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Strategic Capacity Planning Problems in Revenue-Sharing Joint Ventures

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We study strategic capacity investment problems in joint ventures (JVs) with fixed-rate revenue-sharing contracts. We adopt a game-theoretical approach to study two types of JVs depending on how individual resources determine the effective capacity of a JV. With *complementary* resources, the effective capacity of a JV is constrained by the most scarce resource. We show that multiple Nash equilibria could exist. Nevertheless, there exists a unique Strong Nash equilibrium. We show that there is an efficient and fair fixed-rate revenue-sharing contract which induces the system optimal outcome in the Strong Nash equilibrium. On the other hand, with *substitutable* a resource, the effective capacity of a JV is measured by aggregating individual contributions. We show that there does not exist a fixed-rate revenue-sharing contract that induces the system optimum. We quantify that the efficiency of a JV which decreases with the number of participants, the cost asymmetry and the cost margin of the JV. We propose provably-good fixed-rate revenue-sharing contracts with performance guarantees. We also propose a simple modified contract to achieve the channel coordination. Finally, we fit our model with historical data to shed some insights on two JV examples in the motion picture industry.

Key words: capacity planning; joint venture; revenue sharing; game theory; efficiency; coordination
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1. Introduction

This paper studies revenue-sharing joint ventures. A *joint venture* (JV) takes place when two or more business partners pool resources and expertise to achieve a particular goal for a contractual period of time. We focus on fixed-rate revenue-sharing JVs where participating partners are rewarded according to a fixed and pre-negotiated percentage of the revenue. Facing demand uncertainty, the business partners make individual investment decisions, which determine the *effective capacity* of the JV. One main goal of this paper is to quantify the efficiency of a JV, that is, comparing the effective capacity and the total profit achieved in a JV to their system optimal counterparts, where investment decisions are made centrally (as if the participating firms acted as a single entity). We distinguish two types of JVs, depending on the nature of resources that affects how a JV's effective capacity is determined. With *complementary* resources, the final product or service of a JV requires contribution from every individual partner. Thus, the effective capacity of a JV is constrained by the most scarce resource. On the other hand, when resources are *substitutable*, the final product can be made with any available resources. Thus, the effective capacity of a JV is determined by aggregating individual contributions.

Examples of JVs abound. Sanofi (a French drug company) and Verily (a subsidiary of Google) established Onduo, a joint venture integrating medicine and microelectronics to tackle diabetes (see Wall Street Journal 2016). IMAX, the entertainment technology company, has launched JVs with theaters since 2008, where it installs its proprietary entertainment systems in theaters at minimal charge, in return for a portion of box-office receipts. Both are examples of complementary resource sharing: Onduo combines Sanofi's drug development knowledge with Verily's expertise in data analytics, software, and miniaturized devices to create tools for diabetes care; IMAX contributes the projection systems while the theaters provide the physical space and labor.

Meanwhile, there are also many examples of JVs with substitutable resource sharing, e.g., airline alliances such as Sky Team, Star Alliance, OneWorld (see Vinod 2005) as well as car rental JVs between Avis and Shanghai Automotive Industry Corp (see Auto Rental News 2013). In the motion picture industry, theater operators also turn to JVs to develop and operate their theaters together, e.g., Wanda Group with Reliance MediaWorks Ltd (see Reuters 2012a).

When firms agree to a partnership, disparate interests often remain as each firm is more concerned with its own return. The misaligned incentives could be extremely detrimental to the health of a partnership: Bamford et al. (2004) studied over 5000 JVs and concluded that more than half of them eventually failed. The authors argued that many failures could be prevented had more effort been spent on aligning the partners' interests to create a coherent organization. With that in mind, in this study, we attempt to address the following research questions: *How can we design efficient revenue-sharing contracts to better align the incentives of participating firms in a JV?* We focus on the fixed-rate revenue-sharing contract due to its merits of relatively simple structure compared to other coordinating contracts. For some cases, when such a simple scheme is not capable of achieving coordination, we would address a follow-up question: *How much efficiency is lost due to incentive misalignment?* The analysis helps us pinpoint the key factors that cause inefficiency in a JV. Moreover, if the loss of efficiency is proven to be substantial, it provides evidence to advocate more complex and costly contracts for JVs.

1.1. Main Results and Contributions

We propose a game-theoretical approach to analyze the performance of resource sharing where firms in a JV split the return via a fixed-rate revenue sharing contract. Our model differentiates between two types of resource sharing, namely, complementary versus substitutable resources.

With complementary resource sharing, we show that multiple equilibria exist, as the strategy space of each firm also depends on others' strategies. Specifically, when one firm lowers his or her investment from an existing equilibrium level, this action triggers off under-investment from all other firms and establishes a new equilibrium. Nevertheless, we show that for any given fixed-rate revenue-sharing contract, there exists a unique Strong Nash equilibrium, which can be interpreted as the "best" Nash outcome for inducing the highest effective capacity among other Nash equilibria. We show that there exists a fixed-rate revenue-sharing contract which induces the system optimal outcome in the Strong Nash equilibrium setting. This contract rewards each firm proportionally to its share of marginal cost evaluated at the optimal capacity level. Besides efficiency, we also show that this contract embodies a notion of *proportional fairness*.

With substitutable resources, we show the existence and uniqueness of a Nash equilibrium. In such a JV, we show that only the most "efficient" firms will actively participate and it is measured in terms of the marginal cost as well as the positive externality to other participants. We show that there does not exist any fixed-rate revenue-sharing contract that is efficient. In particular, JVs tend to under-invest in their effective capacities compared to the system optimal setting. We measure a JV's efficiency by quantifying a worst-case performance bound on the aggregate profit with respect to its system optimal counterpart. In a two-player setting, we identify fixed-rate revenue-sharing contracts with a worst-case performance guarantee of 2. We show that such schemes reward participants inverse proportionally to their costs, that is, the firms with higher (lower) cost receive less (more). In an n-player setting, we show that efficiency generally decreases with the number of participants, cost asymmetry among the participants, and the cost margin of the JV. We also propose a simple modified contract which rebalances the cost though subsidies or penalties in order to achieve the channel coordination.

We consider an extension where the spillover effect exists which could stem from the transfer of knowledge, skilled labor and technology among participating firms. For complementary resources, we show that having the spillover effect changes the nature of the resources from perfect complements to imperfect complements with partial substitutes. Thus, the resulting JVs inherit the properties of substitutable resource sharing (e.g., a fixed rate revenue-sharing contract can no longer coordinate a JV). Meanwhile, for substitutable resources, having spillovers turns the perfectly substitutable resources into imperfect substitutes. Key drivers behind a JV's performance will need to be modified to capture the externality impact due to spillovers. For instance, firms with high cost might remain active in a JV, conditioned on providing high positive externality to others.

In a numerical case study, we collect historical data to fit our models and evaluate the two aforementioned JV examples in the motion picture industry. To study the JV between IMAX and theater operators (i.e., complementary resources sharing), we utilize the companies' financial data and movie box office revenues between 2009 and 2013. We compare the optimum suggested by our model and the actual performance that took place. Based on our fitted model, we show that the optimal revenue-sharing ratio is very close to what IMAX claims to be receiving from such deals. Moreover, the model suggests that there is still significant room for growth for this type of JVs. We also fit the model to study a JV between two theater operators (i.e., substitutable resource sharing). We observe that such a JV partnership might not be desirable due to the low profit margin and the risky nature of the business. This result provides some explanations to the prevalence of mergers and acquisitions instead of JVs among theater chains in recent years.

1.2. Related Literature

In the literature on joint ventures, a plethora of theories have been employed (see Kogut 1988 for an excellent review on theoretical and empirical perspectives of JVs). The most notable approaches include the theory of transaction cost economics (e.g., Hennart 1991, Williamson 1981), organization theories (e.g., Borys and Jemison 1989, Yiu and Makino 2002 and theories on how strategic behavior influences the competitive positioning of the firm (e.g., Balakrishnan and Koza 1993, Vickers 1985). This study utilizes the approach of using strategic behavior to explain the formation and the performance of JVs. In terms of positioning, our work is closely related to two research streams in the operations management literature, namely, strategic capacity planning and coordination using revenue-sharing contracts.

Strategic capacity planning. The newsvendor model which studies a firm's capacity decision (before observing the demand) provides a central theme in the literature on capacity investment (see Cachon and Netessine 2006, Van Mieghem 2003). Cachon and Lariviere (1999) considered the manufacturer's capacity investment and allocation problem when downstream retailers have private information. Netessine et al. (2002) addressed the impact of demand correlations on a firm's resource investment decision. Caldentey and Wein (2003) analyzed the capacity decisions in a production–inventory system and propose coordinating contracts that incorporate backorder, inventory, and capacity levels. Boyaci and Özer (2010) studied the strategy of using advanced sales to obtain more accurate demand information and evaluated the capacity investment decisions under various market and operating conditions. Kim and Tomlin (2013) focused on technological systems that potentially face an outage and study investment decisions in recovery capacity and/or failure prevention so as to enhance system availability. Chen et al. (2014)studied capacity allocation model in supply chains with distributors as information intermediaries. Besides the single-location, single-period models, Van Mieghem and Rudi (2002) introduced newsvendor networks to study the stochastic capacity investment decisions. Shumsky and Zhang (2009) examined a multiperiod dynamic capacity allocation model with product substitution. By contrast, our work focuses on how players in a JV make capacity decisions depending on the resource type (complementary or substitutable) and how effective revenue-sharing mechanisms coordinate their capacity decisions.

When the resource type is complementary, our model closely resembles the well-studied assembly systems in the literature. Wang and Gerchak (2003, 2004) investigated capacity games in assembly systems with uncertain demand and designed optimal revenue-sharing contracts to achieve the channel coordination. Tomlin (2003) also studied price-only contracts in supply chain capacity procurement games and showed that they could arbitrarily allocate the supply chain profit. Bernstein and DeCroix (2004) considered modular assembly systems (which involves an extra layer of subassemblers) and characterized equilibrium price and capacity choices. Carr and Karmarkar (2005) considered a decentralized assembly system with price sensitive and deterministic demand. Wang (2006) considered an assembly system with multiple suppliers who sell perfectly complimentary products to a retailer with a price sensitive uncertain demand. Gumani and Gerchak (2007) considered an assembly system with random component yield, and characterized the conditions under which system coordination is achieved while respecting participation constraints. Nagarajan and Sošić (2009) studied dynamic alliance among suppliers with complementary products in a decentralized assembly system and characterized the coalition structure with respect to different power structures in the market. Yin (2010) studied how market demand conditions drive coalition formation among complementary suppliers. While the efficiency result on the coordinating contract resembles Wang and Gerchak (2003, 2004), we also draw an interesting connection from cooperative games to show the fairness of the contract. More importantly, we also analyze the settings with substitutable resources as well as spillovers (i.e., from perfect complementary/substitutable resources to imperfect complements/substitutes), which significantly depart from this line of literature.

In the context of JV models, Chevalier et al. (2013) analyzed the capacity investment decisions in a JV, using a two-period model. In the first period, without knowing the demand, the JV partners decide on their investments in a common resource which incurs a linear cost. In the second period, after demand is realized, the partners choose how to allocate the joint capacity so as to fulfill their respective demands. They proposed two contractual arrangements that allow JV partners to lease the joint capacity and studied how such contracts can align the incentives of the partners. Roels and Tang (2017) proposed two types of bidirectional contracts, namely, the ex-post transfer payment contract and the ex ante capacity reservation contract, in a co-production and co-distribution JV. They showed that either contract can improve the JV's total profit in equilibrium but the capacity reservation contract is preferred as it is Pareto-improving (leading to an increase in the profits of both firms). Chen and Özer

(2017) studied the strategic capacity planning problem in the context of newsvendor competition and information leakage prevention, and proposed a mechanism to characterize and categorize a variety of contracts. By contrast, our work focuses on the effectiveness of revenue-sharing contracts for their widespread usage and popularity in practice (e.g., Lafontaine and Slade 2012) depending on different types of resources.

Coordination via revenue-sharing mechanisms. There has also been a large body of literature on using revenue-sharing contracts to achieve coordination in supply chains. Dana and Spier (2001), Cachon and Lariviere (2005), Yao et al. (2008), Linh and Hong (2009) showed that revenue-sharing contracts are capable of coordinating competing retailers in a supply chain via a Stackelberg game setup. Kunter (2012) analyzed a royalty payment contract for a supply chain, which is a more complex form of revenue-sharing contract that includes wholesale price and marketing effort. More recently, Kong et al. (2013) showed that revenue-sharing contracts mitigate the negative effects of potential information leakage when there is information asymmetry among downstream competing retailers. While the majority of the literature has focused on vertical channel coordination (i.e., upstream supplier and downstream retailers), our paper focuses on the horizontal coordination among participating firms as they simultaneously determine their investment decisions which impact the effective capacity in a JV. In particular, we show that the ability of revenue-sharing contracts to align the incentives of individual firms depends on the type of resources.

Besides supply chain settings, revenue-sharing contract has also found its way to many other applications: e.g., car-sharing platforms (e.g., Cohen and Zhang 2017), franchising (e.g., Lal 1990, Mathewson and Winter 1985), video rental (e.g., Dana and Spier 2001, Giannoccaro and Pontrandolfo 2004, Mortimer 2008, Van der Veen and Venugopal 2005), the airline alliance (e.g., Fu and Zhang 2010, Hu et al. 2013, Wright et al. 2010). In particular, revenue sharing is especially ubiquitous in the motion picture industry, e.g., actors being paid a share of revenues or profits of their movies (Chisholm 1997), studios receiving a share of box office revenues from the movies they produce and distributors sharing box office revenues with theaters (see Filson et al. 2005, Hanssen 2002). Our work supplements this series of studies by analyzing two types of JVs that use revenue sharing in the motion picture industry, that is, JV between theaters and the projection system provider (IMAX), and JV between two theater operators.

The rest of the study is organized as follows. Section 2 describes the model formulation and key assumptions. We analyze our results for the complementary and substitutable resources-sharing models in sections 3 and 4, respectively. We present empirical studies to highlight two types of JVs in the motion picture industry in section 6. We conclude our paper in section 7. All proofs are delegated to the appendix.

