The Role of Classroom Instruction in the Development of Early Number Skills

by

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DEDICATION

To all the teachers, children, and parents missing out on business as usual due to COVID-19.

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ABSTRACT

The relation between mathematical achievement in early childhood and future academic success is well established. However, the effect of instruction on mathematical performance is less well-documented and often reliant upon self-report instruction measures and standardized achievement measures. Therefore, this study seeks to use observational data to examine the role of classroom mathematics instruction in the growth of adaptive early mathematical skills across the kindergarten school year. The first research aim identified the relation between children's math skills at school entry and the rate at which their math skills grew during kindergarten. The second research aim determined whether student characteristics, such as age or sex of the child, predicted different mathematical skills. The third research aim examined the effect of observed classroom mathematics instruction on the growth of children's early mathematical skills.

Four schools, fourteen classrooms, and 98 children were recruited to investigate these research aims. Children completed counting and addition measures three times during the school year from an individualized assessment novel to the United States, called Math Garden. Teachers recorded an entire school day using an audio recorder in the middle of the school year. First, longitudinal multilevel models were used to identify the relation between school-entry skills and the rate of growth on counting and addition capabilities. Second, multilevel models determined whether student characteristics predicted counting or addition abilities at the beginning of the school year, or growth across the kindergarten year. Last, multilevel models assessed whether content, quantity, and quality of kindergarten classroom mathematics instruction predicted the growth of children's counting and addition skills.

In this sample, children grew significantly in both counting and addition skills across the kindergarten year. However, the rate of growth for addition capabilities was four times that of counting skills. Children with low entry skills showed a larger rate of growth in both skills across the school year. Children's age in the beginning of school and sex of the child did not predict entry or growth in counting abilities, however, boys performed better than girls on addition in the beginning of the school year. The quantity, quality, and content of classroom mathematics instruction did not predict growth in counting or addition abilities across the kindergarten year.

This study was one of the first to use audio recordings to investigate kindergarten classroom mathematics instruction and its contribution to early mathematical growth. Children with the lowest levels of math skills grew the most across the kindergarten year, suggesting a focus on basic skills continues to consume early grades. Inconsistencies in sex differences on early mathematical tasks highlight the need for future work to address skill proficiency as an essential context. The results suggesting classroom mathematics instruction did not contribute to children's growth in early mathematical skills highlighted important methodological differences between previous research, as well as a need for more robust measurement in assessing classroom mathematics instruction for future work.

CHAPTER I

Introduction

For the last few decades, scientifically robust research has shown the importance of early schooling in cognitive and achievement outcomes for children (Claessens, Duncan, Engel, 2009; Clements et al., 2013; Siegler et al., 2012; Weiland & Yoshikawa, 2013). Children in the United States spend a large portion of their early years in school, beginning around five-years of age. This early age of schooling is based on compulsory education laws that are intended to maximize children's rapid skill development, regardless of socioeconomic status, race, or age (Coffield et al., 2008; Stephens, Yang, 2014). In this way, formal education serves as a basic form of cognitive and achievement intervention across the country. However, schools are responsible for developing more than just achievement-related skills, such as socio-emotional skills that help them control their behavior inside and outside of the classroom, as well as foster positive relationships with other children and adults. Thus, the challenge for any district, school, or class is to consider emphatic placement on various skills, whether academic or socioemotional. Therefore, the early experiences of formal schooling helps children to lay a foundation for their overall future success across a comprehensive set of essential skills that lead to success in adulthood. More recently due to strong federal policy emphasis on achievement, academic skills such as literacy and numeracy, generally receive the strongest curricular emphasis.

Two Foundational Skills: Literacy and Mathematics

Today, the strongest emphasis in educational policy has focused on literacy development (Pew Research Center, OECD, PISA, 2015). The emphasis on literacy interventions are not new in the United States with some experts finding reference to it as far back as the 19th-century (Scammacca et al., 2016). More recently, states have been enacting laws that are intended to keep children who are considered below grade level in literacy from advancing to the next grade until they are proficient in the skill. For example, in the state of Michigan, a new law demands, "*the department shall do all to help ensure that more pupils will achieve a score of at least proficient in English language arts on the grade 3 state assessment"* (Read by Grade Three Act of 2016). Thus, by third grade children must demonstrate proficiency at their appropriate grade reading level. Otherwise, in the state of Michigan, children will repeat third grade if they are more than one grade level behind in reading.

Even prior to these new laws being enacted regarding literacy, teachers tended to spend a significant amount of time in their classrooms focusing on literacy instruction (Engel, Claessens, & Finch, 2013). Thus, it will not be surprising if the push for children to reach certain literacy levels in school creates a situation where teachers are more focused on subject areas that relate to literacy and focus less on other areas, such as mathematics instruction. Unlike literacy, there are no similar benchmarks for student mathematical achievement levels and no laws that dictate what level of proficiency must be achieved in mathematics in order to transition to the next grade level. Indeed, even though there are many literacy interventions that are targeted at children with reading difficulties in the early elementary school, there is no comparable federal, state, or local interventions that target mathematics. Thus, children with poor mathematical abilities will advance to the next grade level, even when they are having difficulty in mathematical concepts and applications.

Although children in the U.S. continue to struggle in many areas of achievement (Lee, Grigg, & Donahue, 2007), the lack of attention to mathematics may be of particular concern. More specifically, individual performance in mathematics is a well-established predictor of future success in schooling and on the job market beyond that of literacy achievement (Duncan et al., 2007; Watts et al., 2014). Research shows that children's early mathematical skills predict high school mathematic course attainment (Claessens et al., 2009; Davis-Kean et al., *under review*). For example, Siegler et al. (2012) use a longitudinal dataset to demonstrate children grouped by third-grade mathematical skill proficiencies predicted high school algebra attainment. Moreover, research has also shown that the mathematic courses children take in high school predict college matriculation and graduation (Murnane, Willett, & Levy, 1995; National Mathematics Advisory Panel, 2008). Interestingly, even beyond college, longitudinal studies have shown that early mathematical abilities are also related to future income and employment (Parsons, Bynner, & Brewer, 2005; Rose & Betts, 2004). Hence, above and beyond literacy achievement, children who develop stronger mathematical skills early on experience higher academic achievement overall than do their peers with weaker numeracy skills.

Furthermore, children better equipped with a basic understanding of mathematical knowledge (e.g., counting and number recognition) when they enter school are also more prepared to learn advanced skills (e.g., arithmetic and other numerical operations). Similar to literacy, proficiency in early mathematical skills provide children with a solid foundation to continue building on their abilities in school when compared to children not equipped with basic numerical knowledge (Claessens, Engel, & Curran, 2014; Griffin & Case, 1996). Thus, these early differences in mathematical skills from as early as three years of age consistently show a contribution to the existing achievement gap in schools (Case, Griffin, & Kelly, 1999; Davis-Kean et al., *under review*; Lee & Burkam, 2002).

Dissertation Study Overview

Accordingly, the goal of this dissertation was to examine the initial level of mathematical skills that may capture what children were exposed to in their environment prior to formal schooling and assess the overall growth of early mathematical skills across kindergarten, children's first formal year of school. Further, this dissertation will also examine how schooling contributes to the growth of those skills.

These goals are essential to further the field and provide a better understanding of the mechanisms through which schooling, or classroom instruction, promotes the development of children's early mathematical skills above those they had at prior to school entry. As mentioned above, early mathematical skills play a crucial role in later school success, and robust research suggests that children who enter school with low skills usually also remain low throughout education (Bodovski & Farks, 2007; Jordan et al., 2009). Thus, this dissertation will examine whether certain aspects of classroom instruction related to more robust growth across the school year. Consequently, results from this dissertation could inform teachers, researchers, and policymakers of this pivotal time in a child’s academic career.

Moreover, this dissertation uses a unique methodological approach to examine both the general development of mathematical skills, as well as different aspects of kindergarten classroom instruction. In the past, many studies did not include individualized methods for mathematical skills or naturalistic methods for classroom instruction. For example, previous research focusing on the development of early mathematical skills tend to focus on standardized measures that tell very little about dynamic, complex individual differences. Similarly, research focusing on classroom instruction often rely on teacher-report questionnaires and very rarely use observational methods in kindergarten. Thus, this dissertation will provide empirical evidence through a novel perspective regarding the role of formal schooling and teaching for the foundational development of early mathematical skills.

Thus, there are three aims of this dissertation. The first aim investigates the relation between foundational mathematical skills at school entry and the growth of these number skills. Then, the second aim examines whether student characteristics predict the growth of these early number skills. Finally, the third aim explores the role of school instruction in the growth of these early number skills. In this dissertation I will be using novel methodologies for collecting data on children’s early mathematical skills across kindergarten, as well as collecting teacher data using a non-intrusive digital audio recorder that allows for intensive data on teacher-child interactions without an interviewer being in the classroom. The dissertation, then, will provide important information on how mathematical skills in kindergarten develop and are potentially enhanced by classroom instruction in the first formal year of schooling.

Mathematical Achievement in the United States

Even after decades of interventions targeted at reducing the achievement gap, significant achievement gaps still exist in the United States (Bryk & Raudenbush, 1989; Lee & Burkam, 2002; Reardon, 2003). A more global achievement gap becomes even starker when comparing the average achievement of students in the United States to those in other countries (PISA, Program for International Student Assessment; OECD, 2001; TIMSS, Trends in International Mathematics Science Study; Mullis et al., 2000). According to the 2015 PISA, children in the United States are slightly above the global average on science and reading but are significantly below in mathematics (Pew Research Center, OECD, PISA, 2015). Nationally, the proficiency in mathematical skills remains consistently low, demonstrating no significant change from 2015 to 2017 in the NAEP average mathematics scores for 4th or 8th graders (NAEP, National Assessment of Educational Progress; NCES, 2018). Thus, issues of achievement in United States schools, especially related to performance in mathematics, remains a severe challenge that emphasizes the need for research to continue to understand the role of learning and education.

The Development of Mathematical Achievement

Similar to literacy, research on mathematical development suggests skills develop linearly. Later skills are built on earlier skills, creating a sort of staircase in achievement. Unfortunately, children display differential mathematics abilities from a very young age at the lower steps of this staircase. These gaps in early skills cast a long shadow over children's educational experiences (Claessens et al., 2009; Duncan et al., 2007; Watts et al., 2014). Furthermore, analyses on a wide range of populations using multiple measurement strategies consistently indicate that preschoolers' facility in knowing numbers, counting, and using arithmetic relates closely with students' mathematical achievement throughout their school careers (Claessens & Engel, 2013; Duncan et al., 2007; Geary, 2013; Jordan et al., 2009; Stevenson & Newman, 1986; Watts et al., 2014; Watts et al., 2017). For example, in a longitudinal dataset, Davis-Kean et al. (*under review*), use a classification technique to group children by mathematical skills at 4.5 years old. Results suggested four groups of children emerged from this study: children with no mastery, children who count, children who can count and add, and children who can count, add, and subtract. These early childhood mathematical mastery groups also predicted entry to college such that 76% of children who could count, add, and subtract at 4.5 years old enrolled in a four-year college. This number is stark when compared to only 26% of the young adults from the group who had not mastered any skills in preschool attended a four-year college (Davis-Kean et al., *under review*). Thus, findings from multiple studies suggest a pivotal point in time to examine indicators of long-term schooling success is before school entry.

It is not surprising then that several studies suggest stimulation and support from caregivers in the home are some of the most substantial factors related to children's developing cognitive skills (LeFevre et al., 2009; Roberts, Jurgens, & Burchinal, 2005). The home environment provides a significant influence on children's early mathematical development that can explain the heterogeneity of skills children bring to school. Parents engage in a variety of mathematical activities with their children at home, sometimes not recognizing that they are mathematic specific. For example, when asked, parents reported spending time on counting objects and sorting things by size or color at home (LeFevre et al., 2009), fostering and mastering early mathematical skills, such as counting, with their children. However, research has shown that these activities vary in amount. A study by Levine et al. (2010) used behavioral observations to examine the number of words and number of elicitations parents use with their children. Families in their study ranged from a total of four to 257 number words across five 90-minute visits. These findings suggest large variability in a child's environment, which is essential for development. Establishing variability was the first step to understanding the complex home environment, and building on that, research also suggests early exposure to more instances of mathematical talk is related to children's mathematical performance a year later, even after controlling for maternal education (Susperreguy & Davis-Kean, 2015). Thus, the number of mathematical words in the home environment varies, and this diversity can set children's mathematical learning trajectories at different levels before schooling has even started.

Another factor related to mathematical learning trajectories is the sex of the child. Research focusing on sex differences in mathematical competencies in areas such as adding and subtracting is mixed and often leans toward a sex equality hypothesis (Hutchison, Lyons, Ansari, 2019). However, a sex difference in attitudes towards mathematics remains prominent such that girls report negative feelings toward mathematics and perceive it as more of a "male subject" (Nosek, Banaji, & Greenwald, 2002; Nosek & Smyth, 2011).

It is important to note, however, that sex-related genetic and physiological factors may contribute to these differential preferences and attitudes towards mathematics. For example, studies focusing on the influence of sex hormones have found considerable differences between male and female spatial abilities (Hampson, 2007; Maki & Sundermann, 2009). Spatial skills serve as an important early indicator of later mathematical abilities (Shea, Lubinski, & Benbow, 2001). Research examining sex-related factors have suggested individuals with high levels of early sex-related hormones (androgens, often seen in males) would be more likely than typical females to express more interest in their spatial world (Berenbaum, Bryk, & Beltz, 2012). Thus, there is an interaction between the individual child's hormones and their social environment. Hormones may facilitate the learning of spatial skills and, thus, influence the toys or activities that children gravitate towards, influencing their interest in mathematical skills early on. Therefore, aspects of the individual's sex may also contribute to differences in mathematical achievement and attitudes toward mathematics.

Overall, previous studies have generally found that different mathematical practices before schooling, both individual and environmental, are related to the skills children bring to school. Thus, this is in line with previous research that shows children come to school with diverse levels of expertise (Chiatovich & Stipek, 2016; Engel, Claessens, & Finch, 2013). However, the growth trajectory for mathematical skill-building remains unclear. Unlike reading, there is no clear consensus of a hierarchy for children's mathematical skill development. Many researchers have attempted to layout possible learning trajectories for mathematical development

(Rittle-Johnson et al., 2016; Sarama & Clements, 2004; Siegler, 2016), but unfortunately, there remains no clear consensus and no framework for educators and policymakers.

Theoretical Framework

Given the importance of the development of mathematical skills, it is important to understand what skills children have at the entry to schooling as well as what foundational skills may lead to development of more advanced skills for later mathematical achievement success. In the past, the field has approached this holistic understanding in multiple ways. In some cases, researchers conduct theory-based interventions focused on building a solid mathematical foundation for children (Clements & Sarama, 2007, Bryant et al., 2008; Bryant et al., 2011; Fuchs et al., 2006). With the use of interventions, scholars attempt to determine causal predictors of mathematical achievement. Other researchers have tried to unravel the cognitive processes responsible for the development of early mathematical skills (Merkley & Ansari, 2016; Siegler & Lortie-Forgues, 2014). For example, many researchers examine the role of nonsymbolic and symbolic magnitude comparison skills to understand the order in which skills underlie other abilities in the development of mathematical achievement (De Smedt, Noël, Gilmore, & Ansari, 2013; Xenidou-Dervou et al., 2017). Finally, research that indicates children's skills emerge early in development has inspired researchers to hone in on understanding the role of the home context and parent attributes as essential mechanisms for developing mathematical skills before schooling (LeFevre et al., 2010; Levine et al., 2010; Susperreguy & Davis-Kean, 2015). Thus, prior research has approached the development of mathematical achievement from multiple angles. However, little research focuses on the relation between the mathematical skills children bring to school and the role of schooling factors in the development of these early skills.

Beyond foundational mathematical skill theory, several diverse systems contribute to differences in children's mathematical achievement levels, including community, family, and individual characteristics (Berch & Mazzocco, 2007; Bronfenbrenner & Morris, 2006). These systems are responsible for the skills children present early in life. As reviewed above, research also suggests that these sources provide children with individual differences in number skills at an early age creating differing foundational mathematical skills at the onset of schooling (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Watts, Duncan, Siegler, & Davis-Kean, 2014). Thus, the complex interaction between an individual and their environment contributes to their developmental trajectory. Although this dissertation does not capture all possible sources contributing to mathematical skill development, it hones in on one, the school environment.

The school environment provides children with a uniformed system that can aid in the growth of early skills. However, these individual differences that exist before schooling are significant for implications in the school. Based on Vygotsky's Zone of Proximal Development (ZPD) theory, children who receive instruction above or below their skill level will not be as successful when compared to children who receive education within their range of skills (Vygotsky, 1978). Thus, content and level of instruction should play an essential role in fostering early skills in the classroom.

Two main mathematical theories ground the understanding of early mathematical skill development in young children. First, the learning trajectory theory posits that sequence matters (Clements & Sarama, 2004; Clements & Sarama, 2007). Similar to how children learn to crawl, then walk, then skip, children follow a developmental progression when learning mathematics. There are specific skills in mathematics that children must first master before they can move forward. The second theory proposes that an understanding of numerical magnitudes is what underlies this developmental progression (Siegler & Lortie-Forgues, 2014). Thus, the sequence in which children learn mathematical skills is essential, and an understanding of magnitudes is an underlying theme in which children can progress through the learning trajectory. Therefore, it is not that teachers can instruct children through each stage, but it is also imperative for children to have a strong foundational understanding of magnitude as they move through each step.

The hypothesis of this dissertation is grounded in the idea that certain aspects of instruction influence a child's skill levels. Thus, it is not surprising that the theoretical basis of this dissertation aligns closely with that of theory and research in reading. Recent research in literacy has discovered that the effectiveness of instruction relies heavily on a child's skill level (Connor, Morrison, Katch, 2004). Students benefit the most from education that matches their entering skill level, thus referred to as the "child x instruction interaction" or CXI for short (Connor et al., 2004). Taken with the previous theory on mathematical learning, the instruction that children receive underlies the key to the most substantial gains in mathematics. Therefore, this dissertation is grounded in the combination of instructional and developmental theories intertwining within every day, business as usual, schooling environment.

Assessing Individual Mathematical Skills

Historically, researchers have relied upon a variety of methodologies to examine specific mathematical skills and their development. Most commonly used in the literature of mathematical achievement is a range of broad, validated mathematical measures that often consist of behavioral assessments providing researchers with an overall mathematics score (Clements, Sarama, & Liu, 2008; Ginsburg & Baroody, 2003; Woodcock, McGrew, & Mather, 2001). In such measures, questions are arranged in a ranked format, starting with simple questions and ending more complex. Children’s responses to those questions then rank children on their overall mathematical abilities. These measures are typically intended to reflect children’s general mathematical ability and are essential for measuring children’s relative performance. However, an overall mathematical ability score in tasks such as the Woodcock-Johnson III Tests of Achievement does not examine the heterogeneity of children’s mathematical skills individually.

Conversely, with a shift in focus on the importance of early mathematics achievement, other mathematical assessments seek to examine more specific components of mathematical abilities (Purpura & Lonigan, 2015). For example, in Purpura, Schmitt, & Ganley (2016), children’s early numeracy skills were measured and assessed through 12 different tasks. These tasks examined specific early numeracy skills such as subitizing, verbal counting, numeral identification, number order, and many others. The specificity in skills and concepts is essential for teachers and researchers to identify and understand the mechanisms of success in mathematical achievement. This newer approach to assessing mathematical skills allow for evaluations of the mathematical learning trajectory. However, research on specific components of mathematical abilities still does not capture the heterogeneity of individual differences that exist within these early skills.

Item Response Theory

Interestingly, one approach to assessing the heterogeneity of individual differences in mathematical skill development is the use of item response theory (IRT) methodologies. Based on statistical models, IRT relates responses to the abilities that the items measure (Lord & Novik, 1968). These responses are then modeled and can be used to create computer-adaptive tests (CAT; Van der Linden & Hambelton, 1997). The purpose of using CAT is to understand and determine the ability level of a person dynamically. CAT techniques administer items dependent on the child's previous response; if the child answered the question quickly and correctly, the next question is more complicated than the former. Thus, IRT presents children with a test specifically tailored to their ability level, and CAT techniques are of high quality and high-frequency measurements that can provide rich information required to examine individual development in detail and answer fundamental questions about cognitive development and learning.

Just as previous research has focused on diving deeper into subcomponents of overall mathematical achievement, it is also essential to use methods of measurement that can hone in on a child's global capability. Thus, methods like a paper and pencil test do not provide an individualized description as accurately as methods that take into account varying aspects of the question posed. For example, previous studies that have used IRT to examine children's dynamic mathematical skills capture two more aspects about the underlying ability of the child beyond that of a standardized assessment. Individualized assessments using include questions based on the accuracy of the question, IRT similar to standardized or paper pencil assessments as mentioned above. However, beyond accuracy, individualized assessments also include, response time, and difficulty of the previous problem answered. Thus, the use of dynamic methods, like IRT, provide researchers with a more nuanced approach for understanding and examining not only subcomponents of mathematical achievement, but also the development of children's early mathematical learning trajectories.

Mathematical Skill Growth

Although the measurement of mathematical skills varies across the United States, there is an overall consensus on the importance of fostering and developing these early skills. As mentioned previously, poor achievement in mathematics is a significant concern in the United States. Overwhelming amounts of research has supported the idea that mathematical competencies are cumulative and follow a developmental progression (Baroody, 2003; Clements, 2007; Clements & Sarama, 2007; Gersten & Chard, 1999). That is, many mathematical difficulties later in life can be traced back to weaknesses in essential whole number competencies (Gersten, Jordan, & Flojo, 2005; Malofeeva, Day, Saco, Young, & Ciancio, 2004; National Mathematics Advisory Panel, 2008). Mastery of fundamental and efficient counting skills early on aids children's learning of number relations, which then leads to more robust mathematical competencies in the future (Siegler & Shrager, 1984). Thus, focusing on foundational mathematical skills is promising, and number sense (broadly defined as understanding numbers and operations; Siegler & Jenkins, 2014) in kindergarten is a core marker for persistent learning disabilities in mathematics (Mazzocco, Feigenson, & Halberda, 2011).

Specifically, children's growth of early mathematical skills can provide an accurate and thorough estimation of the development of these first processes. The mechanisms that influence the growth of early mathematical skills and the sources of individual differences are still relatively unknown (Geary, 1994). However, research has shown that children who begin school behind their peers in necessary mathematical skills such as counting and arithmetic are more likely to stay behind throughout schooling (Duncan et al., 2007). Similarly, one nationally representative study found students who begin kindergarten with low mathematical achievement also show the least growth through grade three (Bodovski & Farkas, 2007). Considering school serves as an intervention on these early skills, the idea that children who start with little expertise in mathematical skills might not benefit from an intervention comes as a surprise.

Beyond children's school entry mathematical skills, previous research has also examined the relation between children's rate of mathematical growth during a school year, and their later mathematical achievement. For example, Jordan, Kaplan, Ramineni, & Locuniak (2007) examined children's new number competencies in kindergarten and found that the actual rate of growth of these early skills predicted mathematical performance level in 3rd grade. Accordingly, children who started with low number sense abilities and made moderate gains by the middle of kindergarten had higher mathematical performance in 3rd grade than the children who started with similar number sense abilities but no increases by the middle of kindergarten. This finding would suggest that both overall growth and rate of growth during kindergarten contribute to overall mathematical achievement. Thus, information regarding which specific activities and instruction type best foster growth of mathematical skills in young children is a question that still requires more research before adequately addressed.

Mathematics Instruction

Relatively few studies have investigated whether and to what extent classroom and school-level factors contribute to early mathematical achievement. More specifically, very few studies have examined the role of specific instruction on subcomponents of early mathematical skills (Desimone & Long, 2010; Palardy & Rumberger, 2008). One reason this may be the case could be due to the difficulty of collecting instructional data in schools. Research in education incorporates both the voices of researchers and teachers. Too often, teacher-researcher collaborations produce benefits for researchers exclusively. Establishing and maintaining a bidirectional relation in these collaborations are imperative for instructional data collection (Mitchell, Reilly, & Logue, 2009; Ulichny & Schoener, 1996).

Further, previous research that has examined the aspects of instruction have used assessment methods that do not adequately capture the complexity of teacher-child interactions. For example, studies that have examined the “business as usual” curriculum often assess classroom mathematics instruction using teacher reports (Engel, Claessens, Finch, 2013; Morgan, Farkas, Maczuga, 2015). There are certain advantages to using teacher reports to assess instructional practices. First, teacher reports provide researchers with a generalizable characterization of overall instructional training from the teacher perspective who has the most experience with the topics of interest. Relatedly, teacher reports are an efficient, low-cost option for researchers interested in assessing instruction. However, despite these advantages, methods of self-report show bias based on what teachers plan to practice, rather than what their day allows. Thus, a more efficient measurement tool that can capture and analyze a variety of aspects in the classroom environment would provide researchers with a more accurate method of assessment.

