Incentive Mechanisms for Managing and Controlling Cyber Risks: The Role of Cyber Insurance and Resource Pooling

by

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To Maman, Baba, Mahya and Minoo

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ABSTRACT

Faced with a myriad of costly and frequent cyber threats, organizations not only invest in software security mechanisms such as firewalls and intrusion detection systems but increasingly also turn to cyber insurance which has emerged as an accepted risk mitigation mechanism and allows purchasers of insurance policies to transfer their risks to the insurer.

Insurance is fundamentally a method of risk transfer, which in general does not reduce the overall risk and may provide disincentives for firms to strengthen their security; an insured may lower its effort after purchasing coverage, a phenomenon known as *moral hazard*. As cyber insurance is a common method for cyber risk management, it is critical to be able to use cyber insurance as both a risk transfer mechanism and an incentive mechanism for firms to increase their security efforts.

This is the central focus and main goal of this dissertation. Specifically, we consider two features of cybersecurity and their impact on the subsequent insurance contract design problem.

- The first is the interdependent nature of cybersecurity, whereby one entity's state of security depends not only on its own effort but also on the effort of others in the same eco-system (e.g., vendors and suppliers).

- The second is our ability to perform an accurate quantitative assessment of security posture at a firm-level by combining recent advances in Internet measurement and machine learning techniques.

The first feature, i.e., the risk interdependence among firms is an interesting aspect that makes this contract problem different from what is typically seen in the literature: how should policies be designed for firms with dependent risk relationships? We show security interdependence leads to a profit opportunity for the insurer, created by the inefficient effort levels exerted by the insureds who do not account for risk externalities when insurance is not available. Security pre-screening then enables effective premium discrimination: firms with better security conditions may get a discount on their premium payment. This type of contract allows the insurer to take advantage of the profit opportunity by incentivizing insureds to increase their security effort and improve the state of network security. We show this conclusion holds even when an insurer has the ability to seek loss recovery when an incident can be attributed to a third party. By embedding these concepts in a practical rate-schedule based underwriting framework we show that these results can be readily implemented in existing practice.

While pre-screening is an effective method to incentivize effort, the insureds may lower their efforts after the pre-screening and post-contract, within the policy period, in yet another manifestation of moral hazard. We show that this can be mitigated through periodic screening combined with premium adjustment, effectively resulting in an *active policy* that has built-in contingencies, and the actual premium payable is realized over time based on the screening results.

Outside the context of insurance, the study of inefficient security investment and how to design incentives is commonly formulated as an interdependent security game. In a departure from typical taxation and subsidy based mechanisms, we consider resource pooling as a way to incentivize effort in a network of interdependent agents, by allowing agents to invest in themselves as well as in other agents. We show that the interaction of strategic and selfish agents under resource pooling improves the agents' efforts as well as their utilities.

CHAPTER 1

Introduction

1.1 Motivation and background

Cyber technologies have brought enormous benefit to society over the past decades and made people and communities more connected. On the other hand, these technologies also provide opportunities for data breaches and large-scale cyber attacks. These incidents can compromise personal and sensitive data and cause business interruptions and ruin companies' reputation and assets. Many large companies, such as Marriot, Facebook, Google, Amazon, Equifax, and Capital One, Target, JP Morgan Chase, have been the victims of cyber attacks and data breaches. For instance, Google had to shut down its Google+ service in April 2019 as personal information of 52 million users were at risk due to the software vulnerabilities. In July 2019, Capital One experienced a cyber incident where the personal information of 100 million individuals was compromised. The average cost of a data breach in the US is estimated to be $8.19 million, more than twice the global average [3].

Firms and companies typically protect themselves from cyber attacks by using advanced and sophisticated cyber defense technologies as well as privacy preserving algorithms [45, 88–90]. Technology based solutions have their limitations, however. For instance, Equifax was aware of its software vulnerability at least two months prior to the 2017 data breach; similarly, two-factor authentication, had it been implemented on its servers, could have prevented hackers from gaining access to JP Morgan Chase's data. In yet another example, the 2014 Target breach was caused by a compromised HVAC contractor who was given VPN access, while Target had all the top-of-the-line security technologies and its intrusion detection system actually triggered an alarm in this case which was not followed up on. These examples all point to the critical human factor in our defenses, and point to the need to consider incentive mechanisms that may lead to better and more judicious security investment and the adoption of better security practices and human due diligence.

Investment in security is generally considered a non-excludable public good with positive externalities. Its benefit is two-fold: the security investment by a firm not only improves its own security posture but also benefits other firms in the same eco-system. This is because a compromised user/entity can be used for propagating attacks against other entities, and a protected entity decreases the chance of attack propagation. This dependency arises in different domains such as financial networks [11], transportation systems [19], and cyber-physical systems [10].

How to incentivize the provision of public goods has a rich literature, some of which we review later in this introduction, including those based on taxation and subsidy. This dissertation will primarily focus on the use of cyber insurance, which has emerged in practice as an accepted risk management and mitigation mechanism that allows purchasers of insurance policies to transfer their risks to the insurer. In 2018, the global cyber insurance market size was $2.92 billion, and is expected to reach $29.8 billion by the end of 2025 [1]. This growing market [26, 31] has motivated extensive literature (see e.g. [8, 29, 37, 38, 50, 55, 61, 64, 73, 76–79, 87]), which aims to understand the unique characteristics of these emerging contracts, and their effect on the insureds' security expenditure.

Cyber insurance is fundamentally a method of risk transfer (from the insured to the insurer), which in general does not reduce the overall risk; in particular, an insured may lower its effort after purchasing coverage, a phenomenon known as *moral hazard* detailed in the next section. In the context of cybersecurity, this means that while it is important for a firm to have insurance to protect against potentially large losses from data breaches, it may also provide disincentives for firms to strengthen their security. In other words, even though the insured enjoys a higher utility by purchasing a policy, its state of cyber risk may now be worse, see, e.g., [58, 79].

A central theme of this dissertation then, is to explore how cyber insurance can be used not only as a risk transfer tool, but also as an incentive mechanism for firms to increase their security efforts/lower their cyber risks, in the presence of risk dependencies, positive externality, and information asymmetry.

For the remainder of this introduction, we will give an overview of cyber insurance as a form of contract followed by an overview of the dissertation and our main contributions.

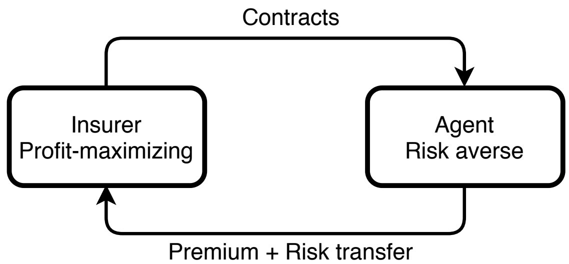


Figure 1.1: Illustration of an insurance contract as a principal-agent problem

1.2 Cyber insurance

Mathematically, the design of an insurance policy can be cast as an instance of the principal-agent problem [17], often studied as a two-stage game with a first mover (principal) and a follower (agent). As shown in Figure 1.1, an insurance policy is a contract between the insurer (principal) and a risk-averse insured (agent), who pays a premium in exchange for coverage in the event of a loss. In the first stage the insurer designs and discloses how the contract terms shall be calculated; in the second stage the insured decides on his effort towards self-protection (against potential loss), which is not observed by the insurer. A market for such contracts exists because of the agent's risk aversion: if the agent is risk neutral then he wouldn't be seeking to transfer his risk and the insurer will not be able to make a profit. For this reason, even though in practice the insurer is also risk-averse (a primary reason for the existence of re-insurance), for the purpose of this dissertation, we shall treat the insurer as risk-neutral and profit maximizing.

There exists an information asymmetry in this setting concerning the contract design: the agent typically possesses more information about his conditions than the principal does. Two common and well-known issues associated with this are *moral hazard* and *adverse selection*. Moral hazard refers to unobservable actions taken by the agent; adverse selection refers to unobservable type (e.g., the extent of risk aversion) of the agent. In the context of cyber insurance, moral hazard translates to the fact that the insured may lower his effort after purchasing insurance, as he is now protected from its consequences, thereby leading to a worse state of security, while adverse selection means that those with higher risk-aversion are more likely to seek insurance/risk protection.

To mitigate moral hazard, which exists in all insurance, a widely employed concept is *premium discrimination*, i.e., an agent/insured who exerts higher effort pays less premium. This, however, relies on the insurer's ability to assess the effort exerted by the insured. There are generally two types of assessment: pre-screening and post-screening. Pre-screening occurs before the insured enters into a contract and can be done at the beginning of each contract period; the result of this process gives the insurer an estimate of risk on the insured, which then factors into the contract terms. Post-screening involves at least two contract periods, whereby the second-period premium is increased if a loss event occurs during the first period. Prior work shows that both pre-screening and post-screening are generally effective in mitigating moral hazard and increasing the insured's effort in other more mature areas of insurance such as home, property, and auto.

The effect of premium discrimination on users' security investments depends on the features of the underlying environment and may improve the state of security. A number of studies focused on a monopolistic insurance market and showed that a monopolistic market in the presence of a profit neutral cyber insurer can improve the network security as compared to a scenario without insurance using premium discrimination [29, 55, 67], while others studied a competitive cyber-insurance market and showed that under certain conditions it is impossible to improve network security using a cyber-insurance contract [67, 78].

How effective risk assessment and premium discrimination are in cyber insurance will be examined closely in this dissertation, given the additional unique characteristics and challenges detailed below.

1. There is a lack of actuarial data and domain knowledge to accurately determine risk and liability. This can lead to inconsistent perceptions of risk. Particularly, this is the case when even though cyber incidents collectively have become commonplace, they remain relatively rare for a given organization.

2. Cyber risks are heavily interdependent due to complex business and vendor relationships among organizations; the cyber risk one faces is no longer the result of one's own actions but the results of all of one's vendors' and suppliers' actions. A prime example is when a service provider is attacked, its customers suffer business interruption loss.

3. Cyber risks are fast-changing compared to other of risks typically under insurance coverage. The threat landscape can change dramatically following events ranging from vulnerability disclosure, development of exploit kits, to geopolitical, cyber-vigilantism, and copycat actions.

The above leads us to rethink how insurance contracts have traditionally been designed and how they should be designed meet the unique characteristics of cyber risks.

1.3 Overview of the dissertation and main contributions

This dissertation sets out to address the challenges outlined in the previous section as we elaborate below.

1.3.1 Overview of the chapters

Chapter 2 focuses on the first two challenges. Specifically, it considers a contract scenario involving multiple risk-dependent insureds/agents as illustrated in Figure 1.2, where the state of security of an agent depends not only on his investment but also on the other's, and where the insurer is able to perform an imperfect risk assessment or *pre-screening* of the agents at the beginning of each contract period. The risk dependency is modeled by assuming that the loss each agent incurs is drawn from a normal distribution whose parameters are functions of a linear combination of both agents' effort levels.

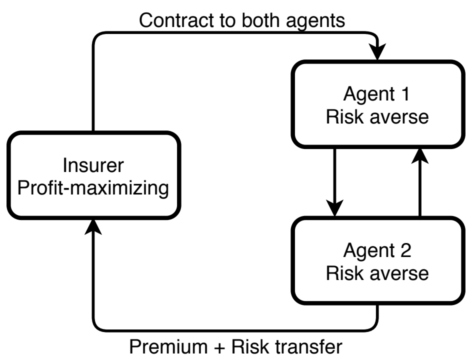


Figure 1.2: Interaction of an insurer with two interdependent agents

We shall show that security interdependency leads to a profit opportunity for the insurer, created by the inefficient effort levels exerted by agents who do not account for the risk externalities when insurance is not available; this is in addition to risk transfer that an insurer typically profits from. Security pre-screening then allows the insurer to take advantage of this additional profit opportunity by designing the appropriate contracts which incentivize agents to increase their effort levels, effectively selling each other's commitment to the agents in addition to insuring their risks. We further identify conditions under which this type of contract leads to not only increased profit for the insurer, but also an improved state of security. Chapter 2 includes work that has appeared in [41, 42, 44].

Chapter 3 again focuses on the first two challenges mentioned in Section 1.2 but examines the contract design problem within a common underwriting framework used in practice. This chapter introduces a second model where security investment affects the probability of loss events and that loss to one agent may be attributed to another agent whom the former depends on. Specifically, we consider a service provider (SP) and its customers; a breach to the service provider may cause a business interruption to the customers but not the other way around. Of particular interest is the analysis and comparison among three different policy portfolios shown in Figure 1.3. In portfolio A, the insurer underwrites only the service provider. In portfolio B, the insurer underwrites both the service provider and its customers. Portfolio C is a case where the insurer underwrites only the customers and seeks compensation from the service provider's insurer when loss can be attributed to the latter.

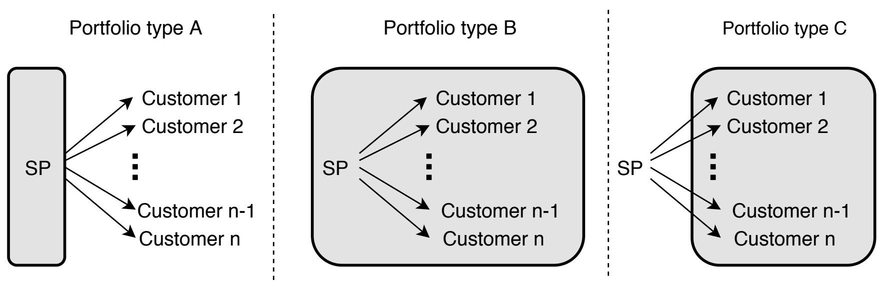


Figure 1.3: Three portfolio types consisting of a service provider (SP) and n customers: shaded areas indicated entities insured.

We shall show that Portfolio B is the insurer's best strategy in terms of profit: by incentivizing the SP (through premium discrimination) to increase its effort in reducing risk, it also benefits from the reduced (spill-over) risk to the customers. Moreover, Portfolio B also results in the highest social welfare. This chapter includes work that has appeared in [39, 40, 43].

Chapter 4 focuses on the first and third challenges and proposes a viable risk assessment method to overcome those challenges. Specifically, we examine the effectiveness of premium discrimination in the presence of rare and extremely large cyber losses. This chapter also considers the fast-changing nature of cyber risks and proposes a type of *active policy* using periodic pre-screening and embedded contingencies, which is able to overcome this challenge. This chapter includes work that has appeared in [46].

In Chapter 5, we take a step back and consider other incentive mechanisms for risk-dependent agents. The study of inefficient security investment and how to design incentives is commonly formulated as an interdependent security (IDS) game with networked agents and positive externalities, where each agent chooses an effort/investment level for securing itself [14, 25, 49, 53]. One main consequence of positive externality is under-investment, i.e., agents under-invest in security to take advantage of other agents' effort and investment. Prior work has extensively studied mechanisms to induce more socially desirable investment levels [52]; these include incentive mechanisms that encourage security investment through taxation/subsidies and with voluntary participation [21, 24, 62, 85], and dictation mechanisms that are implemented by a social planner (e.g., government) who can dictate decisions using regulations [32, 49].

In a departure from such mechanisms, this chapter considers *resource pooling* as a way to incentivize effort in a network of interdependent agents, by allowing agents to invest in themselves as well as in other agents. We shall show that the interaction of strategic and selfish agents under resource pooling improves the agents' efforts as well as their utilities. This chapter includes work that has appeared in [47, 48].

1.3.2 Main contributions of the dissertation

This dissertation contributes to the literature of the economics of information security, incentive mechanisms and contract theory. Our main contributions are summarized as follows.

1. Cyber insurance as an incentive mechanism to improve network security (Chapter 2): this chapter considers a profit-maximizing insurer with the voluntary participation of the agents/clients. The following are the contributions of this chapter.

    - It considers a profit-maximizing risk-neutral insurer, and it shows that the market exists for cyber insurance in the presence of risk-neutral, interdependent agents/insureds. Specifically, it shows that interdependency among risk-neutral agents provides a profit opportunity for the insurer, and without interdependency, there is no market for cyber insurance.
    - It shows that premium discrimination using pre-screening in the presence of a single risk averse agent always improves the network security as compared to a contract without premium discrimination. It further shows that cyber insurance in the presence of a

single risk averse agent cannot improve the network security as compared to a scenario without insurance.

- It shows that the premium discrimination using pre-screening may improve network security as compared to the no insurance scenario in a network of risk-averse, interdependent agents. It further provides sufficient conditions under which this type of premium discrimination improves network security.
- It further considers risk averse insurer and identifies sufficient conditions under which cyber insurance contracts are able to improve the security investment.

2. Controlling risk dependency through cyber insurance contracts (Chapter 3): the chapter's goal is to answer the following question: when faced with risk dependency (between a service provider (SP) and its clients, is it better for an insurer to underwrite all or only the clients and leave the SP to someone else to underwrite the dependency, with the ability to recover all or part of the loss attributable to the SP? The followings are the main findings of the third chapter,

    - It shows that when the insurer insures both SP and its customers, she is able to incentivize the SP to improve his security. As the SP provides positive externality for his customers, all agents benefit from this security improvement, and the chance of cyber incident decreases.
    - Further, it shows that indeed there is a benefit for the insurer in insuring both SP and customers: the insurer obtains higher profit in doing so by taking advantage of the risk dependency, and incentivizing the SP to improve his security. Then, the security improvement increases the insurer's profit as she pays less coverage in a network with a better state of security.

3. Effective premium discrimination in the presence of rare loss incidents (Chapter 4): This chapter tries to identify the effective premium discrimination method when the cyber loss is rare but extremely large.

    - Even though post-screening and pre-screening can be effective methods for premium discrimination and mitigating moral hazard in general, post-screening cannot be useful at all in the presence of a rare loss incident. It further implies that the pre-screening is an effective method for mitigating moral hazard if the cyber loss is rare but extremely large.

- It proposes *active policies* to prevent the insureds from lowering their efforts in the middle of the policy period after pre-screening is done. It shows that pre-screening has to be performed more often, and premium should be adjusted after each screening. This effectively means that the insurance contract should be an active policy with contingencies on periodic screening.

4. The role of resource in an InterDependent Security (IDS) game (Chapter 5): This chapter's goal is to address under-investment issue through resource pooling, i.e., we allow agents to have the ability to both invest in themselves as well as in other agents. The main findings can be summarized as follows.

   - At the unique Nash equilibrium of the game with resource pooling, every agent obtains higher utility as compared to that under the Nash equilibrium of the game without resource pooling.
   - The social welfare (measured by total utility) at the Nash equilibrium of the game with resource pooling is higher than that under the socially optimal outcome of the game without resource pooling induced by Vickrey-Clarke-Groves (VCG) mechanism.
   - It shows that the resource pooling satisfies voluntary participation, i.e., no agent has an incentive to opt out of resource pooling unilaterally.
   - Lastly, it shows that community-based resource pooling, where each agent is able to pool his resources within the community that he belongs to, is able to improve the social welfare and agents' utilities.

CHAPTER 2

Designing Cyber Insurance Policies: The Role of Pre-Screening and Security Interdependence

2.1 Introduction

As mentioned in Section 1.2, in addition to moral hazard and adverse selection, there are three challenges for a cyber insurer. This chapter focuses on the first two challenges:

- Limited data and information to determine insured's risk.
- Interdependence of cyber risks, i.e., security standing of an entity often depends not only on its own effort towards implementing security metrics, but also on the efforts of other entities interacting with it within the eco-system [35, 36, 54, 59].

The first challenge can be partially addressed using recent advances in Internet measurements combined with machine learning techniques which allow us to perform quantitative security posture assessments at a firm level [56]. This can be used as a tool to perform an initial security audit, or *pre-screening*, of a prospective client to mitigate moral hazard by premium discrimination and the design of customized policies.

The second challenge is crucial for the insurer's contract design problem, as the insurer may need to offer coverage to each insured for both its losses due to direct breaches, as well as indirect losses caused by breaches of other entities.

In this chapter, we focus on pre-screening and risk dependency and design a cyber insurance contract to incentivize insureds to improve network security. To distinguish the effect of pre-screening and risk dependency on the cyber-insurance contract design problem, we begin by considering a single-agent; this allows us to remove the effects of risk interdependence and focus on the role of pre-screening. We consider both risk-neutral and risk-averse agents. We first show that when the agent is risk-neutral, a market for cyber-insurance does not exist, this is consistent with previous results, see e.g. [8, 57]. For the risk-averse agent on the other hand, a cyber-insurance market exists. We show that the agent's effort inside the contract increases as the quality of pre-screening increases, that is, the insurer can use pre-screening to mitigate moral hazard. Nevertheless, we show that even with perfect pre-screening, the agent's effort inside the contract remains below his effort before the introduction of insurance. In other words, for a single-agent, and even in the absence of moral hazard, the introduction of cyber-insurance deteriorates the state of network security.

We will next analyze the effect of risk interdependence by considering the design of cyber-insurance contracts for two interdependent agents. We again consider both risk-neutral and risk-averse agents. Here, in contrast to the single agent case, we obtain a rather surprising result: an insurance market *exists* even for two risk-neutral agents. As there is no risk-transfer between the agents and the insurer in this scenario, we conclude that the emergence of a market is due to the agents' interdependence. We intuitively interpret this finding as follows. The interdependency among agents leads them to under-invest in security at the no-insurance equilibrium; this is commonly referred to as *free-riding*, see e.g., [54]. The under-investment issue provides a profit opportunity for the insurer, and she is able to encourage the agents to pay premiums by addressing the under-investment issue and coordinating them. In particular, the insurer can use pre-screening to offer a pair of contracts that incentivize the agents to improve their levels of effort. In return for improving his effort level as prescribed by the contract, an insured is not only offered coverage in case of a loss, but further the commitment of the other agent to higher effort, which will lead to further reduction in the insured's risks. Consequently, network security under these contracts is higher than the no-insurance equilibrium, which further benefits the insurer by lowering the risks of the insureds in its portfolio.

We will then consider the combined effect of risk transfer, interdependence, and security pre-screening, by considering a network of two interdependent risk-averse agents. Similar to the risk-neutral case, the interdependence leads to free-riding by agents in the absence of insurance. Consequently, the insurer can extract profit from both fronts: risk transfer, and taking advantage of the efficiency gap by incentivizing agents to exert higher effort. We identify a sufficient condition under which insurance leads to the improvement of network security compared to the no-insurance scenario. We illustrate these results in both a two-heterogeneous-agents model and an

N-homogeneous-agents model. Lastly, we discuss the effects of correlation in agents' losses, as well as a risk-averse insurer, on the cyber-insurance contracts, and illustrate our findings through numerical simulations.

2.1.1 Main findings

Our main finding is that security interdependence among agents seeking cyber-insurance leads to a profit opportunity for the insurer. A cyber-insurer profits not only from risk-transfer, but also from *coordinating* interdependent agents: each agent will be required to improve its levels of investment in security, in return for the guarantee that other agents will do so as well. Security pre-screening allows the insurer to take advantage of this additional profit opportunity, by designing the appropriate contracts which incentivize agents to increase their effort levels. Together, these contracts can lead to an improvement in the state of network security.

Our analysis is primarily based on a two-agent model. While technically limited in scope, this simple model offers substantial conceptual insights, some of which are more generally applicable. We also use numerical examples to highlight where conclusions are expected to hold under more relaxed assumptions.

2.1.2 Chapter organization

The remainder of this chapter is organized as follows. We review related work in Section 2.2. We present the single agent model in Section 2.3, followed by the analysis in Section 2.4. We present the two-agent model and analysis in Section 2.5. We discuss an N-homogeneous-agent case in Section 2.6, present numerical results in Section 2.7, and conclude in Section 2.8.

2.2 Related work

We provide an overview of existing literature that is most closely related to this chapter. These studies have considered either competitive or monopolistic insurers, as well as either mandatory or voluntary adoption by the insured. The works in [64, 76–79, 87] consider competitive insurance markets under compulsory insurance, and analyze the effect of insurance on agents' security expenditures. The authors of [78, 79] consider a competitive market with homogeneous agents, and show that insurance often deteriorates the state of network security as compared to the no-insurance scenario. [76, 77] study a network of heterogeneous agents and show that the introduction of insurance cannot improve the state of network security. Ogut *et al.* [64] study the impact of the degree of agents' interdependence, and show that agents' investments decreases as the degree of interdependence increases. Yang *et al.* [87] study a competitive market under the assumption of voluntary participation by agents, with and without moral hazard. In the absence of moral hazard, the insurer can observe agents' investments in security, and hence premium discriminates based on the observed investments. They show that such a market can provide incentives for agents to increase their investments in self protection. However, they show that under moral hazard, the market will not provide an incentive for improving agents' investments.

The impact of insurance on the state of network security in the presence of a monopolistic welfare maximizing insurer has been studied in [9,29,37,38,67]. In these models, as the insurer's goal is to maximize social welfare, assuming compulsory insurance, agents are incentivized through premium discrimination, i.e., agents with higher investments in security pay lower premiums. As a result, these studies show that insurance can lead to improvement of network security. An insurance market with a monopolistic profit maximizing insurer, under the assumption of voluntary participation, has been studied in [55], which shows that in the presence of moral hazard, insurance cannot improve network security as compared to the no-insurance scenario.

Our assumptions on the model, namely a profit-maximizing insurer and voluntary participation, are similar to [55]. Our work differs from [55], as well as other existing work, in that we illustrate (i) the role of pre-screening in mitigating moral hazard, and (ii) the possibility of designing contracts that leverage sufficiently accurate pre-screening and agents' interdependence to improve the state of network security.

2.3 Model and preliminaries: single agent

We begin by considering the single-period contract design problem between a risk-neutral insurer and a single agent[1]; we refer the interested reader to [57] for an overview of contract theory. The analysis of the single-agent case allows us to study solely the role of pre-screening by excluding the interdependency, and later, in conjunction with the analysis of Section 2.5.2 and 2.5.3, to uncover the role of interdependency.

[1] Throughout the chapter, we use she/her and he/his to refer to the insurer and agent(s), respectively.

An agent exerts *effort* $e \in [0, +\infty)$ towards securing his system, incurring a cost of c per unit of effort. Let L_e denote the loss, a random variable, that the agent experiences given his effort e. We assume L_e has a normal distribution,[2] with mean $\mu(e) \geq 0$ and variance $\lambda(e) \geq 0$. For ease of exposition, we assume that $\lambda(e)$ is sufficiently small compared to $\mu(e)$, so that $\Pr(L_e < 0)$ is negligible. We assume both $\mu(e)$ and $\lambda(e)$ are strictly convex, strictly decreasing, and twice differentiable. The decreasing assumption implies that increased effort reduces the expected loss, as well as its unpredictability. The convexity assumption suggests that while initial investment in security leads to considerable reduction in loss, the marginal benefit decreases as effort increases. In other words, it is not possible to reduce risk from cyber attacks to zero even if the agent exerts very large effort [34, 53]. We further preclude the possibility of misclaims by assuming that the realized loss is observed perfectly by both the insurer and the agent.

In general, the effort exerted by an agent is not observable by the insurer; this information asymmetry is formally referred to as moral hazard. We assume that in order to reduce this asymmetry and attain better information about the agent, the insurer can conduct a *pre-screening* of the agent's security standing. Through pre-screening, the insurer obtains *pre-screening outcome* $S_e = e + W$, where W is a zero mean Gaussian noise with variance σ^2. We assume both agent and insurer know the distribution of W; such assessment can be obtained through a range of possible methods and (Internet) measurement techniques, information from initial surveys filled out by the agent, external audits, or internal audits conducted by a third party firm. We assume S_e is conditionally independent of L_e, given e. The pre-screening outcome S_e will be used by the insurer in determining the terms of the contract.

2.3.1 Linear contract and the insurer's payoff

We consider the design of a set of *linear* contracts. Specifically, the contract offered by the insurer consists of a base premium p, a discount factor α, and a coverage factor β. The agent pays a premium $p - \alpha \cdot S_e$, and receives $\beta \cdot L_e$ as coverage in the event of a loss. We let $0 \leq \beta \leq 1$, i.e., coverage never exceeds the actual loss. The insurer's utility (profit) is given by:

$$V(p, \alpha, \beta, e) = p - \alpha \cdot S_e - \beta \cdot L_e \,. \tag{2.1}$$

[2]The normal assumption on L_e is to some extent justified by the fact that L_e is meant to capture the sum total of losses from a variety of sources, such as hacking, malware, insider threats, etc.

The insurer's expected profit is then given by $\overline{V}(p,\alpha,\beta,e) = p-\alpha e-\beta\mu(e)$.

2.3.2 Risk-neutral agent

The utility of a risk-neutral agent without insurance is given by,

$$\begin{aligned} U(e) &= -L_e - ce \\ \overline{U}(e) &\overset{\Delta}{=} E(U(e)) = -\mu(e) - ce \end{aligned} \tag{2.2}$$

If the agent chooses not to enter a contract, he bears the full cost of his effort as well as any realized loss. Therefore, the optimal effort (m) of the uninsured agent is $m = \arg\min_{e\geq 0} \ \mu(e)+ce$ and his expected utility outside the contract is $u^o := \overline{U}(m)$.

On the other hand, if the agent purchases a contract (p,α,β) from the insurer, then his utility, and expected utility, are given by:

$$\begin{aligned} U^{in}(p,\alpha,\beta,e) &= -p+\alpha S_e - L_e + \beta L_e - ce \\ \overline{U}^{in}(p,\alpha,\beta,e) &\overset{\Delta}{=} E(U^{in}(p,\alpha,\beta,e)) = -p+(\alpha-c)e+(\beta-1)\mu(e) \end{aligned} \tag{2.3}$$

2.3.3 Risk-averse agent

For simplicity we shall use the same notation for risk-averse agents as for risk-neutral agents. The utility of a risk-averse agent without insurance is given by:

$$U(e) = -\exp\{-\gamma\cdot(-L_e - ce)\}\ , \tag{2.4}$$

where γ denotes the *risk attitude* of the agent; a higher γ implies more risk aversion.[3] We assume γ is known to the insurer , thereby eliminating adverse selection and solely focusing on the moral hazard aspect of the problem.

Using basic properties of the normal distribution, we have the following expected utility for the agent:

$$\overline{U}(e) = E(-\exp\{-\gamma\cdot(-L_e - ce)\}) = -\exp\{\gamma\cdot\mu(e) + \frac{1}{2}\gamma^2\lambda(e) + \gamma ce\}\ . \tag{2.5}$$

[3]Exponential utility exhibits constant absolute risk aversion (CARA).

Using (2.5), the optimal effort for an agent outside the contract is given by $m := \arg\min_{e\geq 0}\left\{\mu(e)+\frac{1}{2}\gamma\lambda(e)+ce\right\}$. Again, let $u^o = \overline{U}(m)$ denote the maximum expected payoff of the agent without a contract.

If a risk-averse agent accepts a contract (p,α,β), his utility is given by:

$$U^{in}(p,\alpha,\beta,e) = \\ -\exp\{-\gamma\cdot(-p+\alpha\cdot S_e - L_e + \beta\cdot L_e - ce)\}\ . \tag{2.6}$$

Noting that S_e and L_e are conditionally independent, his expected utility is

$$\overline{U}^{in}(p,\alpha,\beta,e) = -\exp\{\gamma(p+(c-\alpha)e+\frac{1}{2}\alpha^2\gamma\sigma^2+(1-\beta)\mu(e)+\frac{1}{2}\gamma(1-\beta)^2\lambda(e))\}\ . \tag{2.7}$$

2.3.4 The insurer's problem

The insurer designs the contract (p,α,β) to maximize her expected payoff. In doing so, the insurer also has to satisfy two constraints: Individual Rationality (IR), and Incentive Compatibility (IC). The first stipulates that a rational agent will not enter a contract with expected payoff less than his outside option u^o, and the second that the agent chooses an effort level maximizing his utility given contract parameters. The interaction between the insurer and the agent is sequenced as follows. First the insurer announces the contract parameters, and the agent commits to effort e given the contract parameters. Then the insurer observes S_e, and final premium $p-\alpha S_e$ is calculated. Formally, the insurer's problem can be written as follows,

$$\begin{aligned} \max_{p,\alpha\geq 0, 0\leq\beta\leq 1} \quad & \overline{V}(p,\alpha,\beta,e) = p-\alpha\cdot e-\beta\cdot\mu(e) \\ \text{s.t.} \quad \text{(IR)} \quad & \overline{U}^{in}(p,\alpha,\beta,e)\geq u^o \\ \text{(IC)} \quad & e\in\arg\max_{e'\geq 0}\ \overline{U}^{in}(p,\alpha,\beta,e') \end{aligned} \tag{2.8}$$

The above optimization problem can be simplified, for risk-neutral and risk-averse agents, respectively. As the base premium is a constant in the contract, the (IC) constraint for a risk-neutral agent can be rearranged as:

$$e\in\arg\min_{e'\geq 0}\ (c-\alpha)e'+(1-\beta)\mu(e')\ . \tag{2.9}$$

Similarly, the (IC) constraint for a risk-averse agent can be rewritten as:

$$e \in \arg\min_{e' \geq 0} (c-\alpha)e' + (1-\beta)\mu(e') + \frac{\gamma}{2}(1-\beta)^2\lambda(e') \tag{2.10}$$

Next, we can simplify the (IR) constraint using the following lemma; proofs can be found in the appendix.

Lemma 2.1 *The (IR) constraint is binding in the optimal contract.*

By lemma 2.1, the (IR) constraint of a risk-neutral agent can be written as $p = -u^o - (c-\alpha)\cdot e - (1-\beta)\mu(e)$

and, for a risk-averse agent,

$$p = w^o - (c-\alpha)e - \frac{\gamma}{2}\alpha^2\sigma^2 - (1-\beta)\mu(e) - \frac{\gamma}{2}(1-\beta)^2\lambda(e), \tag{2.11}$$

where $w^o := \frac{\ln(-u^o)}{\gamma} = \min_{e\geq 0}\{\mu(e) + \frac{1}{2}\gamma\lambda(e) + c\cdot e\}$.

Using the above expressions to substitute for the base premium p in the objective function in (2.8), and using the simplified expressions for the (IC) constraints, we re-write the insurer's contract design problem as follows.

Insurer's problem with a risk-neutral agent:

$$\begin{aligned} &\max_{\alpha\geq 0, 0\leq\beta\leq 1} \quad -u^o - \mu(e) - c\cdot e \\ &\text{s.t., } e = \arg\min_{e'\geq 0} \ (c-\alpha)e' + (1-\beta)\mu(e') \end{aligned} \tag{2.12}$$

Insurer's problem with a risk-averse agent:

$$\begin{aligned} &\max_{\alpha\geq 0, 0\leq\beta\leq 1, e\geq 0} \ \ w^o - \mu(e) - \tfrac{\gamma}{2}(1-\beta)^2\lambda(e) - ce - \tfrac{\gamma}{2}\alpha^2\sigma^2 \\ &\text{s.t., } \ e = \arg\min_{e'\geq 0}(c-\alpha)e' + (1-\beta)\mu(e') + \tfrac{\gamma}{2}(1-\beta)^2\lambda(e') \end{aligned} \tag{2.13}$$

2.4 Role of pre-screening for a single agent

We now solve the optimal contract problem posed in (2.12) and (2.13), respectively.

2.4.1 Risk-neutral agent (problem (2.12))

In this case, the objective function of the insurer is given by $-u^o - \mu(e) - c \cdot e$. However, note that $u^o = \max_{e \geq 0}\{-\mu(e) - ce\}$, and therefore the insurer's profit is at most zero. A contract with $(p = 0, \alpha = 0, \beta = 0)$ will yield a payoff of zero, making it an optimal contract. We thus conclude that it is optimal for the insurer to not offer a contract to a risk-neutral agent. Also note that in this case the quality of pre-screening, or indeed the availability of pre-screening regardless of the quality, plays no role in either the insurer's or agent's decisions.

2.4.2 Risk-averse agent (problem (2.13))

We start with the following theorem on the state of network security, defined as the effort exerted by the agent, before and after the purchase of a contract.

Theorem 2.1 *Assume that* $(\hat{\alpha}, \hat{\beta}, \hat{e})$ *solves optimization problem* (2.13)*. Then* $\hat{e} \leq m$*, where* m *is the level of effort outside the contract; in other words, insurance decreases network security.*

Proof. Assume that $(\hat{\alpha}, \hat{\beta}, \hat{e})$ solves optimization problem (2.13), and that, by contradiction, $\hat{e} > m \geq 0$.

First, recall that the agent's optimal effort m outside the contract is given by $m := \arg\min_{e \geq 0}\left\{\mu(e) + \frac{1}{2}\gamma\lambda(e) + ce\right\}$. For m to be the minimizer, we should have $c + \mu'(m) + \frac{1}{2}\gamma\lambda'(m) \geq 0$. Next, consider the following two cases:

(i) $\hat{\alpha} = 0$. Starting from the first order condition (FOC) on the (IC) constraint, we have,

$$\begin{aligned} &(1-\hat{\beta})\mu'(\hat{e}) + \tfrac{1}{2}\gamma(1-\hat{\beta})^2\lambda'(\hat{e}) + c = 0 \\ &\Rightarrow \mu'(\hat{e}) + \tfrac{1}{2}\gamma\lambda'(\hat{e}) + c < 0 \\ &\Rightarrow \mu'(m) + \tfrac{1}{2}\gamma\lambda'(m) + c < 0 \end{aligned} \tag{2.14}$$

Here, the second line follows from the decreasing nature of $\mu(\cdot)$ and $\lambda(\cdot)$, and the third line follows from their convexity. The last inequality is impossible given the optimality of the effort m outside the contract. This contradiction shows that we cannot have $\hat{e} > m$.

(ii) $\hat{\alpha} > 0$. Given the assumption that $\hat{e} > m$, and $\mu(\cdot)$ and $\lambda(\cdot)$ are strictly convex, we have,

$$\begin{aligned} 0 &\leq c + \mu'(m) + \tfrac{1}{2}\gamma\lambda'(m) \\ &\leq c + \mu'(m) + \tfrac{1}{2}\gamma(1-\hat{\beta})^2\lambda'(m) \\ &< c + \mu'(\hat{e}) + \tfrac{1}{2}\gamma(1-\hat{\beta})^2\lambda'(\hat{e}) \end{aligned} \tag{2.15}$$

Therefore, if the insurer decreases $\hat{\alpha}$, the agent decreases his effort (this can be seen from the IC constraint), and consequently the insurer's utility increases, as the objective function of the insurer, $w^o - \mu(e) - \frac{1}{2}(1-\hat{\beta})^2\lambda(e) - ce - \frac{1}{2}\gamma\alpha^2\sigma^2$, is decreasing in e and α at $e = \hat{e}, \alpha = \hat{\alpha}$. Therefore, $(\hat{\alpha}, \hat{\beta}, \hat{e})$ is not the optimal contract. Again by contradiction, we conclude that the agent's effort in the optimal contract should be less than or equal to m. ■

Theorem 2.1 illustrates the inefficiency of cyber insurance as a tool for improving the state of security. Existing work [67, 78] have reached a similar conclusion when studying competitive/unregulated cyber insurance markets. Note also that Theorem 2.1 holds regardless of the pre-screening quality. We next examine the role of pre-screening in this model. We first analyze its impact on the insurer's profit.

Theorem 2.2 *Let $v(\alpha,\beta,e,\sigma^2)$ denote the payoff of the insurer, at a contract (α,β) when the agent exerts effort e, and the noise of pre-screening is σ^2. Let $z(\sigma^2) :=$ $\{\max_{\alpha\geq 0, 0\leq\beta\leq 1, e\geq 0}\ v(\alpha,\beta,e,\sigma^2)$, s.t. (IC)$\}$ be the principal's payoff under the optimal contract as a function of the pre-screening noise. We then have $z(\sigma_1^2) \leq z(\sigma_2^2)$, $\forall \sigma_1^2 \geq \sigma_2^2$. That is, $z(\sigma^2)$ is a decreasing function of the pre-screening noise.*

Proof. Let $v(\alpha,\beta,e,\sigma^2)$ be the payoff of the insurer, at a contract (α,β), when the agent exerts effort e and the noise of pre-screening is σ^2, and let $z(\sigma^2)$ be the insurer's profit at the optimal contract as a function of the pre-screening noise. We have,

$$\begin{aligned} z(\sigma_1^2 + \sigma_2^2) &= \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, \text{IC}} v(\alpha,\beta,e,\sigma_1^2+\sigma_2^2) \\ &\leq \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, \text{IC}} v(\alpha,\beta,e,\sigma_1^2) + \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, \text{IC}} \{-\frac{1}{2}\alpha^2\gamma\sigma_2^2\} \qquad (2.16) \\ &\leq \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, \text{IC}} v(\alpha,\beta,e,,\sigma_1^2) = z(\sigma_1^2) \qquad (2.17) \end{aligned}$$

Therefore, $z(\sigma_1^2 + \sigma_2^2) \leq z(\sigma_1^2)$, $\forall \sigma_2^2$. That is, $z(\sigma^2)$ is a decreasing function of the pre-screening noise. ■

The above result is intuitively to be expected, as a strategic insurer can leverage improved pre-screening to better mitigate moral hazard and attain a higher payoff. The more interesting observation is on the effect of pre-screening on the state of network security. The following theorem presents a sufficient condition under which the availability of a pre-screening assessment improves network security, compared to the no pre-screening scenario. Note that we use $\sigma = \infty$ for evaluating the no pre-screening scenario. The equivalence follows from the fact that, as shown in the appendix, by setting $\sigma = \infty$, the insurer's optimal choice will be $\alpha = 0$, which removes the effects of pre-screening.

Theorem 2.3 *Let e_1, e_2, e_∞ denote the optimal effort of the agent in the optimal contract when $\sigma = \sigma_1$, $\sigma = \sigma_2$ and $\sigma = \infty$, respectively. Let $k(e,\alpha) = \frac{\mu'(e)+\sqrt{\mu'(e)^2-2\gamma(c-\alpha)\lambda'(e)}}{-\gamma\lambda'(e)}$. If $k(e,\alpha_1)^2\lambda(e) - k(e,\alpha_2)^2\lambda(e)$ is non-decreasing in e for all $0 \le \alpha_1 \le \alpha_2 \le c$, then $e_1 \ge e_2$ if $\sigma_1 \le \sigma_2$. In other words, better pre-screening improves network security. In addition, if $k(e,0)^2\lambda(e) - k(e,\alpha)^2\lambda(e)$ is non-decreasing in e for all $0 \le \alpha \le c$, then $e_1 \ge e_\infty$. That is, the availability of a pre-screening improves network security over the no pre-screening scenario.*

Sketch of Proof. The proof proceeds in the following steps:

• We first show that $0 \le \alpha_i \le c$ using the KKT conditions for the (IC) constraint of (2.13), given by

$$\begin{aligned} &(1-\beta_i)\mu'(e_i) + \tfrac{1}{2}\gamma(1-\beta_i)^2\lambda'(e_i) + c - \alpha_i - v_i = 0 \\ &v_i \cdot e_i = 0,\ e_i \ge 0 \end{aligned} \tag{2.18}$$

• We next show that $\alpha_1 \ge \alpha_2$; this follows from the inequalities determining the optimality of the contracts at their respective pre-screening noises. In other words, as pre-screening noise decreases, the insurer offers higher discount factor.

• We then proceed by contradiction, assuming $0 \le e_1 < e_2$. As $e_2 > 0$, by (2.18) we have,

$$\begin{aligned} &(1-\beta_2)\mu'(e_2) + \frac{\gamma(1-\beta_2)^2\lambda'(e_2)}{2} + c - \alpha_2 = 0 \\ &1-\beta_2 = \frac{\mu'(e_2)+\sqrt{\mu'(e_2)^2-2\gamma(c-\alpha_2)\lambda'(e_2)}}{-\gamma\lambda'(e_2)} := \quad k(e_2,\alpha_2) \end{aligned} \tag{2.19}$$

In addition, as $e_1 < e_2$ and $\alpha_1 \ge \alpha_2$, we can show that $\alpha_1 > 0$ and $e_1 > 0$. With $e_1 > 0$, by (2.18) we have,

$$\begin{aligned} &(1-\beta_1)\mu'(e_1) + \frac{\gamma(1-\beta_1)^2\lambda'(e_1)}{2} + c - \alpha_1 = 0 \\ &1-\beta_1 = \frac{\mu'(e_1)+\sqrt{\mu'(e_1)^2-2\gamma(c-\alpha_1)\lambda'(e_1)}}{-\gamma\lambda'(e_1)} := k(e_1,\alpha_1) \end{aligned} \tag{2.20}$$

• Lastly, we show that if $(k(e,\alpha_2)^2 - k(e,\alpha_1)^2)\lambda(e)$ is non-decreasing, then α_1 and e_1 are not the maximizer of the insurer's profit when $\sigma^2 = \sigma_1^2$. This is a contradiction. Therefore, we conclude that $e_1 \geq e_2$. ■

Several instances of $\mu(e)$ and $\lambda(e)$, e.g., $(\mu(e) = \frac{1}{e}, \lambda(e) = \frac{1}{e^2})$, and $(\mu(e) = \exp\{-e\}, \lambda(e) = \exp\{-2e\})$, satisfy the condition of Theorem 2.3.

2.4.3 Comparison

By comparing the contracts in the risk-neutral and risk-averse agent cases, we observe that a market exists and the insurer makes profit only when offering a contract to a risk-averse agent. This is indeed to be expected, as insurance is primarily a method for risk transfer; risk-averse agents are willing to pay premiums that are higher than their expected loss, in order to reduce the uncertainty in their loss, consequently allowing the risk-neutral insurer to make a profit. We further observe that when the market exists, the introduction of pre-screening benefits the insurer (Theorem 2.2) as well the state of network security (Theorem 2.3).

2.5 Model and analysis for two agents

We next study the contract design problem between the insurer and two agents. In particular, we analyze the impact of interdependency and pre-screening on the optimal contract and agents' effort, in the case of two risk neutral and two risk averse agents, respectively, with the former allowing us to exclude the effect of risk aversion and focus on the effect of interdependence.

2.5.1 A model of two agents

The two agents are interdependent, in that the effort exerted by one agent affects not only himself, but also the loss that the other agent experiences. We model the interdependence between these two agents as follows:

$$L_{e_1,e_2}^{(i)} \sim \mathcal{N}(\mu(e_i + x \cdot e_{-i}), \lambda(e_i + x \cdot e_{-i})) \, . \tag{2.21}$$

Here, $\{-i\} = \{1,2\} - \{i\}$, and $L_{e_1,e_2}^{(i)}$ is a random variable denoting the loss that agent i experiences, given both agents' efforts. The *interdependence factor* is denoted by $x \in [0,1)$. Note that this is not a unique modeling choice and is indeed a simplification; a more general way of expressing correlated risks would be to model the losses as jointly distributed; more on extensions is discussed in Section 5.9.

We assume the agents' utilities are again given by (2.2) and (2.4) for risk-neutral and risk-averse agents, respectively, with the loss distributions replaced by the above expression. We allow the two agents to have different effort cost c_1, c_2, as well as different risk attitudes γ_1, γ_2.

The insurer can again conduct a pre-screening assessment, $S_{e_i} = e_i + W_i$, on each agent i, where W_i is a zero mean Gaussian noise with variance σ_i^2. We assume that W_1 and W_2 are independent [4], and that $S_{e_1}, S_{e_2}, L^{(1)}_{e_1,e_2}, L^{(2)}_{e_1,e_2}$ are conditionally independent given e_1, e_2.

Similar to the single agent case, we need to evaluate the agents' outside options from purchasing a contract. These will then be used to impose the individual rationality constraints in determining the terms of the contracts. However, compared to the single agent case, the outside option of one agent is now influenced by the participation choice of the other agent as well. Specifically, we need to evaluate the agents' utilities as well as potential contracts in the following three scenarios:

(i) neither agent enters a contract;

(ii) one enters a contract, while the other opts out; and

(iii) both purchase contracts.

Here, Case (ii) is the outside option for agents in Case (iii), and Case (i) is the outside option for agents in Case (ii). Therefore, in order to evaluate the participation constraints of agents when both purchase insurance contracts (Case (iii)), we first need to find the optimal contracts and agents' payoffs in Cases (i) and (ii). We therefore evaluate the agents' utilities for each case, and subsequently solve the insurer's contract design problem, in Sections 2.5.2 and 2.5.3 for risk-neutral and risk-averse agents, respectively.

