Three Essays in Macro Finance

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Economics)
in The University of Michigan
2019

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ACKNOWLEDGMENTS

First and foremost, I would like to express my deepest gratitude to my advisors Matthew Shapiro, Julio Andres Blanco, Pablo Ottonello, and Toni Whited. It is a great honor for me to have Matthew and Toni on my committee. They have the busiest schedule on Tappan Street, but they have always been responsive to my appointment emails and been patient to my endless, sometimes naive, questions during our meetings. They taught me how to think, write, and present as a professional economist.

The most significant positive shock to my research productivity during the past six years was the coming of Andres and Pablo. As teachers, they taught me techniques to solve challenging problems. As coauthors, they inspired me with their insights and persistence. As friends, they cheered me up from frustration and helped me keep confident when negative shocks hit.

I would like to especially thank Professor John Leahy for his feedback and advise during the development of my job market paper. I would never forget the meeting with John in Andres' office on June 4th, 2018. Without his encouragement in his signature humor, this dissertation might be delayed to the next year.

It was a big fortune for me to have many great friends. It was a great experience to share the office with Xinwei Ma, Kenichi Nagasawa, and Elchin Suleymanov. They make Lorch 107 the most hard-working and productive office in the basement of Lorch Hall. As my roommate, undergraduate classmate, and Ph.D. classmate, Hang Yu helped me a lot in daily life during the past ten years. I also feel lucky to have Yifei Wang as my friend after coming to Michigan. I learned a lot from his talents in research and wisdom in life.

Last but not least, I feel deeply indebted to my parents Jianbao Guo, Guizhen Zhang, and my fiancée Mengyao Ruan. My parents don't have high education degrees but they are open-minded. They sent me out for school when I was three years old and invested a lot in my education. Without their unconditional support and love, it would be impossible for me to go this far. Meeting my fiancée is the luckiest thing since I came to Michigan. Her mental support has been the most important source of my power to keep moving forward.

Looking back, I had six bumpy years. In the first two years, I was struggling with English. In the third year, I had a hard time finding advisors. In the fourth and fifth years, I had many failed trials and finally learned what research was actually about. Then, I had to rush up and push myself out to the job market. At this very moment of writing down this paragraph, I know that it is close to the time for me to leave Ann Arbor. Besides the words written to express my gratitude for my advisors, friends, and families, I also want to write some words for myself to keep in mind in the future.

I cannot remember how many nights when I woke up in the midnight and started running Kalman Filter on my diary book: given what happened today, does it imply that I am a low-type agent or just an agent hit by a temporary negative shock? When I open my old diary and read these analyses now, they look noisy and do not make too much sense. However, from these non-sense analyses, I get the most important reflection about myself over my Ph.D. years. I wish myself to keep the following words in mind from now on:

1. It takes time to build human capital, so please be patient. You should keep learning yourself time to time, but don't run the Kalman Filter at a high-frequency because it can not improve your information set much but waste too much energy you should use alternatively for solving your policy.

2. It is good to have ambitions, but please be practical first. Don't let the goal go beyond your capacity; otherwise, you will be emotional and possibly get trapped in the hole of self-enforcing failure. Nothing can be perfect at any time, and the only thing you should do is to act dynamically: respect rules of the game, make something exist first, then keep improving it.

3. Keep running with gratitude; don't stop no matter you get a punch or a hug.

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ABSTRACT

This dissertation consists of three connected chapters on macro finance.

The first chapter studies the aggregate relevance of financial market fluctuations in driving firms' investment fluctuations. Different with the previous studies, this paper identifies the aggregate shocks based on an estimated general equilibrium model with firm-level heterogeneity. The model includes financial frictions and eight aggregate shocks: two financial shocks capturing the exogenous variation in firms' financing conditions, and six non-financial shocks capturing the exogenous variation in firms' investment profitability. The quantitative results imply that aggregate financial shocks only contribute 1% of the total variation of U.S. public firms' investment. This negligible aggregate relevance of financial shocks mainly results from the interaction between firm-level heterogeneity and general equilibrium effects. If the model is degenerated to a representative-firm model, the implied aggregate relevance of financial shocks will be 50 times larger, which indicates the importance of micro-level heterogeneity in identifying the source of business cycle fluctuations.

The second chapter studies the transmission of monetary policy through firms' equity financing. At the aggregate level, we document that firms respond to monetary expansions by increasing equity issuance, and that the response of equity flows is quantitatively as large as that of debt flows. At the micro level, we show that monetary stimulus significantly mitigates the stock price drop associated with announcements of equity issuance, suggesting a reduction in the asymmetric-information premium in equity markets. We construct a model of firms' equity financing under asymmetric information that can rationalize these findings and study its aggregate implications. Monetary policy leads to "amplification through selection" or a feedback between the issuance choices of high-quality firms and the asymmetric information premium. This channel implies a scope of policy linked to stabilizing informational frictions over the cycle.

The third chapter studies the source of observed inertia of households' behaviors in adjusting their portfolio. I incorporate both observation costs and adjustment costs into a portfolio choice model (Merton (1969)), study the implied policy function, and derive a set of testable

implications. Qualitatively, the policy generated by the model has both time-dependent and state-dependent components. Quantitatively, this model does not support the assumption that households follow a time-dependent policy, which is a common assumption in the macroeconomic finance literature. There are three testable implications of the model: (1) decrease in inattentive duration indicates a higher probability of adjustment; (2) households with higher risky share tend to pay more attention to their portfolio; (3) among the households investing relatively more in risky assets, those with higher risky shares adjust their portfolio less frequently.

CHAPTER I

Financial Shocks and Investment Fluctuation: Small Firms vs. Large Firms

I.1 Introduction

Financial market turbulence is often referred to as a source of firms' investment fluctuation, but how much it matters remains an open question. The observed variation in firms' investment is a joint result of the unobservable shocks to their financing conditions and the unobservable shocks to their investment profitability. This paper quantifies how much of the observed variation in firms' investment is driven by the financial shocks, based on an estimated general equilibrium model with financial frictions and a continuum of heterogeneous firms.

The key feature of this structural model is the firm-level heterogeneity in their investment and financing behaviors. This feature is motivated by the micro-level empirical evidence: within U.S. public firms, there is a large difference between small and large firms in terms of how they finance their investment. For large firms, most of their investment is financed from their retained earnings. But for small firms, a large part of their investment is financed from external financing markets, especially the equity financing market.

Guided by these facts, I build up a general equilibrium model with three components: a block of heterogeneous firms facing financial frictions, a block of representative agents with New Keynesian setup, and eight aggregate shocks. The block of heterogeneous firms is designed to model the firms' investment and financing choices. Firms make the choices based on their idiosyncratic states and aggregate economic conditions. The block of New Keynesian agents is designed to model the endogenous variation in the aggregate quantities and prices faced by the firms. These eight aggregate shocks are the sources of fluctuation in this model: two of them are financial and they capture the exogenous variation in firms' financing conditions; the other six are non-financial and they capture the exogenous variation in firms' investment profitability.

In the heterogeneous firm block, firms are different in their size, leverage ratio and idiosyncratic productivity. Their idiosyncratic productivity follows an exogenous mean reverting process and their production technology features decreasing returns to scale. These two features lead to a finite optimal size for firms, and heterogeneous investment demand across firms: small firms have higher investment demand than large firms. The firms can finance their investment from cash, debt and equity. Both debt and equity financing are frictional: there is a collateral constraint imposed on their debt issuance, and a cost associated with their equity issuance. These financial frictions lead to a "pecking-order" in firms' financing choices. With the above setup for the firms' life-cycle dynamics and financial frictions, this model can endogenously generate the heterogeneous investment and financing behaviors as observed in the data.

The block of New Keynesian representative agents features sticky prices and wages, external habit formation in consumption, and adjustment costs in capital good production. Within these aggregate shocks, the two financial shocks are the shock to the tightness of collateral constraint, and the shock to the cost of equity issuance. The other six aggregate shocks are the shocks to aggregate productivity, price markups, wage markups, investment good production technology, households' inter-temporal substitution preference, and monetary policy. The New Keynesian frictions and these six non-financial aggregate shocks give this model the ability to match the observed time-variation in aggregate quantities and prices.

Given the core aim of this paper is to project the observed variation in firms' investment into the contribution from different types of aggregate shocks, I need to quantify the model to match the observed time-variation. I first calibrate the parameters excluding the ones governing the shock processes such that the cross-time average investment and financing flows of small and large firms in this model match their corresponding empirical moments. Then I use a Bayesian likelihood method to estimate the parameters governing the shock processes to match the time-variation in both the macroeconomic quantities and prices on the aggregate level, and the U.S. public firms' financing choices on the distributional level. With the estimated model, I quantify the effects of financial shocks on both the disaggregate and aggregate level.

On the disaggregate level, financial shocks are only important to explain the variation in small firms' investment. Financial shocks contribute 30% of the small public firms' investment variation, but only 6% of the large public firms' investment variation. This difference directly results from the fact that small firms' investment relies more on external financing. On the aggregate level, financial shocks contribute little to the aggregate investment variation: only 1% of the variation in U.S. public firms' aggregate investment is driven by financial shocks.

This negligible aggregate relevance of financial shocks is mainly due to the interaction between firm-level heterogeneity and general equilibrium effects. Within the firm population, there are two endogenously determined types of firms: financially-constrained and financially-unconstrained firms. Following a contractionary financial shock, the constrained firms are hit directly and they have to cut their investment. The lower investment demand will dampen the capital good price, and the lower capital good price will motivate the unconstrained firms to increase their investment. In aggregation, their responses largely cancel each other and imply a negligible aggregate relevance of the financial shocks.

As a comparison, in the representative firm model, there is only one firm and this firm is constrained. After the financial shocks hit the economy, there is no unconstrained firm to rebalance the direct effects from the financial shocks. Therefore, the quantitative studies based on the representative firm model typically imply a larger aggregate relevance of the financial shocks. To evaluate the magnitude of the difference between the implications from these two types of models, I degenerate the heterogeneous firm model back to a representative firm model, and repeat the same quantitative analysis with the same data. The quantitative study based on the representative firm model implies that 55% of the U.S. public firms' aggregate investment variation is driven by the financial shocks, which is 50 times as large as the result from the model with heterogeneous firms.

This paper contributes to the literature by incorporating micro-level heterogeneity into the identification of the source of business fluctuation. By incorporating micro-level heterogeneity, this paper delivers two takeaways for the literature. First, this paper quantifies the disaggregate-level relevance of financial shocks and shows that financial shocks are important driving forces for small firms' investment fluctuation. Second, the paper compares the aggregate relevance of financial shocks implied by the model with and without the micro-level heterogeneity, and shows that incorporating the micro-level heterogeneity does make a significant difference for our understanding of macroeconomic dynamics.

Related Literature This paper is directly related to the literature which focuses on identifying financial shocks, e.g. Jermann and Quadrini (2012)Jermann and Quadrini (2012), Justiniano, Primiceri and Tambalotti (2011)Justiniano, Primiceri and Tambalotti (2011) and Eisfeldt and Muir (2016). Both Jermann and Quadrini (2012) and Justiniano, Primiceri and Tambalotti (2011) are based on the representative firm DSGE model and aggregate data. Jermann and Quadrini (2012) identify the financial shock to firms' debt financing capacity and find this shock as an important driving force of the aggregate output fluctuation. Justiniano, Primiceri and Tambalotti (2011) identify the shock to marginal efficiency of investment as the most important driver of business cycle fluctuation, and based on their argument, this shock proxies the functioning of financial markets. Eisfeldt and Muir (2016) use a partial equilibrium model with heterogeneous firms to identify the shock to firms' marginal external financing cost. Building upon these studies, this paper incorporates heterogeneous firms into a general equilibrium setup and identifies the financial shocks from both macro-level and micro-level time-variation.

Similar setups which incorporate a continuum of heterogeneous firms into a general equilibrium framework can also be found in Khan and Thomas (2013), Buera and Moll (2015), and Ottonello and Winberry (2018). Unlike these studies, this paper uses this type of setup not only for modeling the transmission channel of the aggregate shocks, but also for identifying these shocks. The setup of the model in this paper is also different from those previous studies in multiple details. An important difference is that the firms in this paper are allowed to issue equity, which is required by the fact that this paper is quantified based on the data of public firms and equity financing is the major source of external financing for the financially constrained public firms.

Given that the micro-level setup in this model is targeted to characterize the public firms' choice, this paper is also related to the structural corporate finance literature, e.g. Gomes (2001) and Hennessy and Whited (2007). The studies in this literature focus on using the structural model to explain the heterogeneity in firms' investment and financing behavior in steady state. This paper follows their setup of the firm-level heterogeneity and embeds it into a general equilibrium framework, so the model can be used to study both cross-time and cross-sectional variation in firms' investment and financing.

The motivational empirical fact in this paper is the difference between small and large firms in terms of how much their investment relies on external financing. Similar facts have been documented by a large body of literature since Gertler and Gilchrist (1994). In a recent study, Crouzet and Mehrotra (2018) investigate the difference between the small and large firms' sensitivity to business cycle fluctuation based on restricted data, and conclude that the difference is not caused by financial frictions. Unlike these studies, the empirical evidence in this paper is based on public firms, but not the firm population in U.S. economy. Given the focus on the sample of public firms, this paper is more related to empirical studies in corporate finance, e.g. Covas and Den Haan (2011) and Begenau and Salomao (2018), which document the pattern of debt and equity financing flows of small and large firms over the business cycle. This paper has a slightly different empirical focus: the reliance of different firms' investment on debt and equity financing. The measurement of this reliance follows the spirit of the measurement used by Zetlin-Jones and Shourideh (2017). The results in this paper can also be supported by previous studies. The disaggregate level implication in this paper is consistent with the findings in Gilchrist and Himmelberg (1998), Baker, Stein and Wurgler (2003), Warusawitharana and Whited (2016) and Mclean and Zhao (2014). Gilchrist and Himmelberg (1998) identifies the financial shock by a panel VAR and they found that the identified financial shocks play a much more important role in driving small U.S. public firms' investment. Baker, Stein and Wurgler (2003) provide reduced-form evidence showing that stock market misvaluation can significantly affect the investment of equity-dependent firms. Given the high fraction of small firms that are equity-dependent firms, their results are also consistent with the result in this paper. Warusawitharana and Whited (2016) quantify the effects of stock market misvaluation with a structural model and find that stock market misvaluation only has significant effects on small firms' investment.

In terms of the aggregate level implication, Zetlin-Jones and Shourideh (2017) have similar findings with this paper. Zetlin-Jones and Shourideh (2017) study how the financial shocks to firms' debt financing capacity affect aggregate output, based on a general equilibrium model with both private firms and public firms. Due to general equilibrium feedbacks, financial shocks trigger the reallocation between private and public firms and the aggregate effects of financial shocks are largely dampened.

This paper is also related to the literature about the computational methods of solving heterogeneous agent models. Due to the high dimensionality of this model, it is technically challenging to implement the likelihood estimation in this paper. I adopt two techniques to make the estimation feasible. First, I solve the model with an algorithm combining a projection method and perturbation method as proposed by Reiter (2009). Second, I approximate the distribution by a quadrature as proposed by Algan, Allais and Den Haan (2008) and Winberry (2018).

Road Map The remainder of this paper is organized as follows. Section I.2 presents stylized facts about small firms' investment and financing which guide the model setup. Section I.3 introduces the model setup. Section I.4 presents the calibration and estimation of the model. Section I.5 discusses the main findings, and Section I.6 concludes.

I.2 Guiding Facts: Investment and Financing by Firm Size

The main idea of this paper is to use the observed variation in firms' investment and financing flows to identify the unobserved shocks. In this section, I will present some stylized facts about firms' investment and financing to highlight the special feature of small firms: small firms' investment depends more on external financing, especially the financing from equity market. These empirical facts will guide the setup and quantification of the structural model.

This section advances in three steps. First, I will introduce the data source and discuss the construction of sample. Second, I will present some basic facts separately about the investment and financing flows of small and large firms, with an emphasis on the difference between small and large firms in terms of the magnitude and composition of their external financing. Third, I will connect the firms' financing to their investment by measuring the fraction of their investment financed from different source of financing, which highlights the difference between small and large firms in terms of the relative dependence of their investment on different external financing sources.

I.2.1 Data

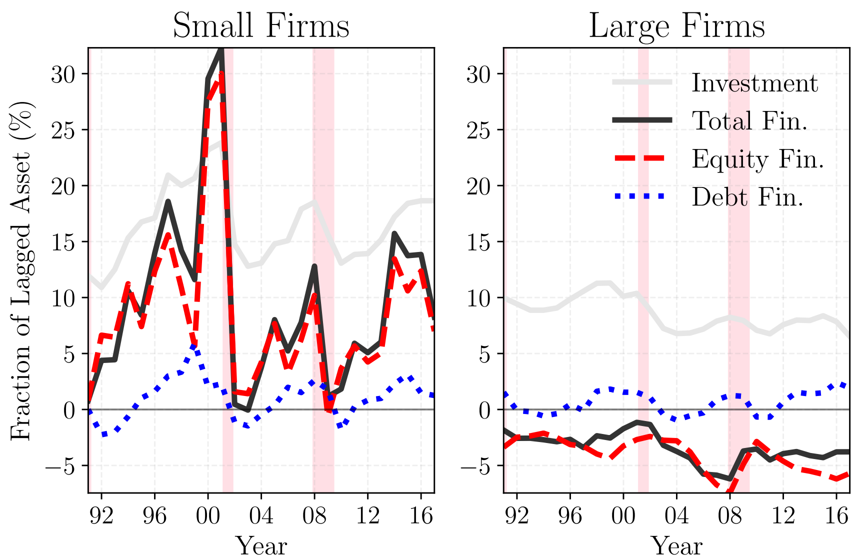
The primary data source is the 2016 CompuStat Annual dataset. Firms from financial sectors (SIC 6000-6999), regulated utility sectors (SIC 4900-4999) and quasi-governmental sectors (SIC 9000-9999) are removed from the sample. To avoid the impact from the change of accounting rules in 1988 and ensure that the tax environment faced by firms is stable, the sample period started from 1989. All the nominal values are converted to real values by the PPI with 2010 as the base year. Besides the standard data cleaning procedure (see Appendix I.7.1 for more details), we also discard the observations with Merge and Acquisitions (M&A) larger than 5% of their book value asset since M&A can significantly change the capital structure of firms.

I.2.2 Investment and Financing Flows

Measurement The investment flow is measured as the sum of capital expenditure and R&D expenditure. The debt financing flow is measured as the sum of net long-term debt issuance and the net change of current debt. The equity financing is measured as the difference between the issuance of common and preferred stocks and the sum of dividend and stock repurchase. The total external financing is measured as the sum of equity financing and debt financing.

Here, the issuance of common and preferred stocks is not directly measured by the item `sstk` reported in Compustat because a large part of the stock issuance reported in this item actually comes from the exercises of stock options by employees. These options are typically viewed as compensation with many years between grants and exercises. To be consistent with the model where the financing flow is determined by the managerial decision in current period, I eliminate this employee-driven equity issuance by merging the data with the equity

Figure I.1. Investment and Financing Flows of US Public Firms



Note: In each period, the firm population is split into small and large firms by the population median size. The relative flow of investment or financing is measured as the total flow of each firm group normalized by the total size, which is measured by the firms' lagged asset, of the corresponding firm group.

offerings in Security Data Company (SDC) database (see Appendix I.7.1 for more details).

The size of each firm is measured by its lagged book value asset. In each year, the *small firms* are defined as the firms whose size is below the population median size in that year, and the *large firms* are defined as the firms larger than the median size. Within each size group, I aggregate the firms' investment and financing flows and normalize them by the aggregate size of the corresponding size group. The time-series of these normalized investment and financing flows for each of the firm groups are depicted in Figure I.1 and the mean of these time-series are summarized in Table I.1.

Table I.1. Time-series Average Investment and Financing Flows

	Small	Large
Investment	0.13	0.05
Total External Financing	0.07	-0.04
Equity Financing	0.06	-0.04
Debt Financing	0.005	0.003

Note: This table reports the mean of the flows shown in Figure I.1 across the sample period 1989-2016.

Facts The investment rate of small firms kept being both higher and more volatile than that of large firms. The key objective of this paper is to understand to what extent this volatile investment of small firms is driven by the financial shocks. In terms of the financing flow, these two groups of firms are different in terms of both the magnitude and the composition.

As for the magnitude of financing flow, small firms are raising funding from the rest part of economy but large firms are paying out over the whole sample period. This difference implies that small firms rely more on the external financing than large firms because large firms do not raise any funding from external financing market on average. In terms of the composition of financing flow, most of the financing of small firms comes from equity market but the financing of large firms, if they have any, mostly comes from debt financing. This fact implies that different firms are mainly financing from different financial markets: small firms are mostly financing from equity market and large firms are mainly financing from debt market, if they raise any external funding.

I.2.3 Fraction of Investment from Equity and Debt Financing

The previous facts about firms' investment and financing are documented separately. Now I measure the fraction of investment financed from different financing sources to understand to which extent the firms' investment are depending on their external financing.

Measurement An immediate candidate for this measure is the ratio between the aggregate financing flows and the investment flow of a specific firm subgroup. But under this measure, some individual firm observations with large negative financing flow could affect the measure and make the measure hard to be interpreted in the designed way. To deal with this problem, I construct a conceptual measure which is based on the truncated ratio between financing flow and investment on the individual level. Similar measure is also used in Zetlin-Jones and Shourideh (2017).

In period t, the fraction of the investment of an individual firm i financed from the funding source $\mathcal{F}$ is measured as:

$$\texttt{Frac}^{\mathcal{F}}_{i,t} = \begin{cases} \frac{\mathcal{F}_{i,t}}{I_{i,t}} & \text{if } \mathcal{F}_{i,t} > 0 \text{ and } I_{i,t} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{I.1}$$

where the $I_{i,t}$ and $\mathcal{F}_{i,t}$ denote the investment and the funding raised from source $\mathcal{F}$. For a firm group $\mathcal{I}_t$[1], the fraction of their investment financed from financing source $\mathcal{F}$ is calculated

[1] Under the classification criteria used in this paper, the sample of small and large firms change over time, so, the group set is indexed by the time. The set of small and large firms in t will be denoted as $\texttt{Small}_t$ and $\texttt{Large}_t$.

as the weighted average of the individual measure:

$$\texttt{Frac}^{\mathcal{F}}_{\mathcal{I}_t,t} = \sum_{i\in\mathcal{I}_t} \texttt{Frac}^{\mathcal{F}}_{i,t} \cdot \omega_i, \text{ where } \omega_i \equiv \frac{I_{i,t} \cdot \mathbb{1}^+(I_{i,t})}{\sum_{i\in\mathcal{I}_t} I_{i,t} \cdot \mathbb{1}^+(I_{i,t})} \tag{I.2}$$

I first measure the fraction of investment financed from equity and debt market for small and large firms in each period by (I.2). Then I calculate the average of the time-series of these measures and the average fractions are summarized in Table I.2.

Table I.2. Fraction of Investment from Different Financing Sources

	Small	Large
Equity	0.50	0.03
Debt	0.07	0.13

Note: This table reports the mean of $\texttt{Frac}^{\mathcal{F}}_{\text{Small}_t,t}$ and $\texttt{Frac}^{\mathcal{F}}_{\text{Large}_t,t}$ over the sample period 1989-2016.

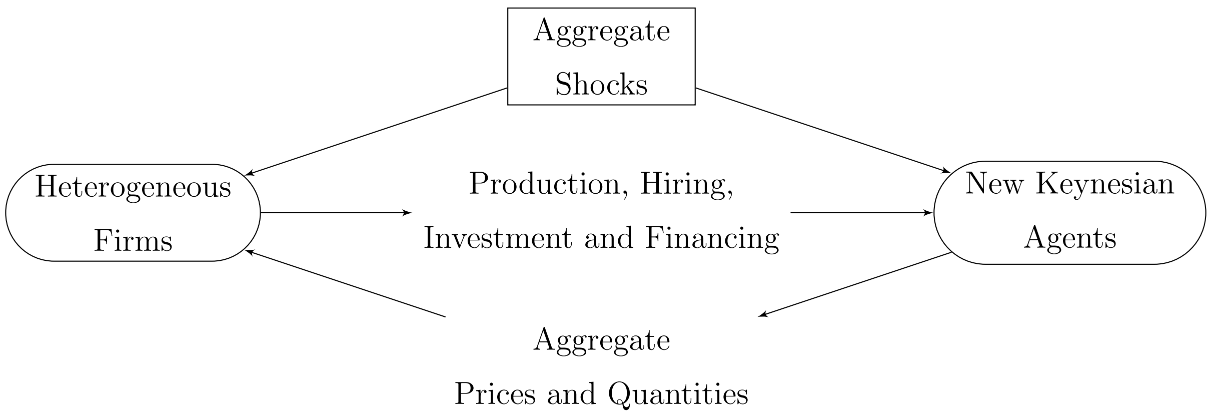
Facts The results presented in Table I.2 highlight which external financing source matters most for the investment of small and large firms. For small firms, half of their investment is financed from equity market but only 7% is financed from debt market. This implies the dominant role of equity market as the financing source for small firms' investment. For large firms, conditional on raising external funding, debt market is much more important than equity market in financing their investment. This implies that debt market is more relevant than equity market for large firms.

Certainly this measure is not perfect and it does not consider other potential use of the external funding. For instance, this measure does not take liquidity accumulation into consideration, which might overstate the importance of external financing to investment. But the point to be highlight here is the relative importance of different financing sources to firms' investment. The defects of this measure would not really change the implication about the relative importance of different financing source to small and large firms.

I.3 Model

The model includes three components: a block of heterogeneous firms, a block of New Keynesian representative agents (see e.g. Justiniano, Primiceri and Tambalotti, 2010), and eight aggregate shocks. The heterogeneous firm block is designed to endogenously generate the firms' heterogeneous investment and financing choices. The New Keynesian block is designed to determine the aggregate quantities and prices faced by the firms. The aggregate shocks are designed to capture the exogenous variation in firms' financing conditions and investment profitability.

Figure I.2. Model Structure

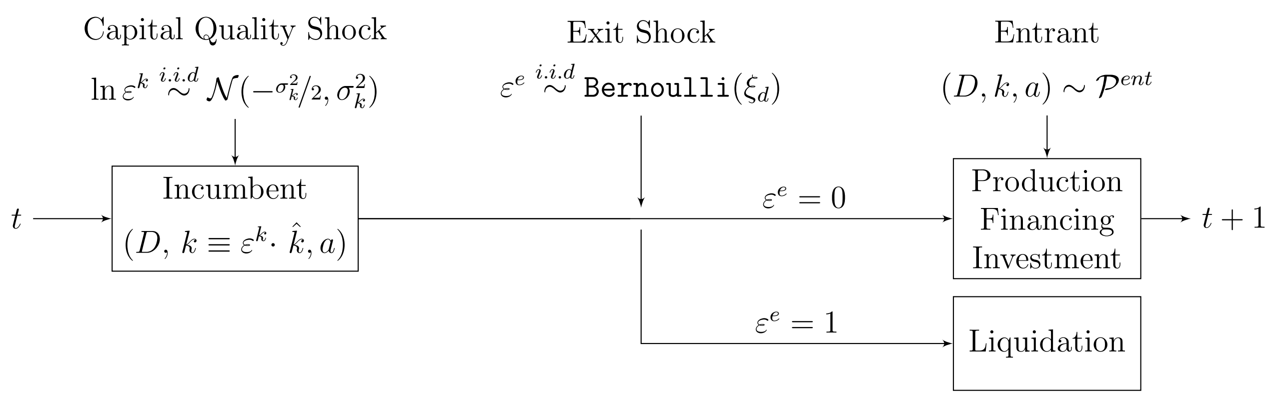


I.3.1 Heterogeneous Firm Block

Setup

In this economy, there is a continuum of heterogeneous firms indexed by $i \in [0, 1]$. They produce homogeneous intermediate goods and sell them in a competitive market.

Figure I.3. Decision Timing for Heterogeneous Firms



Idiosyncratic State For incumbent firm i, there are three individual state variables revealed at the beginning of each period t: $(D_{i,t}, k_{i,t} \equiv \varepsilon^k_{i,t} \cdot \hat{k}_{i,t}, a_{i,t})$. Here, the capital stock

$\hat{k}_{i,t}$ and the nominal debt stock $D_{i,t}$ are inherited from period $t-1$. The capital quality shock $\varepsilon^k_{i,t}$ and idiosyncratic productivity $a_{i,t}$ are exogenous. The evolution of idiosyncratic productivity follows

$$\ln a_{i,t} = \rho_a \cdot \ln a_{i,t-1} + \sigma_a \cdot \varepsilon^a_{i,t},\ \varepsilon^a_{i,t} \overset{i.i.d}{\sim} \mathcal{N}(0,1) \tag{I.3}$$

The i.i.d. quality shock $\varepsilon^k_{i,t}$ is distributed as $\texttt{LogNormal}(-\frac{\sigma_k^2}{2}, \sigma_k^2)$ and it transforms the predetermined capital stock $\hat{k}_{i,t}$ into the effective capital stock $k_{i,t}$[2].

Exit and Entry After the realization of their individual states, firm i will receive an i.i.d. exogenous exit shock $\varepsilon^e_{i,t} \sim \texttt{Bernoulli}(\xi_d)$. If $\varepsilon^e_{i,t} = 1$, firm i has to be liquidated and the shareholders exit the market with the liquidation value $\mathcal{LV}_{i,t} \equiv (1-\delta) \cdot k_{i,t} \cdot Q_t - D_{i,t} \cdot R_{t-1}$, where Q_t is the nominal price for capital good. If $\varepsilon^e_{i,t} = 0$, firm i can keep in operation. Firms in operation can produce, invest and raise funding from financial markets.

