Agent-Based Models for Analyzing Strategic Adaptations to Government Regulation

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Computer Science and Engineering)
in The University of Michigan
2020

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For my parents, my Lili, and my Gavin

ACKNOWLEDGEMENTS

This thesis stands on the shoulders of many family, friends, and colleagues, who have provided me with enough support to move mountains.

I would like to thank my advisor, Michael P. Wellman, for his steadfast guidance whether I was caught in a tempest of activity or in a deathly lull. I relied on him for his honesty, wisdom, and ethical character while navigating a myriad of personal and academic obstacles. I am a better researcher, family man, and citizen because of his example. I will carry his specter wherever I go, and I consider our relationship to be an irreplaceable harvest.

I would also like to thank my collaborators. Without their contributions and tutelage, I certainly would not have gone far past the starting line. Thanks to Kareem Amin, not only for his active role in training me during my embryonic stage, but also for inspiring me with his infectious amicability and rigorous professionalism. He has, perhaps, seen me in a wider range of situations than any person on Earth, and I consider it an honor to call him a friend. Thanks also to Yagil Engel for helping me to step outside my comfort zone with his grounded enthusiasm. I hope to one day be able to approach his ability to perform impactful research within complex corporate landscapes. And thanks to Junming Liu for showing me firsthand the tireless work ethic of a true computer scientist (and for driving me home on those painfully cold wintry nights at 3AM).

Thank you to my intern host at Google NYC, Alex Kulesza, for showing me how a professional research organization is run. Thanks also to Jennifer Gillenwater and

Sergei Vassilvitskii for your support and discussions. And thanks to my fellow intern Chansoo Lee for valuable insights and discussion.

Thanks to my labmates Erik Brinkman, Travis Martin, Bryce Wiedenbeck, Elaine Wah, Mason Wright, Xintong Wang, Megan Shearer, Arunesh Sinha, Thanh Nguyen, Zun Li, and Yongzhao Wang for contributing to a top-notch research environment.

Thanks to my friends in New York and Michigan for helping me weather personal events and listening to my research problems: Elizabeth Hou, Mikhail Yurochkin, Kareem Amin, Megan Shearer, Erik Brinkman, Karry Lu, Brad Ruofei Zhao, Jerry Chao, and Andrew Han.

And finally, thanks to my entire family, who prioritized my progress above anything else. Thanks to my parents, who always believed in me and have always been on my side. Thanks to Matthew, whose lightheartedness kept me grounded. Thanks to Lili's parents, whose hard work sheltered me from the New York lifestyle. And thanks to Lili, who sacrificed more than anyone could expect to give me a home and family of my own. I could always retreat from my battles to find respite and warmth at our dinner table. I carry with me a deep contentment knowing that there is life beyond my research, which has carried me to complete this thesis. Lastly, thanks to Gavin, who carries my wishes for a better future in his tiny hands.

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ABSTRACT

In many economic systems, participants are unable to internalize social costs. These can be anything from pollution to default risk in financial systems. To deal with these costs, regulators impose limits on the behavior of market participants. These regulations do not always have straightforward effects, and for new regulations a model is required to evaluate them. In this thesis I will perform this modeling task in several domains using a computational agent-based approach. This approach affords two advantages. First, agent-based models can handle intricate models of participant behavior. This is often necessary when participants are operating in a complex domain, using large modeling and computational resources of their own. Second, agent-based models combined with empirical game theoretic analysis (EGTA) can calculate Nash equilibria under new regulations. This addresses in part the Lucas critique of models with regulation, which stipulates that agents can adapt their behavior in ways that break fixed assumptions about agent behavior.

I evaluate the overall effects of regulation using metrics appropriate to each domain I study. Using two models of the financial system, one based on an asset market and one based on a debt market, I study Basel regulations which have been criticized for being too simplistic and for actually being counterproductive. I find that in fact, when accounting for the strategic adaptations of banks, Basel regulations are largely beneficial for financial stability. I then examine recent EPA regulations that allow the trading of emissions credits in an attempt to bring down the cost of reducing emissions. I find that while the cost of reducing pollution is reduced as desired, costs to consumers are increased by firms that use emissions trading to coordinate price

hikes. In all cases, the use of game-theoretic analysis was crucial to evaluating the effect of regulation.

CHAPTER I

Introduction

Markets are the primary way our society allocates goods and services. But when a market fails to achieve acceptable social outcomes, governments are called upon to modify the rules of engagement. In some cases, these regulations achieve their intentions and become standard. An example of this is American bank deposit insurance implemented by the FDIC. During the Great Depression, it became clear that bank runs were a result of strategic behavior by depositors. Each individual's withdrawal of deposits made it more likely that the bank would default, making it rational for all individuals to withdraw their funds at once in response to bad financial news. This was a self-fulfilling prophecy as banks with no deposits were almost guaranteed to go out of business. Regulators proposed deposit insurance, which stipulated that deposits would be guaranteed by the government. This changed the incentives of depositors; since they would get their deposits back either way, they no longer felt the collective need to withdraw all deposits, which kept he banks in business when there was bad news. Most importantly, the deposit insurance never actually had to be paid! This was intended, as the government could not afford to pay much deposit insurance during the Depression. Changing the strategic calculus of market participants using regulation is an effective, and sometimes necessary, way of ensuring markets operate smoothly.

However, the strategic response of economic agents is not always straightforward to predict. Modern markets consist of sophisticated agents interacting in a complex environment. When these markets fail, new models are required to explain what went wrong and suggest the best policy response. But governments are compelled to respond with lawmaking immediately, to the best of their ability. The problem with this urgency is that if market participants adapt to new rules in surprising ways, the regulatory response becomes at best obsolete and at worst actively harmful. A prominent example is that of the Phillips curve model, which predicted that high inflation would lead to low unemployment. Stimulus in many countries was rolled out in the 1970s with this model in mind. But instead of the economic growth predicted by the Phillips curve, the surprising development of stagflation, a stagnant economy with high inflation, came about as a result of firms decreasing hiring due to higher *expected* prices. Thus when strategic adaptations by market participants are not anticipated correctly, amendments to the original policy fix become necessary, which themselves may not be well understood by current economic models. Without the ability to predict strategic response, regulators are left constantly playing catch-up to past mistakes.

Even a partial understanding of how market participants respond strategically to regulation can help break this cycle. I advocate a computational agent-based approach to modeling markets and understanding the impact of regulation. This allows for models with an increased ability to handle complex agents and markets, compared with analytically-tractable models. To ensure that I am taking the strategic behavior of these agents into account, I use ***empirical game-theoretic analysis*** (EGTA), a framework for gaining deeper insights into the strategic adaptations of agents in agent-based models. EGTA is outlined in Section 1.2.

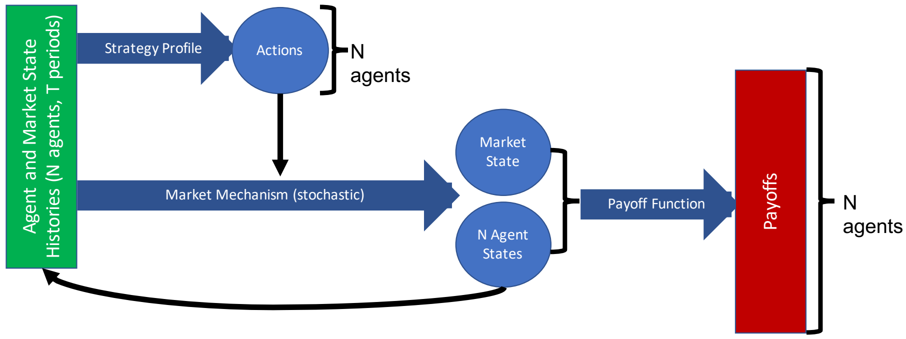


Figure 1.1: Illustration of the components of an agent-based model.

1.1 Agent-based models and methods

The central tool I use in my analysis of markets is the ***agent-based model***, which models economic systems by simulating the behavior of many individual agents. In these models, agents are given the task of optimizing their ***payoffs*** by choosing ***actions***. The market mechanism processes these actions, adds stochasticity, and publishes ***agent states*** and a ***market state***. These states, when input to a ***payoff function***, determine payoffs for each agent. An overall view of how these two processes interact is given in Figure 1.1. One can run an agent-based model by following the figure, starting with a ***state history*** and ending with new market and agent states. The new states are added to the ***state history***, and the process iterates. This act of iterating represents the passage of time, and thus we call each iteration a ***period***.

Dynamic models approximate persistent markets where agents update their decisions over time. In these markets, agents adapt their actions to states revealed to them by the market mechanism in past periods. The information encoded by these states include:

1. Realized randomness from the market clearing process.

2. The effect of agent actions on market state outcomes.

3. The effect of agent actions on agent state outcomes.

Depending on the modeled economic scenario, any or all of these historically realized variables can be input into an agent's ***strategy*** as data. A strategy is set once and for all by an agent before the first period of running the model, and specifies how agents translate state histories into actions that can be submitted to the market mechanism. The set of all possible strategies is called the ***strategy space***. This set is constructed by the modeler and can, in general, be different for each agent. A ***pure strategy profile*** consists of the strategies that are followed by each agent through a complete run of the model. It is an N-tuple, with an entry indexed by i consisting of a single member of agent i's strategy space.

The other process in an agent-based model is the market mechanism, whose main function is to translate the actions of agents into states, which in turn set payoffs. The market mechanism may contain parameters that can be used to emulate different types of economic environments. For example, a noise parameter might specify the size of a typical stochastic shock to output states, representing volatility level in a market. The market mechanism can use information from the state history, most commonly historical market states, to produce new states. This dependence stems from the fact that market mechanisms include regulations that attempt to achieve some goal on market states. Regulators may periodically measure historical market states to inform how stringent future regulations should be.

Figure 1.1 is abstract in that it does not depict how each individual agent's actions lead to an eventual payoff. In fact, the agent-based models in this thesis share the property that any agent's actions can affect any other agent's payoff through any of the market mechanism's output states. Since agents follow strategies that process historical states to decide on actions, this means a particular agent's choices directly affects the actions taken by other agents, which in turn affect the original agent and its payoffs. Therefore, it is of paramount importance that agents consider the strategic impact they may have on the pool of agents when committing to a strategy.

Accounting for this strategic behavior leads us to the original economic insights in this thesis.

The first step towards capturing important strategic behavior is the specification of a strategy space. Sometimes there are actual strategies that are known and widely employed in the market we are studying. Populating the strategy space with these is usually a good idea, as it anchors the resulting analysis and often gives a flavor of real strategic tensions that exist in the market. In general, we want to devise a strategy space that provides agents a diverse set of options corresponding to different ways that they may impact their short and long term payoffs. When we solve for which strategic profiles payoff-seeking agents prefer using (methodology explained in Section 1.2), this results in a clear economic story of how the balancing of strategic tensions leads to an overall economic outcome in the model.

1.2 Equilibrium and empirical game-theoretic analysis

The outcome of an agent-based model is highly sensitive to the combination of strategies chosen by agents. Recall that this is a decision agents must make at the beginning of the model. Their payoffs, and thus their choice of strategy, depends on the strategies chosen by all other agents. How should agents approach this complex decision?

The entire dynamic agent-based model can be understood as a function mapping pure strategy profiles into payoffs for each agent. This function defines a ***game*** that agents are playing. While models require a pure strategy profile as input, an agent is not limited to choosing a single strategy in this game. Instead, it may randomize over its strategy space in pursuit of better payoffs, only choosing a single strategy once the simulated model begins. This randomized strategy selection is a ***mixed strategy***, which is a distribution over an agent's strategy space, corresponding to the probabilities that the agent will pick each member of the space for use over a run of the model. A collection of mixed strategies, one for each agent, constitutes a ***mixed strategy profile***.

A mixed strategy profile is a ***Nash equilibrium*** if no agent can increase their expected payoff by deviating to a different mixed strategy. A Nash equilibrium is always guaranteed to exist, but may be hard to compute [19]. This computational problem is exacerbated by the fact that the agent-based models used in this thesis require a multi-step procedure to go from a particular strategy profile to a payoff. A further complication is that the market mechanism contains stochasticity, so payoffs may be different for different runs of the model for the same strategy profile.

EGTA is a suite of tools that automates the process of finding Nash equilibria in agent-based models [10]. Given black-box access to an agent-based model and a discrete set of strategies, it selects the necessary strategy profiles that must have their payoffs evaluated in order to find an equilibrium. It then schedules simulations to evaluate these payoffs. When payoffs are stochastic, multiple simulations for each profile will be scheduled and an average payoff will be evaluated. The total number of profiles is exponential in the number of strategies as well as the number of agents in the model, so EGTA offers tools to reduce the number of profiles sampled. One technique is called ***deviation-preserving reduction*** [70]. This allows the modeler to specify how many *reduced players* to use. The number of profiles to evaluate decreases to be commensurate with this number instead of actual number of players, at the cost of no longer being able to find exact equilibria. This is effective when the number of players in the game is large. Another technique is to first check for equilibria in subgames that are played with a subset of the full strategy set. This is called ***quiescing*** and can be effective for games with a large amount of strategies but which have equilibria in a small subset.

Once enough payoffs have been simulated, EGTA uses replicator dynamics to find an equilibrium to the game [59]. This equilibrium becomes our prediction for how agents choose strategies in a specified regulatory environment, and the question of how regulations might trigger strategic adaptations is addressed.

1.3 Summary of contributions

Recall that the regulator's problem in setting policies for markets is that it is difficult to predict the strategic response of market participants. I address this difficulty by first building dynamic agent-based models that provide strategic flexibility to agents. I then use EGTA to predict how agents given this flexibility might respond to regulation. This helps to circumvent the game of catch-up that happens when strategic behavior does not inform regulation.

A significant portion of this thesis analyzes financial markets. I was originally motivated by the fact that the 2008 financial crisis did not result in any philosophical changes in regulation. Rather, the same Basel regulatory guidelines were preserved, and made more stringent. This poses the interesting economic question of whether these more stringent regulatory requirements will motivate financial institutions into behavior that leads to better social outcomes. Novel insights into how agents in the financial system adapt strategically have real-world policy implications, and demonstrate how economic analysis of regulations may be able to catch up to real regulated markets.

The second market we study is the automobile market. This market consists of several interrelated products produced by a few manufacturers. This leads to an oligopoly market structure where pricing or production quantity decisions made for each product, by each firm, affects the decisions made by every other firm and product combination. It is analytically intractable to solve for an equilibrium given this market structure, so we must develop a computational approach even for an unregulated market. Adding regulatory constraints only makes this approach more salient. We develop a strategic model of this market that can be used to evaluate the effect of emissions regulations on consumers and firms.

The overall goal of building an agent-based model and applying EGTA to analyze the impact of regulation can be broken down into three stages: defining a modeling framework, building a model, and applying EGTA. My work in this thesis focuses on the appropriate stage(s) given context and existing work, but all three stages are represented. A summary and guide to this thesis is provided below.

1. An extended analysis of an existing agent-based model of financial markets in Chapter II. The existing model predicted that regulations introduced after the 2008 crisis would lead to financial instability. I identify a possible strategic response to regulation in this model. I use EGTA to calculate equilibria that account for this response, with and without regulation. In the new regulated equilibrium, more funds stay out of default and banks lose less capital, reversing the findings of the original study. Presented at the ACM Conference on Economics and Computation 2017 [13].

2. A framework for representing and analyzing network structure in financial markets, called ***financial credit networks*** (FCNs) in Chapter III. I define this framework and develop algorithms that make it possible to represent financial contracts and route payments [12]. FCNs make it possible to build models where the ramifications of local credit events are felt globally, which will be key to my work in Chapter IV. Presented at the ACM Conference on Economics and Computation 2016.

3. An FCN-based model of interbank debt formation, used to analyze the effect of debt market dynamics on financial stability in Chapter IV. This model reproduces several important systemic risks endogenously from profit-seeking bank behavior. These risks have previously only been studied in static networks. At equilibrium, Basel is able to reduce systemic risk significantly despite having

access to only aggregated balance sheet data (as opposed to metrics based on the network structure between banks). A preliminary version of this model was presented at the AI^3 workshop during AAMAS 2018 [14].

4. An investigation of how automobile manufacturers respond strategically to emissions regulation in Chapter V. The model I use incorporates the oligopolistic structure of the car market. This analysis revealed that emissions trading provides a clear benefit to consumers and producers of cars, but that producers benefit disproportionately by using the trading platform to coordinate price increases. Presented at the International Joint Conference on Artificial Intelligence 2019 [11].

1.4 Possible extensions

The most obvious extension to this work is a continuation of modeling the banking system using EFCNs. While my model produces (qualitatively) some important features of the 2008 crisis, the degree to which the evaluated model settings in Chapter IV match data from the crisis is unknown. There are also many real-life parties that are not modeled, such as firms in the real economy and homeowners. EFCNs allow the modeling of dynamic debt relations between these parties, which may illuminate further the ways in which banking sector collapse affects the real economy.

EFCNs themselves may also be extended to include other types of financial relationships. For instance, shared ownership of assets (i.e., equity) is a major way in which participants in modern economies invest in each other, that is abstracted away by necessity in my credit cycle model due to its absence from EFCNs. In addition, nodes in an EFCN have no ability to influence each other through non-debt investment activities, and cannot liquidate their debt through a market. These are all important features of the real banking system that could be added to a new modeling framework.

In my work on EPA regulations, I have assumed a fixed technological profile. In reality, both gas and electric engine technologies are advancing rapidly, and it would be informative to evaluate different scenarios that include these advances. This could be handled with minimal changes to the modeling framework, but would require carefully selected scenarios that are preferably based on data.

CHAPTER II

Banking regulation and the leverage cycle

2.1 Introduction

The financial crisis in 2008 marked a seismic shift in perceptions of how markets operate, and its reverberations are still being felt in debt markets around the world [61]. Our understanding of how financial crises arise time and again in the midst of sophisticated and logically motivated actors is still incomplete. After an initial rush in academic circles to develop new economic models to reflect forensic evidence from the crash, and to recommend regulation to prevent future crashes, we have been left with a plethora of different viewpoints.

One approach that has gained influence even in crisis-unrelated research [31] focuses on the fact that buyers and sellers do not have the same access to information about assets, especially in the context of debt markets. Based on this asymmetry, Bernanke et al. [6] showed that periodic, deep financial crashes are endemic due to an overreaction to interest rate changes. The recommendation is clear: force sellers to be more transparent about their wares. Almost everyone agrees that making more information available is a good policy, and regulators commonly work towards this goal.

There are many other convincing perspectives on the crisis. The irrational exuberance [62] narrative posits a deep behavioral reason for market malfunction. It is also popular to point to systemic factors, like lax lending standards and oversight, leading to market participants whose priorities move away from the proper valuation of assets [15]. Another body of work points to the seizure of credit markets at the most critical juncture of the crisis for amplifying the crisis. This failure has been attributed to either too much refinancing during normal times [55], or a run on liquidity [33] in response to financial panic. Agent-based simulations elaborating on traditional economic models have also been used. Bookstaber et al. [7] introduce one where agents have fixed fire-sale behavior, and the spread of the crisis can subsequently be measured through different pathways. All of these models can explain some aspects of the financial crisis, and suggest various emphases for macroeconomic policy and regulation.

However, given the complexity of the financial system it is difficult to draw conclusions directly from underlying causes for ideal policies. An alternative perspective is to start from the policies, and model the situations where they are beneficial or harmful. I take this perspective in this chapter, focusing on international banking regulations in the ***Basel framework***. Basel's salience follows from its central role in global financial policy, and its force in governing the lending policies of major financial institutions.

Basel regulations consist chiefly of a limit to ***leverage***, or the ratio of gross investment to wealth (capital for banks). By taking on debt, banks can make this ratio arbitrarily large if unregulated by investing on credit. An institution with high leverage cannot pay its obligations if its investments underperform even slightly, so it seems natural that limiting leverage may help control default risk. The fact that leverage levels turned out to predict the financial crisis better than interest rates evidences the centrality of this variable to financial stability.[1] Regulator belief in limiting leverage is so strong that their most prominent operational regulation after

[1]A study by Geanakoplos et al. [26] established a link between financial crashes and leverage levels that successfully predicted data from 2.2 million American households.

the crisis was to simply implement a stricter version of Basel. But as such policies had failed to immunize the financial system, there is some uncertainty about how much the stricter version will help in the future.

The ***leverage cycle*** model of Geanakoplos [24] describes the ebb and flow of leverage, shedding light on how aggregate leverage may increase to dangerous levels. Importantly, the model *endogenizes* leverage, showing how it evolves through agent decision making. Several key predictions made by this model were borne out by asset behavior during the crisis. First, price fluctuations were disproportionate to fundamental value changes. Second, the failure of a few highly leveraged investors had an outsize impact on prices. Third, the severe tightening of short term credit markets contributed to a deeper crash. All of these predictions were entailed by the model, with leverage choices at Walrasian equilibrium under perfect information.

While the model of Geanakoplos [24] has the advantage of being fully analytical, it extended only to three periods and thus could not accommodate changing financial regulation over time. In a discrete agent-based extension to the leverage cycle model, Thurner et al. [64] and Poledna et al. [52] introduced a Basel-style regulator that imposes a leverage limit on financial firms. As in the latest round of Basel financial regulations, the leverage limit in the model is more stringent in times of high asset price volatility. Using this model, Poledna et al. [52] argue that Basel regulations could, counter to their purpose, contribute to financial instability. In their model under certain settings, market participants (or ***funds***) defaulted more frequently and produced more volatile asset prices when a Basel-based leverage limit was implemented. Market participants also made less profit. However, funds that defaulted were of a smaller size, so the cost per default decreased. A follow-up study by Aymanns and Farmer [3] suggests that an inverted Basel regulation would be more effective at preventing financial crises.

My contribution is to observe that a certain fixed assumption in these models, the ***aggression distribution***, might naturally be considered a strategic choice by agents. I extend the model of Poledna et al. [52] to a game, where the agents strategically choose their aggression level in response to the regulation regime. I employ simulation-based methods to analyze this game, identifying aggression distributions that are in approximate Nash equilibrium. Using this approach, I reverse in aggregate the finding that Basel causes more defaults. Losses that are due to default decrease further, and agent profits decrease less. The overall case against Basel is thus weakened by my findings.

In Section 2.2, I describe the model used by Poledna et al. [52], and explain the effect of Basel regulations on the leverage cycle in this model. I also describe implementation details and assumptions used in the original work that carry over into my independent implementation. In Section 2.3, I argue that agent aggression levels in this model should be treated as strategic variables. I propose an approach to analyze the strategic adaptations of agent aggression using Nash equilibrium, while continuing to clear markets in each period, using prices. Finally, I design an experiment that distinguishes the effect of Basel on the leverage cycle for a fixed set of agents (studied in previous work) from its effect in equilibrium, due to strategic adaptations of agents. I measure the effect of Basel on financial stability by looking at default rate, agent profits, capital losses, and price volatility. The strategic adaptation of agents on its own improves every measure except price volatility, which remains unchanged. In aggregate, I find that Basel decreases default rate and capital losses, while also decreasing agent profits and increasing price volatility.

2.2 An agent-based model of the leverage cycle

Following Poledna et al. [52], we adopt and extend the basic leverage cycle model of Geanakoplos [25]. This model has been credited with predicting characteristics of the 2008 financial crisis years in advance. By treating leverage as an endogenous decision, it characterizes equilibrium in financial markets in terms of leverage taken by each agent, in addition to the asset price, and the interest rate on debt. Since Basel targets leverage as a policy variable, the leverage cycle model seems like a natural candidate for evaluating recent financial regulation.

However, the original leverage cycle model is limited in its ability to express real-world complexity. For example, it cannot be extended for an arbitrary number of periods, and short asset positions are not considered. Basel regulation, since it responds to the historical volatility of asset prices, is not easily incorporated. To address such issues, Poledna et al. [52] develop a discrete agent-based model (ABM). At a high level, this model clears the market in each period for persistent financial agents. Agents carry over their wealth to each new period, where demand undergoes a random shock and new prices are formed. Basel regulation is easily expressed in this new model since a regulator may act in each period with knowledge of historical prices. The flip side is that agents are no longer coming to an intertemporal equilibrium as in the leverage cycle model. But leverage remains an endogenous decision by each market participant.

2.2.1 Agent-based model

Poledna et al. [52] include four types of agent, who interact in a market for a single risky asset. The asset has an unchanging ***fundamental value***, V, and a market ***price*** $p(t)$ at time t, determined by the cumulative demand of all agents. The central actors are informed value investors called ***fund managers***. Fund managers know the fundamental value and adjust their demand for the asset based on a ***mispricing signal*** $m = V - p_t$. The fund managers exhibit heterogeneous demand as a function of m, reflecting their differing levels of ***aggression*** in pursuing investment opportunities. The more aggressive a fund is, the more leverage it will take to pursue a given mispricing signal. Aggression reflects factors such as a fund's confidence that the asset price will return to its fundamental in short order, and its tolerance for risk. A fund manager that buys an asset while it is priced under its fundamental is betting that the price will move back up and make her a profit.

To buy or short-sell assets, fund managers may take loans from the ***bank***, treated as an agent in the ABM of Poledna et al. [52]. The bank provides credit to funds but requires a set amount of ***collateral*** per unit debt. In effect, this collateral requirement imposes a ***leverage limit***. To see this, consider how a leveraged investor keeps her books. She knows that to borrow a dollar from the bank she must commit some amount of wealth as collateral. Call this collateral requirement X. But she only has so much wealth, W, to commit in total. Thus there is a maximum amount she can borrow, W/X, which is constant given W. The ratio of this maximum amount to her current wealth is $1/X$. If the bank decides to increase X, it effectively decreases the leverage limit. In practice, we refer to the bank setting a leverage limit without relating explicitly to collateral. In general, the investor is taking on debt in order to take a larger investment position, which she is free to do until she is over the leverage limit. Then she must wind down her investment position and reduce her debt until she is in compliance with the bank.

Thus, in any given time period, funds are limited in the size of the asset position they can take, either long or short, in response to asset mispricings. Note that the bank may be forced to take losses when fund managers default. These losses are recorded but the bank itself never defaults, as it is assumed that there is an unlimited bailout fund.

Mispricings are made possible by stochastic, weakly mean-reverting asset demand from aptly named ***noise traders***. The noise traders represent a collection of ill-informed investors. Their collective behavior exerts a random shock to total asset demand, which in turn shifts the market-clearing asset price.

The final agent type is the ***fund investor***. The fund investor's role is simply to move capital from fund managers with poor (historical) performance, toward those with good performance.

2.2.2 The leverage cycle and Basel: intuition

It is easy to see that without any limits on leverage, the price of the asset would always return to its fundamental value, as funds would just increase their position against the direction of the shock until the mispricing disappears. It requires just one fund able to take unlimited size positions on the asset to ensure that there is never any price volatility around V. But with leverage limits, the system exhibits complete leverage cycles over time. The qualitative steps to the cycle are shown in Figure 2.1.

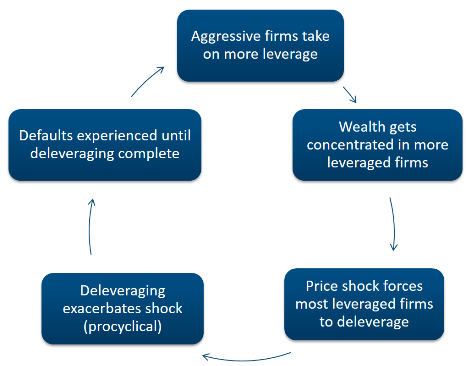


Figure 2.1: Stages of the leverage cycle modeled by Poledna et al. [52].

When shocks are small enough, wealth in the system becomes more and more concentrated in the most aggressive funds as they take more effective leverage in pursuing mispricing opportunities. This gradually increases the sensitivity of total wealth in the system to random price shocks until some investors reach their leverage limit. At that point, a decrease in the asset price would push the leverage ratio over the limit,[2] and so the investor would be forced to sell to get back in compliance when she would actually rather be buying the asset. This action is ***procyclical***, since the

[2]For example, consider the investor with \$1 who borrows \$5 to buy assets worth \$6. Her leverage ratio is currently 6. If the asset value decreases to \$5.5, the investor still owes \$5 to the bank, so her wealth is now \$0.5. Her leverage ratio increases to 11, which may be out of compliance with the bank.

act of selling further depresses the price, which can lead to further deleveraging by other funds. It may even induce a mass sell-off (or buy-off for short positions), as other, formerly less leveraged funds struggle to satisfy the leverage limit due to the acceleration of the price movement against their positions. Eventually, enough funds have either defaulted or deleveraged that collectively, they have become insensitive to further price movements, and the cycle begins again. Price volatility, probability of investor default, and losses on defaulted loans are all elevated due to this leverage cycle.

Within this model we can also incorporate Basel-style regulation, which makes leverage limits more stringent during periods of high volatility, and less stringent otherwise. Under fixed fund behavior, this increases the probability that investors get into procyclical situations, since the leverage limits themselves will adapt to be procyclical. Intuitively, higher leverage limits in volatile times could allow funds to provide a voice of reason and return the price to the asset's fundamental value. So Basel, by doing the opposite, potentially makes volatility and default more likely. On the other hand, Basel may reduce the losses suffered by the bank on defaulted loans, since the doomed funds were forced to deleverage more quickly before their default.

