EXPERIMENTAL FINDINGS AND NUMERICAL SIMULATIONS
ON
GASOLINE FUEL INJECTOR SYSTEM
by
Steven J. Wright
and
E. Benjamin Wylie
FINAL PROJECT REPORT
to
Bendix Research Labs
Southfield, Michigan
Jan. 17, 1980
Department of Civil Engineering
The University of Michigan
Ann Arbor, MI 48109
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1. INTRODUCTION

This report provides documentation of the experimental and numerical investigations performed at The University of Michigan using the Bendix gasoline fuel injection system. The investigations were intended to examine the propagation of pressure waves in the fuel delivery system which are generated as a result of the fuel injector operation. Objectives of the study included a verification of the matched impedance approach to the reduction of pressure wave reflections in the system. Experimental and numerical simulations were conducted to demonstrate the approach. Another objective of the study was to examine the influence of the unsteady nature of the flow on the frictional resistance. Experiments in both laminar and turbulent flow were performed to consider the unsteady effects for the range of conditions expected in actual injector operation. Comparisons of the experimental data collected are made with computed numerical simulations and the results are critically examined.

This report describes the test apparatus used and its operating characteristics. Also presented are the equations used for the development of the numerical model. Included are the computer programs developed, and results of the program outputs are presented for comparison with the experimental data obtained.
2.0 EXPERIMENTAL APPARATUS

Description of System

The experiments were performed on the system supplied by Bendix. A complete description of all system components is not given herein. The apparatus basically consisted of a single fuel injector mounted on a length of stainless steel tubing with orifices and accumulators at the upstream and downstream ends. Fig. 2.1 is a schematic diagram of the system, with pertinent dimensions as listed. Several orifices were supplied by Bendix so that different flow conditions could be produced. Pressure signals were monitored by means of the two pressure transducers; one mounted directly above the fuel injector and the other was located approximately 6 inches upstream from the downstream orifice. The outputs from the transducers were connected to a dual beam storage oscilloscope and the corresponding outputs were recorded by taking a Polaroid photograph of the oscilloscope screen display. A typical record is given in Fig. 2.2 and consists of a trace of the electronic signal to operate the fuel injector solenoid (upper trace) and pressure traces from the two transducers (The middle trace is from the injector transducer and lower one is from the downstream transducer). The experiments were performed primarily with Stoddard solvent (specific gravity = 0.77 and kinematic viscosity = 1.17 centistokes at 25°C) as the working fluid.

A number of preliminary tests were conducted to establish the operating characteristics of the various system components. These were to provide the required inputs into the numerical
Lines shown are 0.305 in I.D.
all other lines in system are 0.312 in I.D.

Fig. 2.1. Schematic of experimental apparatus
simulations and to provide a basis for the interpretation of the experimental results.

The pressure transducers were calibrated so that pressure changes could be determined from the photographs. The calibration procedure consisted of connecting the transducer to a dead weight tester, setting the oscilloscope on a fairly slow sweep period, lowering the known weight onto the test apparatus, and noting the deflection. This was repeated at several points, and the calibration factor was obtained for each point. The oscilloscope was calibrated against millivolt of the test instrument. The deflection at the injection point was used to determine the actual expansion.

The discharge coefficients used in the experiments were measured for use in the numerical simulations. Since it was not possible to measure the orifice diameter accurately, the product $C_d A_o$ in the expression

$$
C_d = \frac{p_1 - p_2}{\gamma g A_o}
$$

was determined since only the product was required for the numerical simulation. Here, $p_1$ is the discharge pressure, $p_2$ is the specific weight of the fluid, and $A_o$ is the pressure difference across the orifice. This quantity was determined by disconnecting the tubing from the orifice to be tested at that point. The pump pressure was

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Fig. 2.2. Typical oscilloscope output.
simulations and to provide a basis for the interpretation of the experimental results.

The pressure transducers were calibrated so that pressure changes could be determined from the Polaroid photographs. The calibration procedure consisted of connecting the transducer to a dead weight tester, setting the oscilloscope to a fairly slow sweep period, lowering the known weight onto the test apparatus and storing the oscilloscope trace. This was repeated at several different pressure levels and the calibration factor was computed as the average of several tests. This procedure gave a calibration factor of 1.22 psi per millivolt of oscilloscope deflection for the transducer located at the injector and a factor of 1.05 psi/mv for the downstream transducer for the ranges of pressures examined during the actual experiments.

The discharge coefficients for the various orifices used in the experiments were measured for use in the numerical simulations. Since it was not possible to measure the orifice diameter accurately, the product $C_D A_o$ in the expression

$$Q = C_D A_o \sqrt{\frac{\Delta P}{2g\gamma}}$$

was determined since only the product was required for the numerical simulation. Here $Q$ is the discharge, $\gamma$ is the specific weight of the fluid, and $\Delta P$ is the pressure difference across the orifice. This quantity was determined by disconnecting the tubing from the system at the upstream end and installing the orifice to be tested at that point. The pump pressure was
adjusted to various levels and the orifice was discharged to atmospheric pressure. For each condition, the pressure level just upstream from the orifice was noted from the pressure gage and the discharge was determined by weighing a sample of fluid discharged in a given time period. All discharge coefficients \( C_{DA_o} \) were determined by an average of several tests and these were used in the numerical simulations.

Injector Characteristics

Other preliminary tests of interest were directed at determining the characteristics of the injector operation. The discharge coefficient (as given above) for the injector was determined in a similar manner. The orifice at the upstream end of the system was removed and replaced by a standard fitting so that relatively little energy loss would occur in the system. The downstream end was blocked off so that no flow would occur except through the injector which was operated in a continuously open condition. Again, by noting the upstream pressure, weighing a volume of fluid passed through the injector, and neglecting other losses, the discharge coefficient could be computed directly. The results of four tests at different pressure levels gave a discharge coefficient \( C_{DA_o} \) of \( 2.6 \times 10^{-6} \) ft\(^2\) with a very small variation (less than 5 percent) between the different tests. This quantity was also used in the numerical simulations.

Another requirement for the simulation was the opening and closing characteristics of the injector. These were determined by running tests at different pressure levels across the system and obtaining test results for the pressure output from
the transducer located at the injector. Results are presented
in Fig. 2.3 a–c for pressure differences across the system of
8, 28, and 68 psi respectively. The different tests show no
detectable variation in injector operation with pressure level
so it was assumed that the opening and closing characteristics
were independent of pressure level. The interpretation of the
exact characteristics were hindered by the oscillations
(discussed below) in the pressure trace, but the following
conditions were observed; the opening was in a linear fashion
(discharge increasing linearly) over a period of approximately
0.8 msec; the closure was also essentially linear except at the
beginning of the closure when it appeared to close more slowly.
For the purposes of the simulation, it was judged to be adequate
to represent the closure as a linearly decreasing discharge over
a time period of 0.3 msec. The description for the numerical
simulations presented later was thus taken for any pulse width
as an initial opening period of 0.8 msec where the injector
opening increased linearly, a fully open period, and a 0.3 msec
closure period with injector opening decreasing linearly to fully
closed.

Even though the opening and closing characteristics of the
injector appeared to be independent of pressure level, other
operating parameters did appear to be strongly influenced by it.
For example, the pressure oscillations mentioned above and visible
in all of the figures occurred at different points of the injector
operation sequence with different rail pressures. A sequence
of experiments over a wide range of pressures is presented in
a) Upstream pressure - 40 psi. Downstream pressure - 32 psi.

b) Upstream pressure - 60 psi. Downstream pressure - 32 psi.

Fig 2.3. Experiments to observe opening and closing characteristics of fuel injector (.5ms/div. horizontal, 2 mv/div. vertical)
c) Upstream pressure - 100 psi. Downstream pressure 32 psi.

Fig. 2.3. (continued)
Fig. 2.4 with the conditions given. Various time intervals for the injector operation are indicated schematically in Fig. 2.5. The results for the experiments in Fig. 2.4 and additional experiments are presented in Table 2.1 and Fig. 2.6. Of particular interest to this study is the time interval $\Delta t_4$. This time interval decreases fairly rapidly with increasing pressure which results in a shorter open time for a particular pulse width. The reason for the injector behavior noted here is not understood since all of the mechanical characteristics have not been explored thoroughly.