One approach previously used to capture the mathematics instruction environment accurately is that of video observations. Multiple studies have used videotapes to code content for literacy activities (Connor et al., 2009; Connor et al., 2014; Connor et al., 2019). However, much less work has included full-day observations of mathematical activities in elementary school. For example, Connor et al. (2018) videotaped mathematic lessons in second-grade classrooms to examine the fidelity of instruction for a CXI mathematical intervention. These videos, however, did not capture an entire day of education, and thus, lack the possibility of obtaining mathematics instruction outside of a mathematics block (integrated mathematics instruction).

Further, Jenkins et al. (2015), used kindergarten classroom live coder observations to explore reasons for preschool program fade-out. However, in this case, the content of classroom instruction was not examined; rather the focus of this study assessed the quality of mathematical pedagogy (Jenkins et al., 2015). Connor et al. (2019), on the other hand, used live coders to develop a coding system that examines the multiple learning experiences of individual children within classrooms. Thus, although researchers coded the content of mathematics instruction (e.g., numbers, operations, geometry) in this case, the purpose of this study focused on the learning opportunities present for a child in 30 minute observation windows. Therefore, an entire day of classroom instruction was still not examined. In contrast to literacy research, very little research has focused on the content of mathematics instruction in elementary school, and many rely on live coder observation systems.

Mathematics Instruction Measurement

There have been several attempts to operationalize effective instruction. For example, effective teaching may include careful planning, motivational phrases, use of appropriate materials, or providing helpful feedback (Cohen et al., 2003; Shouse, 2001). Further, studies demonstrate that language interactions are particularly important for children's development (Clifford, Yazejian, Cryer, & Harms, 2020). Although effective instruction is not clearly defined, an essential aspect of understanding children's mathematical skill development comes from the knowledge that components of education are related to learning (Hausken & Rathbun, 2004). This study focuses on three elements of kindergarten instruction based upon previous literature reviews: content, quantity, and quality.

The importance of the classroom environment and variation in teacher effectiveness in fostering early skills has been well-established (Hanushek, Kain, O'Brien, & Rivkin, 2005; Harris & Sass, 2008; Rockoff, 2004). This relation remains across autoregressive and multi-level growth modeling approaches (Bodovski & Farkas, 2007; D'Agostino, 2000). However, the necessity to examine instruction from multiple facets comes from a large existing mixed literature. The heterogeneity in methodologies being used for studying classroom instruction merits further exploration as to whether one domain (e.g., quantity, quality, content) is better than another in aiding the growth of early mathematical skills.

Quantity

Many studies that examine the amount of time teachers spend in mathematics instruction are based on the theory that in order for mathematics learning to occur, a significant amount of time must be devoted to mathematics (Wang, 2010). One obvious inference of this theory is that teachers spend a reasonable amount of time in mathematics. However, previous literature suggests kindergarten students are engaged in mathematics instruction for a small proportion of their day (Hausken & Rathbun, 2004). Thus, many studies do not assess the contribution of kindergarten business as usual mathematics instruction.

More often in literature, articles assess early mathematical interventions to examine more diversity in the amount of mathematical instruction time in kindergarten. For example, one meta-analysis reviewed articles on the effectiveness of early mathematical interventions (Wang, Firmender, Power, & Byrnes, 2016). This article assessed multiple questions, among which were questions about the quantity and content of instruction. Wang and colleagues (2016) evaluated whether the programs that devoted more significant amounts of time in mathematics instruction produced larger effect sizes than programs that devoted less time. Although the results suggested that there was a tendency for more substantial effects in classrooms that spent more time in mathematics, there was no statistically significant difference in the contrasts between three groups (23-60 min, 63-90 min, 120-150 min in mathematics instruction). The authors suggested multiple explanations for this finding, one of which was that ideal mathematics instruction includes more aspects that just the amount of time, perhaps higher quality and more diverse content as well.

Content

Based on prior literacy research, the main effect of instruction is not what should theoretically provide growth in early skills, alternatively the content of instruction is important for growth. More specifically, it is important to personalize, or individualize instruction (Connor, Morrison, Fishman, Schatschenider, & Underwood, 2007; Connor et al., 2011) such that the content of the instruction children receive is optimally aligned with their achievement level. These child-by-instruction interactions are well established in literacy research, but not as widely examined in mathematics.

In one study, Connor et al. (2017) created an intervention for second grade children in which individualized mathematics instruction was created to examine whether there were similar results to prior reading research. In their intervention, results suggested that the focus on individualizing mathematics instruction, rather than overall mathematics instruction, demonstrated significant improvements on children’s individual mathematics achievement. Thus, ideally focusing on meeting the child at their individual mathematical skill level is best for growth in their achievement. Further, theoretically, these results may also translate to an idea such that classrooms that cover a more diverse array of content areas should teach to more ability levels. Therefore, perhaps classrooms that simply include more content areas in mathematics may provide a similar outcome as personalizing instruction.

Previous research has further examined intervention programs and the content of mathematics instruction present. For example, Wang and colleagues (2016) assessed whether more substantial effects of mathematical intervention programs were associated with targeting multiple content strands or targeting a single content strand. Although the results from the meta-analysis suggested that there was a tendency for more substantial effects for programs that targeted individual content strands, there was no significant difference between the two. However, an area of consideration in this case, was that the outcome variables in which the study used focused on the specific content strand it was training. Thus, if an outcome variable assesses a single content strand of mathematics instruction, then it would presumably be better for the program to focus on only one content strand as well.

Other studies have used nationally representative data to examine the content of mathematics instruction that was not part of an intervention. Bodovski & Farkas (2007) using the Early Childhood Longitudinal Study-Kindergarten Cohort (ECLS-K) examined the contribution of the content in kindergarten mathematics instruction on achievement. The study collected data from 3,151 classrooms across the United States, and teachers responded to questionnaires regarding elements of their mathematical curriculum and instruction practices. Results showed teachers spent time in eight dimensions of content in kindergarten classrooms (e.g., basic numbers and shapes, advanced counting, practical mathematics, advanced practical mathematics, writing numbers, single-digit operations, two-digit operations, and data/approximations). Of these content codes, time spent on advanced counting, practical mathematics, and single-digit operations showed significant positive associations with achievement growth in kindergarten. Instructional time addressing numbers and shapes showed negative associations with achievement. Thus, instruction based on rudimentary content in kindergarten may have served as review for many children, thus resulting in lower achievement.

Interestingly, rudimentary content may only be detrimental for kindergarten mathematical achievement, but not the following school years. Ribner (2020) also used the ECLS-K to examine mathematics instruction from kindergarten to third grade. Results replicated previous findings that advanced content instruction was related to the development of mathematical skills in kindergarten, whereas basic content instruction was unrelated. However, Ribner (2020) also compared the contribution of basic and advanced instruction to first and second-grade growth in mathematics, and kindergarten was the only grade that showed a relation between advanced instruction and mathematical skill development. In first grade, neither advanced nor basic instruction related to children's mathematical skill development across the years. In second grade, the opposite result from kindergarten was true such that only basic, not advanced content instruction related to growth across the year. Thus, findings suggest that advanced content, specifically, is beneficial for kindergarten mathematical achievement.

Quality

Previous literature examining mathematical development has often relied on findings from literacy literature to serve as a foundation for policy initiatives. Suggestions to encourage parents and teachers to spend more time on mathematical concepts, first came from research suggesting more exposure to vocabulary words by teachers or mothers resulted in faster growth in children's vocabulary (Huttenlocher et al., 1991). However, language researchers have been arguing about this finding and suggest that it is not the quantity that matters, but the quality (Rowe, 2012).

Thus, the assessment of the quality of mathematics instruction is a relatively novel concept. Some studies have examined the quality of the classroom by assessing access and use to specific resources (Baird, 2012). However, the quality of mathematics instruction is once again grounded in language theory.

Recent evidence has suggested the quality of the classroom language environment likely contributes to children's development. For example, some studies have used transcriptions methods to better assess the quality a child's learning environment (Cabell et al., 2015; Justice et al., 2018). Cabell et al. (2015) used transcriptions of interactions between teachers and children and found open-ended questions was associated with positive vocabulary growth for preschool children. However, many of these studies only examine brief sections of the school day.

Language researchers have assessed the quality of the home environment for an entire day using the Language ENvironment Analysis (LENA) system. The LENA is a digital language processor (DLP) developed to monitor the language and audio environment of young children. It records up to sixteen hours of the sound environment at a time and then processes the data into three adult-child variables. One variable, in particular, conversational turn count (CTC), has been shown to predict cognitive outcomes later in life (Gilkerson et al., 2018, Romeo et al., 2018). Thus, although based in the theory of language development, the LENA system may be a useful tool to characterize a classroom's productive learning environment beyond brief segments of transcribed instruction.

Further, the advantage of using of a non-intrusive method in the classroom, the LENA device, is crucial to develop an understanding of the mechanisms behind instruction domains for kindergarteners. Examining aspects of everyday classroom instruction through naturalistic observations will provide a more nuanced approach for understanding the classroom factors and processes related to mathematical achievement. Moreover, this approach could provide the information necessary to formulate recommendations for teachers, schools, and policymakers on how to improve early mathematical success best.

Research Questions and Hypotheses

The goal of this dissertation is to understand the growth and development of early mathematical skills during the first formal year of schooling, kindergarten. This school-based investigation brings together methods of examining individualized mathematical skills from educational and psychological theory. The study included behavioral assessments to identify the growth of specific mathematical skills, and an examination of the variability of mathematics instruction input in kindergarten classrooms, and whether the two were associated. There are three specific research questions in this dissertation.

Research Question 1

Do children start school with various early mathematical skills, and do their skills grow throughout the school year? Based on previous literature examining the home environment and the importance of early number skills for future achievement, I hypothesize children will enter kindergarten with a variety of early mathematical skills, and I expect those skill levels to differ. The first research question focuses on examining the intraindividual differences of the students by examining the relation between the mathematical skills children bring to school and how those mathematical skills grow throughout the school year. Children's early mathematical skills were assessed by both an individualized, item-response theory based assessment, as well as a standardized mathematical assessment in this dissertation. The individualized assessment uses higher quality and higher frequency measurements than standardized mathematical assessments. Thus, I will use the individualized assessment to examine growth in children’s kindergarten mathematical skills. Based on previous literature, I expect children to show growth on both counting and addition assessments. Moreover, based on previous research, I hypothesize a positive relation between the skill children bring to school and their rate of growth, such that children who start school with higher mathematical skills show more growth than the children who start school with lower mathematical skills.

Research Question 2

Do individual characteristics predict children's skills at the beginning of kindergarten and growth in mathematics during the kindergarten year? Substantial variations in children's early individual experiences relate to their rapid mathematical skill development. Thus, to examine these interindividual differences, I will explore the functional form of early mathematical skills across time, and consider how individual characteristics relate to that growth across the kindergarten year. Based on previous literature, I expect children's skills at school entry and rate of growth across kindergarten to reflect individual differences in early number skills. Moreover, I expect children’s age at testing to predict changes in mathematics skills at the beginning of kindergarten, but not growth across kindergarten (Johnson & Kuhfeld, 2020). On the other hand, based on the previous literature, I do not expect sex of the child to predict mathematics skills at school entry or growth across kindergarten (Hutchison, Lyons, & Ansari, 2019).

Research Question 3

Do different aspects of mathematics instruction contribute to children's growth in mathematical skills? Building on the second research question, multiple studies have examined the contribution of education to early mathematical skills (Bodovski & Farkas 2007; Engel et al., 2013; Ribner, 2019). However, very few studies have used observational data to examine classroom mathematics instruction. Thus, the third research question in this dissertation will examine classroom mathematics instruction through observational measures, and assess whether quality, quantity, or content of mathematics instruction predict growth in children's mathematical skills across kindergarten. Based on prior research, I hypothesize all domains of teacher instruction will play a role in the growth of early number skills. However, based on the child by instruction interaction theory, I believe the content of mathematics instruction will drive this relation.

Chapter II

Method

Participants

Kindergarten children (*N* = 98, *M*(age) = 5.55 years, 53% male, see Table 1) were recruited from four local elementary schools across 14 kindergarten classrooms in the greater southeast area of Michigan. These four schools participated in previous research collaborations with the University of Michigan in the years before this study. Thus, recruitment consisted of a brief email to the principal asking if they would be interested in continuing a research-school collaboration for a new research study. The schools serve children with a range of socioeconomic backgrounds based on percentages of free and reduced-price lunch (FRPL; 3%, 32%, 45%, and 66% respectively).

Participant recruitment was accomplished by sending invitation letters and consent forms home in children's backpacks. Every child in the 14 classrooms took home invitation letters. Parents were informed that all aspects of participation would take place in the school, and they could also complete a family questionnaire for monetary compensation. Of 311 children in all 14 classrooms, 98 children returned signed parent consent forms for participation in the study. Parent consent forms were returned for students from all classrooms. All children who brought back signed consent forms were invited to participate. Children received small gifts for their participation each time they were taken out of the classroom, such as a small stuffed animal or slinky. Attrition was low throughout the kindergarten school year. Only 5.1% of children did not participate in the second round of data collection. In the final round of data collection in the schools, attrition percentages increased to 7.1%. Thus, overall attrition in the sample of children was low, however, those who did leave the study also left the school and in some cases moved to another state.

Teachers in each classroom were also invited to participate in one aspect of the study focusing on school mathematics instruction. They were asked to record one entire school day in the middle of the school year (February) using the LENA audio recorder and to complete two questionnaires, one at the beginning of the year and one at the end of the year. Of the 14 teachers, all agreed to complete the survey, and 11 participated in the use of the LENA audio recorder for one day during the school year. Some teachers who had participated in research studies prior to this one had been videotaped for classroom observations and would prefer to not be recorded. All teachers were reassured that this would not be videotaping, but rather audio recording for research purposes and the focus was on understanding the growth of children's early mathematical skills. However, three teachers, who had also refused observations in previous years, opted out of the audio recordings. All teachers received monetary compensation for completing the questionnaires and recording one school day. This study was reviewed and approved by the University of Michigan Institutional Review Board.

Ten percent (n = 10) of the children in this sample were attending the school that qualified for 66% FRPL (school 1). Thirty-five percent (n = 34) of the children were attending the school that qualified for 45% FRPL (school 2). Twenty-four percent (n = 24) of the children were attending the school that qualified for 32% FRPL (school 3). Finally, thirty-one percent (n = 30) of the children were attending the school that qualified for 3% FRPL (school 4). The unequal participant percentage distribution may be because the number of kindergarten classrooms varied across schools. For more information, see Table 2.

Eleven of the teachers (78.6%) participated in the LENA portion of the study in which they recorded one full day of instruction. However, all fourteen teachers (100%) participated in both study questionnaires. On average, teachers in the sample taught for 14.14 years and taught kindergarten for 7.43 years. A majority of the teachers held a master's degree (78%). For more information, see descriptive statistics from the teacher questionnaires in Table 3.

Less parents than originally planned (58%) completed the family questionnaire portion of the study. Thus, any questions regarding the home environment were not able to be included for further analyses. Of those participants whose parents completed the questionnaire, 56.14% were White, 8.77% were Black, 7.02% were Hispanic, 15.79% were Asian, and 12.28% were multi-racial. Most of the children attended preschools (98%), such as in-home daycare or center-based preschool. The average household income of the sample that completed the survey was approximately $107,517, and 38.46% of the parents held a postgraduate or professional degree. More questionnaires were completed by parents whose children attended schools with lower FRPL percentages. For more information from the parent questionnaire, see Table 4.

Due to the low percentage of parents that completed the survey, future analyses included school FRPL percentages as a socioeconomic status proxy. Although research has shown FRPL is not the best measure for educational disadvantages, (Domina et al., 2018), the lack of information on socioeconomic status of the parent does not allow for a robust analyses using the parent data. In the sample, FRPL and income were ($r(57) = -.34$, $p < .05$). Though the relation is moderate, it provides some broad information on the financial condition of the families in this study, thus, FRPL percentages were used in all further analyses.

Procedure

At the time of recruitment, invitation letters sent home to the parents informed parents that their child would be participating in a study of different mathematical activities throughout their kindergarten year. The participation was voluntary, and each time children were taken out of class, they were told that they could withdraw their assent at any time in the study without penalty.

Data were collected through direct child assessments, teacher questionnaires, parent questionnaires, and teacher recordings. Child assessments took place five times throughout the school year, teachers were asked to fill out questionnaires twice, parents were asked to fill out a survey once, and teacher recordings were collected one day during the middle of the school year. For more information, see Figure 1.

During the child assessments, the researchers visited classrooms and asked teachers if it would be okay to take participants out of class for our research study. If the teachers said it was not a good time, researchers would return at a later time and visit another classroom. When teachers said it was okay, researchers walked participants to a designated research area in the school. Three times throughout the school year, these same procedures were used to take children from classes in groups of three or four for the group behavioral task with two trained research assistants. Twice throughout the school year, the child was escorted independently from the classroom for one-on-one behavioral assessment sessions with a trained research assistant. All assessments took place in the school setting in an unoccupied area such as a classroom, library, or quiet hallway.

The teacher recordings took place during the middle of the nine-month school year, in approximately February. On the day of the recording, researchers would bring the teacher a coffee or tea and orient the teacher to the LENA device. Teachers were assured that the LENA device could record for up to sixteen hours, so they were told not to turn it on or off the entire day. A few teachers asked if they could leave the LENA in the classroom if they had to use the bathroom, and researchers assured them that they could. However, teachers were asked to wear the device during all instructional periods. Teachers wore a pocket lanyard around their necks, similar to an ID holder that was purchased off of Amazon, in which they put the LENA recorders. Due to the lightweight, simplicity of the LENA, teachers often forgot about wearing a recorder after a couple of minutes. Thus, the device allowed the study and the research team to avoid the intrusiveness of other observational methods (e.g., videotaping the classroom). Teachers wore the LENA at the beginning of the day while children were arriving and took it off at the end of the day when the researcher came and turned the LENA off and log it back in at the University of Michigan. The researcher administered a brief three-question survey at pick-up regarding the content they covered at school that day and the typicality of the day. For more information, see Table 3.

Child Measures

Children's early mathematical skills were collectively assessed six times throughout the school year. Thus, their individual, dynamic skills were evaluated three times, their basic mathematical achievement skills were evaluated twice, and their magnitude knowledge was evaluated once. Midway through the study, it was also recommended to assess children's early self-regulation skills, as research has emphasized the link between the two (Morgan et al., 2019; Nguyen & Duncan, 2019). Thus, children's executive function skills were assessed at one point at the end of the school year.

Individual Mathematics Measures

Applied Problems. At two time points during the year, children completed the Woodcock-Johnson III Tests of Achievement, Applied Problems subtest (WJ-AP; Woodcock, McGrew, & Mather, 2001). The WJ-III Tests of Achievement are standardized administrative tasks that were designed to provide information about a child's abilities in comparison to the national average. The WJ-AP subtest is a task in which children are presented with a set of questions to assess overall, broad mathematics abilities. This task is brief and can determine a wide range of mathematics abilities. It is also widely used in many nationally representative databases.

Number Line Estimation. Children's numerical magnitude ability was measured using a number line estimation task (Thompson & Siegler, 2010). Participants received a 20 cm long paper number line labeled 0 and 20 at the left and right ends, respectively. Their task was to draw a vertical line indicating where a given random integer between 1-20 fell on the number line. The administrator held a flipbook that presented the child with the number they were to place on the line. The books contained 10 randomly chosen integers (16, 4, 1, 13, 17, 9, 8, 19, 6, 10). A new number line was used in each trial so that only one number was placed on each line. Participants completed 10 test trials on the 20 cm line without feedback. Children's performance was scored as the absolute value of the difference (in centimeters) between the correct placement and the participants' placement of the number. These ten scores were then averaged to give an average error.

Group Mathematics Measure

Math Garden. Three times during the year, the children completed Math Garden in groups of two-four. Math Garden is an IRT web-based CAT technique and monitoring system that introduces a challenging environment for children to practice arithmetic (Klinkenberg, Straatemeier, van der Maas, 2011). The web-based monitoring system includes each mathematic operation presented as a different game in which children are working to grow a garden. The game offers a learning platform for children in which each domain (counting or addition) has a plant, and the plant grows as mathematical ability increases. For example, in the game assessing addition, children work to keep their cattail plants alive by answering questions quickly and correctly. Counting, on the other hand, is represented by the growth of blue and orange daisies. Variables measured by the task are response time, the given answer, the correctness (0, 1), and a timestamp at administration. This assessment can help investigate the existence of distinct mathematical skills and help categorize early individual differences. Each domain is comprised of a block that has ten questions presented for a maximum of 20 seconds per question.

Math Garden was created at the University of Amsterdam to accurately measure children's early mathematical knowledge (Klinkenberg, Straatemeier, van der Maas, 2011). This web-based visual format allows children to remain motivated and interested in a task that provides a more nuanced measurement of mathematical skill and is essential for educational approaches, based explicitly on specialized learning.

Counting. In the counting condition of the Math Garden task, children were presented with a screen that had fish on the left-hand side and symbolic numbers on the right-hand side. Children were then directed to push the number that shows how many fish are on the screen. If they were correct, the symbolic number would turn green, and another question would be presented. If their responses were wrong, the number would turn red, and the correct answer would turn green. Children completed four blocks of the counting condition three times during the year.

Addition. The addition condition of the Math Garden task, children were presented with a symbolic addition problem at the top of the screen and were presented with six possible answers. Children were directed to push the number that shows the solution to the question displayed. Similar to counting, if the responses were correct, it would turn green, and if they were incorrect, it would turn red, and the correct answer would turn green. Children completed three blocks of the addition condition three times throughout the year.

Self-Regulation

Working Memory. Children completed the Digit Span subtest of the Wechsler IQ test (Wechsler, 1991) toward the end of the school year. This task was composed of two sections: first, the administrator says a list of numbers at a slow rate, and the participant was asked to recite the numbers back to the administrator. In the second section, the administrator says a list of numbers at a slow rate, and the participant is then asked to recite the numbers back to the administrator backward. The list of numbers increases by one item for every correct response. If the participant answered incorrectly twice in a row, the administrator moved onto the next section (Nesbitt et al., 2013). A score was assigned based upon the largest set at which the child successfully reported.

Teacher Measures

Teacher Questionnaire

Teachers completed two questionnaires designed and used for this study. The first survey asked teachers questions specific to mathematical skills and instruction in the classroom. For example, teachers were asked on a typical day, “how long do you spend on mathematics instruction?” Teachers also ranked children’s mathematical skills on a scale of 1-10 in the beginning and at the end of the year. The second survey asked teachers to assess children’s self-regulation in the spring of the academic year. Teachers completed the approaches to learning (ATL) scale which consisted of seven items related to children’s learning approaches in the classroom. Items included questions such as children’s ability to keep belonging organized, work independently, adapt to changes in routine, etc. Items were adapted from the Social Skills Rating System (Gresham & Elliot, 1990). Responses to the ATL scale were rated on a 1-4 scale (e.g., Never, Sometimes, Often, Very Often). Teachers provided emails on their consent forms for the questionnaires. Teachers received both verbal and email reminders if they had not filled out the surveys.

Classroom Instruction Measures

Teachers wore the LENA recording system for one day in the middle of the school year in February. This recording provided a means to measure aspects of the classroom and the teacher mathematics instruction. Three aspects of the classroom instruction were examined. In essence, the quality of the classroom environment, the quantity of time spent teaching mathematics, and the content of mathematics instruction were used to investigate the role of education in the development of children's mathematical skills during the school year.

Quality. Many researchers have attempted to measure the quality of interactions and classroom environments. In this study, the LENA devices include software that automatically processes the audio recordings to provide specific variables. The software exports variables such as adult word exposure, child vocalizations, and turn-taking interactions throughout the day based on algorithmic analysis (Xu, Yapanel, Gray, Gilkerson, Richards, & Hansen, 2008). Studies have found that turn-taking interactions relate to better language outcomes, brain structure, and long-term outcomes such as IQ (Gilkerson et al., 2018; Romeo et al., 2018). Thus, for this study, the quality of the classroom environment was captured using the LENA's turn-taking variable called the conversational turn count (CTC). The CTC is often used to quantify adult-child vocal initiations with responses that occur within 5 seconds. Both intentional spoken replies and accidental vocal responses can be included in the final CTC (Romeo et al., 2018). The reliability and validity of the CTC measure have been extensively reported (Oller et al., 2010; Xu, Yapanel, Gray & Baer, 2008; Xu et al., 2014; Zimmerman et al., 2009). In theory, the CTC variable captures interactive talk, providing students with a deeper understanding and higher quality of instruction beyond time or topic. To assess the overall quality of the classroom, the CTC will cover the entire recording, rather than specifically during any mathematics instruction time.