My study starts by reimplementing this ABM and replicating the original results. The remainder of this section describes model details; my extension to incorporate strategic response is described in Section 2.3. To implement the model, I use as references the description of Poledna et al. [52] together with code provided by these authors in response to our queries. Unless otherwise stated, all of the following modeling decisions and parameters are as implemented in the prior work. I attempt to keep the model as close as possible to prior work in order to isolate the effect that our equilibrium concept has on the evaluation of Basel policies. Assumptions were justified in the original work by drawing parallels with real data on financial crashes.

2.2.3 Schedule

The state of the market is defined by an equilibrium asset price and the resulting holdings and debt levels of the fund managers. The agent-based simulation iterates between a price formation phase and a wealth update phase. I describe these phases at a high level below to give a sense of the scheduling of tasks.

Price formation

1. The demand from each fund manager is a fixed function of the mispricing signal and the fund's current wealth. These demand functions depend on the maximum leverage allowed by banks as well as the fund's idiosyncratic aggression parameter.

2. Noise traders demand the asset according to a stochastic process such that without fund managers, the asset price would weakly mean revert around the fundamental value of the asset (an Ornstein-Uhlenbeck process). The noise trader's demand is what makes the mispricing signal nonzero in each period.

3. The price of the asset $p(t)$ is set at a level that clears the market given the collection of demand functions submitted by noise traders and fund managers. This is a Walrasian equilibrium within period t.

Wealth update of fund managers

1. The new asset price as determined in the price-formation phase induces revised wealth levels for each of the fund managers based on the market value of their holdings.

2. The fund investors obtain a new datapoint regarding the profitability of each fund manager by observing the wealth gained or lost in the update. They withdraw or deposit capital into funds accordingly.

3. Banks enforce leverage limits on funds. Funds that exceed leverage limits are subject to a ***margin call***, requiring they change their position in the asset to comply with the limit. Note that the violation of leverage limits can be triggered by a change in asset price, by the movement of capital by fund investors, or some combination.

2.2.4 Basic definitions

We define the key variables describing the state of a fund h at time t.

1. $D_h(t)$ is the amount of the asset that the fund holds at t. It may be positive, meaning the fund owns a positive amount of the asset, or negative, indicating a short position.

2. $M_h(t)$ is the cash position of the fund. The cash balance is changed each period based on purchases or sales of the asset. A negative $M_h(t)$ indicates a debt position.

3. The wealth of a fund is defined by

$$W_h(t) \equiv D_h(t)p(t) + M_h(t),$$

   recalling that $p(t)$ is market price of the asset. All nondefaulted funds have $W_h(t) > 0$. This entails in particular that a short asset position is always accompanied by a positive cash position.

4. The ***leverage*** taken by each firm is defined as

$$\lambda_h(t) \equiv \begin{cases} D_h(t)p(t)/W_h(t) & \text{if } D_h(t) \geq 0 \text{ (long)} \\ M_h(t)/W_h(t) & \text{if } D_h(t) < 0 \text{ (short).} \end{cases}$$

Recalling that $W_h(t)$ within a period is constant and positive, and therefore the numerator is always positive, $\lambda_h(t) \in [0, \infty)$. It can in principle be made arbitrarily large by taking a position of sufficient magnitude, although due to leverage restrictions, the funds are constrained in the positions they can take.

These definitions allow for a fund to have ***short*** and ***long*** positions on an asset. For a given wealth, a fund with a long position has a positive value of asset holdings $D_h(t)p(t)$ with cash $M_h(t)$ unrestricted. A common scenario is negative $M_h(t)$, meaning the fund has taken on debt to finance their long position. This necessarily makes $D_h(t)p(t) > W_h(t)$. Thus any fund indebted to the bank must have $\lambda_h(t) \geq 1$.

On the other hand, a fund with a short position owes future shares to its counterparty ($D_h(t) < 0$), but holds the cash it obtained from the sale of these future shares ($M_h(t) > 0$). If the price of the asset $p(t)$ increases, the wealth of a fund that is long on the asset goes up while the wealth of a fund that is short goes down. The opposite occurs if $p(t)$ decreases.

2.2.5 Regulatory environments

In the somewhat misnamed ***unregulated environment***, $\lambda_h(t)$ is constrained to be less than a parameter $\lambda_{\max}$ for all h. The ***Basel environment*** adaptively selects maximum leverage $\lambda_{\max}^{\sigma(t)}$ between periods based on price volatility $\sigma(t)$. $\lambda_{\max}^{\sigma(t)}$ is calculated at the beginning of every timestep t according to the formula

$$\lambda_{\max}^{\sigma(t)} = \min\{\lambda_{\max}, \max\{1, \frac{\sigma_b}{\sigma(t)}\}\}, \tag{2.1}$$

where $\sigma_b = 0.0118$ is a set benchmark level of volatility, and $\sigma(t)$ is measured as the average volatility of the log asset price in the previous 10 periods. Before 10 periods are available, all price data is used. Thus, maximum leverage requirements are adjusted downwards in periods of high volatility and are allowed to reach $\lambda_{\max}$

when volatility is low. In the unregulated case, $\lambda_{\max}$ is still a hard limit on leverage but it does not adapt to market conditions.

2.2.6 Fund demand

Leverage limits effectively constrain the maximum and minimum demand for each fund manager. Individually, each fund manager h with aggression parameter β_h has a demand $D_h(t)$ of the following form, at time t:

$$D_h(t) \equiv \begin{cases} (1 - \lambda_{\max}^{\sigma(t)}) W_h(t-1)/p(t) & m(t) \leq (1 - \lambda_{\max})/\beta_h \\ \lambda_{\max}^{\sigma(t)} W_h(t-1)/p(t) & m(t) > \lambda_{\max}/\beta_h \\ \beta_h m(t) W_h(t-1)/p(t) & \text{otherwise} \end{cases}$$

Here the price $p(t)$ at time t is the free variable and all other quantities are fixed. The fundamental value is set at $V = 1$ and $W_h(t-1)$ denotes the total wealth accumulated by h in the last period. In the original study [52], the aggression parameter β_h is fixed to be $h \times 5$ where $h \in \{1, \ldots, 10\}$ is the index of the fund, of which there are ten in total. In our strategic analysis we allow the funds to choose the β_h's, but for now they are fixed. Notice that in the lower and upper regions of price, demand does not depend on β_h. In particular, demand is fixed in these regions. This comes from the fact that funds have hit their leverage limit and cannot borrow further to pursue investment opportunities. An example of the demand function for two funds with different β_h is shown in Figure 2.2.

2.2.7 Market clearing price

Every period, prices are used to coordinate asset allocation. Each fund submits their demand function truthfully and an equilibrium is found after the noise traders generate stochastic demand. Each period gets its own independent price. The only thing that transfers information between periods is the wealth that each fund ends

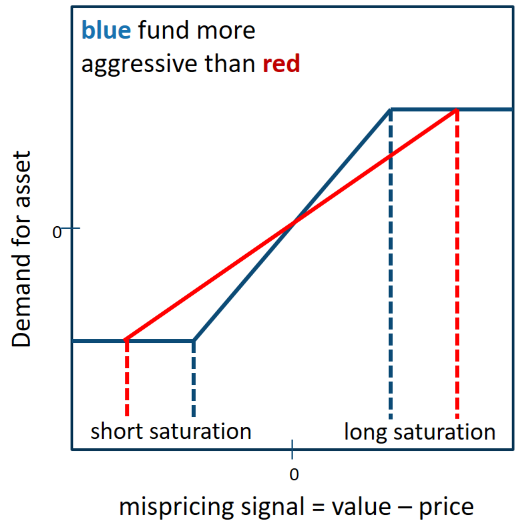


Figure 2.2: Demand as a function of mispricing. The mispricing values beyond short saturation and long saturation on either side result in no additional demand for the asset because leverage limits have been reached.

up holding.

Given the demand functions of all funds as well as the noise trader demand $C(t)$, prices are formed via the market clearing condition

$$\sum_h D_h(t) = N - C(t), \tag{2.2}$$

where $C(t)$ is generated by

$$\log C_n(t) = \rho \log C_n(t-1) + \sigma_n \chi(t) + (1-\rho)\log(N).$$

The noise parameter σ_n is set to 0.035 and $\chi(t)$ is an i.i.d. standard Gaussian draw. Parameter ρ is set to 0.99, representing extremely weak mean reversion. This is the Ornstein-Uhlenbeck process, a standard model in finance for asset price dynamics.

We set $N = 10^9$. Equation (2.2) is a piecewise linear function of $p(t)$ which can be solved using standard methods.

2.2.8 Wealth update

Using the $p(t)$ (recall it is embedded in the demand functions $D_h(t)$) obtained as solution to (2.2), the wealth of each fund $W_h(t)$ can now be updated. Assume for now that $D_h(t-1)$ is positive, that is, fund h takes a long position. There are three possible sources of wealth change in the model: market value of assets, interest payments, and equity flows.

The increase in asset value for each fund is simply $D_h(t-1)[p(t)-p(t-1)]$, and reflects the return on an investment made in the previous period. This is the fund's profit or loss.

In general, funds finance their investment opportunities using debt from the bank. If the amount of cash $M_h(t) = W_h(t) - D_h(t)p(t)$ is negative, then the fund has borrowed from the bank to buy $D_h(t)$ shares of the asset. In this case the fund pays a fixed interest rate $S = 0.015\%$. After paying interest, the fund's total available assets for withdrawal is now

$$M_h^*(t) = D_h(t-1)p(t) + M_h(t-1)(1+S).$$

The other source for changes in wealth is withdrawals or deposits made by fund investors. These make their decisions based on recent performance of each fund. Fund investors examine each fund's rate of return (estimated using an exponential moving average) $r_h(t)$ and withdraw/deposit an amount $F_h(t)$ as follows:

$$F_h(t) = \max\{-1, b(r_h(t) - r_b)\} \max\{0, M_h^*(t)\},$$

where the benchmark return r_b is 0.003 and b, controlling how strongly fund investors react to historical performance, is set at 0.15. The overall motion equation for wealth is then

$$W_h(t) = W_h(t-1) + D_h(t-1)[p(t) - p(t-1)] + F_h(t) + M_h(t-1)(1+S).$$

Analogously, for a short position we have

$$M_h^*(t) = D_h(t-1)p(t) + M_h(t-1) + D_h(t-1)S,$$
$$W_h(t) = W_h(t-1) + D_h(t-1)[p(t) - p(t-1)] + F_h(t) + D_h(t-1)p(t-1)S.$$

Once all fund wealths are updated, the time step is complete and we go back to price formation.

2.2.9 Default

Each fund starts with $W_0 = 2 \times 10^6$ in wealth. Default occurs when wealth goes below a critical value $W_e = 2 \times 10^5$. In this case, the fund's wealth, demand, and cash positions become zero for 100 timesteps, after which the fund is resuscitated at the same level of aggression β_h and given W_0 to start operations again.

2.3 Systemic risk evaluation

The motivation for using this model is to see what effect regulation can have on the leverage cycle, and how this impacts systemic risk. We first develop an approach for evaluating the effect of regulation when fund managers are given the opportunity to adapt strategically according to changing external conditions. Then we define metrics for measuring systemic risk and perform an experiment measuring the effects of the strategic adaptations on systemic risk.

2.3.1 Strategic aggression behavior

Recall from Section 2.2.6 that each fund is assigned an aggression level. We call these assignments the ***aggression distribution***. Poledna et al. [52] assume that the aggression distribution is fixed. This assumption is problematic for several reasons, but fundamentally because it limits the responsiveness of fund behavior to environmental conditions, either of the market or the regulatory system.

We focus on this assumption in particular because the entire model appears to be highly sensitive to the aggression distribution. For example, if all funds have low aggression then leverage will almost never be high enough to induce defaults. As circumstantial evidence for this, in all the experiments described in Section 2.3.4, no fund at an aggression level of 5 ever defaulted. This may be considered an extreme setting, but it is disturbing that one can generate almost any story one wants by manipulating this fixed assumption. It is far better to derive the aggression distribution from something more fundamental, like strategic choice.

We therefore treat aggression level as an *endogenous* variable, taking account of the dependence of each fund's payoff on the aggression of other players. That is, we cast the entire scenario as a game played among funds, where aggression distributions are strategy profiles and the strategy set is comprised of different aggression levels. Aggression levels are chosen by all funds simultaneously. This allows us to search through the space of possible aggression distributions using EGTA to find a Nash equilibrium where no fund can make more profit by changing their aggression level. This approach, notably, leaves each fund's aggression level (and thus demand function) in a given period fixed. Within a given time period we still have perfect information and complete markets, so clearing the asset market still produces a Walrasian equilibrium. The only intertemporal choice funds make is aggression level, which is chosen for all periods simultaneously. Thus our strategic analysis is limited to aggression levels, that is, we do not consider other strategic market behavior.[3] This preserves the spirit of the leverage cycle story for a given equilibrium choice of aggression distribution. We see below that the aggregate effect of equilibrating prices in each period and aggression over all periods has a substantial effect on financial stability.

2.3.2 Experimental setup

In our approach we evaluate the introduction of Basel into the model on two dimensions. First, given a particular aggression distribution, we measure the effect of regulatory environment on systemic risk measures. This is meant to confirm the findings of Poledna et al. [52] by isolating the effect of Basel on the leverage cycle model given fixed aggression levels. Second, and more novel, we look at the effect of shifting between equilibrium aggression distributions given a particular regulatory environment. An equilibrium aggression distribution is a Nash equilibrium induced by a regulatory environment (i.e., Basel versus unregulated). This approach isolates the effect of Basel on the strategic choice of aggression levels while leaving its effect on the leverage cycle constant.

Strategies and Nash equilibrium First, some implementation details. A pure profile in this model is an H-dimensional vector β containing a single aggression level, or strategy, for each of H funds. For our study we chose a number of funds

$$H \in \{10, 21\}$$

and a set of 7 possible aggression levels/strategies

$$\Gamma = \{5, 10, 15, 20, 30, 40, 50\}$$

[3]Since funds are not atomic, there could be room for exercising market power. In our setting we view the market as competitive enough at that level to ignore such options.

Thus a particular fund's mixed strategy is a 7-dimensional simplex, corresponding to the probabilities that the fund will pick each of the seven strategies for use over the entire simulation. A mixed strategy profile D is a probability distribution over pure profiles. Drawing from Γ using the multinomial distribution specified by D assigns a β_h to each of the H funds.

Our goal is to use EGTA to find a D^* that is a Nash equilibrium. To do this, we first need to define how the game is played and how payoffs are generated.

Payoffs and game definition Each run of the model provides a ready made payoff: the fee earned by each fund. To find this fee, we first run the model for $T = 50,000$ periods using $H \in \{10, 21\}$ funds and a given β vector representing the aggression level for each fund. At the end of the simulation, we annualize into 1000 years containing 50 periods each as before. In each year i starting at periods t, we calculate the average size of fund h, $W_h(i) = \sum_t^{t+50} W_h(t)/50$ as well as the profit $\zeta_h(i) = W_h(t+50) - W_h(t) - \sum_t^{t+50} F_h(t)$, which is simply the change in wealth net of investor withdrawals and deposits over the course of the year. The fee for year i is then defined as $0.2\zeta_h(i) + 0.02W_h(i)$ and the total fees over the entire simulation for fund h are

$$\sum_{i=1}^{1000} 0.2\zeta_h(i) + 0.02W_h(i).$$

This is the specification used by Poledna et al. [52], based on standard practice in the financial industry. It captures the common 2-and-20 fee structure, which pays hedge funds 2% of assets under management and 20% of value returned to investors annually. This annualization (1000 years that are each 50 periods long) is done according to the original paper [52], where the model was calibrated to real yearly data. Note that the payoff for a given mixed strategy D depends on whether or not Basel is active as well as $\lambda_{\max}$ and is an average over many 50,000-period simulations where pure profiles β are generated from D.

Environmental settings In summary, we can view the model input as choice of regulation R, a particular β vector of strategies representing a pure profile, and parameter $\lambda_{\max}$. The model itself can be viewed as a black box function, mapping to the model output which is a vector of payoffs, or fees, for each fund. In reality, remember that payoffs are generated by relying on separate Walrasian equilibria in each of the 50,000 periods to set prices.

Finding the Nash equilibrium distribution $D^*(R, \lambda_{\max})$ of such a system requires searching through input β and evaluating fees until an equilibrium is found. This is where EGTA can be used to great effect, which we do by using a suite of tools developed by Cassell and Wellman [10]. We were able to obtain approximate $D^*(R, \lambda_{\max})$ for $R \in \{\mathit{Basel}, \mathit{Unregulated}\}$ and $\lambda_{\max} \in \{8, 20\}$. These levels of $\lambda_{\max}$ were chosen to reflect a wide range of conditions.

Note that I chose to show results for the same set of strategies Γ as that used by Poledna et al. [52]. This was mainly for comparability, but I did test lower minimum (down to 1), higher maximum (up to 70), and a finer resolution in between for aggression levels. I found that the results were not qualitatively different. For example, a β of 1 was not adopted in the mixed strategy equilibrium when added to the reported set of strategies. In all tested cases, Basel encouraged less average aggression in equilibrium.

Empirical game-theoretic analysis The empirical game approach imposes two major approximations. First, the set of available strategies Γ is restricted to a modest number of enumerated choices. As noted above, I limited attention to $|\Gamma| = 7$ choices of aggression level, following the study by Poledna et al. [52]. Since the number of pure strategy profiles is exponential in the number of strategies, and in the worst case EGTA will need to evaluate payoffs for all pure strategy profiles, I have no choice but to impose such a limit. Second, for the $H = 21$ setting I used a technique called

deviation preserving reduction [70], which employs aggregation to model the 21-player game in terms of a reduced six-player game.

2.3.3 Systemic risk metrics

Our metrics for systemic risk differ from those provided by Poledna et al. [52] in one key respect. Rather than focusing on the risk of the most aggressive fund manager, we instead evaluate risk aggregated over all fund managers. The aggregate measure reflects a more direct evaluation of the entire economy. Moreover, as we allow distribution of aggression to vary based on endogenous choices, we need a way to evaluate the riskiness across situations with different maximum aggression levels. This difference is only relevant for measuring probability of default and capital losses.[4]

Probability of default To calculate probability of default of a single fund for a single run of the model, I record the agent-years during which a default occurred. Then I divide this number by the number of agent-years during which default was possible, K_h. $K_h \leq 1000$ because defaulted funds go out of operation for 2 years, or 100 time steps, as specified in Section 2.2.3. Thus, there are entire agent-years during which default is impossible, since the agent has already defaulted. Note that there are also years during which default is only partially possible as the fund is still in a defaulted state at the beginning of the year. We choose to exclude these years from K_h as well. The overall probability of default is then

$$\sum_h \#\mathit{defaults}_h/(K_h H).$$

Here, recall that H is the total number of funds.

[4] We also calculated the original study's metrics and verified that our implementation matched the results reported by Poledna et al. [52].

Capital losses to banks Capital loss is the amount of loss banks suffer when a fund manager defaults. It is defined as the wealth $W_h(t)$ of the defaulting fund at default. This is an important metric for systemic risk because these losses are often paid for through bank bailouts, which detract from social welfare.

Capital losses were measured on an annual basis, in the same way as for probability of default. Yearly capital losses are totaled, then averaged across the number of funds and number of years during which default was possible.

Price volatility Price volatility is detrimental to the extent it contributes to financial uncertainty. One of the functions of financial markets is efficient price discovery, and one might hope that in a single asset market where some investors have perfect information about the fundamental value of the asset, this function would be performed well. Price volatility is a good proxy for how well price discovery is performed. Indeed, if we allow $\sum_h D_h(t)$ to be large compared to $C(t)$ for all t, we would expect price volatility to be near zero as the effect of the noise traders is minimized. Price volatility is the average of the annual variances of $p(t)$. This metric does not depend on K_h like the other two, since prices can be formed regardless of the default status of any fund.

2.3.4 Systemic risk results

Our first notable result is that in every setting evaluated, the strategy profile that maximized total fees was for every fund to use $\beta_h = 5$. This is the least aggressive strategy available, and is the most limiting in the range of investment choices available to the fund. The only benefit to playing this strategy is that it mitigates the leverage cycle and may prevent crashes from happening. This is circumstantial evidence that defaults are very harmful not only to banks and investors, but also to the funds themselves. It also provides a basis for believing that strategy profiles with low aggression may be better for systemic risk.

Regulation	numFunds	λ_{max}	5	10	15	20	30	40	50	$\beta_{weighted}$
Unregulated	21	20	35%	4%	0%	0%	0%	0%	60%	32.3
Basel	21	20	40%	12%	6%	0%	0%	0%	42%	25.3
Unregulated	21	8	32%	15%	7%	2%	0%	0%	43%	26.3
Basel	21	8	46%	11%	9%	5%	10%	15%	4%	16.7
Unregulated	10	20	46%	5%	0%	0%	0%	0%	49%	27.4
Basel	10	20	51%	14%	8%	0%	0%	0%	26%	18.4
Unregulated	10	8	40%	21%	8%	0%	0%	0%	31%	20.8
Basel	10	8	53%	18%	7%	1%	11%	10%	0%	13.0

Figure 2.3: Mixed-strategy symmetric equilibria for different regulatory regimes and leverage restrictions.

Mixed strategy Nash equilibria Shown in Figure 2.3 are approximate mixed strategy Nash equilibria for each of 8 environmental settings. Also shown are $\beta_{weighted}$, which are the average values of β_h drawn from each mixed strategy profile. Notice that decreasing $\lambda_{\max}$ and imposing Basel both decrease the $\beta_{weighted}$ of the Nash equilibrium. When Basel is imposed, it is no longer as lucrative for funds to be aggressive since they will reach their leverage limits immediately. Decreasing $\lambda_{\max}$ has a similar effect. This strategic response has ramifications on our systemic risk measures.

Interestingly, all equilibria are far from the social optimum. This is because it takes coordination to prevent defaults and maximize profits. Unilaterally playing a less aggressive strategy has positive externalities, namely a lower default rate and crash rate for *all* agents, that are not internalized by fund payoffs.

Systemic risk under equilibria When Basel is imposed on a previously unregulated market, two things happen in equilibrium. First, the mechanics of fund behavior change as their leverage limits become more stringent in volatile periods. Second, funds adapt their aggression to fit the regulation, shifting to less aggresive (on average) Nash equilibria over mixed strategies as in Figure 2.3. To evaluate these two effects on financial stability, we first select a mixed strategy equilibrium. Next, we take 200 draws from the distribution. Each draw is a valid strategy profile. We calculate the metrics from Section 2.3.3 on each of these 200 strategy profiles both with and without Basel. The averages are presented in Figure 2.5 and Figure 2.4.

↑	Volatility	Setting	
Profile	λ_{max} = 20	Basel	Unregulated
	Basel	0.0222	0.0182
	Unregulated	0.0221	0.018

↑	Volatility	Setting	
Profile	λ_{max} = 8	Basel	Unregulated
	Basel	0.0338	0.0205
	Unregulated	0.0341	0.0208

↓	Default Probability	Setting	
Profile	λ_{max} = 20	Basel	Unregulated
	Basel	3.67%	2.99%
	Unregulated	5.02%	3.73%

↓	Default Probability	Setting	
Profile	λ_{max} = 8	Basel	Unregulated
	Basel	1.43%	2.20%
	Unregulated	3.11%	3.55%

↓	Capital Loss	Setting	
Profile	λ_{max} = 20	Basel	Unregulated
	Basel	2.66E+06	3.42E+06
	Unregulated	3.21E+06	3.88E+06

↓	Capital Loss	Setting	
Profile	λ_{max} = 8	Basel	Unregulated
	Basel	5.45E+05	2.13E+06
	Unregulated	9.15E+05	2.76E+06

↓	Fees	Setting	
Profile	λ_{max} = 20	Basel	Unregulated
	Basel	1.80E+05	1.90E+05
	Unregulated	1.78E+05	1.87E+05

↓	Fees	Setting	
Profile	λ_{max} = 8	Basel	Unregulated
	Basel	1.43E+05	2.03E+05
	Unregulated	1.37E+05	1.93E+05

Figure 2.4: Three systemic risk metrics evaluated at two levels of λ_{max} for 21 funds. Analagous table for 10 funds in Figure 2.5. Within a table, each row fixes an equilibrium mixed strategy and applies a different regulatory setting, each column fixes a regulatory setting and switches to a different mixed strategy equilibrium. Down arrows signify that the metric decreased when Basel was applied in the study by Poledna et al. [52] while up arrows signify that the metric increased. Blue lettering means changing that property results in a metric change that agrees with Poledna et al. [52] while red means changing that property results in a metric change that goes against their finding. Black means that there was no significant change in the variable. Example: For $\lambda_{max} = 20$'s default probability table, up arrow means that Poledna et al. [52] found Basel increased default probability. Fixing profiles and changing settings agrees with this finding, indicating successful replication, while fixing setting and changing profiles goes against this finding.

Each table shows two-dimensional variation across regulatory environments. Along rows, we keep the mixed strategy equilibrium fixed and change the environment. Along columns, we keep the regulatory setting fixed and change the equilibrium. For every metric and setting we tested, shifting between Basel and unregulated envi-

↑	Volatility	Setting	
Profile	$\lambda_{max} = 20$	Basel	Unregulated
	Basel	0.0232	0.0188
	Unregulated	0.0229	0.0188

↑	Volatility	Setting	
Profile	$\lambda_{max} = 8$	Basel	Unregulated
	Basel	0.0317	0.0205
	Unregulated	0.0313	0.0219

↓	Default Probability	Setting	
Profile	$\lambda_{max} = 20$	Basel	Unregulated
	Basel	2.24%	2.11%
	Unregulated	4.17%	3.65%

↓	Default Probability	Setting	
Profile	$\lambda_{max} = 8$	Basel	Unregulated
	Basel	0.52%	1.35%
	Unregulated	1.23%	2.46%

↓	Capital Loss	Setting	
Profile	$\lambda_{max} = 20$	Basel	Unregulated
	Basel	1.43E+06	2.24E+06
	Unregulated	1.91E+06	2.79E+06

↓	Capital Loss	Setting	
Profile	$\lambda_{max} = 8$	Basel	Unregulated
	Basel	3.09E+05	1.41E+06
	Unregulated	3.52E+05	1.56E+06

---	Fees	Setting	
Profile	$\lambda_{max} = 20$	Basel	Unregulated
	Basel	4.25E+05	4.34E+05
	Unregulated	4.28E+05	4.26E+05

↓	Fees	Setting	
Profile	$\lambda_{max} = 8$	Basel	Unregulated
	Basel	3.48E+05	4.63E+05
	Unregulated	3.28E+05	4.34E+05

Figure 2.5: 10 funds. See Figure 2.4 for key.

ronments under a fixed equilibrium resulted in the same effect on the metric as that reported by Poledna et al. [52]. This should not be surprising since with a fixed aggression distribution, I am essentially repeating their experiment with slightly different aggression settings. This is essentially just confirmation that we have a successful replication.

Something interesting happens when we examine these tables along columns. This is equivalent to fixing the regulatory environment and switching between mixed strategy equilibria. We have already seen that the average β_h for Basel equilibria is substantially lower. It turns out that less aggressive funds increase financial stability by decreasing the probability of default and bank capital losses. For example, at $\lambda_{\max} = 20$, even though Basel makes default more likely for a fixed aggression distribution, the shift towards the Basel equilibrium counteracts and reverses this effect for both $H = 10$ and $H = 21$.

In aggregate, the story has changed. Basel decreases probability of default in every measured case. Capital losses, which were already decreasing in the original study, decrease even further. Price volatility still increases with Basel, since the shift in equilibrium did not have an effect. Fees are lower in the Basel setting, but by less than was predicted by Poledna et al. [52]. Shifting between equilibria increases fees in most cases to cancel out some of the fee loss found in the original study.

2.3.5 Discussion

So what explains Basel's performance in decreasing systemic risk? We can break this question into two parts. First, how does Basel allow less aggressive funds to thrive compared to aggressive ones? And second, why do less aggressive funds allow for better financial stability?

Consider this model with a single fund manager. Suppose she is currently operating in the unregulated environment with $\lambda_{max} = 20$ and she hears that Basel will soon be implemented. This can only lower her leverage limit. We posit that even without any strategic interactions with other funds, she will be inclined to lower her aggression levels. A fund gets more fees by generating higher returns, and higher returns are gotten by taking high leverage when there are large mispricing signals. When leverage limits are high, an aggressive fund will just snap up all the mispriced assets and profit off of all of them. But when leverage limits become more stringent under Basel, too much aggression will quickly lead to paralysis due to hitting the leverage limit. In contrast, less aggression will preserve some investment capacity for the truly large mispricing signals, generating a better return.

Now consider two funds who each know that Basel is coming. If they both decide to be aggressive, they can mitigate more price movements, reduce price volatility, and operate as if Basel was never implemented. However, if fund A decides to shirk its price enforcing duties by being less aggressive, Basel will restrict maximum leverage since price volatility will rise. This doesn't affect fund A much since it doesn't take much leverage anyway. In fact, in all of our simulations, the funds playing strategy

$\beta_h \in \{5, 10\}$ never defaulted a single time. But the leverage limit does affect the more aggressive fund B, and will cause it to default eventually. When this happens, A will have the entire market to itself. When this strategic advantage is coupled with the fact that less aggressive funds stand to make more profit regardless of what other funds do, it is easy to see why Basel induces funds to be less aggressive.

Now why does the shift to a less aggressive set of funds result in better default rates, bank losses, and fees while not having any effect on price volatility? Note that here we are only giving intuition for the effect of the shift in aggression distribution, not the aggregate effect of Basel.