An attempt was made to delineate the source of the oscillations shown in the figures above. These oscillations appeared on both the opening and closing sequences of the injector operation and propagated through the system since they can also be seen at the downstream transducer. In fact the occurrence of these oscillations as they reflect through the system provided an accurate means of identifying reflections from either end of the system. It was possible to remove the opening oscillation from the pressure signal if the electronic pulse width was shorter than a time interval approximately equal to 0.3 ms less than the time interval $\Delta t_2$. An example of this situation is given in Fig. 2.7. For longer pulse widths, the oscillations on the injector opening were unchanged. One thing noted was that when this oscillation was removed by shortening of the pulse width an audible click from the injector also vanished. Again, a total explanation of the operation is not understood although it is speculated that the oscillations are due to some
Fig. 2.4. Polaroid photocopy of injector operation characteristics as a function of pressure difference across fuel injection (1 ms/div. horizontal)
c) Injector pressure 105.8 psi.

d) Injector pressure 128.8 psi.

Fig. 2.4. (continued)
Fig. 2.5 Definition sketch for time intervals given in Table 2.1 and Fig. 2.6.
Table 2.1  Operating Characteristics of Fuel Injector as Function of System Pressures (pulse width 3 msec).

<table>
<thead>
<tr>
<th>Upstream Pressure (psi)</th>
<th>Downstream Pressure (psi)</th>
<th>Injector Pressure (psi)</th>
<th>$\Delta t_1$ (msec)</th>
<th>$\Delta t_2$ (msec)</th>
<th>$\Delta t_3$ (msec)</th>
<th>$\Delta t_4$ (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>21.5</td>
<td>1.5</td>
<td>1.8</td>
<td>4.6</td>
<td>2.1</td>
</tr>
<tr>
<td>60</td>
<td>40</td>
<td>51.5</td>
<td>1.6</td>
<td>2.0</td>
<td>4.4</td>
<td>1.7</td>
</tr>
<tr>
<td>110</td>
<td>80</td>
<td>105.8</td>
<td>1.8</td>
<td>2.4</td>
<td>4.2</td>
<td>1.2</td>
</tr>
<tr>
<td>150</td>
<td>100</td>
<td>128.8</td>
<td>2.1</td>
<td>2.8</td>
<td>4.1</td>
<td>0.9</td>
</tr>
<tr>
<td>110</td>
<td>110</td>
<td>110</td>
<td>1.9</td>
<td>2.5</td>
<td>4.2</td>
<td>-</td>
</tr>
<tr>
<td>110</td>
<td>60</td>
<td>88.8</td>
<td>1.9</td>
<td>2.3</td>
<td>4.2</td>
<td>-</td>
</tr>
<tr>
<td>110</td>
<td>40</td>
<td>80.4</td>
<td>1.85</td>
<td>2.3</td>
<td>4.2</td>
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<td>110</td>
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<td>71.9</td>
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<td>4.3</td>
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<tr>
<td>110</td>
<td>7</td>
<td>66.4</td>
<td>1.8</td>
<td>2.2</td>
<td>4.3</td>
<td>-</td>
</tr>
</tbody>
</table>
Fig. 2.6. Injector operation characteristics as a function of pressure difference across fuel injector.

Electronic pulse width: 3 msec
Fig. 2.7. Removal of initial oscillation by decreasing pulse width.

This procedure was performed under various conditions. An analytical balance was used to measure the injections, and the fluid in a set of syringes was weighed in a flask and weighed.

The results of the sets of experiments were consistent with expected behavior. Absorption by the injected fluid was not observed, and no other influence was noted on the measured parameters. The oscillations appeared to be due to frictional motion of the fluid in the system.
mechanical vibrations in the injector needle. These oscillations were observed to persist even if the injector was blocked at the outlet to prevent through-flow.

Repeatability of Injection

Experiments were performed to evaluate the general repeatability of individual injections. Different sets of experiments were performed with the following conditions held constant: 1) temperature (and thus viscosity) of the fluid; 2) pressure difference across injector; and 3) pulse width. A Mettler analytical balance accurate to one milligram was used to measure the injections. The fluid from a set of injections was caught in a flask and weighed.

This procedure was performed several times for various conditions (e.g. 20, 10, 5 pulses per measurement). The results of two sets of experiments are tabulated in Table 2.2. It is expected that the standard deviation of the measurements should increase with decreasing pulses/measurement if the results observed are due to random variation in the injected volume. Since this was not observed in either case, the variation must be due to other influences. Probable explanations are inaccuracies in reading the balance and the major source of variation which appeared to be due to fluid accumulation on the injector nozzle. This latter condition resulted in drops of 2-4 mg periodically shed during the injector operation. The variation of all measurements was basically within this 4 mg range and represents the probable explanation of the approximately constant standard
deviations. The only conclusion that is justified from the experiments with this influence is that the standard deviation of individual injections is probably much smaller than that indicated and less than the resolution of the analytical balance.
Table 2.2 Results of Repeatability Measurements.

<table>
<thead>
<tr>
<th># of pulses</th>
<th># of samples</th>
<th>standard deviation (gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>.0016</td>
</tr>
<tr>
<td>10</td>
<td>21</td>
<td>.0011</td>
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<td>5</td>
<td>21</td>
<td>.0017</td>
</tr>
<tr>
<td>1</td>
<td>21</td>
<td>.0008</td>
</tr>
<tr>
<td>Set 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>12</td>
<td>.0012</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
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<tr>
<td>5</td>
<td>20</td>
<td>.0011</td>
</tr>
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3.0 NUMERICAL SIMULATIONS

The mathematical modeling of physical experiments was accomplished using a numerical simulation based upon the method of characteristics.* Since the method is reasonably well documented in the literature, the entire theory will not be repeated here. The equations in finite difference form as utilized in the computer programs are discussed in this section.

Figure 3.1 shows a schematic diagram of the physical apparatus as modeled, with the steady state hydraulic grade line indicated. The x-t plane is also shown with typical characteristic lines forming a staggered grid in the plane. Equal distance intervals, $\Delta x$, and time steps, $\Delta t$, to satisfy the Courant condition, $dx/dt = a$, in which $a$ is the wave propagation velocity, are shown. The notation in Fig. 3.1 agrees with the notation in the programs listed in the Appendix.

The pipeline is divided into N equal reaches, which must be an even number for the staggered grid. The fuel injector is located at section NJS, which must be odd. The downstream orifice is located at section NS, an odd number equal to N+1. The upstream orifice is located at section 1. In addition to the pipeline and fuel properties, input data include the upstream and downstream pressures (assumed to remain constant) outside the orifices, and the discharge coefficient multiplied by the area of opening for each orifice. These data enable the steady through flow in the system to be computed.

Fig. 3.1 Schematic diagram and xt plane

Fig. 3.2. Schematic diagram of injector flows and hydraulic gradeline.
Transient simulations are carried out by defining the repetitive period, \( \text{PER} = 60/\text{rpm} \), of each injection cycle, and defining the dimensionless injector needle position vs time in tabular form. Thus the pulse width and the nature of the pulse can be defined (whether it is square or triangular in form). The discharge coefficient multiplied by open area of the injector is needed for the wide open position.

Method of Characteristics: Compatibility Equations

The two compatibility equations which relate the hydraulic gradeline elevation, \( H \), and the discharge, \( Q \), at adjacent sections, and which are valid along the \( C^+ \) and \( C^- \) characteristic lines are:\(^1\)

\[
H_i = C_P - B Q_i
\]  \hspace{1cm} (3.1)

\[
H_i = C_M + B Q_i
\]  \hspace{1cm} (3.2)

in which \( B = a/gA_r \), with \( A_r \) the pipe area, and

\[
C_P = H_{i-1} + B Q_{i-1} - H_{fi-1}
\]  \hspace{1cm} (3.3)

\[
C_M = H_{i+1} - B Q_{i+1} + H_{fi+1}
\]  \hspace{1cm} (3.4)

The subscript \( i \) refers to the current time at section \( i \) while \( i+1 \) refers to one time-step earlier at the adjacent upstream (-) and downstream (+) sections. The last term in Eqs. (3.3) and (3.4) represents the viscous loss term. The energy dis-

---

sipation may be described in terms of a loss equivalent to steady-state flow at the same velocity, in which case, for turbulent flow,

\[ H_{f_{i+1}} = R \frac{Q_{i+1}}{g D_{r} \sqrt{A_r^2}} \]  \hspace{1cm} (3.5)

with \( R = f \frac{\Delta x}{2g DA_r^2} \). The Darcy Weisbach friction factor is given by \( f \), and the fuel rail diameter by \( D \). For laminar flow,

\[ H_{f_{i+1}} = 32 \nu \frac{\Delta x}{g D^2 A_r} \]  \hspace{1cm} (3.6)

in which \( \nu \) is the fluid kinematic viscosity. Alternatively, for laminar flow, the dependence of the viscous losses upon the unsteady nature of the velocity profile can be incorporated in the compatibility equations \(^1\). In this case

\[ H_{f_{i+1}} = H_{f_{i+1}} + H_f' \]  \hspace{1cm} (3.7)

in which \( H_f \) is given by Eq. (3.6), the steady-state friction loss equation, and \( H_f' \) is the deviation from the steady-state value due to the unsteady motion of the fluid. Although this latter refinement is incorporated in the program for laminar flow, Appendix A, little modification in computational results occur as a result of the refinement. This is because of the dominance of the orifice losses on the response in this operating system. Additional discussion of frequency dependent friction is included at the end of this section, prior to the introduction of the computer program.