2.5.2 Two risk-neutral agents

We first consider two risk-neutral agents. As mentioned above, in order to evaluate the agents' opt-out options and finding the optimal contract, the insurer's problem and the agents' utilities need to be studied under three different cases. We begin by analyzing these three cases, and then proceed to discussing the role of pre-screening and the contracts' effect on network security.

[4] An example and discussion on correlated pre-screening noises can be found in the appendix.

2.5.2.1 Case (i): neither agent enters a contract

Let G^{oo} denote the game between two risk-neutral agents who have purchased cyber insurance contracts. In this game, agents' efforts e_1, e_2 are their actions, and the expected payoffs of risk-neutral agents, with unit cost of effort $c_1, c_2 > 0$, are given by:

$$\overline{U}_i(e_1, e_2) = -\mu(e_i + xe_{-i}) - c_i e_i \ . \tag{2.22}$$

The best response of each agent is therefore given by

$$B_i^{out}(e_{-i}) = \arg\max_{e_i \geq 0} \ -\mu(e_i + xe_{-i}) - c_i e_i \ . \tag{2.23}$$

The above optimization problem is convex, and has the following solution:

$$\begin{aligned} m_i \quad & = \arg\min\nolimits_{e \geq 0} \ \mu(e) + c_i e, \quad i = 1, 2 \ , \\ B_i^{out}(e_{-i}) \quad & = (m_i - xe_{-i})^+ \ , \end{aligned} \tag{2.24}$$

where $(a)^+ = \max\{a, 0\}$. The Nash equilibrium is given by the fixed point of the best-response mappings $B_1^{out}(e_2)$ and $B_2^{out}(e_1)$:

$$e_1 = (m_1 - xe_2)^+, \text{and } e_2 = (m_2 - xe_1)^+ \tag{2.25}$$

To find a fixed point, we consider three cases,

• $e_1 = 0, e_2 \geq 0$: In this case, $e_2 = m_2$. Also, this case is valid if $m_1 - xm_2 \leq 0$ otherwise e_1 should be nonzero.

• $e_2 = 0, e_1 \geq 0$: similar to previous case, $e_1 = m_1$. This case is valid if $m_2 - xm_1 \leq 0$ otherwise e_2 should be nonzero.

• $e_1 > 0, e_2 > 0$: In this case, we solve the following system of equations:

$$e_1 = m_1 - xe_2, \text{ and } e_2 = m_2 - xe_1 \tag{2.26}$$

The solutions of above equations is given by,

$$\begin{aligned} e_1 &= \tfrac{m_1 - x \cdot m_2}{1 - x^2} \\ e_2 &= \tfrac{m_2 - x \cdot m_1}{1 - x^2} \end{aligned} \tag{2.27}$$

Notice that this case is valid if $\frac{m_1 - x\cdot m_2}{1-x^2} > 0$ and $\frac{m_2 - x\cdot m_1}{1-x^2} > 0$. Therefore, given $0 \le x < 1$, system of equations (2.25) has a unique fixed point. Agent i's effort, $e_i^*(m_i, m_{-i})$, at the unique Nash equilibrium is given by:

$$e_i^*(m_i, m_{-i}) = \begin{cases} \frac{m_i - x\cdot m_{-i}}{1-x^2} & \text{if } m_i \ge x\cdot m_{-i} \text{ and} \\ & \quad m_{-i} \ge x \cdot m_i \\ 0 & \text{if } m_i \le x\cdot m_{-i} \\ m_i & \text{if } m_{-i} \le x \cdot m_i \end{cases} \tag{2.28}$$

Therefore, $u_i^{oo} = \overline{U}_i(e_1^*(m_1, m_2), e_2^*(m_2, m_1))$ is the utility of agent i in the equilibrium when agents do not choose to enter the contract. As we will see shortly, an insurer uses her knowledge of u_i^{oo} to evaluate agents' outside options when proposing optimal contracts.

2.5.2.2 Case (ii): one and only one enters a contract

Assume without loss of generality that agent 1 enters a contract, while agent 2 opts out. We use G^{io} to denote the game between the insured agent 1 and uninsured agent 2. The agents' expected payoff in this case is:

$$\begin{aligned} \overline{U}_1^{in}(e_1, e_2, p_1, \alpha_1, \beta_1) &= -p_1 - (c_1 - \alpha_1)e_1 - (1-\beta_1)\mu(e_1 + xe_2) \\ \overline{U}_2(e_1, e_2) &= -\mu(e_2 + xe_1) - c_2 e_2 \end{aligned} \tag{2.29}$$

Let $B_1^{in}(e_2)$ denote the best response of agent 1. The following optimization problem finds its best response:

$$\begin{aligned} B_1^{in}(e_2) &= \arg\max_{e_1 \ge 0} \overline{U}_1^{in}(e_1, e_2, p_1, \alpha_1, \beta_1) \\ &= \arg\max_{e_1 \ge 0} -p_1 - (c_1 - \alpha_1)e_1 - (1-\beta_1)\mu(e_1 + xe_2)\ . \end{aligned} \tag{2.30}$$

The above optimization problem is convex, and has a solution given by,

$$\begin{aligned} m_1(\alpha_1, \beta_1) &= \arg\min_{e \ge 0}\{(c_1 - \alpha_1)e + (1-\beta_1)\mu(e)\} \\ B_1^{in}(e_2) &= (m_1(\alpha_1, \beta_1) - xe_2)^+ \end{aligned} \tag{2.31}$$

For the uninsured agent 2, it is easy to see that the best-response function is given by $B_2^{out}(e_1)$, the same best response function in game G^{oo}. We can now find the Nash equilibrium as the fixed point of the best-response mappings. Agents' efforts at the equilibrium are $e_1^*(m_1(\alpha_1, \beta_1), m_2)$ and

$e_2^*(m_2, m_1(\alpha_1,\beta_1))$, as defined in (2.28). For notational convenience, we denote these efforts by e_1^*, e_2^*.

Let $\overline{V}^{io}(p_1, \alpha_1, \beta_1, e_1, e_2)$ denote the insurer's utility, when agent 2 opts out and the insurer offers contract (p_1, α_1, β_1) to agent 1, and agents exert efforts e_1, e_2. The optimal contract offered by the insurer to the participating agent is the solution to,

$$\begin{array}{ll} \max_{p_1,\alpha_1,0\leq\beta_1\leq 1,e_1^*,e_2^*} & \overline{V}^{io}(p_1,\alpha_1,\beta_1,e_1^*,e_2^*) = p_1 - \alpha_1 e_1^* - \beta_1 \cdot \mu(e_1^* + x\cdot e_2^*) \\ s.t., & \text{(IR) } \overline{U}_1^{in}(e_1^*,e_2^*,p_1,\alpha_1,\beta_1) \geq u_1^{oo}, \\ & \text{(IC) } e_1^*, e_2^* \text{ are the agents' efforts in NE of } G^{io} \end{array} \tag{2.32}$$

Similar to Lemma 2.1, we can show that the (IR) constraint is binding under the optimal contract. Therefore, we can re-write the insurer's problem by replacing the base premium p_1, leading to,

$$\begin{array}{l} \max_{\alpha_1,0\leq\beta_1\leq 1,e_1^*,e_2^*} -u_1^{oo} - \mu(e_1^* + xe_2^*) - c_1 e_1^* \\ s.t., \text{(IC) } e_1^*, e_2^* \text{ are the agents' efforts in NE of } G^{io} \end{array} \tag{2.33}$$

Let u_2^{io} be the second agent's utility when the insurer offers the *optimal contract* to the first agent and the second agent opts out. The insurer can calculate u_2^{io} by finding the optimal contract in problem (2.33) and the resulting Nash equilibrium of game G^{io}. Similarly, u_1^{oi} denotes the first agent's utility when he opts out and the second agent purchases the *optimal contract*. The insurer uses her knowledge of u_2^{io} and u_1^{oi} in designing a pair of contracts to attract both agents.

2.5.2.3 Case (iii): both agents purchase contracts

Let G^{ii} denote the game between the two agents when they are both in a contract. Assume the insurer offers each agent i a contract (p_i, α_i, β_i). The expected utility of the agents when both purchase contracts is given by

$$\overline{U}_i^{in}(e_1, e_2, p_i, \alpha_i, \beta_i) = -p_i - (c_i - \alpha_i)e_i - (1-\beta_i)\mu(e_i + x\cdot e_{-i}). \tag{2.34}$$

Following steps similar to those in Section 2.5.2.2, B_i^{in}, the best-response function of agent i, is given by

$$B_i^{in}(e_{-i}) = (m_i(\alpha_i,\beta_i) - xe_{-i})^+ , \tag{2.35}$$

where $m_i(\alpha_i, \beta_i)$ is the solution to,

$$m_i(\alpha_i, \beta_i) = \arg\min_{e \geq 0}\{(c_i - \alpha_i)e + (1 - \beta_i)\mu(e)\} \ . \tag{2.36}$$

The agents' efforts at the Nash equilibrium are again the fixed point of the best-response mappings, and will be given by $e_i^*(m_i(\alpha_i, \beta_i), m_{-i}(\alpha_{-i}, \beta_{-i}))$, with $e_i^*(.,.)$ defined in (2.28). For notational convenience, we will denote these as e_i^*.

To write the insurer's problem, note that the outside option of agent 1 (resp. 2) from this game is his utility in the game G^{oi} (resp. G^{io}). Then, the optimal contracts offered by the insurer to the agents is the solution to the following optimization problem:

$$\begin{aligned}
&\max_{p_1, \alpha_1, 0 \leq \beta_1 \leq 1, p_2, \alpha_2, 0 \leq \beta_2 \leq 1, e_1^*, e_2^*} p_1 - \alpha_1 e_1^* - \beta_1 \cdot \mu(e_1^* + x \cdot e_2^*) + p_2 - \alpha_2 e_2^* - \beta_2 \cdot \mu(e_2^* + x \cdot e_1^*) \\
&\text{s.t., (IR) } \overline{U}_j^{in}(e_1^*, e_2^*, p_j, \alpha_j, \beta_j) \geq u_j^{oi}, \ j = 1, 2 \\
&\text{(IC) } e_1^*, e_2^* \text{ are the agents' efforts in NE of } G^{ii}
\end{aligned} \tag{2.37}$$

The (IR) constraints can again be shown to be binding. Therefore, the insurer's contract design problem for two risk-neutral agents is given by,

$$\begin{aligned}
&v^{ii} := \max_{\alpha_1, 0 \leq \beta_1 \leq 1, \alpha_2, 0 \leq \beta_2 \leq 1, e_1^*, e_2^*} -u_1^{oi} - u_2^{io} \\
&-\mu(e_1^* + x \cdot e_2^*) - c_1 \cdot e_1^* - \mu(e_2^* + x \cdot e_1^*) - c_2 \cdot e_2^* \\
&\text{s.t., } e_1^*, e_2^* \text{ are the agents' efforts in NE of } G^{ii}
\end{aligned} \tag{2.38}$$

2.5.2.4 Optimal contracts for two risk-neutral agents

We now analyze the properties of the contracts designed based on the optimization problem (2.38), and their impact on agents' efforts.

Theorem 2.4 *Let e_i^o denote the effort of agent i when insurance is not available, and e_i^{in} denote the effort of agent i in the solution to* (2.38)*, i.e., when purchasing the optimal contract. Also, let $\tilde{e}_i$ denote the effort level of agent i in the socially optimal outcome (i.e, the efforts maximizing the sum of agents' utilities). Then, the insurer offers contracts to both agents, with the following properties,*

(i) $e_i^{in} = \tilde{e}_i$, for $i = 1, 2$. That is, the agents exert socially optimal effort levels in the optimal contract.

(ii) $e_1^{in} + e_2^{in} \geq e_1^o + e_2^o$. That is, when both agents purchase optimal insurance contracts, the overall effort exerted toward security increases compared to the no-insurance scenario.

(iii) $v^{ii} \geq \overline{U}_1(\tilde{e}_1, \tilde{e}_2) + \overline{U}_2(\tilde{e}_1, \tilde{e}_2) - \overline{U}_1(e_1^o, e_2^o) - \overline{U}_2(e_1^o, e_2^o)$*. That is, the principal's profit is higher than the gap between agents' welfare at the socially optimal solution and the no-insurance equilibrium.*

Theorem 2.4, implies the following. Firstly, recall that, as discussed in Section 2.4.3, the insurer cannot make profit from offering contracts to a single risk-neutral agent, as there is no risk transfer from risk-neutral agents to an insurer. However, we observe that the insurer can make profit when offering contracts to interdependent risk-neutral agents. This improvement is due to the agents' interdependency, and can be interpreted as follows. Due to interdependency, agents under-invest in security at the no-insurance equilibrium. This leads to a profit opportunity for the insurer, in which she uses her (accurate) pre-screening assessments to offer premium discounts and (full) coverage of losses, and in turn requires the agents to exert higher effort (in this particular case, the socially optimal levels of effort). This increase in effort is in the insurer's interest, as it lowers the risks of both of its contracts. In addition, this effect can be viewed as the insurer coordinating the agents to address the under-investment issue. That is, the insurer is also providing each agent with the commitment of the other agent to exert higher effort, if he also commits to exerting high effort.

Secondly, Part (iii) of the theorem shows that the profit opportunity for the insurer is even higher than the welfare gap between the socially optimal and Nash equilibrium outcomes. This is due to the fact that the outside option from the contract for agent i is an outcome in which the insurer offers a contract (only) to agent $-i$. The insurer will select this contract in a way that it requires agent $-i$ to exert low effort and get high coverage, effectively forcing agent i to bear the full cost of effort, leading to a utility lower than the no-insurance Nash equilibrium for agent i. Consequently, as agents' (IR) constraints are also binding, it follows that the insurer's profit is in fact the gap between welfare attained under the optimal contract, and the welfare at these low payoff, unilateral opt out outcomes.

Finally, note that the statements of this theorem do not depend on the pre-screening noise $\sigma_i < \infty$. This is because the expected utilities and consequent effort choices of risk-neutral agents are only sensitive to the mean, but not the variances of uncertainties in the problem parameters. As such, under the assumption of zero mean noise in the pre-screening assessments, agents' behavior will be independent of σ.

2.5.3 Two risk-averse agents

We next analyze the case of two risk-averse agents. Again, as discussed in Section 2.5, in order to evaluate the agents' individual rationality constraints and finding the optimal contracts, we need to account for three possible cases based on the agents' participation alternatives.

The ensuing analysis is similar to that presented in Section 2.5.2, by replacing the agent's utility functions with their risk-averse versions and solving the resulting optimization problems. We thus present the details in the appendix. Following the analysis, the simplified insurer's optimization problem is given by

$$\begin{aligned} v^{ii} \quad &= \max_{\alpha_1, 0\leq\beta_1\leq 1, \alpha_2, 0\leq\beta_2\leq 1, e_1^*\geq 0, e_2^*\geq 0} w_1^{oi} + w_2^{io} - \mu(e_1^* + x\cdot e_2^*) - \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1^* + x\cdot e_2^*) \\ &-c_1\cdot e_1^* - \tfrac{1}{2}\alpha_1^2\gamma_1\sigma_1^2 - \mu(e_2^* + x\cdot e_1^*) - \tfrac{1}{2}\gamma_2(1-\beta_2)^2\lambda(e_2^* + x\cdot e_1^*) - c_2\cdot e_2^* - \tfrac{1}{2}\alpha_2^2\gamma_2\sigma_2^2 \\ s.t., \quad & e_1^*, e_2^* \text{ are the agents' efforts in NE of game } G^{ii} \end{aligned} \tag{2.39}$$

where $w_1^{oi} = \frac{\ln(-u_1^{oi})}{\gamma_1}$ and $w_2^{io} = \frac{\ln(-u_2^{io})}{\gamma_2}$.

We now discuss how different problem parameters, particularly the availability of pre-screening, affect the insurer's profit in the optimal contracts, as well as the system's state of security. We first consider the utility of the insurer. Note that the insurer always has the option to not use the outcome of pre-screening by setting $\alpha = 0$ in the contract. Therefore, the insurer's utility in the optimal contract with pre-screening is larger than that in the optimal contract without pre-screening; i.e., the availability of pre-screening is in the insurer's interest.

We now turn to the effect of pre-screening on the state of network security, which we shall measure by the total effort toward security, $e_1 + e_2$.

Theorem 2.5 *Let $m_i = \arg\min_{e\geq 0}\mu(e) + \frac{1}{2}\gamma_i\lambda(e) + c_i e$. Let e_i and e_i^o denote the effort of agent i in the optimal contract and in the no-insurance equilibrium, respectively.*

(i) Assume perfect pre-screening, i.e., $\sigma_1 = \sigma_2 = 0$. Then, $e_1 + e_2 \geq e_1^o + e_2^o$, if,

$$\begin{aligned} &1.\ \mu'(m_i) && < \tfrac{-c_i + xc_{-i}}{1-x^2},\ \ i = 1,2 \\ &2.\ (\mu')^{-1}(\tfrac{-c_i + xc_{-i}}{1-x^2}) && \geq x(\mu')^{-1}(\tfrac{-c_{-i} + xc_i}{1-x^2}),\ \ i = 1,2 \end{aligned} \tag{2.40}$$

That is, under these conditions, insurance improves network security compared to the no-insurance scenario.

(ii) Assume both pre-screening assessments are uninformative. i.e., $\sigma_1 = \sigma_2 = \infty$. Then $e_1 +$

$e_2 \leq e_1^o + e_2^o$. That is, the insurance contract without pre-screening worsens network security as compared to the no-insurance scenario.

The results of Theorem 2.5 can be intuitively interpreted as follows. By Theorem 2.1, with a single risk-averse agent, the insurer profits from the agent's interest in risk transfer. However, the introduction of insurance always reduces network security. In contrast, Theorem 2.5 shows that with interdependent agents network security can improve, while the insurer continues to make profit. Therefore, it is agents' interdependency that plays a key role in the improvement of security. To see why, note that the insurer uses pre-screening and offers premium discounts accordingly in order to incentivize the interdependent agents to increase their effort levels. Providing such incentives is in the insurer's interest, as higher effort exerted by the agent decreases both agents' risk, and consequently, the coverage required by the insurer once losses are realized. Note also that it is the availability of (accurate) pre-screening that provides the required tools for the insurer in designing such incentives; otherwise, as shown in part (ii) of the theorem, improving network security is no longer possible.

The conditions of part of (i) of the theorem can also be interpreted as follows. The first condition imposes a restriction on the derivative of μ, so that the decrease in loss as a function of effort is faster than the normalized cost of effort; as a result, the insurer will have the option to make more profit through loss reduction (by encouraging agents to exert higher effort). The second condition imposes a restriction on the agents' cost of effort and guarantees that both agents exert positive effort (see proof of Theorem 2.5). Specifically, when the two agents' effort costs are sufficiently similar, this condition is satisfied, and both agents exert non-zero effort.

2.6 *N* homogeneous agents, correlated losses, and risk-averse insurer

In this section we show a number of extensions of our results. First, in Section 2.6.1 we study the optimal contracts in a network of N homogeneous risk-averse agents. In Section 2.6.2, we examine the case where the losses of these agents are not only distributionally dependent but also correlated in their realizations; we will also consider the impact of risk aversion on the part of the insurer on the resulting contract.

2.6.1 N-homogeneous risk-averse agents

Consider a network of N homogeneous risk-averse agents given by $\gamma_i = \gamma$, $c_i = c$, and $\sigma_i = \sigma$, $\forall i$. The assumption of homogeneity simplifies the insurer's problem, allowing us to obtain additional insights about the contracts and their impact on network security. Let $\boldsymbol{e} = (e_1, e_2, \ldots, e_N)$ denote the vector of efforts of all agents. The loss of agent i is given by,

$$L_{\boldsymbol{e}}^{(i)} \sim \mathcal{N}(\mu(e_i + x\sum_{j\neq i} e_j), \lambda(e_i + x\sum_{j\neq i} e_j)) \; . \tag{2.41}$$

The agents' expected utility outside the contract is,

$$\overline{U}_i(\boldsymbol{e}) = E(-\exp\{-\gamma(-L_{\boldsymbol{e}}^{(i)} - ce_i)\}) = -\exp\{\gamma(\mu(e_i + x\textstyle\sum_{j\neq i} e_j) + \frac{\gamma\lambda(e_i + x\sum_{j\neq i} e_j)}{2} + ce_i)\} \tag{2.42}$$

Let $m = \arg\min_{e\geq 0} \mu(e) + \frac{1}{2}\gamma\lambda(e) + ce$. Then, the best response mapping of agent i is given by,

$$B_i^{out}(\boldsymbol{e}_{-i}) = (m - x\sum_{j\neq i} e_j)^+ \; , \tag{2.43}$$

where $(x)^+ = \max\{0, x\}$. The Nash equilibrium is the fixed point of the above best response functions, leading to efforts $e = \frac{m}{1+(N-1)x}$ by each agent at the symmetric Nash equilibrium.

When agent i purchases a contract (p,α,β), his expected utility will be given by,

$$\begin{aligned} \overline{U}_i^{in}(\boldsymbol{e}, p, \alpha, \beta) &= E(-\exp\{-\gamma(-p + \alpha\cdot S_{e_i} - L_{\boldsymbol{e}}^{(i)} + \beta L_{\boldsymbol{e}}^{(i)} - c\cdot e_i)\}) \\ &= -\exp\{\gamma(p + (c-\alpha)e_i + \tfrac{1}{2}\alpha^2\gamma\sigma^2 + (1-\beta)\mu(e_i + x\textstyle\sum_{j\neq i} e_j) + \frac{\gamma(1-\beta)^2\lambda(e_i + x\sum_{j\neq i} e_j)}{2})\} \end{aligned} \tag{2.44}$$

Therefore, the best response of agent i, when he enters the contract, is as follows,

$$\begin{aligned} B_i^{in}(\boldsymbol{e}_{-i}) &= (m(\alpha,\beta) - x\textstyle\sum_{j\neq i} e_j)^+ \\ m(\alpha,\beta) &= \arg\min_{e\geq 0}(1-\beta)\mu(e) + \tfrac{1}{2}(1-\beta)^2\gamma\lambda(e) + (c-\alpha)e \; . \end{aligned} \tag{2.45}$$

Similar to the two-agent case, we can write the insurer's contract design problem as follows,

$$\begin{aligned} \max_{\alpha,\beta,e} \quad & N\cdot\{p - \alpha e - \beta\mu(e + x(N-1)e)\} \\ s.t., \quad & \text{(IR) } \overline{U}_i^{in}(\boldsymbol{e}, p, \alpha, \beta) \geq u^{out} \\ & \text{(IC) } \boldsymbol{e} = (e, \cdots, e) \text{ is the effort of the agents at the NE where all are insured} \end{aligned} \tag{2.46}$$

Here, u^{out} denotes the utility of an agent when he is opts out of purchasing a contract, while all other agents purchase contracts. We can again show that the individual rationality constraints in the above problem are binding at the optimal contract. Consequently, the insurer's optimization problem simplifies to:

$$\begin{array}{lll} \max_{\alpha,\beta,m'} & N\cdot\{w^{out}-\mu(m')-\frac{(1-\beta)^2\gamma\lambda(m')}{2}-\frac{c\cdot m'}{1+(N-1)x}-\frac{\gamma\alpha^2\sigma^2}{2}\} \\ s.t., & (IC) \quad m'=\arg\min_{e\geq 0}(1-\beta)\mu(e)+\frac{(1-\beta)^2\gamma\lambda(e)}{2}+(c-\alpha)e \end{array} \tag{2.47}$$

where $w^{out}=\frac{\ln(-u^{out})}{\gamma}$. Note also that problem (2.47) prescribes identical contracts for all agents.

We now analyze the effect of the pre-screening noise, σ, on the state of network security, defined as the sum of all agents' efforts; with homogeneous agents, this is equivalent to each agent's effort.

Theorem 2.6 *Assume N homogeneous agents purchase contracts from an insurer, and let $m=\arg\min_{e\geq 0}\mu(e)+\frac{1}{2}\gamma\lambda(e)+ce$. Let e^o be the effort of an agent in the no-insurance symmetric equilibrium, e' and $\hat{e}$ denote the effort in the optimal contract with perfect pre-screening and no pre-screening, respectively. Then,*

(i) If pre-screening is accurate, i.e., $\sigma=0$, and $m>0$, then $e'\geq e^o$ if and only if $\mu'(m)<-\frac{c}{1+(N-1)x}$. *That is, network security improves after the introduction of insurance with prefect pre-screening.*

(ii) If pre-screening is uninformative, i.e., $\sigma=\infty$, then $e^o\geq\hat{e}$. That is, network security worsens after the introduction of insurance without pre-screening.

Note that this theorem, as well as its interpretation, is similar to the statements of Theorem 2.5 for two heterogeneous agents. In particular, it is straightforward to check that the conditions of part (i) of these theorems are equivalent when setting $c_i=c$ in Theorem 2.5 and $N=2$ in Theorem 2.6.

Finally, the next theorem shows that with sufficiently accurate, yet imperfect pre-screening, the use of pre-screening can lead to improvement of the state of network security compared to the no-insurance equilibrium.

Theorem 2.7 *Assume N homogeneous agents purchase contracts from an insurer. Let $m=\arg\min_{e\geq 0}\mu(e)+\frac{1}{2}\gamma\lambda(e)+ce$, and assume $\mu'(m)<-\frac{c}{1+(N-1)x}$. Let $\hat{e}$ and e^o be the effort level of agents in the optimal contract and at the no-insurance equilibrium, respectively. Let $\tilde{m}$ be the effort* *at which* $\mu'(\tilde{m}) = -\frac{c}{1+(N-1)x}$*. Then, if* $\sigma \leq \frac{\mu(m)+\frac{c}{1+(N-1)x}m-\mu(\tilde{m})-\frac{c}{1+(N-1)x}\tilde{m}}{0.5\gamma c^2}$*,* $\hat{e} \geq e^o$*. That is, introducing pre-screening improves network security as compared to the no-insurance equilibrium.*

2.6.2 The case of risk averse insurer and correlated losses

We next study the problem of designing cyber-insurance policies in a network of N homogeneous risk-averse agents with perfect pre-screening (i.e., $\gamma_i = \gamma$ and $c_i = c$ and $\sigma_i = \sigma = 0$) with correlated losses defined as follows.

Let θ be the covariance between any two losses, that is,

$$Cov(L^i_{\boldsymbol{e}}, L^j_{\boldsymbol{e}}) = \theta, \;\; \forall i \neq j \tag{2.48}$$

We further assume that the insurer is risk-averse, with risk attitude $\delta \geq 0$ and the vector $(L^1_{\boldsymbol{e}}, \cdots, L^N_{\boldsymbol{e}})$ has the multivariate Gaussian distribution. The insurer can conduct a pre-screening of each agent's security posture and receives the pre-screening outcome $S_i = e_i$ as the pre-screening is perfect. Similar to (2.46), we can write the insurer's problem as follows,

$$\begin{array}{ll} \max_{p,\alpha,\beta,e} & E\Big(-\exp\Big\{-\delta(\sum_{i=1,\cdots,N} p - \alpha S_i - \beta L^i_{\boldsymbol{e}})\Big\}\Big) \\ & = -\exp\Big\{N\delta(-p + \alpha e + \beta\mu(e + x(N-1)e) + \frac{\delta\beta^2\lambda(e+x(N-1)e)}{2} + \frac{(N-1)}{2}\delta\beta^2\theta)\Big\} \\ s.t., & \text{(IR) } \overline{U}^{in}_i(\boldsymbol{e}, p, \alpha, \beta) \geq u^{out} \\ & \text{(IC) } \boldsymbol{e} = (e, e, \cdots, e) \text{ is the effort of the agents at the NE where all are insured} \end{array} \tag{2.49}$$

As the (IR) constraint is binding, similar to (2.47), we have

$$\begin{array}{ll} \max_{\alpha,\beta,e} & w^{out} - \mu(m') - \frac{\beta^2\delta+(1-\beta)^2\gamma}{2}\lambda(m') - \frac{c}{1+(N-1)x}m' - \frac{(N-1)}{2}\delta\beta^2\theta \\ s.t., & m' \in \arg\min_{e\geq 0}(1-\beta)\mu(e) + \frac{\gamma(1-\beta)^2\lambda(e)}{2} + (c-\alpha)e \end{array} \tag{2.50}$$

The following theorem characterizes the effect of pre-screening in the presence of a risk averse insurer.

Theorem 2.8 *Let* $m = \arg\min_{e\geq 0}\mu(e) + \frac{\gamma}{2}\lambda(e) + c$ *and assume* $\theta = 0$ *and* $m > 0$*. Then the agents exerts higher effort than their effort outside the contract if and only if* $\mu'(m) + \frac{1}{2}\frac{\delta\gamma}{\gamma+\delta}\lambda'(m) + \frac{c}{1+(N-1)x} < 0$*.*

Note that when an agent increases his effort, all agents benefit from it, and from the insurer's perspective the effective marginal cost of exerting an effort would be lower than c, i.e., $\frac{c}{1+(N-1)x}$ can be considered as the effective marginal effort cost in Theorem 2.8. The condition of Theorem 2.8 implies that if the marginal benefit of effort at $e = m$ is larger than the effective marginal cost $\frac{c}{1+(N-1)x}$, then the agents increase their effort inside the optimal contract. It is worth mentioning that the condition of Theorem 2.8 reduces to the condition of Theorem 2.6 if we set $\delta = 0$. Also, notice that the condition of Theorem 2.8 is more likely to be satisfied for larger values of δ. For instance, if $\delta = \infty$, the condition is always satisfied, and the agents exert higher effort inside the contract. In other words, if the insurer is more risk averse, it is more likely that she encourages agents to exert higher effort as compared to their efforts outside of the contract.

We close this section by characterizing the effect of correlation on agents' efforts given perfect pre-screening.

Theorem 2.9 *Assume $\theta \geq 0$, i.e., positive correlation between losses. Then, agents' efforts inside the contract increase as θ increases.*

Theorem 2.9 implies that if agents' losses are more correlated, a risk averse insurer encourages the agents to exert more effort. This is because with correlated losses, it is more likely for losses to happen simultaneously as compared to a scenario with independent losses. Note that when $\delta = 0$ in (2.50), i.e., when the insurer is risk neutral, the problem becomes independent of θ, meaning that the covariance between any two losses does not affect the optimal contract or the agents' efforts if the insurer is risk neutral.

2.7 Numerical results

We next present numerical examples of the findings of Sections 2.4-2.6. Our main focus is on the impact of pre-screening noise in various scenarios. Throughout the first part of this section we use the following parameters:

$$\mu(e) = \frac{10}{e+1}, \;\; \lambda(e) = \frac{10}{(e+1)^2}, \;\; c = 2, \;\; \gamma = 1 \; . \tag{2.51}$$

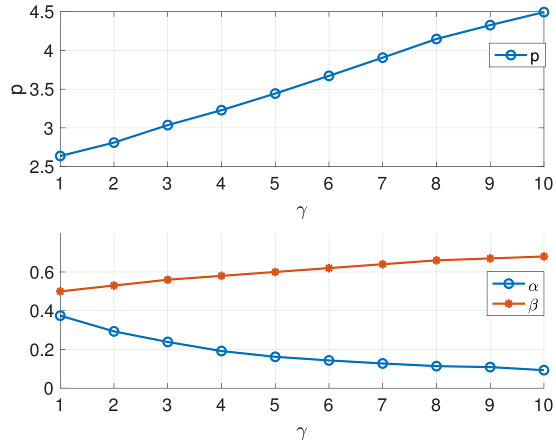


Figure 2.1: Parameters of the optimal contract v.s. risk aversion level γ

2.7.1 Impact of agent's risk attitude γ

Figure 2.1 illustrates the optimal contract as a function of γ. As the agent becomes more risk-averse, the insurer can set a higher base premium p, offer a lower discount factor α, and offer a higher coverage β. In other words, pre-screening becomes less important as the agent's risk-aversion increases, as more risk-averse agents are most interested in transferring more of their risk to the insurer, making their own efforts less important.

Figure 2.2 illustrates network security (agent's effort), both inside and outside of a contract, vs. his risk attitude γ. First, we see that as suggested by Theorem 2.1, the agent's effort in the contract is less that his effort outside of the contract. In other words, insurance decreases network security. Intuitively, as the agent transfers his risk to the insurer, he does not have the incentive to exert high effort. We also observe that the agent's effort in the optimal contract is a decreasing function of γ. This is due to the fact that as shown in Fig. 2.1, as the agent becomes more risk-averse, he transfer more risk to the insurer, and further decreases his effort. Finally, when the agent is outside of the contract, he can only decrease his risks by exerting higher effort. Therefore, we observe that as an agent without insurance becomes more risk-averse, he exerts higher effort.

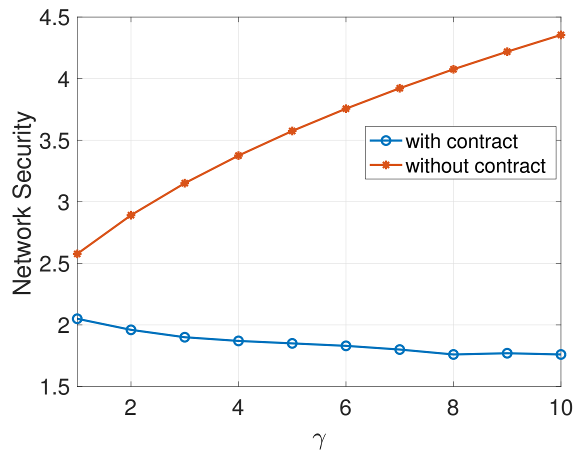


Figure 2.2: Effort of agent vs. risk aversion level γ

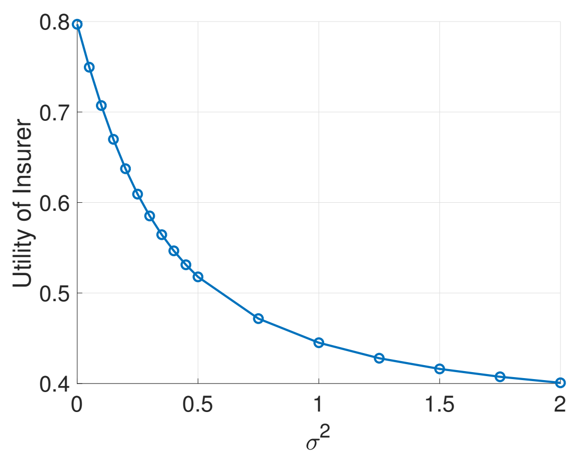


Figure 2.3: Insurer's profit vs. pre-screening noise σ^2 with a single risk-averse agent

2.7.2 Impact of pre-screening noise

A single risk-averse agent: Figure 2.3 illustrates the insurer's profit as a function of the pre-screening noise σ^2. The observation is consistent with Theorem 2.2, which states that the insurer's profit is a decreasing function of σ^2. Figure 2.4 illustrates the effort of the agent inside and outside the contract as a function of σ^2. We see that the effort outside the contract is independent of the pre-screening noise, while it decreases inside the contract as σ^2 increases. This highlights that as the insurer becomes less accurate in her observation of the agent's effort, she starts to place less importance on the pre-screening outcome; as a result, it becomes less beneficial for the agent to exert high effort without receiving sufficient discount. In other words, low quality pre-screening dampens its effectiveness in mitigating moral hazard; consequently, network security worsens. A second observation here is that as the participation constraint is always binding, the constant effort outside the contract also means that the agent's utility remains constant regardless of the pre-screening noise. Thus, it is only the insurer who benefits from pre-screening.

Two homogeneous risk-averse agents: We next consider two homogeneous agents with interdependence factor $x = 0.5$. Figure 2.5 shows the insurer's utility as a function of the quality of pre-screening, which illustrates the insurer's profit decreases when the pre-screening accuracy decreases. Figure 2.6 shows the network security as a function of pre-screening noise. Here, the conditions of Theorem 2.6 is satisfied. As we can see, security under the contract is higher than that without insurance for small values of σ; but as σ increases, security worsens and drops below that without contract.

Two heterogeneous risk-averse agents: We next consider two heterogeneous agents with the following parameters:

$$\begin{aligned} \mu(e) = \tfrac{10}{e+1},\ \ \lambda(e) = \tfrac{10}{(e+1)^2},\ \ c_1 = 1,\ \ c_2 = 1.1 \\ \gamma_1 = 1.2\ \ \gamma_2 = 1,\ \ x = 0.5 \end{aligned} \tag{2.52}$$

We assume that the pre-screening noise (σ^2) is the same for both agents. These parameters together satisfy the condition of Theorem 2.5. Figure 2.7 shows that the introduction of insurance can indeed improve the state of network security provided the pre-screening is sufficiently accurate. Figure 2.8 shows that the insurer's profit decreases as pre-screening becomes less accurate.

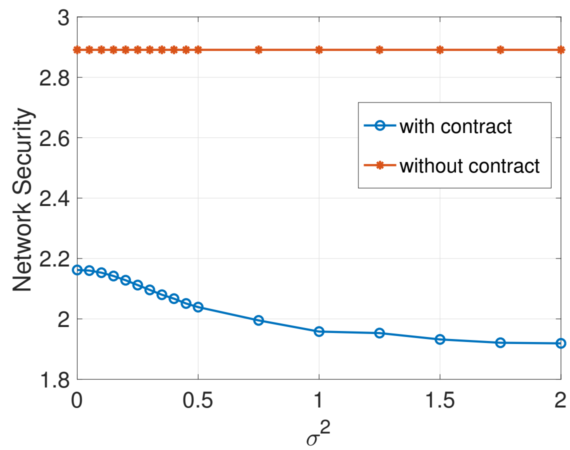


Figure 2.4: Agent's effort vs. pre-screening noise σ^2 with a single risk-averse agent

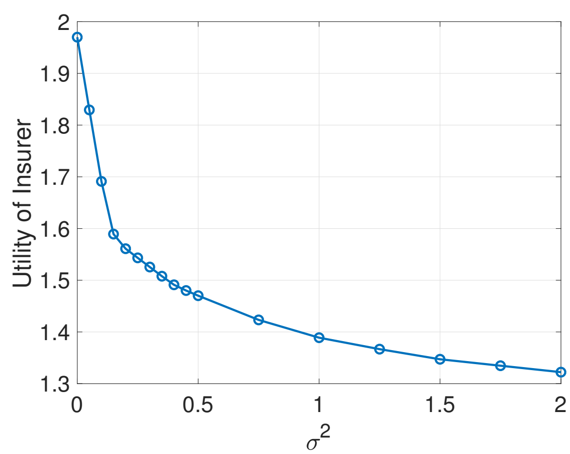


Figure 2.5: Principal's utility vs. σ^2 with two homogeneous risk-averse agents

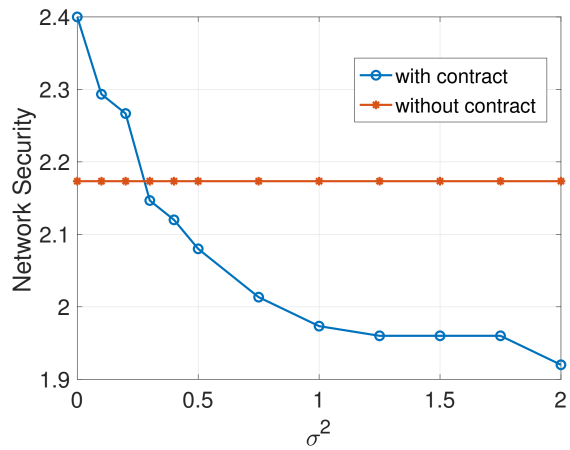


Figure 2.6: Network security ($e_1 + e_2$) vs. σ^2 with two homogeneous risk-averse agents

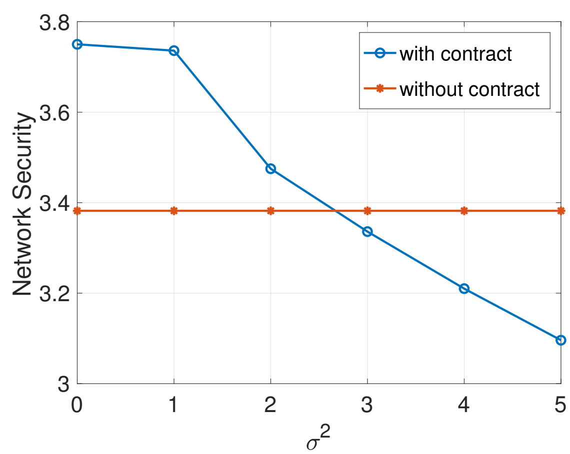


Figure 2.7: Network security ($e_1 + e_2$) vs. σ^2 with two heterogeneous risk-averse agents

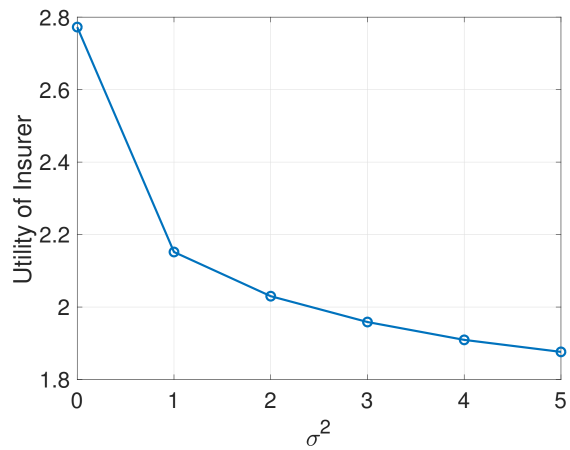


Figure 2.8: Principal's profit vs. σ^2 with two heterogeneous risk-averse agents

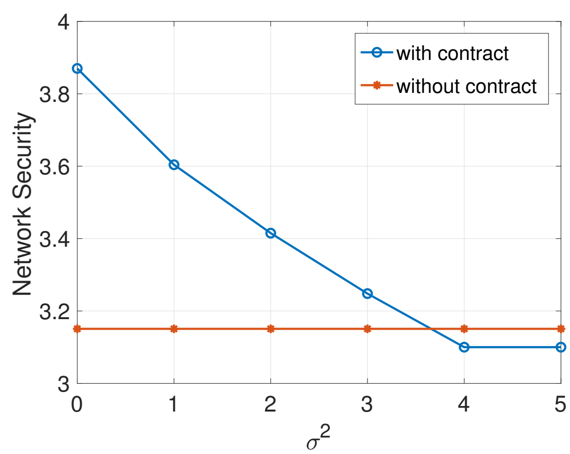


Figure 2.9: Network security $(e_1 + e_2)$ vs. σ^2 with two heterogeneous risk-averse agents. In this example, the conditions of Theorem 2.5 do not hold but network security improves after the introduction of insurance.

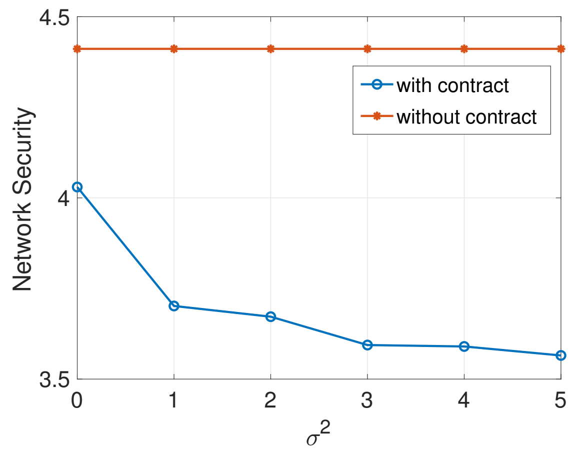


Figure 2.10: Network security ($e_1 + e_2$) vs. σ^2 with two heterogeneous risk-averse agents. In this example, the conditions of Theorem 2.5 do not hold, and network security worsens after introduction of insurance.

2.7.3 On the sufficient conditions of Theorem 2.5

The interpretation of the conditions of Theorem 2.5 was provided in Section 2.5.3, and this section examines these conditions through numerical examples. Consider an example with parameters similar to those given in (2.52), except that $\gamma_1 = 1.5$ and $c_2 = 1.5$. In this case, it can be verified that the conditions of Theorem 2.5 do not hold. However, Figure 2.9 shows that network security improves after the introduction of insurance. This example shows that the sufficient conditions in Theorem 2.5 are not necessary.

Consider again the same parameters given in (2.52), except $x = 0.15$. In this case, it can again be verified that the conditions of Theorem 2.5 do not hold. Figure 2.10 shows that the network security worsens with the introduction of insurance and thus the sufficient conditions are meaningful.

2.7.4 Loss with exponential distribution and pre-screening with uniform distribution: an example

Single Risk-Averse Agent: Throughout our analysis, we assumed that losses and pre-screening outcomes are normally distributed. In this section, we provide a numerical example under the

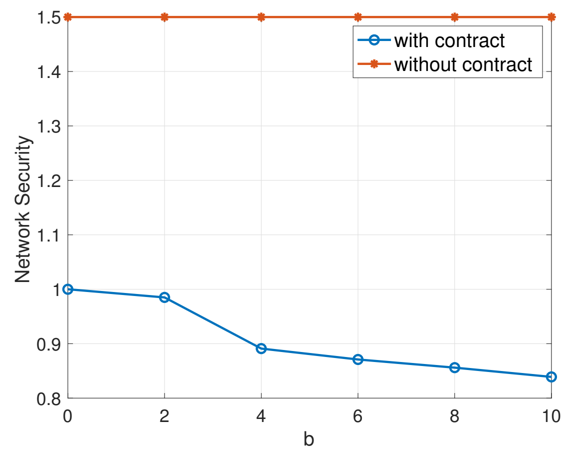


Figure 2.11: Agent's effort vs. σ^2 with a single risk-averse agent and exponentially distributed loss.

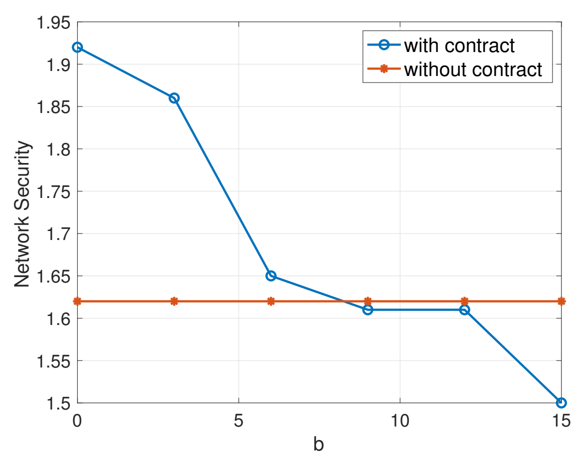


Figure 2.12: Network security $(e_1 + e_2)$ vs. σ^2 with two heterogeneous risk-averse agents with exponentially distributed interdependent losses.

assumption of exponentially distributed losses and uniformly distributed pre-screening outcomes. We illustrate how our previous observations hold in this instance as well. Let,

$$\begin{aligned} \gamma \quad &= 0.9,\ c = 0.25,\ E(L_e) = \mu(e) = \tfrac{1}{1+e}, \\ L_e \quad &\sim \exp(\tfrac{1}{\mu(e)}), \\ S_e \quad &= e + W,\ W \sim\ Unif(-b,b) \end{aligned} \tag{2.53}$$

Figure 2.11 illustrates the agent's effort when pre-screening noise W is uniformly distributed in interval $[-b,b]$. This figure shows that even though the loss and pre-screening outcome are not normally distributed, the agent's effort inside the contract is less than outside the contract; similarly, it remains a decreasing function of b.

Model with two risk-averse agents: We further consider a network of two risk-averse agents with the following parameters,

$$\begin{aligned} \gamma_1 \quad &= \gamma_2 = 0.9,\ c_1 = 0.25,\ c_2 = 0.5, x = 0.5 \\ E(L^i_{e_1,e_2}) \quad &= \mu(e_i + xe_{-i}) = \tfrac{1}{1+e_i+xe_{-i}} \\ L^i_{e_1,e_2} \quad &\sim \exp(\tfrac{1}{\mu(e_i+xe_{-i})}), \\ S_{e_i} \quad &= e_i + W_i,\ W_i \sim\ Unif(-b,b),\ i = 1,2 \end{aligned} \tag{2.54}$$

Where, W_1 , W_2 are independent and uniformly distributed in interval $[-b,b]$.