2. Model and Assumptions

We consider a JV consisting of n firms with asymmetric cost functions. We use a vector $\mathbf{K} = (K_1, \ldots, K_n)$ to denote investment decisions, or resources contributed by individual firms in terms of capacity. Let $f_i(K_i)$ be the cost function associated with investing K_i resources by firm i. We assume that each $f_i(K_i)$ is increasing and convex with $f_i(0) = 0$. We will use its derivative $f_i'(\cdot)$ to denote firm i's marginal cost.

We distinguish two types of resources pooling in JVs based on the notion of *effective capacity*, denoted by $L(\mathbf{K})$. The definitions for two types of resources are introduced as follows.

Definition 1. (Complementary resources). $L(\mathbf{K}) = \min_i K_i$.

With complementary resources, a JV requires contribution from every firm. Thus, the effective capacity of a JV is limited by the most scarce resource, which is known as the bottleneck capacity.

Definition 2. (Substitutable resources). $L(\mathbf{K}) = \sum_{i=1}^{n} K_i$.

These resources are perfect substitutes. A JV can be established with a resource contributed by any firm. The effective capacity in this JV is the sum of the capacity levels invested by each partner.

We denote the average revenue on the product or service produced in a JV as p, which is exogenous. Given the aggregate random demand faced by the JV as D, we can then express the expected revenue generated in this JV as $p\mathbb{E}[\min(D, L(\mathbf{K}))]$, where the expectation is taken over the demand with a cumulative distribution function F_D. Depending on the type of the resources, the JV demand D can take various forms. For instance, with substitutable resources, it is plausible that the JV demand exhibits the pooling effect, that is, $D = \sum_i d_i$, where d_i represents the demand faced by each participant i. We assume the cumulative distribution function $F_D(\cdot)$ is strictly increasing.

In order to evaluate the performance of a JV, we consider a benchmark, that is, the system optimal setting, which produces the highest possible aggregate profit.

Definition 3. (System optimum model). Partners make a collective decision to maximize the joint profit, that is,

$$\max_{\mathbf{K}} \pi_T \triangleq p\mathbb{E}[\min(D, L(\mathbf{K})] - \sum_{i=1}^{n} f_i(K_i).$$

The system optimum mimics the decision-making in a merger, which can be thought as an alternative of resource pooling to JVs. In this model, partners are coordinated to make joint decisions as a single entity. Under the assumptions, there exists a unique solution to the system optimum. We denote the optimal capacity decision, the effective capacity and the corresponding total profit as $\mathbf{K}^*$, L^* and π_T^*.

As JVs stand in the middle ground between perfect competition and a merger, we present a game-theoretic formulation to model the interaction in a JV. To be precise, we denote the fixed, pre-negotiated revenue-sharing ratio that firm i receives as β_i, where $0 \le \beta_i \le 1$ and $\sum_{j=1}^{n} \beta_j = 1$.

Definition 4. (JV model). Each firm determines her capacity investment K_i in order to maximize her share of return, that is,

$$\max_{K_i} \pi_i(\beta) \triangleq \beta_i p\mathbb{E}[\min(D, L(K_i|\mathbf{K_{-i}}))] - f_i(K_i), \quad \forall i$$

where $\mathbf{K_{-i}}$ denotes the individual decisions made by firms other than i.

With this model, we assume the partners behave according to a Nash equilibrium. We denote the equilibrium investment decision, the effective capacity and the total profit generated in this JV as $\mathbf{K}^N$, L^N and $\pi_T^N(\beta)$ respectively, where $\pi_T^N(\beta) = \sum_i \pi_i^N(\beta)$. The existence of Nash equilibrium is assured as the profit function is continuous and concave with respect to each player's strategy and the strategy space is compact and convex. However, we will show in section 3 that the uniqueness of Nash equilibrium might not be guaranteed. In such cases, we will utilize the following equilibrium concept, which can be interpreted as a stronger notion of Nash equilibrium.

Definition 5. (Strong Nash equilibrium (Aumann 1959)). A strategy profile with the property that no coalition of players can deviate in a manner which is profitable to all of its players.

The key difference between the two equilibrium concepts is that while the Nash concept of stability defines an equilibrium only in terms of unilateral deviations, Strong Nash equilibrium allows for deviations by every conceivable coalition. Clearly, every Strong Nash equilibrium must also be a Nash equilibrium. We will use $\mathbf{K}^{SN}$, L^{SN} and π_T^{SN} to differentiate the equilibrium outcome in a Strong Nash equilibrium.

One key focus of this work is to quantify the efficiency of JV models. There are two key quantities of interest, L^N/L^* and $\pi_T^N(\beta)/\pi_T^*$, that is, the comparison of the effective capacity and the total profit generated in a JV with respect to their system optimal counterparts.

Remark 1. The JV model in this study assumes a pre-determined revenue-sharing rate β prior to making capacity decisions. There are many such examples in practice. For example, airline alliance agreements often specify simple revenue-sharing ratios that are fixed in advance, before the reservation period when demand appears and seats are sold (see Shumsky 2006). A typical agreement would allocate 70% of the revenue to the marketing carrier who sells the ticket and 30% to the operating carrier who flies the passengers. Since the value of a flight for an alliance partner depends on the allocation scheme, the capacity allocation decisions and the resulting revenues of the airlines are affected by the pre-determined revenue-sharing rules. For IMAX's JV agreements, it installs free entertainment systems at participating theaters in exchange for approximately 20% of the box-office sales (see Businessinsider 2013).

3. Complementary Resource Sharing

With complementary resources, we first analyze the system optimum model, and then study the JV model and characterize its equilibrium condition. We will show that by appropriately designing the revenue sharing contract, it is possible to induce the system optimal decision in a JV.

3.1. System Optimal Solution

The effective capacity $L(\mathbf{K}) = \min_i K_i$, the system optimal model can be reformulated as follows.

$$\pi_T^* \triangleq \max_{L,\mathbf{K}} p\mathbb{E}[\min(L, D)] - \sum_{i=1}^{n} f_i(K_i), \text{ s.t. } L \le K_i, \\ = 1, \ldots, n. \tag{1}$$

Proposition 1. *With complementary resource sharing, in the system optimal model, every participant invests an equal amount of capacity, that is, $L^* = K_i^*$ for all $i = 1, \ldots, n$, where L^* satisfies*

$$\mathbb{P}(D \le L^*) = 1 - \sum_{i=1}^{n} \frac{f_i'(L^*)}{p}. \tag{2}$$

3.2. Multiple Nash Equilibria in a JV

In a JV with a revenue-sharing ratio β_i, firm i maximizes her profit by choosing her capacity investment level K_i, based on other players' strategies $\mathbf{K_{-i}}$. We can rewrite the JV model as follows.

$$\pi_i^N(\beta) \triangleq \max_{K_i|\mathbf{K_{-i}}} \beta_i p \mathbb{E}[\min(D, L(K_i|\mathbf{K_{-i}}))] - f_i(K_i), \text{ s.t. } L \le K_j, \ j = 1, \ldots, n. \tag{3}$$

PROPOSITION 2. *In a JV with complementary resource sharing, the equilibrium behavior in a JV can be characterized as follows.*

(*a*) *Any* $L^N = K_1^N = \cdots = K_n^N \le \min_i(A_i)$ *is a Nash Equilibrium, where* A_i *solves*

$$\mathbb{P}(D \le A_i) = 1 - \frac{f_i'(A_i)}{\beta_i p}.$$

(*b*) $L^{SN} = K_1^{SN} = \cdots = K_n^{SN} = \min_i(A_i)$ *is a unique Strong Nash equilibrium.*

Proposition 2(a) states the existence of multiple Nash equilibria. With complementary resources, since the expected revenue only depends on the effective capacity, every participant only invests up to the effective capacity. When one player lowers his or her investment to becomes the bottleneck, no player has incentives to unilaterally deviate from it. In other words, the under-investment of one firm triggers a chain-reaction of under-investment by all participating firms. Despite the existence of multiple equilibria, Proposition 2(b) asserts that there exists a unique Strong Nash equilibrium in the JV model. Strong Nash equilibrium can be interpreted as the "best" outcome, as it induces the highest effective capacity in a JV. It is also known as the "dominant equilibrium point" in Van Huyck et al. (1990) as it is not Pareto dominated by any other equilibrium point.

3.3. An Efficient and Fair Revenue-Sharing Contract $\boldsymbol{\beta}^*$

We shall show that there exists a unique efficient revenue-sharing contract, in the sense that it induces the optimal capacity decision as the Strong Nash equilibrium in a JV.

THEOREM 1. *Consider a JV with complementary resource sharing.*

(*a*) *There exists a unique efficient revenue-sharing contract* β^* *where*

$$\beta_i^* = \frac{f_i'(L^*)}{\sum_{j=1}^n f_j'(L^*)}, \quad i = 1, \ldots, n. \tag{4}$$

Under this particular revenue-sharing contract, the Strong Nash equilibrium in a JV is the same as the system optimal solution, that is, $L^{SN}(\beta^*) = L^*$.

(*b*) *In a JV with complementary resource sharing, the unique optimal revenue-sharing contract* β^* *is also proportionally fair. More precisely, under this optimal revenue-sharing contract* β^*, *we have* $L^* = L^{NB}(\beta^*)$, *where* $L^{NB}(\beta^*)$ *is the Nash Bargaining Solution via solving Equation* (5).

Theorem 1(a) shows that there is a way to rely on the fixed-rate revenue-sharing contract to eliminate the incentive misalignment among the firms and induce the system optimal outcome. In particular, it specifies that the marginal revenue share ratio to which firm i is entitled to should be equal to the proportion of her marginal cost to the aggregate marginal costs $f_i'(L^*)/\sum_j f_j'(L^*)$ at the system optimal solution. Under this particular revenue-sharing contract β^*, every firm is willing to invest up to the system optimal capacity level L^* in the equilibrium. We point out that since our model with complementary resources can be considered as a general variant of an assembly system, this efficiency result is expected, which bears a close resemblance to the main results in Wang and Gerchak (2003, 2004) as well as Tomlin (2003) and Bernstein and DeCroix (2004).

More interestingly, we show in Theorem 1(b) that this unique efficient revenue-sharing contract β^* also captures a notion of *proportional fairness*, which is the solution concept of a Nash bargaining problem (Nash 1950). Contrary to the Nash equilibrium notion which is from a *non-cooperative* approach, the Nash bargaining solution is a *cooperative* strategy, that is, players unanimously agree on some solution outcome, under some fairness constraint. More specifically, the fairness constraint states that, under the Nash definition, a transfer of resources between two players is favorable and fair if the percentage increase in the profit of one player is larger than the percentage decrease in profit of the other player. Formally, a Nash bargaining solution (NBS) corresponds to an outcome characterized by a set of Nash axioms, as formalized below.

DEFINITION 6. (NASH BARGAINING SOLUTION (Nash 1950)). An n-player Nash Bargaining game consists of a pair $(\mathcal{N}, \mathbf{v})$, where $\mathcal{N} \subseteq \mathbb{R}_+^n$ is a compact and convex set and $\mathbf{v} \in \mathcal{N}$. Set $\mathcal{N}$ is the feasible set and its elements give utilities that the n players can simultaneously accrue. Point $\mathbf{v}$ is the disagreement point, which gives the utilities that the n players obtain if they decide not to cooperate. Game $(\mathcal{N}, \mathbf{v})$ is said to be feasible if there is a point $\mathbf{w} \in \mathcal{N}$ such that $\mathbf{w} \ge \mathbf{v}$ component-wise. The solution to a feasible game is the point that satisfies the following four axioms:

(a) *Pareto optimality*: No point in $\mathcal{N}$ can weakly dominate v.
(b) *Invariance under affine transformation of utilities*.
(c) *Symmetry*: The numbering of the players should not affect the solution.

(d) *Independence of irrelevant alternatives*: If $\mathbf{w}$ is the solution for $(\mathcal{N}, \mathbf{v})$, and $\mathcal{S} \subseteq \mathbb{R}^n_+$ is a compact and convex set satisfying $\mathbf{v} \in \mathcal{S}$ and $\mathbf{w} \in S \subseteq \mathcal{N}$, then $\mathbf{w}$ is also the solution for $(\mathcal{S}, \mathbf{v})$.

If game $(\mathcal{N}, \mathbf{v})$ is feasible then there is a unique point in $\mathcal{N}$ satisfying the axioms stated above. This is also the unique point that maximizes $\prod_{i=1}^n (w_i - v_i)$ over all $\upsilon \in \mathcal{N}$. This maximizer is referred to as the *Nash Bargaining Solution (NBS)*.

The Nash Bargaining Solution for a JV with complementary resources is presented as follows. Based on a fixed-rate revenue-sharing scheme β, n firms choose their capacity investment levels according to a Nash Bargaining game, that is,

$$\max_{L,K_i} \prod_{i=1}^n (\beta_i p \mathbb{E}[\min(K_i, D)] - f_i(K_i) - v_i), \text{ s.t. } \quad L \leq K_i, i = 1, \ldots, n, \tag{5}$$

where $\mathbf{v} = (v_1, \ldots, v_n)$ is the disagreement outcome or outside option, which represents the payoff that the firms can attain if they decide not to cooperate. Let $L^{NB}(\beta), K_1^{NB}(\beta), \ldots, K_n^{NB}(\beta)$ be the NBS from solving Equation (5). In our setting, it can be thought of as a two step procedure: players first determine the disagreement point $\mathbf{v}$ which corresponds to any Nash Equilibria discussed in section 3.2, and then take part in the Nash bargaining game. This sharing mechanism captures the notion of proportional fairness as mentioned earlier. The result of Theorem 1(b), together with Theorem 1(a), also suggests that the Strong Nash equilibrium is "attainable" via a Nash bargaining framework if the optimal revenue-sharing contract β^* is implemented.

Remark 2. One can view our JV model with the coordinating contract as a two-stage game. In the first stage, the players negotiate their revenue-sharing allocations. In the second stage, based on the agreed revenue allocation scheme, the players determine their respective capacity levels to maximize their own profitability. To solve this two-stage game, we first focus on solving the second stage problem to obtain the equilibrium capacity levels of all players for a given β. We then optimize β to achieve the channel coordination, that is, by choosing β^*, we ensure that the players' capacity decisions in the equilibrium coincide with the system optimum.

