1	Effect of Source Blur on Digital Breast Tomosynthesis Reconstruction
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30 ABSTRACT

Purpose: Most digital breast tomosynthesis (DBT) reconstruction methods neglect the blurring of the projection views caused by the finite size or motion of the x-ray focal spot. This paper studies the effect of source blur on the spatial resolution of reconstructed DBT using analytical calculation and simulation, and compares the influence of source blur over a range of blurred source sizes.

36 Methods: Mathematically derived formulas describe the point spread function (PSF) of source blur on the detector plane as a function of the spatial locations of the finite-sized source and the 37 object. By using the available technical parameters of some clinical DBT systems, we estimated 38 the effective source sizes over a range of exposure time and DBT scan geometries. We used the 39 CatSim simulation tool (GE Global Research, NY) to generate digital phantoms containing line 40 41 pairs and beads at different locations and imaged with sources of four different sizes covering the range of potential source blur. By analyzing the relative contrasts of the test objects in the 42 reconstructed images, we studied the effect of the source blur on the spatial resolution of DBT. 43 Furthermore, we simulated a detector that rotated in synchrony with the source about the rotation 44 45 center and calculated the spatial distribution of the blurring distance in the imaged volume to estimate its influence on source blur. 46

Results: Calculations demonstrate that the PSF is highly shift-variant, making it challenging to accurately implement during reconstruction. The results of the simulated phantoms demonstrated that a typical finite-sized focal spot (~0.3 mm) will not affect the reconstructed image resolution if the x-ray tube is stationary during data acquisition. If the x-ray tube moves during exposure, the extra blur due to the source motion may degrade image resolution, depending on the effective size of the source along the direction of the motion. A detector that rotates in synchrony with the source does not reduce the influence of source blur substantially.

54 **Conclusions:** This study demonstrates that the extra source blur due to the motion of the x-ray 55 tube during image acquisition substantially degrades the reconstructed image resolution. This effect cannot be alleviated by rotating the detector in synchrony with the source. The simulation
results suggest that there are potential benefits of modeling the source blur in image
reconstruction for DBT systems using continuous-motion acquisition mode.

59 Keywords: digital breast tomosynthesis, image reconstruction, x-ray focal spot blur,

60 geometric unsharpness, spatial resolution

61 **1. Introduction**

62 DBT reconstruction methods usually neglect the blurring of the projection views (PVs) caused by the finite size of the x-ray focal spot. In a DBT system, the focal spot of the x-ray tube 63 has a nominal size of around 0.3 mm¹⁻⁴. To date, the U.S. Drug & Food Administration (FDA) 64 has approved four breast imaging systems for tomosynthesis. These systems are SenoClaire (or 65 the new model Pristina) by GE Healthcare, Selenia Dimensions by Hologic, Mammomat 66 Inspiration by Siemens and Aspire Cristalle by Fujifilm. The GE Pristina system operates in the 67 step-and-shoot mode where the x-ray tube essentially stops at each angular location and exposes 68 the projection image. The other three systems operate in a continuous-motion mode where the x-69 ravs are generated within a short pulse at each angle while the gantry is continuously moving 70 during a DBT scan. While the continuous-motion mode can potentially reduce the total scan time 71 72 and the motion blur, it may cause additional source blur along the direction of the source motion. This effect has been found to be an image-quality degrading factor in several studies ⁵⁻⁹. A pure 73 step-and-shoot mode can alleviate this problem. However, the time that the x-ray tube can be 74 75 stationary is always limited. If the x-ray exposure time exceeds the time that the x-ray tube is stationary, there can be some extra source blur although the amount of motion blur is still less 76 than that in continuous-motion DBT systems ^{7, 8}. 77

78 Several studies examined source blur in CT reconstruction. For fan-beam CT, Hofmann et al. studied the effect of modeling the source's ray profile ^{10, 11}. They used a simulated phantom to 79 estimate the critical size for the focal spot that affects the image reconstruction quality and 80 concluded that for common fan-beam CT systems, the size of the focal spot can be neglected in 81 image reconstruction. Tilley et al. studied the effect of modeling the source blur and detector blur 82 for flat-panel cone-beam CT (FP-CBCT)^{12, 13} and demonstrated that modeling the source blur 83 can significantly improve the reconstructed image quality. The reconstruction method proposed 84 85 in their study considered the source blur to be shift-invariant, greatly simplifying its

implementation in the system model. A DBT system also uses cone-beam x-ray and a flat-panel
detector, but the geometry of DBT is very different from that of FP-CBCT. In DBT, the imaged
volume is closer to the detector and the imaged object is much thinner than those in body CT, so
the magnification factor and its variation over the depth of the imaged volume are smaller. The
spatial resolution requirement for DBT is much higher than in CBCT because microcalcifications
have a size range of about 0.1 to 0.5 mm.

This paper describes our study of the effect of source blur on image quality for DBT through 92 analytical calculation and simulation. We first define parameters that describe the geometry of 93 the finite-sized x-ray source. We choose our simulated blurred source sizes based on the range 94 estimated from the three commercial DBT systems that use the continuous-motion data 95 acquisition mode. We then demonstrate by analytical calculation the spatial variance of the 96 source blur over the detector field of view (FOV). Next, we report our CatSim^{11, 14} simulations 97 of DBT imaging systems with a finite-sized focal spot. Two phantoms with line pairs and beads 98 (BB) are configured and imaged with four focal spot sizes for evaluation of the reconstructed 99 image resolution. We analyze the relative contrast curves of these objects in the reconstructed 100 101 DBT when different-sized sources are used to simulate the projections in comparison to those obtained from an ideal point-source DBT system, which can be considered a DBT reconstruction 102 with perfect system modeling to correct for the source blur. For DBT systems with a continuous-103 motion x-ray source and a detector moving in synchrony with the source about the rotation 104 105 center, the source blur may be partly reduced although both the x-ray source and the detector still move relative to the objects being imaged. We compare the source blur of DBT systems with 106 107 moving detector and stationary detector by analyzing the spatial distributions of geometric unsharpness in the imaged volume at different projection angles. These results illustrate 108 109 constraints in designing DBT systems and under what conditions modeling the finite-sized x-ray 110 source may improve the reconstructed image quality.

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2. Materials and Methods

114 2.1 Simplified model for the source blur

Figure 1 shows the geometry of a typical DBT system where the source rotates in a plane 115 tangential to the chest wall of the patient. This study uses a simplified model for source blur that 116 treats the x-ray source as a rectangle with uniform x-ray emission on the anode surface, shown as 117 the blue rectangle. We define x-y-z coordinates for the imaged volume and t-s coordinates for the 118 detector. The origin x, y, z = 0 (marked as O in Figure 1) is the rotation center (the point where the 119 rotation axis intersects with the rotation plane of the source) and $t_{s} = 0$ is its perpendicular 120 projection on the detector. We denote d_{so} and d_{od} the distance from the x-ray source to the 121 rotation center and the distance from the rotation center to the detector, respectively. The center 122 of the finite-sized x-ray source is at the original location of the ideal point source. The rectangle 123 of the focal spot is described with three parameters: its sizes along two directions h_1 and h_2 and 124 the target angle ϕ . ϕ is usually smaller than 45°. Figure 1 shows the case where the projection 125 angle θ is 0°. If the projection angle θ is not 0°, the blue rectangle will tilt by the same angle θ 126 such that the h_2 edge of the rectangle is parallel to the direction that the x-ray source is moving. 127

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129 2.2 Estimation of the h_1 and h_2 for DBT systems with continuous-motion data acquisition

For commercial DBT systems that use a continuous-motion mode, the nominal size of the focal spot, $h_{nominal}$, can be found in their technical documents (see Table A1 in Appendix A). The nominal focal spot size refers to the effective size of the focal spot of the central ray (i.e., the ray perpendicular to the detector plane when the scan angle is 0°) when the source is stationary. Therefore h_1 can be calculated given $h_{nominal}$ and the target angle ϕ :

$$h_1 = h_{\text{nominal}} / \sin \phi \tag{1}$$

The value of h_1 remains the same even when we consider the motion of the source. The effective h_2 , on the other hand, depends on the motion of the source. For DBT systems with continuous-motion x-ray source, the motion during data acquisition results in additional blurring of the finite-sized focal spot and increasing the effective h_2 . Assuming the source is moving with a constant speed, the source blur along the direction of the motion can be approximated by the convolution of two rectangle functions, one with the width of $h_{nominal}$ and the other with the width of the distance that the source moves, denoted as h_{motion} . The result of the convolution is trapezoidal and occasionally triangular (when $h_{\text{motion}} = h_{\text{nominal}}$). For the worst-case scenario, we consider the width of the non-zero part of the convolution result to be the effective h_2 :

$$h_2 = h_{\rm motion} + h_{\rm nominal}.$$
 (2)

Therefore, for simplicity, we simulated the focal spot to be a rectangle at the x-ray anode location (Figure 1) with an effective width of h_2 in the motion direction given by Eq. (2) to approximate the total effect of convolving the focal spot blur function with the motion blur function in the CatSim simulation to produce the projection images used in our study. This rectangular focal spot, however, will produce focal spot point spread function (PSF) that is spatially variant on the detector plane, as described in Section 2.3 and Section 3.1.