Right after the exit of incumbents, a group of entrants enter the market and operate as the same as the surviving incumbents do. The number of entrants is assumed to be equal to the number of exiting incumbents, so the firm population keeps constant over time. In each period, the distribution of the entrants is denoted as $\mathcal{P}^{ent}_t(D,k,a)$, and the distribution of the operating firms, which is made up by the entrants and the surviving incumbents, is denoted as $\mathcal{P}_t(D,k,a)$. The distribution of the entrants is assumed to satisfy that the marginal distributions of the leverage ratio, log of size and log of idiosyncratic productivity are normal and independent with each other:

$$\begin{aligned} \mathcal{P}^{ent}_t(D,k,a) =& \mathcal{P}^{ent}_{lev,t}\left({}^{D}/_{k\cdot Q_t}\right) \cdot \mathcal{P}^{ent}_{k,t}\left(\ln k\right) \cdot \mathcal{P}^{ent}_{a,t}\left(\ln a\right) \\ =& \phi\left(\frac{{}^{D}/_{k\cdot Q_t} - \mu^{ent}_{lev}}{\sigma^{ent}_{lev}}\right) \cdot \phi\left(\frac{\ln k - \mu^{ent}_k}{\sigma^{ent}_k}\right) \cdot \phi\left(\frac{\ln a - \mu^{ent}_a}{\sigma^{ent}_a}\right) \end{aligned} \tag{I.4}$$

where $\phi(\cdot)$ is the PDF of standard normal distribution and $(\mu^{ent}_v, \sigma^{ent}_v),\ \forall v \in \{lev, k, a\}$ are the mean and standard deviation of the corresponding distributions.

Production and Internal Financing Firm i in operation can produce intermediate goods $y_{i,t}$ with a decreasing-return-to-scale technology:

$$y_{i,t} = \exp\left(\eta_{z,t}\right) \cdot a_{i,t} \cdot \left(k^{\alpha}_{i,t} \cdot l^{1-\alpha}_{i,t}\right)^{\theta},\ \alpha \in (0,1),\ \theta \in (0,1) \tag{I.5}$$

[2] The capital quality shock is just a technical assumption made for numerical purpose. Without this shock, there would be many mass points in the distribution and the accuracy of numerical approximation will be affected. In the quantification part, this shock will be calibrated to be small so it will not really affect the conclusion in this paper.

where $\eta_{z,t}$ is the exogenous variation in aggregate productivity and $l_{i,t}$ is the labor input. Firms' profit will be taxed at the constant tax rate of τ^c, and the total source of internal financing available for firm i is

$$\Pi_{i,t} = (1-\tau^c) \cdot \underbrace{\left[y_{i,t} \cdot P_t^I - l_{i,t} \cdot W_t - D_{i,t} \cdot (R_{t-1} - 1)\right]}_{\text{Operating Profit}} + \underbrace{\tau^c \cdot \delta \cdot k_{i,t} \cdot Q_t}_{\text{Tax Rebate from Depreciation}} \tag{I.6}$$

where P^I, W, R and Q are the nominal intermediate good price, wage, gross interest rate and capital good price.

Investment and External Financing Besides the internal financing source, firm i can also finance its investment $I_{i,t} \cdot Q_t$ from debt and equity market. The budget constraint for firm i is

$$I_{i,t} \cdot Q_t = \underbrace{\Pi_{i,t}}_{\text{internal financing}} + \underbrace{E_{i,t}}_{\text{equity financing}} + \underbrace{D_{i,t+1} - D_{i,t}}_{\text{debt financing}} \tag{I.7}$$

The investment will be used to build up the capital stock and the accumulation of capital follows

$$\hat{k}_{i,t+1} = (1-\delta) \cdot k_{i,t} + I_{i,t} - \Phi(\hat{k}_{i,t+1}, k_{i,t}) \tag{I.8}$$

where $\Phi(\hat{k}_{i,t+1}, k_{i,t})$ is the capital adjustment cost which captures the extra managerial effort required for adjusting the scale of production. The capital adjustment cost is constructed as

$$\Phi\left(\hat{k}', k\right) = \frac{\phi^k}{2} \cdot \left(\frac{\hat{k}'}{k} - 1\right)^2 \cdot k \tag{I.9}$$

The definition of equity financing here is consistent with the measurement in Section I.2. when $E_{i,t} \leq 0$, it is counted as dividend payment; when $E_{i,t} > 0$, it is counted as equity issuance. The equity issuance is costly and the costs mainly include the explicit floatation cost (Altinkilic and Hansen, 2000), the implicit adverse selection premium (Myers and Majluf, 1984) and even the market misvaluation (Warusawitharana and Whited, 2016). Given the quantitative purpose of this paper, I abstract from the micro-foundation of the equity issuance cost and model the equity issuance cost in the reduced-form way: there is a proportional cost $\phi_t^e \cdot E_{i,t}$ associated with the issuance, and this cost is paid by the existing shareholders[3]. The equity issuance cost is time-varying, which mainly reflects the potential

[3]This construction of equity issuance cost follows Gomes (2001) and similar setups can be found in many

time-variation in the adverse selection premium (Choe, Masulis and Nanda, 1993) and the market misvaluation (Baker and Wurgler, 2007).

The debt issuance of firm i is subject to a collateral constraint:

$$D_{i,t+1} \leq \phi_t^d \cdot k_{i,t} \cdot Q_t \tag{I.10}$$

The construction of collateral constraint follows Kiyotaki and Moore (1997) and similar setup is widely used in the literature (e.g. Jermann and Quadrini, 2012; Khan and Thomas, 2013). The tightness of collateral constraint ϕ_t^d reflects the supply condition in the debt financing market. The time-variation in ϕ_t^d reflects the time-varying supply condition in the debt financing market (Becker and Ivashina, 2014).

The financial frictions are parameterized as:

$$\phi_t^x = \phi_{SS}^x \cdot \exp\left(\eta_{x,t}/\phi_{SS}^x\right), \ \forall x \in \{e, d\} \tag{I.11}$$

Here, ϕ_{SS}^x denotes the level of financial frictions in steady state, and $\eta_{x,t}$ denotes the exogenous time-variation in the financial frictions.

Decisions

In each period, the firms in operation have to make their decisions in labor hiring, investment, debt financing and equity financing to maximize the total net present value of their future dividend payment. The decision problem can be represented in the standard recursive form:

$$\begin{aligned} V_t(D,k,a) = \max_{l,I,D',E} & -E \cdot \left[1 + \phi_t^e \cdot \mathbb{1}^+(E)\right] \\ & + \mathbb{E}_t\left[\Lambda_{t,t+1} \cdot \frac{P_t}{P_{t+1}} \cdot \left[(1-\xi_d) \cdot V_{t+1}\left(D', e^{\varepsilon^{k'}} \cdot \hat{k}', a'\right) + \xi_d \cdot \mathcal{LV}_{t+1}\right]\right] \\ \text{s.t.}: \quad & \text{production technology (I.5), internal financing source (I.6),} \\ & \text{budget constraint (I.7), capital accumulation (I.8),} \\ & \text{collateral constraint (I.10)} \end{aligned} \tag{I.12}$$

Here, $V_t(\cdot)$ is the value of the firm in period t. The subscript t indicates the dependence of the value function on the aggregate economic conditions, and the aggregate dynamics will

other works in structural corporate finance (e.g. Hennessy and Whited, 2007; Warusawitharana and Whited, 2016). This type of equity financing cost is typically micro-founded by the information asymmetry. Due to the information advantage of internal mangers, equity is typically issued to external investors with a price discount. It can be shown that this price discount is isomorphic to this proportional cost. Please see Appendix I.7.2 for details.

be determined by the New Keynesian block. The value of the firms comes from two parts. The first part is the flow from dividend payment or equity issuance. The second part is the continuation value of the firm. The continuation value is first converted by the aggregate relative price level and then discounted by the real discounting factor (SDF) $\Lambda_{t,t+1}$.

I.3.2 New Keynesian Block

Setup

Retailers There is a continuum of retailers indexed by $j \in [0, 1]$. Retailers produce differentiated retail goods $\hat{y}_{j,t}$ from the homogeneous intermediate goods $y_{j,t}$ with the technology specified as:

$$\hat{y}_{j,t} = y_{j,t} \tag{I.13}$$

Each retailer j has the monopolistic power, and following Calvo (1983), there is a probability of $1 - \xi_p$ for retailer j to get the opportunity to reset its nominal prices in each period.

Final Good Producers There is a representative final good producer who produce final good Y_t by packing retail goods $\{\hat{y}_{j,t}\}_{j\in[0,1]}$ through a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int \hat{y}_j^{\frac{1}{\gamma_p + \eta_{p,t}}} dj \right)^{\gamma_p + \eta_{p,t}} \tag{I.14}$$

where γ_p is the price mark-up in steady state and $\eta_{p,t}$ is the exogenous variation of the price mark-up. The final good market is perfectly competitive and the nominal price of the final goods is denoted as P_t.

Households There is a representative household who consumes final good C_t, supply labor N_t, owns all the firms and save in one-period nominal bond $\frac{B_{t+1}}{R_t}$. The utility function of the household is specified as

$$\sum_{t=0}^{\infty} \beta^t \cdot \exp(\eta_{u,t}) \cdot [\log(C_t - h \cdot C_{t-1}) - \Psi \cdot N_t] \tag{I.15}$$

where β is the discounting factor, $\eta_{u,t}$ is the exogenous variation in the households' intertemporal substitution decision, and h is the parameter controlling the external consumption habit formation. The budget constraint for the household is

$$C_t \cdot P_t + \frac{B_{t+1}}{R_t} = N_t \cdot W_t^N + B_t + T_t \tag{I.16}$$

where W_t^N is the nominal wage to household's labor, and T_t is the lump-sum transfer such that the bond market clears.

Labor Union There is a continuum of labor unions indexed by $s \in [0,1]$ which purchase the homogeneous labor supply $N_{s,t}$ from the representative household and transform it as heterogeneous intermediate labor service $\hat{N}_{s,t}$ with the technology:

$$\hat{N}_{s,t} = N_{s,t} \tag{I.17}$$

Each union s has the monopolistic power and there is a probability of $1 - \xi_w$ for union s to get the opportunity to reset the nominal wage of its specialized labor service.

Labor Packer There is a representative labor packer which packages the heterogeneous types of labor supply $\{\hat{N}_{s,t}\}_{s\in[0,1]}$ as the final labor service L_t with the technology:

$$L_t = \left(\int \hat{N}_{s,t}^{\frac{1}{\gamma_w + \eta_{w,t}}} \right)^{\gamma_w + \eta_{w,t}}$$

where γ_w is the wage mark-up in the steady state and $\eta_{w,t}$ is the exogenous variation in the wage mark-up. The market for final labor service is perfectly competitive and the nominal wage of final labor service is denoted as W_t.

Investment Good Producers There is a representative investment good producer which produces investment good $\hat{I}_t$ from final good Y_t^I with the technology

$$\hat{I}_t = \exp\left(-\eta_{q,t}\right) \cdot Y_t^I \tag{I.18}$$

where $\eta_{q,t}$ is the exogenous variation in the efficiency of transforming final goods into investment goods. The investment good market is competitive and the nominal investment good price is Q_t^I.

Capital Good Producers There is a representative capital good producer which produces capital good I_t from investment good $\hat{I}_t$ with the technology specified as:

$$I_t = \left[1 - S\left(\frac{\hat{I}_t}{\hat{I}_{t-1}}\right)\right] \cdot \hat{I}_t \tag{I.19}$$

where $S(\cdot)$ is the function characterizing the adjustment cost and the adjustment cost function is assumed to satisfy $S(1) = S'(1) = 0$ and $S''(1) > 0$. The investment good market is perfectly competitive and the nominal price of capital good is denoted as Q_t.

Monetary Authority The monetary policy is assumed to follow:

$$\ln R_t - \ln \frac{1}{\beta} = \lambda_R \cdot \left[\ln R_{t-1} - \ln \frac{1}{\beta}\right] + \lambda_\pi \cdot \ln \pi_t + \eta_{m,t} \tag{I.20}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate and $\eta_{m,t}$ is the exogenous variation in the interest rate.

Decisions

The decisions in this block play two roles in this model. First, they build in the general equilibrium feedbacks, which matters for the transmission of the financial shocks. Second, they provide a structure to discipline the dynamics of aggregate quantities and prices, which are important for identifying the financial shocks. Given that the decision problems in this block are close to the ones as specified in Justiniano, Primiceri and Tambalotti (2010), I will directly present the log-linearized results of their decisions without going through all the details. For notational convenience, $\tilde{X}_t$ denotes the log deviation of X_t from its steady state level.

Final Goods Supply and Inflation Dynamics The final good producers maximize their expected total discounted profits by choosing their input of retailed goods. Given the demand from final good producers, retailers maximize their expected total discounted profits by setting the nominal price of their goods. Following Calvo (1983), it is assumed that only a randomly chosen fraction $(1 - \xi_p)$ of the retailers can reset their price in each period. The decisions of final good producers and retailers jointly determine the aggregate supply of final goods and inflation dynamics:

$$\tilde{Y}_t = \tilde{y}_t \tag{I.21}$$

$$\tilde{\pi}_t = \frac{1-\xi_p}{\xi_p} \cdot (1 - \xi_p \cdot \beta) \cdot \left(\tilde{p}_t^I + \eta_{p,t}\right) + \beta \cdot \mathbb{E}_t\left[\tilde{\pi}_{t+1}\right] \tag{I.22}$$

where Y_t is the total output of final goods, y_t is the total output of intermediate goods, and $p_t^I \equiv \frac{P_t^I}{P_t}$ is the intermediate good price in real term.

Labor Demand and Wage Dynamics The labor packer maximizes their expected total discounted profits by choosing their input of differentiated labor services. Given the demand from labor packer, labor unions maximize their expected total discounted profits by setting the nominal wage of their differentiated labor service. It is also assumed that only a randomly chosen fraction $(1-\xi_w)$ of the labor unions can reset their wage in each period. The decisions of labor packer and labor unions jointly determine the aggregate demand for the households' labor and wage dynamics:

$$\tilde{N}_t = \tilde{L}_t \tag{I.23}$$

$$\begin{aligned} \tilde{w}_t =& \frac{\xi_w}{1 + \beta \cdot \xi_w^2 \cdot (1-\xi_w)} \cdot (\tilde{w}_{t-1} - \tilde{\pi}_t) + \frac{(1-\xi_w) \cdot (1 - \beta \cdot \xi_w)}{1 + \beta \cdot \xi_w^2 \cdot (1-\xi_w)} \cdot \left(\tilde{w}_t^N + \eta_{w,t}\right) \\ &+ \frac{(1-\xi_w) \cdot \beta \cdot \xi_w}{1 + \beta \cdot \xi_w^2 \cdot (1-\xi_w)} \cdot \mathbb{E}_t\left[\tilde{w}_{t+1} + \tilde{\pi}_{t+1}\right] \end{aligned} \tag{I.24}$$

where N_t denotes the quantity of households' labor, L_t denotes the total final labor service used by the intermediate good firms, $w_t \equiv \frac{W_t}{P_t}$ and $w_t^N \equiv \frac{W_t^N}{P_t}$ are the final labor service wage and household labor wage in real term. In the equation of wage dynamics, the first term characterizes the backward-looking feature coming from nominal wage rigidity, the second term characterizes the effects from the aggregate wage shocks, and the last term characterizes the forward-looking feature coming from the rational expectation of labor union.

Capital Good Supply and Capital Good Price Dynamics The investment good producer and the capital good producer maximize their expected total discounted profits by choosing their inputs Y_t^I and $\hat{I}_t$. Based on their optimal choice, the price of investment good satisfies:

$$\tilde{q}_t^I = \eta_{q,t} \tag{I.25}$$

The total supply of capital good and the capital good price dynamics are:

$$\tilde{I}_t = \tilde{Y}_t^I - \eta_{q,t} \tag{I.26}$$

$$\tilde{q}_t = \tilde{q}_t^I + S''(1) \cdot \left[\left[\tilde{I}_t - \tilde{I}_{t-1}\right] - \beta \cdot \mathbb{E}_t \left[\tilde{I}_{t+1} - \tilde{I}_t\right]\right] \tag{I.27}$$

where $q_t^I \equiv \frac{Q_t^I}{P_t}$ and $q_t = \frac{Q_t}{P_t}$ denote the investment good price and capital good price in real term.

Labor Supply and Stochastic Discounting Factor Dynamics Representative household maximizes its utility specified in (I.15) subject to their budget constraint in (I.16). The consumption Euler equation and labor supply are:

$$0 = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} + \tilde{R}_t - \tilde{\pi}_{t+1}\right] \tag{I.28}$$

$$\tilde{w}_t^N = \eta_{u,t} - \widetilde{MU}_t \tag{I.29}$$

The dynamics of the real Stochastic Discounting Factor (SDF) is disciplined by:

$$\widetilde{MU}_t = \eta_{u,t} - \left[\frac{1}{1-h} \cdot \tilde{C}_t - \frac{h}{1-h} \cdot \tilde{C}_{t-1}\right] \tag{I.30}$$

$$\tilde{\Lambda}_{t,t+1} = \widetilde{MU}_{t+1} - \widetilde{MU}_t \tag{I.31}$$

I.3.3 Aggregate Shocks

There are eight exogenous variables in this model. Their evolution follow the AR(1) process:

$$\eta_{x,t} = \rho_x \cdot \eta_{x,t-1} + \sigma_x \cdot \varepsilon_{x,t}, \ \forall x \in \{e, d, z, p, w, q, m, u\} \tag{I.32}$$

where the independent exogenous variations $\varepsilon_{x,t} \overset{i.i.d}{\sim} \mathcal{N}(0,1)$ are the aggregate shocks to this economy. Within these eight aggregate shocks, two of them, i.e. $\varepsilon_{e,t}$ and $\varepsilon_{d,t}$, come from financial markets and capture the exogenous variation in firms' financing conditions. The rest six aggregate shocks directly or indirectly capture the exogenous variation in the firms' investment profitability by affecting the production efficiency, prices or preference in this economy.

I.3.4 Equilibrium

An equilibrium of this model is a collection of

1. value function $V_t(k, D, a)$ and the associated policy functions for hiring, production, investment, debt issuance, equity financing and capital holding: $l_t(k, D, a)$, $y_t(k, D, a)$, $I_t(k, D, a)$, $D'_t(k, D, a)$, $E_t(k, D, a)$ and $\hat{k}'_t(k, D, a)$,

2. distribution $\mathcal{P}_t(k, D, a)$,

3. aggregate quantities and prices Y_t, C_t, Y_t^I, I_t, $\hat{I}_t$, N_t, L_t, p_t^I, w_t, w_t^N, q_t, q_t^I, R_t, π_t, Λ_t

such that given the exogenous process of $\eta_{x,t}$, $\forall x \in \{e, d, z, p, w, q, m, u\}$

1. value function $V_t(k, D, a)$ solves the firm's problem in (I.12) with the associated policy functions

2. distribution $\mathcal{P}_t(k, D, a)$ evolves as:

$$\begin{aligned}\mathcal{P}_t(k, D, a) =&(1 - \xi_d) \cdot \int \mathbf{1}\left\{\exp\left(\varepsilon^k\right) \cdot \hat{k}'_{t-1}(k_-, D_-, a_-) = k\right\} \\ &\times \mathbf{1}\left\{D'_{t-1}(k_-, D_-, a_-) = D\right\} \\ &\times \mathbf{1}\left\{\rho_a \cdot \ln a_- + \sigma_a \cdot \varepsilon^a = \ln a\right\} \\ &\times \phi\left(\frac{\varepsilon^k + \frac{\sigma_k^2}{2}}{\sigma_k}\right) \phi\left(\varepsilon^a\right) d\varepsilon^k d\varepsilon^a d\mathcal{P}_{t-1}(k_-, D_-, a_-) \\ &+ \xi_d \cdot \mathcal{P}_t^{ent}(k, D, a)\end{aligned} \tag{I.33}$$

where $\phi(\cdot)$ is the density of the standard normal distribution.

3. the aggregate quantities and prices satisfy the monetary policy specified in (I.20) and the conditions specified in New Keynesian block

4. the markets for final goods, intermediate goods, capital goods and labor all clear:

$$Y_t = C_t + Y_t^I \tag{I.34}$$

$$y_t = \int y_t(k, D, a) d\mathcal{P}_t(k, D, a) \tag{I.35}$$

$$I_t = \int I_t(k, D, a) d\mathcal{P}_t(k, D, a) \tag{I.36}$$

$$L_t = \int l_t(k, D, a) d\mathcal{P}_t(k, D, a) \tag{I.37}$$

I.4 Quantification

The model can be casted into the standard form of rational expectations model:

$$\mathbb{E}_t \left[\mathcal{F} \left(\mathbb{Y}_{t+1}, \mathbb{X}_{t+1}, \mathbb{Y}_t, \mathbb{X}_t | \Theta_{HF}, \Theta_{NK}, \Theta_{SH} \right) \right] = 0 \tag{I.38}$$

where $\mathbb{X}$ is the collection of the state variables, which include the exogenous variables $\eta_{x,t}$, $\forall x \in \{e, d, z, p, w, q, m, u\}$ and the endogenous firm distribution $\mathcal{P}_t$, and $\mathbb{Y}$ is the collection of the control variables, which include the endogenous aggregate quantities, prices, firms' value functions and policy functions. Corresponding to the three components of this model, there are three groups of parameters in this model: Θ_{HF} is the collection of the parameters governing the heterogeneous firm block, Θ_{NK} is the collection of the parameters in the New Keynesian block, and Θ_{SH} collects the persistence and on-impact response to aggregate shocks of the exogenous processes, i.e. ρ_x and σ_x $\forall x \in \{e, d, z, p, w, q, m, u\}$.

The core aim of this paper is to decompose the observed time-variation in firms' investment into the contribution of different aggregate shocks. To make reasonable decomposition results, I need estimate the model to match the observed time-variation first. To estimate the model, I have to solve the model fast enough. But the firm-level heterogeneity significantly increase the dimension of this model and it technically challenging to estimate this model.

In this section, I will first sketch out the algorithm used to solve the model, which is crucial to make the estimation of the model feasible. Then I will present how I pin down the parameter values by calibration and estimation. Within these three groups of parameters, all of the parameters in Θ_{NK}[4] and a part of the parameters in Θ_{HF} are fixed at the values from literature or directly from data. The rest part of the parameters in Θ_{HF} will be calibrated to match the average investment and financing flows as documented in the guiding facts. I will discuss about the firms' policy functions and life-cycle dynamics in steady state as the guidance for the calibration. The parameters in Θ_{SH} are estimated with the Bayesian method to match the observed time-variation in both the quantities and prices on the macro-level, and the firms' financing choices on the distributional level. In the estimation part, I will discuss about the intuition behind the identification of these different aggregate shocks.

[4] For technical reasons, these parameters are fixed but not estimated. In the next round of revision, I will incorporate them into the estimation. In current version, I did the robustness check and found the conclusions are robust to varying these parameter values.

I.4.1 Numerical Solution

I use a hybrid method to solve this model it is solved sufficiently fast that the estimation becomes feasible. This method combines the projection method applied on the micro-level and the perturbation method applied on the aggregate level[5]. The implementation includes two steps:

1. I solve the steady state with the aggregate shocks shut off. This steady state includes the firms' value functions, policy functions, and distributions when the aggregate economic quantities and prices are fixed at the steady state levels. The firms' distribution and the curvature in their policy functions are generated by the idiosyncratic productivity shocks and the model structure.

2. I solve the first-order perturbation solution around the steady state. The solved dynamics characterize the response of firms' policy function, value function, distribution, as well as the aggregate quantities and prices to various aggregate shocks.

I.4.2 Fixed Parameters

Parameters in Θ_{HF} The upper panel of Table I.3 is a collection of the parameters which control firms' operation flow and life-cycle dynamics. The corporate tax rate τ^c is set at 35%, which is the median tax rate as reported in Graham (2000). The share of capital α is set at the standard value 0.30. The return to scale, persistence and conditional standard deviation of the idiosyncratic productivity process are calibrated to match the values used in Begenau and Salomao (2018). The exogenous exit probability is set at 3% to match the average fraction of entrants in Compustat. The capital quality shock in this model only serves for numerical purpose: without this shock, there will be a lot of mass points in the firms' distribution, which will significantly decrease the accuracy of the numerical approximation. To minimize its impact on the quantitative results, I set its standard deviation at 0.1%.

The middle panel of Table I.3 is a collection of the parameters which controls the distribution of entrants. The average size of the entrants is set to match the gap between the 90% quantile and the 10% quantile of the firm size in Compustat. The mean of the entrants' idiosyncratic productivity is set at $-0.5 \times \frac{\sigma_a}{\sqrt{1-\rho_a^2}}$ to be consistent with the fact documented by Foster, Haltiwanger and Syverson (2016) that young firms keep having lower

[5] The method was initially proposed by Reiter (2009). In this paper, to further reduce the dimension of the system, the distribution is approximated as in Algan, Allais and Den Haan (2008) and Winberry (2018). Under the current numerical approximation scheme, the dimension of the system is close to 3000. For more details about the computation, please check Appendix I.7.3.

measured TFP during their early life periods[6]. Since most of the entrant firms have little debt in the data, the mean of the leverage distribution is set at 0. For the similar numerical reason, the standard deviations of these three marginal distribution are set at 1%, which is non-zero and small enough to have negligible impact on the quantitative results.

Parameters in Θ_{NK} The lower panel collects the parameters which control the dynamics in the New Keynesian block[7]. The discount factor is set at 0.98 to match the average real interest rate at 2%. The elasticity of capital good price, the average price and wage markups, the probability of price and wage adjustment, as well as the coefficients of the Taylor rule are set to match the values estimated in Justiniano, Primiceri and Tambalotti (2011).

I.4.3 Calibration and the Steady State

Four parameters in Θ_{HF} are calibrated to match the firms' choices in steady state with the corresponding moments in the data. In this section, I will first present and interpret the firms' policies, distributions, and life-cycle dynamics in the steady state. Then I will elaborate how these parameters are calibrated and the intuition behind the calibration.

Investment and Financing Policy Functions To illustrate the policy functions, I choose $\exp\left(\pm 1.5 \times {}^{\sigma_a}/\sqrt{1-\rho_a^2}\right)$ as the representative high and low idiosyncratic productivity levels, the median size of capital holding and median leverage of the population distribution as the representative levels for capital holding and leverage. The policy functions for investment rate, debt financing and equity financing are depicted in the four panels in Figure I.4. The graphs in each row share the same level of representative productivity level. For the two graphs on the left (right) panel, the leverage ratio (size) are fixed at the representative level. To inform the relevance of the policy functions in different states, I also plot the marginal distribution of the incumbents' size and leverage ratio[8] conditional on the corresponding productivity levels. From these plots, we can tell how these idiosyncratic states affect the firms' investment and financing.

Along the size dimension, there is a significant variation in the investment and equity financing policy, but little variation in the debt financing policy. Given the decreasing return to scale technology and mean-reverting productivity process, at a given productivity, firms

[6]Given the small fraction of entrants in the whole population, the quantitative results of interest are robust to different choice of this parameter.

[7]In each round of calibration, the labor dis-utility parameter Ψ is always calibrated to generate a steady state employment rate 60%.

[8]Given the construction that there are only 3% of firms are entrants and their size are much smaller than the incumbent, I do not show their distribution here.

Table I.3. Fixed Parameters

Parameters		Value	Source
Panel 1: Parameters in Θ_{HF} Controlling Firms' Technology and Life-cycle Dynamics			
τ^c	Corporate tax rate	0.35	Graham (2000)
α	Capital share	0.30	
θ	Return to scale	0.88	Begenau and Salomao (2018)
ρ_a	Persistence of idio. TFP	0.90	–
σ_a	Std. of Idio. TFP Shock	0.06	–
ξ_d	Prob. of firm exit	3.0%	% of new firms in Compustat (CS)
σ_k	Std. of capital quality shock	0.1%	Numerical purpose, small
Panel 2: Parameters in Θ_{HF} Controlling the Distribution of Entrants			
μ_k^{ent}	Mean of size	-1.91	Avg. $\mathtt{Q}_{0.9} - \mathtt{Q}_{0.1}$ of size dist. in CS
μ_a^{ent}	Mean of idio. TFP	-0.07	FHS (2016)
μ_{lev}^{ent}	Mean of leverage	0	Leverage of entrants in Compustat
σ_k^{ent}	Std. of size	0.01	Numerical purpose, small
σ_a^{ent}	Std. of idio. TFP	0.01	Numerical purpose, small
σ_{lev}^{ent}	Std. of leverage	0.01	Numerical purpose, small
Panel 3: Parameters in Θ_{NK}			
β	Discount factor	0.98	Average real interest rate at 2%
$S''(1)$	Elasticity of capital good price	0.65	JPT (2010)
γ_p	Avg. price mark-up	1.1	JPT (2010)
γ_w	Avg. wage mark-up	1.3	JPT (2010)
$1-\xi_p$	Prob. of price adjustment	0.59	JPT (2010)
$1-\xi_w$	Prob. of wage adjustment	0.59	JPT (2010)
λ_R	Taylor rule coefficient	0.85	JPT (2010)
λ_π	Taylor rule coefficient	1.5	JPT (2010)

Note: FHS (2016) refers to Foster, Haltiwanger and Syverson (2016) and JPT(2010) refers to Justiniano, Primiceri and Tambalotti (2010).

have the finite optimal target size. For small firms, they are further away from the optimal size and they have higher demand of investment; for the large firms, they are already around the optimal size and their investment demand is small or even negative. This is the main reason why the investment policy is decreasing with the size. Given the tax benefit of debt in this model, most of the firms bind their collateral constraint. So the firms with median leverage ratio have little capacity to issue new debt if because the median leverage ratio almost reach the limit. Therefore, the debt financing of the firms in the left two graphs is very close to 0 and their equity financing closely follow the investment policy. The most important features in the left two panels in Figure I.4 are that small firms have

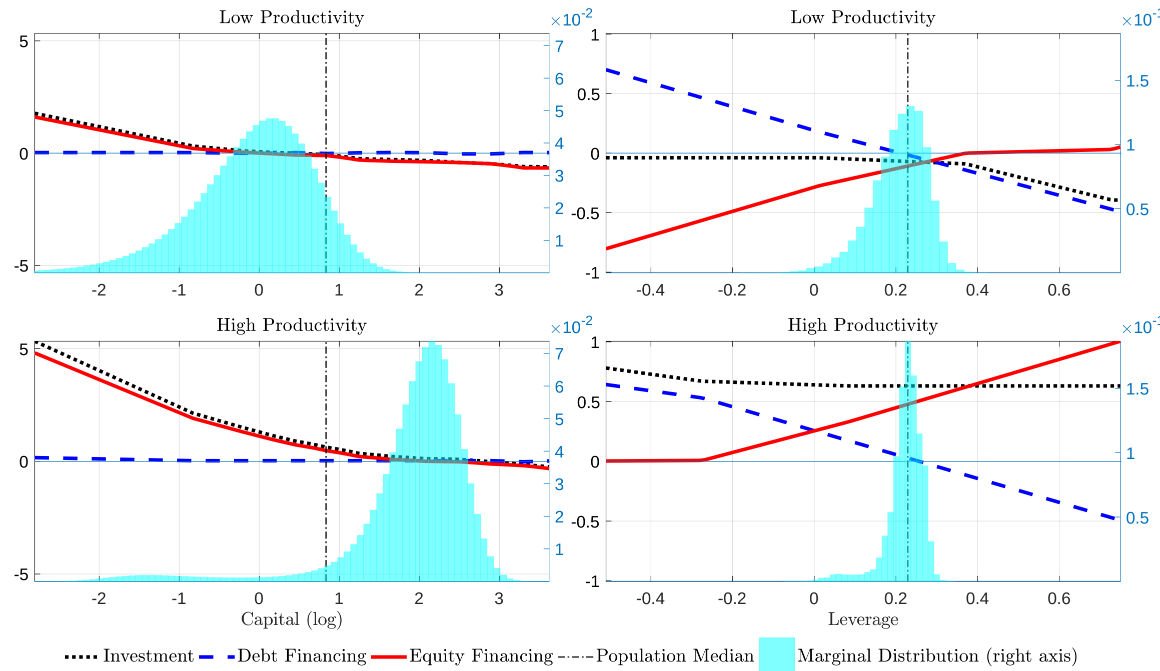


Figure I.4. Policy Function and Conditional Distribution

Note: The distributions are the distribution of incumbents' size and leverage after the realization of capital quality shock and exit shock, but before production, and conditional on each level of idiosyncratic productivity. At each given productivity, the leverage ratio underlying the policy functions along the dimension of firm size is fixed at the median leverage of the firm population; and the size underlying the policy function along the dimension of leverage ratio is fixed at the median size of the firm population. The investment and financing presented here are corresponding flows normalized by the firm size.

higher investment rate and their investment is mainly financed by equity financing, which is consistent with the facts documented in Section I.2.

