ESTIMATION OF ECONOMIC RELATIONSHIPS*

Chairman: Harry Markowitz, The Rand Corporation

THE DYNAMICS OF THE ONION MARKET

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Introduction

THE BEHAVIOR of onion prices and production clearly reflects the operation of powerful dynamic forces. In fact, the systematic annual oscillations of prices and outputs shows the onion market to be almost a textbook example of an agricultural cobweb system. It is the purpose of this paper to capture this system in a three equation econometric model and to analyze its dynamic properties.

The econometric model will be derived from the aggregate behavior of the market, i.e. for the United States as a whole on a crop year basis. This abstraction from the regional nature of onion production, the seasonal timing of crops and the attendant problems of speculation and storage, provides us with only a first approximation to the nature of the underlying market structure. The analysis of the regional structure of the market will be carried forward at a later date. As an example of the kind of analysis required, however, we shall include here some interesting relationships between the early spring crop (South Texas) and the stock of onions in storage on January first, although as will be indicated, the validity of these preliminary results is open to question.

The Model

The model of the aggregate onion market consists of three relationships: (1) a supply schedule relating the quantity of onions available for harvest to prices and costs of the preceding year, (2) a demand equation relating the per capita consumption of onions to farm price and per capita disposable income, and (3) an unharvested crop equation in which the quantity of onions unharvested is related to current price and harvesting cost.

Supply

The fitted supply schedule for onions is the following:

$$\text{(1)} \qquad \text{Log } Q_t = .324 \text{ Log } P^f_{t-1} - .512 \text{ Log } C_{t-1} + .0123t + .134$$

Where Q is number of 50 pound sacks available for harvest, P^f is the

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farm price of onions, C is the prices paid index and t is time in years, measured from 1924 = 0. For ease in computation the price is measured in units of 10 cents, quantity in units of 10 milion sacks, and the prices paid index in units of ten points. Such coding, of course, leaves the regression parameters unaffected, but influences the magnitude of the constant term.

The regression parameters of the supply equation were fitted by least squares to first differences. The constant term was then obtained by fitting the equation to the means of the actual (i.e. undifferenced) values over the period. The method of first differences was used for two reasons. First it serves to avoid any major bias in the estimates that might otherwise arise due to autocorrelation of residuals. Secondly, the prices paid index is a very crude measure of production costs. To the extent that production costs tend to be sticky, the first differences in price alone will tend to produce a useful estimate of price elasticity, whereas any attempt to estimate the price elasticity of supply directly from undifferenced prices alone will necessarily lead to spurious results.[1]

The fitted difference equation was:

$$(1^*) \qquad \Delta \text{ Log } Q_t = \underset{(.06)}{.324\Delta} \text{ Log } P^f_{-1} - \underset{(.3)}{.512\Delta} \text{ Log } C_{-1} + .0123$$

where figures in parentheses are standard errors. The coefficient of multiple correlation was $\bar{R} = .73$. The supply schedule indicates a price elasticity of about .3 and a cost elasticity of about −.5. The coefficient of the time variable indicates a trend over the period of roughly 3 percent per year. It will be noted that the price elasticity is measured with considerable precision, while the cost elasticity is less certain. This uncertainty arises from the sticky nature of the prices paid index as noted above.

The comparison of the actual onion crop with that calculated from equation (1) is given in Figure 1. It will be noted that the performance leaves much to be desired, particularly in the early period.

Demand

Like supply, the estimated demand schedule is in logarithmic form and was obtained by a least squares regression in first differences and then fitted to the averages of the period. The result obtained was:

$$(2) \qquad \text{Log } P^f_t = -2.27 \text{ Log } (D/N)_t + 1.31 \text{ Log } (Y/N)_t + .681$$

where D/N is crop year demand per capita and Y/N is per capita disposable income. Crop year demand was measured as crop less un-

[1] We may note in this connection that the elasticity of supply as estimated from first differences in price only is .30, differing only slightly from that given in equation (1).

