Analysis of Notch Weir Discharge Equations

By
Steven J. Wright Ph.D., P.E.
The University of Michigan
Ann Arbor, Michigan

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SUMMARY

An investigation was performed to examine the ability of standard weir equations developed for contracted weirs to apply to situations in which the weir head is roughly the same as the crest length. These contracted weirs, or notch weirs as they are often referred to, are utilized in the design of flow control structures on stormwater detention basins. In these applications, the weir is typically a notch cut in a vertical riser which may be a section of 12-15 inch-diameter corrugated metal pipe. Typical designs are reported to often yield crest lengths (notch width) on the order of 4-6 inches. The maximum heads on these notch weirs are often much greater than the notch width. This results in a situation where the weir head to notch width ratio is considerably outside the range of the data that were used to generate weir equations that are commonly recommended to design the notch weir. An elementary analysis of the equation recommended in the King and Kitsap County design manuals indicates that physically unrealistic predictions will be obtained in circumstances where the head on the weir is greater than about three times the crest length.

An investigation of the literature was performed to determine the availability of experimental data that could be used to check the weir equations in the ranges of weir head and notch width that are suggested above. One extensive data set was located and was analyzed to determine the suitability of the standard crest length correction. This correction was found to grossly under-estimate the discharge under the conditions of high weir head to notch width. At lower heads, the tendency for the discharge equations recommended in the King County and Kitsap County (State of Washington) Surface Water Design Manuals was to over-estimate the discharge. Removing the weir crest contraction effect from the discharge equation resulted in more consistent discharge predictions but which were generally in the range of ten to twenty percent greater than observed in the experimental data. A subsequent analysis indicated that the weir discharge coefficient employed in the design equation was also inappropriate. The equation for the discharge coefficient very well reproduces the data in the analyzed data set for suppressed weirs (ones in which the weir crest length is the same as the approach channel width). However, it is unsuitable to reproduce the data for contracted weirs which are more representative of the notch weirs that would be used in stormwater retention basin outlet control structures.

An additional modification to the discharge coefficient was found to reproduce the data in the data set to within approximately five percent when used in combination with the uncorrected crest length. The standard weir equations were developed for a notch weir in a flat plate. The application in stormwater detention facilities involves a notch cut in the perimeter of a circular riser and also requires the use of a baffle for oil-water separation. The baffle is simply a length of larger diameter pipe fixed to the riser around the outside of the notch weir. The standard weir equations contain a term that attempts to account for the approach flow conditions. In the riser assembly, there are a number of issues associated with the accounting for the approach flow, including the presence of the baffle and the definition of the height of the weir plate. Each of these is likely to produce deviations from the flows predicted by the standard weir equations and the effects have apparently not been systematically investigated. Given these additional

uncertainties, the five percent uncertainty in the adjusted flow equation is considered to be well within the overall uncertainty of the head-discharge relation for notch weirs. Therefore, the adjusted equation is recommended to be used in place of the standard design equation that indicates major discrepancies in reproducing the available data. This equation is

$$Q = 3.27 L H^{3/2}$$

In which the discharge Q is in cubic feet per second and the crest length L and the weir head H are in feet.

INTRODUCTION

Stormwater detention facilities typically require flow control structures to limit discharges for prescribed hydrological events. A typical design utilizes flow constrictors in a riser connected to the outlet pipe as indicated in Figure 1. The riser and outlet pipe are typically the same diameter and a combination of orifices or weirs installed in the rise are used to regulate the flow. An orifice located at the bottom of the riser is generally used to pass the lower regulatory discharge (e.g. a 2 year, 24 hour storm event). Higher regulatory discharges such as the 10 and/or 100 year events require one or two elbow and orifice assemblies mounted at higher elevations on the riser. In the case that there is insufficient vertical extent to install the elbow/orifice asssemblies on the riser, a notch weir is commonly used to provide for the discharge control at higher discharges. The notch weir is simply a slot cut in the perimeter of the riser pipe at the top; the horizontal length of the slot is designed to pass the required flow. The King and Kitsap County design manuals specify that the slot length cannot exceed more than one-half of the pipe circumference. In addition, an oil-water separator baffle is required. This baffle is simply a section of larger diameter pipe bolted to the riser on the side opposite the weir and extending at least one foot below the weir crest. The only requirement in the design manuals is that the area between the riser and the baffle be sufficient to pass the design flow; it is understood that a common configuration may involve a twelve inch diameter riser and an eighteen inch diameter baffle as an example.