Figure 2.12 illustrates network security in a network of two risk-averse agents with exponentially distributed interdependent losses and uniformly distributed pre-screening outcomes. In this example, when pre-screening is sufficiently accurate (b is sufficiently small), by exploiting agents' interdependence, the insurer can design contracts in a way that network security inside the contract is higher than prior to the introduction of insurance. In contrast, when pre-screening is not accurate enough (b is large), network security inside the contract falls bellow network security outside the contract. Again, these observations are consistent with our results under normally distributed losses and pre-screening.

2.7.5 An example with correlated pre-screening noises

Throughout this chapter, we assumed that pre-screening noises W_i and W_j are independent. Notice that the correlation between W_i and W_j will not affect our results when the insurer is risk-neutral. This is because a risk-neutral insurer is not sensitive to the variance and covariance of the pre-screening outcomes, and therefore a term of the form $Cov(W_i, W_j)$ will not appear in the insurer's expected utility. As a $Cov(W_i, W_j) \neq 0$ will not alter the insurer's behavior, and also since agents' utilities depend directly only on their own pre-screening outcome's noise, having correlated pre-screening noises will not affect the analysis of agents' incentives either. On the other hand, in Section 2.6.2, where we study the insurance market with a risk-averse insurer, $Cov(W_i, W_j)$ can affect the utility of the insurer. Our result in that section is established for perfect pre-screening only, effectively removing the such covariance.

In this part, we conduct a numerical simulation for the case where the insurer is risk averse and pre-screening is imperfect. We observe that network security decreases as the pre-screening noise correlation increase. Figure 2.13 illustrates network security as a function of pre-screening noise correlation. The parameters of this example are as follows,

$$\mu(e) = \frac{100}{2e+1}, \ \lambda(e) = \frac{100}{(2e+1)^2}$$

$$c_1 = c_2 = 1 \ \gamma_1 = \gamma_2 = 0.5 \ \delta = 0.1, \ \sigma^2 = 2, x = 0.5$$

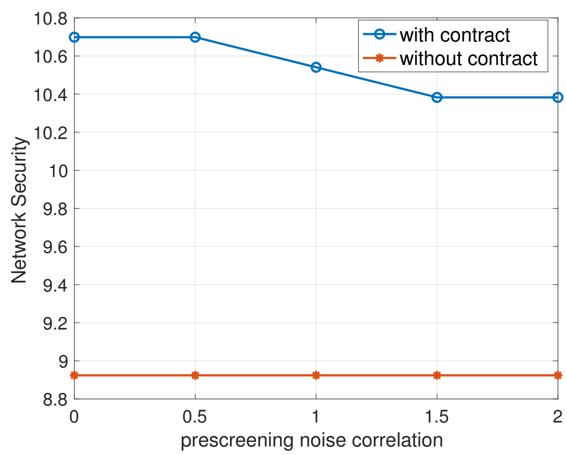


Figure 2.13: Network security as a function of pre-screening noise correlation

2.8 Discussion and conclusion

This chapter studied the problem of designing cyber insurance contracts by a single profit-maximizing insurer, for both risk-neutral and risk-averse agents. While the introduction of insurance worsens network security in a network of independent agents, we showed that the result could be different in a network of interdependent agents. Specifically, we showed that security interdependency leads to a profit opportunity for the insurer, created by the inefficient effort levels exerted by free-riding agents when insurance is not available but interdependency is present; this is in addition to risk transfer that an insurer typically profits from. We showed that security pre-screening then allows the insurer to take advantage of this additional profit opportunity by designing the right contracts to incentivize the agents to increase their effort levels and coordinating interdependent agents. We show under what conditions this type of contracts leads to not only increased profit for the principal and utility for the agents, but also improved state of network security.

There are a number of directions to pursue to extend the above results. As mentioned earlier, all our results are derived under the assumption of perfect information. Studying the problem with pre-screening under partial information assumptions would be an important direction of future research; this would include imperfect knowledge of the agents' type by the principal as well as imperfect knowledge of the interdependence relationship by the agents and the principal. Other modeling choices such as alternative use of pre-screening assessment (as opposed to linear discounts on premiums), and more general ways of capturing correlated risks (e.g., joint distribution of losses as opposed to average loss being a function of joint effort), would also be of great interest. Finally, a competitive market setting and its effects on network security is also worth studying.

CHAPTER 3

Embracing and Controlling Risk Dependency in Cyber-insurance Policy Underwriting

3.1 Introduction

In the previous chapter, we focused on interdependent cyber risks and showed how an insurer can use an imperfect pre-screening to control risk dependency. This chapter will again focus on the same challenges but will examine the contract design problem in a more practical setting. Specifically, we consider a scenario consisting of a service provider (SP, e.g., a cloud platform vendor) and n customers where the security investment affects the probability of a loss incident. In this scenario, interdependence is one-directional; an incident on the service provider's side may cause a business interruption for the customers but not the other way around. We will introduce a second loss model where security investment affects the probability of loss events and that loss to the customers may be attributed to the SP whom the customer depends on. In contrast to Chapter 2, this chapter uses an actual cyber insurance contract and rate schedule to develop an understanding of the cyber-insurance market in the presence of interdependent (and risk-averse) agents in a realistic underwriting setting.

There are two main reasons that we are interested in the SP-customer model. The first is to study simultaneous loss events in a network of interdependent agents, which would threaten the insurer's capital limit or other liquidity requirements. Second, in the event that a data breach or other loss events could be attributed to a third party (i.e., service provider) who may be insured by a different carrier, the insurer of the primary party (i.e., SP's customer) may seek to recover some or all of its losses from the SP's insurer/policy, thereby reducing its own risk exposure. If, on the other hand, the primary party's insurer underwrites both the primary firm and its third party, then even if the loss to the primary could be attributed to the third party, the insurer would effectively be "suing itself" for the losses. All this has led to a strong desire among insurance carriers to minimize this type of risk dependency. However, a proper solution continues to elude insurance carriers, reinsurers, and modeling firms [65].

It is thus of considerable interest to cyber-insurance underwriters to understand how to effectively manage not only individual firm risk, but overall portfolio risk in the presence of interdependent systems among policy holders. One device available to them is the ability to provide incentives (premium discounts) directly to firms that demonstrate improved security posture. While this may help reduce individual firm risk, it is unclear how this may help resolve systemic risk from interdependent business relationships.

Chapter 2 using a contract-theoretic approach showed that contrary to the common dependency-avoidance practice mentioned above, there is an unrealized incentive for an insurer to underwrite dependent risks. Paradoxically, the existence of risk dependency among a network of insureds allows the insurer to jointly design polices that incentivize the insureds to (collectively) commit to higher levels of effort, which can simultaneously result in improved state of security for all as compared to a portfolio of independent insureds, and in improved profits for the insurer.

This chapter further examines whether these observations continue to hold when an insurer can recover a part of the loss suffered by an insured through a third-party liability clause when the loss can be attributed to another insured (the third party) underwritten by a different insurer. Even with this loss recovery as an alternative, conditions exist where it is beneficial both from a security perspective and a profit perspective for an insurer to underwrite both interdependent insureds, precisely because this allows the insurer to control the risk dependency and incentivize both to commit to higher security efforts. In this chapter we analyze different portfolio choices by a underwriter and quantify their impact on the resulting profit, risk reduction, as well as social welfare. Specifically, we use both analytic and computational techniques to model three portfolio alternatives available to the insurance carrier: insure just the service provider, insure both the service provider and its customers, or insure just the service provider's customers. The strategic decision centers on how the insurer can induce the parties to reduce their risk while maximizing its own profits. We examine how these incentives can be used to reduce the direct risk to one party, as well as to reduce indirect risks to dependent firms. We also examine social welfare implications and use data from an actual cyber-insurance policy, as well as one of the only sources of insurance claims data, to calibrate and substantiate our analysis.

3.1.1 Main findings

Our results in this chapter show that the insurer is able to achieve higher profit by insuring all agents (SP and its customers) provided it appropriately incentivizes the SP to improve its state of security. This is because risk reduction by the SP leads to risk reduction for its customers, thus the benefit has a multiplicative effect. This ultimately not only allows the insurer to be able to take on the risk of all agents without hurting its profit, but also leads to higher social welfare.

Overall, our results suggest a novel and improved approach to cyber-insurance policy design that presents a new way of thinking about systemic risk and cyber risk dependency: to embrace and manage these risks, rather than avoid them. While we acknowledge the warranted caution against concurrent and correlated loss events, the emphasis of this chapter is to highlight a clear silver lining behind risk dependency, and an opportunity to actively work toward reducing overall cyber risks in an ever-escalating and interconnected threat landscape.

3.1.2 Chapter organization

The remainder of this chapter is organized as follows. We provide an example of actual cyber insurance policy underwriting in Section 3.2. We then present our model and analysis in Section 3.3, followed by numerical examples in Section 3.4. Section 3.5 discusses different aspects of the presented model, and Section 3.6 concludes the chapter.

3.2 Computing premiums using base rates: examples from an actual underwriter

In this section we briefly describe a common approach to calculating cyber-insurance premiums. The calculation begins by first selecting the *base premium* and a *base retention* (deductible) from previously defined lookup tables. The base premium is then modified through a linear product of additional factors. While different carriers use different values and types of factors in their premium expression, there are a number of commonly used factors.

Below, we provide an example of such a calculation using an actual cyber-insurance policy (see the Appendix to view the full rate schedule), with methods commonly found throughout the insurance industry. First, the base premium and retention are determined using table lookups, where the asset size (for financial institutions) or annual revenue (for non-financial institutions) of the insured maps to assigned values, with both the rate and the retention amounts increasing in asset or revenue size. For instance, a financial institution of asset value up to $100M would be charged a base rate of $5,000 for a base retention of $25,000, while a firm of assets between $500M and $1B would be charged a base rate of $11,000 for a base retention of $100,000, all for a nominal coverage amount of $1M. On the other hand, a non-financial firm with annual revenue between $5M to $10M would be charged a base rate of $7,500 for a base retention of $25,000.

This base rate is then multiplied by a number of factors, with each factor modifying the base rate roughly between −20% and +20% with a few exceptions, as shown below.

- **Industry Factor:** Based on the type of business, an industry hazard is determined, with higher-risk businesses receiving a larger multiplier. For instance, agricultural and construction businesses receive the smallest hazard value (less risky) while web service providers receive the larger hazard value (more risky), as shown in Table 3.1.
- **Retention Factor:** This factor depends on the retention (deductible) that the insured selects. Retention factor decreases as a function of the retention that the insured chooses, as shown in Table 3.2.
- **Increased Limit Factor:** This is a factor driven by the limit of the coverage: it is 1.0 if the insured accepts the default limit (corresponding to the base rate and base retention); it exceeds 1.0 if the insured wants to increase this limit, and falls below 1.0 if the insured asks for a lower coverage limit, as shown in Table 3.3.
- **Co-insurance Factor:** This factor is less than 1.0 if the insured accepts to pay a share of the payment made against a claim. The value of this factor depends on the amount of the share that the insured accepts to pay. Table 3.4 lists some of the co-insurance factors based on the co-insurance percentage.
- **Third-Party Modifier Factors:** This factor depends on the third party service provider. If the insured does not use any third party service, this factor is equal to 1.0. Otherwise, this factor is set based on the third party service and the agreement between the insureds and the service provider, but is not a function of the security posture of the third-party.
- **Optional Coverage Grants:** In addition to the base coverage, the policy holder may purchase coverage for additional exposures, such as privacy costs or crisis management. Each

additional coverage is calculated by multiplying the base rate by a number of factors including an option-specific modifying factor. For instance, the option of privacy notification expense uses a factor of 0.15, while the option of crisis management expense uses a factor of 0.02.

Note that other carriers use similar frameworks for calculating the final premium. We refer the interested reader to [73] for a more complete overview of current insurance policies. This multiplicative formula described above constitutes the basic model used for our analysis in the next section.

Industry	Factor
Agriculture	0.85
Construction	0.85
Not-for-Profit Organizations	1.00
Technology Service Providers	1.2
Telecommunications	1.2

Table 3.1: Industry hazard table

Selected Retention ($)	Base Retention ($)			
	25,000	100,000	500,000	1000,000
25,000	**1.00**	1.16	1.34	1.47
100,000	0.87	**1.00**	1.16	1.27
500,000	0.75	0.87	**1.00**	1.10
1,000,000	0.68	0.79	0.91	**1.00**

Table 3.2: Retention Factor

Example We complete this section by providing an example of how the final premium is calculated using the above tables. Consider a non-financial Technology Service Provider with annual revenue $6M who intends to purchase an insurance policy with retention $100,000, coverage limit $2.5M, and zero percent co-insurance. Moreover, this firm does not use any third party services; it wishes to opt in for additional coverage for privacy notification expense and crisis management expense. Based on the above tables, the following factors will be used in determining the total premium for this company:

Coverage Limit ($)	Increased Limit Factor
1,000,000	1.000
2,500,000	1.865
5,000,000	2.987
10,000,000	4.786
25,000,000	8.925

Table 3.3: Increased limit factor

Co-Insurance (%)	Co-insurance Factor
0	1.000
1.0	0.995
5.0	0.980
10	0.960
20	0.920
50	0.780

Table 3.4: Co-insurance factor

- Base premium: $7,500; Base Retention: $25,000
- Industry Factor: 1.2 (Table 3.1).
- Retention Factor: 0.87 (Table 3.2).
- Limit Factor: 1.865 (Table 3.3).
- Third-Party Modifier Factor: 1.
- Co-insurance Factor: 1 (Table 3.4).
- Privacy notification: 0.15.
- Crisis management: 0.02.

Therefore, the premium for this service provider is calculated as follows,

$$\begin{aligned} \text{Premium} &= 7500 \times 1.2 \times 0.87 \times 1.865 \times 1 \times 1 + 7500 \times (0.15 + 0.02) \\ &= 14602.95 + 1275 = \$15,877.95 \ . \end{aligned} \quad (3.1)$$

3.3 The insurance policy model and analysis

In this section, we model three portfolio alternatives available to the insurance carrier, as depicted in Figure 3.1: insure just the service provider and let someone else insure its customers (*Portfolio type A*), insure both the service provider and its customers (*Portfolio type B*), or insure just the service provider's customers and let someone else insure the SP (*Portfolio type C*).

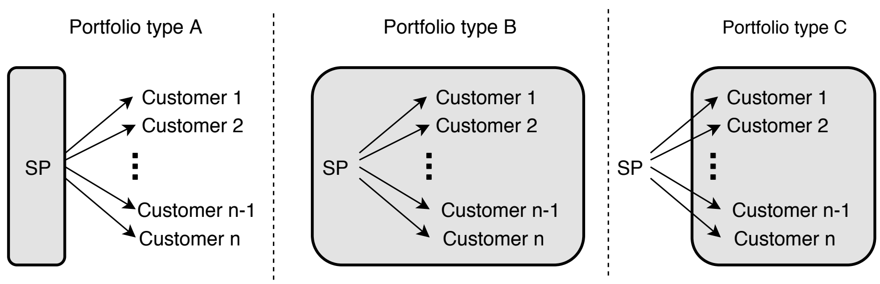


Figure 3.1: Three portfolio types: shaded areas indicate entities insured by the underwriter.

In each case the question we are interested in understanding is to what extent the insurer may be able to induce the parties to reduce their risk while maximizing its own profits. We examine how these policy incentives can be used to reduce the direct and indirect risks to the parties involved. To do so, over the next few subsections we develop a model that formally establishes an insurance carrier's profit, as a function of the insurance policy terms as well as incentives embedded in the policy.

3.3.1 Base premium calculation

Consider an insurer and its prospective insureds (the applicants), which include a service provider (SP, e.g. Amazon cloud services) and its n customers. The insurer charges a base premium b_o to the service provider and base premiums b_i to its customers i, $i = 1, 2, \ldots, n$.

As described, the base premium, b_i, depends on the total assets or revenue of the insureds. The insurer then asks the applicants to fill out a questionnaire describing their information security practices. Based on the completed questionnaire, the insurer modifies the base premiums by a factor f_i, $i = 0, 1, \cdots, n$, as described in the previous section. The insured pays $b_i \cdot f_i$ up front, and the insurer pays the insured $\max\{L_i - d_i, 0\}$ after a loss incident where L_i denotes the loss amount of agent i and d_i is its elected retention/deductible. For the analysis that follows we ignore all the other factors unrelated to cybersecurity, as their inclusion (as additional multipliers) does not affect our model or our conclusion.

This model does not yet consider dependent risks. Specifically, insured i's premium $b_i f_i$, $i = 1, \cdots, n$, is purely a function of its own security posture. While the information security questionnaire used to generate modifier factor f_i may include questions on whether i has a third party supplier, or whether it has proper procedures/policies in place in handling a third party, it does not directly assess the security posture of these third parties. This instead is assessed separately. We refer an interested reader to the Chubb CyberSecurity policy shown in Appendix B.

3.3.2 The security incentive modifier

We now introduce an incentive factor, f'_o, for the SP and subsequently examine its impact on the SP as well as its n customers. Specifically, suppose the insurer is willing to offer the SP a discounted premium in exchange for improved security posture as follows:

- The SP has an initially assessed premium $b_o f_o$, with a security modifier factor f_o.
- The SP agrees to invest more in security such that it could now be assessed at $\tilde{f}_o = f_o - f'_o$, for some $f'_o \in [0, f_o]$, i.e., a reduction in the modifier factor. Therefore, the insured pays $b_o \tilde{f}_o$ as the premium, reflecting a discount given the SP's improved security posture.

Note that here for simplicity of presentation, we have assumed that the insurer is able to assess, and willing to match exactly in discount the amount corresponding to the reduced risk. That is, this SP now enjoys a revised premium equal to that which it would have received had it started at a security level measured at $\tilde{f}_o$ without the incentive. In practice the two need not be equal, i.e., the SP may require more or less in premium discount incentive to reach $\tilde{f}_o$. While this does not affect our qualitative conclusions, it does raise the interesting question as to whether in practice the incentive offered is sufficient for the SP to attain the corresponding risk reduction. In other words, could the SP take the discount amount $b_o f'_o$ and use it toward hiring additional personnel or purchasing products to achieve this goal? We will give an example in the context of the distributions used in our numerical analysis in Section 3.4.

Our subsequent analysis focuses on whether a desirable operating point for the insurer is such that $f'_o > 0$, i.e., offering an incentive to the SP. Obviously, when there is no incentive, $\tilde{f}_o = f_o$, and the problem reverts to the original premium calculation.

3.3.3 Mapping security incentive to probability of loss

The security modifier factor f_i is tied to some underlying assumption of the probability of a cyber incident. This modifier can increase or decrease the base premium; the larger it is, the more likely is a loss event as estimated by the insurer. As far as we can tell by examining the rate schedules of many actual cyber-insurance policies, this factor itself does not appear to be directly tied to the magnitude of a loss; rather we believe the expected loss amount is factored into the base premium which is tied to the sector/industry and the size of the insured. The use of such a factor in the current underwriting practice would suggest that policies are risk priced in additional to being market priced (reflected in the base premium and retention). This aspect however does not affect our analysis since we only consider a single insurer.

To be concrete, let $P_o(\tilde{f}_o)$ denote the the probability of a breach to the SP, which is decreasing in the security incentive factor f'_o and increasing in the overall factor $\tilde{f}_o$. Similarly, we denote by $P_i(f_i)$, $i = 1, \cdots, n$, the probability of a loss incident of customer i *unrelated* to the SP. Both $P_o()$ and $P_i()$ are assumed to be increasing and differentiable. We will assume that if a breach happens to the SP, a business interruption or similar loss event occurs to its customer with probability t, also referred to as the level or degree of dependency. Further, we assume that a business interruption induced by SP and the loss incident unrelated to the SP are independent events.

Putting these together, the probability of a loss event occurring to customer i is given by:

$$P_{li}(\tilde{f}_o, f_i) = P_i(f_i) + t \cdot P_o(\tilde{f}_o) - t \cdot P_o(\tilde{f}_o) \cdot P_i(f_i), \quad i = 1, \cdots, n \tag{3.2}$$

where the loss includes that due to the customer itself, due to business interruption brought on by the SP's breach, or both at the same time.

3.3.4 The insurer's profit function

Next, we derive expressions for the insurer's profit under two portfolio options: when it insures just the service provider (Portfolio A), and then when it insures both the service provider and its customers (Portfolio B).

The insurer's profit (V_o) and expected profit ($\overline{V}_o$) from underwriting *just* the SP are defined as follows, both shown as functions of f'_o given that our focus is on this element under the insurer's control,

$$\begin{aligned} V_o(f'_o) &= b_o \cdot (f_o - f'_o) - I_o \cdot (L_o - d_o)^+ \ ; && (3.3) \\ \overline{V}_o(f'_o) &= E\{V_o(f'_o)\} = b_o \cdot (f_o - f'_o) - l_o \cdot P_o(f_o - f'_o) \ , && (3.4) \end{aligned}$$

where $(x)^+ = \max\{x, 0\}$, L_o is the loss random variable, and $l_o = E((L_o - d_o)^+)$. Note that I_o is a Bernoulli random variable with parameter $P_o(f_o - f'_o)$.

We will assume the customers' security factors $f_i, i = 1, \cdots, n$, are uniformly distributed over some range $[f_{min}, f_{max}]$. The insurer's profit from customer i is then given by the following, again expressed as a function of the controllable f'_o:

$$\begin{aligned} V_i(f'_o) &= b_i f_i - I_i \cdot (L_i - d_i)^+ \ ; && (3.5) \\ \overline{V}_i(f'_o) &= b_i \cdot \frac{f_{min} + f_{max}}{2} - E_{f_i}[P_{li}(f_o - f'_o, f_i)] \cdot l_i \ , && (3.6) \end{aligned}$$

where L_i is the loss random variable of customer i. Again, I_i is a Bernoulli random variable with parameter $P_{li}(f_o - f'_o, f_i)$ and $l_i = E((L_i - d_i)^+)$.

If the insurer chooses to underwrite *both* the SP and its n customers then its expected total profit is given by:

$$\begin{aligned} \overline{V}_{total}(f'_o) &= \overline{V}_o(f'_o) + \sum_{i=1}^{n} \overline{V}_i(f'_o) \ ; && (3.7) \\ \overline{V}_{max} &= \max_{f'_o} \overline{V}_{total}(f'_o) \ . && (3.8) \end{aligned}$$

3.3.5 Analysis of the optimal incentives and carrier profits

Now that we have established expressions for the carrier's profits as a function of security incentives, we next seek to answer two questions: first, what security incentives should the carrier provide to the service provider, and secondly, which portfolio strategy yields higher profit?

We have defined $P_o(\tilde{f}_o)$ to be an increasing function of $\tilde{f}_o$, implying that $P_o(f_o - f'_o)$ is a decreasing function of the incentive f'_o. We assume this to be a strictly convex function of f'_o, reflecting a decreasing marginal return on effort. Note that it is widely accepted to model loss probability as a function of the security investment, see e.g., [34, 54, 58, 68]. Our model here is consistent with this literature since we have assumed that the incentive factor f'_o is proportional to security effort/investment, while allowing us to highlight and express this function in terms of the carrier's controllable in this underwriting framework.

Our first result compares the optimal incentive that an insurance carrier would offer the SP when insuring just the SP (Portfolio A), and insuring both the SP and its customers (Portfolio B). That is, we compare the optimal incentive factor f_o^* that maximizes $\overline{V}_o()$, with the optimal incentive factor f_o^{**} that maximizes $\overline{V}_{total}()$.

Theorem 3.1 *Under the assumption that $P_o(f_o - f'_o)$ is decreasing and strictly convex in f'_o, we have that $f_o^* \leq f_o^{**}$, where $f_o^* = \arg\max_{f'_o} \overline{V}_o(f'_o)$ and $f_o^{**} = \arg\max_{f'_o} \overline{V}_{total}(f'_o)$. In other words, the underwriter offers a higher incentive to the SP when insuring all parties, compared with the incentive offered to the SP as the only insured.*

Proof *The insurer's profit of underwriting the service provider and the customers is given by:*

$$\begin{aligned}
\overline{V}_{total}(f'_o) &= \overline{V}_o(f'_o) + \sum_{i=1}^{n} \overline{V}_i(f'_o) \\
&= b_o \cdot (f_o - f'_o) - l_o P_o(f_o - f'_o) + \sum_{i=1}^{n} b_i \frac{f_{min} + f_{max}}{2} \\
&\quad - l_i \cdot P_o(f_o - f'_o) \cdot (t - tE[P_i(f_i)]) - l_i \cdot E[P_i(f_i)] \,. \qquad (3.9)
\end{aligned}$$

Using the first order optimality condition, we have

$$\frac{\partial \overline{V}_{total}(f_o')}{\partial f_o'} = 0 \tag{3.10}$$

$$\Rightarrow \quad f_o^{**} = \left(f_o - (P_o')^{-1} \left(\frac{b_o}{\left[l_o + \sum_{i=1}^{n} l_i \cdot (t - t \cdot E(P_i(f_i))) \right]} \right) \right)^+ . \tag{3.11}$$

Similarly, we can find the optimal value f_o^* that maximizes $\overline{V}_o$:

$$\frac{\partial \overline{V}_o}{\partial f_o'} = -b_o + l_o \cdot P_o'(f_o - f_o') = 0$$

$$\Rightarrow \quad \arg\max_{f_o'} \overline{V}_o(f_o') \in \left(f_o - (P_o')^{-1}(\frac{b_o}{l_o}) \right)^+$$

$$\Rightarrow \quad f_o^* = \left(f_o - (P_o')^{-1}(\frac{b_o}{l_o}) \right)^+ . \tag{3.12}$$

Because $P_i^{'}()$ is an increasing function and $\frac{b_o}{l_o} > \frac{b_o}{\left[l_o + \sum_{i=1}^{n} l_i \cdot (t - t \cdot E(P_i(f_i))) \right]}$, we have $f_o^* \leq f_o^{**}$.

Theorem 3.1 suggests that if the insurer underwrites both the SP and its customers (Portfolio B), it benefits from a better state of security (induced by higher incentive to the SP) as compared to the optimal level if it only underwrites the SP (Portfolio A). Intuitively, as the SP's risk directly impacts that of its customers, when insuring both, it is in the insurer's interest to control/reduce the SP's risk so the overall, systemic risk it is exposed to is reduced. This obviously means better overall security posture for all parties. The question is whether the insurer will voluntarily choose Portfolio B over A? The next result answers this.

Corollary 3.1 *If parameter values b_i and l_i are such that $\overline{V}_i(f_o^*) > 0$ (i.e., there is expected profit from any single policy when the SP is incentivized at the level f_o^*; this need not be true if b_i is too small and l_i too large, in which case a rational insurer would not underwrite the policy), then we also have the following:*

$$\overline{V}_{total}(f_o^{**}) \underbrace{\geq}_{\text{by the optimality } f_o^{**}} \overline{V}_{total}(f_o^*) \underbrace{\geq}_{\text{by the positivity of } \overline{V}_i(f_o^*)} \overline{V}_o(f_o^*) . \tag{3.13}$$

And similarly,

$$\overline{V}_{total}(f_o^{**}) \underbrace{\geq}_{\text{by the optimality } f_o^{**}} \overline{V}_{total}(f_o^{*}) \underbrace{\geq}_{\text{by the positivity of } \overline{V}_i(f_o^*)} \overline{V}_i(f_o^*) \geq \overline{V}_i(0) \,, \tag{3.14}$$

where the last inequality results from the fact that the risk sustained by customer i is lower when the SP is incentivized at any level $f_o^* > 0$.

The above result suggests that at the right level of incentive for the SP, the insurer enjoys greater profits by insuring both the SP and its customers (Portfolio B), relative to insuring just the SP (Portfolio A), or any subset of its customers.

Note that Theorem 3.1 remains valid even when the assessment is noisy. To see this, let us assume that the SP is assessed at $\tilde{f}_o = f_o - f'_o$, but the true value is $\tilde{F}_o = \tilde{f}_o + N$ where N is a zero-mean random variable. Then we have:

$$\overline{V}_o(f'_o) = b_o \cdot (f_o - f'_o) - E(P_o(f_o - f'_o + N)) \cdot l_o \,. \tag{3.15}$$

Then as long as the function $E(P_o(f_o - f'_o + N))$ is increasing and convex in f_o, the result of Theorem 3.1 is valid. We next show that this is indeed an increasing and convex function. For simplicity of exposition, we will denote this function as $\Pi_o(f_o - f'_o) = E(P_o(f_o - f'_o + N))$, and denote the pdf of N by $g(.)$.

$$\Pi_o(x) = \int P_o(x+s)g(s)ds \quad \rightarrow \quad \Pi'_o(x) = \int P'_o(x+s)g(s)ds \underbrace{\geq}_{P_o(.)\ \text{is increasing}} 0 \,. \tag{3.16}$$

$$\Pi_o(\lambda \cdot x + (1-\lambda)\cdot y) = \int P_o(\lambda \cdot x + (1-\lambda)\cdot y + s)g(s)ds \underbrace{\leq}_{\text{by convexity of} P_o(.)} \tag{3.17}$$

$$\int \lambda \cdot P_o(x+s)g(s)ds + \int (1-\lambda)\cdot P_o(y+s)g(s)ds = \lambda\Pi_o(x) + (1-\lambda)\Pi_o(y) \,. \tag{3.18}$$

3.3.6 Third-party liability

Next we consider third-party liability. This refers to the ability of an injured party to seek redress for losses from an injurer, and is a coverage category commonly found in insurance policies. In the context of our study, this implies that if a firm suffers loss due to business interruption brought on by a breach at its SP, the firm's insurance carrier can, on the firm's behalf, seek redress from the SP's insurer. However, if the same carrier were to underwrite both the firm and the SP, such compensation would obviously not occur. In one of the few datasets that reports actual cyber-insurance claims data, NetDiligence [18] shows that 13% of all data breaches and cyber incidents can be attributed to a third party. Accordingly, we will use a parameter q to represent the probability that a loss can be attributed to an SP.

We define U as the insurer's profit when it underwrites only the SP's customers (Portfolio C). We have:

$$\begin{aligned} U_i(f_o') &= b_i \cdot f_i - J_i \cdot (L_i - d_i)^+; \\ \overline{U}_i(f_o') &= E[U_i(f_o')] \\ &= b_i \cdot \frac{f_{min} + f_{max}}{2} - (E[P_i(f_i)] + (1-q)\,[tP_o(f_o - f_o') - E[P_i(f_i)]tP_o(f_o - f_o')])l_i \,, \end{aligned} \tag{3.19}$$

where J_i is a Bernoulli random variable with parameter $P_i(f_i) + (1-q) \cdot [tP_o(f_o - f_o') \cdot (1 - P_i(f_i))]$; this is the probability that a loss incident happens to customer i and *cannot* be attributed to the SP. In this case the SP is insured by another carrier, referred to as the third-party insurer, whose profit is given by:

$$U_o(f_o') = b_o(f_o - f_o') - I_o \cdot (L_o - d_o)^+ - \sum_{i=1}^{n} K_i \cdot (L_i - d_i)^+; \tag{3.20}$$

$$\begin{aligned} \overline{U}_o(f_o') &= E[U_o(f_o')] \\ &= b_o \cdot (f_o - f_o') - P_o(f_o - f_o') \cdot l_o - \sum_{i=1}^{n} q \cdot [tP_o(f_o - f_o')] \cdot [1 - E[P_i(f_i)]]\, l_i \,, \end{aligned} \tag{3.21}$$

where K_i is a Bernoulli random variable with parameter $q \cdot [tP_o(f_o - f_o')] \cdot [1 - P_i(f_i)]$; this is the probability that a loss incident happens to customer i and it *can* be attributed to the third party successfully. Here we have assumed that whenever losses can be attributed to the SP, the customer's insurer (also referred to as the primary insurer) is fully reimbursed. However, our result in Theorem 3.2 remains valid for partial or fractional compensation as well.

Next, we compare the insurer's profit from underwriting only the SP's customers (with the possibility of recovering losses from the SP's insurer) (Portfolio C), with its profit from underwriting both the SP and its customers (Portfolio B).

We denote the insurer's profit from underwriting only the SP's customers as $\overline{U}_{max} = \sum_{i=1}^{n} \overline{U}_i(f_o^\star)$, where $f_o^\star = \arg\max_{f_o'} \overline{U}_o(f_o')$, and denote the insurer's profit from underwriting both the SP and its customers as $\overline{V}_{max}$ from Eqn. (3.8), where the maximum is attained at f_o^{**}.

Theorem 3.2 *At the right level of incentive for the SP, the insurer enjoys greater profit by insuring both the SP and its customers (Portfolio B), rather than just the SP's customers (Portfolio C). That is,* $\overline{V}_{max} \geq \overline{U}_{max}$, *where* $\overline{V}_{max} = \overline{U}_o(f_o^{**}) + \sum_{i=1}^{n} \overline{U}_i(f_o^{**})$, *and* $\overline{U}_{max} = \sum_{i=1}^{n} \overline{U}_i(f_o^\star)$. *Moreover, given that* $P_o(f_o - f_o')$ *is decreasing and convex in* f_o', *we have* $f_o^\star \leq f_o^{**}$, *which implies that the state of security improves for both the SP and its customers when the insurer underwrites both.*

The first part of the above result is rather trivial: if the primary insurer is compensated by the third-party insurer, it must therefore be profitable to underwrite the SP (otherwise the SP would not be able to obtain a policy in the first place). Thus the insurer of the SP's customers can only gain by insuring the SP itself.

The second part of the result is more interesting and less straightforward. The intuition is that when the insurer underwrites both the SP and its customers (Portfolio B), it is in its best interest to provide stronger incentive to the SP in an attempt to reap the multiplicative effect of risk reduction of the SP on its customers, i.e., the positive externality. In summary, by embracing the risk dependency, the insurer not only gains but also contributes to social welfare.

As in the case of Theorem 3.1, the result of Theorem 3.2 remains valid even when the SP's assessment is noisy, by following the same argument.

3.3.7 Summary of the findings

The findings suggested by the analysis shown in this section are summarized as follows.

- Given the choice between insuring just the SP (Portfolio A), or the SP and all its customers (Portfolio B), an insurance carrier should choose Portfolio B. The reason is that the insurer can incentivize the SP to improve its security posture in exchange for discounted premium. While this reduces the insurer's revenue from the SP, it improves the security posture of the SP and its customers, leading to fewer claims from business interruptions. Collectively this leads to lower overall risk, higher profits for the insurer.

- Given the choice between insuring both the SP and its customers (Portfolio B), or just the SP's customers (Portfolio C) and attributing losses to the SP, an insurance carrier should choose Portfolio B. This is because with Portfolio C the insurer is unable to effectively induce the SP to improve its security posture, which negatively affects all of the provider's customers.

- If an insurer chooses to underwrite only the SP's customers (Portfolio C), it should incorporate the risk condition of the SP into the service provider's customers' premiums. By contrast, current practice often ignores the security posture of the SP (or any third parties) when pricing the customer's policy.

Next, we use data from an actual cyber-insurance policy, as well as insurance claims data provided in [18], to calibrate and substantiate our analysis through numerical examples.

3.4 Numerical examples

In this section we examine closely a number of numerical examples that put the preceding analytical results into context. To do so, we will need to substantiate two elements of our model: the relationship between the security modifying factor, i.e., the function $P(f)$, and the loss distribution governing L. We will also use base premium and retention values found in Section 3.2.

3.4.1 Examples of the loss probability function

We present three examples of $P_o(f_o - f_o')$ as a function of f_o' while fixing $f_o = 1.2$ and $b_o = 52000$; these are illustrated in Figure 3.2 and used later in this section to perform numerical analysis.

$$P_o(f_o - f_o') = \frac{0.05}{\frac{b_o(1.2-(f_o-f_o'))}{1000} + 1} \tag{3.22}$$

$$P_o(f_o - f_o') = \frac{0.05}{(1 + \exp(\frac{b_o \cdot (1.2-(f_o-f_o'))}{1000} - 20))} \tag{3.23}$$

$$P_o(f_o - f_o') = \frac{5}{1000} + 0.05 \cdot \exp(-\frac{b_o \cdot (1.2 - (f_o - f_o'))}{1000}) \tag{3.24}$$

The choice of these functions are somewhat arbitrary: the main intent is to capture a few families of decreasing functions with subtle yet significant differences as explained below, while noting that our conclusion and results hold more generally. More specifically:

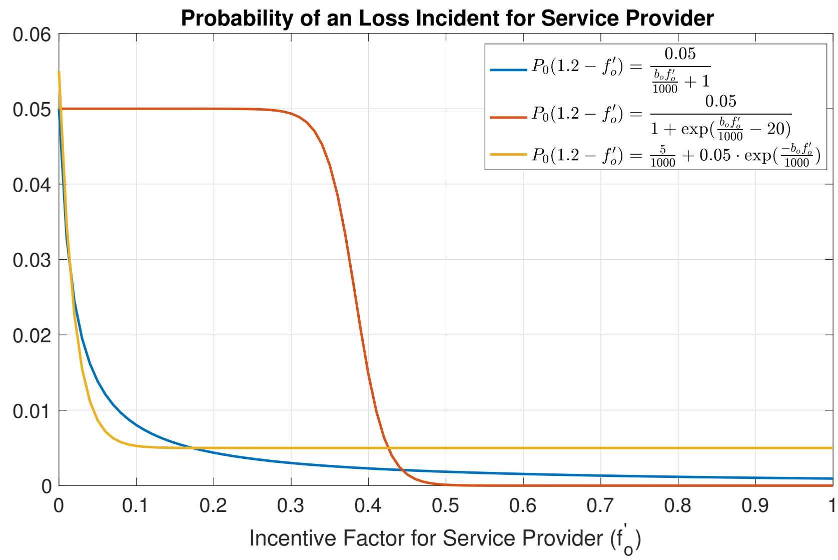


Figure 3.2: $P_o(1.2 - f_o')$, the probability of a loss event to the SP

- The loss given in Eqn (3.22) (the blue curve) is simply a decreasing, convex function which indicates that initial effort in risk reduction results in larger marginal benefits in loss reduction, but that the loss probability will continue to decrease at a diminishing rate. This would apply to a typical firm whose initial investment (say in firewall) is very effective, after which more expensive products (e.g., intrusion detection) continue to reduce risk but at a decreasing rate.

- The loss in Eqn (3.23) (the red curve) suggests the initial effort has to be significant enough (exceeding a threshold) to have any appreciable effect on loss reduction. Equivalently, this may be viewed as modeling a type of firm that only respond to incentives when they are substantial or when they reach a tipping point. Beyond this, the curve similarly exhibits diminishing returns. Note that this loss function is not convex but we show in the appendix the result of theorem 3.1 holds in this case as well.

- Finally, the loss in Eqn (3.24) (the yellow curve) illustrates a scenario where the reduction in loss initially behaves similarly to the first case, but reaches a maximum at a point beyond which no amount of effort can further reduce. This is captures the situation where external factors beyond the insured's control is at significant play, contributing to a non-zero "floor" in the probability of a loss event. This could apply to the case where there is persistent susceptibility to social engineering that no amount of investment or training can completely remove; or, where the firm is simply not able to address all security challenges.

It should be noted that the above examples serve to illustrate the different ways loss probabilities may change as incentives/security investments increase. The actual values used may or may not accurately reflect reality. For instance, in reality the scale of the loss probability could be orders of magnitude larger (0.1 instead of 0.01) or smaller (0.001 instead of 0.1). Unfortunately there is no publicly available data that would allow us to calibrate. As already mentioned, it is unclear how these factor values were derived by an underwriter in the first place.

3.4.2 Examples of the loss distribution

We will use data reported in the cyber-insurance claims study by NetDiligence [18] to obtain breach loss distributions, summarized in Table 3.5. The "Mid Revenue" range contains somewhat unexpected small median and mean values. This appears to be an anomaly: since the sample sizes (number of cases) are small, an oversized or undersized breach can significantly throw off the average.

	Cases	Median ($)	Mean ($)
Nano Revenue (¡ $50M)	52	49,000	215,297
Micro Revenue ($50M - $300M)	31	88,154	487,411
Small Revenue ($300M - $2B)	15	118,671	599,907
Mid Revenue ($2B - $10B)	9	91,457	173,851
Large-Revenue ($10B - $100B)	8	3,326,313	5,965,571

Table 3.5: Cost of data breach between 2016-17 organized based on the breached firm's revenue

3.4.3 Example 1: a service provider and a customer with large revenue

In this example, we consider a SP and a single customer, both of large revenue (e.g., a major web hosting provider and a large corporate customer). Using the rate schedule provided in Section 3.2, we will set the base premium and base retention for the SP and its customer to be $b_o = b_1 = \$52,000$

and $d_o = d_1 = \$250,000$, respectively. We consider the following loss function for the customer: $P_1(f_1) = \frac{0.05}{\frac{b_1 \cdot (1.2 - f_1)}{1000} + 1}$. Moreover, factor f_1 is uniformly distributed over $[0.6, 1.2]$ with $E(f_1) = 0.9$ and as mentioned this depends on the outcome of its information security questionnaire.

Using the NetDiligence data, we will assume that both L_o and L_1 are log-normally distributed with a mean of \$5,965,571 and median \$3,326,313. Moreover, as mentioned earlier NetDiligence reports that 13% of data breaches can be attributed to a third party; we will accordingly set $q = 0.13$. We will assume that the SP was assessed with $f_o = 1.2$.

We will first consider $P_o(f_o - f_o') = \frac{0.05}{\frac{52000 \cdot (1.2 - (f_o - f_o'))}{1000} + 1}$, with results shown in Figure 3.3.

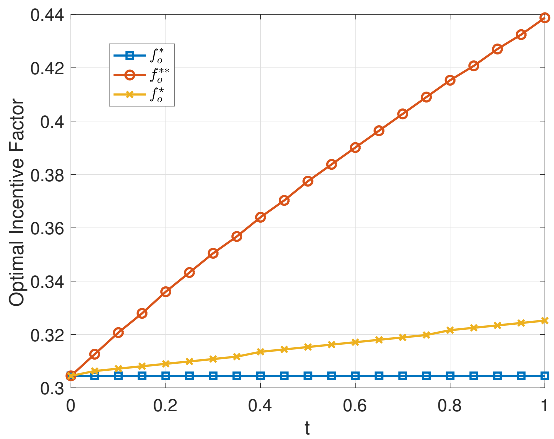
Figure 3.3a plots the optimal incentive factor in different portfolios as a function of the dependency factor t:

$$\text{Portfolio A: } f_o^* = \arg\max_{f_o'} V_o(f_o'),$$
$$\text{Portfolio B: } f_o^{**} = \arg\max_{f_o'} V_{total}(f_o'),$$
$$\text{Portfolio C: } f_o^{\star} = \arg\max_{f_o'} U_o(f_o') .$$

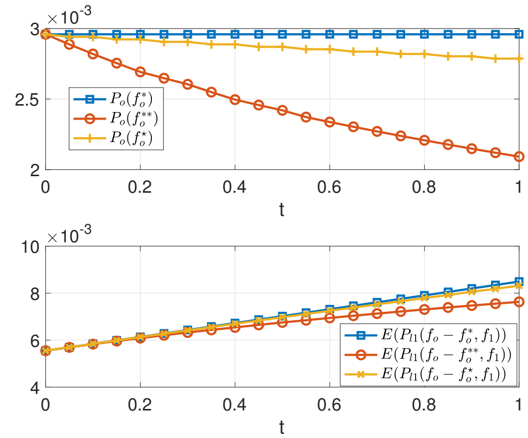
Figure 3.3b illustrates the probability of a loss event to the SP and its customers at optimal incentive factor $(f_o^*, f_o^{**}, f_o^{\star})$ as a function of dependency t.

These figures imply that, if the insurer underwrites only the SP (Portfolio A, blue line), t does not factor into the policy decision and thus the insurer will not offer any incentive to the SP. On the other hand, if the insurer underwrites both, then offering incentive to the SP is now in its interest, and the incentives increases as t increases (Portfolio B, orange line). Finally, if an insurer underwrites only the SP and pays the third-party compensation for its customer's loss (yellow line), the incentive factor is also increasing as a function of t but it increases slower than f_o^{**}. Figure 3.3c shows how much can be gained by taking risk dependency into account, and the higher the dependency the more the insurer gains by jointly designing contracts for both the SP and its customer.

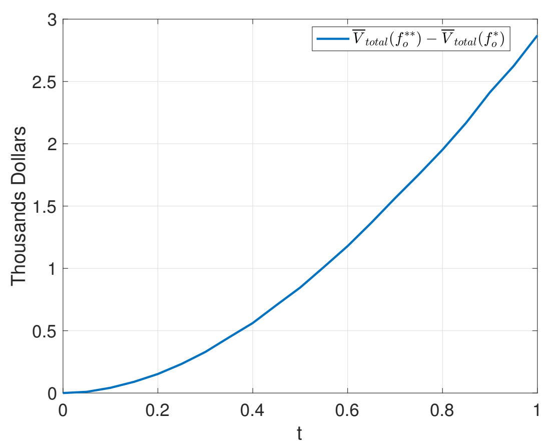
The case $P_o(f_o - f_o') = \frac{0.05}{(1+\exp(\frac{b_o(1.2-(f_o-f_o'))}{1000} - 20))}$ and $P_o(f_o - f_o') = \frac{5}{1000} + 0.05\exp(-\frac{b_o(1.2-(f_o-f_o'))}{1000})$ are shown in Figure 3.4 and 3.5, respectively. We see the similar result as figure 3.3 in these figures as well.



(a) Optimal Incentive Factor as a function of t

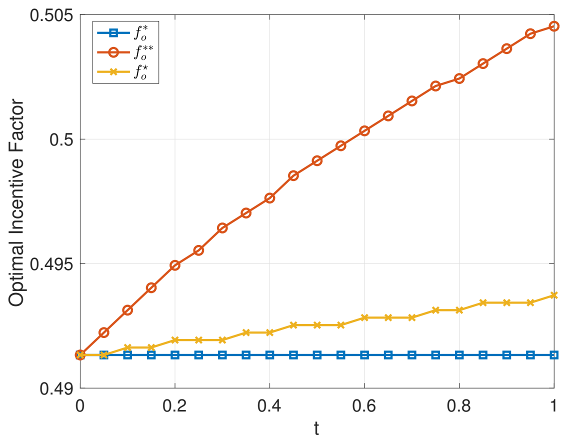


(b) Probability of a loss incident as a function of t

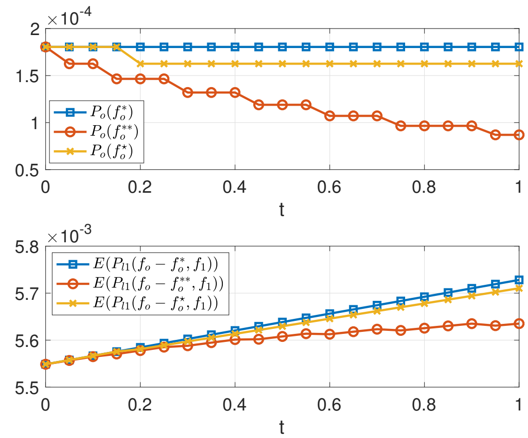


(c) Profit gain as a function of t

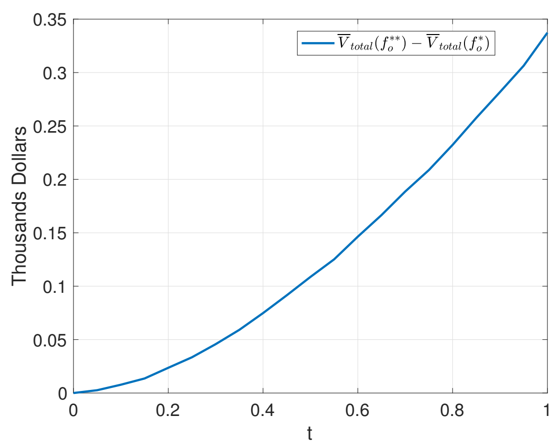
Figure 3.3: Optimal incentive factor and probability of a loss incident and profit gain under loss model (3.22).