Along the dimension of leverage ratio, there is not much variation in the investment policy, but a significant variation in financing policies. As in the previous analysis, the investment demand is mostly fixed at the given level of productivity and size. With the increase in the leverage ratio, the residual capacity of debt financing decreases, so the debt financing is lower and the equity financing will increase correspondingly. Due to the existence of equity issuance cost, there is an "non-issuance" area where the equity financing is zero and investment decreases with the leverage ratio.

Along the productivity dimension, there are two points worth to be highlighted. First, the firms with the higher productivity level have higher investment, and correspondingly, higher equity financing flows. The key assumption which lead to this difference is that the idiosyncratic productivity is persistent. The firms with higher current productivity also hold better perspective about their future investment profitability, so they want to invest more. Given that their debt financing capacity is limited, their equity financing increases with their investment demand. Second, the firms with higher productivity level are larger on average than the firms with lower productivity. This is also due to the persistence of the idiosyncratic productivity process. The reasoning can be illustrated from two different perspectives. Due to the persistence, firms with higher productivity will have a relatively higher target size, so their marginal distribution of size is centered at a higher level. Also, firms with higher current productivity level tend to have a history with more positive productivity shocks, therefore, they are more likely to accumulate larger capital stock.

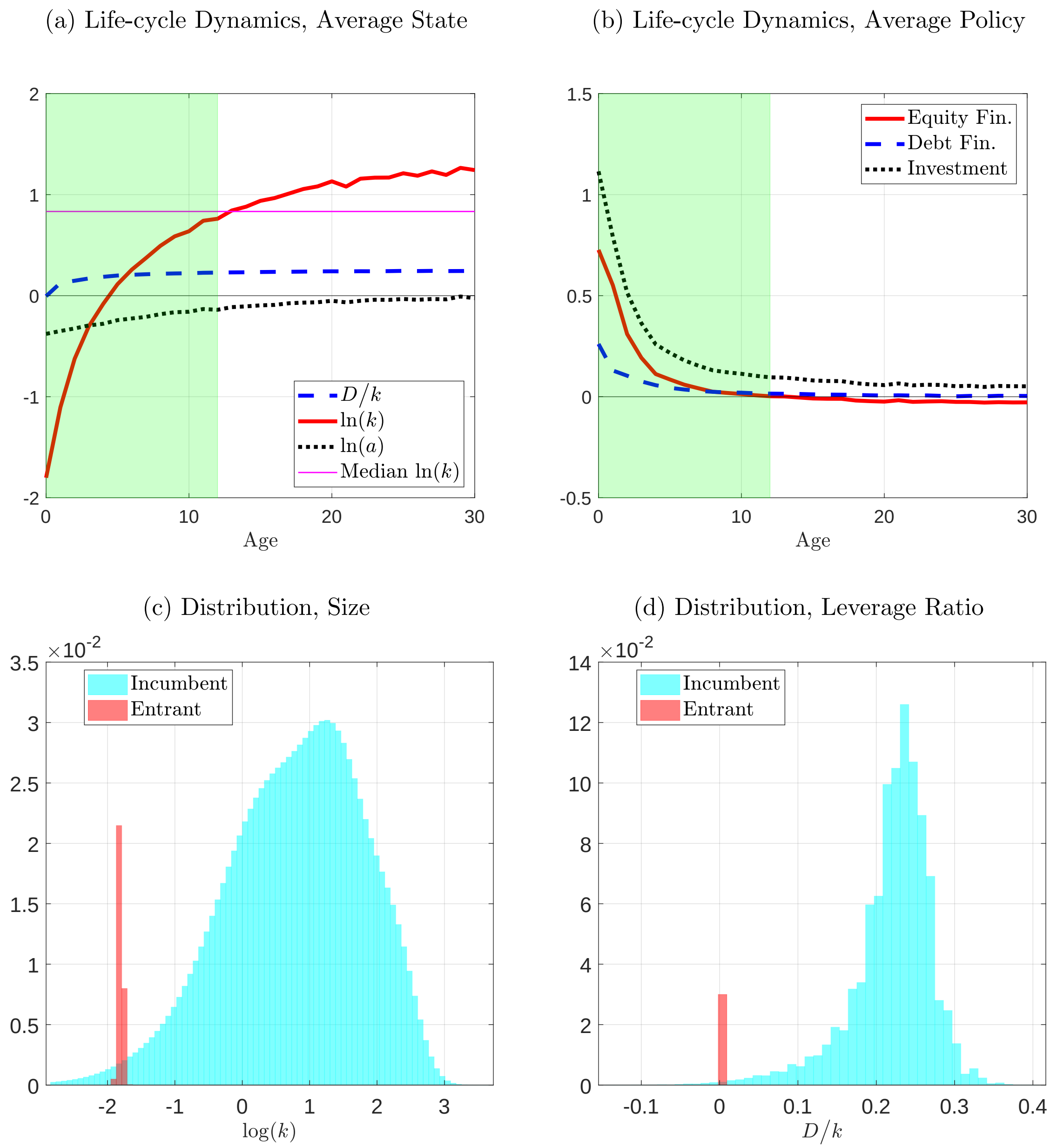
Firms' Life-cycle Dynamics The age profile of firms' average states and policies are summarized in Figure I.5a and I.5b. To compare the entrants and incumbents, the marginal distribution of their size and leverage ratio are summarized in I.5c and I.5d.

As shown in Figure I.5a, firms start their life with smaller size, lower productivity, and little debt, which is consistent with the setup of entrants. With the time going on, they gradually grow up. On average, firms become large at the age of 12, which is close to the cutoff age to classify the firms as matured firms[9]. The age profile of firm size highlight the large overlapping between "being small" and "being young" within this setup. With the firms growing up, their leverage ratio and productivity level also gradually increase. The increase of the leverage ratio is mainly a result of the construction that entrants hold little debt. Due to the tax benefit, firms will try to issue as much debt as they can, but the total debt issuance is limited by their current size. With the firms becoming closer and closer to their target size, their size becomes stable, and their leverage ratio becomes closer and closer to the ratio between their debt issuance and their current size, i.e. the tightness of collateral constraint. The increase of the productivity level is a direct result of the mean reverting feature of the idiosyncratic productivity process. The productivity gradually converges to its unconditional mean.

The age profile of the policies in Figure I.5b is consistent with the age profile of the states in Figure I.5a. The investment rate decreases because the size is growing up with the age and firms are closer to their target size. After the firms become "large firms", their size becomes stable and their investment rate is very close to the depreciation rate. The debt financing decreases because the firms' investment demand is decreasing and the increment of their debt capacity is decreasing with the growth in their size decreasing. The equity financing is decreasing mostly because the investment demand is decreasing. When the

[9]In Haltiwanger, Jarmin and Miranda (2013), 10 is set as the cutoff age between young firms and matured firms.

Figure I.5. Firms' Life-cycle Dynamics and Distribution in Steady State



Note: The life-cycle dynamics are based on a simulated panel with 10,000 firms over 100 years with entry and exit. I.5a plots the average state D/k, $\ln(k)$ and $\ln(a)$ within each age group. I.5b plots the weighted average financing and investment flows within each age group. The firm size is used as the weight for calculating the average policy so it is consistent with the measurement underlying the empirical facts documented in section I.2. The shaded area is the life stage when an average firm is classified as a "small firm'. The distributions in I.5c and I.5d are the ergodic distribution in the steady state with the aggregate shocks shut off. The leverage ratio is the measured at the time after the realization of capital quality shocks but before making the operation decisions.

firms become large, they have little incentive to invest and they start paying dividends to investors.

Calibration There are 4 parameters to be calibrated: the steady state level of equity issuance cost ϕ^e_{SS} and collateral constraint tightness ϕ^d_{SS}, the depreciation rate δ, and the capital adjustment cost ϕ^k. The value of the calibrated parameters and their corresponding moments are summarized in Table I.4.

Table I.4. Calibrated Parameters and the Target Moments

Parameter	Value	Mean	Sample	Data	Model
δ	0.0374	Investment Rate	Large	0.06	0.05
ϕ^k	0.0682	Investment Rate	Small	0.13	0.14
ϕ^d_{SS}	0.2566	Leverage Ratio	Large	0.27	0.24
ϕ^e_{SS}	0.0800	Equity Fin. Rate	Small	0.05	0.05

Note: In each period, the firm population is split into small and large firms by the population median size. The investment rate or financing rate is measured as the total flow of each firm group normalized by the total size of the corresponding firm group. The leverage ratio is measured as the total liability after debt issuance of each firm group normalized by the total size of the corresponding firm group. This table reports the average of these measured rates over the sample period. The moments generated from the model are constructed in the same way.

The steady state level of financial frictions are calibrated to match the firms' average financing choices over the business cycle. As in the analysis of firms' policies, it is mainly the small firms who are issuing equity, so ϕ^e_{SS} is calibrated to match the cross-time average of the small firms' equity financing flow. As in the analysis of firms' life-cycle dynamics, the large firms grow little in their size and their leverage ratio is close to the collateral constraint, ϕ^d_{SS} is calibrated to match the cross-time average of the large firms' leverage ratio.

For the similar reason, large firms' investment is mainly to replenish their depreciated capital, so δ is calibrated to match the cross-time average of large firms' investment rate. For small firms, there is a significant growth in their size and their investment choice is much more subjected to the capital adjustment cost, so ϕ^k is calibrated to match the cross-time average of the small firms' investment.

Another eight non-targeted moments are chosen to check the calibration. The comparison between the moments in data and model is summarized in Table I.5. Overall, the moments about firms' financing choices are well matched. In terms of the fraction of investment from equity and debt financing, the moments generated from the model match the differential results as documented: small firms' investment mostly relies on equity financing and large firms' investment depends relatively more on debt financing. But the model overshoots the fraction of equity financing for large firms and the fraction of debt financing for both small and large firms. Therefore, the model might overstate the importance of the debt financing for both firm groups and the importance of equity financing for the large firms.

Table I.5. Non-Target Moments in Steady State

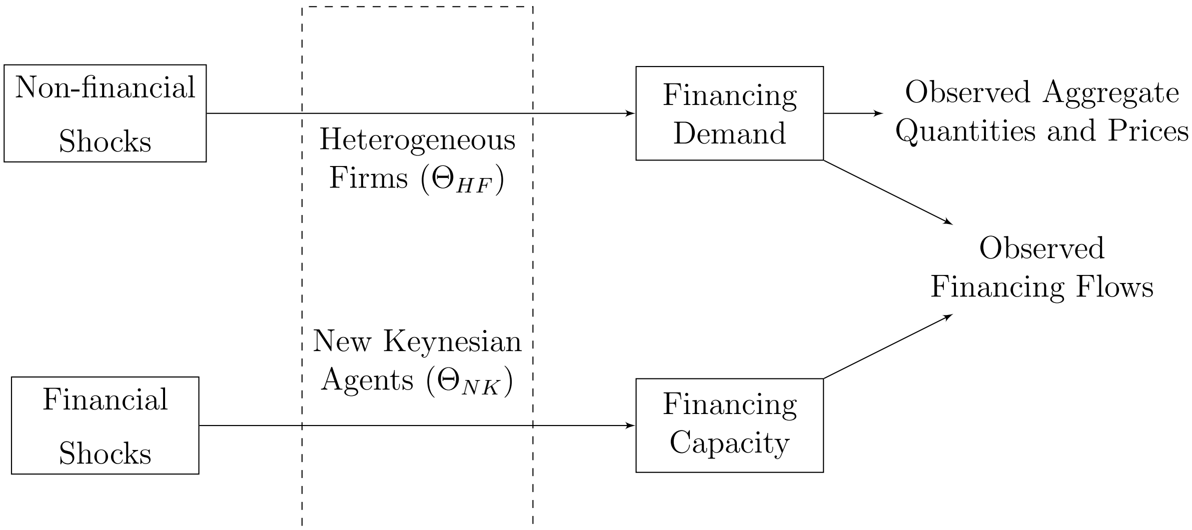
Statistics	Sample	Data	Model
Leverage Ratio	Small	0.18	0.21
Equity Financing Rate	Large	-0.04	-0.03
Debt Financing Rate	Small	0.005	0.023
Debt Financing Rate	Large	0.003	0.003
Fraction of Investment from Equity	Small	0.50	0.47
Fraction of Investment from Equity	Large	0.03	0.16
Fraction of Investment from Debt	Small	0.07	0.20
Fraction of Investment from Debt	Large	0.13	0.20

Note: The leverage ratio and financing flows are measured in the same way as in Table I.4. The fractions of investment from equity and debt are measured in the same way as illustrated in Section I.2.3.

I.4.4 Estimation and the Cyclical Dynamics

Key Idea The parameters in Θ_{HF} and Θ_{NK} determine the sensitivity of different firms' investment to aggregate prices, as well as the sensitivity of aggregate prices to aggregate quantities. With these parameters calibrated, I can estimate the shock parameters in Θ_{SH} such that the model can generate the variation as we observe. The key point here is to identify the financial shocks. The observed financing flows are jointly determined by firms' financing demand and their financing capacity. The firms' financing demand are driven by these six non-financial shocks, which will be identified by the non-financial aggregate time-series. With the financing demand controlled by these non-financial aggregate time-series, the variation in firms' financing capacity will be identified by the observed variation in firms' financing flows. In the following parts, I will illustrate in details how to identify these non-financial shocks from aggregate time-series and how to separately identify different financial shocks using the time-variation in different firms' financing flows.

Figure I.6. Intuition behind the Identification



Note: In a general equilibrium setup, the shocks can induce the variation in any variables. In this diagram, the arrow should be interpreted as the sign indicating the dominant source of variation for an observable. If there is no arrow between shock S and observable O, it does not mean that S does not have any effects on O, but just means that the most part of the variation in O are triggered by the shocks other than S.

Observable Time-series for Estimation There are eight aggregate shocks in this model and eight observable time-series are chosen as the input for the Bayesian estimation:

$$\left\{ \frac{\sum_{i\in\texttt{Small}_t} E_i}{\sum_{i\in\texttt{Small}_t} k_i \cdot Q_t}, \frac{\sum_{i\in[0,1]} D_{i,t+1}}{\sum_{i\in[0,1]} k_{i,t+1} \cdot Q_t}, \Delta \ln Y_t, \ln \pi_t, \Delta \ln w_t, \Delta \ln q_t^I, \ln R_t, \Delta \ln C_t \right\} \tag{I.39}$$

$\frac{\sum_{i\in\texttt{Small}_t} E_i}{\sum_{i\in\texttt{Small}_t} k_{i,t} \cdot Q_t}$ is the equity financing of small firms measured in the same way as in section I.2. $\frac{\sum_{i\in[0,1]} D_{i,t+1}}{\sum_{i\in[0,1]} k_{i,t+1} \cdot Q_t}$ is the weighted average leverage ratio at the end of each period. Output Y_t is measured by the real gross value added by the non-financial corporate sector in Flow of Funds. Consumption C_t is the consumption of non-durable goods and services in real term. The nominal price P_t is measured as the deflator for the non-durable good and service[10] and $\pi_t \equiv {}^{P_t}/_{P_{t-1}}$. The nominal wage W_t is measured as the hourly earnings in the US manufacturing sectors and the real wage w_t is measure as ${}^{W_t}/_{P_t}$. The investment good price Q_t^I is measured by the deflator of the durable-good and private investment and $q_t^I \equiv {}^{Q_t^I}/_{P_t}$. R_t is measured by the federal funds rate. For both Y_t and C_t, the conversion between nominal

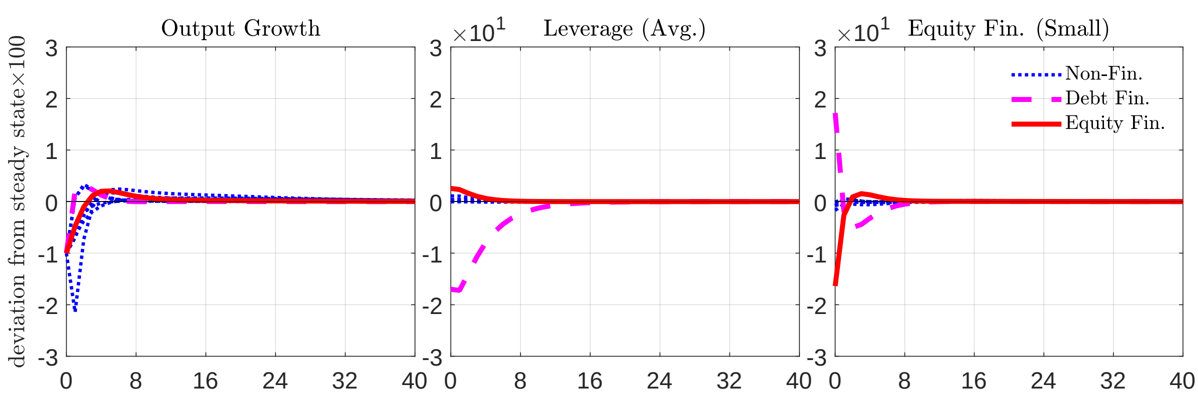
[10] The deflator is the weighted average of the deflator for the non-durable good and the deflator for the services. The values of non-durable good and services are used as the weight. Similar construction also applied to the measure of investment good price. Details about the underlying data source can be found in Appendix I.7.1

values and the real values is based on the price index P_t. The frequency is annual and all the observable time-series are linear-detrended before the estimation.

Identification Within these eight time-series, these six non-financial macroeconomic time-series are standard and they are used to identify the six non-financial aggregate shocks. The rest two time-series about firms' financing choices are used to identify the financial shocks. To illustrate the intuition about how these different shocks are identified, I normalize these eight shocks such that they generate a 1% decrease in output growth rate on impact, then I plot the responses of the weighted average leverage ratio and the small firms' equity financing flow in the middle and right panel of Figure I.7. It is clear that firms' financing choices are much more responsive to financial shocks than to the non-financial shocks. Therefore, with the non-financial shocks controlled by these six macroeconomic non-financial time-series, these two financial shocks will be mainly identified by these two financial time-series.

Within these two financial shocks, the debt financing shock has much larger effects on the leverage than the equity financing shock, so the debt financing shock will be mainly identified by the time-series of average leverage ratio. The small firms' equity financing flow is responsive to both financial shocks, but given that the debt financing shock is mostly identified by the time-series of average leverage ratio. The variation in small firms' equity financing flow will help identify the equity financing shock.

Figure I.7. Impulse Response to Financial and Non-Financial Shocks



Note: The size of shocks is calibrated to generate 1% on-impact decrease in the output growth. The impulse responses are the deviation of the corresponding variables from their steady-state level.

Estimates and Fitting with Data The priors and the key statistics of the estimated posteriors are listed in Table I.6. The moments generated by the model are summarized in Table I.7. Given that the parameters in Θ_{NK} are fixed, the model does a reasonably good job in fitting the data overall. Relatively speaking, the model overshoot the volatility of the aggregate prices, and this indicates that the model might understate the relevance of financial shocks in driving the firms' investment dynamics.

Table I.6. Estimates of the Aggregate Shock Processes Parameters

Parameter	Shock Process	Prior			Posterior		
		Type	Mean	Std.	Mode	5%	95%
Ω: Persistentce							
ρ_e	Equity Financing	Beta	0.6	0.2	0.5440	0.3499	0.6845
ρ_d	Debt Financing	Beta	0.6	0.2	0.7370	0.4381	0.8734
ρ_z	Aggregate TFP	Beta	0.6	0.2	0.1601	0.0300	0.2719
ρ_w	Wage Mark-up	Beta	0.6	0.2	0.9369	0.5802	0.9610
ρ_p	Price Mark-up	Beta	0.6	0.2	0.9470	0.8695	0.9752
ρ_m	Monetary	Beta	0.6	0.2	0.1930	0.0593	0.3226
ρ_q	Investment Good Price	Beta	0.6	0.2	0.7528	0.5269	0.9153
ρ_u	Preference	Beta	0.6	0.2	0.7124	0.3762	0.8331
Σ: Std. of the Shocks							
σ_e	Equity Financing	Exp.	0.02	0.02	0.0420	0.0322	0.0544
σ_d	Debt Financing	Exp.	0.02	0.02	0.0297	0.0243	0.0398
σ_z	Aggregate TFP	Exp.	0.02	0.02	0.0279	0.0238	0.0402
σ_w	Wage Mark-up	Exp.	0.02	0.02	0.0395	0.0316	0.0488
σ_p	Price Mark-up	Exp.	0.02	0.02	0.0476	0.0408	0.0654
σ_m	Monetary	Exp.	0.02	0.02	0.0154	0.0127	0.0190
σ_q	Investment Good Price	Exp.	0.02	0.02	0.0080	0.0068	0.0110
σ_u	Preference	Exp.	0.02	0.02	0.0343	0.0287	0.0493

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm with 10,000 draws.

Recovered Unobservable Shocks Based on the mode of the posterior, I recover the history of the unobserved financial shocks by the Kalman smoother, which is presented in Figure I.8b. For the equity financing shock history, there were significant increases in the equity financing cost during the two recession in the sample periods, and the increase during the burst of "dot com bubble" was much larger than the increase during the recession in 2008. This feature is consistent with the standard perception that the financial crisis in 2001 was mainly featured with the collapse of stock market. In Eisfeldt and Muir (2016), they also estimate an "equity type" external financing cost and I depict their estimated time-series in I.8b as a comparison. The correlation between their estimates and my estimates is as high as 0.73. This high correlation can serve as an external validation of my result.

Table I.7. Moments of the Observable Variables

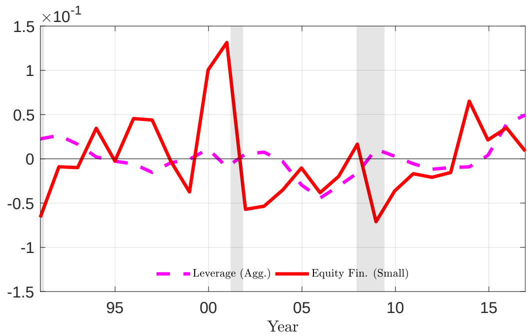
Variable	Standard Deviation				Autocorrelation			
	Data	Model			Data	Model		
		Mode	5%	95%		Mode	5%	95%
Equity Fin. (Small)	0.049	0.056	0.049	0.069	0.243	0.177	0.029	0.283
Leverage (Avg.)	0.020	0.035	0.027	0.049	0.719	0.832	0.685	0.906
Output Growth	0.027	0.040	0.035	0.050	0.414	0.452	0.362	0.478
Consumption Growth	0.011	0.019	0.016	0.023	0.538	0.506	0.335	0.550
Wage Growth	0.010	0.031	0.029	0.041	0.356	-0.015	-0.122	0.028
Inv. Price Growth	0.009	0.009	0.007	0.012	0.107	-0.124	-0.237	-0.042
Inflation	0.009	0.017	0.015	0.022	0.271	-0.053	-0.176	0.026
Nominal Rate	0.015	0.022	0.020	0.030	0.588	0.397	0.327	0.459
	Relative Std.				Cyclicality			
	Data	Model			Data	Model		
		Mode	5%	95%		Mode	5%	95%
Equity Fin. (Small)	1.834	1.422	1.105	1.744	0.460	0.227	0.199	0.289
Leverage (Avg.)	0.762	0.884	0.593	1.224	-0.342	-0.099	-0.218	-0.060
Consumption Growth	0.411	0.468	0.435	0.485	0.715	0.870	0.812	0.908
Wage Growth	0.378	0.795	0.759	0.907	-0.247	0.607	0.512	0.734
Inv. Price Growth	0.332	0.216	0.163	0.311	0.099	-0.006	-0.007	0.002
Inflation	0.326	0.419	0.359	0.529	-0.034	-0.386	-0.541	-0.323
Nominal Rate	0.575	0.552	0.495	0.692	0.148	-0.619	-0.739	-0.559

Note: Here the relative std. refers to the standard deviation normalized by the standard deviation of output growth. The cyclicality refers to the correlation with the output growth. The posterior distribution is obtained using the Metropolis-Hastings algorithm with 10,000 draws.

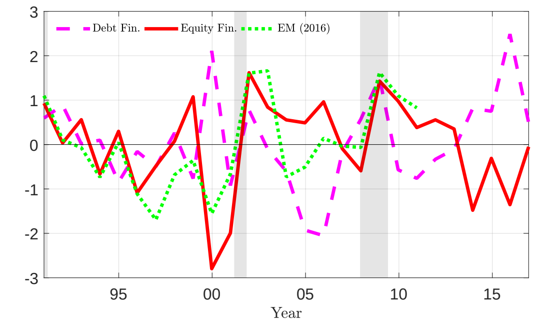
As for the recovered history of debt financing shock, the model implies that the collateral constraint become looser during the recessions. This result is mainly driven by the time-series of the public firms' leverage ratio, which is shown in Figure I.8a. In the data, the average leverage ratio of the U.S. public firms did increase during the recessions. Similar findings are also found by the micro-level study of Halling, Yu and Zechner (2016). The economic mechanism behind this data feature is beyond the scope of this paper, but this feature reflects that the public firms, especially the large public firms, are a special sample of firms in the economy and they have not experienced extreme tough financing conditions.

Figure I.8. History of Observables and Unobservables, and the Decomposition of Observed History

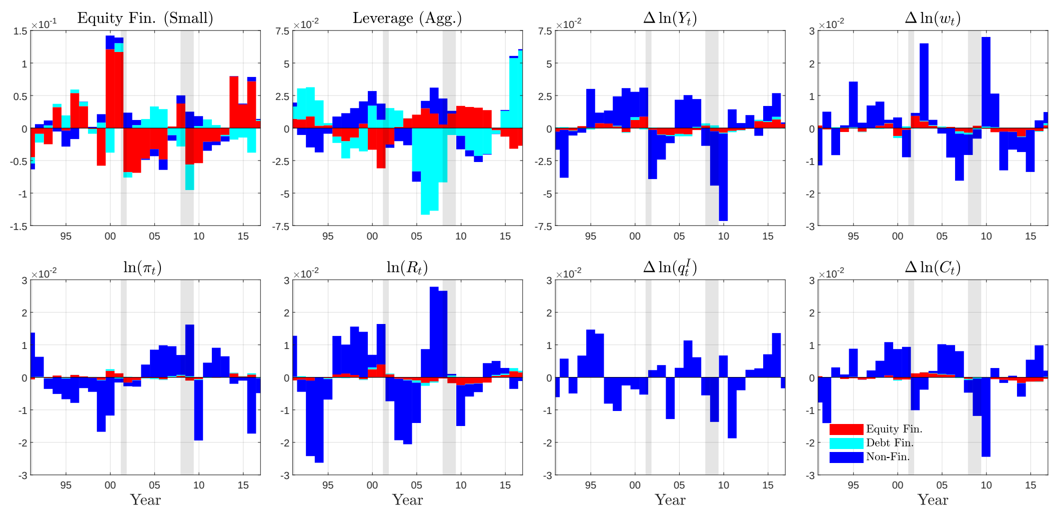
(a) Observed Financial Time Series



(b) Recovered Unobservable Financial Shocks



(c) Decomposition of the Observed History, Financial Shocks vs. Non-financial Shocks



Note: In panel I.8a, the time series is the deviation of the observed time-series from their linear trend. In panel I.8b, the units are the standard deviation of the corresponding shocks. The units in panel I.8c are the same with the data.

History Decomposition Based on the mode of the posterior, I also decompose the observable history into the contribution of different aggregate shocks, which are summarized in Figure I.8c. The history decomposition results confirm the intuition that the financial shock processes are mostly identified by the financial time-series. The macroeconomic non-financial time-series are almost totally driven by the non-financial shocks, so the time-variation in these variables will identify the parameters governing the non-financial shock processes. Conditional on the variation in the aggregate quantities and prices, the financial shocks will be identified by the time-variation in firms' financial choices.

