The Photon Existence Paradox

Armin Nikkhah Shirazi

University of Michigan, Ann Arbor
armin@umich.edu

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Einstein's "Lightbeam Observer Paradox"...

"...a paradox upon which I had already hit at the age of sixteen: If I pursue a beam of light with the velocity c (velocity of light in a vacuum), I should observe such a beam of light as an electromagnetic field at rest though spatially oscillating. There seems to be no such thing, however, neither on the basis of experience nor according to Maxwell's equations. From the very beginning it appeared to me intuitively clear that, judged from the standpoint of such an observer, everything would have to happen according to the same laws as for an observer who, relative to the earth, was at rest. For how should the first observer know or be able to determine, that he is in a state of fast uniform motion? One sees in this paradox the germ of the special relativity theory is already contained[1] *."(p. 49-51)*

[1] Einstein, A.(1979) *Autobiographical Notes: Centennial Edition* Schilpp, A. (ed.): Lasalle and Chicago: Open Court Publishing

...gives rise to a new paradox

Special relativity addresses Einstein's "lightbeam observer paradox" by:

- postulating that c is independent of the motion of its source
- generalizing Galileo's relativity principle, applicable to the laws of mechanics, to all laws of physics.

This transforms Einstein's problem into a new paradox: *The Photon existence paradox*.

Special relativity enjoins lightspeed objects from possessing certain fundamental properties one might naively think are necessary for physically existing objects. Had we not already known of the existence of such objects, we would likely have taken this enjoinment to imply that the existence of lightspeed objects is impossible; and yet, empirically, lightspeed objects such as photons do seem to exist.

Existence of Lightspeed objects as a paradox

Imagine, as vividly as possible, how we would have interpreted special relativity if we were ignorant of the empirical evidence for the existence of lightspeed objects.

- Subluminal objects cannot be accelerated to c
- No observed objects traveling at c
- Theoretical difficulties with lightspeed objects:
  - It is impossible to assign to lightspeed objects a coordinate system origin in space.
  - It is impossible to assign to lightspeed objects a finite observed duration of existence in spacetime.

All of these would lead us to conclude that it is impossible for lightspeed objects to exist. If we subsequently discovered photons, then their observed existence would seem like a paradox!

Three Types of Paradox in Physics

1. **“Error":** theory contains no flaw, paradox due to mistake in assumptions or reasoning about it e.g. misinterpreting/misapplying the theory, or making mistakes in logical or mathematical deductions from it
2. **“Gap":** theory contains a flaw, but the flaw is not fatal to it
3. **“Contradiction":** theory contains a fatal flaw

Examples:

- Special Relativity replaced Newtonian theory because Einstein noticed a “contradiction”
- Twin paradox and similar problems are “errors”.

Which type is the photon existence paradox?

Four Spacetime Principles

Rotations, Translations, Boosts, Conformal Transformations, Dilations leave unaffected

- number of dimensions of transformed object
  $\Longrightarrow$ Invariance of Absolute Dimensionality
- whether a transformed object exists in spacetime or not
  $\Longrightarrow$ Invariance of Spacetime Ontic Value

These operations transform one $3+1$ "slicing" of spacetime only to another $3+1$ slicing:

- physical space has everywhere the same number of dimensions
  $\Longrightarrow$ Homodimensionality of Space
- time has everywhere the same number of dimensions
  $\Longrightarrow$ Homodimensionality of Time

Both are homodimensional $\Longleftrightarrow$ Isodimensionality of Spacetime

Grounding Spacetime Ontic Value in Physics

Postulate: *An object exists in spacetime iff it is characterized by a timelike proper time.*

Cannot derive this existence criterion, but can give plausibility arguments:

- The proper time of timelike objects can unambiguously be interpreted as their duration of existence in spacetime between given spacetime events
- If spacetime ontic value is based on this criterion, it is in fact a physical invariant

Spacetime ontic value:

- 1 if the object satisfies the criterion
- 0 if it fails to satisfy the criterion

The Ontic Equivalence Relation: Setting up the Proof

These 4 principles+special relativity, generalized to $n+1$ dimensions imply
The ontic equivalence relation: Existence in spacetime is an equivalence relation by absolute dimensionality.

- To prove equivalence relation, need to prove **reflexivity**, **symmetry**, and **transitivity**
- Consider an $m-$dimensional object A and suppose that it exists in an $n+1$ dimensional region of spacetime in accordance with the four spacetime principles and special relativity.
- Spacetime ontic value of A in $n+1$ dimensional region is 1.
- What is m?
  - cannot have $m > n$
  - cannot have $m < n$
  - $\Rightarrow$ must have $m = n$
- By the isodimensionality of spacetime, the $n+1$ dimensional region of spacetime in which A exists is, in fact, all of $n+1$ dimensional spacetime.