(a) Optimal incentive factor as a function of t

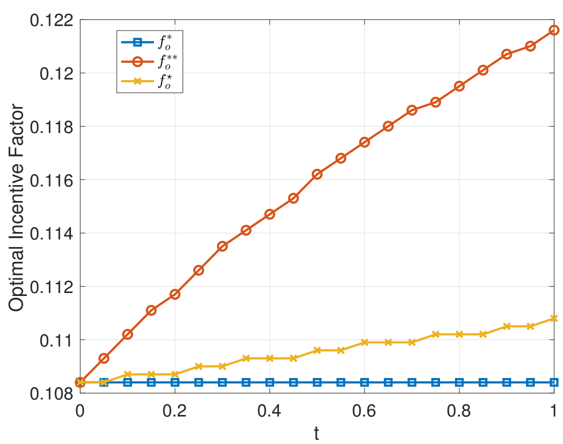


(b) Probability of a loss incident as function of t

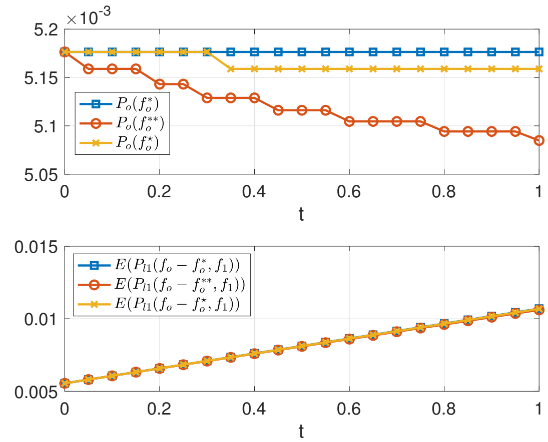


(c) Profit gain as function of t

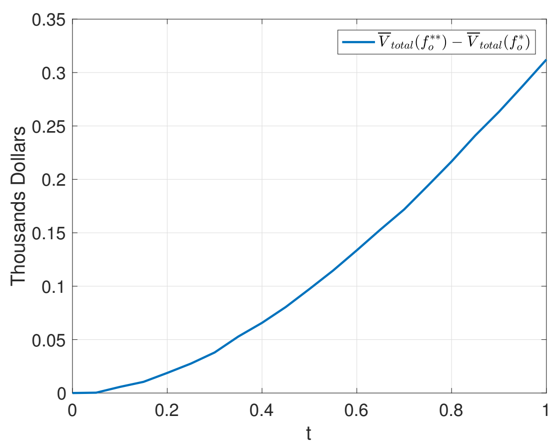
Figure 3.4: Optimal incentive factor and probability of a loss incident and profit gain under loss model (3.23)



(a) Optimal Incentive Factor as a function of t



(b) Probability of a loss incident as function of t



(c) Profit gain as function of t

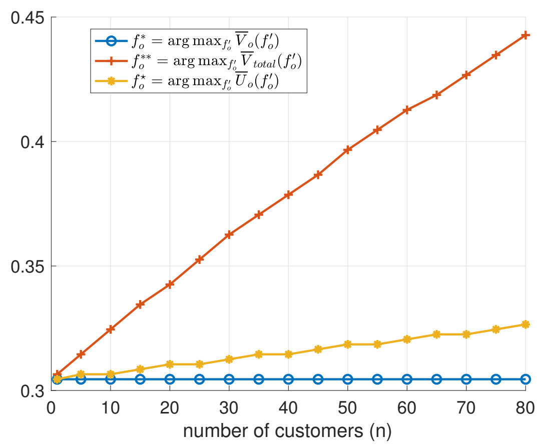
Figure 3.5: Optimal incentive factor and probability of a loss incident and profit gain under loss model (3.24).

3.4.4 Example 2: an SP and multiple customers with smaller revenue

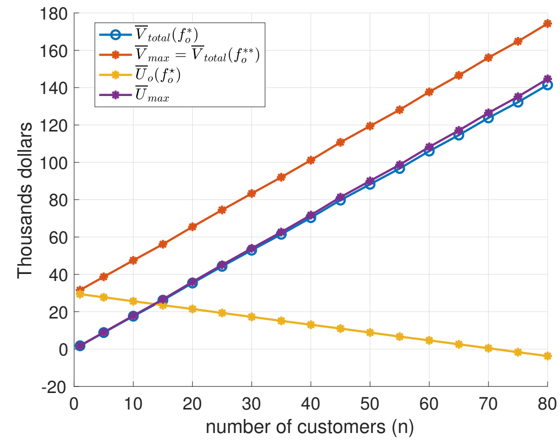
In this example, we consider an SP and n customers with relatively small revenue and study the impact of n on the optimal policy and insurer's utility. Using the rate schedule provided in 3.2, we will set the base rate and retention for the customers at $b_i = \$5,000$, $d_i = \$25,000$, $i = 1,\ldots,n$. The factors f_i, $i = 1,\ldots,n$ are drawn uniformly from [0.6 , 1.2]. Using Table 3.5, the loss random variable L_i, $i = 1,\ldots,n$ has a mean and median of \$599,907 and \$118,671, respectively. Similar as in the previous example, the mean and median of loss L_o are set at \$5,965,571 and \$3,326,313, respectively. We again assume that L_i follows a log-normal distribution. In addition, we set $f_o = 1.2$, $t = 0.5$, and $q = 0.13$. Compared to the previous example, in this example we shall examine the effect of the number of customers (n) on the optimal policy. Moreover, we consider the following loss function for customer i: $P_i(f_i) = \frac{0.05}{\frac{5000(1.2-f_i)}{1000}+1}$. The results are shown in Figure 3.6.

Figure 3.6a illustrates the optimal incentive factors $f_o^*, f_o^{**}, f_o^\star$ as a function of n. This plot implies that as the number of customers increases, the SP's insurer would incentivize the SP more in both portfolio B and C. The reasons behind this is obvious: as the risk spillover impacts more customers, the more the SP can reduce its risk, the greater the benefit to the SP's insurer (e.g., fewer business interruptions). Specifically, given that a breach occurred to the SP, the probability of no upstream business interruption is given by $1-(1-t)^n$, which an increasing function of n. Thus it is in the insurer's interest to reduce the likelihood of loss on the part of the SP. As a result, both f_o^{**} and $f_o^\star$ are increasing in n, while f_o^* is independent of n as it maximizes only $\overline{V}_o$ without considering dependency. Figure 3.6b implies that if the insurer does not gain by underwriting the customers and attributing all or a part of the loss to the SP as compared to the profit by underwriting all of them; we see in some cases the third party's insurer has negative expected profit, in which case a policy is not viable. Figures 3.7 and 3.8 shows similar results for the other two loss functions $P_o(f_o - f_o') = \frac{0.05}{(1+\exp(\frac{b_o(1.2-(f_o-f_o'))}{1000}-20))}$ and $P_o(f_o - f_o') = \frac{5}{1000} + 0.05\cdot\exp(-\frac{b_o\cdot(1.2-(f_o-f_o'))}{1000})$.

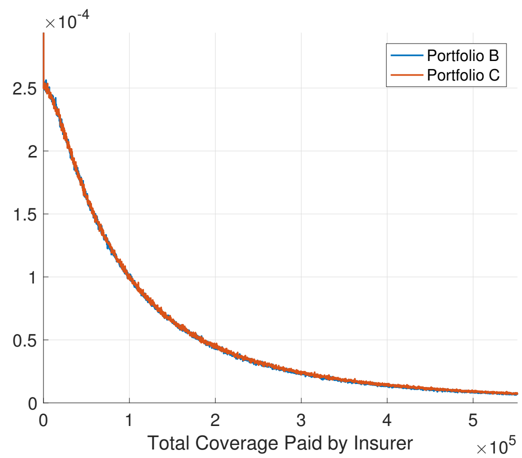
We now comment on Figures 3.6c, 3.7c, and 3.8c, which illustrate the insurer's payout distribution when the SP has $n = 10$ customers. All three show that portfolios B and C are faced with the same payout distributions regardless of the loss model being used. This is in contrast to the earlier comparison when there is only a single customer. This is because more customers leads the insurer to increase its incentive for the SP in order to lower its risk and its customers' risk; this is absent under portfolio C. As a result of this, the two portfolios actually experience the same amount of risk in payout; so again in this case portfolio B is uniformly better than C.



(a) Optimal incentive factor as function of n. Optimal incentive factor is increasing in n.

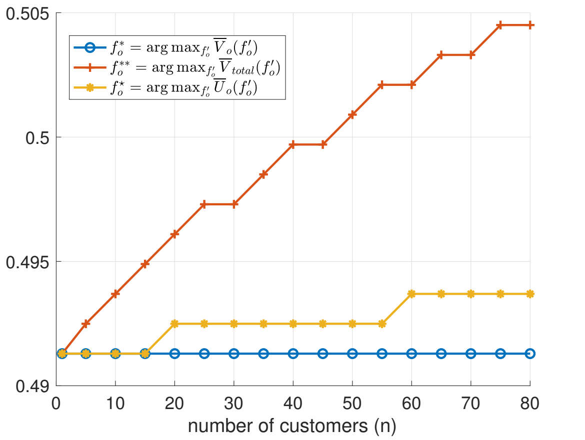


(b) Insurer's profit as function of n. The insurer does not gain by underwriting the SP's customers and attributing the loss to the SP.

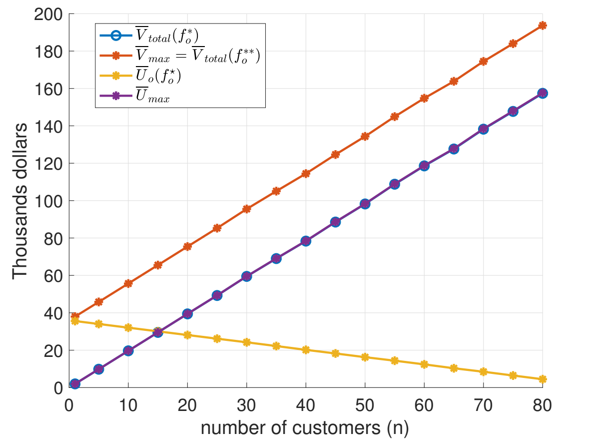


(c) Probability Distribution Function (pdf) of the amount paid out by the insurer in different scenarios.

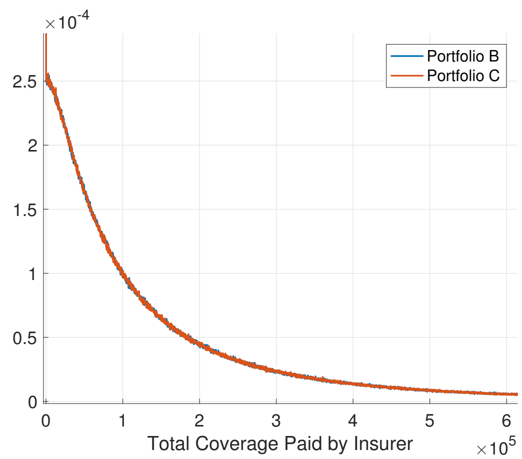
Figure 3.6: Optimal incentive factor and insurer's profit and pdf of the coverage paid by insurer under loss model (3.22)



(a) Optimal incentive factor as function of n. Optimal incentive factor is increasing in n.

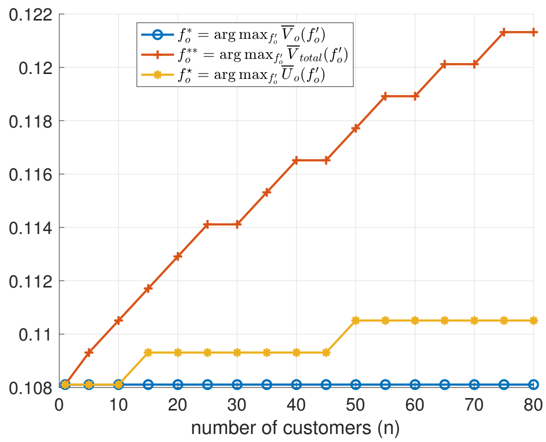


(b) Insurer's profit as function of n. The insurer does not gain by underwriting the SP's customers and attributing the loss to the SP.

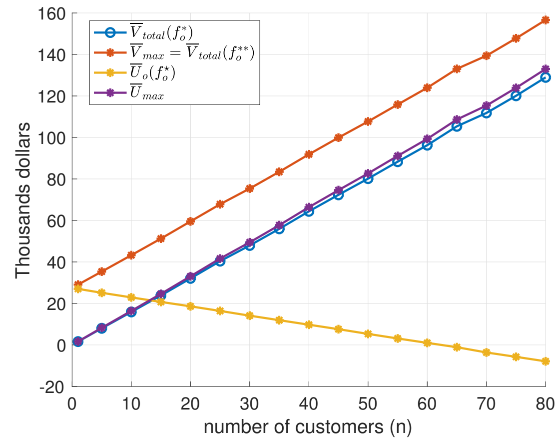


(c) Probability Distribution Function (pdf) of the amount paid out by the insurer in different scenarios.

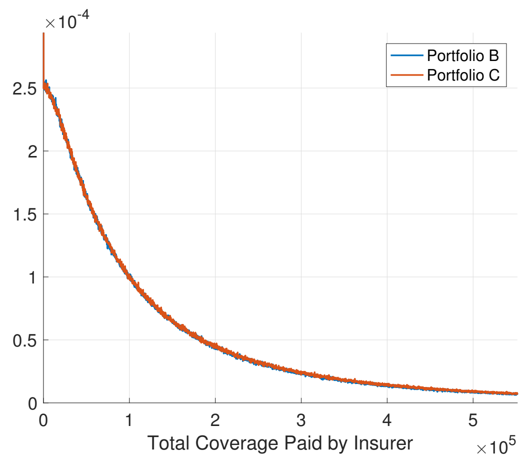
Figure 3.7: Optimal incentive factor and insurer's profit and pdf of the coverage paid by insurer under loss model (3.23)



(a) Optimal incentive factor as function of n. Optimal incentive factor is increasing in n.



(b) Insurer's profit as function of n. The insurer does not gain by underwriting the SP's customers and attributing the loss to the SP.



(c) Probability Distribution Function (pdf) of the amount paid out by the insurer in different scenarios.

Figure 3.8: Optimal incentive factor and insurer's profit and pdf of the coverage paid by insurer under loss model (3.24)

3.5 Discussion

We now discuss further three aspects of the model studied in this chapter.

3.5.1 Is the premium discount sufficient?

Consider a non-financial technology service provider firm with annual revenue between \$5M and \$10M. In this case, the base premium $b_o = \$7,500$. We will assume the firm is assessed with $f_o = 1.2$. Now assume that the insurer sets the incentive factor f'_o to be 0.35. Therefore, the firm pays $b_o \cdot (f_o - f'_o) = \$6375$ as the premium, after receiving $b_o \cdot f'_o = \$2625$ in discount. Using salary surveys such as [2], consider an IT security personnel with a bachelor's degree, 5 years of experience, and commands annual salary $W = \$85K$ for $N = 50$ working weeks. The premium discount the firm receives can be translated into a fraction of this person's compensation:

$$\frac{b_o \cdot f'_o}{W} \times N = \frac{\$2625}{\$85000} \times 50 = 1.5 \text{ weeks.} \tag{3.25}$$

Therefore, the incentive provided by the underwriter is just enough to hire an experienced person for 10 days. It is debatable whether this amount of investment in security is adequate to reduce the firm's cyber risk (by 10^{-9} according to Model (3.23), or by 0.05 according to Model (3.24), by setting $b_o = \$7,500$ in each, respectively). A potential mismatch between what this analysis suggests and reality may be attributed to two factors. Firstly, as already mentioned, the loss values shown in Fig 3.2 could be orders of magnitude different from reality; in other words, if the risk reduction is from a breach probability of 0.1% to 0.07%, then perhaps 10 days' worth of work (say in deploying software patches) is sufficient. Secondly, it may also be argued that the current level of base premium is inconsistent with the underlying cyber risk (and what it takes to reduce the risk) to begin with.

3.5.2 Social welfare

Our study so far has focused on whether it is in the interest of an underwriter to insure risk-dependent insureds, and if so how best to do so. We now turn to the issue of social welfare, i.e., whether by embracing risk dependency the underwriter can also help improve the total utility. We have shown that underwriting both SP and its customers and giving SP more discount on premium improves the insurer profit and decreases the probability of data breach. As a consequence of the latter, the utility of the insureds improves; thus underwriting both SP and its customers improves the social welfare (total utility) in general.

Let $C_o(f'_o)$ and $C_i(f'_o)$ be the total expected cost paid by the SP and its customer, respectively. We have,

$$\begin{aligned} C_o(f'_o) &= \underbrace{b_o \cdot f_o}_{\text{Premium}} + \underbrace{E\{D_o\} \cdot P_o(f_o - f'_o)}_{\text{Expcted uncovered loss}} \\ C_i(f'_o) &= \underbrace{b_i \cdot f_i}_{\text{premium}} + \underbrace{E\{D_i\} \cdot P_{li}(f_o - f'_o)}_{\text{Expected uncovered loss}}, \end{aligned} \tag{3.26}$$

where $D_i = \begin{cases} L_i \text{ if } L_i \leq d_i \\ d_i \text{ o.w} \end{cases}$ is the amount of deductible that insured i pays. Note that we do not consider discount $b_o \cdot f'_o$ in the SP's costs because this is assumed to be used toward its security investment. We define social welfare $SW(f'_o)$ to be the insurer's profit less its costs:

$$SW(f'_o) = V_{total}(f'_o) - C_o(f'_o) - \sum_{i=1}^{n} C_i(f'_o) \tag{3.27}$$

Below we use an example similar to that provided in Section 3.4.3 to illustrate the impact of insurance policy on social welfare.

Consider an SP and a single customer, and assume that both have a large annual revenue (\$10B-\$100B), with a base rate $b_o = b_1 = \$52,000$ and base retention $d_o = d_1 = \$250,000$. We assume that $f_o = 1.2, f_1 = 1$ and $P_o(f) = P_1(f) = \frac{0.05}{1+\frac{b_o(1.2-f)}{1000}}$ and $t = 0.5$. Based on Table 3.5, we assume both L_o and L_1 have log-normal distribution with mean \$5,965,571 and median \$3,326,313.

We now compare two cases. In the first case the insurer ignores the risk dependency and attempts to separately maximize its profit from the SP and its customer, respectively. In the second case the insurer jointly optimizes the two policies.

In the first case, the insurer obtains the discount to the SP as follows:

$$\begin{aligned} &l_o = l_1 = E((L_o - d_o)^+) = \$5,715,600 \\ &\overline{V}_o(f'_o) = b_o \cdot (f_o - f'_o) - l_o P_o(f_o - f'_o) \Rightarrow f^*_o = 0.3057 \end{aligned} \tag{3.28}$$

The insurer's profit and the insureds' costs are as follows:

- Insurer's total expected revenue:

$$
\begin{aligned}
V_{total}(f_o^*) &= b_o \cdot (f_o - f_o^*) - l_o P_o(f_o - f_o^*) + b_1 \cdot f_1 - P_{l1}(f_o - f_o^*) l_1 \\
&= 52000 * (1.2 - 0.3057) - 5715600 * 0.003 + 52000 - 5715600 * 0.0059 = \$47,635
\end{aligned}
$$

- SP's expected cost:

$$
C_o(f_o^*) = b_o \cdot f_o^* + E\{D_o\} P_o(f_o - f_o^*) = +52000 \times 1.2 + 28753 \times 0.003 = \$62,486
$$

- SP's customer's expected cost:

$$
C_1(f_o^*) = b_1 \cdot f_1 + E\{D_1\} \cdot P_{l1}(f_o - f_o^*) = 52000 \times 1 + 28753 \times 0.0059 = \$52,170
$$

- Total utility /Social Welfare (revenue less cost):

$$
SW(f_o^*) = V_{total}(f_o^*) - C_o(f_o^*) - C_1(f_o^*) = \$47,635 - \$62,486 - \$52,170 = -\$67021
$$

In the second case the insurer jointly maximizes the profit from the SP and its customer. It obtains the optimal incentive factor as follows:

$$
\overline{V}_{total}(f_o') = b_o \cdot (f_o - f_o') - l_o P_o(f_o - f_o') + b_1 \cdot f_1 - P_{l1}(f_o - f_o') l_1 \Rightarrow f_o^{**} = 0.3785
$$

The insurer's profit and the insureds' costs are given by:

- Insurer total expected revenue:
- Insurer's total expected revenue:

$$
\begin{aligned}
V_{total}(f_o^{**}) &= b_o \cdot (f_o - f_o^{**}) - l_o P_o(f_o - f_o^{**}) + b_1 \cdot f_1 - P_{l1}(f_o - f_o^{**}) l_1 \\
&= 52000 * (1.2 - 0.3785) - 5715600 * 0.0024 + 52000 - 5715600 * 0.0056 = \$48,993
\end{aligned}
$$

- SP's expected cost:

$$
C_o(f_o^{**}) = b_o \cdot f_o^{**} + E\{D_o\} P_o(f_o - f_o^{**}) = 52000 \times 1.2 + 28753 \times 0.0024 = \$62,469
$$

- SP's customer expected cost:

$$C_1(f_o^{**}) = b_1 \cdot f_1 + E\{D_1\} \cdot P_{l1}(f_o - f_o^{**}) = 52000 \times 1 + 28753 \times 0.0056 = \$52,161$$

- Total utility/Social Welfare (revenue less cost):

$$SW(f_o^{**}) = V_{total}(f_o^{**}) - C_o(f_o^{**}) - C_1(f_o^{**}) = \$48,993 - \$62469 - \$52161 = -\$65,637$$

We see that the total utility or social welfare is higher in the second case, when the insurer takes risk dependency into account and jointly optimizes the two policies. It is interesting to note that the values used in this example lead to negative social welfare, i.e., the total cost born by the insureds exceeds the total profit made by the insurer. The negative total utility is a reflection of the damage inflicted by attackers behind data breaches.

3.5.3 Modeling third party liability

We have assumed that the probability that the insurer can attribute a part of the loss to the third party is a constant (q) and is independent of P_o and P_i and t. An alternative model is to find probability q using P_o, P_i and t. Let q_i be the probability that the insurer of insured i can attribute a part of the loss to its third party. Moreover, define events A_i and B_i as follows:

- A_i: a business interruption occurs to insured i due to a data breach/loss incident on the SP's side.
- B_i: a loss incident occurs to insured i.

We then have:

$$\begin{aligned} &Pr\{A_i \cap B_i\} = P_o(f_o - f_o') \cdot (1 - P_i(f_i)) \\ &P\{B_i\} = P_{li}(f_o - f_o', f_i) = P_i(f_i) + t \cdot P_o(f_o - f_o') - t \cdot P_o(f_o - f_o') \cdot P_i(f_i) \\ &q_i = Pr\{A_i|B_i\} = \frac{Pr\{A_i \cap B_i\}}{Pr\{B_i\}} = \frac{P_o(f_o - f_o') \cdot (1 - P_i(f_i))}{P_i(f_i) + t \cdot P_o(f_o - f_o') - t \cdot P_o(f_o - f_o') \cdot P_i(f_i)} \end{aligned} \quad (3.29)$$

The above equation implies the assumption that the insurer is always able to attribute the loss of insured i to the SP if the latter is the cause of the loss. Under this assumption, Equations (3.19) and (3.20) can be written as follows:

$$\overline{U}_i(f_o') = b_i \cdot \frac{f_{min} + f_{max}}{2} - E[P_i(f_i)] \cdot l_i \,. \tag{3.30}$$

$$\overline{U}_o(f_o') = b_o \cdot (f_o - f_o') - P_o(f_o - f_o') \cdot \left[l_o + t \cdot \sum_{i=1}^{n} (1 - E[P_i(f_i)]) \cdot l_i \right] . \tag{3.31}$$

These two equations are equivalent to Equation (3.19) and (3.20), respectively, by setting $q = 1$ in (3.19) and (3.20). Therefore, all the theorems continue to hold for $q_i = Pr\{A_i|B_i\}$.

Note that the third party liability $t \cdot \sum_{i=1}^{n} (1 - E[P_i(f_i)]) \cdot l_i$ may be large, in which case b_o would also be large, for otherwise insuring SP alone is not profitable for the insurer. If insuring the SP alone is not viable due to high third party liability, then neither portfolio A nor C is viable, and portfolio B becomes the only choice.

3.5.4 Non-monopolistic insurer

Our study has assumed a monopolistic insurer. The modeling choice is aimed at focusing rather singularly on the issue of risk dependency without the interference of competition. Without monopoly the insurer will have to consider giving up its profit, but it does not change the main message of the study. Our analysis simply points to the fact that if the insurer recognizes the risk dependency among the insureds, then with the right incentive it can extract more profit; without monopoly it might have to give up all of this profit. Nonetheless, if there is competition, which often drives profit down to zero depending on the model, it may not be in the interest of the insurer to recognize this risk dependency or incentivize the SP. On the other hand, if one insurer is competing with another who is ignorant of the risk dependency among its prospective clients, then the first insurer now has an advantage in recognizing this and can effectively lower its cost of providing insurance and be able to offer more competitive contracts (with lower premium, i.e., returning a share of the profit to the insureds).

3.6 Conclusion

In this chapter, we applied a principal-agent modeling approach to understanding how an insurance carrier can best manage its portfolio risk of cyber-insurance policies, given interdependent risks across its policy holders. We calibrate our model using a common base rate approach to pricing premiums, and incorporate actual field data. We believe our results are significant because they suggest an alternative and preferred decision strategy for the carrier.

First, we found that insuring interdependent agents (an SP and its customer, Portfolio B) can lead to higher profit, compared with not insuring them simultaneously, the reason being that the insurer can incentivize the SP to increase its security level by offering a discount on its premium. When the SP provides more secure services for its customers, the chance of business interruption for the customers decreases, and the insurer's profit improves. In other words, receiving premiums from all interdependent agents and paying less in coverage due to improved security drives the profit opportunity not present when insuring interdependent agents.

In addition, we considered a scenario where the insurer underwrites only the SP's customers (Portfolio C) and is able to attribute a part of the loss to the SP and receive compensation from SP's insurer due to the third party liability. In this case, the insurer's profit decreases compared with the scenario of insuring both the SP and its customers (Portfolio B). The reason is that the insurer loses the SP's premium and the insurer cannot incentivize the SP to decrease the chance of business interruption for SP's customers. These results identify a countervailing factor against the current practice which avoids insuring interdependent agents.

Finally, we validate our results and theorems by providing numerical examples using real data. We showed the effect of interdependency (t) on the insurer's decision. As the SP and its customers become more interdependent, the insurer must incentivize the SP more in order to use the profit opportunity.

In conclusion, we believe that these results will help insurers and reinsurers better understand and manage systemic risk, while also demonstrating to policy makers how market-based insurance can improve social welfare.

One future direction is to perform various sensitivity analyses of incentive decisions made by an insurer, such as those derived in this chapter, against the actual costs of obtaining accurate information that enables the decisions, including the cost of performing security assessment/audit or continued monitoring to ensure actions by an insured commensurate with the discount it received.

CHAPTER 4

Effective Premium Discrimination with Rare Losses: Periodic Pre-screening and Active Policy

4.1 Introduction

As we mentioned in section 1.2, there are three challenges for a cyber insurer to underwrite entities. Chapter 2 and Chapter 3 focused on the following two challenges: limited data and lack of knowledge for risk assessment, and risk dependency. In this chapter, our goal is to focus on the third challenge, i.e., the fast-changing nature of cyber risks. In this chapter, we assume that data breach and loss incidents are rare for an agent but the amount of loss from a breach is extremely large.[1] Moreover, we assume that there is asymmetry in loss perception between the insurer and insured. We then consider two types of risk assessment: pre-screening and post-screening. As we mentioned before, pre-screening occurs before the agent purchases insurance and is done through internet measurements and available data. Pre-screening gives the insurer an estimate of cyber risk associated with the agent to determine the premium. On the other hand, post-screening implies that the premium in each period depends on past policy periods, and any loss incident in the history of the insured may increase the premium.

It is worth mentioning that rare cyber incidents are different from natural disasters that have been studied in the literature [72, 81]; the latter are also rare incidents with high losses but differ from a cyber incident in the following sense. The agents/insureds cannot prevent natural disasters by exerting effort or their ability to mitigate damages can be limited. The authors in [72, 81] do not

[1]This model is reasonably borne out by recent events such as the Equifax data breach, which affected 143 million American consumers and incurred $68.6 billion in loss for the company [15]; most of these events have been unprecedented in the respective victim's company history.

consider the agent's effort in their models as it does not affect the probability of natural disaster occurrence. On the other hand, an agent can actively and proactively work toward decreasing his chance of being attacked or an attack being successful by investing in security and addressing the vulnerability.

In previous chapters, we showed that pre-screening is an effective method to mitigate moral hazard in cyber insurance market. It has been shown that post-screening also can be effective in general: since an agent faces (potentially significantly) higher payments in the future, there is an incentive for the agent to act responsibly (exert high effort) in the present time to avoid a loss event, see, e.g., Rubinstein *et. al* [74]. The analysis in this study shows, however, that the conclusion becomes more nuanced when loss events are rare, and there is asymmetry in loss perception between the insurer and insured. Specifically, we show that only pre-screening can be effective in such a scenario. We further propose an active policy using periodic pre-screening to overcome the ever-changing nature of cyber risks and prevent the insureds from lowering their effort in the middle of the policy period after the first pre-screening is done.

4.1.1 Main findings

Our main finding in this chapter is that post-screening (which involves at least two contract periods) is not effective at all with rare loss incidents. On the other hand, pre-screening can be an effective method if the agent perceives loss incidents as rarer than the insurer does; in this case *sufficiently accurate* pre-screening can be effective and improves the state of security as well as the insurer's profit as compared to not using premium discrimination.

Moreover, we propose *active policies* to prevent the insureds from lowering their efforts in the middle of the policy period after pre-screening. We show that pre-screening has to be performed more often in the policy period, and premium should be adjusted after each screening, i.e., the insurance contract should be an active policy with contingencies based on periodic screening.

4.1.2 Chapter organization

The organization of this chapter is as follows. Section 4.2 provides an overview of dynamic contract literature. In Section 4.3 we introduce the model and the contract design problem. Section 4.4 summarizes prior results (but recast under our model) on designing cyber insurance policies when incidents are not rare. In Section 4.5 we examine the effect of pre-screening and post-screening on both the state of security and the insurer's profit with rare losses. Section 4.6 studies the active policy using periodic pre-screening, and Section 4.7 discusses the impact of risk dependency on our results. Section 4.8 presents numerical results and Section 4.9 concludes the chapter.

4.2 Related work

Dynamic contract as a form of multi-period contract has been studied in the literature [5–7, 33, 69, 71]. Baron and Besanko [5] study a two-period contract design problem where a regulator specifies quantities of the product based on the reported marginal production cost by the firm. The authors show that if the marginal cost in the second period is independent of the first-period marginal cost, then the two-period contract design problem is equivalent to a single-period problem. This work is extended by [7] and [6] to a scenario with an infinite time horizon and an agent whose type is a Markov process. Dynamic insurance contract is studied in [33, 69]. Janssen and Karamychev [33] consider a multi-period insurance contract (dynamic contract), and show that if the contract parameters at each time step depend on the past performance of the insured, then the insurance improves social welfare. Similarly, Palfrey and Spatt [69] consider repeated insurance contracts and show that long-term repeated contracting solves the under-investment issue associated with moral hazard.

This chapter also considers dynamic insurance contract in the form of multi-period insurance (i.e., premium discrimination using *post-screening*) and demonstrates that it can be effective in mitigating moral hazard, but this fails if loss incidents are rare. On the other hand, we show that single-period contract using *pre-screening* is able to mitigate moral hazard even if loss incidents are rare. Furthermore, we propose an active contract using repeated screening during a single contract period to prevent the insured from lowering his effort over the contract period. In this sense, the proposed active contract is different from the concept of dynamic contracts studied in the literature (cited above) as the latter only applies to multi-period contracts.

4.3 Model

We consider the cyber insurance design, a principal-agent problem, between a profit-maximizing, risk-neutral insurer/principal and a risk-averse insured/agent. The agent exerts effort e toward securing himself, incurring linear cost $c \cdot e$.

Let $p(e)$ denote the probability of a loss incident, assumed to be strictly decreasing and strictly convex. Decreasing and convexity imply that the initial effort toward security leads to a considerable reduction in probability of a loss incident, and strict convexity implies that the probability of a loss cannot be zero even if the agent exerts high effort [34], i.e., it is impossible to achieve perfect protection in reality. Specifically, we assume that probability of a loss incident has the following form,[2]

$$p(e) = t \cdot \exp\{-\alpha \cdot e\}, \tag{4.1}$$

where t is the nominal probability of a successful attack to the agent if he exerts zero effort ($e = 0$) and α is a constant. Larger α implies that investment in security is more effective and $p(.)$ converges to zero faster. Note that t and α both are constants and cannot be modified by the agent or the insurer.

When a loss occurs, the agent suffers the amount of loss l, also a constant. This is obviously a simplification; however, our qualitative conclusions remain the same for a random loss given by a known distribution. The expected utility of the agent without any insurance contract is given by:

$$U(e) = p(e)f(-l - ce) + (1 - p(e))f(-ce), \tag{4.2}$$

where $f(.)$ is a concave function that captures the agent's risk aversion. To make the analysis concrete, we will further assume $f(.)$ is an exponential function with constant absolute risk aversion γ:

$$f(y) = 1 - \exp\{-\gamma \cdot y\}\ , \tag{4.3}$$

where γ is referred to as the agent's risk attitude; the higher the risk attitude the more risk averse the agent.

4.3.1 Agent's effort & utility without insurance

Without insurance, the agent exerts an effort level e^o to maximize his utility:

$$e^o = \arg\max_{e \geq 0} U(e)\ . \tag{4.4}$$

[2] $p(e)$ can be written as $t \cdot (\exp\{-\alpha\})^e$ which is a function consistent with the exponential probability function introduced in [20].

By the first order condition, it is easy to see that if $\gamma c \geq \alpha$, then $e^o = 0$. If $\alpha > \gamma c$, then e^o is given by,

$$\begin{aligned} U(e) &= 1 - t \cdot \exp\{\gamma \cdot l\} \cdot \exp\{(\gamma c - \alpha) \cdot e\} - \exp\{\gamma c e\} + t \exp\{(\gamma c - \alpha) e\} \\ \frac{dU(e)}{de} &= \exp\{\gamma c e\}\left(-\gamma c \ + t \cdot (\alpha - \gamma c) \cdot \exp\{-\alpha \cdot e\} \cdot (\exp\{\gamma l\} - 1)\right) \implies \\ e^o &= \begin{cases} 0 & \text{if } \gamma c \geq \alpha \\ (\frac{1}{\alpha} \ln t \cdot \frac{(\alpha - \gamma c)(\exp\{\gamma l\} - 1)}{\gamma c})^+ & \text{if } \gamma c < \alpha \end{cases} \end{aligned} \tag{4.5}$$

where $(a)^+ = \max\{0, a\}$. As a result, the maximum utility of the agent outside the contract is given by,

$$u^o = U(e^o) = \begin{cases} 1 - \frac{\alpha}{\alpha - \gamma c}(t \cdot \frac{\alpha - \gamma c}{\gamma c}(\exp\{\gamma l\} - 1))^{\frac{\gamma c}{\alpha}} & \text{if } e^o > 0 \\ t \cdot (1 - \exp\{\gamma l\}) & \text{if } e^o = 0\ . \end{cases} \tag{4.6}$$

4.3.2 Contract design

We will assume that in the event of a loss, a contract covers the full amount l. This is again a simplification but it allows us to get to the essence of our analysis in a more straightforward manner without affecting the main qualitative conclusions. Because a loss is covered in full, the agent will exert zero effort after entering an insurance contract. Thus the insurer will have to use premium discrimination to incentivize the insured to exert a higher effort in exchange for lower premium. We next describe in detail the resulting contract design problem under two different methods of premium discrimination: post-screening and pre-screening.

4.3.2.1 Post-screening

In this case the contract design problem is framed in a two-period setting where the insurer is able to assess premium in the second period based on what happens in the first period. Such a contract is given by three parameters (π_1, π_2, π_3): π_1 is the first-period premium; in the second period, the agent pays premium π_2 if a loss happened (and was covered in full) during the first period and pays π_3 otherwise. Obviously $\pi_3 \leq \pi_2$.

In this case, the agent may exert non-zero effort in the first period to decrease the chance of a loss in order to reduce the likelihood of paying a higher premium in the second period. In the second period, on the other hand, the agent will always exert zero effort as the loss is fully covered and he faces no more future punishment.[3]

We assume that when an agent enters such a contract he commits to both periods. The agent's utility inside a contract (π_1, π_2, π_3) with post-screening is thus the summation of his utility in each period:

$$U^{in}(e, \pi_1, \pi_2, \pi_3) = f(-\pi_1 - ce) + p(e)f(-\pi_2) + (1 - p(e))f(-\pi_3), \tag{4.7}$$

where e is the effort in the first period.

The insurer's problem is to maximize her profit subject to the Individual Rationality (IR) constraint and Incentive Compatibility (IC) constraint:

$$\begin{aligned} V &= \max_{\{\pi_1,\pi_2,\pi_3,e\}} \pi_1 - p(e)l + p(e)(\pi_2 - p(0)l) + (1 - p(e))(\pi_3 - p(0)l) \\ \text{s.t.} & \quad (IR) \quad U^{in}(e, \pi_1, \pi_2, \pi_3) \geq 2 \cdot u^o \\ & \quad (IC) \quad e \in \arg\max_{e' \geq 0} U^{in}(e, \pi_1, \pi_2, \pi_3). \end{aligned} \tag{4.8}$$

The (IR) constraint ensures that the agent enters the contract only if he gets no lower utility than his outside option. Note that since the contract covers two periods, the comparison here is between his utility inside the contract over two periods and outside the contracts over two periods. The (IC) constraint suggests that the agent acts in self-interest: his effort level in the first period maximizes his utility given the policy parameters.

Under the contract (π_1, π_2, π_3), by the first order condition, the agent's optimal effort e^{in} in the first period is given by:

$$e^{in}(\pi_1, \pi_2, \pi_3) = \begin{cases} \left(\frac{1}{\alpha+\gamma c} \ln(t \cdot \frac{\alpha}{\gamma c} \frac{\exp\{\gamma\pi_2\} - \exp\{\gamma\pi_3\}}{\exp\{\gamma\pi_1\}})\right)^+ & \text{if } \pi_2 > \pi_3 \\ 0 & \text{if } \pi_2 \leq \pi_3 \end{cases}. \tag{4.9}$$

For notational convenience, we use e^{in} instead of $e^{in}(\pi_1, \pi_2, \pi_3)$, while noting the dependency. We have the following lemma on the (IR) constraint.

Lemma 4.1 *The (IR) constraint in the optimization problem* (4.8) *is binding.*

[3] Our analysis can be extended to a multi-period/infinite-time horizon setting where the premium of each period depends on the agent's history of losses, i.e., the agent's third-period premium depends on his loss events in the first and second periods and so on.

The above lemma implies that at the optimal solution, the agent is indifferent between entering v.s. not entering the contract, as expected.

4.3.2.2 Pre-screening

We now turn to the case of pre-screening. We assume the insurer can conduct a risk assessment prior to determining the contract terms; the determination mechanism is known to the agent so this is again a game of perfect information. We assume the outcome of the pre-screening is given by an assessment $S = e + N$, where N is a zero-mean Gaussian noise with variance σ^2.[4] There are various ways to achieve pre-screening in practice, using surveys, penetration tests, or advanced Internet measurement techniques, see e.g., [56].

The interaction between the agent and the insurer happens in the following order. The insurer offers the agent a contract given by two parameters (π,β), where π is the base premium and β is the assessment-dependent discount factor. After announcing contract parameters (π,β), the agent chooses effort level e, and then the insurer performs an initial security audit and observes pre-screening outcome S. Based on the result of pre-screening, the insurer pays $\pi-\beta S$ in exchange for full coverage in the event of a loss. The agent's total cost inside the contract (π,β) while exerting effort e is:

$$X^{in} = \pi - \beta \cdot S + c \cdot e \,. \tag{4.10}$$

As X^{in} follows a Gaussian distribution, using the moment-generating function the agent's *expected* utility under the contract is given by:

$$U^{in}(\pi,\beta,e) = E(f(-X^{in})) = 1 - \exp\{\gamma\pi + \gamma(c-\beta)e + \tfrac{\gamma^2\beta^2\sigma^2}{2}\}. \tag{4.11}$$

Therefore, the insurer's design problem using pre-screening is as follows:

$$\begin{aligned} \max_{\pi,\beta,e} \quad & E\{\pi - \beta S\} - p(e)\cdot l \\ \text{s.t.} \quad & (IR) \quad U^{in}(\pi,\beta,e) \geq u^o, \\ & (IC) \quad e \in \arg\max_{e'\geq 0} U^{in}(\pi,\beta,e') \end{aligned} \tag{4.12}$$

[4] The analysis can be extended to other noise distributions.

Similar as in Lemma 4.1, we can show that the (IR) constraint is binding in this case. Thus we have the following relation between optimal contract parameters ($w^o = \frac{1}{\gamma}\ln(1-u^o)$):

$$\pi = w^o + \beta e - ce - \frac{\gamma\beta^2\sigma^2}{2} \ . \tag{4.13}$$

Using (4.13), the insurer's problem can be simplified as follows:

$$\begin{aligned} V(\sigma) = \max_{\beta,e} \quad & w^o - ce - \frac{\gamma\beta^2\sigma^2}{2} - p(e)l \\ \text{s.t.} \quad & (IC) \quad e \in \arg\min_{e' \geq 0} (c-\beta)e' + \frac{\gamma\beta^2\sigma^2}{2}, \end{aligned} \tag{4.14}$$

We next summarize (known) results on these two types of premium discrimination in terms of their effectiveness in incentivizing efforts.

4.4 State of security and optimal contract when losses are not rare

Post-screening: Post-screening has been studied in the literature. Rubinstein *et.al.* in [74] showed that post-screening can improve the agent's effort inside the contract compared to the one-period contract without post-screening.

This can be similarly observed in our model. In particular, in Theorem 4.1 below we introduce a sufficient condition under which the agent exerts non-zero effort in the first period of a contract with post-screening. In Section 4.8, we also provide an example where the agent inside a contract with post-screening exerts higher effort as compared to the no-insurance scenario.

Theorem 4.1 *Let* $(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3, \hat{e})$ *be the solution of the optimization problem* (4.8)*. Suppose that* $t = 1$ *and* $\left[\frac{(\alpha-\gamma c)(\exp\{\gamma l\}-1)}{\gamma c}\right] > 1$*, then* $\hat{e} > 0$.

Theorem 4.1 suggests that post-screening can be an effective mechanism to incentivize non-zero effort. Note that the condition $\left[\frac{(\alpha-\gamma c)(\exp\{\gamma l\}-1)}{\gamma c}\right] > 1$ in theorem 4.1 can be satisfied if loss l is sufficiently large.

Pre-screening: Chapter 2 shows that pre-screening can simultaneously incentivize the agent to exert non-zero effort and improve the insurer's utility. This is characterized for the present model in the following theorem.

Theorem 4.2 *Pre-screening incentivizes non-zero effort if and only if*

$$\frac{c}{\alpha \cdot t \cdot l} < 1 \tag{4.15}$$

$$\sigma^2 \leq \frac{-\frac{c}{\alpha} - \frac{c}{\alpha}\ln\frac{c}{\alpha \cdot t \cdot l} + t \cdot l}{0.5 \cdot c^2\gamma}. \tag{4.16}$$

Theorem 4.2 suggests pre-screening is effective if and only if it is sufficiently accurate (Eqn (4.16)) and the expected loss $t \cdot l$ is sufficiently large (Eqn (4.15)). Further, the next theorem identifies the relation between insurer's profit and pre-screening accuracy.

Theorem 4.3 *Let $V(\sigma)$ be the insurer's maximum utility. That is,*

$$V(\sigma) = \max_{\beta,e} w^o - ce - \frac{\gamma\beta^2\sigma^2}{2} - p(e)l \quad s.t. \quad IC\ constraint \tag{4.17}$$

Then, $V(\sigma)$ is decreasing in σ.

4.5 State of security and optimal contract when losses are rare

We next consider the case when loss events are rare, by assuming its likelihood diminishes (i.e., $t \to 0$) but that the loss amount is high in such an event (i.e., $l \to \infty$)[5]. This model is motivated by recent data breaches that result in extremely high losses and damages but remain relatively rare for a single organization as mentioned earlier.

Furthermore, we would like to explicitly capture a common asymmetry in perception between the insurer and the agent, i.e., the latter tends to think of loss as rarer than the former does. Specifically, let t_a and t_p denote the nominal attack probability from the agent and the insurer's perspective, respectively. By our assumption, both t_a and t_p go to zero and l goes to infinity. For tractability, we adopt the following assumptions on t_a and t_p and l,

$$\begin{aligned} &\lim_{\{t_a \to 0, l \to \infty\}} t_a \cdot \exp\{\gamma l\} = \exp\{\gamma l_a\} \\ &\lim_{\{t_p \to 0, l \to \infty\}} t_p \cdot l = l_p, \end{aligned} \tag{4.18}$$

[5]By assuming that t goes to zero, the entire probability of a loss incident (i.e., $p(e) = t\exp(\alpha(e))$) goes to zero.

where l_a and l_p are the perceived expected loss from the agent and the insurer's perspective when the agent exerts zero effort, respectively.[6] It is worth noting that Eqn (4.18) implies that the expected loss is always limited. Otherwise the cyber insurance market may not exist. Moreover, (4.18) implies that $t_a = \frac{\exp\{\gamma l_a\}}{\exp\{\gamma l\}}$ goes to zero exponentially while $t_p = \frac{l_p}{l}$ goes to zero slower than t_a as l goes to infinity, i.e, $t_a > t_p$ as $l \to \infty$. Therefore, the agent thinks the loss is rarer than the insurer does.

4.5.1 Post-screening

With the above rare loss assumptions, we have the following theorem on post-screening.

Theorem 4.4 *Using post-screening and given $t \to 0$,*

1. *the agent always exerts zero effort inside the contract, and*
2. *at the optimal contract we have,*

$$\pi_1 = \pi_3 = \frac{1}{\gamma} \ln[1 - u^o], \;\; \pi_2 \in \mathcal{R}^+$$

.

Theorem 4.4 implies that premium discrimination in the second period based on the first period is not at all effective and the insurer is not able to improve the agent's effort or her utility by post-screening as compared to a contract without premium discrimination.

4.5.2 Pre-screening

For pre-screening, it turns out perception asymmetry makes a difference. The following theorem characterizes the optimal contract and introduces a sufficient condition under which pre-screening can incentivize the agents to exert non-zero effort inside the optimal contract.

[6] If the agent exert effort e, then $l_a \exp\{-\alpha \cdot e\}$ and $l_p \exp\{-\alpha \cdot e\}$ are the perceived expected loss from the agent and the insurer's perspective.

Theorem 4.5 *Pre-screening can incentivize non-zero effort under the rare loss model, if and only if*

$$\frac{c}{\alpha l_p} < 1 \tag{4.19}$$

$$\sigma^2 \leq \frac{-\frac{c}{\alpha} - \frac{c}{\alpha}\ln\left[\frac{c}{\alpha \cdot l_p}\right] + l_p}{0.5 \cdot c^2 \gamma}. \tag{4.20}$$

Note that conditions in Theorem 4.2 reduce to those in Theorem 4.5 if we substitute tl with l_p in (4.15) and (4.16). Theorem 4.5 implies that pre-screening incentivizes effort if and only if the pre-screening is sufficiently accurate and insurer's perceived loss l_p is sufficiently large.

4.6 Contingencies on periodic pre-screening: active policy

So far we have assumed that the agent exerts a one-shot effort level, which applies to the entire policy period. Under this assumption, pre-screening helps incentivize non-zero effort. In reality, keeping risk at a certain level typically requires sustained effort throughout the period, and it is conceivable that the insured may choose to lower his effort after the initial risk assessment (yet another form of moral hazard). If so then our results on pre-screening suggests that it has to be performed more often, whereby premium adjustment is made following each screening. This effectively means that the initial contract is an *active policy* with built-in contingencies, and the actual premium payable is realized over time dependent on the screening results. We illustrate this idea using the following example with one additional, mid-term, screening.