Standard weir equations have been recommended for use in determining the relationship between the weir discharge and the water surface elevation in the detention pond. The equation recommended in the King and Kitsap County design manuals as well as that used in the stormwater design software *WaterWorks* is

$$Q = \left(3.27 + 0.40\frac{H}{P}\right)(L - 0.2H)H^{3/2} \qquad (1)$$

In the above equation, Q is the discharge in cubic feet per second, H is the head on the weir, P is the height of the weir crest above the approach channel bottom in feet, and L is the weir crest length perpendicular to the flow direction, also in feet. The term involving H in the L – 0.2H term may be thought of as a correction to the crest length which compensates for the contraction in the flow passing through the weir. This equation was developed for a weir installed in a flat plate oriented perpendicular to an approach flow. The actual riser/weir configuration deviates from this in several fundamental aspects, calling into question the validity of the discharge equation. Often, the designed notch weir crest length (slot width) is fairly small and less than the weir head (detention pond water surface elevation relative to the weir crest). Standard weir discharge equations call for a correction in the crest length that is associated with the contraction of the flow as the flow passes the sides of the weir. However, this correction was developed from experimental data in which the weir head was less than the crest width. In extreme situations, the discharge equation will result in the prediction of a negative discharge, which is clearly physically unrealistic.

The literature was searched for experimental data in which the weir head to crest length ratio was greater than one in order to test the ability of the standard weir equations to reproduce the data. One such data set was located. This report provides a comparison between the weir equation and this experimental data. Conclusions are drawn from the comparison and recommendations are made for modifying Equation 1 to more accurately represent the experimental data.

BACKGROUND

Figure 2 is a sketch of a standard sharp-crested weir. In this sketch, H is the weir head, which is the elevation difference between the weir crest and the upstream water surface elevation. Other geometrical variables include L, the crest width and P, the weir height, which is the elevation difference between the weir crest and the upstream floor level. The weir discharge Q is generally assumed to be related to these geometric variables. The Handbook of Hydraulics (H.L. King and E.F. Brater, McGraw-Hill) provides several different weir formulas, all of which are empirically derived from some set of experimental data. The weir formula prescribed in the King and Kitsap County design manuals (see Figure 4.4.7 on page 4.4.7-2 of the King County Surface Water Design Manual or Figure 5-22 on page 5-44 of the Kitsap County manual) is Equation (1) provided above. The H/P term in the first set of parentheses in Equation (1) is intended to account for the approach flow conditions and the exact form varies from one weir equation to another. The second term involves a correction of 0.1H for each side of the weir, which is intended to account for the contraction of the flow at the side of the weir. A suppressed weir is one in which the approach channel and the weir have the same width L and the 0.2H correction would not be applied in this situation. This form of the end contraction correction was developed in 1883 by Francis (Lowell Hydraulic Experiments, D. van Nostrand, New York, 1883) and has been stated to be restricted to $L > 3H$ and there are additional restrictions on the width of the approach channel relative to the notch width L.

Inspection of the notch width correction term indicates that the effective width $L - 0.2H$ is zero if $H/L = 5.0$. The predicted discharge would be zero in this case and would be predicted to be negative for $H/L > 5$. Neither of these results are physically realistic and are a consequence of applying the correction outside the range of the experimental data from which it was derived (presumably $H/L < 0.33$ from the above mentioned restriction on applicability). The computer software, Waterworks, utilizes a similar length correction but restricts all discharges to be zero or positive by specifying that a negative predicted Q will be set to zero. However, the above equation predicts a maximum discharge at $H/L = 3$ if the H/P term is neglected and therefore that the discharge will decrease with increasing head above $H/L = 3$. This also is not a physically realistic result and indicates a more severe restriction on the applicability of the weir equation than simply that it predict a non-negative discharge. Since $H/L = 3$ is considerably above the range for recommended applicability of the equation, there may well be even more serious limitations to the use of this length correction in the weir equation.