Assuming a constant speed of the source for continuous-motion DBT systems, we can 150 estimate the speed given the distance from the source to the rotation center, the total acquisition 151 angle and the total exposure time. We obtained the typical total current-time product (mAs) of 152 the three commercial systems for different breast thicknesses from their quality control 153 documents or FDA's summary of safety and effectiveness data (SSED) online. The exposure 154 time per PV can be estimated from the total mAs, the current and the total number of projections. 155 The distance that the source travels during the exposure of one PV (h_{motion}) is the product of the 156 speed of the source and the exposure time per PV. Tables A1 – A4 in Appendix A show the 157 geometric parameters, technical details and the references for the three commercial DBT systems. 158 For most breast thicknesses, the source motion contributes significantly to the effective h_2 , 159 which can be as large as 1.6 mm according to these calculations. Although the technique factors 160 may not be exactly the same as those used clinically, the estimated h_2 values provide a reference 161 range for our study. 162

As seen in Tables A1-A4, the design parameters of commercial DBT systems vary and it is 163 difficult to compare the relative impact of source blur on image resolution in the presence of 164 other confounding factors from different scanning geometries or system design parameters. As it 165 166 is not our purpose to analyze or compare commercial DBT systems, we instead simulate a fixed DBT system geometry that has a range of effective x-ray focal spot sizes covering the potential 167 motion range of the source estimated in the tables. We then demonstrate the spatial variance of 168 source blur and compare the impact of different degrees of source blur on image resolution under 169 170 the same image acquisition and reconstruction conditions.

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172 2.3 Spatial variance of the source blur PSF for DBT system

We used a pinhole array that was parallel to the detector plane to calculate the effective shape 173 174 and size of the focal spot as seen on the detector plane. A pinhole is traditionally used to experimentally measure the x-ray focal spot size ^{15, 16}. The projection of a finite-sized source 175 through the pinhole represents the blurring for a point object at the pinhole's location due to 176 geometric unsharpness and can be considered to be the source blur PSF for the location. Such a 177 178 source blur PSF depends on the distance from the detector and the spatial location of the object on the x-y plane. Therefore, the projection image with source blur cannot be obtained by 179 convolution of a PSF with the ideal projection image of a whole volume. 180

We modeled the imaging geometry of the GE second generation (GEN2) prototype DBT 181 system and the spatial variance of the source blur PSF. Different DBT systems may have 182 183 different geometry (e.g., scan angle, angular increments) but the observed trends of the effects of the source blur PSF should be applicable to other geometries. For this system, the x-ray tube 184 rotates in 3° increments to acquire 21 projection images within $\pm 30^\circ$. The digital detector is 185 stationary during the acquisition, i.e., $h_{\text{motion}}=0$. The system uses a CsI phosphor/a:Si active 186 matrix flat panel detector with a pixel size of $0.1 \times 0.1 \text{ mm}^2$. The distance from the source to the 187 rotation center, denoted as $d_{so,GEN2}$, is 64 cm. The distance from the imaged volume to the digital 188 detector is 2 cm, denoted as $d_{od,GEN2}$. The target angle is $\phi_{GEN2} = 22.5^{\circ}$. 189

During image reconstruction, the x- and y-dimensions of the voxel are chosen to be 0.1 mm², 190 the same as the pixel size of the detector and the z-dimension of the voxel is chosen to be 1 mm. 191 We chose this voxel size because it is typical for DBT reconstruction in the literature and in 192 several of our previous studies $^{17-20}$. Even if reconstruction at smaller pixel size such as 0.05 \times 193 0.05 mm² in-plane resolution can be performed to take advantage of super-resolution ^{19, 21} or for 194 DBT systems with actual detector pixels smaller than $0.1 \times 0.1 \text{ mm}^2$ (Table A1), such high 195 196 resolution has not been implemented in routine clinical use due to consideration of many factors such as data set size and workflow efficiency. In addition, due to geometric magnification the 197 198 Nyquist frequency at a specific plane of the reconstructed volume is higher than the Nyquist frequency of the detector, making a smaller reconstruction voxel size desirable for some 199 200 applications. However, since our purpose is to evaluate the source blur that may affect 201 commercial systems, the study of source blur at high resolution reconstruction is beyond the202 scope of the current study.

We analytically calculate the source blur PSF over the detector plane for the GEN2 System (see Appendix B for the formulas). Instead of using the detector size $192.0 \times 230.4 \text{ mm}^2$ of the system, the detector size is set to be $240.0 \times 300.0 \text{ mm}^2$, which is closer to the detector size of commercial DBT systems ⁴. The nominal size of the x-ray source is 0.3 mm. Therefore we can derive the values for h_1 and h_2 :

$$h_{1,\text{GEN2}} = \frac{0.3}{\sin \phi_{\text{GEN2}}} = 0.78 \text{ mm},$$
 $h_{2,\text{GEN2}} = 0.3 \text{ mm}.$
(3)

Starting from the point t = 10 mm, s = 0 mm, we set up an array of locations every 20 mm along both the t- and s-direction. To illustrate the spatial variations in the source blur PSF, we calculate the PSFs for each location of this array. Using $z_{pinhole}$ to denote the plane of the pinhole array, we study the following two conditions:

212 **Condition A**: $h_1 = 0.78 \text{ mm}$, $h_2 = 0.3 \text{ mm}$, $\phi = 22.5^\circ$, $z_{\text{pinhole}} = -\frac{d_{\text{so,GEN2}}\cos\theta - d_{\text{od,GEN2}}}{2}$ 213 depending on the projection angle θ ;

214 **Condition B**: $h_1 = 0.78$ mm, $h_2 = 0.3$ mm, $z_{pinhole} = -50$ mm.

For Condition A we used a large $z_{pinhole}$ value to illustrate the geometry shape variation of the source PSF over the object plane. In Condition B the source size was chosen to be the typical 0.3 mm. $z_{pinhole}$ was also chosen to simulate a typical depth of the object in a DBT scan.

218 2.4 Configuration of CatSim simulation

As the results in Section 3.1 show, the source blur PSF is highly variant in DBT, making 219 modeling this effect very challenging in image reconstruction. Therefore we used CatSim ^{11, 14} 220 (GE Global Research, NY) to simulate projection images in DBT with finite-sized x-ray sources 221 to study the effect of source blur on the reconstructed images. A range of effective focal spot 222 sizes was used to simulate projections of objects at different spatial locations for a wide range of 223 224 projection angles. The analysis of the resolution of the resulting reconstructed images provides useful information of the limitation of the effective focal spot size (or source motion) on the 225 design of DBT systems and the potential benefits of trying to correct for source blur in DBT 226 reconstruction under certain imaging conditions. 227

We simulated four sets of parameters for the source as specified in Table 1. As a reference 228 point, Source 0 was the ideal point source. Source 1 had the standard nominal size and the target 229 angle of the GEN2 System, as expressed in Eq. (3). For Source 2 and Source 3, we increased the 230 value of h_2 to 1.0 mm and 2.0 mm to simulate the influence of the source motion during the 231 image acquisition, since the effective h_2 could be as large as 1.6 mm according to Table A2 and 232 Table A4. Given the uncertainties in those estimates, we chose $h_2 = 2.0$ mm as an upper bound 233 of the source blur. The comparison of Source 1, Source 2 and Source 3 will demonstrate the 234 effect of the source motion on the reconstructed image resolution, while the comparison between 235 Source 0 and the other three sources will indicate the potential improvement in resolution by 236 237 modeling the source blur in DBT reconstruction.

We configured the geometry of the GEN2 DBT system in CatSim. We simulated a complete 238 set of 21 projections every 3° from -30° to 30°. The detector pixel pitch was $0.1 \times 0.1 \text{ mm}^2$, 239 and had a size of 2400 × 3000 pixels. The x-ray source was an Rh target/Rh filter x-ray tube 240 and the kilovoltage was set to 29 kV. We used an oversampling rate of 10×10 per pixel for the 241 detector. The oversampling rate was the number of rays traced per pixel or per object to simulate 242 a high resolution analog projection image with CatSim^{19, 22}. The oversampling rate for Sources 243 1-3 was set to 6 since our simulation showed that a higher oversampling rate provided negligible 244 improvement in the simulation accuracy. 245

We configured two digital phantoms in this study. The first phantom contained lead line pairs (LP) and lead beads (BBs), referred to as the LPBB phantom. The second phantom only contained BBs of calcium carbonate to simulate the microcalcifications (MC) in DBT, referred to as the MC phantom. Both phantoms were analytically specified in configuration files using the FORBILD syntax ¹¹. The quantum noise, detector blur and noise, and the scattered radiation were turned off (assumed to be 0) and the detector absorbed all incident photons so that we could focus on the investigation of the effects of the source blur on DBT reconstruction.