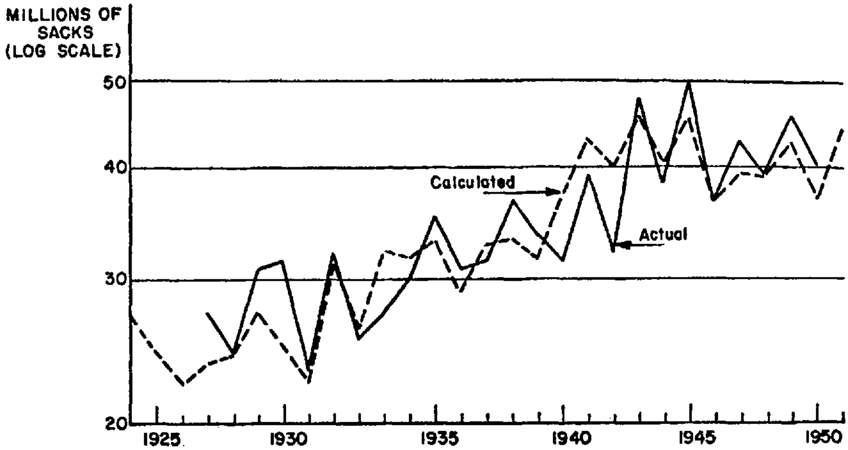

FIG. 1. ONION PRODUCTION, ACTUAL AND CALCULATED FROM EQUATION (1), 1924-1951.

harvested crop less net exports. Disposable income was measured annually on a calendar year basis, and hence actually leads the crop year by roughly one quarter.

The first differences were fitted in a homogeneous form, no allowance being made for a trend in demand. The homogeneous regression in first differences was:

(2*) $$\Delta \text{ Log } P^f_t = -\underset{(.4)}{2.27}\Delta \text{ Log } (D/N)_t + \underset{(.2)}{1.31}\Delta \text{ Log } (Y/N)_t$$

This regression was fitted to the period 1929-1952; the coefficient of multiple correlation was $\bar{R} = .9$.

Equation (2*) compares favorably with that obtained by Shuffett[2] who related farm price to per capita crop and disposable income, his result being, in our symbols,

$$\Delta \text{ Log } P^f_t = -\underset{(.2)}{2.227}\Delta \text{ Log } (Q/N)_t + \underset{(.3)}{1.111}\Delta \text{ Log } (Y/N)_t + .007.$$

When equation (2) is transformed into the usual demand form, it becomes:

(2.1) $$\text{Log } (D/N) = -.44 \text{ Log } P^f + .58 \text{ Log } (Y/N) + .300.$$

It is evident that the price elasticity of demand is about $-.4$ and the income elasticity is approximately .6. The comparison of actual and calculated price is shown in Figure 2.

[2] D. Milton Shuffett, *The Demand and Price Structure for Selected Vegetables*, U. S. Department of Agriculture, Technical Bulletin No. 1105, December 1954, p. 116.

