areas that implement MADM approach for decision making. It is tough to mention all the areas of application of this method, but some of the more noticeable fields are mentioned below:

a) Public sector

WPM, due to its relatively simple and easy-to-apply methodology has been extensively used in the public sector. For the selection of home appliances to ranking the districts, there have been numerous applications in this field. Some of the important applications are:

- Developing a Decision support system to rank the elementary school in each region to determine the quality of schools [86].
- Selection of best location for small hydro power project [92].
- Acoustic signature based optimal route selection to facilitate good quality service to the users [93].

b) Networking and communication

WPM method is used most in this field for selecting and ranking alternatives to assist in decision making. Few examples are:

- Performance Evaluation of Multi-Criteria Vertical Handover for Heterogeneous Wireless Networks [88].
- Comparison between Vertical Handoff Decision Algorithms for Heterogeneous Wireless Networks [89].
- Network selection in heterogeneous wireless networks [90].

c) Management and business

Another area which has been dominated by the application of MADM methods for decision support. Application of WPM method ranges from a selection of best personnel to rank assessment of employee based on performance to a selection of the right project. One of such relatively latest application of WPM is:

- Developing an e-commerce performance assessment model [91].

2.4.6 WASPAS (Weighted Aggregated Sum-Product Assessment) method

2.4.6A Background

The Weighted Sum Model (WSM) is one of the best known and often applied Multiple Attribute Decision Making (MADM) tool for evaluating multiple alternatives [27 & 93]. The secret behind its popularity is its operation simplicity and accuracy of the results. Its quantitative approach based on the weighted average using the arithmetic mean has been approved as an effective way to assess MADM problem.

On the other hand, Weighted Product Method (WPM), which is similar to the WSM method [WPM-1], also functions on the linear transformation of the decision criterion. However, unlike WSM, it works on multiplicative exponential principle [27 and 86]. Both the methods avoid deficiencies that arise from the typical linear criterion form [93]. In the year 2009, Zavadskas *et al.* [WASPAS-1], came up with an idea to formulate a joint criterion based on a weighted mean approach which combines weighted aggregation procedures of additive and multiplicative methods for constructing the generalized criterion. Thus, it led to the proposal of Weighted

Aggregated Sum-Product Assessment (WASPAS) method for ranking of alternatives even though this approach has not negated the typical linear criterion deficiencies but has proved to improve the accuracy of ranking by 1.3 times when compared to WPM and 1.6 times as compared to WSM [94].

2.4.6B WASPAS: As a MADM method

MADM problems relate to making preferential decisions concerning choice, assessment or ranking of decision alternatives in relation to chosen decision attributes. Assessments (criterion) of each alternative in relations to attribute are most commonly presented in the form of decision matrix [95].

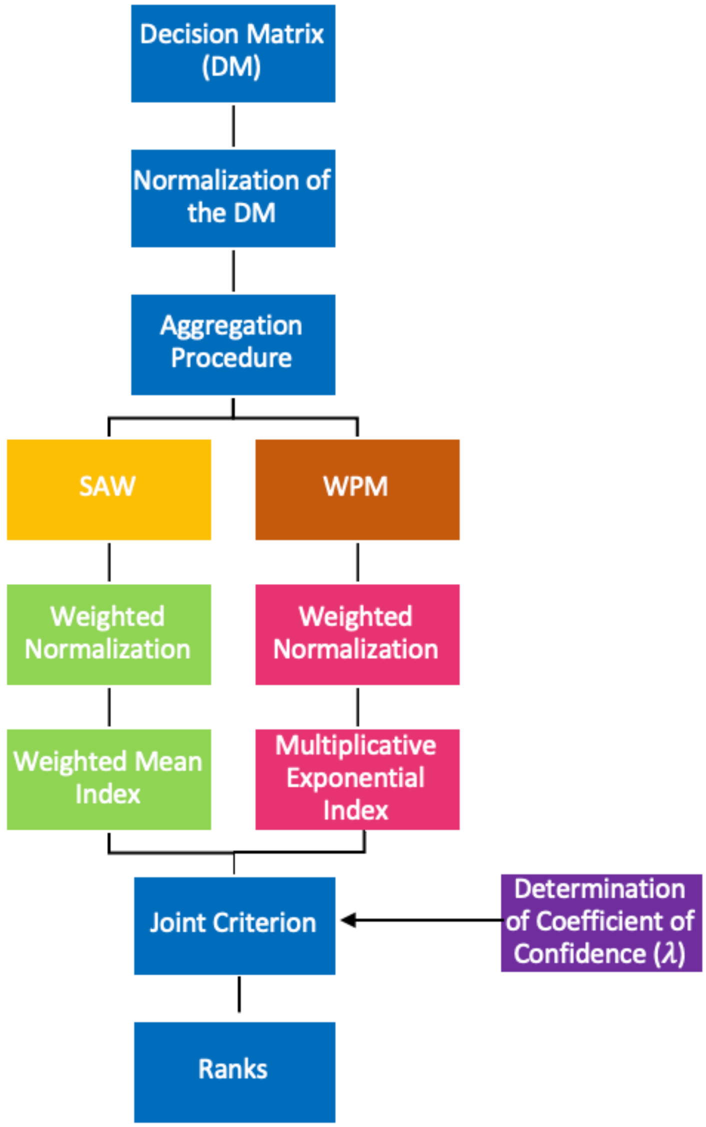


Figure 10: General modus operandi of WASPAS method.

The WASPAS method responds to the MADM problem similarly as all the previous MADM methods in this literature review, that is by triggering the formation of Decision Matrix. The integration of the Weighted Sum Model (WSM) and Weighted Product Model (WPM) occurs in the aggregation phase, where the individual aggregation methods of each model are combined, and a joint criterion is formulated to give the final preference score for the alternatives. Figure 10 shows the flow-chart of WASPAS method.

Mathematical Representation of Joint Criterion [93 & 94]:

$$P_i^{(WASPAS)} = (\lambda)P_i^{(WSM)} + (1-\lambda)P_i^{(WPM)} \quad \text{………… (32)}$$

Where,

"$P_i^{(WASPAS)}$" is the Joint Criterion of "i^{th}" alternative.

"$P_i^{(WSM)}$" is the Weighted Mean aggregation of WSM.

"$P_i^{(WPM)}$" is the Multiplicative exponential aggregation of WPM.

"λ" is the Coefficient of Confidence [93].

2.4.6C Optimal values of coefficient of confidence "$\boldsymbol{\lambda}$" [94]

The accuracy of WASPAS is based on initial criteria accuracy ($P_i^{(WSM)}$ & $P_i^{(WPM)}$) and Coefficient of Confidence (λ)= 0, ..., 1. When λ = 0, WASPAS is transformed to WPM; and when λ = 1, WASPAS is transformed to WSM [94].

Assuming that errors of determining the initial criteria values are stochastic, the variance (σ^2) or standard deviation (σ) is a measure of dispersion in the sample distribution [94].

Suppose, there is a function:

$$y = \xi(x_1, x_2, \dots x_n) \quad \dots\dots\dots\dots (33)$$

The standard deviations of the function's arguments (Eq. 2) are:

$$\sigma(x_1), \sigma(x_2), \dots, \sigma(x_n) \quad \dots\dots\dots\dots (34)$$

The variance of function "y" is determined as follows:

$$\sigma(y^2) = \sum_i^n \left(\frac{\partial\xi}{\partial x_i}\right)^2 \sigma^2(x_i) \quad \dots\dots\dots\dots (35)$$

Where,

$\left(\frac{\partial\xi}{\partial x_i}\right)$ is a partial derivative of a function in respect of every argument.

Now to calculate optimal values of "λ", i.e., to find minimum dispersion and to assure maximal accuracy of estimation. Optimal values of "λ" can be found when searching extreme of function. Extreme of function can be found when the derivative of Eq. (1) in regard to "λ" is equated to zero, and finally the optimal values for "λ" can be calculated as:

$$\lambda = \frac{\sigma^2\left(P_i^{(WPM)}\right)}{\sigma^2\left(P_i^{(WSM)}\right) + \sigma^2\left(P_i^{(WPM)}\right)} \quad \dots\dots\dots\dots (36)$$

Optimal "λ" should be calculated for every alternative before applying WASPAS. Optimal "λ" may vary depending on the ratio of $\sigma^2\left(P_i^{(WSM)}\right)/\sigma^2\left(P_i^{(WPM)}\right)$ in every particular case [94].

2.4.6D Different attribute weighting methods employed with WASPAS

The weighting of attributes is the process of defining the importance of the criteria concerning the decision problem, which involves multiple objectives and constraints. Weighting can be categorized into three groups: Subjective methods, provides a systematically designed mathematical framework for the decision-makers (DM) to assign importance ratings to the attributes; Objective methods, which does not require DM in determining the importance of the criteria and, the Combined weighting scheme of the two previous groups [26].

a) Subjective methods

The subjective methods, where decision-makers (DM) play a crucial part in assigning the importance ratings (or weights) to the attributes, have been successfully applied in WASPAS method to attain the weights. Most commonly applied subjective weighting techniques are: Analytical Hierarchy Process (AHP) [101, 102, 104, 114 & 116], Step-wise Weight Assessment Ratio Analysis (SWARA) [105], and Factor Relationship (FARE) [100].

b) Objective methods

Most commonly applied objective methods in association with WASPAS method are Shannon's Entropy method and Criteria Importance Through Inter-criteria Correlation (CRITIC).

2.4.6E Steps involved Weighted Aggregated Sum-Product Assessment (WASPAS) method [94, 96, 97, 98 & 99]

Step-1: Normalization of the Decision Matrix

Normalization of each score/utility (x_{ij}) is done to convert dimensional attributes to non-dimensional attributes [42].

Different normalization techniques can be applied to achieve the desired purpose. However, most commonly applied technique is Linear Normalization method [94, 95 and 99].

For a Maximizing Attribute:

$$N_{ij}^{(LN)} = \frac{x_{ij}}{x_j^{max}} \quad \ldots\ldots\ldots\ldots (37)$$

, for *i=1, 2……, M; & j=1, 2……, N.*

For a Minimizing Attribute:

$$N_{ij}^{(LN)} = \frac{x_j^{min}}{x_{ij}} \quad \ldots\ldots\ldots\ldots (38)$$

, for *i=1, 2……, M; & j =1, 2……, N.*

Step-2: WPM Aggregation procedure to obtain the preference index (Multiplicative Exponential Criterion)

2.1 Weighted Normalization of Decision Matrix:

It is a crucial step in this method, where relative importance ratings or weights of each criterion is multiplied to the normalized scores. Mathematically, it is represented as:

$$U_{ij}^{(WPM)} = \left(N_{ij}^{(LN)}\right)^{w_j} \quad \ldots\ldots\ldots\ldots (39)$$

, for *j=1, 2…., N.*

Where,

"U_{ij}" is the weighted normalized value of i^{th} alternative with respect to j^{th} attribute.

2.2 Calculating Preference index for each alternative:

Multiplying each weighted normalized value (U_{ij}) will give the over-all performance score of each alternative. The mathematical representation is:

$$P_i^{(WPM)} = \prod_j^N U_{ij}^{(WPM)} \quad \ldots\ldots\ldots\ldots (40)$$

, for all *i=1, 2…., M.*

Step-3: WSM Aggregation procedure to determine the preference index (Weighted Mean Criterion)

3.1 Weighted Normalization of Decision Matrix:

It is a crucial step in this method, where relative importance ratings or weights of each criterion is multiplied to the normalized scores. Mathematically, it is represented as:

$$U_{ij}^{(WSM)} = w_j * N_{ij}^{(LN)} \quad \ldots\ldots\ldots\ldots (41)$$

, for *j=1, 2…., N.*

Where,

"U_{ij}" is the weighted normalized utility/score.

3.2 Calculating the performance index of each alternative

It is done by summation of weighted normalized scores of each alternative:

$$P_i^{(WSM)} = \sum_{j=1}^{N} U_{ij}^{(WSM)} \quad \ldots\ldots\ldots\ldots (42)$$

Step-4: Computing of Optimal Coefficient of Confidence "λ" values

"λ" is calculated for each alternative to obtain an accurate ranking/assessment of the alternatives using the following formula:

$$\lambda = \frac{\sigma^2\left(P_i^{(WPM)}\right)}{\sigma^2\left(P_i^{(WSM)}\right)+\sigma^2\left(P_i^{(WPM)}\right)} \quad \ldots\ldots\ldots\ldots (43)$$

, for i^{th} alternative.

Step-5: Determination of Joint Criterion for each alternative

"Joint Criterion" or over-all performance index for each attribute is calculated by combining the weighted mean criterion and the multiplicative exponential criterion. Coefficient "λ" plays a pivotal role in improving the accuracy of the performance index.

Joint Criterion for each alternative is given by:

$$P_i^{(WASPAS)} = (\lambda)P_i^{(WSM)} + (1-\lambda)P_i^{(WPM)} \quad \ldots\ldots\ldots\ldots (44)$$

Which can be written as:

$$P_i^{(WASPAS)} = (\lambda)\sum_{j=1}^{N} U_{ij}^{(WSM)} + (1-\lambda)\prod_{j}^{N} U_{ij}^{(WPM)} \quad \ldots\ldots\ldots\ldots (45)$$

Step-6: Ranking the Alternatives

Alternatives are ranked in the descending order meaning the alternative with highest performance score is ranked as the best, and similarly, the alternative with the lowest score is regarded as the worst alternative.

2.4.6F Extension of WASPAS method

Most decisions made in the real world take place in an environment in which the goals and constraints, because of their complexity, are not known precisely, and thus, the problem cannot be exactly defined or precisely represented in a crisp value (Bellman *et al* and Zadeh *et al.*, 1970) [289]. To deal with the uncertainties in the decision problem, Zadeh *et al.* (1965) established fuzzy mathematics in the 1960s, Deng *et al.* developed grey systems theory and Pawlak *et al.* (1982) advanced rough set theory in the 1980s, etc. All these works represent some of the most important efforts in the research of uncertain systems. From different angles, these works provide the theories and methodologies for describing and dealing with uncertain information [83].

a) Fuzzy set theory

Uncertainty implies that in a certain situation, a person does not possess the information which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics [105]. Fuzzy set theory was developed to solve problems, taking into account the uncertainty arising from imprecision, vagueness, or lack of information. Pioneering and outstanding works on fuzzy sets are done by Zadeh *et al.* (1965).

The development of the fuzzy concept has led to the provision of models which have the flexibility to control and display uncertainty arising from low accuracy due to lack of knowledge of experts and inadequate data. Many developments have been made to better address inadequate and inaccurate data. Atanassov *et al.* developed intuitive fuzzy sets (IFSs). These sets included the membership function, the non-membership function, and the hesitancy function. Zadeh *et al.* introduced a type-2 fuzzy set which allowed expressing the membership of the components in the form of a fuzzy set [104].

b) WASPAS extension to Interval-valued intuitionistic fuzzy sets (IVIFSs)

An extension of IFSs, Interval-valued intuitionistic fuzzy sets (IVIFSs) has received much attention in different areas and numerous issues related to decision making [113]. The interval-valued intuitionistic fuzzy numbers are a generalized form of fuzzy sets considering a non-membership degree in addition to ordinal membership degree of fuzzy numbers and show these degrees in intervals. This form allows having a better imagination from the real ambiguity and uncertainty of the environment [112]. The WASPAS-IVF method (proposed by Zavadskas *et al.*, Hajiagha *et al.* and Hashem *et al.*) [104] combines the strengths of interval-valued intuitionistic fuzzy sets in handling uncertainty with the enhanced accuracy of multiple criteria decision making [112]. This extension of WASPAS has many applications in real-time decision-making problem such as:

- Ranking derelict buildings' redevelopment decisions and investment alternatives are applying WASPAS-IVIF [for more information refer 112].
- An uncertain decision-making problem of reservoir flood control management policy is implemented with interval-valued intuitionistic fuzzy information [for more information refer 113].

c) WASPAS extension to fuzzy-sets

Turskis *et al.*, Zavadskas *et al.*, Antucheviciene *et al.* and Kosareva *et al.*, proposed a novel fuzzy multi-attribute performance measurement framework using the merits of both a novel Weighted Aggregated Sum-Product Assessment method with Fuzzy values (WASPAS-F) and fuzzy Analytical Hierarchy Process (AHP) [104]. This led to the creation of WASPAS-F method.

d) Extended WASPAS method with Interval Type-2 fuzzy set

The extended WASPAS method with Interval Type-2 fuzzy (IT2FSs) set that uses a group decision-making procedure for handling the MCDM problem with interval type-2 fuzzy sets. A linear normalization method is used in the process of the classical WASPAS method. We may have IT2FSs with zero elements in the decision matrix. Moreover, a modification is performed in the WPM process of the proposed approach to avoid calculations with complex numbers. Using

IT2FSs gives more degrees of flexibility to the decision-makers to express their preferences in an uncertain environment [107 & 108]. Some application of these methods are mentioned below:

- Application in Green Supply Chain Management such as the selection of suppliers in a supply chain according to environmental criteria [107].
- Selection of car-sharing stations in Istanbul [for more details refer to 108].

e) Hesitant fuzzy soft decision-making method based on WASPAS

Wang *et al.* presented the hesitant fuzzy soft sets by aggregating hesitant fuzzy set with soft set and developed an algorithm to solve decision-making problems [106] Peng *et al.* and Dai *et al.* proposed a novel method integrating Hesitant fuzzy soft set (HFSS) and classical WASPAS. This method proceeds as the classical WASPAS method with the exception of utilizing hesitant fuzzy soft values in place of crisp numbers as inputs. An application of the proposed method is presented for "Selection of a software development project to invest" [for more details refer 106].

f) Grey Theory

Grey theory deals with uncertain systems with partially known information through generating, excavating, and extracting useful information from what is available. So, systems' operational behaviors and their laws of evolution can be correctly described and effectively monitored. In the natural world, uncertain systems with small samples and poor information exist commonly. That fact determines the wide range of applicability of grey systems theory [83]. Julong Deng introduced the concepts of grey theory from a grey set by combining concepts of system theory, space theory, and control theory [109].

Zavadskas *et al.*, Turskis *et al.*, Antucheviciene *et al.*, proposed a novel multiple attributes Weighted Aggregated Sum-Product Assessment with the grey attributes scores (WASPS-G) method.

Some applications of this method are:

Applications in the Construction Industry

MADM methods often have many applications in the construction industry, and this method is no different in this regard. Some example of its application in the construction industry are:

- "Efficient Contractor Selection model using WASPAS-G" (for more details refer [109]).
- "Assessing the Redevelopment strategical alternatives using WASPAS-G" (for more details refer [110]).
- "Selection of most appropriate personal protection device using WASPAS-G" (for more details refer [111]).

g) Rough set theory

The purpose of the fuzzy technique in the decision-making process is to enable the transformation of crisp numbers into fuzzy numbers that show uncertainties in real-world systems using the membership function. As opposed to fuzzy sets theory, which requires a subjective approach in determining partial functions and fuzzy set boundaries, rough set theory determines set boundaries based on real values and depends on the degree of certainty of the decision maker. The rough set theory deals solely with internal knowledge, i.e., operational data, there is no need to rely on assumption models. In other words, when applying rough sets, only the structure of the given data is used instead of various additional/external parameters [114]. A very important advantage of using rough set theory to handle vagueness and uncertainty is that it expresses vagueness using the boundary region of a set instead of the membership function. Also, the integration of rough numbers in MCDM methods gives the possibility to explore the subjective and unclear evaluation of the experts and to avoid assumptions, which is not the case when applying fuzzy theory [114].

A Rough WASPAS method was proposed by Stojic *et al.*, Stevic *et al.*, Antucheviciene *et al.*, Pamuca *et al.* and Vasiljevic *et al.* [114], for the "Selection of suppliers in a company producing polyvinyl chloride (PVC) carpentry" (for more details refer [114]).

h) Neutrosophic sets

Neutrosophic logic was introduced by Smarandache *et al.* in 1995 [SVNS-1]. It is a logic in which each proposition is estimated to have a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F). A Neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy, and falsity respectively and which lies between [0, 1], the non-standard unit interval. Unlike in intuitionistic fuzzy sets (IFS), where the incorporated uncertainty is dependent of the degree of belongingness and degree of non-belongingness, here the uncertainty present, i.e., the indeterminacy factor, is independent of truth and falsity values. Neutrosophic sets are indeed more general than IFS as there are no constraints between the degree of truth, degree of indeterminacy, and degree of falsity. All these degrees can individually vary within [0, 1] [116].

In 2005, Wang *et al.* introduced an instance of a neutrosophic set known as single-valued neutrosophic sets (SVNS) which was motivated from the practical point of view, and that can be used in real scientific and engineering applications [116].

Zavadskas *et al.*, Bausys *et al.* and Lazauskas *et al.* [116] proposed an Extension of the WASPAS Method with Single-Valued Neutrosophic Set (WASPAS-SVNS). The WASPAS-SVNS method is developed, applying the framework of the single-valued neutrosophic set [116]. Some of the applications of this method are:

In the article "Sustainable Assessment of Alternative Sites for the Construction of a Waste Incineration Plant", WASPAS-SVN method was employed to determine the site for a waste incineration plant with sustainable construction as a focal point (for more details refer [116]).

Garage location Selection for Residential House by WASPAS-SVNS method (for more details refer [117]).

2.4.7 ROV (Range of Value) method

2.4.7A Background

There are many multiple attribute decision-making (MADM) methods available in the decision-making spectrum. However, the search for a simple, understandable, and usable approach for solving MADM problems is still going on. There is a wide range of the MADM methods that focus on various aspects to assess the alternatives, but during the process pile up many complexities in their operation. Contradicting this course, Yakowitz *et al.* [119] proposed a simple and easy to employ a method to solve MADM problems. It later came to be known as Range of Value (ROV) method.

2.4.7B The motivation behind the development of the method

As quoted by Yakowitz *et al.* in his initial works on the approach:

"This work is prompted by the need for tools that can be easily and quickly understood and sensibly applied to multiple attribute decision making situations that occur in land management" [119].

2.4.7C Introduction

Yakowitz *et al.* [119] proposed the Range of Value (ROV) method for the first time in 1993 [119,120 & 121]. The ROV method, allows decision-maker (DM) to quickly compute the range of values from the best to the worst for a multiple attribute problem under the assumption of an additive value function [119 & 120]. It also allows the DM to quickly assess the most optimistic and most pessimistic DM viewpoint, given the multiple importance orders of the attributes at any stage of the decision problem [119].

The best and the worst additive values are computed for each alternative using two closed form solutions aggregated from a simple linear program, that maximize and minimize the objective of the MADM problem [ROVM-0.5]. (for more details please refer [119 & 120])

2.4.7D Characteristics of ROV method

The basic characteristics that make ROV method unique compared to other MADM methods are:

- The basic ranking or assessing framework of ROV method is based on a simple mathematical operation [123].
- Best and worst additive values for each alternative can be found without requiring the decision maker to set specific weights for each of the criteria [120].
- This method requires only ordinal specification of criteria importance (weights) from a decision maker. Thus, in situations where decision makers are facing problems in supplying

quantitative weights, the application of the ROV method can be particularly useful [121, 123 & 118].

- The application of the ROV method to a decision problem is very efficient even in the cases where quantitative weights are employed [123].

2.4.7E Range of Value approach to a MADM problem

The application of any MADM method for solving a decision-making problem usually involves the three main steps; those are [121]:

1. Determination of the relevant conflicting attributes and feasible alternatives.
2. Measurement of the relative importance (or weights) of the considered attributes and impact of the alternatives on those attributes.
3. Determination of the performance measures (or performance indices) of the alternatives for ranking.

Range of Value (ROV) method is a ranking method that assesses the alternatives based on performance ratings (or decision scores) concerning each attribute in a MADM problem. The procedure to solve a decision problem using ROV method involves the following steps [120, 121, 122, 123 & 125]:

Before starting the procedure, it is advisable to segregate maximizing attributes and minimizing attributes in the decision matrix for ease of operation.

Step-1: Normalization of the Decision Matrix:

Linear max-min normalization method is employed in this method to transform multi-dimensional performance ratings (or decision scores) into non-dimensional entities [42 and 120, 121, 122 & 123].

For maximizing/beneficial criteria,

$$N_{ij_{(b)}}^{LM} = \frac{x_{ij}-x_j^{min}}{x_j^{max}-x_j^{min}} \quad \ldots\ldots\ldots\ldots (46)$$

, for all *i=1,2……, M; & j=1,2……., N.*

For minimizing/non-beneficial criteria,

$$N_{ij_{(n-b)}}^{LM} = \frac{x_j^{max}-x_{ij}}{x_j^{max}-x_j^{min}} \quad \ldots\ldots\ldots\ldots (47)$$

, for all *i=1,2……, M; & j=1,2……., N.*

Where,

"N_{ij}^{LM}" is the normalized element, and "LM" indicates the type of normalization.

Step-2: Weighted Normalization of the Decision Matrix

In this step, the weights obtained from an objective or subjective (or a combination of the both) approaches, is multiplied to the normalized scores of the Decision Matrix.

Mathematically represented as:

$$\{W_j \times N_{ij}^{(LM)}\} \quad \ldots\ldots\ldots\ldots (48)$$

, for $j=1, 2 \ldots., N$.

Step-3: Determination of Best and Worst Utility of each alternative

This is achieved by maximizing and minimizing a utility function. For a linear additive model, the best utility (U_i^+) and the worst utility (U_i^-) of i^{th} the alternative is obtained using the following equations:

$$U_i^+ = \sum_j^N W_j \times N_{ij_{(b)}}^{LM} \quad \ldots\ldots\ldots\ldots (49)$$

for all $i=1,2 \ldots\ldots, M;$ & $j=1,2 \ldots\ldots., N$.

where,

"U_i^+" is the linear additive of weighted average of beneficial criteria, of an alternative.

$$U_i^- = \sum_j^N W_j \times N_{ij_{(n-b)}}^{LM} \quad \ldots\ldots\ldots\ldots (50)$$

Where,

" U_i^- " is the linear additive of weighted average of non-beneficial criteria, of an alternative.

If ($U_i^- > U_i^+$), then alternative i outperforms alternative i' regardless of the actual quantitative weights. If it is not possible to differentiate the options on this basis, then a scoring (enabling subsequent ranking) can be attained from the midpoint [121, 122, 123, 124, 125 & 126].

Step-4: Determination of the scores for each alternative.

The over-all score or utility for each alternative is calculated as a mean of the best and worst utility values.

Mathematically represented as:

$$U_i = \frac{U_i^- + U_i^+}{2} \quad \ldots\ldots\ldots\ldots (51)$$

Step-5: Ranking the alternatives

In this final step, the complete ranking of the alternatives is obtained based on "U_i" values. Thus, the best alternative has the highest ".U_i" value and the worst alternative has the lowest "U_i" value**.**

2.4.7F Applications and extensions of the ROVM

The ROV method has found very limited applications as a decision-assisting tool. Some of its applications are presented below:

- Applied to the problem of evaluating farm or rangeland management systems concerning both economic and environmental criteria (for more details refer [119]).
- Application to Environmental decision making, to evaluate the alternatives and select the best site by determining a rangeland health index (for more details refer [120]).
- Selection of cutting fluids for a machining process, considering multiple attributes and alternative to determine the optimum cutting fluid (for more details refer [121]).
- Application as one of the ranking methods in a Multiple criteria analysis (MCA) framework for evaluating decision options in Water Management problem (for more details refer [122]).

- For the selection of the most appropriate apple among the alternatives in the fresh market for a food company that produces apple juice concentrate (for more details, refer [123]).
- For the selection of the most appropriate non-traditional machining processes (NTMP) for a machining application (for more details refer [124]).
- The application of entropy-ROV methods to formulate Global Performance for selecting the best Automotive Suppliers in Morocco (for more details refer [125]).
- Supplier selection application in Manufacturing environment (for more details refer [126]).

An extension of ROV method to incorporate Taguchi Methodology was proposed by Magic Milos, known as the ROV-based Taguchi Methodology to deal with a multi-objective optimization problem on Laser cutting process (for more details refer [118]).

2.4.8 WEDBA (Weighted Euclidean Distance-Based Approach) method

2.4.8A Introduction

In mathematics, the Euclidean distance or Euclidean metric is the straight-line distance between two points in Euclidean space [LM]. Euclidean geometry has its share of applications in Multiple criteria decision making (MCDM) approaches over the years. One of the most popular Multiple Attribute Decision Making (MADM) methods known as Technique of ranking Preferences by Similarity to the Ideal Solution (TOPSIS), utilizes the concept of Euclidean distance to determine the shortest distance between an alternative from the ideal solution [265]. However, the concept of Euclidean distance is limited in this method.

Rao *et al.* [127], developed a method that incorporates the concept of Euclidean distance, to assess the alternatives based on their distance from the Ideal (or best) and Anti-ideal (or worst) solutions in a MADM framework.

2.4.8B Weighted Euclidean Distance-Based Approach (WEDBA)

The weighted Euclidean distance-based approach (WEDBA) is based on the weighted distance of alternatives from the most and the least favorable situations, respectively. In this method, the most favorable situation is represented by the ideal point (i.e., optimum point), and the least favorable situation is represented by the anti-ideal point (i.e., non-optimum point). For practical purposes, the ideal and anti-ideal points are defined as the best and worst values respectively, which exist within the range of values of attributes [127]. The ideal point is simply the alternative that has all the best values of attributes, and the anti-ideal point is simply the alternative that has all the worst values of attributes. It may happen that a certain alternative has the best values for all attributes or worst values for all attributes. Therefore, in this method, the ideal and anti-ideal points are considered as feasible solutions and are used as a reference to quantitatively compare other alternatives [127 & 128].

The relative numerical differences resulting from the comparison represent the effectiveness of alternatives known as the index scores of the alternatives. The decision problem is to find a feasible solution, which is as close as possible to the ideal point and simultaneously keeping the distance of solution farther from the anti-ideal point [127 &128].

2.4.8C The procedure of Weighted Euclidean Distance-Based approach [127 & 128]

Step-1: Normalization of the Decision Matrix

Linear normalization technique has been applied in this method to normalize the decision score (or values) in the Decision Matrix.

For maximizing attributes:

$$N_{ij}^{(LN)} = \frac{x_{ij}}{x_j^{max}} \quad \ldots\ldots\ldots\ldots (52)$$

, for *i=1,2……, M; & j=1,2……, N.*

For minimizing attributes:

$$N_{ij}^{(LN)} = \frac{x_j^{min}}{x_{ij}} \quad \ldots\ldots\ldots\ldots (53)$$

, for *i=1,2……, M; & j=1,2……, N.*

Where,

$N_{ij}^{(LN)}$ is the normalized score (or value) and "LN" represents linear normalization method.

Step-2: Standardization of Decision Matrix

The values of a standardized attribute data (decision scores) are also known as standard scores. The important property of standard score is that it has a mean of zero and a variance of 1 (i.e., standard deviation equals to 1), which accounts for the name standardized. The Standardization of decision matrix is done by the following formula:

$$Z_{ij} = \frac{N_{ij}^{(LN)} - \mu_j}{\sigma_j} \quad \text{...........} \ (54)$$

, for all *j= 1,2,3, ……., N.*

Where,

"Z_{ij}" is the standard score,

"μ_j" is the mean of the "j^{th}" attribute, and calculated using the formula:

$$\mu_j = \frac{1}{M} \sum_{i=}^{M} N_{ij}^{(LN)} \quad \text{...........} \ (55)$$

"σ_j" is the standard deviation of the j^{th} attribute, and calculated using the formula:

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{M} (N_{ij}^{(LN)} - \mu_j)^2}{M}} \quad \text{...........} \ (56)$$

Step-3: Determination of Ideal and anti-ideal points

The ideal point is the set of attribute values, ideally (most) desired. The anti-ideal point is the set of attribute values ideally not desired at all or least desirable. The ideal point, denoted by 'a^+' and anti-ideal point, denoted by 'a^-' are determined from standardized decision matrix.

$$a^+ = max\{Z_j\} \quad \text{...........} \ (57)$$

$$a^{-} = min\{Z_j\} \quad \ldots\ldots\ldots\ldots (58)$$

where, *j= 1,2,3, …, N.*

Step-4: Calculating Weighted Euclidean distance (WED):

Euclidean distance is the shortest distance between two points. Weighted Euclidean distance (WED) between an alternative 'i' and ideal point. 'a^{+}' is denoted by "WED_i^{+}"

$$WED_i^{+} = \left[\sum_{j=1}^{N}\{W_j(Z_j - a^{+})\}^2\right]^{1/2} \quad \ldots\ldots\ldots\ldots (59)$$

, for *i= 1,2, …., M.*

moreover, between an alternative 'i' and anti-ideal point. 'a^{-}' is denoted by "WED_i^{-}"

$$WED_i^{-} = \left[\sum_{j=1}^{N}\{W_j(Z_j - a^{-})\}^2\right]^{1/2} \quad \ldots\ldots\ldots\ldots (60)$$

, for *i= 1,2, …., M.*

In the later papers, Dr. R. Venkata Rao, proposed improvement over the WEDBA method to incorporate integrated weights (a combination of subjective and objective weights) [128]. Further application of the method validated it when objective or subjective methods were employed.

Step-5: Computing Index Score for each alternative

$$(Index\ score)_i = \frac{WED_i^{-}}{WED_i^{+} + WED_i^{-}} \quad \ldots\ldots\ldots\ldots (61)$$

Step-6: Ranking the alternatives

The alternative for which the value of the index score is highest is the best choice for a decision-making problem. Higher is the index score, higher the rank of that alternative and vice versa.

2.4.8D Applications of the WEDBA method as a MADM tool

R. Venkata Rao, in his book "Decision Making in the Manufacturing Environment Using Graph Theory and Fuzzy Multiple Attribute Decision, Making Methods", Volume 2 (for more details refer [129]), proposed the following applications for this method:

- Material Selection of a Flywheel: Assessing and ranking 10 material alternatives for the flywheel design using WEDBA method, and also validating the WEDBA method results with ELECTRE and VIKOR methods using Spearman's rank correlation coefficient.
- Robot Selection for a Given Industrial Application: Ranking optimum robot by assessing various selection criteria for an industrial application.

Moreover, other manufacturing environment decision-making applications such as:

- Flexible Manufacturing System Selection.
- Optimum Parameters Selection of Green Electric Discharge Machining.
- Selection of Best Product End-of-Life Scenario.
- Facility layout design selection (for more details refer [128]).
- Machine Tool Selection (for more details refer [127]).

Others:

Optimal selection of E-learning websites using multi-attribute decision-making approaches (for more details refer [130]).

2.4.9 MOORA (Multi-Objective Optimization on the basis of Ratio Analysis) method

2.4.9A Origins

Brauers *et al.* [131], a professor and an economist, has many scientific publications credited to his name, which includes 12 books and more than 100 papers and reports. He is a pioneer who developed the Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) (2004) method along with Zavadskas *et al.* Initially, the MOORA method was seen as a Multi-objective decision making (MODM) method, but many successful applications in both MODM and Multi-attribute Decision Making (MADM) approaches by famous researchers such as Zavadskas *et al.*, Chakraborty *et al.*, Ginevicius *et al.* and Brauers *et al.* himself etc., explored the method's utility to over-all MCDM ideology. The basic concept of the MOORA method is evolved from the Multiple Objective Utility Theory (MOUT), which is developed by Brauers *et al.* and his mentioned in detail in his book "Optimization Methods for a Stakeholder Society: A Revolution in Economic Thinking by Multi-objective optimization" [131]. The MOORA method is introduced as a comprehensive framework to deal with a decision problem as a multiple objectives case. Initially, MOORA method begins with an embrace of already familiarized concept of the ratio system and later, a new concept of reference point is used to improve the robustness of the method [131]. And later, Brauers *et al.* introduced an extension to the MOORA method, known as the full multiplicative form of MOORA or simply "MULTIMOORA". This approach was inspired from his paper "The Multiplicative Representation for Multiple Objectives Optimization with an Application for Arms Procurement" [169] which was published in the year 2001. The upcoming sections will provide a detail account of the general procedure, applications and further extensions of the MOORA method.

2.4.9B The Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) method

Multi-objective optimization also known as multi-criteria or multi-attribute optimization, is the process of simultaneously optimizing two or more conflicting attributes (objectives) subject to certain constraints [132 & 135]. Maximizing profit and minimizing the cost of a product; maximizing performance and minimizing fuel consumption of a vehicle; and minimizing weight while maximizing the strength of a particular engineering component are the typical examples of multi-objective optimization problems [132].

Initially, MOORA method was proposed as a combination of two separate aggregation approaches resulting in two set of ranking with high correlativity i.e., (1) the Ratio System Approach, producing dimensionless ratios, (2) the Reference Point Approach, but still based on scores with the problem of the choice of scores (decision scores). Later, Brauers incorporated the ratios found in the ratio system instead of the independent scores to the Reference Point Approach. In this way dimensionless measures were obtained again [132]. The synthesis of these two approaches was called as MOORA.

2.4.9C Detail structure of the method

MOORA method starts with the definition of the elements "x_{ij}" where, 'i' represents the alternative and 'j' indicates the objectives (attributes). The responses of each alternative 'i' to each objective 'j' is recorded in the form of a Matrix, known as Decision Matrix [131].

Ratio Analysis approach

In a ratio analysis system, each response of an alternative on an objective (attribute) is compared to a denominator, which is representative for all alternatives concerning that objective. For this denominator the square root of the sum of squares of each alternative per objective is chosen [131], which is mathematically represented as:

Step-1: Normalization

$$N_{ij}^{(V)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \text{………… (62)}$$

, for *i*= *1, 2, 3, …, M; & j= 1, 2, 3, …., N.*

Where,

"$N_{ij}^{(V)}$" is the normalization of the decision score of i^{th} alternative in response to j^{th} attribute/objective, and the superscript "V" represents Vector normalization method.

Step-2: Identifying the Maximizing and Minimizing responses (scores):

Maximizing Entity: A score or a response which strengthens the objective of the decision problem by causing a positive contribution to it, is known as Maximizing entity/score. It is closely related to beneficial criteria. For example, all the values or score that fall in the beneficial criteria contributes positively to strengthen the objective of the problem. So, all the responses of each beneficial criterion are maximizing entities/score.

Similarly, a Minimizing entity is a response (score) that cause a negative impact to the objective of the decision problem. All the responses of a non-beneficial criterion will fall under this category.

Step-3: Aggregation of the Normalized score

For optimization, all the normalized score of the decision matrix are added in case of maximization and subtracted in case of minimization [131]:

In case of Maximization [131, 132, 133 & 134]:

$$(Max)_i = \sum_{j=1}^{G} N_{ij}^{(V)} \quad \text{...........} \; (63)$$

, for *i= 1, 2, 3, ..., M;* & *j= 1, 2, 3, ..., G.*

In case of Minimization [MOORA-1,2,3,4]:

$$(Min)_i = \sum_{j=G+1}^{N} N_{ij}^{(V)} \quad \text{...........} \; (64)$$

, for *i= 1, 2, 3, ..., M;* & *j= G+1, G+2, ..., N.*

Finally, the Performance Index of i^{th} the alternative is given by [132]:

$$I_i^{RS} = (Max)_i - (Min)_i \quad \ldots\ldots\ldots\ldots (65)$$

An additional step can be included right after the Normalization step to integrate importance ratings or weights of the attributes (objectives). The MOORA method does not require weights to optimize a decision problem, but it is not limited to include them.

Then, the optimization equations change to,

$$I_i^{RS} = \sum_{j=1}^{G} W_j N_{ij}^{(V)} - \sum_{j=G+1}^{N} W_j N_{ij}^{(V)} \quad \ldots\ldots\ldots\ldots (66)$$

Where,

"W_j" indicates the weight of i^{th} attribute (objective), and "RS" superscript refers to Ratio System.

The relation between "Objective" and "Attribute"

In order to define objectives better, we have to focus on the notion of Attribute, which is best understood with an example of the objective "reduce sulfur dioxide emissions" to be measured by the attribute "tons of sulfur dioxide emitted per year". It signifies that an objective and a correspondent attribute always go together. Therefore, when the text mentions "objective", the corresponding attribute is meant as well [131].

A set of ranking based on a ratio system is obtained in this way, but the MOORA method does not end here. A reference point theory is explored to make the MOORA methodology more compact and thorough.

Reference Point approach

The reference point approach starts from the already normalized ratios as defined in the Ratio Analysis approach [131]. In the reference point approach, the normalized decision score of an alternative is chosen concerning each objective (attribute), and the selecting criteria depend on the two categories [131]:

1) In the case of maximization, the highest valued normalized score is selected.

2) In the case of minimization, the lowest valued normalization score is selected.

The reference point approach is more realistic and non-subjective as the score, which is selected for the reference point, are realized in one of the candidate alternatives [132].

Procedure: [131, 132, 135, 137 & 141]

Step-1: Normalization of the elements "x_{ij}" of the decision matrix

The normalization follows the ratios from the ratio analysis approach.Type equation here.

$$N_{ij}^{(V)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \ldots\ldots\ldots\ldots (67)$$

, for *i= 1, 2, 3, …, M;* & *j= 1, 2, 3, …., N.*

Step-2: Identifying the Ideal reference point for each attribute (objective)

In case of beneficial criteria,

$$r_j^b = max_i\{x_{ij}\} \quad \ldots\ldots\ldots\ldots (68)$$

Where,

the superscript “b” represents beneficial criteria category

In the case of non-beneficial criteria,

$$r_j^{nb} = min_i\{x_{ij}\} \quad \ldots\ldots\ldots\ldots (69)$$

Where,

the superscript “nb” represents non-beneficial criteria category

Step-3: Determining the distance between each normalized score concerning the ideal reference point

$$d_i = (r_j - N_{ij}^{(V)}) \quad \ldots\ldots\ldots\ldots (70)$$

Step-4: Aggregation of the distances

In this step, the maximum value of the relative distance “d_i” of each alternative is determined, which is the final performance index for each alternative.

$$I_i^{RP} = \max d_i \quad \ldots\ldots\ldots\ldots (71)$$

Where,

the superscript “RP” refers to a Reference point.

Step-5: Ranking the alternatives

Alternatives are ranked in ascending order because the index refers to the distance from the ideal score, and an alternative that has the lowest distance from it will be the best and alternative furthest away from the ideal score is the worst.

2.4.10 MULTIMOORA method

MULTIMOORA is the extension of the original MOORA method. Like the MOORA, MULTIMOORA consists of Ratio system, and Reference point approaches, in addition to a novel aggregation procedure which is called as "Full Multiplicative form. The MOORA methodology with the full multiplicative form was proposed as a new decision-aiding framework by Brauer *et al.* [169] in his book "Optimization methods for a Stakeholder society" in 2004. However, the full multiplicative form was proposed in the year 2002 as an effective way to solve multi-objective problems (for more details refer [169]).

2.4.10A Background

Mathematical economics is familiar with the multiplicative models like in production functions and demand functions. In the year 1957, Allen *et al.* [169] launched the "bilinear and quadratic form", introducing interrelations between weights and objectives but only when examining two by two [169].

Keeney *et al.* and Raiffa *et al.* (1993, p. 234 [169]) besides additive utilities, a utility function may also include multiplication of the attributes besides. The two dimensional u(y,z) can then be expressed as a multilinear utility function. This representation mixes additive and multiplicative parts (Brauer *et al.*, 2004a, p. 228 [169 & 170]).

For Keeney *et al.* (1973, p. 110 [169]) the additive form is rather a limiting case of the multiplicative utility function, for us the SAW method as explained earlier.

The danger exists that the multiplicative part becomes explosive. The multiplicative part of the equation would then dominate the additive part and finally would bias the results. Considering these shortcomings, preference will be given to a method that is non- linear, non-additive, does not use weights and does not require normalization. Such multiplicative form for multi-objectives was introduced by Miller *et al.* and Starr *et al.* (1969, pp. 237– 239 [169 & 170]) and further developed by Brauer *et al.* (2004a, pp. 227–245 [169]).

2.4.10B Full multiplicative approach [169, 170 & 171]

Step-1: Normalization of the Decision Matrix

The normalization technique followed in this approach is similar to what is followed in Ratio system, and Reference point approaches, i.e., vector normalization.

$$N_{ij}^{(V)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \ldots\ldots\ldots\ldots (72)$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

Step-2: Weighted Normalization of the Decision Matrix

In this step, weights are integrated with the normalized score using multiplicative exponential principal, which is also followed in the weighted product method (WPM). Mathematically represented as:

$$u_{ij} = \left(N_{ij}^{(V)}\right)^{W_j} \quad \text{........... (73)}$$

, for *j=1, 2...., N.*

Where,

"W_j" is the weight of j^{th} attribute, which can be determined using subjective or objective (or combination of both) methods.

Step-3: Identifying the Maximization and Minimization scores/responses

Segregating the two extremities is advisable because it makes further calculations easier and confusion-free. Unlike, in the Ratio system analysis where maximizing and minimizing scores are added, in MULTIMOORA, they are multiplied.

In the case of Maximizing entities,

$$(max)_i = \prod_{j=1}^{G} u_{ij} \quad \text{........... (74)}$$

, for all *i=1,2........., M; & j= 1,2,3, ..., G.*

In case of Minimizing entities,

$$(min)_i = \prod_{j=G+1}^{N} u_{ij} \quad \text{........... (75)}$$

, for all *i=1, 2, ..., M; & j= G+1, G+2, ..., N.*

Step-4: Aggregation to compute the preference index of each alternative

The preference index for each alternative is calculated using the following formula:

$$I_i = \frac{(max)_i}{(min)_i} \quad \ldots\ldots\ldots\ldots (76)$$

The maximizing attribute scores of an alternative is divided by the minimizing attribute scores of the same alternative.

Step-5: Ranking the Alternative

The order of ranking follows the descending order, where the alternative with the lowest preference index (I_i) is ranked lowest in the order and the alternative with the highest index value (I_i) is ranked at the top.

2.4.10C Characteristics of MOORA

MOORA as an approach satisfies the seven conditions to be looked foreseen over other MADM or MODM techniques, such as [138]

- MOORA method utilizes Cardinal data and not ordinal, given the weak points of an ordinal approach as demonstrated by Arrow *et al.* in his work "General economic equilibrium: purpose, analytic techniques (1974)" and Brauers *et al.* in his published work in the International Journal of Management and Decision Making (2007).
- All the objectives (attributes) are considered and respected as a potential difference maker in a decision problem.
- All interrelations between objectives and alternatives are looked upon at the same time instead of pairwise considerations;

- It deals with discrete cases facing a set of a limited number of alternatives, whereas continuous cases concerning with alternatives generated out of a set of continuous and numerous alternatives are ignored.
- It perceives the decision process in a non-subjective viewpoint, no normalization by subjective weights but by non-subjective dimensionless measures (scores).
- MOORA method does not require weights or importance ratings, but not limited to their incorporation.

2.4.10D Merits of the MOORA approach [132]

The MOORA method is credited as one of the best MADM approaches, due to the following reasons:

i. High simplicity in its application to the problem,
ii. less computational time to determine the best alternative, and
iii. uncomplicated mathematical calculations.
iv. There is no usage of extra parameters like for example "v" in VIKOR, "ξ" in GRA method or "λ" in WASPAS.
v. MOORA was proven to be a more robust approach than Minkowski, Euclidean distance metric, tchebycheff min max-metric.
vi. Ability to incorporate subjective, objective, or a combination of both into the process to evaluate the alternatives.

2.4.10E Applications of MOORA [131-161, 172-175, 176 & 177; 178 & 162]

Although the MOORA is a relatively new method, it has been applied to solve many economic, managerial, construction, material selection, and other decision problems. This section presents a detailed account of the applications of MOORA and MULTIMOORA to solve decision problems in various sectors of the market. Figure 11 is the pie-chart representing the areas of application of MOORA based on papers published.

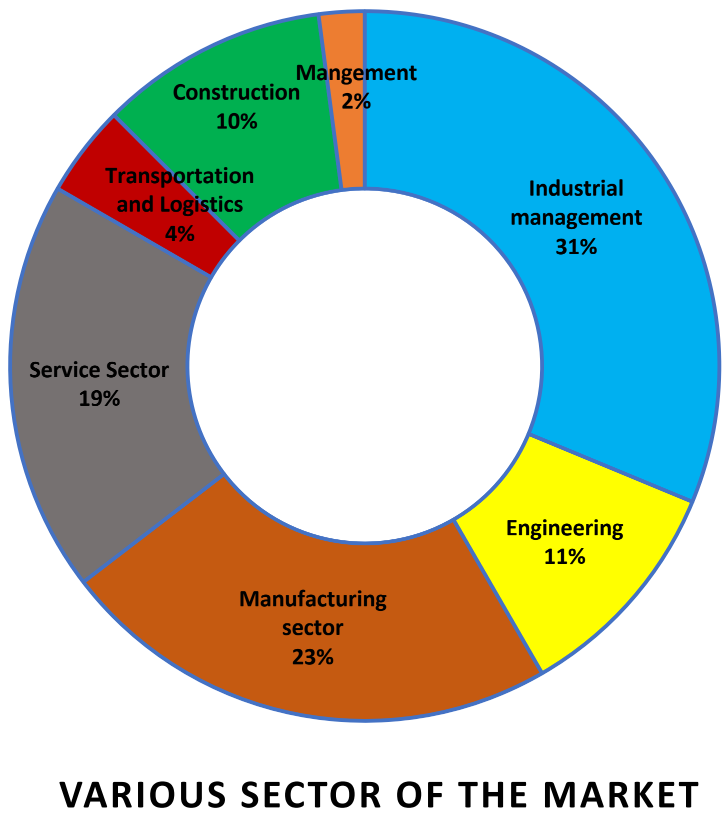


Figure 11: Areas of application of MOORA/MULTIMOORA based on the number of papers published.

a) Industrial Management

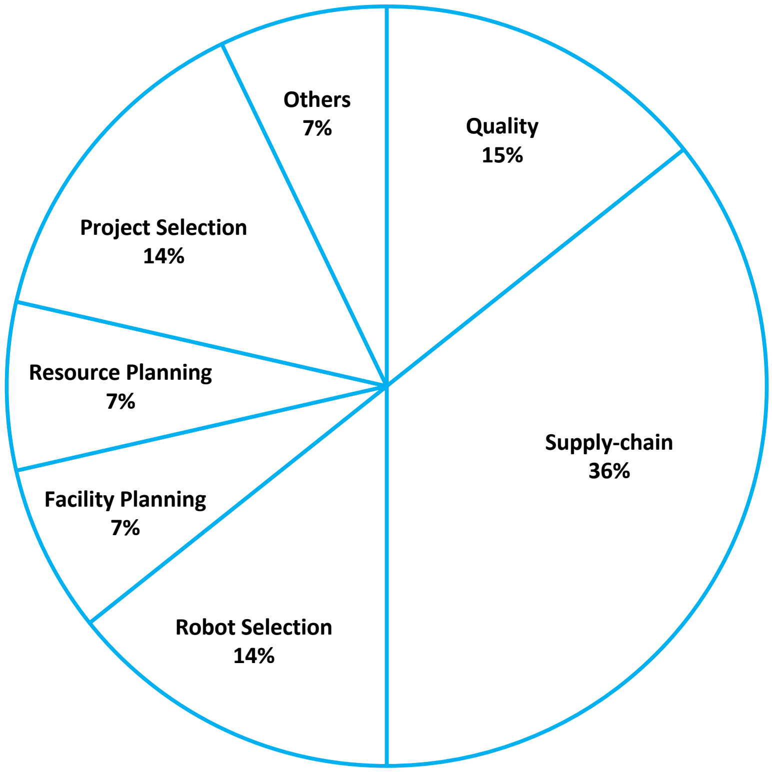


Figure 12: Sub-division wise segregation of the applications of the MOORA/MULTIMOORA.