To study the location dependence of the source blur, we placed multiple groups of highcontrast LPs and BB pairs at different locations. We first configured a group of objects called the base group (Figure 2). Then we shifted the base group to different locations to generate multiple groups of the same objects (Figure 3).

Figure 2 shows the base group of the LPBB phantom containing 15 sets of objects. The distance from each object to the bottom of the imaged volume was chosen to be 50.6 mm so that

the objects were located approximately at the center of the in-focus slice (slice 51 from the 259 bottom of the imaged volume or the breast support plate) when the DBT was reconstructed at a 260 slice thickness of 1 mm. Each set contained a pair of small BBs with their center-to-center line 261 262 oriented at 45° to the x-direction and two sets of line pairs along the x- and y-direction with the same spatial frequency. Each group of line pairs consisted of five lead bars and four spacings, i.e., 263 4.5 line pairs, with the width of the lead bar the same as the width of the spacing. The line pairs 264 were used to study the spatial resolution along the two directions under various source blur 265 conditions. The two spheres were arranged along a 45° line relative to the pixel grid to 266 demonstrate the spatial resolution for small objects, at a representative angle (e.g., diagonal) to 267 the voxel grid, which combined the effect of the spatial resolution in the x- and y-directions. 268 269 Table 2 shows the line pair frequency and the sizes of the individual bars and spheres. The background material was configured as breast tissue with 50% glandular/50% fat based on the 270 data from ICRU report 46²³. The thickness of the background material was set to be 6 cm. The 271 thickness of the lead line pairs is configured to be 0.03 mm in our simulation, similar to the 272 thickness of commercial lead line pair phantoms for testing spatial resolution of mammography 273 systems. 274

Figure 3 shows the LPBB phantom with five groups of test objects. Group 1 was the base 275 group centered at y=0. The other four "derived" groups were obtained by shifting Group 1 to 276 different locations on the plane; Group 2: x-shift =75 mm, y-shift = -48 mm; Group 3: x-shift = 277 75 mm, y-shift=+48 mm; Group 4: x-shift = 150 mm, y-shift = -48 mm; Group 5: x-shift = 150 278 mm, y-shift = +48 mm. We chose these shift distances such that all groups were within the 'valid 279 area' of the slice, which we defined as the area where an object would be imaged within the 280 detector FOV at all projection angles. If an object was too far from the rotation center (outside 281 the valid area), its image would be projected outside the detector FOV at some or all of the 282 283 projection angles. Their reconstructed images would be in the region of truncation artifacts that would affect its contrast ¹⁸. The combined effect of source blur and reconstruction truncation 284 artifacts is out of the scope of this study. 285

The MC phantom contained only BBs of calcium carbonate (CaCO₃) to simulate MCs in DBT. Similar to the LPBB phantom, we configured 15 pairs of BBs for this phantom at 50.6 mm from the bottom of the imaged volume with 50% glandular/50% fat tissue background. The diameters of the BBs were identical to those in the LPBB phantom. Figure 4 shows the base 290 group of objects and the four derived groups. The x-shift locations of the four derived groups 291 were the same as those in the LPBB phantom but the y-shift was ±56 mm. The y-dimension of 292 each group was smaller in the MC phantom than that of the LPBB phantom so that the four 293 groups could be separated farther along the y-direction to fully use the "valid area".

Due to the discrete sampling in digital imaging, the alignment of the objects relative to the 294 pixel grid of the detector affects the resolution and contrast of the reconstructed object images, 295 296 especially for objects of sizes close to the pixel size. The alignment affects the different objects 297 in the phantom to different degrees because of their different locations relative to the pixel grid. To compare different amount of source blurs, it is more useful to study the "average" effect 298 299 when objects are imaged by a DBT system without knowledge of their imaged location relative to the pixel grid. We simulated this average effect by generating projections with the test patterns 300 placed at 5×5 locations with respect to the pixel grid, each of which was shifted by 1/5 pixel 301 (0.02 mm) along either the x- or y-direction. We then reconstructed the DBT at each shift 302 303 location and calculated the line pair contrasts from the reconstructed images. The contrasts of the same line pair were averaged over the different alignments. More details were described in our 304 305 previous study of the segmented separable footprint projector for DBT reconstruction ¹⁹.

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307 2.5 Figures of merit

To quantitatively analyze the image quality with different source blurs, we defined figures of merits (FOM) for the line pairs and BBs, similar to those in our previous study ¹⁹. For each set of line pairs, we extracted nine profiles at the central part of the line pairs and took the average. For each pair of BBs, we extracted one profile through the line that passed through the centers of the two spheres, which were calculated from the analytical locations of the objects as defined in the configuration of the phantom.

To calculate the contrast of the line pairs, we first calculated the ideal profile of the corresponding line pair in the high resolution phantom to identify the spatial boundaries of the peak and valley regions of the line pairs, as shown in the examples in Figure 5. The blue curves show the reconstructed profile and the magenta curves show the ideal profile with a normalized voxel value of 1 in the peak regions. As seen from the line profile that was well resolved in the reconstructed images (Figure 5(b)), the peaks and valleys of the reconstructed profile matched well with those of the ideal profile. The peak and valley regions in the ideal profile were used to

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define these regions where the mean peak and valley values should be calculated in the 321 reconstructed profile even when they were not well resolved, as shown in Figure 5(a). The 322 323 contrast was then calculated as the difference between these two mean values, normalized to the contrast value of the line pairs in the ideal profile, which had the same constant value for all line 324 pair frequencies as the lead line pairs had a constant thickness of 0.03 mm. The calculated ideal 325 326 contrast might not be accurate due to factors such as beam hardening in our simulation. However, the inaccuracy would not affect the relative contrast comparisons in this study because all curves 327 being compared used the same phantom setup and were normalized to the same reference value. 328

We calculated the BB contrast based on the detected peaks along the profile. If 2 peaks and 1 valley were detected, we used the following equation to define the relative contrast of the BB:

Relative Contrast =
$$\frac{(p_1 + p_2)/2 - v}{\max(p_1, p_2) - b'}$$
 (4)

where p_1 and p_2 were the values at two peaks, v was the value at the valley and b is the 331 background voxel value. Otherwise, the contrast was considered to be 0. We used the relative 332 contrast instead of the absolute contrast because BBs with different diameters have different 333 334 thicknesses along the z-direction and some might be split into more than one slice. There are large differences between the absolute contrasts of BBs of different diameters, making the 335 contrast-versus-diameter curve less meaningful. As defined in Eq. (4), the relative contrast 336 represents whether the two BBs can be resolved and a perfectly separate BB pair will have the 337 338 maximum value of 1. When the two peaks are not equal, we used the larger one of the two peaks in the denominator to be conservative in estimating the relative contrast. For simplicity, the 339 relative contrast is referred to as "contrast" in the following discussion. 340

These contrast-versus-frequency curves are similar to the commonly used modulation transfer function (MTF) in x-ray imaging, but they are calculated with rectangular waves instead of sinusoidal functions. Despite the difference, these curves still reflect the relative spatial resolution of the reconstruction with the influence of source blur and other factors.

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346 2.6 Comparison of source blur effects between moving detector and stationary detector

To discuss the influence on source blur of a moving detector compared with a stationary detector, we use the geometry of the Hologic Selenia Dimensions system, which uses a moving detector, as an example. For this system, the distance between the source rotation center and the detector is 0 such that the rotation axis is within the detector plane ¹. Our simulation rotates the detector synchronously with the source about the rotation axis by the same angle of the source so the central ray of the x-ray beam remains normal to the detector plane during image acquisition.

We investigated the influence of the moving detector on source blur by a simplified model 353 using a point source. We simulated 1.3 mm source motion during the exposure of each projection, 354 corresponding to the maximum motion estimated in Table A2. Therefore, the effective focal 355 spot is a 1.3-mm-wide one-dimensional line source parallel to the source motion. At the central 356 projection angle, the line source is parallel to the y-direction. Given that the distance from the 357 source to the rotation center is 700 mm, a source size of ± 0.65 mm corresponds to an angular 358 span of $\pm 0.053^{\circ}$ and the detector also rotates by 0.106° during the exposure of each projection. 359 The projected location of a point in the imaged volume on the detector plane will change with 360 361 the small source motion. Geometrically calculating this location before and after the motion, leads to the distance between these two points. This "blurring distance" represents the amount of 362 363 blurring for one point in the imaged volume due to the source motion. The blurring distance can be calculated as a distribution in the imaged volume for the moving detector or for the stationary 364 365 detector. Such a comparison indicates the effect of the moving detector on the source blur.