Quantity. In addition to examining the quality of the classroom, the amount of mathematics instruction in classrooms was also considered. Quantity of the classroom mathematics instruction was measured by the amount of time (in minutes) teachers spent in the classroom instructing mathematic-related topics. Trained research assistants listened to the entire voice recording and tracked the time teachers spent in mathematic topics, according to a coding scheme (described below in Content). Reliability of the amount of time research assistants used the coding scheme described below was exported from the Noldus Observer XT software and was adequate ($\boldsymbol{\kappa}$ = .99).

To account for integrated mathematics instruction, the research assistants coded the entire day of recording. Thus, they were able to distinguish certain times during the day when teachers would instruct mathematic topics during a specific mathematic lesson, and when teachers would teach mathematic topics outside of a formal mathematic lesson. Frequent periods outside of a mathematic lesson where a teacher may integrate mathematics instruction in kindergarten include morning meetings (e.g., calendar time, counting money), or discussing the daily schedule (e.g., Gym time is at 11:30 am, Lunchtime is at 12:30 pm). Therefore, if teachers were not able to instruct a full lesson on mathematics on the day of the recording, it was still possible for students to receive some amount of mathematics instruction on that day.

Content. Finally, in addition to the quality and quantity of mathematics instruction, the content of mathematics instruction was also examined. Content of mathematics instruction was coded through trained research assistants using a well-established coding system to label the topics covered during instruction. The coding scheme used in this study was based on an Individualized Student Instruction (ISI) coding system, originally designed to assess literacy instruction (Connor et al., 2009). The ISI coding system was developed to examine the relationship between the skill level of the individual child and the amount of time they spend in specific types of instructional activities. Thus, focusing on a child X instruction (CXI) interaction (Connor et al., 2009). Therefore, although the original coding system concentrates on literacy, it has been adapted to include other instructional activities (e.g., mathematics) and noninstructional activities (e.g., transitions, planning, off-task behavior). In this dissertation, research assistants were only trained on the updated mathematical content codes from the ISI system, and thus, those were the only content codes used.

The content of mathematics instruction was also examined using the Noldus Observer XT 13 software (Noldus Information Technology, 2013). The Noldus software requires a .wav file to be uploaded to use the coding system. Rather than using videotapes like most of the other ISI studies, this dissertation used audio recordings from the LENA devices in the Noldus software. Thus, some codes that required visualization (e.g., number line) were estimated based on audio.

***Mathematical Content Coding Categories*.** The ISI coding scheme codes both the duration of time the children in the classroom experienced a specific type of mathematics instructional activity and the content of the particular mathematics instruction. Each code in the ISI coding scheme was only recorded if it lasted at least 15 seconds. The instruction was divided into six general headings of categories, which included 37 subheadings of possible codes:

1. Number sense, concepts, and operations (14 subheadings)
    a. Number Writing and Recognition
    b. Oral Counting
    c. Number Line
    d. Patterns (#s)
    e. Counting Sets
    f. Number Relations
    g. Estimating (#s)
    h. Addition
    i. Subtraction
    j. Multiplication
    k. Division
    l. Place Value
    m. Fractions
    n. Decimals
2. Geometry (5 subheadings)
    a. Shapes
    b. Lines
    c. Transformations
    d. Coordinate Geometry
    e. Spatial Geometry
3. Algebra (3 subheadings)
    a. Patterns (not #s)
    b. Expressions and Equations
    c. Inequalities
4. Measurement (8 subheadings)
    a. Time

    b. Temperature
    c. Money
    d. Length
    e. Circumference, Perimeter, Area
    f. Weight
    g. Capacity
    h. Quantity
5. Data Analysis (3 subheadings)
    a. Data Collection
    b. Data Representation
    c. Data Analyzing
6. Probability (4 subheadings)
    a. Certain, Likely, Impossible
    b. Likelihood
    c. Predict an Outcome
    d. Conduct an Experiment

Examples and a detailed description of the coding system can be found in Appendix B. This Appendix also contains specific instructions for how research assistants used Noldus, as well as how and where the final coded files were saved.

The original ISI coding system based on Connor et al. (2009) also included an extra subheading underneath each broader mathematics instructional heading that was titled “Multiple Components.” This code was usually used when a variety of combined subheadings occurred within a 15 second instruction period. For example, if a teacher asked students to locate the number 20 on a hundred’s chart (e.g., number writing and recognition), then count aloud from 0 to 20 (e.g., oral counting), and then count to 20 by 2’s (e.g., patterns) all within 15 seconds. Thus, this code was categorized separately from the other six subheadings as each content topic were discussed briefly.

***Mathematical Content Coding Reliability*.** All coders were trained before coding the audio recordings from the actual classrooms to ensure coders were using the same criterion while coding the content of the mathematics instruction in classrooms. Coders watched two videotaped recordings from classes recorded from a previous study to have examples of the different codes, and practice their knowledge of the coding scheme. After students were familiar with the coding system, 30% of the audio recordings for the present study were independently coded by two trained research assistants, and Kappas were computed for all subheadings from the coding system (e.g., subtraction, patterns, measurement, etc.). The average Kappa across these recordings, across all categories of the ISI coding scheme, was adequate ($\boldsymbol{\kappa}$ = .93; $\boldsymbol{\kappa}$ *range* = .88 - .97). Once coders reached reliability, they continued to code the rest of the recordings, and questions or disagreements were discussed among the group until resolved.

Family Variables

Parent Questionnaire

The demographic aspects of the families and parenting variables were assessed with one emailed questionnaire. Parents completed a set of general questions pertaining to the background information of children and their families (e.g., maternal education, preschool experiences, income). The questionnaire also included specific questions about mathematical activities in the home, based on LeFevre et al. (2009). These questions were focused around home numeracy experiences that are related to children’s early competence in school. Some examples included the frequency of which the family does mathematical activities (e.g., mathematic workbooks, puzzles, connect-the-dots) or plays number games (e.g., “This Old Man” or “1, 2, Buckle My Shoe”). Responses to these questions were Likert-type that ranged from (1) *almost never* to (5) *daily*. For more information about the parent questionnaire see Appendix C. Parents provided emails on their consent forms for the questionnaires. Parents received email reminders to complete the survey throughout the school year until the end of June.

Data Analysis Plan

In order to address the questions in this dissertation, a framework for examining change over time was used (Singer & Willett, 2003). Due to the clustered nature of the data (children within classrooms), multilevel modeling was chosen as the best method to address the lack of independent in the data. Timepoints were nested within 98 students, and students were nested within 14 classrooms.

A three step process of testing models was used in order to answer the hypothesized changes in the data (Singer & Willett, 2003). In the first step unconditional means and growth models were used to address the first research question: do children start school with various early mathematical skills and are their skills in the fall related to their growth overall? Unconditional multilevel growth models examine whether the intercept and rate of slope for the counting and addition tasks significantly differ from zero. In this case, the models can also determine whether the data fit a model that allows for fixed slopes or random slopes, which is especially useful for determining whether children’s mathematical skill growth in kindergarten varies by child. Unconditional means models determine whether the skills children bring to school are significantly different from zero, thus testing the hypothesis that children enter school with mathematical skills.

The second research question examined whether individual characteristics predicted children's mathematical skills at the beginning of kindergarten or growth in mathematical skills across the kindergarten year. Thus, conditional means and growth models were used to examine whether child-level characteristics (e.g., age at testing and sex) were significant predictors of skills in the fall and growth over the year. These models were compared to the models in the first research question to assess their explanation of variance and fit of the data.

Finally, the third research question examined whether aspects of mathematics instruction predicted children's growth in mathematical skills. Thus, conditional growth models built on the previous models from the second research question to examine whether teacher-level instruction characteristics (quantity, quality, or content) and school free and reduced-price lunch (FRPL) percentages were significant predictors of skill in the fall or growth in mathematical skills. These final models were also compared to the previous models to assess variance explanation and fit of the data.

These three questions resulted in a series of multilevel models in which counting, and addition skills were modeled as a function of an intercept, and a slope (time). First, unconditional means models were estimated to partition the variance present in the outcomes across levels. Then, a series of two-level multilevel models were estimated in which classroom instruction variables were treated as time-invariant characteristic predictors (TIC) of counting and addition skills. A sample two-level model is presented here:

LEVEL 1 (within-person level):

$$Counting/Addition_{ij} = \beta 0_j + \beta 1_j(\text{TIME}) + r_{ij}$$

LEVEL 2 (between-person level):

$$\beta 0_j = \gamma_{00} + \gamma_{0n}\ (\text{AGE/SEX/FPRL/QUAL/QUANT/CONT}) + u_{0j}$$
$$\beta 1_j = \gamma_{11} + \gamma_{1n}\ (\text{AGE/SEX/FPRL/QUAL/QUANT/CONT}) + u_{1j}$$

After sex of the child was dummy-coded and FRPL % was rank-ordered, data were imported from Microsoft Excel into RStudio and analyzed. Two-level multilevel models were estimated using the NLME package (version 3.1.140) in RStudio (version 1.1.456). Little's Missing Completely at Random test (MCAR) was not significant, suggesting data were missing at random ($\chi 2 = 27.97$, $p = .12$).

Models were estimated using restricted maximum likelihood estimation methods (REML). REML is more accurate at predicting random effects when the number of Level 2 groups is less than 50 (Snijders & Bosker, 2011). The NLME function in RStudio estimated the fixed and random effects as well as the fit statistics, reliability, and correlation coefficients.

CHAPTER III

Results Research Question 1

Math Garden Growth

Counting

Children's counting skills were assessed at three timepoints. Children's skills grew across each timepoint during the school year. At the first timepoint, on average the children in the sample scored $M(SD)$ = 3.41 (1.03) on the Math Garden counting scale. At the second timepoint midway through the school year, the average counting score was $M(SD)$ = 3.78 (.94). Finally, at the third timepoint in the spring, the average counting score was $M(SD)$ = 3.85 (.96). To see the variability in individual counting trajectories, see Figure 2.

Addition

Children's addition skills were also assessed at three timepoints. Children's addition skills also progressively grew throughout the school year. At the first timepoint, on average the children in the sample scored $M(SD)$ = -1.34 (2.23) on the Math Garden counting scale. The negative score meant that children were not yet considered proficient in addition skills. At the second timepoint midway through the school year, the average addition score was $M(SD)$ = -.49 (2.17). Finally, at the third timepoint in the spring, the average addition score was $M(SD)$ = .43 (1.98), which was reached the criteria for some proficiency in addition. Variability in individual addition trajectories can be seen in Figure 3.

Relation Between Mathematical Measures

Before addressing the first research question in this dissertation, it is important to ensure that the novel mathematical measure (Math Garden) is related to other mathematical measures used in the study. As mentioned above, Math Garden is an adaptive dynamic test designed to examine mathematical skill level as individualized as possible. Thus, although different, Math Garden was expected to moderately correlate with all other mathematical measures in the study. In Table 5, the correlations between all child measures are reported.

Math Garden and Applied Problems

The standardized mathematical measure used, WJ-AP, was measured at the beginning of the school year, and at the end of the school year. Math Garden was also measured at the beginning and the end of the school year. In the beginning of kindergarten, children who performed better on the WJ-AP task also performed better on the Math Garden counting ($r(98) = .52^{***}$ $p < .001$). and addition tasks ($r(98) = .53^{***}$, $p < .001$). Similarly, in the spring, children who performed better on the WJ-AP task also performed better on the Math Garden counting ($r(91) = .59^{***}$, $p < .001$) and addition tasks ($r(91) = .76^{***}$, $p < .001$). Not surprisingly, the tasks in the beginning of the school year also correlated with tasks in the end of the school year (WJAP2 & MGCount1: $r(92) = .62^{***}$, $p < .001$), (WJAP2 & MGAdd1: $r(91) = .64^{***}$, $p < .001$), (MGCount3 & WJAP1: $r(91) = .42^{***}$, $p < .001$), (MGAdd3 & WJAP1: $r(91) = .66^{***}$, $p < .001$).

Although these assessments related strongly, they were not collinear, thus the individualized assessment provided a more qualitative examination of mathematical skills. See Figures 2-4 to compare variability in mathematical measures. Although the Math Garden counting task used IRT based methodology, it looks similar to the WJAP graph in that most kids do not change much from the first timepoint to the last timepoint. However, the variability in children's addition scores is shows in Figure 3.

Math Garden and Number Line Estimation

Similar to the WJ-AP, the Number Line Estimation (NLE) task is also a well-established measure. NLE was only measured at the third and final timepoint of the study, however, the relation between these tasks at all three timepoints of the Math Garden were examined. At all timepoints, better (lower) scores on the NLE tasks had small to moderate associations with higher scores on the Math Garden counting ($r(92) = -.43^{***}$, $p < .001$, $r(91) = -.44^{***}$, $p < .001$, $r(91) = -.34^{**}$, $p < .01$) and addition tasks ($r(92) = -.27^{**}$, $p < .01$, $r(91) = -.41^{***}$, $p < .001$, $r(91) = -.36^{***}$, $p < .001$).

Research Question 1

Do children start school with various early mathematical skills, and do their skills grow throughout the school year? Now that it has been established that the measures relate to each other across time but are assessing different skills, the primary questions of the study can be answered. In order to address the question of variability in early mathematical skills, unconditional means and slope models for both the counting and addition conditions were examined. Unconditional means models determine whether children's skills significantly differed from zero, and unconditional growth models ascertain whether children's skills grew across the school year. No predictor variables were included in these models, as RQ1 focused on the types of early mathematical skills that children started school with whether these skills grew across time.

Unconditional Means Models

Unconditional means models were estimated to partition the variance in counting and addition skill outcomes across both levels of analysis. Intraclass correlations (ICC) indicated that 54% of the variance in the counting skills model and 65% of the variance in the addition skills model occurred within participants. Given the substantial variance accounted for at each level, full level 1 and level 2 models were then tested. Although ICCs in educational research with cross-sectional designs ranges anywhere from .05 to .20 (Snijder & Bosker, 1999), the relatively high ICCs in this study are probably due to the longitudinal nature of the data given that the same measure was assessed repeatedly from the same student over time. These ICCs mimic those of previous longitudinal MLMs with academic achievement such as (Galla et al., 2014). Model fit for the unconditional means models are reported as the null models in Table 6 for counting and Table 7 for addition.

Using the NLME function using REML in RStudio, an unconditional means model was estimated for both the addition and counting skills. The resulting models (Model 0a and 0b) served as a baseline fit for any further statistical approaches. The unconditional means model for counting indicated that, on average, at Time = 0 in the fall, the mean of the entire sample was significantly different from zero (β *(SE)* = 3.67 (.08), t = 43.45, p <.05). Scores on the addition condition also significantly differed from zero (β *(SE)* = -.54 (.20), t = -2.69, p <.05).

Unconditional Growth Models

Also, in the NLME function using REML in RStudio, unconditional slope models were estimated for counting and addition tasks. Unconditional slope models were examined as both a random and fixed slope model to determine which best fit the data. Model fit indices were used to determine whether the slope should be fixed or vary randomly. Thus, Model 1 and 2 were compared to Model 0 in each condition. When using REML estimation procedures, the deviance statistics for determining whether there is a statistically significant improvement in model fit are only meaningful when comparing random-effects models that share the same fixed effects. Therefore, deviance statistics for model change were only used when comparing the addition of random slopes to the fixed effects models.

Based on previous literature, in both cases, it was expected that the random slope would fit the data better, as children’s skills grow at different rates. In both cases, results indicated that Model 2, the random slope model, suited the counting and the addition conditions best. Thus, only random slope models are reported below. In all future models, confidence intervals around the standard deviation for the variance component were used to infer significance. Confidence intervals that do not include zero indicate that an effect is significantly different from zero (Field, Field, & Miles, 2013).

Counting. The average child's counting score significantly increased at each timepoint *(β (SE)* = .21 (.05), t = 4.16, p <.05, see Model 2a). Further, children's starting points varied almost one point on the counting scale (β *(SE)* = .85 (.56), p <.05), and children's slopes also varied (*Random Slope (SD)* = .30). The correlation between children's random intercept and slope for the counting task was significant such that children that started with higher counting scores improved less, and children who started with lower counting scores improved more across the school year (r(98) = -.46, [95% CI: -.70 to -.12]).

Addition. Similar to the unconditional counting model, results suggested that the average child's score significantly increased at each timepoint (β *(SE)* = .85 (.09), t = 9.71, p <.05, see Model 2b). However, this average value was much larger than the average child's counting value. Further, children's kindergarten starting points in the addition condition varied more than their counting scores, over two points on the addition scale (β *(SE)* = 2.07 (.86), p <.05), and their slopes also varied more (*Random Slope (SD)* = .59, [95% CI: .42 to .83]). The correlation between children's random intercept and slope for the addition task mimicked the counting condition such that the relation was also significant and negative (r(98) = -.47, [95% CI: -.67 to -.20]). Thus, children who started school with higher addition scores improved less, and children who started school with lower addition scores grew more.

CHAPTER IV

Results Research Question 2

Research Question 2

Are there individual characteristics that contribute to children's starting point and rate of growth during the kindergarten year? Due to the two significant random-effect slope standard deviations from the first research questions for Model 2a and Model 2b, it is plausible to examine further potential individual-related predictors that may account for the variation in mathematical skill intercepts and growth rates. Thus, the second research question focuses on individual characteristic variables related to mathematical achievement. Conditional means and growth models were estimated using time-invariant characteristics (TICs) collected in the study. Individual variables are TICs because the values of these variables stay the same across the study. TICs in this study included children's age at testing and children's sex. Conditional means models were estimated to determine the effect of TICs on children's entry skill levels, and conditional growth models were estimated to assess the effect of TICs on children's rate of growth in counting and addition skills.

Conditional Growth Models: Student-Level Characteristics

Using the NLME function with REML in RStudio, two conditional means models were examined: Model 3a included age and sex of the child as predictors of children's starting point in counting skills, and Model 3b used the same TICs for addition capabilities. Based on previous literature, age was expected to predict children's starting point for both tasks. Although the research reviewing sex differences is mixed, sex of the child was not hypothesized to predict differences in children's counting and addition starting points. See Tables 6 and 7 for model coefficients and model fit information.

Counting. One average, children's entry counting score (*β (SE)* = .96 (1.69), $t = 0.57$, $p > .05$), and rate of change in counting (*β (SE)* = .62 (.87), $t = 0.7$, $p > .05$) no longer remained significantly different from zero (see Model 3a). Thus, the inclusion of the student characteristics explained the changes in counting skills across the kindergarten year. Differing from the previous literature, age at entry was not a significant predictor of children's counting skills at Time = 0, the fall of kindergarten (*β (SE)* = .47 (.30), $t = 1.54$, $p > .05$). Further, consistent with our hypotheses, sex of the child was not a significant predictor of entry counting skills (*β (SE)* = -.18 (.20), $t = -0.89$, $p > .05$, see Figure 5). Moreover, age at testing and sex of the child did not significantly predict children's growth in counting skills across the kindergarten year (*Ageβ (SE)* = .47 (.30), $p > .05$., *Sexβ (SE)* = -.18 (.20), $p > .05$). However, children's slopes and mathematical skills at school still significantly varied from child to child.

The correlation between children's random intercept and slopes was also significant and negative ($r(98) = -.47$, [95% CI: -.71 to -.13]). Thus, children with higher counting skills at the first timepoint grew less than children with lower counting skills. The random intercept showed that children's starting points varied almost one point on the counting scale (*β (SE)* = .84 (.56), [95% CI: .69 to 1.03]).

Addition. On average, children's addition score significantly differed from zero (*β (SE)* = -7.61 (3.60), $t = -2.11$, $p < .05$, see Model 3b). However, similar to counting, the average child's rate of growth no longer remained significantly different from zero (β *(SE)* = .42 (1.50), $t = 0.28$, $p > .05$). Inconsistent with the hypotheses, age at testing was not a significant predictor of children's counting skills at Time = 0, (β *(SE)* = 1.23 (.65), $t = 1.91$, $p > .05$). Also inconsistent with the hypotheses, though related to the mixed literature, sex of the child was a significant predictor of children's addition skills at the beginning of kindergarten, such that males scored higher than females on addition skills at the first timepoint (β *(SE)* = -1.23 (.43), $t = -2.89$, $p < .05$, see Figure 6). Neither age at testing, nor sex of the child, were significant predictors of children's growth in early addition skills (*Age*β *(SE)* = .07 (.27), $t = 0.27$, $p > .05$., *Sex*β *(SE)* = .08 (.18), $t = 0.42$, $p > .05$).

The correlation between children's random intercept and slope was also significant and negative ($r(98) = -.49$, [95% CI: -.69 to -.23]). Thus, children with higher addition skills at the first timepoint grew less than children with lower addition skills. Similar to counting, the random intercept coefficient showed that children's starting points varied almost two points on the addition scale (β *(SE)* = 1.95 (.86), [95% CI: 1.65 to 2.30]). Children's slope also varied from child to child at a higher rate than their counting slopes (*Random Slope (SD)* = .61, [95% CI: .44 to .85]).

CHAPTER V

Results Research Question 3

Research Question 3

Do different aspects of mathematics instruction contribute to a student's mathematical skill growth rate? Now that the individual growth of mathematical skills in counting and addition have been established, MLM was used to explore the contribution of different domains of instruction (quantity, quality, and content) and school to early mathematical skill development. Two MLMs were conducted with level-1 variables, including measurement of children's early mathematical skills, and rather than merely using a teacher code at level-2, the quantity, content, and quality of the teacher's mathematics instruction were assessed for their classroom.

Teacher Report Classroom Instruction

For a summary of the teacher questionnaire, see Table 3. Eighty percent of teachers in this sample reported obtaining a master's degree or higher. On average, the teachers in this sample taught for 8.33 years, and 7.43 of those years were spent teaching kindergarten. Teachers, on average, reported spending 50 minutes a day on mathematical topics. In comparison, they reported spending 115 minutes on average on literacy. These results are consistent with the previous literature that shows teachers spend much less time in the mathematics domain than in literacy (Engel et al., 2013).

On average, teachers wore the LENA recorder for 6 hours and 54 minutes on the recording day. All teachers wore the recorder before school started in the morning, and took it off at the end of the day. On average, 407 conversational turns, 25,158 adult words, and 735 child vocalizations were analyzed by the LENA device. A range of two to 42 minutes of television was observed in individual classrooms, but an average of 15 minutes of TV was recorded. Most of the minutes captured by the TV variable in the school included YouTube videos that the children would watch for brain breaks. Out of the total recorded time, proportions of self-report and observed variables were calculated. On average, teachers reported spending 13% of their day in mathematics activities, and 29% of their day in literacy activities. However, after coding mathematics instruction using the LENA recorders, the teachers on average only spent 6% of their day in mathematical activities. Consistent with previous literature, and serving as a robustness check, teacher report of time spent on literacy was used as a control in the following MLMs.

Quantity of Mathematics Instruction in the Classroom

The total quantity of mathematics instruction in the classroom was calculated by trained research assistants listening to the entire classroom day recording, and coding periods where the teachers engaged in any mathematics lesson. For example, some teachers would present a *math check* as part of their morning routine. Although this may only consist of a brief (5 minutes) period in the classroom, those 5 minutes were then added to any other mathematics instruction times the teacher presented that day. Thus, total mathematics instruction time was not just during an organized mathematics block. Mathematics instruction was also included when teachers referenced mathematical concepts throughout the day for longer than 15 seconds. The total time spent in mathematical activities was then exported from Noldus Observer XT (Noldus Information Technology, 2013) by seconds and divided by 60 to report time in minutes. As shown in Table 3, on average, teachers spend approximately 25 minutes on mathematics instruction on an average day (*M(SD)* = 26.16 (13.85)).

When compared to the amount of time teachers reported they spent on mathematics instruction, this number was smaller than expected. Table 3 also shows the amount of time that teachers reported spending in mathematics instruction compared to the amount of time captured on the recording that was spent on mathematics instruction. Figure 7 also shows the comparison of reported versus observed. Although this was only one day of recording, these numbers suggests that having teachers self-report on the amount of instruction they are providing during the day may overrepresent the actual amount of instruction in the classroom.