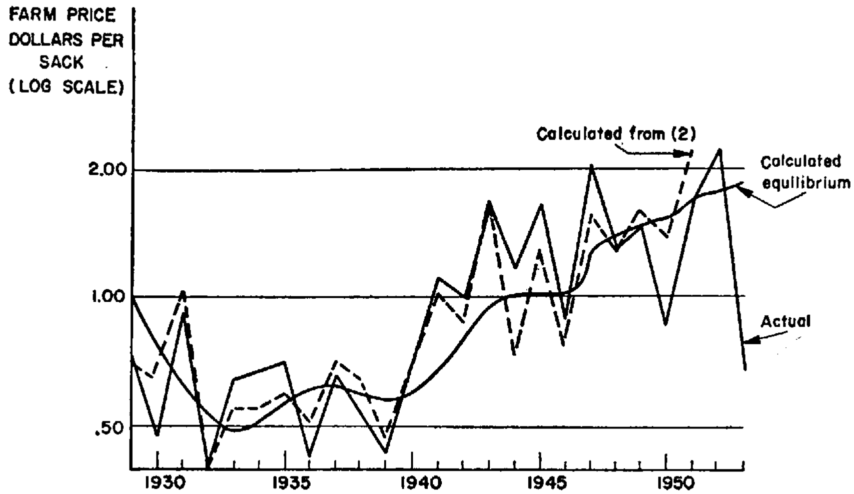

FIG. 2. FARM PRICE; ACTUAL, CALCULATED FROM EQUATION (2), AND CALCULATED EQUILIBRIUM.

Unharvested crop

The final equation in the aggregate model relates the unharvested crop (U) to current price (P^s), harvesting cost (W), the composite farm wage rate, and the quantity available for harvest (Q). The relatively small number of years for which unharvested amounts were reported precluded the use of first differences in the investigation of this relationship, and the equation was fitted to undifferenced values of the (logarithmic) variables.

The price variable used to explain the unharvested crop was the average New York wholesale price for the third quarter of the calendar year. This decision was made on the ground that the average price of onions for the *crop* year necessarily includes prices received for crop sold

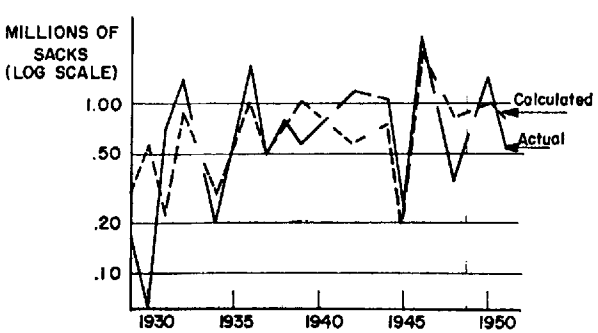

FIG. 3. UNHARVESTED CROP, ACTUAL AND CALCULATED.

long after the decision to harvest must be made. Such prices clearly cannot enter into the determination of the unharvested crop.

Since price and unharvested quantity are mutually determining variables in the onion market, the method of instrumental variables was employed to estimate the relationship. For this purpose, per capita onion production, per capita disposable income, and the industrial wage rate were taken as instrumental variables.

The resulting relationship obtained was:

$$\text{(3)} \qquad \text{Log } U_t = -1.71 \text{ Log } P_t^3 + .22 \text{ Log } W_t^f + 2.56 \text{ Log } Q_t + 1.84$$

Since the method of instrumental variable was employed, no standard errors are available. However, as the comparison of actual and calculated unharvested crop of Figure 3 shows, the result is of reasonable reliability. Taken at its face value, equation (3) indicates that a one percent decline in the average market price of the third quarter is accompanied by a 1.7 percent decrease in the unharvested crop, while one percent increases in farm wage rate and crop available for harvest tend to increase the unharvested quantity by .2 percent and 2.6 percent, respectively.

Equation (3) relates price to the unharvested portion of the onion crop, given the total crop. It may therefore be used to derive the elasticity of market supply given the crop of onions available for market. This elasticity may be derived as follows:

If X is the quantity of onions supplied to the market, then by definition the price elasticity of supply is

$$\text{(3.1)} \qquad E = \frac{P}{X}\frac{dX}{dP} \cdot \text{ Moreover, Q, X and U are related by}$$

$$\text{(3.2)} \qquad X = Q - U. \quad \text{Whence, given Q,}$$

$$\text{(3.3)} \qquad \frac{dX}{dP} = -\frac{dU}{dP}$$

Now from (3), given Q and W^f,

$$\text{(3.4)} \qquad \frac{1}{U}\frac{dU}{dP} = -1.71(1/P)$$

substituting (3.3) and (3.4) in (3.1) then yields

$$\text{(3.5)} \qquad E = 1.71U/X$$

It will be noted that the market supply elasticity, given crop, approaches zero as price approaches the level at which the entire crop is harvested ($U = 0$). The elasticity of market supply tends to rise as price falls below this level. At any realistic value for U/X, however, the elasticity of market supply is well below the elasticity of crop supply. In fact equality between the two elasticities would be reached at a value for U/X of about 20 percent i.e. at a price at which about 16 percent of the total crop would be left unharvested. This may be compared with a maximum observed unharvested crop of about 5 percent of total available.