At all interior computational sections in the fuel rail, the instantaneous hydraulic gradeline and flow rate are given

by a simultaneous solution of Eqs. (3.1) and (3.2)

\[ H_i = \frac{C_p + C_M}{2} \]  \hspace{1cm} (3.8)

\[ Q_i = \frac{H_i - C_M}{B} \]  \hspace{1cm} (3.9)

Boundaries. Upstream Orifice.

Positive flow through the upstream orifice is described by

\[ Q_1 = C_D A_u \sqrt{2g(H_u - H_i)} \]  \hspace{1cm} (3.10a)

and negative flow, by

\[ Q_1 = -C_D A_u \sqrt{2g(H_i - H_u)} \]  \hspace{1cm} (3.10b)

The fuel rail response during transient flow is conveyed to the orifice along the \( C^\rightarrow \) characteristic line and is described by Eq. (3.2). For positive flow simultaneous solution of Eqs. (3.2) and (3.10a) yields

\[ Q_1 = -C_V B + \sqrt{(C_V B)^2 + 2C_V(H_u - C_M)} \]  \hspace{1cm} (3.11a)

and for negative flow, Eqs. (3.2) and (3.10b) yield

\[ Q_1 = C_V B - \sqrt{(C_V B)^2 - 2C_V(H_u - C_M)} \]  \hspace{1cm} (3.11b)

in which \( C_V = g(C_D A_u)^2 \). Examination of these equations shows that Eq. (3.11a) will yield positive flow only when \( (H_u - C_M) > 0 \), and Eq. (3.11b) will yield negative flow only when \( (H_u - C_M) < 0 \). Thus by setting \( S = 1 \) for the first case and \( S = -1 \) for the second case the following equation handles both conditions.
\[ Q_1 = S(-C_v^2B + \sqrt{(C_vB)^2 + 2SC_v(H_u - C_M)}) \tag{3.12} \]

Once \( Q_1 \) is determined in Eq. (3.12), the hydraulic gradeline at section 1 is calculated using Eq. (3.2).

**Downstream Orifice.**

Positive flow through the downstream orifice is given by
\[ Q_{NS} = C_D^2A_D' \sqrt{2g(H_{NS} - H_D)} \tag{3.13a} \]

and negative flow by
\[ Q_{NS} = -C_D^2A_D' \sqrt{2g(H_D - H_{NS})} \tag{3.13b} \]

Simultaneous solution of each of these equations with the \( C^+ \) compatibility Eq. (3.1), with \( S = 1 \) for \((C_p - H_D) > 0\) and \( S = -1 \) for \((C_p - H_D) < 0\), yields
\[ Q_{NS} = S(-C_v^2B + \sqrt{(C_v^2B)^2 + 2SC_v^2(C_p - H_D)}) \tag{3.14} \]

In Eq. (3.14) \( C_{vD} = g(C_D^2A_D)^2 \). Equation (3.1) is used to find the hydraulic grade line in the fuel rail at the downstream orifice, \( H_{NS} \), after \( Q_{NS} \) is determined in Eq. (3.14).

**Fuel Injector.**

Flow through the fuel injector, Fig. 3.2, is given by
\[ Q_{inj} = C_D^2A_i \sqrt{2g(H_{NJS} - H_I)} \tag{3.15} \]

Inasmuch as the injector needle may not open instantaneously, and therefore an intermediate needle position may be needed, a
dimensionless ratio is used to define the needle position

\[ \tau = \frac{C_D A_I}{(C_D A_I)_0} \]  \hspace{1cm} (3.16)

The subscript \( o \) refers to the product of the discharge coefficient and area of opening in the wide open position. The combination of Eqs. (3.15) and (3.16), with \( C_I = (C_D A_I)_o \sqrt{2g} \), yields

\[ Q_{inj} = \tau C_I \sqrt{H_{NJS} - H_I} \]  \hspace{1cm} (3.17)

A common pressure is assumed in the fuel rail at the injector, \( H_{NJS} \), and at any instant continuity of flows at the injector may be stated, Fig. 3.2,

\[ Q_{NJS} = Q_{inj} - Q_{NJS} = 0 \]  \hspace{1cm} (3.18)

When Eqs. (3.17), (3.18), Eq. (3.1) for the upstream side of the injector, and Eq. (3.2) for the downstream side of the injector, are combined, and solved for the hydraulic grade line:

\[ H_{NJS} = (A_1 - \sqrt{A_1 - (C_p + C_M)^2 - B^2 C_I \tau^2 H_I}) / 2 \]  \hspace{1cm} (3.19)

In Eq. (3.19), \( A_1 = C_P + C_M + (BC_I \tau)^2 / 4 \). When the injector needle is closed, \( \tau = 0 \), and Eq. (3.19) reduces to Eq. (3.8). Once \( H_{NJS} \) has been determined the individual flows at the injector may be determined from Eqs. (3.17), (3.1), and (3.2).
Frequency-Dependent Friction

Inasmuch as energy loss due to fluid friction is highly dependent upon the rate of change of velocity, it follows that waves that contain higher harmonics attenuate much faster than low frequency components. For unsteady laminar flow Zielke\(^2\) developed a procedure to correct the steady-state friction term in the method of characteristics. In Eq. (3.7),\(H'_f\), which represents the deviation from the steady-state value of friction loss, is evaluated based on the history of the velocity at the section. In integral form the correction is expressed

\[
H'_f = \frac{16\nu}{gD^2A_r} \int_0^t \frac{dQ}{dt} (u)W(t-u)du \tag{3.20}
\]

in which \(W\) is a weighting function which is a function of time. For convenience it is expressed in terms of dimensionless time \(\tau\),

\[
\tau = \frac{4\nu}{D^2} t \tag{3.21}
\]

and is calculated by the use of two different series of terms, one for values of \(\tau > 0.02\), and the other for values of \(\tau < 0.02\). The two series provide overlapping values in the vicinity of \(\tau = 0.02\).

\[
W(\tau) = e^{-26.3744\tau} + e^{-70.8493\tau} + e^{-135.0198\tau} + e^{-218.9216\tau} + e^{-322.5544\tau} \quad \tau > 0.02 \tag{3.22a}
\]

\[ W(\tau) = 0.282095 \tau^{-12} - 1.25 + 1.057855\tau^{12} + 0.9375\tau + 0.396696\tau^{3/2} - 0.351563\tau^2 \quad \tau < 0.02 \] (3.22b)

In the application of the method of characteristics, Eq. (3.20) is evaluated from a first-order approximation by using the history of the velocity and the weighting function in the following manner.

\[ H_{fi,k} = \frac{16\nu}{gD^2A_r} \sum_{j=1}^{k_m} (Q_{i,j+1} - Q_{i,j}) W((2(k_m + 1 - j) - 1)\Delta t) \]

\[ = \frac{16\nu}{gD^2A_r} \sum_{j=1}^{k_m} (Q_{i,k_m+2-j} - Q_{i,k_m+1-j}) W((2j-1)\Delta t) \] (3.23)

By referring to the grid in Fig. 3.3, Eq. (3.23) can be written

\[ H_{fi,k} = \frac{16\nu}{gD^2A_r} (Q_{i,k_m+1} - Q_{i,k_m}) W(\Delta t) + (Q_{i,k_m} - Q_{i,k_m}) W(3\Delta t) \]

\[ + \ldots + (Q_{i,2} - Q_{i,1}) W((2k_m - 1)\Delta t) \] (3.24)

The weights \( W \) are calculated from Eqs. (3.22a) and (3.22b) as functions of the dimensionless parameter \( \tau \). The numerical value of \( W(\tau) \) approaches zero as \( \tau \) becomes very large. If a value of \( \tau_{\text{max}} \) is selected such that all weights \( W(\tau) \) for \( \tau > \tau_{\text{max}} \) are negligible, then the number of terms in Eq. (3.24) never exceeds \( \tau_{\text{max}} D^2/(8\nu\Delta t) \).