4. Substitutable Resource Sharing

With substitutable resources, the effective capacity is determined by aggregating individual capacity investment. We first characterize the investment decisions in the system optimum model and the JV respectively. Contrary to the complementary resource-sharing model in the previous section, we will show that there does not exist a fixed-rate revenue-sharing contract that is capable of coordinating a JV. We then focus on quantifying the efficiency of a JV so as to address the question: *Compared to the optimal setting, how much profit is lost due to the lack of coordination in a JV?* To accomplish that, we first examine a 2-player setting with quadratic cost functions and propose fixed-rate revenue-sharing contracts that have the worst-case performance guarantee in terms of efficiency. We then analyze the n-player setting with general convex cost functions.

4.1. System Optimal Solution

When resources are substitutable, the effective capacity, which is also known as the *pooling capacity*, is the sum of the individual capacity invested by each player, that is, $L = \sum_i K_i$. The central planner in the system optimal model maximizes the aggregate profit by collectively choosing the capacity investment $\mathbf{K}$, that is,

$$\pi_T^* \triangleq \max_{K_i} p\mathbb{E}\left[\min\left(\sum_i K_i, D\right)\right] - \sum_{i=1}^n f_i(K_i), \tag{6}$$

Proposition 3 characterizes the system optimal solution $\mathbf{K}^*$.

Proposition 3. *With substitutable resource sharing, in the system optimal model,*

(*a*) *for any pair of firms i and j such that $f_i'(x) < f_j'(y)$ for all $x, y > 0$, then player j must be inactive, that is, $K_j^* = 0$;*

(*b*) *for any pair of active firms (i.e., $K_i^* > 0$ and $K_j^* > 0$), their marginal costs at optimality must be the same, that is, $f_i'(K_i^*) = f_j'(K_j^*)$. In addition, K_i^* and K_j^* solve $\mathbb{P}(D \leq \sum_{k=1}^n K_k^*) = 1 - f_i'(K_i^*)/p = 1 - f_j'(K_j^*)/p$.*

Proposition 3 states that in the system optimal setting, the marginal cost of every active firm must be the same. Intuitively, as the resources are perfectly substitutable, only the most cost-efficient firms should be involved. Later in section 5, when we consider spillover effects, resources become imperfectly substitutable and more firms other than "the cheapest" can also take part in the optimal setting. We would like to point out that the "cost efficiency" is measured in terms of each firm's marginal cost $f_i'(K_i^*)$, as opposed to its total cost $f_i(K_i^*)$.

4.2. Nash Equilibrium in a JV

In a JV with substitutable resources, player i maximizes her profit by choosing her capacity investment level K_i by solving the following problem, that is,

$$\pi_i^N(\beta) \triangleq \max_{K_i} \beta_i p \mathbb{E}\left[\min\left(D, \sum_j K_j | \mathbf{K_{-i}}\right)\right] - f_i(K_i), \quad (7)$$

We characterize the Nash equilibrium solution, $\mathbf{K}^N$, in a JV with substitutable resources.

PROPOSITION 4. *In a JV with substitutable resource sharing, there exists a unique equilibrium outcome* $\mathbf{K}^N$ *which satisfies the following condition,*

$$\mathbb{P}(D \leq \sum_i K_i^N) = \frac{\beta_i p - f_i'(K_i^N)}{\beta_i p}, \text{ for all } i = 1, \ldots, n.$$

In a JV, all participants are active, whereas only the most cost-efficient firms remain in the system optimum. Even with symmetric firms, comparing the conditions in Propositions 3 and 4, it is clear that there does not exist a fixed-rate revenue-sharing contract that can coordinate a JV with substitutable resources. This is in contrast to the complementary resource-sharing case, where an efficient fixed-rate revenue-sharing contract can be determined.

Besides inefficiency in costs, another source of inefficiency comes from under-investment in the JV's capacity as shown in the following result.

PROPOSITION 5. *In a JV with substitutable resource sharing, under any nontrivial revenue-sharing scheme (i.e.,* $0 < \beta_i < 1$*), the effective capacity is no greater than that in the system optimum, that is,* $L^N = \sum_{i=1}^n K_i^N \leq \sum_{i=1}^n K_i^* = L^*$.

We want to highlight that Proposition 5 does not depend on the demand distribution or the cost symmetry among the players. While a lower capacity leads to lower revenue in a JV, lower investments also mean lower costs. It is not immediately clear how these two factors will influence the aggregate profit in a JV, which will be our focus for the next subsection.

4.3. Efficiency of Revenue-Sharing Contracts in a JV

The purpose of this section is twofold. First, given a fixed-rate revenue-sharing contract, we will measure the efficiency of a JV, compared to the system optimum. Second, we will investigate how to design a revenue-sharing scheme that comes with a performance guarantee. We will begin with a two-player model and quadratic costs, and we will generalize our results to an n-player model with general convex costs.

4.3.1. A Two-Player Game with Quadratic Costs. We consider a two-player model, where the cost functions are quadratic, that is,

$$f_1(K_1) = \frac{a_1(K_1 + b_1)^2}{2} + c_1, \quad f_2(K_2) = \frac{a_2(K_2 + b_2)^2}{2} + c_2. \quad (8)$$

Without loss of generality, assume that $a_1 \geq a_2 > 0$. We define $\bar{K}_1 = K_1 + b_1$ and $\bar{K}_2 = K_2 + b_2$, and their corresponding modified effective capacity levels in the JV setting and the system optimum can be shown as follows,

$$\bar{L}^N = L^N + b_1 + b_2, \quad \bar{L}^* = L^* + b_1 + b_2.$$

We have seen from Proposition 5 that a JV with substitutable resources always under-invests, that is, $\frac{\bar{L}^N}{\bar{L}^*} \leq 1$. The following result measures the extent of this under-investment. Since $a_1 > a_2$ (i.e., player 1 is less cost-efficient), we restrict to $\beta_1 \leq 0.5$.

PROPOSITION 6. *In a two-player substitutable resource-sharing model, under any demand distribution D and quadratic cost functions, the effective capacity in a JV is bounded by*

$$\frac{\bar{L}^N}{\bar{L}^*} \geq \frac{\beta_1 a_2 + \beta_2 a_1}{a_1 + a_2},$$

for all revenue-sharing contracts with $\beta_1 \leq 0.5$.

Proposition 6 states that in a two-player model with quadratic costs, the effective capacity is at least half of the corresponding system optimum. The worst case, that is, $\bar{L}^* = 2\bar{L}^N$ occurs under two circumstances: (i) equal revenue sharing (i.e., $\beta_1 = \beta_2$) irrespective of cost asymmetry between the two players, and/or (ii) asymmetric revenue sharing with symmetric players (i.e., $\beta_1 \neq \beta_2$ with $a_1 = a_2$).

The next main result Theorem 2 quantifies the worst-case performance of a JV by comparing its total profit π_T^N to the optimum π_T^*. Given that there does not exist an efficient fixed-rate revenue-sharing contract, we will propose a revenue-sharing contract $\tilde{\beta}$ that induces the highest possible profit in a JV, with respect to cost parameters and demand uncertainty. To establish Theorem 2, we derive the following lemma which shows that the expected profit attained in a JV is concave in the capacity limit.

LEMMA 1. *Define an auxiliary function*

$$g(\hat{L}) \triangleq \max_{K_i} p\mathbb{E}[\min(L, D)] - \sum_{i=1}^n f_i(K_i), \text{ s.t. } L \leq \hat{L}.$$

Then $g(\hat{L})$ *is concave in* $\hat{L}$ *where* $\hat{L}$ *is the constraint on effective capacity investment.*

THEOREM 2. *In a two-player substitutable resource-sharing model, under any demand distribution D and quadratic cost functions, the efficiency of a JV is bounded by*

$$\frac{\pi_T^N(\beta)}{\pi_T^*} \geq \frac{1}{2}, \quad \text{for all } \beta_1 \in \left[\frac{mp + a_2}{2mp + a_1 + a_2}, \frac{1}{2}\right],$$

where m is the mode of demand distribution, that is, $\mathrm{f}_D(L) \leq m$ for all $L \geq 0$. Moreover, the revenue-sharing scheme $\tilde{\beta}_1$ that maximizes the total profit in a JV falls in the following interval,

$$\tilde{\beta}_1 \in \left[\frac{a_2}{a_1 + a_2}, \frac{mp + a_2}{2mp + a_1 + a_2}\right].$$

In Theorem 2, we propose an interval of revenue-sharing contracts and establish a performance guarantee. That is, we establish that the total profit generated in a JV is at least half of the optimal profit, when the revenue is divided according to the interval. We want to emphasize that this result holds under any demand distribution. The interval β_1 depends on the cost asymmetry between the two players and the mode of demand. It shrinks as the cost structures become more similar. With symmetric players as an example, the interval converges to $1/2$, that is, the revenue-sharing scheme with performance guarantee asks for an equal division of the revenue. In addition, the interval β_1 also narrows down as the mode of demand m increases. In particular, consider two demand distributions with the same support, the range of provably good revenue-sharing contract is smaller for the distribution that is concentrated around a small region (i.e., high mode).

Another interesting observation of Theorem 2 is on the guidance of the provably "good" revenue-sharing contracts. Note that in Theorem 1 for the complementary resources, the optimal revenue-sharing rule compensates each player proportionally for his or her share of the marginal cost. That is, if $a_1 \geq a_2$, the optimal way to share revenue must follow that $\beta_1 \geq \beta_2$. Theorem 2 implies the exact opposite, that is, if $a_1 \geq a_2$, a provably good contract with performance guarantee should satisfy $\beta_1 \leq \beta_2$. Intuitively, with complementary resources, the effective capacity is constrained by a bottleneck firm. To induce an optimal effective capacity L^* in a JV, these firms have to be awarded such that they are willing to produce up to L^*. In contrast, with substitutable resources, every firm can contribute to the effective capacity. Therefore, players who are cost-efficient are encouraged to produce more while the inefficient players are discouraged from producing, which is reflected by a lower (higher) revenue-sharing ratio for the less (more) cost-efficient firm.

4.3.2. An *n*-Player Game with General Convex Costs.

In this section, we consider a more general setting, that is, an n-player model with asymmetric convex cost functions. The analysis involves several steps where we utilize Lemmas 2 and 3 which are stated below.

Lemma 2. *With substitutable resource sharing, the efficiency on the total profit of a JV is lower bounded by*

$$\frac{\pi^N(f)}{\pi^*(f)} \geq \frac{\pi^N(\bar{f})}{\pi^*(\bar{f})},$$

where $\bar{f} = (\bar{f}_1, \ldots, \bar{f}_n)$ are linear cost functions such that $\bar{f}_i = \alpha_i \times K_i$ where $\alpha_i = f_i'(K_i^N)$.

Lemma 3. *With substitutable resource sharing, the efficiency on the total profit of a JV is lowered bounded by*

$$\frac{\pi^N(\bar{f})}{\pi^*(\bar{f})} \geq \tilde{\alpha}\frac{\tilde{L}^N}{\tilde{L}^*},$$

where the cost asymmetry factor is given by $\tilde{\alpha} = (\min_i \alpha_i)/(\max_i \alpha_i) \leq 1$.

In Lemma 2, we show that the efficiency of the original JV with nonlinear costs can be lower bounded by the efficiency of a *modified* model with linear costs, which have the same marginal cost as in the original model. Next, in Lemma 3, we show that the efficiency ratio can be further lower bounded by comparing the effective capacity in the modified model to its optimal counterpart, which is denoted as $\tilde{L}^N/\tilde{L}^*$ (we use $\tilde{L}$ to note that the effective capacity in the modified model) with a factor that measures the asymmetry in the players' cost functions.

With substitutable resources, equal revenue sharing induces equal marginal costs for every player in a Nash equilibrium, since $\beta_i = \alpha_i / \sum_{j=1}^n \alpha_j$. Thus, $\tilde{\alpha} = 1$, and the comparison between the profit in the two settings can be reduced to a comparison between the total investment level, i.e, $\frac{\pi^N(\bar{f})}{\pi^*(\bar{f})} \geq \frac{\tilde{L}^N}{\tilde{L}^*}$.

Our next result quantifies the efficiency of a JV with substitutable resource sharing.