Because the unharvested crop is an unimportant portion of the market, this relationship was suppressed in the analysis of the market dynamics.

Dynamics of the Market

We may now combine the demand and supply equations together to obtain a difference equation (5) expressing the dynamic behavior of onion prices. This equation can then be analysed to determine: (5.2) the equilibrium value that price would approach, other factors remaining stationary, (5.5) the course that price would tend to follow in approaching this equilibrium, and the speed with which it would tend to approach it, (5.7) the trend of this equilibrium price if other factors tend to follow prescribed trends, and finally (5.8) the complete dynamic expression for the tendency of price behavior given the trends in other factors.

Combining the demand and supply equations together requires a slight adjustment. It will be noted that the supply equation is in terms of total crop, while the demand is in terms of per capita disappearance. To enable us to make the required substitution, therefore, we calculated the average relationship between these two measures over the period 1929-1952:

$$\text{(4)} \qquad \text{Log}\,(D/N) = \text{Log}\,Q - \text{Log}\,N - .0178$$

Inserting (1) and (4) in (2) yields the following difference equation in farm price P^f:

$$\text{(5)} \qquad \text{Log}\,P^f_t = -\,1735\ \text{Log}\,P^f_{t-1} + 1.162\ \text{Log}\,C_{t-1} - .028t + 2.27\ \text{Log}\,N_t + 1.31\ \text{Log}\,(Y/N)_t + .416$$

The symbols have the meanings proviously assigned. It will be recalled that $t = 0$ in 1924.

Equation (5) may be looked upon as the price forecasting equation based on the fitted supply and demand schedules. The performance of this forecasting equation is shown in Figure 4. It must be borne in mind that at no point in the analysis have we made any allowance for the influence of the war on the behavior of the model. In view of this fact the forecasting equation may be judged to have performed reasonably well for the period after 1935, since, although it badly underestimates the level of prices during the war period, its estimate of year-to-year changes is good.

To determine the equilibrium value of P^f under the conditions prevailing at time t, we may set $P_t{}^f = P_t{}^f{}_{-1} = P_e{}^f$ (t) and solve to obtain

$$\text{(5.2)} \qquad \text{Log}\,P^f_e(t) = .67\ \text{Log}\,C_{t-1} - .016t + 1.31\ \text{Log}\,N_t + .76\ \text{log}\,(Y/N)_t + .24$$

where P_e^f (t) is the value of farm price that would be approached if the conditions prevailing in year t were to continue indefinitely. Among the conditions of year t is specifically included the time indicator t itself. It may appear at first sight, therefore, that we are simultaneously assuming that time is to continue indefinitely and is also to stand still. Such is not the case. The supply of onions is influenced by a host of technical factors whose improvement over the period must be taken into account.

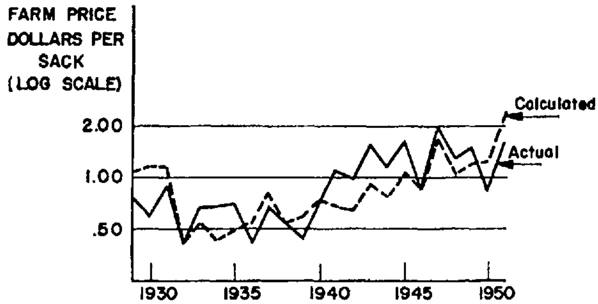

FIG. 4. FARM PRICE, ACTUAL AND CALCULATED FROM EQUATION (5).