I.5 Results and Analysis

The research question of this paper is: how much of the observed variation in firms' investment is driven by the financial shocks. To answer this question, I decompose the forecast error variance of firms' investment, on both disaggregate and aggregate level, into the contribution of different aggregate shocks based on the estimated model, and summarize the results in Table I.8.

Table I.8. Variance Decomposition of Investment

	On Impact			Average		
Shocks	Aggregate	Small	Large	Aggregate	Small	Large
Financial	2.1	97.3	31.8	**1.1**	**29.1**	**6.5**
Equity	1.9	93.5	28.8	1.1	27.3	6
Debt	0.2	3.8	3	0	1.8	0.5
Non-Financial	97.9	2.7	68.2	98.9	70.9	93.5
Price Markup	67.9	1.8	43.7	77.1	57.6	73.6
Wage Markup	7.8	0.3	4.7	12.3	9.2	11.2
TFP	10.8	0.2	10.9	1.7	0.7	1.9
Preference	8.6	0.1	6.1	6.7	3	5.7
Monetary	1.7	0.2	2.3	0.3	0.2	0.4
Investment Good Price	1.2	0.1	0.6	0.8	0.2	0.7

Note: Here, the variance of the forecast error is decomposed at two different horizon: on impact and over the whole sample history. The variance decomposition is based on the mode of estimated posterior.

On the disaggregate level, financial shocks play a much more important role in explaining the small firms' investment variation, no matter on impact, or in the long-run. This result is relatively intuitive because small firms' investment depends more on the external financing as shown in Table I.2 and I.5. Another feature worths to be mentioned is that financial shocks are the dominant source of the variation in small firms' investment in short-run, but their importance quickly decays over the time. This is consistent with the estimates in Table I.6: the process of the equity financing shock, which is the major financial shock, has a large standard deviation for its innovation but a low persistence for its propagation.

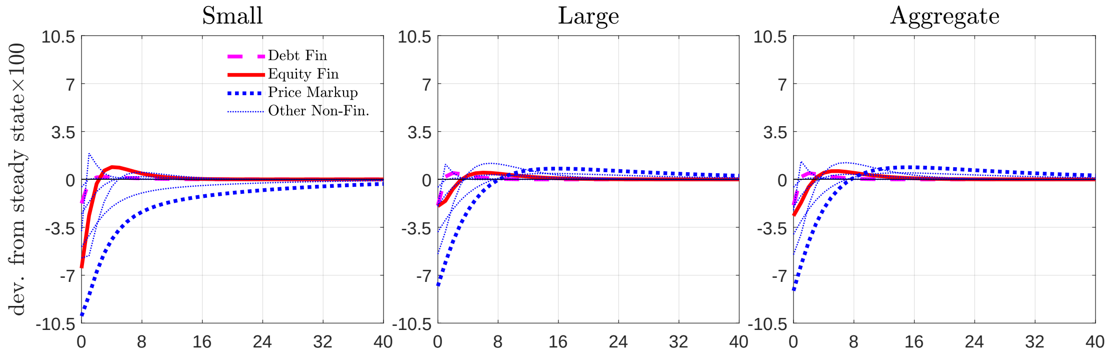
On the aggregate level, financial shocks contribute little to the aggregate investment variation, no matter on impact or in the long-run. In the standard partial equilibrium with heterogeneous firms, aggregate financial shocks could also have little aggregate effect for two reasons: financial shocks are mainly affecting the small firms and the total investment of small firms only count a small fraction of the aggregate investment. Both of these factors still present in this paper, but if they are the only reasons here, the relative importance of financial shocks to the aggregate investment should be a weighted average of their relative importance to small and large firms, i.e. it should be close and a little bit higher than their relative importance to the large firms' investment. However, as shown in Table I.8, the aggregate level relative importance of financial shocks is even smaller than their relevance to the large firms' investment. To understand this negligible aggregate effect of the financial shocks, I need highlight another economic mechanism within the general equilibrium model with heterogeneous firm: the reallocation of investment across different firms.

To explain and highlight the importance of this reallocation channel, I first shut-off the general equilibrium feedbacks and present the impulse response of firms' investment to different aggregate shocks within the partial equilibrium setup in Figure I.9a. Then I will resume the general equilibrium feedbacks, present the general equilibrium feedbacks through the responses of the aggregate prices in Figure I.9c, and discuss how they change the responses of different firms' investment to these shocks in Figure I.9b. The input shocks underlying these impulse responses are all contractionary and their size are all calibrated to the estimated one standard deviation as the posterior modes in Table I.6.

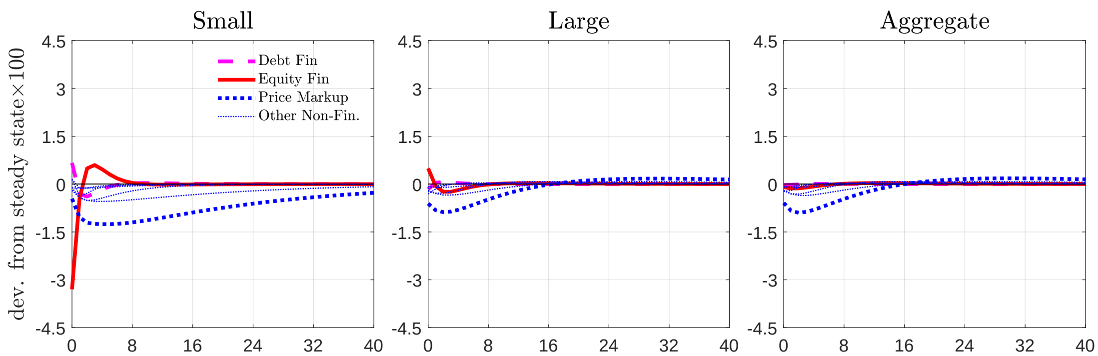
Direct Effects Without the general equilibrium feedbacks, both the small and large firms' investment decreases following the contractionary financial shocks. There are two features worth to be emphasized here. First, the responses of small firms' investment is larger than the responses of the large firms' investment for almost any type of aggregate shock. Small firms are more responsive to the equity financing shock because they rely more on the equity financing. The difference in their responses to the non-financial shocks are mainly due to the amplification effect of the financial frictions. Second, the responses of the aggregate investment are very close to the responses of the large firms' investment, which is due to the

Figure I.9. Impulse Response to Different Aggregate Shocks

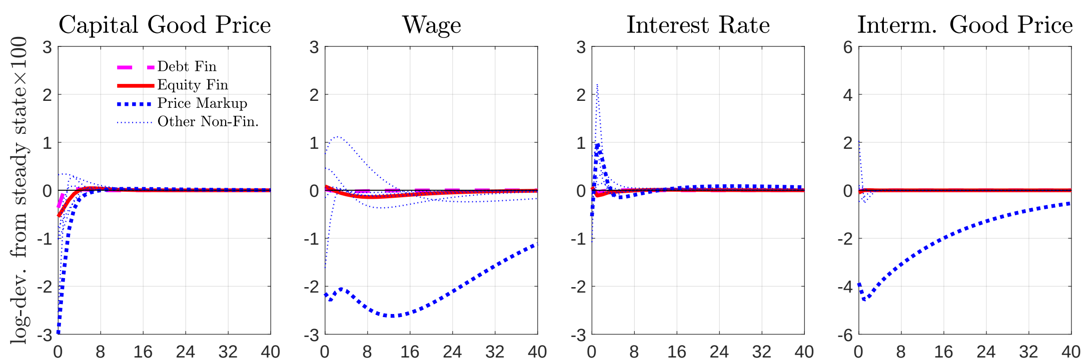
(a) Investment of Different Firm Groups, without General Equilibrium Feedbacks



(b) Investment of Different Firm Groups, with General Equilibrium Feedbacks



(c) Aggregate Prices, with General Equilibrium Feedbacks



Note: The impulse responses are to the one-standard-deviation contractionary shocks. The impulse responses of investment are the deviation of the measured investment from their steady state level. The investment is measured as $\sum_{i\in\mathcal{I}} I_i / \sum_{i\in\mathcal{I}} K_i$, where $\mathcal{I}$ is the indicator for different firm groups and $\mathcal{I} \in \{\text{small, large, population}\}$. The impulse responses of the aggregate prices are the log-deviation of real capital good price, real wage, real gross interest rate and real intermediate good price from their corresponding steady-state levels.

dominance of large firms' total size[11].

General Equilibrium Feedbacks There are four aggregate prices whose dynamics feature the general equilibrium feedbacks: capital good price (q_t), wage (w_t), real interest rate ($\frac{R_{t-1}}{\pi_t}$), and intermediate good price (p_t^I). Their responses to different aggregate shocks within the general equilibrium are summarized in Figure I.9c. By comparing the responses of different aggregate prices to financial shocks, we can find that financial shocks only trigger significant responses in the capital good price. In the following part, I will focus on the general equilibrium effect through the response of the capital good price when I explain the indirect effects of financial shocks.

Indirect Effects By comparing the impulse responses in Figure I.9b and I.9a, we can tell that general equilibrium feedbacks significantly dampen the effects of the non-financial shocks, and they even alter the direction of firms' investment responses to the financial shocks on the disaggregate level. The component of small and large firms is the key to understand the direction of their responses to financial shocks. There are three endogenously generated types of firms in this model, financially unconstrained firms, i.e. the firms who are paying dividend, the financially constrained and debt-dependent firms, i.e. the firms who are neither paying dividend nor issuing equity, and the financially constrained and equity-dependent firms, i.e. the firms who are issuing equity. The fraction of each types' total size within the small and large firms in steady state are summarized in Table I.9.

Table I.9. Size Composition of Small and Large Firms in Steady State (%)

	Constrained		Unconstained
	Equity Dependent	Debt Dependent	
Small	51.43	6.57	41.99
Large	20.52	23.04	56.44

Following an equity financing shock which increases the equity issuance cost, the equity-dependent firms will be hit directly and they have to cut their investment. The lower investment demand will dampen the capital good price, and the lower capital good price will motivate the unconstrained firms to increase their investment. Given that the equity-dependent firms are mainly concentrated in small firms and the unconstrained firms are

[11]In the data, the total investment from the small firms counts for 5% of the aggregate investment. In the model, the fraction of small firms' investment within the aggregate investment is 25%. But this does not change the conclusion of this paper that the negligible aggregate effects of financial shocks are mainly due to the reallocation of investment across different firms.

mainly concentrated in the large firms, the response of small firms' investment is negative and the response of large firms' investment is positive. In the aggregation, their responses largely cancel each other.

Following a debt financing shock which tightens the collateral constraint, the debt-dependent firms will be hit directly, they cut their investment and induce the decrease in the capital good price. At this time, the lower capital good price will motivate both the unconstrained firms and the equity dependent firms increase their investment[12]. Given that there is quite few debt-dependent firms in the small firms, the indirect effects of the debt financing shock will be more loaded on the small firms and the response of their investment is positive. Within the large firms, there are significant fraction of debt-dependent firms and the other types of firms, so the direct effect of the debt financing shock is more loaded on the large firms and there is a negative response in their investment. Because there is also a significant fraction of unconstrained firms within the large firms, the magnitude of large firms' investment response is not very large. But given the size dominance of the large firms, their slight negative response is able to eat out the large positive response of the small firms' investment, and on the aggregate effects of the debt financing shock are also negative but negligible.

Representative Firm Model vs. Heterogeneous Firm Model As discussed above, the negligible aggregate relevance of financial shocks is a result of the interaction between firm-level heterogeneity and general equilibrium feedbacks. Comparing with the result in this paper, the previous studies(e.g. Jermann and Quadrini, 2012), which use the general equilibrium model with a representative firm, implied a much larger aggregate relevance. A natural question here would be: is there such a big difference between the representative firm model and the heterogeneous firm model in terms of their aggregate implication?

To make a reasonable comparison, I degenerate the heterogeneous firm model in this paper back to a representative model by removing the idiosyncratic productivity shocks. Then I estimate the model to match the time-variation in both aggregate quantities and prices, as well as the aggregate financing choices of U.S. public firms. Based on the estimates, I quantify the aggregate effects of financial shocks on the aggregate investment (details about the model and quantification can be found in Appendix I.7.4). As summarized in Table I.10, the representative firm model implies a much larger aggregate effect of financial shocks: financial shocks contribute 55.4% of the variation in the aggregate investment and they become the dominant driving force of the aggregate investment!

[12] The increase in equity-dependent firms' investment is mainly due to the construction that the issuance cost is proportional.

Table I.10. Variance Decomposition of Aggregate Investment

	On Impact		Average	
Shock	Rep. Firm	Hete. Firm	Rep. Firm	Hete. Firm
Financial	11.8	2.1	55.4	1.1
Non-Financial	88.2	97.9	44.6	98.9

As discussed above, the aggregate effects of financial shocks result from the combination of their direct effects on the financially constrained firms, the induced general equilibrium feedbacks, and the indirect effects on the financially unconstrained firms. In the representative firm model, there is only one firm and this only firm is financially constrained because there is no idiosyncratic shocks and the aggregate shocks are too small to give it enough precautionary motive. Therefore, there will be no financially unconstrained firms to rebalance the direct effects from the financial shocks, and the representative firm model implies a much larger aggregate relevance of the financial shocks.

I.6 Concluding Remarks

This paper studies the financial shocks' effects on both disaggregate and aggregate level. To be consistent with the micro-level evidence about the heterogeneity across firms in terms of how they finance their investment, I incorporate a continuum of heterogeneous firms into a general equilibrium model with financial frictions. To separately capture the exogenous variation in firms' financing conditions and investment profitability, this model is constructed with two financial and eight non-financial aggregate shocks. To identify the source of firms' investment fluctuation, this model is estimated by a Bayesian likelihood method to match the time-variation in both the macroeconomic quantities and prices on the aggregate level, and the firms' financing choices on the distributional level. With the estimated model, I quantify the effects of financial shocks on both disaggregate and aggregate level.

At the disaggregate level, financial shocks are only important for the small firms' investment. At the aggregate level, financial shocks contribute little to the aggregate investment variation. The negligible aggregate relevance of financial shocks is mainly due to the reallocation of investment between financially-constrained and financially-unconstrained firms, which is absent within the representative firm setup. If I collapse the heterogeneous firm model back to a representative model and repeat the same quantitative analysis with the same data, the implied aggregate relevance of financial shocks will be much larger.

This paper contributes to the literature by incorporating the micro-level heterogeneity into the identification of the source of business cycle fluctuation. There are two direct benefits from using the structural model with heterogeneous firms: we can match the micro-level data and study the disaggregate level implication of the aggregate shocks. Beyond these direct benefits, this paper also studies the difference between the heterogeneous firm model and the representative firm model in terms of their implication about the aggregate relevance of financial shocks. The results in this paper imply that the quantitative studies based on the representative firm model might overstate the aggregate relevance of the financial shocks.

I.7 Appendix: Additional Details

I.7.1 Data Details

Data from Compustat The CompuStat annual dataset is downloaded from WRDS. The variables used in the empirical evidence are listed in Table I.11. The data cleaning procedure has following steps:

1. Only keep US firms: `fic`='USA'
2. Discard the observations from financial, utility and quasi-governmental sectors, i.e. the observations with `sic` in $6000-6999$, $4900-4999$ and $9000-9999$.
3. Discard the four giant firms which were mainly affected by the 1988 accounting rule change: GE (`gvkey`==005047), Ford (`gvkey`==004839), Chrysler (`gvkey`==003022), GM (`gvkey`==005073).
4. Only keep the records with standard format, i.e. `datafmt`='STD'
5. Only keep the records with SCF format code 7, i.e. `scf`=7
6. Only keep the observations listed on US stock markets, i.e. `exchg` in $0-4$ and $11-20$.
7. Discard the observation with M&A larger than 5% of its book value asset.
8. Drop observations with missing value for the book value asset.
9. Discard observations with real book value asset smaller than 10 million (1982 $)

Data in SDC used to Refine Stock Issuance (`sstk`) I extract the IPO and SEO deals in SDC issued between 1989 and 2016. Only the deals which are issued in US markets are kept. The deals in SDC are merged with CompuStat data mainly by the first 6 digit of CUSIP code. For the rest unmatched deals, I use their ticker symbol to match with the firms in CompuStat.

Aggregate Data The aggregate variables are extracted from Flow of Funds and FRED and the details are listed in Table I.12.

I.7.2 Derivation of Firms' Decision Problem

Objective of Managers In each period, the managers make production, financing and investment decisions to maximize the expected total discounted payoff to the existing shareholders. When firm i issues equity, to raise funding $E_{i,t}$, a fraction of $s_{i,t} \equiv s_t(E_{i,t}; k_{i,t}, D_{i,t}, a_{i,t})$ of the ownership has to be issued to the external investors. For any given sequence of $\{E_{i,t+l}, k_{i,t+l}, D_{i,t+l}, a_{i,t+l}\}_{l=0}^{\infty}$, the expected total discounted payoff to the existing shareholders is:

$$\max -E_{i,t} \cdot \mathbb{1}^+(E_{i,t}) + \sum_{l=1}^{\infty}(1-\xi_d)^{l-1}\mathbb{E}_t\left[\Lambda_{t,t+l} \cdot \frac{P_t}{P_{t+l}} \cdot \bar{S}_{t,t+l} \cdot \mathcal{CF}_{i,t+l}\right] \tag{I.40}$$

where Λ is the real stochastic discounting factor (SDF) determined by households and $\bar{S}_{t,t+l} \equiv \left[\prod_{u=1}^{l-1}(1-s_{i,t+u})\right]$ is the fraction of ownership in period $t+l$ owned by the block of shareholders in period t. There are two possible outcomes for the cash flow to all of the shareholders in period $t+l$: $\max\{0, -E_{i,t+l}\}$ if the firm does not exit, and $\mathcal{LV}_{i,t+l}$ if the firm exits. Then the corresponding expected cash flow $\mathcal{CF}_{i,t+l}$ will be:

$$\mathcal{CF}_{i,t+l} = (1-\xi_d) \cdot \left[-E_{i,t} \cdot \mathbb{1}^+(-E_{i,t})\right] + \xi_d \cdot \mathcal{LV}_{i,t+l} \tag{I.41}$$

When managers maximize the existing shareholders' payoff specified in (I.40), their choice is constrained by the production technology (I.5), budget constraint (I.7), capital accumulation rule (I.8), and the financial frictions.

Collateral Constraint The debt contract is an one-period contract collateralized by the firms' capital. In each period t, firm i has to pay back the existing debt $D_{i,t}$ before issuing new debt $D_{i,t+1}$. Due to the problem of limited enforcement, the new debt cannot excess a

Table I.11. Data Source of Micro-level Evidence

Variable Name	CompuStat Variable
M&A	aqc
book value asset	at
Capital Expenditure	capx
R&D Expenditure	xrd
Net long-term debt issuance	dltis−dltr
Net change of current debt	dlcch
Issuance of common and preferred stocks	sstk
Dividend	dv
Stock repurchase	prstkc

Table I.12. Data Source of Aggregate-level Evidence

Variable	Data Source	Variable ID
Gross value added of non-financial corporate sector	Flow of Fund	FA106902501.A
Consumption, non-durable good	FRED	PCNDA
Consumption, service	FRED	PCESVA
Consumption, durable good	FRED	PCDGA
Investment, private	FRED	GPDIA
Deflator, non-durable good	FRED	DNDGRD3A086NBEA
Deflator, service	FRED	DSERRD3A086NBEA
Deflator, durable good	FRED	DDURRD3A086NBEA
Deflator, private investment	FRED	A006RD3A086NBEA
Hourly earnings in the US manufacturing sectors	FRED	USAHOUREAAISMEI
Federal fund rate (monthly)	FRED	FEDFUNDS

fraction ϕ_t^d of the current value of the capital stock:

$$D_{i,t+1} \leq \phi_t^d \cdot k_{i,t} \cdot Q_t \tag{I.42}$$

Costly Equity Issuance For any given equity contract $s_t(E; k, D, a)$, denote the corresponding value of the firm to existing shareholders as $V_t(k, D, a)$, which is the optimal value of the objective specified in (I.40). Then there exits a recursive representation of the value function as

$$V_t(k, D, a) = \max_{l,I,E,D',\hat{k}'} \underbrace{\max\{0, -E\}}_{\text{Cash flow from Dividend}} + \underbrace{(1 - s_t)}_{\text{Remaning Ownership}} \cdot \underbrace{\hat{V}_t(\hat{k}', D', a)}_{\text{Post-Issuance Value}} \tag{I.43}$$

where the post-issuance value $\hat{V}_t(\hat{k}', D', a)$ comes from the expected cash flow from continuing operation and liquidation

$$\hat{V}_t(\hat{k}', D', a) \equiv \mathbb{E}_t \left[\Lambda_{t,t+1} \cdot \frac{P_t}{P_{t+1}} \cdot \left[(1 - \xi_d) \cdot V_{t+1}\left(e^{\varepsilon^{k'}} \cdot \hat{k}', D', a'\right) + \xi_d \cdot \mathcal{LV}_{t+1}\right]\right] \tag{I.44}$$

If the equity financing market is frictionless, then the value of the shares issued to the external investors should be equal to their supplied funding:

$$s_t \cdot \hat{V}_t(\hat{k}', D', a) = E \cdot \mathbb{1}^+(E)$$

However, when firms are issuing equity in reality, their shares are typically sold at a discounted price due to the underwriting fees or information asymmetry:

$$\frac{s_t \cdot \hat{V}_t(\hat{k}', D', a)}{1 + \phi_t^e} = \max\{0, E\} \tag{I.45}$$

where $\phi_t^e > 0$ is the price discount and it is referred as the cost of equity issuance.

Bellman Equation If we plug (I.45) into (I.43), the manager's decision problem can be recast as

$$\begin{aligned} V_t(k, D, a) = \max_{l,I,D',E} & - E \cdot \left[1 + \phi_t^E \cdot \mathbb{1}^+(E)\right] \\ & + \mathbb{E}_t \left[\Lambda_{t,t+1} \cdot \frac{P_t}{P_{t+1}} \cdot \left[(1 - \xi_d) \cdot V_{t+1}\left(e^{\varepsilon^{k'}} \cdot \hat{k}', D', a'\right) + \xi_d \cdot \mathcal{LV}_{t+1}\right]\right] \end{aligned} \tag{I.46}$$

The optimization is subject to the production technology (I.5), budget constraint (I.7), rule of capital accumulation (I.8) and the collateral constraint (I.10). The associated policy functions for hiring, production, investment, debt holding, equity financing and capital holding are denoted as $l_t(k,D,a)$, $y_t(k,D,a)$, $I_t(k,D,a)$, $D'_t(k,D,a)$, $E_t(k,D,a)$ and $\hat{k}'_t(k,D,a)$.

I.7.3 Sketch of the Numerical Algorithm

Transformed Bellman Equation System The Bellman equation system can be rewritten as

$$\begin{aligned}
&V_t(\tilde{x},k,a)\\
=&\max_{\tilde{e},ki,\tilde{d}',\tilde{k}'} -\tilde{e}\cdot k\cdot q_t\cdot\left(1+\mathbb{1}_{\tilde{e}>0}\cdot\phi_t^E\right)\\
&+\mathbb{E}_t\left[\Lambda_{t,t+1}\cdot\left[(1-\xi_d)\cdot V_{t+1}\left(\tilde{x}',\varepsilon^{k'}\cdot\tilde{k}'\cdot k,a'\right)+\xi_d\cdot\tilde{lv}_{t+1}\left(\tilde{d}',\varepsilon^{k'}\cdot\tilde{k}',a'\right)\right]\right]
\end{aligned}$$

subject to

$$\begin{aligned}
ki =&\tilde{x}+\tilde{e}+\tilde{d}'-(1-\delta)\\
\tilde{k}'+\Phi^K(\tilde{k}'\cdot k,k)\frac{1}{k} =&\tilde{x}+\tilde{e}+\tilde{d}'\\
\tilde{d}' \le&\phi_t^D\\
\tilde{lv}_{t+1}(\tilde{d}',\varepsilon^{k'}\cdot\tilde{k}',a') =&(1-\delta)\cdot q_{t+1}\cdot\varepsilon^{k'}\cdot\tilde{k}'\cdot k-\frac{1+i_t}{1+\pi_{t+1}}\cdot\tilde{d}'\cdot k\cdot q_t\\
\tilde{x}' =&(1-\tau)\cdot\iota\cdot\frac{1}{q_{t+1}}\cdot w_{t+1}^{\frac{-\zeta}{1-\zeta}}\cdot\left(p_{t+1}^I\cdot Z_{t+1}\right)^{\frac{1}{1-\zeta}}\cdot(a')^{\frac{1}{1-\zeta}}\cdot\left(\varepsilon^{k'}\cdot\tilde{k}'\cdot k\right)^{\tilde{\theta}-1}\\
&+(1-\tilde{\delta})-\frac{1+(1-\tau)\cdot i_t}{1+\pi_{t+1}}\cdot\frac{q_t}{q_{t+1}}\cdot\frac{\tilde{d}'}{\tilde{k}'\cdot\varepsilon^{k'}}
\end{aligned}$$

where $\zeta\equiv\theta(1-\alpha)$, $\iota\equiv\zeta^{\frac{\zeta}{1-\zeta}}-\zeta^{\frac{1}{1-\zeta}}$, $\tilde{\theta}\equiv\frac{\alpha\theta}{1-\zeta}$ and $\tilde{x}\equiv\frac{x}{k\cdot q_t},\tilde{d}'\equiv\frac{d'}{k\cdot q_t},ki\equiv\frac{I}{k},\tilde{k}'\equiv\frac{\hat{k}'}{k},\tilde{e}\equiv\frac{e}{k\cdot q_t}$.

Value Function Approximation The grid for the idiosyncratic productivity a is constructed based on Tauchen (1986): there are 3 grid points in total chosen over the scale $[-1.5\times{}^{\sigma_a}/\sqrt{1-\rho_a^2}, 1.5\times{}^{\sigma_a}/\sqrt{1-\rho_a^2}]$. For each grid point of idiosyncratic productivity state, $V_t(\tilde{x},k,a)$ is approximated by the spline with the order of 2 at each dimension of $\tilde{x}$ and k. On the dimension of $\tilde{x}$, there are 10 grid points evenly spread between 0.01 and 2. On the dimension of k, there are 25 grid points spread between $\exp(-5)\times\left[\frac{A_{1,SS}\cdot\exp\left(1.5\times\sigma_a/\sqrt{1-\rho_a^2}\cdot\frac{1}{1-\zeta}\right)}{\delta+\frac{1}{\beta}-1}\right]^{\frac{1}{1-\tilde{\theta}}}$ and $1.1\times\left[\frac{A_{1,SS}\cdot\exp\left(1.5\times\sigma_a/\sqrt{1-\rho_a^2}\cdot\frac{1}{1-\zeta}\right)}{\delta+\frac{1}{\beta}-1}\right]^{\frac{1}{1-\tilde{\theta}}}$, where $A_{1,SS}=(1-\tau)\cdot\iota\cdot\frac{1}{q_{SS}}\cdot w_{SS}^{\frac{-\zeta}{1-\zeta}}\cdot\left(p_{SS}^I\cdot Z_{SS}\right)^{\frac{1}{1-\zeta}}$. The grid for k is distributed such that $\frac{\log k}{0.5}$ are evenly spread. The value function in steady state is solved by collocation method.

Distribution Approximation Since the entrant distribution is fixed over the sample period, I approximate the distribution of firms after the realization of capital quality shocks and exit shocks in two parts: a quadrature with fixed weights for entrants and a quadrature with varying weights for the incumbents. In stead of approximating over $(d \equiv \frac{D}{k \cdot q_t}, k, a)$, I approximate the distribution over a transformed equivalent space (dx, k, a) where $dx \equiv A_{1,SS} \cdot a^{\frac{1}{1-\zeta}} \cdot k^{\tilde{\theta}-1} + (1-(1-\tau)\cdot\delta) - \frac{1+(1-\tau)\cdot i_{SS}}{1+\pi_{SS}} \cdot \frac{1}{q_{SS}} \cdot d$. By this transformation, the geometry of the distribution support at each grid point of a will be more regular and at each grid of a, the approximation scheme follows the quadrature based method as used in Algan, Allais and Den Haan (2008) and Winberry (2018) with the highest order of moments set at 4.

I.7.4 The Comparable Representative Firm Model

Setup There is continuum of representative firms $i \in [0, 1]$. Comparing with the setup of the heterogeneous firm model, there are only two changes in this representative firm model. First, after the incumbents exit, a group of entrants will take over their capital and enter the operation. Second, equity financing friction is modeled as a time-varying wedge, which captures the marginal funding cost of the firms.

Decision Problem

$$\begin{aligned} V_t(D, k) = \max_{l,I,D',E} & -E \cdot [1 + \phi^e_t] \\ & + \mathbb{E}_t \left[\Lambda_{t,t+1} \cdot \frac{P_t}{P_{t+1}} \cdot [(1-\xi_d) \cdot V_{t+1}(D', k') + \xi_d \cdot \mathcal{L}\mathcal{V}_{t+1}] \right] \end{aligned} \tag{I.47}$$

s.t. : production technology (I.5), internal financing source (I.6),
budget constraint (I.7), capital accumulation (I.8),
collateral constraint (I.10)

Quantification All of the non-estimated parameters are still fixed at the same level, except for the value of ϕ^e_{SS}. ϕ^e_{SS} is calibrated to match the cross-time average level of the aggregate equity financing flow. Then the parameters in Θ_{SH} are estimated based on the new model. There is only one input observable which is different with the previous choice: the small firms' equity financing flow is replaced with the aggregate equity financing flow. Based on the mode of the posterior, the variation decomposition results are summarized in Table I.13.