Theorem 3. *In the substitutable resource-sharing model, for an n-player game with general demand and general convex functions, the contract efficiency on the total profit of a JV is lower bounded by*

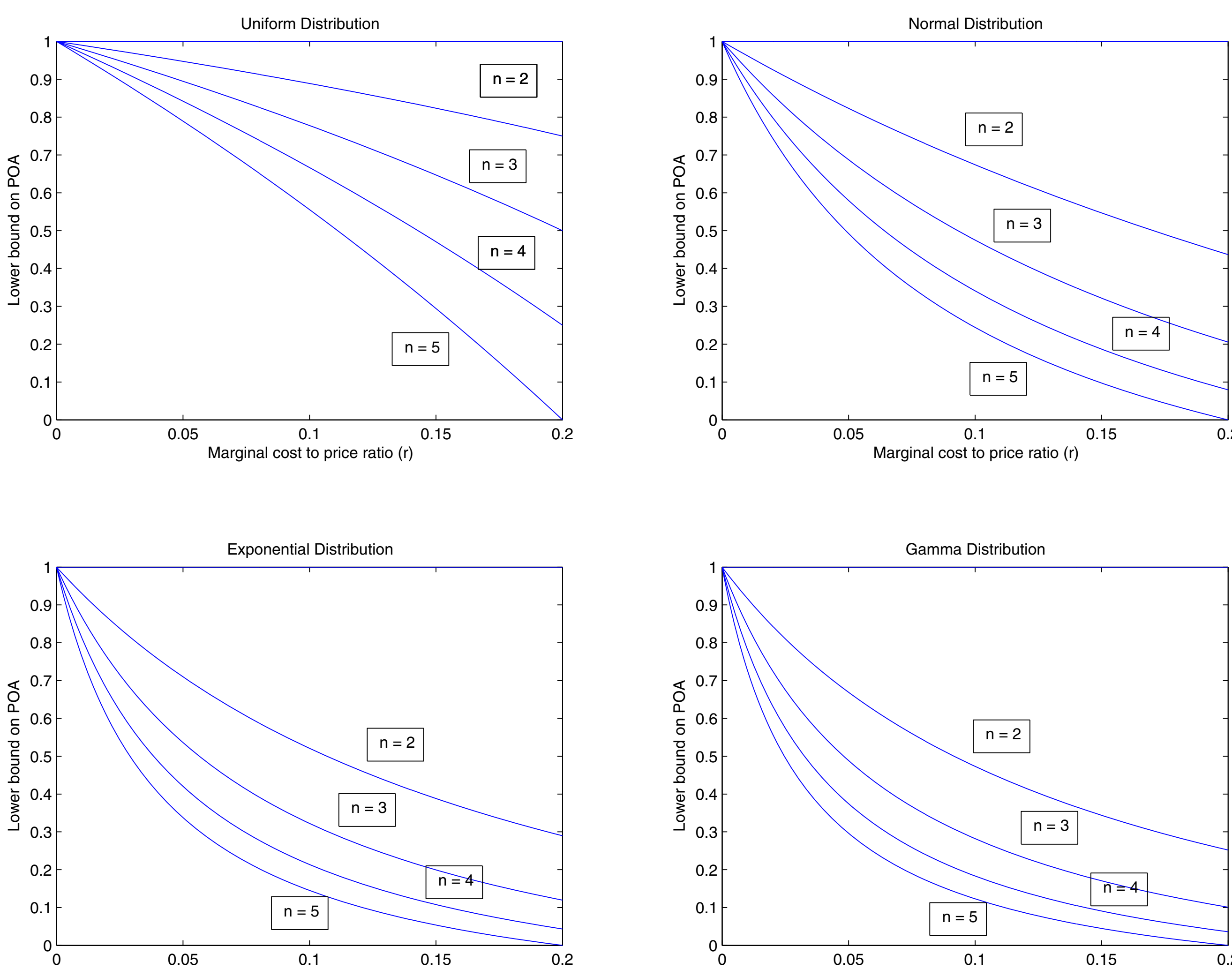
$$\frac{\pi_T^N}{\pi_T^*} \geq \tilde{\alpha}\frac{1 - n\bar{r}}{1 - n\bar{r} + (n-1)\bar{r}\tilde{\theta}},$$

where $\bar{r} = \max_i f_i'(K_i^N)/p$ is the marginal cost to revenue ratio, $\tilde{\alpha} = \min_i f_i'(K_i^N)/\max_i f_i'(K_i^N) \leq 1$ as the cost asymmetry, and $\tilde{\theta}$ is the demand spread, that is, $\tilde{\theta} = \frac{\max \mathrm{f}_D(x)}{\min \mathrm{f}_D(y)} \geq 1$, for $x \leq \tilde{L}^N \leq y \leq \tilde{L}^*$.

Theorem 3 suggests that the efficiency decreases with the number of firms, n, which is intuitive as disparate self-interests increase. Since the cost asymmetry $\tilde{\alpha} \le 1$ by definition, Theorem 3 also states that the efficiency also decreases as the cost structures of the players become more asymmetric. This is not surprising since in the system optimum setting, only the most cost-efficient players should be active. Moreover, the efficiency decreases as the marginal cost to revenue ratio increases. In other words, a JV is more (less) efficient when it has a higher (lower) profit margin. To explain this, we see that the inefficiency in JV is manifested as under-investment. With a higher profit margin, players tend to invest in more capacity than in a JV with a lower margin. Lastly, Theorem 3 also suggests that the efficiency of a JV decreases as the demand spread increases. One explanation is that when the distribution is more concentrated around some regions (i.e., high demand spread), it is more likely for the JV to miss the bulk of demand due to under-investment, resulted in a large profit loss. To illustrates this impact, we plot the efficiency bound with respect to four types of demand distributions in Figure 1.

The demand distributions are uniform, normal distribution **N**(400, 100), exponential distribution with rate 400, and a gamma distribution with **Gamma** (2, 400), respectively. Note that in the case with uniform demand, the demand spread $\tilde{\theta} = 1$, the bound in Theorem 3 is simplified to $\frac{\pi_T^N}{\pi_T^*} \ge \tilde{\alpha}\frac{1-n\bar{r}}{1-\bar{r}}$. Figure 1 clearly illustrates the relationship between the efficiency of a JV and the number of players and the marginal cost to revenue ratio. We also observe that the efficiency has a steeper rate of decrease when the demand spread is higher. Note that in this simulation, for the given input parameters, the gamma distribution has the highest demand spread with $\tilde{\theta} = 8.91$, followed by the exponential distribution ($\tilde{\theta} = 7.35$), the normal distribution ($\tilde{\theta} = 3.86$), and lastly, the uniform distribution ($\tilde{\theta} = 1$).

Figure 1 Lower Bounds on the Efficiency for Uniform, Normal, Exponential and Gamma Demand Distributions [Color figure can be viewed at wileyonlinelibrary.com]



4.4. Cost Rebalancing Contract

As we have established that the simple fixed-rate revenue-sharing scheme is not capable of achieve JV coordination, we now propose what-we-call a *cost rebalancing contract*, to align the incentives of the JV participants. To be precise, given a revenue-sharing allocations β, each JV participant will receive a subsidy or penalty defined as follows:

$$\hat{f}_i(K_i) = f_i(K_i) - \beta_i \sum_{j=1}^{n} f_j(K_j),$$

which is the difference between her own cost and the β_i fraction of the total cost.

Under this new contract, player i maximizes her profit by choosing her capacity investment level K_i by solving

$$\begin{aligned}\pi_i^N(\beta) &\triangleq \max_{K_i} \beta_i p \mathbb{E}\left[\min\left(D, \sum_j K_j | \mathbf{K_{-i}}\right)\right] - f_i(K_i) + \hat{f}_i(K_i), \\ &= \max_{K_i} \beta_i p \left(\mathbb{E}\left[\min\left(D, \sum_j K_j | \mathbf{K_{-i}}\right)\right] - \sum_{j=1}^{n} f_j(K_j)\right).\end{aligned} \tag{9}$$

This simple cost rebalancing contract essentially achieves *profit-sharing allocation* β, which induces behavior perfectly coinciding with the system optimal solution. We formalize the result in the following proposition and omit its proof.

Proposition 7. *Consider a JV with substitutable resource sharing. Given any revenue-sharing ratio* $\beta \in [0, 1]$, *the cost rebalancing contract is coordinating, that is,* $L^N(\beta) = L^*$. *Moreover, this contract allows for an arbitrary revenue (as well as profit) allocation* β *among players.*

The key source of inefficiency in JV with substitutable resources occurs when some cost-inefficient (cost-efficient) players over-produce (under-produce). In order to encourage a player to produce at the system optimal capacity level, she should be subsidized by $\hat{f}_i(K_i)$ if this term is positive, and penalized if $\hat{f}_i(K_i)$ is negative, that is, the β_i fraction of the total cost can be viewed as her "fair" share of the cost that a JV participant should bear (as she receives β_i fraction of the total revenue).

5. An Extension with Spillovers

One extension which we shall consider is the presence of spillovers for JVs, which typically result from transfer and exchange of knowledge, skilled labor and technology among participating firms (see, e.g., De Bondt 1997, Morton et al. 1992). In this case, spillovers from one firm might enable other firms to improve its product, accelerate innovation and enhance efficiency. Following Katsoutacos and Ulph (1998) and Atallah (2007), we model spillovers as an affine function of individual firms' investment.

Definition 7. (Spillover). *Denote* γ_{ij} *as the spillover that player* j *contributes to player* i.

- Complementary resources: $L(\mathbf{K}) = \min_i \left(K_i + \sum_{j \neq i} \gamma_{ij} K_j\right)$;
- Substitutable resources: $L(\mathbf{K}) = \sum_{i=1}^{n}(K_i + \sum_{j \neq i} \gamma_{ij} K_j)$, where $\gamma_{ij} \geq 0$ for all $j \neq i$.

5.1. Complementary Resources

With the presence of the spillover effect, the effective capacity for a JV with complementary resources is given by $L(\mathbf{K}) = \min_i \left(K_i + \sum_{j \neq i} \gamma_{ij} K_j\right)$, where $\gamma_{ij} \geq 0$ for all $j \neq i$. The following result captures the capacity decision in the system optimal setting.

Proposition 8. *With complementary resources with spillover, the joint capacity level* L^* *in the system optimal setting satisfies*

$$p\mathbb{P}(D \geq L^*) = \mathbf{e}^T \mathbf{\Gamma}^{-T} \nabla \mathbf{f}, \text{ where } \mathbf{e} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix},$$

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & 1 & \cdots & \gamma_{2n} \\ & & \ddots & \\ \gamma_{n1} & \gamma_{n2} & \cdots & 1 \end{bmatrix}, \text{ and } \nabla \mathbf{f} = \begin{bmatrix} f_1'(K_1^*) \\ f_2'(K_2^*) \\ \vdots \\ f_n'(K_n^*) \end{bmatrix}.$$

Note that when there is no spillover, that is, $\mathbf{\Gamma}$ becomes an identity matrix, Proposition 8 is reduced to Proposition 1. With the presence of spillover, Proposition 8 requires the existence of the inverse of matrix $\mathbf{\Gamma}$. One such condition to guarantee its existence is symmetry of the matrix ($\gamma_{ij} = \gamma_{ji}$) and strict diagonal dominance ($\sum_{j \neq i} \gamma_{ji} < 1$). The latter implies that the total impact from spillover must be less than one's own capacity contribution.

Corollary 1. *With fully symmetric players, that is,* $f_i(K_i) = f(K)$ *for all* i *and* $\gamma_{ij} = \gamma$ *for all* $j \neq i$, *the joint capacity level* L^* *satisfies* $p\mathbb{P}(D \geq L^*) = \frac{nf'(K^*)}{1 - \gamma + \gamma n}$.

In Proposition 1, we obtain $p\mathbb{P}(D \geq L^*) = nf'(K^*)$ when spillover effect is absent. As $(n-1)\gamma < 1$, each individual participants could contribute a lower level while maintaining the same effective capacity level L^*. This is an intuitive advantage of having spillover.

At the same time, spillover also leads to other subtle changes to a JV. In the absence of spillover, every player contributes the same capacity level which is equal to the effective capacity, that is, $L_i^* = L^*$ for all i, regardless of whether the players are symmetric or asymmetric. The argument is that since the resources are perfectly complementary, every player only invests up to the effective capacity level. On the other hand, when the spillover effect is present, it is possible that some player i's contribution is strictly higher than the effective capacity, $L_i^* > L^*$. However, one cannot lower i's contribution, as some player j's capacity contribution which includes i's spillovers is binding, that is, $L_j^* = L^*$. Nevertheless, the slack in player i's contribution suggests inefficiency in the investment decision in such a JV.

Remark 3. One can, in fact, construct examples to show that with spillovers, even for fully symmetric players, there does not exist a coordinating revenue-sharing contract. The fact that the resource contributed by player i can now be shared with player j implies that spillover has turned fully complementary resources into partially substitutable. Therefore, the resulting JV inherits the properties of a substitutable resource-sharing JV which was studied in section 4, which include that the simple revenue-sharing contract is no longer capable of aligning the incentives of JV participants. We do want to clarify that having spillover does not mean the JV becomes less efficient compared to the case without spillover. Our results only suggest that it is less efficient than what it could have been in a scenario if decisions were made centrally.

5.2. Substitutable Resources

With substitutable resources, when the spillover effect is present, the individual capacity invested by each player is defined as $L_i = K_i + \sum_{j \neq i} \gamma_{ij} K_j$. Under this formulation, it effectively turns the perfectly substitutable resources which we have analyzed in section 4 to imperfect substitutes, depicting a more realistic setting.

Recall in Proposition 3, without the spillover effect, when resources are fully substitutable, then cost is the only consideration for JV participation, that is, only the most cost-efficient firms should be active. On the other hand, in the presence of spillover, it is possible that a firm with a higher marginal cost remains active in a JV, as long as its "discounted" marginal cost after taking spillover cost into consideration is small as shown in the following corollary.

Corollary 2. *In the system optimal model with spillover, all active firms must have the same discounted marginal costs,* $\min_i \frac{f_i'(x)}{1+\sum_{l \neq j} \gamma_{li}}$, *which is lower than that of the inactive firms.*

When there is no spillover, that is, $\gamma_{ij} = 0$ for all $j \neq i$, Corollary 1 reduces to Proposition 3, which implies that every active player participating in a JV must have the same marginal costs at optimality, that is, $f'(K_i^*) = f'(K_j^*)$. The ratio $\frac{f_i'(x)}{1+\sum_{l \neq j} \gamma_{li}}$ can be interpreted the "discounted marginal cost" of having firm i, that is, the numerator refers to the marginal cost of producing x units by i, and the denominator quantifies the positive externality effect of having i which measures the spillover benefit to other active firms. This term which captures the marginal cost to benefit of individual firms plays a central role in the analysis with spillover, as many results proved under the setting without spillover can be carried over by replacing the marginal cost with this term.

Corollary 3. *In a JV with substitutable resource sharing and spillover, there exists a unique equilibrium outcome* $\mathbf{K}^N$ *which satisfies the following condition,*

$$\mathbb{P}(D \leq L^N) = 1 - \frac{f_i'(K_i^N)}{\beta_i(1+\sum_{j \neq i} \gamma_{ji})p}, \text{ for all } i.$$

Corollary 4. *In the substitutable resource-sharing model with spillover, for an n-player game with general demand and general convex functions, the contract efficiency on the total profit of a JV is lower bounded by*

$$\frac{\pi_T^N}{\pi_T^*} \geq \tilde{\alpha} \frac{1 - n\bar{r}}{1 - n\bar{r} + (n-1)\bar{r}\tilde{\theta}},$$

where the parameters

$$\bar{r} \triangleq \max_i \frac{\alpha_i}{(1+\sum_{j \neq i} \gamma_{ji})p}, \quad \tilde{\alpha} \triangleq \frac{\min_i \alpha_i/(1+\sum_{j \neq i} \gamma_{ji})}{\max_i \alpha_i/(1+\sum_{j \neq i} \gamma_{ji})}$$

denote the maximum discounted marginal cost margin and the modified cost asymmetry, respectively.

Corollary 4 is the analogous version of Theorem 3, which quantifies the efficiency of JV for a general setting of n-players and convex cost. One the key difference is that the cost asymmetry $\tilde{\alpha}$ is defined as the ratio between the minimum and the maximum discounted marginal costs.