Less aggressive funds simply do not take much leverage before the mispricing signal disappears randomly. Thus, they never get to the point where a price shock can cause them to lose all their wealth. Again we quote the result that not a single low-aggression fund defaulted in our experiments. Thus, default rates decrease when funds are less aggressive.

Funds that are aggressive under equilibrium that default also cause less capital loss to banks. Recall the feedback cycle that occurs with Basel. First, aggressive firms accumulate all the wealth in good times since Basel hasn't kicked in yet. Then, a price shock happens that forces the aggressive funds to wind down their positions to comply with collateral requirements. This procyclical unwinding deepens the initial shock, increasing volatility and forcing Basel to make leverage limits more stringent. When there are many less aggressive funds, none of this volatility can be absorbed and the Basel requirements will get extremely stringent. Thus, the funds that end up defaulting will have been trying to reach a lower leverage limit. This reduces their size when they default.

Fees are slightly helped by less aggression for the same reason funds decided to be less aggressive in the first place. And price volatility is definitely increased in normal times by less aggressive funds, but the decrease in default rate reflects a lower incidence of financial crashes, which helps price volatility. It seems these two effects cancel out.

2.4 Summary

When evaluating the effect of financial regulation on markets, we should always be aware that agents in this arena are highly sophisticated and resourceful. It is a good bet that any possible path to profit will eventually be explored. So when economic models make fixed assumptions on behavior, we should always be on guard for the day these assumptions start break down. To become more robust, models can endogenize important behavioral assumptions so that agent response to the environment is as realistic as possible.

In this chapter I argued for endogenizing the aggression level of funds in an agent based leverage cycle model used to evaluate Basel regulations. I then proposed a strategic analysis that endogenizes an important fixed assumption, the aggression distribution. After making appropriate approximations to make finding equilibria feasible, I performed a series of experiments to measure the aggregate effect of Basel on financial stability under our new equilibrium concept. My findings suggest that the pessimism surrounding Basel's exacerbation of leverage cycles may be overstated. I find that irrecoverable losses on the part of banks and defaulted funds are reduced substantially under Basel. Although fund profits fall, my new equilibrium concept lessens the fall compared to the one without endogenous aggression. Volatility remains a problem, as funds still cannot prevent day to day fluctuations in price effectively.

There is still much work to be done, both in evaluating Basel and in building richer economic models. For the former, the obvious question is how to improve on Basel so that we get its benefits (less aggressive agents on average) without its costs (agents constrained from taking volatility out of prices). There has been some work on this, but not under endogenous aggression levels [3, 50, 51].

There are also many other directions where computer science techniques can aid in understanding Basel regulations. A notable fixed assumption in the leverage cycle model is that funds may borrow at fixed terms from banks, and that funds have no credit operations of their own. In reality, interest rates and leverage limits are dynamically evolving according to the strategy employed by banks. In addition, the separate roles of banks (who extend credit) and funds (who invest in assets) are not so clear cut; major financial players tend to be active in both arenas. In fact, this omission of credit policies by leverage cycle models causes several phenomenon observed during the 2008 crisis to be ignored, including:

1. The seizing up of credit markets, causing liquidity shortages.

2. The widespread effects of isolated defaults, dubbed contagion.

3. The simultaneous downgrading of credit by ratings agencies and credit issuers.

The infrastructure for building a model incorporating these phenomena requires a new type of agent-based model, which we will build from the ground up in the upcoming chapters.

CHAPTER III

Financial credit networks and strategic payments

In this chapter I detail a general framework for representing the state of a financial system as a graph, called ***financial credit networks*** or FCNs [12]. This framework can be used as a basis to construct agent-based models where strategic loan decisions that are made locally can have global implications on financial stability, as in Chapter IV. I use an abstract agent-based model in this chapter in order to establish basic properties of the FCN framework, from which I can build economic intuition in more realistic models. I will show that a strategic tension exists in how agents choose to utilize the network to make payments, and use EGTA to establish equilibrium behavior under different environmental conditions.

3.1 Introduction

The key functions of a financial system are to allocate funds to productive uses and support transactions across a heterogeneous set of agents. These functions often interact; for example banks provide payment services to consumers in order to attract deposits, which they can then lend out. At the core of both these functions is the management of financial obligations between parties. I therefore view expression of such obligations as prerequisite to comprehensive financial modeling, and introduce here the FCN framework based on these relations. My focus in this chapter is on payment operations, which serve as a foundation for general economic transactions such as the purchase of routine products and services, lending and saving, and capital investment.

I extend an existing abstract model of ***credit networks***: weighted directed graphs that represent the capacity of agents (each represented as a node in the graph) to transact with each other. The credit network model was proposed independently by several distinct groups of researchers who were motivated to capture distributed trust in different contexts. Ghosh et al. [27] aimed to support distributed payment and multi-user credit checking for multi-item auctions. Karlan et al. [36] wanted to construct an economic model of informal borrowing networks. In both of these cases, the concept of credit is financial in nature. In two other cases [20, 45], credit serves as an accounting mechanism to limit computational actions. Moreno-Sanchez et al. [46] demonstrated a privacy-preserving payment protocol for credit networks, and also suggest that some real distributed payment systems like Ripple are based on the credit network model.

The effectiveness of credit networks for distributed transactions was most powerfully demonstrated by Dandekar et al. [17], who established several propositions indicating that transaction failures are unlikely given sufficient network connectivity. That is, credit networks provide a high degree of ***liquidity***: the ability to transact at any time at prevailing terms. In particular they showed for several classes of graphs, the transaction failure probability goes to zero as either network size, link density, or credit capacity increases, holding the other two parameters constant. Computational experiments further demonstrate that even networks small in size or overall credit capacity exhibit high transaction success rates if they are sufficiently well-connected.

A follow-up study addressed the question of whether self-interested agents would issue sufficient credit to form such high-performing networks [18]. Issuing credit entails a tradeoff between increasing the prospect of valuable transactions at the cost of exposure to risk of defaulting counterparties. The study found, across a range of experimental environments, that if there is sufficient transaction profit to be earned, a network will form to extract a sizable fraction of that surplus. However, the credit networks formed in equilibrium are still suboptimal, as might be expected given the positive externality in credit issuance. Namely, when an agent issues a credit line, the entire network benefits from the liquidity it provides while only the issuer bears the risk that the borrower may default.

To provide proper incentives to issue credit, a natural approach is to allow creditors to charge interest on outstanding obligations. Pricing credit is of course standard in actual financial systems, and so extending the credit network formalism to support interest charges would also make them more suitable as a modeling substrate for this domain. The extension to FCNs developed here is driven primarily by the requirement for handling interest rates. I show how the representation of obligations must be refined to accommodate interest on outstanding debt, and demonstrate how to realize payment operations in the extended model.

In §3.2 I introduce the basic concepts of FCNs, through an extended example. §3.3 presents the formal FCN model, and defines what constitutes a feasible solution to the payment routing problem. Next (§3.4) I describe a polynomial algorithm that solves for maximum flow on financial credit networks when interest rates are restricted to the set of contract interest rates. I then show that this is equivalent to solving the problem under unrestricted interest rates. This algorithm is due to my coauthor[1], but is included in this chapter for readability and completeness. In §3.5 I define several payment mechanisms that select among multiple feasible routing solutions. In §3.6 I evaluate the liquidity of each mechanism experimentally and find that there is a positive relationship between how much interest agents pay and how much liquidity is available in the network. I find that several mechanisms exhibit liquidity performance

[1]Kareem Amin

close to the no interest rate setting. The ability to pay interest rates above that offered by lenders plays a large role in this good performance. Finally (§3.7), I consider strategic choice by payers over routing mechanisms. I find that the socially optimum payment mechanism is not strategically stable, but that agents are willing to pay a limited amount in exchange for maintaining liquidity on a financial credit network.

3.2 Financial Credit Networks

A prerequisite for any payment is *trust*. Transactions between unknown parties are often enabled by third parties (e.g., banks or credit card issuers) that assure successful execution by mediating a transfer of obligations tantamount to a flow of funds. In the early days of Internet commerce, platforms like eBay introduced reputation mechanisms to facilitate the development of trust necessary to overcome lack of direct experience with counterparties [54]. Others provided more direct mediation. For example, Alibaba offered an escrow service whereby two parties that mutually trust Alibaba can transact with each other, relying on Alibaba to make them whole if the counterparty defaults [72].

In the basic credit network formalism, a's trust of b is represented by a directed weighted edge from a to b, where weight w denotes the ***capacity*** of credit a offers to b. This credit is interpreted as an obligation for a to accept up to w units of IOUs from b in exchange for commensurate service. These IOUs may be returned by a to b at a later time, in exchange for service from b. The real power of credit networks, though, comes from transactions along paths, achieving an effective transitivity of trust. If a offers credit to b and b to c, then c can transact with a by routing its payment through b: specifically, for a unit transaction c sends one of its IOUs to b, and b sends one of its IOUs to a. The net result is a payment of one unit from a to c. Node b has exchanged one its own IOUs for one of a's, and thus its balance of obligations is unchanged.

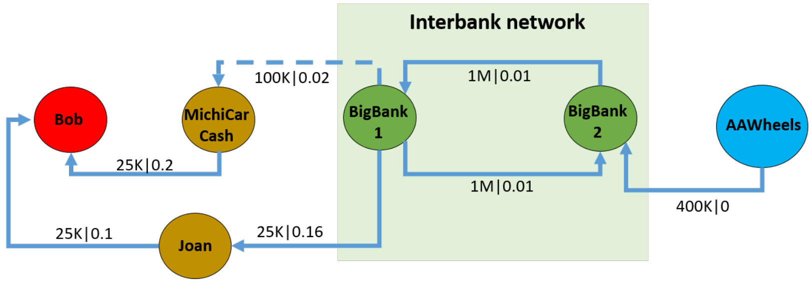
3.2.1 Illustrative example

Chains of payment are common occurrences in everyday commerce. Suppose Bob wishes to buy a new car from his local dealership, AAWheels. He negotiates a deal to purchase his favorite model for $25,000. Bob however does not carry that much cash, and AAWheels does not trust him directly. Anticipating this issue, before car-shopping Bob had applied for a credit line from MichiCarCash, a prominent consumer lender, who after some research decided to issue a credit line of $25,000 at a healthy interest rate (20%). MichiCarCash has a checking account with BigBank1, with a current balance of $100,000. This deposit is essentially a loan to the bank (at a relatively smaller interest rate, 2%), so we can think of MichCarCash as holding 100,000 BigBank1 IOUs. AAWheels maintains a no-interest checking account with BigBank2, currently with zero balance, however it is willing to hold up to $400,000 there. This can be represented as a credit line from AAWheels to BigBank2.

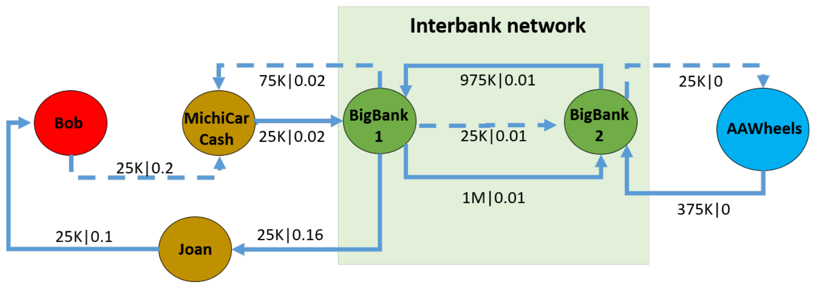
The situation as described thus far comprises part of the FCN depicted in Figure 3.1a. BigBank1 and BigBank2 are connected through the interbank network, which for present purposes I model as a complete subgraph of high-capacity credit lines with nominal interest rates (1%). In the figure, credit edges are indicated with solid arrows, and holdings of IOUs (i.e., actual loans) are indicated with dashed arrows. Whereas in the original credit network formalism credit lines and IOU holdings are treated uniformly, in FCNs we must distinguish them because IOUs accrue interest payments and unused credit lines do not. The rates of interest associated with credit and debt edges are annotated along with the capacities.

To purchase the car, Bob routes a payment of $25,000 to AAWheels. He can do so by drawing on his credit line with MichiCarCash, who in turn returns a like number of IOUs to BigBank1 (i.e., withdraws from its BigBank1 checking account), which then sends its own IOUs through the interbank network[2] to BigBank2, which credits the

[2]I do not model the actual network here, but capturing the structure of the broader financial



(a) Initial credit network



(b) Credit network after 25K payment from Bob to AAWheels through MichiCarCash

Figure 3.1: Financial credit networks before and after the transaction. Arrows indicate obligations: solid for credit and dashed for IOUs. An arrow from x to y with marking $c \mid r$ denotes an obligation from x to y with capacity c at an interest rate r. The direction of payment flow is against the arrows.

checking account of AAWheels (i.e., grants AAWheels 25,000 BigBank2 IOUs). As in the basic credit network $a \to b \to c$ example above, the source has decreased its net capacity balance by the payment amount, and the destination accrues a corresponding increase. The FCN after executing this payment is shown in Figure 3.1b.

3.2.2 Example continued: interest considerations

We can verify from the figure that intermediate nodes on the payment path experience no change in net capacity. For FCNs, however, I must also consider the effect of interest rates. MichiCarCash receives new IOUs from Bob, in exchange for returning BigBank1 IOUs. It is happy to do so though, because the Bob IOUs carry a higher interest rate than the BigBank1 (interest checking) rate. Similarly, BigBank1

system is a long-term goal motivating this research. For instance, I could model cash as IOUs from a central bank, which everyone trusts with high capacity at zero interest.

accepts the payment routing because the rate on its checking IOUs exceeds the interbank rate. Finally, BigBank2 effectively gets the interbank rate on the BigBank1 IOUs, and pays no interest on the checking account of AAWheels. The payment path satisfies *interest rate monotonicity*, and so I deem it *feasible*; no agent routes this payment at a loss.

I next consider other potential payment paths in this network. Bob has a friend Joan, who also (coincidentally) trusts him for $25,000 and requires only 10% interest. Joan, however, has no bank deposits, and her only way to route a payment through the banking system is to use her BigBank1 credit card, which carries a 16% interest rate and $25,000 credit limit. Routing a payment through Joan at the ***contract rate*** (interest rates associated with issued credit lines) would violate monotonicity, and is therefore infeasible. However, if Bob were to borrow from Joan at a higher rate (say 16%), she would cover her credit card interest and still offer Bob a better payment deal than he is getting from MichiCarCash. This keeps everyone satisfied, so I consider paths that use credit lines above contract rates to be feasible as well.

Suppose instead of this credit line, Bob had been holding 25,000 IOUs from Joan at a 10% interest rate. Joan cannot be expected to take these IOUs back to route a payment that will cost her a higher rate. Moreover, unlike the situation with credit lines, there is no obvious way to restore monotonicity by increasing a rate. Bob has no means to compensate Joan other than by returning her IOUs (e.g., he cannot route her a side payment), which merely saves her interest at the specified rate. I therefore consider interest on IOUs to be fixed at the rate at which they were incurred, and disallow payments along paths where a debt link would violate interest rate monotonicity.

3.3 Model

Credit networks are formally specified by a capacitated graph $G = (V, E)$ where vertices represent agents, and edges represent credit relationships. A directed edge $e = (u, v)$ of capacity $c(e)$ indicates that u is willing to lend up to $c(e)$ units to v. Dandekar et al. [17] observed that to determine whether agent s can pay agent t a sum of $X > 0$ units, one needs only check whether there exists an $s - t$ flow of value X in the network derived by flipping the direction of edges in G.

To extend the basic framework with interest rates, we must first capture the additional constraint that intermediary agents not incur a net interest cost. Second, the introduction of interest requires that we distinguish capacity on outstanding credit lines from capacity based on actually incurred debts (the ***IOU holdings***).

3.3.1 Credit networks with interest rates

Formally, a ***financial credit network*** (FCN: credit network with interest rates) is specified by a directed network $G = (V, E)$. Each edge $e \in E$ is associated with three values. The first value is e's obligation type, indicated by $\tau : E \rightarrow \{\text{credit}, \text{iou}\}$. Second is the edge's capacity, given by the function $c : E \rightarrow \mathbb{R}^+$. As in the interest-free setting, edge e's capacity indicates the credit limit associated with that edge (if $\tau(e) = \text{credit}$), or the number of IOUs held, (if $\tau(e) = \text{iou}$). Finally, a third value $r : E \rightarrow \mathbb{R}^+$ represents the edge's ***contract interest rate***. The values τ, c, r at an edge e can be interpreted as follows. An edge $e = (u, v)$ with $\tau(e) = \text{credit}$, means that u is willing to lend $c(e)$ units to v at an interest rate of $r(e)$. Edge e with $\tau(e) = \text{iou}$ means that u owes $c(e)$ units to v, on which v is charging an interest rate of $r(e)$.

One reason that credit edges must be distinguished from iou edges is to assess periodic interest obligations. Another is that credit lines provide greater flexibility on interest rates than do IOU holdings. For a credit line, willingness to lend at an interest rate $r(e)$ implies willingness to lend at any interest rate $r' > r(e)$. In contrast, the interest rate on debt represented by an IOU edge is fixed once the debt is incurred.

This flexibility turns out to be valuable for maximizing the liquidity of the network. In particular, our model assumes that intermediaries along a payment path will not route a payment at a loss. This amounts to requiring that payments be routed along paths of *monotonically nonincreasing* interest rates. By paying a larger interest rate on one of its immediate credit lines, an agent s may be able to route a payment to another agent t that would otherwise be infeasible.

In describing our routing algorithm, it is convenient to refer to the ***reverse network*** of G where edges point in the direction that payments occur, rather than the direction of credit obligations. Define $G^\dagger = (V, E^\dagger)$, the network identical to G but with edges flipped. When applied to edges in $E^\dagger$, the functions τ, c, and r take the same values as they would on the unflipped edges in E. We also define for each $v \in V$, $In(v) = \{(u, v) \mid (u, v) \in E^\dagger\}$ and $Out(v) = \{(v, u) \mid (v, u) \in E^\dagger\}$. Note that G is in fact a multigraph, as there may be many edges between two vertices, differing in types or interest rates.[3]

3.3.2 Payment routing problem

Given a financial credit network $N = \{G, \tau, c, r\}$, we now define what it means for a payment of $X > 0$ units from agent s to t to be ***feasible***. We call the problem of determining whether a payment is feasible the ***payment routing problem***.

As in an interest-free credit network, we demand that there exists an $s - t$ flow $f : E^\dagger \to \mathbb{R}^+$ of value at least X in $G^\dagger$, where f respects the capacity constraints denoted by c. In order for an FCN payment to be feasible, we also impose requirements on the interest rates. Informally, we demand that payments between s and t flow along

[3]Thus, the use of the functions $\tau(e), r(e), c(e)$ is a slight abuse of notation. A single pair $e = (u, v)$ may, for example, be assigned multiple types. An edge in G is more accurately written as a 5-tuple (u, v, c, r, τ). Nevertheless, for clarity, we write τ, r, and c unless this distinction is important.

edges of non-increasing interest rate, which we call the ***monotonicity condition***. This condition is complicated by the fact that realized interest rates are not always equal to contract rates. Whereas the contract interest rates on iou edges are fixed, rates on credit edges can be increased to satisfy monotonicity. Payments that respect this invariant are said to have ***valid realized interest rates***.

Let a sequence of $s-t$ paths $P = (P^{(1)}, \ldots, P^{(K)})$ and corresponding nonnegative quantities $F = (f^{(1)}, \ldots, f^{(K)})$ be a ***consistent decomposition*** of a flow f if for any edge e, $f(e) = \sum_{k=1}^{K} \mathbf{1}[e \in P^{(k)}] f^{(k)}$. In other words, f can be thought of as the finite union of paths in $G^{\dagger}$, where each path $P^{(k)}$ is assigned flow value $f^{(k)}$.

For each such path $P^{(k)}$, which we take as a set of edges, we can define a function $\hat{r}^{(k)} : P^{(k)} \to \mathbb{R}^{+}$ which assigns a ***realized interest rate*** to each edge along the path. For example, if edge (u, v) belongs to path $P^{(k)}$, $\hat{r}^{(k)}((u, v))$ corresponds to the interest rate charged to u by v on the $f^{(k)}$ units of payment routed along edge e in path $P^{(k)}$. Consistent decompositions can be used to state the aforementioned monotonicity and validity conditions more formally.

Definition III.1 (Validity)**.** Given a consistent decomposition P, F, for some flow f, and realized interest rate s $\{\hat{r}^{(k)}\}$, we say that the realized interest rates are ***valid*** if, $\forall k \in \{1, \ldots, K\}$ and $e \in P^{(k)}$, $\hat{r}^{(k)}(e) = r(e)$ if $\tau(e) = \text{iou}$ and $\hat{r}^{(k)}(e) \geq r(e)$ if $\tau(e) = \text{credit}$.

Definition III.2 (Monotonicity)**.** Given a consistent decomposition P, F, for some flow f, and realized interest rates $\{\hat{r}^{(k)}\}$, we say that the realized interest rates are *monotonic* if, $\forall k \in \{1, \ldots, K\}$ with $P^{(k)} = (e_1^{(k)}, e_1^{(k)}, \ldots, e_{n_k}^{(k)})$, and $i \in \{0, \ldots, n_k - 1\}$, $\hat{r}^{(k)}(e_i) \geq \hat{r}^{(k)}(e_{i+1})$.

Suppose there exists an $s-t$ flow f of value X that admits a decomposition P, F with valid and monotonic realized interest rates $\{\hat{r}^{(k)}\}$. A payment of X units can be executed by routing, for each k, $f^{(k)}$ units along the $s-t$ path $P^{(k)}$. Assigning each edge e on that path an interest rate of $\hat{r}^{(k)}(e)$ ensures that no agent along the path $P^{(k)}$ is routing this payment at a loss, as a consequence of monotonicity. Finally, the validity of $\hat{r}^{(k)}$ ensures that the realized interest rates are consistent with existing rates on IOUs, and no smaller than the contract rate on credit lines employed.

Definition 1 (Feasible Payment). For FCN N with $s, t \in V$, we say that a payment of amount $X > 0$ from source s to destination t is *feasible* if there exist (1) an $s - t$ flow $f : E^\dagger \to \mathbb{R}^+$ of at least X, in the standard flow network defined by $G^\dagger$ and c, and (2) a *consistent decomposition* P, F of f, and corresponding *realized interest rates* $\{\hat{r}^{(k)}\}$ that are *valid* and *monotonic*.

3.4 Payment routing algorithm

Given an instance of the payment routing problem (N, s, t, X), our goal is to find a flow f of value at least X in $G^\dagger$, and corresponding realized interest rates that are valid and monotonic, or output that no such flow exists. In general, realized interest rates may be any nonnegative real number. We start by restricting the rates to come from some finite set $\mathcal{I} = \{I_1, \ldots, I_l\} \subset \mathbb{R}^+$. We show that this ***$\mathcal{I}$-restricted payment routing problem*** can be solved via linear programming. We then observe that taking $\mathcal{I}$ to be the set of contract interest rates $\mathcal{I} = \{r(e) \mid e \in E\}$ suffices to solve the *unrestricted* payment routing problem. In other words, payment (N, s, t, X) is feasible if and only if it is feasible with realized interest rates restricted to the set of initial contract rates.

3.4.1 $\mathcal{I}$-restricted payment routing: monotonicity

The key construct for solving the $\mathcal{I}$-restricted routing problem is a function $f' : E^\dagger \times \mathcal{I} \to \mathbb{R}^+$, which we call an ***interest-flow***. We can think of f' as indicating for each edge $e \in E^\dagger$ and interest rate level $I \in \mathcal{I}$ the amount of payment along edge e routed at a realized interest rate level I. Cast in this way, the problem is reminiscent of a *multicommodity flow* [22], where payment routed at an interest rate I corresponds to a distinct commodity. Such problems are known to be NP-hard. In our problem, however, a unit of flow entering a vertex v at interest rate I can exit as a unit of flow at any interest rate $I' \leq I$ (i.e., monotonicity). This relaxation proves crucial for developing a polynomial-time algorithm.

We say an interest flow is *valid* in the analogous way to realized interest rates. Namely f' is valid if for any e, I, $f'(e, I) \geq 0$ only if $\tau(e) = \text{iou}$ and $I = r(e)$ or $\tau(e) = \text{credit}$ and $I \geq r(e)$. The main result of this section establishes that the monotonicity of the realized interest rates can be compactly represented as a collection of local inequalities on the interest-flow. We see that monotonicity is equivalent to the requirement that for every interest rate level I, and vertex v, the flow into v at interest rates $I' \leq I$ is no more than the flow out of v at interest rates $I' \leq I$.

Lemma 1. Let $f : E^{\dagger} \to \mathbb{R}^{+}$ be an arbitrary $s - t$ flow. f admits a consistent decomposition, along with a set of $\mathcal{I}$-restricted, valid, monotonic realized interest rates if and only if there exists a valid interest-flow satisfying the following conditions.

$$\text{For every } e \in E^{\dagger}\text{:} \qquad \sum_{I \in \mathcal{I}} f'(e, I) = f(e), \text{ and} \tag{3.1}$$

for every $I \in \mathcal{I}$ and vertex $v \in V, v \notin \{s, t\}$:

$$\sum_{e \in In(v)} \sum_{I' \in \mathcal{I}, I' \leq I} f'(e, I') \leq \sum_{e \in Out(v)} \sum_{I' \in \mathcal{I}, I' \leq I} f'(e, I'). \tag{3.2}$$

Proof. In the first direction, we suppose that f admits a consistent decomposition with valid, monotonic interest rates, then show that there exists a valid interest-flow satisfying the conditions (3.1) and (3.2). Fix an $s - t$ flow f, a consistent decomposition F, P, and realized interest rates $\{\hat{r}^{(k)}\}$. Define $f'(e, I) \equiv \sum_{k=1}^{K} \mathbf{1}[e \in P^{(k)}]\mathbf{1}[\hat{r}^{(k)}(e) = I] f^{(k)}$. That is, $f'(e, I)$ simply aggregates the flow values along edges e that were assigned interest rate I. We now check condition (3.2). Fixing a vertex

v, and interest rate I,

$$\sum_{e \in In(v)} \sum_{I' \leq I} f'(e, I') = \sum_{e \in In(v)} \sum_{I' \leq I} \sum_{k=1}^{K} \mathbf{1}[e \in P^{(k)}] \mathbf{1}[\hat{r}^{(k)}(e) = I'] f^{(k)}. \tag{3.3}$$

If there exists some path $P^{(k)}$ containing an edge (u, v) with $\hat{r}^{(k)}((u, v)) = I'$, then the very next edge (v, w) in $P^{(k)}$ must satisfy $r^{(k)}((v, w)) \leq I'$, by monotonicity. Thus, for every summand in the right hand side of (3.3) equal to $f^{(k)}$, there exists a distinct summand in $\sum_{e \in Out(v)} \sum_{I' \leq I} \sum_{k=1}^{K} \mathbf{1}[e \in P^{(k)}] \mathbf{1}[\hat{r}^{(k)}(e) = I'] f^{(k)}$ also equal to $f^{(k)}$. Since $\sum_{e \in Out(v)} \sum_{I' \leq I} \sum_{k=1}^{K} \mathbf{1}[e \in P^{(k)}] \mathbf{1}[\hat{r}^{(k)}(e) = I'] f^{(k)} = \sum_{e \in Out(v)} \sum_{I' \leq I} f'(e, I')$, condition (3.2) is satisfied.

From the definition of consistent decomposition, we know that $f(e) = \sum_{k=1}^{K} \mathbf{1}[e \in P^{(k)}] f^{(k)} = \sum_{I \in \mathcal{I}} \sum_{k=1}^{K} \mathbf{1}[e \in P^{(k)}] \mathbf{1}[\hat{r}^{(k)}(e) = I] f^{(k)}$ which is equal to $\sum_{I \in \mathcal{I}} f'(e, I)$ by how we have defined f', and so (3.1) is satisfied as well. Finally, the validity of f' follows immediately from the validity of the realized interest rates.

In the other direction, fix f, and let f' be a valid interest-flow satisfying conditions (3.1) and (3.2). We use f' to reconstruct a consistent decomposition P, F of f with valid and consistent interest rates. Consider a vertex $v \in V$. We assign interest rates to the flow entering and exiting v according to f'. That is, for each edge e containing v, with flow value $f(e)$, we take $f'(e, I)$ units of that flow and assign it a realized interest rate of I.

If interest rates are assigned in this manner, all flow entering v can be routed out of v while respecting monotonicity. In particular, order the interest rates $I_1 < \cdots < I_l$. (3.2) implies $\sum_{e \in In(v)} f'(e, I_1) \leq \sum_{e \in Out(v)} f'(e, I_1)$ which in turn implies that all incoming flow at interest rate I_1 can be routed out of some edge at rate I_1. Now fix some I_{k-1} and suppose for induction that there is a way to route all incoming flow at $I' \leq I_{k-1}$ out of v while respecting monotonicity. To route all incoming flow at level $I' \leq I_k$, we first assign all flow entering v at level $I' \leq I_{k-1}$ to outgoing edges, which by induction we can do while respecting monotonicity. To monotonically route the $\sum_{e\in In(v)} f'(e, I_k)$ units of flow entering at exactly level I_k there needs to be enough remaining capacity on the outgoing edges at interest rate $I' \leq I_k$. In other words, it needs to be the case that $\sum_{e\in In(v)} f'(e, I_k) \leq \sum_{e\in Out(v)} \sum_{I'\leq I_k} f'(e, I') - \sum_{e\in In(v)} \sum_{I\leq I_{k-1}} f'(e, I')$. Rearranging we get

$$\sum_{e\in In(v)} \sum_{I'\leq I_k} f'(e, I') \leq \sum_{e\in Out(v)} \sum_{I'\leq I_k} f'(e, I')$$

which is implied by (3.2).