Multi-criteria decision making (MCDM) methods have a high number of applications to deal with industry related decision-problems. Moreover, the above chart approximately represents the applications to various sub-divisions of industrial management sector. Some specific applications which are illustrated below will present insights into the problem areas dealt by the MOORA/MULTIMOORA method: Figure 12, is the pie-chart of sub-division under industrial management sector.

Quality

Some of the applications of the MOORA or its extensions in the quality sector are:

- Yusuf *et al.* and Yildirim *et al.* have proposed MOORA-based Taguchi optimization framework for improving product or process quality [162].
- Optimization of WEDM process parameters using standard deviation and MOORA method
- Muniappan *et al.* with his team of engineers applied the Standard deviation based MOORA method to optimize the process parameters of Wire electrical discharge machining (WEDM) process [141].

Supply-chain

Supply-chain decision problems usually include non-crisp data due to the subjective nature of the attributes involved in the problem. The fuzzy MOORA method was used for the first time in privatization themed study in subsistence economy by Brauer *et al.* and Zavadskas *et al.* in the year 2006 [163] Later many additions to the simple Fuzzy MOORA came into existence such as:

- Arabsheybani *et al.* have proposed an integrated fuzzy MOORA method and FMEA technique for selection of supplier considering quantity discounts and supplier's risk [165].
- Perez-Dominguez *et al.* with his team proposed Intuitionistic fuzzy MOORA for supplier selection [164].

Robot and machinery assessment

Decision problems involving industrial robots and machinery always involves a lot of complications and risks. To deal with the evaluation of the robots, Datta *et al.*, Sahu *et al.* and Mahapatra *et al.* came up with a novel grey-MULTIMOORA approach (MULTIMOORA-G) [167]. Moreover, Chakraborty *et al.* applied MOORA comprehensively to several industrial based decision problems, which also includes robot selection [135].

b) Manufacturing sector

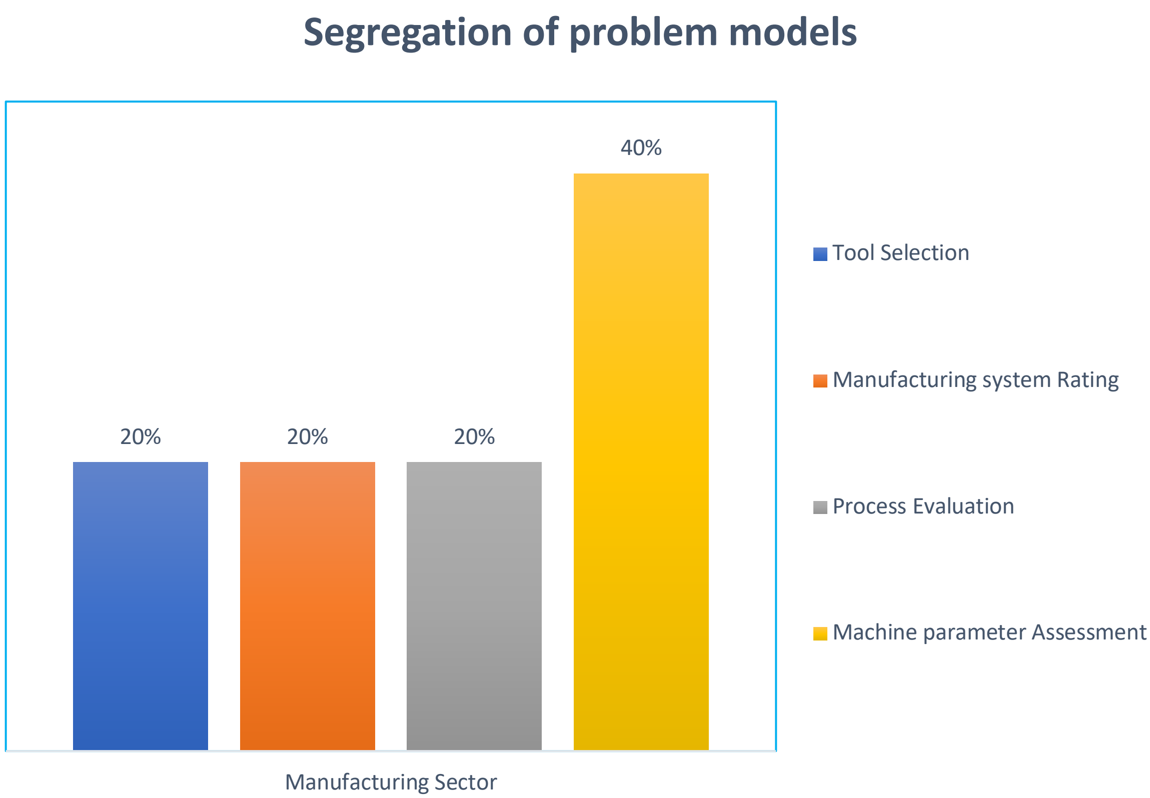


Figure 13: Application of MOORA in various fields in the manufacturing sector.

MOORA/MULTIMOORA has found many applications in the manufacturing sector. The above figure shows an approximate distribution of the application of four major decision problem types that are commonly seen in this sector. This sector, similar to industrial management deals with attributes which cannot be expressed as crisp values. Perez-Dominguez *et al.*, Rodriguez-Picon *et al.*, Alvarado-Iniesta *et al.*, Cruz *et al.* and Xu *et al.*, came up with a unique approach to deal with a manufacturing decision problem known as MOORA under Pythagorean Fuzzy Set [169]. Incorporating Pythagorean Fuzzy set to express the qualitative data, made the MOORA method more robust. Moreover, further, this method was applied to solve the supplier selection problem.

c) Engineering

Material selection

MOORA/MULTIMOORA has been extensively utilized to deal with material selection problem for various engineering design-related problems such as Mondal *et al.* has applied MOORA to select the best material for an Automobile Wheel [159], Karande *et al.*, Chakraborty *et al.* applied to select the optimal material for a fly-wheel, cryogenic storage tank, sailing boat mast and for designs operating in high-temperature oxygen-rich environment [132]. MOORA was also successfully applied to solve the material selection problem for bio-medical applications [171].

Zavadskas *et al.* with his team proposed an extension to the MULTIMOORA method by incorporating neutrosophic set to build a compact decision support system that can aid in material selection problem for residential house [168].

MOORA and MULTIMOORA have been applied to many decision problems in the construction industry, service, and baking sectors, etc.

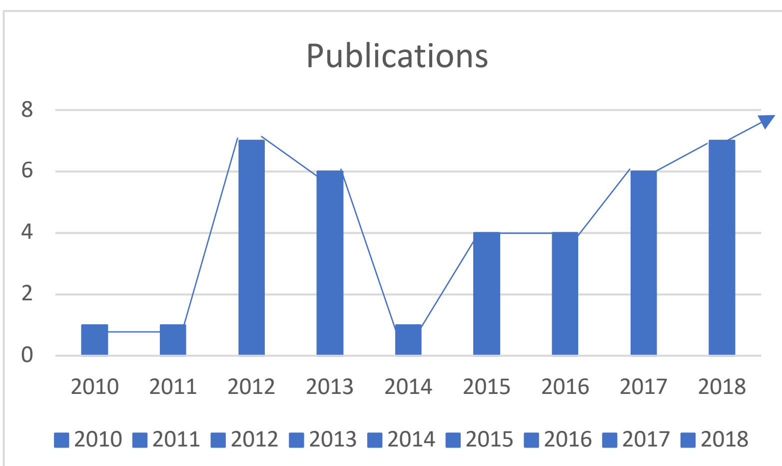


Figure 14: A chart showing the year-wise publications (based on my limited research)

The above chart (figure 14) shows the growing applications of this method over the years, and with further research focused on this method, its application is only expected to rise for the coming years.

2.4.11 MOOSRA (Multi-Objective Optimization of Simple Ratio Analysis) method

2.4.11A Background

Brauer *et al.* proposed the Multi-Objective Optimization based on Ratio Analysis (MOORA) method and later extended it by incorporating reference point approach and the full multiplicative form, to deal with multiple objective optimization (or attribute optimization) problems [132]. The MOORA method has achieved tremendous success as a Multiple Objective Decision Making (MODM) and Multiple Attribute Decision Making (MADM) tool with numerous applications in almost all the sectors of the industry. The characteristics such as easy to understand and apply, less application time and simple procedure, etc., can be credited as the reasons for the success of this method [135]. However, some researchers questioned the robustness of this method dealing with (i) large variations in the criteria values, and (ii) possibility of negative performance scores as a result of aggregation [172 & 173].

Das *et al.*, who has been an author and practitioners of many MADM approaches (including MOORA method) and published numerous articles on their application to different areas. He proposed an improvement of MOORA method that abates the drawbacks of it up to some extent [172]. This method was called a Multi-Objective Optimization of Simple Ratio Analysis (MOOSRA) method.

2.4.11B MOOSRA

This method was proposed in the year 2012 in an international journal "Applied Decision Sciences", of volume-5 and issue-2. It is a multi-objective optimization method [MOOSRA-1] and can be considered as an extension of MOORA method. The application of the MOOSRA method to MADM problems up to some extent is similar to MOORA method. As every MADM approach, the MOOSRA method begins with the construction of the decision matrix (DM). The rows of the DM are filled by the alternatives and the columns by attributes. Assigning the weights to each attribute is an essential and inherent part of MADM theory. Once the preliminaries are completed, MOOSRA method is executed to mathematically assess the alternatives and propose an optimal solution(s) to the decision problem.

The MOOSRA method adopts the ratio system for normalization of the performance ratings (scores) of each alternative concerning each criterion [172 & 173]. The vector normalization method is utilized to convert the dimensional element (score) into a dimensionless entity, which is mathematically represented by:

$$N_{ij}^{(V)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \ldots\ldots\ldots\ldots (77)$$

, for *i= 1,2,3, ..., M;* & *j= 1,2,3, ..., N.*

Where,

"$N_{ij}^{(V)}$" is the normalization of the decision score of i^{th} the alterative in response to j^{th} attribute/objective, and the superscript "V" represents Vector normalization technique.

An additional step can be added to the procedure at this point if the weights of the criteria are assigned. The weighted normalization is done by multiplying weights to the normalized scores [MOOSRA-1].

$$(W_j \times N_{ij}^{(V)}) \quad \ldots\ldots\ldots\ldots (78)$$

Where,

"W_j" is the weight of j^{th} attribute. ($\sum_{j=1}^{N} W_j = 1$).

The next step is to determine the performance score (A_i) of each alternative. The performance score is obtained from the weighted average (mean) method [172, 173 & 176]. It can be performed using the following formula:

For Beneficial/Maximizing Attribute

$$A_i = \sum_{j=1}^{G} W_j N_{ij}^{(V)} \quad \ldots\ldots\ldots\ldots (79)$$

For Non-beneficial/Minimizing Attribute:

$$A_i = \sum_{j=G+1}^{N} W_j N_{ij}^{(V)} \quad \ldots\ldots\ldots\ldots (80)$$

Finally, the overall performance index (Y_i) is calculated. The MOOSRA method uses a simple ratio of the sum of normalized performance scores for beneficial criteria to the sum of normalized performance score for non-beneficial criteria [173] to calculate "Y_i" for each alternative. The formula used is,

$$Y_i = \frac{\sum_{j=1}^{G} W_j N_{ij}^{(V)}}{\sum_{j=G+1}^{N} W_j N_{ij}^{(V)}} \quad \ldots\ldots\ldots\ldots (81)$$

Once the overall performance index is determined, the alternatives are ranked in descending order, were alternative with highest "Y_i" value is ranked as the best alternative and alternative with the lowest "Y_i" value is at the bottom of the list.

2.4.11C Merits of MOOSRA

Some of the merits of the MOOSRA are presented below:

- Easy to understand and apply.
- Less application time.
- Simple mathematical calculations are involved.
- Does not produce negative performance scores during evaluations, which is a major concern in the MOORA method
- More Robust than MOORA approach, as it is less sensitive to large variation in the values of the criteria as compared to MOORA.

2.4.11D Application of MOOSRA to various MADM problems

The MOOSRA method is relatively new as compared to other MADM methods and has very limited applications. One of the major application areas for this method is manufacturing sector, where author Sarkar *et al.* with his team, developed an efficient decision support system for non-traditional machine selection by collaborating MOORA and MOOSRA approaches (for more details, refer [176]). Following the same lead, author Jagadish *et al.* employed MOOSRA method to evaluate cutting fluids based on minimal environmental impacts (for more information, refer

[174]). One more notable area of application is material selection, Dr. Chakraborty *et al.* combined Entropy and MOOSRA method to develop a reliable material selection framework and test it on a real-time case study which dealt with the selection of the most appropriate material for crackable connecting rod (for more details, refer [172]). MOOSRA method also found application in supply-chain management area, for example, Sahu *et al.* developed a knowledge-based decision support system for appraisement of sustainable partner under fuzzy/non-fuzzy information where as a part of the system used MOOSRA and Reference Point Approach (RPA) to evaluate appraisement modules (for more information, refer [175]). These are some of the application areas of MOOSRA, which is based on my limited literature survey.

2.4.12 PSI (Preference Selection Index) Method

2.4.12A Background

To excel in the present scenario and maintain a competitive edge over the others, organizations have become more focused on finding efficient ways to provide a service or develop a product. In the industrial sector, the design of a product is very critical for the organization's business. An effective design comprises of several factors, and material selection is one of the crucial factors. Gone are the days, where cost was considered the sole criteria for material selection. Discovery of new materials and the already abundant material library has given engineers multiple options to choose an optimal material for a specific design. However, this has also made their lives more complicated and made their job very tough. There are very few standardized approaches to aid an engineer to deal with this problem, one of the more reliable approaches that are available is Ashby's approach [16] But this method has a major drawback dealing with multiple selection criteria. Several approaches were used to address this issue, but most of them had a limited effect on it. Finally, the multiple attribute decision making (MADM) perceptive had a breakthrough in dealing with multiple selection criteria decision problems in material selection [177].

Over the years, many MADM methods have been applied successfully to evaluate material alternatives for given product design. For example, simple additive weighted (SAW) method, weighted product method (WPM), technique for order preference by similarity to ideal solution (TOP- SIS), Valse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR) method, analytical hierarchy process (AHP), graph theory and matrix representation approach (GTMA), ELimination and Et Choice Translating REality (ELECTRE) etc. [177], all of these methods have yielded good results in assessing the alternatives over multiple selection criteria. However, there is one downside in their application to the material selection problem, i.e., the reliability on the weights or importance ratings of attributes. Weights play a crucial role in the MADM problem for addressing the particular worth of each attribute with respective to a decision problem. This dependency makes these MADM methods venerable to inconsistency in their evaluations. Thus, there is a need for a MADM method that is self-dependent to solve a material selection problem in a MADM viewpoint.

2.4.12B Introduction

Professor Bhatt *et al.* and Assistant Professor Maniya *et al.* has proposed a novel MADM method to deal with a material selection problem, known as Preference Selection Index (PSI) method [177-179]. This method was published in the year 2010 in a journal. As mentioned earlier, most of the MADM method that has been applied to a material selection problem has a prerequisite to assign relative importance between attributes (attribute weights) and which further requires many complex calculations [177 & 178]. In this method, it is not necessary to assign relative importance between attributes. Instead, the overall preference value (or score) of the attributes are calculated using the concept of statistics [177]. This method is very useful in situations where there is a conflict in deciding the relative importance between attributes [179]. Using overall preference value, the preference selection index for each alternative is calculated, and alternative with a higher value of PSI is selected as the best alternative.

In the coming section, the detail layout of the Preference Selection Index (PSI) method is laid out with a step-by-step procedure to solve a MADM problem.

2.4.12C The PSI methodology [177-183]

A MADM problem begins with the construction of the decision matrix with the determination of decision priorities (elements of the matrix) for each alternative concerning the attributes. The PSI method proceeds after the formation of the decision matrix. The following sequence of steps define the PSI method for solving a MADM problem:

Step-1: Normalization of the Decision Matrix

The process of transforming attributes value into a range of (0–1) is called normalization, and it is required in the MADM methods to transform performance rating with different data measurement unit in a decision matrix into a compatible unit [177].

For a Maximizing Attribute:

$$N_{ij}^{(LN)} = \frac{x_{ij}}{x_j^{max}} \quad \ldots\ldots\ldots\ldots (82)$$

, for *i=1,2……, M; & j=1,2……, N.*

For a Minimizing Attribute:

$$N_{ij}^{(LN)} = \frac{x_j^{min}}{x_{ij}} \text{ .... (83)}$$

, for *i=1,2......, M; & j=1,2......, N.*

Where,

"$N_{ij}^{(LN)}$"is the normalized value of i^{th} alternative for j^{th} attribute and "LN" indicates linear normalization method.

Step-2: Computation of Preference Variation value (PV_j)

The preference variation value (PV_j) for each attribute is determined with the concept of sample variance analogy using the following equation:

$$PV_j = \sum_{i=1}^{M}[N_{ij}^{(LN)} - \bar{N}_j^{(LN)}]^2 \quad \text{ .... (84)}$$

Where,

"$\bar{N}_j^{(LN)}$" is the mean of the normalized value of j^{th} attribute, and is mathematically represented as:

$$\bar{N}_j^{(LN)} = \frac{1}{M}\sum_{i=1}^{M} N_{ij}^{(LN)} \quad \text{ .... (85)}$$

Step-3: Determination of Over-all Preference Value (ψ_j)

The overall preference value (ψ_j) for each attribute is determined by calculating the deviation (Φ_j) in preference values (PV_j). Over-all Preference Value is obtained using the following formula:

$$\psi_j = \frac{\Phi_j}{\sum_{j=1}^{N} \Phi_j} \quad \text{........... (86)}$$

Where,

"Φ_j" is the deviation of j^{th} attribute, which can be calculated by:

$$\Phi_j = 1 - PV_j \quad \text{........... (87)}$$

The summation of the over-all preference value of all the attributes must be one, i.e., $\sum_j \psi_j = 1$.

Step-4: Obtaining Preference Selection Index (PSI)

The PSI for each alternative is determined using the equation:

$$(PSI)_i = \sum_{j=1}^{N} (N_{ij}^{(LN)} \times \psi_j) \quad \text{........... (88)}$$

Step-5: Ranking the alternatives

Once the PSI for each alternative is obtained, alternatives are ranked in the descending order, i.e., the alternative with the lowest PSI is termed as the worst, and the alternative with the highest PSI is interpreted as the best.

2.4.12D Adding weights to the Preference Selection Index (PSI) approach

PSI approach does not require attribute weights (or importance rating). However, weights can be incorporated into this methodology to strengthen it. Weights obtained by objective means (using Shannon's Entropy or Standard deviation methods etc.) or subjective means (using Analytical Hierarchy process etc.) or a combination of both, can be employed in the PSI method.

Weights are added after the Normalization procedure in the following way:

$$U_{ij} = W_j * N_{ij}^{(LN)} \quad \ldots\ldots\ldots\ldots (89)$$

Where,

"W_j" is the weight of j^{th} attribute and "U_{ij}" represents the utility value.

The remaining steps in the procedure remain the same with an exception that means and preference variance (PV_j) are calculated using the utility (U_{ij}) scores.

2.4.12E Implementation of the PSI approach to various MADM problems

Even though the PSI method was introduced as a Material selection aiding tool, its applications are not limited to any one particular type of decision problem, which is further established in this section. PSI method is a relatively new MADM method, and so far, has very limited applications. Maybe few in numbers, but it is applied to a wide range of decision problems stretching from material selection to evaluation of marketing areas for used laptops. Some of the applications are listed below for understanding the versatility of this approach.

- Bhatt *et al.* and Maniya *et al.* applied the PSI method to select an optimal material for Gear used in fuel pump (for more details refer [177]).
- Chamoli *et al.* conducted a Performance evaluation of V down perforated baffle roughed rectangular channel on multiple selection criteria (for more details refer [184]).

PSI method is used to assess process parameters of the various machining process by many engineers and researches such as:

- Determination of optimal parameters to improve the Lase cutting process- by Magic *et al.* and his team (for more details refer [185]).
- Selection of optimal Turning process parameters- by Prasad *et al.* and his team (for more details refer [186]).
- Evaluation of Solar thermal collector parameters- by Chauhan *et al.* and his team (for more details refer [187]).
- Selection of optimal process parameters of Fused Deposition Modelling (FDM) for Polylactic Acid Material- by Patel *et al.* and his team (for more details refer [179]).

Management and service sectors

Vahdani *et al.* applied a PSI method to evaluate different fuel buses alternatives. This decision problem involved criteria of both quantitative and qualitative nature. So, to deal with linguistic variables, Vahdani *et al.* with his research team developed a novel PSI method that incorporated a Fuzzy set to deal with the uncertainty in the data. The method is known as Fuzzy Preference Selection Index (FPSI) method [190].

Vahdani *et al.* proposed another extension to the PSI method that integrates Interval-Valued Fuzzy sets (IVFSs), aiming at solving complex decision-making problems. It is called an Interval-Valued Fuzzy Preference Selection Index (IVF-PSI) and to validate this methodology; it was applied to rate the performance of candidates from the viewpoint of human resource managers [190].

Another extension of the PSI method was proposed by Borujeni *et al.* and Gitinavard *et al.*, known as hesitant fuzzy preference selection index (HFPSI) method to evaluate the sustainable mining contractor selection problems where the imprecise data obtained from decision-makers (DMs) is dealt with hesitant fuzzy sets (HFSs) [194].

Sahir *et al.* with, a team of 12 researchers, evaluated the marketing area concerning various parameters (criteria) to determine the optimal location of Used Laptops [182].

2.4.12F PSI method: An objective weighting method

The PSI method determines criteria weights only by using the information provided in the decision matrix, i.e.; it uses an objective approach to determine criteria weights like standard deviation (SD) or Shannon's entropy method [178]. PSI measures weights according to the degree of convergence in the performance rating of each attribute. The motive and rationale of this objective weighting method have not been explained by authors. While in-contrast to this, Shannon's entropy and SD methods calculate weights according to the degree of divergence in the performance rating of each attribute. Therefore, decision-makers should be aware of this great contrast when they decided to adopt this approach to obtain the objective weights [26].

One major flaw to this theory is that, in PSI, it is possible for Preference variance (PV) to be greater than one and consequently, it results in negative weights, while the negative amount is not acceptable for showing the degree of importance in MCDM [26].

This concludes the literature review of the PSI method.

2.4.13 GRA (Grey Relation Analysis) method

2.4.13A Background

Most decisions made in the real world take place in an environment in which the goals and constraints, because of their complexity, are not known precisely, and thus, the problem cannot be exactly defined or precisely represented in a crisp value (Bellman *et al.* and Zadeh *et al.*, 1970) [Uncertainity-1]. To deal with the uncertainties in the decision problem, Zadeh *et al.* (1965) established fuzzy mathematics in the 1960s, Deng *et al.* developed grey systems theory and Pawlak (1982) advanced rough set theory in the 1980s, etc. All these works represent some of the most important efforts in the research of uncertain systems. From different angles, these works provide the theories and methodologies for describing and dealing with uncertain information [179].

2.4.13B Introduction: Grey Systems Theory (GST)

Grey System theory was introduced in 1982 by Deng *et al.* [180]. The systems which lack information, such as structure message, operation mechanism, and behavior document are referred to as Grey Systems. For example, the human body, agriculture, economy, etc. [180] These examples represent systems that involve uncertainties to define their state or operation. Usually, on the grounds of existing grey relations, grey elements, or grey numbers, one can identify the Grey System. The word "grey" in the name represents a characteristic difference between black and white. Where, "black" indicates "needed information" is not exactly available, conversely "white" means "needed information" is completely available. "Grey" system proposition establishes a connection between black with white. With the established connection, the correct properties of systems are discovered under poorly-informed situations [180 & 193]. The goal of

Grey System and its applications is to bridge the gap existing between social science and natural science. Thus, one can say that the Grey System theory is interdisciplinary, cutting across a variety of specialized fields [180 & 193]. Therefore, the Grey System theory seeks only the intrinsic structure of the system possessing limited data. Grey System theory has five major components. These are Grey Prediction, Grey Relational Analysis, Grey Decision, Grey Programming, and Grey Control [193].

The concept of the Grey System, in its theory and successful application, is now well known all around the world [180]. The fields of application of the Grey System involve agriculture, ecology, economy, medicine, history, geography, industry, earthquake, geology, hydrology, irrigation strategy, traffic, management, material science, environment, biological protection judicial system, etc.

2.4.13C Grey Relational Analysis (GRA)

Grey Relational Analysis, which is used for analyzing relations between the discrete data sets, is one of the popular methods. It is an impact evaluation model that measures the degree of similarity or difference between multiple sequences based on the grade of relation. GRA possesses the merit of point set topology, and so, the global comparison between the sets of data is undertaken instead of local comparison by measuring the distance between the points [179]. It is based on geometrical mathematics, which compliance with the principles of normality, symmetry, entirety, and proximity in dealing with sets of data [184]. GRA is an effective tool to make system analysis and also lays a foundation for modeling, forecasting, clustering of grey systems [184]. GRA is also used as a decision-making tool in multi-attribute cases.

GRA contains four steps to establish a global comparison among the alternatives [179]:

1. Preparation of factor compatibility (normalization of the variables).
2. Derivation of reference sequences (Normalized Sequence).
3. Calculation of grey relational coefficient (GRC).
4. Determination of grey relational grade (GRG).

This method proposes, theoretically, a dependence to measure the correlation degree of factors. Accordingly, this means the more similarity, the more factor correlation. Grey Relational Analysis uses Grey Relational grade to measure the relation degree of factors (Kung *et al.* & Wen *et al.*, 2007: 843). In this respect, Grey Relational theory provides efficient management of uncertainty [193]

The major advantages of GRA are based on original data, easy calculations, and being straightforward and one of the best methods to decide in a business environment. Grey Relational Analysis compares the factors quantitatively in a dynamic way using information from the Grey System. This approach contacts established relations among the factors (or variables) based on the level of similarity and variability [193].

2.4.13D Multi-attribute Grey Relational Analysis

Grey relational analysis (GRA) is part of grey system theory [5], which is suitable for solving a variety of multiple attribute decision-making (MADM) problems with uncertain information. GRA solves MADM problems by aggregating incommensurate attributes for each alternative into a single composite value, while the weight of each attribute is subject to the decision maker's judgment or "Voice of Data" (Objective weights). When such information is unavailable, equal weights seem to be a norm [179].

2.4.13E The sequence of operation [179-190]

Step-1: Normalization of Decision Matrix

Firstly, all alternatives are transformed to a comparability sequence in Grey Relational Analysis. This transformation is called Grey Relational generating. In this step, data are normalized and transformed to values in (0-1) interval.

A Linear Max-Min normalization procedure is used to convert decision matrix elements into dimensionless entities [179-190 & 43].

For maximizing attributes,

$$N_{ij}^{LM} = \frac{x_{ij}-x_j^{min}}{x_j^{max}-x_j^{min}} \quad \ldots\ldots\ldots\ldots (90)$$

, for all *i=1,2……, M; & j=1,2……., N.*

For minimizing attributes,

$$N_{ij}^{LM} = \frac{x_j^{max}-x_{ij}}{x_j^{max}-x_j^{min}} \quad \ldots\ldots\ldots\ldots (91)$$

, for all *i=1,2……, M; & j=1,2……., N.*

Where,

"N_{ij}^{LM}" is the normalized element, and "LM" indicates the type of normalization.

Step-2: Generate the Reference Sequence (R_j)

Once the Normalized sequence is obtained, a reference sequence is then computed using it. The reference (ideal target) sequence is the largest normalized value of each criterion and is calculated as:

$$R_j = Max_{i=1}^{M}\{N_{ij}^{LM}\} \quad \text{...........} \ (92)$$

Step-3: Calculating the Deviation Sequence

The Deviation sequence is defined as the distance between the Reference sequence and Normalized sequence and is mathematically obtained by:

$$\Delta_{ij} = (R_j - N_{ij}^{LM}) \quad \text{...........} \ (93)$$

The Deviation Sequence:

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} \cdots & \Delta_{1N} \\ \vdots & \ddots & \vdots \\ \Delta_{M1} & \Delta_{M2} \cdots & \Delta_{MN} \end{bmatrix}$$

Step-4: Determine Grey Relational Coefficient (γ_{ij})

The Grey Relational Coefficient (γ_{ij}) is calculated between the reference sequence and all comparability sequences using the following formula:

$$\gamma_{ij} = \frac{\Delta_{min} + \zeta\Delta_{max}}{\Delta_{ij} + \zeta\Delta_{max}} \quad \text{...........} \ (94)$$

Where,

$$\Delta_{min} = \min_i \min_j \{\Delta_{ij}\} \quad \text{...........} \ (95),$$

$$\Delta_{max} = \max_i \max_j \{\Delta_{ij}\} \quad \text{………… (96)}$$

and

ζ= is the distinguished coefficient { $\zeta \epsilon (0, 1)$}

Step-5: Computation of Grey relational grade (Ψ_i)

Finally, Grey Relational grade (Ψ_i) between the reference sequence and comparability sequences is calculated according to the grey relational coefficient as:

$$\Psi_i = \sum_{j=1}^{N} (W_j \times \gamma_{ij}) \quad \text{………… (97)}$$

Where,

"W_j" is the importance rating or the weight of the j^{th} attribute.

The grey relational grade that indicates the magnitude of correlation measured between the reference sequence and the i^{th} data sequence.

Step-6: Ranking the alternatives

Therefore, using the grey relational grade, the alternatives can be prioritized, and the one with the highest degree of correlation will be identified as the best alternative that represents the optimal solution.

2.4.13F Application of GRA to various MADM problems

GRA is suitable for solving complicated interrelationships between multiple factors and variables and has been successfully applied to cluster analysis, robot path planning, project selection, prediction analysis, performance evaluation, and factor effect evaluation and multiple criteria decision [182]. Some of the noteworthy applications of GRA method as MADM approach is presented in this section.

a) Design and material

Material selection is a crucial and laborious task in the field of engineering and manufacturing. An improper selection can negatively affect productivity, profitability, and undermine the reputation of an enterprise because of the growing demands for extended manufacturer responsibility and stifling competition [179]. Hence, it led to the utilization of the MADM method. Many MADM methods have been successfully implemented to deal with the decision problems related to material selection for various industries such as automotive, aerospace, etc. GRA is one such method that has received overwhelming acceptance for evaluating different material alternatives and nominating an optimal material based on the requirements of the problem. Some of the applications of GRA in this field are mentioned below:

Chan *et al.* and Tong *et al.* have applied GRA method to select the optimal material for the dust bin of a vacuum cleaner and also evaluated end of life (EOL) strategies for the same product (for more details refer [179]).

Satapathy *et al.*, Bijwe *et al.* and Kolluri *et al.* have assessed the contribution of fiber to the performance of friction materials based on various possible combinations of organic fibers using GRA method (for more details refer [188]).

Similarly, Patnaik *et al.* and Chauhan *et al.* have proposed the Optimization of tribological properties of cement kiln dust-filled brake pad using grey relation analysis (for more details refer [187]).

b) Supply-chain management

In the current scenario, markets are no longer localized. Globalization has engulfed the local markets and transformed them to global markets where there is a constant interaction between customers and suppliers belonging to different parts of the world. In such a context, the need for reviewing and assessing the resilience of suppliers as one of the new concepts in supply chain management has been prioritized. In addition, supplier selection, supplier-chain network evaluation, green supply chain management, etc., have been observed keenly by the industries to improve their environmental and economic performance [192]

Many engineers have dealt with the decision problems related to the supply-chain by implementing grey systems theory due to the uncertain nature of the information recorded. GRA method has many applications in this field with several extensions to accommodate the peculiarity of the tasks. Some published works to support the above claims are listed below:

Sari *et al.*, Baynal *et al.,* and Ergul *et al.* applied GRA method to evaluate suppliers of a food manufacturing company and select the best suppliers based on multiple criteria (for more details refer [185]).

In a similar case as above, Malek *et al.*, Ebrahimnejad *et al.,* and Tavakkoli-Moghaddam *et al.* have proposed a novel method that combines Grey-TOPSIS and GRA methods to deal with the Green Resilient Supply-Chain Network Assessment problem (for more details refer [192]).

Hou *et al.* developed a framework to facilitate the utilization of the GRA method for multiple attribute decision making with intuitionistic fuzzy information, to deal with the supplier selection in supply chain management (for more details refer [195]).

These are some of the prominent applicational fields of the GRA method, which does not indicate its limitations to other fields.

Here ends the literature survey of the Grey Relational Analysis (GRA) method.

2.4.14 COPRAS (Complex Proportional Assessment) method

2.4.14A History

SAW (Simple Additive Weighting) is one of the simplest and most widely used multi-attribute evaluation methods. It demonstrates the idea of integrating the quantitative values of the criteria and its weights (importance ratings) into a single estimating value — the criterion of the method. However, SAW uses only maximizing ("the higher, the better") evaluation criteria, while minimizing ("the lower, the better") evaluation criteria has to be converted into the maximizing entities by one of the normalization techniques before their application [212]. The maximizing (beneficial) and the minimizing (non-beneficial) criteria must be evaluated separately because of their contradictory nature in a decision problem [212]. This is one of the critical drawbacks of the SAW method.

In the year 1996, Zavadskas *et al.* and his disciple professor Kaklauskas *et al.* of Vilnius Gediminas Technical University, proposed a novel methodology that treats maximizing and minimizing criteria separately [211 & 212] and has more robust evaluation structure than the SAW method. It is known as COPRAS (Complex Proportional Assessment).

2.4.14B Introduction

COmplex PRoportional ASsessment (COPRAS) method is a preference ranking method, assumes direct and proportional dependences of the significance (or the degree of satisfaction attained by that alternative w.r.t each criterion) and utility degree of the available alternatives under the presence of mutually conflicting criteria [215]. It takes into account the performance of the alternatives concerning different criteria and the corresponding criteria weights. This method chooses the best decision alternative considering both the ideal and the ideal-worst solutions [213, 223 & 225]. The COPRAS uses a stepwise ranking and evaluating the procedure for the assessment of the alternatives in terms of their significance and utility degree [210 & 215].

2.4.14C The COPRAS approach [215-220]

The application of the COPRAS method to a Multiple Attribute Decision Making (MADM) model begins with the construction of the decision matrix and allocation of the performance ratings (preferably in quantitative format) for each alternative concerning the multiple attributes. Once the decision matrix is completed, COPRAS method proceeds in the following way:

Step-1: Normalization of the Decision Matrix

Normalization is a technique to convert a dimensional criterion to dimensionless entity [N-1]. For a maximizing (beneficial) attributes, i.e., "the larger, the better" kind of criteria.

Normalization formula is:

$$N_{ij}^{(LN)} = \frac{x_{ij}}{x_j^{max}} \quad \text{………… (98)}$$

, for *i=1,2……, M; & j=1,2……, N.*

For minimizing attributes, i.e., "the smaller, the better" kind of criteria, normalization formula is:

$$N_{ij}^{(LN)} = \frac{x_j^{min}}{x_{ij}} \quad \text{………… (99)}$$

, for *i=1,2……, M; & j=1,2……, N.*

Where,

$N_{ij}^{(LN)}$ is the normalized score/value/utility and "LN" represents linear normalization technique.

Step-2: Weighted Normalization of Decision Matrix (DM)

This is a very important step in which performance ratings and weights are integrated to translate the objectives of the decision problem.

$$R_{ij} = W_j * N_{ij}^{(LN)} \quad \text{………… (100)}$$

, for *j=1, 2…., N.*

Where,

"W_j" is the weight (or the importance ratings) of the j^{th} attribute.

"R_{ij}" is the weighted normalized utility/score.

Step-3: Sum of Weighted Normalized values

In this step, the maximizing (beneficial) and minimizing (non-beneficial) attributes are separately summed up.

Maximizing attributes:

$$S_i^+ = \sum_{j=1}^{G} R_{ij} \quad \ldots\ldots\ldots\ldots (101)$$

for all *i= 1, 2……, M.*

Minimizing attributes:

$$S_i^- = \sum_{j=G+1}^{N} R_{ij} \quad \ldots\ldots\ldots\ldots (102)$$

for all *i= 1, 2……, M.*

Step-4: Summation of the "S_i^+" and "S_i^-"

Here, the individual summation of the weighted normalized values of beneficial and non-beneficial criteria of each alternative is added together in the following way:

Summation of Maximizing attributes:

$$S_{+} = \sum_{i=1}^{M} S_i^{+} \quad \text{………… (103)}$$

Summation of Minimizing attributes:

$$S_{-} = \sum_{i=1}^{M} S_i^{-} \quad \text{………… (104)}$$

Step-5: Determining the Relative Significance (Q_i) of each alternative

The relative significance value of an alternative conveys the degree of satisfaction attained by the alternatives. The alternative with the highest relative significance value (Q_{max}) is the best choice among the candidate alternatives.

Relative significance value (priority), "Q_i" of i^{th} alternative can be calculated using the formula below:

$$Q_i = S_i^{+} + \frac{(S_i^{-})_{min} \times (S_{-})}{S_i^{-} \times \sum_{i=1}^{M} \frac{(S_i^{-})_{min}}{S_i^{-}}} \quad \text{………… (105)}$$

Where,

"$(S_i^{-})_{min}$" is the lowest value from the obtained S_i^{-} among all the alternatives.

Step-6: Calculate Quantitative Utility (U_i)

The Quantitative utility of an alternative is directly linked with its relative significance value ("Q_i"). The degree of an alternative's utility, leading to a complete ranking of the candidate alternatives, is determined by comparing the priorities of all the alternatives with the most efficient one and can be denoted as below:

$$U_i = \left(\frac{Q_i}{Q_{max}}\right) \times 100 \quad \text{………… (106)}$$

Step-7: Ranking of the alternatives

The alternative with the highest "U_i" value is ranked as the best and the alternative with lothe west "U_i" value is placed at the bottom of the list.

2.4.14D Implementation of the COPRAS method to various MADM problems

Complex Proportional Assessment (COPRAS) method in one of the most applied MADM methods to evaluate various decision alternatives, which can be contributed to the fact that it is a simple method to understand and utilize [217]. COPRAS method has a very wide range of applications stretching from engineering to domestic problems. Many economic and engineering researchers have modified the classical COPRAS method in order to solve different decision models, which led to the extension of classical COPRAS method. A brief account of the applications and corresponding extensions of the COPRAS method are mentioned below to strengthen the above claims.

a) Economics

MADM ideology originally developed to deal with decision problems associated with economics. Moreover, COPRAS method was also proposed to solve economic and engineering related problems initially and later extended to various fields. COPRAS method has numerous applications in this field, such as:

Social

Author Podvezko *et al.* has applied COPRAS method to conduct a comprehensive evaluation of economic and social development of Lithuanian regions, in his comparative analysis of MCDA methods SAW and COPRAS (for more details refer [212]).

b) Engineering

Design and material

Chatterjee *et al.* Athawale *et al.* and Chakraborty *et al.* tested the capability and applicability of COPRAS method in material selection domain with two illustrated examples of Material selection for a cryogenic storage tank for transportation of liquid nitrogen and selection of most appropriate work material for a particular product designed to operate in a high-temperature oxygen-rich environment. (for more details refer [215]). Whereas Mousavi-Nasab *et al.* and Sotoudeh-Anvari *et al.* applied COPRAS method for selecting optimal material for an auxiliary tool (for more information refer [217]

MADM problems usually involves uncertainties and incompleteness in the data. So, to deal with such problems especially in material selection domain, authors such as Maity *et al.*, Chatterjee *et al.* and Chakraborty *et al.*, utilized grey complex proportional assessment (219) method to deal with incompleteness or uncertainty in data obtained to evaluate Cutting tool material (for more information refer [220]) and Xia *et al.* and his team have proposed an improved COPRAS Method altogether.

Manufacturing

Following in the steps of their peers, Vahdani *et al.* and his team proposed a novel COPRAS method that utilizes interval-valued fuzzy data to deal with both subjective and objective modes of information for robot selection (for more details refer [217]).

Industrial

Ghorabaee *et al.* and his team taken a leaf from their peers to propose an interval type-2 fuzzy numbered (based on the centroid of fuzzy sets) COPRAS method to evaluate suppliers in a supply-chain management problem to cover both qualitative and quantitative data (for more information refer [217]).

Civil engineering

Zavadskas *et al.* and Kaklauskas *et al.* proposed an extension of COPRAS method by combining it with grey relations methodology to deal selection of effective dwelling house walls (refer [213] for more information). These are some of the applications of the COPRAS and its extension to various fields.

2.4.15 TOPSIS (Technique for the Order of Preference by Similarity to Ideal Solution) method

2.4.15A Introduction

TOPSIS (Technique for the order of preference by similarity to ideal solution) method, is one of the most popular and extensively utilized Multiple Attribute Decision Making (MADM) methods.

It is highly appreciated and regarded decision aiding methods in the world of economics and other branches associated with it. Some famous authors and researchers such as Zavadskas *et al.*, Mardani *et al.*, Turskis *et al.*, Jusoh *et al.*, and Nor *et al.*, claimed that the TOPSIS method is the second most popular method among MADM; and Shih *et al.*, Shyurand *et al.* and Lee *et al.* in their work credited it as an important branch of decision making [245].

Hwang *et al.* and Yoon *et al.* have collaborated to develop the TOPSIS method, and it was finally published in the year 1981. However, that was only the beginning, as other researchers embraced this technique and also contributed to it by adding their own ideas leading the expansion of TOPSIS method to cope with different data types, uncertainties, and decision problems. This made the TOPSIS a robust decision aiding methodology.

2.4.15B Principle

TOPSIS defines an index called similarity (or relative closeness) to the positive-ideal solution by combining the proximity to the positive-ideal solution (PIS) and the remoteness from the negative-ideal solution (NIS). Then the method selects an alternative with the maximum similarity to the positive-ideal solution. TOPSIS assumes that each attribute takes either monotonically increasing or monotonically decreasing utility. That is, the larger the attribute outcome, the greater the preference for benefit attributes, and the less the preference for cost attributes [229].

2.4.15C Methodology

TOPSIS is a multi-attributes decision-making method which converts multi-response values (performance ratings) into a single performance response value (utility) [236]. In this method, the alternatives are ranking according to their distances from ideal (PIS) and anti-ideal solutions (NIS), i.e., the best alternative will be the one which has the shortest distance from the ideal solution and simultaneously, the largest distance from the anti-ideal solution [230]. Then the preference order is ranked based on their relative closeness to the ideal solution and the combination of these two distance measures as well. The ideal solution here is a hypothetical solution (alternative) for which all attribute values correspond to the optimum scores in the database comprising the satisfying solutions to the decision problem, and the anti-ideal solution is the hypothetical solution for which all attribute values correspond to the minimum attribute scores in the database. TOPSIS thus gives a solution that is not only closest to the hypothetically best but also the farthest from the hypothetically worst [231].

2.4.15D Measurement of distance between Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

The distance between an attribute concerning the positive ideal (or simply ideal) and negative ideal (or anti-ideal) solutions is measured using Euclidean distance [231]. A Euclidean distance can be simply defined as the straight-line distance between two points [232] in Euclidean space or simply metric space. The Euclidean distance between two points can be calculated using Pythagoras theorem in 2-dimensional space, but to calculate the same in 3-dimensional space, the formula is little modified as:

The Euclidean distance between two points in 3-D space is given as [LM]:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad \ldots\ldots\ldots\ldots (107)$$

Similarly, the distance between PIS and NIS are determined using Euclidean distance. An optimal alternative will have the shortest Euclidean distance from the ideal solution, and the farthest from the negative ideal solution [231].

2.4.15E TOPSIS approach [228-236]

TOPSIS is robustly constructed method with a systematic approach involving multiple steps to break-down the decision problem and simultaneously solve at the same time. The steps involved in this method are laid down below for a detailed understanding of the approach.

Step-1: Normalization of the performance ratings

Initially, the pay-off matrix or the decision matrix is constructed with the allocation of performance ratings by objective or subjective ways for multiple attributes of each alternative. Then, the normalization of the ratings is done to convert the dimensional entities to dimensionless [42]. TOPSIS method utilizes a vector normalization technique for this purpose.

For Maximizing attributes:

$$N_{ij}^{(V)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \qquad \text{...........} (108)$$

, for *i= 1,2,3, ..., M; & j= 1,2,3, ..., N.*

For Minimizing attributes:

$$N_{ij}^{(V)} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \qquad \text{...........} (109)$$

, for *i= 1,2,3, ..., M; & j= 1,2,3, ..., N.*

Where,

"$N_{ij}^{(V)}$" is the vector-normalized rating of i^{th} alternative and j^{th} attribute.

"x_{ij}" is the performance rating of i^{th} alternative and j^{th} attribute.

Step-2: Weighted Normalization of the ratings

In this step, the weights or the importance ratings of the attribute concerning the decision problem is multiplied to the normalized performance ratings to translate the objectives of the problem. The weights of the attributes can be determined using objective methods or subjective methods or by the combination of the two. Mathematically the integration between the performance ratings and importance ratings is represented as:

$$r_{ij} = W_j \times N_{ij}^{(V)} \quad \text{……….. (110)}$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

Where,

"W_j" is the weight of j^{th} attribute. ($\sum_{j=1}^{N} W_j = 1$).

"r_{ij}" is the weighted-normalized rating of i^{th} alternative and j^{th} attribute.

Step-3: Determination of PIS and NIS for each attribute

The composite of all the best attribute ratings attainable is the positive-ideal solution (PIS), whereas the negative-ideal solution (NIS) is composed of all worst attribute ratings attainable.

$$S^{+} = \{r_1^{+}, r_2^{+}, \dots \dots .., r_N^{+}\} = \left\{\max_j r_{ij} \mid \forall\, j = 1,2,3, \dots, N.\right\} \quad \dots\dots\dots\dots (111)$$

$$S^{-} = \{r_1^{-}, r_2^{-}, \dots \dots .., r_N^{-}\} = \left\{\min_j r_{ij} \mid \forall\, j = 1,2,3, \dots, N.\right\} \quad \dots\dots\dots\dots (112)$$

Where,

"S^{+}" is denotes PIS and "S^{-}" denotes NIS.

Step-4: Calculation of the separation distance

Here, the Euclidean distance is measured between each alternative's performance score concerning attribute to the "S^{+}" and "S^{-}" using the following formula:

$$D_i^{+} = \sqrt{\sum_{j=1}^{N}\left(r_{ij} - r_1^{+}\right)^2} \quad \dots\dots\dots\dots (113)$$

, i=1, 2, 3,…., M.

$$D_i^{-} = \sqrt{\sum_{j=1}^{N}\left(r_{ij} - r_1^{-}\right)^2} \quad \dots\dots\dots\dots (114)$$

, i=1, 2, 3, ……., M.

Where,

"D_i^{+}" is the separation distance between each alternative to PIS.

"D_i^{-}" is the separation distance between each alternative to NIS.

Step-5: Calculation of Relative Closeness

In this step, the relative closeness to the ideal solution is calculated using the following formula:

$$C_i = \frac{D_i^{-}}{D_i^{+} + D_i^{-}} \quad \dots\dots\dots\dots (115)$$

, i= 1,2, 3,……M.

Step-6: Ranking of the Alternatives

The ranking is done in the descending order, i.e., the alternative with highest "C_i" is at the top of the ranking order and the alternative with the lowest "C_i" will remain at the bottom of the list.

2.4.15F Merits of TOPSIS method [232]

TOPSIS method is one of the most widely applied methods to solve decision problems in various fields, which can be credited to its positive features that makes researchers and practitioners exercise it. Some of the merits of this method according to one of the most revered author and practitioner Zavadskas *et al.* and Turski *et al.* are listed below [232]:

1. The performance is slightly affected by the number of alternatives and rank discrepancies are amplified to a lesser extent for increasing values of the number of alternatives and the number of criteria.
2. Easy decision making using both negative and positive criteria.
3. It is relatively easy to implement, understandable, and fast, provides a well-structured analytical, systematic process.
4. It is useful for qualitative and quantitative data.
5. A number of criteria can be applied during the decision process.
6. The output (based on a well-structured analytical framework) can be a preferential ranking of the feasible alternatives based on a numerical value which provide a better understanding of differences and similarities among alternatives.
7. It has very high flexibility in the definition of the choice set.

8. TOPSIS has been proved to be one of the best methods in addressing the rank reversal issue, which is the change in the ranking of alternatives when a non- optimal alternative is introduced.

2.4.15G Demerits of TOPSIS method [232]

In his intense research in the TOPSIS methodology, Zavadskas *et al.* listed out some demerits that, according to him, limits its potential. They are mentioned below:

1. The first drawback is the operation of a normalized decision matrix in which the normalized scale for each criterion is usually derived a narrow gap among the performed measures. That is, a narrow gap in the TOPSIS method is not good for ranking and cannot reject the true dominance of alternatives.
2. The second drawback is that we never considered the risk assessment for a decision maker in the TOPSIS method.
3. The third drawback is that similarly to the AHP method, TOPSIS presents the problem of ranking reversal (the final ranking can swap when new alternatives are included in the model. As it can be found from research results, TOPSIS also exhibits certain failure rates. However, these rates are higher than those of the methods examined earlier in this Trianthaphylou research WSM, AHP, and revised AHP.
4. This analysis indicates that a contradiction may occur when the method is used, and a nonoptimal alternative is replaced by a worse one, or nonoptimal alternative is added (in a similar manner as the AHP and WPM methods).