6. Case Study: Motion Picture Industry

In the earlier sections, we have characterized the analytical solutions to quantify the performance of JVs with respect to the system optimum. In this section, we will turn to the motion picture industry and present two case studies that highlight the different types of JVs in this industry. We will begin with a JV case study on complementary resource sharing between IMAX and theater operators. Next, we will present a case by considering substitutable resource sharing between two theater operators.

6.1. JV between IMAX and Theater Operators

The IMAX Corporation (IMAX), founded in 1968, is one of the leading entertainment technology companies, specializing in immersive motion picture technologies. While IMAX has been synonymous with superior film experiences for decades, it historically struggled to reach a mass audience because of the high cost of its equipment, since the upfront cost of an IMAX projection system is about $1.2 m (IMAX 2015). Seeking a solution to overcome the barriers that curtailed its growth, IMAX introduced a revenue-sharing joint venture strategy with theater operators in 2008. Under this arrangement, IMAX installs its proprietary systems in participating theaters at no charge, in return for a portion of the theater's box-office receipts. By offering arrangements in which theaters do not need to invest the substantial initial capital, IMAX has been able to expand its theater network at a significantly faster pace than it had previously. According to the company's annual report (IMAX 2015), its overall network and commercial network have increased by 255% and 437% respectively since the beginning of 2008. As of December 31, 2015, IMAX entered into joint revenue-sharing arrangements for 741 theater systems worldwide, approximately 77% of its entire commercial network. We will utilize historical data between 2009 and 2013 and conduct a backtesting to compare IMAX's existing JV strategy in the United States with the optimal strategy and discuss its implications.

We denote IMAX and a theater operator in the United States as player 1 and 2, respectively. We let the capacity K to denote the number of IMAX systems signed in a JV. To conduct the backtesting, we first need to fit the model parameters in Equation (3), which includes the cost functions for the two players, the uncertain demand distribution and the average revenue of the JV, (i.e., $f_1(\cdot), f_2(\cdot), D$ and p).

6.1.1. Cost and Revenue Estimation.

We first consider the IMAX's cost in a JV. While an IMAX system is priced around $1.2m through outright sales, based on IMAX annual reports between 2009 and 2013, the average cost of goods sold per IMAX screen is $0.5 m. The joint revenue-sharing arrangements are typically for 10–13 years with renewal options. For simplicity, we assume that the lifespan of an IMAX system is 10 years. Under straight-line depreciation, the amortized cost for a period of 5 years per screen is $0.25 m. Therefore, for K such systems, IMAX's cost ($m) is at $f_1(K) = 0.25K$.

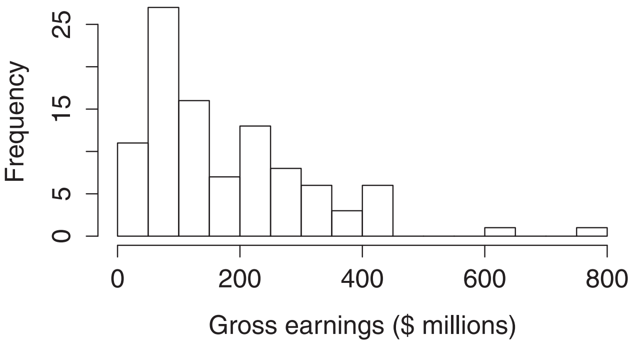
To extrapolate the cost of running a theater from 2009 to 2013, we investigate the financial data of a large public movie theater chain, AMC Theaters. Based on its annual reports, the operating expenses total $3.30b during the past 5 years. Meanwhile, during the same time, the number of screens under AMC ranges from 4347 to 4976 with a coefficient of variation of 0.064, implying a relatively small fluctuation across the time period. Therefore, we take the average number of AMC screens, 4720, and estimate the operating cost per screen for the past 5 years as $0.7 m. To operate K such screens, the cost ($m) is $f_2(K) = 0.7K$.

Next, we need to estimate the average return on an IMAX screen during the 5-year period, p. Table 1 shows the number of theaters from National Association of Theater Owners (NATO). The total box office revenue between 2009 and 2013 is valued at $53,094 m. As the number of screens stays rather stable throughout the years (the coefficient of variation is merely 0.006), we take the average and estimate $1.36 m as the 5-year revenue received by a screen in the United States. Meanwhile, the premium sight and sound experiences offered by IMAX allow theaters to impose a premium, driving better economics for theater operators and the studios while also delivering audience a superior experience. According to the AMC annual report, compared to the average ticket price of $7.88, on average, IMAX pricing premiums amount to $4.34 per patron (an increase of 55.1%).

Table 1 Theatrical Numbers from National Association of Theater Owners (NATO)

Year	Box office ($million)	Attendance (million)	Average ticket price ($)	No. of theaters	No. of indoor screens
2013	10,921	1343	8.13	5281	39,264
2012	10,837	1361	7.96	5317	39,056
2011	10,174	1283	7.93	5331	38,974
2010	10,566	1339	7.89	5399	38,902
2009	10,596	1413	7.50	5561	38,605

Figure 2 Histogram of Box Office for the 99 Movies between 2009 and 2013



Therefore, we estimate the 5-year revenue received by an IMAX screen by scaling the \$1.36 m with a factor of 1.551–\$2.11 m.

6.1.2. Demand Estimation. IMAX theaters can only show movies that are in IMAX format (which can be converted via post-processing or filmed with IMAX cameras). From IMAX's website and its annual reports, we identified 99 IMAX movies that were released in the United States between 2009 and 2013. We excluded educational films and documentaries that were exclusively shown in museums, re-releases of the "classics" in IMAX format (e.g., *Titanic* from 1997, *Indiana Jones and the Raiders of the Lost Ark* from 1981), as well as foreign movies that were not released in the United States.

We collected the domestic box office numbers from Box Office Mojo (www.boxofficemojo.com). For any movie, it reports the *total* gross earning derived from both the conventional film format and IMAX version. The histogram of the box office for the 99 movies is shown in Figure 2. It is apparent that the distribution of box office revenue is skewed to the right, that is, the median and mean are \$127 m and \$176 m respectively. The distribution exhibits the fat-tailed nature, as the majority of films earn little and a few blockbusters achieve stellar box office results, which is a well-documented phenomenon in the motion picture industry[1] (e.g., Clauset et al. 2009, Pan and Sinha 2010).

Taking into account the skewness and the fat-tail of the empirical distribution, we fit the data with three distributions, that is, Weibull, lognormal and gamma. To measure the goodness of fit, we use the Kolmogorov–Smirnov test which compares the empirical distribution to the fitted distribution. The null hypothesis is that the data follow the fitted distribution. It is rejected if the p-value is lower than the significance level. We performed the test for each fitted distribution and the p-value is given by 0.4028 and 0.3835 and 0.4204 for Weibull, lognormal and gamma distributions respectively. For all three distributions, we are unable to reject the null hypothesis because the p-value is sufficiently higher than the significance levels usually referred in the statistical literature.

The comparative results based on the goodness of fit test are not surprising as shown in Figure 3, where we plot the fitted and the empirical distributions. For the rest of the analysis, we select the gamma distribution as it appears to be superior in fitting the tail distribution compared to the other two alternatives (see the Q-Q plot in Figure 3). The fitted distribution for the box office earning in \$million is given by

$$Y \sim \mathbf{Gamma}(k = 1.866, \theta = 0.0106),$$

where k and θ represent the shape and the rate parameters. We assume the earning of a movie is an i.i.d. variable from this gamma distribution. Therefore, the total earnings from 99 movies between 2009 and 2013 is a sum of 99 independent gamma distributions, which still follows a gamma distribution with parameters k and 99θ, that is,

$$\bar{Y} = Y_1 + Y_2 + \cdots + Y_{99} \sim \mathbf{Gamma}(1.866, 1.048).$$

Note that $\bar{Y}$ refers to the aggregate demand in \$m earned by both the IMAX and non-IMAX showings. We need to estimate how much demand is contributed by IMAX screens. In terms of physical locations, there are 380 IMAX screens in the United States as of December 2013, approximately 1% of the total screen count. However, the "IMAX experience" has been proven to draw higher attendance levels despite its fewer locations and the premium pricing. For example, during the opening weekend of the space epic *Gravity* in 2013, the IMAX showings accounted for 20% of ticket sales (Businessinsider 2013). An IMAX presentation can also help salvage a stinker of a film. The 2012 flop, *John Carter*, pulled in \$30.2 m during its opening on 3749 screens, but an impressive \$5 m or 17% of the box office on a mere 289 IMAX screens (Time 2012).

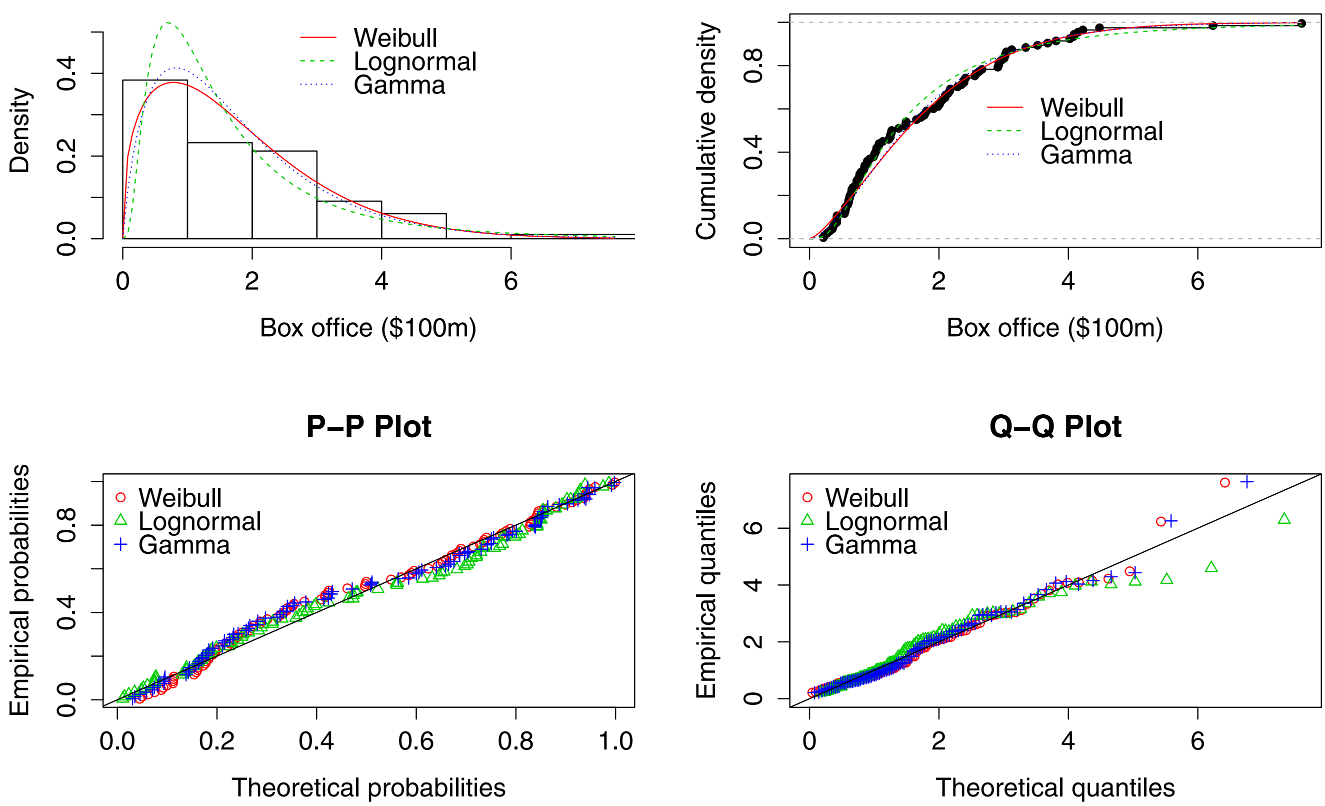
Denote by D the demand measured in terms of IMAX screens. To "extrapolate" D using the total earning data, we use x to denote what-we-call the *demand penetration* for IMAX to vary between 1% and 25% of the total earnings. As we have estimated that an IMAX screen earns \$2.11 m over the past 5 years, we can obtain D by appropriately scaling $\bar{Y}$, that is,

$$D = \frac{x}{2.11}\bar{Y} \sim \mathbf{Gamma}(1.866, 0.497x),$$

where x as the demand penetration, $x \in [1\%, 25\%]$.

While there is qualitative evidence that IMAX's JVs with theaters experience spillovers in terms of accelerated adoption of the digital technology,[2] we are unable to recover this value from the available

Figure 3 Fitting the Empirical Box Office Numbers with Weibull, Lognormal and Gamma Distributions [Color figure can be viewed at wileyonlinelibrary.com]



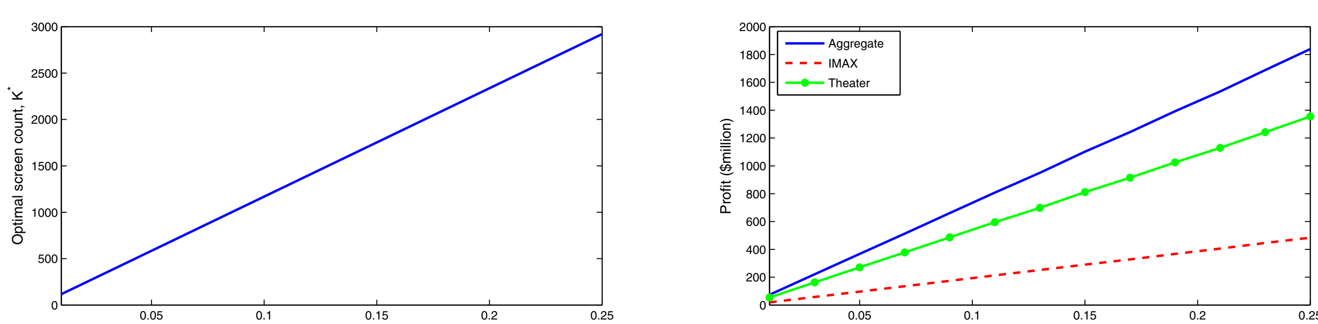
public financial data, we exclude this term from the analysis.