Quality of the Classroom

As mentioned above in the methods, each teacher that participated in the LENA recordings had individual CTC measures from their day. These values are also presented in Table 3. On average, teachers used about 407 conversational turns during the school day (*M(SD)* = 407.09 (173.98)). However, some classrooms used a minimum of 191 conversational turns, and some classrooms used a maximum of 731 conversational turns. Thus, the standard deviation was large. Table 3 also includes descriptive statistics of all LENA software variables available, including adult word count (AWC) and child vocalization count (CVC). Recent research suggests CTC is associated with children's language abilities more robustly than AWC, even when examining within-classroom variability (Duncan et al., *in press*). Thus, although AWC showed a more uniformed relation between quantity (r (77) = .53, $p < .05$) and content (r (77) = .55, $p < .05$), the CTC remained the quality variable of interest for this dissertation.

Content of the Classroom

The content of the mathematics instruction was also coded using the ISI coding system (Connor et al., 2009). This coding system included a range of topics that could be covered in a classroom. Of the possible 36 content codes, only 21 were used in the kindergarten classrooms represented in this study. The most common content topics included addition, time, and counting sets. The codes that did not occur in any classes included multiplication, division, decimals, lines, transformations, coordinate geometry, expressions and equations, length, circumference, capacity, quantity, data collection, identifying, and describing events. These codes may not have occurred in kindergarten as the ISI coding system was created to code a variety of grade levels, up to second grade, so many of the codes not observed were above grade-level. Table 8 includes descriptive statistics of all content codes that were coded in the kindergarten classrooms.

The Relation Between Classroom Instruction Categories

Once all coding was complete, relations between the classroom instruction subcategories were explored. In Table 5, the correlations between the quantity, quality, and content of teacher mathematics instruction are reported. The relation between quality and quantity was small and negative ($r(77) = -.03$, $p > .05$). The relation between quality and content was also moderate and negative ($r(77) = -.22$, $p > .05$). The relation between quantity and content, was significant and strong, suggesting teachers who spent more time in mathematics, also covered more content topics ($r(77) = .90$, $p < .05$). Multicollinearity, was therefore an issue for quantity and content of the mathematics instruction variables, as indicated by the correlations between predictors (see Table 5). Thus, in all further models, content and quality were examined, and quantity and quality were examined. However, quantity and content were not examined in the same models.

Conditional Growth Models: Classroom-Level Characteristics

The third research question focused on the contribution of mathematics instruction to the growth of children's counting and addition skills. The NLME function with REML in RStudio was used to assess this question using a multilevel model. All models below examined the effects of instruction, age and sex of child, and FRPL % on counting and addition skills over time. Model 4a and 4b examined the contribution of quality and quantity predicting mathematical skills, and Model 5a and 5b examined the contribution of quality and content predicting mathematical skills. Similar to previous models, age was expected to predict mathematical skills at school entry in both abilities, and the sex of child was not. FRPL% and instruction were also expected to predict growth in both skills. More specifically, mathematics instruction variables were expected to predict growth in skills. Frequency of literacy instruction was included to ensure mathematics instruction was uniquely related to mathematical skill development.

Counting. A longitudinal multilevel model was used to examine the within-person effect of quality and quantity of instruction and school-level variables on counting performance. Results of the multilevel model are presented in Table 6 (see Model 4a). Consistent with previous models, sex of the child and child's age did not predict entry, or growth in counting skills (Intercept: *Sexβ (SE)* = -.21 (.21), t = -0.98, p > .05, *Ageβ (SE)* = .47 (.30), t = 1.54, p > .05; Slope: *Sexβ (SE)* = -.02 (.11), t = -0.18, p > .05, *Ageβ (SE)* = -.19 (.15), t = -1.21, p > .05). Consistent with the hypotheses, children who attended schools with lower percentages of FRPL showed stronger counting skills in the beginning of kindergarten (*β (SE)* = -.02 (.01), t = -3.86 p < .05, see Figure 8). However, FRPL percentages did not predict growth in counting skills (*β (SE)* = -.00 (.00), t = 1.88, p > .05).

Interestingly, quality and quantity of mathematics instruction did not significantly predict mathematical skills at the first timepoint, or growth on children's early counting skills (Intercept: *Qualityβ (SE)* = -.00 (.00), t = -0.14, p > .05, *Quantityβ (SE)* = .01 (.01), t = 0.85, p > .05; Slope: *Qualityβ (SE)* = .00 (.00), t = 0.91, p > .05, *Quantityβ (SE)* = -.00 (.00), t = -0.89, p > .05). Similarly, the frequency of literacy content instruction was not related to mathematical scores at the initial timepoint, or growth in counting skills (Intercept: *Literacyβ (SE)* = .00 (.00), t = 0.41, p > .05; Slope: *Literacyβ (SE)* = -.00 (.00), t = -1.22, p > .05).

At the between-person level, children's starting points still varied by almost one point on the counting scale (*β (SE)* = .73 (.58), [95% CI: .56 to .95]). Children's slopes also significantly varied from one another, which was present across all previous models (*Random Slope (SD)* = .18, [95% CI: .05 to .69]).

Further, the within-person effect of quality and content of instruction and school-level variables on counting performance was examined. Results of the multilevel model are presented in Table 6 (see Model 5a). Consistent with Model 4a reported above, results remained similar across all individual characteristics. However, similar to quantity, the content of mathematics instruction also did not predict scores at the initial timepoint, or growth in counting skills (Intercept: *Contentβ (SE)* = .03 (.04), $t = 0.68$, $p > .05$; Slope: *Contentβ (SE)* = -.01 (.02), $t = -0.29$, $p > .05$).

Addition. A longitudinal multilevel model was used to examine the within-person effect of quality and quantity of instruction and school-level variables on addition performance. The results of the multilevel model are presented in Table 7 (see Model 4b). Consistent with previous models, sex of the child remained a significant predictor of children's addition skills at the initial timepoint, such that males scored higher (*β (SE)* = -1.29 (.35), $t = -3.29$, $p < .05$). However, consistent with Model 3b, age of child at testing did not predict children's addition scores at the initial timepoint, and neither sex of the child, nor age of the child predicted children's growth in addition skills (Intercept: *Ageβ (SE)* = .72 (.57), $t = 1.27$, $p > .05$.; Slope: *Ageβ (SE)* = .19 (.30), $t = 0.63$, $p > .05$., *Sexβ (SE)* = -.07 (.21), $t = -0.31$, $p > .05$). Consistent with the hypotheses, children who attended schools with lower percentages of FRPL showed stronger addition skills at the first timepoint (*β (SE)* = -.06 (.01), $t = -5.07$, $p < .05$, see Figure 9). FRPL percentages, however, did not significantly predict growth in addition skills (*β (SE)* = .01 (.01), $t = 1.09$, $p > .05$).

Similar to counting skills, at the within-person level, quality and quantity of the mathematics instruction variables did not significantly predict scores at the initial timepoint, or growth on early addition skills (Intercept: *Qualityβ (SE)* = -.00 (.00), t = -1.71, p > .05, *Quantityβ (SE)* = .03 (.02), t = 1.58, p > .05; Slope: *Qualityβ (SE)* =.00 (.00), t = 1.00, p > .05, *Quantityβ (SE)* = -.00 (.01), t =-0.43, p > .05). The frequency of literacy content instruction was also not related to mathematical skills at school entry or growth in addition skills (Intercept: *Literacyβ (SE)* = -.00 (.01), t = -0.49, p > .05; Slope: *Literacyβ (SE)* = .00 (.00), t = 0.31, p > .05).

At the between-person level, children's starting points still varied by one and a half points on the addition scale (*β (SE)* = 1.53 (.78), [95% CI: 1.24 to 1.87]). Children's slopes also still significantly varied from one another, which was present across all previous models (*Random Slope (SD)* = .67, [95% CI: .48 to .93]).

Further, the within-person effect of quality and content of instruction and school-level variables on addition performance was examined. The results of the multilevel model are presented in Table 7 (see Model 5b). Consistent with Model 4b reported above, results showed similar trends across all characteristics. However, similar to the quantity and quality variables, the content of mathematics instruction also did not predict scores in the beginning of kindergarten, or growth in addition skills (Intercept: *Contentβ (SE)* = .15 (.08), t = 1.88, p > .05; Slope: *Contentβ (SE)* = -.01 (.04), t = -0.15, p > .05).

CHAPTER VI

Post-Hoc Analyses

Instruction and Standardized Mathematical Achievement

In both previous models, quality, quantity, and content instruction did not predict growth in counting or addition skills. What is less clear, however, is whether these aspects of instruction predict changes on a standardized mathematical achievement test. For example, Math Garden examines very specific subcomponents of abilities including speed and question difficulty. Thus, the question remains whether aspects of instruction predict changes in overall mathematics. Assessing the contribution of mathematics instruction on a standardized achievement test may provide further clarification. Thus, two post-hoc regression analyses were performed to examine the classroom instruction variables, student characteristics, and school FRPL percentages predicting changes on the Woodcock-Johnson, Applied-Problems subtest.

Standardized Mathematical Achievement

Interestingly, age at testing, sex of the child, and FRPL % were not significant predictors of changes in children's Woodcock-Johnson Applied Problems score across the year (see Table 9). Further, no classroom instruction variables predicted change in the test either. Quality, however, showed a larger effect size, though negative and not significant (β *(SE)* = -.22/-.24 (.00)). Thus, perhaps with larger sample sizes, quality of instruction would significantly predict the change in the standardized achievement test.

Self-Regulation and Mathematical Achievement

Previous research emphasizes the relation between self-regulation and mathematical achievement (Bull & Lee, 2014; Jacob & Parkinson, 2015; Ribner, 2020). Thus, two self-regulation measures were included in later data collection for this dissertation. However, as self-regulation shows change throughout the kindergarten year, these measures could not be included in multilevel models unless they were included as time-varying characteristics with multiple timepoints. Thus, as a post-hoc approach to assessing whether self-regulation explains more variance at the end of year mathematical achievement that is not already explained by prior mathematical achievement, a regression analysis was performed. These results could inform future study designs examining growth in early mathematical skills.

Self-Regulation and Counting

Children’s counting skills in the middle of the school year predicted counting at the end of the school year. Thus, children's scores at the beginning of the school year no longer predicted counting at the end of the year (see Table 10, Model 1). The two self-regulation measures showed different results such that children's working memory skills, or cognitive self-regulation, did not predict counting skills at the end of the year after accounting for previous counting skills. However, children's scores on the approaches to learning scale, or behavioral self-regulation, did predict their counting scores at the end of school after accounting for prior achievement.

Self-Regulation and Addition

*S*imilar to counting skills, addition skills in the middle of the school year were most predictive of addition capabilities at the end of the year (see Table 10, Model 2). Also similar to counting skills, the approaches to learning scale, or behavioral self-regulation, predicted addition scores at the end of the school year above and beyond prior achievement. The cognitive self-regulation task, backward digit span, showed a similar effect size as the approaches to learning scale, although it was not significant. Thus, perhaps in a larger sample, cognitive self-regulation would also predict addition skills at the end of the year.

CHAPTER VII

Discussion

Children in the United States perform slightly above the global average on science and reading but are significantly below the global average in mathematics (Pew Research Center, OECD, PISA, 2015). Nationally, the proficiency in mathematical skills remains consistently low, showing no significant change from 2015 to 2017 (NAEP, National Assessment of Educational Progress; NCES, 2018). Many point to poor instruction in the schools as a source of this disparity. Thus, using novel assessment methods, this dissertation explored the role of mathematics instruction in developing mathematical skills of children as they transition into their first formal year of schooling, kindergarten. Examining the transition into kindergarten allows for the testing of skills that were developed in the home and preschool environment prior to that transition into schooling and then examine how these skills grow across the school year. In this way, we can understand what children are gaining in school and if the instructional environment is promoting that gain.

The results from the set of studies in this dissertation indicate that there are some interesting differences in the development of early mathematical skills across the kindergarten year. In the first study, for example, it was found that children’s mathematical skills grew across the school year, and children’s skills varied both at the beginning of the kindergarten school year, and how much they grew. That is, on average, children who started school with lower mathematical skills grew more than children who started with higher mathematical skills across the kindergarten year. In the second study, results suggested boys performed better on addition skills in the beginning of the kindergarten school year, but grew at a similar rate as girls. In the third study, school free and reduced-price lunch percentages predicted children's mathematical skills at the beginning of the kindergarten school year, such that children attending schools with lower percentages showed higher mathematical skills. After accounting for other important demographic variables and earlier achievement, mathematics instruction showed no unique prediction of children's mathematical skills in kindergarten. Further, no school, classroom, or individual characteristics predicted growth in children's mathematical skills.

Growth of Children's Early Mathematical Skills

Previous studies that have examined the growth of children's early mathematical skills found that children who entered school with higher mathematical skills showed increased growth during the kindergarten school year (Jordan et al., 2006). Based on this research, it was hypothesized children would not only show growth in early mathematical skills, but also their starting mathematical skills would relate to the amount of growth in their mathematical skills throughout kindergarten. Using a novel adaptive assessment software that is based on an item response theory framework for examining early mathematical skills, the results showed children's mathematical skills did grow, and the subdomains of mathematics grew at different rates.

This individualized, adaptive assessment approach allows for the understanding of how individual students are acquiring mathematical skills across the school year and the findings highlight the variability in children’s mathematical skill levels. When examining the change in mathematical skills as assessed by a standardized achievement test, less variability can be seen. Thus, an individualized assessment approach allowed children's mathematical scores to reflect: the difficulty of the question, rate of response, as well as the accuracy of the answer. The standardized assessment, on the other hand, mainly focuses on the latter. In this context, the use of an individualized assessment helps to shed more light on the specific skill levels of children in many different ways, such that diverse methods are essential for further unpacking the development of early mathematical skills.

In addition to the variability in children's mathematical skills assessed by individualized assessments, it was also essential to consider the relation between the mathematical skills children bring to school and how much they grow. Contrary to previous literature, children's mathematical skills at school entry related negatively to their rate of growth. One explanation for this result could be that children who start school with higher mathematical skills have less content to gain, and less room to grow across the kindergarten year. For example, children who enter school with the ability to count cannot necessarily improve as much as children who do not know how to count. In this case, children with lower skills would benefit more from instruction in basic mathematics, such as counting, which other research has suggested is one of the primary skills taught in kindergarten (Engel et al., 2013). Thus, although the achievement gap persists, the growth seen in these specific kindergarten mathematical skills suggests an upward trend in addressing the gap for children in these schools, albeit focused on basic counting and addition abilities.

One question that remains, then, is whether a positive relation between mathematical skills children bring to school and the growth of those mathematical skills exists for more advanced mathematical skills, beyond counting and addition. Based on previous literature, the home environment serves as a potential factor in children's mastery of basic mathematical skills prior to the onset of formal schooling (Susperreguy & Davis-Kean, 2015). This would suggest that while schooling serves as a time for children with lower mathematical skills to catch up, the children who enter school with proficiency in these mathematical skills will build on these and acquire more advanced skills (e.g. subtraction and rudimentary multiplication). Thus, it may be possible that had some of these more advanced mathematical domains been assessed, children from the high skill groups may have shown growth in those domains. Thus, although the results from this study showed a negative relation between children's mathematical skills at school entry and growth across the kindergarten year, the positive relation represented in the literature may have been replicated if more advanced skills had been assessed.

In line with previous research, the development of children's mathematical skills in kindergarten showed substantial growth (Burchinal et al., 2002). However, findings from the first research question that examined children's variability in mathematical skills suggested counting and addition skills vary considerably in the amount of growth they exhibit throughout the kindergarten year. That is, children's counting scores in the beginning of the school year were higher than their addition scores. Thus, children on average showed a lower rate of growth across the kindergarten school year for counting skills than addition skills. This result is consistent with literature suggesting that many children have already mastered counting before kindergarten

(Engel et al., 2013), and, the kindergarten time period may be more important for the development of advanced skills, such as addition (Le et al., 2019).

Although children showed significant growth in both counting and addition skills across the kindergarten year, one explanation for why children did not grow as much on counting skills as addition skills could be that children, on average, began school proficient in counting and not proficient in addition. Children’s counting scores in the beginning of the kindergarten school year did grow, but did not differ much from their counting scores at the end of kindergarten for the average child. It could be that children improved on one aspect of the individualized mathematical counting assessment, rather than the underlying counting skill itself. The result that children showed proficient counting abilities prior to kindergarten provides additional support to the literature suggesting the home numeracy environment offers the groundwork for the development of basic mathematical skills (LeFevre et al., 2009; Napoli & Purpura, 2018). Thus, children’s addition skills provided more room for growth at the beginning of kindergarten. Regardless, by the end of the kindergarten school year, the average child ended kindergarten above the proficiency level in counting and addition abilities, though only slightly above proficiency in addition.

Growth in Children's Early Mathematical Skills Accounting for Demographic Variables

The use of an individualized mathematical assessment allowed for a strong methodological approach in observing how diverse child characteristics contribute to the growth of children's counting and addition skills. Based on previous literature, it was hypothesized that children would vary on a variety of individual characteristics before schooling, and during the kindergarten year. For example, children’s age at the beginning of school was hypothesized to contribute to children's early mathematical skills such that younger children would start school with lower mathematical skills. Further, it was also hypothesized that girls and boys would not differ either at the beginning of school or throughout schooling on their mathematical skill abilities. Moreover, at the school-level, free and reduced-price lunch percentages were hypothesized to relate to children's kindergarten entry skills and rate of growth in both counting and addition skills. Including both individual and school-level characteristics, allowed for a more holistic picture of children's mathematical development during the kindergarten school year.

Sex Differences in Mathematical Achievement

Contrary to my hypotheses, boys performed better than girls on addition skills at the beginning of the kindergarten school year. Previous literature showed conflicting findings for the existence of sex differences in early mathematical skills (Lachance & Mazzocco, 2006). However, more recent, literature has suggested that sex of the child does not play a role in early numerical competencies (Bakker et al., 2018; Hutchinson et al., 2017). For example, Bakker et al. (2018) examined 4-5-year-old children in Belgium on a variety of numerical competence subtests. They found support for the gender equality hypothesis for seven of eight mathematical tasks. Thus, although the majority of early skills supported gender equality, in this study, some mathematical tasks still favored boys.

Further, it is important to emphasize that many studies find that the sex of the child does not matter for a majority of early mathematical skills, however, sex of the child does matter in some cases. Thus, sex differences are the exception, not the rule for mathematical development. For example, some key studies find sex differences at the beginning of kindergarten in early mathematical skills (Jordan et al., 2006; Ribner, 2020). Jordan et al. (2006) found sex differences at the end of kindergarten favoring boys for overall number sense and counting skills, but no other skills. However, Ribner (2020) found sex differences in children's school entry level mathematical skills. Thus, one explanation for the finding that boys outperformed girls in addition skills could reflect an adjustment to children’s first formal year of schooling. Kindergarten may present a particular period of transition in which sex differences in mathematical achievement are present, whereas, in future grades, gender equality becomes more stable.

The inconsistency of sex differences in early mathematical skills was replicated in the results of the second research question that examined how demographic variables related to children’s early mathematical skills, such that the sex of the child was not a significant predictor of children’s counting skills, but was a significant predictor of their addition skills. There are many possible explanations for this finding, but one mentioned above could be that all children enter school with some mastery in counting skills (Engel et al., 2013). This possible explanation is also supported by the first research question examining variability in children’s mathematical skills, in which the results show children entered school with counting skills above a threshold of proficiency, whereas, children’s addition scores on average were below zero, suggesting addition was a skill children had not mastered yet. Thus, one explanation for the inconsistency of sex differences in the early mathematical skill literature could be at the core of understanding children's proficiency levels. Perhaps future research should work to unpack proficiency levels in early mathematical subcomponents and the role of children's sex in the development of these proficiencies.

Another possible explanation worth noting of the role of sex differences in children's addition skills could be the use of the individualized mathematical assessment. Previous studies that have assessed sex differences on basic numerical tasks have mostly focused on standardized or uniform measures, rather than item-response theory based methods. Thus, once accounting for features beyond correct or incorrect responses, more differences in child characteristics may emerge. This explanation is consistent with the literature that suggests there is an effect of sex on speed due to individual differences in response styles (Carr & Jessup, 1997). For example, some studies have found that girls use finger counting and overt strategies more so than boys when solving mathematics problems, thus, impacting the speed to submitting a correct answer (Geary et al., 2000). This can be assessed directly with the individualized assessment, as speed of response is one of the factor that constitutes a child's overall score. However, further research should unpack whether males consistently perform better on addition skills across kindergarten, or if this is an artifact of the assessment methodology.

Age Differences in Mathematical Achievement

Contrary to previous literature, children’s age at the beginning of kindergarten did not relate to children's mathematical skills at the beginning of kindergarten, or the growth of their mathematical skills across kindergarten. In past studies, children’s age at testing was positively associated with higher addition skills (Jordan et al., 2006; Ribner, 2020), and it remained positively associated with the growth of those early addition skills. Previous research has suggested when older children start school, they show higher initial achievement and kindergarten growth rates, but normative growth rates in later grades (Johnson & Kuhfeld, 2020). Perhaps one reason children’s age at the beginning of kindergarten did not predict their addition skills could be that the sample size in the current studies was too small. The previous studies mentioned above included large-scale, nationally representative samples of participants in their analyses and thus, had more statistical power to find a significant difference. Some support for the lack of statistical power in the current study, is supported by examining the moderate effect sizes that suggest that a larger sample may have found the relation between children’s age when they were tested and children’s addition skills to be statistically significant.

Further, although this study had a small sample size, one result to note was that of the contribution of age on counting and addition skill growth once instruction was accounted for. Although nonsignificant in this study, the effect size of the results suggested that younger children may have grown more on counting skills with instruction, whereas older children may have grown more on addition skills with instruction. Thus, suggesting a tendency for younger children to develop more counting skills from the instruction in kindergarten, and older children developing more of the advanced, addition skills in kindergarten. Therefore, although statistically nonsignificant, perhaps with a larger sample the difference between the growth of children's counting and addition skills by instruction may have been observed.

School Differences in Mathematical Achievement

As hypothesized, the percentage of those receiving free and reduced-price lunch in a school predicted children's counting and addition skills at the beginning of kindergarten. In other words, the children that attended schools with a lower percentage of children receiving free and reduced-price lunch showed higher counting and addition scores at the beginning of the kindergarten year when compared to children from higher free and reduced-price lunch percentage schools. However, the percentage of free and reduced-price lunch children in the school did not relate to the growth of children's mathematical skills over the kindergarten year. Thus, these findings reinforce the achievement gap literature such that, children from schools with higher free and reduced-price lunch percentages begin kindergarten with lower skills and schooling did not close these early mathematical achievement gap across the kindergarten school year (Reardon, 2011). Although all children's counting and addition skills grew, children from lower socioeconomic status schools started school with lower mathematical skills compared to that of children in higher socioeconomic schools. Although it is the hope that schools serve as an intervention on these early skills, school socioeconomic status showed no unique prediction to the growth of children's counting and addition skills across the kindergarten school year.

The results suggesting that the percentage of children receiving free and reduced lunch, which served as a proxy for socioeconomic status, predicted children’s mathematical skills in the beginning of the kindergarten school year, but not growth over the year, may have important implications for children's development of early mathematical skills in school. In order to close the achievement gap, the schools with more children receiving free and reduced-price lunch would ideally improve children’s growth rate in mathematical skills such that children attending those schools would then make up the gap present prior to transitioning into kindergarten. Interestingly, the study findings regarding the contribution of school instruction, show that growth coefficients are positive, suggesting a step in the right direction. Further, the effect sizes suggest that perhaps with a larger sample, results that children at schools with higher percentage of free and reduced lunch might improve growth in children’s early counting skills across the kindergarten year, but not addition. Thus, it would be important to try and replicate these results in a larger sample to see if these effect sizes stay the same. In this context, it was imperative for the sample to include school of various backgrounds because, without the variety, results may have sharply differed. Thus, moving forward, in order to learn more information about the children’s mathematical trajectories it is essential for future research to continue to examine diverse samples. To examine mechanisms in which these specific skill differences are associated with the economic background of the children and schools, a diverse sample is necessary.

