# Online Appendix: Building on the work of teachers: Augmenting a functional lens to a teacher-generated framework for describing the instructional practices of responding

Amanda Milewski[1] and Sharon Strickland

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This is the online appendix for the following article:

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As described in the paper, one of the primary reasons we elected to analyze a second lesson episode is to provide the reader the opportunity to observe two classes of moves that were not present in the Surface Area lesson excerpt, namely ***confronting-response*** and ***confronting-rejoinder***. In this episode, Humphreys makes use of a total of 18 out of the 28 total moves from the Modified-Responding Framework, and 12 of those moves are unique to this lesson excerpt in that they do not show up in the Surface Area lesson excerpt. Further, this episode is quite a bit longer than the episode in the manuscript, with a total of 44 responding moves, giving the reader

[1] **Amanda Milewski,** amilewsk@umich.edu is an assistant research scientist at the University of Michigan in Ann Arbor. Her primary research focuses on the design and implementation of innovative forms of professional development that enable teachers to see and intervene on their own instructional practices, and the ways in which teacher's instructional practices of responding might make a difference for students.

**2 Sharon Strickland**, strickland@txstate.edu, is an associate professor of mathematics education at Texas State University in San Marcos, Texas. Her primary research focuses on the role of discourse in mathematics classrooms, particularly how teachers respond to students' mathematical thinking. Other areas of interest include preparing future teachers, mathematics majors learning to prove as they transition to upper level coursework, as well as everyone learning more geometry.

the opportunity to see a greater variety of moves—some of which are made on multiple occasions.

From the eighteen responding moves, two discursive moves drawn uniquely from the Eggins-Slade Framework (*detach and register*) and two discursive moves drawn uniquely from the Teachers' Framework (*invite to evaluate, invite to connect*). Collectively the eighteen moves span five of the six functional categories: ***supporting-response***, ***confronting-response***, ***supporting-rejoinder***, ***confronting-rejoinder***, and ***invitational-response***. Finally, the episode contains two segments that received comments from Boaler and five segments that received comments from Humphreys

*Lesson Episode: Fraction Division*

The second lesson excerpt we analyze focused on the concept of division with fractions (Figure A). Again, before sharing our analysis, we describe the content and context of the excerpted episode, which occurs after the discussion of three other division problems (i.e., $4 \div 2$, $6 \div \frac{1}{2}$, $1 \div \frac{1}{3}$). This excerpt begins with Humphreys asking the students to identify the solution for dividing one by two-thirds and describe why their solution-process makes sense. The students share in whole-class as well as in small groups before returning to whole-class again. Whereas in the surface area lesson excerpt Boaler offered the bulk of commentary, Humphreys does more of the commentary for this selected excerpt.

The fraction division lesson excerpt opens with Humphreys asking students to work together in order to describe how they could make sense of one divided by two-thirds (lines 1-2). Following small-group time, Leslie shares an incorrect answer of "six". Humphreys responds with an *uncover*, which is categorized in the Modified-Responding framework as a ***supporting-rejoinder—***purported to sustain the conversation about the idea. When describing the rationale

for her decision, Humphreys shares the value that she sees in students' wrong answers and how she tries to "exploit [such answers] to dig into the thorny issues that arise when thirty students are trying to understand mathematics" (p. 45). We again, find a lot of similarities between Humphrey's description and purposes of *uncover* and the functional categorization of the move in the Modified-Responding Framework.

In the second noted moment (line 13), Humphreys has elected to react to Leslie's contribution with a *register*, which functions as a ***supporting-response***—acknowledging the speaker without promoting the interaction's focus on that particular idea. Humphreys' stated intention aligns well in that she describes waffling between working more with the student's contribution, but ultimately decides to move on to get more ideas into the public space before focusing on any single one.