As the spillover effect between IMAX and movie theaters is hard to quantify and we are unable to recover this value from the available public financial data, we exclude this term from the analysis.

6.1.3. Numeric Analysis. With complementary resource sharing, we have shown in Theorem 1 that there exists a unique coordinating revenue-sharing ratio β^*, which is invariant to demand and only depends on the marginal cost structure of the players. Given the input parameters estimated in the earlier subsections, we compute that the optimal revenue-sharing ratio is $\beta^* = 25\%$ for IMAX, and $1 - \beta^* = 75\%$ for theaters. While IMAX has not disclosed the detailed contract terms, its CEO, Rich Gelfond, revealed in a recent conference (Communacopia 2013), that IMAX receives roughly about 20% of revenue from theaters that participate in joint ventures. Interestingly, this number is quite close to what we estimate as the efficient and fair revenue-sharing scheme.

Figure 4 depicts the optimal number of IMAX screens as the demand penetration increases from 1% to 25%. Intuitively, as more moviegoers turn to IMAX

Figure 4 Optimal Number of IMAX Screens, and Profit under Joint Ventures with Respect to IMAX Demand Penetration [Color figure can be viewed at wileyonlinelibrary.com]



(i.e., the demand penetration increases), the optimal number of IMAX screens also increases to meet the rising demand. In particular, suppose 20% of moviegoers choose to watch a movie in IMAX (such as *Gravity*), then optimally there should be 2336 IMAX screens, or 6% of the total screens in the United States. Given currently only 1% of the screens are IMAX, our analysis suggests that there is considerable room for growth. Figure 4 also shows the optimal profit of the firms in this joint venture. At 20% demand penetration, the model indicates that IMAX could have earned \$386.5 m over the period of past 5 years from revenue-sharing ventures. During the same period, considering all the revenue streams, IMAX's net income is \$206.7 m, implying huge earning potential of this JV. Our analysis suggests that the theaters can also enjoy a tremendous upside from joint ventures. Between 2009 and 2013, the aggregate net income of AMC theaters is \$666.4 m. Having 4620 screens, this is translated into a net income of \$0.141 m per screen. Figure 4 shows that at 20% demand penetration, the total profit earned by the theater from operating 2336 IMAX screens is \$1080.8 m, resulted in \$0.468 m per screen, or an increase of over two fold from \$0.141 m.

6.2. JV between Theater Operators

Besides complementary resource sharing such as the IMAX example, there is also substitutable resource sharing in the motion picture industry, that is, when theater operators form a JV to operate multiple theaters across regions. Nevertheless, resource sharing among theater operators via JVs is much less common than consolidation via mergers and acquisition in this industry.

According to NATO, as of June 2013, there are 92 theater operators in the United States, where the top four chains (Regal Entertainment Group, AMC, Cinemark, and Carmike) represent almost half of the theater screens. Instead of JVs, the large theater chains primarily rely on mergers and acquisitions to increase their market share. For instance, in 2012, Wanda acquired AMC for \$2.6 billion and Regal Entertainment group acquired a total of 25 theaters representing 301 screens from Great Escape Theaters for \$91 m (Reuters 2012b). A year later, Regal bought 43 theaters with 513 screens for \$191 m from Hollywood Theaters (Businesswire 2013). In the same year, Cinemark acquired 32 theaters representing 483 screens for approximately \$240 m from Rave Cinemas theaters (Reuters 2013).

For the case study, we will fit the substitutable resource-sharing model between two symmetric theater operators who decide the number of screens to operate and evaluate the efficiency of such a JV, by comparing to the system optimum, which in theory, can be attained via an acquisition. In section 6.1.1, we have estimated the operating expense of a screen is about \$0.7m over 5 years, or \$0.14 m per year. Thus, with K_i screens, the costs (\$m) are given by $f_i(K_i) = 0.14K_i$, for i = 1, 2. Meanwhile, we have also shown the annual revenue earned by a regular screen (non-IMAX) is \$0.272m, that is, $p = 0.272$.

Pan and Sinha (2010) study 5222 movies released in the United States between 1999 and 2008, and estimate the distribution of the annual box office. Let Z denote the annual box office (\$), then the authors show that Z follows a lognormal distribution, that is, $Z \sim \log N(\mu, \sigma^2)$, where mean $\mu = 16.607$ and standard deviation $\sigma = 1.471$. To convert the demand in dollars to the number of screens, we utilize the result from section 6.1.1, where we have estimated that the average revenue earned by a regular screen (non-IMAX) over the period of 5 years is \$1.36 m, or \$272,000 per year. We let D' to denote the annual demand in terms of movie screens, where $D' = Z/272,000$. We make use of the property of the lognormal distribution: If $Z \sim \log N(\mu, \sigma^2)$, then $aZ \sim \log N(\mu + \log a, \sigma^2)$, that is,

$$\begin{aligned} D' = Z/272,000 &\sim \log N\big(\mu - \log(272,000), \sigma^2\big) \\ &= \log N(4.093, 1.471^2). \end{aligned}$$

Based on these estimated values, our first observation is that theaters naturally will not enter a JV. To see this, for a given revenue-sharing ratio β, a theater's objective function in a JV is given by

$$\pi_i = \max_{K_i|K_{-i}} 0.272\beta\mathbb{E}[\min(K_i + K_{-i}), D')] - 0.14K_i.$$

Given two symmetric players and affine costs, the only feasible revenue-sharing ratio β is 0.5. The marginal revenue of this JV is 0.272β, or \$0.136 m, which is smaller than the marginal cost at \$0.14 m. Therefore, neither theater has the incentive to participate in the JV of operating non-IMAX screens due to the low return.

Suppose the JV is profitable, e.g., we have shown in section 6.1.1 that IMAX screens fetch a premium of 55.1%, with a revenue of \$0.422 m per screen per year. One can show that with $\beta = 0.5$, the profit achieved in this JV is merely 59.3% of the optimum, implying a significant efficiency loss, which can be eliminated in a merger or acquisition.

The results seem to provide some explanations for the prevalent acquisitions among theater operators. JVs, which can be viewed as a potential alternative to acquisitions or mergers, appear as an inferior option to achieve resource pooling. Operating theaters is a risky business with a slim margin: According to McKenzie and Tullock (2012), theaters regularly bid 55% to 95% of their box-office receipts to movie distributors for the rights to show a movie. Moreover, as the success of a movie is very unpredictable, theaters have to assume a great deal of risk. Since joint ventures require partners to share return, it further reduces the attractiveness of such a partnership. Moreover, we have seen that even when the JV is profitable, inefficient investment decision due to incentive misalignment could lead to efficiency loss by leaving money on the table. In addition, as we have shown in Theorem 3, the loss could be further exacerbated by the slim profit margin. All the aforementioned shortfalls of JVs can be largely corrected when a theater operator expands via acquisitions.

7. Conclusion and Future Directions

In this study, we focused on resource sharing in JVs under demand uncertainties. We quantified the efficiency of a JV by comparing its effective capacity and its total profit with respect to its system optimal counterpart. We distinguished two types of resources, that is, complementary versus substitutable, which affects how the effective capacity is determined. (i) When resources are complementary, the effective capacity in a JV is constrained by the bottleneck resource. We have shown that every participant in a JV is committed to making an equal capacity contribution. We have also shown that, there exists a fixed-rate revenue-sharing contract in which an efficient and fair outcome can be induced in a JV by compensating every participant proportionally to her marginal cost. (ii) When resources are substitutable, the effective capacity in a JV is obtained by aggregating the individual's contributions. In contrast to the case with complementary resources, we have shown that there does not exist an efficient fixed-rate revenue-sharing contract. Nonetheless, we have proposed an alternative scheme to share revenue with worst-case performance guarantees. The scheme encourages (discourages) the participation from the cost-efficient (inefficient) players by setting the revenue-sharing ratio which is inversely proportional to the marginal cost of each participant. (iii) We then presented two case studies in the motion picture industry which illustrated complementary and substitutable resource sharing, respectively. With our estimation based on the data between 2009 and 2013, we showed an efficient and fair revenue scheme which was rather close to what IMAX claimed for their JV contracts. Moreover, we also showed that there could be a lot of room for growth for IMAX screens which would benefit both IMAX and theater operators. We also studied substitutable resource sharing between theater operators. Our results provided some explanations to the observation that theater operators primarily relied on mergers and acquisitions to achieve resource pooling instead of JVs.

In conclusion, we briefly point out two plausible avenues for future research. First, it would be very interesting to investigate other game theoretic formulations (potentially leveraging cooperative game theory) where both the revenue-sharing allocation β and the capacity investment levels of all players are *jointly* decided. Second, the present work mainly focuses on fixed-rate revenue-sharing contracts that are used to coordinate JVs. A natural and important research direction is to investigate whether there are other types of contracts that are both efficient and fair.

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Appendix. Proofs of Lemmas, Propositions, and Theorems

Proof of Proposition 1. Assume by contradiction that there exists a pair of players i and j such that $K_i^* < K_j^*$, we can decrease the capacity invested by player j from K_j^* to K_i^*. By doing so, the profit increases, since the revenue stays the same and the cost has decreased. Hence, we reach a contradiction to the optimality of L^*. At system optimality, $L^* = K_i^*$ for all $i = 1, \ldots, n$, and Equation (1) reduces to a single variable optimization with a concave objective function, so that L^* can be obtained by the first-order condition.

Proof of Proposition 2. Assume by contradiction that if there exists a pair of players i and j such that $K_i^N < K_j^N$, then player j can decrease its capacity investment from K_j^N to K_i^N. By doing so, it lowers her cost and improves her profit, without affecting the overall revenue. Thus, at Nash equilibrium, all players must have the same capacity investment level, that is, $L^N(\beta) = K_i^N(\beta)$ for all $i = 1, \ldots, n$.

Now assume that $\min_{1 \le k \le n}(A_k) = A_m$. Now if $A_m < K^N = K_m^N$, player m always has incentives to unilaterally lower her investment level to A_m since A_m is her profit-maximizing level. This forces all players to invest at A_m. Any capacity investment level $\tilde{A}_m$ such that $0 \le \tilde{A}_m \le A_m$ is also a Nash equilibrium since no player has incentives to unilaterally deviate from $\tilde{A}_m$. To further check whether a Nash equilibrium is also a Strong Nash equilibrium, observe that at $\min_i(A_i)$, no coalition of firms can cooperatively deviate in a way that benefits all of its members in the coalition. This Strong Nash equilibrium is also unique, since all players invest at $\min_i(A_i)$ in this equilibrium, which is a singleton point. □

PROOF OF THEOREM 1. (A). By Proposition 2, L^{SN} equals $\min_{1 \le k \le n}(A_k)$, say A_m, which implies

$$\mathbb{P}(D \ge L^{SN}) = \frac{f'_m(L^{SN})}{\beta_m p}. \tag{A1}$$

By Proposition 1, the optimal effective capacity L^* must also satisfy

$$\mathbb{P}(D \le L^*) = 1 - \sum_{i=1}^{n} \frac{f'_i(L^*)}{p}. \tag{A2}$$

Combining Equations (A1) and (A2), if using β^* defined in Equation (4), we have $L^{SN} = L^*$, and every player i is willing to produce exactly at the system optimal level, that is,

$$\mathbb{P}(D \ge L^*) = \frac{f'_i(L^*)}{\beta^*_i p} = \frac{\sum_{i=1}^n f'_i(L^*)}{p}, \quad \text{for } i = 1, \ldots, n. \tag{A3}$$

It remains to show the optimal revenue-sharing ratio must take the form specified in Equation (4) and is unique. Suppose, otherwise, that there exists another optimal revenue-sharing contract $\tilde{\beta}$. Since $\sum_{j=1}^n \tilde{\beta}_j = 1$ and $\tilde{\beta}_j \ge 0$ for all j, then there must exist some other player m' whose revenue-sharing rate

$$\tilde{\beta}_{m'} < \frac{f'_{m'}(L^*)}{\sum_{j=1}^n f'_j(L^*)}. \tag{A4}$$

Note that since L^* equals $A_m = \min_{1 \le k \le n}(A_k)$, we must have

$$\mathbb{P}(D \ge L^*) = \frac{f'_m(L^*)}{\tilde{\beta}_m p} \ge \frac{f'_i(L^*)}{\tilde{\beta}_i p}, \quad \text{for } i = 1, \ldots, n \text{ and } i \ne m. \tag{A5}$$

Combining Equations (A4) and (A5), we have

$$\mathbb{P}(D \ge L^*) \ge \frac{f'_{m'}(L^*)}{\tilde{\beta}_{m'} p} > \frac{\sum_{i=1}^n f'_i(L^*)}{p}, \tag{A6}$$

which contradicts with Equation (A2). This shows the optimal revenue-sharing contract β^* is unique.

PROOF OF THEOREM 1. (B). The optimization problem (5) is equivalent to

$$\max_{L,K_i} \sum_{i=1}^{n} \log(\beta_i p \mathbb{E}[\min(K_i, D)] - f_i(K_i) - v_i), \quad \text{s.t. } L \le K_i,\ i = 1, \ldots, n. \tag{A7}$$

Denote the admissible set of contracts as Λ, where

$$\Lambda = \{\beta : \beta_i p \mathbb{E}[\min(K_i, D)] - f_i(K_i) - v_i > 0 \text{ for all } i = 1, \ldots, n\}.$$

The admissible set only includes the revenue-sharing contracts that satisfy *individual rationality*, that is, yielding positive gain for every player compared to the non-cooperative payoff.