The Ontic Equivalence Relation: the Proof

- **Reflexivity:** A exists in the spacetime in which it exists.
- **Symmetry:** consider an m-dimensional object B. By the same argument as just given for A, it must exist in an $m+1$ dimensional spacetime. Suppose that B exists in the same spacetime as A. That means $m+1 = n+1$, and because equality implies symmetry, symmetry follows at once.
- **Transitivity:** consider an $l-$dimensional object C. By the same argument as before, C must exist in an $l+1$-dimensional spacetime. Now, suppose that A exists in the same spacetime as B, and that B exists in the same spacetime as C. That means $n+1 = m+1 = l+1$, and because equality implies transitivity, transitivity follows at once.

$\therefore$ Existence in a spacetime is an equivalence relation by the number of length dimensions that characterize an object.

Implication of the Ontic Equivalence Relation

- Any equivalence relation on a set partitions that set into equivalence classes
- $\Rightarrow$ ontic equivalence relation partitions the set of all objects in the domain of physics into equivalence classes by the number of length dimensions, such that each equivalence class only exists in a spacetime with same number of length dimensions
- lightspeed objects do not exist in our $3+1$ spacetime, but in a $2+1$ analog, because one of their length dimensions is completely contracted.

To emphasize the equal footing of $2+1$ space with $3+1$ spacetime as a physical space, I will call it *areatime*.

Resolution of the Photon Existence Paradox

Lightspeed objects do not exist in spacetime but in areatime!

- cannot accelerate to c because if you could, you would no longer exist in spacetime
- cannot assign to lightspeed objects the origin of a coordinate system in space because the coordinate system in which this is possible must belong to a 2-dimensional space
- cannot assign to lightspeed objects a finite observed duration of existence in spacetime because they do not exist in spacetime

The Photon Existence Paradox as an “Error”

What was the “error”?

- Absolute Existence: the existence of a physical object can be specified without reference to any $n+1$-dimensional spacetime
  - An object exists $\Rightarrow$ the object exists in spacetime
- Relative Existence: the existence of a physical object must always be specified with reference to a particular $n+1$-dimensional spacetime.

Special Relativity gave us indications that absolute existence is false, but we did not recognize them.

Example:

- Integral of any volume of space in the lightlike direction is zero.

We nearly universally assumed absolute existence in spite of special relativity $\Rightarrow$ “Error”

The Photon Existence Paradox as a “Gap"

What was the “gap"?

- Standard special relativity lacks any reference to the concept of existence
- The theory could only provide indirect “hints” that lightspeed objects do not exist in spacetime

How to conceptualize the resolution as closure of a “gap” in special relativity:

- The existence criterion is necessary to prove the ontic equivalence relation
- But, the existence criterion also allows us to infer directly that lightspeed objects do not exist in spacetime $\Rightarrow$ “Gap” in special relativity

So, whether paradox is due to an “error” or a “gap” depends on what one wants to emphasize:

- Inability to recognize hints from special relativity **vs.** incompleteness of the theory

Photons as Areatime Objects

How do we know photons (or any lightspeed objects) exist at all?

- as the speed of an object approaches c, its direction of motion and its time direction approach lightlike direction.
- In the limit of c, both directions "merge" along that direction.
  $\Rightarrow$ spacetime, considered as a vector space, becomes linearly dependent
  $\Rightarrow 3+1$ spacetime has one dimension too many relative to space in which the two directions are merged
  $\Rightarrow$ reduction of spacetime by one dimension results either in a $3+0$ or a $2+1$ space.
  $\Rightarrow$ lightspeed objects have only two spacelike degrees of freedom, leaving $2+1$ dimensions.

Conclusion

> *"Today, everyone knows, of course, that all attempts to clarify this paradox satisfactorily were condemned to failure as long as the axiom of the absolute character of time...was rooted unrecognized in the unconscious. To recognize clearly this axiom and its arbitrary character already implies the essentials of the solution of the problem[2]." (p.51)*

Once the ontic equivalence relation is widely known and accepted, this quote could apply today by substituting "existence" for time:

- Early 20th century: there is no absolute time.
- Early 21st century: there is no absolute existence

The implications of the latter are only starting to be worked out!

[2] Einstein, A.(1979) *Autobiographical Notes: Centennial Edition* Schilpp, A. (ed.): Lasalle and Chicago: Open Court Publishing