DATA SETS

A literature search was conducted to determine the availability of data sets on weir flows with large H/L ratios. One source was discovered: A Comprehensive Discharge Equation for Rectangular-Notch Weirs, R.W. Carter, M.S. Thesis, Georgia Institute of Technology, June 1956 and a copy of the thesis was obtained from the

Georgia Tech Library system. A total of 346 individual sets of head and discharge measurements were presented for both suppressed and contracted weirs. A wide variety of H/L ratios were investigated, but in particular, a number of individual experiments with performed with L = 0.1, 0.2 and 0.4 ft with weir heads H that would yield H/L values up to a maximum value of 7.1. The above range of crest lengths are apparently within the range of typical values employed in detention pond applications. In addition to the three L values mentioned above, there are more limited data for L values of 0.118, 0.121, 0.281, 0.292, 0.577 and 0.60 ft. Additional data sets with much larger crest lengths were also analyzed to clearly elucidate the end contraction effect. Some of the data mentioned above were for suppressed weirs in which no end contraction correction should be required and these were also analyzed in order to clarify any effects of the end contractions.

DATA ANALYSIS

A number of different analyses were performed on the data in order to attempt to determine the nature of the weir discharge coefficient (The term 3.27 + 0.4 H/P in Equation 1). In general, the prescribed discharge coefficient only moderately well described the experimental data sets. It was found that it described the data fairly well for suppressed weirs, i.e. ones in which the weir crest length is the same as the approach channel width. Figures 3-5 provide comparisons of data for suppressed weirs with different crest lengths ranging from 0.1 to 2.68 feet. In these figures, Equation (1) is employed with the actual crest length L replacing the L – 0.2H term since no end contraction is expected. With the exception of the smallest crest length of 0.1 feet, nearly all data fall within 1.5 percent of the prediction utilizing Equation (1) with no end contraction correction applied. The results in Figure 4 are indicative of an effect due to the boundary layer on the walls of the approach channel. The wall boundary layers should reduce the weir discharge in this application with a very narrow crest length and this is observed in Figure 4 in that the predicted discharge exceeds the observed discharge.

Agreement was less satisfactory for the contracted weir data. Typical results are indicated in Figs. 6– 9. In general, it is observed that Equation (1) (with the crest length correction as given in the equation) tends to over-predict the measured discharge at lower H/L values and then predicts decreasingly lower discharges compared to the observed values as H/L increases. This is also observed with the larger L of 1.19 ft as indicated in Figure 10 but in this data set, the predicted discharge always exceeds the observed due to the range of smaller H/L ratios with the greater crest width. The results of this comparison indicate two things: 1.) The weir coefficient 3.27 + 0.4 H/P is generally too large to predict the correct flow for all experiments; and 2.) The length contraction correction of 0.2 H is much too severe for all flow conditions with small crest lengths, i.e. with L in the range of 0.1 – 0.4. Results are inconclusive for larger crest lengths; this is especially true in Figure 9 in which Equation (1) consistently over-predicted the observed discharges with an increasing discrepancy at larger H/L values or opposite to the trends indicated in Figures 6-8. These disparate findings led to the conclusion that the two separate effects noted above were contributing to the lack of agreement between the predicted and observed discharges.

Further analysis was performed in which no length correction was applied to the data for contracted weirs, i.e. the entire crest length L was used in the weir equation instead of the contracted length. Typical results are indicated in Figs. 10 and 11. Here, it can be seen that the weir equation generally predicts too high a discharge but more importantly, that the discrepancy increases with increasing weir head. Analysis was conducted on individual data sets to determine the correlation between the H/P ratio and the change in the discharge coefficient. The weir coefficient was computed from the ratio $Q/(LH^{3/2})$. In this analysis, the actual crest length was used instead of the length correction suggested by Equation (1). Example data sets are included as Figures 12-14. It can been seen from an inspection of these figures that the weir discharge coefficient generally decreases with H/P for most of the data sets as opposed to the trend in Equation (1) that increases the weir coefficient with H/P. In the few cases that the weir coefficient increases with H/P, a linear coefficient required to fit the data is much less than the value of 0.4 in Equation (1). It appears that the trend of the weir coefficient with H/P is only applicable for suppressed weirs and therefore would not be valid for the types of notch weirs used in stormwater detention control structures. For any of the individual data sets, the variation of the weir coefficient with H/P is fairly small. This led to a final attempt which simply used a constant weir coefficient of 3.27 to describe the data in conjunction with the full crest length to provide a flow equation of:

$$Q = 3.27\,LH^{3/2} \qquad (2)$$

The results from the predictions with Equation (2) are fairly acceptable as indicated by example results in Figures 15-18. In general, Equation (2) can describe all of the available data to within approximately five percent with the exception of the partial contraction data, Figures 16 and 17. The lower B/L (B/L = 1.25) ratio exhibits more discrepancy with Equation (2) than the larger B/L ratio of 2.0. Notch weirs of the type installed as a slot in the circumference of a riser pipe would not lie in this range of B/L values and therefore, this lack of ability to Equation (2) to reproduce those data sets should not be a concern. In any case, Equation (2) can reproduce the data in Figures 16 and 17 much better than can Equation (1) as can be seen by comparing Figures 7 and 8 to Figures 16 and 17, respectively. In order to appreciate the nature of the improvement in predictive capability, Figures 19 and 20 are included which directly compare the predictive ability of Equation (2) compared to Equation (1). The improvement is obvious and dramatic at large relative weir heads.

Another factor that should be considered in any considered applications is that the flow conditions approaching the weir crest will have some influence on the discharge. The data presented are for a weir installed in a flat plate perpendicular to an approaching flow in a laboratory flume. The detention pond application deviates from this condition in at least three potentially important ways:

1.) The presence of the oil-water separator baffle causes a significant deflection of the approach flow and most likely reduces the discharge compared to a direct approach to the weir crest.

2.) The weir is installed in the perimeter of a circular perimeter. In the situation in which L is small compared to the riser diameter D, this effect will likely be small. However, the design manuals indicate that L can be as large as half the pipe perimeter in which case, there would likely be a significant deviation in the flow conditions at the weir crest.

3.) The distance P is poorly defined in the riser assembly. The design manuals specify defining P as the distance from the weir crest to the invert of the outlet pipe. However, this is a relevant distance only for the flow downstream from the weir crest whereas the H/P ratio is intended to reflect approach flow conditions. Presumably the H/P ratio is small in most applications and its effect on the predicted discharge is small, but the actual influence is unknown and probably not consistent with the given weir equation.

Given these three considerations, it is likely that the flow predicted by any standard weir formula could be off by a fair amount due to the geometrical differences between the standard configuration and the riser assembly with oil-water separator baffle. It is reasonable to expect that the differences could be at least ten percent or more. Therefore, in the absence of more detailed information on the effects of the system geometry, the results indicated in Figures 15-18 indicate that an acceptable approach would be to modify the notch weir formula to the form of Equation (2). The formula does not exhibit gross errors at the large H/L values and the predictions are most likely within the uncertainties in weir discharge due to the system geometry and the flow rates derived from hydrological analyses. At the least, the obvious errors introduced in applying the standard weir formula at high H/L values have been removed and only a small (approximately less than five percent) uncertainty remains in the weir discharge formula. This level of uncertainty should be quite adequate for the purposes of designing outlet structures in retention basins.

CONCLUSIONS

A rational analysis of the standard weir formula to be applied for contracted weirs indicates that it predicts physically unrealistic results at large H/L values and therefore cannot be applied for those conditions. The contradictions are not just that negative discharges can be predicted but also that decreasing discharge with increasing weir head will also be predicted for H/L greater than about 3. A comparison of the weir formula with available data indicates the same sort of discrepancy and indicates that the standard weir formula should not be applied for H/L greater than about 1. The weir formula does appear to work well for suppressed weirs, however, but this is not of particular relevance to the notch weirs required in outlet structures in stormwater detention basins.