Table I.13. Variance Decomposition of Aggregate Investment, Details

	On Impact		Historical Average	
Shock	Rep. Firm	Hete. Firm	Rep. Firm	Hete. Firm
Financial	11.8	2.1	55.4	1.1
Equity	11.8	0.2	55.4	0
Debt	0	1.9	0	1.1
Non-Financial	88.2	98	44.5	98.9
Price Markup	43.9	67.9	21.7	77.1
Wage Markup	0.7	7.8	1.8	12.3
TFP	42	10.8	19.7	1.7
Preference	1.4	8.6	1.1	6.7
Monetary	0.1	1.7	0	0.3
Investment Good Price	0.1	1.2	0.2	0.8

CHAPTER II

Equity Financing and Monetary Policy

II.1 Introduction

Frictions in firms' access to external finance are central to the transmission of monetary policy to the real economy. However, the effect of monetary policy on the equity side of financing remains largely unexplored. For example, while quantitative corporate finance models consider firms' decisions under parameterized equity costs, these models are not designed to study policy experiments. In addition, macroeconomic models that can be used for policy analysis focus on debt as firms' marginal source of external finance. This analysis cannot speak to the issue of monetary transmission through equity markets, as the frictions underlying debt markets can differ fundamentally from those characterizing equity markets (Tirole, 2006).

We enter this picture with an investigation of the impact of monetary policy on firms' equity financing, and we quantify the extent to which this policy transmission channel matters for aggregate fluctuations. We start by providing evidence of linkages between monetary shocks and firm equity financing. Empirically, we find a large response of aggregate equity flows to shocks to monetary policy. Moreover, monetary expansions mitigate the price drops that typically characterize announcements of equity issuance. Next, we rationalize this evidence by formulating a model of firms' equity financing under asymmetric information. In the model, monetary stimulus decreases outside investors' required returns on equity issuance, so firms with more favorable private information about the quality of their assets in place become more willing to issue equity and invest. The resulting improvement in the composition of issuing firms reduces the asymmetric-information premium associated with issuance. This mechanism in the model constitutes a novel channel of monetary transmission

This chapter is based on the working paper coauthored with Pablo Ottonello and Toni M. Whited.

that we call amplification through selection. The reduction in the asymmetric-information premium further induces other high-quality firms to invest, which further reduces informational frictions. This channel implies that monetary policy can play a role of stabilizing inefficiencies from informational frictions over the cycle.

The paper begins by empirically documenting the relevance of equity flows in the financing of firms' investment following monetary stimulus. To this end, we combine data on firms' aggregate equity and debt financing flows from the U.S. Flow of Funds with the monetary-policy shocks from Romer and Romer (2004). We show that the increase in equity flows following a monetary expansion is similar in magnitude to the increase in debt flows. This pattern is robust, as we observe it in both corporate and noncorporate sectors, as well as for the active margins of equity financing such as the number of SEOs.

Motivated by this aggregate evidence, we then empirically study potential mechanisms driving the response of equity financing to monetary shocks. Our empirical strategy builds on the large literature in corporate finance documenting stock-price drops following the release of information about firms' equity issuance, a fact often interpreted as evidence of the relevance of informational frictions in equity markets. Using individual SEO-deal issuance dates and stock-price data around these events, we show that monetary-policy stimulus mitigates the price drop experienced by firms filing for issuance. The effects are quantitatively large, implying that a 25 basis-point cut in policy rates mitigates 18% of the average cumulative abnormal-return drop experienced by the issuing firms.

To further our understanding of this empirical evidence, we construct a model of firms' investment under frictional equity financing, and we study its aggregate implications. The model embeds a market with equity issuance with asymmetric information (Myers and Majluf, 1984) into a framework with monetary shocks. When issuing equity to finance new investment projects, firms have private information on the quality of their existing capital stock. Given that the market cost of equity issuance reflects the average quality capital of firms participating in the issuance, firms with higher quality of capital prefer not to issue equity. In this environment, an announcement of equity issuance signals that a firm has low quality of assets in place, so it suffers a stock price drop. Monetary stimulus decreases investors' required returns, which makes firms with high-quality capital more willing to issue equity, increasing the average quality of issuing firms, and reducing the price drop associated with announcements of equity issuance, as observed in our empirical analysis.

The model implies that monetary-policy transmission generates amplification through selection. Because informational frictions fall when firms with higher capital quality issue equity, more firms with higher capital quality also willing to issue. This channel implies a scope of policy linked to stabilizing informational frictions over the cycle.

Our paper contributes to four branches of the literature. The first comprises studies of the role of firms' financial frictions in shaping the transmission of monetary policy to the real economy. To date, models have mostly focused on the amplification that arises from the debt side of external financing. For instance, Bernanke, Gertler and Gilchrist (1999) construct a New Keynesian model in which firms finance investment with risky borrowing and show how this leads to a financial accelerator, or feedback between collateral values and investment. Ottonello and Winberry (2018) introduce firm heterogeneity into this framework and show how the distribution of default risk in the economy matters for amplification. Gomes, Jermann and Schmid (2016) study how unanticipated inflation leads to debt overhang and distorts investment choices with nominal long-term debt contracts. Rocheteau, Wright and Zhang (2018) show how pass-through from nominal rates to real rates can arise when firms search for banks' funding in over-the-counter markets. We complement this body of work by studying the amplification that arises from frictions in equity financing, as well as by introducing our novel amplification channel.

Second, our paper is related to the empirical literature that examines the connection between monetary policy and asset prices. For instance, Bernanke and Kuttner (2005) and Gorodnichenko and Weber (2016) show how monetary policy expansions affect stock market prices, while Gürkaynak, Sack and Swanson (2005) and Nakamura and Steinsson (2018) document the effects of monetary policy shocks on risk-free rates over several horizons, and Anderson and Cesa-Bianchi (2019) show how monetary shocks affect the price of corporate bonds in secondary markets. We complement this literature by studying how monetary policy affects the cost of issuing new equity. Our findings also complement event-study evidence of stock price drops that follow equity issuance (see, for example, Masulis and Korwar, 1986; Asquith and Mullins, 1986).

Third, our paper is related to the literature that studies the role of asymmetric information over the business cycle, such as Eisfeldt (2004), Kurlat (2013), and Bigio (2015). We contribute to this literature by introducing nominal rigidities and studying monetary policy. This highlights a novel role of monetary policy, namely expansionary monetary policies can stabilize informational frictions over the business cycle.

Finally, our paper contributes to the quantitative corporate finance literature. An important part of this literature has shown how dynamic models with parameterized costs of equity financing can account for the key cross-sectional patterns of firms' investment and financing (see, for example, Gomes, 2001; Hennessy and Whited, 2007). Another set of studies has shown how time-varying financing frictions are necessary to explain the cyclical pattern of firms' financing behavior and their asset prices (Covas and Den Haan, 2012; Jermann and Quadrini, 2012; Belo, Lin and Yang, 2018). Our paper contributes to this literature by endogenizing a time-varying cost of equity that is based on asymmetric information,as in Myers and Majluf (1984) and Krasker (1986).

The rest of the paper is organized as follows. Section II.2 presents the empirical evidence. Section II.3 presents the model. Finally, Section II.4 concludes.

II.2 Empirical Evidence

We study how changes in monetary policy affect equity financing. We first analyze data on aggregate financing flows, finding that equity is a key margin of firm's financial adjustment following monetary-policy shocks. We then use micro-level data on equity issuance filings to show that expansionary monetary policy mitigates the price drop associated with these events.

II.2.1 Aggregate equity flows and monetary policy

Data and descriptive statistics

We measure firms' aggregate financing flows using quarterly data from the *Financial Accounts of the United States*. Our main variables of interest are the net debt- and equity-financing flows of nonfinancial firms. The aggregate net debt-financing flow is defined as the sum of the net increase in loans and debt securities. The aggregate net equity-financing flows is defined as equity issuance net of dividend payments. Net equity issuance is defined as gross equity issuance net of equity retirement, which is measured as the sum of share repurchases and mergers and acquisitions. Appendix II.5.1 provides more details regarding the construction of these variables. We combine these data with monetary-policy shocks, measured with the narrative approach from Romer and Romer (2004), as extended by Wieland and Yang (2019). These shocks have been widely used to study the effects of monetary policy on macroeconomic variables, providing a relevant benchmark for our study.[1] Our sample starts in 1970, when Romer and Romer (2004) start the measurement of monetary policy shocks, and ends in 2016.

In Table II.1, we provide summary statistics for aggregate net equity and debt flows of nonfinancial firms for our period of analysis. We also report statistics for the different components of equity flows, which is our main variable of interest. Panel (a) shows that on average, firms raise debt and pay out equity, with net flows of around 1.5% of assets per

[1]See, for example Ramey (2016) and references therein. The availability of these shocks since the 1970s makes them particularly suitable for the aggregate analysis of this subsection. In the next subsection, using microeconomic data, we also use a high-frequency identification approach as an alternative measure of monetary-policy shocks.

Table II.1. Descriptive Statistics: U.S. Nonfinancial Firms' Financial Flows

Variable	$\mathbb{E}(X)$ (%)	$\sigma(X)$ (%)	$\rho(X, GDP)$
(a) Aggregate Net Flows			
Equity financing	-1.29	0.34	0.00
Debt financing	1.42	0.85	0.45
Total external financing	0.13	0.78	0.49
(b) Equity Financing Flows by Sector			
Corporate Net Flows	-1.75	0.42	-0.23
Net issuance	-0.52	0.38	-0.25
Dividend payment	1.23	0.16	0.00
Noncorporate net flows	-0.24	0.65	0.28
(c) Corporate Equity Flows by Component			
Gross Issuance	1.17	0.26	0.22
Initial public offerings (IPO)	0.10	0.06	0.13
Seasoned equity offerings (SEO)	0.21	0.05	0.33
Others	0.86	0.19	0.17
Retirement	2.13	0.54	0.14
Repurchase	1.09	0.27	0.10
Mergers and acquisitions	1.04	0.32	0.15

Note: $\mathbb{E}(X)$, $\sigma(X)$, and $\rho(X, GDP)$ denote for variable X, the mean, standard deviation, and correlation with real GDP. Statistics reported correspond to quarterly data for the period 1970–2016, except for Panel (c), which corresponds to the 1996–2016 period because of data availability. Panel (a) refers to data from corporate and noncorporate sectors. Data source: *Financial Accounts of the United States*. Flows are normalized by (lagged) total assets of the corresponding sector. Standard deviations and correlations are computed with detrended data using the filter from Baxter and King (1999).

year. It also shows that equity flows are acyclical on average and less volatile than debt flows. While this finding is at odds with the negative correlation reported in Jermann and Quadrini (2012), the difference is due entirely to the difference in our sample period. In Table II.3 in the Appendix, we present descriptive statistics for the post-1984 period, showing that equity flows are negatively correlated with output. Panel (b) shows that the acyclicality of aggregate equity flows is the result of countercyclical equity flows for the corporate sector and procyclical equity flows for the noncorporate sector. Panel (c) shows that gross-issuance flows of the corporate sector are also procyclical, a pattern also evident in flows from initial public offerings (IPOs) and seasoned equity offerings (SEOs). Interestingly, equity retirements are also procyclical, resulting in the acyclicality of net flows.

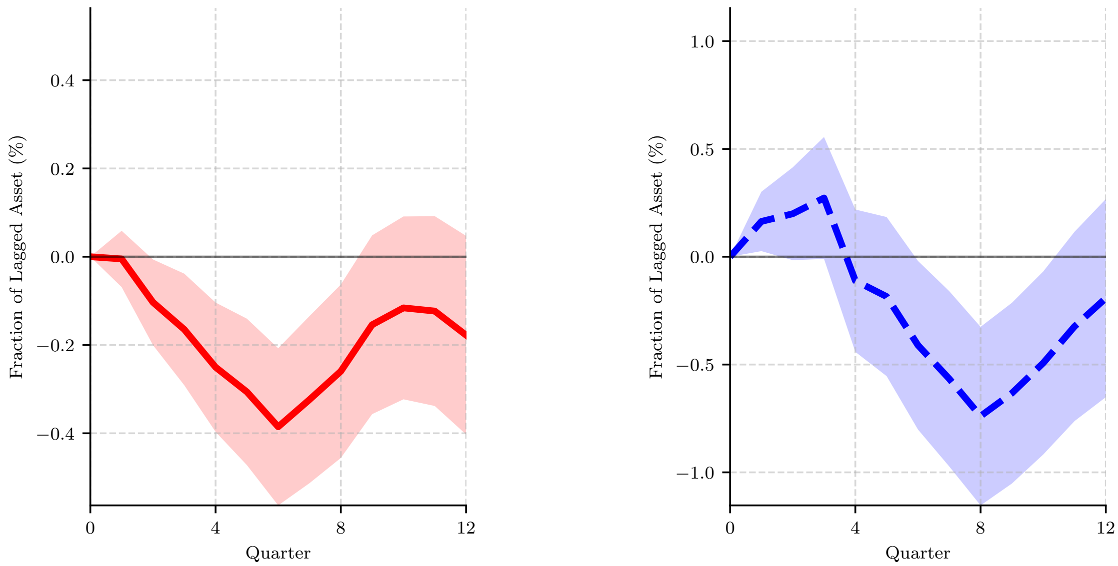
Empirical model

While these unconditional business-cycle moments are of interest, they mask the response of firms financial flows to monetary shocks. To study these responses, we estimate an empirical model that closely follows the approach in Romer and Romer (2004), which measures the effect of monetary policy shocks on economic activity. Specifically, the model is given by:

$$\Delta Y_t = \sum_{\iota=1}^{N_{\mathrm{m}}} \gamma_\iota v_{t-\iota}^{\mathrm{m}} + \sum_{\eta=1}^{N_{\mathrm{y}}} \rho_\eta \Delta Y_{t-\eta} + \beta Z_t + \varepsilon_t, \tag{II.1}$$

where Y_t denotes an aggregate financial flow (equity, debt, or total), v_t^{m} denotes the monetary shock in period t, and Z_t denotes a vector of controls with seasonal dummies. We estimate these regressions with data through 2007Q2 to focus on conventional monetary policy. With these estimates, our object of interest is the impulse response of Y_t to monetary shocks, which can be computed as as $\mathtt{IRF}_\iota = \sum_{j=0}^{\iota} \gamma_j$. Following Romer and Romer (2004), we use three-year lags for the monetary shocks ($N_{\mathrm{m}} = 12$ quarters) and two-year lags for the autoregressive controls ($N_{\mathrm{y}} = 8$).

As a validation of our exercise, Appendix Figure II.6 reports the results from replicating Romer and Romer (2004) with quarterly data for our period of analysis. It also shows the results of estimating (II.1) with output and investment as the dependent variables at a quarterly frequency. Consistent with other empirical studies, these validation exercises indicate that a one percent increase in monetary-policy rate in our sample leads to a 1.5% decline in economic activity and a 5% decline in investment, which peaks between one and two years after the shock.



(a) Equity

(b) Debt

Figure II.1. Responses of Net Financing Flows to Monetary Policy Shocks

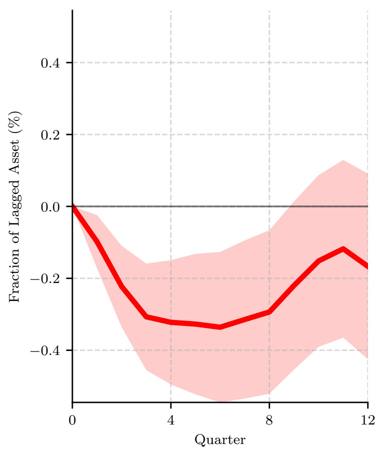
Note: Aggregate net equity- and debt-financing flows of nonfinancial U.S. firms normalized by (lagged) total assets. The impulse-response function is based on Equation (II.1) and the sample spans 1970Q1–2007Q2. The shaded area indicates the one standard error (68%) confidence interval of the estimates.

The effect of monetary policy on aggregate financing flows

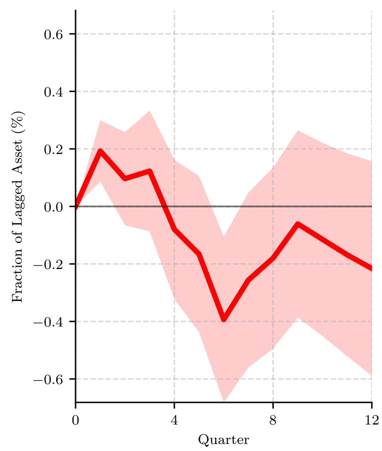
Figure II.1 shows the response of net equity and debt flows to a contractionary monetary-policy shock. Both sources of financing contract following an increase in monetary policy rates, with a peak response between 4 and 8 quarters after the shock, which is consistent with the contraction of firms' investment, which we document in Appendix Figure II.6. The main takeaway from this exercise is that equity-flow contraction following an increase in interest rates is of the same magnitude as that of debt flows, around 0.4% of assets or close to one-standard deviation of these flows. This result suggests that equity financing can be as relevant as debt financing for understanding firm financing after monetary-policy changes.

Additional results

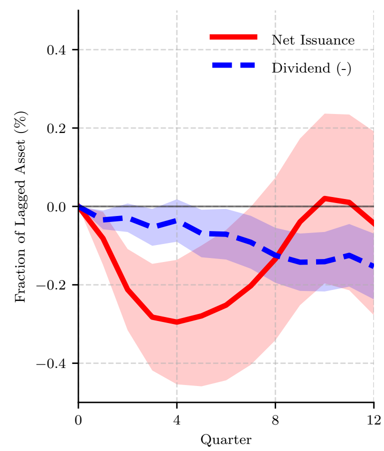
Figure II.2 shows that the decrease in equity flows following a contractionary monetary stimulus is apparent across different sectors and types of flows. First, Panel (a) and (b) of Figure II.2 show that the contraction in equity flows following a monetary contraction is observed both in the noncorporate and corporate sectors, although it is more pronounced in the latter. Second, Panel (c) shows that for corporate-sector equity financing, the response



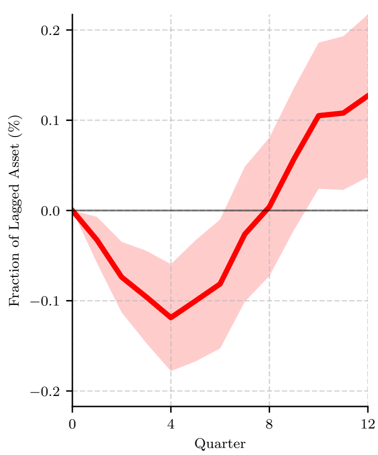
(a) Net Flow, Corporate



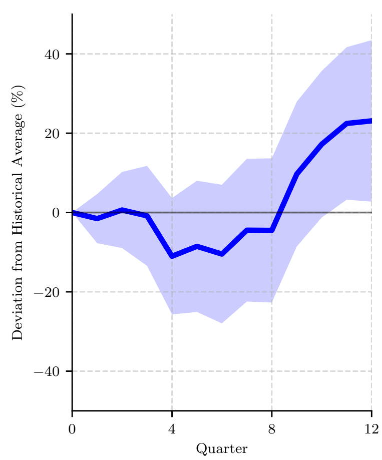
(b) Net Flow, Noncorporate



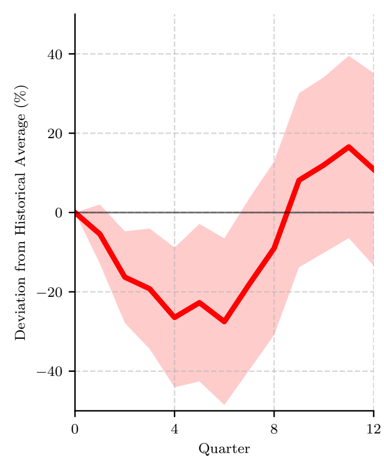
(c) Net Issuance vs. Dividends, Corporate



(d) Gross Issuance, Corporate



(e) IPO Count, Corporate



(f) SEO Count, Corporate

Figure II.2. Response of Equity Flows to Contractionary Monetary Shocks

Note: In Panel (a), the net equity-financing flow is defined as the net change in proprietors' equity, and it is normalized by the lagged total assets of U.S. nonfinancial noncorporate businesses. In Panels (b) to (f), the flows are normalized by the lagged total assets of U.S. nonfinancial corporate businesses. In Panel (d), gross equity issuance includes the common and preferred equities issued by U.S. nonfinancial corporations, downloaded from `http://pages.stern.nyu.edu/~jwurgler/main.htm`. In Panels (e) and (f), the total number of IPOs and SEOs are from `https://site.warrington.ufl.edu/ritter/ipo-data/`. The numbers of IPOs and SEOs are normalized by their quarterly average during the sample period, 82 and 83. The impulse response function is based on the regression specified as Equation (II.1), and the sample spans from 1970Q1 to 2007Q2 to focus on the conventional monetary policies. The shaded area indicates the one standard error (68%) confidence interval of the estimates.

of dividend payments is relatively modest. Most of the response in equity flows following monetary-policy changes are driven by net issuance. Panel (d) shows that gross equity issuance significantly contracts following monetary-policy tightening. Panels (e) and (f) show that in comparison to IPOs, SEOs respond more significantly to the monetary shock.[2]

II.2.2 Stock price drops in equity issuance and monetary policy

The second part of our empirical work builds on the large literature in corporate finance that documents stock-price drops following the release of information on firms' equity issuance, a fact often interpreted as evidence of the relevance of informational frictions in equity markets (e.g. Masulis and Korwar, 1986; Asquith and Mullins, 1986; Myers and Majluf, 1984). We extend this set of findings by using data on stock-price dynamics around issuance events to show that monetary-policy stimulus mitigates this price drop.

Data and descriptive statistics

Our analysis of stock-price dynamics around issuance events follows standard practices in the literature, combining data from multiple sources. First, we extract SEO-deal issuance dates from SDC Platinum. For each deal, we focus on the filing date provided by SDC, that is, the date when the issuance is filed with the SEC for the first time. For shelf-registered deals, the filing date is the first filing date of the original shelf registration. For deals without shelf registration, the filing date is the launch date, which is date when the registration of the offering was filed. When the launch date is unavailable, the announcement date is used. For a more detailed description of the variable definitions, see `http://mergers.thomsonib.com/td/DealSearch/help/nidef.htm`.

Our sample of deals starts in 1983Q1, when the filing-date data become available, and ends in 2007Q2, which is consistent with the empirical exercise in the previous section. For this period, we obtain a sample of over 3,800 issuance events. SDC also contains useful information about deal characteristics, which we use in the empirical analysis: the size of the issuance (filed amount of issuance), type of shares (only primary shares, only secondary shares, or a combination of primary and secondary shares), and whether the issuance is shelf

[2]The IPO and SEO numbers are from `https://site.warrington.ufl.edu/ritter/ipo-data/` and constructed mostly based on SEC filing records. Compared with these time series, Thomson and Reuters' Security Data Company (SDC) has less comprehensive coverage of the IPO and SEO deals, especially during the early 1970s. A drawback of these time series is that they do not exclude financial firms' issuances. As a robustness check, we aggregate the IPOs and SEOs from nonfinancial firms in SDC and repeat the same analysis. The responses of both the aggregate flows and number of issuance look similar to the results shown in Figure II.2. The details can be found in Appendix II.5.2.

registered under SEC Rule 415. Appendix II.5.1 provides more details on the sample of deals used in the analysis.

Our main variable of interest in the empirical analysis is the stock-price change around each SEO filing event, which we extracted from CRSP and link to the SDC events through CUSIP codes and ticker symbols. For a given issuance event i by firm j at date t, we measure the cost of equity issuance through the cumulative abnormal return ($\mathcal{CAR}$) within the event window $[t-\delta^-, t+\delta^+]$, $\mathcal{CAR}_{ijt} \equiv \sum_{\tau=t-\delta^-}^{t+\delta^+} \mathcal{AR}_{j\tau}$, where δ^- and δ^+ define the event-window width, and $\mathcal{AR}_{j\tau}$ is the daily abnormal return for a firm j's that issues at date τ. The abnormal return, in turn, follows a standard construction given by $\mathcal{AR}_{j\tau} \equiv (\mathcal{R}_{j\tau} - \mathcal{R}^f_\tau) - \alpha_j - \beta_j \left(\mathcal{R}_{\texttt{S\&P500},\tau} - \mathcal{R}^f_\tau\right)$, where $\mathcal{R}_{j\tau}$ denotes the stock market return of firm j on date τ, $\mathcal{R}^f_\tau$ denotes the risk-free rate, measured by the effective Federal Funds rate, and $\mathcal{R}_{\texttt{S\&P500},\tau}$ denotes the return on the S&P 500 index at date τ. The coefficients α_i and β_i are estimated by running the one-factor model $\mathcal{R}_{j\tau} - \mathcal{R}^f_\tau = \alpha_j + \beta_j \left(\mathcal{R}_{S\&P500,\tau} - \mathcal{R}^f_\tau\right) + \varepsilon_{j\tau}$ over the period $[t+\Delta^-, t+\Delta^+]$, where we set $\Delta^- = 10$ and $\Delta^+ = 160$ following Choe, Masulis and Nanda (1993).

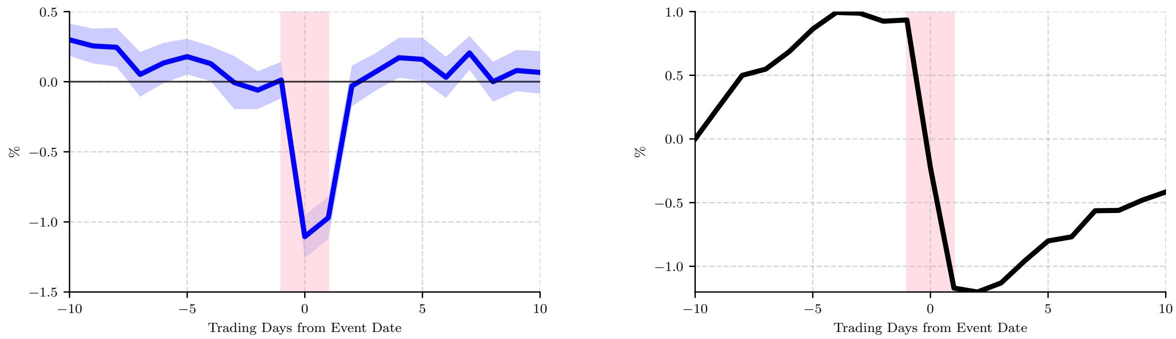
Figure II.3 illustrates the average stock price dynamics experienced by firms that file for equity issuance in our sample. Panel (a) shows that, on average, the stock price of issuing firms displays a significant drop within a narrow window around the filing date, which validates our use of the filing date forour event study. Panel (b) shows that, on average, the stock price drops by about 2% between the day before and the day after the filing date, which is consistent with the evidence provided in the previous literature (e.g., Autore, Kumar and Shome, 2008). Appendix Table II.4 provides more descriptive statistics on the price dynamics associated with filing events, showing that 65% of the events are characterized by negative abnormal returns.

Finally, for each filing event, we collect balance-sheet data about the issuing firm from Compustat, which we use as controls to measure the observable characteristics of issuing firms. These variables include leverage, sales growth, size, and industry. For details regarding these variables see Appendix II.5.1.

Empirical model and results

We use the following empirical model to study the effect of monetary policy on the abnormal returns associated with equity issuance:

$$\mathcal{CAR}_{ijt} = \alpha + \beta v_t^{\mathrm{m}} + \mathbf{\Gamma}'_Y Y_t + \mathbf{\Gamma}'_X X_{ijt} + \varepsilon_{ijt}. \quad \text{(II.2)}$$



(a) Abnormal Return (b) Cumulative Abnormal Return

Figure II.3. Average Abnormal Stock Price Movement Around Filing Date

Note: In Panel (a), the solid line is the average abnormal return on each event date, and the shaded area around the solid line is the one-standard-error bar. Panel (b) documents the average cumulative abnormal return. For each issuance, the cumulative abnormal return on each event date is the cumulative sum of the abnormal return from 10 days before the event date. In both panels, the vertical shaded area indicates the window from the day before the event date to the day after the event date. The sample period is 1983Q1 to 2007Q2.

In (II.2) v_t^{m} denotes the sum of the monetary policy shocks between $t-90$ and t, Y_t denotes a vector of macroeconomic controls for time t, including calendar year fixed effects, as well as the growth rate of GDP, the inflation rate, the unemployment rate, the effective Federal funds rate, and the one-quarter lagged average stock market return and volatility. X_{ijt} denotes a vector of deal-firm controls typically included in the literature to absorb firm-specific sources of stock price movements around the issuance events, such as firm characteristics and deal characteristics. For details regarding the variables included in the regression, see Appendix II.5.1. Our coefficient of interest, β, measures how a one standard deviation monetary-policy shock affects the cumulative abnormal return associated with filing events.

Table II.2 shows the results from estimating II.2 across different specifications using the Romer and Romer (2004) monetary shocks. Column (1) shows that an unexpected decrease of 1% in interest rates in the quarter before the deal reduces the cumulative abnormal return by 1.15%. Given the average cumulative abnormal return of −1.88%, this result implies that a one-standard deviation shock to monetary policy (35 basis points) reduces the average effect by 21%. Columns (2) and (3) show that these effects are stronger when we control for the macroeconomic conditions and the firm–deal characteristics. Overall, the results shown in Table II.2 indicate that monetary stimulus can significantly mitigate the price drops associated with equity issuance.

Table II.2. Response of Cumulative Abnormal Returns around Filing Date to Monetary Shocks

	(1)	(2)	(3)
Monetary shock (v_t^{m})	-1.15*	-1.27**	-1.38**
	(0.65)	(0.64)	(0.63)
Aggregate Controls	No	Yes	Yes
Firm-deal Controls	No	No	Yes
R^2	0.010	0.013	0.031
Observations	3896	3896	3452

Note: The sample period starts with 1983Q1 and ends with 2007Q2. The monetary shocks at each FOMC meeting date are constructed following Romer and Romer (2004). The exposure of each date t to the monetary shocks v_t^{m} is measured as the sum of the monetary shocks on the FOMC meeting dates between $t-90$ and t. Standard errors are clustered by calendar year. *, **, *** indicate significance levels at 0.1, 0.05, and 0.01.

II.3 Model

II.3.1 Environment

Time is discrete, and there are two periods, $t = 0, 1$. The economy is populated by households and firms. There are two types of goods, final goods and capital goods, and two types of financial securities, stocks and bonds.