By lines 26-31, Humphreys had heard from two students (i.e., Leslie and Claire), both who have justified their thinking with procedural explanations (lines 9-25, e.g., invert-and- multiply-type rationales) rather than the conceptual ones Humphreys had requested. She reacts to these two ideas with a pair of *orients* (a ***confronting-rejoinder***) in which she asks the students to reconsider their contributions in light of the goal of the lesson to make sense of division (see Boaler & Humphreys, 2005, p. 41). Sandwiched between these two *orients*, Humphreys also uses a *non-evaluative revoice,* a ***supporting-rejoinder***, in which she restates for the class what both students have contributed. In the attached commentary, we see Humphreys explaining that she purposely did not want to declare which student was correct. Instead, she decided to send the class off into their groups after offering clarification for what she wanted the class to consider—how they can use sense-making as a strategy to determine what is correct. As a **confronting-**

***Figure A***. Excerpt from the Division of Fractions Lesson

| Line | Speaker | Transcription | Responding Move | Function | Commentary |
|---|---|---|---|---|---|
| 1 - 2 | TEACHER: | Without doing a rule that you know, like, see if you can make sense of why, what one divided by two-thirds is. | | | |
| 3 - 4 | | *[class works in small groups and then teacher calls them together]* | | | |
| 5 - 6 | TEACHER: | All right, let's hear what some people have to say. Leslie. | | | |
| 7 | LESLIE: | What is the answer? | | | |
| 8 | TEACHER: | Yeah, say what you think it is. | | | |
| 9 | LESLIE: | I think it's six. | | | |
| 10 | TEACHER: | **And you think it's six because . . .** | Uncover | Supporting Rejoinder | Humphreys: "When [Leslie] said, 'Six,' I was thankful ... I have found the value of wrong answers to be inestimable as sites for learning in mathematics … As often as I can, **I try to exploit these ideas to dig into the thorny issues that arise when thirty students are trying to understand mathematics from their own unique experiences and perspectives** …" (bolded emphasis ours, p. 44 - 45) |
| 11 - 12 | LESLIE: | Because a third goes into one three times and then two times three is six. | | | |
| 13 | TEACHER: | **All right.** | Register | Supporting Response | Humphreys: "I noticed that her explanation (one-third goes into one three times, and two times three equals six) was largely procedural; I wondered why she multiplied two times three, but in the kind of split-second decision making |

| Line | Speaker | Transcription | Responding Move | Function | Commentary |
|---|---|---|---|---|---|
| | | | | | that teachers deal with every day, **I decided that before we examined this more closely it would be good to get some other ideas into the public arena**" (bolded emphasis ours, p. 45). |
| 14 | TEACHER: | Um, other thoughts about this? Um, Claire. | Resolicit | Confronting Response | |
| 15 - 16 | CLAIRE: | I got one and a half. I switched around the three and the two. | | | |
| 17 | TEACHER: | I can't hear. | | | |
| 18 - 19 | CLAIRE: | Um, I got one and a half because I made the three the numerator and the two the denominator . . . | | | |
| 20 - 21 | TEACHER: | **You used the reciprocal of three-halves of two-thirds**. | Add on | Supporting Rejoinder | |
| 22 | CLAIRE: | Yeah. | | | |
| 23 | TEACHER: | **Why?** | Uncover | Supporting Rejoinder | |
| 24 - 25 | CLAIRE: | Because on the other one we switched around the numerator and denominator . . . | | | |
| 26 - 31 | TEACHER: | **Yeah, and so see what's, here's what, here's where I'm going with this today. Before the end of class today you're going to have a rule and some people already know the rule, but I don't care about the rule right now. What I want to know is, can anybody make sense of this?** | Orient | Confronting Rejoinder | Humphreys: "When Claire said she got one and a half (the right answer) because she switched the numerator and denominator, I used her explanation to **send a strong message to the students about how important it was that they make sense of the problem rather than just follow a rule** ... **today I wanted them to bring reasoning to whatever they were doing**." (bolded emphases ours, p. 45). |