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367 **3. Results and discussions**

368 3.1 Spatial Distribution of Source Blur PSF

369 3.1.1 Condition A – Illustration of Spatially Variant Shape

We projected the focal spot through a pinhole array to the detector plane to illustrate the 370 spatially variant shape of the focal spot PSF. To facilitate visualization, we enlarged each 371 projected focal spot by a factor of 20 while fixing its center at the original projected location in 372 the figures. Figure 6(a) shows the source blur PSF at the projection angle $\theta = 0^{\circ}$. As expected, 373 the distribution of the PSF is symmetrical along the s = 0 axis. The PSF closest to the central ray 374 at t = 10 mm, s = 0 mm is approximately the shape of a square. This is reasonable considering 375 that the nominal focal spot size is measured with the central beam at t = 0 mm, s = 0 mm. For 376 most PSFs that are not close to the rotation axis, their shape is more similar to a parallelogram. 377 378 The area of the PSF decreases when t increases. Figure 6(b) shows the source blur PSF at a projection angle $\theta = 30^{\circ}$. Most PSFs are of the shape similar to a parallelogram but their two 379

sides perpendicular to the anode-cathode axis are not necessarily parallel to the s-axis. It can be observed that the PSF of the source blur changes gradually throughout the detector plane and is highly shift-variant.

383

384 3.1.2 Condition B – Typical Focal Spot Size in DBT Systems

Condition B shows the shape of each of the PSFs of a type focal spot of size 0.3 mm. The PSF is similar to that at the same location in Figure 6 except that the actual projected size is plotted. Figure 7 and Figure 8 show the PSF at four locations for the projection angles $\theta = 0^{\circ}$ and $\theta = 30^{\circ}$ in, respectively. The PSFs in Figure 8 are generally larger than that of Figure 7, since the distance from the source to the detector is smaller for Figure 8, resulting in greater geometric unsharpness.

391 Figure 7 and Figure 8 show that the size of the PSFs is on average about 0.04 mm along one direction. For a system with a detector pixel size of 0.1 mm, the source blur PSF will not 392 strongly affect the projection images for DBT systems if the effective h_2 stays as 0.3 mm such as 393 an ideal step-and-shoot system. On the other hand, for DBT systems designed with continuous 394 scanning motion and pulsed x-ray exposure during the acquisition of the projections, the 395 effective h_2 can be as large as 1.6 mm, as shown in Table A2 and Table A4. For these systems, 396 the effect of the source blur on image reconstruction may not be negligible, as discussed in the 397 next section. 398

399

400 3.2 Simulating the Effect of Source Blur with CatSim

We quantitatively analyzed the objects reconstructed from projection images simulated with different source sizes. DBT reconstruction was performed with the simultaneous algebraic reconstruction technique (SART) with five iterations for all conditions¹⁷. Three types of objects (horizontal line pairs, vertical line pairs and BBs) were analyzed. The FOMs described in Section 2.5 were calculated. The plotted curves were the average of all the shifted locations for the same objects imaged under the same conditions. The mean contrast curves were compared for the different test objects and different source blur conditions.

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409 3.2.1 Horizontal line pairs in the LPBB phantom

Figure 9 shows the contrast as a function of spatial frequency for the horizontal line pairs in 410 the reconstructed in-focus slice of the LPBB phantom DBT. The horizontal line pairs are 411 412 perpendicular to the source motion direction. For all sources studied (Source 0 to Source 3), the line pairs in the different groups of objects had similar contrast at each spatial frequency, 413 indicating that the contrast does not depend on the locations. We plotted only Source 0 and 414 Source 3 in Figure 9(a) and 9(b), respectively, as examples. The resolution of the horizontal line 415 pairs is mainly affected by the focal spot size in the source motion direction, which changes 416 relatively slowly (see Figure 7 and Figure 8 that shifted by 140 mm) within the ±48 mm shifts in 417 locations between Group 2 and Group 3, or between Group 4 and Group 5. Although the 418 419 effective focal spot size changes rapidly along the direction of the anode-cathode axis, it does not affect the horizontal line pairs as they are constant in this direction. As a result, for the same 420 421 spatial frequency, the contrast of a set of horizontal line pairs does not change much among different groups of objects. Because of the limited "valid" region that is free of truncation 422 423 artifacts, we are not able to compare the horizontal resolution in the regions near the two ends of the imaged volume, so it is unknown whether this observation still holds in those regions. 424

Figure 9(c) and 9(d) show the dependence of the line pair contrast on the source for the horizontal line pairs in Group 1 and Group 5. The contrast of horizontal line pairs is almost identical for Source 0 and Source 1 at different frequencies and spatial locations. Since Source 1 has a typical focal spot size of a DBT system (~0.3 mm) if the source is stationary at exposure, Figure 9(c) and 9(d) indicate that treating the 0.3 mm source as a point source has a negligible effect on the reconstructed quality for the horizontal line pairs if the pixel size of the detector or at reconstruction is 0.1 mm.

Figure 9(c) and 9(d) also show that the contrast of horizontal line pairs decreases if Source 2 or Source 3 is used. For Source 3, the contrast of the horizontal line pairs becomes negative at spatial frequencies higher than about 4 line pairs/mm, indicating that the reconstructed line pairs has a phase shift of about 180° compared with the ideal profile of the line pairs. In other words, the negative contrast indicates that the peaks and valleys of the line pairs reverse their polarity in the reconstructed images. The difference between Source 0 and Source 2 is smaller than the difference between Source 2 and Source 3. In summary, the spatial resolution in the direction of source motion is sensitive to the extra source blur from the motion. It can be substantially degraded in the range of pulsed exposure time used by DBT systems with continuous-motion acquisition mode.

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443 3.2.2 Vertical line pairs in LPBB phantom

Figure 10 shows the contrast curves as a function of the frequency for the vertical line pairs 444 in the reconstructed in-focus slice of the LPBB phantom DBT. Figure 10(a) shows the 445 dependence of the contrast of the vertical line pairs on the group location with Source 0 used in 446 the simulation of the projection images. It can be seen that the curves of Group 2 and Group 3 447 are not distinguishable. The curves of Group 4 and Group 5 are also almost identical. However, 448 the contrast curve of Group 1 is very different from those of Group 2 and Group 3, as well as 449 those of Group 4 and Group 5. Group 4 and Group 5 have negative contrast for spatial 450 frequencies higher than about 3 line pairs/mm. Generally, Figure 10(a) shows that the vertical 451 452 line pairs of high spatial frequencies are less resolvable if they are farther away from the chest wall even though the focal spot dimension perpendicular to the line pairs decreases as the 453 454 distances from the chest wall (x-direction) increases. The rapid reduction in resolution in this direction is likely caused by the reconstruction leakage from the diverging cone-beam x-rays. 455 456 Due to the finite thickness of the reconstructed slices, the intensity of high-contrast objects would leak to the adjacent voxels along the ray path, thus reducing the contrast of the line pairs. 457 458 The influence on the adjacent voxels increases with increasing distance from the chest wall because the angle of the x-ray path intersecting the DBT slice increases. Another possible cause 459 460 of the rapid reduction in resolution is the increasingly sparse sampling in these planes due to the cone-beam geometry as their distances from the chest wall increase. A future study to explore 461 462 this possibility using a Defrise phantom may be of interest.

Figure 10(b) - 10(d) show that, unlike the horizontal line pairs, the contrast of the vertical line pairs is essentially independent of the source blur. This is expected because, in comparison to Source 1, the extra blur caused by the source motion as simulated by Source 2 and Source 3 is mainly along the vertical direction. Blurring the vertical line pairs along the vertical direction does not affect its contrast.

The only noticeable difference among the sources can be observed in Figure 10(b), where the contrast curve for Source 0 is slightly higher than the overlapping contrast curves for Sources 1 to 3. The finite-sized sources have the same target angle ϕ and size h_1 (Table 1), which cause the same amount of source blur along the horizontal direction that affects the vertical line pairs. The difference between the point source and the finite-sized sources diminishes for Group 3 (Figure 10(c)) and Group 5 (Figure 10(d)) because the effective source blur along the horizontal direction is smaller for locations farther away from the chest wall.

In summary, if the source is of a typical focal spot size (~0.3 mm) and is stationary during exposure, treating the finite-sized source as a point source does not affect the reconstructed quality for the vertical line pairs if the pixel size of the detector or at reconstruction is 0.1 mm. Even if the source is not stationary such that the effective size of the source blur is as large as 1 mm (Source 2) or 2 mm (Source 3) along the source scanning direction, there is essentially no change in the reconstructed contrast of vertical line pairs.

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482 3.2.3 BBs in LPBB phantom and MC phantom

Figure 11 shows the dependence of the contrast of BBs on the group location for Source 0 483 and Source 3 in the LPBB phantom and the MC phantom. For both sources in either phantom, 484 the contrast of the BBs has strong dependence on the group locations. Generally speaking, the 485 contrast of the BBs is higher in Group 1 than in Group 2/Group 3 and it further decreases in 486 Group 4/Group 5, indicating that the contrast of the BBs decreases as their distance from the 487 488 chest wall plane increases. The dependence of the contrast of the BBs on the group location is not as strong as that of the vertical line pairs shown in Figure 10(a) but much stronger than that 489 in Figure 9(a), where the contrast of horizontal line pairs is almost independent of the group 490 location. This is expected because the BBs are two dimensional objects that are affected by the 491 492 resolution of the imaging system in both the horizontal and the vertical directions.