2.4.15H Areas of application of the TOPSIS method

The wide range of real-world applications of the TOPSIS method by various practitioners and researchers has imposed a strong motivation for categorizing applications across different fields and specific sub-areas [229]. Application research studies include stats and records from state-of-the-art surveys and research papers that revolve around the comprehensive analysis of the TOPSIS method.

The categorization of the areas or the fields of application are listed below (courtesy state-of-art survey by Behzadian *et al.*, for more details, refer [229]):

a) Supply-chain management and logistics

Supply Chain Management and Logistics are considered vital for the success of an industrial. Moreover, coincidently is one of the most popular topics associated with TOPSIS applications. Supply chain and logistics management cover several specific sub-areas, including supplier selection, transportation, and location problem, etc. A detail list of applications presented in [235]).

b) Design, engineering and manufacturing systems

Design, Engineering, and Manufacturing Systems issue is a broad area in the TOPSIS publications. The area typically includes papers in modern manufacturing systems, automation, material engineering, mechatronics, product design, and quality engineering. An intense research on applications of TOPSIS with regards to this section is presented in [235] *c) Business and marketing management*

Business and Marketing Management is the third most popular area in TOPSIS applications. It covers applications that use TOPSIS for organizational performance, financial measurement, investment projects, customer satisfaction, and competitive advantages. Approximately 12.3% of all papers fall under the business and marketing management category [229].

These are the most important areas where TOPSIS method has the highest number of applications as per articles and papers published [229]. For more detailed account of the applications and extensions of the TOPSIS method, please refer "A state-of-the art survey of TOPSIS applications" by author Behzadian *et al.* [229] and "A bibliometric-based survey on AHP and TOPSIS techniques" by author Zyoud *et al.* [235].

Here ends the literature review of the TOPSIS approach.

2.4.16 VIKOR: A compromise ranking method

2.4.16A Background

The concept of the ideal solution was introduced simultaneously by Yu *et al.* (in the year 1973 [CP-1]) and economist Zeleny *et al.* (in the year 1973 [335]). Once this point been defined, it was possible to establish that the best-compromise solution as the nearest solution concerning the ideal, considering the basic postulate that the decision maker (DM) prefers solutions as close as possible to the ideal [335]. These simple but ingenious ideas became the basis for a famous multiple criteria analysis technique, known as compromise programing method (CP).

Compromise Programming (CP) is an interactive method used to resolve a multiple linear objective problem. CP identifies solutions which are closest to the ideal solution as determined by some measure of distance. The compromise solutions are those identified as being closest to the ideal one, and it constitutes the compromise set [335]. Figure 15 describes the principle behind the compromise programing with a visual representation of the decision-making scenario and a set of feasible and non-feasible solutions to choose from [251].

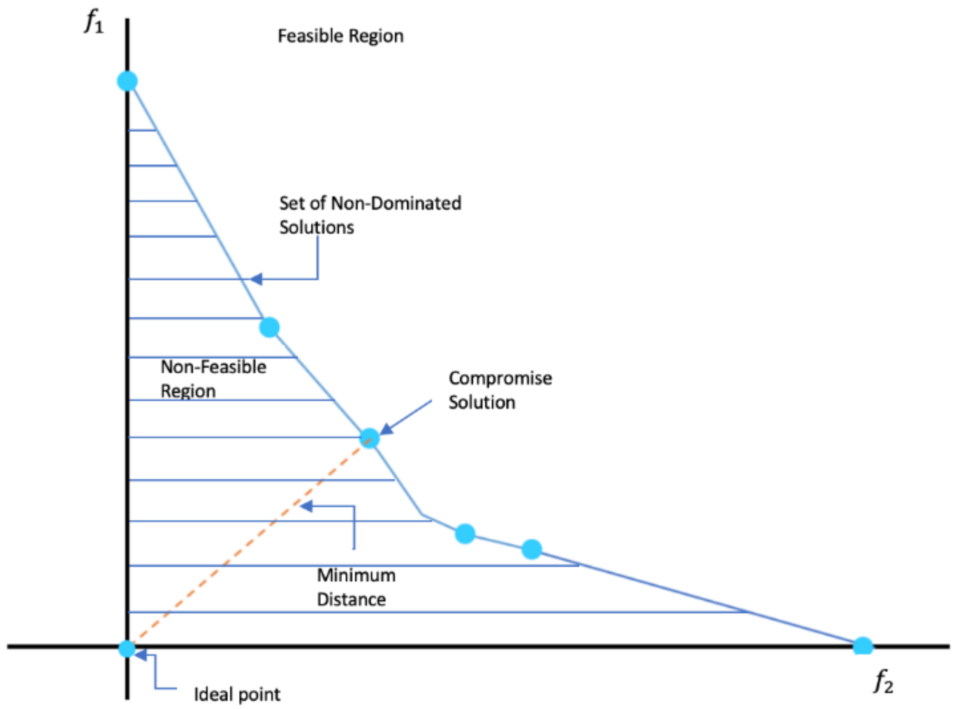


Figure 15: Decision-making scenario [251].

To understand the meaning of the two concepts mentioned above, let us consider the concept of an ideal solution and the appropriate used distance. The vector $z^* = \{z_1, z_2, \ldots, z_i\}$, defines the ideal solution and the z_i are the solution of the following problem [335]

$$Max\ z_i$$

Subject to $x \in X$, for all $i= 1, 2, 3, \ldots, M$.

In general, the ideal solution x^* is not feasible; however, it is considered as a standard for evaluation of the attainable non-dominated solution. The set $z^*(x^*) = \{z_1^*(x^*), z_2^*(x^*), \ldots, z_i^*(x^*)\}$, define the non- dominated set (in objective space) consisted of only one point. The evaluation of non-dominated point consists in measuring how these points come close to the ideal solution. The distance measure (the most frequently used) utilized in Compromise Programming is the family of L_p-metrics (or Lebesgue metric). It is defined in two equivalent ways [251]:

$$L_s = \left[\sum_i^M \alpha_i^s \,(z_i^* - z_i(x))^s\right]^{1/s} \quad \ldots\ldots\ldots\ldots (116)$$

(Or)

$$L_s = \sum_i^M \alpha_i^s \,(z_i^* - z_i(x))^s \quad \ldots\ldots\ldots\ldots (117)$$

Where,

$One \leq s \leq \infty$,

α_i= criteria weights.

If we note the compromise solution as x_s^*, this solution corresponds to the resolution of the following program [335]:

$$Min\ L_s\,(x) = L_s(x_s^*) \quad \ldots\ldots\ldots\ldots (118)$$

Subject to $x \in X$.

The final expression after all the transformation is as follows [335 & 251]:

$$Min\left\{L_s(x) = \sum_{i=1}^{M} \alpha_i^s \left(\frac{z_i^* - z_i(x)}{z_i^* - z_i^{**}}\right)^s\right\} = L_s(x_s^*) \qquad \text{……….. (119)}$$

Subject to $x \in X$.

Where,

z_i^{**} = min z_i ($x \in X$, for all i= 1, 2, 3, …., M.)

Based on the concept of compromise solution and inspired by the compromise programing algorithm to identify the best compromise solution and to use the L_p-metrics to measure the distance between non-feasible ideal solution and the best compromise solution [251 & 252], author and Opricovic *et al.* had developed the VIKOR method (the Serbian name is 'Vlse Kriterijumska Optimizacija Kompromisno Resenje' which means multi-criteria optimization (MCO) and compromise solution) in his Ph.D. dissertation in 1979 [262] which has been highly appreciated and extensively used to resolve complex Multiple Attribute Decision Making (MADM) problems.

2.4.16B Methodology

VIKOR is a compromise ranking method, developed to solve MCDM problems which usually involves conflicting and non-commensurable (attributes with different units) criteria, invested in the concept of a compromise solution to determine the best alternative to resolve a conflict. In a realistic decision-making situation, the chance for an ideal solution or alternative is zero. So, usually the decision maker wants a solution that is the closest to the ideal solution, and the alternatives can be evaluated according to all the established criteria [252 & 255] This philosophy led to the foundation of the VIKOR methodology.

The compromise ranking method (known as VIKOR) is introduced as one applicable technique (of the compromise programing) to be implemented within MCDM [253]. This method inherits the approach of the Compromise Programing (CP) up to some extent, which can be observed in the next section, which layout its algorithm [251 & 252].

2.4.16C The VIKOR approach [251, 252, 253, 255 and 256]

As mentioned in the previous sections, the VIKOR methodology resolves a MADM problem with multiple and often conflicting criteria by identifying the compromise solution (one or more) by measuring the distance between the ideal solution and other feasible solutions. The solution with the shortest distance from the ideal is chosen as the compromise solution. To measure this distance L_p-metrics is employed [251, 252, 253, & 255].

Unlike the CP method, VIKOR utilizes both the utility and regret measures to formulate ranking measure for each alternative. Where, the solution obtained by minimum utility measure is with a maximum group utility (‘‘majority’’ rule), and the solution obtained by minimum regret measure is with a minimum individual regret of the ‘‘opponent’’ [256].

The VIKOR algorithm begins with the formulation of decision matrix (DM), listing the attributes and identifying the objectives of the decision-problem. Once the DM is constructed, the algorithm proceeds in the following way:

Step-1: Identify the best and worst values of all the criteria

In this step, the best and the worst values of the list of criteria are identified. The best performance value of the criteria is denoted by "x_i^+" and the worst value is denoted by "x_i^-";

Where,

$$x_i^+ = \max_j x_{ij} \quad \text{………. (120)}$$

, for *j= 1, 2, …., N.*

Moreover,

$$x_i^- = \min_j x_{ij} \qquad W_j \frac{(x_i^+ - x_{ij})}{(x_i^+ - x_i^-)} \quad \text{………. (121)}$$

Step-2: Calculate the utility and the regret measure

In the VIKOR method, the closeness of each alternative concerning an imaginary ideal solution is measured using the Lebesgue metric ($L_p - metric$). The utility measure is denoted by "S_i" and the regret measure by "R_i", are the relative measures between each criterion and the ideal solution. There are obtained using the following formulas:

$$S_i = \sum_{j=1}^{N} W_j \frac{(x_i^+ - x_{ij})}{(x_i^+ - x_i^-)} \quad \text{………. (122)}$$

Moreover,

$$R_i = \max\left[W_j \frac{(x_i^+ - x_{ij})}{(x_i^+ - x_i^-)}\right] \quad \text{………. (123)}$$

Step-3: Determine the relative preference index

The relative preference index is performance evaluation of each alternative with the ideal solution and is measure using the cumulative utility and regret measures. The formula to calculate the relative preference index is:

$$Q_i = \upsilon \frac{(S_i - S^+)}{(S^+ - S^-)} + (1-\upsilon) \frac{(R_i - R^+)}{(R^+ - R^-)} \quad \ldots\ldots\ldots\ldots (124)$$

Where,

$S^+ = \max_i S_i$ ………… (125),

$S^- = \min_i S_i$ ………… (126),

$R^+ = \max_i R_i$ ………… (127),

$R^- = \min_i R_i$ ………… (128),

$\upsilon \in [0,1]$ *is the , maximum group utility.*

Step-4: Ranking the alternatives

Alternatives can be ranked using “S_i”, “R_i” or “Q_i” values in decreasing order. But for a best compromise ranking “$Q_{i.}$” values are used.

2.4.16D Applications

VIKOR method characteristically has a simple computational procedure and offers a systematic and logical approach to arrive at the best decision [255] by considering a compromise solution to a complex problem with conflicting attributes. These features made it an effective MADM tool, which resulted in numerous applications in various fields. The figure below exhibits the range of application areas for the VIKOR method. The applications of the VIKOR method mentioned in this section are based only on the published works and articles.

Figure 16: Distribution of papers based on main application areas.

The stats mentioned in figure 16 is taken directly from a state-of-the-art literature review of VIKOR and its fuzzy extensions by Gul *et al.* and his team which is based on 343 published papers (for more details of the application of VIKOR, refer [262]).

The most popular areas of application of the VIKOR method are:

a) Design and manufacturing

b) Business management

a) Design and manufacturing

This is the most popular application area for VIKOR methods based on the papers published. It is a broad category which can be dissolved into sub-categories such as material selection, robot selection, new product development, and machine tool selection, as these found to be some of the more prevalent sub-application areas in this category [262]. Still, material selection stands out as the most applied field.

Material selection

Material selection has become a crucial part of engineering. To select the optimal material among a wide variety of material alternatives is a tedious job. Various attributes are considered while selecting assessing material, such as mechanical properties, functional properties, physical properties, material cost and availability, processing and environment impact, etc. Under such circumstances, an engineer or a decision-maker in most cases look for a solution that is the best compromise considering all the attributes. This led to the extensive employment of the VIKOR method in this field. For example, Jahan *et al.* and his team have applied the VIKOR method in a biomedical area to select an optimal material (for more information, refer [265]). Whereas author and Liu *et al.* with his team developed an extension of the classical VIKOR method with DANP for developing a decision-making model for material selection (for more details, refer [263]). There are many other applications in this category for this approach.

b) Business management

It is the 2nd most popular category after design and manufacturing. It is divided into sub-categories such as human resources, personnel appraisal, marketing and performance evaluation, etc. Some examples of applications in these sub categories are: Yalcin *et al.* and her team have employed VIKOR method to evaluation financial performance of Turkish manufacturing industries (for more details, refer [263]). Ghadikolaei *et al.* evaluated the financial performance of Iranian companies using Fuzzy VIKOR (for more information, refer [264]). Moreover, Wang *et al.* assessed the brand marketing strategies using the VIKOR method (for more details, refer [265]). There are other applications related to this field, and for a comprehensive application account of VIKOR method, please refer [270 & 261].

2.4.17 SBA (Similarity-Based Approach) method

2.4.17A Background

Multiple Criteria Decision Making (MCDM) approach has established itself as a game changer in the world of operations research. This ideology has been successfully applied in almost all the fields that deals with decision-making problems such as manufacturing, engineering, supply-chain management, business management, and public sector, etc. The expansion of MCDM approach has led to the development of various methodologies that utilizes this platform and employs specific aggregation techniques to evaluate a list of eligible alternatives concerning multiple and often conflicting attributes to come up with an optimal solution to a decision problem. There are many MCDM/MADM methods such as Simple additive weighting (SAW), Technique of ranking preferences by similarity to the ideal solution (TOPSIS), Compromise ranking method (VIKOR), Grey relation Analysis (GRA), Superiority and Inferiority Ranking (SIR), Evaluation based on Distance from Average Solution (EDAS) and Preference selection index (PSI) etc. Out of these methods, the TOPSIS method has received special recognition as a technique that fully inherits the MADM ideology [230, 231, 236 & 238].

The TOPSIS approach is developed based on the idea that the best alternative will be the one which has the shortest distance from the ideal solution and simultaneously, the largest distance from the anti-ideal solution [235 & 229]. This concept was widely accepted in the industrial universe, and as a result, it has found numerous applications in the field of economics, management, and engineering, etc. However, later, some researchers have found that under some circumstances, counter-intuitive outcomes may occur while comparing two alternatives (vectors) just based on the distance between them and the ideal solution [259]. Deng *et al.* have postulated that mathematically, the relative similarity (closeness) between each alternative and the ideal solution is better represented by the magnitude of the alternatives and the degree of conflict between them [SBA-1]. This led to the development of a novel MADM methodology called Similarity Based approach that is considered as an improvement over TOPSIS approach [262, 263 & 264].

2.4.17B Theory

This section is dedicated to the theory behind the Similarity-based approach, which includes the concept of Degree of Conflict and Degree of Similarity between the alternatives.

Degree of conflict

Decision making in the present scenario has become a complex and rigorous task that involves numerous factors (criteria) to be considered for making a shrewd judgment. Dealing with such decision problems inevitably involves multiple criteria which are often conflicting by nature. The performance of the alternatives in all evaluations concerning criteria are in complete concordance, is a rare or unrealistic outcome. Therefore, discordance (or conflict) arising between the alternatives is a common phenomenon, and its measure plays a very crucial in selecting an optimal solution [260].

There are different ways to measure the conflict between two alternatives in multi-attribute decision problems [260]. Among them, the concept of the alternative gradient to represent the conflict of decision is the mostly common one. Using this method, a conflict index between two alternatives is calculated to show the degree of conflict between the alternatives [260].

Derivation of degree of conflict [260]

This section is referred from the origin paper that proposed the Similarity based-approach by Deng *et al.* (please refer [260] for more details).

Assuming that A_i and A_j are the two alternatives considered in a given MADM problem, these two alternatives can be considered as two vectors in the m- dimensional real space. The angle between A_i and A_j in the m-dimensional real space is a good measure of conflict between them. As shown in Figure 17 below, there is no conflict between A_i and A_j if $\theta_{ij} = 0$, the conflict is possible if $\theta_{ij} \neq 0$, i.e. $\theta_{ij} \in (0, \pi/2)$. This is so because when $\theta_{ij} = 0$ the gradients of both the alternatives A_i and A_j are simultaneously in the same increasing direction and there is no conflict between them. The situation of conflict occurs when $\theta_{ij} \neq 0$, i.e. when the gradients of A_i and A_j are not coincident.

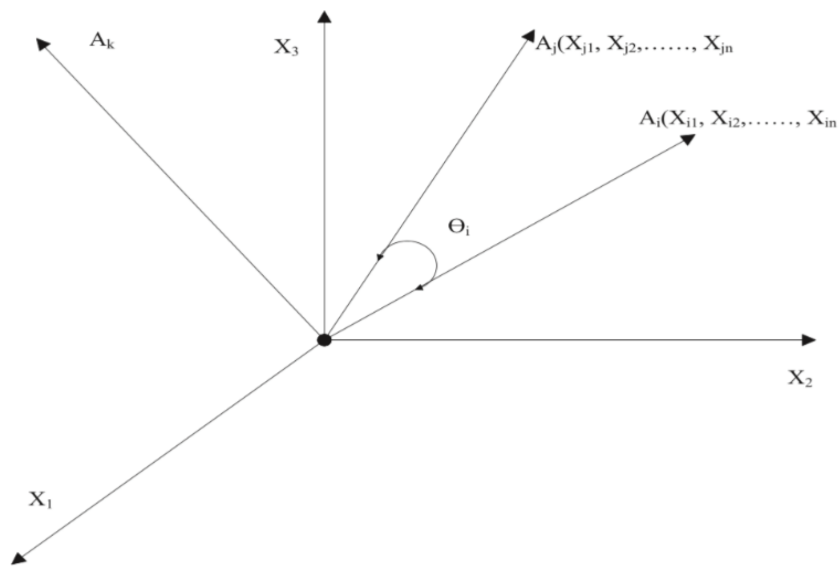


Figure 17: Degree of conflict between alternatives by gradients.

The degree of conflict between alternatives A_i and A_j is determined by

$$\cos\theta_{ij} = \frac{\sum_{k=1}^{M} x_{ik}\, x_{jk}}{\left[\left(\sum_{k=1}^{M} x_{ik}^2\right)\left(\sum_{k=1}^{M} x_{jk}^2\right)\right]^{1/2}} \quad \ldots\ldots\ldots\ldots (129)$$

Where θ_{ij} is the angle between the gradients of the two alternatives, and $(x_{i1}, x_{i2}, x_{i3}, \ldots\ldots, x_{in})$ and $(x_{j1}, x_{j2}, x_{j3}, \ldots\ldots, x_{jn})$ are the gradients of two alternatives A_i and A_j respectively.

The conflict index equals to one characterized by $\theta_{ij} = 0$ as the corresponding gradient vectors lie in the same direction of improvement. Similarly, the conflict index is zero characterized by θ_{ij} = *π/2* which indicates that their gradient vectors have the perpendicular relationship between each other.

Based on the degree of the conflict between the alternatives, the degree of similarity between the two alternatives can be calculated. The degree of similarity between alternative A_i and A_j is denoted by S_{ij}, it measures the relative similarity (closeness) alternative A_i and A_j, given as:

$$S_{ij} = \frac{\left(\sum_{k=1}^{M} x_{ik}^2\right)^{1/2} \cos\theta_{ij}}{\left(\sum_{k=1}^{M} x_{jk}^2\right)^{1/2}} \quad \ldots\ldots\ldots\ldots (130)$$

where θ_{ij} is the angle between alternative A_i and alternative A_j as represented in the derivation of the degree of conflict above. The larger the S_{ij} is, the higher the degree of similarity between alternative A_i and A_j.

2.4.17C Deng's Similarity-Based Approach

Similarity-Based approach (SBA) is a distance method similar to TOPSIS. It is proposed by Deng *et al.* in the year 2007. It employs the concept ideal solution in such a manner that the most preferred alternative should have the highest degree of similarity concerning it. It proposes evaluating the alternatives using the conflict gradient between two alternatives to show the degree of conflict between the alternatives [263 & 265].

Deng presented SBA as a better exponent of the concept of similarity with a sturdy distance measuring formulation [268] over the TOPSIS methodology, but it has become synonymous as a modification of TOPSIS.

In the next section, the Similarity- Based approach's algorithm is presented in a detailed manner.

2.4.17D Modus operandi [258-262]

Step-1: Normalization of the elements of Decision Matrix (DM)

Normalization is performed to transform all the criteria into dimensionless entities [42] and also to ensure all the criteria are in maximizing form [258]. Vector normalization technique opts for this purpose.

For maximizing attributes:

$$N_{ij}^{(V)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \text{………..} \ (131)$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

For minimizing attributes:

$$N_{ij}^{(V)} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \text{………..} \ (132)$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

Where,

"$N_{ij}^{(V)}$" is the vector-normalized rating of i^{th} alternative and j^{th} attribute.

"x_{ij}" is the performance rating of i^{th} alternative and j^{th} attribute in the DM.

Step-2: Weighted Normalization of the ratings

Weights are an important aspect of the MADM ideology. They translate the objectives of the decision problem to the performance evolution scheme applied by a MADM approach. In this step, weights are multiplied to normalized elements of the DM. Mathematically, this integration is represented as:

$$r_{ij} = W_j \times N_{ij}^{(V)} \quad \text{………… (133)}$$

, for *i= 1,2,3, …, M;* & *j= 1,2,3, …., N.*

Where,

"W_j" is the weight of jth attribute. ($\sum_{j=1}^{N} W_j = 1$).

"r_{ij}" is the weighted-normalized rating of i^{th} alternative and j^{th} attribute.

Step-3: Determination of the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

All the best attribute values that benefit the objectives of the decision problem are considered as PIS, and similarly, all the attribute values that weaken the performance of the alternatives concerning the objectives of the problem are considered as NIS.

$$A^{+}{}_{j} = \{r_1^{+}, r_2^{+}, \dots \dots \dots, r_M^{+}\} = \left\{\max_{j} r_{ij} \,|\forall\, j = 1,2,3, \dots, N.\right\} \quad \text{………… (134)}$$

$$A^{-}{}_{j} = \{r_1^{-}, r_2^{-}, \dots \dots \dots, r_M^{-}\} = \left\{\min_{j} r_{ij} \,|\forall\, j = 1,2,3, \dots, N.\right\} \quad \text{………… (135)}$$

Where,

"A^{+}" is denotes PIS and "A^{-}" denotes NIS.

Step-4: Calculate the Degree of Conflict between each alternative concerning PIS and NIS.

The degree of conflict is measured for each alternative concerning PIS and NIS and is calculated using the following formulas:

$$\cos\theta_{i+} = \frac{\sum_{j=1}^{N} r_{ij} \cdot A^{+}{}_{j}}{(\sum_{j=1}^{N} r_{ij}{}^{2} . \sum_{j=1}^{N} A^{+}{}_{j}{}^{2})^{1/2}} \quad \text{………… (136)}$$

Moreover,

$$\cos\theta_{i-} = \frac{\sum_{j=1}^{N} r_{ij} \cdot A^{-}{}_{j}}{(\sum_{j=1}^{N} r_{ij}{}^{2} . \sum_{j=1}^{N} A^{-}{}_{j}{}^{2})^{1/2}} \quad \text{………… (137)}$$

Where,

Positive degree of conflict is denoted by "$\cos\theta_{i+}$" which represents the distance measure between each alternative and PIS. And, Negative degree of conflict is denoted by "$\cos\theta_{i-}$" which represents the distance measure between each alternative and NIS.

Step-5: Compute the degree of similarity between each alternative and the PIS and the NIS.

The main constituent of the degree of similarity is the degree of conflict. The degree of similarity between each alternative and the positive ideal solution (PIS) and the negative ideal solution (NIS) can be determined by:

$$S_i^{+} = \frac{(\sum_{j=1}^{N} r_{ij}{}^{2})^{1/2} \cos\theta_{i+}}{(\sum_{j=1}^{N} A^{+}{}_{j}{}^{2})^{1/2}} \quad \text{………… (138)}$$

Moreover,

$$S_i^- = \frac{(\sum_{j=1}^{N} r_{ij}{}^2)^{1/2} \cos\theta_{i-}}{(\sum_{j=1}^{N} A^-{}_j{}^2)^{1/2}} \quad \text{………… (139)}$$

Step-6: Determine the Over-all performance index

The overall performance index for each alternative is calculated using the degree of similarity in the following way:

$$P_i = \frac{S_i^+}{S_i^+ + S_i^-} \quad \text{………… (140)}$$

, *i= 1, 2, ……, M.*

Step-7: Ranking the alternatives

Ranking of the alternative is done in the descending order, where the alternative with largest index is ranked at the top, and the alternative with the lowest index is at the bottom of the list.

2.4.17E Applications

Even though the Similarity-Based approach (SBA) is quite recently developed as compared to most other MADM methods, it has versatile but limited applications so far. Major portion of the applications are associated with the manufacturing sector, and particularly in the assessment of manufacturing processes and tool geometry. Abhishek *et al.,* and his team utilized the SBA for evaluating drilling parameters and drilling responses in case of providing optimal combination for dealing with the drilling of composites (for more details, refer [259]). Later, they also applied the similarity-based approach to assess drilling parameters for GFRP (Glass fiber Reinforced Plastics) composites. Moreover, Senthil kumar *et al.* with his team evaluated process parameters to optimize drilling operation for CFRP/Ti6Al4V stacks using a similarity-based approach (for more information, refer [263]).

SBA was also successfully applied as a decision aide in the software industry, defense sector, and business management. Singh *et al.,* and Tyagi *et al.*, published a paper on evaluation strategy of multiple services for reliability estimation of Service-oriented architecture (SOA) (for more details, refer [264]). A decision problem related to business management often involves qualitative responses for evaluations. In the year 2014, Moradi *et al.* with his disciple proposed a decision-aiding framework for the economic evaluation of companies based on Corporate Governance (CG) Measures combining Fuzzy Analytic Hierarchy Process (AHP) and Similarity-Based Approach (for more information, refer [263]). These are some of the applications based on articles and papers published.

2.4.17F Extension of Similarity-Based Approach

In the year 2013, Safari *et al.* with his team proposed an extension (or improvement) of Deng's Similarity-based approach, called Modified Similarity method. They pointed out an error in the calculation of negative similarity measure and proposed an altered equation that involves derivation of the negative degree of conflict (for more details, refer [267]). Safari *et al.* validated the method with numerical examples and compared the results of both Deng's method and his own proposed approach to establish it as an improvement over the older approach.

This method found some applications in the manufacturing sector and other areas. Chakraborty *et al.* applied the modified similarity-based method for assessing cutting fluids on multiple parameters for selecting an optical fluid (for more details, refer [268]).

2.4.18 PROMETHEE method

2.4.18A Outranking theory

The concept of outranking in the decision theory was established in the early 1960s [260]. The outranking approach was developed and flourished in France and later spread all across Europe and other continents. The birth of this idea can be contributed to the failure of the utility (value) function theory [260 & 264]. The outline of this theory can be stated as, in a multiple criteria decision-making (MCDM) problem, one alternative is said to outrank another, if it outperforms the other on enough criteria (or attributes) of sufficient importance (or criteria weights) and is not outperformed by the other option in the sense of recording a significantly inferior performance on any one criterion [OR-4]. All options are then assessed in terms of the extent to which they exhibit sufficient outranking concerning the full set of options being considered as measured against a pair of threshold parameters [264].

One striking feature of outranking approach is that under certain conditions, it is plausible for two alternatives to be classified as ‘incomparable’ (or ‘difficult to compare’). Incomparability of two alternatives is not the same as indifference between two options and might, for example, be associated with missing information at the time the assessment is made. This is not an unlikely occurrence in many decision-making exercises [264].

2.4.18B Outranking relations

In order to understand the concept of outranking, it is important to know the nomenclature and outranking relations that define the approach [260].

Let “A” be the set of alternatives considered for a decision problem and “C” be the family of ‘n’ criteria (or attributes) defined. The assessment of the alternative is done concerning the actions (or performance) performed by each alternative on each criterion. Let $g_j(a)$ is the j^{th} performance of ‘a’ and it is a real number (even if it reflects a qualitative assessment). Then, the outranking relations are:

a) $\forall\, a' \in A$ and $a \in A$, $g_j(a') \geq g_j(a) \Rightarrow a'$ is at least as good as ‘a’ for j^{th} criterion.

b) $g_j(a') = g_j(a) \Rightarrow a'$ is indifferent to ‘a’.

c) $g_j(a') > g_j(a) \Rightarrow a'$ is preferred to ‘a’ and if the $g_j(a') - g_j(a)$ is sufficiently significant, then a' is strictly preferred to ‘a’.

These relations govern the outranking approach and many novel methods in the recent times have been developed using the outranking concept. Some examples of such methods are PROMETHEE: family of methods which includes PROMETHMEE-I, PROMETHEE-II, PROMETHEE-III, PROMETHEE-IV, PROMETHEE-V and PROMETHEE-VI etc., EXPROM-II (Extension of PROMETHEE-II), Superiority and Inferiority Ranking (SIR) method etc. In the coming sections, some of the above-mentioned methodologies are demonstrated with their procedure and applications for better understanding of Outranking concept and its evolution as the time passed.

2.4.18C Introduction

PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations), which was developed by Brans *et al.* in the year 1982 and further extended by Vincke *et al.* and Brans *et al.* himself in the year 1985 [260], is a family of methods based on the concept of outranking relations [260, 261, 262 & 265]. PROMETHEE methodology has been introduced to present a better case of outranking approach and improve on the limitations of the ELECTRE method (developed by Roy *et al.* in the mid 1960s).

The major drawback of ELECTRE is that, when comparing two alternatives while considering a criterion, even though one alternative does not dominate the other quantitatively, there is a possibility that the decision-maker may regard it as a better alternative. Therefore, ELECTRE method ignores the quantitative difference between the alternatives to establish the dominance. To avoid this, PROMETHEE proposes a preference function, representing the intensity of preference of an alternative concerning another for a criterion. Some test results exhibit that PROMETHEE is more stable than ELECTRE-III method [260, 262, 265 & 267].

2.4.18D Preference function

The PROMETHEE methods do not allocate an absolute intrinsic utility (utility function) to each alternative, neither globally nor on individual criteria like in the SAW method or TOPSIS method. The preference structure of PROMETHEE methods is based on pairwise comparisons. In this case, the deviation between the evaluations of two alternatives on a particular criterion is considered. For small deviation, the decision maker would allocate a small preference to the best alternative and even possibly no preference if the decision maker considers that this deviation is negligible. The larger the deviation, the larger the preference. These preferences are real numbers varying between 0 and 1. This means that for each criterion, the decision maker considers the following preference function [261]

$$P_j(a,b) = F_j\left[d_j(a,b)\right] \qquad \forall\, a,b \in A \quad \ldots\ldots\ldots\ldots (141)$$

Where,

$P_j(a, b)$ is the preference function between two alternatives “a” and “b”,

$d_j(a, b)$ is the amplitude of deviation between evaluations of the two alternatives with respect to jth criterion (or attribute) [261] i.e.

$$d_j(a, b) = g_j(a) - g_j(b) \quad \text{………..} \ (142)$$

Moreover, “F_j” is the preference function.

There are six types of preference functions proposed by Brans *et al.*, to select an optimal function for each attribute. There are:

Type-1: Usual Criterion

$$P(d) = \begin{cases} 0, & d \leq 1 \\ 1, & d > 1 \end{cases} \quad \text{………..} \ (143)$$

Type-2: U-shape Criterion

$$P(d) = \begin{cases} 0, & d \leq q \\ 1, & d > q \end{cases} \quad \text{………..} \ (144)$$

Where,

“q” is a threshold of indifference, which is the largest deviation which can be considered as negligible by the decision maker (DM) [262 & 263].

Type-3: V-shape Criterion

$$P(d) = \begin{cases} 0, & d \leq 0 \\ \frac{d}{p}, & 0 \leq d \leq p \\ 1, & d > p \end{cases} \qquad \text{………… (145)}$$

Where,

"p" is a threshold of strict preference, which is the smallest deviation which can be considered as sufficient to generate a full preference [262 & 263].

Type-4: Level Criterion

$$P(d) = \begin{cases} 0, & d \leq q \\ \frac{1}{2}, & q < d \leq p \\ 1, & d > p \end{cases} \qquad \text{………… (146)}$$

Type-5: V-shape with indifference criterion

$$P(d) = \begin{cases} 0, & d \leq q \\ \frac{d-q}{p-q}, & q < d \leq p \\ 1, & d > p \end{cases} \qquad \text{………… (147)}$$

Type-6: Gaussian Criterion

$$P(d) = \begin{cases} 0, & d \leq 0 \\ \left(1 - e^{-\frac{d^2}{2s^2}}\right), & d > 0 \end{cases} \qquad \text{………… (148)}$$

Where,

"s" is a parameter that must be fixed by the DM. It has an intermediate value between q and p.

All the parameters mentioned above allows the DM to express his/her preference for each attribute by fixing their values.

The family of PROMETHEE consists of PROMETHMEE-I(partial ranking method) and PROMETHEE-II (complete ranking method), proposed for the first time in 1982 Brans *et al.*; PROMETHEE III (ranking based on intervals) and PROMETHEE IV (continuous case) methods were introduced by Brans *et al.* and Mareschal *et al.*; in 1992 and 1994 respectively, they further suggested two extensions of PROMETHEE method, i.e., PROMETHEE V (MCDM including segmentation constraints) and PROMETHEE VI (representation of the human brain) [262]. The latest addition to the PROMETHEE is PROMETHEE-GAIA (Geometrical Analysis for Interactive Aid).

In this literature review, PROMETHEE I and II are elucidated briefly.

2.4.18E PROMETHEE I and II

In PROMETHEE I, partial ranking is obtained by calculating the positive and the negative outranking flows. The positive outranking flow represents how each alternative dominates the rest of the alternatives, and the negative outranking flow represents how each alternative is dominated by the other alternatives. Both the flows do not usually convey the same set of rankings [264]. In the situations, where the decision-maker (DM) desires to know these individual sets of ranks to understand the precise behavior of each alternative concerning one another, PROMETHEE-I stands strong. However, the decision makers usually want to have complete rankings at their disposal to make a constructive decision. For such cases, PROMETHEE-II is employed. PROMETHEE method for the most of its operation follows the same procedural steps as the

PROMETHEE-I, but unlike it, PROMETHEE-II formulates an aggregation step to combine the partial rankings (positive and negative outranking flows) into a single complete ranking. The complete rankings obtained from the PROMETHEE-II provides a complete picture to the DM to analyze how each alternative dominates and gets dominated and its over-all preference index.

In the next section, the step-by-step approach of the PROMETHEE-II to the MADM model is presented.

2.4.18F Methodology [260, 261 & 263]

Step-1: Determination of the deviation between each alternative by pair-wise comparisons

PROMETHEE-2 method begins with the pair-wise comparison of the alternatives concerning their attributes. Let '$g_j(a)$' be the performance rating of alternative 'a' corresponding to criterion 'j' and '$.\,g_j(b)$' be the same for alternative 'b' corresponding to same criterion 'j', then deviation is calculated as the difference between '$g_{j.}(a)$' and '$g_j(b)$'. For a better understanding it is represented mathematically as:

$$d_j(a,b) = \; g_j(a) - g_j(b) \quad \text{………… (149)}$$

Where,

"$d_j(a,b)$" denotes the difference between the evaluations of a and b for each criterion.

Step-2: Application of the Preference function

For each criterion, the preference function translates the difference between the evaluations obtained by pair-wise comparison of two alternatives into a preference degree ranging from zero to one. In order to facilitate the selection of a specific preference function, authors J.P. Brans and P. Vincke proposed six basic types: (1) usual criterion, (2) U-shape criterion, (3) V-shape criterion, (4) level criterion, (5) V-shape with indifference criterion and (6) Gaussian criterion [261]. This feature allows the decision-maker to customize their preferences for each criterion.

Mathematical representation:

$$P_j(a,b) = F_j[d_j(a,b)] \quad \text{...........} \ (150)$$

, for *j=1, 2, ..., N.*

Where,

"$P_j(a,b)$" represents the preference of the alternative 'a' with regard to alternative 'b' on each criterion, "F_j" is a non-decreasing function of the observed deviation "d_j".

Step-3: Calculation of the global preference index

Global preference index is defined as the weighted sum of "$P_j(a,b)$" for each criterion and its important ratings (or weights). It is represented as:

$$\pi(a,b) = \sum_{j=1}^{N} P_j(a,b) \, . W_j \quad \text{...........} \ (151)$$

$, \forall \ a, b \in A.$

Where,

"$\pi(a,b)$" denotes Global preference index. And,

"W_j" is the weight of j^{th} attribute. ($\sum_{j=1}^{N} W_j = 1$).

Step-4: Compute the Positive and Negative outranking flows for each alternative

The positive outranking flow of an alternative shows its preference strength, whereas the negative outranking flow indicates the short falling of that alternative as compared with other alternatives in the list. These are determined using the following formulas:

$$\phi^+(a) = \frac{1}{M-1} \sum_{x \in A} \pi(a, x) \quad \text{……….. (152)}$$

Moreover,

$$\phi^-(a) = \frac{1}{M-1} \sum_{x \in A} \pi(x, a) \quad \text{……….. (153)}$$

Where,

"M" indicates the total number of alternatives.

"$\phi^+(a)$" denotes positive outranking flow for each alternative and "$\phi^-(a)$" represents the negative outranking flow.

Step-5: Determination of the net outranking flow

The net outranking flow for each alternative gives the complete ranking. It is the simple difference between the positive and negative outranking flow of each alternative.

$$\phi(a) = \phi^+(a) - \phi^-(a) \quad \text{……….. (154)}$$

Where,

"$\phi(a)$" denotes net outranking flow for an alternative.

Step-6: Ranking the alternatives

Alternatives are ranked in Descending order with an alternative with higher net outranking flow placed at the top of the ranking, and consequently, the alternatives with lower net outranking flow descend at the bottom.

2.4.18G Areas of application

This is a very limited survey on the applications of PROMETHEE, as it includes twenty published works based on articles and research papers taken from international journals, that exercise this methodology (to be precise 260 and 261) as a decision aide to solve different MADM problems. Even though limited in number these are explicitly picked, such that they up to some extent represent the wide range of applications of the PROMETHEE. The major areas of applications, as indicated from this review are:

a) Mechanical design and material selection
b) Water resource management
c) Manufacturing and logistics
d) Business Management

Some of the positive characteristics that influenced a large number of applications can be summarized as:

- The simplicity of the approach.
- Easy to understand.
- Assign more weight to the decision-makers (DM) preference by introducing the concept of preference function.
- Flexible to incorporate quantitative and qualitative criterion responses with discrete alternatives, etc.

a) Mechanical design and material selection

Mechanical design and material selection can be regarded as a single area, because both are interdependent and often cannot be dealt with separately. PROMETHEE method has found an ample number of applications in this field. Some of the well-known application related to this area is listed below:

- Lan *et al.,* and his team have applied PROMETHEE -II for assessing multiple material alternatives for an optimal selection in case of mass-produced non-heat-treatable cylindrical cover material and wing-spar material for a human-powered aircraft (for more information, refer [260]).
- Chakraborty *et al.* has utilized the PROMETHEE-II to evaluate tool steel materials concerning nine criteria and proposed two best choices from the list of ten alternatives (for more details, refer [261]).
- Gul *et al.,* with his team proposed fuzzy PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) method based on trapezoidal fuzzy interval numbers that can be applied to the selection of materials for an automotive instrument panel (for more information, refer [267]

b) Water resource management

Water resource management encompasses the topic such as sustainable water resources planning, water management strategies assessment, and irrigation planning. In this survey, a total of six papers were dedicated to water resource management that captures a various aspect of it. Some of these articles are presented below with a brief description.

- Dragovic *et al.* have conducted multi-criteria decision analysis for a sub-watershed ranking using PROMETHEE-II (for more details, refer [263]).
- Kumar *et al.,* and Raju *et al.,* applied PROMETHEE-II for assessing irrigation plans for prestigious Sri Ram Sagar Irrigation Project in India (for more information, refer [264]).
- Raju *et al.,* and Pillai *et al.,* employed PROMETHEE-2 to select the best reservoir configuration for the case study of Chaliyar river basin, Kerala, India (for more information, refer [265]).

c) Manufacturing, transportation, and logistics

Operations Research is an integral part of industrial engineering, and MADM tools have always found their applications in the manufacturing, transportation and logistics sectors to a high degree, because these sectors constitute a crucial part in an industry. PROMETHEE is one of the most popular MADM tools has been inevitably applied numerous times in these fields. For example, Anand *et al.,* and Kodali *et al.,* applied PROMETHEE to evaluate and select the best lean Manufacturing System (LMS) out of the alternatives (for more details, refer [266]); and Araz *et al.,* conducted supplier evaluation considering supplier’s co-design capabilities and overall performances and also presented potential reasons for differences in performance of supplier groups and laid out performances improvement strategies by applying supplier development programs using PROMETHEE (for more details, refer [272]).

d) Business management

Business management deals with economic activities of a company which includes human resource management, marketing, product management, etc. PROMETHEE has been successfully applied to various sectors of this broad field. Babic *et al.,* and Plazibat *et al.,* proposed a methodology to evaluate enterprises based on business efficiency established by multiple criteria using a combined AHP (analytic hierarchy process) and PROMETHEE-II methods (for more details, refer [274] Following the similar path, Bilsel *et al.,* and his team developed a quality evaluation model for measuring the performance of hospital Web sites utilizing Fuzzy PROMETHEE combined with Fuzzy AHP (for more details, refer [275]). There are many more papers published on the application of this method in the business management area.

Here, ends the literature review of PROMETHEE methodology, for a better account of applications, please refer [277].

2.4.19 EXPROM method

2.4.19A Background

PROMETHEE and ELECTREIII are most known and widely applied techniques belonging to the family of multicriteria outranking methods [EXPROM-0]. They are based on the construction of an outranking relation in the set of actions examined and provide partial preorders, such that some of these actions can remain incomparable. The information included in a partial preorder of alternative solutions, especially about incomparability, is very useful for the decision-making process. The PROMETHEE method is an improvement over ELECTRE III, which answered most of the questions regarding preference establishment by quantitatively expressing the dominance over each pair of alternatives considered. Moreover, it also advances the exploitation of the outranking relations to a step forward, providing a complete preorder of the actions, which excludes incomparability between them [PROMETHEE-1 & EXPROM-0]. Such a preorder constitutes a more concrete, but also a less realistic tool for the decision maker [264]

PROMETHEE develops both partial and complete preorders expressing the preferences using an ordinal scale (solutions are rank-ordered), so in the case where two actions are neither equal nor incomparable, we cannot quantitatively express the preference between them [264]. Consequently, a quantitative measure of the global preference of the decision maker is not determined. This constitutes the main drawback of the PROMETHEE methodology.

In the year 1989, Diakoulaki *et al.,* and Koumoutsos *et al.,* of national technical university of Athens, proposed an extension of the PROMETHEE method superseding these limitations by ranking actions in a cardinal scale, a process that makes apparent the intensity of preference [264 & 265]

2.4.19B EXPROM

The abbreviation of EXPROM is Extension of PROMETHEE. It follows the concept of outranking relations like PROMETHEE, but with a little twist, it incorporates the idea of ideal and anti-ideal solutions to determine the relative performance of alternatives concerning these two extreme solutions [264]. This results in the definition of two preference indices, namely: Strict preference index and Weak preference index. The weak preference index is based on the aggregated preference function taking into account the criteria weights as determined in the PROMETHEE II method. The second one is a strict preference index based on the notion of ideal and anti-ideal solutions. Ideal and anti-ideal solutions are directly derived from the decision matrix, and it reflects the extreme limits for each criterion. A total preference index is then computed by adding the strict and the weak preference indices, which gives an accurate measure of the intensity of preference of one alternative over the other considering all the criteria [264-266].

The step-by-step procedure of EXPROM is presented below: [264-266]

Step-1: Construction of Decision matrix [DM]

Formation of the Decision Matrix is the initial step in any Multiple Attribute Decision Making (MADM) approach which includes a selection of the alternatives, short listing the criteria and expressing the performance of each alternative concerning each criterion quantitatively.

Step-2: Allocation of importance ratings (weights) to each attribute

Allotting weight to each criterion is a high priority task, as the objectives of the decision problem are translated via importance ratings that the decision maker assign. There are three ways to assign the weights,

1) Subjective
2) Objective
3) Combination of both,

Once the Decision matrix and weights are assigned, the actual EXPROM procedure begins.

Step-3: Pair-wise comparison of alternatives for determination of deviation between them

This method begins with the pair-wise comparison of the alternatives concerning their attributes. Let '$g_j(a)$' be the performance rating of alternative 'a' corresponding to criterion 'j' and '$.g_j(b)$' be the same for alternative 'b' corresponding to same criterion 'j', then deviation is calculated as the difference between '$g_{j.}(a)$' and '$g_j(b)$'. For a better understanding it is represented mathematically as:

$$d_j(a,b) = g_j(a) - g_j(b)$$

Where,

"$d_j(a,b)$" denotes the difference between the evaluations of a and b for each criterion.

Step-4: Application of the Preference function

For each criterion, the preference function translates the difference between the evaluations obtained by pair-wise comparison of two alternatives into a preference degree ranging from zero to one. In order to facilitate the selection of a specific preference function, authors J.P. Brans and P. Vincke proposed six basic types: (1) usual criterion, (2) U-shape criterion, (3) V-shape criterion, (4) level criterion, (5) V-shape with indifference criterion and (6) Gaussian criterion [265]. This feature allows the decision-maker to customize their preferences for each criterion.

Mathematical representation:

$$P_j(a,b) = F_j[d_j(a,b)] \quad , \text{for } j=1,\ 2,\ \ldots\ldots,\ N.$$

Where,

"$P_j(a,b)$" represents the preference of the alternative 'a' with regard to alternative 'b' on each criterion, "F_j" is a non-decreasing function of the observed deviation "d_j".

Step-5: Calculation of Weak preference index

The weak preference index is established by multiplying the preference function with the particular criterion weight. It is achieved using the following formula:

$$WP(a,b) = \frac{\left[\sum_{j=1}^{N} W_j \times P_j(a,b)\right]}{\sum_{j=1}^{N} W_j} \quad \text{...........} \ (155)$$

$$, \forall \ a, b \in A$$

Step-6: Computation of the strict preference function

The strict preference function is based on the comparison of the difference values (dm_j) with the range of values as defined by the evaluation of the whole set of alternatives for a criterion.

$$SP_j(a,b) = \frac{\left[\max\left(0, d_j - L_j\right)\right]}{\left[dm_j - L_j\right]} \quad \text{...........} \ (156)$$

Where,

L_j= limit of preference (0 for usual criterion preference function, and indifference values for other five preference functions)

dm_j= difference between ideal and anti-ideal values of j^{th} criterion.

Step-7: Calculation of Strict preference index

Strict preference index is the weighted average of strict preference functions and is mathematically represented as:

$$SP(a,b) = \frac{\left[\sum_{j=1}^{N} W_j \times SP_j(a,b)\right]}{\sum_{j=1}^{N} W_j} \quad \text{…………} (157)$$

Step-8: Computation of Total preference index

It is defined as the summation of strict and weak preference indices to gives a measure of the intensity of preference of one alternative over the other, for all criteria [266]. It is calculated by:

$$TP(a,b) = Min[1, WP(a,b) + SP(a,b)] \quad \text{…………} (158)$$

Step-9: Calculation of the Positive (Leaving) and Negative (entering) flows for each alternative

The positive outranking flow of an alternative shows its preference intensity, whereas the negative outranking flow indicates the short falling of that alternative as compared with other alternatives in the list. These are determined using the following formulas:

Leaving flow:

$$\phi^{+}(a) = \frac{1}{M-1} \sum_{a,b \in A} TP(a,b) \quad \text{…………} (159)$$

Entering flow:

$$\phi^{-}(a) = \frac{1}{M-1} \sum_{a,b \in A} TP(a,b) \quad \text{…………} (160)$$

Where,

“M” indicates the total number of alternatives.

Step-5: Determination of the net outranking flow

The net outranking flow for each alternative gives the complete ranking; it is the simple difference between leaving and entering flows of each alternative.

$$\phi(a) = \phi^{+}(a) - \phi^{-}(a) \quad \ldots\ldots\ldots\ldots (161)$$

Where,

"$\phi(a)$" denotes net outranking flow for an alternative.

Step-6: Ranking the alternatives

Alternatives are ranked in descending order with an alternative with higher net outranking flow placed at the top of the ranking, and consequently, the alternatives with lower net outranking flow descend at the bottom.