We first show that in a Nash bargaining model with $\beta \in \Lambda$, the capacity invested by each partner is the same, that is, $L^{NB}(\beta) = K_i^{NB}(\beta)$ for all $i = 1, \ldots, n$. To see this, the constraint set enforces that $L^{NB} \le K_i^{NB}$ for all $i = 1, \ldots, n$. Assume by contradiction, if there exists a player j such that $L^{NB} < K_j^{NB}$, then player j can decrease her capacity investment from K_j^{NB} to L^{NB} thereby strictly lowering her cost and improving her profit. Thus, at optimality, $L^{NB} = K_i^{NB}$ for all $i = 1, \ldots, n$. This then implies that Equation (A6) reduces to the following single variable optimization problem

$$\max_{L} \sum_{i=1}^{N} \log(\beta_i p \mathbb{E}[\min(L, D)] - f_i(L) - v_i). \tag{A8}$$

It is straightforward to verify by Equation (A8) that, for each parnter i, her payoff function $\beta_i p \mathbb{E}[\min(L, D)] - f_i(L) - v_i$ is concave in L. Since $\log(\cdot)$ is concave and non-decreasing, her logarithm payoff function is concave in L. Thus, the objective function in Equation (A8) is concave in L (since summation preserves concavity), which can be uniquely solved using the first-order condition as follows,

$$\sum_{i=1}^{N} \frac{\beta_i p \mathbb{P}(D \ge L^{NB}) - f'_i(L^{NB})}{\beta_i p \mathbb{E}[\min(L^{NB}, D)] - f_i(L^{NB}) - v_i} = 0. \tag{A9}$$

On the other hand, by Equation (A3), we also have

$$\beta^*_i p \mathbb{P}(D \ge L^*) - f'_i(L^*) = 0, \text{ for all } i = 1, \ldots, n. \tag{A10}$$

Comparing the conditions (A9) and (A10), it shows that L^* also solves Equation (A9).

PROOF OF PROPOSITION 8. The system optimal solution can be solved by using the KKT conditions.

$$\pi_T = \max_{L,\mathbf{K}} p\mathbb{E}[\min(D,L)] - \sum_{i=1}^{n} f_i(K_i)$$

$$\text{s.t.} \quad L \leq K_i + \sum_{j\neq i} \gamma_{ij} K_j \quad \text{for all } i, \qquad (\mu_i)$$

$$K_i \geq 0 \quad \text{for all } i. \qquad (v_i)$$

are given by the following,

$$-pP(L \leq D) + \sum_i \mu_i = 0, \qquad \text{(A11)}$$

$$f_i'(K_i) - \mu_i - \sum_{j\neq i} \gamma_{ji}\mu_j - v_i = 0 \quad \text{for all } i. \qquad \text{(A12)}$$

While the complementary slackness requires that

$$\mu_i(L - K_i + \sum_{j\neq i} \gamma_{ij} K_j) = 0 \quad \text{for all } i,$$

$$v_i K_i = 0 \quad \text{for all } i.$$

The last condition requires for all active participants with $K_i > 0$, then $v_i = 0$. Thus, Equation (A12) becomes $f_i'(K_i) = \sum_j \gamma_{ji}\mu_j$, where $\gamma_{ii} = 1$ for all i. Stack the equations into a matrix form, we obtain $\nabla \mathbf{f} = \mathbf{\Gamma}^T \boldsymbol{\mu}$, or $\boldsymbol{\mu} = \mathbf{\Gamma}^{-T}\nabla \mathbf{f}$, where $\boldsymbol{\mu}$ is a column vector of μ_i. Substituting this condition into Equation (A11), we obtain the desired result, that is, $pP(L \leq D) = \mathbf{e}^T\mathbf{\Gamma}^{-T}\nabla\mathbf{f}$. □

Proof of Corollary 1. Assuming that the inverse of the spillover matrix

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & \gamma & \dots & \gamma \\ & & \ddots & \\ \gamma & \dots & \gamma & 1 \end{bmatrix}$$

exists, the key step of the proof is to show that the inverse can be expressed as $\mathbf{\Gamma}^{-1} = \frac{1}{1-\gamma}\mathbf{I} - \frac{\gamma}{(1-\gamma)(1-\gamma+\gamma n)}\mathbf{H}$ via the series expansion, where $\mathbf{I}$ is an identify matrix and $\mathbf{H}$ is a matrix of all 1s. To do so, one needs to expand $\mathbf{\Gamma} = (1-\gamma)\mathbf{I} + \gamma\mathbf{H}$. We refer readers to Sun (2006) for more details on the inverse expansion.

Since the players are fully symmetric, then $K_i^* = K^*$ and $f_i'(K_i^*) = f'(K^*)$ for all i. It is straightforward to show that $\mathbf{e}^T\mathbf{\Gamma}^{-1}\nabla\mathbf{f} = f'(K^*)\mathbf{e}^T\mathbf{\Gamma}^{-1}\mathbf{e} = \frac{nf'(K^*)}{1-\gamma+\gamma^* n}$, once we substitute the expression derived for $\mathbf{\Gamma}^{-1}$ previously. □

Proof of Proposition 3. To show (a), we see that if the *marginal* cost function of j dominates i across the entire feasible region, that is, $f_j'(y) > f_i'(x)$ for all $x,y \geq 0$, then firm j's capacity can be invested by firm i at a lower cost, without affecting the revenue. (b) follows from the first-order condition of Equation (6), which is $\mathbb{P}(D \leq \sum_k K_k^*) = 1 - f_i'(K_i^*)/p = 1 - f_j'(K_j^*)/p$, for all i and j's.

Proof of Proposition 4. The equilibrium condition follows from the first-order condition of (7), which is

$$\mathbb{P}(D \leq \sum_i K_i^N) = \frac{\beta_i p - f_i'(K_i^N)}{\beta_i p}, \text{ for all } i = 1, \dots, n.$$

Next, we argue that $\mathbf{K}^N$ to the above system of equations is unique. First, we rewrite the above system as

$$\bar{F}_D\left(\sum_{i=1}^n K_i^N\right) = \mathbb{P}(D > \sum_i K_i^N) = \frac{f_1'(K_1^N)}{\beta_1 p} = \dots = \frac{f_n'(K_n^N)}{\beta_n p}, \qquad \text{(A13)}$$

whhere $\bar{F}_D(\cdot)$ is the complementary cumulative distribution function (CCDF) of D. Now suppose there exists an alternative solution $\tilde{\mathbf{K}}^N$ that is different than $\mathbf{K}^N$. Without loss of generality, let us assume $\tilde{K}_1^N > K_1^N$. Because $f_1'(\cdot), \dots, f_n'(\cdot)$ are all increasing and both $\mathbf{K}^N$ and $\tilde{\mathbf{K}}^N$ are feasible solutions to Equation (A13), we have

$$\frac{f_n'(K_n^N)}{\beta_n p} = \dots = \frac{f_1'(K_1^N)}{\beta_1 p} < \frac{f_1'(\tilde{K}_1^N)}{\beta_1 p} = \dots = \frac{f_n'(\tilde{K}_n^N)}{\beta_n p},$$

which implies that $\tilde{K}_2^N > K_2^N, \dots, \tilde{K}_n^N > K_n^N$. Thus, we must have $\sum_{i=1}^n \tilde{K}_i^N > \sum_{i=1}^n K_i^N$. However, again by the fact that both $\mathbf{K}^N$ and $\tilde{\mathbf{K}}^N$ are feasible solutions to Equation (A13), we have

$$\bar{F}_D\left(\sum_{i=1}^n K_i^N\right) = \frac{f_1'(K_1^N)}{\beta_1 p} < \frac{f_1'(\tilde{K}_1^N)}{\beta_1 p} = \bar{F}_D\left(\sum_{i=1}^n \tilde{K}_i^N\right).$$

Since $\bar{F}_D(\cdot)$ is decreasing, we must have $\sum_{i=1}^n K_i^N > \sum_{i=1}^n \tilde{K}_i^N$, which leads to a contradiction. A symmetric argument applied to the case $\tilde{K}_1^N < K_1^N$. Hence, we must have $\tilde{\mathbf{K}}^N = \mathbf{K}^N$, which completes the proof. □

Proof of Proposition 5. If $K_i^N \leq K_i^*$ for all $i = 1, \dots, n$, it is clear that $\sum_{i=1}^n K_i^N \leq \sum_{i=1}^n K_i^*$. Suppose now that, without loss of generality, there exists one player j such that $K_j^N \geq K_j^*$, we still want to show that $\sum_{i=1}^n K_i^N \leq \sum_{i=1}^n K_i^*$. By Proposition 4 and $f_j'(K_j^N) \geq f_j'(K_j^*)$, we have

$$F_D\left(\sum_{i=1}^{n} K_i^N\right) = \frac{\beta p - f_j'(K_j^N)}{\beta p} \le \frac{\beta p - f_j'(K_j^*)}{\beta p} \le \frac{p - f_j'(K_j^*)}{p} = F_D\left(\sum_{i=1}^{n} K_i^*\right),$$

where $F_D(x) = \mathbb{P}(D \le x)$. Take F_D^{-1} on both sides (i.e., F_D^{-1} is monotonically increasing and the sign does not change), we have $\sum_{i=1}^{n} K_i^N \le \sum_{i=1}^{n} K_i^*$. □

Proof of Proposition 6. We will prove this proposition with an intercept argument. With cost functions given in Equation (8), using the optimality conditions, we have

$$\mathbb{P}(D \le K_1^N + K_2^N) = \frac{\beta_1 p - a_1(K_1^N - b_1)}{\beta_1 p} = \frac{\beta_2 p - a_2(K_2^N - b_2)}{\beta_2 p}.$$

By changing of variables, where $\bar{K}_1 = K_1 + b_1$ and $\bar{K}_2 = K_2 + b_2$,

$$\mathbb{P}(D + b_1 + b_2 \le \bar{K}_1^N + \bar{K}_2^N) = 1 - \frac{a_1 \bar{K}_1^N}{\beta_1 p} = 1 - \frac{a_2 \bar{K}_2^N}{\beta_2 p}.$$

Then $\beta_2 a_1 \bar{K}_1^N = \beta_1 a_2 \bar{K}_2^N$ and we have

$$\bar{L}^N = \frac{\beta_1 a_2 + \beta_2 a_1}{\beta_1 a_2} \bar{K}_1^N, \quad \text{or} \quad \bar{L}^N = \frac{\beta_1 a_2 + \beta_2 a_1}{\beta_2 a_1} \bar{K}_2^N$$

Thus, we have

$$\mathbb{P}(D + b_1 + b_2 \le \bar{L}^N) = 1 - \frac{1}{p}\left(\frac{a_1 a_2}{\beta_1 a_2 + \beta_2 a_1}\right)\bar{L}^N. \quad \text{(A14)}$$

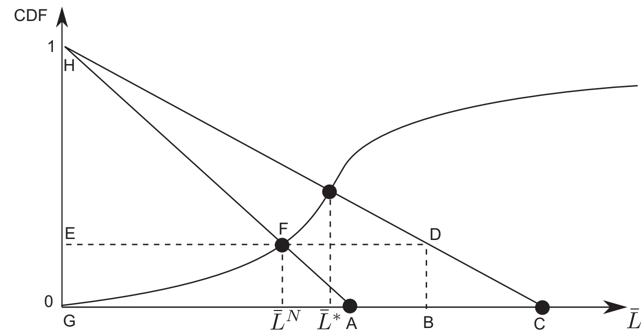
Using a similar transformation on the first-order condition of the system optimum, we have

$$\mathbb{P}(D + b_1 + b_2 \le \bar{L}^*) = 1 - \frac{1}{p}\left(\frac{a_1 a_2}{a_1 + a_2}\right)\bar{L}^*. \quad \text{(A15)}$$

As shown in Figure A1, the horizontal axis is the modified total capacity investment level and the vertical axis is the cumulative distribution function of the demand. The upward sloping curve (cumulative distribution function) represents the left hand sides of Equations (A14) and (A15), and the two downward sloping lines represent the right hand sides of Equations (A14) and (A15). Thus, $\bar{L}^N$ and $\bar{L}^*$ can be identified graphically. We also observe that

$$\frac{\bar{L}^*}{\bar{L}^N} \le \frac{B}{\bar{L}^N} = \frac{C}{A} = \frac{a_1 + a_2}{\beta_1 a_2 + \beta_2 a_1}.$$

Figure A1 A Graphical Proof of Proposition 6



Notes. The horizontal axis is the modified total capacity investment level. The curve represents the cumulative distribution function of the demand. The two downward sloping lines represent the right hand sides of Equations (A14) and (A15). The intersection points of the curve with the two lines determine the modified total capacity investment levels in the Nash and system optimal settings.

where the first equality holds because

$$\frac{\text{HE}}{\text{HG}} = \frac{\text{EF}}{\text{GA}} \quad \text{and} \quad \frac{\text{HE}}{\text{HG}} = \frac{\text{ED}}{\text{GC}} \quad \text{and} \quad \text{EF} = \bar{L}^N, \quad \text{GA} = A, \quad \text{ED} = B, \quad \text{GC} = C$$

by identifying two sets of similar triangles, namely, (HEF and HGA) and (HED and HGC), and the second equality holds because the points C and A are the x-intercepts which can be readily evaluated from Equations (A14) and (A15). □

Proof of Lemma 1. Suppose L^* is the optimal solution to the system problem. It is easy to see that for all $\hat{L} \ge L^*$, $g(\hat{L}) = \pi_T^*$. For all $L < L^*$, the budget constraint becomes tight. It suffices to show that

$$h(\hat{L}) = \min_{K_i} \sum_{i=1}^{n} f_i(K_i), \text{ s.t. } \sum_{i=1}^{n} K_i = \hat{L}.$$

is convex in $\hat{L}$. For any $\lambda \in [0, 1]$, we can write

$$h(\lambda \hat{L}_1 + (1-\lambda)\hat{L}_2) = \min_{K_i, K_i'} \sum_{i=1}^{n} f_i(\lambda K_i + (1-\lambda)K_i') \text{ s.t. } \sum_{i=1}^{n} K_i = \hat{L}_1, \ \sum_{i=1}^{n} K_i' = \hat{L}_2,$$

$$\lambda h(\hat{L}_1) + (1-\lambda)h(\hat{L}_2) = \min_{K_i, K_i'} \lambda \sum_{i=1}^{n} f_i(K_i) + (1-\lambda) \sum_{i=1}^{n} f_i(K_i') \text{ s.t. } \sum_{i=1}^{n} K_i = \hat{L}_1, \ \sum_{i=1}^{n} K_i' = \hat{L}_2.$$

By convexity of function f_i for $i = 1, \ldots, n$, for any K_i, we know that

$$f_i(\lambda K_i + (1-\lambda)K_i') \geq \lambda f_i(K_i) + (1-\lambda) f_i(K_i').$$