Let's assume that the agent exerts effort e before the first screening, resulting in assessment outcome $S = e + N$ as before, and then he lowers the effort to e'. Accordingly, let $S' = e' + N'$ be the outcome of the second, mid-term screening, where N' is a zero-mean Gaussian noise with variance σ^2. We assume that N, N' are independent random variables. Below we show that the insurer is able to incentivize the agent not to decrease the effort level through the second screening, i.e., to ensure $e' = e$. The interaction between the agent and the insurer under an active policy consists of the following steps. First, the insurer offers the agent an active contract with three parameters (π,β,β'), where β' is a penalty factor, β a discount factor, and π the base premium. Then, the agent chooses effort level e before the first pre-screening. The insurer then observes pre-screening outcome S and charges the agent $\pi-\beta S$. After the first pre-screening, the insured may choose to lower his effort to e'. In the middle of the contact period, the insurer conducts another screening and observes S', and charges the agent $\beta'(S-S')$. Effectively, the insured pays $\pi-\beta\cdot S+\beta'(S-S')$ as the final premium, and the total cost of the agent is given by,

$$X^{in} = \pi - \beta\cdot S + ce + \beta'(S-S') - b(e-e'), \tag{4.21}$$

where $0 \le b \le c$ and b is the benefit of lowering the effort, and $\beta'(S-S')$ is the penalty that the insured would pay after the second risk assessment.[7]

Similar to (4.11), the agent's expected utility under contract (π,β,β') is:

$$\begin{aligned} &U^{in}(\pi,\beta,e,\beta',e') = E(f(-X^{in})) = \\ &1-\exp\{\gamma\pi+\gamma(c-b+\beta'-\beta)e+\gamma(-\beta'+b)e'+\gamma^2\sigma^2\frac{(\beta-\beta')^2+(\beta')^2}{2}\}. \end{aligned} \tag{4.22}$$

The insurer's problem can be written as follows:

$$\begin{aligned} R(\sigma) = \max_{\{\pi,\beta,e,\beta',e'\}} \quad & [E\{\pi-\beta S+\beta'(S-S')\}-p(e')l] \qquad (4.23)\\ \text{s.t.} \quad & (IR) \quad U^{in}(\pi,\beta,e,\beta',e') \ge u^o \\ & (IC) \quad (e,e') \in \arg\max_{\tilde{e},\tilde{e}'\ge 0} U^{in}(\pi,\beta,\tilde{e},\beta',\tilde{e}'),\ e' \le e \end{aligned}$$

The following theorem shows that the second risk assessment is effective in preventing the agent from lowering his effort.

Theorem 4.6 *Let $\hat{e}$ and $\hat{e}'$ be the agent's effort level at the solution to* (4.23)*, and $\overline{e}$ be the optimal effort level in optimization problem* (4.14)*. Then, we have $\hat{e}=\hat{e}'$, and the optimal contract parameters are $\beta=c$ and $\beta'=b$ if $\hat{e}>0$ otherwise they are $\beta=\beta'=0$. Moreover, if $\overline{e}>0$, then $\hat{e}=\overline{e}$. Lastly, we have $V(\sigma)\le R(\sigma)$, where $V(\sigma)$ is obtained from* (4.14) *by assuming there is only one pre-screening and the agent does not lower his effort afterward, with equality achieved if $b=c$.*

The last part of the theorem above suggests that performing the second screening helps the insurer

[7] Note that equation (4.21) is valid if $e'\le e$. If $e'>e$, then the total cost is given by $X^{in}=\pi-\beta\cdot S+ce+\beta'(S-S')-c\cdot(e-e')$. We do not consider the case where $e'>e$, but similar to the proof of Theorem 4.6, we can show that e' is not larger than e in the optimal contract.

to improve profit even when the agent may be assumed not to lower his effort. This is because second pre-screening decreases the variance and uncertainty in agent's utility. Therefore, a risk averse agent is willing to pay more premium when the uncertainty and variance on his side decreases.

4.7 Discussion

So far we have assumed that the probability of a loss incident is solely determined by the effort of the agent. On the other hand, risk dependency is a unique feature of cyber risks: the incident probability for an agent may depend on the effort levels of other agents (the former's vendors or service providers, etc.). In Chapter 2, we considered a cyber insurance market in the presence of risk dependency, and showed that the insurer can achieve higher profit as compared to a network of independent agents; moreover, pre-screening in such a case increases the agents' efforts as compared to the no insurance scenario. If we introduce security dependency into our rare loss model (i.e., the probability of a loss incident for agent $i \in \{1,2\}$ depends on $e_i + xe_{-i}$ where e_i is the effort of agent i and x is the interdependence factor), similar to the analysis of this chapter, it can be shown that post-screening is not able to incentivize non-zero effort while pre-screening can. Table 4.1 summarizes the role of dependency and rare loss on the agents' effort, where $(*)$ indicates the associated result holds under certain conditions.

	Pre-screening	Post-screening
Rare Loss, dependent agents	$e^{in} > e^{o}$ $(*)$	$e^{in} = 0$
Rare Loss, independent agents	$e^{in} > e^{o}$ $(*)$	$e^{in} = 0$
Frequent loss, dependent agents	$e^{in} > e^{o}$ $(*)$	$e^{in} > e^{o}$ $(*)$
Frequent loss, independent agents	$e^{o} \geq e^{in} \geq 0$	$e^{in} > e^{o}$ $(*)$

Table 4.1: Comparing agent's effort inside (e^{in}) and outside (e^{o}) a contract

4.8 Numerical result

We show a number of numerical examples with the following parameters $\gamma = c = 1$, $\alpha = 1.5$.

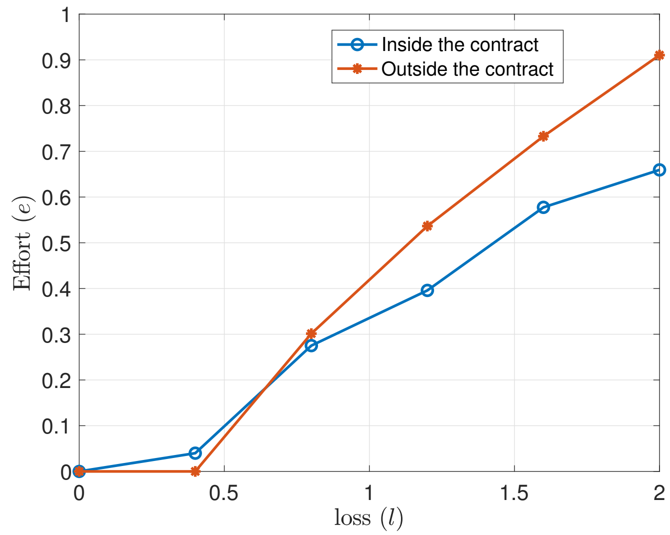


Figure 4.1: Post-screening: agent's effort v.s. loss (l)

4.8.1 Frequent losses: post-screening

Our first example shows when post-screening may be effective in incentivizing the agent to exert higher effort as compared to the no-insurance scenario.

Consider a scenario where the nominal probability of attack $t = 1$. Figure 4.1 illustrates the agent's effort in the first period as a function of loss l. We note that post-screening can be an effective mechanism to incentivize the agents to exert non-zero effort inside a contract with full coverage. In this example, the agent exerts higher effort as compared to the no insurance scenario when $l \leq 0.7$. This is because since the loss is relatively low, even without insurance the agent is not willing to exert substantial effort as the cost of effort is higher than the actual loss. Within a contract, the insurer is able to incentivize the agent to exert higher effort by imposing a large penalty (a much higher premium in the second period).

4.8.2 Rare losses: pre-screening

Our second example examines the effect of pre-screening on the agent's effort. Consider a scenario where t_a, t_p go to zero and l goes to infinity. Moreover, assume $l_a = 5$ and $\sigma = 0.1$. Figure 4.2 illustrates the agent's effort inside and outside the insurance contract with pre-screening. We see

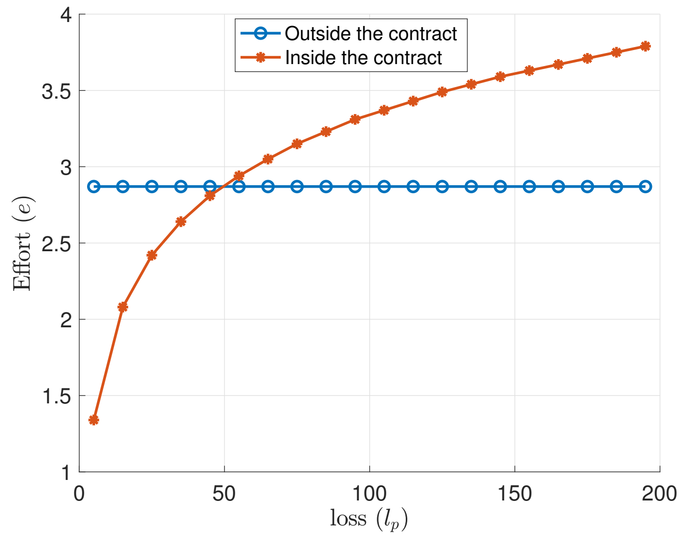


Figure 4.2: Pre-screening: agent's effort v.s. loss (l_p)

that the agent exerts non-zero effort inside the insurance contract and the effort increases as l_p increases. Note that outside a contract the agent's effort is a function of his perceived loss l_a and does not change with l_p. On the other hand, inside the contract, as the insurer's perceived loss l_p increases, the insurer incentivizes the agent to increase his efforts using premium discrimination (high premium for low pre-screening outcomes).

Figure 4.3 illustrates the insurer's utility as a function of l_p. This figure implies that the insurer's utility is negative for $l_p \geq 85$. Therefore, she does not insure the agent if $l_p \geq 85$. Also, as expected, the insurer's utility decreases as the perceived expected loss l_p increases. The reason is that as the perceived expected loss increases, the insurer expects to pay more coverage and make less profit.

4.9 Discussion and conclusion

We studied the problem of designing cyber-insurance contracts between a single profit-maximizing insurer and two risk-averse agents. First, we showed that multi-period contract is an effective method of premium discrimination if the loss incidents are not rare. Second, we studied cyber insurance contracts in the presence of very rare loss incident which is the common feature of the cybersecurity. In this case, we showed that multi-period contract is not effective to improve

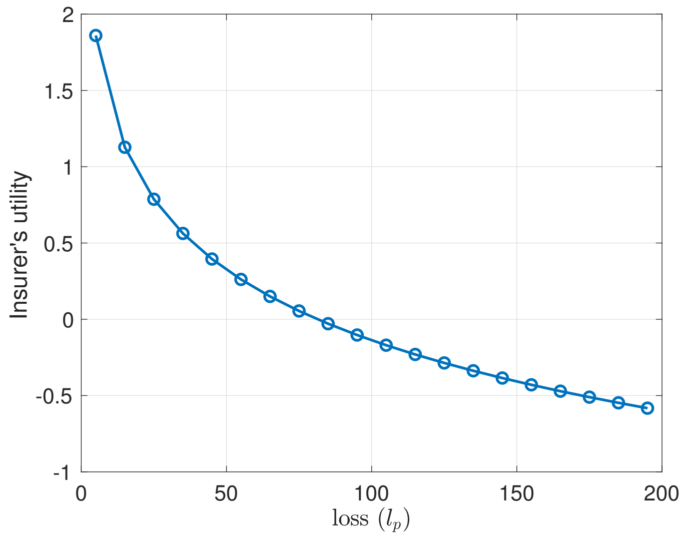


Figure 4.3: Insurer's utility v.s. loss (l_p)

the agent's effort and the agent exerts zero effort inside a contract with full coverage. Then, we proposed pre-screening which allows insurer to predict the agent's state of security and premium discriminates properly. We showed that premium discrimination by the use of pre-screening is an effective method to improve agents' efforts in the presence of rare loss incidents.

To continue this work, we are interested in generalizing our results, specially evaluating the multi-period/infinite-time horizon contract (more than two periods).

CHAPTER 5

Resource Pooling for Shared Fate: Incentivizing Effort in Interdependent Security Games through Cross-investments

5.1 Introduction

In Chapter 2 and Chapter 3, we studied interdependent cyber risks and showed how an insurer can control risk dependency through cyber insurance contracts. In this chapter, we take a step back and focus on the under-investment issue by considering other incentive mechanisms for risk-dependent agents. Specifically, in a network of interdependent entities, the attempt toward improving the state of security by an agent provides positive externality for other entities as the probability of attack propagation from protected entities reduces significantly. Decision making in such a scenario has often been modeled as an interdependent security (IDS) game [53]. The free-riding issue (i.e., the under-investment) arises when an entity tries to take advantage of others' efforts by under-investing in security. As a result, the Nash equilibrium (NE) in IDS games is inefficient, and individuals' investment in security is below the optimum [85].

To address the free-riding issue, various incentive mechanism has been proposed [24, 32, 62]. In particular, Naghizadeh and Liu [62] propose the Pivotal (VCG) and Externality mechanisms (both are in the form of a taxation/subsidy mechanism) to induce socially optimal outcome in IDS games. This type of mechanism design turns out to be more challenging than in other resource allocation contexts because security is a *non-excludable* public good and individuals continue to benefit from others' effort even if they unilaterally opt out of the mechanism. Indeed, whether one can find a taxation mechanism that can simultaneously achieve social optimality and satisfy both weak budget balance and voluntary participation constraints depends on the choice of the utility function. They show that this is in general impossible for a strictly concave utility function. We provide an example in Appendix D which shows that their result can be extended to the quadratic utility model considered in this chapter.

In contrast to the existing literature, in this chapter we are interested in finding a mechanism which improves social welfare and security investment without the existence of a social planner. In order to do so, absent of such a central entity, we instead model the presence of resource pooling (RP) by allowing agents to have the ability to both invest in themselves as well as in other agents, so that they can choose to not only improve their own but also others' state of security. This modeling choice leads to a different IDS game, referred to as the RP-augmented IDS game, or simply RP-IDS game. In addition, we will focus on a quadratic utility function, which has been commonly used in the literature, see e.g., [12, 13, 16, 66, 68].

In practice, exerting efforts on other agents' behalf has context dependent interpretations, such as providing product/service discounts to customers by a service provider, as well as funding open source development. Note that both IDS game and RP-IDS are non-cooperative games where agents selfishly choose their action to maximize their own utility. Our model is different from the existing literature considering a cooperative game [82–84] where the players form coalitions and choose an action to maximize the utility of the coalition that they belong to. A cooperative game is able to improve network security as compared to a non-cooperative scenario if the cost of forming coalition is low enough, but forming coalition is not always possible due to cultural, economical, or social reasons [75].

We study the IDS game with a weighted total effort and quadratic cost model under two scenarios: (i) no RP (the original IDS game), where each agent exerts effort only to improve his own security; and (ii) with RP (RP-IDS), where selfish agents pool their resources. We then compare these two scenarios to understand the effect of resource pooling on agents' utilities, agents' efforts, and social welfare.

5.1.1 Main findings

Our main findings are summarized as follows.

1. Both games have a unique NE. At the NE of the RP-IDS game, every agent obtains higher utility as compared to that under the NE of the IDS game.

2. The social welfare (measured by total utility) at the NE of the RP-IDS game is higher than that under the *socially optimal* outcome of the IDS game, induced by mechanisms such as VCG and externality mechanisms [62]. In other words, as a mechanism, RP outperforms these tax-based mechanisms.

3. While the VCG and externality mechanisms cannot guarantee voluntary participation while imposing budget balance [62], we show that in the RP-IDS game no agent will unilaterally opt out of resource pooling (while continues to be part of the IDS game), thereby ensuring voluntary participation.

4. We further consider community-based resource pooling, where each agent is able to pool his resources within the community that he belongs to. We show that community based resource pooling is able to improve the social welfare and agents' utilities.

5.1.2 Chapter organization

In the remainder of this chapter, Section 5.2 reviews the related work to this chapter. Then, we present the IDS game model without RP, and the RP-IDS game model, and their associated analysis, in Section 5.3 and 5.4, respectively. Section 5.5 discusses the best response dynamics for both game IDS and RP-IDS game. We study the voluntary participation property of RP-IDS game in Section 5.6, and present the community-based resource pooling in Section 5.7. A number of discussions are given in Sec. 5.8. Sec. 5.9 concludes the chapter.

5.2 Related literature

5.2.1 Distributed mechanism design

Distributed mechanism framework has been proposed to induce socially optimal outcome in a distributed manner, i.e., message transmission is performed locally, and mechanism/tax functions depend on messages from neighboring agents [27, 28, 80]. Even though distributed mechanisms are viable options to implement socially optimal outcome without a central planner, they cannot be used in IDS games because they are in the form of taxation mechanism and not able to satisfy the notion of voluntary participation [62]. Moreover, they rely on message passing among agents and result in communication overhead.

5.2.2 IDS games

Outside the incentives context, IDS games have been extensively studied in the literature [4, 22, 23, 30, 34, 51, 60]; we reference some of the more relevant ones below. Miura-Ko *et al.* [60] consider a linear influence network and find a condition on the dependence matrix to guarantee the existence and uniqueness of the NE. Hota and Sundaram in [30] consider IDS games under behavioral probability weighting and show that security risk can be reduced by such weighting strategies. Jiang *et al.* in [34] show that the price of anarchy in an IDS game can increase with the network size regardless of security technology improvement, while a repeated security game can decrease the price of anarchy and make the resulting NE more efficient. Amin *et al.* [4] show that the under-investment issue similarly exists in a two-stage game model. La in [51] examines the relationship between risk exposure and agents' degrees in the dependence graph. Finally, the effect of network structure on the existence and uniqueness of an NE has been studied in the more general context of network games, of which IDS games are a special case, see e.g., [12, 63, 70].

5.3 Interdependent security game without resource pooling (IDS)

Consider n agents on a directed, weighted graph denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, X)$, where $\mathcal{V} = \{1, 2, \cdots, n\}$ is the set of n agents, $\mathcal{E} \subseteq \{(i,j) | i, j \in \mathcal{V}\}$ the set of edges between them, and $X = [x_{ij}]_{n\times n}$ the adjacency weight matrix of this graph, where $x_{ij} > 0, i \neq j$, $(i,j) \in \mathcal{E}$ is the edge weight, $x_{ij} = 0, (i,j) \notin \mathcal{E}$, and $x_{ii} = 0, i \in \mathcal{V}$. An edge $(i,j) \in \mathcal{E}$ indicates that agent i *depends on* agent j (or agent j *influences* i) with the degree of dependence given by edge weight x_{ij}. Dependence need not be symmetrical, i.e., $x_{ij} \neq x_{ji}$ in general. Agent i exerts effort $e_i \geq 0$ towards securing himself, incurring cost $b_i \cdot e_i^2$ ($b_i > 0$ a constant). Given effort profile $\boldsymbol{e} = [e_1, e_2, \cdots, e_n]^T$, agent i has utility

$$u_i(e_i, e_{-i}) = -l_i + a_i \cdot e_i + e_i \cdot \left(\sum_{j=1}^{n} x_{ij} e_j \right) - b_i \cdot e_i^2, \tag{5.1}$$

where e_{-i} denotes all elements in $\boldsymbol{e}$ excluding e_i, $-l_i$ a nominal loss agent i suffers without any effort, $a_i \cdot e_i$, $a_i \geq 0$, the benefit it derives from effort e_i, and $e_i \cdot x_{ij} \cdot e_j$ the benefit it derives from other agents' efforts. This last term indicates a case of positive externality between agents i and j;

see e.g., [22] for IDS games with negative externalities. Second and third terms together in (5.1) imply that with zero effort, agent i cannot benefit from other agents' efforts, i.e., it cannot solely rely on the others. This is a form of the quadratic utility function widely used in the literature of network games [12, 16] and IDS games [13, 66, 68]; it provides a second-order approximation to higher order concave utility functions while preserving the properties of them [68].

The interaction of agents induces a game, denoted as $G = \{\mathcal{V}, \{u_i(.)\}_{i\in\mathcal{V}}, A = [0,+\infty)^n\}$, where A is the action space. In the rest of the chapter, we shall use the terms *exerted effort*, *actions* and *security investments* interchangeably. For convenience of notation, when comparing two games given by the same $\mathcal{V}, \mathcal{E}$ but different weight matrices X_1 and X_2, we will denote the resulting games as $G(X_1)$ and $G(X_2)$, respectively. Next we analyze the equilibrium of game G.

5.3.1 Equilibrium Analysis

Let $Br_i(e_{-i})$ denote the best response function of agent i. Using the first order condition we have

$$\begin{aligned} Br_i(e_{-i}) &= \arg\max_{e\geq 0} \ u_i(e, e_{-i}) \\ &= \max\left\{\frac{a_i}{2b_i} + \frac{1}{2b_i}\sum_{j=1}^{n} x_{ij}e_j,\ 0\right\} \\ &= \frac{a_i}{2b_i} + \frac{1}{2b_i}\sum_{j=1}^{n} x_{ij}e_j\ . \end{aligned} \tag{5.2}$$

We will primarily focus on pure strategy Nash equilibrium (NE), and for simplicity of expositions for the rest of the chapter Nash equilibrium refers to a pure strategy NE. An NE is the fixed point of the best response mapping. Let $\hat{\boldsymbol{e}}$ denote the agents' effort at the NE of game G; then $\hat{\boldsymbol{e}}$ satisfies the following equations:

$$\begin{aligned} 2b_i\hat{e}_i - \sum_{j=1}^{n} x_{ij}\hat{e}_j &= a_i, \ \ i = 1, 2\cdots, n \\ \text{or} \ \ (2\cdot B - X)\cdot\hat{\boldsymbol{e}} &= \boldsymbol{a}, \end{aligned} \tag{5.3}$$

where B is a matrix with b_i's on its main diagonal and zeros everywhere else, and $\boldsymbol{a} =$

$[a_1, a_2, \cdots, a_n]^T$.

We make the following assumption on cost b_i to ensure that the effort levels are bounded at the NE. More discussion on this assumption is provided in Section 5.8.1.

Assumption 5.1 $2b_i > \sum_{j=1}^n x_{ij}, \ \forall i \in \mathcal{V}$.

Under Assumption 5.1, we have the following lemma on the best response mapping and the NE of game G.

Theorem 5.1 *Under Assumption 5.1, matrix $(2B - X)$ is invertible and $\hat{\boldsymbol{e}} = (2 \cdot B - X)^{-1} \cdot \boldsymbol{a}$ is the unique NE of game G.*

It is worth mentioning that under assumption 5.1, $\frac{1}{2}B^{-1}X$ is sub-stochastic, and $(2 \cdot B - X)^{-1}$ can be written as follows:

$$(2 \cdot B - X)^{-1} = \frac{1}{2} \cdot B^{-1}(I - \frac{1}{2}B^{-1}X)^{-1} = \frac{1}{2} \cdot B^{-1} \sum_{i=0}^{\infty} (\frac{1}{2}B^{-1}X)^i. \tag{5.4}$$

As all entries of $B^{-1}X$ are non-negative, $(2 \cdot B - X)^{-1}$ is a non-negative matrix too, and $\hat{\boldsymbol{e}} = (2 \cdot B - X)^{-1} \cdot \boldsymbol{a}$ is a non-negative vector.

Theorem 5.1 and the fact that $(2B - X)^{-1}$ is a non-negative matrix lead to the following corollary.

Corollary 5.1 *Let X and $\tilde{X}$ be two adjacency matrices over the same $\mathcal{V}$ and $\mathcal{E}$. Consider the games $G(X)$ and $G(X + \tilde{X})$, and their respective NE $\hat{\boldsymbol{e}}$ and $\tilde{\boldsymbol{e}}$. If $2b_i > \sum_{j=1}^n \left[x_{ij} + \tilde{x}_{ij}\right]$, then $\tilde{\boldsymbol{e}} \geq \hat{\boldsymbol{e}}$.[1] In other words, agents exert higher effort at the NE given stronger externality.*

5.3.2 Socially optimal outcome

We now consider the socially optimal effort levels for the IDS game. Denote by $\boldsymbol{e}^* = [e_1^*, e_2^*, \cdots, e_n^*]$, the socially optimal effort profile maximizes the total utility:

$$\boldsymbol{e}^* \in \arg\max_{\boldsymbol{e} \in A} \sum_{i=1}^{n} u_i(e_i, e_{-i}) \, . \tag{5.5}$$

[1] $\boldsymbol{v} = [v_1 \cdots v_n]^T \geq \boldsymbol{\theta} = [\theta_1 \cdots \theta_n]^T$ means that $v_i \geq \theta_i, \ \forall i$.

To ensure the existence of a socially optimal strategy, we make the following assumption (see Section 5.8.1 for more discussion).

Assumption 5.2 $2b_i > \sum_{j=1}^{n} \left[x_{ij} + x_{ji}\right], \ \forall i \in \mathcal{V}.$

Theorem 5.2 *Let $\hat{\boldsymbol{e}}$ be the effort level at the NE of game G and $\boldsymbol{e}^*$ be the socially optimal effort level. Then under Assumption 5.2 we have:*

1. $\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$;

2. $e_i^* \geq \hat{e}_i, \ \forall i.$

That is, every agent exerts higher effort at the socially optimal solution compared to the NE.

Remark: The above shows that the socially optimal effort profile of game $G(X)$, given by $\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$, also happens to be the NE of game $G(X + X^T)$. Also note that for game $G(X)$, while the total utility under $\boldsymbol{e}^*$ is higher than that under the NE $\hat{\boldsymbol{e}}$, this may or may not be true for agents' individual utility, as the following example shows.

Example 5.1 *Consider the following IDS game:*

$$\begin{aligned} n &= 2, \ b_1 = b_2 = 1, \ a_1 = a_2 = 1 \\ x_{12} &= 0.1, \ x_{21} = 0.9, \ l_1 = l_2 = 1 \\ \hat{\boldsymbol{e}} &= (2B - X)^{-1} \cdot \boldsymbol{a} = [0.5371 \ 0.7417]^T \\ u_1(\hat{\boldsymbol{e}}) &= -0.7115, \ u_2(\hat{\boldsymbol{e}}) = -0.4499 \\ \boldsymbol{e}^* &= (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = [1 \ 1]^T \\ u_1(\boldsymbol{e}^*) &= -0.9000, \ u_2(\boldsymbol{e}^*) = -0.1000 \end{aligned} \tag{5.6}$$

In this example, agent 1 has higher influence on agent 2 ($x_{21} > x_{12}$); agent 2 benefits from socially optimal effort ($u_2(\boldsymbol{e}^*) > u_2(\hat{\boldsymbol{e}})$), while agent 1's utility worsens even though it exerts higher effort under $\boldsymbol{e}^*$.

Example 5.2 *Consider the following IDS game where both agents benefit from the socially optimal outcome:*

$$
\begin{aligned}
n &= 2,\ b_1 = b_2 = 1,\ a_1 = a_2 = 1 \\
x_{12} &= x_{21} = 0.5,\ l_1 = l_2 = 1 \\
\hat{\boldsymbol{e}} &= (2B - X)^{-1} \cdot \boldsymbol{a} = [\frac{2}{3},\ \frac{2}{3}]^T \\
u_1(\hat{\boldsymbol{e}}) &= u_2(\hat{\boldsymbol{e}}) = -\frac{5}{9} = -0.5555 \\
\boldsymbol{e}^* &= (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = [1\ 1]^T \\
u_1(\boldsymbol{e}^*) &= u_2(\boldsymbol{e}^*) = -0.5
\end{aligned} \tag{5.7}
$$

These examples show that socially optimal outcome is not necessarily desirable to all agents. Mechanism design in the context of IDS games aims to incentivize agents to exert higher effort than that under the NE. In the next section, we will examine the impact of introducing resource pooling as a mechanism to improve agents' effort and social welfare.

5.4 Interdependent security game with resource pooling (RP-IDS)

Consider the same IDS game setting. Let $\boldsymbol{e}_i = [e_{i1}, e_{i2}, \cdots, e_{in}]^T$ be the action of agent i where $e_{ij} \geq 0$ denotes the effort exerted by agent i on behalf of agent j. Moreover, agent i incurs cost $b_j \cdot e_{ij}^2$ by exerting effort e_{ij} on behalf of agent j. Let $E = [\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_n]^T$ be an $n \times n$ matrix that denotes the effort profile, and let $E_i = \sum_{j=1}^{n} e_{ji}$ denote the total effort exerted on behalf of agent i. Agent i's utility given profile E is:

$$
\begin{aligned}
v_i(\boldsymbol{e}_i, \boldsymbol{e}_{-i}) &= -l_i + a_i(\sum_{j=1}^{n} e_{ji}) - \sum_{k=1}^{n} b_k \cdot e_{ik}^2 \\
&+ (\sum_{j=1}^{n} e_{ji}) \cdot \left(\sum_{k=1}^{n} x_{ik} \cdot (\sum_{r=1}^{n} e_{rk}) \right) \\
&= -l_i + a_i E_i + E_i \cdot \sum_{j=1}^{n} x_{ij} E_j - \sum_{k=1}^{n} b_k \cdot e_{ik}^2.
\end{aligned}
$$

The interaction of agents induces the RP-IDS game $G_{rp} = \left\{\mathcal{V}, \{v_i\}_{i\in\mathcal{V}}, A_{rp} = [0,+\infty)^{n^2}\right\}$, where A_{rp} is the action space under resource pooling. By the first order condition, the best response function of agent i satisfies the following:

$$\begin{aligned}
\boldsymbol{e}_i &= Br_i(\boldsymbol{e}_{-i}) \\
e_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^{n} x_{ik}\cdot E_k}{2b_i} \\
e_{ij} &= \frac{x_{ij}\cdot E_i}{2b_j},\ \forall j \neq i
\end{aligned} \tag{5.8}$$

Let $\hat{E} = [\hat{e}_{ij}]_{n\times n}$ be the NE of game G_{rp} and $\hat{E}_i = \sum_{j=1}^{n} \hat{e}_{ji}$ the total effort exerted on behalf of agent i at the NE. We have the following lemma on effort profile $\hat{E}$.

Lemma 5.1 *Assume that game G_{rp} has at least one Nash equilibrium. The effort profile $\hat{E}$ at the NE satisfies the following system of equations,*

$$(2B - X - X^T)\cdot\begin{bmatrix}\hat{E}_1\\ \vdots \\ \hat{E}_n\end{bmatrix} = \boldsymbol{a}\ .$$

Proof *As $\hat{E}$ is the fixed point of the best response mapping, we have,*

$$\begin{aligned}
\hat{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^{n} x_{ik}\cdot \hat{E}_k}{2b_i} \\
\hat{e}_{ji} &= \frac{x_{ji}\cdot \hat{E}_j}{2b_i}\ \forall j \neq i \implies \\
&\textit{by adding above equations:} \\
2b_i\cdot\hat{E}_i &= a_i + \sum_{j=1}^{n}(x_{ij}+x_{ji})\hat{E}_j\ \ \forall i\in\mathcal{V} \\
\implies \boldsymbol{a} &= (2B - X - X^T)\cdot\begin{bmatrix}\hat{E}_1\\ \vdots \\ \hat{E}_n\end{bmatrix}
\end{aligned} \tag{5.9}$$

Theorem 5.3 *Under Assumption 5.2, $(2B - X - X^T)$ is invertible and game G_{rp} has a unique NE* *given as follows:*

$$\begin{bmatrix} \hat{E}_1 \\ \vdots \\ \hat{E}_n \end{bmatrix} = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$$

$$\hat{e}_{ii} = \frac{a_i}{2b_i} + \frac{\sum_{k=1}^{n} x_{ik} \cdot \hat{E}_k}{2b_i}$$

$$\hat{e}_{ij} = \frac{x_{ij} \cdot \hat{E}_i}{2b_j}, \ \forall j \neq i \tag{5.10}$$

Proof *Similar to the proof of Theorem 5.1, we can show that if* $2b_i > \sum_{j=1}^{n} x_{ij} + x_{ji}, \forall i$*, then all eigenvalues of matrix* $(2B - X - X^T)$ *are non-zero. Therefore, matrix* $(2B - X - X^T)$ *is invertible. Similar to* (5.4)*, we can show that all entries of* $(2B - X - X^T)^{-1}$ *are non-negative and* $\left[\hat{E}_1 \cdots \hat{E}_n\right]^T = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$ *is a non-negative vector. Moreover, by best response mapping provided in* (5.8)*, we know that* $\hat{e}_{ij}$ *can be calculated by the following,*

$$\hat{e}_{ii} = \frac{a_i}{2b_i} + \frac{\sum_{k=1}^{n} x_{ik} \cdot \hat{E}_k}{2b_i} \geq 0$$

$$\hat{e}_{ij} = \frac{x_{ij} \cdot \hat{E}_i}{2b_j} \geq 0, \ \forall j \neq i \tag{5.11}$$

Therefore, the fixed point of the best response mapping is non-negative and unique, implying the NE of game G_{rp} *is unique and can be found by* (5.10)*.*

Remark: It is worth pointing out that for the same weight matrix X, the *total* effort exerted by each agent, $[\hat{E}_1, \hat{E}_2, \cdots, \hat{E}_n]$, at the NE of the RP-IDS game G_{rp} is the same as the socially optimal effort of the IDS game G. That is,

$$\begin{bmatrix} \hat{E}_1 \\ \vdots \\ \hat{E}_n \end{bmatrix} = (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = \boldsymbol{e}^* \underbrace{\geq}_{\text{By Theorem 5.2}} \hat{\boldsymbol{e}} \,. \tag{5.12}$$

In other words, the introduction of resource pooling incentivizes agents to boost their effort to the socially optimal levels for game G. Note that the game G_{rp} has its own socially optimal solution as we discuss in Section 5.8.2.

Next we show that every agent at the NE of game G_{rp} obtains a higher utility than that attained at the NE of game G, i.e., resource pooling improves the utility for all agents.

Theorem 5.4 *Let $\hat{E} = [\hat{e}_{ij}]_{n\times n}$ be the NE of G_{rp} and $\hat{\boldsymbol{e}}$ be the effort profile at the NE of game G. Under Assumption 5.2, we have:*

$$v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}}), \quad \forall i \in \mathcal{V} \ . \tag{5.13}$$

Proof *Let $\tilde{\boldsymbol{e}}_i$ be a vector with length n and all its elements are zero except entry i which is equal to $\hat{e}_i$ (effort level of agent i at NE of game G). By definition of Nash equilibrium we have,*

$$v_i(\hat{E}) \geq v_i(\tilde{\boldsymbol{e}}_i, \hat{\boldsymbol{e}}_{-i}). \tag{5.14}$$

As $\hat{E}_i \geq \hat{e}_i$, $\forall i$, by (5.10) *and* (5.3) *we have $\hat{e}_{ii} \geq \hat{e}_i$. Moreover,*

$$\begin{aligned} v_i(\tilde{\boldsymbol{e}}_i, \hat{\boldsymbol{e}}_{-i}) = -l_i + a_i \cdot \hat{e}_i + a_i \sum_{k\neq i} \hat{e}_{ki} - b_i \cdot (\hat{e}_i)^2 \\ +(\hat{e}_i + \sum_{k\neq i} \hat{e}_{ki}) \cdot \sum_{j=1}^{n} \left(x_{ij} \cdot (\sum_{k\neq i} \hat{e}_{kj}) \right) \geq \\ -l_i + a_i \cdot \hat{e}_i - b_i \cdot (\hat{e}_i)^2 + \hat{e}_i \cdot \sum_{j=1}^{n} x_{ij} \cdot \hat{e}_j = u_i(\hat{e}_i, \hat{e}_{-i}) \end{aligned} \tag{5.15}$$

By (C.14) *and* (C.15)*, $v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}})$ $\forall i \in V$.*

The following theorem shows that social welfare at the NE of game G_{rp} is higher than the maximum social welfare of game G, even though the total effort exerted by each agent is the same under both as noted earlier.

Theorem 5.5 *Let $\hat{E}$ be the effort profile at the NE of game G_{rp} and $\boldsymbol{e}^*$ be the socially optimal effort profile in game G. Under Assumption 5.2 we have,*

$$\sum_{i=1}^{n} v_i(\hat{E}) \geq \sum_{i=1}^{n} u_i(\boldsymbol{e}^*) \ .$$

Proof

$$\sum_{i=1}^{n} v_i(\hat{E}) = \sum_{i=1}^{n} \left(-l_i + a_i \hat{E}_i - b_i \cdot \left[\sum_{j=1}^{n} \hat{e}_{ji}^2 \right] + \hat{E}_i \cdot \left[\sum_{j=1}^{n} x_{ij} \cdot \hat{E}_j \right] \right)$$

By (5.12), $(e_i^*)^2 = \hat{E}_i^2 = (\sum_{j=1}^{n} \hat{e}_{ji})^2 \geq \sum_{j=1}^{n} (\hat{e}_{ji})^2$, *and* $\hat{E}_i = e_i^*$. *Therefore,*

$$\begin{aligned} \sum_{i=1}^{n} v_i(\hat{E}) &\geq \sum_{i=1}^{n} \left(-l_i + a_i \hat{E}_i - b_i \cdot \hat{E}_i^2 + \hat{E}_i \cdot \left[\sum_{j=1}^{n} x_{ij} \cdot \hat{E}_j \right] \right) \\ &= \sum_{i=1}^{n} u_i(\boldsymbol{e}^*). \end{aligned}$$

We conclude this section by highlighting the role of resource pooling in the IDS game.

- At the NE, with resource pooling (game G_{rp}) agents exert higher effort (for themselves and on others) and experience higher utility than without (game G); e.g., $\hat{E}_i \geq \hat{e}_i$, and $v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}})$.
- Resource pooling induces agents to exert socially optimal levels of effort (under game G), while improving the social welfare as it allows more judicious spreading of efforts; e.g., $\hat{E} = \boldsymbol{e}^*$ and $\sum_{i=1}^{n} v_i(\hat{E}) \geq \sum_{i=1}^{n} u_i(\boldsymbol{e}^*)$.

5.5 Best response dynamics for IDS game (game G) and RP-IDS game (game G_{rp})

Based on Theorem 5.1 and 5.3, we have to calculate an inverse of a matrix to find the Nash equilibrium of game G and G_{rp}. In this section, we develop an iterative best response dynamic which converges to the Nash equilibrium without calculating an inverse of a matrix.

5.5.1 Best response dynamics for IDS game (game G)

As game G is a game with quadratic utility functions, the best response dynamic shown in Algorithm 1 converges to the Nash equilibrium. In the next theorem we prove the convergence of Algorithm 1.

Theorem 5.6 *Under assumption 5.1, Algorithm 1 converges to $\hat{\boldsymbol{e}}$, the Nash equilibrium of game G.*

Proof *Let $\hat{\boldsymbol{e}}^{(t)} = [e_1^{(t)}, e_2^{(t)}, \cdots, e_n^{(t)}]^T$. First we show that $e_k^{(1)} \leq e_k^{(0)}, \forall k$.*

$$
\begin{aligned}
& e_k^{(1)} - e_k^{(0)} = \\
& \frac{a_k}{2b_k} - \left(\max_i \frac{a_i}{2b_i - \sum_{j=1}^n x_{ij}} \right) \frac{2b_k - \sum_{j=1}^n x_{kj}}{a_k} \frac{a_k}{2b_k} \\
& \leq \frac{a_k}{2b_k} - \frac{a_k}{2b_k} = 0
\end{aligned} \tag{5.16}
$$

Next we show that if $\hat{\boldsymbol{e}}^{(t)} \leq \hat{\boldsymbol{e}}^{(t-1)}$, then $\hat{\boldsymbol{e}}^{(t+1)} \leq \hat{\boldsymbol{e}}^{(t)}$.

$$
\begin{aligned}
\hat{\boldsymbol{e}}^{(t)} &\leq \hat{\boldsymbol{e}}^{(t-1)} \\
e_k^{(t+1)} &= \frac{a_k}{2b_k} + \frac{1}{2b_k} \sum_{j=1}^n x_{kj} e_j^{(t)} \\
&\leq \frac{a_k}{2b_k} + \frac{1}{2b_k} \sum_{j=1}^n x_{kj} e_j^{(t-1)} = e_k^{(t)}
\end{aligned} \tag{5.17}
$$

By equations (5.16) *and* (5.17)*, we conclude that $\hat{\boldsymbol{e}}^{(t)}$ is a decreasing sequence. Moreover, $\hat{\boldsymbol{e}}^{(t)}$ is a positive sequence and bounded below. Therefore, $\hat{\boldsymbol{e}}^{(t)}$ converges. Notice that the convergence point has to be the fixed point of the best response mapping and has to be the Nash equilibrium.*

Algorithm 1 Finding Nash equilibrium for game G using best response dynamics

---

Initialization: set $e_k^{(0)} = \max_i \frac{a_i}{2b_i - \sum_{j=1}^n x_{ij}}$, $\forall\ k \in \mathcal{V}$

for t= 1,2,$\cdots$, $\mathcal{T}$ **do**

  $e_i^{(t)} = \frac{a_i}{2b_i} + \frac{1}{2b_i} \sum_{j=1}^n x_{ij} e_j^{(t-1)}, \forall\ i \in \mathcal{V}$

end

Output: $[e_1^{(\mathcal{T})}, e_2^{(\mathcal{T})}, \cdots, e_n^{(\mathcal{T})}]$

5.5.2 Modified best response dynamics for RP-IDS game (game G_{rp})

Best response dynamics for Game G_{rp} can be computationally expensive. At each iteration, we have to calculate the best response function given by a system of linear equations defined in (5.8). In order to avoid solving a system of equation for each agent every iteration, we try to find the total effort exerted on behalf of each agent iteratively. Algorithm 2 is the modified best response dynamics for game G_{rp}.

Theorem 5.7 *Under assumption 5.2, the output of algorithm 2 is the effort profile of the agents at the NE of game G_{rp}.*

Proof *Based on Theorem 5.6, we know that $E_i^{(t)}, t = 0, 1, \cdots$, defined in Algorithm (2), converges to the effort of agent i at the Nash equilibrium of game $G(X + X^T)$. Moreover, we know that the total effort exerted on behalf of agent i at the NE of game G_{rp} is equal to the effort of agent i at the Nash equilibrium of game $G(X + X^T)$. Therefore, $E^{(t)_i}, t = 0, 1, \cdots$ converges to the total effort exerted on behalf of agent i at the NE of game G_{rp}. By theorem 5.3, the output of Algorithm 2 would be the equilibrium of game G_{rp}.*

Algorithm 2 Finding Nash equilibrium for game G_{rp} using modified best response dynamics

---

Initialization: set $E_k^{(0)} = \max_i \frac{a_i}{2b_i - \sum_{j=1}^n x_{ij} + x_{ji}}$, $\forall\ k \in \mathcal{V}$

for *t= 1,2,$\cdots$, $\mathcal{T}$* **do**

  $E_i^{(t)} = \frac{a_i}{2b_i} + \frac{1}{2b_i} \sum_{j=1}^n (x_{ij} + x_{ji}) E_j^{(t-1)}, \forall\ i \in \mathcal{V}$

end

$\hat{e}_{kk} = \frac{a_k}{2b_k} + \frac{\sum_{j=1}^n x_{kj} \cdot E_j^{(\mathcal{T})}}{2b_k}, \forall k \in \mathcal{V}$

$\hat{e}_{kk'} = \frac{x_{kk'} \cdot E_k^{\mathcal{T}}}{2b'_k}$, $\forall k \neq k'$

Output: $\hat{E} = [\hat{e}_{ij}]_{n \times n}$

5.6 Voluntary participation in RP

As investment in security is a non-excludable public good, an agent can benefit even if it chooses not to participate in an incentive mechanism. As a result, designing a mechanism which incentivizes the agents to voluntarily participate and exert socially optimal effort levels is not straightforward, and it is important to check whether agents will voluntarily participate in resource pooling. In [62] and Appendix D, it was shown that no taxation mechanism is able to implement the socially optimal solution while guaranteeing both weak budget balance and voluntary participation. For this reason, in what follows, we first define this notion and then show that under resource pooling the voluntary participation property is satisfied.

Definition 5.1 (Voluntary Participation (VP)) *Consider game G^k_{rp} where agent k opts out of RP and only invests in himself and nobody else invest in agent k ($e_{kj} = e_{jk} = 0,\ \forall j \neq k$), while other agents participate in RP. Let $\mathring{E} = [\mathring{e}_{ij}]_{n\times n}$ be the NE of game G^k_{rp} and $v_i(\mathring{E})$ be the utility of agent i at the NE. We say that resource pooling has the voluntary participation property with respect to agent k, if*

$$v_k(\mathring{E}) \leq v_k(\hat{E}), \tag{5.18}$$

where $\hat{E}$ is the effort profile at the NE of game G_{rp}.[2] If the above is true for all $k \in \mathcal{V}$, then we say that resource pooling has the voluntary participation property.

The following theorem suggests that resource pooling always satisfies the VP property.

Theorem 5.8 *If Assumption 5.2 holds, then agent i achieves higher utility at the NE of game G_{rp}, than his utility at the NE of game G^i_{rp} for all $i \in \mathcal{V}$. That is, resource pooling always satisfies the VP property.*

It is worth noting that resource pooling is able to satisfy a stronger notion of voluntary participation defined as follows.

Definition 5.2 (Stronger Notion of Voluntary Participation (SVP)) *Consider game $\overline{G}^k_{rp}$ where agent k opts out of RP and only invests in himself ($e_{kj} = 0,\ \forall j \neq k$), while the other agents participate in RP and may choose to invest in agent k. In other words, while agent k chooses not to exert any effort on behalf of other agents, he may receive resources from other agents in game $\overline{G}^k_{rp}$ if it is in the other agents' self interest to do so.*

Let $\acute{E} = [\acute{e}_{ij}]_{n\times n}$ be the NE of game $\overline{G}^k_{rp}$ and $v_i(\acute{E})$ be the utility of agent i at the NE. We say that resource pooling has the strong voluntary participation property with respect to agent k, if

$$v_k(\acute{E}) \leq v_k(\hat{E}), \tag{5.19}$$

[2] Under Assumption 5.2, both G_{rp} and G^k_{rp} have an NE.

where $\hat{E}$ is the effort profile at the NE of game G_{rp}. If the above is true for all $k \in \mathcal{V}$, then we say that resource pooling has the strong voluntary participation property.

It is worth noting a crucial difference between the definition of an NE and the VP property. An NE in game G_{rp} implies that $v_k(\hat{\boldsymbol{e}}_k, \hat{\boldsymbol{e}}_{-k}) \geq v_k(\boldsymbol{e}_k, \hat{\boldsymbol{e}}_{-k})$, $\forall \boldsymbol{e}_k \in R^n$. In words, this definition says that at the NE, agent k cannot improve his utility by changing his action while other agents do not change their effort and keep the same action. On the other hand, Equations (5.18) and (5.19) imply that agent k is not able to improve his utility if he chooses not to pool his resources and the other agents best respond to his decision and choose their actions accordingly.

The following theorem shows that resource pooling is able to satisfy the SVP property defined in Definition 5.2.

Theorem 5.9 *If Assumption 5.2 holds, resource pooling always satisfies the SVP property defined in Definition 5.2.*

5.7 Community based resource pooling

So far we have assumed that each agent can pool his resources with all other agents in the network. We next consider a more realistic setting where each agent is able to pool resources within a community that he belongs to. Specifically, we assume that agents form m disjoint communities and they are allowed to pool their resources within the communities they belong to. Let $C_1, \cdots, C_m$ denote the m communities, where $\cup_{k=1}^{m} C_k = \mathcal{V}$ and $C_k \cap C_{k'} = \emptyset, \forall k, k'$. Moreover, let $I(i)$ be the index of the community that agent i belongs to, i.e., $i \in C_{I(i)}$. Let G_{rp}^c denote the game induced by the interaction of the agents who are allowed to pool their resource within their communities. Let $\boldsymbol{e}_i = [e_{ij}], j \in C_{I(i)}$ be the action of agent i and $E_i = \sum_{j \in C_{I(i)}} e_{ji}$ be the total effort exerted on behalf of agent i in game G_{rp}^c. The utility of agent i is given by:

$$\begin{aligned} v_i(\boldsymbol{e}_i, \boldsymbol{e}_{-i}) &= -l_i + a_i E_i + E_i \left(\sum_{j=1}^{n} x_{ij} E_j \right) \\ &- \sum_{j \in C_{I(i)}} b_j e_{ij}^2 \, . \end{aligned} \tag{5.20}$$

Let $\check{\boldsymbol{e}}_i = [\check{e}_{ij}]$, $j \in C_{I(i)}$ be the effort profile of agent i at the NE of game G_{rp}^c and $\check{E}_i = \sum_{j \in C_{I(i)}} \check{e}_{ji}$. We have the following result.