2.4.19C Implementation of EXPROM

EXPROM has been applied to numerous decision problems covering almost all the sectors of industry. However, there are some areas that implemented and adopted this approach successfully more than the rest. One of such areas includes mechanical design, Chakraborty *et al.,* has been masterful in applying MADM approaches to deal with the material selection problem and also conducted many comparative studies to promote the use of MADM approaches to this genre of problems. In 2012, Chakraborty *et al.,* in collaboration with Chatterjee *et al.,* proposed material selection methodology for gears using EXPROM (for more details, refer [264]). Another popular application area is sustainable water resource management, and there have been many papers and article published related to it. Raju *et al.,* and author Duckstein *et al.,* have collaborated to work on many memorable pieces on water conservation and management. In the years 2001, they published an article in an international journal on sustainable water resource planning for an irrigation area in Spain, where they employed EXPROM-II for assessing various irrigation systems (for more information, refer [271]). In the very next year, they assessed multiple sub-irrigation systems for the prestigious Sri Ram Sagar irrigation project using EXPROM-II and PROMETHEE-II methods (for more details, refer [267]). Apart from these technical areas, EXPROM has many applications in management and finance sectors. Gorecka *et al.,* in 2011 proposed an extension of EXPROM II by stochastic dominance (SD) rules to solve the project selection and ordering problem (for more information, refer [270] Later, in the year 2013, She applied the same methodology to assess different countries to identify the optimal market for expanding international company (for more details, refer [271]).

These are just some of the notable mentions to the applications of the EXPROM method.

2.4.20 SIR (Superiority and Inferiority Ranking) method

2.4.20A Introduction

Rebai *et al.,* proposed the theory of fuzzy bags for Multi-Criteria Decision Making (MCDM) in early 1990s. This theory gave birth to the concept of superiority and inferiority scores [268]. These scores were introduced through the comparison between criteria values. Collaborating these concepts with the outranking theory, Xu *et al.,* in the year 2001 proposed Superiority and Inferiority Ranking (SIR) methodology [269].

Xu *et al.,* embraced the idea of PROMETHEE and applied it to this method by adopting the six basic types of criterion preference function as proposed by Brans *et al.*, for the decision-maker to choose from for each attribute [269]. This approach follows the PROMETHEE methodology initially for establishing pair-wise comparison and preference index for each alternative, then the paths get separated thereafter. Due to these reasons, this method can be considered as an extension of PROMETHEE methods [269].

Once the preference index is established, the superiority matrix and the inferiority matrix are constructed to calculate superiority index and inferiority index respectively for each alternative with regard to each criterion [269]. These matrices inherit the concept of superiority and inferiority scores. Some aggregation procedure is then applied to derive the superiority and inferiority flows for each alternative. Different aggregation techniques result in different kinds of flows. Therefore, SIR method is not a single method, but in fact it represents a family of methods [269]. The most widely used aggregation techniques are simple additive weighting (SAW) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). When SAW aggregation is employed, this method coincides with the second step of PROMETHEE method i.e., the derived superiority and inferiority flows exactly matches the positive preference flow and negative preference flows respectively as defined by Brans *et al.,* [268-269]. However, there are plenty of choices available here, for example when TOPSIS aggregation is applied the relative distance between the both the flows are calculated to derive partial rankings [268]. So, the choice is left for the decision-maker to select appropriate aggregation method for obtaining partial or complete rankings of the alternatives.

The figure below, is a flow chart of the SIR method presenting the basic step-by-step procedure,

Where,

‘a_i’ and ‘a_k’ are two alternatives.

$g_j(a_i)$ and $g_j(a_k)$ are the performance ratings of alternative ‘a’ and ‘b’ respectively with regards to jth criterion.

f_j is the preference function for j^{th} criterion. And,

$P_j(a_i, a_k)$ is the preference intensity of ‘a_i’ with respect to ‘a_k’.

The aggregation procedure is applied to derive the superiority and inferiority flow and later to construct partial or complete rankings of the alternatives.

Figure 18 displays the modus operandi of the SIR method using a flow chart.

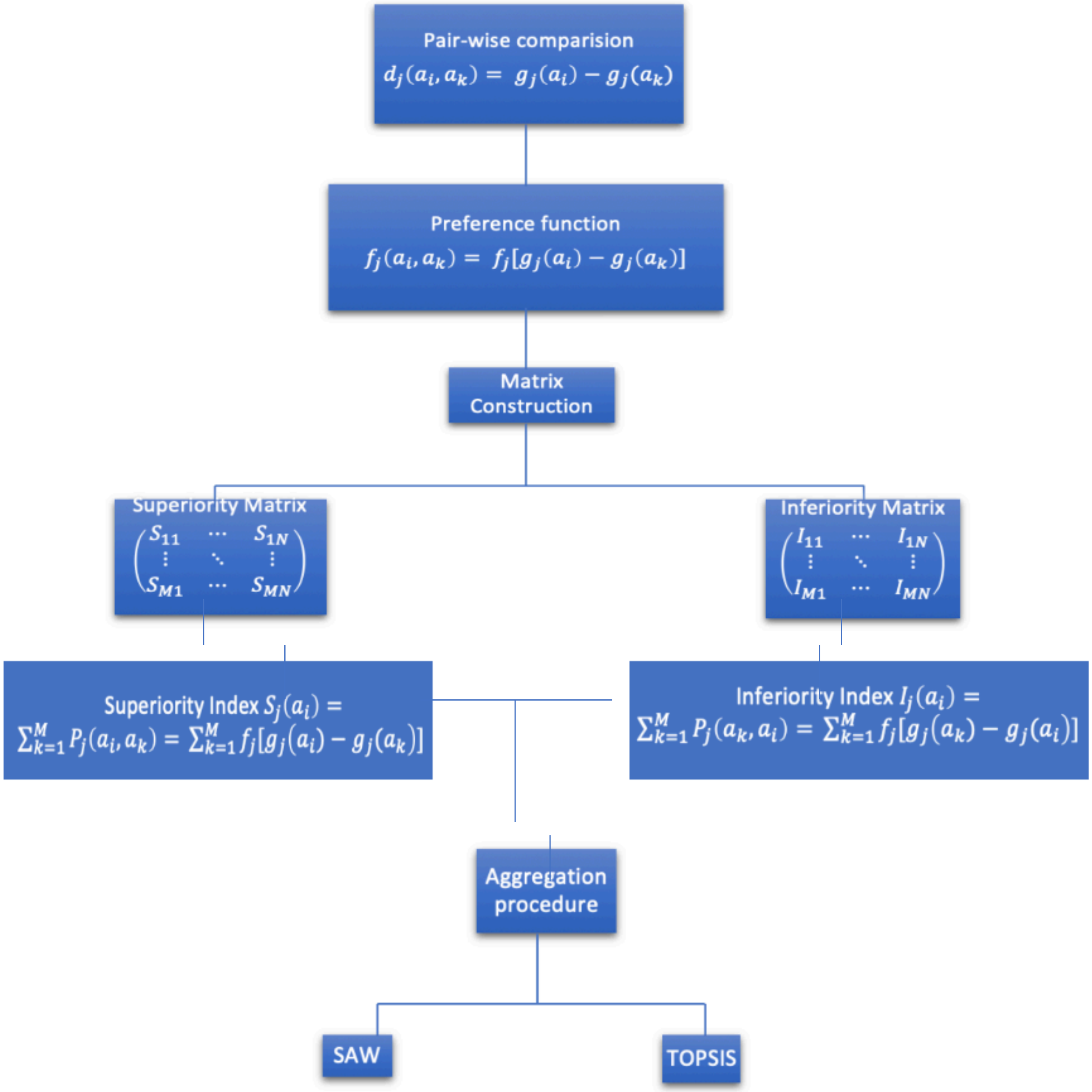


Figure 18: Flowchart of SIR method.

2.4.20B Aggregation procedure in SIR method

The aggregation procedure is applied to achieve two purposes,

1) To derive the superiority and inferiority flows.

2) To calculate complete rankings.

Once the superiority matrix and inferiority matrix are constructed, the next step is applying one of the standard MCDM aggregation procedures [268-269]. As mentioned above, the aggregation procedures are employed to aggregate the superiority and inferiority indices into two types of global preference indices: the superiority flow and the inferiority flow, which represent the global intensity of superiority and inferiority of each alternative [268 & 269].

Let ‘V’ be the aggregation function, then for each alternative ‘a_i’, its superiority flow and inferiority flow are defined as [268]:

Superiority Flow:

$$\varphi^{>}(a_i) = V\left[S_1(a_i), \dots \dots \dots, S_j(a_i), \dots \dots \dots, S_N(a_i)\right] \quad \dots\dots\dots\dots (162)$$

And,

Inferiority Flow:

$$\varphi^{<}(a_i) = V\left[I_1(a_i), \dots \dots \dots, I_j(a_i), \dots \dots \dots, I_N(a_i)\right] \quad \dots\dots\dots\dots (163)$$

The higher the $\varphi^{>}$ (S-flow) and the lower the $\varphi^{<}$ (I-flow), the better the alternative. The choice of an appropriate aggregation procedure should be governed by some guidelines by taking some important aspects, such as: the decision-maker’s attitudes towards compensation, trade-off, the global preference structure (e.g., the types of the output information).

The two most common aggregation methods are 1) SAW and 2) TOPSIS. Whenever an aggregation procedure is applied with the name, for example, XXX, then the method is called as SIR·XXX [268].

In the next section, a detailed framework of SIR method with two aggregation procedures, namely, SIR·SAW, and SIR·TOPSIS is presented.

2.4.20C Superiority and Inferiority Ranking (SIR) framework

As all the MADM methods begin with the formation of Decision Matrix (DM) which constitutes alternatives as rows and attributes as columns, and performance ratings allotted to each alternative concerning each criterion, the SIR method approaches no differently. Then, the next step is assigning weights to each criterion which be performed using objective methods, subjective methods, or a combination of both. Once this protocol is completed, the method proceeds in the following manner:

Step-1: Pair-wise comparison

The outranking relations, as proposed by Brans *et al.,* recommends a paired comparison of an alternative to evaluate the pairs and establish the type (preference or indifference) relation between them. As the outranking theory emphasizes on the intensity of the relation between each alternative pair by taking into the account of the amplitude of their difference (in case of cardinal criteria values). Let $g_j(a_i)$ be the performance rating of alternative 'i' corresponding to criterion 'j' and $g_j(a_k)$ be the same for alternative 'k' corresponding to same criterion 'j', then the amplitude of difference between '$g_j(.a_i)$' and '$g_j(a_k)$' is mathematically represented as:

$$d_j(a_i, a_k) = g_j(a_i) - g_j(a_k)$$

Where,

"$d_j(a_i, a_k)$" denotes the difference between the evaluations of '$g_{j.}(a_i)$' and '$g_j(a_k)$' for each criterion.

Step-2: Application of the Preference function

To calculate the preference intensity between the alternative pairs, SIR method employs preference function as described by Brans *et al.,* and Vincke *et al*. They proposed six basic types: (1) usual criterion, (2) U-shape criterion, (3) V-shape criterion, (4) level criterion, (5) V-shape with indifference criterion and (6) Gaussian criterion [270]. This feature allows the decision-maker to customize their preferences for each criterion.

Mathematical representation:

$$P_j(a_i, a_k) = F_j[d_j(a_i, a_k)] \qquad \text{, for } j=1, 2, \ldots\ldots, N.$$

Where,

"$P_j(a_i, a_k)$" represents the preference of the alternative '$a_{i.}$' with regard to alternative 'a_k'. on each criterion, "F_j" is a non-decreasing function of the observed deviation "d_j".

Step-3: Calculation of the Superiority index and the Inferiority index

The superiority index is defined as the weighted sum of "$P_j(a_i, a_k)$" for each criterion and its important ratings (or weights). It is represented as:

$$S_j(a_i) = \sum_{k=1}^{N} P_j(a_i, a_k) \quad \text{………..} (164)$$

$$, \forall\, a, b \in A.$$

The inferiority index is defined as the weighted sum of "$P_j(a_k, a_i)$" for each criterion and its important ratings (or weights). It is represented as:

$$I_j(a_i) = \sum_{k=1}^{N} P_j(a_k, a_i) \quad \text{………..} (165)$$

$$, \forall\, a, b \in A.$$

Where,

$S_j(a_i)$ denotes Superiority index.

$I_j(a_i)$ denotes the Inferiority index.

Step-4: Construction of Superiority Matrix and Inferiority Matrix

The Superiority Matrix constitutes the superiority index of each alternative across all the criteria, and similarly, an inferiority matrix constitutes the inferiority index for each alternative across all the criteria.

Superiority Matrix:

$$S = \begin{bmatrix} S_1(a_1) & \cdots & S_N(a_1) \\ \vdots & \ddots & \vdots \\ S_1(a_M) & \cdots & S_N(a_M) \end{bmatrix}$$

Inferiority Matrix:

$$I = \begin{bmatrix} I_1(a_1) & \cdots & I_N(a_1) \\ \vdots & \ddots & \vdots \\ I_1(a_M) & \cdots & I_N(a_M) \end{bmatrix}$$

Step-5: Application of Aggregation Procedures

a) SIR·SAW:

SAW (i.e., weighted average method) is the best known and most widely used aggregation procedure due to its simplicity in application. If SAW is used as the aggregation method, then the S- flow is given by:

$$\varphi^{>}(a_i) = \sum_{j=1}^{N} W_j \times S_j(a_i) \quad \ldots\ldots\ldots\ldots (166)$$

moreover, the I- flow:

$$\varphi^{<}(a_i) = \sum_{j=1}^{N} W_j \times I_j(a_i) \quad \ldots\ldots\ldots\ldots (167)$$

Where,

"W_j" is the weight of j^{th} attribute. ($\sum_{j=1}^{N} W_j = 1$).

The net outranking flow:

The net outranking flow for each alternative gives the complete ranking. It is the simple difference between the positive and negative outranking flow of each alternative.

$$\phi(a) = \phi^{>}(a_i) - \phi^{<}(a_i) \quad \ldots\ldots\ldots\ldots (168)$$

Where,

"$\phi(a)$" denotes net outranking flow for an alternative.

Ranking the alternatives:

Alternatives are ranked in Descending order with an alternative with higher net outranking flow ($\phi(a)$) placed at the top of the ranking and consequently, the alternatives with lower net outranking flow ($\phi(a)$) descend at the bottom.

b) SIR·TOPSIS:

TOPSIS method works on the principle that the alternative with the shortest distance from the ideal solution is the best alternative. This approach can be used to derive the superiority and inferiority flows.

Derivation of Ideal and Anti-ideal solutions for superiority flow:

Identifying the ideal and anti-ideal solution form superiority matrix is an easy job, the ideal solution will be the highest superiority index for each alternative, and the anti-ideal solution will be the lowest superiority index for each alternative.

Ideal solution:

$$A_s^+ = \left(\max_i S_1(a_i), \dots., \max_i S_N(a_i)\right) = (S_1^+, \dots \dots, S_N^+) \quad \dots\dots\dots\dots (169)$$

Anti-ideal solution:

$$A_s^- = (\min_i S_1(a_i), \ldots., \min_i S_N(a_i)) = (S_1^-, \ldots \ldots, S_N^-) \quad \ldots\ldots\ldots\ldots (170)$$

The Superiority flow:

$$\varphi^>(a_i) = \frac{S^-(a_i)}{S^-(a_i)+S^+(a_i)} \quad \ldots\ldots\ldots\ldots (171)$$

Where,

$$S^-(a_i) = \left\{\sum_{j=1}^{N} \left|W_j(S_j(a_i) - S_j^-)\right|^{\lambda}\right\}^{1/\lambda} \quad \ldots\ldots\ldots\ldots (172)$$

Moreover,

$$S^+(a_i) = \left\{\sum_{j=1}^{N} \left|W_j(S_j(a_i) - S_j^+)\right|^{\lambda}\right\}^{1/\lambda} \quad \ldots\ldots\ldots\ldots (173)$$

Here, the Minkowski distance

$$d^{\lambda}(\alpha, \beta) = \left\{\sum_{j=1}^{N} \left|a_j - b_j\right|^{\lambda}\right\}^{1/\lambda} \qquad (1 \leq \lambda \leq \infty) \quad \ldots\ldots\ldots\ldots (174)$$

Between two vectors $\alpha = (a_1, \ldots \ldots., a_N)$ and $\beta = (b_1, \ldots \ldots., b_N)$ is used.

Similarly, the ideal solution and anti-ideal solution for the inferiority matrix is defined as:

Ideal solution:

$$A_i^+ = (\min_i I_1(a_i), \ldots., \min_i I_N(a_i)) = (I_1^+, \ldots \ldots, I_N^+) \quad \ldots\ldots\ldots\ldots (175)$$

Anti-ideal solution

$$A_I^- = \left(\max_i I_1(a_i), \dots, \max_i I_N(a_i)\right) = (I_1^-, \dots\dots, I_N^-) \quad \dots\dots\dots\dots (176)$$

The Inferiority flow:

$$\varphi^<(a_i) = \frac{I^+(a_i)}{I^+(a_i)+I^-(a_i)} \quad \dots\dots\dots\dots (177)$$

Where,

$$I^+(a_i) = \left\{\sum_{j=1}^N \left|W_j(I_j(a_i) - I_j^+)\right|^\lambda\right\}^{1/\lambda} \quad \dots\dots\dots\dots (178)$$

Moreover,

$$I^-(a_i) = \left\{\sum_{j=1}^N \left|W_j(I_j(a_i) - I_j^-)\right|^\lambda\right\}^{1/\lambda} \quad \dots\dots\dots\dots (179)$$

Special cases of SIR·TOPSIS:

i. When $\lambda = 2$, we will use Euclidean distance to derive the S-flow and the I-flow.
ii. When $\lambda = 1$, we will use block distance.

Ranking the alternatives:

Ranking of the alternatives can be using the Superiority flow ($\varphi^>(a_i)$), Inferiority flow ($\varphi^<(a_i)$) or the relative flow ($\varphi_r(a_i)$).

In case of Superiority flow ($\varphi^{>}(a_i)$), the alternatives are ranked in descending order with alternative with highest superiority flow ($\varphi^{>}(a_i)$) sits at the top of the rankings and alternative with lowest superiority flow ($\varphi^{>}(a_i)$) is at the bottom.

In the case of the Inferiority flow ($\varphi^{<}(a_i)$), the alternatives are ranked in descending order with alternative with highest inferiority flow ($\varphi^{<}(a_i)$) sits at the top of the rankings and alternative with lowest inferiority flow ($\varphi^{<}(a_i)$) is at the bottom.

The relative flow ($\varphi_r(a_i)$):

$$\varphi_r(a_i) = \frac{\varphi^{>}(a_i)}{\varphi^{>}(a_i)+\varphi^{<}(a_i)} \quad \ldots\ldots\ldots\ldots (180)$$

When considering relative flow ($\varphi_r(a_i)$), alternatives are ranked in descending order, where alternative with highest relative flow ($\varphi_r(a_i)$) value prevails as the best solution for the decision problem and vice versa.

2.4.20D Areas of application

The literature review presented in this section is based on twenty papers, including article and research pieces published in reputed international journals. The major areas of application, according to this survey, are:

a) Environmental management
b) Transport and logistics
c) Supply-chain management

a) Environmental management

Environmental management has become a hot topic in the industry nowadays, as the local and global regulations and policies are ensuring that companies and firms commit themselves to the conservation of the environment. Environmental management can be decomposed into sub-categories such as waste management, sustainable energy, water management, and pollution control, etc. Companies and organizations often face difficulties to ensure effective strategy in dealing with decisions related to this category. Tan *et al.,* and her team proposed a novel methodology by extending the classical SIR method to incorporate inexact fuzzy stochastic programming (IFSP) model to address complexities in municipal solid waste management systems and assess waste management planning strategies (for more information, refer [271]). Zhao *et al.,* and her team proposed a novel extension of SIR, namely hesitant fuzzy linguistic prioritized superiority and inferiority ranking method to deal with uncertainties associated with decision-making while evaluating sustainable energy technologies (for more information, refer [271]). These are some of the applications related to this category.

b) Transportation and logistics

With globalization rapidly connecting different parts of the world, the emphasis on robust logistic strategies and transportation planning has increased ten folds in the past decade for efficient operation of an industry. Evaluation of the transportation routes, logistic strategies or logistic provider, etc., is a tough job. SIR method has been applied as a decision aide dealing with logistics related problems. Tavana *et al.,* has developed a hybrid model integrating the analytic network process (ANP) and the intuitionistic fuzzy- grey superiority and inferiority ranking (IFG-SIR) method to assist an industrial production group in selecting a third-party reverse logistics provider (for more details, refer [275]). Similarly, Sharma *et al.,* has employed the SIR method to rank the best recovery alternatives in reverse logistics (for more information, refer [272]).

c) Supply-chain management

Gupta *et al.,* with a team of three, developed an extension of SIR method integrating intuitionistic fuzzy (IF) scores to improve superiority and inferiority flows of the existing SIR method. Then, applied the modified intuitionistic fuzzy SIR approach to a supplier selection problem which usually details with uncertainty in data (for more information, refer [276]). For a similar kind of problem, Chou *et al.,* and Ongkowijoyo *et al.*, collaborated to develops a decision aid for selecting on-site ready-mix concrete (RMC) unloading type in decision making situations involving multiple stakeholders and evaluation criteria using stochastic superiority and inferiority ranking methods. Chou *et al.,* applied Grey relation aggregation procedure to derive superiority and inferiority flow in this decision aide tool (for more details, refer [269]).

2.4.21 OCRA (Operational Competitiveness Rating) method

2.4.21A Origins

A production unit (PU) is a purposeful system that converts resources (inputs) into products and services (outputs). Individuals, groups, departments, plants, etc., can be considered as a PUs. In order to understand and control a PU's operational performance, construction of a meaningful relationship between the inputs consumed and the outputs produced by that PU, which would be the basis of a measurement procedure. This relationship should tie the two-mean performance-related components, cost, and revenue efficiency [275].

There are two prominent approaches to the problem of measuring operational performance. The first approach involves the calculation of ratio measures of input and output quantities. The second approach attempts to construct an underlying relationship between the observed inputs and outputs. If a hypothesized functional form is employed, then it is called Parametric frontier analysis. Moreover, if no particular functional form is used to establish the underlying relation, then the approach is known as Non-parametric analysis [275]. Data Envelopment Analysis (DEA) is one of the popular non-parametric approaches available. Both the approaches suffer from their inherent limitations, but non-parametric analysis approaches received more acceptance.

Parkan *et al.* noticed the wide acceptance of the use non-parametric analysis in the context of consumer and production theories [275] to assess the assumptions and developed a novel method to measure the operation competitiveness of PUs. This method, unlike most other non-parametric approaches, proved to be simple, flexible, and intuitively appealing [275].

It is called Operational Competitiveness Rating (OCRA) method, which is an operation performance measurement (OPM) procedure [278]. Parkan *et al.,* further established the approach with successive articles strengthening the stability and applicability to various related problems such as performance profile of manufacturing type production unit (PU) [275], rating competitiveness of bank personnel (for more details, refer [276]), etc. In the year 1997, Parkan *et al.* published his research work titled "equivalence of operational performance measurements (OPM) and multi-attribute decision making (MADM)" [277], where he proved that MADM and OPM are "essentially the same process" [277]. This opened the OCRA approach to MADM problems and became an essential part of this theory.

2.4.21B A MADM model of Operational Competitiveness Rating (OCRA) method

When applying to a MADM problem, the basic characteristics of the OCRA approach remains intact [273]. A production unit (PU) resembles the alternative, and the competitiveness parameters coincide with multiple and often conflicting criteria of a MADM problem [273]. The operational procedure of the OCRA with its aggregation steps is modified slightly to solve this type of problems. First, the preference ratings concerning non-beneficial (or input) criteria are determined; then in the second step, the preference ratings concerning beneficial (or output) criteria are computed and finally, the overall preference ratings or the global preference ratings are derived for each alternative [277]. The OCRA uses an intuitive method for incorporating the decision maker's preferences about the relative importance (weights) of the criteria. Therefore, the preference ratings of the alternatives in OCRA method reflect the decision maker's preferences for the criteria [279-282].

2.4.21C The operational procedure of OCRA to a MADM problem [284, 285 and 286]

Step-1: Computation of the Preference ratings (I_i) with respective to non-beneficial criteria

In this step, OCRA method is only concerned with the scores that various alternatives receive for the input criteria without considering the scores received for the beneficial (output) criteria. The lower values of the non-beneficial criteria are more preferable. The aggregate performance of i^{th} alternative with respect to all the input criteria is calculated using the following equation:

$$I_i = \sum_{j=G+1}^{N} W_j \frac{\max(x_{ij})-x_{ij}}{\min(x_{ij})} \quad \ldots\ldots\ldots\ldots (181)$$

, for *i= 1, 2, ……, M; & j= G+1, G+2, ……., N.*

Where,

"I_i" is the measure of relative performance of i^{th} alternative,

"x_{ij}" is the performance score of i^{th} alternative with respect to j^{th} criteria (or attribute),

"W_j" is the calibration constant in the above equation, known as relative importance rating or weight of j^{th} criterion ($\sum_{j=1}^{N} W_j = 1$),

j= G+1, G+2, ………N. represents the number of non-beneficial criteria of the problem.

Step-2: Calculation of the linear preference rating for the input criteria

The linear scaling of preference rating is done to assign a zero rating to least preferred an alternative. It is done using the following formula:

$$I_i^* = I_i - \min(I_i) \quad \text{………… (182)}$$

Where,

"I_i^*" represents the aggregate preference rating for i^{th} alternative with respect to the input criteria.

Step-3: Computation of Preference ratings concerning output criteria

The aggregate performance for i^{th} alternative on all the beneficial (or output) criteria is measured using the following expression:

$$O_i = \sum_{j=1}^{G} W_j \frac{x_{ij} - \min(x_{ij})}{\min(x_{ij})} \quad \text{………… (183)}$$

, for *i*= *1, 2, ……, M;* & ***j***= *1, 2, ……., G.*

Where,

"O_i" is the measure of relative performance of i^{th} alternative,

j= 1, 2, ……., G indicates the number of beneficial (or output) criteria.

Step-4: Calculation of the linear preference rating for output criteria.

It is done using the following expression:

$$O_i^* = O_i - \min(O_i) \quad \text{……….. (184)}$$

Step-5: Determination of the Overall (or Global) preference rating for each alternative

The overall preference rating for each alternative is calculated by scaling the sum $(I_i^* + O_i^*)$, so that the least preferable alternative receives a rating of zero.

The overall preference rating ($\overline{P_i}$) is calculated as follows:

$$\overline{P_i} = (I_i^* + O_i^*) - \min(I_i^* + O_i^*) \quad \text{……….. (185)}$$

Step-6: Ranking the alternatives

The alternatives are ranked according to the values of the overall preference rating ($\overline{P_i}$). The alternative with the highest overall preference rating receives the first rank.

2.4.21D Merits of the OCRA approach [284-290]

The OCRA method has seen a rise in its application from the year 1997. This was when the author established the equivalency between the operation performance measurement procedure (OPM) and multi-attribute decision making (MADM) approach. The rise in the applications can be credited to two main reasons:

a) Adaptation to MADM problems
b) The overwhelming merits that OCRA possesses.

In this section, the merits of OCRA are listed.

1. OCRA is simple and easy to apply the method for analyzing different sectors and comparing different decision units (alternatives) [291].
2. OCRA can handle both cardinal and ordinal data forms to evaluate alternatives [291].
3. It inherits the ability to compare and monitor the performance of a decision unit (alternative) over time [291].
4. This method has the advantage of treating alternatives concerning maximization (beneficial) and minimization (non-beneficial) criteria separately [292].
5. Another advantage of the OCRA method is that it is a nonparametric approach, i.e. calculation procedure is not affected by the introduction of any additional parameters such in the case of the WASPAS [292], VIKOR or CODAS (Combinative Distance-based Assessment), etc.
6. One of the main advantages of OCRA method is that it can deal with those MCDM situations when the relative weights of the criteria are dependent on the alternatives, and different weight

distributions are assigned to the criteria for different alternatives as well as some of the criteria do not apply to all the alternatives [282].

2.4.21E Applications to MADM problems

The OCRA method has been applied to numerous decision problems involving multiple alternatives and criteria to evaluate. This literature survey on applications of OCRA is limited and is based on articles and research papers published in reputed international journals only. Twenty such articles and papers are referred to construct this survey.

OCRA, as a MADM approach, has a wide range of applications from material selection to laptop selection. In the material selection domain, it has seen a remarkable number of applications. Chatterjee *et al.,* and Chakraborty *et al.,* collaborated to employ OCRA method to evaluate several alternatives for a gear material problem (for more information, refer [277]). Following their lead, Darji *et al.,* and Rao *et al.,* applied OCRA method to assess various alternatives of pipe material in a sugar industry (for more details, refer [278]). Upping the ante, Gomez *et al.,* showcased the versatility of OCRA method by employing it to two complexly contrasting application, 1) to select an optimal material for designing a multi-tubular packed-bed Fischer-Tropsch reactor (MPBR) (for more information, refer [275]); 2) to select optimal cookware material (for more details, refer [282]).

Apart from material selection domain, OCRA has been applied to finance and business management sector. Stanujkic *et al.,* with a team of four, developed an improved OCRA method based on the use of interval Grey numbers and used it to select the best capital investment project (for more details, refer [285]). Chakraborty *et al.* [285], utilized OCRA method to evaluated various alternatives with respect to multiple attributes to select best facility location for distribution centers (for more information, refer [285]). OCRA method has also found application in manufacture sector and one such application is by Antucheviciene *et al.,* and her team evaluated process parameters for laser cutting operation to propose best conditions using OCRA approach (for more details, refer [288]).

As mentioned at the beginning of this section, this survey is limited, and above-mentioned applications projects a small sample of overall applications.

2.4.22 DIA (Distance to Ideal Alternative) method

2.4.22A Background

The technique for the order of preference by similarity to ideal solution (TOPSIS) is undoubtedly one of the best if not the best multi-attribute decision making (MADM) approaches available. The wide spectrum of applications to various domains ranging from manufacturing to design, business management to environmental management and supply-chain management to simple domestic issues [229], etc., is a proof of its acceptance and popularity.

In September 2008, Tran *et al.,* and Boukhatem *et al.* [279] presented their collaborated work on a comparative study of top MADM approaches namely simple additive weighting (SAW) method, Weighted product (WP) and TOPSIS for interface selection in heterogeneous wireless networks [279]. In their research, they found the limitations of all the three methods mentioned above. They concluded that TOPSIS suffers from "ranking abnormality" problem, whereas SAW and WP provide less accuracy in identifying the alternative ranks [279]. They comprehensively discussed the reasons behind the rank abnormality occurrence in TOPSIS and presented the following explanation:

1. When one of the alternatives is removed from the candidate's list, the calculation of the weighted normalized decision scores of thus transformed decision matrix will change and consequently the best and worst values for each of the attributes to change. TOPSIS calculates the m-dimensional Euclidean distance of attributes from the respective positive ideal and negative ideal values [279]
2. When an alternative is removed, the Euclidean distance calculation for each alternative will be based on the new positive ideal, and new negative ideal values, and this distance changes non-uniformly with the alternatives. Therefore, the relative closeness to the ideal solution based on these new distance values will change non-uniformly and, as a result, the calculation of the preference order (the separation distance from positive and negative ideal solution respectively) can provide a different ranking order than the prior one [279].

To ensure no rank abnormality and also providing a good accuracy in identifying the alternative ranks [279], Tran *et al.,* and Boukhatem *et al.,* presented a novel distance-based MADM approach called Distance to Ideal Alternative (DIA).

2.4.22B Distance to Ideal Alternative (DIA) method

DIA is a distance-based MADM approach similar to TOPSIS. Both the methods invest in the concept of ideal solution and share a common principle that the best alternative has the shortest distance concerning positive ideal and negative ideal solutions. However, both differ in their approach to attain this distance, TOPSIS employs the Euclidean Distance to calculate the distance between each alternative and the corresponding criterion positive ideal and negative ideal solutions; whereas DIA utilizes Manhattan distance to achieve the same. This (Manhattan approach) allows these distances to change uniformly when an alternative is removed out of the list of candidates and therefore avoiding rank abnormalities, which is a major limitation to TOPSIS approach [279& 280]. Moreover, the positive ideal alternative (PIA) which has the minimum distance to the positive ideal attribute and maximum distance to the ideal negative attribute is determined, and the best "actual" alternative has the shortest distance to the PIA instead of the relative closeness to the ideal solution as in TOPSIS [279].

2.4.22C DIA method's operational framework

The first step of any MADM approach is the construction of decision matrix with screened out alternatives as to the rows and short-listed criteria filling the columns. Then, the task is to assign weights to each criterion based on the objectives of the decision problem. The modes to attain weights can be through subjective or objective or combination of both. Once these preliminaries are completed, the actual procedure begins as follows:

Step-1: Construction of Normalized Decision Matrix

Normalization is done to convert dimensional quantity into dimensionless [43] so that further aggregation can be performed on the performance scores.

Vector normalization is utilized for this purpose, and it is expressed as:

For beneficial (maximizing) criteria

$$N_{ij}^{(V)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \text{………… (186)}$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

For non-beneficial (minimizing/cost) criteria:

$$N_{ij}^{(V)} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{M} x_{ij}^2}} \quad \text{………… (187)}$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

Where,

"$N_{ij}^{(V)}$" is the vector-normalized rating of i^{th} alternative and j^{th} attribute.

"x_{ij}" is the performance rating of i^{th} alternative and j^{th} attribute.

Step-2: Construction of the Weighted normalized Decision Matrix

Weighted normalization of the decision matrix is done using the following mathematical expression:

$$r_{ij} = W_j \times N_{ij}^{(V)} \quad \text{………… (188)}$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

Where,

"W_j" is the weight of j^{th} attribute. ($\sum_{j=1}^{N} W_j = 1$).

"r_{ij}" is the weighted-normalized rating of i^{th} alternative and j^{th} attribute.

Step-3: Determination of positive ideal values and negative ideal values for each attribute

The positive ideal attribute (S^+) value of a criterion is the best performance rating, whereas the negative ideal attribute (S^-) value is the worst performance score.

$$S^+ = \{r_1^+, r_2^+, \dots \dots .., r_N^+\} = \left\{\max_j r_{ij} \mid \forall\, j = 1,2,3, \dots, N.\right\} \quad \dots\dots\dots\dots (189)$$

$$S^- = \{r_1^-, r_2^-, \dots \dots .., r_N^-\} = \left\{\min_j r_{ij} \mid \forall\, j = 1,2,3, \dots, N.\right\} \quad \dots\dots\dots\dots (190)$$

Step-4: Calculation of the Manhattan distance to the S^+ and S^-.

Here, the Manhattan distance (M_i) is measured between each alternative's performance with respect to attribute to the "S^+" and "S^-" using the following formula:

$$M_i^+ = \sum_{j=1}^{N} \left| r_{ij} - r_j^+ \right| \quad \dots\dots\dots\dots (191)$$

, *i=1, 2, 3, ……., M.*

$$M_i^- = \sum_{j=1}^{N} \left| r_{ij} - r_j^- \right| \quad \dots\dots\dots\dots (192)$$

, *i=1, 2, 3, ……., M.*

Where,

"M_i^+" is the separation distance between each alternative to S^+.

"M_i^-" is the separation distance between each alternative to S^-.

Step-5: Determination of the Positive Ideal Alternative (PIA)

The PIA is the alternative which has the minimum Manhattan distance concerning $S^+$and maximum Manhattan distance with respective S^-.

$$PIA = \{\min(M_i^+), \max(M_i^-)\} \quad \text{………… (193)}$$

, for *i=1, 2, 3, …….., M.*

Step-6: Determination of distance of an alternative concerning the PIA

The distance between each alternative and PIA is calculated using the following formula:

$$V_i = \sqrt{(M_i^+ - \min(M_i^+))^2 + (M_i^- - \max(M_i^-))^2} \quad \text{………… (194)}$$

Step-7: Ranking the alternative

Alternatives are ranked in ascending order, where alternative with lowest V_i value sits at the top of the list.

2.4.22D Applications of DIA method

DIA is a fairly new and unexplored MADM method. Its applications are very limited and mostly focused on the communication sector. Tran *et al.,* and Boukhatem *et al.,* introduced this method to solve the interface selection problem in heterogeneous wireless networks (for more details, refer [279]). Following their lead, Almutairi *et al.,* tested for the optimum pairing of attribute weighting techniques used with DIA method applied to wireless network vertical handover (for more information, refer [280]). DIA has received much appreciation in the field of communication and has been seen as a benchmark approach for solving network related multi-criteria problems. Adib *et al.,* and his team used DIA method to validate their novel method in a network selection scenario (for more details, refer [280]).

These are some of the applications of DIA based on my limited literature survey, which included research papers and articles picked from international journals.

2.4.23 EDAS (Evaluation Based on Distance from Average Solution) method

2.4.23A Introduction

In the compromise Multi-Attribute Decision Making (MADM) methods such as VIKOR and TOPSIS, the best alternative is determined by calculating the distance of each alternative from ideal and anti-ideal solutions. The desirable alternative has a minimum distance from an ideal solution and maximum distance from the nadir solution. In contrast to this, Ghorabaee *et al.,* in the year 2015, proposed a novel methodology which evaluates the alternatives based on the distance from the average solution (AV) [282]. This eliminates the need to calculate ideal and nadir solutions. This method is called as Evaluation Based on Distance from Average Solution (EDAS).

The EDAS method measures the desirability of the alternatives based on two distances. The first measure is the positive distance from average (PDA), and the second is the negative distance from average (NDA) [282]. These measurements demonstrate the difference between each alternative and the average solution [EDAS-1]. The EDAS methodology is based on the principle that the desirable alternative(s) must possess higher values of PDA and lower values of NDA [282]. Higher values of PDA and lower values of NDA indicates that the alternative is at least better than average solution. This constructs a reasonable compromise when dealing with multiple and often conflicting criteria. Moreover, measures (PDA/NDA) are calculated according to the type of criteria (beneficial or non-beneficial) [283].

2.4.23B Calculation of Positive Distance from Average (PDA) and Negative Distance from Average (NDA)

The PDA and NDA measures are calculated using the average performance score of the criteria, and the distance concerning this point derives these measures. PDA and NDA measures are calculated differently when attending maximizing (beneficial) and minimizing (non-beneficial) attributes. These distances are computed using the following expressions:

$$PDA = [PDA_{ij}]_{M\times N} \quad \ldots\ldots\ldots\ldots (195)$$

$$NDA = [NDA_{ij}]_{M\times N} \quad \ldots\ldots\ldots\ldots (196)$$

For a maximizing attribute,

$$\mathrm{PDA_{ij}} = \frac{\max(0,(x_{ij}-AV_j))}{AV_j} \quad \ldots\ldots\ldots\ldots (197)$$

$$\mathrm{NDA_{ij}} = \frac{\max(0,(AV_j-x_{ij}))}{AV_j} \quad \ldots\ldots\ldots\ldots (198)$$

, for *i= 1,2,3, …, M; & j= 1,2,3, …., N.*

For a minimizing attribute,

$$\text{PDA}_{\text{ij}} = \frac{\max(0,(AV_j - x_{ij}))}{AV_j} \quad \text{...........} (199)$$

$$\text{NDA}_{\text{ij}} = \frac{\max(0,(x_{ij} - AV_j))}{AV_j} \quad \text{...........} (200)$$

, for *i= 1,2,3, ..., M;* & *j= 1,2,3, ..., N.*

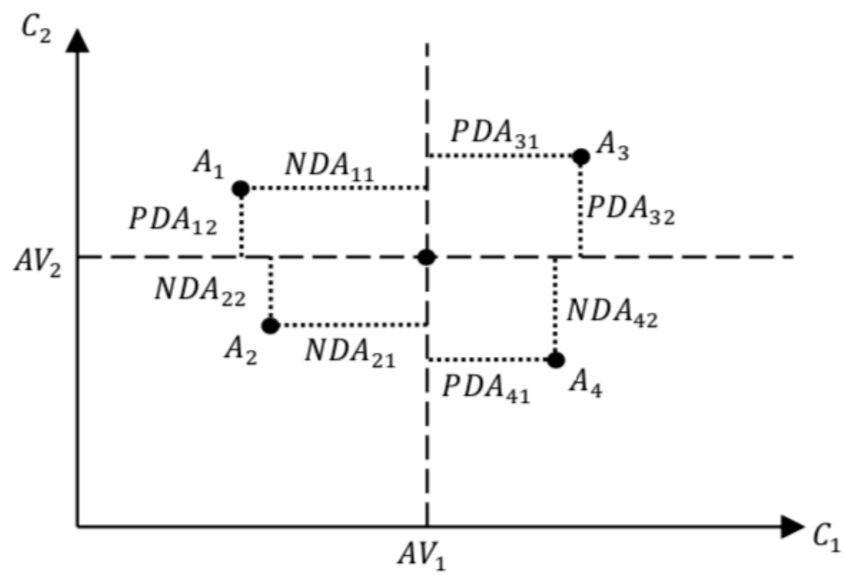


Figure 19: Representation of PDA and NDA values in a simple situation (courtesy [282]).

where PDA_{ij} moreover, NPDA_{ij} Denotes the positive and negative distance of i_{th} alternative from average solution in terms of j_{th} criterion, respectively [282]. The graphical representation of PDA and NDA values in a sample condition with four alternatives and two beneficial criteria is shown in the above figure 19.

2.4.23C Merits

EDAS method possesses some unique merits such as:

A MADM problem usually involves multiple and usually conflicting criteria, and obtaining an optimal alternative is a complex task. Evaluating alternatives based on hypothetically ideal solution often leads to bias. So, assessment achieved through the average solution is more pragmatic and reduces bias against the list of alternatives.

It deals with maximizing/beneficial and minimizing/non-beneficial criteria separately [282].

When decision makers (DMs) are assigning the performance ratings to each alternative under uncertainty in a group decision making (GDM), it should be taken into account that not all the DMs have the same level of knowledge, background, and experience. What makes EDAS stand out is that the result is obtained from the average solution, which in turn somehow eliminates the risk of biasedness of experts towards an alternative. The result being derived from an average solution already normalizes the data, which limits the chances of deviation from the best solution to the far great extent. So, it provides a better and accurate solution than that of TOPSIS or VIKOR to the real problems [284].

2.4.23D Modus operandi of EDAS approach [282-284]

Step-1: Shortlisting the most important attributes as described in the objectives of the problem.

Step-2: Constructing the decision matrix (X):

Decision matrix (X) contains alternatives (i=1, 2, 3, ….., M) in rows and attributes (j=1, 2, 3, ……, N) in columns.

$$X = [x_{ij}]_{M\times N} = \begin{bmatrix} x_{11} & x_{12}\cdots & x_{1N} \\ \vdots & \ddots & \vdots \\ x_{M1} & x_{M2}\cdots & x_{MN} \end{bmatrix}$$

Step-3: Determining the average solution according to all criteria:

$$AV = [AV_j]_{1\times N} \quad \ldots\ldots\ldots\ldots (201)$$

Where,

$$\mathrm{AV}_j = \frac{\sum_{i=1}^{M} x_{ij}}{M} \quad \ldots\ldots\ldots\ldots (202)$$

M= number of alternatives.

Step-4: Calculating PDA and NDA:

PDA and NDA are computed using the following mathematical expressions:

$$PDA = [PDA_{ij}]_{M\times N} \quad \ldots\ldots\ldots\ldots (203)$$

$$NDA = [NDA_{ij}]_{M\times N} \quad \ldots\ldots\ldots\ldots (204)$$

For a beneficial criterion (attribute),

$$\mathrm{PDA_{ij}} = \frac{\max\,(0,(x_{ij}-AV_j))}{AV_j} \quad \ldots\ldots\ldots\ldots (205)$$

$$\mathrm{NDA_{ij}} = \frac{\max\,(0,(AV_j-x_{ij}))}{AV_j} \quad \ldots\ldots\ldots\ldots (206)$$

, for $i= 1,2,3, \ldots, M;$ & $j= 1,2,3, \ldots., N.$

For a non-beneficial criterion (attribute),

$$\mathrm{PDA_{ij}} = \frac{\max\left(0,(AV_j - x_{ij})\right)}{AV_j} \quad \ldots\ldots\ldots\ldots (207)$$

$$\mathrm{NDA_{ij}} = \frac{\max\left(0,(x_{ij} - AV_j)\right)}{AV_j} \quad \ldots\ldots\ldots\ldots (208)$$

, for $i= 1,2,3, \ldots, M;$ & $j= 1,2,3, \ldots., N.$

Step-5: Determining the weighted sum of PDA and NDA for all the alternatives:

It is achieved using the following expressions:

Weighted Sum of PDA (SP_i),

$$SP_i = \sum_{j=1}^{N} W_j \times \mathrm{PDA_{ij}} \quad \ldots\ldots\ldots\ldots (209)$$

Weighted Sum of NDA (SN_i),

$$SN_i = \sum_{j=1}^{N} W_j \times \mathrm{NDA_{ij}} \quad \ldots\ldots\ldots\ldots (210)$$

Where,

W_j is the weight of j^{th} criterion. ($\sum_{j=1}^{N} W_j = 1$).

Step-6: Normalizing the SP and SN values for all the alternatives:

$$NSP_i = \frac{SP_i}{\max_i(SP_i)} \quad \ldots\ldots\ldots\ldots (211)$$

Moreover,

$$NSN_i = 1 - \frac{SN_i}{\max\limits_i(SN_i)} \quad \text{........... (212)}$$

Where,

"NSP_i" is the normalized SP value of i^{th} alternative and "NSN_i" is the normalized SN value of i^{th} alternative.

Step-7: Determining the Appraisal Score (AS) for the alternatives

The appraisal score is the overall performance score of individual alternatives and is calculated suing the following formula:

$$AS_i = \frac{1}{2}(NSP_i + NSN_i) \quad \text{........... (213)}$$

Where $0 \leq AS_i \leq 1$.

Step-8: Ranking the alternatives:

Alternatives are ranked based on appraisal scores (AS_i), alternative with lowest AS_i score the is placed at the bottom and alternative with highest the AS_i score is at the top of the rankings.

2.4.23E Extensions and applications of EDAS approach

The EDAS method has found numerous applications in various types of MADM situations. The classical EDAS method is equipped to deal with crisp information, but the nature of most the decision problems are not so straight forward to express the performance of each alternative in terms of crisp values all the times. Many theories have been developed to deal with uncertainties, imprecisions, and incompleteness of the information, which is crucial to solve any type of decision problem. The Fuzzy sets theory, Grey theory, Rough sets theory, and Neutrosophic sets theory, etc., are the examples of such theories. These theories have been incorporated into most of the MADM approaches transforming them to solve extremely complex problems involving imperfections. EDAS method has also been extended to include such theories to improve its range of solving different types of decision problems.

a) Fuzzy EDAS method

Ghorabaee *et al.,* in the year 2016, proposed an extension to his classical EDAS method to incorporate Fuzzy sets to develop Fuzzy EDAS (F-EDAS) method [282]. This extended version was applied in a supplier selection problem, and sensitivity analysis was conducted to test its stability. The results were positive and inevitably found many applications. Zavadskas *et al.,* who is also the co-author of the EDAS method, applied the F-EDAS method to evaluate carpenter manufacturers on a set of criteria in a domestic problem (for more details, refer [282]).

b) Grey EDAS method

Ghorabaee *et al.* developed Grey EDAS method, which integrated classical EDAS method with Grey numbers to expand its realms in decision-solving arena in early 2017 [282].

Interval-valued Intuitionistic Fuzzy (IVIF) EDAS method

Ghorabaee *et al.,* proposed an extension to classical EDAS method to accommodate Interval-Valued Intuitionistic Fuzzy sets (IVIF). IVIF is an improvement over simple Fuzzy sets as it includes a degree of hesitation due to non-membership of an element to fuzzy sets. This method is called as Interval-Valued Intuitionistic Fuzzy sets EDAS (IVIF-EDAS). Later, Ghorabaee *et al.* applied this novel method for evaluating disposal sites for solid wastes (for more information, refer [282]).

c) Interval Type-2 Fuzzy (IT2FS) EDAS method

Ghorabaee *et al.* proposed an improvement to his previous methods, i.e., Fuzzy EDAS and IVIF EDAS with novel Interval Type-2 Fuzzy Sets (IT2FS) EDAS in early 2017. The type-2 fuzzy sets were introduced as an extension of fuzzy sets by Zadeh *et al.,* in 1975. This type of fuzzy sets can capture more degrees of uncertainty and lead to a more rational model in an uncertain environment. In the following, some of the important descriptions related to this type of fuzzy sets are presented [EDAS_IT2F-1]. To validate this method, he later applied to evaluate suppliers and assess order allocation concerning environmental criteria (for more details, refer [282]).

d) Interval-valued Neutrosophic (IVN) EDAS method

Karasan *et al.* proposed a new extension to EDAS by incorporating Interval-Valued Neutrosophic Sets (IVN) to cope with indeterminacy degree of experts while assigning the performance rating of alternatives. None of the above method extensions considers indeterminacy and inconsistency with regards to information to such a great extent as IVN [282]. Later, Ali *et al.,* applied this new method for prioritization of the United Nations national sustainable development goals (for more information, refer [282]).

e) Stochastic EDAS method

Ghorabaee *et al.* proposed yet another extension to EDAS, called Stochastic EDAS (S-EDAS). Ghorabaee *et al.* argued that because of the importance and applicability of the normal distribution, we assume that the performance values of alternatives on each criterion follow the normal distribution [282]. Moreover, believed that utilizing the stochastic process will actually make the method more efficient [282]. Then to validate his claims, he applied this novel approach to assess the performance of bank branches on various criteria and presented results (for more details, refer [282].

These are some well-known extension of EDAS presented in this section.

2.4.23F Applications of classical EDAS method

The original EDAS method has found many applications to various types of decision problems involving multiple and usually conflicting criteria to evaluate a finite number of alternatives [282]. This section presents applications of EDAS based on a limited literature survey conducted on twenty articles signifying the application of EDAS to real-time problems. These articles are taken from reputed international journals.

Based on the survey, the most prominent areas of application are:

a) Industrial Management

Mathewa *et al.,* and Sahua *et al.,* used EDAS and other MCDM methods to evaluate material handling equipment alternatives on six conflicting criteria (for more information, refer [284]).

Ecer *et al.,* solved a supply-chain management problem using an integrated model of Fuzzy AHP and EDAS. In this model, Fuzzy AHP is used to calculate criteria weights, and different third-party logistics (3pls) providers are assessed and ranked using EDAS method (for more details, refer [285]).