Another interesting observation in Figure 11 is that, for either source or with either phantom, 493 494 the contrast of the BBs in Group 3 is higher than that in Group 2, and the contrast of BBs in Group 5 is also consistently higher than that in Group 4. Note that Group 2 and Group 4 are in 495 496 the upper half while Group 3 and Group 5 are in the lower half of the imaging field (Figure 4). The center-to-center lines of all BB pairs are oriented in the same direction. The center-to-center 497 498 lines of the BBs in Group 2 and Group 4 are generally more in line with the cone-beam x-ray 499 paths of all projections. Similar to the contrast loss of the vertical line pairs discussed above, the lower contrast of the BBs in Group 2 and Group 4 may be attributed to the leakage along the x-500

ray paths of a high-intensity object to the adjacent voxels in the reconstructed slice, thus reducingthe valley between the pair of BB.

503 Figure 12 compares the contrast of the BBs obtained with the four sources for Group 1 and Group 5. Figures 12(a) and 12(b)) show that in Group 1 the BB pairs with a diameter larger than 504 about 0.15 mm are highly resolvable with a contrast close to or higher than 0.8 and the difference 505 506 among the four sources is small. For BBs with a diameter smaller than 0.15 mm, the decrease in contrast with Source 2 and Source 3 becomes noticeable, especially with Source 3. For example, 507 in the LPBB phantom, the contrast of the 0.1-mm-diameter BBs is 0.347 for Source 0. The 508 contrast decreases by 12% to 0.306 for Source 2 and by 37% to 0.219 for Source 3. Figure 12(c) 509 and 12(d) show that the contrast of the BBs in Group 5 is much lower than the corresponding 510 pairs in Group 1. The difference between Source 0 and Source 2 is smaller than the difference 511 between Source 2 and Source 3. Comparing the contrast curves for Source 0 and Source 2, for 512 the BBs of diameters from 0.053 mm to 0.125 mm, the contrast is reduced by 16% to 33% in the 513 LPBB phantom and by 5% to 33% in the MC phantom. Overall, the dependence of the resolution 514 of the BBs on the spatial location on the image plane is stronger than the dependence on the 515 source blur over the range of source sizes studied. 516

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518 3.3 Comparison of source blur effects between moving detector and stationary detector

To compare the influence of moving detector and stationary detector on source blur effect, 519 the distributions of the blurring distance for two projection angles (0° and 7.5°) and two y-z 520 521 planes (x = 0 and x = 200 mm) are calculated and shown in Figure 13 and Figure 14. Similar to Figure 1, we still use the rotation center as the origin of the coordinate system. The z-coordinate 522 of the imaged volume then starts from -25 mm, since the distance from the rotation center to the 523 imaged volume is 25 mm according to Sechopoulos et al.¹. The sizes of the imaged volume along 524 525 the y- and z-directions are 290 mm and 100 mm, respectively, assuming that the thickness of the imaged volume is 100 mm and that the imaged volume has the same size as the detector in image 526 reconstruction¹. 527

The first rows of Figure 13 and Figure 14 show the distribution of the blurring distance with a stationary detector. The second rows show the distribution with a moving detector. The third rows show their relative difference calculated by subtracting the first rows from the second rows (moving detector – stationary detector) and dividing the results by the maximum blurring distance with the stationary detector. A negative value in the third rows therefore indicates that
the moving detector reduces the blurring distance. The same color bar settings were used in
Figure 13 and Figure 14.

Figure 13 shows the distribution of the blurring distance for the central projection angle. As 535 expected, the distribution is symmetric about y = 0 for both detectors on both y-z planes. For the 536 stationary detector, the distribution of the blurring distance does not depend on the x- or y-537 coordinate because the 1-D line source blur is parallel to the detector plane for the stationary 538 detector at the central projection angle. The blurring distance increases when the location is 539 farther away from the detector plane, reaching a maximum value of 0.28 mm at z = -125 mm, 540 which corresponds to the top of a 10-cm-thick breast. This is expected considering that the 541 geometric unsharpness increases as the object-to-detector distance increases. For a moving 542 detector, for the x = 0 plane at the chest wall, the blurring distance reduces by 0% to 29.3% 543 compared with the stationary detector. The average relative reduction of the blurring distance is 544 8.4%. 545

As shown in the second row of Figure 13, the blurring distance is not negligible even with the moving detector, especially for the top slices. The maximum blurring distance is 0.28 mm at y = 0, z = -125 mm, which is the same as that for the stationary detector. The blurring distance also increases for the planes farther away from the chest wall. At x = 200 mm, the blurring distance of the moving detector can exceed that of the stationary detector in the bottom slices, as indicated by a positive relative difference. On average, the moving detector reduces the blurring distance by 3.2%.

Figure 14 shows the comparison for projection angle $\theta = 7.5^{\circ}$ (the maximum projection 553 angle of the Hologic DBT system). For the x = 0 plane, the moving detector can reduce the 554 blurring distance by as much as 52.0%, as observed in the upper-left corner in the third row of 555 Figure 14(a). The average relative reduction of the blurring distance is 9.1%. The maximum 556 blurring distance with the moving detector is 0.29 mm, which is slightly larger than that at the 557 central projection angle. For the x = 200 mm plane, the blurring distance of the moving detector 558 is larger than that of the stationary detector on the right half of the plane, as shown in the third 559 560 row of Figure 14(b). The average reduction of the blurring distance is 4.1%, mainly contributed by the left half of the plane. As a result, at this projection angle, the moving detector reduces the 561

source blur more than that at the central projection angle, but the variation of the source blur overthe imaged volume is large and asymmetric.

Figure 15 compares the moving detector and the stationary detector in an x-y plane at z = -564 105 mm, which is 80 mm from the bottom of the imaged volume. At the central projection angle 565 shown in Figure 15(a), the distribution of the blurring distance for the stationary detector is 566 uniform. This is because the equivalent finite-sized source is 1-D and is parallel to the detector, 567 as explained above for first row of Figure 13. On the other hand, the blurring distance is non-568 uniform with the moving detector, decreasing from the center to the two sides of the FOV. The 569 average reduction of blurring distance is 9.2%. At a projection angle of 7.5°, the average 570 reduction is 11.4%, but the blurring distance actually increases locally by more than 5% in the 571 lower-right corner in Figure 15(f). 572

In summary, these calculations indicate that the additional source blur caused by the motion of the x-ray tube during data acquisition cannot be neglected even when using a detector moving in synchrony with the source. It is likely that the general trends observed in our analysis of spatial resolution with the CatSim simulation (Section 3.2) using the stationary detector also apply to a moving detector, although this conjecture needs to be confirmed in a future study.

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579 **4. Discussion**

580 4.1 Summary of the Influence of Source Blur

Our simulation results indicate that for a stationary source of a typical focal spot size (~0.3 581 mm), treating the finite-sized source as a point source has negligible effect on the reconstructed 582 583 image resolution in both the directions parallel and perpendicular to the source motion direction as shown by the horizontal and vertical line pairs and BBs. If the source is not stationary such 584 that the effective size of the source blur (h_2) increases to about 1 mm (Source 2), the spatial 585 resolution in the direction parallel to the source motion (the relative contrast of horizontal line 586 pairs) and BBs will degrade noticeably. If the effective size of the source blur is 2 mm (Source 3), 587 588 the contrast of horizontal line pairs and BBs will decrease substantially and the degradation increases from the chest wall to the anterior of the FOV. How much source blur is tolerable 589

depends on the specific imaging task and other factors in the imaging and reconstructionprocesses.

592 Although we estimated the potential source blur of the commercial DBT systems (Tables A1-593 A4) based on the published system parameters, typical exposure techniques, and simple constant motion of the x-ray source, we did not investigate the many possible combinations of parameters 594 for the various systems. For example, the number of PVs, acquisition angle, detector pixel size, 595 reconstruction voxel size and reconstruction algorithm etc. differ among systems. The Hologic 596 system uses a moving detector (non-stationary) and the Fujifilm system uses a detector with 597 hexagonal elements, which are very different from our CatSim simulation. The design of a DBT 598 system involves many factors besides minimizing the source blur. In addition, we did not include 599 other image quality degrading factors such as detector blur, noise or scattered radiation, making 600 601 it more difficult to predict the relative influence of source blur on the reconstructed image quality and the overall benefit of modeling the source blur in image reconstruction in practice for a 602 specific system. Nevertheless, we will make some general discussion based on our simulation 603 results as a reference that might be helpful for other researchers and DBT manufacturers. 604

For DBT systems that use a step-and-shoot scanning mode such as the GE SenoClaire or Pristina DBT system, our simulation shows that treating a finite-sized source as a point source causes minimal loss in resolution if the focal spot size is about 0.3 mm, the detector has a pixel size of 0.1 mm and the reconstructed voxel size is $0.1 \times 0.1 \times 1$ mm³. Neglecting the source blur may not affect the reconstructed image quality. The benefit of modeling the source blur in reconstruction for this type of systems appears to be limited.