Mathematics Instruction and Growth in Children's Early Mathematical Skills

The role of mathematics instruction in assisting the growth and development of children's early mathematical skills is a crucial aspect of understanding how children acquire mathematical education. Previous research has explored how mathematics instruction contributes to children's early mathematical skills, and found that classrooms that spend more time on math activities in more advanced contexts show better mathematical achievement (Engel et al., 2013; Connor et al., 2019; Ribner, 2020). In study three examining classroom instruction contributions, an approach different from previous studies was used to examine the measurement of classroom mathematics instruction. Results from this naturalistic, more representative methodological approach using audio digital recorders provide insight into diverse aspects of mathematics instruction and whether they contribute to the development of children's early mathematical skills across the kindergarten year. In particular, contrary previous studies, results suggested the quality, quantity, and content of mathematics instruction did not predict skills in the beginning of kindergarten, or growth in counting or addition capabilities as measured

Classroom Instruction Sampling

Although mathematics instruction did not predict growth in counting or addition skills as measured, it should be noted that the takeaway is not that teaching does not matter. Studies examining education have the difficulty of capturing many aspects of input with limited methods. For example, teachers interact with students for nine months, and researchers only accumulate a few days of data. One possibility as to why mathematics instruction did not predict counting or addition skills could be that they did not accurately capture the typical variety of mathematical activities present in kindergarten classrooms. Audio recordings only took place over one school day, and thus, it is important to remember that almost nine months of instruction were not captured.

Building on the fact that teachers were only observed for one day, the mathematics instruction used in this study assumed that teachers delivered instruction exactly the same across the kindergarten school year. This was another limitation to only recording one day. Thus, perhaps in future studies, study designs should assess whether kindergarten mathematics instruction remains stable across the kindergarten school year. For example, following the common core curriculum, one could imagine that content of mathematics instruction should change throughout the school year (National Governors Association, 2010). Although there remain certain staples, such as addition, across the school year, it could be that content in the fall of kindergarten focuses more on counting, whereas the content in the spring of kindergarten introduces more advanced concepts, such as subtraction. Hence, future studies examining the contribution of mathematics instruction on early mathematical skills should assess whether mathematics instruction is a time-varying characteristic.

Content of Mathematics Instruction and Mathematical Achievement

Contrary to the study hypotheses, variety in the content of mathematics instruction did not predict children's growth in early addition or counting skills across the kindergarten year. The content measure was intended to capture the diverse topics These results were inconsistent with the individualized instruction theory suggesting children who receive literacy instruction close to their literacy skill level improve more (Connor et al., 2004). However, one important note was that this study examined the overall main effect of instruction on the growth of early mathematical skills. Thus, in some ways the results remain consistent with Connor et al. (2004) such that more information regarding the individualized aspects of instruction were needed. Put more simply, including the variety of total classroom instruction was not the same as assessing individualized instruction.

One explanation for why total instruction did not accurately capture individualized instruction could be due to the assumption that the content of mathematics instruction remained the same across the kindergarten school year, such that, teachers who instructed three topics on the observed day continuously instructed little content throughout the year. Further, rather than simply counting the variability in content topics covered in the kindergarten classroom, an area for future research may be to examine the type of mathematical content being covered. Similar to the individualized instruction theory for literacy that focuses on code-focused versus meaning-focused instruction (Connor et al., 2007), mathematics instruction, especially in kindergarten, should further examine the extent to which the interaction between advanced and basic instructional content and skill level matter for early mathematical growth. This is one of many limitations and an area for future research to focus on.

Quality of Mathematics Instruction and Mathematical Achievement

The quality of mathematics instruction is one relatively understudied, however, based on previous language development literature, quality was hypothesized to relate to children's counting and addition skills. Findings suggested that quality of mathematics instruction did not relate to children's mathematical skills. One possible reason why the quality of the mathematics instruction did not predict mathematical skills could be that the teachers wore the audio recorder. Perhaps, had each child worn the recording device, more variability in the quality variable would be present, thus speaking more to the quality of that child's specific environment. Previous research suggests there is more variability in classroom conversational turn counts when the child is wearing the device (Duncan et al., *in press*). This approach also would have allowed for more power at the instruction level in the multilevel model. However, perhaps the best approach to capturing the classroom environment for the quantity of mathematics instruction might be to assess the quality of the time teachers spent on advanced content topics compared to basic content topics. Combining quality, quantity, and content of mathematics instruction in these ways are open questions that would benefit from future research that examines interactions between the different features of mathematics instruction.

Classroom Mathematics Instruction: Reported vs. Observed

In the beginning of the kindergarten school year, teachers were asked to report how long they spend in mathematics instruction on an average day, and this was compared to the observed amount of mathematics instruction from the recorded day. Previous research that has found significant effects of the quantity of mathematics instruction on children's mathematical achievement have relied solely on self-report measures (Bodovski & Farkas, 2007; Ribner, 2019). However, findings from the third study suggests that teachers often overestimate the amount of time they spent in mathematics instruction on a typical school day. Thus, one possible explanation for why the quantity of mathematics instruction did not relate to children's mathematical skills could be due to measurement error. This finding provides some evidence for caution surrounding self-report measures. Teachers plan their schedules based on blocks of time; however, it may not be possible to expect teachers to predict precisely how their days will play out. Thus, future researcher should consider approaching this methodological concern in two ways: first, if possible, observational methods should be used to assess instructional contributions. However, should observations not be an option, it may be beneficial for the field to interpret self-report measures with caution.

Further, the findings on mathematics instruction, contrary to previous literature (e.g. Engel et al., 2013), suggests that kindergarten teachers are teaching more advanced mathematical content and not just basic content areas during mathematics instruction, such as counting and shapes. The content topics coded in these audio recordings included 22.6% that focused on addition concepts, whereas oral counting only accounted for 4.1%. Using the digital audio recorder allowed for the recording of an entire day in the classroom, which is uncommon in much of the educational research on classrooms due to the burden placed on teachers and students. Even though this was an improvement in measurement in the classroom setting, the results may have been even more robust if data was gathered at varying timepoints throughout the kindergarten year to capture the depth and richness of the school environment, and potentially more mathematical concepts being taught. It would also assist with the often small sample sizes of classrooms obtained in education research by providing for multiple data points within a school year.

Limitations

This study was a year-long, multiple time point design that included novel approaches to examining mathematics instruction and individualized assessments of mathematical skills. However, even with the attempt to improve on some of the quality and robustness of the research, there were several limitations that need to be considered in the interpretation of these results. Specifically, there were sample limitations, limitations in using audio recorders, and limitations in study design. These three broad categories are addressed below.

Sample Limitations

Although four diverse schools were included in this sample, they are not fully representative of public schools in the area. Thus, those who signed up to participate were not completely random. By examining four diverse schools, it became more apparent how important it is for future studies to include schools that are representative of the diversity of public school education. In this study, the percentage of children receiving free and reduced lunch was, predictive of children's counting and addition skills at the beginning of kindergarten. Thus, to better understand individual trajectories in mathematical skill development, a more diverse sample will allow for the understanding of how early child characteristics relate to achievement at the beginning of school and beyond for the breadth of students in schooling.

Another limitation of the study was the absence of information from the home environment. Multiple studies have shown the importance of the home environment in the development of mathematical skills that could be contributing to the diverse mathematical skill levels children bring to school (Susperreguy & Davis-Kean, 2015). One limitation to collecting data in the school environment is the lack of communication and responses from parents. All parents in the study received the parenting questionnaire, but only half responded. Thus, questions about the home environment were not able to be used for these analyses. The findings suggest that many of the differences between children are occurring prior to schooling and may related to socioeconomic differences at school entry, thus, understanding the role of the home environment in the development of mathematical skills should be a focus of future research.

Finally, the small sample of teachers that participated in data collection served as a limitation in analytic power. Of the four schools, there were a total of fourteen kindergarten classrooms. Thus, although the study was powered to assess the within-person level differences with 98 children, the sample was not equipped to evaluate the between-person level differences with only fourteen teachers. Regardless of the power issue at the between-person level, findings did show interesting differences across classroom variability, such as the variety in reported time and observed time in mathematics instruction.

Audio Recording Limitations

Digital audio recorders were employed to observe the naturalistic instruction of teachers across an entire day of schooling. Prior research assessing and capturing mathematics instruction in the classroom have either videotaped their classrooms or used teacher report measures (Connor et al., 2018; Engel, Claessens, & Finch, 2013). Videotaping the class usually requires the researcher to be present for the length of the recording, which would not have allowed for the naturalistic instruction to occur. In situations where the researcher was not present, the video camera could only capture the instruction in the classroom. Thus, no instruction that may have taken place in other areas of the school such as on the playground, cafeteria, or computer lab could have been captured. Thus, the use of audio recorders to examine education in a non-obtrusive approach allowed for the richness of naturalistic data to be present in this study throughout all areas of the school.

Nevertheless, audio recorders were not without their challenges. One of the more obvious limitations to using audio recorders over videotapes was the inability to observe footage of the mathematics instruction taking place. For example, one teacher gave verbal instructions to put a monkey on the string, as part of mathematics instructions for a child participating in an "add-on" game. Trained research assistants felt they were able to assess that the teacher was focusing on addition skills quickly; however, this may have been easier with actual visualization of what the children were doing. Thus, although audio recordings provided enough information for the variables in this study, it may be helpful for future studies to combine audio and video recordings as a best practice approach.

Furthermore, in light of fostering teacher-researcher collaborations, only one day of mathematics instruction was recorded during the school year. Many teachers also participated in a previous study in which videotapes captured the classroom environment. In this previous study, videotaping only happened once during the year for a portion of the school day, thus, the previous layout design was mimicked in this study as to not surprise or overwhelm teachers. Upon completion of this study, teachers were able to experience the audio device recorders and they have reported back that they would be willing to wear the recorder multiple times throughout the school year. Thus, studies in the future would benefit from getting several recorded instruction days to assess a more holistic and representative picture of the classroom environment and instruction.

Finally, the quality of mathematics instruction is complex and challenging to measure. One benefit to using the audio recorders over video observations was that the LENA device provided a pre-coded quantitative measure regarding the language environment. If quality were to be measured using video observation data, one would have to create a coding scheme and manually code those data. However, there are still multiple aspects of the classroom that may contribute to the quality of classroom mathematics instruction. For example, resources the children have access to, especially for mathematical activities, may differ by school or classroom. Thus, perhaps the quality of mathematics instruction would better be assessed by approaching both language and resource quality in the school that could be captured by using both audio recorders and videotapes. Future studies should work to use both methodological approaches to compare and contrast the advantages and disadvantages for coding the quality of instruction in schools.

Study Design Limitations

Though the study design for data collection was carefully planned ahead of time, certain aspects served as limitations. To begin, assessing the growth of mathematical skills in one year may lead to biased results. Previous research has suggested that kindergarten presents a period of time that differs from other school years as it serves as the first year of formal schooling (Johnson & Kuhfeld, 2020; Jordan et al., 2006; Ribner, 2020). However, it is also important to emphasize that this study only focused on one school year, and children's mathematical skill growth continues through adulthood. Thus, only examining mathematical skill growth in kindergarten might show results that do not replicate across future grades, such as the findings suggesting sex differences in addition skills. Previous literature has also found small sex differences in kindergarten, but often these differences are no longer present later in school (Jordan et al., 2006; Ribner, 2020). Therefore, the results of this study should not be generalized to all years of schooling, or to the amount of growth in mathematical skills.

As the main research questions focused specifically on the development of early mathematical skills, no data were collected on other important skills, like the development of early literacy skills. This presents a significant limitation when assessing mathematics instruction. Ideally, this study would have been more beneficial and robust if a literacy outcome could also suggest that the mathematics instruction examined also did not predict children's growth in literacy abilities, as was done for mathematical development. Thus, for future research, it may be beneficial to include multiple academic measures, including literacy scores.

On the other hand, two variables collected to assess self-regulatory skills in children included the approaches to learning scale and the digit span. Prior research has shown robust links between children's performance in mathematical and self-regulation skills (Jacob & Parkinson, 2015; Purpura, Schmitt, & Ganely, 2017). However, the cognitive and behavioral self-regulation measures were collected toward the end of the school year. Based on descriptive statistics in nationally representative datasets, both of these skills show change across the kindergarten school year (Ribner, 2020). Thus, using these self-regulation measures in a multilevel model as time-invariant variables would be theoretically implausible. Nevertheless, post-hoc analyses suggested both the cognitive and behavioral self-regulation measures provided information about later individualized mathematical achievement that earlier mathematics could not. This result would suggest that children’s self-regulation abilities potentially contribute to their development of early mathematics. Hence, to assess the contribution of these self-regulation abilities, future studies should work to include multiple measurements of these early skills or should be sure to include these assessments at the beginning of the study design.

Finally, although this is a longitudinal study, causality cannot be inferred. These data are correlational, and it could be possible that results found no effects of mathematics instruction on children's growth in mathematical skills because mathematics instruction across kindergarten classrooms was not manipulated. Although one strength of the third research question is that it assesses mathematics instruction in a business as usual setting, one approach to establishing more causal effects would be with the use of interventions. Without intervening, it is more challenging to determine which features of mathematics instruction individually contribute to differences in mathematical skills. Thus, if intervention groups were created, and they were designed to receive different types, amounts, and quality of mathematics instruction, it may be easier to infer causality in the association between education and skills.

Future Directions

This dissertation considered how kindergarten mathematics instruction contributed to the growth of children's early mathematical skills. The set of studies presented provided further insight into kindergarten mathematics instruction and its role in mathematical development. With the use of novel methodological approaches in these studies, findings advanced the field of educational and psychological perspectives regarding classroom instruction. However, there are still many questions and avenues for future directions in research.

Future research should investigate mathematics instruction across numerous days of school throughout the year to better assess and compare many of the population datasets that include self-report measures. With more classroom instruction data across time, researchers could do a deeper dive into specific aspects and variability of mathematics instruction across the school year. For example, variability in mathematics instruction may fluctuate throughout the year. Thus, with multiple instruction timepoints, piecewise latent growth models could be used to examine whether there are certain times throughout the kindergarten year, where particular aspects of mathematics instruction matter more for mathematical skill growth. Moreover, future studies should also aim to be powered at the teacher level to examine these between-person differences in instructional approaches and skill growth across different points of the school year.

Alternatively, future research should also examine how mathematics instruction variables are combined by teachers across classrooms. For example, a more productive approach to use observational classroom instruction data may be to include the intersection of all three instructional aspects. The combination of quality, quantity, and content may be essential to assess together such that children in classrooms that spend more time on advanced topics with higher conversational turn counts may grow more than children in classes with less time on advanced topics. Thus, future research should consider the significance of these intersecting variables as they are not mutually exclusive.

Future studies should also include measures beyond the focus of mathematics, such as literacy and self-regulation measures. Well-rounded assessments would allow the researcher to robustly focus on the development of mathematics in general, rather than mathematical development in isolation. Many cognitive skills develop concurrently, thus, should researchers examine the interaction of how language skills and self-regulation abilities foster mathematical learning, the field would have a better grasp of children's mathematical development trajectory. For example, previous research suggesting children's executive functioning facilitates learning from mathematics instruction in kindergarten (Ribner, 2020) indicates that explicitly focusing on the co-development of mathematics and self-regulation is an essential next step. Thus, an appealing future study might examine the growth of children's mathematical skills and changes in their self-regulation skills, along with observed mathematics instruction across the kindergarten year. This approach would likely lead to more precise estimates of the relation between education and cognitive skills.

Finally, the current study found that boys outperformed girls on addition skills, but not their early counting skills. Much of the previous literature found findings mixed concerning sex differences in early mathematical skills. However, recent research shows that a male advantage in foundational numerical skills is the exception rather than the rule (Hutchison, Lyons, & Ansari, 2018). Interestingly, these results capture that statement rather well and show how difficult it is to claim there are no significant sex differences in mathematical abilities. Boys and girls performed equally on the counting task, and also similarly grew in counting skills. However, all children reached proficiency on the counting assessment prior to entering kindergarten. Thus, a question becomes whether or not there were sex differences in the beginning of the kindergarten year for addition because children had not yet achieved proficiency in this skill. It could be that proficiency levels in specific mathematical skills such as counting, or addition drives the finding of sex differences favoring boys over girls—especially when using adaptive testing techniques that include reaction time in individualized results. Therefore, more research using multiple methodological approaches and robust statistical analyses are needed to focus on these varieties of sex differences in the early proficiencies of mathematical skills.

General Implications

Results from this dissertation provide the educational research field with a variety of general implications that shed light on the relation between the growth of early mathematical skills and the role of formal education. First, findings show that children are exposed to even less mathematics instruction than found in self-report methods. Second, although most children had mastered counting prior to kindergarten, counting skills still showed growth across the kindergarten school year. Finally, the findings from this dissertation challenge the field's understanding of the contribution of mathematics instruction to the growth of early mathematical skills. Moreover, these implications provides insight into educational and psychological perspectives of mathematical development.

First, as noted in previous literature, mathematics is not a prominent domain in kindergarten instruction. For example, teachers reported spending, on average, 13% of their classroom time on mathematical activities and 29% of their time on literacy activities. This statistic is consistent with previous results that teachers do not spend an equal amount of time teaching both core domains (Engel, Claessens, & Finch, 2013). However, results suggest instructional time in mathematics might be even less prominent than initially discussed. Based on the recordings of instruction in the classroom, only 6% of the average recording day was spent in the mathematical domain. Thus, only half of the time that teachers reportedly spent in mathematics was observed in the recordings.

Consequently, this result suggested children are exposed to even less mathematics than initially reported, and perhaps that is why mathematics instruction did not have as meaningful of a contribution to mathematical skills as initially hypothesized. Theoretically, if a more significant percentage of time was spent in mathematics, more content topics could be covered with better quality mathematics instruction. Thus, a more substantial impact of education may be seen in early mathematical growth.

Despite the small amount of time spent on mathematics instruction in kindergarten classrooms, results suggested children's mathematical skills still grew across the kindergarten school year. For example, children's addition skills increased significantly from the beginning of the kindergarten school year to the end of the school year. Children's counting skills, on the other hand, suggested many children had already become proficient in counting; however, there was still a significant increase in their scores on the counting task by the end of the kindergarten year. Thus, one crucial implication of these results would be that early mathematical skills continue growing, even after they may be considered proficient. Although it may not be essential to continue teaching children to count in kindergarten (Engel, Claessens, & Finch, 2013), these skills continue to enhance and build on top of the other topics teachers instruct. These findings are supported by the idea that early mathematical skills are like building blocks (Clements & Sarama, 2007), children's early counting skills provide a foundation for the development of their addition skills. Thus, one crucial implication of these results is that the emphasis or shift to more advanced content areas still fosters growth in basic mathematical skills.

Finally, inconsistent with previous work, no aspects of mathematics instruction contributed to children's mathematical growth (Claessens et al., 2013; Le et al., 2019; Ribner, 2020). Thus, findings would suggest perhaps previous studies examining the amount of mathematics instruction may be limited in their assessments of self-report measures. In general, the lack of relation between mathematics instruction and growth in mathematics in this sample could be due to the minimal amount of time teachers spent in mathematical activities. However, based on the limitations of this study, the results would also need to be replicated in a more robust and diverse sample.

Conclusion

The present study sought to examine the role of classroom mathematics instruction in the development of mathematical skills across the first year of formal schooling. The consequences of poor mathematical skills are well established in the field (Duncan et al., 2007; Siegler et al., 2012), however, whether instruction serves as a potential protective factor for these skills is not as well researched (Alcock et al., 2016; Connor et al., 2018). Children's growth in mathematical skills throughout the kindergarten year was related to the skills they brought to school with them. Student demographics predicted mathematical skills differently, supporting previous research suggesting there are inconsistencies in the literature (Hutchison et al., 2019). Further, as measured, no classroom instruction variables predicted children's growth in mathematical skills across the kindergarten school year.

Based on these results, although formal schooling is viewed as an intervention for improving one's skills, no variables in this study related to children's growth in early math skills. Thus, although children's counting and addition skills both grew across the kindergarten year, school, sex of the child, age of the child, and classroom instruction did not predict the growth in those skills. Further, children who came in with high mathematical skills did not grow at the same rate as children with lower mathematical skills. Thus, providing further evidence that the kindergarten school year serves as review for many children with early mathematical skills (Engel et al., 2013). Future research should further investigate the trajectories of students who improve most from the kindergarten school year and possible explanations for why these students do or do not respond to certain instructional practices.

Further, this dissertation highlighted the limitations of previous classroom instruction measurement approaches. The amount of time in which kindergarten teachers spend in mathematics instruction was less than originally reported. Given the importance of classroom instruction for understanding schooling as an intervention on early academic skills, examining, and measuring the utility of diverse methodological approaches of education is of practical importance.

More specifically, if educational practices in place for literacy use child by instruction theory to specialize instruction at the child’s level (Al Otaiba et al., 2011; Connor et al., 2013), and thus, show the best gains in early literacy skills, it is imperative to also consider the same concept for the second core foundational skill: mathematics. If differences in instructional practices are the key to improving children’s skill development, it is imperative to first assess the measurement mayhem of the methodological approaches surrounding classroom mathematics instruction, then enforce educational practices in place for mathematics.

TABLES

Table 1

Child Descriptive Statistics

Variable	N	M (SD)	Min	Max
Age	98	5.55 (0.33)	4.92	6.67
Female	98	0.47 (0.50)	0	1
FRPL %	98	31.1 (20.92)	3	66
Counting				
Fall	98	3.41 (1.03)	-0.98	4.98
Winter	93	3.78 (0.94)	1.23	5.8
Spring	91	3.85 (0.96)	0.7	5.84
Adding				
Fall	98	-1.34 (2.23)	-5.43	4.7
Winter	93	-0.49 (2.17)	-4.16	5.34
Spring	91	0.43 (1.98)	-3.75	5.08
Math Achievement				
Fall	98	18.7 (4.27)	2	28
Spring	92	21.82 (4.11)	11	30
Number Line Estimation	92	2.71 (1.53)	0.72	9.26
Working Memory	92	2.89 (1.48)	0	6
Approaches To Learning	96	3.17 (0.69)	1.33	4

Table 2

School Descriptive Statistics

	n(students)	***n(teachers)***	**FRPL %**
School 1	30	4	2
School 2	24	3	61
School 3	34	5	68.5
School 4	10	2	71.9

Table 3

Teacher Recording and Questionnaire Descriptive Statistics

Variable	N	M (SD)	Min	Max
Math Instruction				
Quality	11	407.09 (173.98)	191	731
Quantity	11	26.16 (13.85)	3.48	47.45
Content	11	5.73 (2.80)	1	10
Other Recording Variables				
Adult Word Count	11	25158 (5750.39)	18292	33378
Child Vocalization Count	11	735.73(482.88)	238	1821
Recording Length	11	411.36 (31.56)	350	442
Teacher Questionnaire				
Math Minutes	14	50 (15.19)	30	90
Literacy Minutes	14	115 (44.81)	60	180
Years Teaching	14	14.14 (9.09)	2	30
Years Teaching Kindergarten	14	7.43 (5.68)	2	19
Masters Degree	14	0.79 (0.43)	0	1

Table 4

Parent Questionnaire Descriptive Statistics (N = 57)

Variable	Proportion or Mean
Household Income M(SD)	$107,517.04 (94,078.07)
Child Race	
% White	56.14
% Black	8.77
% Hispanic	7.02
% Asian	15.79
% Multiracial	12.28
Parent Education	
% Some High School	1.92
% High School Diploma	3.85
% Some College	11.54
% 2 year college	17.31
% 4 year college	26.92
% Postgraduate or Professional	38.46
Parent Response Rate	
% School 1	63
% School 2	75
% School 3	53
% School 4	20

Table 5

Correlations for Individual and Classroom Variables

	N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Age	98	-															
2. Female	98	-0.03	-														
3. FRPL %	98	-0.06	0.11	-													
Math Garden																	
4. Counting Fall	98	0.19	-0.06	-0.34 ***	-												
5. Counting Winter	93	0.08	-0.2	-0.29 **	0.61 ***	-											
6. Counting Spring	91	0.16	-0.08	-0.29 **	0.51 ***	0.65 ***	-										
7. Adding Fall	98	0.2 *	-0.28 **	-0.42 ***	0.47 ***	0.43 ***	0.46 ***	-									
8. Adding Winter	93	0.18	-0.3 **	-0.33 **	0.6 ***	0.61 ***	0.56 ***	0.78 ***	-								
9. Adding Spring	91	0.25 *	-0.29 **	-0.3 **	0.59 ***	0.56 ***	0.58 ***	0.69 ***	0.83 ***	-							
Measures of Math Achievement																	
10. WJ Applied Problems Fall	98	0.13	-0.24 *	-0.22 *	0.52 ***	0.49 ***	0.42 ***	0.53 ***	0.7 ***	0.66 ***	-						
11. WJ Applied Problems Spring	92	0.11	-0.15	-0.34 ***	0.62 ***	0.61 ***	0.59 ***	0.64 ***	0.78 ***	0.76 ***	0.77 ***	-					
12. Number Line Estimation	92	-0.23 *	-0.07	0.17	-0.43 ***	-0.44 ***	-0.34 ***	-0.27 ***	-0.42 ***	-0.36 ***	-0.51 ***	-0.5 ***	-				
Measures of Self-Regulation																	
13. Working Memory	92	0.11	-0.1	-0.27 *	0.52 ***	0.39 ***	0.37 ***	0.4 ***	0.49 ***	0.53 ***	0.5 ***	0.66 ***	-0.39 ***	-			
14. Approaches To Learning	96	0.24 *	0.24 *	-0.22 *	0.42 ***	0.32 **	0.39 ***	0.29 **	0.39 ***	0.45 ***	0.36 ***	0.44 ***	-0.31 **	0.35 ***	-		
Classroom Math Instruction																	
15. Quality	77	0.01	0.16	-0.02	-0.03	0.05	0.03	-0.23 *	-0.2	-0.15	0.02	-0.15	-0.04	-0.06	0.02	-	
16. Quantity	77	-0.13	0.05	0.08	-0.02	0.13	-0.02	0.13	0.18	0.12	0.06	0.15	-0.09	0.11	0.09	-0.03	-
17. Content	77	-0.2	-0.08	0.16	-0.08	0.14	-0.01	0.17	0.22	0.17	0.04	0.17	-0.04	0.11	-0.04	-0.22	0.9 ***