Thus, f can be decomposed into P, F, where the interest rates $\{r^{(k)}\}$ assigned along the paths are derived from the above procedure. By (3.1) all flow is accounted for, and since f' is valid, $\{r^{(k)}\}$ is also valid. □

3.4.2 A linear program for $\mathcal{I}$-restricted payment routing

With Lemma 1 in hand, we can derive a linear program for solving the $\mathcal{I}$-restricted payment routing problem (Algorithm MaxInterestFlowLP). The first four conditions of the LP make it so that the ordinary flow f derived from the interest-flow f' (via (3.1)) is both a valid flow and routes X units of payment from s to t. The remaining conditions specify that f' also induces valid, monotonic, interest rates, which is a direct consequence of Lemma 1.

Theorem III.3. *Given an instance (N, s, t, X) of the $\mathcal{I}$-restricted routing problem, the payment is feasible if and only if the LP solved by Algorithm* MaxInterestFlowLP *has a solution of value at least X. Furthermore, the LP has a total of $\mathcal{O}(|\mathcal{I}||E|)$ variables and $\mathcal{O}(|\mathcal{I}|(|V| + |E|))$ constraints.*

3.4.3 A solution to the unrestricted problem

We now prove that the payment (N, s, t, X) is feasible with unrestricted interest rates if and only if there is a feasible payment for the $\mathcal{I}$-restricted routing problem, when taking $\mathcal{I} = \{r(e) \mid e \in E\}$, the set of initial contract interest rates. As a consequence, Algorithm MaxInterestFlowLP gives us an algorithm for the unrestricted payment routing problem.

Algorithm MaxInterestFlowLP:

$$\max_{f_{e,I},X} X \text{ s.t.} \tag{Objective}$$

$$\forall e \in E^{\dagger} : f_e = \sum_{I \in \mathcal{I}} f_{e,I} \tag{Total Flow}$$

$$X + \sum_{e \in In(s)} f_e = \sum_{e \in Out(s)} f_e, \qquad \sum_{e \in In(t)} f_e = X + \sum_{e \in Out(t)} f_e \tag{Flow Value}$$

$$\forall v \notin \{s,t\} : \sum_{e \in In(v)} f_e = \sum_{e \in Out(v)} f_e \tag{Flow Conservation}$$

$$\forall v \notin \{s,t\}, \forall I : \sum_{e \in In(v)} \sum_{I' \leq I} f_{e,I'} \leq \sum_{e \in Out(v)} \sum_{I' \leq I} f_{e,I'} \tag{Monotonicity}$$

$$\forall e \in E^{\dagger}, \tau(e) = \text{credit}, \forall I < r(e) : f_{e,I} = 0$$

$$\forall e \in E^{\dagger}, \tau(e) = \text{iou}, \forall I \neq r(e) : f_{e,I} = 0 \tag{Valid Interest Rates}$$

$$\forall e \in Ef, I : 0 \leq f_{e,I}, \forall e : 0 \leq f_e \leq c(e) \tag{Capacity}$$

First note that if there is no solution to the general (non-restricted) payment routing problem, then clearly there cannot be a solution to the $\mathcal{I}$-restricted payment routing problem (for any $\mathcal{I}$). The following lemma states that the other direction holds as well, when taking $\mathcal{I}$ to be the original set of contract interest rates.

Lemma 2. Given an instance of the payment routing problem, let $\mathcal{I} = \{r(e) \mid e \in E\}$. There exists a feasible payment for the payment routing problem if and only if there exists a feasible payment for the $\mathcal{I}$-restricted payment routing problem.

Proof. As described above, one direction is immediate, and so we prove that the existence of a solution to the non-restricted payment routing problem implies a solution to the $\mathcal{I}$-restricted payment routing problem.

Let f be the flow solution to the non-restricted problem, with consistent decomposition P, F, and valid, monotonic realized interest rates $\{\hat{r}^{(k)}\}$. We define a new set of realized interest rates $\{\hat{r}_{\mathcal{I}}^{(k)}\}$ which take values only in $\mathcal{I}$, but are still valid and monotonic. f is still a flow for the restricted problem, and F, P a consistent decomposition, so this is sufficient to prove the lemma.

To define $\{\hat{r}_{\mathcal{I}}^{(k)}\}$, fix a k, and consider the path $P^{(k)} = (e_1^{(k)}, \ldots, e_{n_k}^{(k)})$. For any edge e_i such that $\tau(e_i) = \text{iou}$, we leave the interest rate unchanged. That is, we set $\hat{r}_{\mathcal{I}}^{(k)}(e_i) = \hat{r}^{(k)}(e_i)$ which is also equal to $r(e_i)$ since $\{\hat{r}^{(k)}\}$ are valid.

Next consider edges e_i such that $\tau(e_i) = \text{credit}$. At a high level, we define $\hat{r}_{\mathcal{I}}^{(k)}(e_i)$ by taking $r^{(k)}(e_i)$ and increasing it to the contract rate of the preceding iou edge along the path. In detail, if e_i and e_j, for $i < j$, are consecutive iou edges in $P^{(k)}$, then for all $e_{i'}$, $i < i' < j$ we define $\hat{r}_{\mathcal{I}}^{(k)}(e_{i'})$ by letting $\hat{r}_{\mathcal{I}}^{(k)}(e_{i'}) = r(e_i)$. The validity and monotonicity of $\hat{r}^{(k)}$ tells us that $r(e_i) = \hat{r}^{(k)}(e_i) \geq \hat{r}^{(k)}(e_{i+1}) \geq \cdots \geq \hat{r}^{(k)}(e_j) = r^{(k)}(e_j)$. And therefore $r(e_i) = \hat{r}_{\mathcal{I}}^{(k)}(e_i) = \hat{r}_{\mathcal{I}}^{(k)}(e_{i+1}) = \cdots = \hat{r}_{\mathcal{I}}^{(k)}(e_{j-1}) \geq \hat{r}_{\mathcal{I}}^{(k)}(e_j) = r(e_j)$. Similarly, if e_i is the last iou edge, we set $\hat{r}_{\mathcal{I}}^{(k)}(e_{i'}) = r(e_i)$ for $i < i' \leq n_k$. Finally, if e_j is the first iou edge, we set $\hat{r}_{\mathcal{I}}^{(k)}(e_{i'}) = \max\{r(e_1), r(e_2), \ldots, r(e_j)\}$, and therefore $\hat{r}_{\mathcal{I}}^{(k)}(e_1) = \cdots = \hat{r}_{\mathcal{I}}^{(k)}(e_{j-1}) \geq \hat{r}_{\mathcal{I}}^{(k)}(e_j) = r(e_j)$. In each of these cases $\hat{r}_{\mathcal{I}}^{(k)}(\cdot)$ is monotonically nonincreasing over the subsequence in question, and is therefore nonincreasing along the entire path.

The $\{\hat{r}_{\mathcal{I}}^{(k)}\}$ are also valid since the realized interest rate for iou edges are unchanged (compared to $\hat{r}^{(k)}$), and the realized interest rate for credit edges only increase, so the validity of $\{\hat{r}^{(k)}\}$ also ensures that $\hat{r}_{\mathcal{I}}^{(k)}(e_i) \geq \hat{r}^{(k)}(e_i) \geq r(e_i)$ for any edge e_i. Observing that the construction ensures the each $\hat{r}_{\mathcal{I}}^{(k)}$ takes values in $\mathcal{I}$ concludes the proof. $\square$

Lemma 2 tells us that the $\mathcal{I}$-restricted problem is equivalent to the non-restricted problem when $\mathcal{I} = \{r(e) \in E\}$. Thus, we can state our main algorithmic result as a corollary of this lemma and Theorem III.3.

Corollary 1. Let $\mathcal{I} = \{r(e) \mid e \in I\}$ be the set of contract interest rates. Given an instance (N, s, t, X) of the payment routing problem, the payment is feasible if and only if the the LP solved by Algorithm MaxInterestFlowLP finds a solution of value at least X for the $\mathcal{I}$-restricted routing problem.

3.4.4 Routing multiple payments

The preceding demonstrates how to efficiently compute whether a payment is feasible for some static instance (N, s, t, X) of the payment routing problem in an FCN. To route multiple payments in sequence, we update the FCN N after each to reflect the state of obligations between agents. An illustration of such an update is provided in Figure 3.1a. Here we describe this update formally. In order to do so, we must be explicit about the fact that G contains multi-edges described by 5-tuples (u, v, τ, c, r).

A feasible payment is given by a flow f in $G^\dagger$, a consistent decomposition $F = \{f^{(1)}, \ldots, f^{(K)}\}$, $P = \{P^{(1)}, \ldots, P^{(K)}\}$, and realized interest rates $\{\hat{r}^{(1)}, \ldots, \hat{r}^{(1)}\}$. Given such a payment $N = (G, \tau, c, r)$ is updated as follows. For each $k \in \{1, \ldots, K\}$, consider each edge $e = (u, v) \in P^{(k)}$.

If $\tau(e) = \text{iou}$, the payment was routed through edge e by having v relinquish $f^{(k)}$ IOUs back to u. Thus, there exists some edge (v, u, iou, c, r) in G, for some c and r. We update this edge to $(v, u, \text{iou}, c - f^{(k)}, r)$. At the same time, if there exists a credit line from u to v represented by $(u, v, \text{credit}, c', r')$, we update this edge to $(u, v, \text{credit}, c' + f^{(k)}, r')$, (allowing $c' = 0$ if the credit line between u and v is saturated).

If $\tau(e) = \text{credit}$, the payment was routed through edge e by drawing upon a line of credit that v extends to u. Thus, there exists some edge $(v, u, \text{credit}, c, r)$ in G, for some values of c and r. We update this edge to $(v, u, \text{credit}, c - f^{(k)}, r)$. Drawing upon this credit creates a debt that u owes v at realized interest rate $r^{(k)}(e)$. Thus, if there exists an edge $(u, v, \text{iou}, c', r^{(k)}(e))$, we update this edge to $(u, v, \text{iou}, c' + f^{(k)}, r^{(k)}(e))$, otherwise we create a new iou edge given by $(u, v, f^{(k)}, r^{(k)}(e))$. Note that in both these cases, the newly created IOU is given the realized interest rate for the credit line.

3.5 Payment mechanisms

In general, there may be several feasible ways to route a payment between two agents. In this section, I discuss ***payment mechanisms***: rules for choosing flows to achieve a designated payment.

3.5.1 Choice of payment paths

In basic credit networks, the selection of payment paths has no bearing on long-term liquidity. Dandekar et al. [17] showed that if a sequence of unit flows defined by source/sink pairs $\{s_1, t_1\}, \ldots, \{s_k, t_k\}$ is feasible using corresponding payment paths $\{P_1, \ldots, P_k\}$ on a network with unit capacities, they remain feasible if P_i, for any i, is changed to an arbitrary feasible payment path P_i'. Given this invariance, and the lack of any differential costs, selecting among feasible payment paths has not been a pressing issue in the basic model without interest rates.

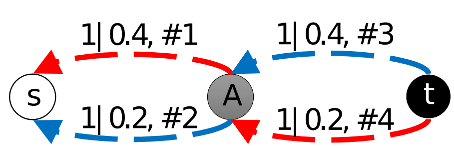


Figure 3.2: Dotted arrows represent IOUs, and edges are identified by $\#num$ labels. The max flow from s to t is two, but a unit payment on the (red) path (#1,#4) blocks any subsequent flow.

In FCNs, however, interest rates do pose differential costs, and moreover the liquidity invariance property does not hold. Consider the simple example of Figure 3.2. The max flow between s and t is 2, but if a payment of one unit is first routed along the red path, no further flow is achievable. Payment on the path comprising edges #2 and #3 in the residual FCN would be infeasible due to the interest monotonicity constraint. So in general the success of a sequence of transactions may depend on which feasible payment paths are chosen. As I show below, alternative path selection mechanisms exhibit systematically different long-term liquidity properties.

3.5.2 Mechanism definitions

One way to define alternative payment mechanisms is by refining the objective function of MaxInterestFlowLP. Instead of maximizing flow, I specify the requested flow as a constraint and insert criteria for choosing among feasible flows in the objective. For example, I could choose based on length of paths, or some function of the interest rates on the included paths.

The monotonicity constraint ensures that intermediate nodes accrue nonnegative net interest, but alternative paths may differ on the amount of positive net interest. The payment source generally pays positive interest. I term the interest associated with the first edge on a payment path the ***originating rate***. To the extent the transaction initiator has influence over paths chosen, it may be particularly concerned with minimizing this rate.

For mechanisms below, fix $\mathcal{I}$ to be the set of initial contract interest rates, and let $I^+ \triangleq \max(\mathcal{I})$ be the maximum possible interest rate in the network. Let s be the source, and $Out(s)$ the outgoing edges from s in the reverse network $G^\dagger$, as defined in §3.3.1. The mechanisms are defined by replacing the objective function of MaxInterestFlowLP with those exhibited, fixing the flow to the requested amount, and in one instance adding additional constraints. The optimization variables and remaining constraints in MaxInterestFlowLP are unchanged.

$$\min \sum_{e \in Out(s), I \in \mathcal{I}} f_{e,I} \times I \qquad \textbf{Minimize source cost (MinSrc)}$$

The MinSrc mechanism minimizes the originating rate. I place a cost equal to the interest rate that would be paid on all potential outgoing flows from the source.

$$\min \sum_{e \in Out(s), I \in \mathcal{I}} f_{e,I} \times (I + (I^{+} + 1) \times \mathbb{1}\{\tau(e) = \text{credit}\})$$

$$\textbf{Minimize credit usage (MinCred)}$$

The MinCred mechanism prioritizes use of IOUs relative to credit lines. The idea is that since credit edges have flexible realized interest rates, their capacity should be preserved for future situations. The originating rate is minimized secondarily. The flexibility provided by a credit line could enable additional payment paths by raising the originating interest rate when necessary. The $I^{+} + 1$ term is simply a way to ensure that IOU status has priority in the objective over originating rate.

$$\min \sum_{e \in Out(s), I \in \mathcal{I}} f_{e,I} \times I \qquad \textbf{Restrict to contract rates (NoMarkup)}$$

$$\forall e, I > r(e) : f_{e,I} = 0$$

For flow $f_{e,I}$ on a credit line, I refer to the difference $I - r(e)$ as the ***interest markup***. The markup is additional interest above the contract rate assessed on credit edges in order to make a payment path monotone. The NoMarkup mechanism minimizes originating rate, subject to constraints disallowing any agent from drawing on credit lines at anything other than their contract rates. NoMarkup is unique among the mechanisms I consider in that it imposes additional constraints beyond flow feasibility. This allows us to evaluate the effect of flexible markup policies (which all our other mechanisms allow) on liquidity.

$$\max \sum_{e \in E, I \in \mathcal{I}} f_{e,I} \times \mathbb{1}\{I = I^+\} \qquad \textbf{Maximize total interest paid (MaxIR)}$$

Given an FCN initially consisting of credit edges, liquidity is maximized by always assessing the highest interest rate possible. In fact, this MaxIR mechanism provides liquidity equivalent to that of basic credit networks (Theorem III.5). Note that MaxIR is sensitive to the maximum interest rate I^+, and thus is not very robust. I can peg the interest paid to an arbitrarily high rate r^* by introducing a single credit edge e' with $r(e') = r^*$, regardless of how much credit exists at reasonable rates. I include MaxIR to provide an upper bound on liquidity and demonstrate the tradeoff between liquidity and interest rates.

3.6 Steady-state liquidity analysis

With respect to a given transaction, any payment mechanism that does not restrict feasible payments (i.e., all those listed above except NoMarkup) offers the same prospects for success. As illustrated in §3.5.1, however, how a payment is routed can affect the network's configuration, which changes the prospects for subsequent transactions. Alternative payment mechanisms may affect network configurations systematically, and thus have a qualitative impact on liquidity. I measure liquidity by the long-term failure rate of transactions once the network has reached a steady-state distribution over network configurations. I call this the ***steady-state transaction failure rate***. I show in §3.6.2 that given our experimental setup, such a steady state exists.

3.6.1 Experimental setup

Our experimental setup is designed as an extension of the no-interest liquidity analysis of Dandekar et al. [17]. I initialize an Erdos-Renyi graph [21] with a chosen average degree between 5 and 35, and size of 200 nodes. Each directed edge represents a credit line with initial capacity 10. Edge orientation is a fair coin toss and interest rates are generated uniformly from a specified set $\mathcal{I}$. Namely, if I choose to have four interest rates, each edge is assigned contract rates of 0.01, 0.02, 0.03, or 0.04 uniformly, and so on for different $|\mathcal{I}|$. I generate transactions by selecting source and destination nodes uniformly at random and attempting to route 10 units of flow between them.

For each mechanism, I attempt to route transactions sequentially while recording any failures. Following a failure, I do nothing to the graph. After 9000 transaction attempts, I check the failure rate among the first 4500 against the second 4500 transactions. If the two failure rates are within 0.002 of each other, I stop and record the failure rate of the entire history as the steady-state failure rate. Otherwise, I generate another transaction and move our observation window forward by one, that is, I compare observations 2 through 4501 with 4502 through 9001. This continues until I satisfy our steady-state criterion. I perform the whole process ten times, averaging the results.

Note that for purposes of this liquidity analysis, I do not consider actual interest payments. Interest plays a role here only for transaction feasibility and selection of flows by payment mechanisms.

3.6.2 Steady-state liquidity of FCNs

A network configuration, or ***state***, is a single specification of the tuple $\{G, \tau, c, r\}$. In our simulations, the initial state is set at $\tau(e) = \text{credit}$, c maps to a constant integer, r maps to an element of a set of constant size, and G is an Erdos-Renyi graph, for all $e \in E$. The number of reachable states is in general unbounded, since a flow on an edge may take on any value between 0 and $c(e)$, leaving any real value as a possible residual capacity. However, if $|\mathcal{I}|$ is finite, payment amounts are integers, and all flows are restricted to be integral, then the number of states is finite. In all the simulations reported here, I route constant integer payment amounts under bounded $|\mathcal{I}|$. As the linear program solver software I used (CPLEX) maintains integer solutions, the set of states is finite and discrete [32].

The generation of random transactions induces a ***transition probability matrix*** between states. The resulting stochastic process is an ergodic Markov chain and therefore has a steady-state distribution.

Theorem III.4. *Let $h, h' \in T$ where T is the finite state space over states of the FCN, when routing integer valued flows of amount X. Let $P(h, h')$ be the probability of moving from state h to state h'. These probabilities are induced by the transaction probability matrix Λ which picks any source and sink node pair (s, t) with positive probability, between which a payment of amount X is attempted. This stochastic process χ is an ergodic Markov chain.*

Proof. First note that χ satisfies the Markov property. Given I are in state h, $P(h, h')$ is independent of any other future or historical state for any state h'. To calculate this probability, I can just sum the probabilities of every transaction that will get us from h to h'.

To show that χ is ergodic, first I need that h is reachable from h' with nonzero probability (irreducibility), for any $h, h' \in T$. I use the fact that any pair of nodes can be selected for a transaction with nonzero probability, since I choose the source and sink uniformly at random. So any series of pairwise transactions also has nonzero probability. In particular, neighbors in the graph can be selected with positive probability. Given h, I can always find a series of pairwise transactions between neighbors to reach h'. The reason for this is that any one-hop path can be routed ignoring the monotonicity constraint, since the source is willing to pay any originating rate. So first I can route flows on a sequence of one-hop paths that cancels all existing IOUs and returns to G to the all-credit edge graph. I can then route a sequence of transactions that result in the exact configuration of h'. So χ is *irreducible*.

A version of the ergodicity theorem says that for a finite-state irreducible Markov chain, I need only one state to be aperiodic in order for all states to be aperiodic [60]. Consider a state h, with two agents s and t such that a payment between s and t is not feasible. Since there is a non-zero probability of picking s and t for the next transaction, there is a non-zero probability that h transitions to itself. Therefore, the state h is aperiodic, which implies that the Markov chain is aperiodic. Such a state is always reachable by considering a sequence of states that fill all feasible payment paths from s to t to capacity. □

This means that after routing enough transactions, the overall probability of being in a particular state is invariant. Since failure probability is a function of network state, this probability is also invariant. Following Dandekar et al. [17], I use this steady-state failure probability as our measure of *liquidity*. I expect the procedure of §3.6.1 to yield the correct liquidity if our error tolerance is appropriately chosen. Thus, I can evaluate the liquidity when using each of our payment mechanisms. Notably, the MaxIR mechanism maximizes liquidity.

Theorem III.5. *Initialize an FCN $G = (V, E)$ such that $\forall e \in E$, $\tau(e) =$ credit. If mechanism MaxIR is used for every payment in G with contract rates $\mathcal{I}$, each transaction is infeasible iff it is infeasible under a basic (interest-free) credit network* *with the same graph structure.*

Proof. If a transaction is infeasible in the interest-free network, then adding additional constraints by including interest rates will not cause it to succeed. In the other direction, no matter how many transactions are routed on G under $\mathcal{I}$, there will never exist an interest rate on an IOU that is not I^+. That is, $\forall e \in E. \tau(e) = \text{IOU} \implies r(e) = I^+$, by definition of the mechanism. Furthermore, the mechanism always opts for the maximum markup on every credit edge. Thus, realized interest rates are constant, at the value I^+, and monotonicity is trivially satisfied. If a payment of amount X is not feasible it must be because there was no $s - t$ flow of amount X, which implies that the payment is not feasible in the interest-free network. □

3.6.3 Markup flexibility on credit lines

Figure 3.3 indicates that steady-state failure rate converges and becomes small at around degree 25 regardless of how many interest rates are available and regardless of which payment mechanism I use. The number of available interest rates is much more influential when the NoMarkup mechanism is used. Especially striking is how much allowing markups increased liquidity at lower degrees. The liquidity of MinSrc is almost as good as the basic credit network case (i.e., one interest rate) at degrees as low as 10. $|\mathcal{I}|$ has minimal effect on liquidity. For comparison, NoMarkup has almost three times the failure rate of the basic credit network at degree 10. This suggests that accounting for interest has modest effect on liquidity as long as agents are allowed to increase rates on credit lines.

3.6.4 Tradeoff between high markups and better liquidity

I already know that FCN liquidity is maximized by forcing every agent to pay the maximum possible interest rate on every edge. The relationship between higher interest rates and improved liquidity is also exhibited more generally. In Figure 3.4b, I

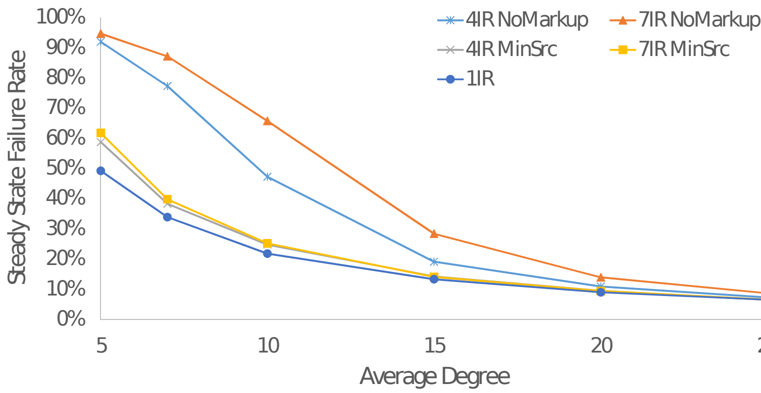
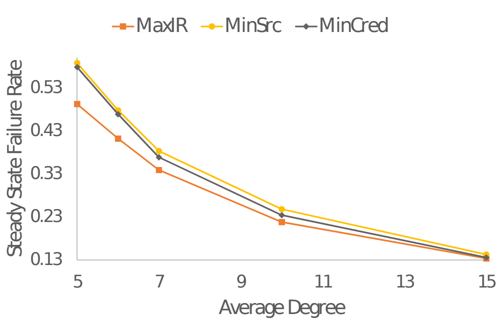
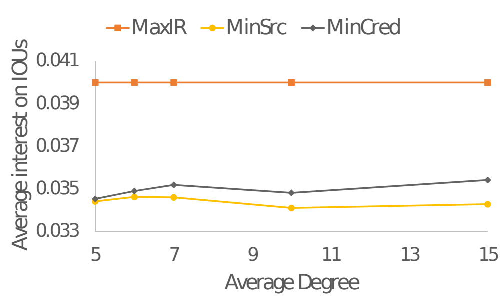


Figure 3.3: Steady-state failure probabilities for NoMarkup and MinSrc mechanisms at degrees 5 to 25, Erdos-Renyi graphs with 200 nodes. "xIR" means $|\mathcal{I}| = x$.

see that compared to MinSrc, the MinCred mechanism yields a slightly higher average interest rate on IOUs. Correspondingly, it has a slightly lower steady-state failure rate (Figure 3.4a). Many other heuristic mechanisms were tested and the relation between interest and failure rates held true. For brevity, I omit detailed descriptions, and show only average interest and failure rates (Figure 3.5). Conditional on average degree of the graph, almost all variation in liquidity among mechanisms can be explained by the average interest rate paid by agents when the mechanism is employed. The tradeoff is clear: better liquidity comes with higher interest rates. This tradeoff diminishes when the graph is well-connected, as almost all transactions succeed.



(a) Liquidity



(b) Interest rate level

Figure 3.4: Failure rate and average interest rate level over all IOUs in the final graph, obtained after reaching steady state. Starting from degree 7, difference in failure rate between MinSrc and MinCred is statistically significant.

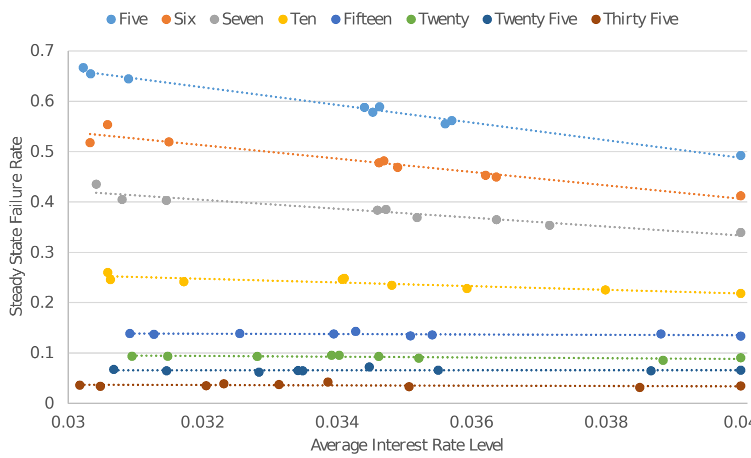


Figure 3.5: Each point represents a heuristic payment mechanism's steady state failure rate at a certain average interest rate level on Erdos-Renyi graphs of fixed average degree. Each dotted line represents a collection of mechanisms whose liquidity was tested at a fixed average degree.

The explanation for this is straightforward. Given the monotonicity constraint, an IOU at low interest restricts the completing paths. Thus, higher interest on IOUs promotes greater transaction capacity for the network.

3.7 Strategic routing game

The relationship between interest rates and liquidity presents a strategic dilemma for choice among mechanisms. A low originating interest rate minimizes costs for the payer, but imposes an externality on the rest of the network in the form of reduced liquidity. I explore the conflict between individual incentives and global effectiveness in FCNs by defining a *game*, where the source of each transaction chooses a payment mechanism. I evaluate the game by simulation, over a range of network settings and payment mechanisms similar to those explored in the liquidity study above.

Our scenario employs 100 agents initialized to a random-graph FCN, who attempt to execute a randomly generated sequence of transactions. Specifically, the steps of each simulation run are: (1) Assign strategies to agents according to a specified *strategy profile*. Strategies in this context are simply payment mechanisms as defined in §3.5.2. (2) Generate a directed Erdos-Renyi graph with 100 nodes and a specified average degree. Assign agents to these nodes. Set the capacity on each edge to 100, and the contract interest uniformly at random from the set $\{0.01, 0.02, 0.03, 0.04\}$. Initialize the payoff for each agent to zero. (3) Choose a source and sink uniformly at random. (4) Attempt to route 100 units from source to sink using the source's fixed strategy. If successful, update the FCN based on the chosen flows, and increment the source's payoff by the amount $xVal$. (5) Repeat steps (3) and (4) 2000 times. (6) Calculate net interest income for each node, and add this to the corresponding payoff.

The interest income is defined under the assumption that IOUs resulting from the transaction sequence remain outstanding for one period. Let $IN_x = \{e_{uv} \in E : v = x, \tau(e_{uv}) = \text{iou}\}$ denote the set of incoming IOUs of node x, and similarly $OUT_x = \{e_{uv} \in E : u = x, \tau(e_{uv}) = \text{iou}\}$ the outgoing IOUs of x. Node x's net interest income is then $\sum_{e \in IN_s} c(e) \times r(e) - \sum_{e \in OUT_s} c(e) \times r(e)$. An agent's overall payoff is the cumulative value from successful transactions plus net interest income.

In our analysis the only scenario parameter I vary is the average degree of the initial random graph.[4] I explored three settings: 8, 15, and 22.

Since promoting liquidity comes with positive externalities, I would not expect to see social welfare maximized in equilibrium. Our hypothesis was that agents would choose to pay a lower originating rate than is socially optimal. To evaluate this hypothesis, I performed simulations over strategy profiles combining three mechanisms: MinSrc, MinCred, and MaxIR. These represent three points on the spectrum between minimizing interest cost (MinSrc) and maximizing liquidity (MaxIR).