As stated earlier, the introduction of the frequency-dependent losses in this particular problem made little difference in the
Fig. 3.3. Notation for unsteady friction.
calculated response for laminar flow. This is because the pipeline losses are much smaller than other system losses, namely the orifice losses, which totally dominate the system dissipation.

The relative importance of the frequency dependent friction effects for typical fuel injection systems can be estimated by considering the magnitude of the friction term to other terms in the characteristic equations. Specifically consider the ratio

\[
\frac{gH_f A_r}{\frac{dQ}{dt}}
\]

(3.25)

for the case of a linear drop (or rise) in pressure in time \(\Delta t\) (e.g. associated with opening and closing of injector). This results in change of flow \(\Delta Q\) in the same time period. Take a typical value of \(\Delta t\) on the order of 1 msec, \(v\) of \(1.0 \times 10^{-5}\) ft\(^2\)/sec; and \(D\) of 0.3". Then the major contribution to the unsteady friction term would be associated with the maximum change in discharge

\[
H_f' \sim \frac{16v}{gD^2 A_r} \Delta Q W(\tau)
\]

(3.26)

For \(\tau = 4v\Delta t/D^2 = 6.4 \times 10^{-5}\) then \(W(\tau) = 34\). and the unsteady friction term is much larger in magnitude than the steady state friction term provided that \(\Delta Q\) and \(Q\) are of the same order of magnitude or if \(\Delta Q\) is greater than \(Q\). The ratio in Eq. (3.25) is thus given approximately by

\[
\frac{gH_f A_r}{\frac{dQ}{dt}} \sim W(\tau) \tau
\]

(3.27)
The maximum value of this ratio is for \( \tau \) on the order of .02 and is 0.072. For the value of \( \tau \) above, the value is .0088. Thus it can be seen for the first reflection where only one or two terms in the unsteady friction are important, that the unsteady friction is small relative to other terms in the characteristic equations, and it should be expected that significant unsteady friction effects should not become apparent until there have been several reflections through the system. Although this conclusion is only valid for laminar flow, it is presumed that a similar conclusion would be justified for turbulent flow. Since there is no theoretical development for frequency dependent friction effects in turbulent flow, this hypothesis cannot be verified mathematically.

Two programs in FORTRAN are provided in the Appendix. The first is for laminar flow and includes the frequency dependent losses. The second is essentially the same program except it is for turbulent flow and includes viscous losses evaluated using the Darcy-Weisbach equation, with the friction factor evaluated as a function of the local mean velocity utilizing the Colebrook Equation. A sample output is provided along with a list of variables in the programs.
4.0 THE FUEL RAIL WITH MATCHED IMPEDANCES AT THE TERMINALS

The equations that describe the transient behavior of the fuel rail are identified below, and the variables and controlling parameters in the problem are discussed.

Transient Pipeline Equation: Since viscous losses along the pipeline are minor compared with losses at the terminal orifices, the flow delivered when the injector opens is derived equally from each side of the injector. If the injector flow is \( 2\Delta Q \) then \( \Delta p = \pm \gamma B \Delta Q \) describes the transient behavior in the fuel rail during fuel injection. \( \Delta p \) is the pressure drop, \( \gamma = \rho g \) is the unit weight of the fluid with \( \rho \) the mass density. \( B = a/gA_r \) is the characteristic impedance of the pipeline in which \( a \) is the wave propagation velocity and \( A_r \) is the pipe cross-sectional area. In terms of the variables shown in Fig. 4.1 the wave propagation in the fuel rail is described by

\[
H_0 - H = B \Delta Q \quad (4.1)
\]

Orifices: Flows through the orifices are described by the following relationships. In each case the coefficient \( C = KA_o \sqrt{2g} \), in which \( K \) is the orifice discharge coefficient and \( A_o \) is the flow area.

Injector:

\[
2 \Delta Q = C_I \sqrt{H - H_I} \quad (4.2)
\]
Fig. 4.1. Fuel Rail
Upstream orifice:

\[ Q_o = C_u \sqrt{H_1 - H_0} \]  
\[ Q_o + \Delta Q = C_u \sqrt{H_1} - H \]  

\[ 4.3 \]

\[ 4.4 \]

Downstream orifice:

\[ Q_o = C_d \sqrt{H_0 - H_2} \]  
\[ Q_o - \Delta Q = C_d \sqrt{H - H_2} \]  

\[ 4.5 \]

\[ 4.6 \]

Equations (4.4) and (4.6), with \( H \) defined in Fig. 4.1, are valid only for the condition of no reflection at the orifice.

Controlling Parameters and Variables: There are eleven variables and parameters that dictate the behavior of the fuel injector.

- \( B \): the pipeline impedance = \( a/gA_r \)
- \( C_i \): the injector coefficient = \( K_i A_{oi} \sqrt{2g} \)
- \( C_u \): the upstream orifice coefficient = \( K_u A_{ou} \sqrt{2g} \)
- \( C_d \): the downstream orifice coefficient = \( K_d A_{od} \sqrt{2g} \)
- \( Q_o \): the fuel rail flow rate
- \( \Delta Q \): one-half of the injector flow rate
- \( H_o \) or \( (p_o/\gamma+Z) \): the hydraulic grade line elevation along the fuel rail
- \( H_1 \) or \( (p_1/\gamma+Z) \): the hydraulic grade line on the upstream side of the upstream orifice
- \( H_2 \) or \( (p_2/\gamma+Z) \): the hydraulic grade line on the downstream side of the downstream orifice
- \( H \) or \( (p/\gamma+Z) \): the hydraulic grade line in the fuel rail after passage of injector pulse
- \( H_i \) or \( (p_i/\gamma+Z_i) \): the hydraulic grade line outside the injector orifice
For a given fuel rail, the pipeline impedance \( B \) is determined. With six independent equations and 10 variables, four of them can be arbitrarily specified. In a typical design, once the pipeline has been selected, as well as the required injector flow rate \( AQ \), the designer is likely to identify the three external pressures. Then the remaining six variables \( H_0, H, Q_0 \), and the three orifice sizes are determined to provide a system free of terminal reflections. An instantaneous opening and closure of the injector needle is assumed as well as a time of opening less than the reflection time between injector and nearest terminal orifice. Should \( Q_0 \) be one of the specified variables then one of the delivery pressures cannot be specified.

In the laboratory apparatus the injector orifice size and injector delivery pressure are fixed. With particular upstream and downstream orifices, only one operating condition exists to provide the reflectionless system. A number of tests are described in the next section to illustrate the behavior of the system with matched as well as with unmatched conditions.
5.0 EXPERIMENTAL AND NUMERICAL RESULTS

Procedure

A number of different experiments were performed with the important results given in this section. Numerical simulations for the given experimental conditions are also presented. The general procedure adopted for a given experiment was as follows:

A particular pulse width and frequency were selected and set with the injector controls. The choice of these was somewhat arbitrary; generally pulse widths were selected as 3, 5 or 8 msec and frequencies of 20 - 35 Hz for the injector operation were used.

A given flow condition was then set by installing particular orifices at the upstream and downstream ends of the line and adjusting system pressures to obtain the required flow condition. The upstream pressure was controlled by adjustment of the pump vane setting while the downstream pressure was established by an adjustable regulator.

The steady flow rate through the system was then measured as was the flow with the injector in operation. This was done by weighing a sample of fluid passed through the system in a given amount of time. The flow rate through the injector in its operating condition was also measured.

The experimental conditions were noted and a Polaroid photograph of the repeating oscilloscope trace was obtained. The oscilloscope scale factors were also noted.