Note. WJ = Woodcock-Johnson III Tests of Achievement.
*p <.05, **p <.01, ***p <.001

Table 6

Multilevel Models for Math Garden Counting Task

	Null			**Unconditional Slope**						**Conditional Slope: Individual and Classroom-Level Covariates**								
	Mean			Fixed			Random			Individual Covariates			Quality & Quantity			Quality & Content		
	Model 0a			Model 1a			Model 2a			Model 3a			Model 4a			Model 5a		
	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]
Fixed Effects																		
Intercept	3.67 (.08)	43.45	[3.49 to 3.83]	3.46 (.10)	36.37	[3.27 to 3.65]	3.46 (.10)	34.48	[3.26 to 3.66]	.96 (1.69)	0.57	[-2.36 to 4.29]	1.57 (1.83)	0.86	[-2.05 to 5.19]	1.51 (1.90)	0.79	[-2.24 to 5.26]
Student Level																		
Age at Testing										.47 (.30)	1.54	[-0.13 to 1.06]	.47 (.30)	1.54	[-0.14 to 1.08]	.48 (.31)	1.56	[-0.13 to 1.09]
Sex										-.18 (.20)	-0.89	[-0.57 to 0.22]	-.21 (.21)	-0.98	[-0.62 to 0.21]	-.19 (.21)	-0.89	[-0.61 to 0.23]
Classroom Level																		
Quality													-.00 (.00)	-0.14	[-0.00 to 0.00]	.00 (.00)	0.04	[-0.00 to 0.00]
Quantity													.01 (.01)	0.85	[-0.01 to 0.03]			
Content																.03 (.04)	0.68	[-0.06 to 0.12]
Teacher Report Literacy													.00 (.00)	0.41	[-0.00 to 0.01]	.00 (.00)	0.26	[-0.01 to 0.01]
School Level																		
FRPL %													-.02 (.01)	-3.86	[-0.04 to -0.01]	-.03 (.01)	-3.84	[-0.04 to -0.01]
Slope				.21 (.05)	4.56	[0.12 to 0.31]	.21 (.05)	4.16	[0.11 to 0.32]	.62 (.87)	0.7	[-1.11 to 2.34]	1.19 (.92)	1.29	[-0.64 to 3.02]	1.08 (.96)	1.11	[-0.83 to 2.99]
Student Level																		
Age at Testing										-.07 (.16)	-0.45	[-0.38 to 0.24]	-.19 (.15)	-1.21	[-0.49 to 0.12]	-.19 (.16)	-1.17	[-0.49 to 0.13]
Sex										-.03 (.10)	-0.28	[-0.23 to 0.18]	-.02 (.11)	-0.18	[-0.23 to 0.19]	-.02 (.11)	-0.21	[-0.24 to 0.19]
Classroom Level																		
Quality													.00 (.00)	0.91	[-0.00 to 0.00]	.00 (.00)	0.76	[-0.00 to 0.00]
Quantity													-.00 (.00)	-0.89	[-0.01 to 0.01]			
Content																-.01 (.02)	-0.29	[-0.05 to 0.04]
Teacher Report Literacy													-.00 (.00)	-1.22	[-0.00 to 0.00]	-.00 (.00)	-0.91	[-0.00 to 0.00]
School Level																		
FRPL %													.01 (.00)	1.88	[-0.00 to 0.01]	.01 (.00)	1.82	[-0.00 to 0.01]
Random Effects																		
Slope (SD)							0.3		[0.19 to 0.48]	0.31		[0.19 to 0.49]	0.18		[0.05 to 0.69]	0.18		[0.05 to 0.67]
Intercept	0.73		[0.61 to 0.88]	0.74		[0.62 to 0.88]	0.85		[0.70 to 1.04]	0.84		[0.69 to 1.03]	0.73		[0.56 to 0.95]	0.73		[0.56 to 0.95]
Correlation							-0.46		[-0.70 to -0.12]	-0.47		[-0.71 to -0.13]	-0.19		[-0.73 to 0.50]	-0.2		[-0.73 to 0.48]
Residual Error	0.67		[0.60 to 0.74]	0.63		[0.57 to 0.71]	0.56		[0.49 to 0.65]	0.56		[0.49 to 0.65]	0.58		[0.49 to 0.68]	0.58		[0.49 to 0.68]

Note. Fixed effects parameters are unstandardized regression coefficients, and random effects parameters are estimates of variance. Missing data were handled using restricted maximum likelihood.

Table 7

Multilevel Models for Math Garden Adding Task

	Null			**Unconditional Slope**						**Conditional Slope: Individual and Classroom-Level Covariates**								
	Mean Model 0b			Fixed Model 1b			Random Model 2b			Individual Covariates Model 3b			Quality & Quantity Model 4b			Quality & Content Model 5b		
	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]	β (SE)	*t*	[95% CI]
Fixed Effects																		
Intercept	-.54 (.20)	-2.69	[-0.93 to -0.14]	-1.35 (.21)	-6.45	[-1.77 to -0.94]	-1.35 (.22)	-6.03	[-1.79 to -0.91]	-7.61 (3.60)	-2.11	[-14.71 to -0.50]	-2.01 (3.42)	-0.59	[-8.78 to 4.75]	-2.96 (3.52)	-0.84	[-9.92 to 3.99]
Student Level																		
Age at Testing										1.23 (.65)	1.91	[-0.05 to 2.51]	.72 (.57)	1.27	[-0.41 to 1.86]	.83 (.57)	1.45	[-0.31 to 1.97]
Sex										-1.23 (.43)	-2.89	[-2.08 to -0.39]	-1.29 (.39)	-3.29	[-2.08 to -0.51]	-1.22 (.39)	-3.12	[-1.99 to -0.44]
Classroom Level																		
Quality													-.00 (.00)	-1.71	[-0.01 to 0.00]	-.00 (.00)	-1.34	[-0.00 to 0.00]
Quantity													.03 (.02)	1.58	[-0.01 to 0.06]			
Content																.15 (.08)	1.88	[-0.01 to 0.32]
Teacher Report Literacy													-.00 (.01)	-0.49	[-0.01 to 0.01]	-.00 (.00)	-0.56	[-0.01 to 0.01]
School Level																		
FRPL %													-.06 (.01)	-5.07	[-0.08 to -0.04]	-.06 (.01)	-5.21	[-0.08 to -0.04]
Slope				.85 (.08)	11.11	[0.70 to 1.00]	.85 (.09)	9.71	[0.68 to 1.03]	.42 (1.50)	0.28	[-2.54 to 3.38]	-.67 (1.80)	-0.37	[-4.22 to 2.88]	-.79 (1.87)	-0.42	[-4.49 to 2.90]
Student Level																		
Age at Testing										.07 (.27)	0.27	[-0.46 to 0.60]	.19 (.30)	0.63	[-0.40 to 0.78]	.20 (.30)	0.64	[-0.41 to 0.79]
Sex										.08 (.18)	0.42	[-0.28 to 0.43]	-.07 (.21)	-0.31	[-0.48 to 0.35]	-.07 (.21)	-0.33	[-0.48 to 0.34]
Classroom Level																		
Quality													.00 (.00)	1	[-0.00 to 0.00]	.00 (.00)	0.93	[-0.00 to 0.00]
Quantity													-.00 (.01)	-0.43	[-0.02 to 0.01]			
Content																-.01 (.04)	-0.15	[-0.09 to 0.08]
Teacher Report Literacy													.00 (.00)	0.31	[-0.01 to 0.01]	.00 (.00)	0.51	[-0.00 to 0.01]
School Level																		
FRPL %													.01 (.01)	1.09	[-0.01 to 0.02]	.01 (.01)	1.06	[-0.01 to 0.02]
Random Effects																		
Slope (SD)							0.59		[0.42 to 0.83]	0.61		[0.44 to 0.85]	0.67		[0.48 to 0.93]	0.67		[0.49 to 0.94]
Intercept	1.8		[1.52 to 2.14]	1.84		[1.58 to 2.16]	2.07		[1.76 to 2.44]	1.95		[1.65 to 2.31]	1.53		[1.24 to 1.87]	1.51		[1.23 to 1.86]
Correlation							-0.47		[-0.67 to -0.20]	-0.49		[-0.69 to -0.23]	-0.34		[-0.61 to -0.01]	-0.35		[-0.61 to -0.03]
Residual Error	1.33		[1.21 to 1.48]	1.04		[0.94 to 1.15]	0.86		[0.74 to 0.99]	0.86		[0.74 to 0.99]	0.78		[0.66 to 0.92]	0.78		[0.65 to 0.92]

Note. Fixed effects parameters are unstandardized regression coefficients, and random effects parameters are estimates of variance. Missing data were handled using restricted maximum likelihood.

Table 8

Math Instruction Individualized Student Instruction Code, Descriptive Statistics (N = 146)

ISI Code	Proportion of Total Coded
1. Multiple Components	**1.37**
Number Sense, Concepts, & Operations	**58.89**
2. Number Writing and Recognition	6.85
3. Oral Counting	4.11
4. Number Line	2.74
5. Patterns (#s)	2.06
6. Counting Sets	9.59
7. Number Relations	3.42
8. Estimating	0.68
9. Addition	22.6
10. Subtraction	5.48
11. Multiplication	-
12. Division	-
13. Place Value	0.68
14. Fractions	0.68
15. Decimals	-
Geometry	**10.27**
16. Shapes	9.59
17. Lines	-
18. Transformations	-
19. Coordinate Geometry	-
20. Spatial Geometry	0.68
Algebra	**2.74**
21. Patterns (not #)	-
22. Expressions and Equations	-
23. Inequalities	2.74
Measurement	**17.79**
24. Time	14.38
25. Temperature	2.05
26. Money	0.68
27. Length	-
28. Circumfrence	-
29. Weight	0.68
30. Capacity	-
31. Quantity	-
Data Analysis	**8.9**
32. Data Collection	-
33. Data Representations	4.79
34. Analyzing Data	4.11
Probability	**0**
35. Certain, Likely, Possible	-
36. Likelihood	-
37. Predict an Outcome	-
38. Conduct an Experiment	-

Table 9

Post-Hoc Regression of Classroom Instruction Predicting Change in Standardized Math Achievement (N = 92)

	Model 1	**Model 2**
Variable	β (SE)	β (SE)
Age	-.02 (1.02)	-.00 (1.03)
Sex	.20 (.72) †	.21 (.72) †
FRPL %	-.08 (.02)	-.08 (.02)
Classroom Instruction		
Quality	-.22 (.00) †	-.24 (.00) †
Content	.03 (.15)	
Quantity		.09 (.76)

Note. †p <.10

Table 10

Post-Hoc Analyses Self-Regulation Predicting Math Garden Spring Scores (N = 91)

	Counting	**Adding**
	Model 1	**Model 2**
Variable	β (SE)	β (SE)
Working Memory	.07 (.06)	.13 (.09) †
Approaches to Learning	.18 (.13)*	.13 (.19)*
Prior Math Garden		
Counting Time 1	.08 (.13)	
Counting Time 2	.52 (.10)***	
Adding Time 1		.09 (.08)
Adding Time 2		.65 (.09)***

*Note. †p <.10, *p <.05, **p <.01, ***p <.001*

Time of Year	Group Behavioral	Individual Behavioral	Classroom
October 2018	Math Garden		Teacher Survey
Nov/Dec 2018		WJAP	
February 2019	Math Garden		Teacher LENA
April/May 2019		WJAP + NLE	
May 2019	Math Garden		Teacher ATL

Figure 1.
Data Collection Schedule.

WJAP = Woodcock-Johnson, Applied Problems. ATL = Approaches To Learning

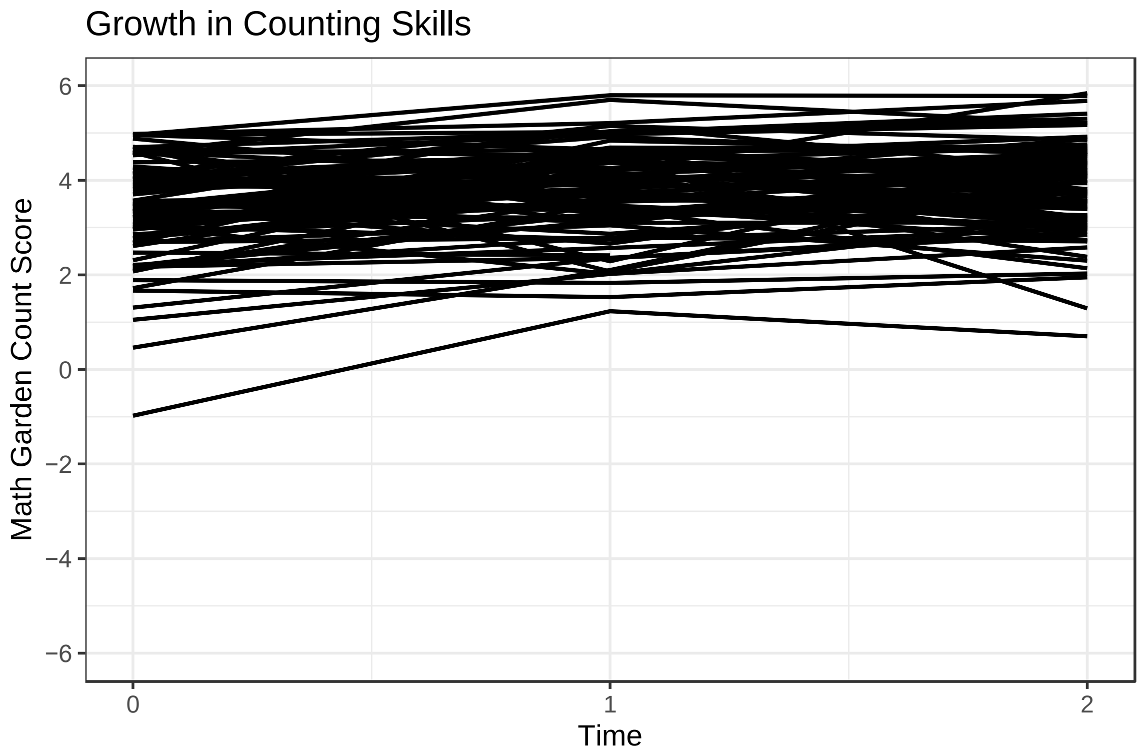

Figure 2. Children’s Math Garden counting trajectories over the kindergarten school year

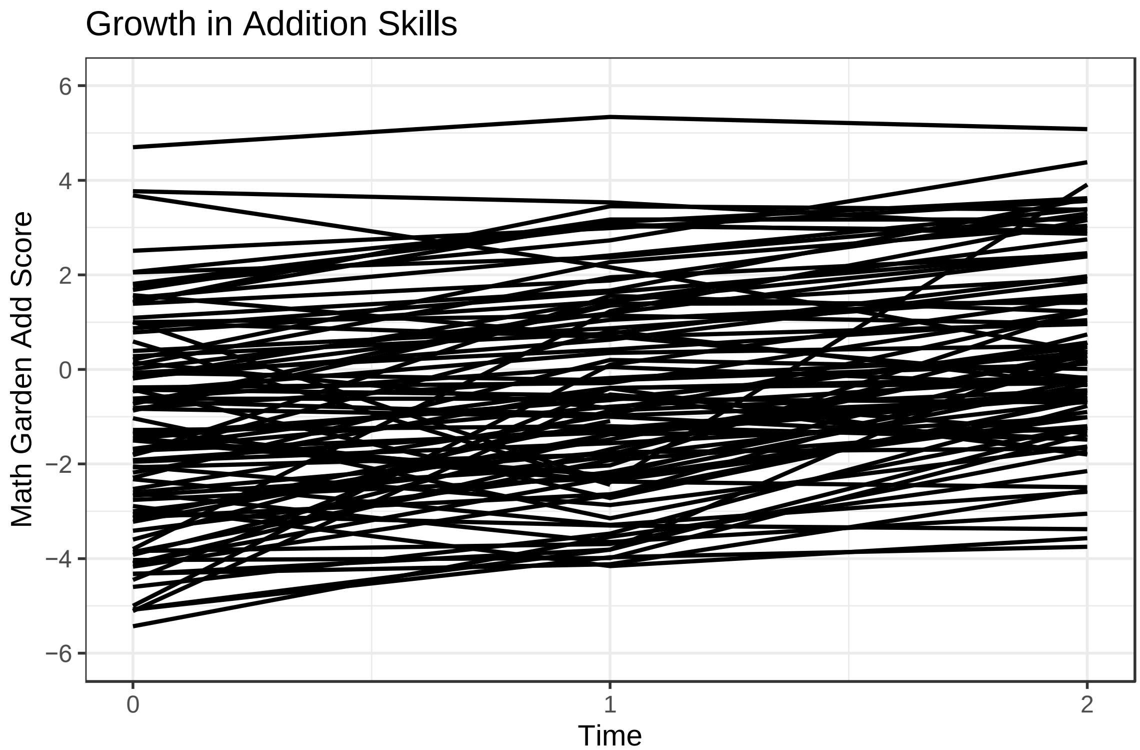

Figure 3. Children’s Math Garden addition trajectories over the kindergarten school year

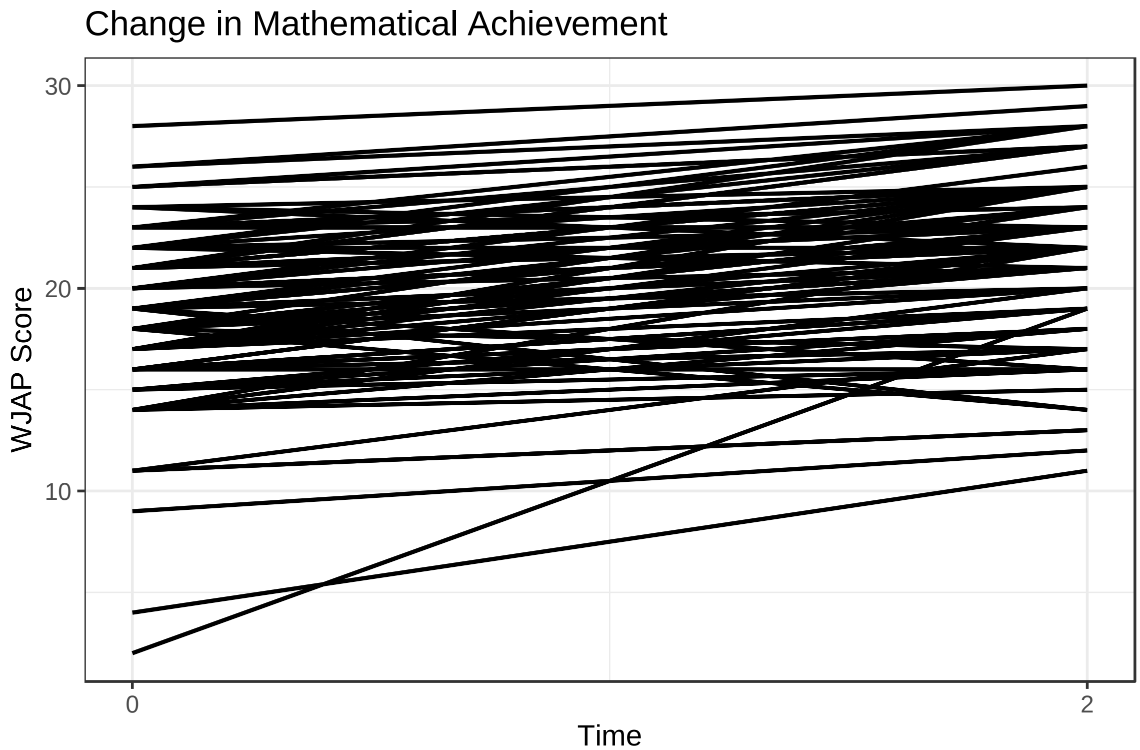

Figure 4. Children's standardized mathematical achievement change scores across the kindergarten year.

(WJAP = Woodcock-Johnson Applied Problems)

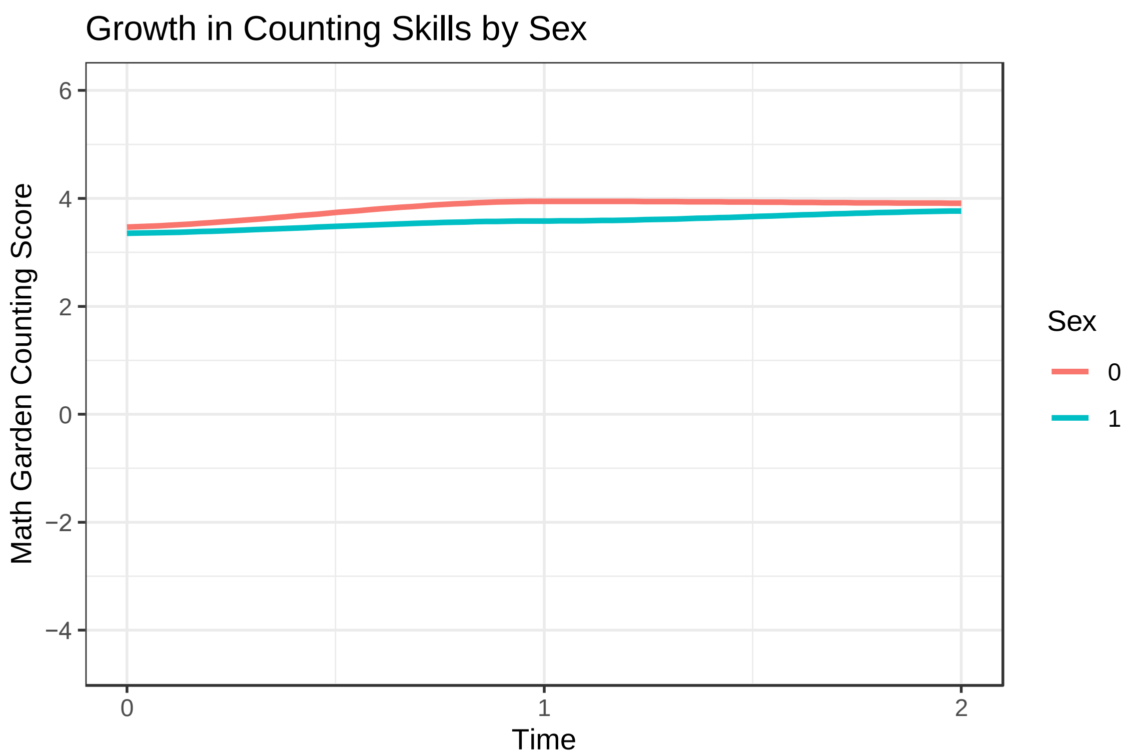

Figure 5. Children's Math Garden counting trajectories by sex across the kindergarten school year

(0 = male, 1 = female)

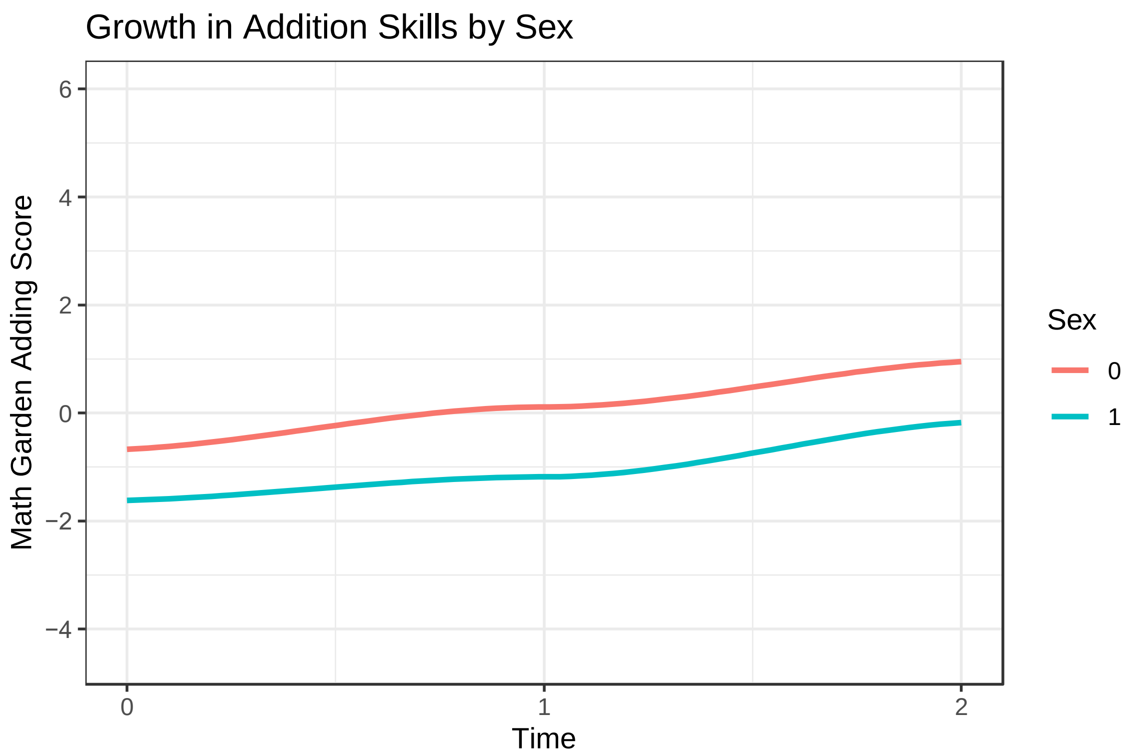

Figure 6. Children's Math Garden addition trajectories by sex across the kindergarten school year

(0 = male, 1 = female).

