Existence in Spacetime and the Lorentz Transformations

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Integrating Existence into Physics

- This talk introduces a novel equivalence relation which directly connects the concept of existence to physics.
- Why might that be useful?
  - Arguably, our understanding of reality has become so deep that in order to make further progress, we should try to integrate the concept of existence into physics
- The main result is the derivation of what I call the *ontic equivalence relation*

Order of Presentation

1. Introduce a novel reinterpretation of the Lorentz Transformations
2. The reinterpretations bring attention to 4 spacetime principles which seem to have gone unappreciated so far
3. From the 4 principles, derive the proposition that existence in Minkowski space, as defined below, is an equivalence relation by absolute dimensionality
4. Discuss a few implications

An Informal Preview of the Reinterpretation of Length Contraction

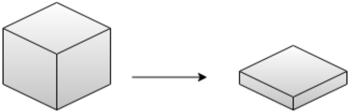

Figure: A length-contracted object has a stronger 2-dimensional character

- **Dimensional abatement:** As an object is length-contracted it attains an greater two-dimensional character, up until in the limit of c, when the contraction is complete and the object is dimensionally reduced.
- Will now be made mathematically precise

Lorentz Contraction as Dimensional Abatement I

Definition

Absolute Dimensionality: *The absolute dimensionality of an object is a dimensionless natural number that refers to the independent length dimensions which characterize it.*

Definition

Volume-Boundary ratio: *The Volume-Boundary ratio of a compact object with absolute dimensionality $n > 1$ is the ratio of its n-dimensional volume to its $n - 1$-dimensional boundary.*

Lorentz Contraction as Dimensional Abatement II

Definition

Relative Dimensionality: *Relative Dimensionality is the dimensionless ratio of the Volume-Boundary ratio of a compact object with absolute dimensionality* $n > 1$ *to that of a compact reference object, also with absolute dimensionality* n.

$$\dim_{\text{rel}}(a/b) = \frac{\frac{\int dV_a}{\int dA_a}}{\frac{\int dV_b}{\int dA_b}}$$ where

- a is the comparison object, $\int dV_a$ its volume, $\int dA_a$ its surface area
- b is the reference object, $\int dV_b$ its volume, $\int dA_b$ its surface area
- $\dim_{\text{rel}}(a/b)$ is the relative dimensionality of a to b in three space dimensions, a dimensionless measure in the interval $[0, 1]$.

Lorentz Contraction as Dimensional Abatement III

Definition

Dimensional Diminution: *For an n−dimensional compact object, dimensional diminution is the decrease of its relative dimensionality compared to its original state to a real number in the open interval* $(0, 1)$.

Definition

Dimensional Reduction: *For an n−dimensional compact object (n>1), dimensional reduction is the decrease of its absolute dimensionality to* $n - 1$.

Lorentz Contraction as Dimensional Abatement IV

Definition

Dimensional Abatement: *A less specific umbrella term which can either refer to Dimensional Diminution or to Dimensional Reduction.*

Proposition

Lorentz contraction can be conceptualized in terms of dimensional abatement. More specifically, it signifies dimensional diminution for $0 < v < c$ ***and dimensional reduction for*** $v = c$***.***

Proof: Consider a compact body B moving in a frame S and a moving frame S' in which B is at rest. We imagine B in S' as being made out of infinitesimal cubical volume elements oriented, without loss of generality, such that the direction of contraction in S will be normal to one of the sides. It is trivial to show that the Lorentz contraction of each cubical element in S causes it to be dimensionally abated. Since this is true of every infinitesimal volume element of B, it is true of B. ■

An Informal Preview of the Re-interpretation of Time Dilation

- **Time** is reinterpreted from an external parameter to a property of an object: Its *duration of existence in spacetime* between two spacetime events.
- **Proper time** is reinterpreted as the observed duration of existence of an object in spacetime between two spacetime events.
- **Coordinate time** is reinterpreted as the observed duration of existence of the observer in spacetime between two spacetime events (or of a class of clocks at rest with respect to the observer).
- **Ontochronic abatement:** As an object is time-dilated, its duration of existence in spacetime between two given spacetime events is diminished, up until in the limit of c, when time dilation is complete and its duration of existence in spacetime between spacetime events is exactly zero.

A Criterion for Physical Existence in Spacetime

Arguably, our understanding of nature has become so deep that in order to make further progress, we need to incorporate the concept of existence into physics. The following existence criterion, presented as an axiom, is an attempt to do so:

Criterion

A physical object exists in Minkowski spacetime if and only if it is characterized by a timelike spacetime interval.