In using t for this purpose, it becomes a *dummy* variable representing the stage of technology. Thus in holding t constant in equation (5.2) we are supposing the level of technology to remain unchanged. We do this with precisely the same justification with which we may suppose population or per capita disposable income to remain unchanged.

The equilibrium values associated with each time t are plotted together with actual farm prices in Figure 2. With the exception of the war period we observe that current prices tend to fluctuate around the relatively smooth equilibrium series.

Equation (5) enables us to analyze the fluctuations of current price around its equilibrium value. Holding all other things constant (5) becomes

(5.3) $$\text{Log } P_t^f = -.735 \text{ Log } P_{t-1}^f + H$$

where H is the value of the remaining variables on the right of (5).

Since the equilibrium values of (5.2) also satisfy (5.3) we may subtract to obtain

(5.4) $$\overset{*}{P}_t = -.735\overset{*}{P}_{t-1}$$

where P_t^* is the deviation of current (log) farm price from equilibrium at time t. The solution of this difference equation is clearly

(5.5) $$\overset{*}{P}_t = \overset{*}{P}_0(-.735)^t$$

where P_t^* is the deviation from equilibrium t years after the deviation P_0^*.

We may conclude that current farm price tends to perform a damped oscillation around its equilibrium value. The period of this oscillation is clearly two years, a price above equilibrium tending to be followed by one below. Each full cycle tends to bring the (log) price half way toward its equilibrium position, and 90 percent of the deviation from equilibrium is recovered in about 7 years.[3]

Finally we may relax our assumption that other things remain equal and ask what the course of onion prices would tend to be if other things follow specified trends. For this purpose let us assume a secular increase in per capita disposable income of 3.0 percent per annum, in population 1.8 percent per annum, and in supply technology at the observed rate of 3.0 percent per annum. The prices paid index will be taken as fixed. Translating into logarithmic form we then have

$$\text{Log}\,(Y/N)_t = \text{Log}\,(Y/N)_0 + .013t$$
$$\text{Log}\,N_t = \text{Log}\,N_0 + .008t$$
$$C_t = C_0$$

where the indicator t represents number of years after the initial position $t = t_0$.

Substituting these values in (5) yields an equation of the form

$$\text{Log}\,P_t^f + .735\,\text{Log}\,P_{t-1}^f = H_0 + .007t \tag{5.6}$$

where the right side represents the combined trend influence of the other variables t years after an initial position at $t = 0$. The moving equilibrium of (5.6) is readily determined to be

$$\text{Log}\,\hat{P}_f = \frac{H_0 + .735}{1.735} + .004t \tag{5.7}$$

This represents an upward trend in price of slightly less than one percent per year. In view of the nature of the statistical equations from which this result is derived, it is hard to credit it with significance; but we are surely safe in concluding that the general trend in onion prices under the reasonable assumptions made is of a very low order of absolute magnitude and clearly negligible in any short period.

The combination of (5.7) and (5.4) gives us the complete dynamic equation of onion prices;

$$\text{Log}\,P_t^f = \frac{H_0 + .735}{1.735} + .004t + P_0^*(-.735)^t \tag{5.8}$$

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[3] To obtain this result we set

$$(.735)^t = .10 \quad \text{so}$$
$$t\,\log .735 = \log .1 \quad \text{or}$$
$$-.134\,t = -1.0 \quad \text{and} \quad t = 7.46$$

where P_0^* is the deviation of (log) price from moving equilibrium at some initial time $t = 0$. (5.8), of course, represents the oscillation of (5.5) around the slightly rising trend in prices of (5.7).

"Fine structure" Dynamics: An Example

The foregoing analysis has been entirely based on the behavior of the entire market, averaged over a complete crop year. But the fact that onions are grown in different regions, the crops maturing at different times, means that there is a fine structure to the dynamics of the market, defined by the relationships among these regions and harvest periods. It is our hope that we can ultimately capture the nature of this fine structure in a more detailed model of the market. At present we are able to present only a particular example of the "fine" structure of the dynamics.