Preferences, Technologies, and Information Households have preferences over consumption of the final good given by

$$c_0 + \beta c_1 \tag{II.3}$$

At the beginning of period 0, households are endowed with y_0 units of the final good and with the ownership of firms, which takes the form of shares. Firms are endowed with k_0 units of the capital good, which we can think of as assets in place. They have access to a technology to accumulate capital $k_{j1} = a_j k_0 + i_j$, where $a_j > 0$ denotes the quality of assets in place for firm j, and i_j denotes investment in terms of final goods of firm j. This investment occurs in projects of fixed size, which we normalize to one, so $i_j \in (0, 1)$. We assume that a_j is private information of firm j. Its distribution is public information, with p.d.f. denoted by $f(a_j)$. For analytical purposes, we assume a_j is uniformly distributed over $[0, \bar{a}]$ with $\bar{a} > 0$, for all $j \in [0, 1]$. In period 1, firms have access to a linear technology to produce final goods $y_{j1} = Zk_{j1}$ where Z is an exogenous productivity term, which is public information and known in period 0.

Financial securities Two types of financial securities are traded in competitive markets: stocks and bonds. Stocks are issued by firms, and they specify the share of profits, $1/s$, transferred in period 1 to the shareholder. The economy also features risk-free bonds that are only traded by households. Purchasing a unit of the risk-free bond in period 0 gives its holder R units of the final good in period 1.

II.3.2 Optimization

Firms' problem To finance their investment projects, firms have access only to equity issuance. We denote by x the mass of shares that firms must issue to finance the (unit) cost of an investment project. Formally, x is defined as $x = \frac{1}{\mathcal{P}(1)}$, where $\mathcal{P}(1) > 0$ is the stock price of a firm that is issuing equity to finance its investment project. As firms take the price $\mathcal{P}(1)$ as given, we can characterize the firm's problem in terms of x for $\mathcal{P}(1) > 0$. We refer to x as the cost of issuing equity. Because we normalize the mass of shares held by initial shareholders to one, if a firm issues a unit of equity at the cost of x, initial shareholders maintain a fraction $\frac{1}{1+x}$ of the dividends in period 1, while investors that participate in the equity issuance receive a fraction $\frac{x}{1+x}$ of the dividends.

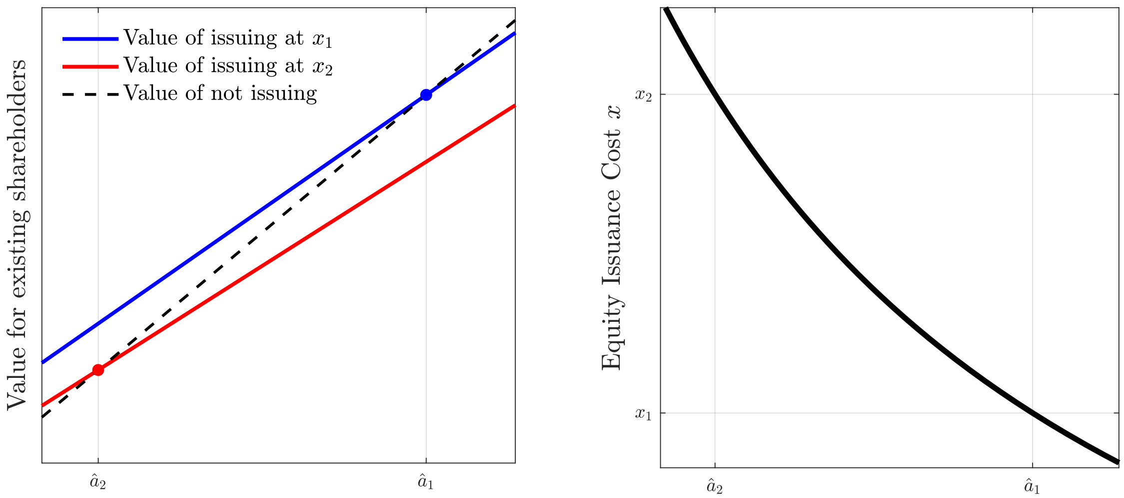
In period 0, taking the cost of issuing the equity contract, x, as given, firms choose investment to maximize their current shareholders' expected dividends:

$$\max_{i \in \{0,1\}} \beta \left[i \frac{1}{1+x} Z \left[ak + i\right] + (1-i) Zak \right]. \tag{II.4}$$

Denoting the policy function associated with problem (II.4) as $\iota(a)$, we define the set of firms who are willing to take the equity contract as $\mathcal{I} \equiv \{a \in [0, \bar{a}] : \iota(a) = 1\}$. Given any equity issuance cost, x, firms of type a are willing to take the equity financing contract if and only if

$$\frac{1}{1+x} \left[ak + 1\right] \geq ak. \tag{II.5}$$

As illustrated in panel (a) of Figure II.4, the value of issuing is greater than that of not issuing for $a = 0$, $\forall x > 0$. However, the slope of issuing increases with a by less than that of not issuing, because firms only keep part of their assets in place when issuing new equity. Therefore, firms' optimal policies can be characterized by a cutoff rule. If $a \leq \hat{a} \equiv \dfrac{1}{xk}$, the firms take the contract and invest, so $\mathcal{I} = [0, \hat{a}]$. Next, panel (b) in Figure II.4 shows that when x increases from x_1 to x_2, the value for existing shareholders if the firm chooses to issue will be shifted downward as shown in panel (a). Then the cutoff $\hat{a}$ will decrease from $\hat{a}_1$ to $\hat{a}_2$, which reflects that as the cost of equity issuance increases, firms with a high outside option find it less profitable to invest. Note that monetary policy only affects this policy through the cost of issuing equity x, which is taken as given by firms.



(a) Value of Issuing vs. Not Issuing

(b) Cutoff Type and Equity Price

Figure II.4. Firms' Optimality

Note: In Panel (a), the solid lines depicts the value for existing shareholders of firms as a function of capital quality a for given equity issuance costs x_1 and x_2, with $x_2 > x_1$. The blue solid line corresponds to the equity issuance cost, x_1, and the red solid line corresponds to the equity issuance cost x_2. The dotted line depicts the value of not issuing equity. Panel (b) shows how the cutoff type, $\hat{a}$, varies with the equity issuance cost x.

Households' problem The budget constraints for the representative household are given by:

$$c_0 + \int_{j\in[0,1]} s_j \mathcal{P}(i_j)\mathrm{d}j + b_1 = \int_{j\in[0,1]} (div_{j0} + \mathcal{P}(i_j))\mathrm{d}j + y_0 \tag{II.6}$$

$$c_1 = \int_{j\in[0,1]} s_j \frac{div_{j1}}{1 + i_j x_j}\mathrm{d}j + Rb_1 \tag{II.7}$$

where $s_j \geq 0$ denotes the purchase of firm-j's shares in period 0 (claims for dividends distributed in period 1), $\frac{div_{j1}}{1+i_j x_j}$ denotes dividends per share distributed by firm j in period 1, and b denotes the purchase of risk-free bonds. The household's problem is to choose $\{s, b\}$ to maximize expected utility, (II.3), subject to (II.6) and (II.7). Because the entrepreneurs only produce in period 1, we have $div_{j0} = 0$ and $div_{j1} = \iota(a_j) \cdot Z \cdot (ak+1) + (1-\iota(a_j)) \cdot Z \cdot ak$.

Thus, consumers' optimal portfolio choices imply that for any firm with $s_j > 0$:

$$\frac{\frac{1}{1+x}\mathbb{E}\left[Z\left(ak+1\right)|a \in \mathcal{I}\right]}{\mathcal{P}(1)} = \frac{\mathbb{E}\left[Z\left(ak\right)|a \notin \mathcal{I}\right]}{\mathcal{P}(0)} = R. \tag{II.8}$$

In addition, the optimal choice of bonds implies that, in any interior solution, $R = \frac{1}{\beta}$.

II.3.3 Equilibrium

We assume that $\beta Z \geq 1$, that is, the investment project has a non-negative net present value. Otherwise an equilibrium with firms issuing equity cannot exist, and the market shuts down. If this condition holds, an equilibrium is defined as follows.

***Definition* 1** An equilibrium is a set of policies $(i_j)_{j\in[0,1]}$, allocations $\{(s_j)_{j\in[0,1]}, b\}$ and prices $\{R, x\}$ such that

1. Given prices, firms' policies solve the their optimization problem (II.4), allocations solve household' maximization of (II.3) subject to (II.6) and (II.7).

2. Equity and bond markets clear, i.e., $s_j = 1 + i_j x$ for all j, and $b = 0$.

Next, the following result characterizes the equilibrium cost of issuing equity:

***Lemma* 1** The equilibrium cost of issuing equity, x^*, satisfies

$$\frac{x^*}{1+x^*}\beta \cdot Z\left(\mathbb{E}[a|a \leq \hat{a}(x^*)] \cdot k + 1\right) = 1 \tag{II.9}$$

Proof. See Appendix II.5.3. ∎

We now provide conditions under which the economy features a separating equilibrium, which can be linked to our empirical results in Section II.2.2.

***Proposition* 1** The economy can display two types of equilibrium:

1. **Separating equilibrium**: If $\beta Z \leq \dfrac{\bar{a}k+1}{\frac{\bar{a}}{2}k+1}$, the equilibrium issuance cost is $x^* = \frac{\beta Z - \frac{1}{2}}{1-\beta Z}$. In this equilibrium, only the firms with $a \leq \hat{a}^* \equiv \frac{1-\beta Z}{(\beta Z - \frac{1}{2})k}$ choose to issue equity, where $\frac{\partial x^*}{\partial \beta} < 0$, $\frac{\partial x^*}{\partial Z} < 0$, $\frac{\partial \hat{a}^*}{\partial \beta} > 0$, and $\frac{\partial \hat{a}^*}{\partial Z} > 0$.

2. **Pooling equilibrium**: If $\beta Z > \dfrac{\bar{a}k+1}{\frac{\bar{a}}{2}k+1}$, all firms choose to issue equity and the equilibrium issuance cost is $x^* = \dfrac{\beta Z}{\frac{\bar{a}}{2}k+1-\beta Z}$, where $\frac{\partial x^*}{\partial \beta} < 0$ and $\frac{\partial x^*}{\partial Z} < 0$.

Proof. See Appendix II.5.3. ■

Next, we show that in the separating equilibrium, firms that issue equity experience a drop in the valuation of their businesses, as documented in Section II.2.2.

***Proposition* 2** In the separating equilibrium, the abnormal return associated with equity issuance is negative, i.e. $\mathcal{AR} \equiv \ln \mathcal{P}(1) - \ln \mathcal{P}(0) < 0$.

Proof. See Appendix II.5.3. ■

The intuition for this result is simple. Given that only firms with sufficiently low quality of capital are willing to issue, any issuance signals lower capital quality, which is reflected in the stock price upon issuance.

II.3.4 The Effects of Monetary Policy

The next result shows that in our model, an increase in the risk-free rate leads to a reduction in the price drop for firms that issue equity, consistent with the evidence of shrinkage of cumulative abnormal returns after a monetary policy shock, documented in Section II.2.

***Proposition* 3** In the separating equilibrium, the abnormal return associated with equity issuance increases with the discount rate, i.e. $\frac{\partial \mathcal{AR}}{\partial \beta} > 0$.

Proof. See Appendix II.5.3. ■

Figure II.5 illustrates the intuition behind Proposition 3. The solid lines in the left panel depict for a given $\hat{a}$, the equity issuance cost, x, that makes households indifferent between investing in equity and debt, which offers the risk-free rate of return, R. This upward-sloping relationship implies that when the risk-free rate is higher, households require more shares to participate in equity investment, for a given fixed pool of firms who participate in the equity markets. As in Figure II.4, the right panel again shows the capital quality threshold, $\hat{a}$, below which firms issue new equity at a given equity issuance cost x. The initial equilibrium is represented by the point (R_0, x_0) in the left panel, and by the point $(\hat{a}_0, x_0)$ in the right panel.

Consider now an exogenous increase in the discount rate, β, which leads to a decline in the risk-free rate from R_0 to R^*. Fixing the capital quality threshold, $\hat{a}$, this decrease in the interest rate implies a movement along the original (topmost) indifference curve in the left panel from x_0 to x_1. In the right panel, this decline in equity issuance cost leads to an increase in the capital quality threshold, from $\hat{a}_0$ to $\hat{a}_1$. At this new point, firms with higher capital quality are now willing to take equity contracts, so the expected quality of

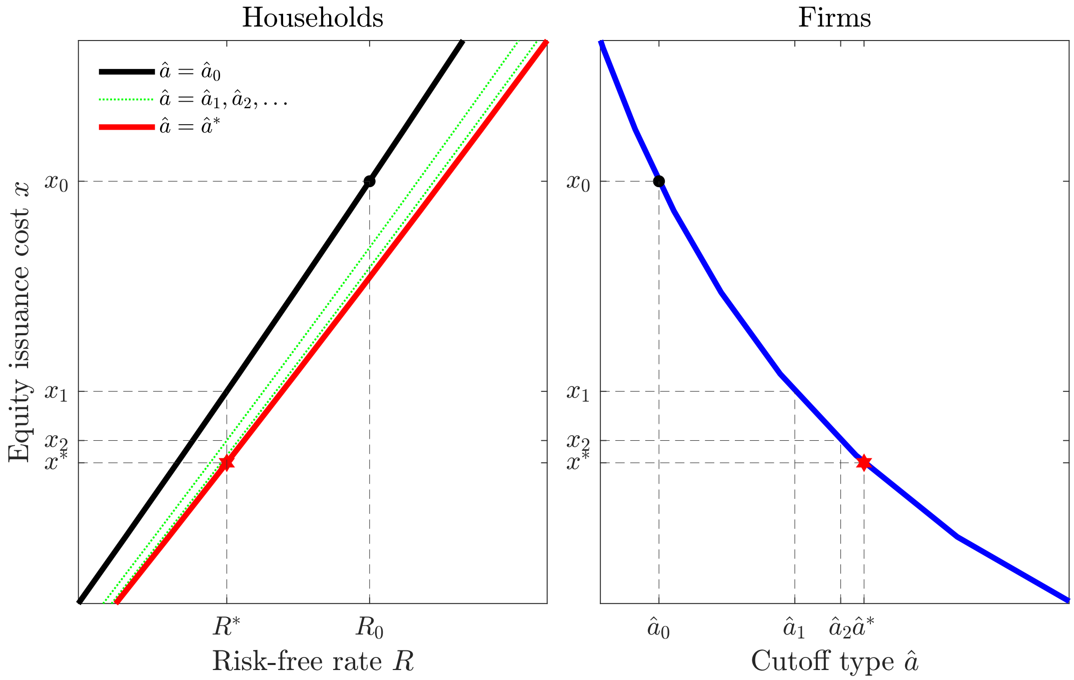


Figure II.5. Monetary Transmission Through Equity Markets

Note: In the left panel, the solid lines depict the equity issuance cost that makes households indifferent between debt and equity at different levels of the gross interest rate, R, for a given $\hat{a}$. The line in the right panel depicts the equilibrium relation between th cutoff type $\hat{a}$ and the equity issuance cost x.

firms that issue equity is higher. This positive selection leads to a shift of the indifference curve in the left panel to the southeast, from the initial black solid line to the dotted green line. The result is that for any level of the interest rate, the equity issuance cost required by the households is lower because they get additional compensation from higher expected profits generated by relatively higher capital quality. The decrease in the equity issuance cost in turn leads to an additional increase in the capital quality threshold, which further displaces the households' indifference curve. We refer to this novel amplification mechanism of monetary policy as "amplification through selection." Better firms choose to participate in equity contracts, which leads to decreases in the premium of equity issuance and further increases in the capital quality of firms that are willing to participate in the equity market.

II.4 Conclusion

In this paper, we study how the transmission of monetary policy to the real economy is shaped by equity financing. In the data, equity flows are an important component of firms' responses to changes in monetary policy. Furthermore, monetary policy expansions reduce the price drops associated with the announcements of equity issuance, suggesting that monetary stimulus induce firms with better quality to participate in equity markets. Motivated by this evidence, we constructed a model in which firms finance investment by issuing equity under asymmetric information. The model is consistent with the patterns observed in the data. Moreover, it shows that informational frictions amplify the response of investment to monetary policy, through a process of selection in which firms with the better-quality projects are selected to participate in the market. This selection then reduces the asymmetric information penalty. This channel implies a scope of policy linked to stabilizing informational frictions over the business cycle. We plan now to extend the model to a quantitative environment, which allows us to quantify the relevance of this amplification channel through equity financing.

II.5 Appendix: Additional Details

II.5.1 Data Appendix

Variables used in the macro-level analysis

1. *Net equity-financing flows:* These variables are obtained from the *Financial Accounts of the United States*. For corporate firms, they equal net equity issuance (measured by `FA103164103.Q`) net of dividend payments (measured by `FA106121075.Q`). For noncorporate firms, they equal the net change in proprietors' equity in noncorporate businesses (measured by `FA112090205.Q`). At the aggregate level, they equal the sum of flows of the corporate and noncorporate business.

2. *Net debt-financing flows:* These variables are obtained from the *Financial Accounts of the United States*. For corporate firms, they equal the sum of net debt-security issuance (`FA104122005.Q`) and loan issuance (`FA104123005.Q`). For noncorporate firms, they equal net loan issuance (`FA114123005.Q`). At the aggregate level, they equal 5, the sum of the corporate and noncorporate flows.

3. *Gross equity issuance and retirement, corporate:* These variables are obtained from the Federal Reserve Board. Gross equity issuance equals the value of funds raised through the sale of equity by publicly and privately held nonfinancial firms. The IPO series measures the funds raised by new public equity offerings. The SEO series captures the funds raised from new equity issuance by firms that are already traded publicly at the time of issuance. Equity retirements measure the value of nonfinancial firms' equity that is retired each quarter. The channels of retirement captured by this series comprise equity repurchases and retirements through mergers and acquisitions (M&A). Repurchases represent the value of equity repurchased by public nonfinancial firms through share buyback programs. Equity retirements through M&A activity measure the value of cash-financed transactions by domestic acquirers plus the value of both cash- and equity-financed transactions by foreign acquirers.

4. *Gross equity issuance, corporate*: This variable is obtained from Baker and Wurgler (2000)[3]. It includes the issuance of common and preferred stocks, but excludes private placements.

5. *Number of IPOs and SEOs:* These variables are obtained from Jay Ritter's website. The monthly number of IPOs is downloaded from `https://site.warrington.ufl.`

[3] `http://www.stern.nyu.edu/~jwurgler/data/Equity_Share_3.xls`

edu/ritter/files/2019/01/IPOALL_2018.xlsx, and the detailed description of the data construction can be found in Ibbotson, Sindelar and Ritter (1994). The monthly number of SEOs is downloaded from https://site.warrington.ufl.edu/ritter/files/2017/01/SEO-Monthly-Counts-70-15.pdf. We aggregate these variables to quarterly time series.

6. *IPO and SEO issuance*: Aggregated from SDC with four restrictions. First, firms must be incorporated in the United States. Second, the currency must be U.S. dollars. Third, the exchange must be either "NASDAQ", "New York", "American" or "OTC." Fourth, the SIC code must not be between 6000 and 6999.

Variables used in the micro-level analysis

- Macro controls (Y_t):

  1. *GDP growth rate:* Fred A191RL1Q225SBEA.
  2. *Inflation:* Constructed from the GDP deflator (Fred GDPDEF).
  3. *Unemployment rate:* Aggregated from the monthly unemployment rate (Fred UNRATE).
  4. *Effective federal funds rate:* Aggregated from the monthly effective federal funds rate (Fred FEDFUNDS).
  5. *Average stock market return and volatility:* Aggregated from the daily history of the S&P500 index (WRDS-CRSP dsp500).

- Firm–deal controls (X_{ijt}):

  1. *Leverage:* The one-quarter lagged ratio of total liabilities (Compustat ltq) to total assets (Compustat atq).
  2. *Sales-growth dummies:* Dummies for whether firm i's one-period lagged sales (Compustat saleq) growth rate is within the top 25% or bottom 25% of Compustat's industrial-firm population in the same quarter.
  3. *Size dummies:* Dummies for whether firm i's one-period laggedassets (Compustat atq) fall within the top 25% or bottom 25% Compustat's industrial-firm population in the same quarter.
  4. *Industry dummy:* Dummies based on the Fama and French five-industry categorization.

5. *Issuance-type dummy:* Dummies for whether the issuance has only secondary shares or whether the issuance includes both the primary and secondary shares.

6. *Shelf-registered-issuance dummy:* Dummy for whether the issuance is shelf-registered under SEC rule 415.

7. *Issuance size:* Ratio of the filed size to the one-quarter laggedtotal assets of firm i (Compustat `atq`).

- Conversion between nominal and real values: All nominal values are converted to real terms using the quarterly PPI (Fred `PIEAMP01USQ661N`).

II.5.2 Additional Tables and Figures

Table II.3. Descriptive Statistics post-1984: U.S. Nonfinancial Firms' Financial Flows

Variable	$\mathbb{E}(X)$ (%)	$\sigma(X)$ (%)	$\rho(X, GDP)$
Equity flows	-1.52	0.35	-0.13
Debt flows	1.17	0.81	0.17
Total external financing flows	-0.36	0.64	0.15

Note: $\mathbb{E}(X)$, $\sigma(X)$, and $\rho(X, GDP)$ denote for variable X, the mean, standard deviation, and correlation with real GDP. Statistics reported correspond to quarterly data for the period 1970–2016, except for Panel (c), which corresponds to the 1996–2016 period because of data availability. Panel (a) refers to data from corporate and noncorporate sectors. Data source: *Financial Accounts of the United States*. Flows are normalized by (lagged) total assets of the corresponding sector. Standard deviations and correlations are computed with detrended data using the filter from Baxter and King (1999).

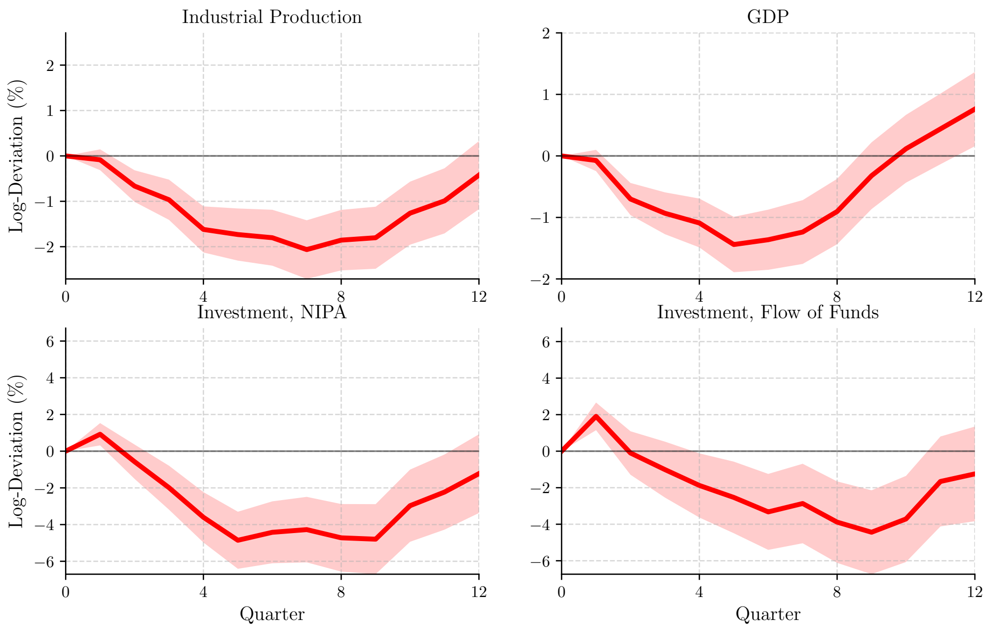


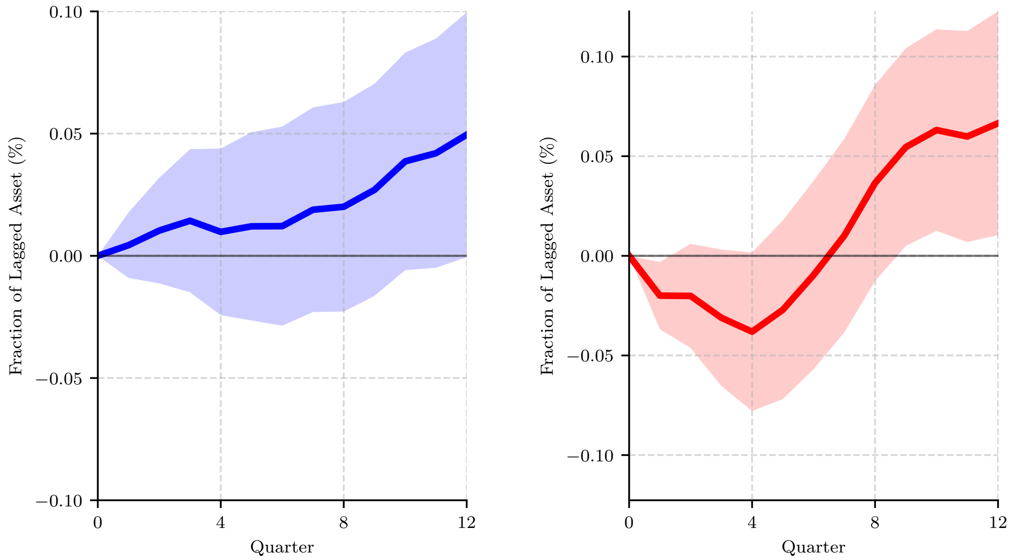
Figure II.6. Replication and Extension of Romer and Romer (2004)

Note: Aggregate net equity- and debt-financing flows of nonfinancial U.S. firms normalized by (lagged) total assets. The impulse-response function is based on Equation (II.1) and the sample spans 1970Q1–2007Q2. The shaded area indicates the one standard error (68%) confidence interval of the estimates.

Table II.4. Summary Statistics for Price Changes: Unconditional Moments

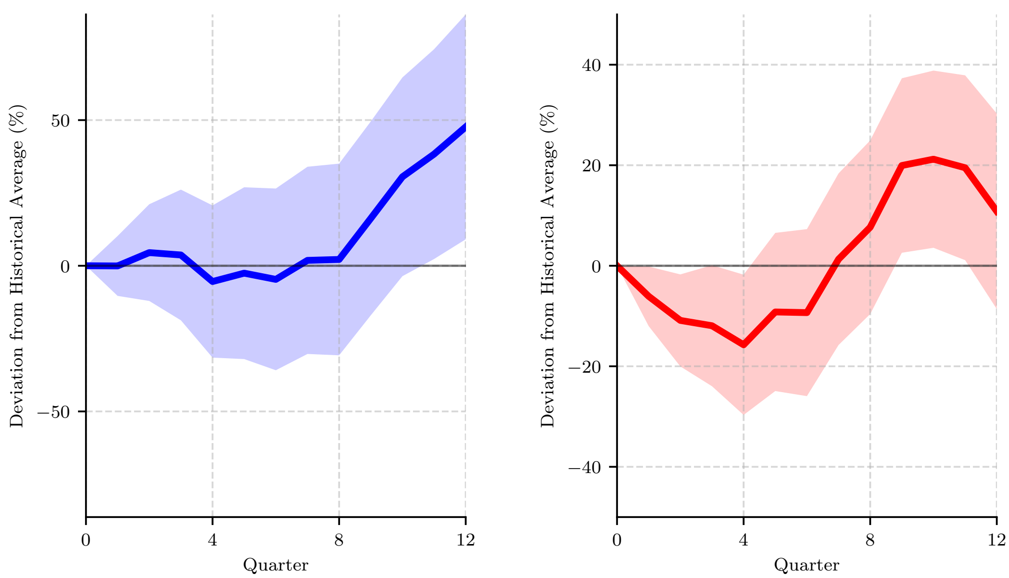
Cumulative Abnormal Return (%) between $t-1$ and $t+1$	
Median	-1.75
Mean	-1.88
Std	6.38
Pr(≤ 0) (%)	64.58
Obs	3896
Cumulative Return (%) between $t-1$ and $t+1$	
Median	-1.80
Mean	-2.04
Std	6.54
Pr(≤ 0) (%)	67.12
Obs	3896

Note: The sample period is 1983Q1–2007Q2.



(a) Aggregate Flow of IPO

(b) Aggregate Flow of SEO



(c) Total Number of IPO

(d) Total Number of SEO

Figure II.7. Response of Public Equity Issuance In SDC to Contractionary Monetary Shocks

Note: In Panels (a) and (b), the IPO and SEO flows are aggregated from SDC with the sample restricted to deals issued by U.S. nonfinancial, non utility and non quasi-governmental firms and traded on the NASDAQ, NYSE, American Exchange, or OTC . The aggregate flows are normalized by the lagged total assets of U.S. nonfinancial corporate firms. The total number of IPOs and SEOs used in Panels (e) and (f) are aggregated from SDC, with the same restrictions used to construct the sample for panels (a) and (b). The numbers of IPOs and SEOs are normalized by their historical averages, which are 52 and 71 per period during the sample period. The impulse-response function is based on the regression specified as Equation (II.1) and the sample period is 1970Q1–2007Q2. The shaded area indicates the one standard error (68%) confidence interval of the estimates.

Table II.5. Response of Cumulative Abnormal Return around Filing Date to Monetary Shocks (High-Frequency)

	(1)	(2)	(3)	(4)
Monetary shock (v_t^{m})	-1.48***	-1.26***	-1.34***	-1.10*
	(0.32)	(0.31)	(0.45)	(0.61)
Aggregate Controls	No	Yes	Yes	Yes
Firm-deal Controls	No	No	Yes	Yes
Year Fix Effects	Yes	Yes	Yes	No
R^2	0.010	0.013	0.041	0.028
Observations	2542	2542	2286	2286

Note: The sample period is 1994Q1–2007Q2. The monetary shocks at each FOMC meeting date are constructed following Gorodnichenko and Weber (2016), and the exposure to the monetary shocks v_t^{m} is measured as the sum of the monetary shocks on ninety days preceding each FOMC meeting date. To make the results comparable to the results in Table II.2, the monetary shocks are normalized so that a 1% increase in the monetary shock is associated with the same magnitude of the actual change in federal funds rate, relative to a 1% increase in the monetary shocks constructed following Romer and Romer (2004). The standard errors are clustered by calendar years. *, **, *** represent significance levels of 0.1, 0.05, and 0.01, respectively.