Theorem 5.10 *Under Assumption 5.2, game G_{rp}^c has a unique Nash equilibrium and the effort profile of agents at the equilibrium is given by:*

$$\begin{aligned}
\begin{bmatrix} \check{E}_1 \\ \vdots \\ \check{E}_n \end{bmatrix} &= (2B - X - X_c^T)^{-1}\boldsymbol{a}, \\
\check{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{j=1}^n x_{ij}\check{E}_j}{2b_i}, \\
\check{e}_{ij} &= \frac{x_{ij} \cdot \check{E}_i}{2b_j}, \forall j \neq i, j \in C_{I(i)} \qquad (5.21)
\end{aligned}$$

where entry (i, j) of matrix X_c is equal to x_{ij} if $j \in C_{I(i)}$, otherwise it is zero.

Thus both games G and G_{rp}^c have a unique Nash equilibrium under Assumption 5.2. The next theorem compares the NE of games G and G_{rp}^c.

Theorem 5.11 *Under Assumption 5.2 we have the following results.*

- *Community based resource pooling improves the effort and utility of each agent as compared to those at the NE of game G. That is,*

$$\begin{aligned}
\check{E}_i &\geq \hat{e}_i, \forall i, \\
v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}) &\geq u_i(\hat{e}_i, \hat{e}_{-i}), \forall i.
\end{aligned}$$

- *$\check{E}_i \leq e_i^*, \forall i$. That is, the total effort exerted on behalf of each agent is less than the socially optimal effort level in game G.*

Next theorem characterizes the effect of merging communities on agents' efforts and utilities.

Theorem 5.12 *Consider game $\overline{G}_{rp}^c$ a community based resource pooling game with the following communities:*

$$C_1, C_2, \cdots, C_{m-2}, C_{m-1} \cup C_m$$

Moreover, let $\check{\overline{\boldsymbol{e}}}_i$ be the strategy of agent i at the NE of game $\overline{G}_{rp}^c$. Moreover, let $\overline{I}(i)$ be the index of the community that agent i belongs to in game $\overline{G}_{rp}^c$.

We have,

$$\begin{aligned} \bar{\check{E}}_i &\geq \check{E}_i, \forall i, \\ v_i(\bar{\check{\boldsymbol{e}}}_i, \bar{\check{\boldsymbol{e}}}_{-i}) &\geq v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}), \forall i, \end{aligned}$$

where, $\bar{\check{E}}_i = \sum_{j \in C_{\bar{I}(i)}} \bar{\check{e}}_{ji}$*. In other words, merging two communities improves agents' utilities as well as agents' efforts.*

While the social welfare at the NE of game G^c_{rp} is higher than that at the NE of game G, it may or may not be higher than the maximum social welfare of game G. Next we provide a numerical example to highlight the impact of community based resource pooling.

Consider a network with $n = 10$ agents and the following parameters:

$$\begin{aligned} a_i &= 1, \forall i, \;\; b_i = 2, \forall i \\ x_{ij} &= \begin{cases} 1 & \text{if } j = i+1 \text{ and } i \text{ is odd.} \\ 1 & \text{if } j = i-1 \text{ and } i \text{ is even.} \\ 0 & \text{if } i = j \\ 0.1 & \text{o.w.} \end{cases} \end{aligned}$$

Without loss of generality, we will set $l_i = 0, \forall i$; as l_i is a constant, this will not affect agents' decision. Given this set of parameters, we divide the agents to $m, m = 1, \cdots, 10$ communities using spectral clustering method [86] as follows.

$$\begin{aligned} m = 1,\ C_1 &= \mathcal{V} \\ m = 2,\ C_1 &= \{1,2,3,4,9,10\},\ C_2 = \{5,6,7,8\} \\ m = 3,\ C_1 &= \{1,2,3,4,7,8\},\ C_2 = \{5,6,7,8\}, \\ C_3 &= \{9,10\} \\ m = 4,\ C_1 &= \{1,2\}, C_2 = \{3,4\}, C_3 = \{5,6,9,10\}, \\ C_4 &= \{7,8\} \end{aligned}$$

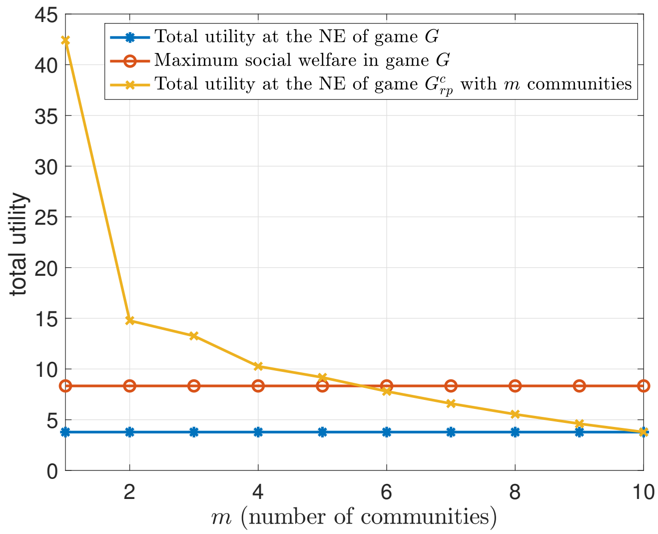


Figure 5.1: The total utility at the NE of game G^c_{rp} with m communities. Resource pooling within fewer and large size communities is more effective in improving social welfare.

$$
\begin{aligned}
m = 5,\ C_1 &= \{1,2\}, C_2 = \{3,4\}, C_3 = \{5,6\}, \\
C_4 &= \{7,8\}, C_5 = \{9,10\} \\
m = 6,\ C_1 &= \{1\}, C_2 = \{2\}, C_3 = \{3,4\}, C_4 = \{5,6\}, \\
C_5 &= \{7,8\}, C_6 = \{9,10\} \\
m = 7,\ C_1 &= \{1,2\}, C_2 = \{3\}, C_3 = \{4\}, \\
C_4 &= \{5,6\}, C_5 = \{7,8\}, C_6 = \{9\}, C_7 = \{10\} \\
m = 8,\ C_1 &= \{1\}, C_2 = \{2\}, C_3 = \{3\}, C_4 = \{4\} \\
C_5 &= \{5,6\}, C_6 = \{7\}, C_7 = \{8\}, C_8 = \{9,10\} \\
m = 9,\ C_1 &= \{1\}, C_2 = \{2\}, C_3 = \{3\}, C_4 = \{4\}, \\
C_5 &= \{5,6\}, C_6 = \{7\}, C_7 = \{8\}, C_8 = \{9\}, \\
C_9 &= \{10\} \\
m = 10,\ C_k &= \{k\},\ \forall k
\end{aligned}
$$

It is easy to see that $m = 1$ corresponds to the case without community as studied earlier in the chapter, whereas $m = 10$ corresponds to the case where resource pooling is not allowed.

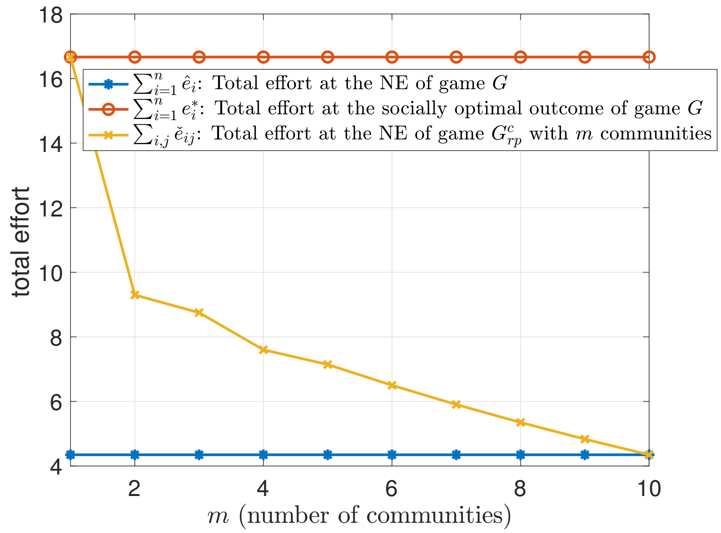


Figure 5.2: The total effort at the NE of game G_{rp}^c with m communities. Resource pooling within fewer and large size communities is more effective in incetivizing agents to invest more in security.

Figure 5.1 illustrates the total utility at the NE using community based resource pooling as the number of communities m increases. These results verifies our theoretical finding that resource pooling even limited within communities always leads to higher total utility. Furthermore, we see that when $m \geq 6$, the total utility at the NE of game G_{rp}^c falls below the maximum social welfare in game G, suggesting that resource pooling is more effective with fewer and larger communities ($m \leq 5$).

Figure 5.2 illustrates the total effort at the NE of G_{rp}^c as a function of the number of communities (m). First, we note that the total effort at the socially optimal outcome of game G is the same as the total effort at game G_{rp} ($m = 1$), as expected. Also, consistent with the previous figure, we see that the total investment decreases as a function of the number of communities m.

5.8 Discussion

5.8.1 On assumption $2b_i > \sum_{j=1}^n x_{ij}$

Throughout the analysis we have used the following assumptions:

- Existence and uniqueness of NE for game G: $2b_i > \sum_{j=1}^{n} x_{ij}, \forall i$
- Existence and uniqueness of socially optimal strategy profile in game G: $2b_i > \sum_{j=1}^{n} x_{ij} + x_{ji}, \forall i$
- Existence and uniqueness of NE profile in game G_{rp}: $2b_i > \sum_{j=1}^{n} x_{ij} + x_{ji}, \forall i$

The reason behind these assumptions is to prevent the model from becoming pathological: if the cost of effort is sufficiently low, then there may not exist NE or socially optimal strategy, and it may be beneficial for the agents to exert very high effort with unbounded utility.

Example 5.3 *Consider a network with* $x_{ii} = 0,\ x_{ij} = \frac{1}{n-1}\ \forall i, j \in V,\ i \neq j$ *and* $b_i = 1$. *Under these parameters Assumption 5.2 does not hold. Moreover, set* $e_i = r,\ \forall i \in V$. *We have:*

$$\sum_{i=1}^{n} u_i(\boldsymbol{e}) = \sum_{i=1}^{n} \left(-l_i + (r)a_i - b_i \cdot r^2 + r^2 \sum_{j=1}^{n} x_{ij} \right)$$

$$= \left(-\sum_{i=1}^{n} l_i \right) + r \cdot \left(\sum_{i=1}^{n} a_i \right),$$

which is a linear function in r and is unbounded. In this case the socially optimal effort does not exist.

5.8.2 On the socially optimal outcome of game G_{rp}

While the NE of the RP-IDS game G_{rp} achieves socially optimal levels of effort defined for the IDS game G, the introduction of resource pooling means that each agent now has a bigger action space, thereby giving rise to a different social optimum for this new game. We next show how this new optimum can be computed.

Let $E^* = [e^*_{ij}]_{n\times n}$ be the socially optimal effort profile for the RP-IDS game:

$$\begin{aligned} E^* &= \arg\max_{E\in R_+^{n\times n}} \sum_{i=1}^{n} v_i(E) \\ &= \arg\max_{E\in R_+^{n\times n}} \sum_{i=1}^{n} \left[-l_i + a_i E_i - b_i (\sum_{j=1}^{n} e_{ji}^2) + E_i \sum_{j=1}^{n} x_{ij} E_j \right]. \end{aligned}$$

The assumption below ensures the existence of a solution.

Assumption 5.3 $2b_i > n \cdot \sum_{j=1}^{n}(x_{ij} + x_{ji}),\ \forall i \in \mathcal{V}$

Under Assumption 5.3, it is easy to check that $g(E) = \sum_{i=1}^{n} v_i(E)$ is strictly concave in E. By the first order condition, E^* satisfies the following:

$$
\begin{aligned}
\frac{\partial g(E)}{\partial e_{ii}}|_{E=E^*} &= a_i - 2b_i e^*_{ii} + \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j = 0 \\
\frac{\partial g(E)}{\partial e_{ki}}|_{E=E^*} &= a_i - 2b_i e^*_{ki} + \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j = 0 \\
&\implies n \cdot a_i - 2b_i E^*_i + n \cdot \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j = 0,\ \forall i \in \mathcal{V}, \\
&\implies (2B - n \cdot (X + X^T)) \cdot \begin{bmatrix} E^*_1 \\ \vdots \\ E^*_n \end{bmatrix} = n \cdot \boldsymbol{a}\ . \qquad (5.22)
\end{aligned}
$$

Similar as before, we can show that under Assumption 5.3, $(2B - n \cdot (X + X^T))$ is invertible. Thus the optimal outcome E^* is given by:

$$
\begin{aligned}
\begin{bmatrix} E^*_1 \\ \vdots \\ E^*_n \end{bmatrix} &= n \cdot (2B - n \cdot (X + X^T))^{-1} \cdot \boldsymbol{a} \\
e^*_{ki} &= \frac{a_i}{2b_i} + \frac{\sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j}{2b_i},\ \forall k, i \in \mathcal{V} \qquad (5.23)
\end{aligned}
$$

By Corollary 5.1, we have $E^*_i \geq \hat{E}_i$, $\forall i$, i.e., the total effort exerted on behalf of agent i improves under the social optimum compared to that under the NE of game G_{rp}. As before, not all agents may attain higher individual utility under E^* as compared to their utility under NE effort profile $\hat{E}$. Following examples show the effect of socially optimal effort level on agents' utility in game G_{rp},

Example 5.4

$$
\begin{aligned}
n &= 2,\ b_1 = 1.5,\ b_2 = 1,\ a_1 = a_2 = 1 \\
x_{12} &= 0.1,\ x_{21} = 0.8,\ l_1 = l_2 = 1 \\
\begin{bmatrix} \hat{E}_1 \\ \hat{E}_2 \end{bmatrix} &= (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = \begin{bmatrix} 0.5588 \\ 0.7514 \end{bmatrix} \\
\hat{E} &= \begin{bmatrix} 0.3520 & 0.0279 \\ 0.2004 & 0.8006 \end{bmatrix} \\
v_1(\hat{E}) &= -0.5858 \quad v_2(\hat{E}) = -0.6138 \\
\begin{bmatrix} E_1^* \\ E_2^* \end{bmatrix} &= 2 * (2B - 2X - 2X^T)^{-1} \cdot \boldsymbol{a} = \begin{bmatrix} 2.7536 \\ 3.4783 \end{bmatrix} \\
E^* &= \begin{bmatrix} 1.3768 & 1.7391 \\ 1.3768 & 1.7391 \end{bmatrix} \\
v_1(E^*) &= -3.1566,\ v_2(E^*) = 4.2725
\end{aligned}
\tag{5.24}
$$

In this example, we can see that $E^* \geq \hat{E}$, but the first agent experiences lower utility in the socially optimal outcome of game G_{rp} compared to its NE.

Example 5.5

$$
\begin{aligned}
n &= 2,\ b_1 = 1,\ b_2 = 1,\ a_1 = a_2 = 1 \\
x_{12} &= x_{21} = 0.25,\ l_1 = l_2 = 1 \\
\begin{bmatrix} \hat{E}_1 \\ \hat{E}_2 \end{bmatrix} &= (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} \\
\hat{E} &= \begin{bmatrix} 0.5833 & 0.0833 \\ 0.0833 & 0.5833 \end{bmatrix} \\
v_1(\hat{E}) &= v_2(\hat{E}) = -0.5694 \\
\begin{bmatrix} E_1^* \\ E_2^* \end{bmatrix} &= 2 * (2B - 2X - 2X^T)^{-1} \cdot \boldsymbol{a} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \\
E^* &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\
v_1(E^*) &= v_2(E^*) = 0
\end{aligned}
\tag{5.25}
$$

In this example, both agents experience higher utility in the socially optimal outcome of game G_{rp} as compared to the NE.

5.9 Conclusion

This chapter considered an IDS game with positive externality, and introduced a resource pooling augmented IDS game, the RP-IDS game, to examine the effect of using resource pooling as a mechanism to incentivize higher effort levels by interdependent agents. It showed that (1) resource pooling increases the total effort exerted on behalf of each agent as compere to no resource pooling, (2) each agent experiences higher utility under resource pooling as compared to no resource pooling, (3) social welfare at the NE of the RP-IDS game is higher than the optimal social welfare under the IDS game, and (4) agents voluntarily participate in resource pooling.

CHAPTER 6

Conclusion

6.1 A brief review

In this thesis, we studied the impact of cyber insurance contracts on network security. Specifically, we considered a profit-maximizing insurer with the voluntary participation of the insureds/agents. We showed that positive externalities among insureds provide a profit opportunity for the insurer, created by the inefficient effort levels exerted by the insureds who want to take advantage of others' investments and efforts when insurance is not available. Then we showed that the insurer is able to use this profit opportunity through premium discrimination based on pre-screening. As a result of the premium discrimination, the agents exert higher effort in the presence of cyber insurance as compared to a scenario without insurance.

We then investigated the following question: when faced with risk dependency (say between a vendor/service provider (SP) and its customers/clients), is it better for an insurer to underwrite all or only the SP's clients and leave the SP to be insured by someone else, with the ability to recover all or part of the loss attributable to the SP from the latter's policy? The conventional wisdom in the underwriting industry is that dependency is to be avoided as it could lead to simultaneous loss events, which could threaten one's capital limit and liquidity. However, we showed that there is a benefit in insuring both, whereby the insurer obtains higher profit in doing so by taking advantage of the risk dependency: when the insurer underwrites the SP and its clients, she can incentivize the SP to improve its security. Because of the positive externality, the clients benefit from this security improvement, and the risk decreases for both the SP and its customers. This security improvement increases the insurer's profit as she pays less coverage in a network with a better state of security.

Next, we considered a scenario of rare cyber incidents in the presence of a profit-maximizing insurer and a single risk-averse agent. In this scenario, we identified an effective method of premium discrimination. Specifically, we considered two methods of assessing risk in order to implement premium discrimination. Pre-screening occurs before the insured enters into a contract and gives the insurer an estimate of the security posture of the insured, which then determines the premium and other contract parameters. On the other hand, the post-screening mechanism includes at least two policy periods whereby the second-period premium is determined based on the first period, i.e., the premium goes up in the second period if there is a loss incident in the first period. We showed in the presence of rare and extremely large cyber losses, post-screening is not an effective method of risk assessment. Moreover, we demonstrated that pre-screening is indeed effective to incentivize the insured to improve its security investment and increase the insurer's profit.

Lastly, we addressed the under-investment issue in interdependent security (IDS) games. Due to the positive externality among the players in an IDS game, they under-invest, leading to a poorer state of security. In order to incentivize effort and investment in these games, we proposed resource pooling, i.e., we considered the possibility of allowing agents/players to invest in their security as well as in other agents' security. We showed that the interaction of strategic and selfish agents under resource pooling improves the agents' security investments as well as their utility. Moreover, we showed that resource pooling satisfies voluntary participation, i.e., no one can enhance his utility by unilaterally opting out of resource pooling.

6.2 Future directions

We first discuss additional directions for studying the cyber insurance market. As we mentioned in Chapter 2, our results are derived under the assumption of perfect information and positive externality. Looking at the contract design problem with pre-screening under partial information is an important future direction; this would include imperfect knowledge of the agents' type by the principal as well as imperfect knowledge of the interdependence relationship by the agents and principal. Other modeling choices such as considering negative externality among insureds instead of positive externality, alternative use of pre-screening assessment (as opposed to linear discounts on premiums), and more general ways of capturing correlated risks (e.g., joint distribution of losses as opposed to average loss being a function of joint effort), and a competitive market setting would also be of great interest.

In Chapter 3, we showed how interdependency could be taken into account in designing cyber insurance contracts to manage and mitigate cyber risks and increase the insurer's profit. As we mentioned earlier, we did not consider the cost of performing an accurate risk assessment, which enables the insurer to monitor the insured to ensure the insured's security posture is commensurate with the discount he received. Therefore, one of the possible future direction is to study the contract design problem when the cost of risk assessment is proportional to its accuracy. Another possible future direction is to consider the cost of security improvement in our model. Chapter 3 assumes that any amount of discount on the premium can be used directly for security improvement and decreasing the chance of a loss incident. On the other hand, it is possible that the discount is either not sufficient or cannot be used directly toward improving security. Therefore, designing a cyber insurance contract by taking the actual security cost into account is also an interesting future direction.

We further studied the insurance contract design problem in the presence of rare loss incidents in Chapter 4 and showed that pre-screening is an effective method of risk assessment when loss incidents are rare and extremely large. In this chapter, we assumed that the accuracy of the risk assessment through pre-screening is time-invariant, which may not always be valid. Therefore, considering pre-screening with time-variant accuracy is a possible future direction for this study. Moreover, in this study, we removed the effects of risk interdependence from our model and purely focused on the rare loss incidents by considering a single risk-averse insured. Therefore, another possible extension of this study is to analyze the cyber insurance market in the presence of multiple interdependent insured and correlated cyber risks.

In Chapter 4, we proposed resource pooling in interdependent security games to overcome the under-investment and free-riding issue. We considered interdependent agents with quadratic utility functions and an unlimited investment budget. Studying resource pooling in an IDS game with non-quadratic utility functions as well as players with limited investment budget would be possible extensions of this study.

Finally, we are keenly aware that regulatory institutions (e.g., governments) can and have from time to time imposed new regulations and established security standards. Such regulatory moves can have significant impact on the behavior of both the insurer and the insured. The models proposed and studied in this dissertation do not necessarily capture the existence of a regulator or emergent behavior on the part of firms. As an example, the EU's General Data Protection Regulation (GDPR) implemented in May 2018 applies to any company processing personal data of European customers. This regulation is updated annually and affects data collection and data use processes and give users an authority to access, modify, and delete their personal information stored by any organization. Under this regulation, if a European citizen complains about an international company who is subsequently found liable, the company has to pay up to 4% of its annual revenue. GDPR is such a cybersecurity regulatory example, whose potentially substantial impact on insurance policy underwriting has yet to be carefully analyzed and understood. Modeling user behavior under these regulations and designing an insurance contract that is compatible with these emergent regulations could be challenging and needs further investigation.

APPENDIX A

Designing Cyber Insurance Policies: The Role of Pre-Screening and Security Interdependence

Proofs

Proof (Lemma 2.1) *Proof by contradiction. Assume that in the optimal contract* (p,α,β)*, the IR constraint (2.8) holds with strict inequality. If this is the case, the insurer can increase her payoff by increasing p; this increase will not violate the (IR) constraint, or impact the (IC) constraint as the (IC) constraint in* (2.8) *is independent of p. This means that* (p,α,β) *is not an optimal contract. Hence, the (IR) constraint should be binding in the optimal contract.*

Proof (Theorem 2.1) *Assume that* $(\hat{\alpha},\hat{\beta},\hat{e})$ *solves optimization problem (2.13), and that, by contradiction,* $\hat{e} > m \geq 0$*.*

First, recall that the agent's optimal effort m outside the contract is given by

$$m := \arg\min_{e\geq 0}\left\{\mu(e)+\frac{1}{2}\gamma\lambda(e)+ce\right\} .$$

For m to be the minimizer, we should have $c+\mu'(m)+\frac{1}{2}\gamma\lambda'(m)\geq 0$*.*

Next, consider the following two cases:

(i) $\hat{\alpha}=0$*. Starting from the FOC on the (IC) constraint, we have,*

$$\begin{aligned}
&(1-\hat{\beta})\mu'(\hat{e})+\tfrac{1}{2}\gamma(1-\hat{\beta})^2\lambda'(\hat{e})+c=0\\
&\Rightarrow \mu'(\hat{e})+\tfrac{1}{2}\gamma\lambda'(\hat{e})+c<0\\
&\Rightarrow \mu'(m)+\tfrac{1}{2}\gamma\lambda'(m)+c<0
\end{aligned}$$

Here, the second line follows from the decreasing nature of $\mu(\cdot)$ and $\lambda(\cdot)$, and the third line follows from their convexity. The last inequality is impossible given the optimality of the effort m outside the contract. This contradiction shows that we cannot have $\hat{e} > m$.

(ii) $\hat{\alpha} > 0$. Given the assumption that $\hat{e} > m$, and $\mu(\cdot)$ and $\lambda(\cdot)$ are strictly convex, we have,

$$\begin{aligned} 0 &\leq c + \mu'(m) + \tfrac{1}{2}\gamma\lambda'(m) \\ &\leq c + \mu'(m) + \tfrac{1}{2}\gamma(1-\hat{\beta})^2\lambda'(m) \\ &< c + \mu'(\hat{e}) + \tfrac{1}{2}\gamma(1-\hat{\beta})^2\lambda'(\hat{e}) \end{aligned}$$

Therefore, if the insurer decreases $\hat{\alpha}$, the agent decreases his effort (this can be seen from the IC constraint), and consequently the insurer's utility increases, as the objective function of the insurer, $w^o - \mu(e) - \frac{1}{2}(1-\hat{\beta})^2\lambda(e) - ce - \frac{1}{2}\gamma\alpha^2\sigma^2$, is decreasing in e and α at $e = \hat{e}, \alpha = \hat{\alpha}$. Therefore, $(\hat{\alpha}, \hat{\beta}, \hat{e})$ is not the optimal contract. Again by contradiction, we conclude that the agent's effort in the optimal contract should be less than or equal to m.

Proof (Theorem 2.2) *Let $v(\alpha,\beta,e,\sigma^2)$ be the payoff of the principal, at a contract (α,β), when the agent exerts effort e and the noise of pre-screening is σ^2, and let $z(\sigma^2)$ be the insurer's profit at the optimal contract as a function of the pre-screening noise. We have,*

$$\begin{aligned} z(\sigma_1^2 + \sigma_2^2) = &\quad \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, IC} v(\alpha,\beta,e,\sigma_1^2+\sigma_2^2) \\ &\leq \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, IC} v(\alpha,\beta,e,\sigma_1^2) + \\ &\quad \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, IC}\{-\tfrac{1}{2}\alpha^2\gamma\sigma_2^2\} \leq \\ &\quad \max_{\alpha, 0\leq\beta\leq 1, e\geq 0, IC} v(\alpha,\beta,e,,\sigma_1^2) = \\ &\quad z(\sigma_1^2) \end{aligned}$$

Therefore, $z(\sigma_1^2 + \sigma_2^2) \leq z(\sigma_1^2), \ \forall \sigma_2^2$. That is, $z(\sigma^2)$ is a decreasing function of the pre-screening noise.

The following lemma will be used in the proof of Theorem 2.3.

Lemma A.1 *In an optimal contract, α is nonnegative and less then or equal to c.*

Proof *Assume (α,β,e) is the optimal solution of (2.13).*

The KKT condition for the (IC) constraint is as follows,

$$\begin{aligned} &(1-\beta)\mu'(e) + \tfrac{1}{2}\gamma(1-\beta)^2\lambda'(e) - v = \alpha - c, \\ &v \cdot e = 0, v, e \geq 0\ . \end{aligned} \tag{A.1}$$

In the above equation, the left hand side is negative, as both $\mu(\cdot)$ and $\lambda(\cdot)$ are decreasing, and the slack variable is $v \geq 0$. Therefore, we need $\alpha \leq c$ to make the right hand side positive as well.

Next, assume that $\alpha < 0$. In this case, we must have $v = 0$. To see why, note that otherwise, if $v > 0$, we can decrease v and increase α by the same amount (while still keeping $\alpha < 0$) in the KKT condition (A.1)*, without changing β and e. This would increase the objective function of the insurer (note that we have decreased α^2), contradicting the optimality of the solution. Therefore, if $\alpha < 0$, we have $v = 0$.*

Setting $v = 0$ in (A.1)*, we have,*

$$c + (1-\beta)\mu'(e) + \frac{1}{2}\gamma(1-\beta)^2\lambda'(e) = \alpha < 0 \tag{A.2}$$

From this, we get

$$-\mu'(e) - \frac{1}{2}\gamma(1-\beta)^2\lambda'(e) - c \geq -c - (1-\beta)\mu'(e) - \frac{1}{2}\gamma(1-\beta)^2\lambda'(e) > 0$$

That is, $-\mu(e) - \frac{1}{2}\gamma(1-\beta)^2\lambda(e) - ce$ is an increasing function of e.

Next, note that as $\mu(\cdot)$ and $\lambda(\cdot)$ are convex, the LHS of (A.2) *. Therefore, we can find $0 > \alpha' > \alpha$ and $e' > e$ such that*

$$c + (1-\beta)\mu'(e') + \frac{1}{2}\gamma(1-\beta)^2\lambda'(e') = \alpha' \ . \tag{A.3}$$

As $-\mu(e) - \frac{1}{2}\gamma(1-\beta)^2\lambda(e) - ce$ is an increasing function of the effort, and $e' > e$, and also as $0 > \alpha' > \alpha$, the new solution (α', β, e') improves the objective function of the insurer in comparison with (α, β, e). From this contradiction, we conclude that α cannot be negative.

Proof (Theorem 2.3) *Assume $\sigma_1^2 \leq \sigma_2^2$. Let (α_i, β_i) and e_i be the parameters of optimal contract and the optimal effort of the agent in that contract, respectively, when the pre-screening noise is $\sigma^2 = \sigma_i^2$.*

First we show that $\alpha_2 \leq \alpha_1$. In contradiction, assume $\alpha_2 > \alpha_1$. First note that from the optimality of (α_1, β_1) when $\sigma = \sigma_1$, we have

$$-\mu(e_1) - \tfrac{1}{2}\gamma(1-\beta_1)^2\lambda(e_1) - ce_1 - \tfrac{1}{2}\gamma\sigma_1^2\alpha_1^2$$
$$\geq$$
$$-\mu(e_2) - \tfrac{1}{2}\gamma(1-\beta_2)^2\lambda(e_2) - ce_2 - \tfrac{1}{2}\gamma\sigma_1^2\alpha_2^2 \ .$$

In addition, if $\alpha_2 > \alpha_1$, we have

$$\frac{1}{2}\gamma\alpha_1^2(\sigma_1^2 - \sigma_2^2) > \frac{1}{2}\gamma\alpha_2^2(\sigma_1^2 - \sigma_2^2) \; .$$

Summing the two inequalities, we get

$$\begin{gathered} -\mu(e_1) - \tfrac{1}{2}\gamma(1-\beta_1)^2\lambda(e_1) - ce_1 - \tfrac{1}{2}\gamma\sigma_1^2\alpha_1^2 + \tfrac{1}{2}\gamma\alpha_1^2(\sigma_1^2 - \sigma_2^2) \\ > \\ -\mu(e_2) - \tfrac{1}{2}\gamma(1-\beta_2)^2\lambda(e_2) - ce_2 - \tfrac{1}{2}\gamma\sigma_1^2\alpha_2^2 + \tfrac{1}{2}\gamma\alpha_2^2(\sigma_1^2 - \sigma_2^2) \end{gathered}$$

The expressions on both sides of the inequality simplify to the objective function of the insurer when $\sigma^2 = \sigma_2^2$, and imply that (α_1,β_1) outperforms the optimal contract (α_2,β_2) when $\sigma^2 = \sigma_2^2$. From the contradiction, we conclude that we must have $\alpha_2 \le \alpha_1$.

Next, by the KKT condition for the (IC) constraints we have,

$$\begin{aligned} &(1-\beta_i)\mu'(e_i) + \tfrac{1}{2}(1-\beta_i)^2\gamma\lambda'(e_i) + c - \alpha_i = v_i, \\ &e_i v_i = 0 \; , \; v_i, e_i \ge 0 \; . \end{aligned} \tag{A.4}$$

We proceed by contradiction. Assume that $0 \le e_1 < e_2$. As e_2 is strictly positive, we can set $v_2 = 0$ in equation (A.4), leading to

$$(1-\beta_2)\mu'(e_2) + \frac{1}{2}(1-\beta_2)^2\gamma\lambda'(e_2) + c - \alpha_2 = 0 \; . \tag{A.5}$$

We can use the above to solve for β_2 as a function of α_2 and e_2 as follows:

$$1 - \beta_2 = \frac{\mu'(e_2) + \sqrt{\mu'(e_2)^2 - 2\gamma(c-\alpha_2)\lambda'(e_2)}}{-\gamma\lambda'(e_2)} := k(e_2,\alpha_2)$$

Therefore, e_2 solves the following optimization problem:

$$\max_{e\ge 0, k(e,\alpha_2)\le 1} w^o - \mu(e) - \frac{1}{2}\gamma k(e,\alpha_2)^2\lambda(e) - ce - \frac{1}{2}\alpha_2^2\gamma\sigma_2^2$$

By the optimality of e_2, we conclude that,

$$-\mu(e_2) - \tfrac{1}{2}\gamma k(e_2,\alpha_2)^2\lambda(e_2) - ce_2 \ge -\mu(e_1) - \tfrac{1}{2}\gamma k(e_1,\alpha_2)^2\lambda(e_1) - ce_1 \tag{A.6}$$

Now consider three cases, based on whether α_1, e_1, or both, are non-zero.

• $\alpha_1 = 0$.

We know that $\alpha_2 \leq \alpha_1$. Therefore, $\alpha_2 = 0$. In this case, it follows from the insurer's optimization problem that $\beta_2 = \beta_1$ and $e_2 = e_1$. This is however a contradiction, as we assumed that $e_1 < e_2$. We therefore must have $\alpha_1 > 0$.

• $\alpha_1 > 0$ *and* $e_1 = 0$.

Take the (IC) constraint of the agent.

$$\begin{aligned}&(1-\beta_1)\mu'(e_1) + \tfrac{1}{2}(1-\beta_1)^2\gamma\lambda'(e_1) + c = v_1 + \alpha_1,\\ &e_1 v_1 = 0\end{aligned}$$

If α_1 is non-zero, then insurer can decrease $\alpha_1 = 0$, instead increasing v_1; increasing v_1 is possible as when $e_1 = 0$, v_1 can be strictly positive. Decreasing α_1 in this way increases the insurer's payoff without affecting β_1, e_1, contradicting the optimality of the contract. Therefore, this case is not possible either.

• $\alpha_1 > 0$ *and* $e_1 > 0$.

From the (IC) constraint at $\sigma^2 = \sigma_1^2$, we have,

$$(1-\beta_1)\mu'(e_1) + \frac{1}{2}(1-\beta_1)^2\gamma\lambda'(e_1) + c - \alpha_1 = 0\ . \tag{A.7}$$

From this, we find β_1 as a function of α_1 and e_1,

$$1-\beta_1 = \frac{\mu'(e_1) + \sqrt{\mu'(e_1)^2 - 2\gamma(c-\alpha_1)\lambda'(e_1)}}{-\gamma\lambda'(e_1)}\ .$$

Therefore, e_1 solves the following optimization problem,

$$\max_{e\geq 0, k(e,\alpha_1)\leq 1}\ \ h(e) := w^o - \mu(e) - \tfrac{1}{2}\gamma k(e,\alpha_1)^2\lambda(e) - ce - \tfrac{1}{2}\sigma_1^2\gamma\alpha_1^2 \tag{A.8}$$

Re-write $h(e_1)$ as follows:

$$h(e_1) = w^o - \mu(e_1) - \frac{1}{2}\gamma k(e_1,\alpha_2)^2\lambda(e_1) - ce_1 - \frac{1}{2}\sigma_1^2\gamma\alpha_1^2 + \frac{1}{2}\gamma(k(e_1,\alpha_2)^2 - k(e_1,\alpha_1)^2)\lambda(e_1)$$

Then $h(e_2) - h(e_1)$ is given by,

$$h(e_2) - h(e_1) =$$

$$(-\mu(e_2) - \frac{1}{2}\gamma k(e_2,\alpha_2)^2\lambda(e_2) - ce_2) - (-\mu(e_1) - \frac{1}{2}\gamma k(e_1,\alpha_2)^2\lambda(e_1) - ce_1) \tag{A.9}$$

$$+\frac{1}{2}\gamma[(k(e_2,\alpha_2)^2 - k(e_2,\alpha_1)^2)\lambda(e_2) - (k(e_1,\alpha_2)^2 - k(e_1,\alpha_1)^2)\lambda(e_1)] \tag{A.10}$$

First, note that (A.9) *is non-negative by* (A.6)*. Next, take $0 \le \alpha_2 \le \alpha_1 \le c$; $\alpha_2 \le \alpha_1$ follows from the proof at the beginning of this theorem, and the lower and upper bounds follow from Lemma A.1. Assume $(k(e,\alpha_2)^2 - k(e,\alpha_1)^2)\lambda(e)$ is non-decreasing. Then,* (A.10) *is non-negative as well. Therefore, $h(e_2) \ge h(e_1)$, which is a contradiction, as e_1 is the maximizer of $h(e)$ in the optimization problem* (A.8)*. Therefore, we conclude that $e_1 \ge e_2$.*

For the second part, we compare the presence of any pre-screening quality $\sigma = \sigma_1$, to the case of $\sigma = \infty$ of uninformative pre-screening. We want to show that when $k(e,0)^2\lambda(e) - k(e,\alpha)^2\lambda(e)$ is non-decreasing, then $e_\infty \le e_1$. Note that when σ is infinity, then the optimal discount factor is zero. As a result, the proof follows the proof of the first part of the theorem, with $\alpha_2 = 0$.

Proof (Theorem 2.4) *First, recall that the principal's contract design problem is as follows,*

$$\begin{aligned}
&\max_{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{e}} && -u_1^{oi} - u_2^{io} - \mu(e_1 + xe_2) - \mu(e_2 + xe_1) - c_1e_1 - c_2e_2 \\
&s.t., && \\
&m_i(\alpha_i,\beta_i) && = \arg\min_{e\ge 0}(1-\beta_i)\mu(e) + (c_i - \alpha_i)e \quad i = 1,2 \\
&e_i && = e_i^*(m_i(\alpha_i,\beta_i), m_{-i}(\alpha_{-i},\beta_{-i})) \quad i = 1,2
\end{aligned} \tag{A.11}$$

We first observe that α_i and β_i do not appear in the objective function of optimization problem (A.11)*. Therefore, by choosing $\alpha_i = c_i$ and $\beta_i = 1$, any non negative number will satisfy the constraint $\arg\min_{e\ge 0}(1-\beta_i)\mu(e) + (c_i - \alpha_i)e$. As a result, the principal can incentivize agents to choose her desired level of effort by offering such contract. Consequently, the insurer's problem simplifies to,*

$$\max_{e_1\ge 0, e_2\ge 0} -u_1^{oi} - u_2^{io} - \mu(e_1 + xe_2) - \mu(e_2 + xe_1) - c_1e_1 - c_2e_2$$

This is equivalent to maximizing $\overline{U}_1(e_1,e_2) + \overline{U}_2(e_1,e_2)$ with constraints $e_1 \ge 0, e_2 \ge 0$. Consequently, the socially optimal strategies maximize the principal's profit in the optimal contract. In *other words,* $e_i^{in} = \tilde{e}_i$, $i = 1,2$. *This establishes part (i) of the theorem.*

We now prove part (ii) of the theorem. The socially optimal efforts of agents are given by the solution to,

$$\begin{aligned}(\tilde{e}_1, \tilde{e}_2) &= \arg\max_{e_1 \geq 0, e_2 \geq 0} \overline{U}_1(e_1, e_2) + \overline{U}_2(e_1, e_2) \\ &= \arg\max_{e_1 \geq 0, e_2 \geq 0} -\mu(e_1 + xe_2) - c_1 e_1 - \mu(e_2 + xe_1) - c_2 e_2\end{aligned} \tag{A.12}$$

Let $g_i(e_{-i})$ *be the effort level* e_i *maximizing the objective function above, as a function of the other agent's effort* e_{-i}. *Recall that from the viewpoint of agent i, the optimal value of* e_i *maximizing* $\overline{U}_i$, *as a function of* e_{-i}, *is given by* $(m_i - xe_{-i})^+$, *where* $m_i = \arg\min_{e \geq 0} \mu(e) + c_i e$.

We next show that $g_i(e_{-i}) \geq (m_i - xe_{-i})^+$. *We proceed by contradiction. Assume that* $g_i(e'_{-i}) < (m_i - xe'_{-i})^+$ *for a given value of* e'_{-i}. *Note that* $\overline{U}_{-i}$ *is an increasing function in* e_i. *Also,* $\overline{U}_i$ *is maximized at* $e_i = (m_i - xe'_{-i})^+$. *As a result, with* $e_i = (m_i - xe'_{-i})^+$ *instead of* $g_i(e'_{-i})$, *both* $\overline{U}_1(e_i, e'_{-i})$ *and* $\overline{U}_2(e_i, e'_{-i})$ *increase, which would in turn imply that* $g_i(e'_{-i})$ *is suboptimal. Therefore,* $g_i(e_{-i}) \geq (m_i - xe_{-i})^+$.

Next, assume that $(\tilde{e}_1, \tilde{e}_2)$ *solves the optimization problem* (A.12). *We know that* $g_i(\tilde{e}_{-i}) = \tilde{e}_i \geq (m_i - x\tilde{e}_{-i})^+ \geq m_i - x\tilde{e}_{-i}$. *Therefore, we have,*

$$\begin{aligned}&\tilde{e}_i \geq m_i - x\tilde{e}_{-i} \;\Rightarrow\; \tilde{e}_i + x\tilde{e}_{-i} \geq m_i \\ &\Rightarrow\; \tilde{e}_i + \tilde{e}_{-i} \geq m_i\end{aligned}$$

In other words, network security in the socially optimal solution is higher that both m_1 *and* m_2.

In addition, we have,

$$\begin{aligned}&\tilde{e}_1 \geq m_1 - x\tilde{e}_2, \textit{ and, } \tilde{e}_2 \geq m_2 - x\tilde{e}_1 \\ &\Rightarrow\; \tilde{e}_1 + \tilde{e}_2 \geq \tfrac{m_1 + m_2}{1+x}\end{aligned}$$

That is, network security in the socially optimal solution is higher than $\frac{m_1+m_2}{1+x}$.

Let e_1^o, e_2^o *denote that efforts of the agents in the Nash equilibrium when both are outside of the contract. By* (2.28), *we know that*

$$e_1^o + e_2^o = \begin{cases} m_1 & \textit{if } xm_1 \geq m_2 \\ m_2 & \textit{if } xm_2 \geq m_1 \\ \frac{m_1+m_2}{1+x} & o.w \end{cases}$$

We have shown that $\tilde{e}_1 + \tilde{e}_2 \geq \max\{m_1, m_2, \frac{m_1+m_2}{1+x}\}$. *Therefore,* $\tilde{e}_1 + \tilde{e}_2 \geq e_1^o + e_2^o$. *This establishes* *part (ii) of the theorem, as we have shown that efforts inside the contract (which are at the socially optimal level), are higher than the Nash equilibrium efforts prior to the introduction of insurance.*

Lastly, we prove part (iii) of the theorem. First we show that u_2^{io} is lower then u_2^{oo}; recall that u_2^{io} denotes the utility of agent 2 *when agent* 2 *is outside of the contract and agent* 1 *purchases an optimal contract, and u_2^{oo} is the utility of the second agent when both agents opt out.*

Consider the case where agent 1 is inside the contract while agent 2 opts out. The principal's problem is as follows,

$$\begin{aligned}
&\max_{\alpha_1,\beta_1,e_1,e_2} && -u_1^{oo}-\mu(e_1+xe_2)-c_1e_1\\
&s.t.,\\
&m_1(\alpha_1,\beta_1) && =\arg\min_{e\geq 0}(1-\beta_1)\mu(e)+(c_1-\alpha_1)e\\
&m_2 && =\arg\min_{e\geq 0}\mu(e)+c_2e\\
&e_1 && =e_1^*(m_1(\alpha_1,\beta_1),m_2)\\
&e_2 && =e_2^*(m_2,m_1(\alpha_1,\beta_1))
\end{aligned} \tag{A.13}$$

where u_1^{oo} is the utility of the first agent when both agents opt out.

We note that the objective function of the principal's problem is independent of α_1,β_1. As a result, an optimal contract for the principal is to select $\alpha_1=c_1$ and $\beta_1=1$, in which case, any non negative effort level satisfies the agent 1's (IC) constraint. We therefore substitute the first constraint in (A.13) *with $m_1(c_1,1)\geq 0$.*

We next note that under $(\alpha_1=c_1,\beta_1=1)$, any effort level e_1, and the corresponding best response of the second agent to e_1, will constitute a Nash equilibrium. We therefore re-write the principal's problem as,

$$\begin{aligned}
&\max_{m_1(c_1,1)\geq 0} && -u_1^{oo}-\mu(\max\{m_1(c_1,1),xm_2\})-c_1(\min\{\tfrac{m_1(c_1,1)-xm_2}{1-x^2},m_1(c_1,1)\})^+\\
&s.t.,\\
& && m_1(c_1,1)\geq 0\\
& && m_2=\arg\min_{e\geq 0}\mu(e)+c_2e
\end{aligned} \tag{A.14}$$

Let $m_1=\arg\min_{e\geq 0}\mu(e)+c_1e$. We show that m_1^*, the solution to the optimization problem (A.14)*, is lower than m_1. In other words, the first agent exerts lower effort when he enters the contract, as $e_1^*(m_1,m_2)$ is decreasing in m_1. Consequently, we conclude that $u_2^{io}\leq u_2^{oo}$.*

We proceed by contradiction. Assume that the first agent exerts strictly higher effort than his *effort in the no-insurance equilibrium when he enters the contract (game G^{io}). That is, assume $m_1^* > m_1$. We consider three cases,*

Case i: $m_1^* > \frac{m_2}{x}$

In this case, the objective function of the principal's problem in (A.14) *is given by,* $-u_1^{oo} - \mu(m_1^*) - c_1 m_1^*$. *The first derivative of this function is* $-\mu'(m_1^*) - c_1$ *which is negative as,*

$$\begin{aligned} & m_1 = \arg\min_{e\geq 0} \mu(e) + c_1 e \;\Rightarrow \mu'(m_1) + c_1 \geq 0 \\ & \Rightarrow\; \mu'(m_1^*) + c_1 > 0 \;\; (\textit{by strictly convexity of } \mu) \end{aligned}$$

Therefore, m_1^* is not optimal in this case, as its decrease would improve the principal's objective function. By the contradiction, we conclude that under the assumption of this case, we should have $m_1^* \leq m_1$.

Case ii: $\frac{m_2}{x} \geq m_1^* > x \cdot m_2$

In this case, the objective function of principal's problem in (A.14) *is,* $-u_1^{oo} - \mu(m_1^*) - c_1 \frac{m_1^* - x m_2}{1-x^2}$. *The first derivative is given by* $-\mu'(m_1^*) - \frac{c}{1-x^2}$, *which is negative as,*

$$\begin{aligned} & m_1 = \arg\min_{e\geq 0} \mu(e) + c_1 e \;\Rightarrow \mu'(m_1) + c_1 \geq 0 \\ & \Rightarrow -\mu'(m_1^*) - \frac{c}{1-x^2} < 0 (\textit{by strictly convexity of } \mu) \end{aligned}$$

Therefore, m_1^* is not optimal in this case and the decrease in m_1^* improves principal's objective function. Therefore, we contradict the assumption of $m_1^* > m_1$ in this case as well.

Case iii: $x \cdot m_2 \geq m_1^*$

In this case, the first agent exerts zero effort in both the no-insurance equilibrium and in the game G^{io}. This again contradicts $m_1^* > m_1$.