The problem to be analyzed is the relationship between the stock of onions on January 1 and the early spring crop in south Texas which ordinarily becomes available for market late in the first quarter or early in the second quarter of the year. In as much as the stock of onions on hand represents potential competition for early onions, and since growers are informed about the magnitude of this stock at the time planting is done, we should expect the early supply of onions to be influenced by the stock on hand.

To determine this influence, a supply schedule was fitted to the south Texas region, relating the early onion crop, (Q_t^e) to last year's farm price in the region (P_{t-1}^e), to last year's farm price of Texas carrots (C_{t-1}^e) and the per capita stock, January 1, $(S/N)_t$. The price of carrots was used as an opportunity cost to marginal onion producers.

In fitting the regression the extreme values for the year 1941 were deleted from the data, the period 1929-1953 being used. The resulting supply equation for south Texas was

$$(6) \quad \text{Log } Q_t^e = \underset{(.099)}{.437} \text{ Log } P_{t-1}^e - \underset{(.112)}{.353} \text{ Log } C_{t-1}^e - \underset{(.149)}{.520} \text{ Log } (S/N)_t + .070$$

The elasticities of price and cost in this supply equation compare favorably with those obtained for the nation as a whole. Moreover we see that the influence of the stock of onions on south Texas production is highly significant, and indicates that an increase of one percent in the per capita stock of January 1 tends to reduce the south Texas onion crop by one half of one percent.

The equation was fitted to undifferenced values but a test of the residuals fails to show significant auto correlation.

We may now ask whether there is a reverse influence between the early crop of onions and the sale of the stock. The argument runs as follows. Although the stock of onions on January 1 is historically given, the time distributions of its sale is subject to the decision of the holder. If a large early crop is anticipated, the stock will tend to be sold early to avoid the falling price which the competition of the spring onions would cause. On the other hand the expectation of a small crop should tend to postpone the sale of stock to take advantage of the improved market. Thus there should be a significant relationship between the shipment of stock during the first quarter (before the early crop is marketed) and the size of the early crop. For the years 1936 to 1948 the percent of stock shipped by months, January through May, have been estimated.[4] By simple addition one can calculate the percent of shipment that occur in the first quarter. The relationship between this percentage (I), the stock on hand (S) and the early spring crop (Q^e) was determined to be

$$\text{(7)} \qquad \text{Log } I = .106 \underset{(.03)}{\text{Log } Q^e} - .021 \underset{(.05)}{\text{Log } S} + .915$$

This relation says that the percentage of stock that will be supplied to the market during the first quarter of the year is significantly related to the size of the approaching harvest (which will come on the market during the second quarter) and not significantly related to the actual size of the stock.

It is tempting at this point to combine equations (6) and (7) together with a supply schedule for late summer onions (from which the stock is derived) and the demand, to work out the dynamic structure of this part of the market. That we have not done so is due to the serious doubts we have regarding the validity of equation (7).

Equation (7) is clearly compatible with the hypothesis we had in mind in deriving it. Our doubts concern the data employed in fitting it. Although the figures used purported to be estimates of the percentage of *stock* shipped, we are inclined to believe they were obtained from a percentage distribution of *shipments.* In that case the percentage of shipments occurring in the first quarter is directly determined by the amount shipped thereafter and equation (7) is compatible with another, and in our opinion more likely hypothesis: that onions are shipped from storage until storage onions can no longer sell in competition with the new harvest, at which time they are dumped.

Given the data, equation (7) cannot distinguish between the two hypotheses. Nor, since both are doubtless valid to some extent, can it separate one effect from the other.

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[4] *Production and Marketing of Commercial Onions,* Production & Marketing Admin., Fruit & Vegetables Div., January 27, 1949, Mimeo., Table 9.