Zavadskas *et al.,* and his team tackled contractor contracts evaluation problem based on quality assurance using multiple MADM approaches, which also included EDAS (for more information, refer [285]).

b) Environmental Management

Zavadskas *et al.* developed an MCDM system consisting of different weighting and ranking method including EDAS for assessing twenty-one Vilnius neighborhoods on "healthy and safe built Environment" model according to sustainable development principles (for more details, refer [283]).

c) Material Selection

Chatterjee *et al.,* and Chakraborty *et al.,* collaborated to develop a hybrid meta-model for material selection using Design of Experiments (DOE) and EDAS Method (for more details, refer [282]).

Here ends the literature review of EDAS method.

2.4.24 CODAS (Combinative Distance-based Assessment) method

2.4.24A Background

Multi-attribute decision making (MADM) model has been hailed as one of the best methodologies to deal with a decision situation involving a catalog of alternatives each possessing characteristic potential to solve the problem, and evaluation parameters set by the decision-maker(s) which are usually conflicting in nature. The above situation pretty much expresses a real-time decision-making crisis in industry. The MADM methods such as SAW, TOPSIS, VIKOR, WASPS, MOORA, and EDAS, etc., have all been tremendous as a decision-aiding tool for the decision-makers over the years. They all possess certain inherent advantages and flaws which may be credited to the different alternative appraisal criterion that each of them exhibits (the alternative appraisal criterion is the unique mathematical aggregation procedures followed by each approach to rank the alternatives). Even though many MADM methods have been developed, there are still some unexplored features that have not been considered in other MADM tools [285].

Ghorabaee *et al.* proposed a new methodology (in the year 2016) to fill in this gap [285]. This method was called as Combinative Distance-based Assessment (CODAS). One of the well-known MADM methods is TOPSIS, which employs a Euclidean measure to calculate the distance between the alternatives and the positive and negative ideal solution [269]. Whereas, the DIA (distance to the ideal alternative) utilizes the Manhattan measures to obtain these distances [285]. Both of these distance measures have their advantages and limitations. Ghorabaee *et al.,* proposed method the CODAS use both the Euclidean and Taxicab (or Manhattan) measure to calculate the distance of the alternatives from the negative ideal solution and hence, enjoying the benefits of both the methods.

2.4.24B Introduction

Combinative Distance-based Assessment method is one of the distance-based MADM methods. In this method, the desirability of alternatives is determined by using two measures. The primary measure is related to the Euclidean distance of alternatives from the negative-ideal solution. Using this type of distance requires an l^2-norm indifference space for criteria [285]. The secondary measure is the Taxicab distance which is related to the l^1-norm indifference space. If the Euclidean distances of two alternatives are very close to each other, the Taxicab distance is used to compare them [285]. The degree of closeness of Euclidean distances is set by a threshold parameter [285]. The l^2 and l^1 indifference space for criteria is explained by author Yoon *et al.,* in [KY-0]. Since the desirability of an alternative depends on its distance with respect to the negative ideal solution, the alternative with has greater distance has greater desirability [285].

The detail step-by-step procedure of the CODAS method is presented below: [285]

Step-1: Normalize the Decision-Matrix (DM).

Normalization is a process transforming a dimensional entity into a dimensionless one [N-1]. Normalization is a key step in the MADM model since the aggregation procedure usually involves combining each criterion value of an alternative in one way or another to produce an overall preference index that assists the decision-maker in making a sensible decision. The CODAS method employs a linear normalization technique to achieve this task [285].

For beneficial attributes:

$$N_{ij}^{(LN)} = \frac{x_{ij}}{x_j^{max}} \qquad \ldots\ldots\ldots\ldots (214)$$

, for *i=1,2……, M;* & *j=1,2……, N.*

For non-beneficial attributes:

$$N_{ij}^{(LN)} = \frac{x_j^{min}}{x_{ij}} \quad \ldots\ldots\ldots\ldots (215)$$

, for *i=1,2……, M;* & *j=1,2……, N.*

Where,

$N_{ij}^{(LN)}$ is the normalized score and “LN” abbreviates to linear normalization. And,

“x_{ij}” is the performance score of i^{th} alternative with respect to j^{th} criterion.

Step-2: Weighted Normalization of the ratings.

Criterion Weight can be defined as the relative importance score of each criterion (attribute) concerning the set of criteria chosen for a decision problem. This relative worth is derived from the objectives of the decision problem. Criterion weights can be obtained by subjective approach, objective approach, or by a combination of both.

In this step, the criterion weights are multiplied to each normalized score belonging to that particular criterion [285]. It is mathematically expressed as:

$$r_{ij} = W_j \times N_{ij}^{(V)} \quad \ldots\ldots\ldots\ldots (216)$$

, for *i= 1,2,3, …, M;* & *j= 1,2,3, …., N.*

Where,

"W_j" is the weight of j^{th} attribute. ($\sum_{j=1}^{N} W_j = 1$).

"r_{ij}" is the weighted-normalized rating of i^{th} alternative and j^{th} attribute.

Step-3: Determine the Negative Ideal Solution (NIS) for each criterion.

The negative-ideal solution (NIS) is composed of all worst attribute ratings attainable.

$$S^{-} = \{r_1^{-}, r_2^{-}, \dots \dots \dots, r_N^{-}\} = \left\{\min_j r_{ij} \mid \forall\, j = 1,2,3, \dots, N.\right\} \quad \dots\dots\dots\dots (217)$$

Where,

"S^{-}" denotes NIS.

Step-4: Calculate the Euclidean and Taxicab distance between each alternative and the NIS.

The Euclidean distance is measured between each alternative's performance score concerning an attribute to the "S^{-}" using the following formula:

$$E_i = \sqrt{\sum_{j=1}^{N} \left(r_{ij} - r_1^{-}\right)^2} \quad \dots\dots\dots\dots (218)$$

, *i=1, 2, 3, ……., M.*

Where,

Similarly, the Taxicab (or Manhattan) distance is measured between each alternative's performance score concerning an attribute to the "S^{-}" using the following formula:

$$T_i = \sum_{j=1}^{N} \left| r_{ij} - r_j^{-} \right| \quad \dots\dots\dots\dots (219)$$

$, i=1, 2, 3, \ldots\ldots, M.$

Step-5: Construct the Relative Assessment (RA) matrix.

$$RA = [h_{ik}]_{N\times N} \quad \ldots\ldots\ldots\ldots (220)$$

Where,

$$h_{ik} = (E_i - E_k) + (\psi(E_i - E_k) \times ((T_i - T_k)) \quad \ldots\ldots\ldots\ldots (221)$$

$k \in \{1, 2, \ldots\ldots, M\}$,

"ψ" denotes a threshold function to recognize the equality of the Euclidean distances of two alternatives, and is defined as follows [CODAS-0]:

$$\psi(x) = \begin{cases} 1 & if \quad |x| \geq \tau \\ 0 & if \quad |x| < \tau \end{cases} \quad \ldots\ldots\ldots\ldots (222)$$

In this function, τ is the threshold parameter that can be set by the decision- maker. It is suggested to set this parameter at a value between 0.01 and 0.05. If the difference between Euclidean distances of two alternatives is less than τ, these two alternatives are also compared by the Taxicab distance. And, most commonly assumed value of τ is 0.02 for the calculations. [285 & 286].

Step-6: Determine the Assessment score of each alternative.

The Assessment score (H_i)is calculated using the following formula:

$$H_i = \sum_{k=1}^{M} h_{ik} \quad \ldots\ldots\ldots\ldots (223)$$

Step-7: Ranking the alternatives.

Ranking of the alternatives is done basing on the assessment score (H_i). The alternate with highest H_i scorthe e is ranked as the best among the list.

2.4.24C Areas of application of CODAS

The CODAS is one of the newest MADM approaches, proposed in 2016. Still, has found some considerable amount of applications spread across different areas of the industry. Applications of the CODAS method mentioned in this section are based on the literature survey conducted on twenty articles obtained from international journals.

There is no particular predominant area of application, but rather versatile with many areas. Badi *et al.* has been instrumental in establishing the CODAS method as a decision-aiding tool in various areas of industries. He applied CODAS method to evaluate suppliers for a Libya based steel-making company and in the end presented the best supplier (for more information, refer [286]). Then, he proposed an extension of CODAS method by incorporating Linguistic Neutrosophic Numbers (LNN) to classical CODAS. This, up to some extent, made it possible to eliminate subjective qualitative assessments and assumptions by decision makers in complex decision-making conditions. He tested and validated this novel method by applying it in a case study of the selection of optimal Power-Generation Technology (PGT) in Libya (for more details, refer [285]). Continuing his affair with CODAS method, Badi *et al.,* assessed multiple site locations in Libya for desalination plant installation (for more details, refer [287]).

In the Manufacturing sector, Mathew *et al.,* compared multiple MADM methods including the CODAS in an attempt to present a robust assessment of material handling equipment alternatives and later proposed the best option (for more details, refer [287]). Following his lead, Bolturk *et* *al.,* developed Pythagorean fuzzy CODAS to select the best supplier for a manufacturing firm (for more information, refer [290]).

Ghorabaee *et al.,* extended his classical CODAS method to integrate Fuzzy sets (called CODAS-F) to tackle the uncertainties that exist in the market segment evaluation problems (for more information, refer [291]). Later, he proposed an improvement to the CODAS-F by incorporating the Interval-Valued Intuitionistic Fuzzy sets (IVIF). This method was called as CODAS-IVIF. The inclusion of IVIF sets improves the method to deal with the uncertainties in a better light and also adds a degree of hesitation that elements have to the Fuzzy sets. He used this method (i.e., CODAS-IVIF) to develop an evaluation model based on business intelligence for enterprise systems (for more information, refer [285]).

These are some of the applications and extension of CODAS method, which is based on my limited literature survey.

2.5 Data clustering

2.5.1 Introduction

The world is driven by data. We see people all around us indulge in the activities of procuring or processing data. Basically, data means information. For example, characteristics of a living species, properties of a natural phenomenon, results of a scientific experiment, and dynamics of a running machinery system, etc., constitute data [292]. More importantly, data provide a basis for further analysis, reasoning, decisions, and ultimately, for the understanding of all kinds of phenomena [292]. One of the most important of the myriad of data analysis activities is to classify or group data into a set of categories or clusters. The basic criterion for clustering is a similarity.

Similarity can be established using different attributes or characteristics that individual data objects possess. Data objects that are classified in the same group will display similar properties based on some attributes. Knowing one object of the group gives some basic idea about the rest of the objects in that group. Hence, clustering is extremely important for understanding data and makes it easier too.

The goal of clustering is to separate a finite, unlabeled data set into a finite and discrete set of "natural," hidden data structures, rather than to provide an accurate characterization of unobserved samples generated from the same probability distribution [292].

2.5.2 Hierarchical clustering

Clustering techniques are broadly classified as partitional clustering and hierarchical clustering, based on the properties of the generated clusters [292]. Partitional clustering directly divides data points into some prespecified number of clusters without the hierarchical structure, while hierarchical clustering groups data with a sequence of nested partitions, either from singleton clusters to a cluster including all individuals or vice versa. The former is known as agglomerative hierarchical clustering, and the latter is called divisive hierarchical clustering. Figure 20 is the dendrogram representing the two types of Hierarchical clustering.

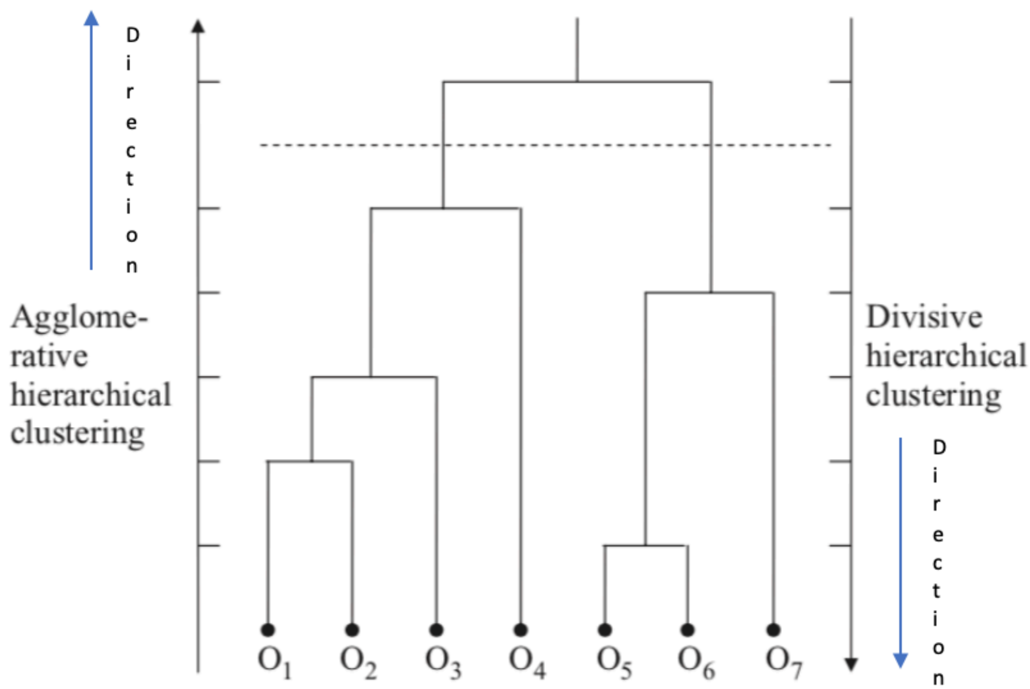


Figure 20: Dendrogram representing the two types of hierarchical clustering [292].

a) Divisive hierarchical clustering

In this type of hierarchical clustering, the entire data set belongs to a cluster, and a procedure successively divides it until all clusters are singletons (individual data-points). For a data set with N objects, a divisive hierarchical algorithm would start by considering $(2^{N-1} - 1)$ possible divisions of the data into two nonempty subsets, which is computationally expensive even for small-scale data sets [292]. Therefore, divisive clustering is not a common choice in practice. However, the divisive clustering algorithms do provide clearer insights of the main structure of the data since the larger clusters are generated at the early stage of the clustering process and are less likely to suffer from the accumulated erroneous decisions, which cannot be corrected by the successive process [292].

b) Agglomerative clustering

Agglomerative clustering starts with N clusters, each of which includes exactly one data point. A series of merge operations is then followed that eventually forces all objects into the same group. The general agglomerative clustering can be summarized by Figure 21 below [292]:

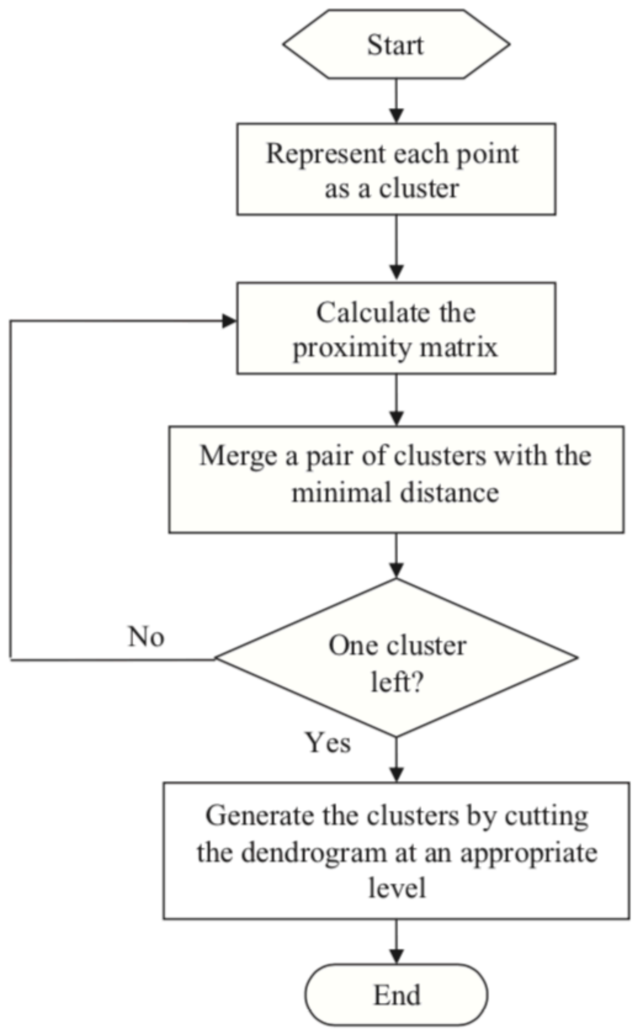


Figure 21: Flowchart of the agglomerative hierarchical clustering algorithm [292].

The merge of a pair of clusters or the formation of a new cluster is dependent on the definition of the distance function between two clusters. There exist a large number of distance definitions between a cluster $C_l$and a new cluster C_{ij} formed by the merge of two clusters C_i and C_j, which can be generalized by the recurrence formula proposed by Lance and Williams [292] as:

$$\mathrm{D}\left(\mathrm{C_l}, \left(\mathrm{C_i}, \mathrm{C_j}\right)\right) = \alpha_\mathrm{i}\, \mathrm{D}(\mathrm{C_l}, \mathrm{C_i}) + \alpha_\mathrm{j}\, \mathrm{D}\left(\mathrm{C_l}, \mathrm{C_j}\right) + \beta\, \mathrm{D}\left(\mathrm{C_i}, \mathrm{C_j}\right) + \gamma\left|\mathrm{D}(\mathrm{C_l}, \mathrm{C_i}) - \mathrm{D}\left(\mathrm{C_l}, \mathrm{C_j}\right)\right| \quad \ldots\ldots\ldots\ldots (224)$$

Where,

D (·, ·) is the distance function and,

α_i, α_j, β, and γ are coefficients that take values dependent on the scheme used.

The parameter values for the commonly used algorithms are summarized table below [292],

Table 1: Lance and Williams' parameters for agglomerative hierarchical clustering (Courtesy [292]).

Clustering algorithms	α_i	α_j	β	γ
Single linkage (nearest neighbor)	1/2	1/2	0	–1/2
Complete linkage (farthest neighbor)	1/2	1/2	0	1/2
Group average linkage (UPGMA)	$\frac{n_i}{n_i+n_j}$	$\frac{n_j}{n_i+n_j}$	0	0
Weighted average linkage (WPGMA)	1/2	1/2	0	0
Median linkage (WPGMC)	1/2	1/2	–1/4	0
Centroid linkage (UPGMC)	$\frac{n_i}{n_i+n_j}$	$\frac{n_j}{n_i+n_j}$	$\frac{-n_i n_j}{(n_i+n_j)^2}$	0
Ward's method	$\frac{n_i+n_l}{n_i+n_j+n_l}$	$\frac{n_j+n_l}{n_i+n_j+n_l}$	$\frac{-n_l}{n_i+n_j+n_l}$	0

U: unweighted; W: weighted; PGM: pair group method; A: average; C: centroid.

Where,

n_i, n_j, and n_l are the number of data points in clustethe r C_i, C_j, and C_l, respectively.

Both Hierarchical clustering types have their own set of advantages and disadvantages. Their application depends purely on the objectives of the clustering problem. (For more information on data clustering and hierarchical clustering, refer [292].)

2.6 Principal Component Analysis (PCA)

A data set obtained from a scientific experiment usually contains variables belonging to different dimensional spaces credited to the fact that these data points are usually measured in different units. Visual representation of this kind of data sets is very difficult, as presenting data beyond 2-dimensional space becomes complicated. Principal component analysis (PCA) deals with the problem of fitting data sets belonging to high-dimensional space to a low-dimensional affine subspace [293]. The central idea of principal component analysis (PCA) is to reduce the dimensionality of a data set consisting of a large number of interrelated variables while retaining as much as possible of the variation present in the data set. This is achieved by transforming to a new set of variables, the principal components (PCs), which are uncorrelated and are ordered so that the first few retain most of the variation present in all of the original variables. [293]

2.6.1 Definition of principal components

Suppose that ‘x’ is a vector of ‘p’ random variables, such that the variances of the ‘p’ random variables and the structure of the covariances or correlations between the ‘p’ variables are of interest. Unless ‘p’ is small, or the structure is very simple, it will often not very helpful to simply look at the ‘p’ variances and all of the $\frac{1}{2}p(p-1)$ correlations or covariances. An alternative approach is to look for a few ($\ll$ p) derived variables that preserve most of the information given by these variances and correlations or covariances [293].

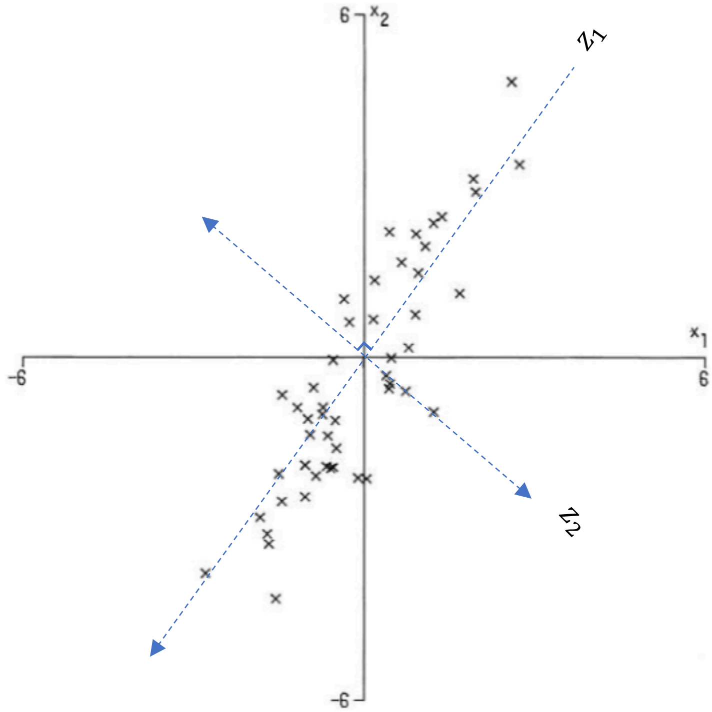


Figure 22: Plot of 50 observations on two variables $x_1$and $x_2$with superimposed PCs $z_1$and z_2.

For establishing principal components (PCs), the first step is to look for a linear function $\alpha_1'x$ of the elements of 'x' having maximum variance, where $\alpha_1$is a vector of 'p' constants $\alpha_{11,}\alpha_{12}, \dots \dots, \alpha_{1p}$, and $'$ denotes transpose [293], so that

$$\alpha_1'x = \alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1p}x_p = \sum_{j=1}^{p} \alpha_{1j}x_j \quad \dots\dots\dots\dots (225)$$

Next, look for a linear function $\alpha_2'x$, uncorrelated with $\alpha_1'x$ having maximum variance, and so on, so that at the kth stage a linear function $\alpha_k'x$ is found that has maximum variance subject to being uncorrelated with $\alpha_1'x, \alpha_2'x, \dots \dots, \alpha_{k-1}'x$. The kth derived variable, $\alpha_k'x$ is the kth PC. Up to 'p', PCs could be found, but it is hoped, in general, that most of the variation in 'x' will be accounted for by 'm' PCs, where m ≪ p. The reduction in complexity achieved by transforming the original variables to PCs will be demonstrated in unrealistic, but a simple case where p = 2. The advantage of p = 2 is, of course, that the data can be plotted exactly in two dimensions [293].

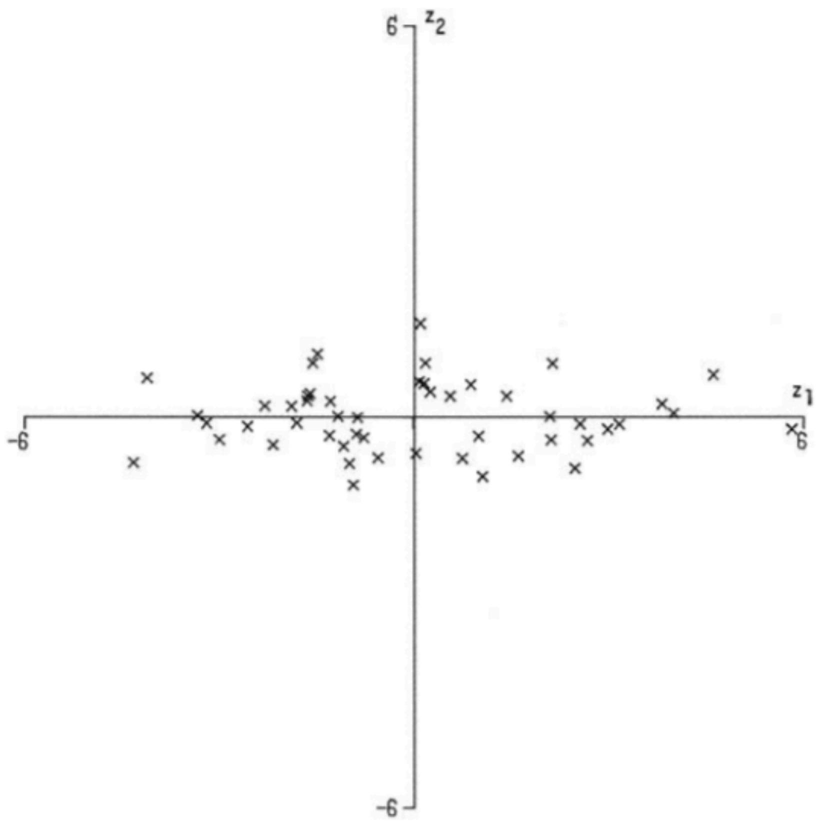


Figure 23: Plot of 50 observations from the previous figure concerning their PC's $z_1$and z_2.

Figure 22 gives a plot of 50 observations on two highly correlated variables $x_1$and x_2. There is considerable variation in both variables, though rather more in the direction of x_2 than x_1. If we transform to PCs $z_1$and z_2, we obtain the plot given in Figure 23.

It is clear that there is greater variation in the direction of z_1 then in either of the original variables, but very little variation in the direction of z_2. More generally, if a set of p (> 2) variables have substantial correlations among them, then the first few PCs will account for most of the variation in the original variables. Conversely, the last few PCs identify directions in which there is very little variation; that is, they identify near-constant linear relationships among the original variables [293].

2.6.2 Derivation of principal components

To derive the form of the PCs, consider first $\alpha_1' x$; the vector α_1 maximizes variance $[\alpha_1' x]$ = $\alpha_1' \sum \alpha_1$, where $\sum$ is the covariance matrix of vector 'x'. It is clear that, as it stands, the maximum will not be achieved for finite α_1, so a normalization constraint must be imposed [293]. The constraint used in the derivation is $\alpha_1' \alpha_1$ = 1, i.e., the sum of squares of elements of α_1 equals 1. Other constraints, for example $Max_j\ |\alpha_{1j}|$ = 1, may more useful in other circumstances, and can easily be substituted later on. However, the use of constraints other than $\alpha_1' \alpha_1$= constant in the derivation leads to a more difficult optimization problem, and it will produce a set of derived variables different from the PCs [293].

To maximize $\alpha_1' \sum \alpha_1$ subject to $\alpha_1' \alpha_1$ = 1, Lagrange multipliers technique is used.

Maximize [293]

$$\alpha_1' \Sigma \alpha_1 - \lambda(\alpha_1' \alpha_1 - 1) \quad \text{………… (226),}$$

Where λ is a Lagrange multiplier. Differentiation concerning α1 gives [PCA-2]:

$$\Sigma \alpha_1 - \lambda \alpha_1 = 0 \quad \text{………… (227),}$$

(or)

$$(\Sigma - \lambda I_p)\alpha_1 = 0 \quad \text{………… (228),}$$

Where,

I_p is the (p × p) identity matrix,

λ is an eigenvalue of Σ, and

α_1 is the corresponding eigenvector.

To decide which of the 'p' eigenvectors gives $\alpha_1' x$ with maximum variance, note that the quantity to be maximized [PCA-2] is:

$$\alpha_1'\sum\alpha_1 = \alpha_1'\lambda\alpha_1 = \lambda\alpha_1'\alpha_1 = \lambda \quad \text{………..} \ (229)$$

So λ must be as large as possible. Thus, α_1 is the eigenvector corresponding to the largest eigenvalue of Σ, and variance of $(\alpha_1' x)$ = $\alpha_1'\sum\alpha_1$ =λ_1, the largest eigenvalue.

In general, the kth PC of 'x' is $\alpha_k' x$ and variance of $(\alpha_k' x)$ = λ_k, where λ_k, is the kth largest eigenvalue of Σ, and α_k is the corresponding eigenvector.

This derivation of the PC coefficients and variances as eigenvectors and eigenvalues of a covariance matrix is standard practice [293].

a) Score Plot

PCA does not discard any samples or characteristics (variables). Instead, it reduces the overwhelming number of dimensions by constructing principal components (PCs). PCs describe variation and account for the varied influences of the original characteristics [293]. In the process of transposing the sample into the new axis, the relative distance of the samples in new axis concerning an original axis is calculated, and a score (or eigenvalue) is given to each variable. Using these values, a plot is constructed, which is known as Score plot. [293]

b) Loading plot

Loading plot defines the correlation between the criteria used to evaluate a sample. The angles between the vectors that determine the criteria indicate the correlation between them [293]. When two vectors are close, forming a small angle, the two variables they represent are positively correlated. If they meet each other at 90°, they are not likely to be correlated. When they diverge and form a large angle (close to 180°), they are negative correlated [293].

c) Outlier Plot

In statistics, an outlier is an observation point that is distant from other observations [293]. An outlier may be due to variability in the measurement or it may indicate the experimental error; the latter are sometimes excluded from the data set. [293] An outlier can cause serious problems in statistical analyses.

Outliers can occur by chance in any distribution, but they often indicate either measurement error or that the population has a heavy-tailed distribution. In the former case, one wishes to discard them or use statistics that are robust to outliers, while in the latter case they indicate that the distribution has high skewness and that one should be very cautious in using tools or intuitions that assume a normal distribution. [293]

d) Biplot

It is a plot which is a combination of score plot and loading. The score plot is superimposed on a loading plot to understand the correlation between them [293].

Chapter 3: Motivation

Currently, the material selection strategy proposed by Michael F. Ashby, which is popularly known as Ashby method, is being used in the industry to select an optimal material for a given design. It ensures that a "right" material is selected for an application by subjecting each candidate to undergo a unique evaluation procedure. However, this methodology faces a crucial limitation, i.e., the complication of it during application increase with the increase in selection attributes. At once, only three or four attributes can be considered during evaluation. This is a major limitation as the material selection has evolved into a complex task that needs to consider multiple attributes while evaluating the materials. This predicament was overcome by the introduction of Multi-attribute Decision Making (MADM) approach. However, this approach is just limited to the ranking of candidates, and there are different MADM approaches available to achieve this purpose. Moreover, Ashby and MADM methods are usually applied to assess designs dealing with structural properties and less prevalent in the case of functional properties. There is no standardized technique or strategy to select a magnetic material for a function-oriented design. So, a need for robust magnetic material selection strategy which can assess multiple selections attributes that the modern-day engineering designs inherit and provide an optimal solution in a constructive and comprehensible way is imminent.

Chapter 4: Methodology

The main idea behind the proposed methodology is to integrate statistical approaches with material selection strategy to aid engineers/decision-makers to screen out a wide spectrum of material alternatives and present a narrow list of candidates ranked in an orderly manner by undertaking a comprehensive assessment on multiple selection criteria. Approaching material selection problem with a statistical mind set ensures that the voice of data is heard. Advancements in statistics have enabled the development of this methodology. Statistical tools such as Multi-attribute Decision Making (MADM) framework, Principal Component Analysis (PCA) and Data Clustering have been extensively utilized in this methodology. Evaluating material alternatives over a finite set of parameters (criteria) motivated by pre-determined selection objective(s) derived from the concept/design requirements constitutes the basic essence of this framework.

In this section, a novel material selection methodology is proposed. The basic framework of this methodology complies with the general material selection protocol, which is presented elaborately in chapter 2 with certain improvisations. The proposed methodology comprises of four tasks: Translation, Ranking, Visualization, and Documentation. Figure 24 shows the crucial steps in the proposed material selection strategy.

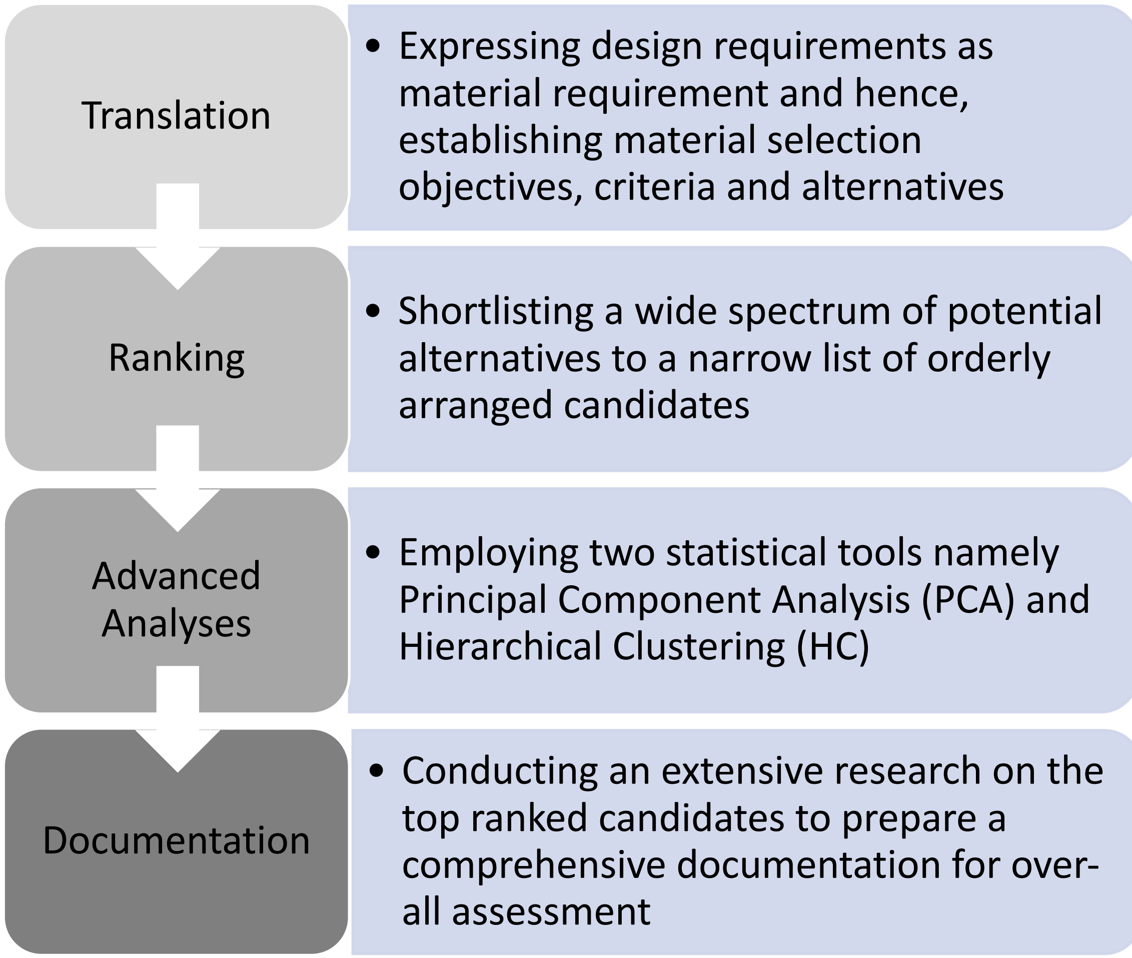


Figure 24: Proposed material selection strategy.

4.1 Translation

The translation is a process of transforming the design/concept requirements into material specifications. It is a multifaceted task which requires collaborative multidisciplinary action. It is a crucial phase which lays the foundation to the entire material selection process as the objectives of the problem are established and selection criteria are derived. Objective(s) signifies the aspirational goals setup by the engineers/decision-makers for the material selection problem. These are usually influenced by organizational goals and market requirements.

Selection criteria are usually expressed as material properties such as density, ultimate tensile strength, modulus of elasticity, magnetic coercivity, electrical resistivity, maximum service temperature, etc. Moreover, the objectives of the problem set the limitations on these criteria. The limitations can be quantitative in nature such as density $\geq 7\ g/cm^3$ or qualitative such as magnetic coercivity must be high or low.

Figure 25 represents a flow chart of the translation phase with its constituent elements. Criteria weights are extracted from the design requirements and are covered aptly in the ranking phase. Once these preliminaries are sorted out, we can go ahead with the next step.

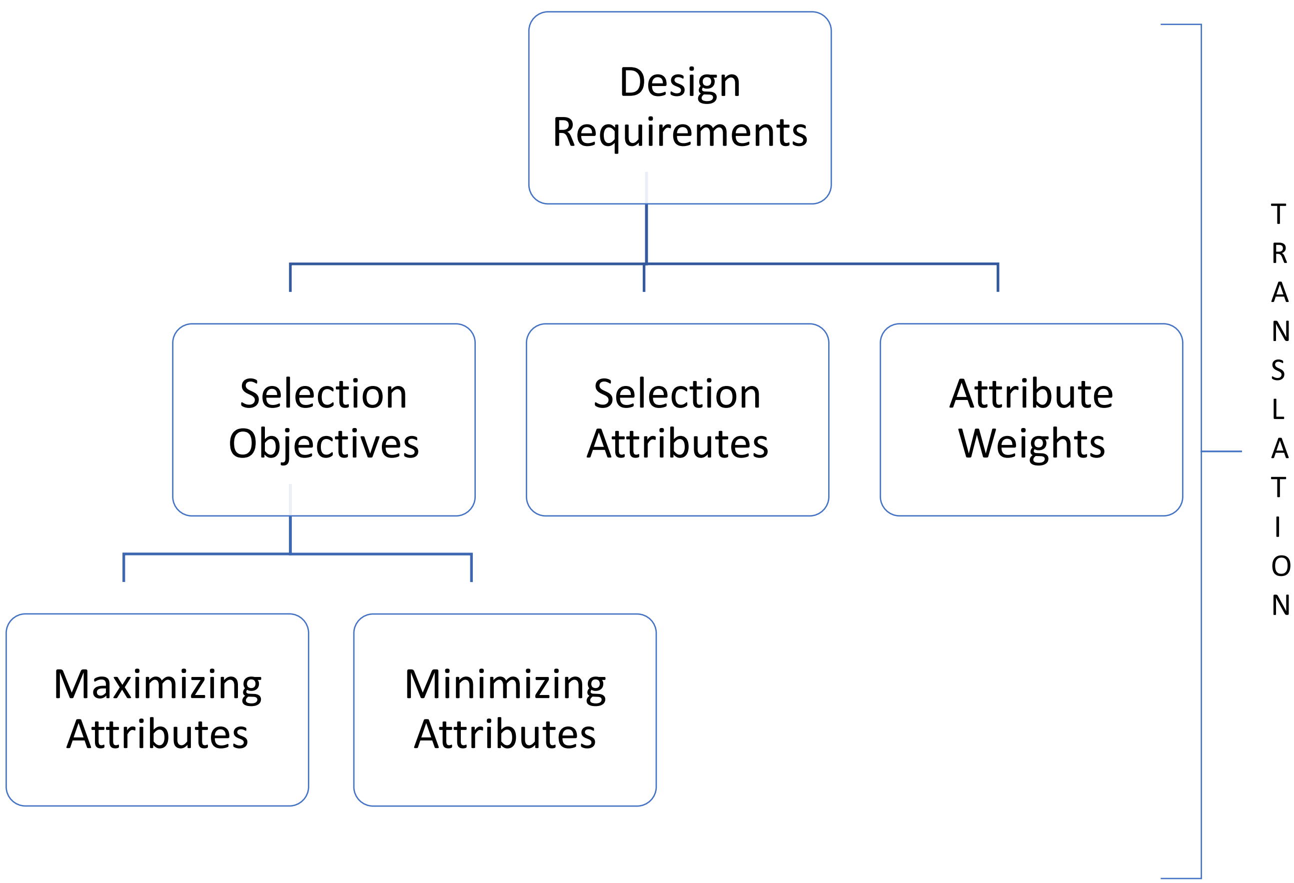


Figure 25: Various stages in translation.

4.2 Ranking

To avoid any bias, it is essential to consider every material candidate as a potential solution to the selection problem. Usually, a traditional material selection strategy must employ a screening process to ensure the shortlisting of candidates from an overwhelming menu of potential alternatives. There is no separate screening phase in this new methodology, as screening and ranking are done simultaneously by applying multiple attribute decision making (MADM) framework. For evaluating and finally ranking the material alternatives, MADM model is employed and its basic components such as Decision Matrix (DM), attributes, alternatives, and attribute weights are computed. Basically, in this step, the material selection problem is modeled into a MADM framework.

For more insight on the MADM framework and its components, please refer chapter-2.

One of the important components of MADM framework is attribute (or criteria) weights. This is a data-driven methodology, so the weights of the attributes are obtained using Shannon's Entropy method, which is undoubtedly the most accepted and reliable objective weighting techniques.

There are many distinct MADM methods available for ranking the alternatives. Most of the MADM methods proceed similarly as shown in the flowchart (figure 7). However, the distinction lies in the way they aggregate the information provided in the decision matrix and produce ranks. Each MADM method presents a unique aggregation strategy for evaluating the set of alternatives.

In order to take advantage of the unique aggregation techniques employed by different MADM methods, twenty-five methods belonging to distinct classes/categories have been chosen to accomplish the task of ranking the material alternatives.

Different MADM methods applied in this methodology are listed in the table below:

Table 2: List of MADM methods applied.

S/N	Name of the Method	Classification
1	TOPSIS	Distance-based
2	EDAS	Distance-based
3	SBM	Distance-based
4	CODAS	Distance-based
5	DIA	Distance-based
6	WEDBA	Distance-based
7	VIKOR	Compromise
8	SIR-SAW	Outranking
9	SIR-TOPSIS	Outranking
10	PROMETHEE	Outranking
11	EXPROM	Outranking
12	SAW	Weighted-mean
13	SMART	Weighted-mean
14	MEW	Exponential Weighting
15	MULTIMOORA	Exponential Weighting
16	WASPAS	Combinative
17	GRA	Grey-theory
18	MOORA	Objective optimization
19	MOOSA	Objective optimization
20	ARAS	Other
21	ROVM	Other
22	LFA	Other
23	COPRAS	Other
24	PSI	Other
25	OCRA	Other

Each of the above-mentioned MADM methods exhibits a unique mathematical aggregation procedure to rank the alternatives. Application of such distinct aggregation procedures generates a robust set of ranks of the candidates, which will strengthen the results and make them more reliable. However, the ranks produced by each method may not completely coincide with the other sets of ranks, but they will be co-relatable. Spearman's correlation coefficient method is used to establish the correlation quantitatively between the different sets of ranks and in the process, streamline the results obtained from these different MADM methods to make ranking more robust. Figure 26 shows an overview of the ranking phase.

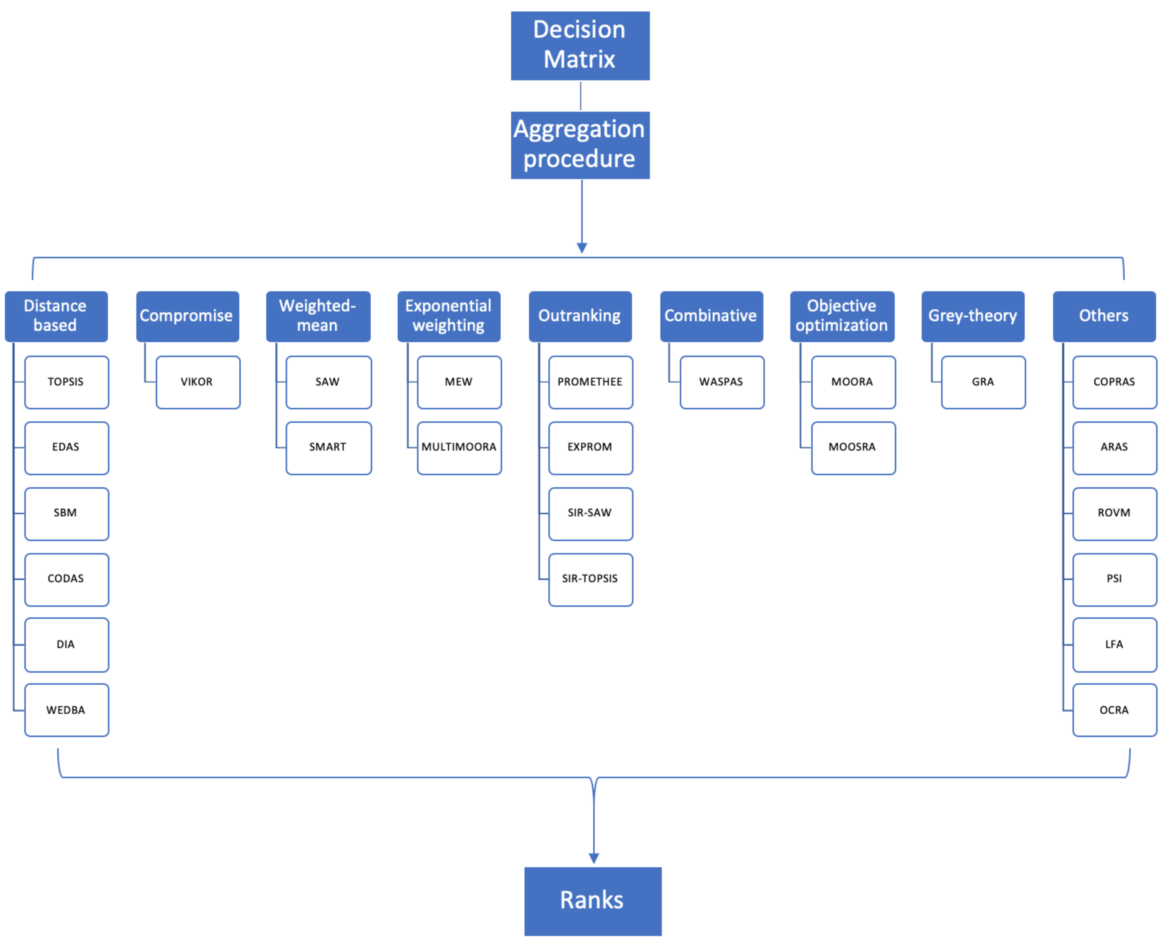


Figure 26: Overview of the ranking phase.

Once ranks are obtained from the application of different MADM methods, correlation is performed to streamline the results.

The ranks produced in this phase will be further analyzed using data analysis techniques such as hierarchical clustering (HC) and principal component analysis (PCA) to provide a visual representation of the outcomes.

4.3 Advanced analyses

Advance analyses has been conducted to serve two purposes a) to visual represents the results obtained in the ranking phase, b) to conduct a comprehensive analysis of the material candidates with respect to the material properties, in order to provide valuable information to the decision-maker/engineer that will assist in the final selection.

For accomplishing the first motive, the ranks obtained from various MADM techniques are analyzed using a statistical tool called Principal Component Analysis (PCA). In statistics, data indicating ranks are called ordinal data. So, PCA is conducted on the ordinal data obtained from the ranking phase to visual represents the final rank order of the material candidates. Presenting results using visual aids such as graphs, charts, or dendrogram, helps the decision-maker(s) to understand the results better.

Now, for achieving the second motive, statistical tools PCA and Hierarchical Clustering (HC) are employed to analyze material candidates with respect to the material properties (attributes). In statistics, data indicating quantities such as properties are called cardinal data. Hierarchical clustering assists in grouping materials into clusters based on their relative similarity and graphically presents them using a dendrogram. Similarly, PCA represents the results in a reduced dimensional space using a PCA plot. A pictorial representation of the flow-chart of advanced analyses operation is shown in figure 27.

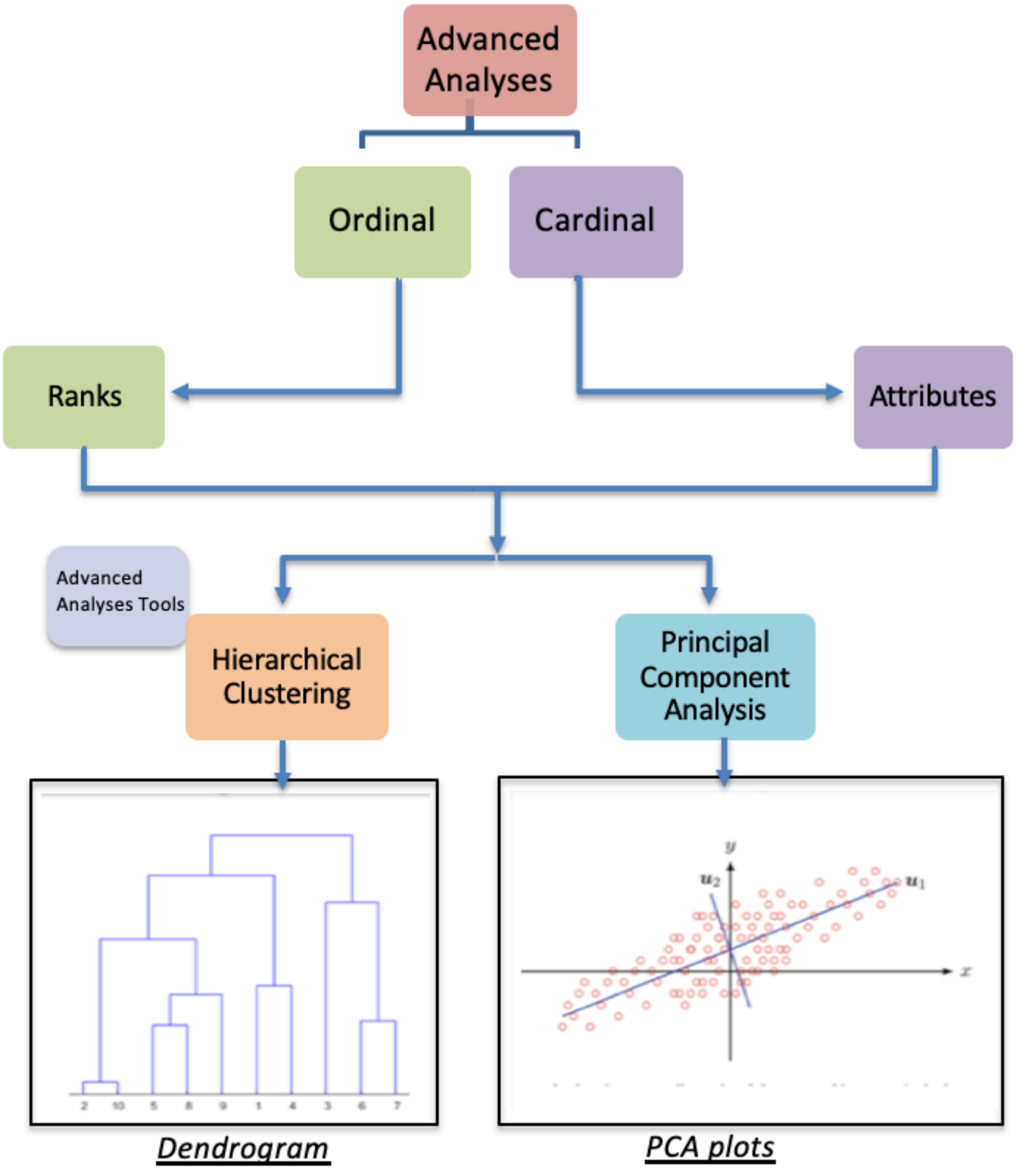


Figure 27: Process layout of the advanced analyses operation.