We therefore conclude that $m_1^* \leq m_1$. That is, we have shown that the first agent exerts less effort than the no-insurance equilibrium, when only agent 1 enters the contract. This in turn leads to less utility for the second agent, as compared to the case when both agents are outside of the contract. Therefore, we have,

$$u_2^{oo} \geq u_2^{io}$$

Similarly, we can show that,

$$u_1^{oo} \geq u_1^{oi}$$

Next, we show that the principal can obtain positive profit when offering the optimal contracts.

Notice that we have established $u_1^{oi} + u_2^{io} \le u_1^{oo} + u_2^{oo}$. *In the optimal contract, when both agents purchase contracts, the principal's objective value is*

$$\begin{aligned} v^{*ii} &= \max \quad -u_1^{oi} - u_2^{io} - \mu(e_1 + xe_2) - \mu(e_2 + xe_1) - c_1 e_1 - c_2 e_2 \\ &= -u_1^{oi} - u_2^{io} + \overline{U}_1(\tilde{e}_1, \tilde{e}_2) + \overline{U}_2(\tilde{e}_1, \tilde{e}_2) \\ &\ge \overline{U}_1(\tilde{e}_1, \tilde{e}_2) + \overline{U}_2(\tilde{e}_1, \tilde{e}_2) - \overline{U}_1(e_1^o, e_2^o) - \overline{U}_2(e_1^o, e_2^o) \,. \end{aligned}$$

This establishes the lower bound on the profit of the insurer (part (iii) of the theorem), and concludes our proof.

We will use the following lemma in the proof of Theorem 2.5.

Lemma A.2 *Assume perfect pre-screening* ($\sigma_1 = \sigma_2 = 0$)*. Then, the optimal effort level of agents satisfies,*

$$\begin{aligned} e_1 &= \frac{(\mu')^{-1}\left(\frac{-(c_1-v_1)+x(c_2-v_2)}{1-x^2}\right) - x(\mu')^{-1}\left(\frac{-(c_2-v_2)+x(c_1-v_1)}{1-x^2}\right)}{1-x^2} \\ e_2 &= \frac{(\mu')^{-1}\left(\frac{-(c_2-v_2)+x(c_1-v_1)}{1-x^2}\right) - x(\mu')^{-1}\left(\frac{-(c_1-v_1)+x(c_2-v_2)}{1-x^2}\right)}{1-x^2} \\ e_1 v_1 &= 0, e_2 v_2 = 0, \; e_1 \ge 0, e_2 \ge 0, \; v_1 \ge 0, v_2 \ge 0 \,. \end{aligned} \tag{A.15}$$

Proof *When pre-screening is perfect, then insurer's problem is as follows,*

$$\begin{aligned} \max_{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{e}} \quad & w_1^{oi} + w_2^{io} - \textstyle\sum_{i=1,2} \mu(e_i + xe_{-i}) + \frac{\gamma_i(1-\beta_i)^2}{2}\lambda(e_i + xe_{-i}) + c_i e_i \\ s.t., \quad & \\ & m_i(\alpha_i, \beta_i) = \arg\min_{e \ge 0} (1-\beta_i)\mu(e) + \tfrac{1}{2}\gamma_i(1-\beta_i)^2\lambda(e) + (c_i - \alpha_i)e \\ & e_i = e_i^*(m_i(\alpha_i, \beta_i), m_{-i}(\alpha_{-i}, \beta_{-i})) \; for \; i = 1, 2 \end{aligned} \tag{A.16}$$

where $e_i^*(.,.)$ *is defined in* (2.28). *When* $\sigma_1 = \sigma_2 = 0$*, the optimal contract is* $(\alpha_i, \beta_i) = (c_i, 1)$*. This is because* α_i *does not appear in the objective function of* (A.16)*, and on the other hand, by setting* $(\alpha_i, \beta_i) = (c_i, 1)$*, the term* $-\frac{1}{2}\gamma_i(1-\beta_i)^2\lambda(e_i + xe_{-i})$ *vanishes from the objective function, improving the insurer's objective function. By setting* $(\alpha_i, \beta_i) = (c_i, 1)$*, any non negative effort level is the minimizer of the agents' (IC) constraints. In other words, when* $(\alpha_i, \beta_i) = (c_i, 1)$*, the utility functions of agents are independent of their effort, and agents will be (weakly) incentivized to exert a level of effort desired by the insurer. In order to find this optimal effort level, the insurer solves* *the following optimization problem,*

$$\max_{e_1\geq 0, e_2\geq 0} -\mu(e_1+xe_2)-\mu(e_2+xe_1)-c_1e_1-c_2e_2$$

As the above optimization problem is convex, we can find a solution by using the KKT conditions,

$$\begin{aligned}
&\mu'(e_1+xe_2)+x\mu'(e_2+xe_1)+c_1-v_1=0\\
&x\mu'(e_1+xe_2)+\mu'(e_2+xe_1)+c_2-v_2=0\\
&e_1\geq 0, e_2\geq 0,\ v_1\geq 0, v_2\geq 0
\end{aligned}$$

We rewrite the above conditions as,

$$\begin{aligned}
&\mu'(e_1+xe_2)=\tfrac{-(c_1-v_1)+x(c_2-v_2)}{1-x^2}\\
&\mu'(e_2+xe_1)=\tfrac{-(c_2-v_2)+x(c_1-v_1)}{1-x^2}\\
&v_1e_1=0, v_2e_2=0, e_1\geq 0, e_2\geq 0, v_1\geq 0, v_2\geq 0
\end{aligned}$$

The solution of the above system of equations is given by (A.15).

Proof (theorem 2.5) *(i) Under the second condition of the Theorem ($(\mu')^{-1}(\frac{-c_i+xc_{-i}}{1-x^2})\geq x(\mu')^{-1}(\frac{-c_{-i}+xc_i}{1-x^2})\ \ i=1,2$) we conclude that the solution to* (A.15) *is as follows,*

$$\begin{aligned}
&e_1=\frac{(\mu')^{-1}(\frac{-c_1+xc_2}{1-x^2})-x(\mu')^{-1}(\frac{-c_2+xc_1}{1-x^2})}{1-x^2}\\
&e_2=\frac{(\mu')^{-1}(\frac{-c_2+xc_1}{1-x^2})-x(\mu')^{-1}(\frac{-c_1+xc_2}{1-x^2})}{1-x^2}\\
&v_1=v_2=0
\end{aligned}$$

The first condition of the theorem ($\mu'(m_i)<\frac{-c_i+xc_{-i}}{1-x^2}$), on the other hand, guarantees that $e_i+xe_{-i}\geq m_i$. Therefore,

$$\begin{gathered}
e_1+xe_2\geq m_1 \Rightarrow e_1+e_2\geq m_1\\
e_2+xe_1\geq m_2 \Rightarrow e_1+e_2\geq m_2\\
e_1+xe_2\geq m_1, e_2+xe_1\geq m_2 \Rightarrow e_1+e_2\geq \tfrac{m_1+m_2}{1+x}
\end{gathered}$$

In the absence of insurance, network security is given by,

$$S(m_1,m_2) = \begin{cases} \frac{m_1+m_2}{1+x} & \text{if } m_1 \geq x\cdot m_2,\ m_2 \geq x\cdot m_1 \\ m_2 & \text{if } m_1 \leq x\cdot m_2 \\ m_1 & \text{if } m_2 \leq x\cdot m_1 \end{cases}$$

By comparing the two, we conclude that $e_1 + e_2 \geq S(m_1,m_2)$. Therefore, network security improves under insurance.

(ii) Recall that the insurer's problem is given by,

$$\begin{aligned} \max_{\alpha_1,0\leq\beta_1\leq1,\alpha_2,0\leq\beta_2\leq1,e_1\geq0,e_2\geq0} \quad & w_1^{oi} + w_2^{io} \\ & -\mu(e_1^* + x\cdot e_2^*) - \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1 + x\cdot e_2) \\ & -c_1\cdot e_1 - \tfrac{1}{2}\alpha_1^2\gamma_1\sigma_1^2 \\ & -\mu(e_2 + x\cdot e_1) - \tfrac{1}{2}\gamma_2(1-\beta_2)^2\lambda(e_2 + x\cdot e_1) \\ & -c_2\cdot e_2 - \tfrac{1}{2}\alpha_2^2\gamma_2\sigma_2^2 \\ s.t., \quad & \\ & (IC)\ e_1,e_2 \text{ are the agents' effort in the NE of game } G^{ii} \end{aligned}$$

When setting $\sigma_1 = \sigma_2 = \infty$, we have $\alpha_1 = \alpha_2 = 0$. Therefore, we modify the insurer's problem as follows,

$$\begin{aligned} \max_{0\leq\beta_1\leq1,0\leq\beta_2\leq1,e_1^*\geq0,e_2^*\geq0} \quad & w_1^{oi} + w_2^{io} \\ & -\mu(e_1^* + x\cdot e_2^*) - \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1^* + x\cdot e_2^*) - c_1\cdot e_1^* \\ & -\mu(e_2^* + x\cdot e_1^*) - \tfrac{1}{2}\gamma_2(1-\beta_2)^2\lambda(e_2^* + x\cdot e_1^*) - c_2\cdot e_2^* \\ s.t., \quad & \\ & (IC)\ m_i' = \arg\min_{e\geq0}(1-\beta_i)\mu(e) + \tfrac{1}{2}(1-\beta_i)^2\lambda(e) + c_i e \\ & e_i = e_i^*(m_i', m_{-i}') \end{aligned}$$

Note that if m_1', m_2' solve the above optimization problem, then $m_i' \leq m_i$. This is because $m_i' = \arg\min_{e\geq0}(1-\beta_i)\mu(e) + \frac{1}{2}(1-\beta_i)^2\lambda(e) + c_i e$, and $\mu(\cdot), \lambda(\cdot)$ are strictly decreasing and strictly convex.

If e_1, e_2 solves the insurer's problem, we have,

$$
\begin{aligned}
&e_1^o = (m_1 - xe_2^o)^+ \geq (m_1' - xe_2^o)^+ \geq m_1' - xe_2^o \Rightarrow e_1^o + e_2^o \geq m_1' \\
&e_2^o = (m_2 - xe_1^o)^+ \geq (m_2' - xe_1^o)^+ \geq m_1' - xe_1^o \Rightarrow e_1^o + e_2^o \geq m_2' \\
&e_1^o \geq m_1' - xe_2^o, e_2^o \geq m_1' - xe_1^o \Rightarrow e_1^o + e_2^o \geq \tfrac{m_1' + m_2'}{1+x}
\end{aligned}
$$

In the absence of pre-screening, network security is given by,

$$
S(m_1', m_2') = \begin{cases} \frac{m_1' + m_2'}{1+x} & \text{if } m_1' \geq x \cdot m_2',\ m_2' \geq x \cdot m_1' \\ m_2' & \text{if } m_1' \leq x \cdot m_2' \\ m_1' & \text{if } m_2' \leq x \cdot m_1' \end{cases}
$$

By comparing the two expressions, we conclude that $e_1^o + e_2^o \geq S(m_1', m_2')$. Therefore, network security worsens under insurance without pre-screening.

Proof (Theorem 2.6) *(i) Consider N symmetric agents, with $\gamma_i = \gamma$, $\forall i$, and $c_i = c$, $\forall i$, and assume that insurer offers these agents identical contracts. At equilibrium, each agent exerts effort $e' = \frac{m'}{1+(N-1)x}$ in the Nash equilibrium, where,*

$$
m' = \arg\min_{e \geq 0} (1-\beta)\mu(e) + \tfrac{1}{2}(1-\beta)^2 \gamma \lambda(e) + (c - \alpha)e
$$

Therefore, the insurer's problem, when she offers identical contracts to the agents, and $\sigma = 0$, is as follows,

$$
\begin{aligned}
&\max_{\alpha, 0 \leq \beta \leq 1, m'} \quad N \cdot \{w^{out} - \mu(m') - \tfrac{1}{2}\gamma(1-\beta)^2 \lambda(m') - \tfrac{c}{1+(N-1)x} m'\} \\
&s.t., \\
&\qquad (IC)\ m' = \arg\min_{e \geq 0} (1-\beta)\mu(e) + \tfrac{1}{2}(1-\beta)^2 \gamma \lambda(e) + (c-\alpha)e
\end{aligned}
$$

In the above optimization problem, as α does not appear in the objective function, then insurer can choose her desired α; one choice is to set $\alpha = c$, in which case the insurer can encourage users to exert her desired choice of level of effort. In addition, note that setting $\beta = 1$ will improve the objective function. Therefore, when $\sigma = 0$, the optimal contract is $(\alpha = c, \beta = 1)$. In this contract, any non-negative effort level satisfies the agents' (IC) constraints. Therefore, the optimum effort in *the contract can be found as follows,*

$$m' = \arg\max_{e \geq 0} w_1^{out} - \mu(e) - c \cdot \tfrac{e}{1+(N-1)x} \ , \tag{A.17}$$

and it will be given by $e' = \frac{m'}{1+(N-1)x}$.

Furthermore, when none of the agents enter a contract, their equilibrium efforts are given by

$$\begin{aligned} m &= \arg\min_{e \geq 0} \mu(e) + \tfrac{1}{2}\gamma\lambda(e) + ce \\ e^o &= \tfrac{m}{1+(N-1)x} \end{aligned} \tag{A.18}$$

We will next show that $m' \geq m$ *if and only if* $\mu'(m) < -\frac{c}{1+(N-1)x}$.

We assume $m > 0$*. Then, by* (A.18)*, we have,*

$$\begin{aligned} f(e) &= \mu'(e) + \tfrac{1}{2}\gamma\lambda'(e) + c \\ f(m) &= 0 \end{aligned}$$

Let $l(e) = \mu'(e) + \frac{c}{1+(N-1)x}$*. Notice that* $f(e)$ *and* $l(e)$ *both are strictly increasing, and have a single root. We know that m is the roof of* $f(\cdot)$*. Therefore, the root of* $l(e)$ *is larger than that of* $f(e)$ *if and only if* $f(m) > l(m)$*, or equivalently* $\mu'(m) + \frac{c}{1+(N-1)x} < 0$*. Therefore, the optimal value of* m' *in* (A.17) *is larger than m, if and only if* $\mu'(m) + \frac{c}{1+(N-1)x} < 0$.

(ii) Next, we show that when pre-screening is uninformative, i.e., $\sigma = \infty$*, then network security worsens as compared to the no-insurance scenario. We again proceed by contradiction. Assume* $(\hat{\alpha}, \hat{\beta}, \hat{m})$ *solves the insurer's optimization problem and* $\hat{e} = \frac{\hat{m}}{1+(N-1)x}$ *and* $\hat{m} > m \geq 0$*. Note that* $\sigma = \infty$ *implies that* $\hat{\alpha} = 0$*. We show that* $(\hat{\alpha}, \hat{\beta}, \hat{m})$ *cannot be the optimal contract for the insruer if* $\hat{m} > m \geq 0$*.*

By the agents' (IC) constraints and the fact that $\mu(\cdot)$ *is a decreasing and convex function, we have (notice that* $\hat{m} > 0$*),*

$$\begin{aligned} (1-\hat{\beta})\mu'(\hat{m}) + \tfrac{1}{2}(1-\hat{\beta})^2\gamma\lambda'(\hat{m}) + c = 0 \Rightarrow \\ \mu'(m) + \tfrac{1}{2}\gamma\lambda'(m) + c < 0 \ . \end{aligned}$$

This is a contradiction because from (A.18) we have,

$$\mu'(m) + \frac{1}{2}\gamma\lambda'(m) + c \geq 0 \ .$$

Therefore, we conclude that $\hat{m} \leq m$*, i.e., network security worsens as compared to the no-insurance scenario when pre-screening is uninformative.*

Proof (Theorem 2.7) *When agents are homogeneous and the insurer offers them identical contracts, it is straightforward to show that they exert effort* $e = \frac{m'}{1+(N-1)x}$ *in the Nash equilibrium, where,*

$$m' = \arg\min_{e\geq 0}(1-\beta)\mu(e) + \tfrac{1}{2}(1-\beta)^2\gamma\lambda(e) + (c-\alpha)e$$

The insurer's problem is as follows,

$$\begin{aligned}
&\max_{\alpha, 0\leq\beta\leq 1, m'} \quad N \cdot \{w^{out} - \mu(m') - \tfrac{1}{2}\gamma(1-\beta)^2\lambda(m') - \tfrac{c}{1+(N-1)x}m' - \tfrac{1}{2}\alpha^2\gamma\sigma_1^2\} \\
&s.t., \\
&\qquad (IC)\ m' = \arg\min_{e\geq 0}(1-\beta)\mu(e) + \tfrac{1}{2}(1-\beta)^2\gamma\lambda(e) + (c-\alpha)e
\end{aligned} \tag{A.19}$$

Notice that if $\arg\min_{e\geq 0}\mu'(e) + \frac{1}{2}\gamma\lambda'(e) + c = 0$*, then agents exert zero effort outside of the contract, in which case insurance cannot worsen the network security as compared to the no-insurance equilibrium. Therefore, we assume that* $m > 0$*. Let* $\mu'(\tilde{m}) = -\frac{c}{1+(N-1)x}$*, and assume* $\mu'(m) + \frac{1}{2}\gamma\lambda'(m) + c = 0$*.*

Now assume that the pre-screening noise is such that $\sigma^2 \leq \frac{\mu(m)+\frac{c}{1+(N-1)x}m-\mu(\tilde{m})-\frac{c}{1+(N-1)x}\tilde{m}}{0.5\gamma c^2}$*, and that given this pre-screening accuracy,* $(\hat{\alpha},\hat{\beta},\hat{m})$ *solves the optimization problem* (A.19)*, with* $\hat{e} = \frac{\hat{m}}{1+(N-1)x}$*.*

We show that network security improves under the optimal contract, i.e., $\hat{m} \geq m$*. We proceed by contradiction. Assume that* $\hat{m} < m$*. If this is true, then a contract with* $(\alpha = c, \beta = 1, m' = \tilde{m})$ *would provides better utility for the insurer. To see why, first notice that* $(\beta = 1, \alpha = c, m' = \tilde{m})$ *satisfies the (IC) constraint.*

Next, by optimality of $(\hat{\alpha},\hat{\beta},\hat{m})$ *at* σ^2 *we have,*

$$\mu(\hat{m}) + \tfrac{1}{2}(1-\hat{\beta})\gamma\lambda(\hat{m}) + \tfrac{c}{1+(N-1)x}\hat{m} + \tfrac{1}{2}\gamma\sigma^2\hat{\alpha}^2 \leq \mu(\tilde{m}) + \tfrac{c}{1+(N-1)x}\tilde{m} + \tfrac{1}{2}\gamma\sigma^2c^2 \tag{A.20}$$

Also, by the assumption that $\sigma^2 \leq \frac{\mu(m)+\frac{c}{1+(N-1)x}m-\mu(\tilde{m})-\frac{c}{1+(N-1)x}\tilde{m}}{0.5\gamma c^2}$*, we have,*

$$\mu(\tilde{m}) + \tfrac{c}{1+(N-1)x}\tilde{m} + \tfrac{1}{2}\gamma\sigma^2c^2 \leq \mu(m) + \tfrac{c}{1+(N-1)x}m \tag{A.21}$$

From (A.20) *and* (A.21)*, we get,*

$$\mu(\hat{m}) + \tfrac{1}{2}(1-\hat{\beta})\gamma\lambda(\hat{m}) + \tfrac{c}{1+(N-1)x}\hat{m} + \tfrac{1}{2}\gamma\sigma^2\hat{\alpha}^2 \le \mu(m) + \tfrac{c}{1+(N-1)x}m\ .$$

The last inequality can not hold, due to the fact that $\mu(e) + \frac{c}{1+(N-1)x}e$ *is strictly decreasing at* $e = m$ *(recall that* $\mu'(e) < -\frac{c}{1+(N-1)x}$*) and* $\hat{m} < m$*. This contradiction implies that* $\hat{m} \ge m$ *and network security increases if* $\sigma^2 \le \frac{\mu(m)+\frac{c}{1+(N-1)x}m-\mu(\tilde{m})-\frac{c}{1+(N-1)x}\tilde{m}}{0.5\gamma c^2}$.

Proof (Theorem 2.8) *Let* $a(\beta) = \delta\beta^2 + \gamma(1-\beta)^2$*. For choosing optimal value of* β*, the insurer first minimizes* $a(\beta)$*. Notice that the minimizer of* $a(\beta)$ *is* $\frac{\gamma}{\gamma+\delta}$ *and the minimum of the* $a(\beta)$ *is* $\frac{\gamma\delta}{\gamma+\delta}$*. Therefore, the insurer's choose* α *such that it encourages the agents to exert the effort* $\frac{m'}{1+(N-1)x}$ *maximizing following function,*

$$w^{out} - \mu(m') - \frac{1}{2}\frac{\gamma\delta}{\gamma+\delta}\lambda(m') - \frac{c}{1+(N-1)x}m' \tag{A.22}$$

By the first order condition we have,

$$\mu'(m') + \tfrac{1}{2}\tfrac{\gamma\delta}{\gamma+\delta}\lambda'(m') + \tfrac{c}{1+(N-1)x} = 0 \tag{A.23}$$

Because $b(m') = \mu'(m') + \frac{1}{2}\frac{\gamma\delta}{\gamma+\delta}\lambda'(m') + \frac{c}{1+(N-1)x}$ *is an increasing function,* $m' > m$ *if and only if* $b(m) < 0$*. In other words,*

$$\mu'(m) + \frac{1}{2}\frac{\gamma\delta}{\gamma+\delta}\lambda'(m') + \frac{c}{1+(N-1)x} < 0 \tag{A.24}$$

Proof (Theorem 2.9) *Let* (α_i,β_i,m_i) *are the solution of the optimization problem* (2.50) *when the covariance between the losses is* θ_i*. Let's assume* $\theta_1 \ge \theta_2 \ge 0$*. First we show that* $\beta_1 \le \beta_2$*.*

By the optimality (α_i,β_i,m_i) *at* θ_i *we have,*

$$\begin{gathered} w^{out} - \mu(m_1) - \tfrac{\beta_1^2\delta+(1-\beta_1)^2\gamma}{2}\lambda(m_1) - \tfrac{c}{1+x}m_1 - \tfrac{\delta+\gamma}{2}\alpha_1^2\sigma^2 - \tfrac{(N-1)}{2}\delta\beta_1^2\theta_1 \\ \ge \\ w^{out} - \mu(m_2) - \tfrac{\beta_2^2\delta+(1-\beta_2)^2\gamma}{2}\lambda(m_2) - \tfrac{c}{1+x}m_2 - \tfrac{\delta+\gamma}{2}\alpha_2^2\sigma^2 - \tfrac{(N-1)}{2}\delta\beta_2^2\theta_1 \end{gathered} \tag{A.25}$$

$$\begin{gathered} w^{out} - \mu(m_2) - \tfrac{\beta_2^2\delta+(1-\beta_2)^2\gamma}{2}\lambda(m_2) - \tfrac{c}{1+x}m_2 - \tfrac{\delta+\gamma}{2}\alpha_2^2\sigma^2 - \tfrac{(N-1)}{2}\delta\beta_2^2\theta_2 \\ \ge \\ w^{out} - \mu(m_1) - \tfrac{\beta_1^2\delta+(1-\beta_1)^2\gamma}{2}\lambda(m_1) - \tfrac{c}{1+x}m_1 - \tfrac{\delta+\gamma}{2}\alpha_1^2\sigma^2 - \tfrac{(N-1)}{2}\delta\beta_1^2\theta_2 \end{gathered} \tag{A.26}$$

If we add the above equations we get,

$$(\beta_2^2 - \beta_1^2)(\theta_1 - \theta_2) \geq 0 \Rightarrow \beta_2 \geq \beta_1 \tag{A.27}$$

Now we should show that $m_1 \geq m_2$. We proceed by contradiction. Let's assume $m_1 < m_2$. Because $\beta_1 \leq \beta_2$ and $m_1 < m_2$, then $\alpha_1 < \alpha_2$ by (IC) constraint.

Because increase or decrease in α_2 does not improve the insurer's objective function, we conclude that the first derivative of insurer objective function at $(\alpha_2, \beta_2, m_2, \theta_2)$ is zero. Also, in the proof of the theorem 2.8 we showed that the optimal coverage factor for $\theta = 0$ is $\beta = \frac{\gamma}{\gamma+\delta}$. Notice that $a(\beta) = \beta_1^2\delta + (1-\beta_1)^2\gamma$ is decreasing for $\beta \leq \frac{\gamma}{\gamma+\delta}$. We have,

$$\begin{array}{l} -\mu'(m_2) - \frac{\beta_2^2\delta+(1-\beta_2)^2\gamma}{2}\lambda'(m_2) - \frac{c}{1+x} = 0 \Rightarrow \\ -\mu'(m_1) - \frac{\beta_1^2\delta+(1-\beta_1)^2\gamma}{2}\lambda'(m_1) - \frac{c}{1+x} > 0 \end{array} \tag{A.28}$$

The last equation implies that the insurer can improve her profit at $(\alpha_1, \beta_1, m_1, \theta_1)$ by increasing α_1. This is the contradiction. Therefore, $m_1 \geq m_2$.

∎

Outside Options for Two Risk-Averse Agents

Case (i): Neither agent enters a contract

Let G^{oo} be the game between two agents, neither of which have purchased cyber insurance contracts. In this game, their actions are their effort level, and the expected payoffs of these agents, with unit costs of effort $c_1, c_2 > 0$, are given by,

$$\bar{U}_i(e_1, e_2) = -\exp\{\gamma_i\mu(e_i + x\cdot e_{-i}) + \tfrac{1}{2}\gamma_i^2\lambda(e_i + x\cdot e_{-i}) + \gamma_i\cdot c_i\cdot e_i\}$$

The best-response of each agent, when both opt out, can be found by solving the following optimization problem,

$$\begin{aligned} B_i^{out}(e_{-i}) &= \arg\max_{e_i\geq 0} -\exp\{\gamma_i\mu(e_i + x\cdot e_{-i}) + \tfrac{1}{2}\gamma_i^2\lambda(e_i + x\cdot e_{-i}) + \gamma_i\cdot c_i\cdot e_i\} \\ &= \arg\min_{e_i\geq 0} \mu(e_i + x\cdot e_{-i}) + \tfrac{1}{2}\gamma_i\lambda(e_i + x\cdot e_{-i}) + c_i\cdot e_i\ . \end{aligned} \tag{A.29}$$

The above optimization problem is a convex optimization problem and has a unique solution.

The solution to (A.29) is given by

$$\begin{aligned} m_i \quad &:= \arg\min_{e\geq 0}\{\mu(e) + \tfrac{1}{2}\gamma_i\lambda(e) + c_i \cdot e\} \\ B_i^{out} \quad &= (m_i - xe_{-i})^+ \end{aligned}$$

The Nash equilibrium is given by the fixed point of the best-response mappings $B_1^{out}(e_2)$ and $B_2^{out}(e_1)$. Similar to section 2.5.2.1, $e_i^*(m_i, m_{-i})$, the effort of agent i at the *unique* Nash equilibrium can be calculated by (2.28).

Therefore, $u_i^{oo} = \overline{U}_i(e_1^*(m_1, m_2), e_2^*(m_2, m_1))$ is the utility of agent i in the equilibrium when agents do not choose to enter the contract.

Case (ii): One of the agents enters a contract

Consider the game G^{io} between the insured agent 1 and uninsured agent 2. The expected payoffs of agents in this game are as follows,

$$\begin{aligned} &\overline{U}_1^{in}(e_1, e_2, p_1, \alpha_1, \beta_1) = E(-\exp\{-\gamma_1 \cdot (-p_1 + \alpha_1 S_{e_1} + (\beta_1 - 1)L^{(1)}_{e_1,e_2} - c_1 e_1)\}) = \\ &-\exp\{\gamma_1 \cdot (p_1 + (c_1 - \alpha_1)\cdot e_1 + \tfrac{1}{2}\alpha_1^2 \cdot \gamma_1\sigma_1^2 + (1-\beta_1)\mu(e_1 + xe_2) + \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1 + xe_2))\} \\ &\overline{U}_2(e_1, e_2) = E(-\exp\{-\gamma_2(-L^{(2)}_{e_1,e_2} - c_2 e_2)\}) = -\exp\{\gamma_2\mu(e_2 + xe_1) + \tfrac{1}{2}\gamma_2^2\lambda(e_2 + xe_1) + \gamma_2 c_2 e_2\} \end{aligned} \tag{A.30}$$

Let $B_i^{in}(e_{-i})$ denote the best response of agent i. We have,

$$\begin{aligned} B_1^{in}(e_2) \quad &= \arg\max_{e_1\geq 0} -\exp\{\gamma_1(p_1 + (c_1 - \alpha_1)e_1 \\ &+ \tfrac{1}{2}\alpha_1^2\gamma_1\sigma_1^2 + (1-\beta_1)\mu(e_1 + xe_2) + \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1 + xe_2))\} \\ &= \arg\min_{e_1\geq 0}(c_1 - \alpha_1)\cdot e_1 + (1-\beta_1)\mu(e_1 + x\cdot e_2) + \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1 + x\cdot e_2) \end{aligned}$$

Similar to section 2.5.2.2, $B_1^{in}(e_2)$ can be calculated as follows,

$$\begin{aligned} m_1(\alpha_1, \beta_1) \quad &= \arg\min_{e\geq 0}(c_1 - \alpha_1)e + (1-\beta_1)\mu(e) + \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e) \\ B_1^{in}(e_2) \quad &= (m_1(\alpha_1, \beta_1) - xe_2)^+ \end{aligned}$$

For the uninsured agent 2, it is easy to see that the best-response function is exactly the best response function $B_2^{out}(e_1)$ in game G^{oo}.

We can now find the Nash equilibrium as the fixed point of the best-response mappings. Agents' efforts at the equilibrium are $e_1^*(m_1(\alpha_1, \beta_1), m_2)$ and $e_2^*(m_2, m_1(\alpha_1, \beta_1))$ which are defined in (2.28). For notational convenience, we denote these efforts by e_1^*, e_2^*.

Let $\overline{V}^{io}(p_1,\alpha_1,\beta_1,e_1,e_2)$ denote the insurer's utility, when she offers contract (p_1,α_1,β_1) to agent 1, and agents exert efforts e_1,e_2. The optimal contract offered by the insurer is the solution to the following optimization problem:

$$\begin{aligned}
&v^{io} = \max_{p_1,\alpha_1,\beta,e_1^*,e_2^*} \quad \overline{V}^{io}(p_1,\alpha_1,\beta_1,e_1^*,e_2^*) = p_1 - \alpha_1 e_1^* - \beta_1 \cdot \mu(e_1^* + x \cdot e_2^*) \\
&s.t., \\
&\qquad\qquad\qquad \text{(IR)}\overline{U}_1^{in}(e_1^*,e_2^*,p_1,\alpha_1,\beta_1) \geq u_1^{oo}, \\
&\qquad\qquad\qquad \text{(IC) } e_1^*,e_2^* \text{ are the agetn's effort in NE of game } G^{io}
\end{aligned}$$

Notice that, if insurer wants to ensure that the contract is not profitable for second agent, it is sufficient to set p_2 (premium of second contract) large enough.

We first re-write the (IR) constraint for agent 1 as follows,

$$p_1 + (c_1 - \alpha_1) \cdot e_1^* + \tfrac{1}{2}\alpha_1^2\gamma_1\sigma_1^2(1-\beta_1)\mu(e_1^* + xe_2^*) + \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1^* + xe_2^*) \leq w_1^{oo} \ ,$$

where $w_1^{oo} = \frac{\ln(-u_1^{oo})}{\gamma_1}$.

Similar to Lemma 2.1, we can conclude that (IR) constraint is binding in the optimal contract. Therefore, we can re-write the insurer's problem by replacing for the base premium p, and we get following optimization problem,

$$\begin{aligned}
&\max_{p_1,\alpha_1,\beta,e_1^*,e_2^*} \quad w_1^{oo} - \mu(e_1^* + xe_2^*) - \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1^* + xe_2^*) - c_1 e_1^* - \tfrac{1}{2}\alpha_1^2\gamma\sigma_1^2 \\
&s.t., \\
&\qquad\qquad \text{(IC) } e_1^*,e_2^* \text{ are agent's effort in NE of game } G^{io}
\end{aligned} \tag{A.31}$$

Now, let u_2^{io}, the expected payoff of agent 2 when he opts out while agent 1 purchased the optimal contract, and it can be calculated by (A.30) and (A.31).

Similarly, u_1^{oi} denotes expected payoff of agent 1 when he opts out while agent 2 purchased the optimal contract.

Case (iii): Both agents purchase contracts

Assume the insurer offers each agent i a contract (p_i,α_i,β_i). The expected utility of agents when both purchase contracts is given by,

$$\overline{U}_i^{in}(e_1,e_2,p_i,\alpha_i,\beta_i) = E(-\exp\{-\gamma_i(-p_i+\alpha_i S_{e_i} + (-1+\beta_i)L^{(i)}_{e_1,e_2} - c_i e_i)\})$$
$$= -\exp\{\gamma_i \cdot (p_i + (c_i-\alpha_i)\cdot e_i + \tfrac{1}{2}\alpha_i^2 \cdot \gamma_i \sigma_i^2 + (1-\beta_i)\mu(e_i + xe_{-i}) + \tfrac{1}{2}\gamma_i(1-\beta_i)^2\lambda(e_i+xe_{-i}))\}$$

Similar to Section 2.5.2.3, the best-response function of player i, is B_i^{in}, is given by,

$$\begin{aligned} m_i(\alpha_i,\beta_i) &= \arg\min_{e\geq 0}(1-\beta_i)\mu(e) + \tfrac{1}{2}\gamma_i(1-\beta_i)^2\lambda(e) + (c_i-\alpha_i)\cdot e\ . \\ B_i^{in} &= (m_i(\alpha_i,\beta_i) - xe_{-i})^+ \end{aligned}$$

Agents' efforts at the *unique* Nash equilibrium are $e_i^*(m_i(\alpha_i,\beta_i), m_{-i}(\alpha_{-i},\beta_{-i}))$, with $e_i^*(.,.)$ defined in (2.28). For notational convenience, we simply denote these by e_i^*.

To write the insurer's problem, note that the outside option of agent 1 (resp. 2) from this game is the game G^{oi} (resp. G^{io}). The IR constraints can again be shown to be binding, simplifying the insurer's problem to,

$$\begin{aligned} v^{ii} = \max_{\alpha_1, 0\leq\beta_1\leq 1, \alpha_2, 0\leq\beta_2\leq 1, e_1^*\geq 0, e_2^*\geq 0} \quad & w_1^{oi} + w_2^{io} \\ & -\mu(e_1^* + x\cdot e_2^*) - \tfrac{1}{2}\gamma_1(1-\beta_1)^2\lambda(e_1^* + x\cdot e_2^*) \\ & -c_1\cdot e_1^* - \tfrac{1}{2}\alpha_1^2\gamma_1\sigma_1^2 \\ & -\mu(e_2^* + x\cdot e_1^*) - \tfrac{1}{2}\gamma_2(1-\beta_2)^2\lambda(e_2^* + x\cdot e_1^*) \\ & -c_2\cdot e_2^* - \tfrac{1}{2}\alpha_2^2\gamma_2\sigma_2^2 \\ s.t., \quad & \\ & e_1^*, e_2^* \text{ are the agents' effort in NE of game } G^{ii} \end{aligned}$$

where $w_1^{oi} = \frac{\ln(-u_1^{oi})}{\gamma_1}$ and $w_2^{io} = \frac{\ln(-u_2^{io})}{\gamma_2}$.

APPENDIX B

Embracing and Controlling Risk Dependency in Cyber-insurance Policy Underwriting

Proof of Theorem 3.2

Proof (Theorem 3.2) *Notice that* $\overline{U}_o(f'_o) + \sum_{i=1}^{n} \overline{U}_i(f'_o) = \overline{V}_o(f'_o) + \sum_{i=1}^{n} \overline{V}_i(f'_o) = \overline{V}_{total}(f'_o)$. *We assume that* $\overline{U}_o(f^\star_o) \geq 0$, *otherwise no insurer underwrites the SP. By the optimality of* f^{**}_o *for* $\overline{V}_{total}(f'_o)$ *we have,*

$$\overline{V}_{max} = \overline{V}_{total}(f^{**}_o) \geq \overline{V}_{total}(f^\star_o) = \overline{U}_o(f^\star_o) + \sum_{i=1}^{n} \overline{U}_i(f^\star_o) \geq \sum_{i=1}^{n} \overline{U}_i(f^\star_o) = \overline{U}_{max} \tag{B.1}$$

Moreover, similar to the proof of theorem 3.1, by the first order condition we can show that,

$$f^\star_o = \left(f_o - (P'_o)^{-1} \left(\frac{b_o}{\left[l_o + q \cdot \sum_{i=1}^{n} l_i \cdot (t - t \cdot E(P_i(f_i))) \right]} \right) \right)^+ \tag{B.2}$$

Also, from the proof of theorem 3.1, we have,

$$f^{**}_o = \left(f_o - (P'_o)^{-1} \left(\frac{b_o}{\left[l_o + \sum_{i=1}^{n} l_i \cdot (t - t \cdot E(P_i(f_i))) \right]} \right) \right)^+ \tag{B.3}$$

Because $P'_i(.)$ *is an increasing function and* $\frac{b_0}{\left[l_0 + q \cdot \sum_{i=1}^{n} l_i \cdot (t - t \cdot E(P_i(f_i))) \right]} > \frac{b_0}{\left[l_0 + \sum_{i=1}^{n} l_i \cdot (t - t \cdot E(P_i(f_i))) \right]}$, *we have* $f^*_o \leq f^{**}_o$.

Examples of the Loss Probability Function and Optimal Incentive Factors

Let's assume that $P_i(f_i) = \frac{q_i}{\frac{b_i \cdot (a_i - f_i)}{r_i} + 1}$, where q_i, a_i, r_i are constants and $q_i < 1$ and $a_i > f_{max}$. Then we have,

$$E\{P_i(f_i)\} = \frac{q_i \cdot r_i}{b_i \cdot (f_{max} - f_{min})} \ln \frac{b_i \cdot (a_i - f_{min}) + r_i}{b_i(a_i - f_{max}) + r_i} \tag{B.4}$$

Now we find f_o^* and f_o^{**} for the following examples,

- $P_o(f_o - f_o') = \frac{p}{\frac{b_o \cdot (a - (f_o - f_o'))}{r} + 1}$, where p, a, r are constant.

  The optimal incentive factor f_o^* is given by,

  $$f_o^* = (f_o - a + \frac{r}{b_o}(\sqrt{\frac{p \cdot l_o}{r}} - 1))^+ \tag{B.5}$$

  Moreover, we can calculate f_o^{**} as follows,

  $$f_o^{**} = (f_o - a + \frac{r}{b_o}(\sqrt{\frac{p \cdot \left[l_o + \sum_{i=1}^{n} l_i \cdot (t - t \cdot E(p_i(f_i)))\right]}{r}} - 1))^+ \tag{B.6}$$

  Notice that $f_o^{**} \geq f_o^*$.

- $p_o(f_o - f_o') = \frac{p}{(1 + \exp(\frac{b_o \cdot (a - (f_o - f_o'))}{r}))}$ where p, a, r are constants. Notice that this faction is not convex but we will show that $f_o^{**} \geq f_o^*$ in this case as well.

  The optimal incentive factor f_o^* is given by,

  - If $\frac{p \cdot l_o}{r} < 4$, then $f_o^* = 0$
  - If $\frac{p \cdot l_o}{r} > 4$ and $\frac{\frac{p \cdot l_o}{r} - 2 + \sqrt{(2 - \frac{p \cdot l_o}{r})^2 - 4}}{2} < \exp(\frac{b_o \cdot (a - f_o)}{r})$, then $f_o^* = 0$.
  - Otherwise, f_o^* satisfies following equation:

  $$\exp(\frac{b_o \cdot (a - (f_o - f_o^*))}{r}) = \frac{\frac{p \cdot l_o}{r} - 2 + \sqrt{(2 - \frac{p \cdot l_o}{r})^2 - 4}}{2} \tag{B.7}$$

  If the insurer underwrites the service provider and customers, then optimal incentive factor f_o^{**} is given by,

- If $\frac{p\cdot\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right]}{r} < 4$, then $f_o^{**} = 0$
- If $\frac{p\cdot\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right]}{r} > 4$ and $\frac{\frac{p\cdot\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right]}{r}-2+\sqrt{(2-\frac{p\cdot\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right]}{r})^2-4}}{2} < \exp(\frac{b_o\cdot(a-f_o)}{r})$, then $f_o^{**} = 0$
- Otherwise, f_o^{**} satisfies following equation:

$$\exp(\frac{b_o\cdot(a-(f_o-f_o^{**}))}{r}) = \frac{\frac{p\cdot\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right]}{r}-2+\sqrt{(2-\frac{p\cdot\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right]}{r})^2-4}}{2} \tag{B.8}$$

Because $\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right] \geq l_o$, then $f_o^{**} \geq f_o^*$ in this case as well.

- $p_o(f_o - f_o') = q + p\exp(-\frac{b_o\cdot(a-(f_o-f_o'))}{r})$, where p, q, r, a are constant and $p+q<1$ and $f_o < a$.

  By the first order condition we have,

$$f_o^* = \left(f_o - a - \frac{r}{b_o}\ln\frac{r}{p\cdot l_o}\right)^+ \tag{B.9}$$

  Moreover, f_o^{**} is given by,

$$f_o^{**} = \left(f_o - a - \frac{r}{b_o}\ln\frac{r}{p\cdot\left[l_o+\sum_{i=1}^{n} l_i\cdot(t-t\cdot E(P_i(f_i)))\right]}\right)^+ \tag{B.10}$$

All of the above examples imply that the insurer should offer higher discount factor when she underwrites the SP and the customers as compared to the optimal incentive factor which maximizes $\overline{V}_o(f_o')$.

A Cyber-insurance policy: CyberSecurity by Chubb

Premium Calculation

Premiums are calculated by size and type-of-operation, and coverage characteristics associated with e-risks. The policy premium is the sum of the Cyber Liability premium (Coverage A); the premium for any selected optional coverage grants (Coverages B through H); and the premium for the Premier Privacy rate bearing endorsement, if selected.

The limit for any selected optional coverage grant may vary from the Cyber Liability coverage limit, but must be less than or equal to the Cyber Liability coverage limit. Similarly, the retention for any selected optional coverage grant may vary from the Cyber Liability coverage retention (without further restriction).

<u>The Cyber Liability premium is calculated as follows:</u>

(Section 1 Base Rate) x (Section 2 Industry Factor) x (Section 3.1 ILF) x (Section 3.2 Retention Factor)
x (Section 3.3 Coinsurance Factor) x (Section 6 Third-Party Modifier Factors)

<u>The premium for optional coverage grants B and C1 are calculated as follows:</u>

(Section 1 Base Rate) x (Section 2 Industry Factor) x (Section 3.3 Coinsurance Factor)
x (Section 4.#.I Base Rate Factor) x (Section 4.#.II ILF) x (Section 4.#.III Retention Factor)
x (Section 6 Third-Party Modifier Factors) *where # corresponds to the respective coverage grant letter*

<u>The premium for optional coverage grant C2 (Reward Expenses) is calculated as follows:</u>

(Section 2 Industry Factor) x (Section 3.3 Coinsurance Factor) x (Section 4.C2 Base Premium)
x (Section 6 Third-Party Modifier Factors)

<u>The premium for optional coverage grants D, E, F, G and H are calculated as follows:</u>

(Section 1 Base Rate) x (Section 2 Industry Factor) x (Section 3.3 Coinsurance Factor)
x (Section 4.#.I Base Rate Factor) x (Section 4.#.II ILF) x (Section 4.#.III Retention Factor)
x (Section 7 First-Party Modifier Factors) *where # corresponds to the respective coverage grant letter*

<u>The premium for the Premier Privacy endorsement is calculated as follows:</u>

(Section 1 Base Rate) x (Section 2 Industry Factor) x (Section 3.1 ILF) x (Section 3.2 Retention Factor)
x (Section 3.3 Coinsurance Factor) x (Section 5 Premier Privacy Endorsement Factor)
x (Section 6 Third-Party Modifier Factors)

CyberSecurity by Chubb[SM]

Federal Insurance Company

Section 1: Base Rates: Base rates for Financial Institutions use Asset Size as the exposure base, while base rates for all other industries use Annual Revenue as the exposure base.

A1. Cyber Liability – Financial Institutions

Base rates are for a $1,000,000 limit and a base retention based on Asset Size. Applicants with Asset Sizes above $100 billion will be 'a' rated.

Asset Size			Base Rate	Base Retention
	to	$100,000,000	$5,000	$25,000
$100,000,001	to	$250,000,000	$7,000	$25,000
$250,000,001	to	$500,000,000	$8,500	$50,000
$500,000,001	to	$1,000,000,000	$11,000	$100,000
$1,000,000,001	to	$2,500,000,000	$14,000	$150,000
$2,500,000,001	to	$5,000,000,000	$16,500	$250,000
$5,000,000,001	to	$10,000,000,000	$20,000	$250,000
$10,000,000,001	to	$25,000,000,000	$26,000	$500,000
$25,000,000,001	to	$50,000,000,000	$35,000	$500,000
$50,000,000,001	to	$75,000,000,000	$41,000	$1,000,000
$75,000,000,001	to	$100,000,000,000	$45,000	$1,000,000

A2. Cyber Liability – Other than Financial Institutions

Base rates are for a $1 million limit of liability and a base retention based on Annual Revenue. Applicants with Annual Revenue above $1 billion will be 'a' rated.

Annual Revenue			Base Rate	Base Retention
	to	$5,000,000	$5,000	$25,000
$5,000,001	to	$10,000,000	$7,500	$25,000
$10,000,001	to	$25,000,000	$11,500	$25,000
$25,000,001	to	$50,000,000	$16,500	$50,000
$50,000,001	to	$75,000,000	$20,500	$50,000
$75,000,001	to	$100,000,000	$22,500	$50,000
$100,000,001	to	$150,000,000	$24,000	$100,000
$150,000,001	to	$200,000,000	$27,000	$100,000
$200,000,001	to	$300,000,000	$31,000	$100,000
$300,000,001	to	$400,000,000	$33,500	$150,000
$400,000,001	to	$500,000,000	$37,000	$150,000
$500,000,001	to	$750,000,000	$40,000	$250,000
$750,000,001	to	$1,000,000,000	$43,500	$250,000

Section 2: Industry Factors: The appropriate factor should be applied multiplicatively.

Industry – Non-Financials	Factor
Accounting Firms	0.85
Advertising Firms	0.85
Agriculture	0.85
Construction	0.85
Consulting Firms	0.85
Entertainment Industry (PCI* Level 3 – 4)	0.85
Hospitality Industry (PCI* Level 3 – 4)	0.85
Human Resources Firms	0.85
Law Firms	0.85
Manufacturing	0.85
Media Firms	0.85
Other Professional Services Firms	0.85
Publishing Firms	0.85
Retail Merchants; such as Clothing, Grocery (PCI* Level 3 – 4)	0.85
Technology Developers (Software and Hardware Providers)	0.85
Transportation Companies	0.85
All Other Non-Financials	1.00
Energy; such as Oil & Gas, Natural Resources (excluding Utilities)	1.00
Entertainment Industry (PCI* Level 1 – 2)	1.00
Hospitality Industry (PCI* Level 1 – 2)	1.00
Labor Management Trusts	1.00
Not-for-Profit Organizations	1.00
Unions	1.00
Bio-Technology / Pharmaceutical	1.20
Data Aggregators	1.20
Educational Institutions (Schools, Colleges, Universities)	1.20
Gaming (including Online)	1.20
Government Agencies	1.20
Medical / Healthcare Related Services	1.20
Municipalities (Local, County, State)	1.20
Payroll Processing	1.20
Retail Merchants (PCI* Level 1 – 2)	1.20
Technology Service Providers (ASP, Portals, Web Search Engines)	1.20
Telecommunications (including Cable, Internet Service Providers)	1.20
Utilities	1.20

*PCI: Payment Card Industry Data Security Standards

Industry – Financials	Factor
Banks (Assets ≤ $10 Billion)	0.85
Hedge Funds	0.85
Insurance Companies (non-Life)	0.85
Investment Advisors / Asset Managers	0.85
Venture Capital firms	0.85
All Other Financials (Assets ≤ $500 Billion)	1.00
Credit Unions	1.00
Mutual Funds	1.00
Real Estate Investment Trusts	1.00
All Other Financials (Assets > $500 Billion)	1.20
Credit card Processors	1.20
Investment banks	1.20
Life Insurance Companies	1.20
Security Broker /Dealers	1.20
Stock exchanges	1.20

Section 3: Increased Limit Factors, Retention Factors and Coinsurance Factors

1. **<u>Increased Limit Factors</u>:** The appropriate factor should be applied multiplicatively.

   a. For a limit ≤ $1 million the factors in the chart below apply.