| Line | Speaker | Transcription | Responding Move | Function | Commentary |
|---|---|---|---|---|---|
| 32 - 38 | TEACHER: | Let's just, so right now we have two different answers. **We have the answer six and Leslie's saying it's because you divide, there's three one-thirds in one and then two times three is six. And then we have the other theory that the . . .** Put your hands down . . . **The other theory is that the answer's one and a half, but we don't know why except the rule works.** | Revoice | Supporting Rejoinder | |
| 39 - 46 | TEACHER | **So, the rule working is not good enough today. All right, so why does it make sense? Why does Leslie's answer make sense? Why does one and a half make sense? I want to know why it makes sense . . .** OK. I've got almost one hand up in every group but not quite. **I want every group to come up with what makes sense. Some reason, why.** Put those heads together. | Orient | Confronting Rejoinder | |
| 47 - 48 | | *[Students work in groups, while teacher walks around an listens in.]* | | | |
| 49 - 62 | | *[Teacher selects one student to go to the board and describe his thinking. Various students react to this with questions and comments. Cheryl comes forward share that you can think of the problem as a number sentence and rewrite that number sentence using an inverse operation]* | | | |
| 63 | TEACHER | **Why does that make sense? Or does it make sense?** | Invite to Evaluate | Invitational Response | |
| 64 - 65 | CHRISTINE | I don't really understand, like, I still think it's six because, um, I... | | | |
| 66 - 67 | TEACHER | **Oh wait, Christine. And I know you've had your hand up and I want to call on you about why it's six** | Table | Supporting Rejoinder | Boaler: "A move of particular interest to me that can be seen in the teaching extract came about when a student offered an idea **that was** |

| Line | Speaker | Transcription | Responding Move | Function | Commentary |
|---|---|---|---|---|---|
| | | | | | **not one that Cathy thought would help the class at that time.** This happened when Cheryl offered an explanation for the division problem using the inverse of fractions. Cathy asked the class why it made sense and Christine answered 'I don't really understand like, I still think it's six because …' Cathy stopped Christine saying, 'Oh wait, Christine. And I know you've had your hand up and I want to call on you about why it's six but just, what method is Cheryl using here for this one?' **Cathy knew that Christine needed to talk, but she also knew that Cheryl's method required some unpacking … Less experienced teachers may have let Christine continue speaking, but Cathy knew she could employ a pedagogical practice of asking Christine to wait**" (bolded emphases ours, p. 52) |
| 68 | TEACHER | **but just, what method is Cheryl using here for this one? . . .** | Invite to Connect | Invitational Response | |
| 69 - 73 | TEACHER | OK. Since people aren't remembering, **remember yesterday we talked about that fam . . . , the four related number sentences? Can anybody, well, OK, so you chose another number sentence to show why that would be true.** | Connect | Supporting Rejoinder | |
| 74 | TEACHER | **OK. Um, Christine, and you still think it's six?** | | | |
| 75 - 76 | CHRISTINE | **Um, two times three equals six and um, six times one equals six.** | | | |
| 77 | TEACHER | **And why does that make sense, Christine? (1)** | Orient | Supporting Rejoinder | |

| Line | Speaker | Transcription | Responding Move | Function | Commentary |
|---|---|---|---|---|---|
| 78 | CHRISTINE | **I don't . . . um . . .** | | | |
| 79 - 80 | TEACHER | Leslie, **what do you think about, what do you think about your original answer?** | Orient | Supporting Rejoinder | |
| 81 - 82 | LESLIE | Well, I thought it was six but then I don't think it can be that big. | | | |
| 83 | TEACHER | Ah, **why don't you think it could be that big?** | Uncover | Supporting Rejoinder | |
| 84 - 86 | LESLIE | Well, because you're doing, like, one divided by two-thirds, and it would be less than six. I don't know any other way to do it. | | | |
| 87 - 88 | TEACHER | **So you were trying to figure out, and you're trying to figure out, something to do with numbers.** | Comment | Supporting Response | |
| 89 - 91 | CHRISTINE | I was thinking about Cheryl's method and I don't really understand it that much, but I'm trying to, I was looking at it and it makes sense now. | | | |
| 92 - 93 | TEACHER | **Okay**, we've got two people who are just about having a fit over here. Would you call on a few people? | Register | Supporting Response | |
| 94 | CHRISTINE | Um, Amy. | | | |
| 95 - 96 | AMY | Well, if you multiply six by two-thirds, it wouldn't equal one. | | | |
| 97 - 106 | TEACHER | Okay. **So I'm going to stop this right now** because here's something that really, I think is the biggest danger in math is that you follow rules, you do things with numbers. It's kind of like waving your magic wand … | Detach | Confronting Response | Humphreys: "With all of these ideas on the table, there were many teaching moves I could have made. Amy's argument was very sophisticated, and I could have moved the class discussion to a more abstract level by examining it more closely. I could have had a more detailed examination of Cheryl's use of |