To evaluate a profile, I average over at least 1500 simulation runs, to produce accurate payoff estimates. This takes 50-100 core-hours, depending on average node

[4] I explored two $xVal$ settings, but observed no interesting differences, so report results only for $xVal = 10$.

	Pure Strategy Profile Payoffs				
Average Degree	MaxIR	MinCred	MinSrc	PSNE	Dominated Strategies
8	145	139	132	MinSrc, MinCred	MaxIR
15	180	178	177	MinCred	MaxIR
22	193	192	192	MinCred	MaxIR, MinSrc

Figure 3.6: Payoff and symmetric equilibria information for varying average degree.

degree. Even with only three strategies, exhaustive simulation of profiles for 100 players is not feasible. Exploiting symmetry, there are 5151 profiles in the full game, which would take too long to cover with available resources. I therefore employed *deviation preserving reduction* [71] to approximate the 100-player game with a reduced 4-player version. This required simulation of only 30 full-game (100-agent) profiles to estimate each game model.

Among the symmetric pure-strategy profiles (i.e., where all players choose the same strategy) shown in Figure 3.6, the profile with the highest social welfare (i.e., sum of all payoffs) in every setting is that where all players play MaxIR. The MinCred profile had lower social welfare, followed by MinSrc. MaxIR was dominated at all settings. I searched for symmetric Nash equilibria using replicator dynamics [59]. I found only pure-strategy Nash equilibria (PSNE) in our experiments, reflecting a coordination benefit for adopting a uniform mechanism.

MinSrc and MinCred exhibited similar payoffs in most profiles containing both strategies. At low average degree (8), I found PSNE consisting of both strategies, but at high average degree (15, 22) only MinCred is an equilibrium. At average degree 22, the Mincred PSNE was confirmed statistically using the bootstrapped regret methodology outlined by Wiedenbeck et al. [69]. No other games exhibited solutions with zero regret at the 95th percentile.

Though the socially optimal outcome (i.e., all agents playing MaxIR) is strategically unstable, the fact that MinCred is competitive with MinSrc is encouraging. From Figure 3.4 I know that MinCred trades off higher interest rate for better liquidity compared to MinSrc. This suggests that agents have some willingness to pay higher interest, in conjunction with the coordination benefit, to achieve higher liquidity.

3.8 Summary

I have extended the credit-network model of distributed trust with interest rates and a distinction between credit lines and debt. FCNs provide a standard means to represent incentives for extending credit, and thereby also support direct modeling of real-world financial relationships. The extension to support interest rates raises new issues in routing payments over the network, in the form of constraints to assure the willingness of intermediaries to participate. These are further complicated by a distinction between credit lines and debt, namely that the former may admit flexibility in rates whereas the latter are more rigid. I formalize these constraints, and develop an efficient algorithm for determining a feasible payment flow, as well as a range of mechanisms for choosing among feasible flows.

Given the plethora of routing policies and options, I perform computational studies to explore the implications of alternative mechanisms on network liquidity. I find that performance can vary greatly across mechanisms, and moreover that there may be a strategic tension between preferences of the transaction source and global network effectiveness. I explore this issue through empirical game-theoretic analysis, and find that while this tension does exist, there is evidence that agents will not simply maximize myopic gain and instead consider overall network liquidity in equilibrium.

3.9 Economic modeling using FCNs

In this chapter I have focused on outlining how the ability to make payments is affected by network structure and choice of payment mechanism. I showed through experiments that the failure rate of transactions is drastically reduced by the use of interest rate markups, but equally important to constructing economic models is the amount of the markup. Given an FCN, a payment mechanism, and a payment, the amount of markup given by Algorithm MaxInterestFlowLP reflects in some sense the cost of providing liquidity (remember that liquidity is defined as the likelihood of a successful transaction). A real-world analog of this cost is the liquidity haircut that firms typically take in order to sell illiquid assets (such as long term debt or mortgages). This is the ***liquidity cost*** of making the payment. The experiments done in this chapter show that liquidity costs in an FCN consist of:

1. Remoteness of the payment destination (i.e., how many types of IOUs the destination will accept as payment)

2. Liquidity of the payment source's assets

3. Average degree in the FCN

4. Choice of payment mechanism

All of 1-3 reflect determinants of liquidity cost in real debt markets. An example of 1) is a vendor not accepting personal checks as payment, or requiring a fee. 2) can, for example, reflect how much cash the payment source decides to carry; cash will never incur a liquidity cost because it can be used to make any payment. 3) is more of a macroeconomic condition reflecting overall competition for servicing the payment. High degree implies more possible payment paths, out of which a low markup one can be chosen by the source. 4) does not correspond to real debt markets directly, so I will simply assume all payment source agents are choosing the markup-minimizing mechanism going forward.

In order to establish the basic payments properties of FCNs, two economic activities have been abstracted away in this chapter. First is the motivation for making payments. What good or service is being paid for, and what is the decision process behind making these payments? Second is the origin of the underlying trust modeled by the FCN. Why should there be trust, and how does it form? These questions represent the opportunity to build an agent-based model of the financial system using FCNs, which I proceed to do in Chapter IV.

CHAPTER IV

An agent-based model of the credit cycle

In Chapter IV we evaluated Basel-style leverage regulations using a dynamic model of the asset market. In this chapter, I introduce a new agent-based model that enables further evaluation of such regulation, considering evolving debt relations among banks. Debt as an investment is qualitatively different from investing in single-asset markets since each debt contract entitles the holder to payments from a specific borrower. Thus, in a system with n banks, there are $n-1$ different types of debt for each creditor to invest in (lenders cannot borrow from themselves). To accommodate this heterogeneity, creditors offer highly variable terms based on the perceived riskiness of borrowers. A model of debt dynamics must be able to express these terms. The original FCNs allow creditors to specify a single term feature on debt contracts: the interest rate. I introduce ***extended FCNs*** (or EFCNs) in order to model debt contracts that also specify a ***collateral rate***. This allows creditors to specify how much of their debt they will automatically recoup if a borrower defaults. Setting this rate is an important tactic for lenders to control risk, and allows for two broad types of debt to be issued: ***secured debt***, which has a high collateral rate, and ***unsecured*** debt, which has a low collateral rate. These have been the two basic types of debt for decades, serving crucial and disparate funding needs [4]. EFCNs allow them to be modeled using a network formation process.

By constructing a behavioral model for banks as well as a payment mechanism compatible with EFCNs, I examine experimentally how the collective debt issued by lenders can contribute to systemic risk, and how Basel regulations affect this risk. The model focuses on how the network of obligations among banks changes over time due to profit-maximizing behavior from banks. There are two stages to bank behavior. In the first stage, banks choose over a set of heuristic strategies. Specifically, these strategies include whether to offer secured or unsecured debt as well as how much to tolerate risk when deciding on investment mix and amount. Once a heuristic strategy is chosen, it cannot be changed.

In the second stage, banks execute their heuristic strategy by observing market conditions (such as the cost of funding debt and assets, their own access to funding, and the demand for debt) and change the mix of cash, debt, and assets they hold in their investment portfolios in response. Banks borrow from each other in order to satisfy investment or liquidity needs. This means that the availability of funding as well as the exposure to risks of default are endogenously generated. As a result of this behavior, a ***credit cycle*** emerges in our model as aggressive banks take on leverage until creditors become leery of their exposure to risk, and withdraw their credit lines. This causes either a spate of defaults or a gradual deleveraging, luring creditors to reopen credit lines, starting the cycle anew.

Similarly to the evaluation of Basel in Chapter II, the effect of regulation can be broken down into two parts. First, any particular fixed strategy implemented by a bank will express itself differently in specific actions under the constraints of a new regulation. This is a mechanical effect that can be seen by introducing regulation into the model while fixing banks' choice of heuristic strategy. Second, banks may change the strategies they employ in response to regulation. To analyze these changes, I use empirical game-theoretic analysis to find Nash equilibrium strategy profiles.

Overall, results of this analysis indicate that Basel regulations decrease losses on debt, default rates, and interest rate volatility significantly. The effect on profit levels for banks is mixed. There is some evidence to suggest that Basel induces banks to shift towards more risky strategy profiles, but the majority of equilibria under Basel are qualitatively similar to the unregulated ones. It appears that Basel is able to attenuate the excesses of the credit cycle without inducing a significant strategic response from banks.

4.1 Introduction

One of the driving forces behind the 2008 financial crisis was the interconnectivity between banks [39]. The failure of a single bank, for any reason, can influence the survival of other banks. The failure of Lehman Brothers is the most prominent example of this in recent history. Directly preceding the crisis, Lehman suffered large losses due to its exposure to subprime mortgages. But Lehman on its own was not deemed to control a systemically important amount of assets, and therefore it was not within the purview of regulators to offer a bailout. This decision ultimately led to a deepening of the crisis. The network of credit relationships between banks enabled this deepening in two ways.

First, since other banks were depending on payments owed to them by Lehman to continue operations, many had difficulties meeting obligations to their own creditors. Some of these banks were not able to meet all of their own obligations, in turn causing some of their creditors to become financially stressed. This phenomenon is called ***financial contagion***, and it propagates through tangible debt obligations that must be repaid on penalty of default. These debt obligations are collectively called the ***interbank debt network*** in this chapter.

Second, the same market conditions that led to the failure of Lehman Brothers caused creditors to revise upwards their assessments of the risk of holding debt from each other. In addition, Lehman Brothers themselves could no longer provide credit.

Thus, right as banks needed emergency funding, it became more expensive (or impossible) to borrow funds. This phenomenon is dubbed a ***credit freeze***. Credit freezes take place on the network of *potential* debt contracts at terms set by lenders. These potential contracts are called ***credit lines*** in this chapter, and collectively they form the ***interbank credit network***. It represents the confidence that lenders have that debt will be repaid.

The leverage cycle models employed in Chapter II predict that small external shocks to asset prices can produce amplified swings due to a differential proclivity among market participants to take leverage. This story fits quite well with what happened in the subprime mortgage market that was responsible for Lehman's initial troubles. But in these leverage cycle models, banks interact with each other only through transactions in an asset market. Without explicit treatment of dynamics in the credit market, important systemic risks such as credit freezes and financial contagion are overlooked.

In this chapter I rectify this by modeling the formation of both of the interbank debt network and interbank credit network within the EFCN framework. Both networks form as a result of decisions that banks make to maximize profits. In particular, banks strategize over how and to what extent to manage investment risk. Each bank's profit is a function of actions taken by all other banks in a complex, networked environment. Periodically, the collective actions of all banks produce credit freezes and financial contagion. This model provides an opportunity to evaluate the effect on financial stability that Basel has on a dynamic, strategically reactive debt market.

The rest of this chapter is organized as follows. Section 4.2 covers some basic concepts of credit dynamics captured in the new model. In Section 4.3 I describe specific processes incorporated in the credit cycle model and explain some of its dynamics. I then introduce EFCNs and various notation used throughout the rest of the chapter in Section 4.4. Particulars of the credit cycle agent-based model are specified in Section 4.5. I present experimental setup and results in Section 4.6. Finally, I summarize findings in Section 4.7.

4.2 Background: credit dynamics

Banks and other financial institutions deploy massive resources to make sure that their risk profile fits that of their management and investors. Yet disastrous phenomena like financial contagion and credit freeze have happened repeatedly in recent history. Some prominent policymakers have advocated modeling the complex network of relationships between banks for answers to this conundrum [39]. Subsequently, economists have developed models using techniques from network theory that focus mostly on analyzing financial contagion.

Financial contagion A common strategy for modeling contagion is to fix either the debt network or the credit network and theorize about how a stylized version of the financial system may cause defaults and contagion. This approach has the advantage of being analytically tractable and establishes some important baselines. Roughly speaking, the mechanism for contagion in these models is for a borrower to default due to an unforeseen demand for liquid assets, or a ***liquidity shock***, leaving creditors with a loss that may cause them to default as well, and so on. A classic example comes from Allen and Gale [2]. These authors find that a fixed, ***complete*** (i.e., fully connected and infinite capacity) credit line network facilitates an egalitarian response to deposit shocks and decreases the size of shock needed to trigger contagion compared to an ***incomplete*** network where some banks do not offer credit lines to each other. Another result due to Acemoglu et al. [1] fixes a debt network. In that model, the robustness of complete debt networks drops sharply beyond a certain shock magnitude threshold.

To date, analytically tractable models have been able to incorporate only the most basic features of real financial networks. Gale and Allen assume deposits are shared among connected banks, meaning credit lines must be reciprocal and (functionally) infinite in capacity. Acemoglu et al. restrict attention to debt networks with reciprocal edges and uniform debt sizes. Banks in these models have a binary relation with other banks (trust for infinity or zero credit, borrow a set amount or not). Thus, the graph models are unweighted, and unable to capture degrees and direction of dependence. This is where the expressivity of EFCNs, which allow for credit lines with arbitrary capacity and debt contracts with flexible terms, can shine.

Credit freeze In contrast to contagion, which is tied to the debt network, credit freezes depend crucially on the available terms for *potential debt* in the credit network. These terms are a response to evolving market conditions, so the fixed debt network models used to evaluate financial contagion are inadequate. Instead, the motivations of lenders must be carefully examined and modeled. There have been many empirical studies documenting credit freeze during the financial crisis [34]. There is widely available evidence that liquidity management was the source of the credit shortage, as banks with more illiquid assets on their balance sheet decreased lending by more [16]. Theoretical models point to uncertainty in asset markets and borrower solvency as explanations for credit drying up [63]. Gorton and Metrick [29] cite increased collateral requirements during the crisis as a major reason for credit freeze. Note that none of these studies model interbank credit lines explicitly; debt is instead handled using a monolithic market mechanism.

In my model I include liquidity management, uncertainty over borrower quality, and the volatility of asset values as drivers of credit freeze. I model credit line formation so that the interbank credit network is made explicit. As a result, a unique driver of credit freeze arises: when one bank decides to decrease the amount it wants to lend out, other banks see a decrease in the maximum they may invest, which induces them to decrease the credit they extend as well. The opposing effect is also true, resulting in temporary credit bubbles.

Towards dynamic network models Network models are particularly appealing for modeling the financial system because several phenomena observed in the last financial crisis can be modeled. The interbank debt and credit networks both may include pairs of banks that know nothing about each other and have no direct credit or debt relationship. Yet they can still influence each other indirectly through the network. Even if somehow the entire network were known to every bank, extracting the relevant information to manage risk is still a hard problem that is beyond current financial practice. It is conceivable that the incongruity between the risk practices of banks and the reality of systemic risks in the financial system is due to this opaqueness of network information. Financial contagion is an example of this gap in practice. If banks were aware of how the state of the network affected their own profit, they could gauge how exposed they were to contagion and take the appropriate measures. But because the true network is not known, default cascades are not checked and can take banks by surprise.

Another way for crisis to happen in network models with risk-aware banks is when systemic risks are not adequately represented in the banks' profit. Since each bank can exacerbate or alleviate risks in the network for all other banks, it is possible for profit-seeking banks to collectively ignore the damage they are doing to the financial system. If this type of externality happens at equilibrium, then banks can be locked into a cycle of repeating crises. A game-theoretic analysis of a dynamic model can reveal this path to crisis.

The model I develop in this chapter models banks as profit-seeking enterprises while incorporating the complexity of interbank debt and credit network formation. Banks make decisions at the most granular level: evaluating whether or not to lend or borrow from specific counterparties. This granularity gives rise to macro-level phenomena naturally, and enables the macroeconomic response to regulation to have foundations in the intricate profit-seeking behavior of banks. However, this type of model demands that bank decisions be made in an extremely high dimensional action space, so a behavioral model is needed to specify reasonable bank reactions to market conditions. In turn, empirical game theoretical analysis must be used to bring the behavioral model to equilibrium. I use this approach to create a rich credit cycle model incorporating both credit freeze and financial contagion that offers new perspectives on the efficacy of Basel regulations.

4.3 Credit cycle overview

As in the Allen and Gale model, N banks each operate in their own region with a specific set of retail customers. These customers put their savings into banks as ***deposits***, effectively creating debt that the bank must repay on demand. Every period a random amount of deposits will either be created (signifying consumers putting more cash into a bank) or withdrawn (signifying consumers withdrawing their cash from banks). The inability of banks to honor large withdrawals is a major reason for default.

A bank's business is to take cheap forms of funding (such as deposits) and transform them into higher-yield investments. Two investments are available: ***assets*** which represent readily purchasable but illiquid securities, and ***interbank debt*** which are contracts between banks to exchange currently available funding for interest and repayment in the future. Both types of investment are funded using a combination of interbank debt, retail deposits, and the bank's own capital.

Assets have high expected return, but also high volatility of returns. They require payment in the current period, and offer a random, unknown payoff in the next period. They lose value if ***liquidated*** to make payments before the next period comes. Asset liquidation is the most expensive way to make payments, and can lead to insolvency.

All properties of the distribution of asset returns are public knowledge, and throughout this chapter I assume an infinite supply of assets, with price normalized to 1 (so that k units of asset costs k dollars). Notice that asset prices in this model do not change as a result of bank demand, as they do in the leverage cycle model in Chapter II. So the sale and purchase of assets does not have a direct effect on other banks; rather it serves as a catalyst for interesting debt and credit network dynamics. This choice makes the causal link between interbank debt dynamics and the economic findings in Section 4.6 more clear, especially as some of these findings mirror results from the leverage cycle literature which highlights asset market dynamics.

An alternative to buying assets is to loan other banks money by extending ***credit lines***. Creditors use these to express terms at which a they are willing to accept a borrower's debt. Debt issued on these lines is subordinate to deposits, so that in the event of a default, deposits will always be paid before interbank debt. In the work of Allen and Gale, as well as in numerous subsequent models of financial contagion, banks are connected to each other via static, homogeneous credit lines. My model departs from this treatment significantly, as credit lines originate from efforts by creditors to procure debt as an investment. Specifically, banks estimate and compare the return and return variance on debt versus assets, and construct an expected utility-maximizing portfolio. But while the the asset's stochastic properties are assumed to be known, those of debt are not analytically tractable and must be approximated throughout the simulation. In addition, debt is not readily available for purchase as assets are. Instead, creditors must adjust their offerings based on market conditions, in the hope of achieving their investment goal. For these two reasons, the market for debt does not naturally come to an equilibrium. Instead, creditors are given a set of heuristic strategies to choose from and the model is solved, using EGTA, for a Nash equilibrium in the set of strategies.

When borrowers utilize credit lines and pay back their debt with ***interest*** in the next period, then the credit line has paid off as an investment. If a borrower defaults, then the creditor loses part of its principal determined by the ***collateral rate*** on the credit line, and the credit line has yielded a negative return. An unused credit line corresponds to zero return. Both collateral rate and interest rate are properties of the credit line, and are set by creditors in a competitive environment.

Finally, banks can keep deposits in the form of cash, forgoing investment returns entirely. This shields them from investment risk, and also preserves their ability to make required payments (such as returning cash to depositors) in a universally accepted currency. Forcing banks to maintain ***cash reserves*** is a major regulatory pillar that will be implemented in the model.

Banks in this model are heterogeneous in the following ways:

1. Their depositors are different, which means each bank gets a different liquidity shock each period.

2. They differ in their investment preferences, and thus their targeted portfolios have different mixes of debt and assets.

3. The set of fellow banks they extend credit to differs (but may overlap), and the terms at which they extend credit is chosen according to differing policies.

2) and 3) together provide the basis for a ***debt market***. 2) splits the population of banks into borrowers and lenders. The typical borrower uses credit lines in order to buy assets, while the typical lender issues credit lines in order to receive stable interest income. 3) results in competition between creditors for a limited pool of available debt investments. The fact that creditors do not consider every potential borrower during every period is a substantial assumption. It stems from the fact that creditors in the real world must bear the cost of evaluating and maintaining relationships with

institutional borrowers. Therefore, there is some stickiness in the process for which borrowers are considered for credit lines.

The modeled debt market described in Section 4.5.6 is a heuristic for allocating debt at set terms to banks who wish to invest in debt. It is not economically efficient or strategically stable, but it allows for a large, realistic set of debt investment options. Every credit line is considered an investment by the issuing bank, and therefore the terms of each credit line are updated according to investment needs.

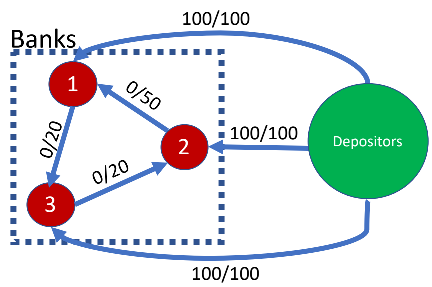
4.3.1 Payments on EFCNs and the balance sheet

A unique aspect of this model, enabled by the use of EFCNs to encode debt and credit relations, is that it considers exchanges of debt that are made possible by ***intermediaries***. Consider the following scenario:

Suppose there are three banks A, B, and C. They each extend a credit line of five dollars to each other, so that each has access to ten dollars of credit. Let A have five dollars of cash, B have five dollars, and C ten dollars. Suppose A wants to make a payment of twenty dollars to its depositors. It can immediately pay fifteen using its ten dollars in credit from B and C and its own cash. This leaves B and A with zero dollars, and C with five dollars. C is unwilling to lend these last five dollars to A to complete the payment. But B can intermediate by borrowing five dollars from C using its own credit line, and lending those five dollars to A. A can incentivize B to do this by offering a higher interest rate than B pays C. The opportunity for B to intermediate is created by the fact that C isn't willing to lend out all its cash to one party. Unlike in previous work, these opportunities are abundant in our model, where credit lines are issued based on profitability.

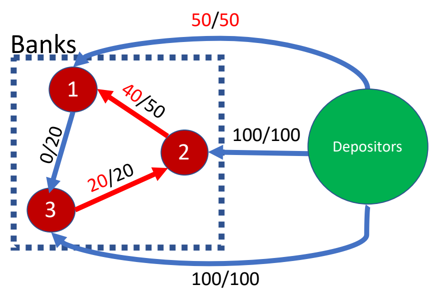
All of these payments between banks with and without intermediaries, and using either credit or debt, are handled naturally by an EFCN. The overall structure of the EFCN, and the way in which it reflects a bank's balance sheet during a liquidity shock, is depicted in Figure 4.1. A payment to depositors through an intermediary is also shown.

Bank	Capital	Deposits	Cash	Assets	Debt
1	10	100	10	100	0
2	10	100	20	90	0
3	10	100	30	80	0



(a) Initial EFCN. Each bank holds 100 in deposits, which they have invested in varying amounts of assets and cash. Banks extend credit lines to each other in different amounts.

Bank	Capital	Deposits	Cash	Assets	Debt
1	10	50	0	100	40
2	10	100	0	90	-20
3	10	100	10	80	-20



(b) FCN after Bank 1 pays its depositors back 50 dollars. Bank 2 lends all of its 20 dollars in cash, and facilitates the lending of another 20 dollars of Bank 3's cash, to Bank 1.

Figure 4.1: Financial credit networks before and after a liquidity shock of 50 dollars. Arrows indicate credit lines. An arrow from x to y with marking x/y denotes a credit line with capacity y and utilization x. The direction of payment flow is against the direction of arrows.

While it may seem that intermediaries are not readily observed or regulated, in reality the sale of a complex debt instrument is functionally equivalent to explicit intermediation. Although the correspondence is not one-to-one, the real life debt network is certainly highly influenced by intermediate lenders and borrowers. Intermediated transactions generate more aggregate debt, which has implications on the level of financial contagion that happens in the event of a default.

4.3.2 The credit cycle

While there are many moving parts in the credit cycle model, its namesake emerges as a fairly consistent dynamic process. At the start of the model, banks invest in a differential manner according to their risk tolerance. For aggressive banks, this means borrowing on the interbank market to buy risky assets. For conservative banks, this means extending credit lines, which are not perceived to be risky, and conserving cash. Over time, creditors adapt to market conditions so that they offer credit lines that are utilized by borrowers, at interest rates that they are willing to pay. Cheaper secured debt is used by all banks to deal with deposit shocks, providing liquidity for a pledge of illiquid assets. Higher-interest unsecured debt is used to increase investment by taking leverage.

Eventually, a large asset shock causes damage to the balance sheets of relatively aggressive borrowers, which have accumulated large amounts of capital by taking more leverage. A default may happen, and its ensuing contagion spreads. Or borrowers may be able to avert catastrophe. Regardless, interest rates skyrocket as creditors revise default risk upwards, making leverage-funded investment less attractive. The entire system undergoes deleveraging until balance sheets recover, and the cycle starts again.

A key feature of bank behavior is that they are constantly monitoring the profitability of investing in debt versus assets. For assets this means monitoring the interest rates paid for buying assets on leverage, and for debt this means monitoring credit line capacities and default rates. If interest rates become high, banks will shift their investment towards debt and if interest rates are low or default rates are high, they will buy assets. During a crisis, the cost of buying assets increases as interest rates rise, which discourages banks from investing. Meanwhile, during boom years assets purchases become cheap to fund as interest rates fall due to lower perceived borrower risk. The credit cycle is powered by this profit-seeking behavior by creditors.

Basel regulation enters fairly naturally into the model as an additional constraint on leverage taken by banks. It imposes a constraint that is more stringent the higher the weighted (by capital) average estimated default rate is.

4.4 Extended FCNs

The FCNs developed in Chapter III are a natural modeling tool for representing interbank network dynamics. Credit edges in an FCN correspond to an offer to invest in a specific borrower's debt at a specific interest rate, up to a limit. Once a set of these offers/credit edges have been established, banks can use them as a source of funding for any payments they choose to make. These payments generate debt edges which are also encoded in the FCN, representing obligations to repay current funding in the future. Collectively, debt edges map to the debt networks which are used in the literature to study financial contagion.

Interest payments are an important pricing tool for banks, but perhaps equally important is the collateral rate. This sets the amount of a loan that the lender recovers in the event that the borrower defaults. Since default is the only risk to lenders, loans at different collateral rates are extremely differentiated products with macroeconomic ramifications. One way to see this is that the aggregate collateral required by lenders defines a leverage limit on the financial system. This leverage limit constrains investment and thus decreases exposure to risky assets. In fact, lenders perform a similar function to regulators by setting collateral rates to limit their exposure to default risk. Endogenously set collateral rates interact with the leverage limits set by regulators to create an overall tolerance for leverage in the financial system.

From the borrower side, credit lines with different collateral requirements requirements are qualitatively different funding opportunities. At the extreme, if a collateral rate of one hundred percent is required on a credit line, borrowing is simply a way to access liquidity using assets that may be difficult or costly to unload immediately. No leverage is taken, at all in this case. On the other hand, a collateral rate of zero is an opportunity to increase leverage and invest more than would be possible otherwise. So borrowers have an entirely different set of considerations in choosing which collateral requirements to take on.

In order to incorporate collateral requirements into a model of credit network formation, FCNs must be extended to include collateral requirements. This extension leads to the construction of a richer payment mechanism where banks choose between debt options at different collateral rates, detailed in Section 4.5.3. It also introduces richness in lending strategy, as banks can set rates according to market conditions and consider how much collateral to require on their loans. This is done given estimates of each borrower's risk of default.

4.4.1 Extended FCNs: notation

To handle realistic debt contracts, the original FCN framework from Chapter III must be extended with collateral rates. To do so, we will use the following set of notation.

An EFCN is represented by a tuple (V, E) where V is a set of nodes indexed by $\{1, \ldots, N\}$ and E is a set of edges on V. The following functions allow agents to access important characteristics of nodes and edges.

- $\tau : E \to \{debt, credit\}$ is the type of the edge.
- $C : E \to \mathbb{R}^+$ denotes the edge's ***capacity***. For credit edges, this is the amount that may be borrowed. For debt edges, this is the amount of the debt.
- $R : E \to \mathbb{R}^+$ is the edge's *interest rate*. For credit, this is the rate at which the lender is willing to lend, or the contract rate. For debt, this is the agreed-upon rate for the debt contract.

- $CR : E \to \mathbb{R}^+$ is the edge's *collateral rate*. For credit edges this specifies, per unit borrowed, how much capital must be set aside as collateral to be seized by the creditor in case the borrower cannot repay its debt. For debt, it is the collateral rate agreed upon at the time of issuance.

- $DR : V \to \mathbb{R}^+$ is the ***default rate*** of the node. $\widehat{DR} : V \to \mathbb{R}^+$ is an estimated default rate.

I slightly overload notation and sometimes refer to $DR(e)$ or $\widehat{DR}(e)$ with $e \in E$. This will be understood to mean the default rate of the borrower for either credit or debt edges.

As far as the basic difference between EFCNs and FCNs, the function CR is the only one. But we will see that this simple addition has large ramifications in terms of the model we may build as well as how payment mechanisms must be constructed.

4.4.2 Nodes as banks: notation

The original FCNs considered interest payments to be the only way of receiving returns by holding debt. In our EFCN-based model, creditors value debt more realistically in two ways. First, the ex-ante default rate of the borrower can be estimated and used to help creditors decide on what terms to offer. Second, creditors can set a collateral rate which controls losses on defaults for themselves, and imposes a constraint on how borrowers use capital. To include these two factors, payment mechanisms on EFCNs must allow for *asymmetrical* valuations of debt by borrowers and creditors. To see this, consider a single piece of debt $e \in E$. The creditor must worry about losing the value of the debt in the event of default. Meanwhile, the borrower values the debt only insofar as it allows it to achieve a payment. Default is a catastrophic event for the borrower, but it has no impact on how it values the cost of using e. A similar asymmetry between creditors and borrowers exists when gauging the value of the collateral rate on debt. In light of this, we define two functions to access the value of using edges to make payments. See Section 4.5.3 for details.