Laminar Flows Results

The majority of the experiments were designed such that the steady flow (without injector operation) through the system
was laminar. This was originally intended to provide a verification of the unsteady friction effects as discussed earlier in this report. The orifices were those originally supplied by Bendix (.0275 in. diameter downstream, .0362 in. diameter upstream). One experimental run and corresponding numerical simulation are given in Fig. 5.1. This particular result is to demonstrate a flow in which the matched impedance condition was not satisfied. This should not be interpreted as a typical flow result since different conditions could show a significantly different pressure trace. It is included to show the pressure fluctuations remaining in the system after the initial pressure pulse has propagated to the ends. A different flow condition with the same orifices but which did satisfy the matched impedance criteria is presented in Fig. 5.2. Although there are some residual pressure fluctuations after the initial pulse has passed the transducers, these are very small in magnitude and no systematic deviations are apparent. The numerical simulation shows essentially the same result. The deviations that are apparent are associated with the leading and trailing edges of the injector pulse where the flow rate is intermediate between the two matched conditions.

Additional experiments were performed by changing orifice diameters and establishing the new matched impedance conditions. These results are given in Fig. 5.3 and again show the validity of the concept as only very minor pressure fluctuations occur after the passage of the initial pressure wave.
Fig. 5.1. Unmatched impedances, laminar flow.
Figure 5.2. Matched impedances, laminar flow

CDAU = $6.34 \times 10^{-6}$ ft$^2$
CDAD = $3.76 \times 10^{-6}$ ft$^2$
PU = 150 psi
PD = 127 psi
Fig. 5.3. Matched Impedances, laminar flow with different orifice.

CDAU = 6.34 x 10^{-6} ft^2  PU = 59 psi
CDAD = 5.05 x 10^{-6} ft^2  PD = 38 psi
Turbulent Flow

Two additional orifices (0.061 and 0.060 inch diameter) were supplied by Bendix and were intended to provide a flow condition in which the steady state flow through the system was turbulent at the matched impedance condition. However after the discharge coefficients were measured for the two orifices ($C_D A = 1.59 \times 10^{-5} \text{ ft}^2$ and $1.71 \times 10^{-5} \text{ ft}^2$ for the 0.060 and 0.061 inch orifices respectively) the computed matched impedance condition required a higher pressure than the pump could supply. The numerical simulation of the matched impedance condition is presented, however, in Fig. 5.4, and shows a similar behavior to the results for laminar flow. In addition, new orifices were obtained so that the matched impedance conditions could be produced experimentally. This condition is given in Fig. 5.5 and indicates the validity of the concept for turbulent flows as well. Fig. 5.6 present an experiment where the matched impedance condition is not satisfied. Again, this result is for comparison purposes only and does not represent a general result for all flow conditions.

Additional Experiments.

Experiments were also performed to examine the necessity of the accumulators in the system. The flow condition considered was the same as that given in Fig. 5.3 with an accumulator and a regulator at both ends of the experimental system. The upstream regulator was added to the system for these particular tests. The experiment was then repeated with these devices progressively removed in the following order with the results presented in Figs. 5.7-5.12:
Fig. 5.4. Matched impedances, Turbulent flow, numerical simulation only.

CDAU = $1.71 \times 10^{-5}$ ft$^2$    PU = 405 psi
CDAD = $1.59 \times 10^{-5}$ ft$^2$    PD = 229 psi
Fig. 5.5. Matched impedances, turbulent flow.
Fig. 5.6. Unmatched Impedances, Turbulent Flow.

CDAU = $1.71 \times 10^{-5} \text{ ft}^2$
CDAD = $1.59 \times 10^{-5} \text{ ft}^2$
PU = 80 psi
PD = 10 psi
Fig. 5.7 accumulators and regulators at both ends of system
Fig. 5.8 upstream regulator removed
Fig. 5.9 upstream accumulator removed
Fig. 5.10 upstream accumulator and regulator removed
Fig. 5.11 downstream accumulator removed
Fig. 5.12 upstream and downstream accumulators removed

The downstream regulator could not be removed with the matched impedance condition reproduced.

The results of these experiments can be summarized as follows: The removal of the regulators does not appear to make any significant difference to the system response. This can be seen in the similarity of both Figs. 5.6 and 5.7 and also Figs. 5.9 and 5.10. Each set of experiments is similar except that the upstream regulator has been removed in the second test. The removal of the accumulators appears to make a minor but detectable difference in response. The magnitude of pressure fluctuations increases by approximately 1-2 psi when the accumulators are removed but tends to be damped out after approximately one reflection and scarcely detectable afterward.

Pressure and Discharge Relations

An additional objective of the experimental study was to evaluate the potential for using the relationship in Eq. 4.1 in conjunction with measuring the pressure to determine the volume injected in a particular injection sequence. The volume injected $\Delta V$ is given as

$$\Delta V = \int 2\Delta Q \ dt = \frac{2}{B} \int (H_0 - H) \ dt$$
Fig. 5.7. All regulators and accumulators in system.
Fig. 5.8. Upstream regulator removed.
Fig. 5.9. Upstream accumulator removed
Fig. 5.10. Upstream regulator and accumulator removed.
Fig. 5.11. Downstream accumulator removed.
The volume of fluid can then be determined directly from the pressure trace.

Fig. 5.12. Both accumulators removed.
The volume of fluid can thus be determined directly from
the pressure trace.

Experiments were conducted to demonstrate the concept. The fuel injection apparatus was tested at two arbitrary operating conditions.

The pulse frequency was kept low to provide sufficient time for pressure transients to attenuate between pulses, since the system was not at matched impedance conditions.

The operating conditions were:

I. Pulse width = 4 msec
   Pressure upstream = 40 psi
   Pressure downstream = 30 psi

II. Pulse width = 3 msec
    Pressure upstream = 55 psi
    Pressure downstream = 45 psi

All measurements were made at one time including 25 photographs of individual pressure traces for case I and 30 photographs for case II. Actual pulse volumes were measured by counting a given number of pulses and weighing the fluid mass associated with them. The average volume per pulse was computed from repeated measurements and determined to be accurate within + 0.5% from the various measurements.

The area above the pressure drop trace was measured with a planimeter to evaluate the pressure-time integral. Figure 5.13 shows an image of several pressure traces which was used to select a procedure to measure the areas in a consistent manner. Interpretation of the area enclosed was as follows:
- Extrapolate a horizontal line across the top from the horizontal portion of the trace at the upper left. (ignore the small bump 2 squares from the left side).

- Extend a vertical line down from where the trace ends as one proceeds from far left to right.

- Extend a vertical line up from the trace where it ends as one proceeds from center to right.

- Pass a line through the center of the oscillations at the bottom of the photo, extrapolating horizontally to the left where information is missing.

These interpretations of the area are supported by reference to Fig. 5.13 which has recorded the high frequency noise at the beginning and end of the injector pulses.

Results of planimeter measurements of several photographs as given in Table 5.1 and corresponding photographs are given in Figs. 5.14 - 5.23. The remaining photographs were not analyzed. For the purposes of the calculations, the following conditions were used:

- specific gravity of fluid = .76
- wave speed = 3940 ft/sec
- pipe diameter = .305 in

yielding a B value of 241160 sec/ft². The pressure transducer recording the pressure traces was taken to have a calibration factor of 1.055 psi/mv; a result of additional experiments beyond those discussed in Chapter 2. The possible error in each of the factors involved in each of the calculation parameters yielded an estimated maximum error of approximately +3-4%. All results in Table 5.1 are within this range and represent excellent agreement of the experiments with the theoretical concept.
Table 5.1 Experiments to compute volume Injected from Measured Pressure Trace.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Integrated Pressure (psi-msec)</th>
<th>Computed Volume ($ft^3 \times 10^{-7}$)</th>
<th>Measured Volume ($ft^3 \times 10^{-7}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.14</td>
<td>26.12</td>
<td>6.58</td>
<td>6.505</td>
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<tr>
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</tr>
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<td>26.25</td>
<td>6.61</td>
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<tr>
<td>5.17</td>
<td>26.29</td>
<td>6.62</td>
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<td>5.18</td>
<td>26.25</td>
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<tr>
<td>5.19</td>
<td>22.92</td>
<td>5.77</td>
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<tr>
<td>5.23</td>
<td>22.95</td>
<td>5.78</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
Fig 5.13 Composite photograph of several Case II injections