PCA conducted on ordinal data (i.e., ranks) does not just provide a visual representation of the final ranked order of the material candidates, but also analyze the ranks obtained from 25 MADM methods to generate a final rank set. Moreover, PCA conducted on raw cardinal data generates multifaceted graphical plots that provide a deeper understanding of the relationship between materials and their properties (or selection criteria). PCA plots such as biplot score, loading, and score (eigen plot) plots provide additional information that on the factors that influence material selection. This information assists the engineer to make a well-versed decision.

4.4 Documentation

Ranking of the material candidates does not conclude the material selection process, nor the principal component analysis or clustering. Materials are ranked based on the limited selection attributes, and there may be some other hidden aspects of these candidates that must be taken into account before making a decision. So, a detailed history of these candidates must be acquired and studied to conclusively decide which material will be the optimal fit for the considered problem. In the documentation phase, a detail investigation of the material is conducted on various factors such as machinability, availability, cost, previous applications, etc.

4.5 Final selection

Finally, a top-ranked material with a strong background is selected for the given application. The selected material may not be ranked number one but will definitely be one of the top-ranked candidates. Figure 28 represents the overview of the proposed methodology.

In the next section, an application of this methodology to a hard-magnetic material selection problem is proposed.

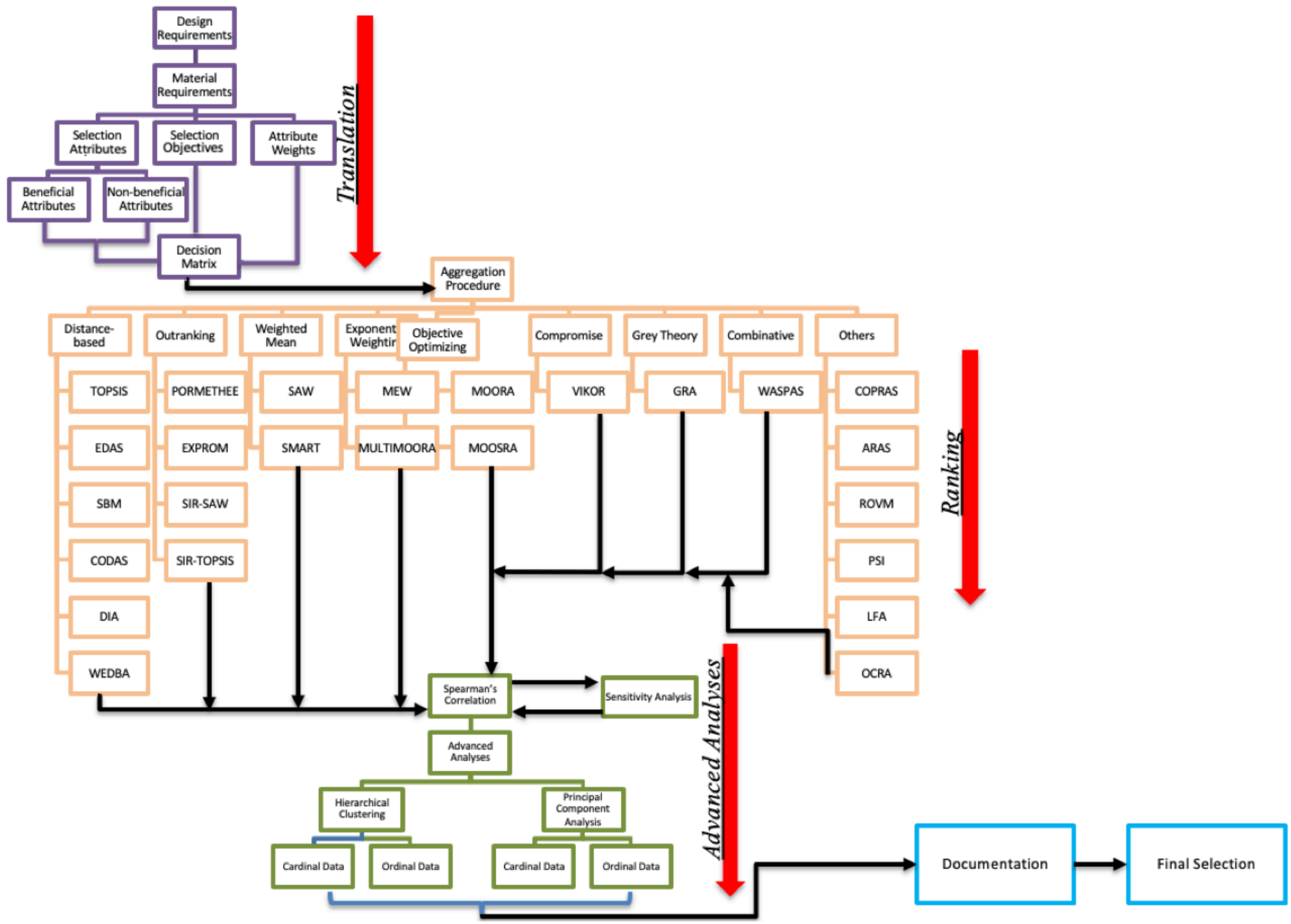


Figure 28: Overview of the material selection methodology.

4.6 Application of the proposed methodology

Potential usage of permanent magnets is a matter of interest in the scientific world today. Growing concern about global warming has scientific and technological implications, and many of these impinge on the use of permanent magnets. Some of the present-day application of the permanent magnets are listed below:

a) Automotive: starter motors, anti-lock braking systems (ABS), motor drives for wipers, injection pumps, fans and controls for windows, seats etc., loudspeakers, eddy current brakes, alternators.

b) Telecommunications: loudspeakers, microphones, telephone ringers, electro-acoustic pick-ups, switches and relays.

c) Data Processing: disc drives and actuators, stepping motors, Printers.

d) Consumer Electronics: DC motors for showers, washing machines, drills, low voltage DC drives for cordless appliances, loudspeakers for TV and audio, TV beam correction and focusing device, compact-disc drives, home computers, video recorders, clocks.

e) Electronic and Instrumentation: sensors, contactless switches, NMR spectrometer, energy meter disc, electro-mechanical transducers, crossed field tubes, flux-transfer trip device, dampers.

f) Industrial: DC motors for magnetic tools, robotics, magnetic separators for extracting metals and ores, magnetic bearings, servo-motor drives, lifting apparatus, brakes and clutches, meters, and measuring equipment.

g) Astro and Aerospace: frictionless bearings, stepping motors, couplings, instrumentation, travelling wave tubes, auto-compass.

h) Bio-surgical: dentures, orthodontics, orthopedics, wound closures, stomach seals, repulsion collars, ferromagnetic probes, cancer cell separators, magnetomotive artificial hearts, NMR / MRI body scanner.

That is not it! Future uses could include their more widespread use in “white goods” such as washing machines, refrigerators etc., in order to improve energy efficiency and hence reduce carbon dioxide emissions [14]. Another potential area of application could be in clean energy production, such as in windmills. The biggest potential, however, is in electric vehicles (EVs) which could be hybrid vehicles or totally driven by electricity in the form of batteries or a fuel cell. There has been an enormous increase in interest and activity in this area over the past 5 years [14].

With such a long list of applications and potential applications, it is quite a challenge to pick an optimal permanent magnet that satisfies the design requirements from a catalog that constitutes over fifty potential candidates available for the engineers to choose from — such circumstances laydown a perfect platform for applying the proposed methodology.

To assess permanent magnets, it is crucial to include all the essential parameters that define the objectives of the design. Parameters here indicates the material properties namely: Magnetic Coercive force, Electrical Resistivity, Magnetic Remanence, Magnetic Maximum Energy Product, Curie Temperature, Maximum Service Temperature, Coefficient of Linear Thermal Expansion and Density. There is a specific reason behind selecting these eight selection criteria i.e., as most of the designs employing permanent magnets tries to amplify the magnetic behavior to the fullest, which is expressed and measured by properties such as Magnetic Coercive force, Electrical Resistivity, Magnetic Remanence and Magnetic Maximum Energy Product. The other four attributes sum up the other crucial physical properties that have been pertinent in the modern-day design.

Attributes such as cost, manufacturability, and environmental factors are not considered for the moment. These attributes will be taken into account during the documentation phase and to be frank, can play an influential role in the final selection.

Translation

As mentioned in the previous section, the first phase of the proposed methodology is Translation, where the design requirements are construed as the material selection objectives and criteria. When dealing with functional materials such as magnetic materials, the design requirements usually focus on elevating their magnetic behavior. Therefore, the objective(s) of the selection problem can be summarized as the evaluation of permanent magnetic materials on various parameters to identify the optimal material candidate. The list of material alternatives with their chemical composition is listed in the table below.

Table 3: Material candidates with chemical composition.

S/N	**Name**	**Chemical Composition**
1	Alnico® 1 (Isotropic, Cast)	59Fe-12Al-21Ni-5Co-3Cu
2	Alnico® 12 (Anisotropic, Cast)	33Fe-6Al-18Ni-35Co-8Ti
3	Alnico® 2 (Isotropic, Cast)	55Fe-10Al-19Ni-13Co-3Cu
4	Alnico® 2 (Isotropic, Sintered)	52Fe-10Al-19Ni-13Co-3Cu-3Ti
5	Alnico® 3 (Isotropic, Cast)	60Fe-12Al-25Ni-3Cu
6	Alnico® 4 (Isotropic, Cast)	56Fe-12Al-27Ni-5Co
7	Alnico® 4 (Sintered)	55Fe-12Al-28Ni-5Co
8	Alnico® 5 (Anisotropic, Cast)	51Fe-8.5Al-14.5Ni-24Co-3Cu
9	Alnico® 5 (Anisotropic, Sintered)	48Fe-8.5Al-14.5Ni-24Co-3Cu-3Ti
10	Alnico® 5-7 (Anisotropic, Cast)	51Fe-8.5Al-14.5Ni-24Co-3Cu
11	Alnico® 5DG (Anisotropic, Cast)	51Fe-8.5Al-14.5Ni-24Co-3Cu
12	Alnico® 6 (Anisotropic, Cast)	46Fe-8Al-16Ni-24Co-3Cu-1Ti

13	Alnico® 6 (Anisotropic, Sintered)	47Fe-8Al-15Ni-24Co-3Cu-3Ti
14	Alnico® 7 (Anisotropic, Cast)	40Fe-8Al-18Ni-24Co-5Cu-5Ti
15	Alnico® 8 (Anisotropic, Cast)	34Fe-7Al-15Ni-35Co-4Cu-5Ti
16	Alnico® 8 (Anisotropic, Sintered)	34Fe-7Al-15Ni-35Co-4Cu-5Ti
17	Alnico® 8HC (Anisotropic, Cast)	29Fe-8Al-14Ni-38Co-3Cu-8Ti
18	Alnico® 8HC (Anisotropic, Sintered)	35Fe-7Al-14Ni-38Co-3Cu-3Ti
19	Alnico® 9 (Anisotropic, Cast)	34Fe-7Al-15Ni-35Co-4Cu-5Ti
20	Carpenter® P6 Alloy	Fe-45Co-6Ni-5V
21	Chromindur® II Alloy	Fe-28Cr-10.5Co
22	Cobalt Samarium 1	67Co-34Sm ($SmCo_5$)
23	Cobalt Samarium 2	67Co-34Sm ($SmCo_5$)
24	Cobalt Samarium 3	67Co-34Sm ($SmCo_5$)
25	Cobalt Samarium 4	77Co-23Sm (Sm_2Co_{17})
26	Cunife® Alloy	20Fe-20Ni-60Cu
27	Ferrite 1, Sintered Iron Barium Oxide	BaO-$6Fe_2O_3$
28	Ferrite 2, Sintered Iron Barium Oxide	BaO-$6Fe_2O_3$
29	Ferrite 3, Sintered Iron Barium Oxide	BaO-$6Fe_2O_3$
30	Ferrite 4, Sintered Iron Strontium Oxide	SrO-$6Fe_2O_3$
31	Ferrite 5, Sintered Iron Strontium Oxide	SrO-$6Fe_2O_3$
32	Ferrite A, Bonded Iron Barium Oxide	BaO-$6Fe_2O_3$ + Organic Binder
33	Ferrite B, Bonded Iron Barium Oxide	BaO-$6Fe_2O_3$ + Organic Binder
34	High Cobalt Steel	Fe-36Co-3.75W-5.75Cr-0.8C
35	Low Cobalt Steel	Fe-17Co-8.5W-2.5Cr-0.7C
36	Neodymium® 27	NdFeB
37	Neodymium® 27H	NdFeB
38	Neodymium® 30	NdFeB
39	Neodymium® 30H	NdFeB
40	Neodymium® 35	NdFeB
41	Neodymium® 35H	NdFeB
42	Platinum-Cobalt Alloy	76.7Pt-23.3Co
43	Tungsten Steel	Fe-6W-0.5Cr-0.7C
44	Vicaloy® I Alloy	Fe-52Co-10V-0.5Mn
45	Vicaloy® II Alloy	Fe-52Co-14V

Objective(s) of a material selection problem expresses the design goals in terms of material parameters by setting up the conditions to achieve these goals. Once the objectives are established, the next step is to derive attributes. Attributes are derived from the design requirements and also compliment the material selection objectives. The attributes for the considered selection problem are presented in the table below:

Table 4: List of attributes.

Property	Unit
1. Magnetic Coercive force ($\boldsymbol{H_c}$)	$Oersted$
2. Magnetic Remanence ($\boldsymbol{M_r}$)	$Gauss$
3. Magnetic Maximum Energy product $(\boldsymbol{BH})_{\boldsymbol{max}}$	$MGOe$
4. Electrical Resistivity (**ρ**)	$Ohm - cm$
5. Curie temperature ($\boldsymbol{T_c}$)	℃
6. Maximum Service temperature ($\boldsymbol{S_t}$)	℃
7. Coefficient of Linear thermal Expansion (**α**)	$\mu m/m$℃
8. Density (***d***).	g/cm^3

Now, to accomplish the material selection objective(s), some of the attributes must be maximized and some must be minimized. These conditions are established by the objective(s) itself. The table 5, lists the attributes that are maximized and the attributes that are minimized.

Table 5: Maximizing attributes and minimizing attributes.

Maximizing Attributes	Minimizing Attributes
1) Electrical Resistivity ($\boldsymbol{\rho}$). 2) Magnetic Coercive force ($\boldsymbol{H_c}$). 3) Magnetic Remanence ($\boldsymbol{M_r}$). 4) Magnetic Maximum Energy product $(\boldsymbol{BH})_{\boldsymbol{max}}$ 5) Curie temperature ($\boldsymbol{T_c}$). **6)** Maximum Service temperature ($\boldsymbol{S_t}$).	1) Density ($\boldsymbol{d}$). 2) Coefficient of Linear Thermal Expansion ($\boldsymbol{\alpha}$).

Determination of the selection objectives, criteria (attributes), and alternatives signals the end of the translation phase.

Ranking

In the ranking phase, the material candidates are ranked using Multiple Attribute Decision Making (MADM) . However, before applying the MADM methods, the material selection problem should be transformed into MADM model. The remodeling of the selection problem is facilitated by the construction of Decision Matrix (DM). The definition and the basic components of DM are covered in chapter-2.

Decision Matrix (DM) for the present problem is constructed below:

Table 6: Decision matrix

Attributes→

Alternatives→

Name	d	α	ρ	H_c	M_r	$(BH)_{max}$	T_c	S_t
Alnico® 1 (Isotropic, Cast)	6900	12.6	0.00000075	37.00352427	0.72	11.064	1053	723
Alnico® 12 (Anisotropic, Cast)	7400	11	0.00000062	63.98028712	0.6	14.01	1133	753
Alnico® 2 (Isotropic, Cast)	7100	12.4	0.00000065	44.96127142	0.75	13.452	1083	813
Alnico® 2 (Isotropic, Sintered)	6800	12.4	0.00000068	44.00634176	0.71	11.9	883	753
Alnico® 3 (Isotropic, Cast)	6900	13	0.0000006	38.0380314	0.7	11.001	1033	753
Alnico® 4 (Isotropic, Cast)	7000	13.1	0.00000075	58.01197676	0.535	10.03	1073	863
Alnico® 4 (Sintered)	6900	13.1	0.00000068	56.02253997	0.52	10.03	1073	863
Alnico® 5 (Anisotropic, Cast)	7300	11.4	0.00000047	51.00915926	1.265	43.78	1133	798
Alnico® 5 (Anisotropic, Sintered)	6900	11.3	0.0000005	49.01972247	1.065	30.009	1133	813
Alnico® 5-7 (Anisotropic, Cast)	7300	11.4	0.00000047	58.01197676	1.335	59.7	1163	773
Alnico® 5DG (Anisotropic, Cast)	7300	11.4	0.00000047	52.99859605	1.33	57.71	1173	773
Alnico® 6 (Anisotropic, Cast)	7400	11.4	0.0000005	61.99085033	1.05	30.964	1133	798
Alnico® 6 (Anisotropic, Sintered)	6900	11.4	0.00000054	63.02535746	0.94	23.084	1133	813
Alnico® 7 (Anisotropic, Cast)	7300	11.4	0.00000058	84.03380995	0.857	30.009	1113	813
Alnico® 8 (Anisotropic, Cast)	7300	11	0.00000053	130.9845182	0.825	42.188	1133	823
Alnico® 8 (Anisotropic, Sintered)	7000	11	0.00000054	118.96832	0.74	31.76	1133	813
Alnico® 8HC (Anisotropic, Cast)	7300	11	0.00000054	151.038041	0.72	39.641	1133	823
Alnico® 8HC (Anisotropic, Sintered)	7000	11	0.00000054	143.0007164	0.67	4.5372	1133	813
Alnico® 9 (Anisotropic, Cast)	7300	11	0.00000053	114.9894464	1.06	71.64	1133	793
Carpenter® P6 Alloy	8160	13	0.00000033	5.013380707	1.44	16	1133	683
Chromindur® II Alloy	7900	21	0.00000075	31.99014356	1.01	16	903	773
Cobalt Samarium 1	8200	12	0.0000005	720.0169625	0.92	170.03	998	523
Cobalt Samarium 2	8200	12	0.0000005	639.9620262	0.86	145.03	998	773
Cobalt Samarium 3	8200	12	0.0000005	534.9993412	0.8	120.04	998	773
Cobalt Samarium 4	8200	12	0.0000005	639.9620262	1.13	240.07	1073	773
Cunife® Alloy	8600	12	0.00000018	44.00634176	0.54	12.02	683	623
Ferrite 1 , Sintered Iron Barium Oxide	4800	10.1	10000	144.9901532	0.22	8.0396	723	673
Ferrite 2 , Sintered Iron Barium Oxide	5000	10.1	10000	174.9908599	0.38	26.984	723	673
Ferrite 3, Sintered Iron Barium Oxide	4500	10.1	10000	240.0056542	0.32	19.98	723	673
Ferrite 4, Sintered Iron Strontium Oxide	4800	10.1	10000	174.9908599	0.4	30.009	723	733
Ferrite 5, Sintered Iron Strontium Oxide	4500	10.1	10000	250.0324156	0.355	24.039	733	673
Ferrite A, Bonded Iron Barium Oxide	3700	10.1	10000	155.0169146	0.214	8.0396	723	368
Ferrite B, Bonded Iron Barium Oxide	3700	10.1	10000	91.99155711	0.14	3.0248	723	368

High Cobalt Steel	8180	20	0.00000027	19.0190157	0.975	7.4028	1163	783
Low Cobalt Steel	8350	20	0.00000028	14.00563499	0.95	5.174	983	763
Neodymium® 27	7400	6.5	10000	739.9909079	1.08	215.08	553	353
Neodymium® 27H	7400	6.5	10000	779.7000662	1.08	215.08	573	373
Neodymium® 30	7400	6.5	10000	796.0134479	1.1	238.08	553	353
Neodymium® 30H	7400	6.5	10000	835.9613386	1.1	223.04	573	373
Neodymium® 35	7400	6.5	10000	835.9613386	1.205	273.82	553	353
Neodymium® 35H	7300	6.5	10000	898.9866961	1.18	248.35	571	353
Platinum Cobalt Alloy	15500	11	0.00000028	354.9951006	0.645	74.028	753	623
Tungsten Steel	8120	6.5	0.0000003	5.888732894	0.95	2.6268	1033	703
Vicaloy® I Alloy	8160	12	0.00000063	16.71126902	1.07	11.144	1128	823
Vicaloy® II Alloy	8160	12	0.00000063	42.01690498	1	28.019	973	773

The formulation of the problem into MADM format is completed with the construction of the DM. The next crucial step before applying MADM methods to rank the material candidates is to determine the weights of each attribute. In the next chapter, the results and discussions arising from the application of this novel methodology are presented in detail.

Chapter 5: Results and Discussions

The results obtained from the application of the proposed methodology to a hard-magnetic material selection problem is discussed in detail in this section.

5.1 Attribute weights

The proposed methodology is primarily data-driven, so to understand the pulse of the data, an objective method is employed to determine the weights of the attributes. Shannon's Entropy method is utilized for this purpose. The weights of each attribute obtained from the Entropy method is presented in the table below:

Table 7: Weights of each attribute as obtained by Shannon's Entropy method.

S.No.	Attributes	Weights
1	Density (d)	0.00986
2	Coefficient of Linear Thermal Expansion (α)	0.01297
3	Electrical Resistivity (ρ)	0.44133
4	Magnetic Coercive Force (H_c)	0.21992
5	Magnetic Remanence (M_r)	0.02967
6	Magnetic Max Energy Product (BH_{max})	0.22929
7	Curie Temperature (T_c)	0.02010
8	Max Service Temperature (S_t)	0.03686

From the figure 29, it is evident that Electrical resistivity has emerged as the most important attribute with the weight of almost 44%, followed by Magnetic maximum energy product and magnetic coercive force, which weigh almost equally. As mentioned earlier, the attributes that elevate the magnetic behavior usually have a greater say in the function-oriented design. Electrical resistivity is the major factor that amplifies the magnetic effects to a higher degree than most factors. The remaining attributes weigh marginal due to the context of the design. Density is weighed the least.

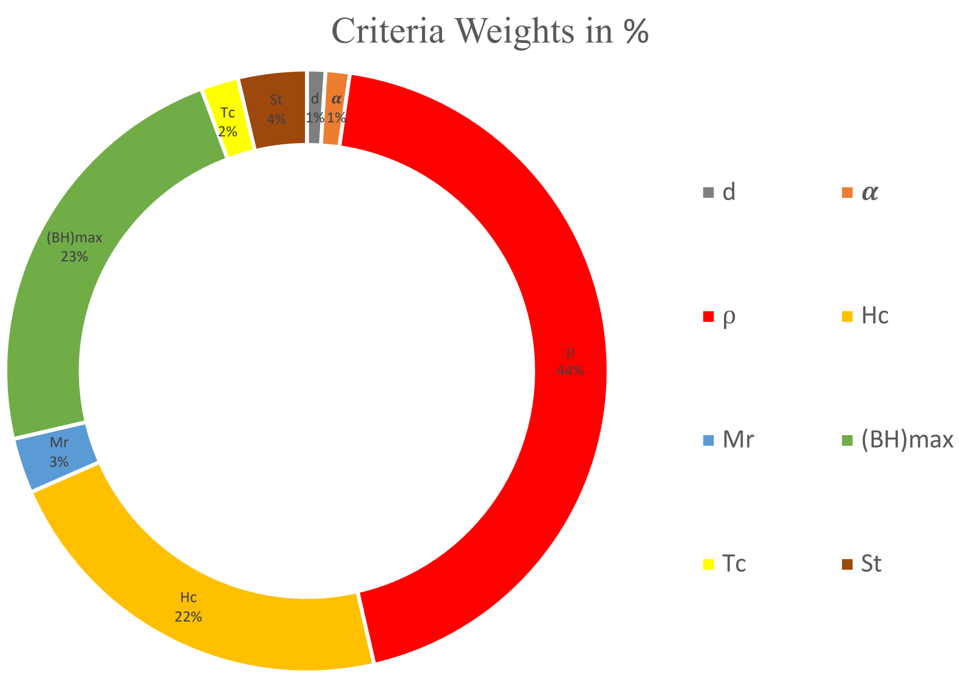


Figure 29: Criteria weights obtained by Shannon's Entropy method.

Once these intrinsic components of the MADM are determined, the ranking of the candidates begins by the application of different MADM tools.

5.2 Ranking

The ranks obtained from different MADM methods are presented in table 7.

Neodymium® 35 is ranked as the best permanent magnet from a list of 45 materials by the majority of MADM methods. Followed by Neodymium® 35H and remaining magnetic materials from Neodymium family take up top 6 positions. The worst magnetic material is a tossup between Cufine alloy, low Cobalt steel, and Tungsten steel.

Table 8: Ranks of the material candidates obtained from 25 MADM methods.

S.No.	Name	SAW	SMART	MEW	WASPAS	TOPSIS	VIKOR	PSI	ARAS	ROVM	LFA	EDAS	SBM	GRA	WEDBA	COPRAS
1	Alnico® 1 (Isotropic, Cast)	40	39	37	40	40	39	40	39	39	39	38	40	41	42	40
2	Alnico® 12 (Anisotropic, Cast)	33	33	31	33	33	33	33	32	33	33	33	32	34	38	32
3	Alnico® 2 (Isotropic, Cast)	34	34	34	34	35	34	34	33	34	34	34	34	35	37	34
4	Alnico® 2 (Isotropic, Sintered)	39	38	36	39	39	38	38	38	38	38	42	38	43	40	38
5	Alnico® 3 (Isotropic, Cast)	41	41	39	41	42	41	41	40	41	41	37	39	42	43	39
6	Alnico® 4 (Isotropic, Cast)	37	36	32	37	36	36	37	36	36	36	35	35	36	41	36
7	Alnico® 4 (Sintered)	38	37	35	38	37	37	39	37	37	37	36	36	37	44	37
8	Alnico® 5 (Anisotropic, Cast)	24	24	26	24	24	24	24	25	24	24	28	25	22	24	25
9	Alnico® 5 (Anisotropic, Sintered)	28	28	30	28	29	28	28	28	28	28	21	28	27	28	28
10	Alnico® 5-7 (Anisotropic, Cast)	20	20	24	20	20	20	20	20	20	20	26	22	20	20	20
11	Alnico® 5DG (Anisotropic, Cast)	21	21	25	21	21	21	21	23	21	21	27	23	21	21	23
12	Alnico® 6 (Anisotropic, Cast)	27	27	27	27	28	27	26	27	27	27	30	27	26	26	27
13	Alnico® 6 (Anisotropic, Sintered)	29	29	29	29	31	29	29	29	29	29	23	31	29	32	29
14	Alnico® 7 (Anisotropic, Cast)	26	26	23	26	27	26	27	26	26	26	29	26	28	29	26
15	Alnico® 8 (Anisotropic, Cast)	23	23	21	23	23	23	22	22	23	23	25	21	23	23	22
16	Alnico® 8 (Anisotropic, Sintered)	25	25	22	25	25	25	25	24	25	25	19	24	25	25	24
17	Alnico® 8HC (Anisotropic, Cast)	22	22	20	22	22	22	23	21	22	22	24	20	24	22	21
18	Alnico® 8HC (Anisotropic, Sintered)	30	30	38	30	26	30	30	30	30	30	22	29	31	30	30
19	Alnico® 9 (Anisotropic, Cast)	19	19	18	19	19	19	19	19	19	19	18	19	19	19	19
20	Carpenter® P6 Alloy	32	32	43	32	32	32	32	35	32	32	44	41	30	27	35
21	Chromindur® II Alloy	35	40	33	35	34	40	36	34	40	40	43	33	39	34	33
22	Cobalt Samarium 1	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
23	Cobalt Samarium 2	16	16	16	16	16	16	16	16	16	16	12	16	16	16	16
24	Cobalt Samarium 3	17	17	17	17	17	17	17	17	17	17	16	17	17	17	17
25	Cobalt Samarium 4	14	14	14	14	14	14	14	14	14	14	10	14	14	14	14
26	Cunife® Alloy	44	44	41	44	41	44	45	41	44	44	45	37	45	45	41
27	Ferrite 1 , Sintered Iron Barium Oxide	12	11	12	12	12	11	11	12	11	11	14	11	11	12	12
28	Ferrite 2 , Sintered Iron Barium Oxide	10	10	9	10	10	9	10	10	10	10	11	9	10	10	10
29	Ferrite 3, Sintered Iron Barium Oxide	8	8	10	9	8	10	8	8	8	8	9	8	8	8	8
30	Ferrite 4, Sintered Iron Strontium Oxide	9	9	8	8	9	8	9	9	9	9	7	10	9	9	9
31	Ferrite 5, Sintered Iron Strontium Oxide	7	7	7	7	7	7	7	7	7	7	8	7	7	7	7
32	Ferrite A, Bonded Iron Barium Oxide	11	12	11	11	11	12	12	11	12	12	13	12	12	11	11
33	Ferrite B, Bonded Iron Barium Oxide	13	13	13	13	13	13	13	13	13	13	17	13	13	13	13
34	High Cobalt Steel	42	43	42	42	43	43	43	43	43	43	40	43	40	36	43
35	Low Cobalt Steel	45	45	44	45	45	45	44	44	45	45	41	44	44	39	44
36	Neodymium® 27	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
37	Neodymium® 27H	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
38	Neodymium® 30	3	3	3	3	3	3	3	3	3	3	3	4	3	3	3
39	Neodymium® 30H	4	4	4	4	4	4	4	4	4	4	4	3	4	4	4
40	Neodymium® 35	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1
41	Neodymium® 35H	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2
42	Platinum Cobalt Alloy	18	18	19	18	18	18	18	18	18	18	20	18	18	18	18
43	Tungsten Steel	43	42	45	43	44	42	42	45	42	42	31	45	38	35	45
44	Vicaloy® I Alloy	36	35	40	36	38	35	35	42	35	35	39	42	33	33	42
45	Vicaloy® II Alloy	31	31	28	31	30	31	31	31	31	31	32	30	32	31	31

Continuation of Table 8..

S.No.	Name	MOORA		MULTIMOORA	MOORSA	OCRA	CODAS	DIA	SIR-SAW		SIR-TOPSIS			PROMETHEE			EXPROM
		RSA	RPA						Qnet	(Qr)	S-Flow	I-Flow	RELATIVE	Net	Positive Outranking flow	Negative Outranking flow	
1	Alnico® 1 (Isotropic, Cast)	40	40	37	38	38	44	40	31	31	30	29	30	31	31	30	32
2	Alnico® 12 (Anisotropic, Cast)	32	32	31	32	32	40	32	25	25	27	22	24	25	23	26	24
3	Alnico® 2 (Isotropic, Cast)	34	34	34	33	35	38	34	27	27	29	25	26	27	25	29	25
4	Alnico® 2 (Isotropic, Sintered)	37	37	36	35	37	43	37	30	30	28	27	29	30	30	31	31
5	Alnico® 3 (Isotropic, Cast)	41	41	39	39	39	42	41	40	40	38	38	39	40	40	40	40
6	Alnico® 4 (Isotropic, Cast)	36	36	32	36	33	34	36	23	23	23	20	21	23	22	23	23
7	Alnico® 4 (Sintered)	38	38	35	37	34	35	38	29	29	26	26	27	29	28	28	28
8	Alnico® 5 (Anisotropic, Cast)	24	24	26	25	27	24	24	39	38	39	39	40	39	38	39	38
9	Alnico® 5 (Anisotropic, Sintered)	28	28	30	28	30	29	28	38	39	40	31	38	38	39	35	39
10	Alnico® 5-7 (Anisotropic, Cast)	20	20	24	20	23	20	20	34	34	36	33	35	34	35	34	33
11	Alnico® 5DG (Anisotropic, Cast)	21	21	25	23	24	21	21	37	37	37	36	37	37	37	38	36
12	Alnico® 6 (Anisotropic, Cast)	27	27	27	27	28	28	27	33	33	34	28	32	33	33	32	34
13	Alnico® 6 (Anisotropic, Sintered)	29	29	29	30	29	30	29	28	28	30	24	28	28	29	25	29
14	Alnico® 7 (Anisotropic, Cast)	26	26	23	26	26	27	26	21	21	19	19	18	21	19	21	21
15	Alnico® 8 (Anisotropic, Cast)	23	23	21	22	21	23	23	22	22	22	18	19	22	18	22	20
16	Alnico® 8 (Anisotropic, Sintered)	25	25	22	24	22	26	25	20	20	21	16	17	20	20	19	22
17	Alnico® 8HC (Anisotropic, Cast)	22	22	20	21	20	22	22	17	17	15	13	14	17	12	17	18
18	Alnico® 8HC (Anisotropic, Sintered)	30	30	38	29	25	25	30	35	36	33	32	34	35	36	36	37
19	Alnico® 9 (Anisotropic, Cast)	19	19	18	18	19	19	19	19	19	18	17	16	19	16	20	19
20	Carpenter® P6 Alloy	33	33	43	34	42	33	33	41	41	42	41	42	41	41	41	41
21	Chromindur® II Alloy	35	35	33	43	36	37	35	24	24	24	21	22	24	26	24	27
22	Cobalt Samarium 1	15	15	15	15	15	15	15	14	14	14	9	12	14	15	15	14
23	Cobalt Samarium 2	16	16	16	16	16	16	16	15	16	16	10	13	15	17	16	16
24	Cobalt Samarium 3	17	17	17	17	17	17	17	18	18	20	12	15	18	21	18	17
25	Cobalt Samarium 4	11	11	14	14	14	14	11	13	13	13	7	9	13	11	13	11
26	Cunife® Alloy	42	42	41	41	40	45	42	42	42	41	42	41	42	42	42	42
27	Ferrite 1 , Sintered Iron Barium Oxide	13	13	12	13	12	11	13	12	12	12	37	23	12	14	12	13
28	Ferrite 2 , Sintered Iron Barium Oxide	10	10	9	11	10	10	10	9	9	9	14	10	9	9	9	9
29	Ferrite 3, Sintered Iron Barium Oxide	8	8	10	8	8	8	8	10	10	10	15	11	10	10	10	10
30	Ferrite 4, Sintered Iron Strontium Oxide	9	9	8	10	9	9	9	7	7	7	8	7	7	7	7	8
31	Ferrite 5, Sintered Iron Strontium Oxide	7	7	7	7	7	7	7	8	8	8	11	8	8	8	8	7
32	Ferrite A, Bonded Iron Barium Oxide	12	12	11	9	11	12	12	11	11	11	35	20	11	13	11	12
33	Ferrite B, Bonded Iron Barium Oxide	14	14	13	12	13	13	14	16	15	17	40	33	16	24	14	15
34	High Cobalt Steel	43	43	42	44	43	36	43	43	43	43	45	43	43	43	43	43
35	Low Cobalt Steel	45	45	44	45	44	39	45	45	45	45	43	45	45	45	45	45
36	Neodymium® 27	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
37	Neodymium® 27H	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
38	Neodymium® 30	3	3	3	3	4	3	3	4	4	4	4	4	4	4	4	4
39	Neodymium® 30H	4	4	4	4	3	4	4	3	3	3	3	3	3	3	3	3
40	Neodymium® 35	1	1	1	1	2	1	1	2	2	2	2	2	2	2	2	1
41	Neodymium® 35H	2	2	2	2	1	2	2	1	1	1	1	1	1	1	1	2
42	Platinum Cobalt Alloy	18	18	19	19	18	18	18	32	32	32	30	31	32	32	33	30
43	Tungsten Steel	44	44	45	42	45	41	44	44	44	44	44	44	44	44	44	44
44	Vicaloy® I Alloy	39	39	40	40	41	31	39	36	35	35	34	36	36	34	37	35
45	Vicaloy® II Alloy	31	31	28	31	31	32	31	26	26	25	23	25	26	27	27	26

The ranks of the material candidates are obtained from the application of the proposed methodology, and the results obtained from it needs to be discussed for understanding this material selection approach holistically. The coming sections in this chapter analyze the important aspects of the methodology and discuss the implications of the results.

Microsoft Excel software is used to obtain the ranks by applying 25 different MADM methods.

5.3 Analysis of the ranks

It is very important to examine the ranks obtained from various MADM methods/tools. Because the ranking order of the material candidates obtained from each MADM method may yield slightly dissimilar results. The reason for this dissimilarity can be credited to the fact that each MADM method ranks the alternatives (materials) using unique aggregation procedure. For example, a distance-based approach, such as the TOPSIS method ranks the alternatives with a premise that the best/optimal alternative has the shortest distance from the ideal solution [228]. Whereas, an outranking approach such as PROMETHEE or SIR, evaluates the alternatives according to the outranking principle, which works with the premise that, in a multiple attribute decision-making (MADM) problem, one alternative is said to outrank another, if it outperforms the other on enough criteria (or attributes) of sufficient importance (or criteria weights) and is not outperformed by the other option in the sense of recording a significantly inferior performance on any one criterion [OR-4]. So, when such diverse methodologies are applied, it expected to produce dissimilar sets of ranks.

5.4 Understanding the graph

Figure 30 presents a graphical representation of the ranks obtained from different MADM tools for the same list of magnetic material candidates. The X-axis and Y-axis of the graph indicate the material alternatives and their ranks, respectively. The curves joining the different set of coordinates on the graph creates a near homogeneous pattern with some irregularities. Each curve is generated by joining the set of coordinates that indicate the ranks of the material alternatives expressed in the 2-dimensional space, procured from each MADM method. To identify each curve easily, they are represented with a unique color.

Most of the curves follow a homogenous pattern indicating the similarities in the sets of ranks. However, some curves deviate from this pattern at certain intervals pointing out the unprejudiced evaluation of the alternatives by this diverse, multifaceted approach.

These kinds of irregularities are not redundant but welcomed in this proposed method because such irregularities encourage the decision-maker/engineer to view the potential candidates with a broader perceptive.

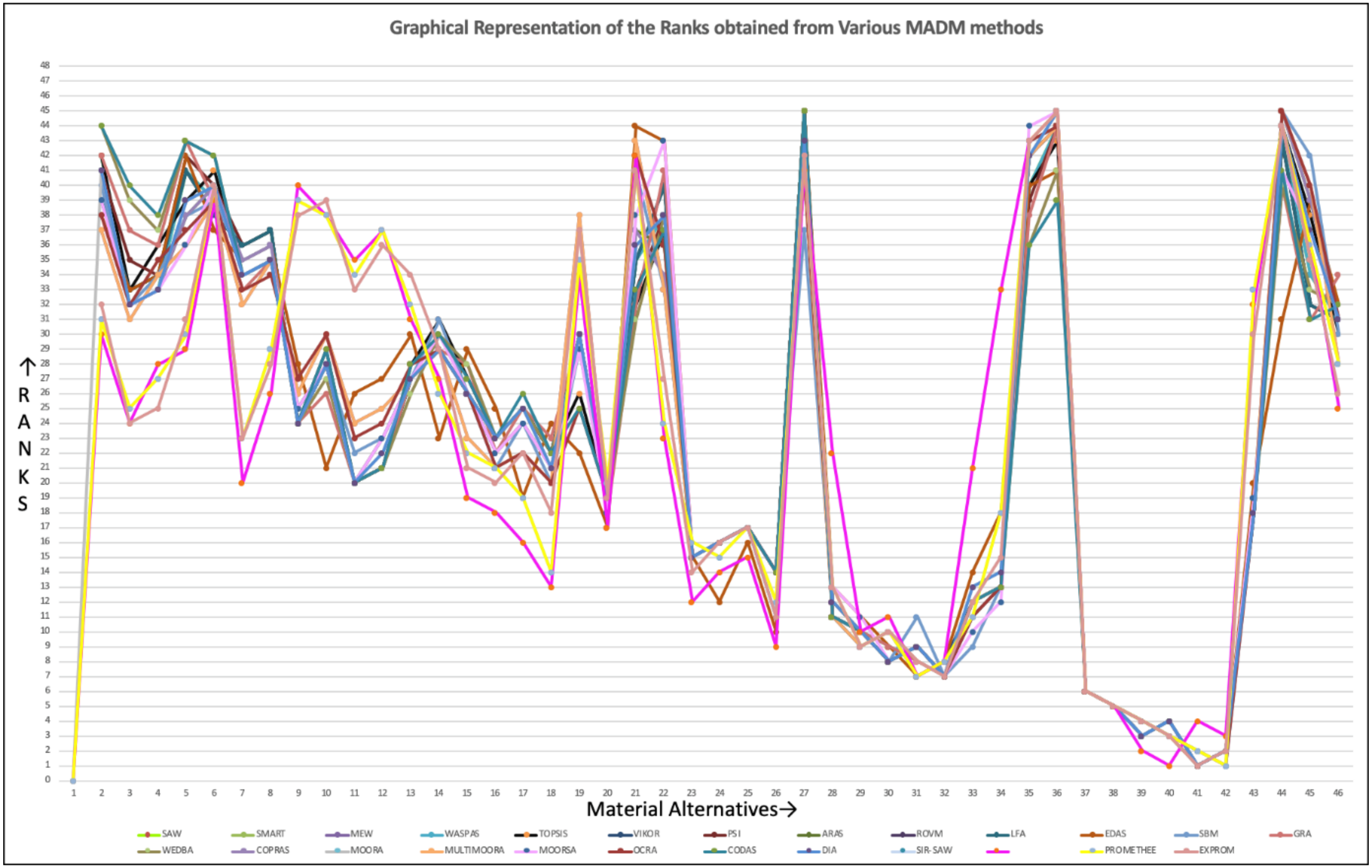


Figure 30: Graphical representation of the ranks of the material candidates obtained from 25 MADM methods.

5.5 Deviation in the graph

If Figure 30 is observed closely, we can notice two patterns being generated. One pattern consists of a stream of curves plotted using the ranks obtained from Distance-based approaches, Compromise-based approach, and other MADM methods which makeup 20 out of 25 methods. The second pattern is plotted primarily using the ranks acquired from outranking methods. As already mentioned in the previous section, the outranking methods look for the solution (i.e., alternative/candidate) which dominates most of the alternatives in majority attributes and gets least dominated by other alternatives. The MADM tools employed in this methodology approach the decision problem differently with unique selection principles. Outranking methods have been very reliable in selecting optimal solutions in many cases, and its theory is well established. The ranks obtained by these outranking methods offer a broader view of the potential candidates and cannot be ignored.

Figure 31, graphically represents the ranks obtained by outranking methods and a mean curve plotted using the average ranks derived from the rest of the methods to show the similarities and dissimilarities between them clearly. This graph (i.e., figure 31) differentiates the general trend of the ranks with a slightly deviated pattern which initially appears to project a trend that is completely different from the general notion of the ranks. By observing the graph, it is clear that there is a single trend that is created by various curves forged by the different sets of ranks with a deviation in the initial stages.

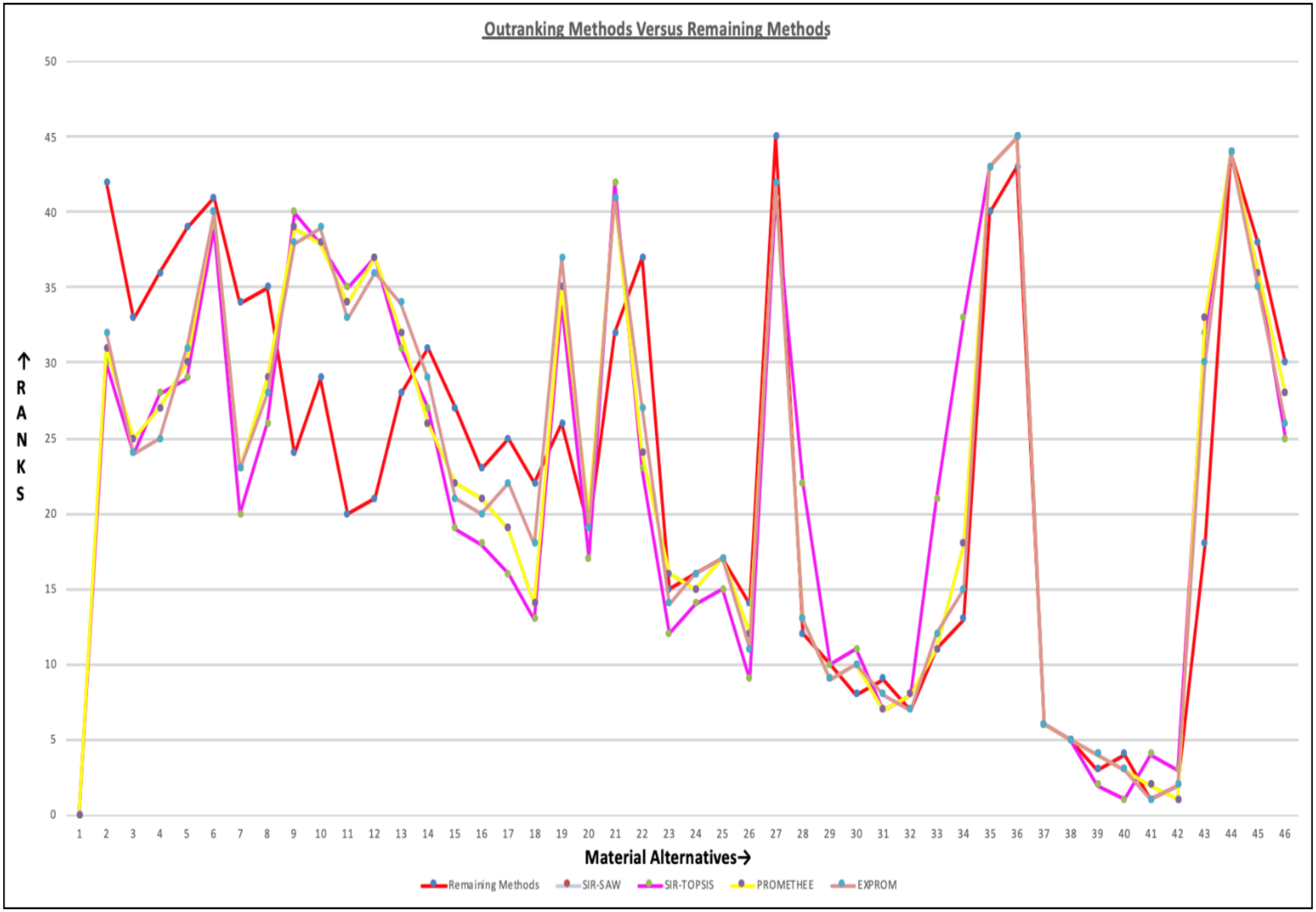


Figure 31: Outranking methods Vs rest of the methods.

5.6 Correlation between the different sets of ranks

Such simple graphs cannot exhibit the true relation between the ranks obtained from 25 different MADM methods. So, to understand their relation, we need to look at this problem statistically. The ranks represent the variables of the material data set, and a statistical approach is needed to unveil the relationship between these variables. Spearman's rank-order correlation method is utilized for this purpose. Spearman’s correlation method is developed especially to deal with ordinal (i.e., rank) data.

The results of Spearman’s correlation method are tabulated in table 9. The numerical values in the table represent the correlation between ranks obtained from each MADM method. The correlation between variables is always expressed in single values confined within the range of (0-1), where “0” indicates no relation and “1” indicates that both the variables are completely coincident.

Table 9: Correlation matrix of the set of ranks.