- $DR : V \to \mathbb{R}^+$ is the ***default rate*** of the node.
- $Val_{in} : E \to \mathbb{R}$ is the ***debt value*** of receiving a payment on a particular edge.
- $Val_{out} : E \to \mathbb{R}$ is the debt value of making a payment on a particular edge.

For our purposes, it will be convenient to also refer to ***payer*** and ***receiver*** banks:

- $Payer : E \to V$ returns the node which can utilize an edge to make a payment. For a credit edge, this is the borrower (by ***issuing debt***), and for a debt edge this is the creditor (by ***debt cancellation***).
- $Receiver : E \to V$ returns the node which can receive a payment on a particular edge. For a credit edge, this is the creditor (who receives the borrower's debt), and for a debt edge this is the borrower (who can have its debt cancelled).

We define the following edge subsets for convenience. With $v \in V$:

- $Out_c(v)$ are credit edges where v is the lender, $Out_d(v)$ are debt edges where v is the borrower, and $Out(v)$ is $Out_c(v) \cup Out_d(v)$. Thus $Out(v)$ consists of all edges that can be used to pay v.
- $In_c(v)$ are credit edges where v is the borrower, $In_d(v)$ are debt edges where v is the lender, and $In(v)$ is $In_c(v) \cup In_d(v)$. Thus $In(v)$ consists of all edges that v can use to pay other nodes.

Banks in this model do not always have perfect information. For uncertain quantities X, ***estimates*** are often used, and are denoted by $\widehat{X}$. Notable estimated quantities are the default rate $\widehat{DR}$ and the expected debt values $\widehat{E}(Val_{in})$ and $\widehat{E}(Val_{out})$, where E is the standard notation for the expected value, and $\widehat{E}$ is an approximation of the expected value.

Finally, in every EFCN discussed in this chapter, there is a market node that accepts payments for deposit shocks and assets. All other nodes in the graph (representing banks), offer a credit edge with unlimited capacity, zero interest, and zero collateral rate to the market node. Debt issued by the market node can be interpreted as ***cash***, since it is completely liquid and is accepted by all banks as payment. None of the credit edges issued to the market node are included in the *In* and *Out* subsets, and the market node is not counted within the N bank nodes.

4.5 Credit cycle model environment and bank behavior

The credit cycle model will consist of banks alternating between making investment decisions, including issuing credit lines, and seeing their investments play out. Each cycle is called a period, and the model ends after T periods. T is chosen to be large enough so that interesting dynamics may play out.

4.5.1 Model schedule

During the ***investment phase***, each bank prepares a balance sheet according to ***targets*** set in the last period, and updates targets for the next period based on new observed returns. Targets are distinguished from actual achieved balance sheet values using brackets $\langle\rangle$. An EFCN consisting of credit lines is originated in this phase, to be utilized for all payments during the rest of the period.

1. Check every bank for overleveraging. See Section 4.5.2.

2. Purchase asset amounts A_t^k according to target amount $\langle A_t^k \rangle$ for all $k \in \{1, \ldots, N\}$ in random order.

3. Update default rate estimates $\widehat{DR}(k)$, for all k. See Section 4.5.4 for details.

4. Set asset investment target $\langle A^k_{t+1} \rangle$, debt investment target $\langle \sum_{e \in In_d(k)_{t+1}} C(e) \rangle$, and target cash reserve amount $\langle m^k_{t+1} \rangle$. This requires updating estimates for the cost of funding, asset/debt returns, and the volatility of asset/debt returns. It also entails requesting credit from other banks. See Section 4.5.6 for details.

5. Each bank issues credit according to their own target debt investment level $\langle \sum_{e \in In_d(k)_{t+1}} C(e) \rangle$, credit requests made by other banks, and their own credit allocation strategy. Credit lines from last period expire. Details in Section 4.5.6.

6. Update all debt valuations for every edge. This includes updating collateral values. Create debt valuation clusters for use in payment algorithm. See Section 4.5.3.

7. End the iteration, $t \leftarrow t + 1$.

During the ***capital update phase***, the balance sheets and the EFCN constructed during the investment phase are shocked stochastically. Debt is also repaid in this phase with interest. All potential default events happen during this phase.

1. A deposit shock for each bank is generated, and if necessary payments are made to depositors. After the shock, debt valuations are updated.

2. Debt is repaid with interest. Debt valuations are then updated.

3. υ_t is generated, and the new asset value $A^k_{t-1}\upsilon_t$ is returned to bank k as cash. Debt valuations are then updated. See Section 4.5.7 for details on υ_t.

4.5.2 Bank state and balance sheet

Each bank k has a balance sheet equality at time t of the following form:

$$m^k_t + hA^k_t + \sum_{e \in In_d(k)_t} C(e) = w^k_t + d^k_t + \sum_{e \in Out_d(k)_t} C(e) \tag{4.1}$$

m_t^k is cash, A_t^k is assets, w_t^k is ***capital***, h is an asset liquidity ***haircut***, and d_t^k is deposits. Banks start with an initial endowment of w_{init}, and initial deposits of d_{init}, which is all granted in cash. A bank must maintain certain conditions on its balance sheet in order to continue operations, as described below.

Overleveraging ***Overleveraging*** occurs when $w_t^k < \sum_{e \in In_d(k)} CR(e)C(e)$. Banks that are overleveraged cannot meet all of their collateral requirements, and thus do not have further access to credit. Typically, banks become overleveraged when their assets lose value. They are particularly susceptible to this when they choose to borrow close to their leverage limit in search of returns.

Banks can ***default*** in two ways: becoming ***insolvent*** or failing to pay a debt. These events are described below.

Insolvency A bank that owes more than its total asset value (equivalently, has negative capital) is ***insolvent***:

$$w_t^k = m_t^k + hA_t^k + \sum_{e \in In_d(k)_t} C(e) - d_t^k - \sum_{e \in Out_d(k)_t} C(e) < 0. \tag{4.2}$$

There are three main ways for banks to lose capital.

1. Assets can lose value.

2. A large deposit shock can cause the bank to liquidate assets at a loss.

3. A borrower can default on their debt and never repay it.

Payment failure If at any point a bank cannot honor its depositors' random withdrawals, or cannot pay back its interbank debts with interest, then it defaults. Overleveraging is a common root cause of this failure, as banks with access to credit can access the liquidity of creditor banks to pay off shocks and debt. Overleveraged banks, however, have only their own cash and assets that, if liquidated, must be sold at a loss.

Consequences of default A defaulted bank first pays back its collateral to the best of its ability. It is then forced to be inactive for a set number of periods ϵ, after which it is reset to an initial state in which it is forgiven of all debt, and granted w_{init} and d_{init}.

Note that there is a growing literature on handling default as an endogenous decision, and settling payments between defaulting banks simultaneously in financial networks. For example, Schuldenzucker et al. [58] explore the simultaneous clearing of default swap obligations. This layer of complexity is largely omitted from this model and we only consider sequential defaults.

4.5.3 Debt valuation

A key feature of EFCNs is the ability for banks to set both interest rates and collateral rates on credit lines. In addition, banks are aware that a portion of the debt owed them will not be paid back, and estimate default rates to account for this. These features distinguish EFCNs from FCNs, and require that each bank be able to make a more complex calculation for the value of debt. Creditors need this value to judge the value of debt as an investment and set terms, and debtors need it in order to choose which credit lines to utilize. All banks need this value to decide whether to participate in a payment chain. Values can be negative, signifying the cost of utilizing debt, or positive, signifying the value of receiving debt.

In this section we specify Val_{in} and Val_{out}, functions that return the value per unit of debt utilizing edge e. If $\tau(e) = credit$, then a borrower needs to know $Val_{out}(e)$ the cost for the act of borrowing, and a creditor needs $Val_{in}(e)$, value for the act of ***holding debt*** as an investment. If $\tau(e) = debt$, then a creditor must have $Val_{out}(e)$, cost for the act of ***debt cancellation*** and a borrower must have $Val_{in}(e)$, the value for having its debt cancelled. $Val_{in}(e)$ returns the value of receiving a payment on edge e, and Val_{out} returns the cost of making a payment.

First, we examine the debt valuation problem from a creditor's point of view, receiving a unit payment on credit edge e and considering what $Val_{in}(e)$ should be. The return on the resulting debt if the debtor does not default is $R(e)$. The return if the debtor does default is $-(1 - CR(e))$. Let the estimated default rate of the borrower be $DRB = \widehat{DR}(Payer(e))$. Then the estimated expected return is $-DRB(1 - CR(e)) + (1 - DRB)R(e)$. So $\widehat{E}(Val_{in}(e))$ takes this value when $\tau(e) = credit$. Here $\widehat{E}$ denotes an *estimated* expected value, since the default rate used in this calculation is not exact. This is explained in Section 4.5.2. This is the value of receiving debt from a borrower on credit edge e. It is also the cost to the creditor for canceling an existing debt owed it (as in when routing a payment). So $\widehat{E}(Val_{out}(e))$ also takes this value when $\tau(e) = debt$.

Now consider a borrower utilizing a credit edge. It will have to fulfill two promises: pay the interest rate $R(e)$ and commit to maintaining enough collateral at rate $CR(e)$. Intuitively, having to commit more collateral is costly, as it hampers investment flexibility. But a collateral requirement that is lax enough will not cost the borrower anything, as it does not constrain the execution of her investment plan. Call the value of the collateral pledged towards a unit of debt on edge e $CV(e) : E \to \mathbb{R}$. This can be positive or negative depending on how high the collateral rate is. Ultimately, the cost of the collateral requirement depends on whether the borrower's investment goals are achieved. It is impossible to consider all of the environmental and strategic factors that contribute to this success or failure, so for this model I use an approximation.

Start with the borrower's ***investment target***:

$$Target = \langle A_t^k \rangle + \langle \sum_{e \in In_d(k)_t} C(e) \rangle.$$

Target is bank k's preferred amount of total investment. Let the expected rate of return on this investment be Ω_t^k. Taking on debt at a stringent collateral rate can cost bank k investment returns if *Target* is not achieved. To find out if this is the case, the first baseline to establish is whether or not the investment target is realistic to achieve, overall. We can do this by calculating the maximum investment possible with current levels of capital. Define $mDebt_t^k = \sum_{e \in In_d(v_t^k)} C(e)(1 + R(e))$, the debt payments owed to node k at time $t+1$. Then the maximum investment is approximately the amount of cash on hand $m_{t-1}^k + A_{t-1}^k \upsilon_t + mDebt_{t-1}^k$ in addition to the amount of credit accessible given current levels of capital w_{t-1}^k. Call the sum of these quantities the ***max payment*** in period t for bank k, Π_t^k. If $\Pi_t^k > Target$, then the investment target is ***feasible***. The baseline expectation for feasible targets is that they will be achieved, so stringent collateral requirements will contribute a negative debt value but lax collateral requirements will be viewed as not affecting debt value. Meanwhile, the baseline expectation for ***infeasible*** targets (i.e., $\Pi_t^k < Target$) is that the they will not be achieved, so lax collateral requirements will contribute positive debt value while stringent ones will be viewed as not affecting debt value.

We rely on two assumptions to calculate the impact on debt value of a particular collateral rate.

Assumption IV.1 (Linear interpolation)**.** *$CV(e)$ is constant for a particular bank and period, and thus does not depend on the amount borrowed, X. Therefore, λX is the **total collateral value** of borrowing $X > 0$ dollars, for constant $\lambda > 0$.*

So given an edge e, two amounts borrowed X and Y, and their corresponding total collateral values A and B, a valid equality under this assumption is that $CV(e) = \frac{A-B}{X-Y}$. Since linear interpolation doesn't hold exactly, the collateral value we calculate using this equality is an approximation, and is thus denoted by $\widehat{CV}(e)$.

To see why linear interpolation is an approximation, note that taking on any single collateral requirement (no matter how stringent or lax) at a low borrowed amount is likely to be inconsequential to the final investment amount achieved. So $CV(e) = 0$ for many small debts, but increases as the debt amount increases.

The second assumption we make is that credit lines are usable to make payments.

Assumption IV.2 (Creditor Liquidity)**.** *The feasible payment level* Π_t^k *is achievable using a combination of credit and cash.*

In reality, not all creditors have enough access to liquidity to help borrowers make investments. In our calculation of the maximum payment, we also refrain from including assets that may be liquidated at a loss or debt that may be exchanged or canceled because these are investments that should only be liquidated in emergencies.

Now we have the ingredients necessary to estimate $CV(e)$. The strategy will be to first note that the total collateral value of utilizing zero dollars on edge e is 0. Then, we will calculate the total collateral value of utilizing all available capital w_{t-1}^k at the collateral rate of $CR(e)$. This total value will be calculated in several different ways depending on the bank's funding situation as compared to their investment goals. Finally, we will use linear interpolation to calculate $\widehat{CV}(e)$.

First, imagine we have a feasible target, so that the bank expects to achieve its investment target. Consider a credit line e with collateral rate $CR(e)$. If the bank used all of its available capital w_t^k as collateral to borrow at collateral rate $CR(e)$, it would end up investing the amount $w_t^k/CR(e)$. If $w_{t-1}^k/CR(e) > \mathit{Target}$, the bank keeps $\widehat{CV}(e) = 0$, since using the credit line keeps it at its baseline of achieving its investment target. The bank would actually be on track to invest more than its target, but it does not prefer to do so, and thus assigns no extra value to the lax collateral terms. However, if $w_t^k/CR(e) < \mathit{Target}$ then the bank is losing out on $\mathit{Target} - w_t^k/CR(e)$ in investment amount and $\Omega_t^k(\mathit{Target} - w_t^k/CR(e))$ in investment returns. So we have that at a utilization amount of 0, the total collateral value is 0, but at a utilization amount of $w_t^k/CR(e)$, the total collateral value is $\Omega_t^k(\mathit{Target} - w_t^k/CR(e))$. Using linear interpolation, we stipulate that the estimated collateral value is $\Omega_t^k \frac{Target - w_t^k/CR(e)}{w_t^k/CR(e)}$.

Now suppose instead that the target is infeasible, so that the bank expects to achieve at best an investment level of $\Pi_t^k < Target$. Then for a credit edge e with collateral rate $CR(e)$, the value of borrowing $w_t^k/CR(e)$ amount is $\Omega_t^k \frac{w_t^k/CR(e) - \Pi_t^k}{w_t^k/CR(e)}$. Note that if $(w_{t-1}^k/CR(e) - \Pi_t^k) < 0$, the bank assigns a penalty to using the credit edge, as the collateral rate is below the rate required to invest Π_t^k. There is no scenario where a collateral constraint is valueless, as the bank which cannot ever achieve its investment goal is always desperate for more collateral.

Finally, consider the collateral value for cancelling debt. When debt edge e is cancelled, bank $k = Receiver(e)$ receives some collateral back from its creditor $Payer(e)$. The value of this collateral for bank k depends on how it is used to achieve bank k's investment target. If $\Pi_t^k > Target$, the extra wealth has no investment value since the target is already achieved. Otherwise, the wealth can be used to increase Π_t^k to be closer to $Target$. Let $(\Pi_t^k)_{CR(e)}$ be the maximum payment achievable by bank k given $CR(e)$ more units of capital. Then the collateral value is given by $\frac{(\Pi_t^k)_{CR(e)} - (\Pi_t^k)}{\Omega}{}_t^k$.

So our ***collateral value function*** $\widehat{CV} : E \to \mathbb{R}^+$, which is the value that having collateral rate $CR(e)$ contributes for utilizing one unit of new debt, is the following for $\tau(e) = credit$:

$$\widehat{CV}(e) = \begin{cases} \Omega_t^k \frac{w_t^k/CR(e) - \Pi_t^k}{w_t^k/CR(e)}, & \text{if } \Pi_t^k < Target \\ 0, & \text{if } \Pi_t^k > Target \text{ and } w_t^k/CR(e) > Target \\ \Omega_t^k \frac{w_t^k/CR(e) - Target}{w_t^k/CR(e)}, & \text{otherwise} \end{cases}$$

and for $\tau(e) = debt$:

$$\widehat{CV}(e) = \begin{cases} 0, & \text{if } \Pi_t^k > Target \\ \Omega_t^k[(\Pi_t^k)_{CR(e)} - (\Pi_t^k)], & \text{otherwise} \end{cases}$$

Now we can sum up the two debt valuation functions. For edge e, $k = Receiver(e)$, and recalling $DRB = DR(Payer(e))$:

$$\widehat{E}(Val_{in}(e)) = \begin{cases} (1 - DRB)R(e) - DRB(1 - CR(e)), & \text{if } \tau(e) = credit \\ R(e) + \widehat{CV}(e), & \text{if } \tau(e) = debt \end{cases}$$

$$\widehat{E}(Val_{out}(e)) = \begin{cases} -(R(e) + \widehat{CV}(e)), & \text{if } \tau(e) = credit \\ -(1 - DRB)R(e) + DRB(1 - CR(e)), & \text{if } \tau(e) = debt \end{cases}$$

4.5.4 Default rate estimation

A bank can default in several ways, outlined in Section 4.5.2. Out of these, insolvency represents the greatest danger to creditors since creditors take losses on their debt when borrowers default due to insolvency. Given that the asset return distribution is known to be lognormal, we can calculate in closed form an approximate insolvency rate. We want the probability that the bank capital of bank k is below zero:

$$P(w_t^k < 0) = P(m_t^k + \upsilon_t A_t^k + \sum_{e \in In_d(k)_t} C(e) - d_t^k - \sum_{e \in Out_d(k)_t} C(e) < 0) \tag{4.3}$$

$$= P(\upsilon_t < \frac{d_t^k + \sum_{e \in Out_d(k)_t} C(e) - m_t^k - \sum_{e \in In_d(k)_t} C(e)}{A_t^k}) \tag{4.4}$$

The CDF of υ_t is known, so this probability may be calculated to estimate the insolvency rate, which in turn can be used as an estimate of the default rate for any bank given their balance sheet information. This estimate $\widehat{DR}(k)$ is publicly available to all banks.

$\widehat{DR}$ is an underestimate of insolvency, as it only considers asset return shocks. It misses financial contagion effects completely, and also fails to capture capital losses due to asset liquidation. Its purpose is to provide banks with a simple, actionable insight that requires only balance sheet information, which in the real world is readily available. In contrast, financial contagion estimation requires knowledge of the entire debt network, and asset liquidation prediction requires knowing the maximum payment a bank can make. The former is not public knowledge in this model or in reality, and the latter requires knowing intimate details about each bank's access to credit.

4.5.5 Payments

Making payments in EFCNs requires a new payment mechanism with newly thought out desiderata. For a payment of X dollars from node s to t in an EFCN to succeed, we require that it must consist of payment paths that satisfy the following properties.

1. The paying bank (source) loses P dollars immediately, and the paid bank (destination) gains P dollars.

2. All intermediary banks net a positive ***debt valuation***, as defined in Section 4.5.3.

3. All new debt contracts on any credit edge pay at least the contract interest rate, but possibly higher.

4. No bank k in the transaction has $\sum_{e \in In_d(k)} CR(e)C(e) > w_t^k$ after the transaction if they had $\sum_{e \in In_d(k)} CR(e)C(e) \leq w_t^k$ before the transaction.

Apart from property 2 and 4, these are all equivalent to conditions satisfied by payment paths in FCNs as described in Chapter III. In words, property 4 stipulates that any bank that was not overleveraged before a payment path is executed cannot become overleveraged after execution. Overleveraging and its consequences are described in Section 4.5.2. Preventing overleveraging guarantees that every bank maintains enough collateral to pay back its creditors if it defaults.

Property 2 is similar to the interest rate monotonicity property in the original FCNs, but with one key difference. Interest rates are a universal valuation for debt accepted by all agents. Debt valuations, on the other hand, are not universal. In fact, for a single piece of debt, the creditor and the borrower will generally differ in how they value it. The interpretation for interest rate markups in EFCNs is that they are payments made to smooth out differences in debt valuations along payment paths.

Algorithm MaxInterestFlowLP must be modified to handle these changes and produce valid payments. Recall that in the original FCNs, Algorithm MaxInterestFlowLP chose from a set of interest rates with limited cardinality. Similarly, Algorithm MaxDebtValueFlowLP will choose from a set of debt values of limited cardinality, $\mathcal{DV}$. Recall that flow variables in Algorithm MaxInterestFlowLP consisted of an edge on which to route the flow as well as an interest rate at which to route the flow. This setup is modified slightly here, as we instead consider flow variables to pairs of edges and *debt values* to the receiver of the flow. This is an arbitrary choice, as setting this value fixes the interest rate paid on the flow, and also fixes the value paid by the sender of the flow.

Define $\mathit{ValClust} : \mathbb{R} \times \{\mathit{sending}, \mathit{receiving}\} \to \mathbb{R}$ as taking a tuple (v, s) and returning the closest member of $\mathcal{DV}$ that is greater than v if $s = \mathit{receiving}$ and the closest member of $\mathcal{DV}$ that is smaller than v otherwise.

In addition, define the functions $\mathit{IRVal} : \mathbb{R} \times E \to \mathbb{R}$, $\mathit{ValIR} : \mathbb{R} \times E \to \mathbb{R}$, and $\mathit{SendVal} : \mathbb{R} \times e \to \mathbb{R}$ as follows:

$$\mathit{IRVal}(v, e) = \frac{(v + (1 - CR(e))\widehat{DR}(e))}{1 - \widehat{DR}(e)} \tag{4.5}$$

$$\mathit{ValIR}(ir, e) = \mathit{ValClust}(ir + CV(e), \mathit{sending}) \tag{4.6}$$

$$SendVal(recVal, e) = ValClust(Val_{out}(e) + IRVal(recVal, e) - r(e)), sending) \quad (4.7)$$

Algorithm MaxDebtValueFlowLP:

$$\max_{f_{e,DV}, X} X \text{ s.t.} \quad \text{(Objective)}$$

$$\forall e \in E^{\dagger} : f_e = \sum_{DV \in \mathcal{DV}} f_{e,DV} \quad \text{(Total Flow)}$$

$$X + \sum_{e \in In(s)} f_e = \sum_{e \in Out(s)} f_e, \qquad \sum_{e \in In(t)} f_e = X + \sum_{e \in Out(t)} f_e$$

$$\text{(Flow Value)}$$

$$\forall v \notin \{s, t\} : \sum_{e \in In(v)} f_e = \sum_{e \in Out(v)} f_e \quad \text{(Flow Conservation)}$$

$$\forall v \notin \{s, t\}, \forall DV \in \mathcal{DV} : \sum_{e \in In(v)} \sum_{DV' \leq DV} f_{e,DV'} \leq \sum_{e \in Out(v)} \sum_{SendVal(DV', e) \leq DV} f_{e,DV'}$$

$$\text{(Monotonicity)}$$

$$\forall e \in E^{\dagger}, \tau(e) = \text{credit}, \forall DV \in \mathcal{DV} s.t.\ IRVal(DV) < r(e)\ or$$

$$DV < Val_{in}(e) : f_{e,DV} = 0$$

$$\forall e \in E^{\dagger}, \tau(e) = \text{iou}, \forall DV \in \mathcal{DV} \neq ValIR(r(e)) : f_{e,DV} = 0$$

$$\text{(Valid Interest Rates)}$$

$$\forall e \in E^{\dagger}, DV : 0 \leq f_{e,DV}, \forall e : 0 \leq f_e \leq c(e) \quad \text{(Capacity)}$$

To make payments, one can simply write down a tuple (X, s, t) representing the amount to route X, payer s and destination t. This is the basic payment algorithm used throughout, and produces payments consisting of valid payment paths. Max flow is achieved if all possible debt valuations are included in $\mathcal{DV}$. Otherwise, some payments that might have been routed are not, due to the rounding up and down of true debt valuations to members of $\mathcal{DV}$.

Note that there are two types of payments in the model: ***discretionary*** and ***necessary***. Buying assets is discretionary as failure to complete the full payment is inconsequential, so if P is more than the max flow, then the max flow is routed for a partial payment instead.

On the other hand, payments to depositors and creditors are necessary to continued operation. For these payments, if X is more than the max flow, then

1. Any non-liquid assets will be liquidated, even at a loss, to attempt to complete the payment.

2. If there is no way to make the payment, no partial payments are made. Instead, the appropriate default proceedings will be triggered.

Payment mechanisms Payment mechanisms will implement flows calculated using Algorithm MaxDebtValueFlowLPȦs a modeling choice, we will replace the objective in Algorithm MaxDebtValueFlowLP for all our payment mechanisms so that the debt value lost by s is minimized.

Denote the payment mechanism for necessary payments as $Pay : R^+ \times V \times V \times EFCN \rightarrow P \in \mathbb{P}$ where $\mathbb{P}$ is the space of all possible payments (i.e., collections of payment flows). Pay takes as arguments a tuple (X, s, t, G) and produces a set of s-t payment paths $P = P^{(1)}, \ldots, P^{(m)}$ such that the sum of their flows is X. The mechanism proceeds to route this payment. If a valid P does not exist, then a max flow payment P will be returned. The bank will proceed to route this payment and assets will be ***liquidated*** at a loss of ρ to pay the difference. If there are not enough assets, then the bank defaults.

$PayAsset : (R^+, V, V, EFCN) \rightarrow P \in \mathbb{P}$ is similar, but does not have to route the full amount X, as it is paying for discretionary investment in assets. It does not proceed with liquidation if X is not feasible, and just routes the max flow.

4.5.6 Investment stage

During the investment stage, each bank buys assets, reserves cash, and issues credit lines to obtain debt investments. To make these decisions, they use a utility function for guidance.

Utility function At the end of period t, $t \in \{1, \ldots, T\}$, bank k decides on a target investment portfolio size $\lambda_{t+1}^k w_t^k$ as well as the proportion of the portfolio to be used on assets, α_{t+1}^k. Naturally, the proportion that is reserved for making loans is $1 - \alpha_t^k$. These quantities are chosen by maximizing an estimated constant relative risk aversion (CRRA) utility function of the following form.

$$u_t^k = \frac{1}{1-\theta^k}\{1+\lambda_t^k((\alpha_t^k\mu^a+(1-\alpha_t^k)\mu^c)-\frac{1}{2}\theta^k(\lambda_t^k)^2[(\alpha_t^k)^2(\sigma^a)^2+(1-\alpha_t^k)^2(\sigma^c)^2]\}^{1-\theta^k}$$

The risk aversion parameter θ^k is strategically chosen at the beginning of the simulation. μ^a and σ^a are the known and constant asset return and asset return variance, respectively. The cost of capital for credit β_t^c, the cost of capital for β_t^a, credit return (μ_t^c) and credit return variance (σ_t^c) are unknown, as they depend on events that transpire during the simulation. They are estimated as described in Section 4.5.8 Note that μ_t^c and σ_t^c are parameters for the entire debt market, and do not reflect the profitability of lending to any single bank.

The utility function boils down to a preference for high returns at low volatility. θ^k governs how the two are balanced for a particular bank. An important assumption made here is that there is no correlation between asset and credit returns that might increase or decrease the total return volatility of the portfolio.

The proportion of capital invested that maximizes utility is

$$(\lambda_t^k)^* = \frac{\mu_a \sigma_c^2 + \mu_c (\sigma^a)^2}{(\sigma^a)^2 (\sigma^c)^2 \theta^k} \tag{4.8}$$

and the optimal proportion of the investment target allocated to buying assets is

$$(\alpha_t^k)^* = \frac{\mu^a (\sigma^c)^2}{\mu^a (\sigma^c)^2 + \mu^c (\sigma^a)^2}.$$

Asset investment The multiplier on capital used to calculate the asset investment target is then $(\lambda_t^k)^* (\alpha_t^k)^* = \frac{\mu^a}{(\sigma^a)^2 \theta^k}$. This quantity is often greater than one, implying that banks would like to take leverage to buy assets. Cash is required to buy assets, which can be taken either from m_t^k or borrowed from other banks. The actual amount of assets bought is

$$PayAsset\{(\lambda_t^k)^* (\alpha_t^k)^* w_{t-1}^k, k, Market\} = A_t^k \leq \langle A_t^k \rangle = (\lambda_t^k)^* (\alpha_t^k)^* w_{t-1}^k, \tag{4.9}$$

with inequality since there may not be enough cash available for bank k to achieve its investment goal. Remember that *PayAsset* does not permit k to become overleveraged as a result of the transaction. For details on the payment mechanism *PayAsset*, see Section 4.5.5.

Cash reserves Banks might choose to hold cash reserves for reasons other than issuing debt. In this model it is actually regulators who mandate that a certain amount of cash be carried in order to service deposits. They do this by setting $mReserve \in [0, 1]$, dictating the percentage of deposits that must be held in cash.

Credit investment The total amount of debt desired by bank k is $(\lambda_t^k)^* (1 - \alpha_t^k)^* w_{t-1}^k$. This debt is obtained by extending credit lines to other banks, and facilitating payments on these credit lines. To do this, creditors reserve a portion of their cash $mDebt_t^k = \min\{(m_t^k - mReserve)(1-\alpha_t^k), (\lambda_t^k)^*(1-\alpha_t^k)^* w_{t-1}^k\}$ so that they can service their credit lines.

In borrowers' eyes, debt is a commodity and is differentiated only by price. Under some conditions, economic theory predicts that there should be a single price of debt established. There are a few reasons that this expectation does not hold in this model.

1. Borrowers have different default rates, which affects the terms on loans.

2. There are two terms on loans that affect value: collateral rate and interest rate. Lenders may strategize over which to focus on, resulting in differing loan terms.