II.5.3 Model Appendix

Proof of Lemma 1 Plug $\mathcal{I} = \left\{a : a \leq \frac{1}{xk}\right\}$ and $R = \frac{1}{\beta}$ into (II.8), we can get the result directly.

Proof of Proposition 1 We first find the necessary condition for the existence of a separating equilibrium. Given the equilibrium cutoff, denoted by $\hat{a}^* \in [0, \bar{a}]$, the equilibrium equity issuance cost, $x^* > 0$, satisfies $x^* = \frac{1}{\hat{a}^* k}$. Plugging this last expression into equation (II.9), we obtain:

$$\beta Z \left[\frac{\hat{a}^*}{2}k + 1\right] = 1 + \hat{a}^* k \quad \Rightarrow \hat{a}^* = \frac{\beta Z - 1}{\left(1 - \frac{1}{2} \cdot \beta Z\right) k}.$$

Because we assume that in equilibrium, $\hat{a}^* \in [0, \bar{a}]$, βZ has to satisfy

$$0 \leq \frac{\beta Z - 1}{\left(1 - \frac{1}{2}\beta Z\right) k} \leq \bar{a}$$

When $\beta Z \geq 2$, $\beta Z - 1 > 1$ and $\left(1 - \frac{1}{2}\beta Z\right) k < 0$, then the above inequality cannot hold. When $1 < \beta Z < 2$ and $\left(1 - \frac{1}{2}\beta Z\right) k > 0$, the above inequality can be transformed into

$$\beta Z - 1 \leq (1 - \frac{1}{2}\beta Z) \cdot \bar{a}k \Rightarrow \quad \beta Z \leq \frac{\bar{a}k + 1}{\frac{\bar{a}}{2}k + 1} \Rightarrow \quad \beta Z \leq \frac{\bar{a}k + 1}{\frac{\bar{a}}{2}k + 1}$$

Under this condition, the separating equilibrium exists and the equilibrium issuance cost is $x^* = \frac{1}{a^* k} = \frac{\beta Z - \frac{1}{2}}{1 - \beta Z}$.

As shown in the above steps, when $\beta Z > \frac{\bar{a}k+1}{\frac{\bar{a}}{2}k+1}$, no separating equilibrium exists. In this case, if the equilibrium is a pooling equilibrium, that is, an equilibrium with $\hat{a}^* > \bar{a}$, we can plug $\hat{a}^* = \frac{1}{x^* k}$ into the equilibrium condition (II.9) to derive:

$$\frac{1}{\hat{a}^* k + 1} \cdot \beta Z \left[\frac{\bar{a}}{2}k + 1\right] = 1 \quad \Rightarrow \quad \hat{a}^* = \frac{\beta Z \left[\frac{\bar{a}}{2}k + 1\right] - 1}{k}$$

Because $\hat{a}^* > \bar{a}$ in the pooling equilibrium, we have

$$\beta Z > \frac{\bar{a}k + 1}{\frac{\bar{a}}{2}k + 1}.$$

Therefore, when $\beta Z > \frac{\bar{a}k+1}{\frac{\bar{a}}{2}k+1}$, only a pooling equilibrium can exist, and the equilibrium equity contract is $x^* = \frac{1}{\beta Z\left[\frac{\bar{a}}{2}k+1\right]-1}$.

Proof of Proposition 2 Because the equilibrium condition II.9 can be rewritten as

$$\beta Z = \frac{x^*}{1+x^*} \cdot \left(\frac{\hat{a}^*}{2}k + 1\right) = \frac{1}{1+\hat{a}^* k} \cdot \left(\frac{\hat{a}^*}{2}k + 1\right),$$

and because $x^* = \frac{1}{a^* k}$, the abnormal return associated with equity issuance can be derived as:

$$\mathcal{AR} = \ln\left[\frac{2\hat{a}^*}{\hat{a}^* + \bar{a}} \cdot \frac{1 + \frac{\hat{a}^*}{2}k}{1+\hat{a}^* k}\right] < 0,$$

where the last inequality holds because $\hat{a}^* \in [0, \bar{a}]$ in the separating equilibrium.

Proof of Proposition 3 Following the proof of Proposition 2, in any separating equilibrium, we have

$$\frac{\partial \mathcal{AR}}{\partial \beta} = \left[\frac{\bar{a}}{\hat{a}^* \cdot (\hat{a}^* + \bar{a})} - \frac{k}{(2+\hat{a}^* k) \cdot (1+\hat{a}^* k)}\right] \cdot \frac{\partial \hat{a}^*}{\partial \beta} \tag{II.10}$$

$$= \frac{(\bar{a}k - 1)k \cdot (\hat{a}^*)^2 + 2\bar{a}k \cdot \hat{a}^* + 2\bar{a}}{\hat{a}^* \cdot (\hat{a}^* + \bar{a}) \cdot (2+\hat{a}^* k) \cdot (1+\hat{a}^* k)} \cdot \frac{\partial \hat{a}^*}{\partial \beta}. \tag{II.11}$$

The denominator of (II.11) is clearly positive, so it remains to show that the numerator, that is the function $\varphi(\hat{a}) = (\bar{a}k - 1)k \cdot \hat{a}^2 + 2\bar{a}k \cdot \hat{a} + 2\bar{a} > 0, \forall \hat{a} \in [0, \bar{a}]$. We consider two cases. First, when $\bar{a}k - 1 \geq 0$, $\varphi(\hat{a})$ is increasing in $[0, \bar{a}]$ and $\varphi(\hat{a}) \geq \varphi(0) = 2\bar{a} > 0$. Second, when $\bar{a}k - 1 < 0$, $\varphi(\hat{a})$ is increasing in $[0, \frac{\bar{a}}{1-\bar{a}k}]$. Because $\frac{\bar{a}}{1-\bar{a}k} > \bar{a}$, $\varphi(\hat{a}) \geq \varphi(0) > 0 \; \forall \hat{a} \in (0, \bar{a}]$. As shown in Proposition 1, we have $\frac{\partial \hat{a}^*}{\partial \beta} > 0$, so we have $\frac{\partial \mathcal{AR}}{\partial \beta} > 0$ in the separating equilibrium.

CHAPTER III

What Generates Households' Inertia in Rebalancing their Portfolio: Attention Cost or Adjustment Cost

III.1 Introduction

Since the seminal work of Merton (1969), it has become a guiding rule that investors should keep a constant risky share which is determined by their belief about the return process and their risk attitude. However, during the following three decades, researchers did not know whether households did follow this rule in their portfolio management due to limited data. Since 2000, with richer data in household finance, economists started investigating how households actually manage their portfolio. A very common phenomenon found by these empirical studies is that households have very strong inertia in rebalancing their portfolio (see e.g. Calvet, Campbell and Sodini, 2009; Brunnermeier and Nagel, 2008). This behavioral feature is important not just because it shows the necessity for improving the canonical model on the micro-decision level, but also because it provides a new mechanism to explain some aggregate effects. For instance, Chien, Cole and Lustig (2012) have shown that the rebalancing inertia of households can significantly impact price and volatility in financial markets. As emphasized in many papers discussing the aggregate effect of rebalancing inertia, to generate a significant aggregate effect, it is necessary to assume that rebalancing behavior of households follows a time-dependent policy.[1] How plausible is this assumption? Without a micro model to provide a structure for empirical test, it is impossible to answer this question. This paper intends to provide such a micro model framework for implementing related empirical tests.

On micro level, inertia behavior is typically rationalized by an attention cost, which

[1] In the literature about sticky price, Caplin and Spulber (1987) has shown that the monetary policy will be ineffective if the price-setting follows a state-dependent policy. Similar logic applies here.

will generate a time-dependent policy, or an adjustment cost, which will generate a state-dependent policy. But which friction is the one that governs the rebalancing behavior of households? So far, empirical evidence has not really answered this question[2]. Following the literature about periodical cash withdrawal and sluggish price adjustment, this paper simultaneously imposes observation costs and adjustment costs on portfolio rebalancing within the framework of Merton's canonical model. One rationale for this construction is to make model more realistic: these two types of costs typically exist simultaneously. Second rationale is from the research motivation: it provides a unified framework under which we can compare these two costs just by varying the parameterization, which will provide us a way to empirically distinguish these two types of frictions in explaining micro data. Since there is no close-form solution existing for this model, the model is solved numerically and the empirical implications are derived through a large number of numerical static comparative studies.

In the general setup, policy function generated from the model is a mix of state-dependent policy and time-dependent policy. This paper proposes an index "synchronization probability", which is defined as the probability of making an adjustment at the initial observation date, to characterize the closeness of the mixed policy to time-dependent policy. Based on the numerical studies, the adjustment cost has to be unrealistically small (less than 1 cent for \$ 1 million in wealth) to make the synchronization probability higher than 0.9. Hence, the model does not really support the assumption of time-dependent policy in previous macroeconomic literature. Through a series of numerical comparative static analyses, this paper develops three main empirically testable implications about households' rebalancing behavior: (1) for each individual, a decrease in inattentive duration indicates higher probability of making adjustment; (2) households with higher risky share tend to observe their portfolio more frequently; (3) if the perceived risk premium is controlled, households with higher risky shares adjust more frequently; if the perceived risk premium is not controlled, among the households investing relatively more in risky asset, those with higher risky shares adjust their portfolio less frequently. For each model implication, empirical testing strategies in different data environments are provided.

The organization of this paper is as follows. Section 2 specifies the model setup and Section 3 solves the model. In Section 4, several testable implications of the model and corresponding empirical testing strategies are presented. Section 5 concludes.

[2]Brunnermeier and Nagel (2008) found that households do not rebalance even after some big changes, which shows the necessity of observation costs, but cannot really rule out adjustment costs.

III.2 Model

III.2.1 Setup

The basic framework of this model follows Merton's canonical portfolio choice model with Cash-In-Advance (CIA) requirement. Households have two types of account: one is transaction account that finances their consumption and its balance is denoted by X. The second is a portfolio account for investment, and its balance is denoted as W. In the transaction account, money earns a constant liquid return rate of l. In portfolio account, there are two type of assets: a risky asset, accounting for α share of the portfolio account balance, and a riskless asset, accounting for the remaining $1-\alpha$. In this model, short and leverage are not allowed hence α is restricted to lie in $[0,1]$. The return rate for the riskless asset is constant at r and the price of risky asset follows geometric Brownian motion parameterized by (μ,σ), i.e.

$$\frac{dP_t}{P_t} = \mu dt + \sigma dB_t$$

where μ and σ are the mean and volatility of stock market return, and B_t is a standard Brownian motion. Here, we assume $\mu - \frac{\sigma^2}{2} > r > l$ to keep the risk premium positive and ensure that investing money in the portfolio account dominates leaving tehm in the transaction account.

The decisions that households need to make are a consumption plan and an investment plan. If there is no friction on the transfer between portfolio account and transaction account, households will never leave any money in their transaction account and the CIA requirement will have no impact on households' decision. Put differently, households can finance their consumption by continuously transferring money from portfolio account to transaction account. If there is no friction on the adjustment of portfolio composition, households continuously rebalance the risky share α to a constant share. In this paper, two frictions are introduced into the model. The first friction is observation cost: to observe X, W and α, households need to pay $\phi_o(X+W)$. The second friction is adjustment cost: to change the portfolio structure, i.e. risky share α, households need to pay $\phi_a W$. Since it is costly to observe, households will not keep monitoring their accounts. Because any adjustment (transfers between two accounts or change of risky share) should be based on the latest information, households have to keep some money in their transaction account to finance their consumption during the inattentive period and they can only rebalance their portfolio at the observation date. Due to the separate cost for adjusting α, rebalancing might not always coincide with observation. At an observation date denoted by time 0, households pay the observation cost, observe the balance after paying observation cost (X, W, α) and they have to determine:

1. the new risky share α_+ (if $\alpha_+ \neq \alpha$, adjustment cost has to be paid).

2. the inattentive duration τ (the next observation will happen at time τ, observation cost also needs to be paid at time τ).

3. the transfer between portfolio account and transaction, or equivalently, the new account balances (X_+, W_+) which satisfy $X_+ + W_+ = X + (1 - \phi_a \mathbb{1}_{\alpha_+ \neq \alpha})W$.

In this model, observation costs are constructed following the literature. As discussed in Dixit (1993), any adjustment cost which is not marginally equal to zero can generate inaction. However, in the context of portfolio management, transaction costs which are proportional to trading volume have very moderate effects on making households inactive given the magnitude of these types of costs in reality. It is the "fixed cost" which mainly determines the inertia in households' behavior. Here, the "fixed cost" is a general concept that includes all the financial and cognitive costs which are required for adjustment. There are two reasons to assume "fixed cost" linear in the total wealth of households: (1) the opportunity cost of time for richer people is relatively higher than that for poorer people; (2) technically, this can induce homogeneity in the value function, which is convenient for tractability. For these reasons, I assume that adjustment costs are proportional to the size of portfolio.

III.2.2 Relation to the Literature

This model follows Abel, Eberly and Panageas (2013) and Alvarez, Guiso and Lippi (2012), but is different from their studies in a variety of dimensions.

Comparing with Alvarez, Guiso and Lippi (2012), this model has:

1. *No Adjustment Cost for Cash Management:* In Alvarez, Guiso and Lippi (2012), whenever the risky share is adjusted or money is transferred to transaction account, adjustment costs are triggered. Since this paper mainly focuses on the rebalancing decision, to make the main logic as clear as possible, transfers between the transaction account and the portfolio account (cash management) are assumed to be perfectly frictionless.

2. *Uncontrolled Risky Share Process during the Inattentive Period:* Alvarez, Guiso and Lippi (2012) do not focus on the rebalancing decision, so they assume the risky share to

be continuously rebalanced to a preset target during the inattentive period. However, the empirical evidence which this model is designed to match shows that a significant part of risky share fluctuation comes from being passively driven by asset price fluctuations. Therefore, in this paper, the risky share is uncontrolled during the inattentive period.

3. *Non-Durable Goods Consumption:* Alvarez, Guiso and Lippi (2012) use durable goods consumption to match the pattern in transaction account balance, but this paper does not focus on the transaction account balance adjustment, therefore, utility flows just come from non-durable goods consumption in this paper.

Comparing with Abel, Eberly and Panageas (2013), this model has:

1. *Simpler Observation Cost:* The ultimate purpose of this paper is to fit the model to data, but employing the general observation cost adopted in Abel, Eberly and Panageas (2013) is computationally infeasible. The proportional observation cost, which is also adopted in Alvarez, Guiso and Lippi (2012), is more empirically feasible and also matches the intuition that it is more expensive for richer people to pay attention since the opportunity cost of their time is higher.

2. *Fixed Adjustment Cost on Rebalancing:* In Abel, Eberly and Panageas (2013), it is costly to make transfers between the transaction account and the portfolio account, but adjustment of the risky share is free. Due to the different focus of this paper, the cost of cash management is removed and new cost is imposed on the adjustment of the risky share. Also, in my model, the transaction cost which is proportional to trading volume is removed. As discussed in the last section, this type of transaction cost can only make the inaction band slightly larger and generate two different resetting risky shares which might be very closed to each other, but cannot significantly change the location and width of the inaction band. Therefore, removing it will make negligible impact on the results, but can make the model much more empirically feasible.

III.2.3 Bellman Equation

After paying the observation cost, households observe their wealth information (X, W, α). Denote the corresponding value function after observation as $V(X, W, \alpha)$. Next, households need to decide whether to adjust their risky share. Denote the value function for not adjusting the risky share as $\bar{V}(X, W, \alpha)$ and the value function for adjusting the risky share as $\hat{V}(X, (1-$

$\phi_a)W)$. The Bellman Equation for this Dynamic Programming problem is:

$$V(X,W,\alpha) = \max\left\{\bar{V}(X,W,\alpha), \hat{V}(X,(1-\phi_a)W)\right\} \tag{III.1}$$

where the value function for non-adjustment has to satisfy:

$$\bar{V}(X,W,\alpha) = \max_{\{c_t|0\le t\le\tau\},\tau} \int_0^\tau e^{-\rho t}u(c_t)dt + e^{-\rho\tau}\mathbb{E}\left[V(X_\tau,W_\tau,\alpha_\tau)\right] \tag{III.2}$$

Subject to:

$$\begin{aligned}
X_\tau &= (1-\phi_o)\left[e^{l\tau}X_+ - \int_0^\tau e^{l(\tau-t)}c_t dt\right],\ X_\tau \ge 0 \\
X_+ &= X + W - W_+ \\
W_\tau &= (1-\phi_o)\left[(1-\alpha)e^{r\tau} + \alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}\right]W_+,\ Z\sim\mathcal{N}(0,1) \\
\alpha_\tau &= \frac{\alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}{(1-\alpha)e^{r\tau}+\alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}
\end{aligned}$$

the value function for adjustment has to satisfy:

$$\hat{V}(X,W) = \max_{\{c_t|0\le t\le\tau\},\tau,\alpha_+} \int_0^\tau e^{-\rho t}u(c_t)dt + e^{-\rho\tau}\mathbb{E}\left[V(X_\tau,W_\tau,\alpha_\tau)\right] \tag{III.3}$$

Subject to:

$$\begin{aligned}
X_\tau &= (1-\phi_o)\left[e^{l\tau}X_+ - \int_0^\tau e^{l(\tau-t)}c_t dt\right],\ X_\tau \ge 0 \\
X_+ &= X + W - W_+ \\
W_\tau &= (1-\phi_o)\left[(1-\alpha_+)e^{r\tau} + \alpha_+ e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}\right]W_+,\ Z\sim\mathcal{N}(0,1) \\
\alpha_\tau &= \frac{\alpha_+ e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}{(1-\alpha_+)e^{r\tau}+\alpha_+ e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}
\end{aligned}$$

In this model, the utility function of households is given as $u(c) = \frac{c^\gamma}{\gamma}$ where $\gamma < 1$.

III.3 Solution to Model

III.3.1 Reduction of State Space

Lemma III.1 (Exhausted Consumption at Observation Date)

Under the assumption that $\mu - \frac{\sigma^2}{2} > r > l$, $X_\tau = 0$ at each observation date. ∥

Proof. With power utility and $\mu - \frac{\sigma^2}{2} > r$, the household will always choose a strictly positive α, which means that the optimal portfolio allocation will strictly dominate holding 100% riskless asset. Due to $r > l$, holding 100% riskless asset dominates leaving money in the transaction account. Therefore, one dollar in the portfolio account (invested optimally) will always dominate one dollar in transaction account. In this decision problem, both the date of next observation τ and the consumption path during the inattentive period $[0, \tau]$ are determined at time 0, and correspondingly, X_τ is also determined at time 0. If there is $X_\tau > 0$ left in the transaction account right before next observation, households can always transfer $e^{-l\tau} X_\tau$ from transaction account to portfolio account at time 0 without changing the risky share, which will neither break any constraints nor affect the consumption path, but can strictly increase the welfare. Now, the state space can be reduced to (W, α) in that X will always be 0 at observation dates. ■

Lemma III.2 (Determined Consumption Path during Inattentive Period)
If the transaction account balance is set at X_+ after an observation and the inattentive duration is set at τ, the optimal consumption path will generate discounted utility as $U(X_+, \tau) = N(\tau)\frac{X_+^\gamma}{\gamma}$, *where* $N(\tau) = \left[\frac{\frac{\rho-\gamma l}{\gamma-1}}{e^{\frac{\rho-\gamma l}{\gamma-1}\tau} - 1}\right]^{\gamma-1}$. ‖

Proof. During the inattentive period, households cannot make transfers between portfolio account and transaction account. The Euler equation for consumption will be:

$$c_0^{\gamma-1} = e^{(l-\rho)t} c_t^{\gamma-1}, \ \forall t \in [0, \tau]$$

from which we can derive $c_t = e^{\frac{\rho-l}{\gamma-1}t} c_0$. From Lemma III.1 , we have the budget constraint as $\int_0^\tau e^{-lt} c_t dt = X_+$. After some simple algebraic manipulation, we have the discounted utility brought by the optimal consumption path as:

$$U(X_+, \tau) = \left[\frac{\frac{\rho-\gamma l}{\gamma-1}}{e^{\frac{\rho-\gamma l}{\gamma-1}\tau} - 1}\right]^{\gamma-1} \frac{X_+^\gamma}{\gamma} \equiv N(\tau)\frac{X_+^\gamma}{\gamma}$$

■

By Lemma III.1 and Lemma III.2, we can rewrite the Bellman Equation as:

$$V(W, \alpha) = \max\left\{\bar{V}(W, \alpha), \hat{V}((1 - \phi_a)W)\right\} \quad \text{(III.4)}$$

where the value function for non-adjustment has to satisfy:

$$\bar{V}(W,\alpha) = \max_{X_+,\tau} N(\tau)\frac{X_+^\gamma}{\gamma} + e^{-\rho\tau}\mathbb{E}\left[V(W_\tau,\alpha_\tau)\right] \tag{III.5}$$

Subject to:

$$W_\tau = (1-\phi_o)\left[(1-\alpha)e^{r\tau} + \alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}\right](W - X_+),\ Z \sim \mathcal{N}(0,1)$$

$$\alpha_\tau = \frac{\alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}{(1-\alpha)e^{r\tau} + \alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}$$

the value function for adjustment has to satisfy:

$$\hat{V}(W) = \max_{X_+,\tau,\alpha_+} N(\tau)\frac{X_+^\gamma}{\gamma} + e^{-\rho\tau}\mathbb{E}\left[V(W_\tau,\alpha_\tau)\right] \tag{III.6}$$

Subject to:

$$W_\tau = (1-\phi_o)\left[(1-\alpha_+)e^{r\tau} + \alpha_+ e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}\right](W - X_+),\ Z \sim \mathcal{N}(0,1)$$

$$\alpha_\tau = \frac{\alpha_+ e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}{(1-\alpha_+)e^{r\tau} + \alpha_+ e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}$$

Lemma III.3 (Homogeneity of Value Function)
Value functions $V(W,\alpha)$, $\bar{V}(W,\alpha)$ and $\hat{V}(W)$, are homogeneous of degree of γ in W. ‖

Proof. Given that both the constraints and frictional costs are linear in the state variable W, it follows that the value functions are homogeneous of degree γ in W, as proved by Stokey (2009). ■

Now, above value functions can be simplified as: $V(W,\alpha) \equiv W^\gamma v(\alpha)$, $\bar{V}(W,\alpha) \equiv W^\gamma \bar{v}(\alpha)$, $\hat{V}(W) \equiv W^\gamma \hat{v}$. Then, the Bellman Equations III.4, III.5 and III.6 can be rewritten as:

$$v(\alpha) = \max\left\{\bar{v}(\alpha), (1-\phi_a)^\gamma \hat{v}\right\} \tag{III.7}$$

where the revised value function for the non-adjustment case[3] is:

$$\bar{v}(\alpha) = \max_{x_+,\tau} N(\tau)\frac{x_+^\gamma}{\gamma} + e^{-\rho\tau}(1-\phi_o)^\gamma(1-x_+)^\gamma \mathbb{E}\left[R(\tau,\alpha)^\gamma v(\alpha_\tau)\right] \tag{III.8}$$

Subject to:

$$R(\tau,\alpha) = (1-\alpha)e^{r\tau} + \alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z},\ Z \sim \mathcal{N}(0,1)$$

$$\alpha_\tau = \frac{\alpha e^{(\mu-\frac{\sigma^2}{2})\tau+\sqrt{\tau}\sigma Z}}{R(\tau,\alpha)}$$

and the revised value function for the adjustment case satisfies:

$$\hat{v} = \max_{\alpha} \bar{v}(\alpha) \tag{III.9}$$

Now, we reduce the state space to a unidimensional space. The policy functions we need to solve are: (1) optimal risky share choice $\alpha_+(\alpha)$; (2) optimal inattentive duration choice $\tau(\alpha)$; (3) optimal consumption funding choice $x_+(\alpha)$.

III.3.2 Typical Policy Function

Since this paper focuses on rebalancing behavior, I will only discuss the policy functions related to the choice of the risky share and inattentive duration. Right after observing α, if $\alpha \notin [\underline{\alpha}, \bar{\alpha}]$, the household pays the adjustment and adjusts the risky share to α^*. If $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, the household will not adjust the risky share. Depending on α, the household will also need determine an inattentive duration τ. To make the results comparable to the results in earlier literatures (Abel, Eberly and Panageas, 2007; Alvarez, Guiso and Lippi, 2012; Abel, Eberly and Panageas, 2013, e.g.)(e.g. Abel, Eberly and Panageas (2007), the parameters are calibrated to a set of values adopted by the earlier literature, which are listed in Table III.1.

Based on the baseline calibration, the policy function $\alpha_+(\alpha)$ and $\tau(\alpha)$ are summarized in Figure III.1. Under this calibration, households will observe their portfolio around every 3 weeks and adjust their portfolio around every half year.

The following properties about the policy function $\tau(\alpha)$ are worth pointing out:

1. $\tau(\alpha) = \tau(\alpha^*),\ \forall\alpha \notin [\underline{\alpha}, \bar{\alpha}]$, because the household will adjust their portfolio share to α^* if $\alpha \notin [\underline{\alpha}, \bar{\alpha}]$.

2. When $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, $\tau(\alpha)$ becomes smaller than $\tau(\alpha^*)$ when α is very close to the bound-

[3] x_+ is the proportion of wealth transferred to transaction account, i.e. $x_+ \equiv \frac{X_+}{W}$

Table III.1. Baseline Calibration

Parameter		Value
μ	Mean of Risky Asset Return	6%
σ	Volatility of Risky Asset Return	16%
r	Riskless Asset Return	2%
l	Return for Liquid Asset	0
ρ	Discount Factor	2%
γ	1-Relative Risk Aversion	-3
ϕ_o	Observation Cost	0.01 bp
ϕ_a	Adjustment Cost	0.05 bp

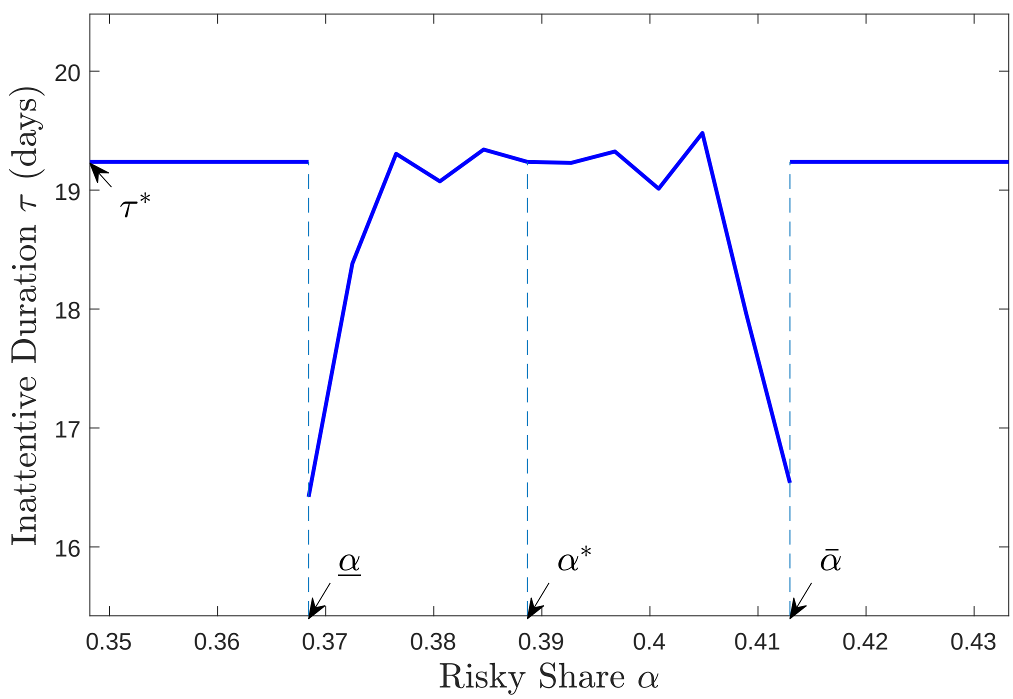


Figure III.1. Typical Policy Function

aries. The intuition for this is that the risky share will hit the boundary soon and households need to monitor the risky share more frequently.

3. Denote the first observation after adjustment as the "*initial observation*" and the duration of the first inattentive period after adjustment as the "*initial inattentive duration*". Then the initial inattentive duration τ^* only depends on the belief of households in that the post-adjustment risky share is always α^*. However, the duration of other inattentive periods also depends on the realized risky asset return in that the realized return affects the risky share α. Hence, the initial inattentive duration has much less noise than other inattentive durations and it can be a better variable to extract the information about household's preferences and beliefs.

III.4 Model Implications to be Empirically Tested

III.4.1 Data Environment

To facilitate the discussion in this section, potential data which can be used to test the model implications is described as follows.

Naturally Occurring Data: The date of each observation (login) of households is recorded and denoted as $\{t_{i,h}\}$, which indexes the i-th observation of household h. For each observation, their login risky share (at the beginning of observation) is denoted as $\alpha_{i,h}$ and logout risky share (at the end of observation) is denoted as $\alpha_{+i,h}$. With this information, we can construct:

1. A dummy variable $A_{i,h}$ recording whether household h makes adjustment at i-th observation: $A_{i,h} = \mathbb{1}[\alpha_{i,h} \neq \alpha_{+i,h}]$.

2. A dummy variable $O_{i,h}$ recording whether i-th observation of household h is an initial observation after adjustment: $O_{i,h} = A_{i-1,h}$.

3. Inattentive duration $\tau_{i,h} = t_{i+1,h} - t_{i,h}$ and total observation times of each household I_h.

4. Adjustment duration $\mathcal{T}_{j,h}$, which means the length of time between the $(j-1)$-th adjustment and the j-th adjustment, and the total adjustment times of each household J_h.

5. The resetting risky share $\alpha^*_{j,h}$ and initial inattentive duration $\tau^*_{j,h}$ after each adjustment.

Constant Frequency Data: Compared with naturally occurring data, it is easier to access constant frequency data, which could even be imputed from household survey data. In each period (month, season or year), the number of observations (logins) and adjustments for household h are recorded as $NO_{t,h}$ and $NA_{t,h}$, and their portfolio risky share at end of each period is recorded as $\alpha_{t,h}$.