	SAW	**SMART**	**MEW**	**WASPAS**	**TOPSIS**	**VOKOR**	**PSI**	**ARAS**	**ROVM**	**LFA**	**EDAS**	**SBM**	**GRA**	**WEDBA**	**COPRAS**	**MOORA**	**MULTIMOORA**	**MOOSRA**	**OCRA**	**CODAS**	**DIA**	**SIR-SAW**	**SIR-TOPSIS**	**PROMETHEE**	**EXPROM**
SAW	1	1	0.98	1	1	1	1	1	1	1	0.95	0.98	0.99	0.99	1	1	0.98	0.99	0.98	0.99	1	0.86	0.8	0.86	0.88
SMART	1	1	0.97	1	0.99	1	1	0.99	1	1	0.96	0.98	0.99	0.99	0.99	1	0.97	0.99	0.98	0.99	1	0.86	0.8	0.86	0.88
MEW	0.98	0.97	1	0.98	0.97	0.97	0.97	0.99	0.97	0.97	0.94	0.99	0.96	0.96	0.99	0.98	1	0.98	0.99	0.95	0.98	0.92	0.87	0.92	0.93
WASPAS	1	1	0.98	1	0.99	1	1	1	1	1	0.96	0.98	0.99	0.99	1	1	0.98	0.99	0.98	0.99	1	0.87	0.8	0.87	0.88

TOPSIS	1	0.99	0.97	0.99	1	0.99	0.99	0.99	0.99	0.99	0.95	0.98	0.99	0.99	0.99	1	0.97	0.99	0.99	0.99	1	0.86	0.8	0.86	0.88
VIKOR	1	1	0.97	1	0.99	1	1	0.99	1	1	0.96	0.98	0.99	0.99	0.99	1	0.97	0.99	0.98	0.98	1	0.86	0.8	0.86	0.88
PSI	1	1	0.97	1	0.99	1	1	0.99	1	1	0.95	0.98	1	0.99	0.99	0.99	0.97	0.99	0.98	0.99	0.99	0.86	0.79	0.86	0.87
ARAS	1	0.99	0.99	1	0.99	0.99	0.99	1	0.99	0.99	0.95	0.99	0.98	0.98	1	1	0.99	0.99	0.99	0.98	1	0.89	0.82	0.89	0.9
ROVM	1	1	0.97	1	0.99	1	1	0.99	1	1	0.96	0.98	0.99	0.99	0.99	1	0.97	0.99	0.98	0.99	1	0.86	0.8	0.86	0.88
LFA	1	1	0.97	1	0.99	1	1	0.99	1	1	0.96	0.98	0.99	0.99	0.99	1	0.97	0.99	0.98	0.99	1	0.86	0.8	0.86	0.88
EDAS	0.95	0.96	0.94	0.96	0.95	0.96	0.95	0.95	0.96	0.96	1	0.95	0.95	0.95	0.95	0.96	0.94	0.96	0.96	0.95	0.96	0.85	0.81	0.85	0.86
SBM	0.98	0.98	0.99	0.98	0.98	0.98	0.98	0.99	0.98	0.98	0.95	1	0.97	0.97	0.99	0.99	0.99	0.99	0.99	0.96	0.99	0.89	0.83	0.89	0.9
GRA	0.99	0.99	0.96	0.99	0.99	0.99	1	0.98	0.99	0.99	0.95	0.97	1	1	0.98	0.99	0.96	0.98	0.97	0.99	0.99	0.84	0.77	0.84	0.86
WEDBA	0.99	0.99	0.96	0.99	0.99	0.99	0.99	0.98	0.99	0.99	0.95	0.97	1	1	0.98	0.99	0.96	0.98	0.97	1	0.99	0.84	0.77	0.84	0.85
COPRAS	1	0.99	0.99	1	0.99	0.99	0.99	1	0.99	0.99	0.95	0.99	0.98	0.98	1	1	0.99	0.99	0.99	0.98	1	0.89	0.82	0.89	0.9
MOORA	1	1	0.98	1	1	1	0.99	1	1	1	0.96	0.99	0.99	0.99	1	1	0.98	0.99	0.99	0.98	1	0.88	0.82	0.88	0.89
MULTIMOORA	0.98	0.97	1	0.98	0.97	0.97	0.97	0.99	0.97	0.97	0.94	0.99	0.96	0.96	0.99	0.98	1	0.98	0.99	0.95	0.98	0.92	0.87	0.92	0.93

MOOSRA	0.99	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.96	0.99	0.98	0.98	0.99	0.99	0.98	1	0.99	0.97	0.99	0.88	0.81	0.88	0.89
OCRA	0.98	0.98	0.99	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.96	0.99	0.97	0.97	0.99	0.99	0.99	0.99	1	0.97	0.99	0.9	0.85	0.9	0.91
CODAS	0.99	0.99	0.95	0.99	0.99	0.98	0.99	0.98	0.99	0.99	0.95	0.96	0.99	1	0.98	0.98	0.95	0.97	0.97	1	0.98	0.83	0.77	0.83	0.85
DIA	1	1	0.98	1	1	1	0.99	1	1	1	0.96	0.99	0.99	0.99	1	1	0.98	0.99	0.99	0.98	1	0.88	0.82	0.88	0.89
SIRSAW	0.86	0.86	0.92	0.87	0.86	0.86	0.86	0.89	0.86	0.86	0.85	0.89	0.84	0.84	0.89	0.88	0.92	0.88	0.9	0.83	0.88	1	0.97	1	0.99
SIRTOPSIS	0.8	0.8	0.87	0.8	0.8	0.8	0.79	0.82	0.8	0.8	0.81	0.83	0.77	0.77	0.82	0.82	0.87	0.81	0.85	0.77	0.82	0.97	1	0.97	0.96
PROMETHEE	0.86	0.86	0.92	0.87	0.86	0.86	0.86	0.89	0.86	0.86	0.85	0.89	0.84	0.84	0.89	0.88	0.92	0.88	0.9	0.83	0.88	1	0.97	1	0.99
EXPROM	0.88	0.88	0.93	0.88	0.88	0.88	0.87	0.9	0.88	0.88	0.86	0.9	0.86	0.85	0.9	0.89	0.93	0.89	0.91	0.85	0.89	0.99	0.96	0.99	1

On further addition of the relative correlation coefficients, the final correlation coefficients for each MADM method are presented in the table below:

Table 10: Relative correlation coefficients.

Methods	SAW	SMART	MEW	WASPAS	TOPSIS	VOKOR	PSI	ARAS	ROVM	LFA	EDAS	SBM
Summation of Coefficients	23.207	23.182	23.144	23.209	23.153	23.174	23.142	23.3	23.182	23.182	22.45	23.131
Rank	7	9	14	6	13	12	16	1	9	9	21	17

Methods	GRA	WEDBA	COPRAS	MOORA	MULTIMOORA	MOOSRA	OCRA	CODAS	DIA	SIR-SAW	SIR-TOPSIS	PROMETHEE	EXPROM
Summation of Coefficients	22.975	22.903	23.285	23.278	23.144	23.198	23.224	22.838	23.278	21.405	20.065	21.405	21.655
Rank	18	19	2	3	14	8	5	20	3	23	25	23	22

Based on the results obtained from the Spearman's correlation, the ARAS method has the highest cumulative correlation coefficient value concerning each method.

5.7 Sensitivity analysis

Sensitivity analysis (SA) is conducted to test the robustness of the results obtained from various MADM methods employed to rank the material candidates. The key benefits of the SA are:

- It helps in decision-making or the development of recommendations for decision makers.
- Bridges the communication gap.
- Increases the understanding or quantification of the system.
- Provides the stability analysis of the decision model.

In practice, there are many different possible ways to conduct SA and observe the decision framework results. To validate the results, the stability of the candidate ranking was examined using the dynamic sensitivity analyses. In the dynamic sensitivity analysis, the priorities (weights) of the criteria are changed to observe the effects of these changes on the ranking order of the material candidates. It is conducted by omitting some material alternatives. The list of alternatives considered for the application consists of different classes or families of magnetic materials such as Neodymium, Alnicos, Samarium Cobalt, Ferrites, Cobalt steels, etc. So, initially, sensitivity analysis is conducted by omitting Neodymium magnets then by excluding Ferrites and finally by removing Samarium Cobalt magnets. Since, we obtain the weights of the attributes by objective weighting method (i.e., Shannon's Entropy method), omitting materials will inevitably affect the weights.

Procedure:

In order to conduct a sensitivity analysis, the following procedure is followed:

1. One of the classes of material is omitted from the candidate list, and Shannon's entropy method is applied again to generate weights of the criteria.
2. All the MADM methods are again applied to produce the sets of ranks for the candidates.
3. To correlate the sets of ranks obtained by initial weights and the newly revised weights, Spearman's correlation method is applied. To make the process simpler, mean ranks are derived for the sets of ranks, and then Spearman's correlation is executed.
4. The correlation coefficient value closer to 1, establishes the robustness of the framework.
5. This procedure is repeated by omitting different classes of materials to produce three cases to test the robustness of the framework.

Case-1: Omitting Neodymium Magnetic materials

As expected, the weights of the attributes have changed due to the omission of Neodymium magnets. The changed weights are tabulated in Table 11 below:

Table 11: Revised weights of the attributes.

d	α	ρ	Hc	Mr	(BH)max	Tc	St
0.01053	0.00731	0.56363	0.18410	0.02987	0.18336	0.00916	0.01205

Figure 32 shows the revised attribute weights in percentage.

The number of alternatives now becomes 39 after removing Neodymium magnetic materials. Using these weights, the remaining material alternates are evaluated using the 25 MADM methods to yield the ranks shown in the table 12.

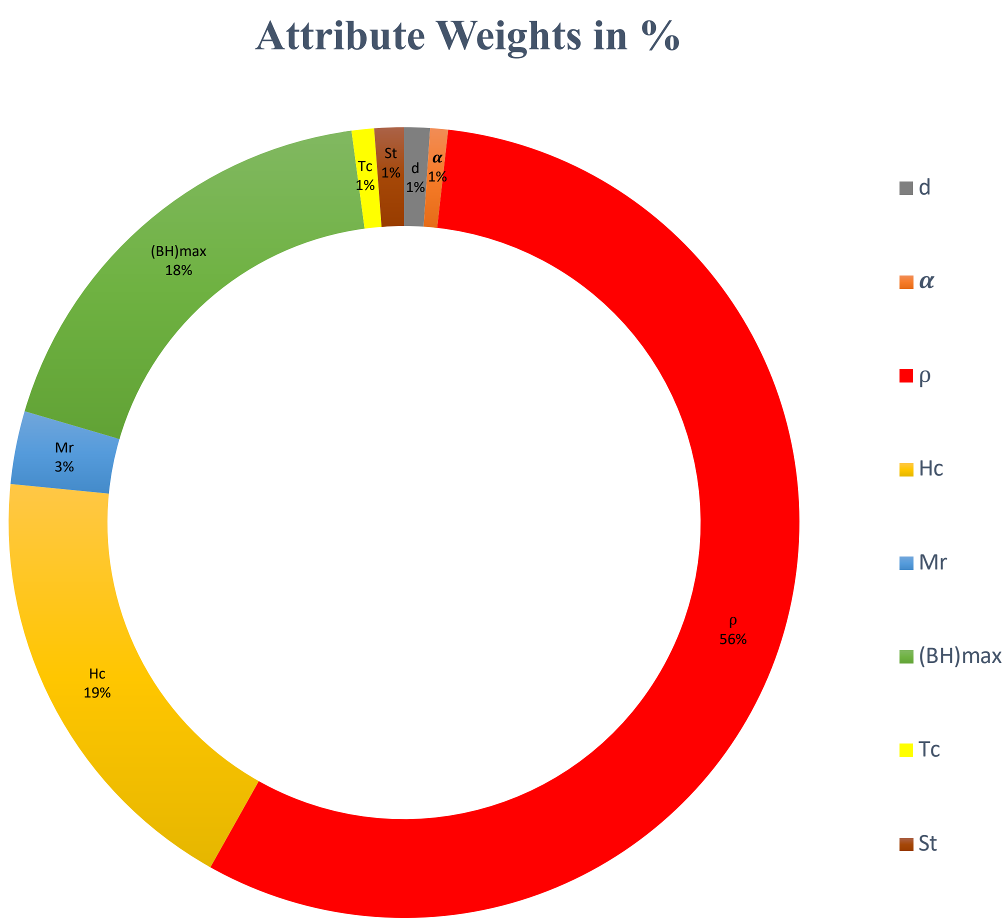


Figure 32: Revised weights after omitting neodymium magnets.

To make the sensitivity analysis simpler, the mean of the ranks is calculated to produce a single rank order for the 39 materials and this mean rank set is compared with the mean ranks obtained from actual sets of rank as shown in table 13. However, when comparing with actual rank sets, the materials are ranked ignoring the Neodymium materials for a fair comparison. In a sense, materials are evaluated using different attribute weights. Spearman's correlation coefficient is used to compare the two sets of ranks obtained from different attribute weights.

Table 12: Ranks of the material candidates after revising the attribute weights.

S.No.	Name	SAW	SMART	MEW	WASPAS	TOPSIS	VIKOR	PSI	ARAS	ROVM	LFA	EDAS	SBM	GRA	WEDBA	COPRAS
1	Alnico® 1 (Isotropic, Cast)	35	33	28	35	36	33	35	34	33	33	36	34	35	38	34
2	Alnico® 12 (Anisotropic, Cast)	27	27	26	27	26	27	27	26	27	27	27	26	29	29	26
3	Alnico® 2 (Isotropic, Cast)	28	28	30	28	29	28	28	27	28	28	28	28	28	30	28
4	Alnico® 2 (Isotropic, Sintered)	33	34	31	33	32	34	33	31	34	34	35	32	37	36	31
5	Alnico® 3 (Isotropic, Cast)	34	35	33	34	35	35	34	33	35	35	31	33	34	37	33
6	Alnico® 4 (Isotropic, Cast)	31	30	24	31	30	30	31	29	30	30	29	29	30	33	29
7	Alnico® 4 (Sintered)	32	31	29	32	31	31	32	30	31	31	30	30	31	34	30
8	Alnico® 5 (Anisotropic, Cast)	18	18	22	18	18	18	18	19	18	18	17	19	17	18	19
9	Alnico® 5 (Anisotropic, Sintered)	22	22	27	22	23	22	22	22	22	22	18	22	21	23	22
10	Alnico® 5-7 (Anisotropic, Cast)	14	14	18	14	14	14	14	14	14	14	14	16	14	14	14
11	Alnico® 5DG (Anisotropic, Cast)	15	15	19	15	15	15	15	17	15	15	15	17	15	15	17
12	Alnico® 6 (Anisotropic, Cast)	21	21	21	21	22	21	21	21	21	21	22	21	20	21	21
13	Alnico® 6 (Anisotropic, Sintered)	24	24	23	24	25	24	23	24	24	24	19	25	23	26	24
14	Alnico® 7 (Anisotropic, Cast)	20	20	16	20	21	20	20	20	20	20	21	20	22	22	20
15	Alnico® 8 (Anisotropic, Cast)	17	17	14	17	17	17	17	16	17	17	13	15	16	17	16
16	Alnico® 8 (Anisotropic, Sintered)	19	19	15	19	19	19	19	18	19	19	20	18	19	19	18
17	Alnico® 8HC (Anisotropic, Cast)	16	16	13	16	16	16	16	15	16	16	16	14	18	16	15
18	Alnico® 8HC (Anisotropic, Sintered)	23	23	32	23	20	23	24	23	23	23	24	24	25	20	23
19	Alnico® 9 (Anisotropic, Cast)	13	13	12	13	13	13	13	13	13	13	12	13	13	13	13
20	Carpenter® P6 Alloy	26	26	35	26	27	26	26	32	26	26	37	35	24	25	32
21	Chromindur® II Alloy	29	32	25	29	28	32	29	28	32	32	34	27	33	27	27
22	Cobalt Samarium 1	9	9	9	9	9	9	9	9	9	9	11	9	9	9	9
23	Cobalt Samarium 2	10	10	10	10	10	10	10	10	10	10	9	10	10	10	10
24	Cobalt Samarium 3	11	11	11	11	11	11	11	11	11	11	10	11	11	11	11
25	Cobalt Samarium 4	8	8	8	8	8	8	8	8	8	8	7	8	8	8	8
26	Cunife® Alloy	39	39	37	39	33	39	39	35	39	39	39	31	39	39	35
27	Ferrite 1 , Sintered Iron Barium Oxide	5	5	5	5	6	5	5	6	5	5	6	2	5	6	6
28	Ferrite 2 , Sintered Iron Barium Oxide	4	4	3	4	4	4	4	4	4	4	3	4	4	4	4
29	Ferrite 3, Sintered Iron Barium Oxide	2	2	4	3	2	3	2	2	2	2	4	5	2	2	2
30	Ferrite 4, Sintered Iron Strontium Oxide	3	3	2	2	3	2	3	3	3	3	1	6	3	3	3
31	Ferrite 5, Sintered Iron Strontium Oxide	1	1	1	1	1	1	1	1	1	1	2	7	1	1	1
32	Ferrite A, Bonded Iron Barium Oxide	6	6	6	6	5	6	6	5	6	6	5	3	6	5	5
33	Ferrite B, Bonded Iron Barium Oxide	7	7	7	7	7	7	7	7	7	7	8	1	7	7	7
34	High Cobalt Steel	36	36	36	36	37	36	36	37	36	36	32	37	32	32	37
35	Low Cobalt Steel	38	38	38	38	38	38	38	38	38	38	38	38	38	35	38
36	Platinum Cobalt Alloy	12	12	17	12	12	12	12	12	12	12	23	12	12	12	12
37	Tungsten Steel	37	37	39	37	39	37	37	39	37	37	33	39	36	31	39
38	Vicaloy® I Alloy	30	29	34	30	34	29	30	36	29	29	25	36	27	28	36
39	Vicaloy® II Alloy	25	25	20	25	24	25	25	25	25	25	26	23	26	24	25

Continuation of Table 12…

S.No.	Name	MOORA		MULTIMOORA	MOORSA	OCRA	CODAS	DIA	SIR-SAW		SIR-TOPSIS			PROMETHEE			EXPROM
		RSA	RPA						Qnet	(Qr)	S-Flow	I-Flow	Relative	Net	Positive Outranking flow	Negative Outranking flow	
1	Alnico® 1 (Isotropic, Cast)	34	34	28	32	32	38	34	16	16	12	7	7	16	18	17	23
2	Alnico® 12 (Anisotropic, Cast)	26	26	26	26	26	30	26	21	22	20	12	14	21	13	24	20
3	Alnico® 2 (Isotropic, Cast)	27	27	30	27	30	33	27	19	19	16	11	12	19	12	23	18
4	Alnico® 2 (Isotropic, Sintered)	32	32	31	28	31	36	32	22	23	14	10	11	22	20	21	24
5	Alnico® 3 (Isotropic, Cast)	33	33	33	33	33	37	33	29	28	27	21	27	29	27	30	27
6	Alnico® 4 (Isotropic, Cast)	30	30	24	29	27	34	30	8	8	8	4	5	8	8	9	13
7	Alnico® 4 (Sintered)	31	31	29	30	28	35	31	18	18	13	8	8	18	16	19	22
8	Alnico® 5 (Anisotropic, Cast)	19	19	22	19	21	18	19	34	34	34	33	34	34	34	34	33
9	Alnico® 5 (Anisotropic, Sintered)	22	22	27	22	24	24	22	30	30	31	23	30	30	33	29	34
10	Alnico® 5-7 (Anisotropic, Cast)	14	14	18	14	17	14	14	31	31	30	28	31	31	31	31	31
11	Alnico® 5DG (Anisotropic, Cast)	16	16	19	17	18	15	16	33	33	32	31	33	33	32	32	32
12	Alnico® 6 (Anisotropic, Cast)	21	21	21	21	22	22	21	28	29	29	17	29	28	29	28	30
13	Alnico® 6 (Anisotropic, Sintered)	23	23	23	24	23	26	23	25	25	25	13	18	25	25	25	25
14	Alnico® 7 (Anisotropic, Cast)	20	20	16	20	20	21	20	12	12	10	18	16	12	10	16	14
15	Alnico® 8 (Anisotropic, Cast)	17	17	14	16	15	17	17	24	24	22	22	20	24	21	22	19
16	Alnico® 8 (Anisotropic, Sintered)	18	18	15	18	16	20	18	13	13	18	15	17	13	15	14	17
17	Alnico® 8HC (Anisotropic, Cast)	15	15	13	15	14	16	15	9	9	11	14	15	9	11	10	12
18	Alnico® 8HC (Anisotropic, Sintered)	24	24	32	23	19	19	24	27	27	28	19	26	27	28	27	28
19	Alnico® 9 (Anisotropic, Cast)	13	13	12	12	13	13	13	17	17	17	20	19	17	14	18	16
20	Carpenter® P6 Alloy	29	29	35	31	36	23	29	35	35	36	35	36	35	35	35	35
21	Chromindur® II Alloy	28	28	25	35	29	27	28	10	10	9	6	6	10	9	11	15
22	Cobalt Samarium 1	9	9	9	9	9	9	9	14	14	21	27	22	14	22	12	9
23	Cobalt Samarium 2	10	10	10	10	10	10	10	15	15	23	29	25	15	23	13	10
24	Cobalt Samarium 3	11	11	11	11	11	11	11	23	21	24	32	28	23	24	15	11
25	Cobalt Samarium 4	8	8	8	8	8	8	8	11	11	19	25	21	11	19	8	8
26	Cunife® Alloy	36	36	37	36	35	39	36	36	36	35	36	35	36	36	36	36
27	Ferrite 1 , Sintered Iron Barium Oxide	6	6	5	7	6	5	6	6	6	6	26	13	6	6	6	6
28	Ferrite 2 , Sintered Iron Barium Oxide	4	4	3	6	4	4	4	3	3	3	3	3	3	3	3	3
29	Ferrite 3, Sintered Iron Barium Oxide	2	2	4	3	2	2	2	4	4	4	5	4	4	4	4	4
30	Ferrite 4, Sintered Iron Strontium Oxide	3	3	2	5	3	3	3	1	1	2	1	1	1	1	1	2
31	Ferrite 5, Sintered Iron Strontium Oxide	1	1	1	2	1	1	1	2	2	1	2	2	2	2	2	1
32	Ferrite A, Bonded Iron Barium Oxide	5	5	6	1	5	6	5	5	5	5	24	9	5	5	5	5
33	Ferrite B, Bonded Iron Barium Oxide	7	7	7	4	7	7	7	7	7	7	34	24	7	7	7	7
34	High Cobalt Steel	37	37	36	38	37	29	37	38	38	39	39	39	38	38	39	38
35	Low Cobalt Steel	38	38	38	39	38	32	38	39	39	38	38	38	39	39	38	39
36	Platinum Cobalt Alloy	12	12	17	13	12	12	12	32	32	33	30	32	32	30	33	29
37	Tungsten Steel	39	39	39	37	39	31	39	37	37	37	37	37	37	37	37	37
38	Vicaloy® I Alloy	35	35	34	34	34	28	35	26	26	26	16	23	26	26	26	26
39	Vicaloy® II Alloy	25	25	20	25	25	25	25	20	20	15	9	10	20	17	20	21

Table 13: Revised ranks with the omission of Neodymium magnetic materials.

S/N	Materials	Actual Ranks	Ranks with new Weigths
1	Alnico® 1 (Isotropic, Cast)	34	34
2	Alnico® 12 (Anisotropic, Cast)	26	26
3	Alnico® 2 (Isotropic, Cast)	28	29
4	Alnico® 2 (Isotropic, Sintered)	33	33
5	Alnico® 3 (Isotropic, Cast)	35	35
6	Alnico® 4 (Isotropic, Cast)	27	27
7	Alnico® 4 (Sintered)	29	30
8	Alnico® 5 (Anisotropic, Cast)	20	20
9	Alnico® 5 (Anisotropic, Sintered)	23	24
10	Alnico® 5-7 (Anisotropic, Cast)	16	16
11	Alnico® 5DG (Anisotropic, Cast)	18	18
12	Alnico® 6 (Anisotropic, Cast)	21	21
13	Alnico® 6 (Anisotropic, Sintered)	22	23
14	Alnico® 7 (Anisotropic, Cast)	19	19
15	Alnico® 8 (Anisotropic, Cast)	15	15
16	Alnico® 8 (Anisotropic, Sintered)	17	17
17	Alnico® 8HC (Anisotropic, Cast)	13	13
18	Alnico® 8HC (Anisotropic, Sintered)	25	25
19	Alnico® 9 (Anisotropic, Cast)	12	12
20	Carpenter® P6 Alloy	32	31
21	Chromindur® II Alloy	30	28
22	Cobalt Samarium 1	9	9
23	Cobalt Samarium 2	10	10
24	Cobalt Samarium 3	11	11
25	Cobalt Samarium 4	7	8
26	Cunife® Alloy	38	37
27	Ferrite 1 , Sintered Iron Barium Oxide	5	6
28	Ferrite 2 , Sintered Iron Barium Oxide	4	4
29	Ferrite 3, Sintered Iron Barium Oxide	3	3
30	Ferrite 4, Sintered Iron Strontium Oxide	2	2
31	Ferrite 5, Sintered Iron Strontium Oxide	1	1
32	Ferrite A, Bonded Iron Barium Oxide	6	5
33	Ferrite B, Bonded Iron Barium Oxide	8	7
34	High Cobalt Steel	36	36
35	Low Cobalt Steel	39	39
36	Platinum Cobalt Alloy	14	14
37	Tungsten Steel	37	38
38	Vicaloy® I Alloy	31	32
39	Vicaloy® II Alloy	24	22

The result of Spearman's correlation is 0.9979.

Case-2: Omitting Ferrite Magnetic materials

Revised weights after omitting ferrite magnetic materials are tabulated in table 14 below:

Table 14: Revised attribute weights.

d	α	ρ	Hc	Mr	(BH)max	Tc	St
0.00	0.01	0.55	0.20	0.01	0.18	0.02	0.03

To understand the revised weights in relation with all the criteria, there are shown in figure 33.

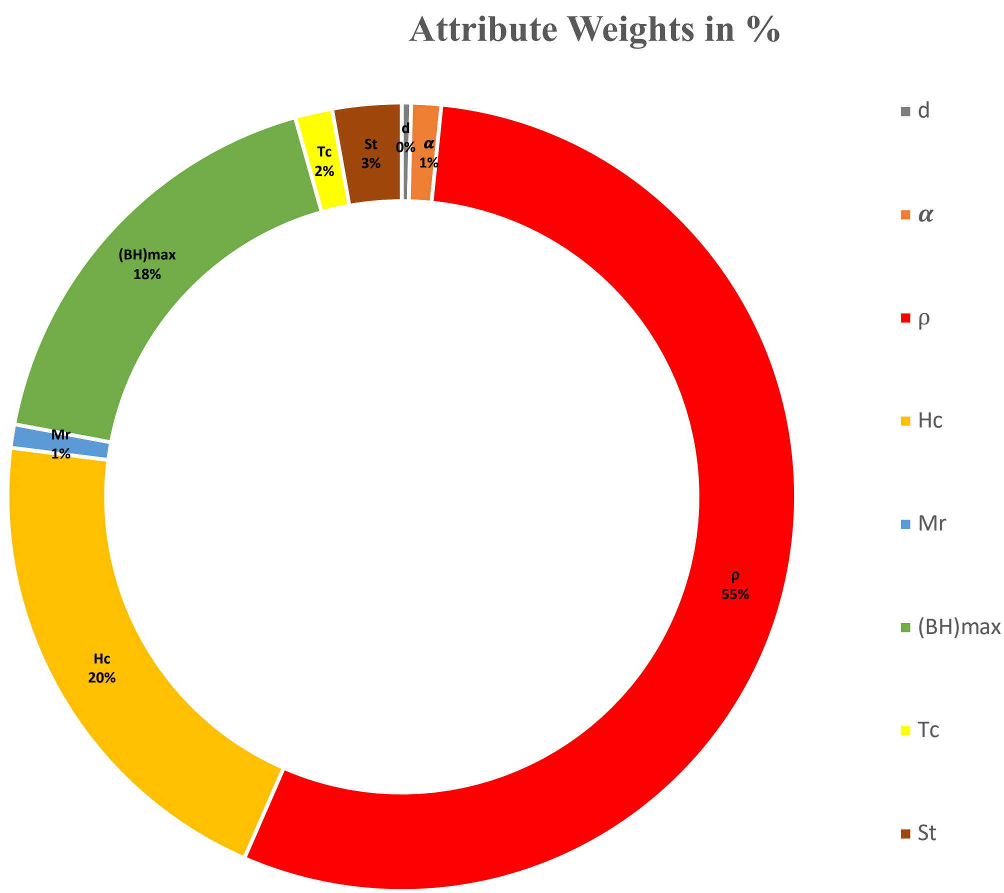


Figure 33: Revised weights after omitting ferrite magnets.

Following the same procedure as in the previous case, the ranks are obtained, and the average ranks are compared. The ranks obtained from the revised weights displayed in table 15,

Table 15: Ranks of the material candidates after revising the attribute weights.

S.No.	Name	SAW	SMART	MEW	WASPAS	TOPSIS	VIKOR	PSI	ARAS	ROVM	LFA	EDAS	SBM	GRA	WEDBA	COPRAS
1	Alnico® 1 (Isotropic, Cast)	33	32	29	33	33	32	34	32	32	32	29	33	33	36	32
2	Alnico® 12 (Anisotropic, Cast)	25	25	24	25	25	25	25	25	25	25	24	25	28	29	25
3	Alnico® 2 (Isotropic, Cast)	28	27	28	28	28	27	28	28	27	27	26	29	27	27	28
4	Alnico® 2 (Isotropic, Sintered)	32	33	30	32	30	33	32	29	33	33	35	31	35	35	30
5	Alnico® 3 (Isotropic, Cast)	30	31	32	30	32	31	30	31	31	31	28	32	32	33	31
6	Alnico® 4 (Isotropic, Cast)	26	26	20	26	26	26	26	26	26	26	25	26	24	24	26
7	Alnico® 4 (Sintered)	27	28	27	27	27	28	27	27	28	28	27	28	25	25	27
8	Alnico® 5 (Anisotropic, Cast)	18	18	23	18	19	18	18	18	18	18	14	18	18	18	18
9	Alnico® 5 (Anisotropic, Sintered)	22	22	26	22	22	22	22	22	22	22	16	22	21	22	22
10	Alnico® 5-7 (Anisotropic, Cast)	15	15	17	15	14	15	15	15	15	15	11	15	15	14	15
11	Alnico® 5DG (Anisotropic, Cast)	16	16	18	16	16	16	16	16	16	16	12	16	16	16	16
12	Alnico® 6 (Anisotropic, Cast)	20	20	21	20	21	20	20	21	20	20	15	21	22	21	21
13	Alnico® 6 (Anisotropic, Sintered)	23	23	22	23	23	23	23	23	23	23	18	23	23	23	23
14	Alnico® 7 (Anisotropic, Cast)	19	19	16	19	20	19	19	19	19	19	22	19	19	20	19
15	Alnico® 8 (Anisotropic, Cast)	14	14	13	14	15	14	14	14	14	14	19	14	14	15	14
16	Alnico® 8 (Anisotropic, Sintered)	17	17	14	17	17	17	17	17	17	17	20	17	17	17	17
17	Alnico® 8HC (Anisotropic, Cast)	13	13	12	13	13	13	13	13	13	13	17	13	13	13	13
18	Alnico® 8HC (Anisotropic, Sintered)	21	21	31	21	18	21	21	20	21	21	23	20	20	19	20
19	Alnico® 9 (Anisotropic, Cast)	12	12	11	12	12	12	12	12	12	12	9	12	12	12	12
20	Carpenter® P6 Alloy	34	30	36	34	34	30	33	35	30	30	37	34	30	32	34
21	Chromindur® II Alloy	31	34	25	31	29	34	31	30	34	34	36	27	36	31	29
22	Cobalt Samarium 1	8	8	8	8	8	8	8	8	8	8	13	8	8	8	8
23	Cobalt Samarium 2	9	9	9	9	9	9	9	9	9	9	8	9	9	9	9
24	Cobalt Samarium 3	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
25	Cobalt Samarium 4	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
26	Cunife® Alloy	38	38	34	38	35	38	38	34	38	38	38	30	38	38	35
27	High Cobalt Steel	35	35	35	35	36	35	35	36	35	35	31	36	31	30	36
28	Low Cobalt Steel	37	37	37	37	38	37	37	37	37	37	33	37	37	37	37
29	Neodymium® 27	6	6	6	6	6	6	6	6	6	6	6	1	6	6	6
30	Neodymium® 27H	5	5	5	5	5	5	5	5	5	5	5	2	5	5	5
31	Neodymium® 30	4	4	4	4	3	3	4	3	4	4	4	4	4	3	3
32	Neodymium® 30H	3	3	3	3	4	4	3	4	3	3	3	3	3	4	4
33	Neodymium® 35	1	1	1	1	1	1	1	1	1	1	2	6	1	1	1
34	Neodymium® 35H	2	2	2	2	2	2	2	2	2	2	1	5	2	2	2
35	Platinum Cobalt Alloy	11	11	15	11	11	11	11	11	11	11	21	11	11	11	11
36	Tungsten Steel	36	36	38	36	37	36	36	38	36	36	32	38	34	34	38
37	Vicaloy® I Alloy	29	29	33	29	31	29	29	33	29	29	30	35	26	28	33
38	Vicaloy® II Alloy	24	24	19	24	24	24	24	24	24	24	34	24	29	26	24

Continuation of Table 15…

S.No.	Name	MOORA		MULTIMOORA	MOORSA	OCRA	CODAS	DIA	SIR-SAW		SIR-TOPSIS			PROMETHEE			EXPROM
		RSA	RPA						Qnet	(Qr)	S-Flow	I-Flow	RELATIVE	Net	Positive Outranking flow	Negative Outranking flow	
1	Alnico® 1 (Isotropic, Cast)	32	32	29	32	31	35	32	16	16	15	14	14	16	14	14	20
2	Alnico® 12 (Anisotropic, Cast)	25	25	24	25	25	29	25	13	13	12	15	13	13	12	15	14
3	Alnico® 2 (Isotropic, Cast)	28	28	28	26	28	27	28	14	15	13	17	15	14	13	18	15
4	Alnico® 2 (Isotropic, Sintered)	29	29	30	29	29	33	29	17	17	14	19	17	17	16	19	21
5	Alnico® 3 (Isotropic, Cast)	30	30	32	33	32	32	30	27	27	27	28	27	27	27	28	27
6	Alnico® 4 (Isotropic, Cast)	26	26	20	27	26	24	26	7	7	7	7	7	7	7	7	8
7	Alnico® 4 (Sintered)	27	27	27	28	27	25	27	11	11	10	12	10	11	11	12	12
8	Alnico® 5 (Anisotropic, Cast)	18	18	23	18	20	19	18	32	33	32	33	32	32	33	32	33
9	Alnico® 5 (Anisotropic, Sintered)	22	22	26	22	23	22	22	29	29	30	29	29	29	30	29	32
10	Alnico® 5-7 (Anisotropic, Cast)	15	15	17	15	16	14	15	30	30	29	30	30	30	29	30	30
11	Alnico® 5DG (Anisotropic, Cast)	16	16	18	16	18	16	16	31	31	31	31	31	31	32	31	31
12	Alnico® 6 (Anisotropic, Cast)	20	20	21	21	21	21	20	28	28	28	27	28	28	28	27	28
13	Alnico® 6 (Anisotropic, Sintered)	23	23	22	23	22	23	23	23	23	22	23	23	23	21	23	24
14	Alnico® 7 (Anisotropic, Cast)	19	19	16	19	19	20	19	10	10	9	13	11	10	8	13	11
15	Alnico® 8 (Anisotropic, Cast)	14	14	13	14	14	15	14	19	19	19	21	19	19	18	21	19
16	Alnico® 8 (Anisotropic, Sintered)	17	17	14	17	15	18	17	12	12	16	11	12	12	15	11	18
17	Alnico® 8HC (Anisotropic, Cast)	13	13	12	13	13	13	13	8	8	11	8	9	8	9	8	9
18	Alnico® 8HC (Anisotropic, Sintered)	21	21	31	20	17	17	21	26	26	26	25	25	26	26	25	26
19	Alnico® 9 (Anisotropic, Cast)	12	12	11	12	12	12	12	18	18	17	20	18	18	17	20	17
20	Carpenter® P6 Alloy	34	34	36	34	35	36	34	34	34	35	34	35	34	34	34	34
21	Chromindur® II Alloy	33	33	25	36	30	31	33	9	9	8	9	8	9	10	9	13
22	Cobalt Samarium 1	8	8	8	8	8	8	8	21	21	21	18	21	21	22	17	10
23	Cobalt Samarium 2	9	9	9	9	9	9	9	22	22	23	16	22	22	23	16	16
24	Cobalt Samarium 3	10	10	10	10	10	10	10	24	24	25	22	24	24	25	22	22
25	Cobalt Samarium 4	7	7	7	7	7	7	7	15	14	20	10	16	15	20	10	7
26	Cunife® Alloy	35	35	34	35	34	38	35	35	35	34	35	34	35	35	35	35
27	High Cobalt Steel	36	36	35	37	36	30	36	37	37	37	38	37	37	37	37	37
28	Low Cobalt Steel	38	38	37	38	37	34	38	38	38	38	37	38	38	38	38	38
29	Neodymium® 27	6	6	6	6	6	6	6	6	6	6	5	6	6	6	6	6
30	Neodymium® 27H	5	5	5	5	5	5	5	5	5	5	6	5	5	5	5	5
31	Neodymium® 30	3	3	4	3	4	4	3	4	4	4	3	3	4	4	4	4
32	Neodymium® 30H	4	4	3	4	3	3	4	3	3	3	1	1	3	3	3	3
33	Neodymium® 35	1	1	1	2	2	1	1	2	2	2	2	2	2	2	2	2
34	Neodymium® 35H	2	2	2	1	1	2	2	1	1	1	4	4	1	1	1	1
35	Platinum Cobalt Alloy	11	11	15	11	11	11	11	33	32	33	32	33	33	31	33	29
36	Tungsten Steel	37	37	38	30	38	37	37	36	36	36	36	36	36	36	36	36
37	Vicaloy® I Alloy	31	31	33	31	33	28	31	25	25	24	26	26	25	24	26	25
38	Vicaloy® II Alloy	24	24	19	24	24	26	24	20	20	18	24	20	20	19	24	23

The same procedure is followed to compare the sets of ranks obtained from revised weights and actual weights. The mean ranks are presented in table 16 below:

Table 16: Ranks comparison.

S/N	Materials	Actual Ranks	Revised Ranks
1	Alnico® 1 (Isotropic, Cast)	33	33
2	Alnico® 12 (Anisotropic, Cast)	25	25
3	Alnico® 2 (Isotropic, Cast)	27	27
4	Alnico® 2 (Isotropic, Sintered)	32	32
5	Alnico® 3 (Isotropic, Cast)	34	34
6	Alnico® 4 (Isotropic, Cast)	26	26
7	Alnico® 4 (Sintered)	28	28
8	Alnico® 5 (Anisotropic, Cast)	19	19
9	Alnico® 5 (Anisotropic, Sintered)	22	22
10	Alnico® 5-7 (Anisotropic, Cast)	15	15
11	Alnico® 5DG (Anisotropic, Cast)	17	17
12	Alnico® 6 (Anisotropic, Cast)	20	20
13	Alnico® 6 (Anisotropic, Sintered)	21	21
14	Alnico® 7 (Anisotropic, Cast)	18	18
15	Alnico® 8 (Anisotropic, Cast)	14	14
16	Alnico® 8 (Anisotropic, Sintered)	16	16
17	Alnico® 8HC (Anisotropic, Cast)	12	12
18	Alnico® 8HC (Anisotropic, Sintered)	24	24
19	Alnico® 9 (Anisotropic, Cast)	11	11
20	Carpenter® P6 Alloy	31	31
21	Chromindur® II Alloy	29	29
22	Cobalt Samarium 1	8	8
23	Cobalt Samarium 2	9	9
24	Cobalt Samarium 3	10	10
25	Cobalt Samarium 4	7	7
26	Cunife® Alloy	37	37
27	High Cobalt Steel	35	35
28	Low Cobalt Steel	38	38
29	Neodymium® 27	6	6
30	Neodymium® 27H	5	5
31	Neodymium® 30	3	3
32	Neodymium® 30H	4	4
33	Neodymium® 35	1	1
34	Neodymium® 35H	2	2
35	Platinum Cobalt Alloy	13	13
36	Tungsten Steel	36	36
37	Vicaloy® I Alloy	30	30
38	Vicaloy® II Alloy	23	23

The result of the Spearman's correlation is 1.00. It means that the ranks are robust even though weights are changed slightly.

Case-3: Omitting Samarium Cobalt Magnetic materials

There are four samarium cobalt magnetic materials in the initial list of permanent magnets. Once these four materials are omitted, the weights of the attributes are shown in table 17.

Table 17: Revised attribute weights.

d	α	ρ	Hc	Mr	(BH)max	Tc	St
0.01040	0.01395	0.39956	0.23682	0.03166	0.24755	0.02155	0.03851

To understand the relative attribute weights, the percentage of each attribute is shown in figure 34.

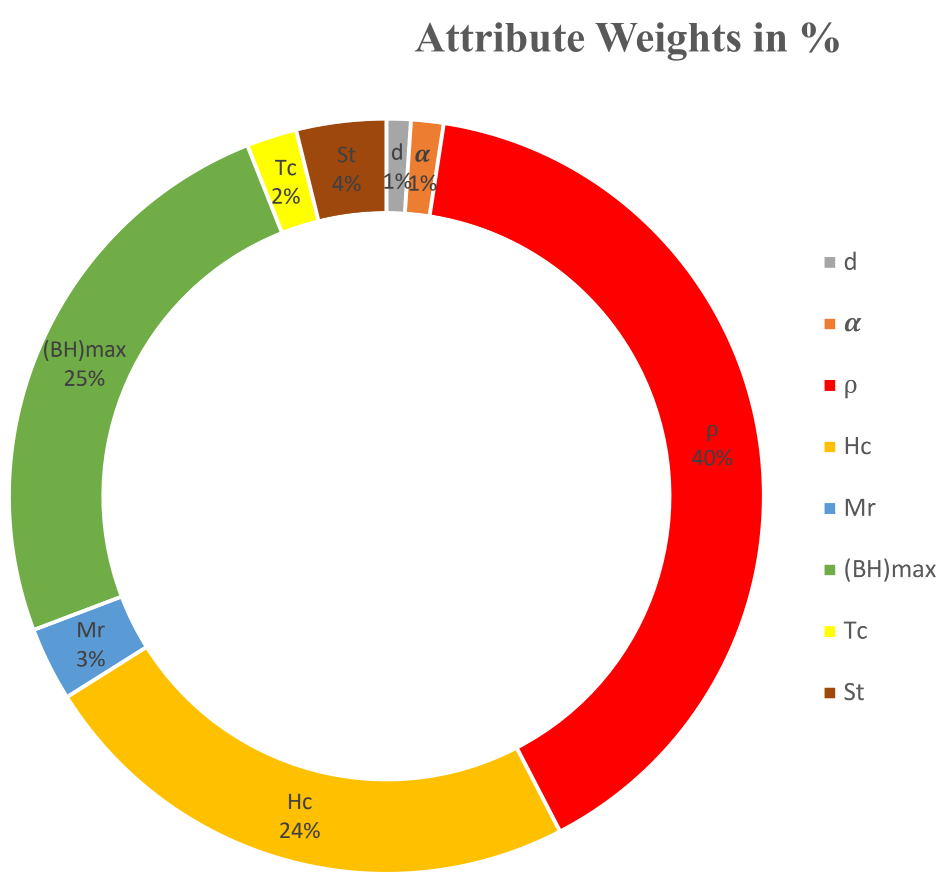


Figure 34: Revised weights after omitting samarium cobalt magnets.

Ranking of the remaining 41 magnetic material is performed using all the 25 MADM methods as performed before. The ranks of the candidates with revised weights are presented in the table 18.

Table 18: Ranks of the material candidates after revising the attribute weights.

S.No.	Name	SAW	SMART	MEW	WASPAS	TOPSIS	VIKOR	PSI	ARAS	ROVM	LFA	EDAS	SBM	GRA	WEDBA	COPRAS
1	Alnico® 1 (Isotropic, Cast)	38	35	33	38	37	35	37	37	35	35	34	36	38	40	37
2	Alnico® 12 (Anisotropic, Cast)	28	28	27	28	28	28	31	28	28	28	29	28	33	35	28
3	Alnico® 2 (Isotropic, Cast)	29	29	30	29	32	29	30	29	29	29	30	30	32	32	29
4	Alnico® 2 (Isotropic, Sintered)	37	37	32	37	35	37	38	33	37	37	38	34	39	39	34
5	Alnico® 3 (Isotropic, Cast)	35	34	35	35	36	34	36	35	34	34	33	35	36	38	35
6	Alnico® 4 (Isotropic, Cast)	32	32	28	32	30	32	32	30	32	32	31	31	30	30	30
7	Alnico® 4 (Sintered)	33	33	31	33	31	33	33	31	33	33	32	32	31	31	32
8	Alnico® 5 (Anisotropic, Cast)	20	20	22	20	20	20	20	21	20	20	24	21	20	20	21
9	Alnico® 5 (Anisotropic, Sintered)	24	24	26	24	25	24	24	24	24	24	21	24	22	25	24
10	Alnico® 5-7 (Anisotropic, Cast)	16	16	19	16	16	16	16	17	16	16	18	18	16	16	17
11	Alnico® 5DG (Anisotropic, Cast)	17	17	21	17	17	17	17	19	17	17	19	19	17	17	19
12	Alnico® 6 (Anisotropic, Cast)	23	23	23	23	24	23	23	23	23	23	26	23	23	22	23
13	Alnico® 6 (Anisotropic, Sintered)	25	25	25	25	27	25	25	26	25	25	23	27	25	26	26
14	Alnico® 7 (Anisotropic, Cast)	22	22	20	22	23	22	22	22	22	22	25	22	24	24	22
15	Alnico® 8 (Anisotropic, Cast)	19	19	17	19	19	19	19	18	19	19	15	17	18	19	18
16	Alnico® 8 (Anisotropic, Sintered)	21	21	18	21	21	21	21	20	21	21	17	20	21	21	20
17	Alnico® 8HC (Anisotropic, Cast)	18	18	16	18	18	18	18	16	18	18	20	16	19	18	16
18	Alnico® 8HC (Anisotropic, Sintered)	26	26	34	26	22	26	26	25	26	26	22	25	26	23	25
19	Alnico® 9 (Anisotropic, Cast)	15	15	14	15	15	15	15	15	15	15	12	15	14	15	15
20	Carpenter® P6 Alloy	31	31	39	31	29	31	29	34	31	31	40	37	28	28	33
21	Chromindur® II Alloy	34	36	29	34	33	36	34	32	36	36	39	29	37	33	31
22	Cunife® Alloy	41	41	37	41	40	41	41	38	41	41	41	33	41	41	38
23	Ferrite 1 , Sintered Iron Barium Oxide	11	11	11	11	12	11	11	11	11	11	11	12	11	11	11
24	Ferrite 2 , Sintered Iron Barium Oxide	10	10	9	10	10	9	10	10	10	10	10	9	10	10	10
25	Ferrite 3, Sintered Iron Barium Oxide	8	8	10	9	8	10	8	8	8	8	9	8	8	8	8
26	Ferrite 4, Sintered Iron Strontium Oxide	9	9	8	8	9	7	9	9	9	9	7	10	9	9	9
27	Ferrite 5, Sintered Iron Strontium Oxide	7	7	7	7	7	8	7	7	7	7	8	7	7	7	7
28	Ferrite A, Bonded Iron Barium Oxide	12	12	12	12	11	12	12	12	12	12	13	11	12	12	12
29	Ferrite B, Bonded Iron Barium Oxide	13	13	13	13	13	13	13	13	13	13	14	13	13	13	13
30	High Cobalt Steel	36	38	38	36	38	38	35	39	38	38	36	39	34	34	39
31	Low Cobalt Steel	40	40	40	40	39	40	40	40	40	40	37	40	40	37	40
32	Neodymium® 27	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
33	Neodymium® 27H	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
34	Neodymium® 30	3	3	4	4	3	3	3	3	3	3	4	4	3	3	3
35	Neodymium® 30H	4	4	3	3	4	4	4	4	4	4	3	3	4	4	4
36	Neodymium® 35	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1
37	Neodymium® 35H	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2
38	Platinum Cobalt Alloy	14	14	15	14	14	14	14	14	14	14	16	14	15	14	14
39	Tungsten Steel	39	39	41	39	41	39	39	41	39	39	28	41	35	36	41
40	Vicaloy® I Alloy	30	30	36	30	34	30	28	36	30	30	35	38	27	29	36
41	Vicaloy® II Alloy	27	27	24	27	26	27	27	27	27	27	27	26	29	27	27

Continuation of Table 18…

S.No.	Name	MOORA		MULTIMOORA	MOORSA	OCRA	CODAS	DIA	SIR-SAW		SIR-TOPSIS			PROMETHEE			EXPROM
		RSA	RPA						Qnet	(Qr)	S-Flow	I-Flow	RELATIVE	Net	Positive Outranking flow	Negative Outranking flow	
1	Alnico® 1 (Isotropic, Cast)	37	37	33	35	34	40	37	32	32	31	27	30	32	32	32	32
2	Alnico® 12 (Anisotropic, Cast)	28	28	27	28	28	36	28	21	21	22	19	20	21	19	23	20
3	Alnico® 2 (Isotropic, Cast)	29	29	30	29	31	33	29	26	26	25	24	26	26	21	29	25
4	Alnico® 2 (Isotropic, Sintered)	35	35	32	32	33	39	35	31	31	26	28	28	31	31	31	31
5	Alnico® 3 (Isotropic, Cast)	36	36	35	36	35	38	36	36	36	36	35	36	36	36	36	36
6	Alnico® 4 (Isotropic, Cast)	30	30	28	30	29	30	30	20	20	20	16	19	20	20	19	22
7	Alnico® 4 (Sintered)	31	31	31	31	30	31	31	28	28	24	26	27	28	28	27	29
8	Alnico® 5 (Anisotropic, Cast)	21	21	22	21	23	20	21	30	30	32	31	32	30	30	30	30
9	Alnico® 5 (Anisotropic, Sintered)	24	24	26	24	26	25	24	33	33	33	29	33	33	33	33	33
10	Alnico® 5-7 (Anisotropic, Cast)	16	16	19	17	19	16	16	23	23	27	20	24	23	24	22	21
11	Alnico® 5DG (Anisotropic, Cast)	19	19	21	19	20	17	19	29	29	30	25	29	29	29	28	27
12	Alnico® 6 (Anisotropic, Cast)	23	23	23	23	24	24	23	27	27	29	22	25	27	25	26	24
13	Alnico® 6 (Anisotropic, Sintered)	25	25	25	26	25	26	25	22	22	28	18	23	22	27	21	26
14	Alnico® 7 (Anisotropic, Cast)	22	22	20	22	22	23	22	17	17	17	15	15	17	17	18	18
15	Alnico® 8 (Anisotropic, Cast)	18	18	17	18	17	19	18	16	16	15	14	14	16	13	17	16
16	Alnico® 8 (Anisotropic, Sintered)	20	20	18	20	18	22	20	15	15	16	12	13	15	15	15	17
17	Alnico® 8HC (Anisotropic, Cast)	17	17	16	16	16	18	17	13	13	13	9	11	13	11	13	14
18	Alnico® 8HC (Anisotropic, Sintered)	26	26	34	25	21	21	26	34	34	34	33	34	34	35	34	34
19	Alnico® 9 (Anisotropic, Cast)	15	15	14	14	15	15	15	14	14	14	13	12	14	12	16	15
20	Carpenter® P6 Alloy	32	32	39	34	38	28	32	37	37	38	37	38	37	37	37	37
21	Chromindur® II Alloy	34	34	29	39	32	34	34	24	24	23	21	21	24	26	24	28
22	Cunife® Alloy	39	39	37	38	36	41	39	38	38	37	38	37	38	38	38	38
23	Ferrite 1 , Sintered Iron Barium Oxide	11	11	11	12	12	11	11	12	12	12	32	17	12	16	12	12
24	Ferrite 2 , Sintered Iron Barium Oxide	10	10	9	10	10	10	10	9	9	9	10	9	9	9	9	9
25	Ferrite 3, Sintered Iron Barium Oxide	8	8	10	8	8	8	8	10	10	10	11	10	10	10	10	10
26	Ferrite 4, Sintered Iron Strontium Oxide	9	9	8	9	9	9	9	7	7	7	7	7	7	7	7	8
27	Ferrite 5, Sintered Iron Strontium Oxide	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	7
28	Ferrite A, Bonded Iron Barium Oxide	12	12	12	11	11	12	12	11	11	11	30	16	11	14	11	11
29	Ferrite B, Bonded Iron Barium Oxide	13	13	13	13	13	13	13	18	18	18	36	31	18	23	14	13
30	High Cobalt Steel	38	38	38	40	39	32	38	39	39	39	40	39	39	39	39	39
31	Low Cobalt Steel	41	41	40	41	40	35	41	40	41	41	39	41	40	41	40	41
32	Neodymium® 27	6	6	6	6	6	6	6	6	6	6	4	6	6	6	6	6
33	Neodymium® 27H	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
34	Neodymium® 30	3	3	4	3	4	3	3	4	4	4	2	2	4	4	4	4
35	Neodymium® 30H	4	4	3	4	3	4	4	3	3	3	1	1	3	3	3	3
36	Neodymium® 35	1	1	1	1	2	1	1	2	2	2	6	4	2	2	2	1
37	Neodymium® 35H	2	2	2	2	1	2	2	1	1	1	3	3	1	1	1	2
38	Platinum Cobalt Alloy	14	14	15	15	14	14	14	19	19	19	17	18	19	18	20	19
39	Tungsten Steel	40	40	41	37	41	37	40	41	40	40	41	40	41	40	41	40
40	Vicaloy® I Alloy	33	33	36	33	37	29	33	35	35	35	34	35	35	34	35	35
41	Vicaloy® II Alloy	27	27	24	27	27	27	27	25	25	21	23	22	25	22	25	23

The mean ranks are presented in the table below:

Table 19: Rank comparison.