Fig 5.14 Case I
Fig 5.15 Case I

Fig 5.16 Case I
Fig 5.17 Case I

Fig 5.18 Case II
Fig 5.19 Case II

Fig 5.20 Case II
Fig 5.21 Case II

Fig 5.22 Case II
The results of this study have demonstrated the concept of diminishing pressure fluctuations by applying the matched impedance principle to a dual injection system. The results of the experimental investigations indicate that when the matched flow conditions are achieved, the pressure fluctuations remaining are due to fuel injection alone and not to turbulence. The experimental work has been compared with an analytical model which has been found to give good agreement. The results indicated that the unsteady friction influences on the flow behavior were negligible for the particular system. This was true because the major pressure drops in the system were in the orifices used to create the matched impedance conditions and the friction effects were negligible. Although the numerical model for injector flow was developed with unsteady friction effects included, the results were not significantly different than if the instantaneous local velocities were used to compute friction losses. In the turbulent flow model, the injection approach was used, and the good agreement of the numerical simulations with experimental data indicate that unsteady influences were in the model.
6.0 SUMMARY

The results of this study have demonstrated the concept of minimizing pressure fluctuations by applying the matched impedance principle to a fuel injection system. The results of the experimental investigations indicate that when the matched flow conditions are achieved, the pressure fluctuations remaining in the system after the initial response to fuel injector operation are not of practical importance. The experimental results were conducted for both laminar and turbulent flow systems. The computer model used to simulate the experimental data was found to be in good agreement, indicating that the numerical model can be a useful tool for predicting the major features of the system response for actual fuel injection systems.

The results also indicated that the unsteady friction influences on the flow behavior were negligible for the particular system. This was true because the major pressure drops in the system were in the orifices used to create the matched impedance conditions and so friction effects were negligible. Although the numerical model for laminar flow was developed with unsteady friction effects included, the results were not significantly different than if the instantaneous local velocities were used to compute friction losses. In the turbulent flow model, this latter approach was used, and the good agreement of the numerical simulations with experimental data indicate that unsteady influences were again minor.
Finally, the principle of the determination of the fuel volume injected by monitoring the pressure response of the fuel delivery system was demonstrated. By integrating the pressure response over time and applying the waterhammer relations, it was possible to compute the volume of fuel passed through the fuel injector. This principle has potential application to various flow control devices on an actual fuel injection system.
7.0 ACKNOWLEDGEMENTS

Mr. Dan Cooper ably performed most of the experimental work and carried out many of the numerical comparisons during this project.

The cooperation of Bendix personnel particularly Mr. Lael Taplin, in providing the experimental apparatus and for considerable technical interaction, is greatly appreciated.
APPENDIX A ...