Taking the minimum with respect to the same constraints preserves the inequality, we have

$$h(\lambda \hat{L}_1 + (1-\lambda)\hat{L}_2) \geq \lambda h(\hat{L}_1) + (1-\lambda) h(\hat{L}_2).$$

This completes the proof. □

Proof of Theorem 2. By Proposition 6, we know that

$$\bar{L}^N = \frac{\beta_1 a_2 + \beta_2 a_1}{\beta_1 a_2} \bar{K}_1^N, \quad \text{or} \quad \bar{L}^N = \frac{\beta_1 a_2 + \beta_2 a_1}{\beta_2 a_1} \bar{K}_2^N.$$

The Nash profit functions can be expressed as functions of $\bar{L}^N$, that is,

$$\pi_T^N(\beta) = p\mathbb{E}[\min(\bar{L}^N - b_1 - b_2, D)] - \left(\frac{a_1 a_2^2 \beta_1^2 + a_2 a_1^2 \beta_2^2}{2(a_2\beta_1 + a_1\beta_2)^2}\right) \bar{L}^{N2} - c_1 - c_2. \quad \text{(A16)}$$

If we impose a budget constraint $L \leq \bar{L}^N$ on the system optimal, the budget-constrained system optimal profit can also be expressed as functions of $\bar{L}^N$that is,

$$g(\bar{L}^N) = p\mathbb{E}[\min(\bar{L}^N - b_1 - b_2, D)] - \left(\frac{a_1 a_2}{2(a_2 + a_1)}\right) \bar{L}^{N2} - c_1 - c_2,$$

Observe that $g(\bar{L}^N) = \pi_T^N(\beta)$ when $\beta_1 = \frac{1}{2}$. However, by Proposition 6, we know that for all $\beta_1 \leq \frac{1}{2}$, $\frac{\bar{L}^*}{\bar{L}^N} \leq 2$. In addition, $g(\bar{L})$ is concave in $\bar{L}$ by Lemma 1. Thus, we have

$$\frac{\pi_T^N(\beta_1 = \frac{1}{2})}{\pi_T^*} = \frac{\pi_T^N(\beta_1 = \frac{1}{2})}{g(\bar{L}^*)} \geq \frac{\pi_T^N(\beta_1 = \frac{1}{2})}{2g(\bar{L}^*/2)} \geq \frac{\pi_T^N(\beta_1 = \frac{1}{2})}{2g(\bar{L}^N)} = \frac{1}{2}.$$

Now let $\bar{D} = D + b_1 + b_2$. Since

$$\mathbb{P}(\bar{D} \leq \bar{L}^N) = 1 - \frac{1}{p}\left(\frac{a_1 a_2}{\beta_1 a_2 + (1-\beta_1)a_1}\right)\bar{L}^N \quad \Rightarrow$$
$$\frac{\mathbf{d}\bar{L}^N}{\mathbf{d}\beta_1} = \frac{-\frac{\bar{L}^N}{p}\left(\frac{a_1 a_2(a_1 - a_2)}{(\beta_1 a_2 + (1-\beta_1)a_1)^2}\right)}{f_{\bar{D}}(\bar{L}^N) + \frac{1}{p}\left(\frac{a_1 a_2}{\beta_1 a_2 + (1-\beta_1)a_1}\right)},$$

By Equation (A16), we have

$$\begin{aligned}\frac{\mathbf{d}\pi_T^N(\beta_1)}{\mathbf{d}\beta_1} &= \frac{-(1-\mathbb{P}(\bar{D} \leq \bar{L}^N))\bar{L}^N\left(\frac{a_1 a_2(a_1-a_2)}{(\beta_1 a_2 + (1-\beta_1)a_1)^2}\right)}{f_{\bar{D}}(\bar{L}^N) + \frac{1}{p}\left(\frac{a_1 a_2}{\beta_1 a_2 + (1-\beta_1)a_1}\right)} \\ &\quad + \frac{a_1^2 a_2^2 (1-2\beta_1)}{(a_2\beta_1 + a_1(1-\beta_1))^3}\bar{L}^{N2} \\ &\quad - \left(\frac{a_1 a_2^2 \beta_1^2 + a_2 a_1^2 (1-\beta_1)^2}{(\beta_1 a_2 + (1-\beta_1)a_1)^2}\right)\bar{L}^N \frac{\mathbf{d}\bar{L}^N}{\mathbf{d}\beta_1} \\ &= \frac{\bar{L}^{N2}}{(\beta_1 a_2 + (1-\beta_1)a_1)^3} \\ &\quad \left(-\frac{a_1^2 a_2^2 (a_1 - a_2)(a_1 + a_2)\beta_1(1-\beta_1)}{p(\beta_1 a_2 + (1-\beta_1)a_1) f_{\bar{D}}(\bar{L}^N) + a_1 a_2} + a_1^2 a_2^2 (1-2\beta_1)\right).\end{aligned}$$

If the highest density of D is m (i.e., $F_D'(L) \leq m, \forall L \geq 0$), then $\pi_T^N(\beta_1)$ is decreasing in β_1 for all

$$\beta \in \left[\frac{mp + a_2}{2mp + a_1 + a_2}, \frac{1}{2}\right],$$

and $\pi_T^N(\beta_1)$ is increasing in β_1 for all

$$\beta \in \left[0, \frac{a_2}{a_1 + a_2}\right].$$

Thus, the revenue-sharing scheme $\tilde{\beta}_1$ which induces the highest profit in a JV lies in the following interval

$$\tilde{\beta}_1 \in \left[\frac{a_2}{a_1 + a_2}, \frac{mp + a_2}{2mp + a_1 + a_2}\right].$$

This completes the proof. □

Proof of Lemma 2. By convexity of f_i for all $i = 1, \ldots, n$, we know that

$$f_i(K_i^*) \geq f_i(K_i^N) + f_i'(K_i^N)(K_i^* - K_i^N). \quad \text{(A17)}$$

Therefore we can write

$$\begin{aligned}\Omega(f) &= \frac{p\mathbb{E}[\min(L^N, D)] - \sum_{i=1}^n f_i(K_i^N)}{p\mathbb{E}[\min(L^*, D)] - \sum_{i=1}^n f_i(K_i^*)} \\ &\geq \frac{p\mathbb{E}[\min(L^N, D)] - \sum_{i=1}^n f_i(K_i^N)}{p\mathbb{E}[\min(L^*, D)] - \sum_{i=1}^n (f_i(K_i^N) + f_i'(K_i^N)(K_i^* - K_i^N))}.\end{aligned} \quad \text{(A18)}$$

Notice that Equation (A17) is true for every K, by putting $K_i^* = 0$, we have

$$0 = f_i(0) \geq f_i(K_i^N) + f_i'(K_i^N)(-K_i^N) \quad \Rightarrow \\ f_i(K_i^N) - f_i'(K_i^N)(K_i^N) \leq 0, \tag{A19}$$

we add Equation (A19) onto both the numerator and denominator of Equation (A18),

$$\Omega(f) \geq \frac{p\mathbb{E}[\min(L^N, D)] + \sum_{i=1}^n (-f_i(K_i^N) + f_i(K_i^N) - f_i'(K_i^N)(K_i^N))}{p\mathbb{E}[\min(L^*, D)] + \sum_{i=1}^n (-f_i(K_i^N) - f_i'(K_i^N)(K_i^* - K_i^N) + f_i(K_i^N) - f_i'(K_i^N)(K_i^N))}$$
$$\geq \frac{p\mathbb{E}[\min(L^N, D)] - \sum_{i=1}^n f_i'(K_i^N)(K_i^N)}{p\mathbb{E}[\min(L^*, D)] - \sum_{i=1}^n f_i'(K_i^N)(K_i^*)}.$$

Now let $\tilde{K}_i^N$ and $\tilde{K}_i^*$ be the Nash equilibrium solution and the system optimal solution with respect to the same problem but with the modified linear cost functions such that $\bar{f}_i = \alpha_i \times K_i$ where $\alpha_i = f_i'(K_i^N)$. Correspondingly, $\tilde{L}^N = \sum_{i=1}^n \tilde{K}_i^N$ and $\tilde{L}^* = \sum_{i=1}^n \tilde{K}_i^*$.

Since $\tilde{K}_i^N = K_i^N$ (having the same set of first-order conditions), we have

$$p\mathbb{E}[\min(L^N, D)] - \sum_{i=1}^n f_i'(K_i^N)(K_i^N)$$
$$= p\mathbb{E}[\min(\tilde{L}^N, D)] - \sum_{i=1}^n \alpha_i \tilde{K}_i^N.$$

Because $\tilde{K}_i^*$ is the optimal capacity investment level for the modified problem, it implies that

$$p\mathbb{E}[\min(L^*, D)] - \sum_{i=1}^n f_i'(K_i^N)(K_i^*) \leq p\mathbb{E}[\min(\tilde{L}^*, D)]$$
$$- \sum_{i=1}^n \alpha_i \tilde{K}_i^*.$$

Thus, we have

$$\Omega(f) \geq \frac{p\mathbb{E}[\min(\tilde{L}^N, D)] - \sum_{i=1}^n \alpha_i \tilde{K}_i^N}{p\mathbb{E}[\min(\tilde{L}^*, D)] - \sum_{i=1}^n \alpha_i \tilde{K}_i^*} \geq \frac{\pi^N(\bar{f})}{\pi^*(\bar{f})} = \Omega(\bar{f}).$$

This completes the proof. □

Proof of Lemma 3. Assume that, without loss of generality, $\alpha_m = \alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_n = \alpha_M$. Define the set $P = \{j | \alpha_j = \alpha_m\}$. If $|P| = s$, then s symmetric players invest in the system optimal solution and therefore $\tilde{L}^* = s\tilde{K}_j^*$ for $i \in P$.

$$\Omega(\bar{f}) = \frac{p\int_0^{\tilde{L}^N} \bar{F}_D(x)dx - \sum_{i=1}^N \alpha_i \tilde{K}_i^N}{p\int_0^{\tilde{L}^N} \bar{F}_D(x)dx + p\int_{\tilde{L}^N}^{\tilde{L}^*} \bar{F}_D(x)dx - \alpha_m \tilde{L}^N}$$
$$\geq \frac{\sum_{i=1}^N \alpha_i \tilde{L}^N - \sum_{i=1}^N \alpha_i \tilde{K}_i^N}{\sum_{i=1}^N \alpha_i \tilde{L}^N + \sum_{i=1}^N \alpha_i (\tilde{L}^* - \tilde{L}^N) - \alpha_m \tilde{L}^*}$$
$$\geq \frac{\sum_{i=1}^N \alpha_i (\tilde{L}^N - \tilde{K}_i^N)}{\sum_{i=1}^N \alpha_i \tilde{L}^* - \alpha_m \tilde{L}^*} \geq \frac{\alpha_m (n-1)\tilde{L}^N}{\alpha_M (n-1)\tilde{L}^*} \geq \tilde{\alpha} \frac{\tilde{L}^N}{\tilde{L}^*}$$

where the cost asymmetry factor $\tilde{\alpha} = \alpha_m / \alpha_M \leq 1$. This completes the proof. □

Proof of Theorem 3. Denote $f = (f_i(K_i))_{i=1}^n$ as general convex cost functions. Let $\pi^N(f)$ and $\pi^*(f)$ be the Nash and system profit of n players with respect to the general cost f, respectively. With Lemmas 2 and 3, we will present how to bound the efficiency ratio. Define

$$\theta_M \triangleq \max_{0 \leq x \leq \tilde{L}^N} F_D'(x) \quad \text{and} \quad \theta_m \triangleq \min_{\tilde{L}^N \leq y \leq \tilde{L}^*} F_D'(x)$$

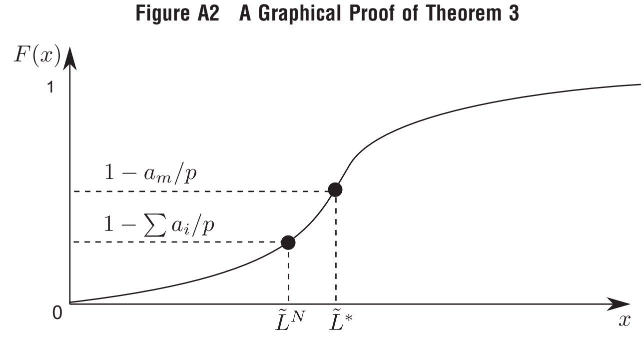
We first obtain a lower bound on the ratio of $\tilde{L}^N$ to $\tilde{L}^*$.

$$\frac{\tilde{L}^N}{\tilde{L}^*} \geq \frac{\tilde{L}^N}{\tilde{L}^N + (\sum_{i=1}^n \alpha_i - \alpha_m)/(\theta_m p)}$$
$$\geq \frac{(1 - \sum_{i=1}^n \alpha_i / p)/\theta_M}{(1 - \sum_{i=1}^n \alpha_i / p)/\theta_M + (\sum_{i=1}^n \alpha_i - \alpha_m)/(\theta_m p)}$$
$$= \frac{p - \sum_{i=1}^n \alpha_i}{p - \sum_{i=1}^n \alpha_i + (\sum_{i=1}^n \alpha_i - \alpha_m)\tilde{\theta}}$$
$$\geq \frac{p - n\alpha_M}{p - n\alpha_M + (n-1)\alpha_M \tilde{\theta}} = \frac{1 - n\bar{r}}{1 - n\bar{r} + (n-1)\bar{r}\tilde{\theta}},$$

where $\bar{r} = \alpha_M / p$.

Combining this result with Lemma 3 which shows that the efficiency ratio in terms of the total profit can be bounded from below by the product of the cost asymmetry factor and a comparison of the effective capacity, we have completed the proof. □

Figure A2 A Graphical Proof of Theorem 3



Notes

[1]Two movies which appear as "outliers" in the histogram are *Avatar* in 2009 and *Marvel's The Avengers* in 2012, which generated $760 m and $623 m in earnings respectively.

[2]Besides being prohibitively expensive, IMAX's traditional large-format film stock and projectors are also cumbersome. It requires screen towering over 70 feet high, leaving very few locations suitable for IMAX installation. The Digital IMAX projector which was developed in 2008, which allows the company to retrofit existing spaces in multiplexes. Digital IMAX projection screens are only about 10 feet wider than traditional multiplex screens (Vanderhoef 2013).

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