III.4.2 Distance between a Mixed Policy and Time-Dependent Policy

Generally, the rebalancing behavior generated by this model follows a mixed policy. When the adjustment cost approaches zero, the policy becomes purely time-dependent[4]. However, how small does the adjustment cost have to be to make a mixed policy close enough to a time-dependent policy? To answer this question, one needs a way to measure the distance between a mixed policy and a time-dependent policy. In this paper, "*synchronization probability*", which is defined as the probability of making an adjustment at the initial observation date, is used to measure this distance. When synchronization probability is closer to 1, a higher proportion of non-adjustment duration will be equal to the initial inattentive duration τ^* and the adjustment behavior is closer to the pattern that households adjust their portfolio with a constant frequency, which means the policy is closer to a time-dependent policy. Therefore, the assumption made in many previous studies (e.g. Chien, Cole and Lustig (2012)) can be justified by whether or not the synchronization probability is close to 1.

Empirically, if one had the naturally occurring data as described above, one could directly calculate this index as $\frac{\sum_{i,h} A_{i,h} O_{i,h}}{\sum_{i,h} O_{i,h}}$. If the index is close to 1, then the mixed policy is close to a time-dependent policy. If one only has constant frequency data, one can calculate the adjustment-observation ratio: $\frac{\sum_{t,h} NA_{t,h}}{\sum_{t,h} NO_{t,h}}$. Even though this index is not exactly the synchronization probability, it is highly correlated with synchronization probability in my simulation study. Therefore, it is also very informative for us to judge whether the rebalancing behavior can be approximated by a time-dependent rule. In a recent study, Sicherman, Loewenstein, Seppi and Utkus (2015) use administrative data from Vanguard and show that the adjustment-observation ratio is very small. This empirical evidence implicates that the adjustment behavior of households might be very far away from a time-dependent policy.

If there is no data available, one can use the model to numerically investigate whether a reasonable adjustment cost level can generate a synchronization probability close to 1. I implement several numerical experiments and illustrate their results in Figure III.2. To

[4]Abel, Eberly and Panageas (2013) have a rigorous proof for this result.

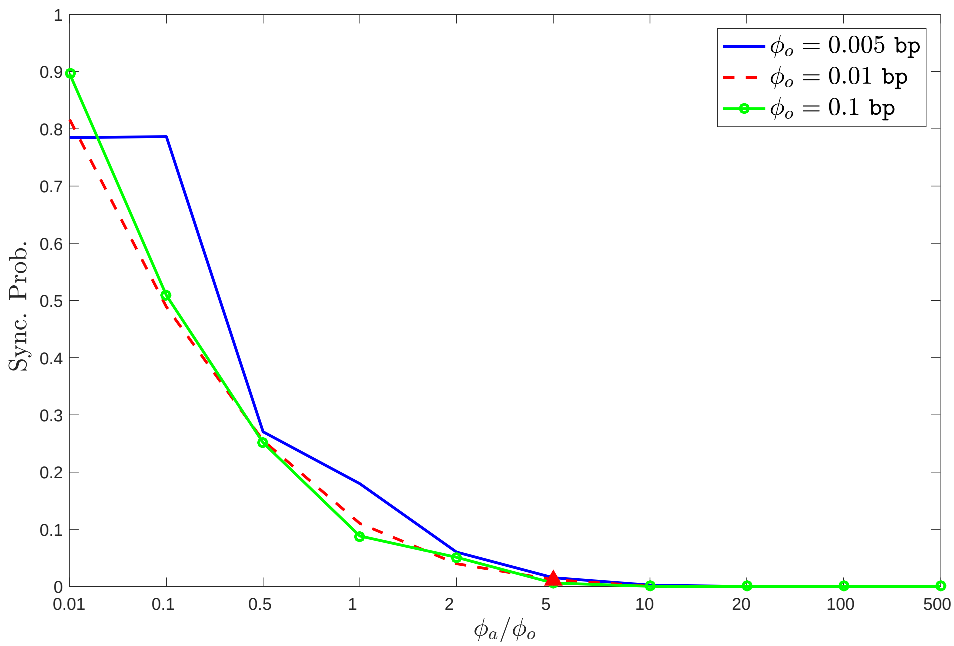


Figure III.2. Synchronization Probability for Different Cost Size Combination

Note: ▲ labels the synchronization probability and cost level under the baseline calibration.

make the synchronization probability higher than 0.9, the adjustment cost has to be no larger than 0.0001 `bp` (1 cent for \$ 1 million wealth). Even though we do not have a good prior for the size of the adjustment cost so far, this magnitude is just too small to justify its economic plausibility. Qualitatively, the model leaves the possibility for time-dependent policy; but quantitatively, the model does not really support time-dependent policy, which is also consistent with the empirical evidence found by Sicherman, Loewenstein, Seppi and Utkus (2015). Therefore, for the remainder of the paper, I follow the baseline calibration and base my discussion on the mixed policy with synchronization probability around 3%.

III.4.3 $\Delta\tau$ as a Leading Indicator for Adjustment

The policy shown in Figure III.1 indicates that when α is close to the boarder of $[\underline{\alpha}, \bar{\alpha}]$, the inattentive duration becomes shorter. This indicates that the decrease in inattentive duration might be an indicator for adjustment. To verify this conjecture, the choice of inattentive duration at different α's and the probability of making adjustment after the corresponding inattentive period, are calculated in the baseline model. A scatter plot shown in Figure III.3 illustrates a very clear negative relationship between those two variables. To provide a clear empirical implication from this model, the adjustment probability and inattentive duration are fitted using logistic regression. In terms of the marginal effect on the adjustment probability, the difference in the predicted probability when τ decreases from τ^* to $\tau^* - 1$ is reported. There are two reasons for choosing this marginal effect as a benchmark:

1. From Figure III.1, we can see that $\tau(\alpha)$ is around τ^* in most of the $[\underline{\alpha}, \bar{\alpha}]$ space. This can be also verified in Figure III.3: the density of points around τ^* is much higher compared with other areas. This implies that in real data, it is very likely that the observed inattentive durations of a specific household might be highly concentrated around τ^*. Therefore, τ^* is a good representative candidate for inattentive duration.

2. From Figure III.3, we can see that the marginal effect around τ^* is relatively smaller than the marginal effect at other τ's. Therefore, the marginal effect at τ^* provides an approximated lower bound for the marginal effect.

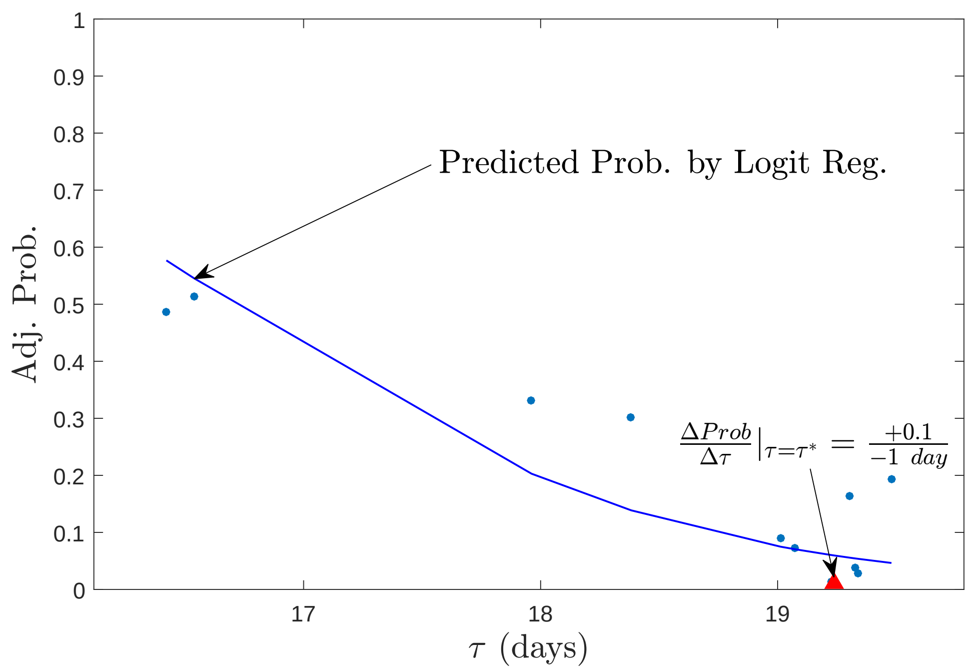
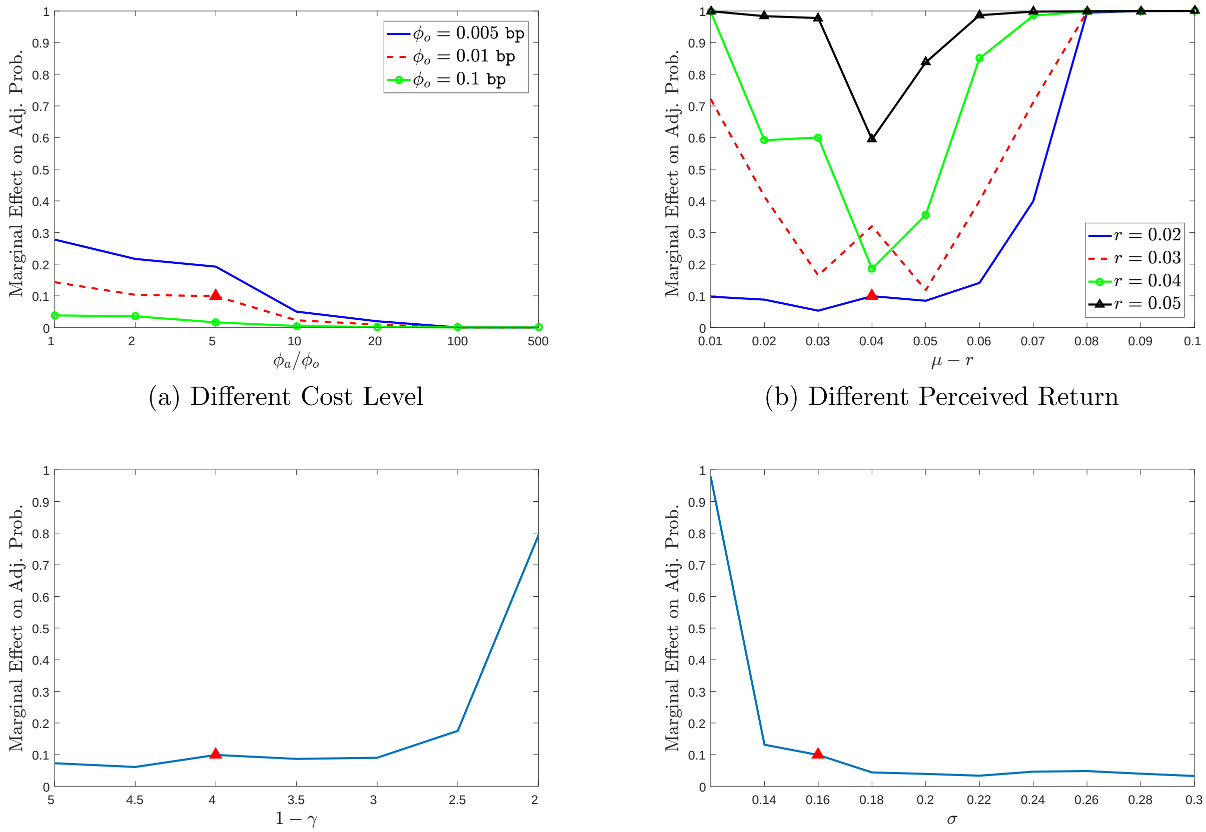


Figure III.3. Probability of Adjustment and Inattentive Duration

Note: ▲ labels the initial inattentive duration and the corresponding adjustment probability under the baseline calibration.

In the baseline model, if a household decreases the inattentive duration by 1 day, the probability of making an adjustment right after the inattentive duration will increase by at least 0.1. To verify the robustness of this prediction, several numerical experiments with different parameter calibrations are implemented and the marginal effect at τ^* are summarized in Figure III.4. From these results, we can conclude that the positive marginal effects are robust and remain economically significant for a large variety of parameter values.



(a) Different Cost Level

(b) Different Perceived Return

(c) Different Risk Aversion

(d) Different Perceived Volatility

Figure III.4. Marginal Effects at τ^* in Different Numerical Experiments

Note: Here, the marginal effects mean the change in adjustment probability when the inattentive duration decreases from τ^* to $\tau^* - 1$ days. ▲ labels the marginal effect and the corresponding parameter value in the baseline calibration.

In empirical study, to test this implication, we have to rely on the naturally occurring data, in which we can run the logistic regression:

$$\mathbb{P}\left[A_{i+1,h} = 1\right] = \frac{1}{1 + \exp(\mathtt{a}_h + \mathtt{b}\tau_{i,h} + \mathtt{control}_{i,h})}$$

the marginal effect of a decrease in $\tau_{i,h}$ should be significantly positive.

III.4.4 Inattentive Duration is Negatively Correlated with Risky Share

Under different parameter calibrations, the combination of optimal resetting risky share α^* and initial inattentive duration τ^* are summarized in Figure III.5, from which we have following findings:

1. Cost level does not change the optimal resetting risky share α^*. Both higher observation and higher adjustment costs can generate a longer initial inattentive duration τ^*, but observation costs have much more significant impact than adjustment costs do.

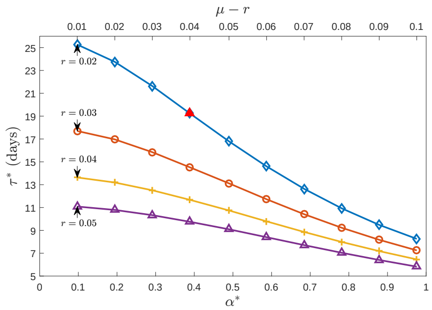
2. The risk-free return has a negligible impact on α^* but higher risk-free returns will generate a shorter initial inattentive duration. The effect of r on α^* can be rationalized by the intuition from Merton's model: only $\mu - r$, σ and γ affect the trade-off between risky and riskless asset, hence change in r will not change α^* if $\mu - r$ and other parameters are kept constant. When r increases, even though α^* does not change, the overall return rate of the portfolio increases and correspondingly the value of portfolio increases, which allows the household to observe more frequently.

3. Higher risk aversion will shift down the optimal resetting risky share, which follows the same intuition from Merton's model, decreasing the optimal value of portfolio and the value of observation will not be as high as before. Therefore, with the risk aversion increasing, α^* decreases and the initial inattentive duration increases.

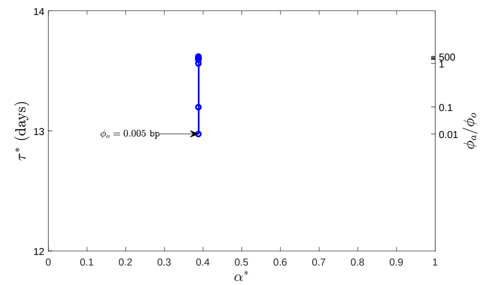
4. When the perceived volatility of risky return increases, α^* will decrease following the trade-off between risky and riskless asset, which decreases the value of the portfolio. Following the same logic, the initial inattentive duration increases with the increase in perceived volatility[5].

Within this model, the variation in (α^*, τ^*) can only come from the variation in parameters $(\phi_o, \phi_a, \gamma, \mu, r, \sigma)$. According to the comparative static analysis results summarized in Figure III.5, the parameter variation which causes the change in τ^* (variation in $\phi_o,\ \phi_a,\ r$) might not really change α^*, but the variation which causes the change in α^* (variation in $\mu - r,\ \sigma,\ \gamma$) will definitely change τ^* and the sign of $\Delta\tau^*$ is always opposite to the sign of

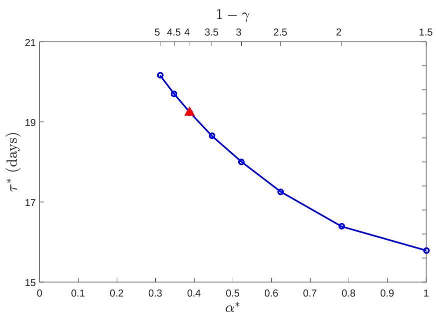
[5] Sicherman, Loewenstein, Seppi and Utkus (2015) find that households login less frequently when VIX is higher, which exactly matches the implication given by this model. They use "ostrich effect" to rationalize this effect. However, in our model, even if the utility does not depend on information directly, this effect still exists due to observation costs.



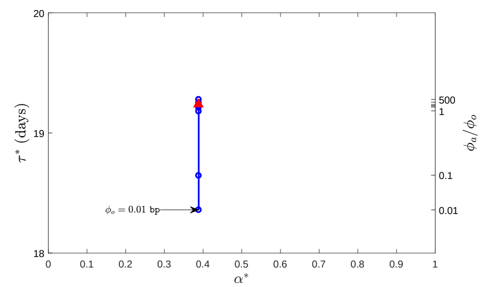
(a) Different Perceived Return



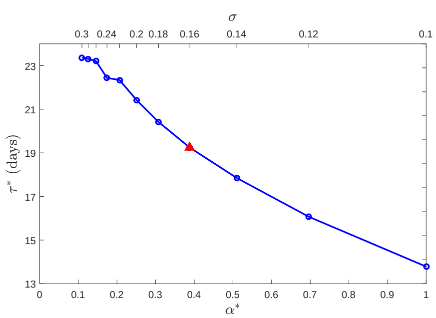
(b) Different Cost Levels, $\phi_o = 0.005$ bp



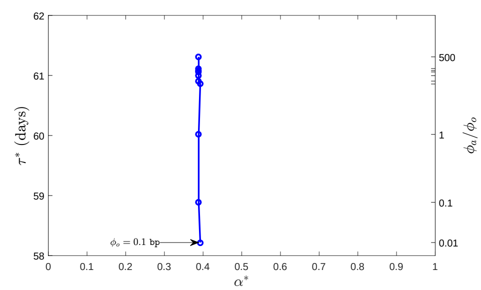
(c) Different Risk Aversion



(d) Different Cost Levels, $\phi_o = 0.01$ bp



(e) Different Perceived Volatility



(f) Different Cost Levels, $\phi_o = 0.1$ bp

Figure III.5. Optimal Resetting Risky Share and Initial Inattentive Duration

Note: Here, ▲ labels the (α^*, τ^*) in the benchmark calibration. Since τ^* is totally determined by belief and preference, we do not need worry about whether different actual return process can affect τ^*. In the simulation study, we found that the relationship between average τ and α^*, or relationship between average τ and average α, is very similar to the results in this figure, which also holds when we fit different actual return process into the simulation.

$\Delta\alpha^*$. In an empirical evaluation with naturally occurring data, if we run a pooled regression

$$\tau^*_{j,h} - \tau^*_{j-1,h} = \mathtt{a}_h + \mathtt{b}(\alpha^*_{j,h} - \alpha^*_{j-1,h}) + \mathtt{control}_{j,h}$$

we should get a negative estimate for $\mathtt{b}$. Here, the change in α^* can serve as a proxy for unobservable changes in belief and preference, and $\mathtt{b}$ captures the relative effect of those changes on τ^*. The constant term $\mathtt{a}_h$ captures household specific change in cost levels and beliefs about the risk-free rate. Because there usually is little variation in these parameters, it is reasonable to expect that $\mathtt{a}_h$ is close to 0, if the model is correct in characterizing the behaviors of households.

Based on the simulation results, I find that average inattentive duration and average risky share also approximately follow the same correlation pattern. Given the facts that the inaction band $[\underline{\alpha}, \bar{\alpha}]$ is narrow and α^* is usually approximately at the mid point of the inaction band, the average observed α will not be very far away from α^*. From the policy shown in Figure III.1, $\tau(\alpha) \approx \tau^*$ for a large scale of $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, which makes the average τ close to τ^* as well. Therefore, the correlation between α^* and τ^* should be inherited by average α and average τ under typical calibrations. Based on this property, one can actually use constant frequency data to test this implication by running the following regression:

$$\frac{1}{NO_{t,h}} = \mathtt{a}_h + \mathtt{b}\alpha_{t,h} + \mathtt{control}_{j,h}$$

One should expect similar estimation results as those discussed above. Generally, this model implies that households with higher risky share tend to pay more attention to their portfolio.

III.4.5 Correlation between α^* and Average Non-adjustment Duration

After computing the policy functions under different calibrations, I performed simulation studies under each calibration and calculated the average non-adjustment duration, which provides another set of implications that could be empirically tested. The average non-adjustment duration and optimal resetting risky share under each calibration are summarized in Figure III.6, from which the following findings emerge:

1. Cost level does not have a significant impact on α^*, but its impact on non-adjustment duration is much huger than its impact on inattentive duration.

2. Beliefs about the risk-free rate do not have any effect on α^* and the non-adjustment duration. This result comes from the fact that r does not affect the trade-off between two assets and the uncontrolled process of α if $\mu - r$ is held constant. For this reason, I only show the results for $r = 0.02$. The increase of perceived risk premium has a monotonic effect on the optimal resetting risky share, but its effect on average non-adjustment duration is not monotonic. To see this, observe that with α approaching 0 and 1, the variation in uncontrolled α process will become much smaller and the non-adjustment duration will be mechanically extended. Sicherman, Loewenstein, Seppi

and Utkus (2015) find a non-monotonic relationship between trading frequency and lagged returns, which is consistent with the implication of this model. This U shaped curve is also consistent with the empirical evidence shown in Calvet, Campbell and Sodini (2009) (Figure II).

3. For households with higher relative risk aversion, α^* will be lower and their average non-adjustment duration will become shorter due to the shrinkage of their inaction band.

4. When the perceived volatility of risky return increases, α^* decreases and the average non-adjustment duration decreases. In simulation, I tried to break the rational expectation assumption and used different return processes to calculate the non-adjustment duration without changing policy function. The results from the non-rational belief case do not deviate from the original result much and the positive correlation between α^* and non-adjustment duration still persist. Hence, one can conclude that this correlation is mainly caused by the shrinkage of the inaction band rather than the higher volatility of the return process.

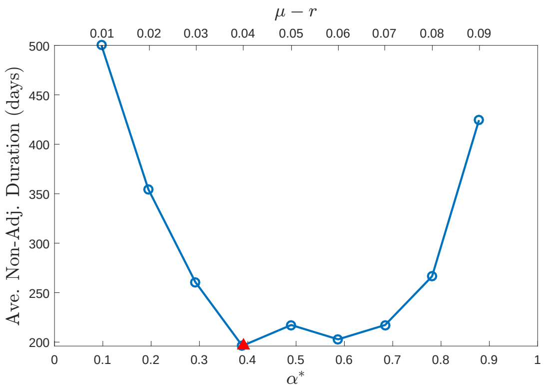
Similar to the implications discussed in last section, implications above can also be tested by regressing the non-adjustment duration on α^*. But the main difference here is that the sign of the regression coefficient will depends on the location of α^* if the variation in α^* is mainly induced by the variation in $\mu - r$. To deal with this new difficulty with naturally occurring data, one can pick S break points[6] in $[0,1]$: $0 = \hat{\alpha}_0 < \hat{\alpha}_1 < \cdots < \hat{\alpha}_s < \hat{\alpha}_{s+1} < \cdots < \hat{\alpha}_S < \hat{\alpha}_{S+1} = 1$, which divide the whole risky share space into $S+1$ non-overlapping intervals. Then one can run the regression:

$$\mathcal{T}_{j,h} = \mathtt{a}_h + \sum_{s=1}^{S+1} \mathtt{b}_s \mathbb{1}_{[\hat{\alpha}_{s-1},\hat{\alpha}_s]}(\alpha^*_{j,h})\alpha^*_{j,h} + \mathtt{control}_{j,h}$$

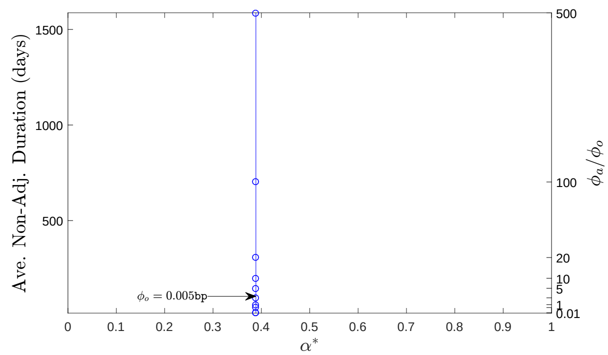
if the model is correct, then estimates for $\mathtt{b}_s$ can be negative only for a few intervals at the left end. Under typical calibrations, average α is close to α^*, therefore, with constant frequency data, we can run following cross-section regression

$$\frac{T}{\sum_{t=1}^{T} NA_{t,h}} = \mathtt{a}_h + \sum_{s=1}^{S+1} \mathtt{b}_s \mathbb{1}_{[\hat{\alpha}_{s-1},\hat{\alpha}_s]}\left(\frac{\sum_{t=1}^{T}\alpha_{t,h}}{T}\right)\frac{\sum_{t=1}^{T}\alpha_{t,h}}{T} + \mathtt{control}_h$$

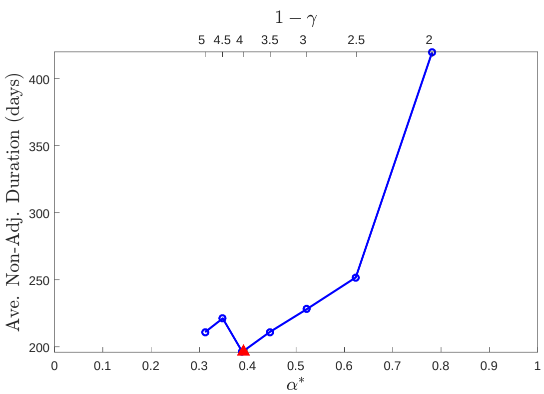
[6] In practice, one usually just needs pick one break point. For many simulation studies, the turning point of the U-shape curve is always around 0.5. Therefore, choosing 0.5 as the break point will be enough in practice.



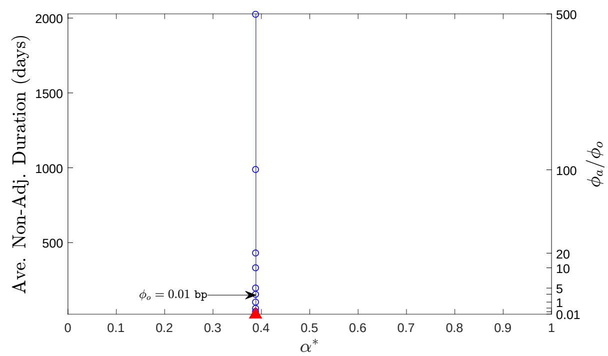
(a) Different Perceived Return, $r = 0.02$



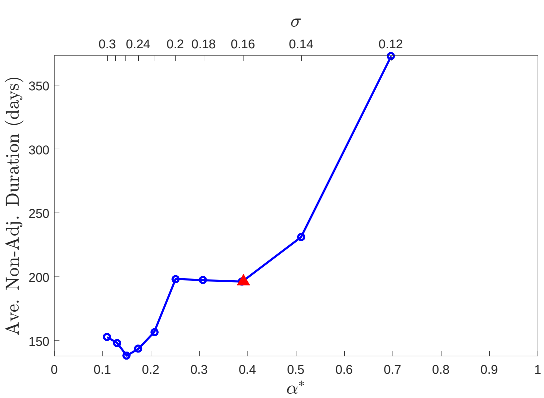
(b) Different Cost Levels, $\phi_o = 0.005$ bp



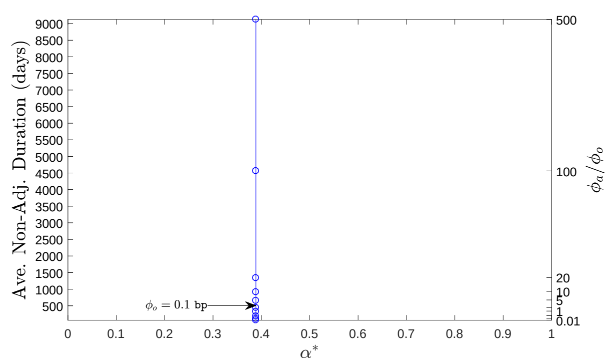
(c) Different Risk Aversion



(d) Different Cost Levels, $\phi_o = 0.01$ bp



(e) Different Perceived Volatility



(f) Different Cost Levels, $\phi_o = 0.1$ bp

Figure III.6. Optimal Resetting Risky Share and Average Non-Adjustment Duration

Note: Here, ▲ labels the α^* and average non-adjustment duration in the benchmark calibration. Here, average non-adjustment duration depends on actual return process. When we vary the belief parameter about return processes, we fit two actual return process into the simulation study: one is regulated by the same parameter as belief; the other one is regulated by the baseline parameter. We found: (1) the relationship between average τ and α^* is very similar to the relationship between average τ and average α; (2) those two relationships do not change much when we change the actual return process.

and we should be able to get similar results as those obtained with naturally occurring data. Even though the correlation between non-adjustment duration and risky share is not monotonic, this model predicts that households with higher risky share (above 50%) tend to adjust their portfolio relatively less frequently.

III.5 Conclusion

With both observation costs and adjustment costs, the rebalancing decision of households follows a policy which is a mixture of time-dependent and state-dependent policy. Given the policy structure, how well a mixed policy can be approximated as a time-dependent policy can be measured by "synchronization probability". Based on typical calibrations for return processes and preferences, one needs extremely low adjustment cost to generate a high "synchronization probability", which is contrary to the assumption made by previous literature that household's rebalancing behavior follows a time-dependent policy. By employing several different comparative static analyses, this paper derives three main testable implications: (1) for each individual household, decrease in inattentive duration indicates higher probability to adjust; (2) households with higher risky share should observe their portfolio more frequently; (3) although the relation between risky share and adjustment frequency might not be monotonic, the model predicts that among the households investing relatively more in risky assets, those with higher risky shares tend to adjust their portfolio less frequently. For each implication, I provide an empirical testing strategy in different data environments. With these testable implications, it becomes feasible to empirically distinguish these two types of friction and this will be the next step for this ongoing research project.

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