| Line | Speaker | Transcription | Responding Move | Function | Commentary |
|---|---|---|---|---|---|
| | | *[Teacher goes on to provide a contextualized example about dividing up a piece of 1 foot lumber into ⅔ foot pieces to provide students with a quantitative rationale about why one and a half was more reasonable of a quantity than six.]* | | | inverse operations ... But I decided instead to **cut the conversation off in order to shine a brighter light on** Leslie's idea that six was 'too big' to be a reasonable answer … I did not want students to leave class that day thinking that the answer could be six. I was surprised that the class discussion itself had not accomplished this, but I was ready to shift my role to contribute an argument. Inherent in the weight of my status as a teacher is the danger that students will disregard the often equally reasonable arguments of their peers. My desire to have students publicly share their thinking is not a perfunctory way of going through the motions before presenting the *real* way. **But it was time to move on** …" (bolded emphases ours, p.46 - 48)<br><br>Boaler: "The end of the video showed Cathy telling the students about a situation that could involve the problem 1 ÷ ⅔: a piece of lumber being divided into sections. **She chose to curtail the conversation in order to give it closure.** One of the most difficult decisions teachers have to make when they have classroom discussions is when to end them—when have students discussed something enough?" (bolded emphasis ours, p. 52 - 53) |

**rejoinder**, we can see how *orient* confronted what she had heard (or did not hear) from both Leslie and Claire to that point—namely neither had discussed sense-making.

It may be more challenging to understand *orient* as a move belonging to the ***rejoinder*** category. Again, it's functional categorization (drawn from the Eggins-Slade Framework) helps us to understand how such a move can have the potential for simultaneously addressing the lack of alignment between a student's contribution and the goal of a lesson/task while remaining open to the potential for sustaining the interaction around the contribution. Indeed, the interaction in this excerpt demonstrates that the discussion around Leslie and Claire's contribution does continue, but in a group conversation rather than in a whole-class conversation. This *orient* allows, not only those two students to keep thinking about their ideas, but it also lets the many other students participate, while refocusing the interaction on the sense-making rather than on the absolute correctness.

After the groups discuss, a student named Cheryl describes her thinking, Humphreys uses an ***invitational-response*** to ask the class whether Cheryl's idea makes sense, and another student named Christine begins to say that she got a different (incorrect) answer than Cheryl (lines 63-65), but here Humphreys reacts to Christine with a *table* (lines 66-67)—a move that puts the contribution aside for later consideration and is classified by the Modified-Responding Framework as a ***supporting-rejoinder***. In her commentary, Boaler, notes that Cathy did something "of particular interest", acknowledging "that Christine needed to talk…..but Cathy knew she could employ a pedagogical practice of asking Christine to wait" (p. 52). We find it reassuring that Boaler's description of the move, coded by the Modified-Responding Framework as *table,* matches quite closely to our description of the move.