FORTRAN program and output, laminar flow with frequency dependent friction.
C FUEL INJECTION SYSTEM. RESERVOIR AND ORIFICE AT EACH PIPE END.
C FUEL INJECTOR AT INTERNAL SECTION.
C LAMINAR FRICTION, staggered grid. N MUST BE EVEN.
C INJECTOR AT SECTION NJS, WHICH MUST BE CCD.
C LAMINAR FREQUENCY DEPENDENT LOSSES, ZIELKE'S WEIGHTING FUNCTION.
C
OF TERMS IN FRICTION TERM IS KMAX.
DIMENSION H(62), Q(62), EL(62), CF(62, 91), W(91), HF(62), HFF(91)
1, TD(15), TD(15), TIME(200), PINJ(200), PDS(200).
NAMELIST/DIN/A, XL, D, G, EL, PU, PD, CD, CIA, CPI, TMAX, N, IPR, VISC
1, KMAX, RHO, PER, NDATA, TAD, TD, NJS, IDIAG, IPLT
10 READ (5, DIN, END=220)
IF(N/2*2.NE.N) N=N+1
IF(KMAX/2*2.NE.KMAX) KMAX=KMAX+1
KMAX=KMAX+1
NS=NS+1
NS2=NS+1.
AR=.7854*D*D
FC=16.*VISC*XL/(G*D*AR*FLOAT(N))
R=32.*VISC*XL/FLOAT(N) G D*AR
B=A/(G*.7854*D*D)
DT=XL/(A*FLOAT(N))
GAM=RHO*G
HU=EL(1)+PU/GAM
HD=EL(NS)+PD/GAM
HI=EL(NJS)+PI/GAM
A1=.5/(G*D*CDAU**2+1./CDAD**2)
CV=G*CDAU*CD AU
CVD=G*CDAD*CDAD
CI=CDAI*SQRT(2.*G)
VCL=0.
C FIND STEADY STATE FLOW AND STORE INITIAL VARIABLES
Q0=.5*(R*N+SRT(R*R*N*N+4.*A1*(HU-HD)))/A1
H(1)=HU-Q0*Q0*.5/CV
DO 20 I=1, NS2
H(I)=H(1)-(I-1)*P*Q0
Q(I)=Q0
20 QF(I,KK)=Q0
QNJ=0.
PP=GAM*(H(NJS)-EL(NJS))/144.
PPP=GAM*(H(N)-EL(N))/144.
CALL WEIGHT(W, DT, KMAX, VISC, 0)
WRITE(6, 30) (W(I), I=1, KMAX)
30 FORMAT(' W=', 10F7.3)
T=0.
TT=0.
K=0
WRITE (6, 40) A, XL, D, VISC, PU, PD, PI, Q0, CIA, CD, CIA, G, TMAX, DT, B, N,
INJS, IPR, NDATA, RHO, PER, TD, TAD, KMAX, =1, NDATA)
40 FORMAT(' A, XL, D, VISC=', 2F8.1, 2F8.2, E12.4/
2. N, NJS, IPR, NDATA=', 4I4/, RHO, PER, TD, TAD=', 2F8.4/(2F8.5))
WRITE(6, 50)
50 FORMAT(' P AND DISCHARGES ALONG THE PIPE'/
1. TIME X/L= 0. INJ 1. QNJ')
C BEGINNING OF TRANSIENT LOOP
60 PI=(H(1)-EL(1))*GAM/144.
P2=(H(NJS)-EL(NJS))*GAM/144.
P3=(H(N)-EL(N))*GAM/144.
C REPOSITION VELOCITIES IN STORAGE AND EVALUATE VISCOUS LOSSES AT
C SECTION.
90 DO 110 I=I11,NS,2
100 QF(I,KK)=QF(I,KK+1)
110 IF(I.EQ.N) HFF(KK)=A1
120 QF(NS2,KK)=QF(NS2,KK+1)
130 IF(TII.EQ.1) WRITE(6,150)HB,HP,HF(N),(KK,HFF(KK),KK=1,KMAX)
150 FORMAT(1H6,150)
160 K=K+1
T=T+DT
C COMPUTATION OF INTERIOR POINTS
180 DO 180 I=I11,N,2
190 IF(I.EQ.NJS) GO TO 180
X=X+1+1
200 CP=H(I-1)+Q(I-1)*B-HF(I-1)
210 QX=Q(I+1)
220 HX=HF(I+1)
230 IF(I+1.NE.NJS) GO TO 170
240 QX=C(NS2)
250 HX=HF(NS2)
260 CM=H(I+1)-QX*B+HX
270 H(I)=.5*(CP+CM)
280 Q(I)=(H(I)-CM)/B
290 CONTINUE
300 IF (II.EQ.3) GO TO 190
310 III=2
320 II=3
330 GO TO 90
C UPSTREAM BOUNDARY
190 CM=H(2)-Q(2)*B+HF(2)
120  S=1.
121  IF(HU-CM.LT.0.)S=-1.
122  Q(1)=S*(-CV*B+SQRT(CV*B*CV*B+S*S2.*CV*(HU-CM))
123  H(1)=CM+B*Q(1).
124  C DOWNSTREAM BOUNDARY
125  CP=H(N)+Q(N)*B-HF(N)
126  S=1.
127  IF(CP-HD.LT.0.)S=-1.
128  Q(NS)=S*(-CV*B+SQRT(CV*D*CV*B*B+S*S2.*CVD*(CP-HD)))
129  H(NS)=CP-B*Q(NS).
130  C INJECTOR BOUNDARY
131  C FINC INJECTOR FLOW
132  TT=TT+DT*2./PER.
133  IF(TT,GT.1.) TT=TT-1.
134  DO 200 I=2,NDATA
135  KK=I-1
136  IF(TD(I).GT.TT) GC TO 210
137  200 CONTINUE
138  210 TA=TAD(KK)+(TT-TV(KK))*(TAC(KK+1)-TAD(KK))/
139  (TDD(KK+1)-TDD(KK))
140  CP=H(NJS-1)+Q(NJS-1)*B-HF(NJS-1)
141  CM=H(NJS+1)-Q(NJS+1)*B+HF(NJS+1)
142  AL=CP+CM*(R*CI*TA)*5**2
143  H(NJS)=.5*(AL-SQRT(AL*AL-(CP+CM)*5**2-(B*C*I*TA)*5**2*HI))
144  Q(NJS)=(-CM+H(NJS))/B
145  Q(NS)=-(CP-H(NJS))/B
146  QIC=QINJ
147  QINJ=Q(NS)-Q(NJS)
148  VCL=VCL+DT*(QIC+QINJ)
149  IF(TA.EQ.1.) QMAX=QINJ
150  IF(K/IPR.EQ.1) 80,50,60
151  220 QAVE=VCL/PER
152  WRITE(6,230) VCL,QAVE,MAX
153  230 FORMAT(' VOLUME INJECTED PER CYCLE=','F12.9/','AVERAGE FLOW RATE OVER CYCLE=','F12.8/','MAX FLOW RATE AT INJECTOR=','F12.8/','IF(IPLT.EQ.1) CALL 4LOT(TIME,PINJ,PDS,L,0.,-14.,-002.8.,10.,7.) STOP CONTINUE
157  158  SUBROUTINE WEIGHT(W,DT,KMAX,VISC,0)
159  DIMENSION W(l)
160  TALM=VISC*4./C*0
161  DO 20 K=1,KMAX
162  TAU=TAU*(2-K-1)*0T
163  ST=SQRT(TAU)
164  IF(TAU.GT.0.) GC TO 10
165  10 W(K)=1./EXP(26.3744*TAU)+1./EXP(70.3493*TAU)+1./EXP(135.0198*TAU)
166  +1./EXP(218.9216*TAU)+1./EXP(322.5544*TAU)
167  20 CONTINUE
168  RETURN
169  END
170  C WEIGHT EVALUATES THE WEIGHTING FUNCTION FOR SPECIFIC VALUES OF TAU.
171  DIMENSION W(l)
172  TAU=VISC*4./C*0
173  DO 20 K=1,KMAX
174  TAU=TAU*(2-K-1)*DT
175  ST=SQRT(TAU)
176  IF(TAU.GT.0.) GC TO 10
177  W(K)=292095/ST-1.2*+1.057855*ST+.9375*TAU+.396696*TAU*ST-.1351563*TAU)
178  10 GO TO 20
179  20 CONTINUE
180  RETURN
181  END
182  Cavin X=20.2,A=4000.,D=225417,G=32.2,EL=2*0.,60*33,23,
183  CDAI=2.6E-6,P=0.4,
184  DU=9568.,PD=5357.,RHC=1.5,CDAU=6.34E-6,CCAD=5.05E-6,VISC=1.26E-5,
185  TMAX=.0053,N=56,NJS=19,IPR=2,KMAX=3C,NDATA=5,IOIG=0,IOPLT=0,
186  PER=.05,TAD=0.,1.,1.,0.,0.,10=Q.,014.,05.,055,l., END
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AVERAGE FLOW RATE OVER CYCLE = 0.00001049
MAX FLOW RATE AT INJECTOR = 0.00022964
APPENDIX B
FORTRAN program
and output, turbulent flow
C FUEL INJECTION SYSTEM, RESEVOIR AND ORIFICE AT EACH PIPE END.
C FUEL INJECTOR AT INTERNAL SECTION.
C TURBULENT FRICTION, STAGGERED GRID. N MUST BE EVEN
C INJECTOR AT SECTION NJS, WHICH MUST BE ODD.
C CELEBRITY FORMULA TO EVALUATE FRICTION FACTOR
C HEAT LOSS EVALUATED IN SUBROUTINE HE.
DIMENSION H(62),Q(62),EL(62),TD(15),TD(15),TIME(200).
CNAMELIST/DIN/A,XL,D,G,EL,PQ,PD,PI,CDU,CDAD,CDAI,TMAX,N,IPR,VISC
C 1,PHO,PER,NODATA,TAD,T,D,NJS,EPS,TPLT
C READ (5,DIN,END=150)
C IF(N/2*2.0E-N.E.N)N=N+1
C NS=NS+1
C DX=X/(N)
C AR=.7854*0*0
C B=A/(G*.7854*0*0)
C DT=X/(A*(N))
C GAM=RHO*G
C HU=EL(I)+PI/GAM
C HD=EL(NJS)+PD/GAM
C HI=EL(NJS)+PI/GAM
C CV=G*CDU+CDU
C CVG=G*CDAD+CDAD
C CI=CDAI*SQRT(2.*G)
C VCL=0.
C A1=0.5/CG+0.5/CVD
C FIND STATIC STATE FLOW AND STORE INITIAL VARIABLES
C Q0=SQRT((H-I)/A1)
C DC 20 I=1,5
C R=HF(Q0,0,A,R,VISC,EPS,DX,G)/Q0/Q0
C 20 Q0=SQRT((H-I)/(A1+R*FLOAT(N)))
C H(I)=H(I)+Q0*Q0*5/CV
C DO 20 I=1,NS
C H(I)=H(I)-(I-1)*R-Q0*Q0
C 30 Q(I)=Q0
C H(NS2)=H(NJS)
C P=P=GAM*((H(NJS)-EL(NJS))/144.
C P=P=GAM*((H(N)-EL(N))/144.
C T=C.
C TT=0.
C K=0
C QINJ=0.
C WRITE(6,40) A,XL,D,VISC,PU,PD,PI,Q0,CDU,CDAD,CDAI,G,TMAX,DT,3,N,
C INJS,IPR,NODATA,RHO,PER,(TD(I),TAD(I),I=1,NDATA)
C 40 FORMAT(1,4F8.1,2E9.2/P,PD,PI,Q0=1,3F8.2,E12.4/
C 1*CDU,CDAD,CDAI=1,3E9.2/5,TMAX,DT,8=1,F8.3,2F8.4,F9.1/
C 2*N,NJS,IPR,NODATA=1,4I4/RHO,PER,TAD=1,2F8.4/(2F8.5))
C WRITE(6,50)
C 50 FORMAT(1) P AND DISCHARGES ALONG THE PIPE'/
C 1'TIME X/L= ? QINJ?)
C BEGINNING OF TRANSIENT LOOP
C 50 1=P1=(H(I)-EL(I))*GAM/144.
C P2=(H(NJS)-EL(NJS))*GAM/144.
C P3=(H(N)-FL(N))*GAM/144.
C Q1=Q(I)/Q0
C Q2=Q(NJS)/Q0
C Q3=Q(N)/Q0
C Q4=QINJ/Q0
C WRITE(6,70) T,P1,P2,P3,P4,P5,P6
COMPUTATION OF INTERIOR POINTS
DO 110 I=II,N,2
IF(I.EQ.NJS) GO TO 110
CP=H(I-1)+Q(I-1)*B-HF(Q(I-1),D,AR, VISC, EPS,OX,G)
QX=Q(I-1)
HX=HF(Q(I+1),D,AR, VISC, EPS,OX,G)
IF(I+1.LE.NJS) GO TO 100
QX=Q(NS2)
HX=HF(Q(NS2),D,AR, VISC, EPS,OX,G)
110 CONTINUE
IF(II.EQ.3) GO TO 120
II=3
GO TO 90
UPSTREAM BOUNDARY
CM=H(2)-Q(2)*B+HF(Q(2),D,AR, VISC, EPS,OX,G)
S=1.
IF(HU-CM.LT.0.)S=-1.
Q(1)=S*(-CV*B+SQR(T(CV*B*CV*B*S2.*CV*(HU-CM)))
H(1)=CM+B*Q(1)
DOWNSTREAM BOUNDARY
CP=H(N)+Q(N)*B-HF(Q(N),D,AR, VISC, EPS,OX,G)
S=1.
IF(CP-HD.LT.0.)S=-1.
Q(NS)=S*(-CV*D+SQR(T(CVD*CVD*B*S2.*CVD*(CP-HD)))
H(NS)=CP-B*Q(NS)
INJECTOR BOUNDARY
IF(TD(I).GT.TT) GO TO 140
130 CONTINUE
140 TA=TAD(KK)+(TT-TD(KK))*TAD(KK+1)-TAD(KK)
1(TD(KK+1)-TD(KK))
CP=H(NJS-1)+Q(NJS-1)*B-HF(Q(NJS-1),D,AR, VISC, EPS,OX,G)
CM=H(NJS+1)-Q(NJS+1)*B-HF(Q(NJS+1),D,AR, VISC, EPS,OX,G)
A1=CP+C4+(B*CI*TA*5)**2
H(NJS)=.5*(A1+SQRT(A1-A1-(CP+CM)**2-(B*CI*TA)**2))
Q(NJS)=(CM+H(NJS))/B
Q(NS2)=(CP-H(NJS))/B
QINJ=Q(NS2)-Q(NJS)
VCL=VCL+DT*(QIC+QINJ)
IF(TA.EQ.1.*) QMAX=QINJ
IF(K/IPR*IPR-K)80,60,80
QAVE=VOL/PER
WRITE(6,160) VOL,QAVE,QMAX
160 FORMAT(' VOLUME INJECTED PER CYCLE=',F12.9/)
172 ' AVERAGE FLOW RATE OVER CYCLE=',F12.8/
123 ' MAXIMUM FLOW RATE AT INJECTOR=',F12.3)
124 IF(IPLT.EQ.1)CALL PLOT(TIME,PINJ,POS,L,0.,-16.,.002,3.,10.,7.)
125 STCP
126 END
127 FUNCTION HF(Q,D,AR,VISC,EPStOX,G)
128 C SUBROUTINE HF EVALUATES FRICTION FACTOR AND HEAD LOSS WITH CCLEBROO
129 C FORMLA
130  C=EPS/D
131  R=C/AR*Q/VISC
132 IF(R.GT.2000.)GO TO 10
133  FF=64./R
134  GC TC 30
135 10 FF=.02
136 C FIND FRICTION FACTOR
137 DC 20 K=1,10
138  F1=SQRT(FF)
139  F=1./F1+0.86*ALOG(C1/3.7+2.51/P/F1)
140  IF(ABS(F/FF).LE.0.01)GO TO 30
141  DF=-0.5/F1*3*(1.+(2.51*.86/R/(C1/3.7+2.51/R/F1)))
142  FF=FF-DF/FF
143 20 CONTINUE
144 30 HF=FF*Q*ABS(Q)*DX/2./G/AR/AR/D
145 RETURN
146 EN
165 EDIN XL=29.2,A=4000.,D=0.25417,G=32.2,EL=2*B0.,60*3.28,
172 T=0.,CDAI=2.6E-6,
133 PU=53368.,PD=32976.,RHO=1.5,CCEAU=1.71E-5,CCAD=1.59E-5,VISC=.7E-5,
134 TMAX=.005,N=56,NJS=19,IPR=2,NDATA=5,EPStOX=5.E-6,IPLT=0,
135 PER=.05,TAD=0.,1.,1.,0.,0.,TD=0.,.01,.05,.055,1., & END
1 /COMPILE
\[ \text{VCLVUE INJECTED PER CYCLE} = 0.000001477 \]
\[ \text{AVERAGE FLOW RATE OVER CYCLE} = 0.00002954 \]
\[ \text{MAXIMUM FLOW RATE AT INJECTOR} = 0.000001477 \]