Materials	Actual Ranks	Revised Ranks
Alnico® 1 (Isotropic, Cast)	36	37
Alnico® 12 (Anisotropic, Cast)	28	28
Alnico® 2 (Isotropic, Cast)	30	30
Alnico® 2 (Isotropic, Sintered)	35	35
Alnico® 3 (Isotropic, Cast)	37	36
Alnico® 4 (Isotropic, Cast)	29	29
Alnico® 4 (Sintered)	31	31
Alnico® 5 (Anisotropic, Cast)	22	22
Alnico® 5 (Anisotropic, Sintered)	25	25
Alnico® 5-7 (Anisotropic, Cast)	18	17
Alnico® 5DG (Anisotropic, Cast)	20	20
Alnico® 6 (Anisotropic, Cast)	23	23
Alnico® 6 (Anisotropic, Sintered)	24	24
Alnico® 7 (Anisotropic, Cast)	21	21
Alnico® 8 (Anisotropic, Cast)	17	17
Alnico® 8 (Anisotropic, Sintered)	19	19
Alnico® 8HC (Anisotropic, Cast)	15	16
Alnico® 8HC (Anisotropic, Sintered)	27	27
Alnico® 9 (Anisotropic, Cast)	14	14
Carpenter® P6 Alloy	34	34
Chromindur® II Alloy	32	32
Cunife® Alloy	40	40
Ferrite 1 , Sintered Iron Barium Oxide	11	11
Ferrite 2 , Sintered Iron Barium Oxide	10	10
Ferrite 3, Sintered Iron Barium Oxide	9	9
Ferrite 4, Sintered Iron Strontium Oxide	8	8
Ferrite 5, Sintered Iron Strontium Oxide	7	7
Ferrite A, Bonded Iron Barium Oxide	12	12
Ferrite B, Bonded Iron Barium Oxide	13	13
High Cobalt Steel	38	38
Low Cobalt Steel	41	41
Neodymium® 27	6	6
Neodymium® 27H	5	5
Neodymium® 30	3	3
Neodymium® 30H	4	4
Neodymium® 35	1	1
Neodymium® 35H	2	2
Platinum Cobalt Alloy	16	15
Tungsten Steel	39	39
Vicaloy® I Alloy	33	33
Vicaloy® II Alloy	26	26

Spearman's correlation is used to compare ordinal data. Moreover, the correlation coefficient obtained is <u>0.999608</u> (approx.1).

The results of the three cases are shown in the figure below:

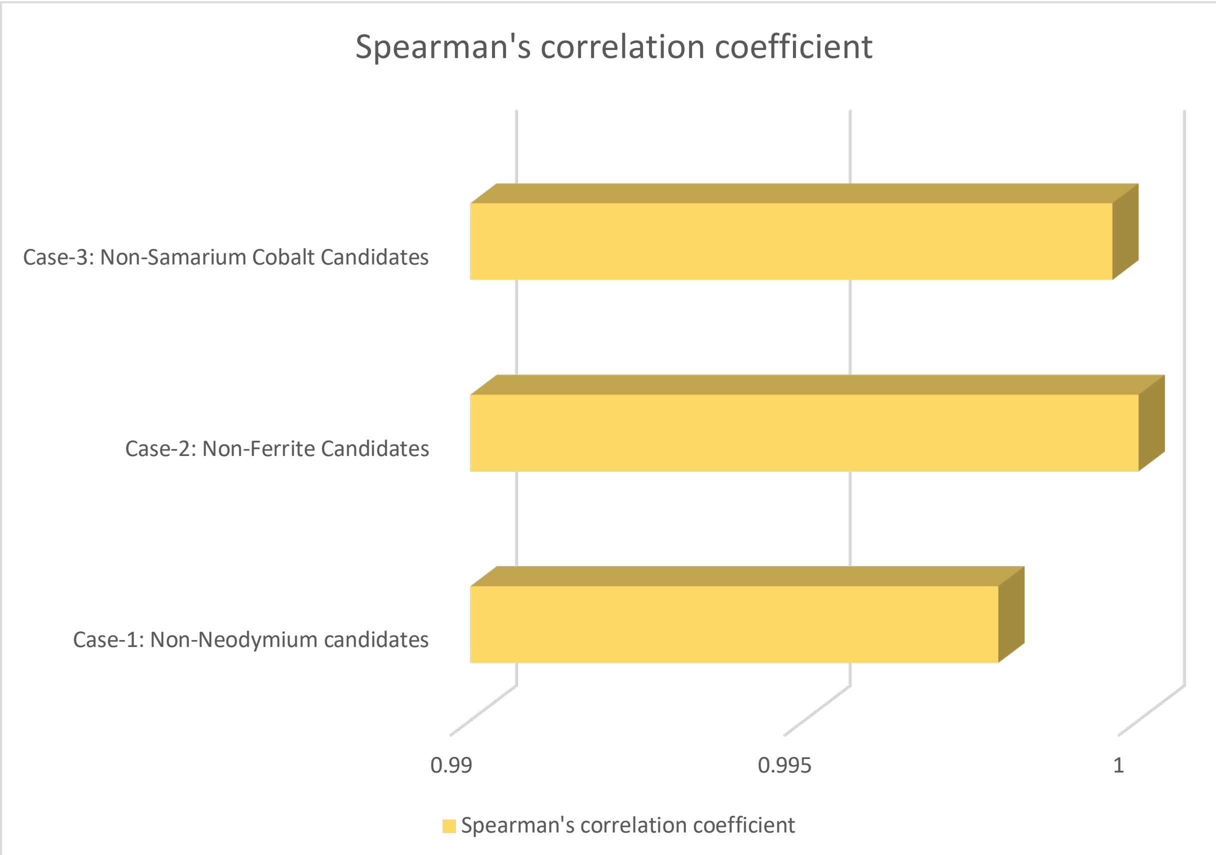


Figure 35: Comparison of Spearman's correlation coefficient values for the above three cases.

The correlation coefficients of the means set of ranks obtained in three separate cases round about 1, which establishes the robustness of the framework.

5.8 Advanced analyses

For plotting PCA graphs and performing hierarchical clustering (HC), Minitab software is employed.

PCA on Ordinal Data

In statistical terms, the rank sets obtained from the various methods are called as Ordinal data. In the visualization phase, these ordinal data points are analyzed presented using hierarchical clustering and Principle component analysis techniques.

The ranks obtained from 25 different MADM methods which are high dimensional datasets (because each set of ranks is obtained by each MADM method employed) can be plotted in a simple 2-dimensional graph using principle component analysis (PCA). PCA is conducted on the ordinal data to generate a score plot and a biplot to present the final ranks of the candidates.

a. Score plot

Score plot projects the high dimensional data in a 2-dimensional space. It is achieved by calculating the distance of each data point with respect to the old axes and positioning them in the new 2-dimensional space by determining the eigen values.

The material at the furthest left of the first principal component is the top ranked material, and the material at the extreme right is the lowest ranked material.

Figure 36 presents the score graph with plots of the materials concerning two principal components which envelopes the maximum variation of the ordinal data.

Figure 36: Score plot with ordinal (rank) data.

Table 20 lists the materials in the ranking order with Neodymium 35 topping the table and low Cobalt steel at the bottom.

Table 20: Ranks of the material candidates according to PCA.

Designations	Magnetic Materials	Final Ranks
N-35	Neodymium® 35	1
N-35H	Neodymium® 35H	2
N-30	Neodymium® 30	3
N-30H	Neodymium® 30H	4
N-27H	Neodymium® 27H	5
N-27	Neodymium® 27	6
F-5	Ferrite 5, Sintered Iron Strontium Oxide	7
F-4	Ferrite 4, Sintered Iron Strontium Oxide	8
F-3	Ferrite 3, Sintered Iron Barium Oxide	9
F-2	Ferrite 2, Sintered Iron Barium Oxide	10
F-1	Ferrite 1, Sintered Iron Barium Oxide	11
F-A	Ferrite A, Bonded Iron Barium Oxide	12
SC-4	Cobalt Samarium 4	13
F-B	Ferrite B, Bonded Iron Barium Oxide	14
SC-1	Cobalt Samarium 1	15
SC-2	Cobalt Samarium 2	16
SC-3	Cobalt Samarium 3	17
AL-9	Alnico® 9 (Anisotropic, Cast)	18
PC	Platinum-Cobalt Alloy	19
AL-8H(C)	Alnico® 8HC (Anisotropic, Cast)	20
AL-8(C)	Alnico® 8 (Anisotropic, Cast)	21
AL-5-7	Alnico® 5-7 (Anisotropic, Cast)	22
AL-8(S)	Alnico® 8 (Anisotropic, Sintered)	23
AL-5DG	Alnico® 5DG (Anisotropic, Cast)	24
AL-7	Alnico® 7 (Anisotropic, Cast)	25
AL-5(C)	Alnico® 5 (Anisotropic, Cast)	26
AL-6(C)	Alnico® 6 (Anisotropic, Cast)	27
AL-6(S)	Alnico® 6 (Anisotropic, Sintered)	28
AL-5(S)	Alnico® 5 (Anisotropic, Sintered)	29
V-II	Vicaloy® II Alloy	30
AL-8HC(S)	Alnico® 8HC (Anisotropic, Sintered)	31
AL-12	Alnico® 12 (Anisotropic, Cast)	32
AL-4(C)	Alnico® 4 (Isotropic, Cast)	33
AL-2(C)	Alnico® 2 (Isotropic, Cast)	34

AL-4(S)	Alnico® 4 (Sintered)	35
C-II	Chromindur® II Alloy	36
V-II	Vicaloy® I Alloy	37
C-P6	Carpenter® P6 Alloy	38
AL-2(S)	Alnico® 2 (Isotropic, Sintered)	39
AL-12	Alnico® 1 (Isotropic, Cast)	40
AL-3	Alnico® 3 (Isotropic, Cast)	41
C-H	High Cobalt Steel	42
T	Tungsten Steel	43
CA	Cunife® Alloy	44
C-L	Low Cobalt Steel	45

The entire PCA score plot can be read using only the PC-1 axis since it accommodates 95.2% of the original data, as shown in figure 21.

Table 21: Distribution of the data among the 25 PCs.

Eigenvalue	**23.804**	0.886	0.093	0.078	0.042	0.034	0.023	0.010	0.010	0.006	0.004
Proportion	**0.952**	0.035	0.004	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000
Cumulative	**0.952**	0.988	0.991	0.994	0.996	0.997	0.998	0.999	0.999	0.999	1.000
Eigenvalue	0.003	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000
Proportion	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Cumulative	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Eigenvalue	0.000	0.000	0.000								
Proportion	0.000	0.000	0.000								
Cumulative	1.000	1.000	1.000								

So, the entire results can be determined by reading the graph concerning PC-1 alone.

Neodymium 35 is ranked number 1, and low Cobalt steel ranked as the worst among the considered list of material candidates when considered the selected attributes.

b. Biplot

A biplot combines both the loading and the score plots and visually represents the relation between the material candidates and the 25 principal components generated in this case (due to 25 MADM methods from which the ranks are obtained). The ranks generated from each method is evaluated on eight selection criteria (i.e., material properties).

Materials away from the individual axis, have low magnitude (i.e., rank) concerning that axis and materials towards or in the direction the individual axis will have high magnitude. The material candidate at the extreme end in the opposite direction to the individual axis has the lowest rank according to that axis and vice-versa. Figure 37 is the biplot plotted on the ordinal data of the materials.

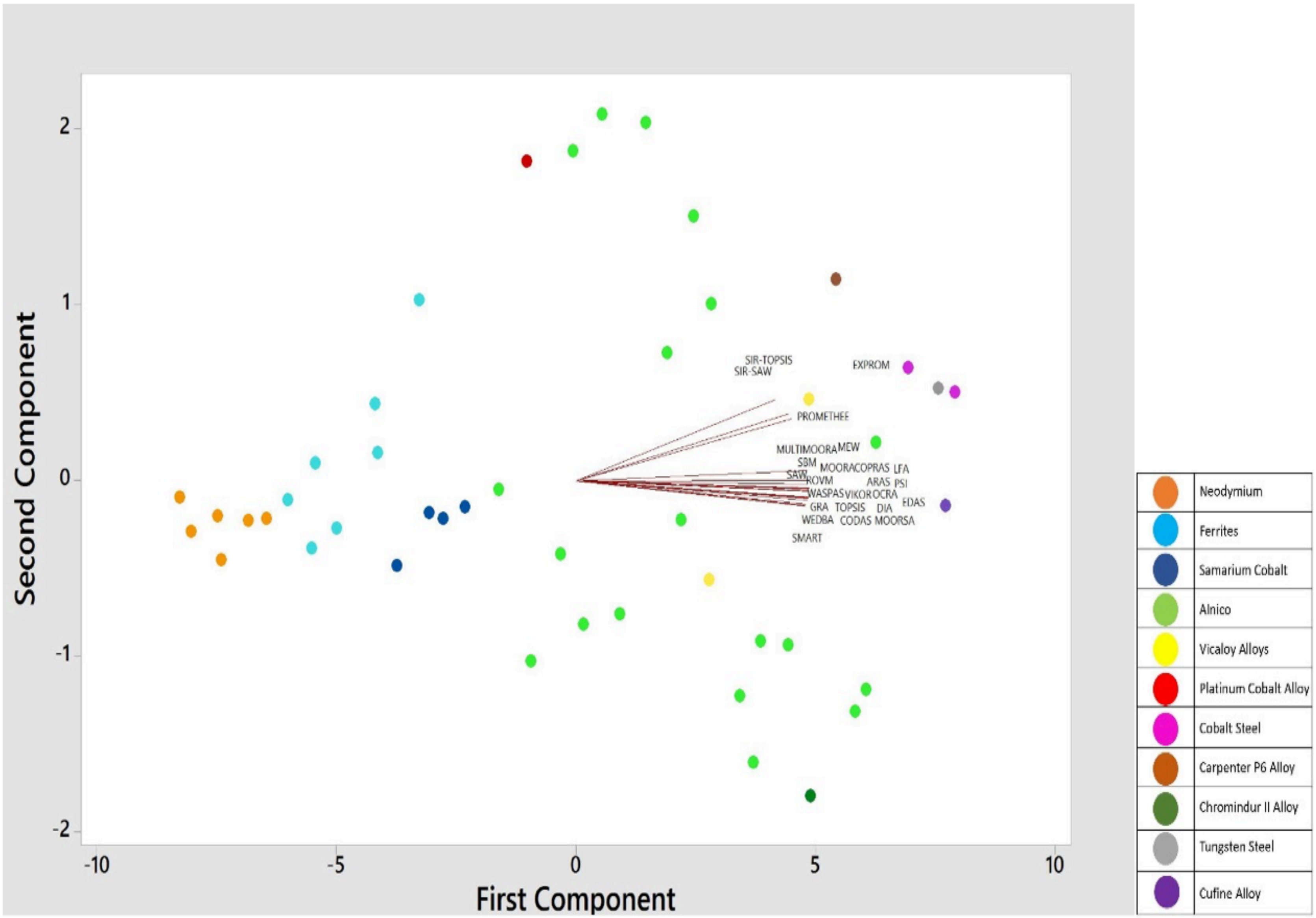


Figure 37: PCA biplot of the ordinal data derived from 25 MADM methods.

PCA on Cardinal Data

Cardinal data refers to quantity. PCA is conducted on the materials concerning eight criteria (i.e., material properties). Therefore, the principal components generated will be eight, each representing a mechanical property. PCA on cardinal data provides additional information on the relationship between material candidates and material properties.

a. Score plot

The cardinal data belonging to a high dimensional space (with eight different criteria) is reduced to a low dimensional space (i.e., 2-D space). The variation of the original data captured by these two PCs cumulatively accounts for nearly 80%, which is reliable. Figure 38 shows the score plot of the cardinal data.

The score graph is plotted using the eigen distance, which is a measure of position of data points in principal components with respect to the original dimensions. The eigen values of the data in score graph is presented in table 22.

Table 22: Distribution of the variation captured by eight PCs

Eigenvalue	4.2585	2.0603	0.6596	0.4863	0.3312	0.1377	0.0498	0.0167
Proportion	0.532	0.258	0.082	0.061	0.041	0.017	0.006	0.002
Cumulative	0.532	0.790	0.872	0.933	0.974	0.992	0.998	1.000

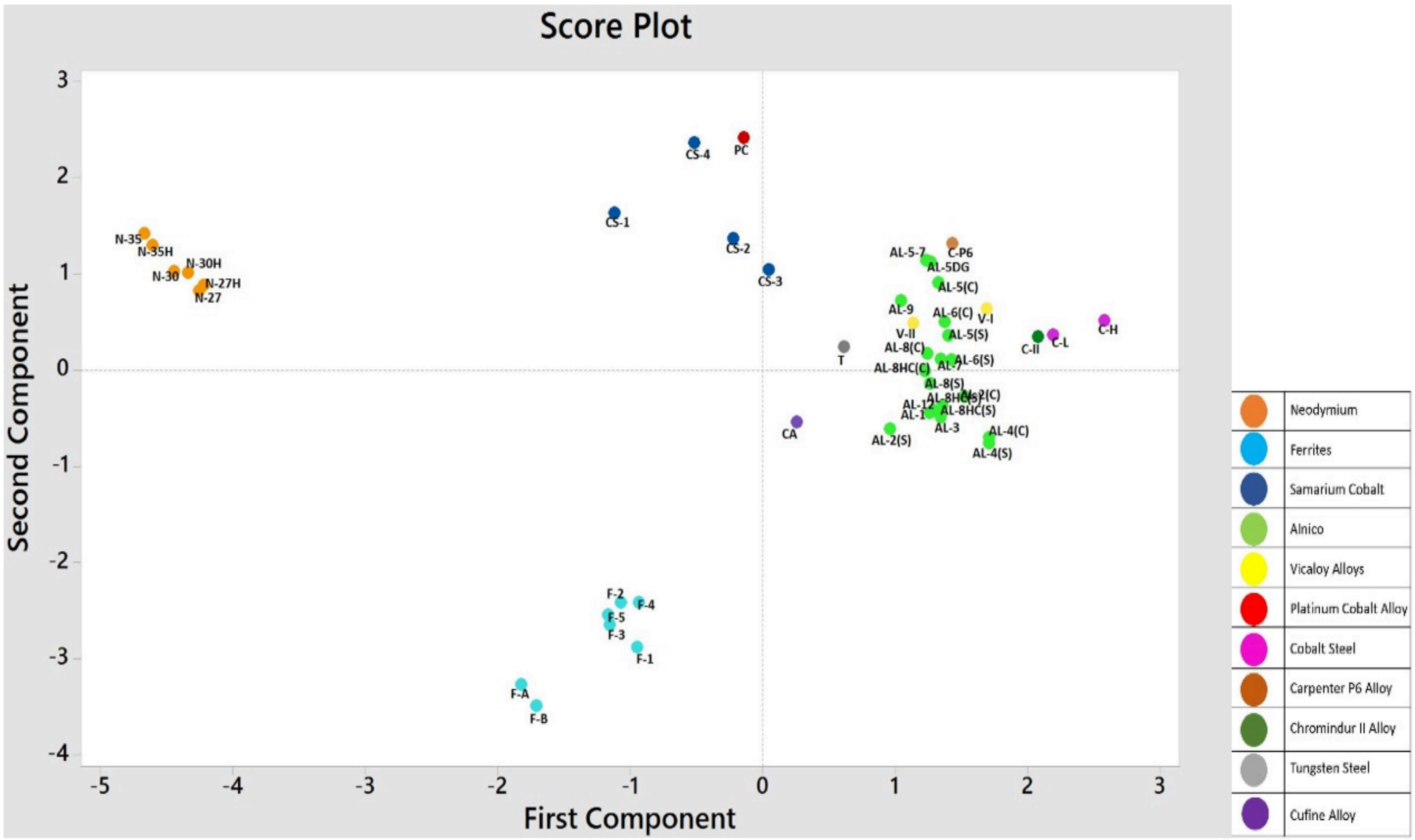


Figure 38: PCA score plot of the cardinal data of the material candidates.

b. Loading plot

The loading plot of the PCA shows the relation between each loading axis (representing each attribute) and the two principal components. The loading axes in parallel or make an acute angle with to the first or second principal component contributes positively to that component. Whereas, the loading axes perpendicular or make an obtuse angle with respect to any principal component, contributes negatively to it.

In the present scenario, the distribution of eight loading axes with respect to each principal component is defined by the following equation:

First Principal Component,

$$0.081\ (d) + 0.358\ (\alpha) - 0.411\ (\rho) - 0.416\ (Hc) - 0.003\ (Mr) - 0.382\ [(BH)max] + 0.427\ Tc + 0.442\ (St)$$

Second Principal Component,

$$0.547(d) + 0.015\ (\alpha) - 0.317(\rho) + 0.273(Hc) + 0.594(Mr) + 0.382[(BH)max] + 0.162(Tc) + 0.025(St)$$

Where,

d= Density

St= Maximum Service temperature

α= Coefficient of Linear thermal Expansion

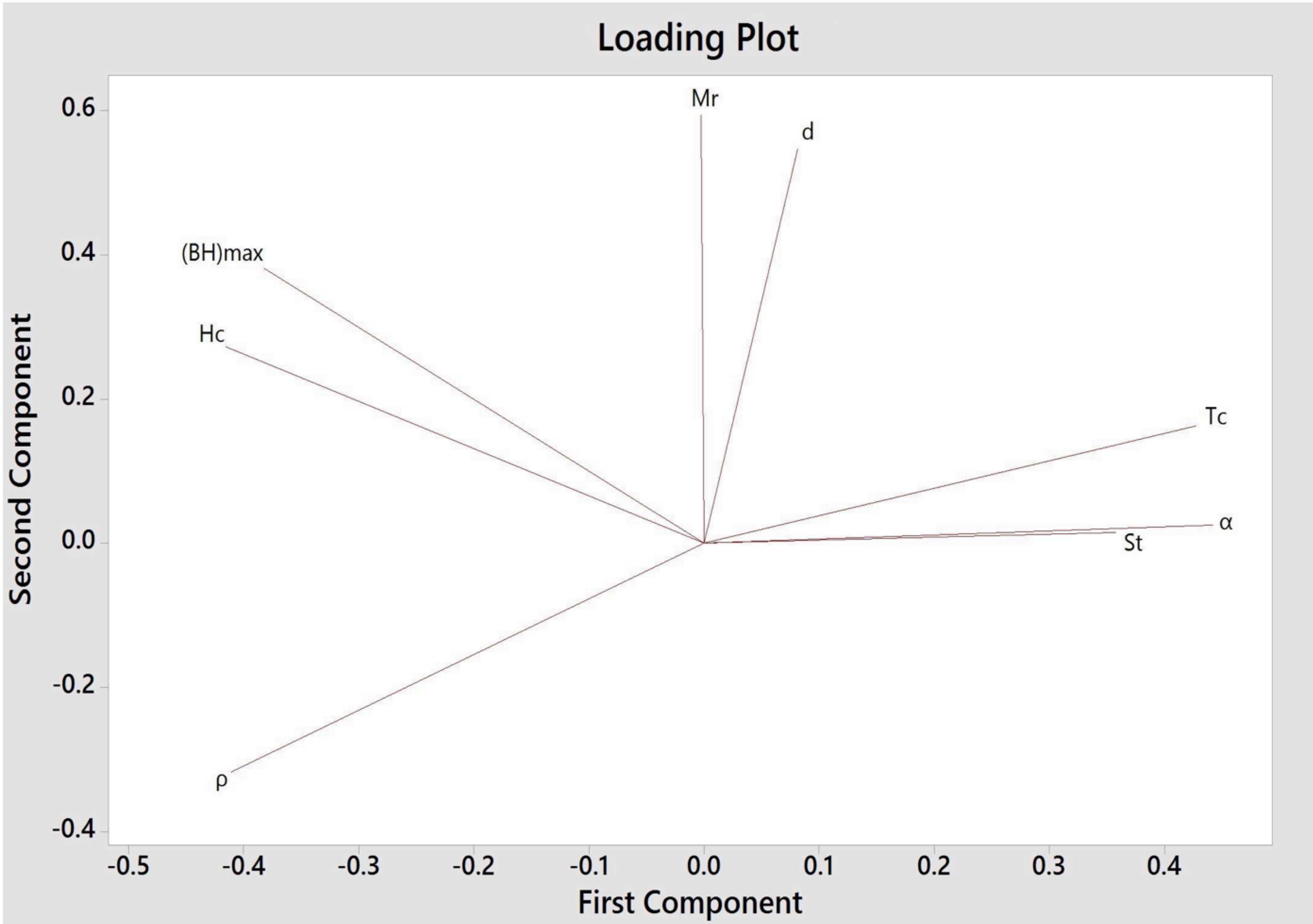
ρ= Electrical Resistivity

Hc= Magnetic Coercive force

Mr= Magnetic Remanence

$(BH)max$= Magnetic Maximum Energy product

Tc= Curie temperature



$\{0.547(d) + 0.015\,(\alpha) - 0.317(\rho) + 0.273(Hc) + 0.594(Mr) + 0.382[(BH)max] + 0.162(Tc) + 0.025(St)\}$

$\{0.081\,(d) + 0.358\,(\alpha) - 0.411\,(\rho) - 0.416\,(Hc) - 0.003\,(Mr) - 0.382\,[(BH)max] + 0.427\,Tc + 0.442\,(St)\}$

Figure 39: PCA loading plot of the cardinal data.

c. Biplot

Biplot provides a very useful information that can assist in the decision-making process. It is a combination of score plot and the loading plot. The biplot graph shows the clusters of the materials formed based on their most significant association with each loading axis. The material candidates clustered around different loading axes define the linear relation between materials and the corresponding attributes.

The following observations are made from figure 40,

- Neodymium magnets are clustered together because they possess very high Magnetic maximum energy product $(BH)_{max}$ and Magnetic coercive force (H_c) properties.
- Ferrites are clustered together because they possess very high Electrical resistivity (ρ).
- Samarium Cobalt and Platinum Cobalt possess high magnetic remanence (M_r)), so they are clustered together.
- Materials clustered away or in opposite direction of the loading axes, have low values with respect to those attributes. For example, Alnicos possess high Curie temperatures (T_c), Coefficient of linear thermal Expansion (α), and Maximum service temperature (S_t). But, they possess low $(BH)_{max}$, H_c, and ρ.

So, engineers looking for materials with high Magnetic Coercivity and maximum energy product can choose materials from Neodymium cluster, or if they need materials with high electrical resistivity, they can choose materials from ferrites cluster. So, the biplot showcase the relation between selection criteria (i.e., properties) and the material alternatives.

Apart from that, these clusters give the decision-makers an option for substituting a material for an existing design. For example, platinum Cobalt alloy is capable of replacing Samarium Cobalt alloys for a design requiring high $(BH)_{max}$, H_c and M_r.

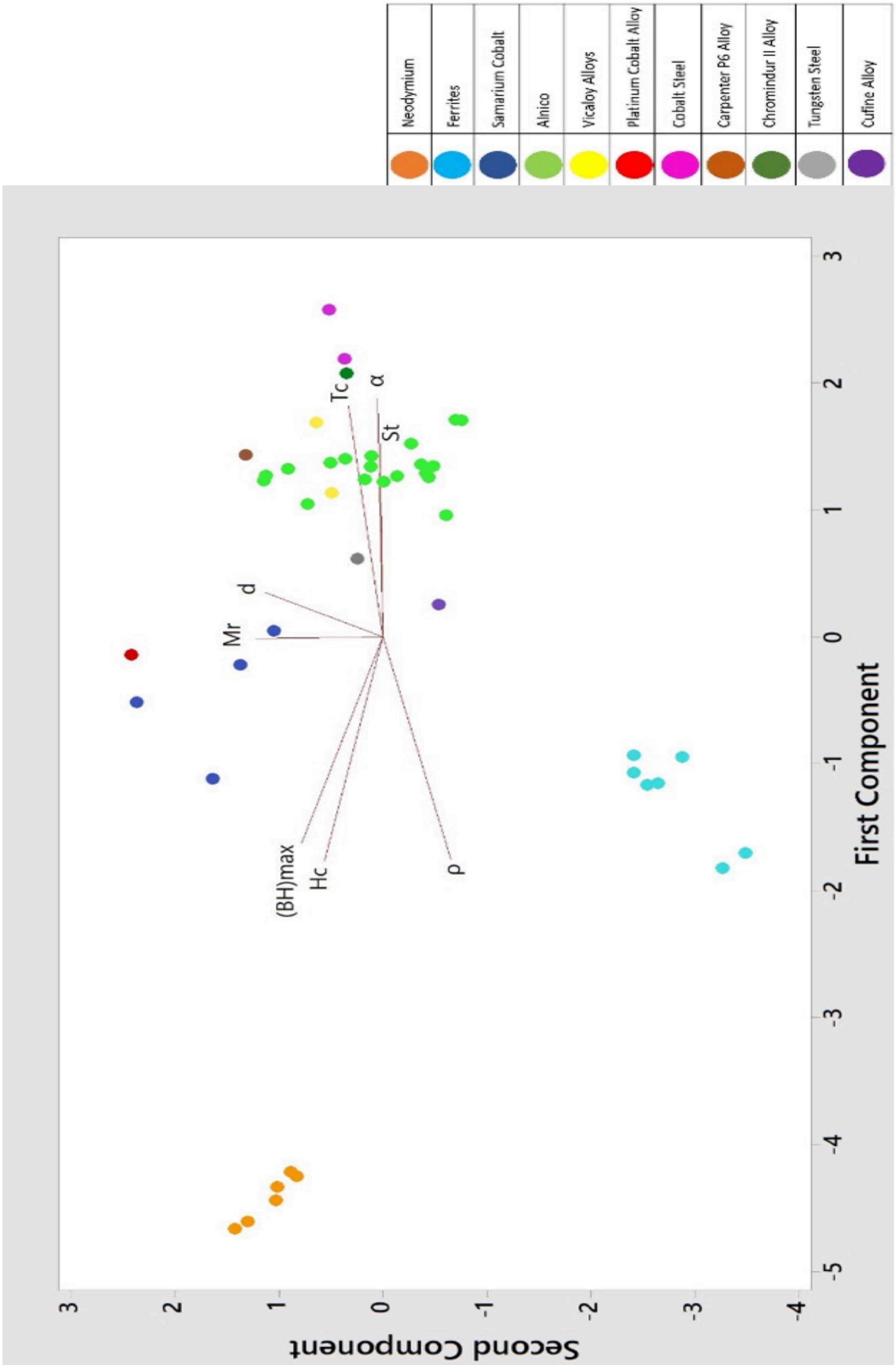


Figure 40: PCA biplot of the cardinal data.

d. Outlier plot

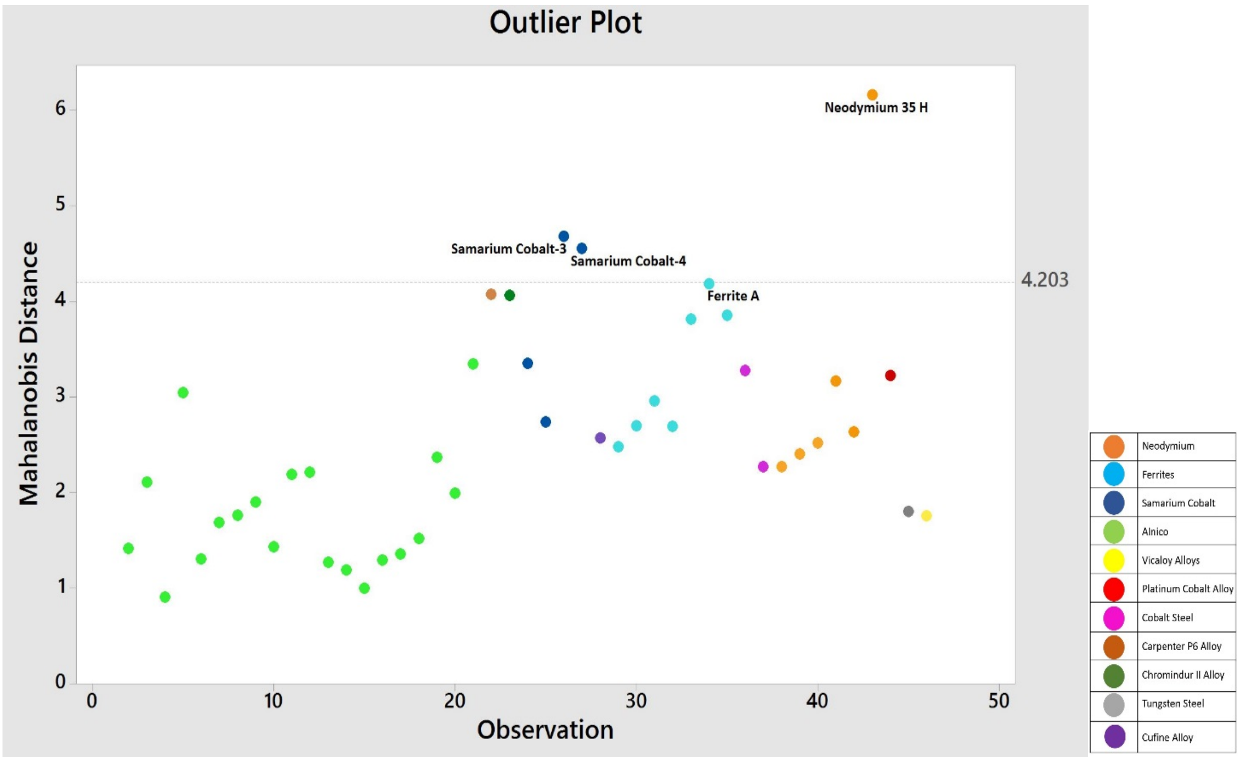


Figure 41: Outlier plot of PCA on the cardinal data.

In statistics, an outlier plays a crucial role in understanding the anomaly in a given sample. The outliers detected from the data (from figure 41) are Neodymium 35H, Samarium Cobalt-3, and Samarium Cobalt-4. These materials outline from the rest of the candidates beyond the similarity threshold of 4.203.

5.9 Hierarchical clustering

Ordinal Data

Cluster analysis is done on the sets of ranks to visually represent the similarities existing between them. For this, an agglomerative hierarchical clustering approach is employed. The type of linkage created between the various clusters is Complete Linkage. In complete linkage, we define the distance between two clusters to be the maximum distance between any single data point in the first cluster and any single data point in the second cluster. Based on this definition of distance between clusters, at each stage of the process, we combine the two clusters that have the smallest complete linkage distance [292]. The similarities between the clusters are calculated using the correlation coefficient distance between the variables (i.e., ranks) of the different MADM methods.

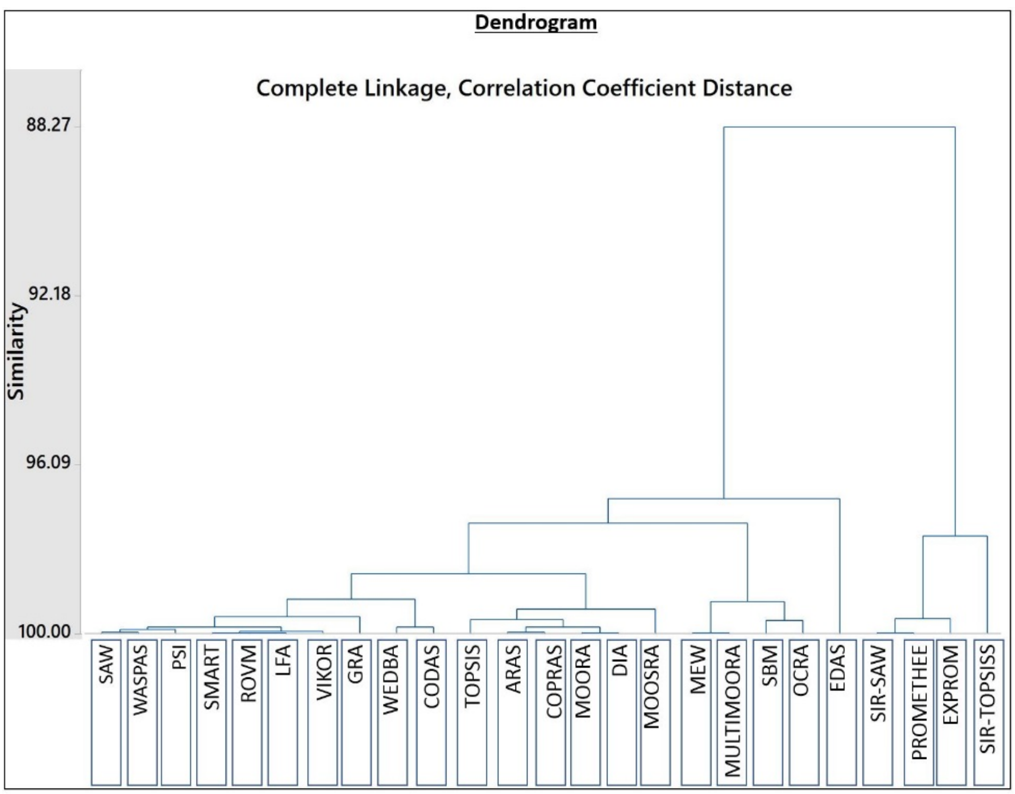


Figure 42: Agglomerative hierarchical clustering of the methods based on ranks.

Figure 42 is a representation of the clusters in the form of a dendrogram. The vertical axis of the dendrogram shows the degree of similarity existing between the clusters. The ultimate cluster is formed between Outranking methods and Rest of the methods, once again proving the vast distinction existing between them. The lowest correlation coefficient distance between the two ultimate clusters is about 88%, which shows the robustness of the ranks obtained.

Cardinal Data

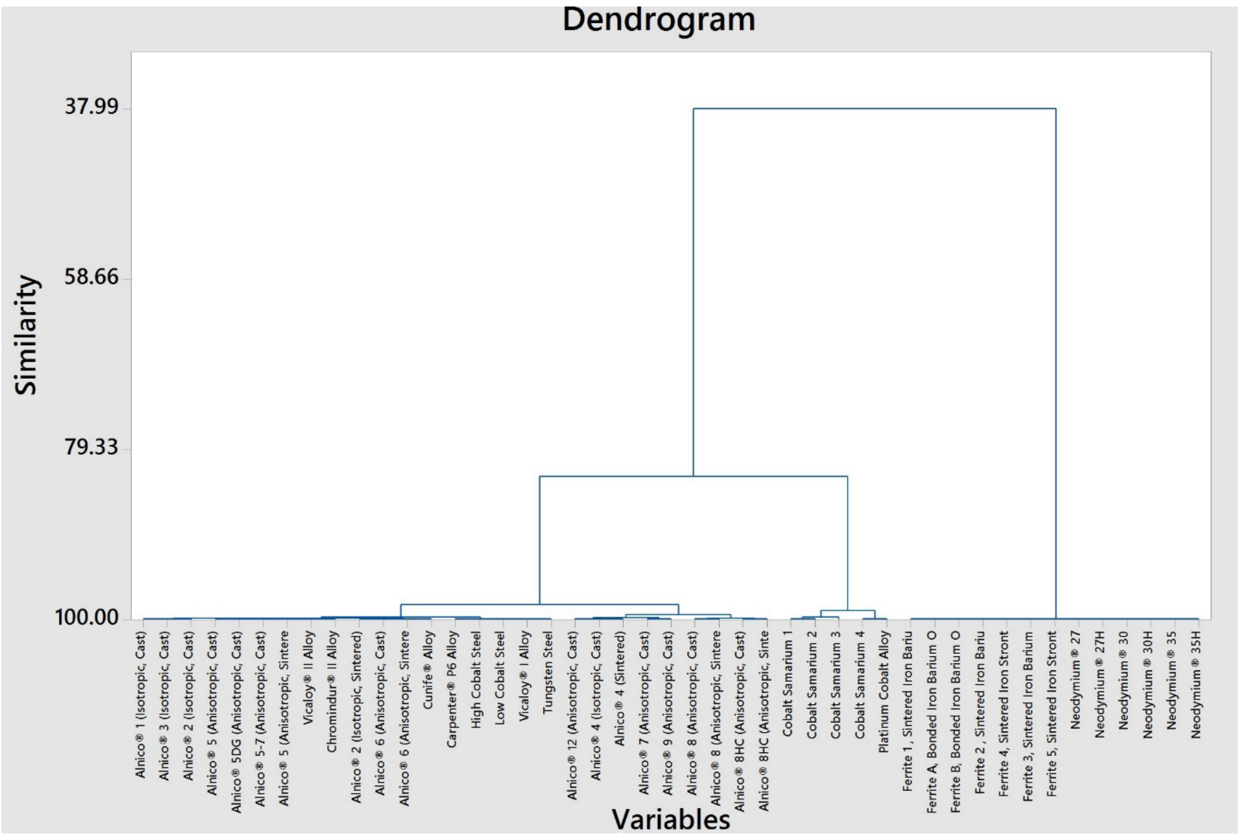


Figure 43: Hierarchical clustering of the material candidates on the basis of the selection criteria.

Hierarchical clustering (HC) of the materials on their properties is conducted to deliver a comprehensive database for the decision-maker to utilize during the material selection. Figure 43 shows the clustering of 45 material candidates in the form of a dendrogram. The major clusters formed between the materials is between Neodymium family/and Ferrites family and the rest of the materials. From the rankings obtained from different MADM methods, top 13 ranks are obtained by materials from Neodymium and Ferrites class. Moreover, HC has reinstated the underlying difference between these materials among the chosen material alternatives (as shown in figure 44).

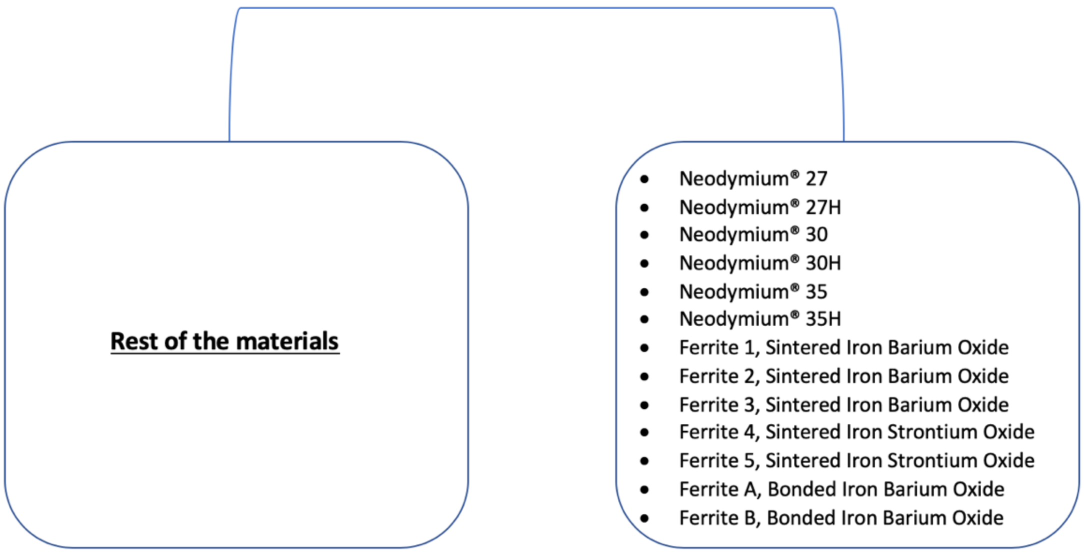


Figure 44: Final cluster of the hierarchical clustering of materials.

Another major cluster is formed between Samarium Cobalt magnets and rest of the candidates excluding Neodymium and Ferrites magnets (as shown in figure 45).

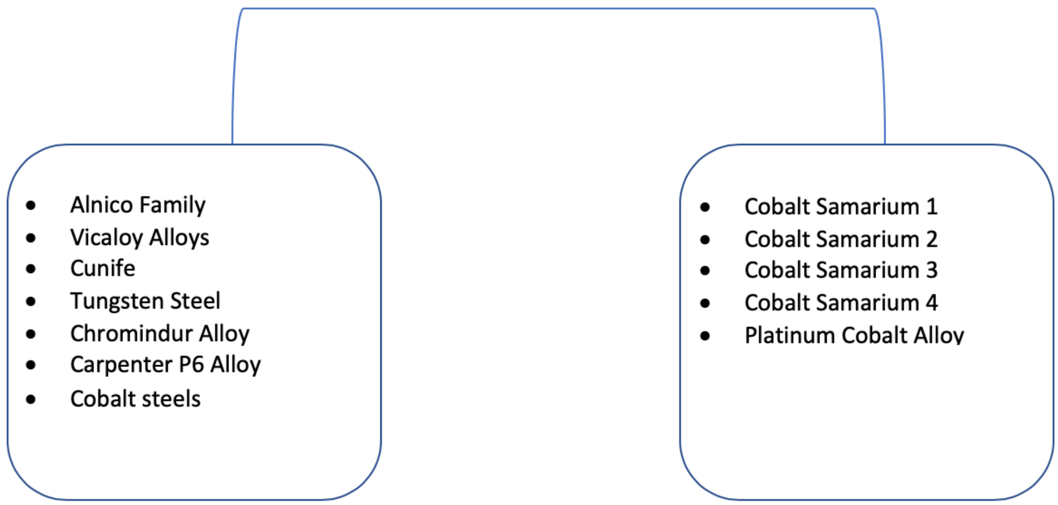


Figure 45: Penultimate cluster of the hierarchical clustering of materials.

5.10 Documentation

The documentation phase allows the decision-maker to research more about the ranked materials to know their strengths and potential weakness with respective attributes that are not considered during the initial material selection process. So, the evaluation of the top-ranked material on the factors such as cost, manufacturability, and environmental factors like carbon footprint, recyclability, etc., gives a comprehensive history of the materials.

The top ranked materials from the results belong to Neodymium and Ferrite families. A detailed history of these materials, which includes previous applications, registered failures associated with them, carbon footprint details, etc., must be acquired. Table (), further evaluates these top-ranked materials on unsubscribed criteria.

5.11 Final selection

Based on the results from ranking, visualization, and documentation phases, the final choice is made.

Chapter 6: Conclusions

The proposed methodology has been successfully applied to a generalized material selection problem involving permanent magnets for a function-oriented design. Based on the results and discussions, there can be two sets of conclusions drawn, 1) conclusions on methodology and, 2) conclusions on material selection.

Methodology:

1. This methodology captures voice of the data which is evident from the employment of Shannon's entropy objective method.
2. The weights obtained from this objective method were extremely reasonable considering the list of criteria.
3. Admission of the objective weighting method has eliminated the prejudice of the experts to some extent.
4. The twenty-five MADM methods have yielded co-relatable rank orders.
5. Spearman's correlation has shown that the ranks obtained from ARAS method is the most co-relatable order among all the twenty-five methods considered.
6. Sensitivity analysis has proven the robustness of the methodology.

7. Cluster analysis on the ranks has clearly segregated the outranking methods and rest of the methods into two clusters. This analysis is conducted on the basis of similarities existing between the methods.
8. Using the Principal Component Analysis on the ordinal data produced a ranking order which was very similar to the ranks obtained from ARAS method.
9. Loading plot of PCA, again reinstated the segregation between outranking and the rest of the methods.

Material Selection:

1. The four properties which inherent for the magnetic behavior namely Electrical Resistivity (ρ), Magnetic Coercive force, Magnetic Remanence and Magnetic Maximum Energy product account for 92% of over-all weight distribution among the eight selection attributes.
2. From the ranks obtained from twenty-five MADM methods, a clear trend of the order can be extracted which shows Neodymium magnets leads the permanent magnets followed by ferrites. If the magnets are grouped into families then, the ranks will be as presented in the table 23.

Table 23: Ranking of the family of materials

Material Family	Ranks
Neodymium	1
Ferrites	2
Cobalt Samarium	3
Alnicos	4

3. The score plot and the loading plot of PCA has shown the clusters of materials formed based on their affinity towards the material attributes.
4. The loading plot of PCA shows relation between material attributes.
5. Neodymium 3H, Samarium Cobalt-3 and Samarium Cobalt -4 have been identified as outliers.
6. Cluster analysis on the cardinal data of the materials have produced clusters of material based on the similarity. The most notable ones out of 45 clusters are the ultimate cluster between cluster comprising Neodymium and ferrite magnets and the cluster containing rest of the magnets and the penultimate cluster segregating Cobalt Samarium and rest of the magnets excluding ferrites and neodymium magnets.
7. The simplicity of this material selection methodology without losing the comprehensiveness to deal with multifaceted decision problem is the key feature.

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