Limit	Factor
$100,000	0.300
$250,000	0.500
$500,000	0.700
$1,000,000	**1.000**

Linear interpolation should be used to determine an Increased Limit Factor (ILF) if the desired limit is not shown above (rounded to 3 decimal places).

   b. For a limit > $1 million the ILF is calculated using the following exponential formula (rounded to 3 decimal places): ILF = (limit ÷ 1,000,000)^0.68

The table below show sample ILFs calculated using the above formula:

Limit	Factor
$1,000,000	1.000
$2,000,000	1.602
$2,500,000	1.865
$3,000,000	2.111
$4,000,000	2.567
$5,000,000	2.987
$7,500,000	3.936
$10,000,000	4.786
$15,000,000	6.306
$20,000,000	7.668
$25,000,000	8.925

2. Retention Factors: The appropriate factor should be applied multiplicatively.

Selected Retention	Base Retention							
	$25,000	**$50,000**	**$100,000**	**$150,000**	**$250,000**	**$500,000**	**$1,000,000**	**$2,500,000**
$10,000	1.10	1.21	1.28	1.35	1.39	1.47	1.62	1.80
$25,000	**1.00**	1.10	1.16	1.23	1.26	1.34	1.47	1.63
$50,000	0.91	**1.00**	1.05	1.12	1.14	1.21	1.33	1.48
$100,000	0.87	0.95	**1.00**	1.06	1.09	1.16	1.27	1.41
$150,000	0.82	0.90	0.94	**1.00**	1.03	1.09	1.20	1.33
$250,000	0.79	0.87	0.92	0.98	**1.00**	1.06	1.17	1.30
$500,000	0.75	0.83	0.87	0.93	0.95	**1.00**	1.10	1.22
$750,000	0.71	0.79	0.83	0.88	0.90	0.95	1.05	1.17
$1,000,000	0.68	0.75	0.79	0.84	0.86	0.91	**1.00**	1.11
$2,000,000	0.63	0.69	0.73	0.78	0.79	0.84	0.93	1.03
$2,500,000	0.61	0.67	0.71	0.76	0.77	0.82	0.90	**1.00**
$5,000,000	0.56	0.61	0.64	0.68	0.70	0.74	0.82	0.91

Linear interpolation should be used to determine a Retention Factor if the desired retention is not shown above (rounded to 3 decimal places).

3. Coinsurance Factors: The appropriate factor should be applied multiplicatively.

Coinsurance %	Factor
0.0%	**1.000**
1.0%	0.995
2.5%	0.990
5.0%	0.980
7.5%	0.970
10%	0.960
15%	0.940
20%	0.920
25%	0.900
30%	0.875
40%	0.830
50%	0.780

Linear interpolation should be used to determine a Coinsurance Factor if the desired coinsurance % is not shown above (rounded to 3 decimal places).

Section 4: Optional Coverage Grants

The following coverage grants are available for an additional premium. The limits selected for each of coverage grants B through H can vary from the limit selected in Section 3.1 for Coverage A, but must be less than or equal to the limit selected for Coverage A. The retentions selected for each of coverage grants B through H can vary from the retention selected in Section 3.2 for Coverage A.

B. Privacy Notification Expenses:

I. Base Rate Factors
Base rate factors are for a $1 million limit of liability and a base retention as determined in Section 1. The appropriate base rate factor should be applied multiplicatively.

Privacy Notification Expenses Factor: 0.15

II. Increased Limit Factors
Select a limit for Coverage B less than or equal to the limit selected for Coverage A. Calculate the ILF using the applicable rules and table/formula of Section 3.1.

III. Retention Factors
Select a retention for Coverage B. Calculate the Retention Factor using the applicable rules and table of Section 3.2.

C1. Crisis Management Expenses:

I. Base Rate Factors
Base rate factors are for a $1 million limit of liability and a base retention as determined in Section 1. The appropriate base rate factor should be applied multiplicatively.

Crisis Management Expenses Factor: 0.02

II. Increased Limit Factors
Select a limit for Coverage C1 less than or equal to the limit selected for Coverage A. Calculate the ILF using the applicable rules and table/formula of Section 3.1.

III. Retention Factors
Select a retention for Coverage C1. Calculate the Retention Factor using the applicable rules and table of Section 3.2.

C2. Reward Expenses:

The Reward Expenses base premium is determined based on the selection of the coverage C2 limit and retention from the table below.

	Limit				
Retention	**$10,000**	**$25,000**	**$50,000**	**$75,000**	**$100,000**
$1,000	$75	$165	$300	$425	$525
$2,500	$70	$150	$275	$375	$475
$5,000	$60	$130	$250	$325	$425
$10,000	$50	$100	$200	$275	$350
$25,000	$40	$95	$175	$250	$300

Linear interpolation should be used to determine a Reward Expenses Base Premium if the desired limit and retention combination is not shown above (rounded to whole $).

D. E-Business Interruption and Extra Expenses:

I. Base Rate Factors

Base rate factors are for a $1 million limit of liability and a base retention as determined in Section 1. The appropriate base rate factor should be applied multiplicatively.

E-Business Interruption and Extra Expenses Factor: 0.25

II. Increased Limit Factors

Select a limit for Coverage D less than or equal to the limit selected for Coverage A. Calculate the ILF using the applicable rules and table/formula of Section 3.1.

III. Retention Factors

Select a retention for Coverage D. Calculate the Retention Factor using the applicable rules and table of Section 3.2.

E. E-Theft Loss:

I. Base Rate Factors

Base rate factors are for a $1 million limit of liability and a base retention as determined in Section 1. The appropriate base rate factor should be applied multiplicatively.

E-Theft Loss Factor: 0.25

II. Increased Limit Factors

Select a limit for Coverage E less than or equal to the limit selected for Coverage A. Calculate the ILF using the applicable rules and table/formula of Section 3.1.

III. Retention Factors

Select a retention for Coverage E. Calculate the Retention Factor using the applicable rules and table of Section 3.2.

F. E-Communicaton Loss:

I. Base Rate Factors

Base rate factors are for a $1 million limit of liability and a base retention as determined in Section 1. The appropriate base rate factor should be applied multiplicatively.

E-Communication Loss Factor: 0.10

II. Increased Limit Factors

Select a limit for Coverage F less than or equal to the limit selected for Coverage A. Calculate the ILF using the applicable rules and table/formula of Section 3.1.

III. Retention Factors

Select a retention for Coverage F. Calculate the Retention Factor using the applicable rules and table of Section 3.2.

G. E-Threat Expenses:

I. Base Rate Factors

Base rate factors are for a $1 million limit of liability and a base retention as determined in Section 1. The appropriate base rate factor should be applied multiplicatively.

E-Threat Expenses Factor: 0.20

II. Increased Limit Factors

Select a limit for Coverage G less than or equal to the limit selected for Coverage A. Calculate the ILF using the applicable rules and table/formula of Section 3.1.

III. Retention Factors

Select a retention for Coverage G. Calculate the Retention Factor using the applicable rules and table of Section 3.2.

H. E-Vandalism Expenses:

I. Base Rate Factors

Base rate factors are for a $1 million limit of liability and a base retention as determined in Section 1. The appropriate base rate factor should be applied multiplicatively.

E-Vandalism Expenses Factor: 0.02

II. Increased Limit Factors

Select a limit for Coverage H less than or equal to the limit selected for Coverage A. Calculate the ILF using the applicable rules and table/formula of Section 3.1.

III. Retention Factors

Select a retention for Coverage H. Calculate the Retention Factor using the applicable rules and table of Section 3.2.

Section 5: Premier Privacy Rate Bearing Endorsement

This endorsement applies to the Cyber Liability coverage section, and is available for an additional premium. The endorsement factor is for a $1 million limit of liability and a base retention as determined in Section 1, and should be applied multiplicatively. The limit and retention selected for Coverage A in Section 3.1 and 3.2, respectively, shall apply to this endorsement.

Premier Privacy Endorsement Factor: 0.10

Section 6: Third-Party Modifiers: The appropriate factors should be applied multiplicatively.

1. **Information Systems Security Policy:** Relevant questions include:

   (1) Does the insured maintain an information systems security policy?

   (2) Is the information systems security policy kept current and reviewed at least annually and updated as necessary?

Answer YES to	Factor
Two of the above	0.80 to 0.90
One of the above	0.95 to 1.05
None of the above	1.10 to 1.20

2. **Laptop Security Policy:**

Does the insured have a laptop security policy?	Factor
Yes	0.80 to 0.90
N/A (insured does not use laptops)	1.00
No	1.10 to 1.20

3. **Website Third Party Service Provider:** Relevant questions include:

   (1) Is a written agreement in place between the insured and the third party provider?

   (2) Does the agreement require a level of security commensurate with the insured's information systems security policy?

   (3) Does the insured review the results of the most recent SAS 70 or commensurate risk assessment?

Answer YES to	Factor
N/A (website infrastructure/content is not hosted/managed by a third party)	1.00
Three of the above questions	0.80 to 0.90
Two of the above questions	0.91 to 0.99
One of the above questions	1.00 to 1.05
None of the above questions	1.06 to 1.15

4. **Application Service Provider (ASP):** Relevant questions include:

   (1) Is a written agreement in place between the insured and the ASP?

   (2) Does the agreement require a level of security commensurate with the insured's information systems security policy?

   (3) Does the insured review the results of the most recent SAS 70 or commensurate risk assessment?

Answer YES to	**Factor**
N/A (insured does not use the services of an ASP)	1.00
Three of the above questions	0.80 to 0.90
Two of the above questions	0.91 to 0.99
One of the above questions	1.00 to 1.05
None of the above questions	1.06 to 1.15

5. **Infrastructure Operations Third Party Provider:** Relevant questions include:

   (1) Is a written agreement in place between the insured and the third party provider?

   (2) Does the agreement require a level of security commensurate with the insured's information systems security policy?

   (3) Does the insured review the results of the most recent SAS 70 or commensurate risk assessment?

Answer YES to	**Factor**
N/A (insured does not outsource infrastructure operations)	1.00
Three of the above questions	0.80 to 0.90
Two of the above questions	0.91 to 0.99
One of the above questions	1.00 to 1.05
None of the above questions	1.06 to 1.15

APPENDIX C

Effective Premium Discrimination with Rare Losses: Periodic Pre-screening and Active Policy

Proofs

Proof (Lemma 4.1) *Proof by contradiction. Let* $(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3)$ *be the solution of optimization problem* (4.8)*, and assume that the (IR) constraint is not binding at the optimal contract* $(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3)$*. Because the (IR) constraint is not binding, the insurer can increase her utility by increasing* $\hat{\pi}_2, \hat{\pi}_3$ *while she keeps* $\exp\{\gamma\hat{\pi}_2\} - \exp\{\gamma\hat{\pi}_3\}$ *fixed. Therefore, based on* (4.9) *the agent's effort inside the contract does not change, but the insurer's profit increases. As a result,* $(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3)$ *is not an optimal contract. This is the contradiction implying that the (IR) constraint is binding.*

■

Proof (Theorem 4.1) *Proof by contradiction: Assume that* $\hat{e} = 0$ *and* $t = 1$ *and* $\left[\frac{(\alpha-\gamma c)(\exp\{\gamma l\}-1)}{\gamma c}\right] > 1$*. First we show that under these assumptions,* $\hat{\pi}_1 = \hat{\pi}_2 = \frac{1}{\gamma}\ln(1-u^o) := w^o$*. Because* $\hat{e} = 0$ *and* $t = 1$*, the optimization problem for finding* $(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3)$ *is as follows,*

$$\begin{aligned}
&\max_{\{\pi_1,\pi_2,\pi_3\}} \pi_1 + \pi_2 - 2l \\
&s.t., \\
&(IR)\ 1 - \exp\{\gamma\pi_1\} + 1 - \exp\{\gamma\pi_2\} = 2u^o \\
&(IC)\ 0 = e^{in}(\pi_1, \pi_2, \pi_3)
\end{aligned} \tag{C.1}$$

By (IR) constraint we have,

$$\frac{1}{\gamma}\ln(2 - 2u^o - \exp\{\gamma\pi_1\}) = \pi_2 \tag{C.2}$$

Therefore, we re-write the optimization problem (C.1) *as follows,*

$$\begin{aligned}
&\max_{\{\pi_1,\pi_2,\pi_3\}} \pi_1 + \frac{1}{\gamma}\ln(2-2u^o-\exp\{\gamma\pi_1\}) - 2l \\
&s.t., \\
&(IC)\ 0 = e^{in}(\pi_1,\pi_2,\pi_3) \\
&\quad \frac{1}{\gamma}\ln(2-2u^o-\exp\{\gamma\pi_1\}) = \pi_2
\end{aligned} \tag{C.3}$$

Because π_3 does not appear in the objective function, we first find π_1 and π_2 such that they maximize the objective function. Then, we pick π_3 such that (IC) constraint is satisfied. By the first order optimality condition for the objective function, we have,

$$\hat{\pi}_1 = \hat{\pi}_2 = \frac{1}{\gamma}\ln(1-u^o) \tag{C.4}$$

Without loss of generality, we set $\hat{\pi}_3 = \frac{1}{\gamma}\ln(\frac{\alpha-\gamma c}{\alpha}(1-u^o))$. By (4.9)*, $\hat{e} = 0$ (Notice that $\frac{\alpha}{\gamma c}\frac{\exp\{\gamma\hat{\pi}_2\}-\exp\{\gamma\hat{\pi}_3\}}{\exp\{\gamma\hat{\pi}_1\}} = 1$ and a slight decrease in $\hat{\pi}_3$, increases the agent's effort based on* (4.9)*).*

Now we show that the decrease in $\hat{\pi}_3$ increases the insurer's payoff. Notice that a slight decrease in $\hat{\pi}_3$, increases the agent's effort (based on (4.9)*) and improves agents' utility and the (IR) constraint is not violated. We write the insurer's objective function as a function of π_3. Therefore, we have (derivatives in the following equation are left derivatives),*

$$\begin{aligned}
h(\pi_3) &= \hat{\pi}_1 - p(e^{in}(\hat{\pi}_1,\hat{\pi}_2,\pi_3))(l-\hat{\pi}_2) + (1-p(e^{in}(\hat{\pi}_1,\hat{\pi}_2,\pi_3)))\pi_3 - l \\
\frac{\partial h}{\partial \pi_3}|_{\pi_3=\hat{\pi}_3} &= \frac{\partial p(e^{in}(\hat{\pi}_1,\hat{\pi}_2,\pi_3))}{\partial \pi_3}\cdot(\hat{\pi}_2 - l) \\
&- \frac{\partial p(e^{in}(\hat{\pi}_1,\hat{\pi}_2,\pi_3))}{\partial \pi_3}\cdot\pi_3 + (1-p(e^{in}(\hat{\pi}_1,\hat{\pi}_2,\pi_3))) \\
&= \left(\frac{\partial p(e^{in}(\hat{\pi}_1,\hat{\pi}_2,\pi_3))}{\partial \pi_3}|_{\pi_3=\hat{\pi}_3}\cdot(-l+\hat{\pi}_2-\hat{\pi}_3) - (1-p(e_1^{in}(\hat{\pi}_1,\hat{\pi}_2,\hat{\pi}_3)))\right)
\end{aligned}$$

Because $\left[\frac{(\alpha-\gamma c)(\exp\{\gamma l\}-1)}{\gamma c}\right] > 1$, (4.5) *implies that e^o is not zero and $\hat{\pi}_2 = \frac{1}{\gamma}\ln(1-u^o) < l$. Moreover, $\frac{\partial p(e^{in}(\hat{\pi}_1,\hat{\pi}_2,\pi_3))}{\partial \pi_3}|_{\pi_3=\hat{\pi}_3} > 0$ implies that $\frac{\partial h}{\partial \pi_3}|_{\pi_3=\hat{\pi}_3} < 0$. Therefore, the decrease in $\hat{\pi}_3$ increases the insurer's payoff. This is a contradiction and the agent exerts non-zero effort in the optimal contract under given assumptions.*

■

Proof (Theorem 4.2) *By* (4.14)*, the agent exerts non-zero effort in a contract if* $\beta = c$*. If the discount factor* $\beta = c$*, then any positive number satisfies the (IC) constraint. Therefore, if* $\beta = c$*, then the desired effort maximizes the insurer's utility. By* (4.14)*, we have,*

$$\overline{e} = \arg\max_{e} w^o - ce - tl\exp\{-\alpha \cdot e\} - \gamma c^2\sigma^2 \tag{C.5}$$

By the first order condition of optimality, the solution of above optimization problem is $\overline{e} = (\frac{1}{\alpha}\ln(\frac{\alpha \cdot t \cdot l}{c}))^+$*. Moreover, if* $\overline{e} > 0$*, then the maximum insurer's profit using pre-screening (i.e.,* $\beta = c$*) is given by,*

$$\left\{ w^o - \frac{c}{\alpha}\ln(\frac{\alpha tl}{c}) - \frac{c}{\alpha} - \frac{\gamma c^2\sigma^2}{2} \right\} \tag{C.6}$$

Without pre-screening (i.e., $\beta = 0$*), the agent exerts zero effort and the insurer's profit is given by,*

$$w^o - t \cdot l \tag{C.7}$$

Therefore, the insurer uses pre-screening if and only if,

$$\begin{aligned} &\frac{1}{\alpha}\ln(\frac{\alpha \cdot t \cdot l}{c}) > 0 \\ &w^o - \frac{c}{\alpha}\ln(\frac{\alpha tl}{c}) - \frac{c}{\alpha} - \frac{\gamma c^2\sigma^2}{2} \geq w^o - tl \end{aligned} \tag{C.8}$$

In other words, the insurer uses pre-screening and the agent exerts non-zero effort if and only if,

$$\begin{aligned} &\frac{\alpha \cdot t \cdot l}{c} > 1 \\ &\sigma^2 \leq \frac{2}{\gamma c^2}(tl - \frac{c}{\alpha}(1 + \ln(\frac{\alpha tl}{c})) \end{aligned} \tag{C.9}$$

■

Proof (Theorem 4.3) *Assume* $\sigma < \sigma'$*.*

Let $g(\beta, e, \sigma) = \left[w^o - ce - \frac{\gamma\beta^2\sigma^2}{2} - p(e)l \right]$*. It is easy to see that* $g(\beta, e, \sigma') \leq g(\beta, e, \sigma)$*. Therefore,* *we have,*

$$\max_{\beta,e,IC\ constraint} g(\beta,e,\sigma') \leq \max_{\beta,e,IC\ constraint} g(\beta,e,\sigma)$$

Therefore, $V(\sigma') \leq V(\sigma)$.

■

Proof (Theorem 4.4)

- *By* (4.9)*, the agent exerts zero effort if* $t_a \frac{\alpha}{\gamma c} \frac{\exp\{\gamma\pi_2\}-\exp\{\gamma\pi_3\}}{\exp\{\gamma\pi_1\}} \leq 1$. *Because* t_a *goes to zero,* $t_a \frac{\alpha}{\gamma c} \frac{\exp\{\gamma\pi_2\}-\exp\{\gamma\pi_3\}}{\exp\{\gamma\pi_1\}}$ *also goes to zero. Therefore, the agent exerts zero effort under any insurance contract.*

- *Because the agent exerts zero effort inside the optimal contract, his utility is given by,*

$$\begin{aligned} &U^{in}(0,\pi_1,\pi_2,\pi_3) = -\exp\{\gamma\pi\} - t_a \exp\{\gamma\pi_2\} - (1-t_a)\exp\{\gamma\pi_3\} \\ &\textit{(IR) is binding and } t_a \to 0 \Rightarrow\ 1-\exp\{\gamma\pi_1\} + 1 - \exp\{\gamma\pi_3\} = 2u^o \end{aligned} \tag{C.10}$$

Therefore, the insurer's problem (4.8) *can be written as follows,*

$$\begin{aligned} &\max_{\pi_1,\pi_2,\pi_3} \pi_1 + \pi_3 - 2\cdot l_p \\ &s.t., \exp\{\gamma\pi_1\} + \exp\{\gamma\pi_3\} = 2 - 2u^o \end{aligned} \tag{C.11}$$

or

$$\max_{\pi_1} \pi_1 + \frac{1}{\gamma}\ln(2-2u^o - \exp\{\gamma\pi_1\}) - 2l_p \tag{C.12}$$

The optimal solution for the above optimization problem is $\pi_1 = \pi_3 = \frac{1}{\gamma}\ln(1-u^o)$ *and also the value of* π_2 *does not affect insurer's or agent's utility and can be any positive value.*

■

Proof (Theorem 4.5) *The proof is similar to the proof of theorem 4.2 except that we should substitute* l_p *for* $t \cdot l$.

■

Proof (Theorem 4.6) *As the (IR) constraint is binding in* (4.23)*, similar to* (4.14) *we can re-write optimization problem* (4.23) *as follows,*

$$
\begin{aligned}
&R(\sigma) = \max_{\{\beta,e,\beta',e'\}}\left[w^o - ce + b(e-e') - \gamma\tfrac{(\beta-\beta')^2\sigma^2+(\beta')^2\sigma^2}{2} - p(e')l\right]\\
&s.t., (IC)(e,e') \in \arg\min_{(\tilde{e}\geq\tilde{e}')} \gamma(c-b+\beta'-\beta)\tilde{e} + \gamma(-\beta'+b)\tilde{e}'
\end{aligned} \tag{C.13}
$$

First we show that $\hat{e} = \hat{e}'$. Proof by contradiction. Assume $\hat{e} > \hat{e}' \geq 0$. Then, $\beta' - \beta = b - c$ since otherwise $\hat{e} = \infty$ or $\hat{e} = 0$. As $b \leq c$, then the objective function of (C.13) *can be improved by decreasing $\hat{e}$ without violating (IC) constraint. This contradiction shows that $\hat{e} = \hat{e}'$.*

By IC constraint, it is easy to see that if $\hat{e} = \hat{e}' > 0$, then $\beta = c$, and $\beta' = b$.

Let $\beta = \bar{\beta}, e = \bar{e}$ be the solution to (4.14)*. According to the IC constraint of* (4.14)*, two cases can happen:*

i) $\overline{\beta} = 0$ and $\overline{e} = 0$. Then, ($\beta = \beta' = e = e' = 0$) satisfies the IC constraint in (C.13) *and is a feasible point. We have,*

$$
\begin{aligned}
w^o - c\overline{e} - \frac{\gamma\overline{\beta}^2\sigma^2}{2} - p(\overline{e})l &=\\
w^o - c\overline{e} + b(\overline{e}-\overline{e}) - \gamma\frac{(\overline{\beta}-\overline{\beta}')^2 + (\overline{\beta}')^2}{2}\sigma^2 - p(\overline{e})l
\end{aligned} \tag{C.14}
$$

ii) $\overline{\beta} = c$. Then ($\beta = c, \beta' = b, e = e' = \overline{e}$) is a feasible point for (C.13) *and satisfies the IC constraint. We have,*

$$
\begin{aligned}
w^o - c\overline{e} - \frac{\gamma c^2\sigma^2}{2} - p(\overline{e})l &\leq\\
w^o - c\cdot\overline{e} + b(\overline{e}-\overline{e}) - \gamma\frac{(c-b)^2 + b^2}{2}\sigma^2 - p(\overline{e})l
\end{aligned} \tag{C.15}
$$

Note that in this case ($\beta = c, \beta' = b, e = e' = \overline{e}$) is the solution to (C.13)*.*

By (C.14) *and* (C.15) *we have, $V(\sigma) \leq R(\sigma)$. Notice that if $b = c$, then* (C.13) *and* (4.14) *are equivalent and $V(\sigma) = R(\sigma)$ as $\hat{e} = \hat{e}'$.*

■

APPENDIX D

Resource Pooling for Shared Fate: Incentivizing Effort in Interdependent Security Games through Cross-investments

On the voluntary participation and budget balance constraints for taxation mechanisms

Neghizadeh and Liu [62] consider a model with a strictly concave utility function and show that the taxation mechanisms may not be able to satisfy the voluntary participation and budget balance constraints simultaneously. In this part, we provide an example to show that their result can be extended to the quadratic utility model. Consider an example with the following parameters,

$$n = 30, \quad x_{ij} = 1 \ \forall i, j, \ i \neq j, \quad b_i = 30, \ a_i = 1, \ l_i = 0 \ \forall i. \tag{D.1}$$

In this example, the social welfare at the socially optimal outcome of game G is given by,

$$\begin{aligned} \boldsymbol{e}^* &= (2B - X - X^T)^{-1} \cdot a = [0.5 \ldots 0.5]^T, \\ u_i(\boldsymbol{e}^*) &= 0.25 \ \forall i, \\ \sum_{i=1}^{n} u_i(\boldsymbol{e}^*) &= 7.5. \end{aligned} \tag{D.2}$$

By the notion of exit equilibrium defined in [62], the agents' effort when agent i unilaterally opts out of the taxation mechanism is given by,

$$
\begin{aligned}
\hat{e}_i^{(i)} &= \arg\max_{e_i \geq 0} u_i(e_i, \hat{e}_{-i}^{(i)}), \\
\hat{e}_{-i}^{(i)} &= \arg\max_{e_{-i} \geq 0} \sum_{j \neq i} u_j(\hat{e}_i^{(i)}, e_{-i}),
\end{aligned} \tag{D.3}
$$

where, $\hat{e}_i^{(i)}$ is the effort of agent i, and $\hat{e}_{-i}^{(i)}$ is a $(n-1)$-dimensional vector denoting effort of the agents excluding agent i at the exit equilibrium. Using the first order condition, the solution to (D.3) satisfies the following system of linear equations,

$$
\begin{aligned}
a_i - 2 \cdot b_i \cdot \hat{e}_i^{(i)} + \sum_{k=1}^{n} x_{ik} \hat{e}_k^{(i)} &= 0, \\
a_j - 2 \cdot b_j \cdot \hat{e}_j^{(i)} + \sum_{k=1}^{n} x_{jk} \hat{e}_k^{(i)} + \sum_{k \neq i} x_{kj} \hat{e}_k^{(i)} &= 0 \; \forall j \neq i,
\end{aligned} \tag{D.4}
$$

or equivalently,

$$
(2B - X - X_{[i]}^T) \cdot \hat{\boldsymbol{e}}^{(i)} = a, \tag{D.5}
$$

where $\hat{\boldsymbol{e}}^{(i)} = [\hat{e}_1^{(i)}, \ldots, \hat{e}_n^{(i)}]^T$, and entry (r, s) of $X_{[i]}$ is equal to x_{rs} if $r \neq i$ and $s \neq i$. Otherwise, it is zero.

In our example, the utility of agent i when he is the outlier, and the other agents are participating in the taxation mechanism is given by,

$$
\begin{aligned}
\hat{\boldsymbol{e}}^{(i)} &= (2B - X - X_{[i]}^T)^{-1} \cdot a, \\
\hat{e}_i^{(i)} &= 0.1564, \\
\hat{e}_j^{(i)} &= 0.2891 \;\; \forall j \neq i, \\
u_i(\hat{\boldsymbol{e}}^{(i)}) &= 0.7338, \\
u_j(\hat{\boldsymbol{e}}^{(i)}) &= 0.1672.
\end{aligned} \tag{D.6}
$$

By symmetry, it is easy to see that $u_i(\hat{\boldsymbol{e}}^{(i)}) = 0.7338, \forall i$.

By [62], if $\sum_{i=1}^{n} u_i(\boldsymbol{e}^*) - \sum_{i=1}^{n} u_i(\hat{\boldsymbol{e}}^{(i)}) < 0$, then there is no taxation mechanism which induces the socially optimal outcome and satisfies both week budget balance and voluntary participation constraints. In our example, we have $\sum_{i=1}^{n} u_i(\boldsymbol{e}^*) - \sum_{i=1}^{n} u_i(\hat{\boldsymbol{e}}^{(i)}) = -14.5143 < 0$ which shows that the result in [62] can be extended to the quadratic utility model.

proofs

Proof (Theorem 5.1)

- *Let $\boldsymbol{v}$ be the eigenvector of matrix $2B - X$ and λ be its eigenvalue corresponding eigenvector $\boldsymbol{v}$. Without loss of generality, we can assume that v_i is the maximum element of $\boldsymbol{v}$ in absolute value ($|v_i| \geq |v_j|, \forall j$). Note that $|v_i| > 0$ as the zero vector cannot be an eigenvector by the definition of eigenvector. We have,*

$$
\begin{aligned}
(2B - X)\cdot \boldsymbol{v} &= \lambda \cdot \boldsymbol{v} \implies \\
|\lambda \cdot v_i| &= |2b_i \cdot v_i - \sum_{j=1}^{n} x_{ij} v_j| \\
&\geq |2b_i \cdot v_i| - |\sum_{j=1}^{n} x_{ij} v_i \geq 2b_i \cdot |v_i| - \sum_{j=1}^{n} x_{ij}|v_j| \\
&\geq \underbrace{(2b_i - \sum_{j=1}^{n} x_{ij})}_{>0} |v_i| > 0 \implies |\lambda| > 0
\end{aligned}
\tag{D.7}
$$

Therefore, eigenvalues of $2B - X$ are non-zero and $2B - X$ is invertible.

- *Let $\overline{e}$ be a constant such that $\overline{e} > \max_i \frac{a_i}{2b_i - \sum_{j=1}^{n} x_{ij}}$. Consider game $G' = \left\{\mathcal{V}, \{u_i\}_{i\in\mathcal{V}}, \overline{A} = [0, \overline{e}]^n\right\}$. Note that $\overline{A}$ (the action space of game G') is convex and compact and utility $u_i(e_i, e_{-i})$ is concave in e_i. Therefore, by Brouwer fixed-point theorem, the best response mapping of game G' has at least one fixed point (Nash equilibrium). Let $\hat{\boldsymbol{e}}'$ be the Nash equilibrium of game G', and $\hat{e}'_i$ be the maximum element in $\hat{\boldsymbol{e}}'$. By (5.2), we know that $\hat{e}'_j \neq 0, \forall j$. We have,*

$$
\begin{aligned}
\frac{d\,u_i(\boldsymbol{e})}{d\,e_i}|_{\boldsymbol{e}=\hat{\boldsymbol{e}}'} &\geq 0 \;\; (\textit{equality holds if } \hat{e}'_i < \overline{e}) \\
a_i - 2b_i\hat{e}'_i + \sum_{j=1}^{n} x_{ij}\hat{e}'_j &\geq 0 \implies \\
(2b_i - \sum_{j=1}^{n} x_{ij}) \cdot \hat{e}'_i &\leq a_i \implies \\
\hat{e}'_i \leq \frac{a_i}{2b_i - \sum_{j=1}^{n} x_{ij}} &< \overline{e}.
\end{aligned} \tag{D.8}
$$

Therefore, $\hat{e}'_i < \overline{e}$ *which implies that* $\hat{\boldsymbol{e}}'$ *is an interior point of set* $\overline{A}$ *and it should be an NE for game G as well. Therefore, game G has at least one Nash equilibrium.*

By (5.3), *the fixed point of best response mapping of game* $G(X)$ *satisfies the following,*

$$
(2B - X) \cdot \hat{\boldsymbol{e}} = \boldsymbol{a}
$$

As $(2B-X)$ *is invertible, the best response mapping has a unique fixed point* $\hat{\boldsymbol{e}} = (2B-X)^{-1} \cdot \boldsymbol{a}$. *As game* $G(X)$ *has at least one Nash equilibrium, and fixed point* $(2B-X)^{-1} \cdot \boldsymbol{a}$ *is the* only *candidate for NE,* $(2B-X)^{-1} \cdot \boldsymbol{a}$ *should be a non-negative vector and a unique NE for* $G(X)$.

Proof (Corollary 5.1) *Let* $\mathbf{0} \in \mathbb{R}^n$ *be a zero vector. By Theorem 5.1, we know that* $(2 \cdot B - X)^{-1} \cdot \tilde{\boldsymbol{a}} \geq \mathbf{0}$ *for any non-negative vector* $\tilde{\boldsymbol{a}}$. *Set* $\tilde{a}_i = 1$ *and* $\tilde{a}_j = 0$, $\forall j \neq i$ *and* $\tilde{\boldsymbol{a}} = [\tilde{a}_1, \cdots, \tilde{a}_n]^T$. *Then,* $(2 \cdot B - X)^{-1} \cdot \tilde{\boldsymbol{a}} \geq \mathbf{0}$ *is the ith column of* $(2 \cdot B - X)^{-1}$. *Because i is arbitrary, all columns of* $(2 \cdot B - X)^{-1}$ *are non-negative. Moreover, we have,*

$$
\begin{aligned}
(2B - X) \cdot \hat{\boldsymbol{e}} &= \boldsymbol{a} \\
(2B - X - \tilde{X}) \cdot \tilde{\boldsymbol{e}} &= \boldsymbol{a} \implies \\
\tilde{\boldsymbol{e}} = (2B - X)^{-1} \cdot \boldsymbol{a} &+ (2B - X)^{-1} \cdot \tilde{X} \cdot \tilde{\boldsymbol{e}} \\
&= \hat{\boldsymbol{e}} + \underbrace{(2B - X)^{-1} \cdot \tilde{X} \cdot \tilde{\boldsymbol{e}}}_{\geq \mathbf{0}} \geq \hat{\boldsymbol{e}}
\end{aligned}
$$

Proof (Theorem 5.2) *Define $f(\boldsymbol{e})$ as follows:*

$$\begin{aligned} f(\boldsymbol{e}) &= \sum_{i=1}^{n} u_i(\boldsymbol{e}) \\ &= \sum_{i=1}^{n} \left(-l_i + a_i \cdot e_i - b_i e_i^2 + e_i \cdot \sum_{j=1}^{n} x_{ij} e_j \right) \end{aligned} \tag{D.9}$$

First, notice that the Hessian of $f(.)$ is $H = -2B + X + X^T$, and H is a symmetric matrix with real eigenvalues. Similar to the proof of Theorem 5.1, we can show that if $2b_i \geq \sum_{j=1}^{n} x_{ij} + x_{ji}$, $\forall i$, then all eigenvalues of H are negative implying that $f(.)$ is strictly concave and H is invertible. Therefore, we can use the first order condition to find $\boldsymbol{e}^*$:

$$\begin{aligned} &\nabla f(\boldsymbol{e}^*) = \boldsymbol{a} - (2B - X - X^T) \cdot \boldsymbol{e}^* = 0 \implies \\ &\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a} \ . \end{aligned} \tag{D.10}$$

Note that $\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$ is the NE of game $G(X + X^T)$, which implies that $(2B - X - X^T)^{-1} \cdot \boldsymbol{a} \geq \boldsymbol{0}$. The result then follows from Corollary 5.1.

Proof (Theorem 5.8) *Consider game G_{rp}^1. In this game $e_{1j} = e_{j1} = 0$ for all $j \in \mathcal{V} - \{1\}$. Let $\mathring{E} = [\mathring{e}_{ij}]_{n \times n}$ be the NE of G_{rp}^1 with $\mathring{e}_{1j} = \mathring{e}_{j1} = 0$, $\forall j \in \mathcal{V} - \{1\}$. Moreover, let $\mathring{E}_i = \sum_{j=1}^{n} \mathring{e}_{ji}$.*

By the first order condition, best response of agent 1 is given by,

$$2b_1 \mathring{e}_{11} - \sum_{j=1}^{n} x_{1j} \mathring{E}_j = a_1 \tag{D.11}$$

Moreover, by best response function of agent $i > 1$, we have,

$$\begin{aligned} &2b_i \mathring{e}_{ii} - \sum_{j=1}^{n} x_{ij} \mathring{E}_j = a_i \\ &2b_j \mathring{e}_{ij} - x_{ij} \mathring{E}_i = 0, j \neq i \end{aligned} \tag{D.12}$$

Similar to (5.9) *and by equation* (D.11) *and* (D.12), $\begin{bmatrix} \mathring{E}_1 \\ \vdots \\ \mathring{E}_n \end{bmatrix}$ *satisfies the following,*

$$(2B - X_{[1]}^T - X)\begin{bmatrix} \mathring{E}_1 \\ \vdots \\ \mathring{E}_n \end{bmatrix} = \boldsymbol{a}, \tag{D.13}$$

where, all the elements of $X_{[1]}$ *are equal to X except its first row and column which are zero vectors.*

Similar to Theorem 5.3, if $2b_i > \sum_{j=1}^{n}\left[x_{ij} + x_{ji}\right]$, $\forall i$*, then game* G_{rp}^1 *has a unique Nash equilibrium, and we have,*

$$\begin{aligned}
\begin{bmatrix} \mathring{E}_1 \\ \vdots \\ \mathring{E}_n \end{bmatrix} &= (2B - X_{[1]}^T - X)^{-1}\boldsymbol{a} \\
\mathring{e}_{11} &= \frac{a_1}{2b_1} + \frac{\sum_{k=1}^{n} x_{1k} \cdot \mathring{E}_k}{2b_1} \\
\mathring{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^{n} x_{ik} \cdot \mathring{E}_k}{2b_i} \ \forall i > 1 \\
\mathring{e}_{ji} &= \frac{x_{ji} \cdot \mathring{E}_j}{2b_i} \ \forall j \neq i, \ i > 1, \ j > 1
\end{aligned} \tag{D.14}$$

By Corollary 5.1 and equation (D.14) *and* (5.10)*, it is easy to see that* $\mathring{E}_i \leq \hat{E}_i$ *and* $\mathring{e}_{ij} \leq \hat{e}_{ij}$, $\forall i, j$.

$$
\begin{aligned}
v_1(\mathring{E}) &= -l_1 + a_1\mathring{e}_{11} - b_1(\mathring{e}_{11})^2 + (\mathring{e}_{11})\sum_{j=1}^{n} x_{1j}\mathring{E}_j \\
&\leq -l_1 + a_1(\mathring{e}_{11} + \sum_{j=2}^{n}\hat{e}_{j1}) - b_1\mathring{e}_{11}^2 \\
&+ \underbrace{(\hat{e}_{11} + \sum_{j=2}^{n}\hat{e}_{j1})\sum_{j=1}^{n}\left(x_{1j}(\sum_{k\neq 1}\hat{e}_{kj})\right)}_{\leq} \\
&\quad \textit{by definiton of NE for } G_{rp} \\
&- \quad l_1 + a_1(\hat{e}_{11} + \sum_{j=2}^{n}\hat{e}_{j1}) \\
&- \sum_{j=1}^{n} b_j\hat{e}_{1j}^2 + (\hat{e}_{11} + \sum_{j=2}^{n}\hat{e}_{j1})\sum_{j=1}^{n} x_{1j}\hat{E}_j \\
&= v_i(\hat{E}) \qquad \text{(D.15)}
\end{aligned}
$$

Therefore, the resource pooling satisfies the voluntary participation with respect to agent 1. *Similarly, we can show that it satisfies the voluntary participation with respect to any agent.*

Proof (Theorem 5.9) *The proof is similar to the proof of theorem 5.8. Consider game* $\overline{G}_{rp}^{1}$. *With the same argument we have,*

$$
\begin{aligned}
\begin{bmatrix} \acute{E}_1 \\ \vdots \\ \acute{E}_n \end{bmatrix} &= (2B - X_{[r1]}^T - X)^{-1}\boldsymbol{a} \\
\acute{e}_{11} &= \frac{a_1}{2b_1} + \frac{\sum_{k=1}^{n} x_{1k}\cdot \acute{E}_k}{2b_1} \\
\acute{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^{n} x_{ik}\cdot \acute{E}_k}{2b_i} \ \forall i > 1 \\
\acute{e}_{ji} &= \frac{x_{ji}\cdot \acute{E}_j}{2b_i} \ \forall j \neq i,\ j > 1 \qquad \text{(D.16)}
\end{aligned}
$$

where all the elements of $X_{[r1]}$ *are equal to* X *except its first row which is a zero vector. By*

Corollary 5.1 and equation (D.16) *and* (5.10)*, it is easy to see that* $\acute{E}_i \leq \hat{E}_i$ *and* $\acute{e}_{ij} \leq \hat{e}_{ij}$, $\forall i, j$. *With the similar procedure as equation* (D.15)*, we can conclude that the resource pooling satisfies the strong voluntary participation defined in Definition 5.2 with respect to agent* 1*. Similarly, resource pooling satisfies the strong voluntary participation with respect to all agents.*

Proof (Theorem 5.10) *Proof is similar to the proof of Theorem 5.3. We use the first order condition to find the best response functions.*

$$\begin{aligned}
\frac{\partial v_i(\boldsymbol{e}_i, \boldsymbol{e}_{-i})}{\partial e_{ii}} &= 0 \implies \\
\check{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik} \cdot \check{E}_k}{2b_i} \forall i \\
\frac{\partial v_j(\boldsymbol{e}_j, \boldsymbol{e}_{-j})}{\partial e_{ji}} &= 0 \implies \\
\check{e}_{ji} &= \frac{x_{ji} \cdot \check{E}_j}{2b_i} \; \forall j \neq i, j \in C_{I(i)} \implies \\
&\textit{by adding above equations:} \\
2b_i \cdot \check{E}_i &= a_i + \left(\sum_{j=1}^n x_{ij} \check{E}_j \right) \\
&+ \left(\sum_{j \in C_{I(i)}} x_{ji} \check{E}_j \right) \; \forall i \in \mathcal{V} \\
\implies \boldsymbol{a} &= (2B - X - X_c^T) \cdot \begin{bmatrix} \check{E}_1 \\ \vdots \\ \check{E}_n \end{bmatrix}
\end{aligned} \tag{D.17}$$

Under assumption 5.2, $(2B - X - X_c^T)$ *is invertible. We have,*

$$(2B - X - X_c^T)^{-1} \boldsymbol{a} = \begin{bmatrix} \check{E}_1 \\ \vdots \\ \check{E}_n \end{bmatrix} \tag{D.18}$$

Moreover, by best response mapping we have,

$$\begin{aligned}
\check{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{j=1}^{n} x_{ij}\check{E}_j}{2b_i}, \\
\check{e}_{ij} &= \frac{x_{ij}\cdot \check{E}_i}{2b_j}, \forall j \neq i, j \in C_{I(i)}
\end{aligned} \tag{D.19}$$

Therefore, the bast response mapping has a unique fixed point implying uniqueness of NE.

Proof (Theorem 5.11)

$$\begin{aligned}
\boldsymbol{e}^* &= (2B - X - X^T)^{-1}\cdot \boldsymbol{a} \\
\hat{\boldsymbol{e}} &= (2B - X)^{-1}\cdot \boldsymbol{a} \\
\begin{bmatrix} \check{E}_1 \\ \vdots \\ \check{E}_n \end{bmatrix} &= (2B - X - X_c^T)^{-1}\cdot \boldsymbol{a} \\
(2B - X) &\geq (2B - X - X_c^T) \\
&\geq (2B - X - X^T) \\
\textit{Corollary 1} \implies \boldsymbol{e}^* &\geq \begin{bmatrix} \check{E}_1 \\ \vdots \\ \check{E}_n \end{bmatrix} \geq \hat{\boldsymbol{e}}
\end{aligned} \tag{D.20}$$

Next we show that $v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}) \geq u_i(\hat{e}_i, \hat{e}_{-i})$. *Let* $\tilde{\boldsymbol{e}}_i$ *be a vector with length* $|C_{I(i)}|$ *and all its elements are zero except* e_{ii} *which is equal to* $\hat{e}_i$ *(effort level of agent i at NE of game G). By definition of NE, we have,*

$$v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}) \geq v_i(\tilde{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}). \tag{D.21}$$

As $\check{E}_i \geq \hat{e}_i$, $\forall i$, *by* (D.19) *and* (5.3) *we have* $\check{e}_{ii} \geq \hat{e}_i$. *Moreover,*

$$
\begin{aligned}
v_i(\tilde{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i}) = -l_i + a_i \cdot \hat{e}_i + a_i \sum_{k \neq i, k \in C_{I(i)}} \check{e}_{ki} - b_i \cdot (\hat{e}_i)^2 \\
+(\hat{e}_i + \sum_{k \neq i, k \in C_{I(i)}} \check{e}_{ki}) \cdot \sum_{j=1}^{n} \left(x_{ij} \cdot (\sum_{k \neq i, k \in C_{I(j)}} \check{e}_{kj}) \right) \geq \\
-l_i + a_i \cdot \hat{e}_i - b_i \cdot (\hat{e}_i)^2 + \hat{e}_i \cdot \sum_{j=1}^{n} x_{ij} \cdot \hat{e}_j = u_i(\hat{e}_i, \hat{e}_{-i})
\end{aligned}
\tag{D.22}
$$

By (D.21) *and* (D.22), $v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}})\ \forall i \in V$.

Proof (Theorem 5.12) *We first show* $\check{\overline{E}}_i \geq \check{E}_i$. *Note that* $\begin{bmatrix} \check{E}_1 \\ \vdots \\ \check{E}_n \end{bmatrix} = (2B - X - X_c^T)^{-1} \cdot \boldsymbol{a}$ *and* $\begin{bmatrix} \check{\overline{E}}_1 \\ \vdots \\ \check{\overline{E}}_n \end{bmatrix} = (2B - X - \overline{X}_c^T)^{-1} \cdot \boldsymbol{a}$, *where, entry* (i, j) *of* $\overline{X}_c$ *is equal to* x_{ij} *if agent i and j belong to the same community after merging community* C_m *and* C_{m-1}. *Otherwise, it is zero. We have,*

$$
\begin{aligned}
(2B - X - X_c^T) &\geq (2B - X - \overline{X}_c^T) \\
\textit{Corollary 1} \implies \begin{bmatrix} \check{\overline{E}}_1 \\ \vdots \\ \check{\overline{E}}_n \end{bmatrix} &\geq \begin{bmatrix} \check{E}_1 \\ \vdots \\ \check{E}_n \end{bmatrix}
\end{aligned}
\tag{D.23}
$$

As $\check{\overline{E}}_i \geq \check{E}_i,\ \forall i$*, by* (D.19)*, we have* $\check{\overline{e}}_{ij} \geq \check{e}_{ij},\ \forall i, j$.

$$\begin{aligned}
v_i(\check{\boldsymbol{e}}_i, \check{\overline{\boldsymbol{e}}}_{-i}) = -l_i + a_i \cdot \check{e}_{ii} + a_i \sum_{k \neq i, k \in C_{\bar{I}(i)}} \check{\overline{e}}_{ki} - \sum_{j=1}^{n} b_j \cdot (\check{e}_{ij})^2 \\
+ (\check{e}_{ii} + \sum_{k \neq i, k \in C_{\bar{I}(i)}} \check{\overline{e}}_{ki}) \cdot \sum_{j=1}^{n} \left(x_{ij} \cdot (\check{e}_{ij} + \sum_{k \neq i, k \in C_{\bar{I}(j)}} \check{\overline{e}}_{kj}) \right) \geq \\
-l_i + a_i \cdot \check{E}_i - \sum_{j=1}^{n} b_j \cdot (\check{e}_{ij})^2 + \check{E}_i \cdot \sum_{j=1}^{n} x_{ij} \cdot \check{E}_j = v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i})
\end{aligned} \tag{D.24}$$

Moreover, by the definition of Nash equilibrium for game $\overline{G}_{rp}^{c}$*, we have* $v_i(\check{\overline{\boldsymbol{e}}}_i, \check{\overline{\boldsymbol{e}}}_{-i}) \geq v_i(\check{\boldsymbol{e}}_i, \check{\overline{\boldsymbol{e}}}_{-i})$*. Therefore,* $v_i(\check{\overline{\boldsymbol{e}}}_i, \check{\overline{\boldsymbol{e}}}_{-i}) \geq v_i(\check{\boldsymbol{e}}_i, \check{\boldsymbol{e}}_{-i})$.

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