For narrow-angle DBT systems that use a continuous-motion scanning x-ray source with a 611 moving detector such as the Hologic Selenia Dimensions system, our simulation shows that the 612 613 source motion blur is substantial and the moving detector does not greatly reduce the source blur, 614 especially if small pixel size such as 0.07 mm is used. If the detector pixel size is binned to 0.14 mm in the reconstructed DBT¹, the relative impact of the source motion blur is reduced. 615 According to our estimates in Table A2, the effective h_2 is about 1.3 mm for 6-cm-thick breasts 616 and 1.6 mm for 8-cm-thick breasts. If we consider the size of the source blur relative to the pixel 617 size, a source blur of 1.3 mm is comparable to a source blur of about 0.8 mm and 1.6 mm is 618 619 about 1 mm (Source 2) in our simulation that uses a pixel size of 0.1 mm. The source blur is therefore not negligible in DBT for slightly above-average to thick breasts and modeling the 620

source blur in reconstruction may be beneficial. The experimental study by Qian et al.²⁴ supports
 our conclusion, where replacing the rotating x-ray tube in the Hologic Selenia Dimensions DBT
 system with a stationary carbon nanotube x-ray source array demonstrates increased system
 spatial resolution.

For wide-angle DBT systems with a continuous-motion scanning x-ray source and a 625 stationary detector, the impact of motion source blur is strong unless the source moves at a 626 relatively slow speed such as the Siemens Mammomat Inspiration system. According to our 627 estimates in Table A3, the effective h_2 is 1.2 mm for thick breasts (thickness ~ 10 cm). The pixel 628 size is 0.085 mm for this system ¹. For a 10-cm-thick breast, an effective h_2 of 1.2 mm is 629 between Source 2 and Source 3 in our simulation. For a 5-cm-thick breast, the effective h_2 is 0.8 630 mm, which is comparable to Source 2. Our simulation shows that the source motion noticeably 631 degrades image quality for average to thick breasts. Modeling the source blur may improve the 632 image quality. Modeling the source blur may also allow the system to scan with faster motion of 633 634 the x-ray source, which would decrease the potential motion blur of the breast and improve the comfort of DBT imaging. 635

For narrow-angle DBT systems with continuous x-ray source motion and a stationary 636 detector the source motion blur can be substantial, especially when the detector is stationary and 637 the pixel size is small such as the Fujifilm Aspire Cristalle system. This system has a detector 638 with hexagonal pixels with a side length of 0.05 mm $^{25, 26}$, which is equivalent by pixel area to a 639 square pixel of 0.08 mm. If we simply assume a square pixel of 0.08 mm for the system, then the 640 effective $h_2 = 1.6$ mm for thick breasts (thickness ~ 9 cm) is comparable to Source 3 in our 641 simulation and could result in substantial degradation in spatial resolution. Modeling the source 642 blur in reconstruction may therefore improve the image quality. In general, reducing the scan 643 speed or reducing the x-ray pulse width will alleviate the problem of source motion blur but it 644 depends on other system design considerations. Furthermore, increasing the total scan time also 645 increases the possibility of motion blur of the breast. 646

In summary, our simulation results indicate that the step-and-shoot approach may suffice to preserve the resolution of objects despite the finite size of the focal spot in typical DBT systems. The continuous motion approach will be the main contributor to the source blur and may cause different levels of image quality degradation depending on the thickness of the breast and other parameters of the DBT system. The latter type of DBT systems may benefit from modeling source blur in reconstruction but the specific gain in image quality should be studied byconsidering other system design and imaging factors that may also affect image quality.

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655 4.2 Limitations of the study

This study compared the relative effects of source blur on the spatial resolution of DBT 656 under the same image acquisition and reconstruction conditions. There are several limitations. 657 First, we used only SART with 21 projections in reconstruction. It may be of interest to study 658 659 DBT systems with different geometries and reconstructions by other algorithms to evaluate how source blur depends on these parameters. Second, we simulated a fixed detector and 660 reconstruction pixel size. Since the pixel size and the reconstruction projector have strong 661 impacts on the reconstructed image resolution ^{19, 20}, it will be useful to study how the effect of 662 source blur may interact with these factors. Third, our simulation neglected quantum noise, 663 664 readout noise, detector blur, scattered radiation and other factors. A comparison between the ideal point source and a finite-sized source taking into account these factors will better gauge the 665 significance of modeling source blur in DBT reconstruction. DBT image quality involves a large 666 number of factors in the imaging chain and reconstruction process but we can only explore a 667 small part of the parameter space in one study. Despite the limitations, we believe that the 668 simulation results improve our understanding and provide some meaningful information on the 669 effects of source blur in DBT reconstruction. 670

671

672 **5. Conclusion**

673 This paper used analytical calculations and CatSim simulations to study the effect of the source blur on the spatial resolution of DBT reconstructions. Our analytical calculations 674 demonstrated that the PSF of source blur is highly shift-variant. The shape of the PSF of the 675 source blur also strongly depends on the spatial location over the image plane, making it 676 challenging to be implemented precisely in a system model. We used CatSim to simulate 677 phantoms containing line pairs and BBs at different locations with sources of four different sizes. 678 679 The reconstructed results of the simulated phantoms demonstrate that a typical finite-sized focal spot (~ 0.3 mm) will not have a substantial impact on the image quality if the x-ray tube is 680 681 stationary during data acquisition. If the x-ray tube is moving, the extra source blur due to the

682 motion may degrade image resolution, depending on the effective size of the source along the 683 direction of the motion. Our simulation results suggest that there are potential benefits of 684 modeling the source blur in image reconstruction for DBT systems using continuous-motion 685 acquisition mode.

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687 ACKNOWLEDGEMENT

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702	Append	ix A. Geometry, scanning parameters and typical exposure techniques for three
703		commercial DBT systems
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705 Appendix B. Analytical Calculation of Source Blur PSF

With the simplified source blur model described in Section 2.1, the projection of the rectangular source through a pinhole can be analytically calculated on the detector plane. We first introduce the following lemma:

Lemma 1: The projection of a straight line segment l_1 on a plane P through a point O is contained in a straight line.

Proof: Let *A* denote an arbitrary point on l_1 . The projection of *A* on the plane *P* through *O* is contained in the plane determined by l_1 and *O*. Let *Q* denote this plane. Obviously the projection of *A* on *P* is contained in *P*. Because the intersection of *P* and *Q* is a straight line and *A* is an arbitrary point on l_1 , the projections of all the points on l_1 are contained in the same straight line.

Because of Lemma 1, the projection of a rectangular source on a plane through a pinhole can be obtained by calculating the projections of only the four corners. We simply need to connect the projections of the four corners to get the shape of the PSF of the source blur.

We derive the locations of the four corners of the rectangular focal spot and their projections. The finite-sized focal spot shown in Figure 1 is enlarged in Figure 16 to illustrate the locations of its corners. Let d_{S0} denote the distance from the center of the source (denoted as *S*) to the rotation center (denoted as *O*) and d_{OP} denote the distance from the rotation center to the origin of the detector (denoted as *P*). The center of the source (*S*) is located at:

$$\vec{r}_S = (0, d_{SO} \sin \theta, -d_{SO} \cos \theta). \tag{5}$$

The locations of the four corners (*A*, *B*, *C* and *D* in Figure 16) of the rectangular source are:

 $\vec{r}_{A} = \vec{r}_{S} - \vec{d}_{1} - \vec{d}_{2},$ $\vec{r}_{B} = \vec{r}_{S} + \vec{d}_{1} - \vec{d}_{2},$ $\vec{r}_{C} = \vec{r}_{S} + \vec{d}_{1} + \vec{d}_{2},$ $\vec{r}_{D} = \vec{r}_{S} - \vec{d}_{1} + \vec{d}_{2},$ (6)

where \vec{d}_1 and \vec{d}_2 are vectors of lengths $\frac{h_1}{2}$ and $\frac{h_2}{2}$ along the h_1 and h_2 directions in Figure 1, shown as red arrows in Figure 16.

The expressions of \vec{d}_1 and \vec{d}_2 are derived based on solid geometry. We have: $\overrightarrow{AB} \parallel \overrightarrow{DC}$ and $\overrightarrow{AD} \parallel \overrightarrow{BC}$. \vec{d}_1 are \vec{d}_2 are along the directions of \overrightarrow{AB} and \overrightarrow{BC} . They are perpendicular to each other and their lengths are $\frac{h_1}{2}$ and $\frac{h_2}{2}$. If we can derive the direction vectors of \overrightarrow{AB} and \overrightarrow{BC} , denoted as $\vec{n}_{\overrightarrow{AB}}$ and $\vec{n}_{\overrightarrow{BC}}$, \vec{d}_1 and \vec{d}_2 can be obtained by multiplying these direction vectors with $\frac{h_1}{2}$ and $\frac{h_2}{2}$.