3. Lenders prefer to offer debt to repeat customers. This modeling choice is inspired by the real-life cost of vetting borrowers.

Although the aggregate debt target is set using aggregate return and volatility estimates, the terms on individual credit lines are adapted using performance data each borrower. The full policy is a complex heuristic, but it is easy to understand the basic idea. At the highest level, creditors must decide on the ***premium*** that they charge for each borrower. Remember that the value of debt edge e for creditors is $R(e)DR(Payer(e)) + (1 - R(e))(1 - CR(e))$. A higher interest rate or a higher collateral rate increase value for the creditor, but decrease value for the borrower. The premium ζ_t^i that creditor i assigns to its credit is thus equal to the value of edge $e \in In_c(i)$:

$$\zeta_t^i = R(e)(1 - DR(Payer(e))) - DR((Payer(e)))(1 - CR(e)). \tag{4.10}$$

Each creditor in our model chooses strategically whether to offer secured debt at a collateral rate of 1 or unsecured debt at a collateral rate of 0 for the duration of model. This simplification reduces the strategy space, but compromises on profit opportunity for lenders. In practice lenders can always use interest rate to differentiate their debt offerings. In addition, these two types of debt make up the majority of real debt instruments.

The strategic parameter $K^i \in \{0, 1\}$ is used to specify the collateral rate offered on all credit lines extended by bank i.

So the terms that are offered on all $e \in Out_c(i)$ with are fixed as:

$$R(e) = \frac{\zeta_t^i + (1 - K^i) DR(Payer(e))}{1 - DR(Payer(e))} \tag{4.11}$$

Creditors choose ζ_t^i every period according to market conditions. Roughly, if credit lines are fully ***utilized*** then the creditor will explore increasing the premium in order to make more profit. Conversely, if credit lines are lightly utilized then the creditor will decrease the premium in the hope of attracting more debt. Utilization U^i is simply:

$$U^i = \frac{\sum_{e \in In_d(i)} C(e)}{\sum_{e \in In_d(i)} C(e) + \sum_{e \in Out_c(i)} C(e)} \tag{4.12}$$

The specific update rule for ζ_t^i is to set a threshold parameter UT and parameters $UpPrem$, $DownPrem$. When $U^i > UT$, $\zeta_t^i = \zeta_{t-1}^i * UpPrem$. Otherwise, $\zeta_t^i = \zeta_{t-1}^i * DownPrem$.

The second lever that creditors use in their credit extension policy is the capacity of extended credit lines. This includes extending zero capacity credit lines to some potential borrowers (i.e., doing borrower selection). The main idea is that at all times, each creditor maintains an ***active set*** AS_t^k of borrowers that receive the bulk of all borrowing capacity. Namely, the active set receives up to $\gamma \langle \sum_{e \in In_d(k)_t} C(e) \rangle$. Borrowers that have consistently low utilization drop out of the active set. Meanwhile, a small budget is set aside to extend credit to an ***exploratory set*** ES_t^k of borrowers. These are chosen randomly, and may enter the active set if they have a consistently high utilization rate. The amount of credit extended to the exploratory set is just $(1-$

$\gamma)\langle\sum_{e \in In_d(k)_t} C(e)\rangle$. The maximum size of the active set is a constant, environmental parameter $|ES|$.

If it is within investment targets, the capacity of credit that the creditor extends to the active set is $\langle C(e)\rangle$, which is the amount requested by the borrower during its investment targeting step. If $\sum_{e:Payer(e)\in AS_t^k}\langle C(e)\rangle > \gamma\langle\sum_{e\in In_d(k)_t} C(e)\rangle$ then credit capacities are normalized: $C(e) = \gamma\langle C(e)\rangle$. All credit $\langle\sum_{e\in In_d(k)_t} C(e)\rangle$ not invested in AS_t^k is invested into credit lines for banks in ES_t^k, proportionately according to their credit requests.

Before G_t is updated with new credit edges, banks record the debt principle and interest that must be repaid in a matrix $\mathbb{D}_{\approx}$.

$$\begin{aligned} &\forall i \in \{1, \ldots, N\} : \\ &\forall j \in \{1, \ldots, N\} : \\ &\quad \mathbb{D}_t(i, j) \leftarrow \sum_{e \in IOU(j,i)} C(e)[1 + R(e)] \end{aligned} \tag{4.13}$$

Since the ***maturity of debt*** is always 1 period in our simulation, we then clear G_t of all debt. This allows each bank to later choose between paying off its matured debt and interest (if paid back using cash) or extending the debt if the updated G_t affords enough credit (by paying using credit). Finally, G_t is updated with new credit edges.

4.5.7 Capital update

Deposit shock A random amount of deposits are withdrawn or deposited into each bank. Unlike asset returns, this is not a common shock. Rather, it is idiosyncratic for each bank, defined by a uniform distribution:

$$\forall k \in \{1, \ldots, N\} :$$

$$\Delta_t^k \sim w_t^k U(-d_{t-1}, d_{t-1}) \tag{4.14}$$

$$d_t^k \leftarrow d_{t-1}^k + \Delta_t^k \tag{4.15}$$

If the shock Δ_t^k is positive, bank k receives Δ_t^k in cash. If the shock Δ_t^k is negative and $Pay(\Delta_t^k, k, Market)$ does not return a solution, so that the depositors' withdrawal is not satisfied, then bank k defaults. See Section 4.5.2 for details on default.

Asset payoff After investments are made, each bank k's matured assets A_t^k generate a random payoff in period $t+1$ according to a lognormally distributed ***return factor*** υ_{t+1}.

$$\forall k \in \{1, \ldots, N\} :$$

$$\log p_t \sim N(\mu_a, \sigma_a^2) \tag{4.16}$$

$$m_t^k \leftarrow m_{t-1}^k + A_{t-1}^k \upsilon_t \tag{4.17}$$

Interbank debt repayment Finally the interbank loan obligations encoded in $\vec{D}_{t-1}$ are paid. These are loans generated on G_{t-1} that are now matured and must be repaid with interest.

$$\forall i \in \{1, \ldots, N\} :$$

$$\forall j \in \{1, \ldots, N\} :$$

$$Pay(\vec{D}_{t-1}(i, j), i, j) \tag{4.18}$$

Again, if $Pay(\vec{D}_{t-1}(i,j), i, j)$ returns no solution, then bank i defaults.

4.5.8 Estimating returns

To make investment decisions, banks require an estimate for μ_t^c, σ_t^c, μ_t^a, and σ_t^a. They also need an estimate for β_t^c and β_t^a, the funding costs for credit and assets respectively. To obtain these, they monitor the state of the entire credit and asset markets, since their individual performance in these markets is not stable from period to period. Funding costs are recorded in real time, right as their associated payments are made. The rest of these calculations are done right before investment targets are set, after banks have already ended an iteration by purchasing assets and finalizing their balance sheet.

Debt market As the performance of debt investments depends on an unpredictable collection of decisions made by banks, all of μ_t^c, σ_t^c, and β_t^c are estimated from history. The following sums are calculated to do this.

$$DebtSum_t = [\sum_{e \in E_t} C(e)\mathbb{1}(\tau(e) = Debt)] + [\max(0, m_t^k - mReserve_t)] \tag{4.19}$$

$$DebtMuSum_t = \sum_{e \in E_t} C(e)\widehat{E}[Val_{in}(e)]\mathbb{1}(\tau(e) = Debt) \tag{4.20}$$

$$DebtSigmaSum_t = \sum_{e \in E_t} C(e)\widehat{Var}[Val_{in}(e)]\mathbb{1}(\tau(e) = Debt) \tag{4.21}$$

Here $DebtSum_t$ captures the total funding spent on procuring debt investments. $DebtMuSum_t$ captures the total expected returns on debt. $DebtSigmaSum_t$ captures the volatility of returns on debt. There is randomness in that we do not know the default status of borrowers. Thus, the expectations and variances of Val_{in} are taken over the random Bernoulli variable indicating default.

In Equation 4.19, the second term represents uninvested cash that was reserved for funding debt. Intuitively, it should be the case that when debt cannot be obtained in exchange for cash, then the estimated return on debt should go down to reflect scarcity. The way to think about this scenario is that the bank invested a portion of its cash in debt, and a portion back into cash. Cash is an investment with sure returns of zero, so it contributes nothing to $DebtMuSum_t$ and $DebtSigmaSum_t$, but is counted as utilized funding in $DebtSum$.

Some approximations are being made here. First, we do not know the true expectation or variance of debt returns because we do not have the true default rate on debt. $\widehat{E}$ is thus an *approximate* expectation, and $\widehat{Var}$ is an approximate variance. Since in general $\widehat{DR} < DR$, $DebtMuSum_t$ is overestimated. In addition, a major assumption embedded in Equation 4.21 is that the return distributions for all debt are mutually independent. This assumption tends to bias $DebtSigmaSum_t$ downwards (since in reality returns on debt are positively correlated due to a common asset price shock and financial contagion), which decreases the variance estimate for debt returns. Thus, debt tends to look over-attractive as an investment to banks.

To calculate the cost of funding for debt, observe that along a payment chain of length L (i.e., consisting of L individual payments) in an EFCN consisting entirely of credit edges, the Lth payment funds the $L-1$st debt investment, the $L-1$st funds the $L-2$nd, and so on up to the second payment funding the source's issued debt. Now consider a single debt cancellation in the middle of a payment path. The receiver of the debt cancellation payment does not hold any new debt as a result of the payment, so the next payment along the path should not count towards the cost of funding debt. However, the payer of the debt cancellation is possibly funding debt that was originated to it in the payment preceding the debt cancellation.

Suppose we have a payment P of amount X given by payment mechanism Pay, consisting of payment paths $P^{(1)}, \ldots, P^{(K)}$. Each payment path may originate debt, and we would like to know the funding cost of this debt so that we may include it in our estimate of the debt market's expected return. Recall that each $P^{(k)}$ consists of a subset of edges on which a flow of $f^{(k)}$ is routed from s to t, and $\sum_{k\in\{1,dots,K\}} f^{(k)} = X$.

Let the total ***cost of funding debt*** during period t be COD_t, and initialize $COD_t = 0$ every period. To update this given a single payment P, iterate through each $P^{(k)} \in P$. Let $|P^{(k)}| = L$. Index the edges $e \in P^{(k)}$ in order using $i \in \{1, dots, L\}$ so that $Receiver(e_i) = Payer(e_{i+1})$. Now start with e_1 and note its type $\tau(e_1)$. Move on to consider e_2. If $\tau(e_1) = debt$, do nothing. Else, update $COD_t = COD_t + X\,Val_{out}(e_2)$. Repeat for all pairs $e_i, e_{i+1}, i \in \{2, \ldots, L-1\}$. Note that the interest rate used to calculate Val_{out} is the *realized interest rate* specified by P. For borrowing, this is the interest rate that will be paid by the borrower. For debt cancellation, this is the interest rate on the cancelled debt. COD_t is updated every time there is a payment for any reason, as debt can be issued on any payment.

Once all four sums have been calculated, the investment parameters are updated:

$$\mu_t^c = \frac{DebtMuSum_t}{DebtSum_t}, \quad \sigma_t^c = \frac{DebtSigmaSum_t}{DebtSum_t}, \quad \beta_t^c = \frac{COD_t}{DebtSum_t} \tag{4.22}$$

These parameters are used by all banks via a utility function optimization to make investment decisions for the next period.

Asset market The asset return distribution is invariant over time so $\mu_t^a = \mu^a$ and $\sigma_t^a = \sigma^a$. The values of μ^a and σ^a are public information, and can be used directly by banks. Meanwhile, the cost of funding used to buy assets varies over time depending on how deposit shocks and the debt market play out. These dynamics are opaque to banks, so historical data is used to estimate β_t^a.

Every payment made for asset purchases is executed using the $PayAsset$ payment mechanism. This mechanism returns a payment P of amount $X' = \min(X, MF)$

where MF is the max flow from s to t and X is the amount requested by s. We want to keep track of the sum of ***costs used to fund asset purchases***, COA_t. In each period, initialize $COA_t = 0$. Then for every payment made using $PayAsset$, do $COA_t = COA_t + \sum_{e \in P | Payer = s} C(e) ValOut(e)$. Here, Val_{out} is again using the realized interest rates that $PayAsset$ returns, which are the actual per-unit payment costs to s.

Finally, update the cost of funding for asset purchases:

$$\beta_t^a = \frac{COA_t}{\sum_{i \in \{1,\dots,N\}} A_t^i}. \tag{4.23}$$

4.5.9 Regulation

Several of the model parameters are assumed to be fixed by a regulator. First, the ***reserve rate*** *mReserve* for every bank is fixed. A maximum leverage ratio $\lambda^{\max}$ is also instituted for the unregulated settings.

Meanwhile, Basel regulation institutes a maximum leverage ratio λ_t^B during period t that takes a minimum value of 1 and a maximum value of $\lambda^{\max}$. Basel regulates based on the overall capital-weighted default rate of all banks, $\widehat{DR}(all_t)$. It is a sigmoid function with input $\widehat{DR}(all_t)\beta + \alpha$ for α, β chosen so that $\lambda_t^B = \lambda^{max}$ when $\widehat{DR}(all_t) = 0$ and $\lambda_t^B = 1$ when $\widehat{DR}(all_t) = crit$.

4.6 Experiments and results

4.6.1 Basic properties of the credit cycle model

To illustrate how the credit cycle looks in practice, I first present aggregate results showing the relationship between several key macroeconomic indicators. Note that the figures in this section specify which profile and environment they are generated with, but the qualitative features of each graph are invariant to these factors. I display just one example per graph type for viewability. Environmental settings are the same as in the credit cycle game in Section 4.6.3.

A note on notation for this section: $x\%(0.2\_0.0), (100-x)\%(0.4\_1.0)$ denotes the mixed strategy profile with weight x/100 on $\theta = 0.2$ and issuing unsecured debt, weight (100-x)/100 on $\theta = 0.4$ and issuing secured debt.

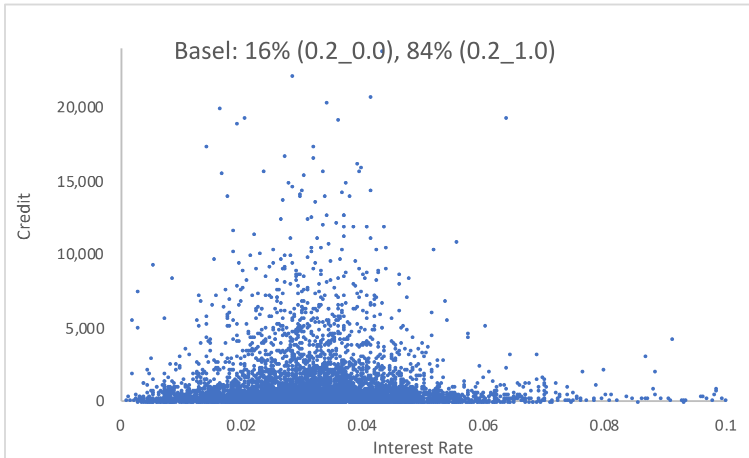


Figure 4.2: Total credit line capacity as a function of interest rates weighted by debt amount

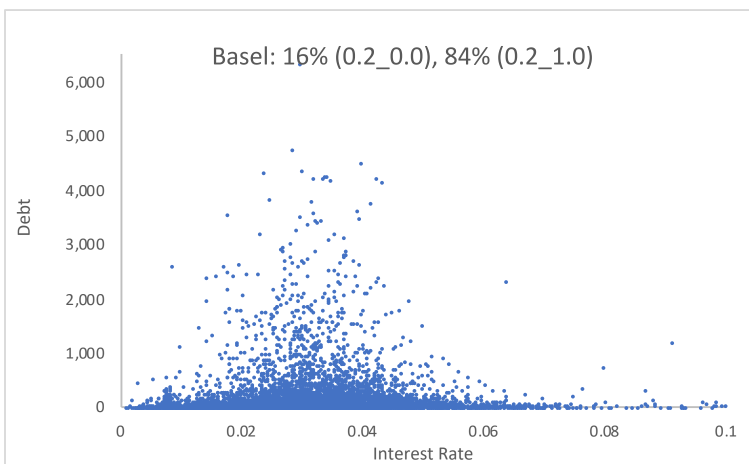


Figure 4.3: Total debt as a function of interest rates weighted by debt amount.

As evidence that the debt market is working as intended, note the non-monotonic relationship between interest rates and credit offered in Figure 4.2. This is indicative of the fact that interest rates are set according to both scarcity (via changing the

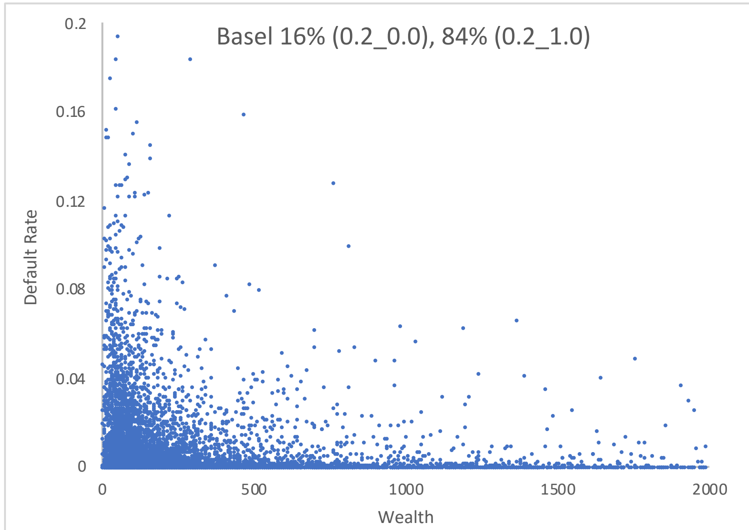


Figure 4.4: Estimated default rates weighted by debt amount as a function of capital.

credit premium) as well as default rate. When premiums increase, more creditors are lured into providing credit lines as debt is seen as a more profitable investment. But as estimated default rates increase, the perceived riskiness of debt increases and banks are more reluctant to extend credit. The first effect dominates at low interest rates and the second effect dominates at high rates. The fact that credit supply decreases at the highest interest rates supports the credit cycle story, where banks withdraw credit in times of high leverage.

Figure 4.3 shows that debt has the same non-monotonic relation with interest rates that credit does. This shows that the credit supply is tracking the demand for debt fairly well. Otherwise, increasing interest rates would lead to a decrease in debt as banks choose to invest less in response.

Figure 4.4 shows that the estimated default rate of banks falls with the amount of capital a bank has. But the estimated default rate is just a function of leverage, and a bank's target leverage should not vary with wealth. The explanation for this oddity is that there is high inequality in size between banks. Banks with a large amount of capital cannot execute high-leverage investment strategies because of a lack of credit issued from surrounding banks. The bigger a bank gets, the more constrained its

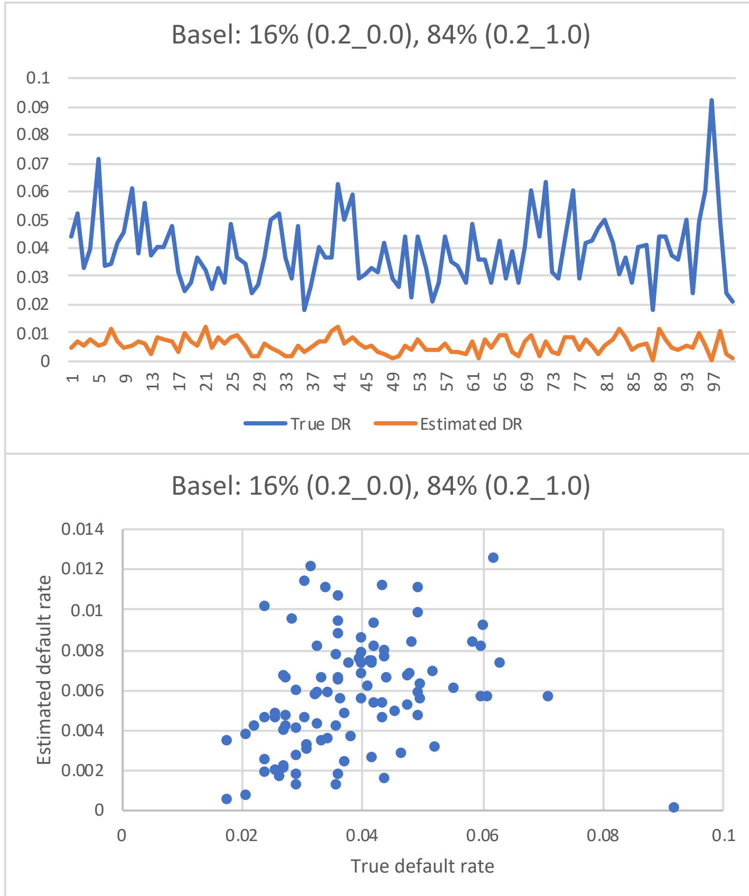


Figure 4.5: Top: default rates and estimated default rates for entire 80 epoch runs, weighted by debt amount. Bottom: Estimated default rate as a function of default rate weighted by debt amount.

leverage, and the healthier its balance sheet in the eyes of lenders.

Figure 4.5 shows that estimated default rates are extremely biased and fairly noisy. Since these estimates correspond to the probability of a default due purely to asset valuation fluctuations, we can surmise that the bias is due to financial contagion pushing real default rates up.

4.6.2 Systemic risk measures

To evaluate the effect of Basel, we need to dictate which measures to use for systemic risk. The first is default rate, which is calculated as the total number of defaults divided by the total number of defaultable periods (i.e., a defaulted bank cannot default again during its ϵ period sojourn). The second is interest rate volatility, calculated as the standard deviation of average interest rate paid each period, weighted by debt amount. Volatile interest rates lead to uncertainty about funding availability, which poses a risk for banks. We also look at losses on debt due to default. This is just an average of sums over each period. This tells us the direct damage caused by default.

4.6.3 Credit cycle game

Experimental setup I run the model for $T = 80$ periods, with the following parameters settings: $N = 10$, $h = 1$, $\epsilon = 7$, $w_{init} = 10$, $d_{init} = 500$, $|DV| = 5$, $\mu^a = 0.05$, $\sigma^a = 0.2$, $mReserve = 0.3$, $UpPrem = 1.05, DownPrem = 0.9, UT = 0.95$, $\gamma = 0.75$, $crit = 0.03$. The strategy set available to banks was $\{(0.2_0), (0.2_1), (0.4_0),$ $(0.4_1), (0.6_0), (0.6_1)\}$. Both $\lambda_{\max} = 40$, where the maximum leverage allowed is 40 regardless of Basel regulations, and $\lambda_{\max} = 10$ are explored.

I present results from our main experiment evaluating Basel regulations. The key takeaways are as follows:

1. Basel regulations taken as an external treatment decrease default rates, interest rate volatility, and debt losses almost across the board in our examined scenarios.

2. Profiles with smaller $|\vec{\theta}|$ and where less secured debt is offered are more dangerous for financial stability (on default rates, debt losses, and interest rate volatility).

3. Although Basel does not encourage strategic shifts towards less risky profiles, in most cases it does not encourage riskier profiles either. The one exception is the 52% $0.2_0.0$, 48% $0.2_1.0$ profile which is, by a thin margin, the riskiest equilibrium seen in the $\lambda_{\max} = 10$ scenario.

4. Basel tends to improve bank profit in riskier profiles, where the benefit to avoiding crisis outweighs the additional leverage constraints.

It is quite surprising that Basel regulations are so effective while acting on extremely noisy and biased estimates of default rate. These estimates are quite useless for giving an idea of actual systemic risk. However, because they are correlated with the real default rate, a policy based on its fluctuations can effectively deter banks from pursuing risky portfolios in times of high systemic leverage. The fact that the estimated default rate is a *leading indicator* of upcoming defaults is good enough for Basel to drastically decrease systemic risk. This finding puts to question the notion that an accurate measure of network risk is necessary to control financial contagion.

Banks pursuing profit are heavily biased towards taking more risks. The benefit to taking more risk is unbounded, while the cost of taking risk is limited to being inactive for ϵ number of periods. This reflects the situation facing banks in the real financial system, and underscores the need for effective regulation of risk.

Default Rate		
Profile	Basel	Unregulated
100% (0.2_0.0)	4.0%	8.0%
92.8% (0.2_1.0), 7.2% (0.4_1.0)	3.8%	5.0%

Interest Rate Volatility		
Profile	Basel	Unregulated
100% (0.2_0.0)	0.100	0.151
92.8% (0.2_1.0), 7.2% (0.4_1.0)	0.013	0.016

Capital		
Profile	Basel	Unregulated
100% (0.2_0.0)	52,442	64,563
92.8% (0.2_1.0), 7.2% (0.4_1.0)	63,382	86,280

Losses on Debt		
Profile	Basel	Unregulated
100% (0.2_0.0)	223	1019
92.8% (0.2_1.0), 7.2% (0.4_1.0)	66	133

Figure 4.6: Four systemic risk metrics evaluated at $\lambda_{\max} = 40$ for 10 funds. Analagous table for $\lambda_{\max} = 10$ in Figure 4.7. Within a table, each row fixes an equilibrium mixed strategy and applies a different regulatory setting, each column fixes a regulatory setting and switches between different mixed strategy equilibrium. Green signifies Basel setting/equilibrium profile, red signifies non-Basel, and blue signifies that the profile appeared in equilibrium undeer both Basel and non-Basel settings.

Default Rate		
Profile	Basel	Unregulated
4.63% (0.2_0.0), 95.37% (0.2_1.0)	3.7%	4.5%
52% (0.2_0.0), 48% (0.2_1.0)	3.9%	7.3%
100% (0.4_0.0)	1.0%	1.1%
16% (0.2_0.0), 84% (0.2_1.0)	3.8%	4.7%

Interest Rate Volatility		
Profile	Basel	Unregulated
4.63% (0.2_0.0), 95.37% (0.2_1.0)	0.025	0.032
52% (0.2_0.0), 48% (0.2_1.0)	0.019	0.078
100% (0.4_0.0)	0.047	0.051
16% (0.2_0.0), 84% (0.2_1.0)	0.028	0.052

Capital		
Profile	Basel	Unregulated
4.63% (0.2_0.0), 95.37% (0.2_1.0)	74,325	69,354
52% (0.2_0.0), 48% (0.2_1.0)	70,039	51,679
100% (0.4_0.0)	21,505	39,428
16% (0.2_0.0), 84% (0.2_1.0)	38,075	55,920

Losses on Debt		
Profile	Basel	Unregulated
4.63% (0.2_0.0), 95.37% (0.2_1.0)	200	368
52% (0.2_0.0), 48% (0.2_1.0)	168	896
100% (0.4_0.0)	52	29
16% (0.2_0.0), 84% (0.2_1.0)	177	435

Figure 4.7: Four systemic risk metrics evaluated at $\lambda_{\max} = 10$ for 10 funds. See Figure 4.6 for key.

4.7 Summary

The model developed in this chapter captures a dynamic process of debt formation at its most granular level. The network that evolves from this debt exhibits both credit freeze and financial contagion endogenously. We find that Basel regulations inserted into the model results in a much more stable financial system. While this seems to encourage slightly more risky behavior from banks, who can rely on regulators to limit them when they invest excessively, the overall effect of Basel is still positive. This finding is surprising in light of the limited information on systemic risk available to regulators. While there has been extensive interest in new network-based risk measures, we found that in this complex network-based model, simple balance sheet information was enough for Basel to reduce systemic risk. These findings supplement those in Chapter II and paint Basel regulations as a useful regulation.

CHAPTER V

Strategic adaptations by automobile firms to the opening of a carbon emissions market

5.1 Introduction

In 1992, the UN established cap-and-trade as the standard regulatory approach to achieve reduction of greenhouse gases (GHG) [8]. This approach allocates pollution rights to large-scale polluters according to a desired emissions cap. Polluters are then permitted to trade these rights, thereby achieving a more efficient economic outcome. Several studies commissioned at the time, as well as numerous subsequent studies, found that this approach resulted in great cost savings as compared to a simple hard-cap approach that does not allow the trading of pollution rights [48, 49].

One domain under the purview of GHG regulation is the vehicle market. The auto market in the US alone represents hundreds of billions of dollars per year in transactions, and environmental regulation has a significant impact on the vehicles produced and sold. A technical challenge in analyzing this market is that vehicles are differentiated products in the eyes of consumers, and the oligopolistic nature of the industry means that each firm has individual pricing power. The demand for each vehicle line depends not only on its own price, but on the price of every other vehicle. In general these prices are set by different firms, resulting in profit functions that are strategically entwined. This is in contrast to the heavily studied power industry, where the product (electricity) is homogeneous. There, the typical scenario has prices that are fixed by an independent party that attempts to maximize social welfare [41]. In the vehicle market, prices are determined as a result of a game that firms play with each other. This complicates the determination of the effects of cap-and-trade, both on overall efficiency and the distribution of costs among firms and consumers.

The innovation in our approach is to cast the cost of pollution to firms as lost opportunities to increase profit due to regulation. We model firms in a vehicle market with many vehicle types. These firms are tied together by consumer demand, which is modeled using a nested-logit form that combines a comprehensive set of vehicle features. This demand model is the product of industrial research, born from the need to make accurate predictions rather than to build analytical models. We compute strategic equilibria among firms in the vehicle market through ***iterative best-response*** (IBR), which is guaranteed to converge for several of the models that we compare. Because our model allows explicit computation of strategic dynamics, we are able to explore counterfactual scenarios such as alternative industrial organization, and different regulation setups.