### P AND DISCHARGES ALONG THE PIPE

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LIST OF VARIABLES

MAIN PROGRAM, LAMINAR FLOW

A: pressure wave propagation velocity
AR: pipe cross-sectional area
AI: miscellaneous collection of variables
B: pipeline characteristic impedance, \( B = \frac{A}{(G\times AR)} \)

CDAD: product of downstream orifice discharge coefficient and flow area
CDAI: product of injector discharge coefficient and flow area
CDAU: product of upstream orifice discharge coefficient and flow area
CI: \( = \frac{CDAI}{2g} \)

CM, CP: collection of variables in \( C^+ \) and \( C^- \) compatibility equations
CV: \( = g(CDAU)^2 \)
CVD: \( = g(CDAD)^2 \)
D: pipe diameter
DIN: namelist name
DT: time-step size \( = \Delta x/A \)
EL: elevation of sections above reference datum
FC: coefficient in the unsteady friction term
G: acceleration of gravity
GAM: unit weight of fluid = \( \rho g \)
H: hydraulic grade line elevation above reference datum, \( H = EL + p/\rho g \)

HB: \( = RQ_N \), normal viscous friction loss at section \( N \)
HD: constant pressure head outside the downstream orifice
HF: total friction loss at a section
HFF: vector to store components of friction term (for printout only)
HI: constant hydraulic gradeline elevation outside the injector

HP: unsteady portion of the total friction loss at section N

HU: constant pressure head on the upstream side of the upstream orifice

HX: temporary storage of HF(NS2) for calculation of CM at the section upstream from injector

I: integer usually used to identify pipe section

IDIAG: = 1 to printout frequency dependent friction diagnostic information
        = 0 for no printout

II: refers to the first upstream section at which interior point calculations are carried out: = 2 when K is odd, = 3 when K is even

III: = II - 1

IPLT: = 1 to obtain a plot of pressures
       = 0 for no plot

IPR: controls number of pairs of time steps to be computed between print out; must be even

K: integer counter identifying the number of time steps

KF,KK: integers

KMAX: number of terms used in unsteady friction

KMI: = KMAX + 1

L: integer that counts the number of points to be plotted

N: number of reaches in system between upstream and downstream orifices

NDATA: number of pairs of values of τ and t to describe the injector pulse

NJS: section number of injector location

NS: = N + 1

NS2: = NS + 1; subscript location to store information at upstream side of injector in main pipe
PD: pressure at downstream side of downstream orifice
PDS: vector to store downstream pressure to plot
PER: period of injector pulse
PI: pressure outside the injector
PINJ: vector to store pressure in fuel rail at the injector
PP,PPP: initial pressure at downstream orifice and injector respectively
PU: constant pressure upstream from upstream orifice
P1,P2,P3: used for print out of pressures
Q: volumetric flow rate at sections
QAVE: volumetric flow rate at injector averaged over entire period
QF: volumetric flow rate stored as a function of time at each section
QINJ: injector volumetric flow rate, current
QIO: injector volumetric flow rate, 2 time steps earlier
QMAX: maximum instantaneous flow rate at injector
QX: temporary storage of Q for calculations of CM
QO: initial steady-state volumetric flow rate
Q1,Q2,Q3,Q4: dimensionless flow rates for print out
R: steady-state resistance factor
RHO: mass density of fluid
S: = ± 1 to designate sign of flow direction
T: time
TA: dimensionless number, τ, to describe the injector needle position
TAD: tabular input values of τ vs time, TD, to describe injector motion during cycle
TD: dimensionless time, t/PER, tabular values paired with TAD
TIME: vector to store time corresponding to PINJ and PDS to plot

TMAX: duration of transient simulation in seconds

TT: dimensionless time during each injector cycle

VISC: kinematic viscosity

VOL: volume of fluid injected during each cycle

W: weighting function used in unsteady friction evaluation

XL: length of pipe between orifices

Subroutine Weight

K: integer counter

ST: $\sqrt{TAU}$

TAU: dimensionless time used in weighting function

$= 4v t / D^2$

$TAU_1: = 4v / D^2$

Main Program; Turbulent Flow

Additional variables not used in laminar flow program

DX: $= XL / N$, length of each reach

EPS: linear dimension of surface roughness of pipe

Function HF, used in Turbulent Flow Program

Cl: $= EPS / D$, dimensionless roughness

DF: derivative of Colebrook equation

F: designation of Colebrook equation

FF: Darcy Weisbach friction factor

FI: $= \sqrt{FF}$

R: Reynolds number $= VD / V$
AIIM SCANNER TEST CHART # 2

Spectra

Times Roman

Century Schoolbook Bold

News Gothic Bold Reversed

Bodoni Italic

Greek and Math Symbols

White

Black

Isolated Characters

e m 1 2 3 a

4 5 6 7 0

8 9 0 h i B

MESH HALFTONE WEDGES

65

85

100

110

133

150