We first derive \vec{n}_{BC} . \vec{BC} is parallel to the y-z plane and perpendicular to \vec{OS} . The direction vectors of the y-z plane and \vec{OS} are:

$$\vec{n}_x = (1,0,0),$$
 (7)

 $\vec{n}_{\overline{OS}} = (0, \sin\theta, -\cos\theta). \tag{8}$

Therefore \vec{n}_{BC} can be obtained by calculating their cross product:

$$\vec{n}_{B\vec{C}} = \vec{n}_x \times \vec{n}_{\vec{OS}} = (0, \cos\theta, \sin\theta).$$
(9)

727 Next we derive $\vec{n}_{\overline{AB}}$. $\vec{n}_{\overline{AB}}$ is perpendicular to $\vec{n}_{\overline{BC}}$. We also know that the angle between $\vec{n}_{\overline{AB}}$ 728 and $\vec{n}_{\overline{SO}}$ is ϕ . Therefore we have the follow equations:

$$\vec{n}_{\overline{AB}} \cdot \vec{n}_{\overline{BC}} = 0, \tag{10}$$

$$\vec{n}_{\overline{AB}} \cdot \vec{n}_{\overline{SO}} = \cos \phi, \tag{11}$$

$$\vec{n}_{\overline{AB}} \cdot \vec{n}_{\overline{AB}} = 1, \tag{12}$$

where Eq. (12) is the constraint for the length of the direction vector. $\vec{n}_{\overline{SO}}$ is the opposite direction of $\vec{n}_{\overline{OS}}$: $\vec{n}_{\overline{SO}} = -\vec{n}_{\overline{OS}}$, where $\vec{n}_{\overline{OS}}$ is known as shown in Eq. (8). $\vec{n}_{\overline{BC}}$ is shown in Eq. (9).

Therefore, by solving Eq. (10) - (12), we have:

$$\vec{n}_{\overline{AB}} = (\sin\phi, -\cos\phi\sin\theta, \cos\phi\cos\theta). \tag{13}$$

Multiplying $\vec{n}_{\overline{AB}}$ and $\vec{n}_{\overline{BC}}$ with $\frac{h_1}{2}$ and $\frac{h_2}{2}$ leads to the expressions of \vec{d}_1 and \vec{d}_2 in Eq. (14):

$$\vec{d}_{1} = \left(\frac{h_{1}}{2}\sin\phi, -\frac{h_{1}}{2}\cos\phi\sin\theta, \frac{h_{1}}{2}\cos\phi\cos\theta\right),$$

$$\vec{d}_{2} = \left(0, \frac{h_{2}}{2}\cos\theta, \frac{h_{2}}{2}\sin\theta\right).$$
(14)

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Using solid geometry, the projection \vec{p} of an arbitrary location \vec{r} on the detector plane is:

$$\vec{p} = \vec{r} + \frac{(\vec{r}_{detector} - \vec{r}) \cdot \vec{n}_{detector}}{(\vec{r}_{pinhole} - \vec{r}) \cdot \vec{n}_{detector}} (\vec{r}_{pinhole} - \vec{r}),$$
(15)

where the operator \cdot denotes inner product, $\vec{r}_{pinhole}$ is the known location of the pinhole and the two vectors that describe the detector plane are:

$$\vec{n}_{\text{detector}} = (0,0,1),$$
 (16)

$$\vec{r}_{\text{detector}} = (0, 0, d_{\text{od}}). \tag{17}$$

With Eq. (5), (6) and (14) – (17), we can analytically calculate the PSF of the source blur given the location of the pinhole $\vec{r}_{pinhole}$.

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 813 814 815 816 817 818 819 820 821 822 823 824 	(d) Source 0, Group 5 Source 1, Group 5 Source 2, Group 5 Source 3, Group 5 Figure 11 (a) Source 0, Group 1 Source 0, Group 2
 813 814 815 816 817 818 819 820 821 822 823 824 825 	(d) Source 0, Group 5 Source 1, Group 5 Source 2, Group 5 Source 3, Group 5 Figure 11 (a) Source 0, Group 1 Source 0, Group 2 Source 0, Group 3
 813 814 815 816 817 818 819 820 821 822 823 824 825 826 	(d) Source 0, Group 5 Source 1, Group 5 Source 2, Group 5 Source 3, Group 5 Figure 11 (a) Source 0, Group 1 Source 0, Group 1 Source 0, Group 2 Source 0, Group 3 Source 0, Group 4
 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 	(d) Source 0, Group 5 Source 1, Group 5 Source 2, Group 5 Source 3, Group 5 Figure 11 (a) Source 0, Group 1 Source 0, Group 1 Source 0, Group 2 Source 0, Group 3 Source 0, Group 4 Source 0, Group 5

829	(b)
830	Source 0, Group 1
831	Source 0, Group 2
832	Source 0, Group 3
833	Source 0, Group 4
834	Source 0, Group 5
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836	(c)
837	Source 3, Group 1
838	Source 3, Group 2
839	Source 3, Group 3
840	Source 3, Group 4
841	Source 3, Group 5
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843	(d)
844	Source 3, Group 1
845	Source 3, Group 2
846	Source 3, Group 3
847	Source 3, Group 4
848	Source 3, Group 5
849	Figure 12
850	(a)
851	Source 0, Group 1
852	Source 1, Group 1
853	Source 2, Group 1
854	Source 3, Group 1
855	
856	(b)
857	Source 0, Group 1
858	Source 1, Group 1
859	Source 2, Group 1

860	Source 3, Group 1
861	
862	(c)
863	Source 0, Group 5
864	Source 1, Group 5
865	Source 2, Group 5
866	Source 3, Group 5
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868	(d)
869	Source 0, Group 5
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Appendix A. Geometry, scanning parameters and typical exposure techniques for three commercial DBT systems

- 3 Table A1. Geometry and scanning parameters of three commercial DBT system using
- 4 continuous-motion scanning mode.

, Li	Hologic Selenia Dimensions	Siemens Mammomat Inspiration	Fujifilm Aspire Cristalle
0	Ref. ^{1, 2, 27}	Ref. ^{1, 3}	Ref. ^{25, 26, 28}
Pixel size	0.07 mm (detector) 0.14 mm (2×2 binning)	0.085 mm	0.05 mm (hexagonal), ~ 0.08 mm (square)
Number of projections	15	25	15
Distance from source to the rotation center (mm)	700	608	650
Acquisition angle (degree)	15	50	15
Total acquisition time (s)	3.7	25	4.0
Total motion of the source (mm)	183	530.6	170
Speed of the source (mm/s)	49.5	21.2	42.5
Nominal focal spot size (mm)	0.3	0.3	0.3

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7 Table A2. Estimation of the source blur h_2 for Hologic Selenia Dimensions system. We used the 8 maximum current of the x-ray tube in the vendor's user guide² as the current for each thickness 9 of the breast, ignoring the possible dependence of the current on kV setting. The Hologic system 10 bins 2 × 2 pixels during the image reconstruction. Therefore, the pixel size is 0.14 mm for this 11 system.

Breast	Kilovoltage	Total	Current	Total	Exposure	Source	Source
Thickness	(kV)	Current-Time	(mA)	Exposure	Time per	Motion of	Blur <mark>h</mark> 2

(mm)		Product		Time (s)	PV (s)	one PV	(mm)
		(mAs)				(mm)	
20	26	32	200	0.160	0.011	0.5	0.8
40	29	43	200	0.215	0.014	0.7	1.0
60	33	60	200	0.300	0.020	1.0	1.3
80	38	81	200	0.405	0.027	1.3	1.6

Table A3. Estimation of source blur h_2 for Siemens Mammomat Inspiration system. The current cannot be found in the technical documents and is therefore estimated with the voltage and the fixed power output of the x-ray tube, which is 5 kW according to the vendor's information³.

Breast	Willowalta an	Total Current-	Comment	Total	Exposure	Source	Source
Thickness	(LV)	Time Product	(mA)	Exposure	Time per	Motion of	Blur <mark>h</mark> 2
(mm)		(mAs)	(IIIA)	Time (s)	PV (s)	one PV (mm)	(mm)
20	25	50	200	0.250	0.010	0.2	0.5
30	26	70	192	0.364	0.015	0.3	0.6
40	26	90	192	0.468	0.019	0.4	0.7
50	27	110	185	0.594	0.024	0.5	0.8
60	28	120	179	0.672	0.027	0.6	0.9
70	29	130	172	0.754	0.030	0.6	0.9
80	30	140	167	0.840	0.034	0.7	1.0
90	30	160	167	0.960	0.038	0.8	1.1
100	31	180	161	1.116	0.045	0.9	1.2

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Table A4. Estimation of source blur h_2 for the Fujifilm Aspire Cristalle system. The current 21 cannot be found in the technical documents and is therefore estimated with the kilovoltage and 22 the fixed power output of the x-ray tube, which is 4.9 kW according to the x-ray tube vendor's 23 information²⁹. The breast thickness is converted from the PMMA phantom used in the Fujifilm 24 quality control manual by interpolating curve of the equivalent breast thickness to the PMMA 25 phantom thickness ²⁸. The digital detector uses an array of hexagonal pixels of a side width of 26 0.05mm. The area of a hexagonal pixel is the same as a square pixel of 0.08 mm, so we estimate 27 the equivalent pixel size to be 0.08 mm. 28

Breast Thickness (kV)	Total Current- Time Product (mAs)	Current (mA)	Total Exposure Time (s)	Exposure Time per PV (s)	Source Motion of one PV (mm)	Source Blur h_2 (mm)
21.0 26	36	188	0.191	0.013	0.5	0.8
33.0 28	32	175	0.183	0.012	0.5	0.8
45.0 30	40	163	0.245	0.016	0.7	1.0
52.5 32	40	153	0.261	0.017	0.7	1.0
60.0 33	42	148	0.283	0.019	0.8	1.1
75.0 36	50	136	0.367	0.024	1.0	1.3
90.0 37	63	132	0.476	0.032	1.3	1.6

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32 Appendix B. Analytical Calculation of Source Blur PSF

With the simplified source blur model described in Section 2.1, the projection of the rectangular source through a pinhole can be analytically calculated on the detector plane. We first introduce the following lemma:

Lemma 1: The projection of a straight line segment l_1 on a plane P through a point O is contained in a straight line.