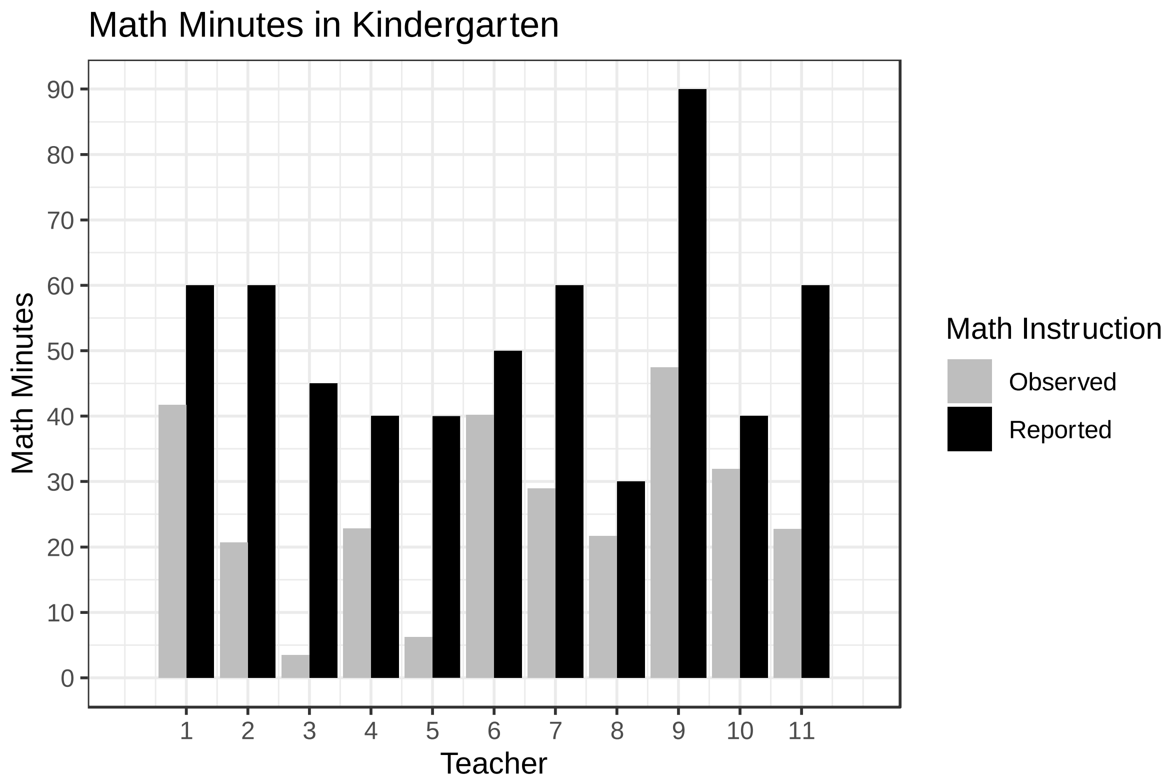

Figure 7. Teacher reported and observed minutes spent in mathematics instruction

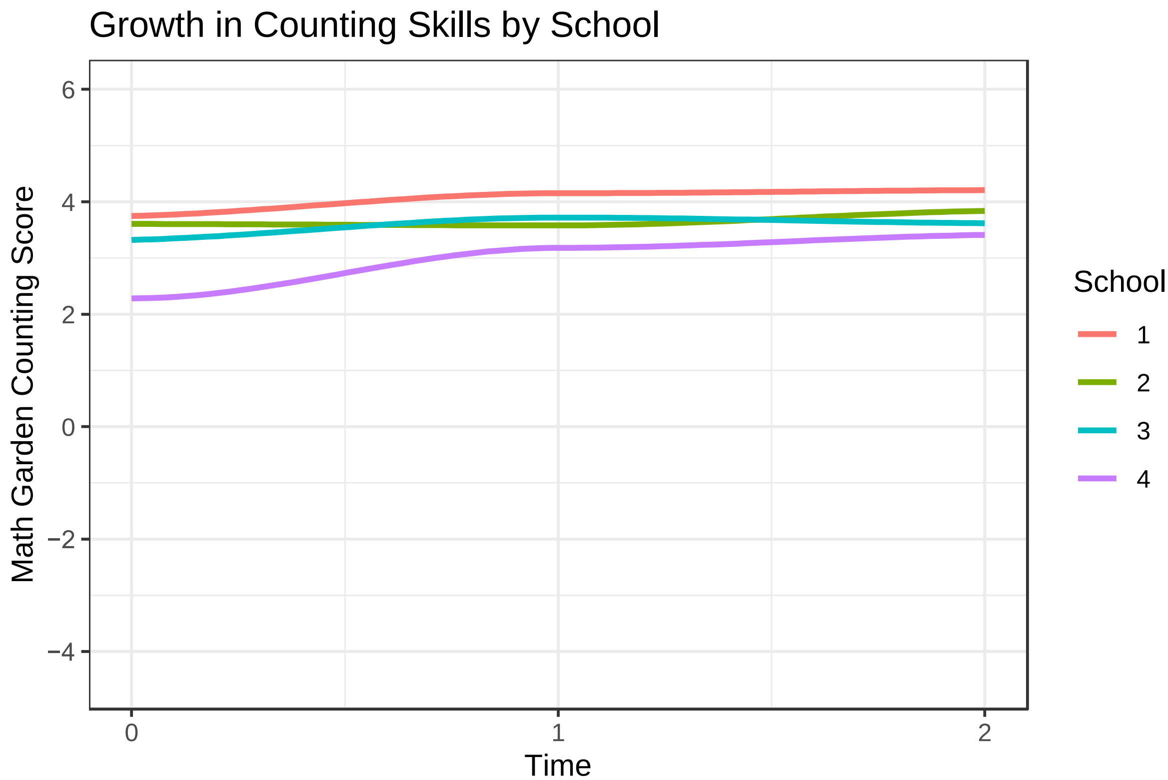

Figure 8. Children's Math Garden counting trajectories by school across the kindergarten school year

(school number corresponds to Table 2, and are rank-ordered based on free and reduced-price lunch percentage)

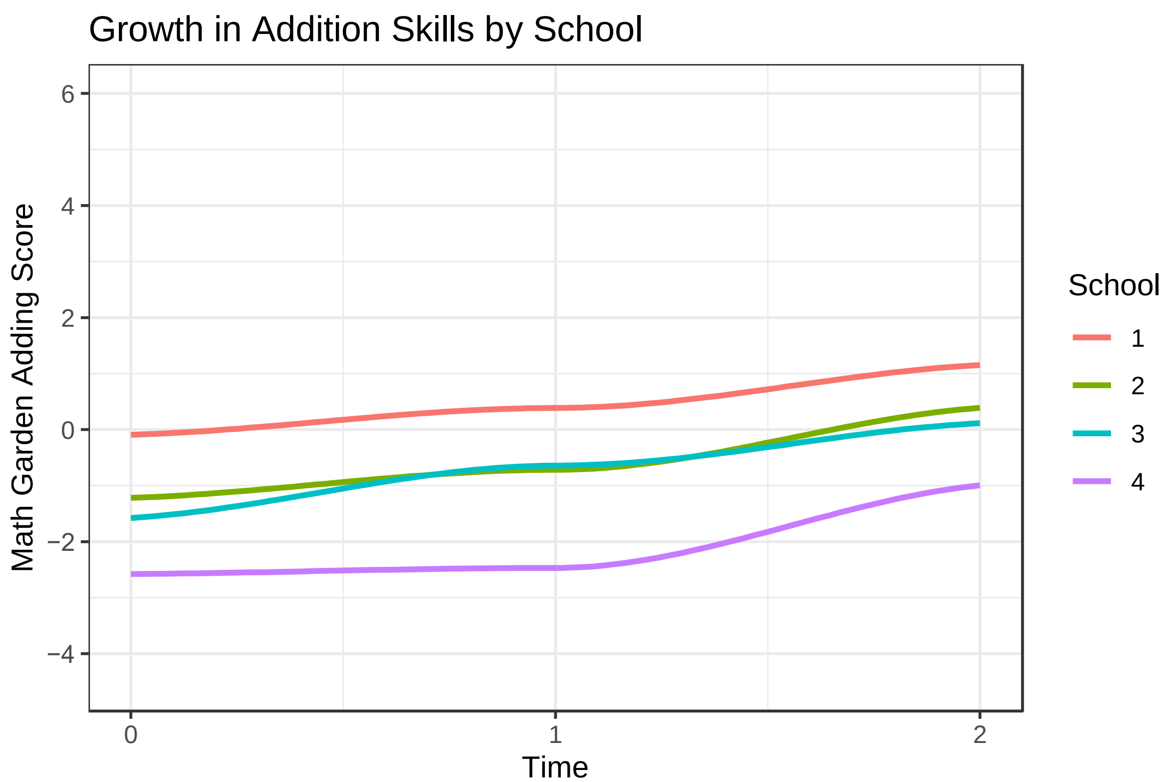

Figure 9. Children's Math Garden addition trajectories by school across the kindergarten school year

(school number corresponds to Table 2, and are rank-ordered based on free and reduced-price lunch percentage)

APPENDICES

APPENDIX A

Teacher Questionnaire

Start of Block: Personal Questions

Q1 What is your name?

____

Q2 Gender:

○ Male (1)

○ Female (2)

Q3 Age:

▼ 20 (1) ... 80 (61)

Q4 Highest Level of Education:

▼ Some High School (1) ... Postgraduate or Professional Degree (e.g., MA, MS, PhD, JD, MD) (7)

Q5 Total number of years teaching:

▼ .5 (1) ... 40 (41)

Q6 Number of years teaching Kindergarten:

▼ .5 (1) ... 40 (41)

End of Block: Personal Questions

Start of Block: Block 3

Q7 Total number students you have in class?

◯ Total: (1) ____

◯ Male: (2) ____

◯ Female: (3) ____

Q8 Do you follow the common core for math K?

◯ Yes (1)

◯ No (2)

Display This Question:

If Q8 = No

Q16 If not, what else do you use?

____

End of Block: Block 3

Start of Block: Classroom Items

*

Q11 In a typical day, how many **minutes** are devoted to **math** instruction?

____

*

Q12 In a typical day, how many **minutes** are devoted to **literacy** instruction?

____

Q13 How do you think, if at all, the 3rd grade reading law (*the department shall do all to help ensure that more pupils will achieve a score of at least proficient in English language arts on the grade 3 state assessment*) in the state of Michigan has **influenced your instruction time?** Please explain.

____

____

____

____

____

Q15 How many students from your class are participating in this study?

▼ 1 (1) ... 10 (10)

End of Block: Classroom Items

Start of Block: Ratings

Q36 For each child, how would you **rank** his/her math skills on a 1-10 scale?

	Well Below Average		Below Average		Average		Above Average		Well Above Average		
	0	1	2	3	4	5	6	7	8	9	10
Child #1 ()											
Child #2 ()											
Child #3 ()											
Child #4 ()											
Child#5 ()											
Child #6 ()											
Child #7 ()											
Child #8 ()											
Child #9 ()											
Child #10 ()											

Q26 How would you **predict** each child's math **performance by the end of the year?**

	Well Below Average		Below Average		Average		Above Average		Well Above Average	
	0	1	2	3	4	5	6	7	8	9 10
Child #1 ()										
Child #2 ()										
Child #3 ()										
Child #4 ()										
Child#5 ()										
Child #6 ()										
Child #7 ()										
Child #8 ()										
Child #9 ()										
Child #10 ()										

Q37 Please rank these from 1 (greatest) to 5 (least) which you think will have the largest impact on your student's math performance?

______ Your attitude toward math (1)
______ The amount of math time in school (2)
______ Genetics (3)
______ How early they were exposed to mathematical concepts (4)
______ The child's parent's attitude toward math (5)

End of Block: Ratings

Teacher Approaches to Learning Scale

For Child #1:

How frequently does this child exhibit the following behaviors or characteristics?

	Never	Sometimes	Often	Very Often	No opportunity to observe
Keeps belongings organized	○	○	○	○	○
Shows eagerness to learn new things	○	○	○	○	○
Works independently	○	○	○	○	○
Easily adapts to changes in routine	○	○	○	○	○
Persists in completing tasks	○	○	○	○	○
Pays attention well	○	○	○	○	○
Loses track during complicated tasks and may eventually abandon these tasks	○	○	○	○	○
Makes place-keeping errors (e.g., skipping or repeating steps)	○	○	○	○	○
Shows incomplete recall of information	○	○	○	○	○

APPENDIX B

Individualizing Student Instruction Classroom Observations Coding Manual - Mathematics

Individualizing Student Instruction
Classroom Observations Coding Manual – Mathematics
Version 09.11.2019
Alexa Ellis

Adapted From:
Version 6 07.02.2014
Carol McDonald Connor
Elizabeth Crowe
Stephanie Glasney
Florida State University and the Florida Center for Reading Research
Sarah Ingebrand
Arizona State University and the Learning Sciences Institute

1. **Coding Protocol (Adapted from Pathways and Connor Code)**
    - **1.1.** Recordings will be captured and assigned prior to starting to code a new round of recordings.
    - **1.2.** Coders should listen to recordings created for the observation of interest. Recordings need to be coded to capture all the activities in which children participate. It is recommended to listen to the recordings in real time in 10 minute increments to become familiar with the instruction.
    - **1.3.** Observations should be coded using the Math_ISI_09_11_19 project. Click on the project once to open it. Choose START OBSERVATION from the Observation pull-down menu to open the Observation Module.
    - **1.4.** A dialog box should open, where you can (a) select an observation which has already been (partially or fully) coded or (b) start a new observation. If starting a new observation, choose the relevant video file (located in terastations) and name your observation using the following format: Teacher ID (4-digit #) (e.g., 6001); Coder's initials. Coders should be consistent in the initials they use. After naming the observation file, click OK. You will be prompted with a Video Selection window to select the relevant file.
    - **1.5.** A new dialog box will open which will allow you to input the Independent Variables. Code the Independent Variables as specified in section below.
    - **1.6.** Once the Independent Variables have been defined, another dialog box will open and ask you to position the tape where you want to begin the observation. You want to start at the beginning of the tape.
    - **1.7.** The Initialize Channels dialog box will open. All subjects should be "initialized" in the Instruction-Null, Dimension-Null, and Content Area-Null. As soon as you begin coding, however, you will need to indicate which subjects are active and in what activity they are involved (i.e., instruction code). Channels should only need to be initialized when a new observation is started.
    - **1.8.** Click the green button that says "Not Recording" to start Record mode. The button should turn red when it is in record mode to begin coding. To avoid errors later on, always make sure you are coding while in record mode.
    - **1.9.** Instructional activities should last at least 15 seconds to be coded. The following guidelines should also be followed:
        - **1.9.1.** If two Behavioral Instructions are equal lengths of time but neither are 15 seconds, the first occurring of the two parts should be coded for both Behavioral Instructions.
        - **1.9.2.** If two separate Behavioral Instructions do not last for 15 seconds and they are not equal lengths, the longer of the two parts should be coded.
    - **1.10.** Periodically save data in the Observation menu when coding.
    - **1.11.** Any incomplete coded files need to be deleted. Only one coding file should exist for each recording. Do not keep multiple files for the same observation on multiple computers.
    - **1.12.** Always code the activity/instruction the teachers are completing over the activity of the student.
    - **1.13.** If a Behavioral Instruction is not covered in the coding manual then write a brief description of the activity/instruction that occurred and note the video file and time in the Code Book Suggestion Notebook and/or discuss at coding meetings.

1.14. If a code is added just before the video ends and does not last 15 seconds, here is what to do: if the code is less than 12 seconds, delete it; if the code is 12+ seconds and the previous code is more than 15 seconds, stretch it into the previous code; if the last code is 12+ seconds but the previous code is only 15 seconds (you have nowhere to stretch it), delete it.

1.15. When you are finished coding, click on the observation menu select SAVE DATA and then click END OBSERVATION in the Observation menu.

2. Coding With Noldus (Adapted from Pathways Code)

2.1. Subjects, instruction/non-instruction behaviors, and type instruction modifiers are changed through instruction/non-instruction behavior (e.g., Geometry>Shapes, "gsh"). *Each behavior/activity must last at least 15 seconds to be coded*; behaviors/activities which are shorter than 15 seconds should either be ignored or coded following protocol (see section 1). Also any time you change an instruction behavior, you should be prompted to also enter a modifier for the instruction (even if the modifier has not changed).

2.2. While coding, if a mistake is made by typing an incorrect code, you can use the CURSOR KEYS (i.e., up, down, left, right) to navigate to the field where the mistake was made. Once in the correct field replace the mistake by typing the correct code. The COMPUTER MOUSE can also be used to navigate for making corrections. "Point-and-click" the mouse cursor/pointer on the desired field, once highlighted, the mistake can be replaced by typing the correct code.

2.3. Comments are added by clicking on the comment field in the observation log. In "The Observer XT" comments should be limited to a maximum length of 256 characters. It is preferable to type only letters in the comment field and to avoid using punctuation.

2.4. For troubleshooting, consult the Noldus reference manual first and if further help is needed e-mail tech support at tech@noldus.com.

4. Independent Variables

The independent variables should be entered prior to beginning coding.

4.1. School ID

The ID number for the school at which the observation is taking place should be retrieved from the database and entered in this field.

4.2. Teacher ID

The ID number for the particular teacher whose classroom is under observation should be retrieved from the database and entered in this field.

4.3. Date Taped

The date on which the observation was filmed should be entered in this field in the form MM/DD/YYYY.

4.4. Date Coded

The date on which the observation began being coded should be entered in this field in the form MM/DD/YYYY.

4.5. Coder

The name of the coder (first and last) should be entered in this field. Names should be entered consistently (i.e., no nicknames).

6. Instruction (Behavioral Class)

The codes within the Instruction behavioral class are used to indicate the content of activities/instruction including those which do not include actual academic content. This behavioral class is divided into various math-related behaviors, non-math literacy codes, and non-instructional behaviors. *All activities/behaviors must last for at least 15 seconds to be coded*; activities/behaviors which are shorter than 15 seconds are ignored and considered part of the prior or next activity/behavior to be coded. As behavioral classes are not mutually exclusive an Instruction code must be designated for each subject at all times.

6.1. Math-Related Codes

6.1.1. Instruction Null (Behavior) inl

Instruction Null should be coded as a means of "turning off" irrelevant instruction codes (i.e., student is absent, student left the class room).

6.2. Number Sense, Concepts, and Operations

6.2.1. Multiple Components (Behavior) nsm

Number Sense>Multiple Component should be coded when a variety of combined *number sense, concepts, and operations* occurred within at least the 15 second instruction minimum and/or for a longer duration. To be coded as Number Sense>Multiple Components all of the activities/instruction occurring together must be part of number sense, concepts, and operations. For example, the teacher may ask students to locate the number 20 on a hundred's chart (i.e., number writing and recognition), then count aloud from 0 to 20 (i.e., oral counting), and then count to 20 by 2's (i.e., patterns) all within 15 seconds. *A brief description of the activity should be noted in the comment field.*

6.2.2. Number Writing and Recognition (Behavior) nnw

Number Sense>Number Writing and Recognition should be coded when students are involved with activities/instruction related to number writing and recognition including; recognition of verbal and/or written number names, numeral writing, ordinal numbers, ordinal position, identifying even and odd numbers, identifying and locating numbers on a hundred chart, reading and identifying numbers to 100, finding missing numbers on a hundreds chart, writing numbers to 100, 1000, 10,000, etc., and/or multiple components (number writing and recognition). For example, a hundred's chart with numbers from 0 to 100 may be posted on the board/classroom wall, and the teacher requests students to identify and locate specific numbers on the hundred's chart. Another example, can occur when the children are expected to write numbers from 0 to 100 on a worksheet/notebook.

6.2.3. Oral Counting (Behavior) noc

Number Sense>Oral Counting should be coded when students are involved with activities/instruction related to oral counting including; oral counting, counting backwards, and/or multiple components (oral counting). An example of this behavior occurs when students are asked to count "aloud" backwards from 100. *During oral counting activities/instruction the students are counting aloud.*

6.2.4. Number Line (Behavior) nnl

Number Sense>Number Line should be coded when students are involved in activities/instruction related to number line including; counting with a number line, drawing a number line, locating points on a number line and/or multiple component (number line). For example, during instruction students are working together to locate points on a number line, as well, as counting with aid of the number line. *Important that during these behaviors a number line must be used.*

6.2.5. Patterns (Behavior) nsp

Number Sense>Patterns should be coded when students are involved in activities/instruction related to patterns including; counting in patterns (i.e., counting by 2's, 3's, 4's 5's, 10's, 100's). Also counting by 5's using a clock, counting on from a given number and multiple components (patterns). For example, the teacher asks students to count from ten to one-hundred using several different patterns (i.e., counting by 2's, 5's, and 10's). Even though students are counting aloud, in this case, since they are counting in *patterns* it is coded as Number Sense>Patterns. If they were only counting aloud, for example, zero to twenty 1-by-1 (i.e., 1, 2, 3, 4, 5, etc.), then this would be coded as Number Sense>Oral Counting.

6.2.6. Counting Sets (Behavior) ncs

Number Sense>Counting Sets should be coded when students are involved in activities/instruction related to counting sets including; counting sets, ordering sets, combining sets, and/or multiple components (counting sets). An example of counting sets can be students combining multiple linking cubes into sets of 10, and then students are asked to calculate the total number of linking cubes by counting the number of sets (i.e., there are 5 sets of 10 linking cubes; 5 sets X 10 cubes = 50 cubes).

6.2.7. Number Relations (Behavior) nnr

Number Sense>Number Relations should be coded when students are involved in activities/instruction related to number relations including; determining/understanding more and less, more and less (using number line), more and less (using spinner), ordering numbers, identifying one more and one less on a hundred chart, and/or multiple components (number relations). Number relations can be, for example, during activity/instruction students are working together to determine which numbers are more and/or less. The students would be doing this with the aid of a spinner and/or number line. In another example, students are asked to order number cards from 0 to 20 from smallest to largest. Then they are asked to point out specific numbers and identify the next number (i.e., point to the number after 7; "What number is it?" The answer is "8").

6.2.8. Estimating (Behavior) nse

Number Sense>Estimating should be coded when students are involved in activities/instruction related to estimating including; estimating a collection, rounding a number to the nearest 10 (for computation), and/or multiple component (estimating). An example of estimating occurs when the teacher and students are working on an estimation of how many paper clips fit into a cup. Each student provides their estimate and then the teacher counts the paper clips to determine which estimate is most correct.