Again, it might be quite challenging for the reader to trust that *table* belongs to the functional categorization of ***rejoinder***. Leaning into the functional categorization from the Eggins-Slade Framework for *table*, we find it predicts well that *table* does indeed prolong the discussion of Christine's ideas, but does so in a way that delays that continuation to some later point (lines 74-76). It may also be challenging for the reader to accept that *table* belongs to the category of ***support***. After all, in this context *table* was interpreted by Boaler as a reaction to a contribution "that was not one Cathy thought would help the class at that time" (p. 52). We do not disagree with Boaler, but we would go further to claim that by using a *table* (as opposed to some other ***rejoinder*** like *orient* or even ***response*** like *evaluate*), Humphreys was communicating that the idea had a place in the discussion at some later moment. Thus, the confrontation one might sense in a move like *table* is not a confrontation of the idea, but rather a confrontation of the timing for the idea—and by default an admission or support of the contribution for later discussion.

After returning to Christine's contribution, Humphreys also hears a string of contributions from Leslie (lines 79-88), Christine—again who aims to build on a previous idea from Cheryl (lines 89-94), and then another student named Amy (lines 95-96). In Humphreys' commentary, she shares that at this point there were many possible moves she could have made in reaction to all of these ideas. Instead, she chose to end the interaction about students' ideas, with a *detach* (lines 97-106)—a move categorized by the Eggins-Slade Framework as a ***confronting-rejoinder*** that we elected to recategorize as a ***confronting-response*** (this choice is described above where we outlined our RQ1 findings). That is, we suspected that a *detach* would act to close the conversation off rather than prolong it. Here we find support for our decision to recategorize this move in the actual excerpt in that the conversation shifted to a contextual

example and sense-making argument for the division of fractions provided by the teacher. We also find support for our decision in the commentary provided by both Humphreys and Boaler with Boaler saying Humphreys "chose to curtail the conversation in order to give it closure." (p. 52-53). Humphreys states she aimed to "cut the conversation off", claiming she was surprised that the conversation had not resolved the incorrectness of six as the answer and so decided that "it was time to move on" (p. 52-53).

We again begin by noting the five most prevalent responding moves (see Figure B).

***Figure B.*** Five most prevalent reacting moves found in the lesson excerpt

| Prevalence of reacting moves (n=44) | Division Lesson |
|---|---|
| n=12, 27% | Uncover (supporting-rejoinder) |
| n=5, 11% | Register (supporting-response) |
| n=5, 11% | Evaluative Revoice (supporting-response) |
| n=4, 9% | Non-Evaluative Revoice (supporting-rejoinder) |
| n=3, 7% | Orient (confronting-rejoinder) |

This lesson made use of 18 of the 28 moves form the Modified-Responding Framework. Two of these (*detach* and *register*) were imported from the Eggins-Slade Framework and two were unique to the Teachers' Framework (from the ***invitational*** categories). Unlike the Surface Area lesson excerpt, this excerpt does include moves representing both ***confronting-responses*** (*resolicit*, *detach*) as well as ***confronting-rejoinder,*** (*orient*), which were used multiple times.

The only functional category absent is ***invitational-rejoinders***.

Finally, we note that 8 of the 28 reacting moves from the Modified-Responding Framework were not observed in either excerpt analyzed for this paper. Notably, only two of the ***confronting-responses*** were used, namely *detach* and *resolicit*, both from the Division of Fractions excerpt. Similarly, only one of the ***confronting-rejoinders*** was used, namely *orient,* and that move was used on four occasions in the *Division of Fractions Lesson* excerpt. The remaining moves in these two functional categories (***confronting-responses:*** *negative evaluate* and *tell;* ***confronting-rejoinders:*** *negative advance, unresolve,* and *rebound*) remained unused by Humphreys in these excerpts. Though not present here, the education literature as well as the baseline videos of the original project teachers who first created the Teachers' Framework, reveal that *negative evaluate* and *tell* are quite common. This suggests that Humphreys' teaching is unique or that these two excerpts are not representative of her teaching. Other moves that went unused by Humphreys include the following: *resolve* (***supporting-rejoinder***) and *invite to revoice* (***invitational-rejoinder***). We saw plenty of *invite to revoice* in the project teachers' practice; but as for *resolve*, it is too new of a move in our work for us to be sure how readily such a move would be used by teachers.