Proof: Let A denote an arbitrary point on l_1 . The projection of A on the plane P through O is

contained in the plane determined by l_1 and O. Let Q denote this plane. Obviously the projection of A on P is contained in P. Because the intersection of P and Q is a straight line and A is an arbitrary point on l_1 , the projections of all the points on l_1 are contained in the same straight line.

Because of Lemma 1, the projection of a rectangular source on a plane through a pinhole can be obtained by calculating the projections of only the four corners. We simply need to connect the projections of the four corners to get the shape of the PSF of the source blur.

We derive the locations of the four corners of the rectangular focal spot and their projections. The finite-sized focal spot shown in Figure 1 is enlarged in Figure 16 to illustrate the locations of its corners. Let d_{so} denote the distance from the center of the source (denoted as S) to the rotation center (denoted as O) and d_{oP} denote the distance from the rotation center to the origin of the detector (denoted as P). The center of the source (S) is located at:

$$\vec{r}_{S} = (0, d_{SO} \sin \theta, -d_{SO} \cos \theta).$$
(5)

44 The locations of the four corners (A, B, C and D in Figure 16) of the rectangular source are:

$$\vec{r}_{A} = \vec{r}_{S} - \vec{d}_{1} - \vec{d}_{2},$$

$$\vec{r}_{B} = \vec{r}_{S} + \vec{d}_{1} - \vec{d}_{2},$$

$$\vec{r}_{C} = \vec{r}_{S} + \vec{d}_{1} + \vec{d}_{2},$$

$$\vec{r}_{D} = \vec{r}_{S} - \vec{d}_{1} + \vec{d}_{2},$$
(6)

45 where \vec{d}_1 and \vec{d}_2 are vectors of lengths $\frac{h_1}{2}$ and $\frac{h_2}{2}$ along the h_1 and h_2 directions in Figure 1, 46 shown as red arrows in Figure 16.

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Figure 16. Derivation of the vectors along the edges of the rectangular source $(\vec{d}_1 \text{ and } \vec{d}_2)$. The blue rectangle shows the location of the digital detector.

The expressions of \vec{d}_1 and \vec{d}_2 are derived based on solid geometry. We have: $\overrightarrow{AB} \parallel \overrightarrow{DC}$ and $\overrightarrow{AD} \parallel \overrightarrow{BC}$. \vec{d}_1 are \vec{d}_2 are along the directions of \overrightarrow{AB} and \overrightarrow{BC} . They are perpendicular to each other and their lengths are $\frac{h_1}{2}$ and $\frac{h_2}{2}$. If we can derive the direction vectors of \overrightarrow{AB} and \overrightarrow{BC} , denoted as $\vec{n}_{\overrightarrow{AB}}$ and $\vec{n}_{\overrightarrow{BC}}$, \vec{d}_1 and \vec{d}_2 can be obtained by multiplying these direction vectors with $\frac{h_1}{2}$ and $\frac{h_2}{2}$. We first derive $\vec{n}_{\overrightarrow{BC}}$. \overrightarrow{BC} is parallel to the y-z plane and perpendicular to \overrightarrow{OS} . The direction vectors of the y-z plane and \overrightarrow{OS} are:

$$\vec{n}_x = (1,0,0),$$
 (7)

$$\vec{n}_{\overrightarrow{os}} = (0, \sin\theta, -\cos\theta). \tag{8}$$

56 Therefore \vec{n}_{BC} can be obtained by calculating their cross product:

$$\vec{n}_{\overline{BC}} = \vec{n}_x \times \vec{n}_{\overline{OS}} = (0, \cos\theta, \sin\theta).$$
(9)

57 Next we derive \vec{n}_{AB} . \vec{n}_{AB} is perpendicular to \vec{n}_{BC} . We also know that the angle between \vec{n}_{AB}

and $\vec{n}_{\overline{SO}}$ is ϕ . Therefore we have the follow equations:

$$\vec{n}_{\overline{AB}} \cdot \vec{n}_{\overline{BC}} = 0, \tag{10}$$

$$\vec{n}_{\overline{AB}} \cdot \vec{n}_{\overline{SO}} = \cos\phi, \tag{11}$$

$$\vec{n}_{\overline{AB}} \cdot \vec{n}_{\overline{AB}} = 1, \tag{12}$$

where Eq. (12) is the constraint for the length of the direction vector. $\vec{n}_{\overline{so}}$ is the opposite direction of $\vec{n}_{\overline{os}}$: $\vec{n}_{\overline{so}} = -\vec{n}_{\overline{os}}$, where $\vec{n}_{\overline{os}}$ is known as shown in Eq. (8). $\vec{n}_{\overline{BC}}$ is shown in Eq. (9). Therefore, by solving Eq. (10) – (12), we have:

$$\vec{n}_{\overline{AB}} = (\sin\phi, -\cos\phi\sin\theta, \cos\phi\cos\theta).$$
(13)

62 Multiplying \vec{n}_{AB} and \vec{n}_{BC} with $\frac{h_1}{2}$ and $\frac{h_2}{2}$ leads to the expressions of \vec{d}_1 and \vec{d}_2 in Eq. (14):

$$\vec{d}_1 = \left(\frac{h_1}{2}\sin\phi, -\frac{h_1}{2}\cos\phi\sin\theta, \frac{h_1}{2}\cos\phi\cos\theta\right),$$

$$\vec{d}_2 = \left(0, \frac{h_2}{2}\cos\theta, \frac{h_2}{2}\sin\theta\right).$$
(14)

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64 Using solid geometry, the projection \vec{p} of an arbitrary location \vec{r} on the detector plane is:

$$\vec{p} = \vec{r} + \frac{(\vec{r}_{detector} - \vec{r}) \cdot \vec{n}_{detector}}{(\vec{r}_{pinhole} - \vec{r}) \cdot \vec{n}_{detector}} (\vec{r}_{pinhole} - \vec{r}),$$
(15)

65 where the operator \cdot denotes inner product, $\vec{r}_{pinhole}$ is the known location of the pinhole and the 66 two vectors that describe the detector plane are:

$$\vec{n}_{detector} = (0,0,1),$$
 (16)

$$\vec{r}_{detector} = (0,0,d_{od}). \tag{17}$$

67 With Eq. (5), (6) and (14) – (17), we can analytically calculate the PSF of the source blur 68 given the location of the pinhole $\vec{r}_{pinhole}$.

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Table 1. X-ray sources simulated in this study. Source 0 simulated an ideal point source although it still had a finite physical size as required by CatSim.

Name	Source 0	Source 1	Source 2	Source 3
Oversampling rate	1	6	6	6
Target angle (ϕ)	22.5°	22.5°	22.5°	22.5°
$h_1 (\mathrm{mm})$	0.001	0.784	0.784	0.784
<i>h</i> ₂ (mm)	0.001	0.3	1.0	2.0

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Table 2. Objects sizes (mm) in the digital phantom. The object set number corresponds to the number next to each box in Figure 2. The center-to-center distance between the two BBs in a pair is equal to the BB diameter.

	Object Set Number	1	4	7	10	13
	line pairs/mm	9.5	8.0	6.5	5.0	3.0
	Line or space width	0.053	0.063	0.077	0.100	0.167
	BB Diameter	0.053	0.063	0.077	0.100	0.167
	Object Set Number	2	5	8	11	14
	line pairs/mm	9.0	7.5	6.0	4.5	2.0
	Line or space width	0.056	0.067	0.083	0.111	0.250
+	BB Diameter	0.056	0.067	0.083	0.111	0.250
	Object Set Number	3	6	9	12	15
4	line pairs/mm	8.5	7.0	5.5	4.0	1.0
	Line or space width	0.059	0.071	0.091	0.125	0.500
	BB Diameter	0.059	0.071	0.091	0.125	0.500



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mp_13801_f2.tif





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