An altered weir formula was applied to analyze the data. This formula is provided above as Equation (2). Two modifications were considered to the presently recommended Equation (1).The first modification was to use the full notch width L in the formula as opposed to the contracted length of L – 0.2 H. This formulation tended to over-predict the discharge, but avoided the physical inconsistencies associated with the standard formulation. The second modification was to make the weir discharge coefficient a constant as opposed to varying significantly with the H/P ratio. The combination of the two modifications led to predictions that were generally within five percent of the observed discharges of the relatively large data set so long as the B/L ratio for the weir was relatively large. This would be more representative of the conditions associated with notch weirs of the type described in the introduction. In the absence of more extensive experimental investigations to specifically resolve these issues, Equation (2) is recommended as a much superior equation for relating the weir head and discharge.

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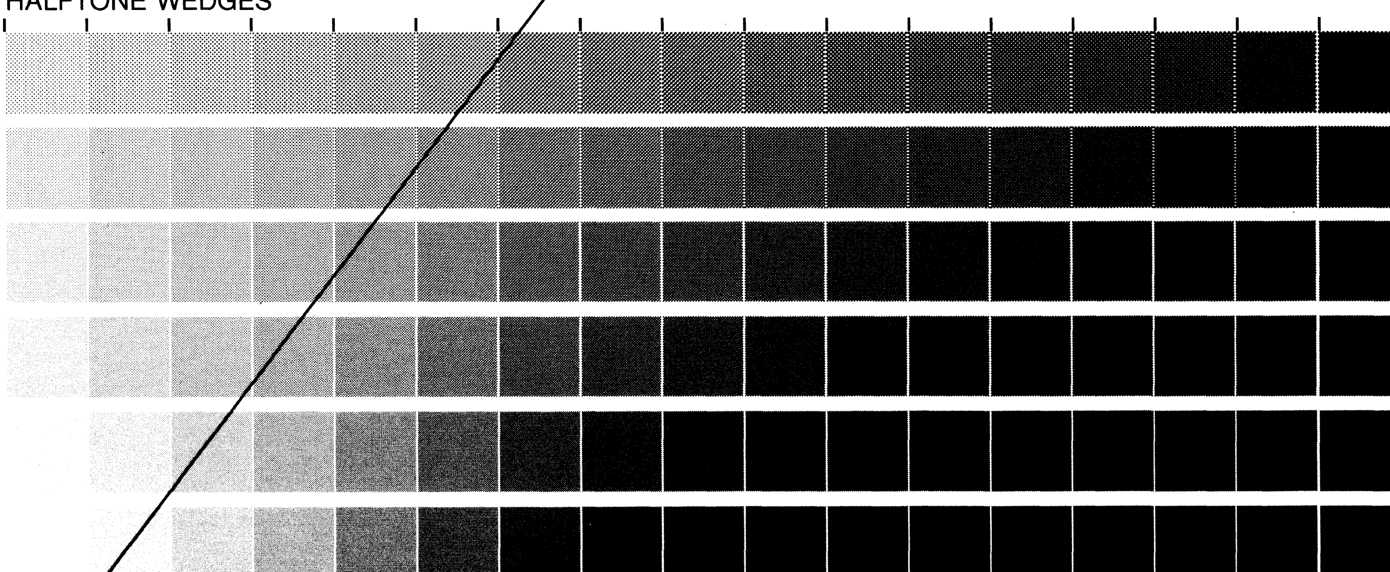
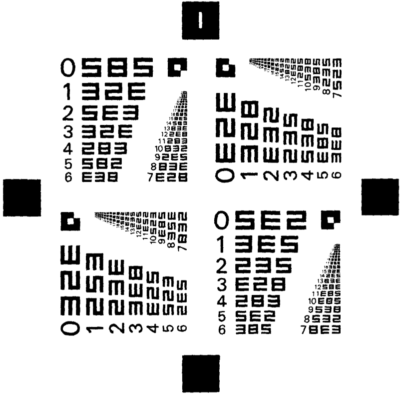


RIT ALPHANUMERIC RESOLUTION TEST OBJECT, RT-1-71



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