The market for trading pollution rights (or ***credits***) introduces another level of strategic behavior for firms, as they consider their utilization of credits to shape their vehicle production, along with their bidding behavior in the market for credits. We model the credits market as an iterative double auction mechanism. Equilibrium analysis, or even best-response calculation, is not apparently feasible for the credits-market behavior alone, let alone considering the coupling of credits allocation with strategic behavior in the vehicle market. We therefore explore heuristic strategies for credit trading, and employ ***empirical game-theoretic analysis*** (EGTA), a simulation-based game reasoning framework, to derive equilibrium behavior in this heuristic strategy space.

Our main contributions are twofold. First, we construct and solve novel models for studying emissions trading in the vehicle market. Second, we produce economic insights for this domain, most notably the finding that the welfare gained by moving from hard-cap to cap-and-trade is disproportionately accrued by firms. In aggregate, firm profits under the cap-and-trade regulation approach those of the unregulated case, thanks to market segmentation opportunities created by emissions regulation. In contrast, consumers absorb the brunt of costs through higher prices for polluting vehicles. This effect is more pronounced with a less fragmented industrial organization. When firms use credit trading to manipulate their position in the vehicle market, this effect is reduced slightly, as firms lose some credit trading volume due to an inability to coordinate trades, and thus are not able to segment as efficiently.

5.1.1 Relation to prior work

The standard tool for analyzing cap-and trade emissions regulation is the marginal abatement cost (MAC) curve [30]. At competitive equilibrium, the marginal cost for abating one more unit of GHG pollution is the same for all firms—otherwise, firms could gain from trading the right to pollute. This facilitates analysis of cap-and-trade in the electric power industry [5, 9, 41], where firms lack pricing power and therefore marginal costs directly determine equilibrium. Prior studies on vehicles [37, 38] likewise apply MAC analysis in a non-strategic manner, by focusing on the cost of improving vehicle efficiency. In our setting, the more relevant abatement cost is foregone opportunity to sell profitable high-polluting vehicles, which calls for a more explicitly strategic analysis.

Previous work has followed different approaches to estimate MAC curves for vehicle markets. Firm behavior has been modeled as fixed and non-optimizing [23, 43], as well as attempting to maximize fuel economy for consumers [56]. Our approach models firms as strategic profit maximizers, and represents consumer utility using a demand function.

Studies using simple analytical models have examined the effect of industrial organization, specifically oligopoly, on emissions regulation [42, 57]. These models suggest that oligopolies are detrimental to overall welfare outcomes under cap-and-trade regulation. We duplicate these results under a more complex model, and in addition find that consumers bear the brunt of this welfare loss.

Strategic credit trading has been studied in experimental laboratory settings as well as through stylized analytical models [28, 47, 68]. In general, prior studies have found that when participants have market power in the emissions credit market, there is a decrease in efficiency. We see the same in our study, with total welfare falling when we allow strategic credit trading, but we find in addition that consumers benefit, while firms make less profit.

This work adds to a growing library of AI research that seeks to uncover economic insights using tools from computational game theory [35, 40, 53, 67].

5.1.2 Background

The US government regulates auto emissions primarily through two programs. The *CAFE* program, administered by the National Highway Traffic Safety Administration, mandates corporate average fuel economy standards. The second program, maintained by the Environmental Protection Agency (*EPA*), directly regulates GHG emissions. We focus exclusively on the EPA regulations, which broadly operate as follows:

1. Each vehicle is assigned a real-numbered rating: negative for polluting, positive for non-polluting.

2. Each firm's annual ***credit balance***, defined as sum of ratings weighted by sales volume, must exceed zero.

3. Positive balances may be carried forward into the next year.

4. Credits may be traded between firms within each year.

Due to a combination of technological, consumer-preference, and other economic factors, non-polluting vehicles tend to be much less profitable than polluting vehicles such as SUVs and trucks. Acquiring credits allows a firm to produce a larger fraction of more profitable vehicles.

5.1.3 Overview of models and assumptions

The regulation market cap-and-trade is implemented by opening a market in compliance credits, secondary to the auto market itself. We are interested in understanding the dynamics of the credits market, particularly to examine possible strategic behavior by automakers, and how the special dynamics of this secondary market affect the auto market.

For this purpose, we model four different environments:

1. *Unregulated*. This represents a world where the auto market operates without constraint on emissions.

2. *Hard-cap*. This serves as a baseline model, where each firm must satisfy EPA requirements individually, with no trading of compliance credits.

3. *Credits trading with price takers*. In this model we derive a single clearing price for emission credits, under competitive (Walrasian) assumptions. These assumptions are similar to the MAC models which dominate the literature.

4. *Trading with strategic bidders*. This model includes a concrete trading mechanism for compliance credits, where firms interact through a bidding process, iterating between actions in the credits market and decisions about vehicle

production. Including strategic bidding captures issues of market power and dynamic information that are reflective of cap-and-trade in practice.

In each setting, the solution concept is Nash equilibrium with respect to the decision variables in that setting.

We assume ***complete information*** in the vehicle market game; that is, each firm knows the set of vehicles and costs of its competitors, and also has an accurate consumer demand model. Though not exactly realistic, we believe this provides a reasonable approximation. Real firms can make an educated guess of their competitors' costs using their own costs, and by observing equilibrium prices over time. Furthermore, consumer demand models can be built using historic transaction data available from commercial sources.

Another assumption we make is that the cost of producing vehicles, the consumer demand for vehicles, and the way regulators assign credit balances do not change over time. This assumption is not made because the methods in this chapter rely on it; rather, we do this mainly for convenience as we do not have the information required to forecast these quantities. If forecasts were provided, the analysis could be updated without any model changes.

Finally, we assume that firms do not price their vehicles below cost. This assumption is salient for non-polluting vehicles whose production generates emissions credits that could be sold. We make this assumption mainly as a reasonable baseline expectation for firm behavior. Our models and methodology do not depend on this assumption, and only the empirical results are affected by it.

5.2 A model of auto emissions trading

5.2.1 Notation

Consider a set of m ***products***, denoted $V = \{v_1, \ldots, v_m\}$. These products are sold in each of T time periods. Each product is produced by a single ***firm*** from set $F = \{f_1, \ldots, f_n\}$ where $|F| = n$. The firm producing a particular product is given by function $f : V \to F$. The products produced by firm i are denoted $V_i \subset V$. The set of indices corresponding to V_i are $I_i \subset \{1, \ldots, m\}$. Product ***prices***, which are set by their producers, are given by vector $\Psi = \{\psi_1, \ldots, \psi_m\} \in \mathbb{R}^m$. The ***demand function*** $\phi : \Psi \in \mathbb{R}^m \to \mathbb{R}^m$ returns the quantity of each product that is sold to a mass of consumers given Ψ. ***Costs***, which are fixed and given per marginal unit of production, are denoted $\Sigma = \{\sigma_1, \ldots, \sigma_m\}$. ***Regulation credits*** are assigned according to a function $\kappa : \phi(\Psi) \in \mathbb{R}^m \to \mathbb{R}^n$. Roughly speaking, these are non-negative for production mixes that achieve a regulatory goal, and negative otherwise.

For convenience, we use V_j to denote a vector selecting V's elements with indices I_j. V_{-j} denotes a vector selecting elements of V with indices from the complement of I_j. It is understood that the union operation, $\cup$, preserves indices such that, for example, $\Sigma_{-j} \cup \Sigma_j = \Sigma$. To select a particular element of vector V with index i, we use $V_{(i)}$. We use superscripts to denote an instantiation of a vector. These instantiations can be of functions or vectors. So for period t, Ψ_j^t is the vector of product prices actually set by firm j. Meanwhile, κ_j^t is equivalent to $\kappa(\phi(\Psi_j^t))$. This is the actual amount of regulation credits assigned to firm j in time t.

To denote elements of a function that are given, we use the | symbol in the following way: $F(x) \mid y, G = x + G(y)$ means that x is a variable, G is a given function, and y is fixed value. It should be clear from context whether functions or constants are being given.

Finally, let θ_j^t denote ***credits retained*** without being sold (by firm j). These

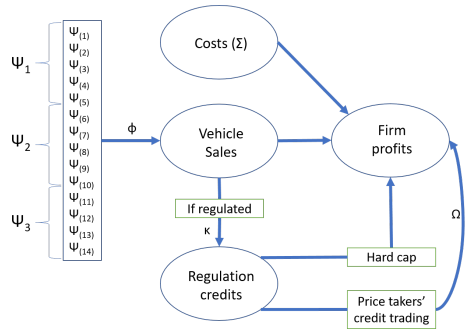


Figure 5.1: Profit generation process.

credits will roll over into the next period, $t + 1$, and must be non-negative because deficits cannot be rolled over. And, let λ_j^t denote ***credit obligations*** held by firm j at time t. Negative λ_j^t means obligations owed (sold) while positive means obligations held (purchased).

5.2.2 The vehicle market model

We model the vehicle market as a Bertrand pricing game, for n multi-product firms with differentiated products. Firm j at time t strategizes over the prices it controls (each element of Ψ_j^t). The profit for firm j made by selling cars at time t is fully determined given a set of vehicle prices $\{\bigcup \Psi^t, t \in \{1, \ldots, T\}\}$, as follows.

$$\Pi_j^{cars}(\Psi^1, \ldots, \Psi^T) \mid \Sigma_j, \phi = \sum_{t=1}^{T} [\Psi_j^t - \Sigma_j]' \phi_j^t. \tag{5.1}$$

The demand function ϕ is a consumer choice model with (nested) logit demand, loosely based on real industry data. Naturally, vehicles in this model are ***gross substitutes*** (increasing the price of a product can only increase the demand for other products). We assume throughout that ϕ is known to all firms.

5.2.3 Unregulated vehicle market

In the unregulated vehicle market, firms set prices in an attempt to maximize Π_j^{cars}. In Equation 5.1, every component of firm payoffs is constant over time. This means that firms treat each period independently and equivalently. Their decision on price is independent of T. Call this constant decision Ψ^0. Then the payoff function can be reduced to

$$\Pi_j^U(\Psi^0) \mid \Sigma_j, \phi = T \times [\Psi_j^0 - \Sigma_j]' \phi_j^0.$$

We can now define a Nash equilibrium for the pricing game in the unregulated market.

Definition V.1 (Unregulated price equilibrium)**.** A set of prices Ψ^* is an unregulated price equilibrium if, for all $j \in \{1, \ldots, n\}$:

$$\Pi_j^U(\Psi^*) \mid \Sigma_j, \phi \geq \Pi_j^U(\Psi_j \cup \Psi_{-j}^*) \mid \Sigma_j, \phi, \forall \Psi_j \in \mathbb{R}^{|I_j|}.$$

Our simulations find Nash Equilibrium in this market using IBR - simulating each automaker's optimal response to the current prices of other automakers. In general games, IBR might not converge, and would instead cycle forever. However, since the game model we employ is a Bertrand pricing game with logit demand model and linear cost function, we can show that IBR converges.

Theorem V.2 (Convergence of Iterative Best Response)**.** *IBR on the market described above converges to a unique (unregulated) price equilibrium. [44]*

5.2.4 Emissions regulation: Hard-cap

Under hard-cap emissions regulation, firms need to satisfy a cap on emissions per vehicle while maximizing vehicle profits. Again letting Ψ^0 be the firms' constant decision, this leads to a constrained profit function in the following form.

$$\Pi_j^{cap}(\Psi^0) \mid \Sigma_j, \phi, \kappa = \begin{cases} -\infty, & \text{if } \kappa_j^0 < 0 \\ T \times [\Psi_j^0 - \Sigma_j]'\phi_j^0, & \text{otherwise} \end{cases}$$

Recall that κ_j^0 is the overall amount of emissions credits assigned to firm j by regulators based on its vehicle sales, and is a weighted sum of ϕ_j^0 where the coefficients are the EPA rating for each vehicle. We can now define Nash equilibrium.

Definition V.3 (Hard-cap price equilibrium)**.** A set of prices Ψ^* is a hard-cap price equilibrium if, for all $j \in \{1, \ldots, n\}$,

$$\Pi_j^{cap}(\Psi^*) \mid \Sigma_j, \phi, \kappa \geq \Pi_j^{cap}(\Psi_j \cup \Psi_{-j}^*) \mid \Sigma_j, \phi, \kappa, \qquad \forall \Psi_j \in \mathbb{R}^{|I_j|}.$$

Although the hard-cap breaks the linearity of the cost function, we can still show (by employing results by [65, 66]) that this market maintains the same useful property of the unregulated case.

Theorem V.4 (Convergence of iterative best response under hard cap)**.** *IBR on the market described above converges to a unique (hard-cap) price equilibrium.*

5.2.5 A price-takers emissions credits market

In an effort to allow firms to meet regulatory constraints as efficiently as possible, credits can be traded on a ***credits market***. This is intended to incentivize firms that have a lower cost for producing credits to take on more of the burden.

In the price-takers market, firms maximize the following profit function.

$\Pi^W_j(\Psi^0)|\Sigma_j, \phi, \Omega, \kappa =$

$$\begin{cases} -\infty, & \text{if } \kappa^0_j + \lambda^0_j < 0 \\ T \times [\Psi^0_j - \Sigma_j]'\phi^0_j + \Omega\lambda^0_j, & \text{otherwise} \end{cases}$$

Recall that λ^0_j is the number of credit obligations bought or sold. These are credits that transfer hands from a firm who produces them by producing non-polluting vehicles, to a firm who needs them to satisfy regulatory requirements.

There are two key assumptions embedded in this payoff function. First, firms take credits prices as given externally. Second, firms do not attempt to manipulate the credits market by retaining credits for the next period, as is allowed by regulation. We will loosen both of these assumptions in our strategic credit trading model.

We can specify a Nash equilibrium given this payoff function. Note that although λ^0_j is a choice made by firms, it is determined mechanically given Ψ^0_j via the constraint $\kappa^0_j + \lambda^0_j < 0$. Thus, λ^0_j is not a strategic variable.

Definition V.5 (Price taker's credit market equilibrium)**.** A set of prices Ψ^* is a price taker's credit market equilibrium if, for all $j \in \{1, \ldots, n\}, \Psi_j \in \mathbb{R}^{|I_j|}$:

$$\Pi^W_j(\Psi^*)|\Sigma_j, \phi, \kappa, \Omega \geq \Pi^W_j(\Psi_j \cup \Psi^*_{-j}) \mid \Sigma_j, \phi, \kappa, \Omega.$$

The convergence result of Iterative Best Response can easily be extended to this setting by adding the cost of compliance to the cost of each vehicle.

Theorem V.6 (Convergence of iterative best response under credit market price)**.** *IBR on the market described above converges to a (credit market) equilibrium.*

Firms in this setting assume they will be able to buy and sell λ^0_j credits without considering who their trading partner will be. In reality, these trades are not all feasible, as setting a particular Ω will usually induce a shortage or surplus in the credits market. So the price taker's Nash equilibrium is only feasible if a ***market clearing condition*** holds:

Definition V.7 (Walrasian credit equilibrium)**.** A tuple (Ψ^*, Ω) is a Walrasian credit equilibrium if Ψ^* is a Price taker's credit market equilibrium, and in addition the following condition holds:

$$\sum_{j\in\{1,\ldots,n\}} \lambda_j^* = 0$$

With credits in hand, a firm has the flexibility to produce more polluting vehicles and earn more profit. By selling credits, a firm has constrained itself to losing profits in the vehicle market since it must produce enough non-polluting vehicles to honor the sale. At a Walrasian credit equilibrium, buyers have determined that their collective profit gain per credit in the Bertrand pricing game from buying a set number of credits is equal to the given price of credits. Similarly, the collective profit loss per credit experienced by sellers by selling a set number of credits is equal to the given price of credits. Thus, the price of credits is analogous to the cost of abatement, and the market clearing condition is analogous to equality of marginal costs of abatement across firms.

5.3 Strategic behavior in the emissions market

The assumption that firms are strictly price takers in the credits market is a simplification of reality. Firms have an incentive to buy and sell credits at prices favorable to themselves, and they have different levels of bargaining power to achieve those prices. In the strategic emissions market, we allow firms to play out this bargaining process using heuristic strategies while trying to maximize estimated profit from both credit obligation transactions as well as the vehicle market.

The mechanism for exchanging credits is a single price multi-unit double auction. Firms submit (price, quantity) bids to either sell or buy credits. After all bids clear, firms observe the outcome of their bids and submit a new round of bids. This process iterates until a fixed horizon is reached. This mechanism does nothing to ensure truth telling, and rich strategic behavior is expected.

Firms in this model strategize over two bidding parameters: χ_j, an ***impatience parameter***, and μ_j, a ***shading parameter***. Roughly speaking, χ_j corresponds to a bid pricing strategy, while μ_j corresponds to a bid quantity strategy. A baseline bidding strategy for each firm is to offer around the historically observed market price of credits at a quantity given by the amount of credits transacted at the associated price taker's credit market equilibrium. This strategy, utilized by choosing a large μ_j and large χ_j, signifies a firm's willingness to be a price taker.

Firms choose their bidding parameters strategically by observing an overall payoff, generated as follows. First, firms submit bids in the credit market. They observe the resulting clearing price and execute any bids that have been matched. They then adjust their bids based on this information, and submit them to the double auction again. They do this for ν ***bidding iterations***, coming away with either positive or negative revenue and, correspondingly, a negative or positive credits balance. Using what they have observed in the credits market, they form a belief about what the future credits price is. Armed with their actual credit balance and their estimated credit price, they enter the vehicle market where iterated best response runs and converges. They then carry forward any unused credits into the next period, and repeat the process. The total ***number of periods*** is an environment setting, called T. The total payoff is the sum of all credit revenues added to the sum of all vehicle profits.

We need to simulate payoffs for many strategy profiles before being able to find a Nash equilibrium in χ and μ. To do this search, we use empirical game-theoretic analysis, which is a tool for sampling payoffs intelligently [10].

The purpose of this model is not to incorporate every possible bidding strategy, but to complement our price-takers credit market analysis. By allowing firms to exercise market power to some degree, we address a gap in our analysis of the credit market for vehicles, since we know that, in reality, there are firms who can bargain successfully. For a deeper, reproducible explanation of the strategic credit trading model, see the s upplemental materials[1].

5.4 Experiments and results

5.4.1 Scenarios and setup

The ***baseline scenario*** in our experiments has $n = 3$ and $m = 14$. ϕ, Σ, and κ are given by our industry sponsor and are based on real data and government guidance. Each firm can produce a mix of emissions credit generating (mostly electric) vehicles and credit consuming (gas) vehicles, at different profitability levels. Firm 1 can produce credit generating vehicles at a lower cost than the others, and will be the seller in the credits market. The other two firms produce highly profitable gas vehicles, and will be buyers in the credits market. Unlike firms 1 and 2, firm 3 does not produce trucks, and produces a much smaller total number of vehicles than the other two.

Two other scenarios examine how extreme types of industrial organization affect the model. The first scenario is a ***monopoly scenario***, where all products are produced by a single firm ($n = 1$,$m = 14$). The second scenario is a ***fragmented scenario***, where every product is produced by a different firm ($n = 14$, $m = 14$). Although the ownership relation from firms to vehicles changes, we keep ϕ, Σ, and κ constant throughout.

For the strategic credit market model, we examine two settings. First, we set $T = 4$ and $\nu = 20$. We call this model setting ***strategic credits (long)***. The

[1] https://www.dropbox.com/s/a2dfjzkfp25ppgv/ijcai2019_supplemental.pdf?dl=0

	Unregulated			Hard Cap			Walrasian credits			Strategic credits (long)			Strategic credits (short)		
Monopoly	12.4	6.3	18.7	10.8	5.1	15.8	-			-			-		
Baseline	10.5	8.9	19.4	7.3	4.8	12.1	10.5	5.9	16.5	10.1	6.0	16.1	10.2	6.0	16.2
Fragmented	10.3	9.1	19.3	3.1	2.8	5.91	10.7	7.8	18.5	-			-		

Figure 5.2: Aggregate welfare in millions. Profit in green followed by consumer surplus in red followed by total welfare in blue.

other strategic model setting, ***strategic credits (short)***, has $T = 5$ and $\nu = 10$. The firms in strategic credits models draw bidding strategy (μ_j, χ_j) from the set $(\chi, \mu) | \chi \in \{0.5, 0.7, 0.9, 0.999\}, \mu \in \{0.01, 0.4, 0.7\}$.

5.4.2 Evaluating welfare

The measure of value we use to judge equilibria is ***total welfare***. Welfare in our case is defined as the sum of ***consumer surplus*** and firm profit. We do not attempt to quantify the benefit to the environment, assuming that emissions regulation is doing its job.

Consumer surplus is a measure of the total savings that consumers get from buying vehicles at an equilibrium price. Given a set of equilibrium prices Ψ^*, consumer surplus for vehicle i (denoted γ_i) is calculated as follows:

$$\gamma_i = \int_{\Psi^*_{(i)}}^{\infty} \phi(\Psi_{(i)} | \Psi^*_{(-i)}) \; d\Psi_{(i)}$$

Total consumer surplus is $\sum_{i=1}^{m} \gamma_i$. In general, consumer surplus decreases with higher equilibrium prices. Calculation is done via numerical integration.

Welfare results are shown in Figure 5.2. Omitted entries for the monopoly row were due to the impossibility of trading with oneself. Instead, since the monopoly controls all vehicle markets, it generates credits in fuel-efficient segments for use in fuel inefficient ones. Thus, the hard-cap model for monopoly is comparable to the credit trading models for other scenarios.

Omitted entries in the competitive setting were due to computational constraints. Since none of these games are symmetric, expanding to 14 firms using the full strategy set requires evaluating payoffs for $12^{14} = 1.28e{+}15$ profiles.

We call attention to several phenomena in Figure 5.2:

1. Monopoly profit is higher than all other scenarios in the unregulated model. In addition, monopoly profit under the hard-cap scenario beats profit under all other regulated scenarios. Both of these results are expected, as monopolies can coordinate between the different vehicle markets more efficiently, both to avoid cannibalization and to navigate regulatory requirements.

2. The credits market allows firms to collectively achieve a near-monopolistic profit level under regulation. The hard-cap monopoly beats the regulated credit trading scenarios by much less than the unregulated monopoly beats all other unregulated scenarios.

3. In the baseline and fragmented scenarios, credit trading results in profit levels at or above those of the unregulated model. Meanwhile, consumer surplus always decreases greatly when regulation with credit trading is applied. Consumers are hurt more than firms by emissions regulation. This effect is least pronounced (though still significant) in the fragmented scenario out of all scenarios.

4. Strategic bidding decreases profits and raises consumer surplus slightly.

The overarching theme is that when credit trading is introduced, firms are able to offset and even overturn losses due to hard-cap emissions constraints by raising prices in tandem. Consumer surplus suffers as a result.

It is the unique industrial organization in the baseline scenario that allows firms to maintain high prices and profits, given the ability to trade credits. Namely, each firm has a very diverse set of vehicles. This allows each firm to strategically exit and enter certain markets based on their credit costs. Compared to the unregulated scenario, firm 1, which must maintain a high credits balance in the credit markets trading scenario, abandons high-pollution trucks. Firm 2, which can buy low-cost credits from firm 1, abandons low-pollution cars. This segmentation gets rid of competitive forces within each vehicle type's market and keeps prices high.

This is in contrast to the fragmented scenario, where firms do not have the option of exiting a vehicle segment. Thus, competitive forces are maintained in each vehicle segment, and consumer surplus is highest in this scenario out of all regulated scenarios. Profit is actually almost at monopoly levels as well. This is because of the firms producing polluting vehicles; the most cost-efficient firms become more dominant, as they can afford to pay for more credits at the same vehicle price. They do this because they know that they can maintain margins even with the credit price factored in. This efficiency results in the highest total welfare out of any regulated scenario. Overall, we find that a more fragmented industrial organization is beneficial to cap-and-trade regulation.

5.4.3 Strategic credit trading analysis

The shift from greater profits to greater consumer surplus in the strategic trading case stems from the fact that fewer credits are traded. Figure 5.4 shows how profit and credit market activity varies among firms, compared to the Walrasian equilibrium baseline. In all cases, fewer credits are traded in the strategic case. The one exception is for Firm 3 in the "long" model, where the credit balance is similar to the Walrasian case. Both large firms (1 and 2) suffer profit losses, while the smaller firm 3 comes out ahead. For an analogous table for the "short" setting, see the supplemental materials.

	v = 20, T = 4		v = 10, T = 5	
	μ*	χ*	μ*	χ*
Firm 1	0.5	0.01	0.5	0.4
Firm 2	0.5	0.01	0.5	0.4
Firm 3	0.9	0.01	0.7	0.7

Figure 5.3: Nash equilibrium bidding parameters

Firm	Vehicle Profit	Total Profit	Absolute Credits Balance	Average Credit Price
1	-2.83%	-4.16%	-4.26%	-1.41%
2	-8.53%	-9.74%	-5.09%	-1.54%
3	3.28%	4.85%	-0.69%	-0.87%

Figure 5.4: Aggregate changes in strategic trading (long) baseline vehicle market as a percentage from Walrasian model.

The root cause of this is the large amount of shading applied at equilibrium (see Figure 5.3). The two large firms are engaged in restricting credits volume for fear of the credit price moving against them if they yield. Thus, they both shade to the maximum level. This leads them to compete with each other in the vehicle market, as firm 1 owes less credit obligations, so can sell more polluting vehicles. A secondary effect is that firm 2 focuses on selling its most profitable polluting vehicle, trucks, since it has limited access to pollution credits. This creates an opening for firm 3, who chooses to shade less in order to buy more credits and sell more SUVs. Overall, there is less segmentation of the market between the two large firms, resulting in a transfer of welfare from firms to consumers. For evidence of this summary of results at the vehicle level, see the supplemental materials.

The biggest welfare effect of the strategic maneuvering in the credits market is more competition in the truck sector in the (long) model. Since firm 3 has a relatively large quantity of credits, it focuses on selling SUVs, crowding the other two firms out into the truck market. The subsequent price war transfers welfare from firms to consumers.

5.5 Policy implications

By analyzing how firms adapt strategically to emissions regulation, we gain insight into the question: Who pays for emissions reduction in the US vehicles market?

The answer is, resoundingly, that consumers pay as firms segment themselves by vehicle type and raise prices to protect themselves against production constraints. Trading emissions credits provides a remarkably efficient way for firms to preserve profits, as even a monopoly facing emissions regulation cannot do appreciably better. Strategic trading provides some relief to consumers, as firms tend to limit credits trading volume and thus compete more. But with this redistribution of welfare comes some deadweight loss. The best way we have found to bring consumer surplus up, and with it, overall welfare, is to fragment the production of vehicles so that no firm has an incentive to focus on one vehicle at the expense of another. Our results suggest that cap-and-trade is definitely an improvement over a hard cap, but that caution must be used to ensure that price hikes do not become an issue for consumers in the oligopolistic vehicle market.

CHAPTER VI

Conclusions

In this thesis I have evaluated new and upcoming regulations in two of the largest and most complex markets in the United States: the financial market and the automobile market. The financial markets consist of two main traded products: debt and assets. Basel regulations changed how both of these markets work by imposing stricter, procyclical leverage requirements. I take an existing model of asset market dynamics with a fixed supply of debt, and show how the strategic adaptation of banks to Basel changes systemic risk measures. These changes are enough to overturn the finding from the study in which the model was originally developed that Basel is destabilizing in asset market.

I then turn to examine the market for debt. An important feature of the debt market is that there are as many types of debt as pairs of agents, and possibly more. These many types of debt form a network through which banks impact each other, unlike the monolithic asset market where they can only impact each other through price movements. Being able to model the full network of debt is important in order to account for phenomena seen during the last crisis, such as credit freeze and financial contagion. As no existing model has the ability to do this, I build my own. I start with specifying EFCNs, a framework for writing down debt and credit networks and making payments on them. I then use EFCNs to build a behavioral model for profit-seeking banks that dynamically updates an EFCN in response to market conditions. I use this model to evaluate Basel regulations, finding that it performs well at reducing instability by using only balance sheet data, despite the fact that deep network insights (such as the exposure of each bank to potential financial contagion) are completely hidden to this type of data.

In the automobile market, I evaluate the impact of emissions regulations that allow the trading of emissions credits. I find that the credits market becomes a platform for firms to maintain profits through price increases. Thus, the majority of the cost of implementing an emissions cap falls on consumers.

Beyond the economic findings, I have also demonstrated that EGTA is a powerful tool in the analysis of agent-based models. Without the ability to solve for an equilibrium, agent-based models can often come to a variety of conclusions depending on how agent behavior is fixed. This is especially true if agents are permitted to execute complex strategies as the range of outcomes of the model becomes quite diverse. EGTA allows agent-based models to be principled in selecting among model outcomes: following an equilibrium profile rather than an arbitrarily chosen one.

As mentioned in the introduction, EFCNs offer a rich opportunity for further modeling of the banking system. A more complex asset market where banks influence each other through their decisions on asset purchases would help connect the model to a large branch of asset market-based research such as the works based on the leverage cycle. Banks with shared equity stakes might behave quite differently when deciding how much to invest.

The economic findings in this thesis reproduce qualitative observations (such as financial contagion in the 2008 crisis and lack of price decreases following EPA cap-and-trade regulations), but no attempt has been made to match quantitative data. This shortcoming could be addressed given access to the right data sources, and would help sharpen findings to be more relevant to policy-making. The scenarios evaluated experimentally in this thesis are chosen using common sense, an approach which could be improved upon using data.

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