Two Plausibility Arguments:

- Proper time is Lorentz invariant, already making it a good candidate for having ontological significance
- Three ways to fail to satisfy the criterion:
  - **Spacelike Interval**: Already considered unphysical due to causality violations/speed of light limit
  - **No spacetime Interval**: It seems reasonable to associate this with non-existence in spacetime
  - **Lightlike Interval:** Will discuss shortly

Time Dilation as Ontochronic Abatement I

Definition

Spacetime Ontic Function: *The spacetime ontic function is a map* $\exists_S : \mathfrak{O} \rightarrow \{0, 1\}$ *where* $\mathfrak{O}$ *is the set of all physical objects taken to be within the domain of physics and* $S \subset \mathfrak{O}$ *is the subset of* $\mathfrak{O}$ *of all objects that exist in spacetime. The spacetime ontic value of an object is determined by whether it satisfies the existence criterion* $(\exists_S(x) = 1)$ *or not* $(\exists_S(x) = 0)$*.*

Definition

Ontochronicity: *Ontochronicity is the quality of having a duration of physical existence.*

Time Dilation as Ontochronic Abatement II

Definition

Relative Ontochronicity: *Relative ontochronicity is the dimensionless ratio of the the observed duration of existence of an object compared to that of a reference object, usually the observer.*

$\text{ont}_{\text{rel}}(a/b) = \frac{\int d\tau_a}{\int d\tau_b}$ where

- a is the comparison object and $\int d\tau_a$ will turn out to be its proper time
- b is the reference object and $\int d\tau_b$ will turn out to be coordinate time.
- $\text{ont}_{\text{rel}}(a/b)$ is the relative ontochronicity of a to b

Note: When b is an observer observing a, we can write $\int \tau_a = \tau$, $\int \tau_b = t$ and thus $\text{ont}_{\text{rel}}(a/b) = \frac{\tau}{t}$ which is similar to, but distinct from $\gamma^{-1} = \frac{d\tau}{dt}$. In situations in which the context is clear, the definition may be relaxed to subsume γ^{-1}.

Time Dilations as Ontochronic Abatement III

Definition

Ontochronic Diminution: *Ontochronic diminution is the decrease of the observed duration of existence of an object in a given time interval by a dimensionless factor in the open interval* $(0, 1)$.

Definition

Ontic Reduction: *Ontic reduction is the reduction of the ontic value of an object to* 0.

Time Dilation as Ontochronic Abatement IV

Definition

Ontochronic Abatement: *Ontochronic abatement is a less specific umbrella term which can either refer to ontochronic diminution or to ontic reduction.*

Proposition

Relativistic time dilation can be conceptualized in terms of ontochronic abatement. More specifically, it signifies ontochronic diminution for $0 < v < c$ ***and ontic reduction for*** $v = c$***.***

Proof: Follows trivially from re-interpreting the proper time of an object as its observed duration of existence in spacetime, and coordinate time as the duration of existence in spacetime of the observer, between two given spacetime events.■

Four Unappreciated Spacetime Principles

The reinterpretation focuses attention on two invariance and two symmetry principles:

1. **Invariance of Absolute Dimensionality:** *The absolute dimensionality of any compact body is invariant under spacetime coordinate transformations.*
2. **Homodimensionality of Space:** *The dimensionality of every (maximally dimensional) space-like hypersurface of Minkowski spacetime is everywhere the same.*
3. **Invariance of Spacetime Ontic Value:** *The spacetime ontic value of any compact body is invariant under spacetime coordinate transformations.*
4. **Homodimensionality of Time:** *The dimensionality of every timelike hypersurface of Minkowski spacetime is everywhere the same.*

The Relationship Between the Spacetime Principles and the LT

1. The Lorentz transformations ensure that an object observed to be 3-dimensional in one spacetime frame will never observed to be 2-dimensional in any other spacetime frame, and vice versa.
   - Lorentz contraction obeys the invariance of absolute dimensionality.
2. The Lorentz transformations ensure that an object observed to have a finite duration of existence in spacetime in one spacetime frame will never observed to have a zero duration of existence in spacetime in any other spacetime frame, and vice versa.
   - Time Dilation obeys invariance of spacetime ontic value

Physical Existence in Spacetime as an Equivalence Relation I

- **Principles 1 and 3 together couple absolute dimensionality to spacetime ontic value.** Propositions 1 and 2 together already show that dimensional and ontochronic diminution couple to each other exactly as Lorentz contraction and time dilation couple to each other, but the two invariance principles together extend this to dimensional and ontic reduction.
- **Principles 2 and 4 together ensure that the coupling of absolute dimensionality to spacetime ontic value holds globally.** In a spacetime in which the homodimensionality of space or of time fails to hold, there could conceivably be regions in which spacelike or timelike hypersurfaces have a different dimensionality inside the region than outside, and in such regions absolute dimensionality and ontic value could decouple. The two homodimensionality principles together ensure that this does not happen.

Definition

Isodimensional: *A Spacetime is isodimensional if and only if both its timelike and maximally dimensional spacelike hypersurfaces are homodimensional.*

Physical Existence in spacetime as an Equivalence Relation II

Proposition

Physical existence in Minkowski spacetime is an equivalence relation by absolute dimensionality.

Proof: An equivalence relation is determined by the properties of reflexivity, symmetry and transitivity. Consider an n-dimensional compact object A subject to the above principles. By the the coupling of ontic value to absolute dimensionality, it must exist in an $n+1$ dimensional Minkowski spacetime region. By the isodimensionality of Minkowski spacetime, this region is, in fact, all of $n+1$ dimensional spacetime. In particular, A exists in the $n+1$-dimensional Minkowski spacetime in which it exists. This proves reflexivity. Now consider an m-dimensional compact object B. By the same argument as given for reflexivity, it must exist in an $m+1$ dimensional spacetime. Suppose A exists in the same spacetime as B. This requires that $n+1=m+1$, and, consequently, that $n=m$. But that means B has the same absolute dimensionality as A, and therefore exists in the same spacetime as A. This proves symmetry. Finally, consider an l-dimensional compact object C. By the same argument as given for reflexivity, it must exist in an $l+1$-dimensional spacetime. Now suppose that B exists in the same spacetime as C, and that A exists in the same spacetime as B. This requires $m+1=l+1$ and $n+1=m+1$, respectively, from which it follows that $n=m=l$, so A has the same absolute dimensionality as C and therefore exists in the same spacetime as C. This proves transitivity. ■

A Partition on all Things that Physically Exist *per se*

The ontic equivalence relation considered here partitions the set of all objects that physically exist *per se* into *ontic equivalence classes* such that for each $n + 1$ dimensional Minkowski spacetime, there is a corresponding equivalence class of n-dimensional objects that exist in it.

0+1 Spacetime	1+1 Spacetime	2+1 Spacetime	3+1 Spacetime	4+1 Spacetime	...
0-dimensional objects	1-dimensional objects	2-dimensional objects	3-dimensional objects	4-dimensional objects	...

Figure: A partition of all physically existing objects into ontic equivalence classes by absolute dimensionality.

Speed-of-light objects belong to a different ontic equivalence class than spacetime observers

- Can be used as an *explanation* for the impossibility of transforming to the rest frame of a speed-of-light object: If a spacetime observer could transform to a speed-of-light rest frame, he or she would no longer be a *spacetime* observer.
- That speed of light objects belong to a different ontic equivalence class than spacetime observers implies that **it is possible for something to exist without existing in spacetime**.
- The mathematics of special relativity gave us hints:
  - Duration of existence between spacetime events is always zero
  - Cannot obtain 4-volume by integrating over 3 volume in lightlike direction
  - No spacetime observer (=object) can transform to a c rest frame
- Could not indicate this directly because standard SR lacks concept of existence

Where do speed of light objects exist?

- Completely length-contracted $\rightarrow$ 2-dimensional $\rightarrow$ exist in $2+1$ dimensional spacetime
- To denote its own place as a repository of existence beside spacetime, I will call this areatime
- The mathematics of special relativity gave us hints:
  - As an object approaches c, in its frame r and t direction become more lightlike. In the limit of c, they are both lightlike $\rightarrow$ in such a frame, spacetime is a linearly dependent $\rightarrow$ in such a frame, spacetime has too many dimensions
  - In such a frame, there are only two independent spacelike directions

The Ontic Equivalence Relation May Be Applied More Generally

1. We tacitly assume the four spacetime principles apply to spacetimes other than Minkowski because spacetimes which violate them would seem sufficiently weird for us to notice the violation.
    1. Ordinarily, we take it as part of the *definition* of a pseudo-Riemannian manifold (M, g) that it is isodimensional. If M is isodimensional, then the two homodimensionality principles hold in M.
    2. The invariance of absolute dimensionality and spacetime ontic value seem to be assumed without an accompanying definition.
2. If the homodimensionality and invariance principles hold in M, then the ontic equivalence relation also holds in M.
3. **But:** It appears that singularities (e.g. inside Black holes) denote point-like spatial regions in which the four spacetime principles may not hold.

Definition

Isodimensional almost everywhere: *A spacetime is isodimensional almost everywhere if and only if it is isodimensional up to a countable number of point-like (in space) singularities .*

Can modify proof of the ontic equivalence relation so that it holds ‘almost everywhere‘ in M.

Conclusion

- This talk presented the ontic equivalence relation
- It is easy to derive, easy to understand, but has profound implications for our fundamental scientific worldview
- Why it should be taken seriously:
    - Based on one of the best-tested theories in all of science
    - Derived from four fairly self-evident principles
    - Mathematical Derivation extremely straightforward

Thank you!