6.2.9. Addition (Behavior) nsa

Number Sense>Addition should be coded when students are involved in activities/instruction related to addition including; 1+1 addition (with manipulative use), 1+1 addition (algorithm), addition (i.e., adding 0, 1, 2-9, 10), adding 10 to a single-digit/two-digit number, adding 10 to a multiple of 10, addition (doubles with sums to 18), addition (doubles plus one), adding 2 to an even/odd number, sums below 8, sums of 8 to 18, finding a sum by counting on, illustrating and writing addition number sentences, identifying the associative/commutative property of addition, adding three or more single-digit numbers, 2+2 addition (with or w/out regrouping) using coins, 2+2 addition (w/regrouping) algorithm, doubling a number, adding two-digit numbers with sum greater than 100, adding 3 two-digit numbers (w/regrouping), adding 3 two-digit numbers with a sum greater than 100, estimating sums, adding two and/or three-digit numbers and money amounts, adding objects or pictures, addition word problems, repeated addition with number sentences, using a calculator to add, and/or multiple components (addition). For example, the teacher guides students through an addition exercise by adding zero to a number, adding one to a number, and later practicing "doubles" addition using the *Saxon "Learning Wrap-Ups"* manipulative. In another example, the teacher explains the commutative property of addition; instructing students that changing the order of addends does not affect the sum. Also students may be asked to solve an addition word problem; for example, "Charlie had eight pencils and Mary gave him two more pencils. How many pencils does Charlie have now?"

6.2.10. Subtraction (Behavior) nss

Number Sense>Subtraction should be coded when students are involved in activities/instruction related to subtraction including; 1-1 subtraction (with manipulative use), 1-1 subtraction (algorithm), subtraction – subtracting number from itself, subtraction (subtracting 0 to 10), subtracting a number from 10, subtraction word problems, subtracting half of a double, mental computation – subtract 10 from a two-digit number, subtraction – minuends greater than 10, subtracting 2 from an even/odd number, illustrating and writing subtraction number sentences, 2-2 subtraction (w/out regrouping), 2-2 subtraction (with regrouping) using coins, 2-2 subtraction (w/regrouping) algorithm, subtracting three-digit numbers and money amounts, estimating differences, using a calculator to explore addition, subtraction, and skip counting, repeated subtraction (via word problems), using a calculator to subtract, and/or multiple components (subtraction). For example, the teacher writes the number 10 on the board and then students subtract numbers from 10 (i.e., 10-2=8, 10-5=5, 10-10=0, etc.). Also, students illustrate and write out a subtraction number sentence, for example, a word problem states "William has ten balloons and then gives five balloons to Sarah. How many balloons does William have now?" Students first illustrate the subtraction word problem and then write out the number sentence (i.e., 10-5=5). In another example, the teacher guides students through a worksheet activity/instruction where they use a calculator to solve subtraction problems.

6.2.11. Multiplication (Behavior) nmt

Number Sense>Multiplication should be coded when students are involved in activities/instruction related to multiplication including; multiplication (i.e., multiplying 0, 1-5, 10, 100), making and labeling an array, writing number sentences for arrays, making and using a multiplication table, drawing pictures and writing multiplication sentences to show groups, identifying multiples of a number (i.e., multiples of 2, 3, 4, 5), using a calculator to multiply, and/or multiple components (multiplication). For example, the teacher guides students through multiplying numbers by zero, one, and five (i.e., 0x2=0, 1x2=2, and/or 5x2=10). Also, in order to better conceptualize multiplication the students create a rectangular array with four columns of five squares each column. Then a number sentence is derived from this array (i.e., 4x5=20). Continuing the array activity/instruction the teacher works with students to determine the product of three equal groups consisting of four items per group (i.e., 3x4=12). Another example of multiplication can be when students use a multiplication table to aid in their completion of a worksheet.

6.2.12. Division (Behavior) ndi

Number Sense>Division should be coded when students are involved in activities/instruction related to division including; dividing a set of objects into equal parts, dividing a set of objects by sharing, repeated subtraction (via word problems), illustrating and writing number sentences for "equal-groups" story problems, division by 2, using a calculator to divide, and/or multiple components (division). For example, students are given twenty blocks and asked to divide them into equal groups, so they divide the blocks into two groups of ten, then five groups of four, and then four groups of five. Also, students are asked to illustrate and write a number sentence for a story problem, for example, “The teacher has sixty cookies and needs to equally divide them between three groups of students. How many cookies will each group receive?” Students first illustrate the division problem and then write out a number sentence (i.e., 60÷3=20; each group receives 20 cookies).

6.2.13. Place Value (Behavior) npv

Number Sense>Place Value should be coded when students are involved in activities/instruction related to place value including; understanding the place value of a number, illustrating the place value of a number, using concrete/pictorial models to represent numbers, expressing a number in expanded form (verbally), writing a number in expanded form, and/or multiple components (place value). For example, the teacher writes several numbers on the board (i.e., 22, 68, 155, and 2010), then students are asked to separate the digits in each number with a line and write the place value above each digit (i.e., ones, tens, hundreds, thousands). Also, students are completing a worksheet to write numbers in their expanded form (i.e., 155=100 + 50 + 5), and then follow up by reviewing their answers aloud.

6.2.14. Fractions (Behavior) nfr

Number Sense>Fractions should be coded when students are involved in activities/instruction related to fractions including; dividing a shape into its fractional parts, dividing an object into halves, identifying one half of a whole, identifying numerator and denominator, comparing fractional parts of a whole, identifying if a fractional part of a whole is closer to 1, ½, or 1, writing a unit fraction using fraction notation (i.e., ½), identifying/naming/picturing a fractional part of a set, writing a fraction to show a part of a set, finding one half of a set with even/odd number of objects, representing and writing mixed numbers, and/or multiple components (fractions). For example, students are completing a worksheet where they are asked to divide shapes into fractional parts (i.e., dividing a square into equal halves). In another example, the teacher explains both proper and improper fractions to students. It is pointed out that fractions are made up of two numbers; a top number called the numerator and a bottom number called the denominator. Later the teacher and students go on to write proper fractions, as well as, change improper fractions into mixed numbers.

6.2.15. Decimals (Behavior) nde

Number Sense>Decimals should be coded when students are involved in activities/instruction related to decimals including; the function of decimals within money, writing monetary amounts using the decimal, identifying decimal portions of shaded figures, converting fractions to decimals, and/or multiple components (decimals). For example, the teacher explains and demonstrates the function of decimals when used to separate dollars from cents for representing monetary amounts (i.e., $1.45, $10.50, and $100.75). Later students are asked to complete a worksheet where they use decimal points to separate dollars from cents in monetary amounts. The same activity/instruction can occur when explaining and demonstrating how to separate a whole number from the decimal part of a number.

6.3. Geometry

6.3.1. Multiple Components (Behavior) gmc

Geometry>Multiple Component should be coded when a variety of combined *Geometry* behaviors occur within at least the 15 second instruction minimum and/or for a longer duration.

To be coded as Geometry>Multiple Components all of the activities/instruction occurring together must be part of Geometry. For example, during an activity/instruction students first plot a set of ordered pairs onto a coordinate plane (i.e., coordinate geometry), then draw a shape from these plotted points (i.e., shapes), and finally do a rotation and reflection with the shape (i.e., transformations), all within 15 seconds. *A brief description of the activity should be noted in the comment field.*

6.3.2. Shapes (Behavior) gsh

Geometry>Shapes should be coded when students are involved in activities/instruction related to shapes including; identifying basic shapes, number of sides and angles of basic shapes, covering designs with pattern blocks/tangram pieces, identifying and creating overlapping geometric shapes, identifying and creating similar shapes and designs, identifying and making congruent shapes, drawing congruent shapes and designs, identifying and sorting common geometric shapes by attribute, identifying and drawing polygons, making polygons on a geo-board, identifying the angles of a polygon, identifying pentagons, describing and classifying plane figures, identifying geometric solids (i.e., cones, spheres, cubes, cylinders, rectangle prisms), identifying/describing/comparing geometric solids, identifying and drawing a line of symmetry, creating a symmetrical design, dividing a solid in half, cutting a geometric shape apart and making a new shape, combining shapes to create new shapes, and/or multiple components (shapes). For example, students are working independently to complete an activity/instruction using pattern blocks/tangram pieces to cover various shapes (i.e., rectangles, squares, and/or triangles). Also, during a center activity students use a geo-board to create a variety of polygon shapes; including making congruent shapes. In another example, the class is working together to divide a solid into a half and also to identify and draw a line of symmetry through several shapes.

6.3.3. Lines (Behavior) gli

Geometry>Lines should be coded when students are involved in activities/instruction related to lines including; identifying parallel lines, line segments, intersecting lines, perpendicular lines, horizontal/vertical/oblique lines, right angles, acute/obtuse angles and/or multiple components (lines). For example, the teacher spends time explaining the differences between parallel and perpendicular lines including; drawing examples on the board. Also, the teacher and students discuss what makes a line horizontal, vertical, or oblique. In another example, students independently complete a worksheet where they are expected to determine types of angles (i.e., acute, obtuse, or right angle).

6.3.4. Transformations (Behavior) gtr

Geometry>Transformations should be coded when students are involved in activities/instruction related to transformations including; exploring transformations (i.e., slides, turns, flips), identifying and showing transformations (i.e., translations, rotations, reflection), and/or multiple components (transformations). For example, in order for students to better understand the concept of geometric transformations the teacher is at the board using shapes to demonstrate a variety of transformations (i.e., flips, reflections, rotations, slides, translations, and/or turns). Often a geometric transformations activity/instruction involves the use of a coordinate plane (i.e., coordinate graph).

6.3.5. Coordinate Geometry (Behavior) gco

Geometry>Coordinate Geometry should be coded when students are involved in activities/instruction related to coordinate geometry including; locating and graphing points on a coordinate graph, and/or multiple components (coordinate geometry). For example, the teacher prepares a coordinate plane (i.e., coordinate graph) on the board and then asks student volunteers to locate points on the graph using specific ordered pairs (i.e., (x, y), (3, 5), etc.). After sufficient points have been correctly located, then the teacher demonstrates how students can graph line segments, angles, and/or shapes.

6.3.6. Spatial Geometry (Behavior) gsp

Geometry>Spatial Geometry should be coded when students are involved in activities/instruction related to spatial geometry including; identifying right, left, between, middle, inside, outside, and/or multiple components (spatial geometry). For example, the teacher asks the students to raise their right or left hand to determine if students understand the concept of right and left. Also, during a worksheet activity/instruction students are asked to place a check mark "√" in specific locations as they relate to shapes on the page (i.e., place a check mark "√" next to the right side of the triangle; place a check mark "√" on the inside of the circle, etc.).

<u>6.4. Algebra</u>

6.4.1. Multiple Components (Behavior) amc

Algebra>Multiple Components should be coded when a variety of combined *Algebra* behaviors occur within at least the 15 second instruction minimum and/or for a longer duration. To be coded as Algebra>Multiple Components all of the activities/instruction occurring together must be part of Algebra. For example, the teacher and students are working to determine missing addends/subtrahends (i.e., expressions and equations) and then using comparison symbols (<, >, or =) to designate which equations and/or expressions are greater than, less than, or equal to others (i.e., inequalities) all within 15 seconds. *A brief description of the activity should be noted in the comment field.*

6.4.2. Patterns (Behavior) apa

Algebra>Patterns should be coded when students are involved in activities/instruction related to patterns including; sorting common objects, identifying attributes of pattern blocks, sorting by one attribute, creating a color pattern (including shapes), continuing a repeated pattern, creating a repeated pattern, continuing a growing pattern, identifying a number between two numbers, and/or multiple components (patterns). For example, students complete a worksheet activity/instruction where they are expected to continue repeated patterns (i.e., circle/square/triangle/circle/square/triangle, etc.). Also, the teacher and students spend time identifying attributes of pattern blocks (i.e., shape, color, size), then sort the pattern blocks according to an attribute. After sorting the blocks they then create repeated patterns (i.e., blue/red/yellow, circle/square/triangle, etc.).

6.4.3. Expressions and Equations (Behavior) aee

Algebra>Expressions and Equations should be coded when students are involved in activities/instruction related to expressions and equations including; identifying a missing addend identifying a missing subtrahend, and/or multiple components (expressions and equations). For example, the class is working together to determine the missing addends/subtrahends in several algebraic expressions and/or equations (i.e., $4 + x = 6$; $10 + 5 = 5 + x$). In order for students to better conceptualize this activity/instruction the teacher has written several expressions/equations on the board that lack either an addend or a subtrahend. Student volunteers come to the board and complete the expressions/equations with the correct addend or subtrahend.

6.4.4. Inequalities (Behavior) ain

Algebra>Inequalities should be coded when students are involved in activities/instruction related to inequalities including; using comparison symbols (<, >, or =) with two single-digit numbers/with two-digit numbers or greater/with fractions/with two or more equations, greater/less/and equal to, and/or multiple components (inequalities). For example, the teacher is guiding students through an activity/instruction in order for them to better understand the concept of using comparison symbols (i.e., <, >, or =). To do so, they review the relationship of greater than, less than, and/or equal to for several pairs of single-digit and two-digit numbers. In another example, students are working in small groups on an activity/instruction and they are expected to determine if a number is greater than (>), less than (<), or equal to (=) another number.

6.5. Measurement

6.5.1. Multiple Components (Behavior) mmc

Measurement>Multiple Components should be coded when a variety of combined *Measurement* behaviors occur within at least the 15 second instruction minimum and/or for a longer duration. To be coded as Measurement>Multiple Components all of the activities/instruction occurring together must be part of Measurement. For example, the teacher may ask the students to identify the days of the week/months of the year (i.e., time), identify the day as cold/cool/warm/hot (i.e., temperature), and identify various coins, such as, pennies/nickels/dimes/quarters (i.e., money) all within 15 seconds. *A brief description of the activity should be noted in the comment field.*

6.5.2. Time (Behavior) mti

Measurement>Time should be coded when students are involved in activities/instruction related to time including; identifying month/date/year, writing the date using digits, identifying days of week/months of year, time in day terms, clock, numbering a clock face, telling/showing time to hour/half-hour/quarter-hour/minute/five-minute intervals, identifying a.m. and p.m./noon and midnight, elapsed time (i.e., one hour ago, one hour from now, one half hour ago), elapsed time word problems by hours, ordering events by time, identifying activities that take one hour/one minute/one second, and/or multiple components (time). For example, the class participates in the "beginning-of-the-day" math routine which is focused around a classroom meeting board. As part of this daily routine, the teacher and students, identify and recite the days of the week and the months of the year (i.e., Monday – Friday, January – December). Also, each day during their math routine students mark the date on a calendar and then write out the date (i.e., 10-25-2010; October 25, 2010). In another example, students are completing a worksheet where they are asked to show the time (i.e., show half past nine) by first writing it out using digits (i.e., 9:30) and then by numbering a clock face.

6.5.3. Temperature (Behavior) mte

Measurement>Temperature should be coded when students are involved in activities/instruction related to temperature including; identifying cold/cool/warm/hot temperatures, reading a thermometer to 10 degrees, estimating temperature to the nearest 10 degrees, reading a thermometer to the nearest 2 degrees Fahrenheit, and/or multiple components (temperature). For example, the teacher introduces new vocabulary to students which relates to temperature; such as, thermometer, hot, warm, cool, and cold. After discussing the similarities/differences between various temperatures the class completes a worksheet marking various temperatures they have measured (i.e., the classroom's air temperature and/or the water temperature from the cup of water, etc.) onto a "thermometer" illustration. Also, each day the students spend time estimating the temperature outdoors (i.e., cold, cool, warm, or hot).

6.5.4. Money (Behavior) mmo

Measurement>Money should be coded when students are involved in activities/instruction related to money including; counting pennies/nickels/dimes/quarters/combined coins, trading pennies for dimes, showing money amounts using coins, identifying similarities/differences among coins, paying for items using dimes and pennies/quarters-dimes-nickels-and pennies, writing money amounts using the Cent symbol, identifying one/five/ten/twenty dollar bills, using bills to pay for items to $20, writing money amounts using a dollar sign, showing/counting back change for $1.00, and/or multiple components (money). For example, the class participates in the "beginning-of-the-day" math routine which is focused around a classroom meeting board. As part of this daily routine, the teacher and students, identify and count money with the aid of "meeting board coins" (i.e., enlarged images of coins; e.g., pennies, nickels, dimes, quarters). In another example, students are asked to answer questions on a worksheet which requires them to count various stacks of coins to determine the total(s). Also, the teacher and students work together on a "General Store" activity/instruction where various priced items are listed on a worksheet and students must determine which coins are necessary to pay for the items (i.e., the toy costs $1.56; that is four quarters, five dimes and six pennies).

6.5.5. Length (Behavior) mle

Measurement>Length should be coded when students are involved in activities/instruction related to length including; measuring length (nonstandard units), comparing nonstandard units, estimating length (nonstandard units), creating a measuring tool, measuring with one-inch color tiles, measuring to nearest standard unit (i.e., inches, feet, centimeters, etc.), measuring using feet and inches, measuring and drawing line segments to the nearest inch/half-inch, comparing/ordering objects by length/height/width, selecting an appropriate tool for measuring length, estimating/measuring distances using feet, identifying metric units of length, using a ruler to draw a line segment, measuring/drawing line segments to the nearest centimeter, and/or multiple components (length). For example, during a center activity student "walk the room" using a ruler to measure various objects (i.e., a book, desk, pencil, and/or table) in customary/standard units (i.e., feet and inches). Also, for a worksheet activity student uses the ruler to draw line segments to specified lengths (i.e., ½ inch, one inch, 50 centimeters, etc.). In another example, students use non-standard units (i.e., an unsharpened pencil) to measure the length of various objects and once complete order the objects by length (i.e., shortest to longest).

6.5.6. Circumference, Perimeter & Area (Behavior) mpa

Measurement>Circumference, Perimeter, and Area should be coded when students are involved in activities/instruction related to circumference/perimeter/area including; finding area using one-inch color tiles, finding the area of shapes using pattern blocks, comparing/ordering objects by size (area), finding circumference/perimeter, and/or multiple components (circumference, perimeter, and area). For example, students are using non-standard units of measure (i.e., different sized pattern blocks) to find the area of a shape (i.e., a parallelogram). To complete this activity/instruction students determine how many of each differently sized pattern block is needed to cover the shape. In another example, students are asked to complete a worksheet where they find the area of various sized squares. To do so, they are using one-inch color tiles as a means of finding the area of each square.

6.5.7. Weight (Behavior) mwe

Measurement>Weight should be coded when students are involved in activities/instruction related to weight including; identifying lighter/heavier, comparing/ordering objects by weight, estimating/weighing objects (nonstandard units), measuring weight using customary units, exploring standard units of mass, selecting the appropriate tool to measure mass, measuring weight using metric units, and/or multiple components (weight). For example, groups of students are working together to identify which in a group of objects is lightest/heaviest, as well as, estimate the weight of the objects in non-standard units (i.e., pennies). First, students use a balance to determine which object; a crayon, an eraser, or a pencil, is the lightest, heaviest, etc. Next, after ordering the objects lightest to heaviest, the students estimate the weight of each object in non-standard units (i.e., pennies). Finally, again using a balance they determine the actual weight of the items in pennies.

6.5.8. Capacity (Behavior) mcp

Measurement>Capacity should be coded when students are involved in activities/instruction related to capacity including; estimating the capacity of containers, ordering containers by capacity, identifying standard units of capacity, identifying gallon/half-gallon/quart/cup/liter containers, estimating and measuring the capacity of a container in cups, selecting the appropriate tool to measure capacity, identifying 1-cup and ½-cup measuring cups, identifying tablespoon/teaspoons/and ½ teaspoons, and/or multiple components (capacity). For example, the students are working on an exercise to estimate/order the capacity of containers. First, the teacher shows the class five containers and asks which container they believe to be the smallest/largest, etc. After ordering the containers by size; next, the teacher asks students to estimate the capacity (i.e., volume) of each container. Finally, the class measures the true capacity of each container by filling them with cups of water and learning if their estimates and the capacity order they decided upon are correct.

6.5.9. Quantity (Behavior) mqu

Measurement>Quantity should be coded when students are involved in activities/instruction related to quantity including; dividing a set of objects into groups of two (pairs), identifying pairs/most and fewest/dozen and half dozen, and/or multiple components (quantity). For example, the class reviews the amount that makes up a dozen and a half dozen. Following this discussion, the teacher asks students to separate piles of pattern blocks into groups of two and then put three pairs of two together to make a half dozen and six pairs of two together to make a dozen. Focus is placed on dividing objects into pairs and identifying the quantity of a dozen and half dozen.

6.6. Data Analysis

6.6.1. Multiple Components (Behavior) dmc

Data Analysis>Multiple Components should be coded when a variety of combined *Data Analysis* behaviors occur within at least the 15 second instruction minimum and/or for a longer duration. To be coded as Data Analysis>Multiple Components all of the activities/instruction occurring together must be part of Data Analysis. For example, the class quickly reviews a few survey questions (i.e., data collection), next the teacher constructs a bar graph using students' responses to a question (i.e., data representation), and then the students are asked to identify most/fewest on the graph (i.e., analyzing data) all within 15 seconds. *A brief description of the activity should be noted in the comment field.*

6.6.2. Data Collection (Behavior) dco

Data Analysis>Data Collection should be coded when students are involved in activities/instruction related to data collection including; choosing a survey question/choices, collecting/sorting data, vocabulary/definitions, and/or multiple components (data collection). For example, the teacher reviews vocabulary related to graphing; such as, what are graphs, pictographs, and bar graphs. Also, the teacher and students brainstorm in order to come up with survey questions for an upcoming data collection activity. In another example the students take big handfuls of blue or red pattern blocks out of a bag. The teacher asks them to sort the blocks by color; either blue or red in preparation for graphing.

6.6.3. Data Representations (Behavior) dre

Data Analysis>Data Representations should be coded when students are involved in activities/instruction related to data representations including; tallying, properties of graphs, placing an object on a graph, graphing a picture on a pictograph, drawing a pictograph/with scale of 2, using data to construct a bar-type graph, creating a bar graph/with scale of 2, graphing data in a chart, creating a Venn diagram, representing data using a graph, recording information on a graph, and/or multiple components (data representations). For example, earlier in the day the teacher asked students; "What is your favorite ice cream flavor?", and the teacher is now making *tally* marks on the board representing the students' favorite ice cream flavors. After tallying is complete the teacher then creates a pictograph to represent the responses from students, for example, ten ice cream cones in the vanilla flavor column, twelve cones in the chocolate flavor column, and three cones in the strawberry flavor column.

6.6.4. Analyzing Data (Behavior) dan

Data Analysis>Analyzing Data should be coded when students are involved in activities/instruction related to analyzing data including; identifying most/fewest on a graph, reading a pictograph/with scale of 2, reading a bar graph/with scale of 2, identifying how many more on a graph, reading a Venn diagram, writing observations about a graph, identifying the median of a set of numbers/mode and range of a set of data, using a calculator to compare data, and/or multiple components (analyzing data). For example, the class works together to analyze data from a bar graph. The teacher has created a bar graph which represents the number of AR books that former students read during the past school year. The teacher asks students to analyze the bar graph data, for example, identify the student who read the most/fewest AR books, and identify the mean/median/mode of the number of AR books read.

6.7. Probability

6.7.1. Multiple Components (Behavior) pmc

Probability>Multiple Components should be coded when a variety of combined *Probability* behaviors occur within at least the 15 second instruction minimum and/or for a longer duration. To be coded as Probability>Multiple Components all of the activities/instruction occurring together must be part of Probability. For example, students are involved in an activity/instruction that combines probability behaviors including; identifying events, describing likelihood of an event, and predicting the outcome of a probability experiment. The probability behaviors are happening rapidly and in concert, as a result, it is best to code the activity/instruction as Probability>Multiple Component. *A brief description of the activity should be noted in the comment field.*

6.7.2. Identifying Events as Certain, Likely, or Impossible (Behavior) pec

Probability>Identifying Events as Certain, Likely, Impossible should be coded when students are involved in activities/instruction related to identifying events as certain, likely, or impossible. For example, the teacher places 10 blue cubes into a bag and asks students if it is *certain* a blue cube will be chosen from the bag. Next the teacher explains it is *impossible* to pull a red cube from the bag because there are no red cubes currently in the bag. Finally, the teacher puts five yellow cubes in the bag and students decide if it is more *likely* to pull a blue or a yellow cube from the bag.

6.7.3. Describing the Likelihood of an Event (Behavior) ple

Probability>Describing Likelihood of an Event should be coded when students are involved in activities/instruction related to describing the likelihood of an event. For example, the teacher and students spend time describing the likelihood of an event. This can occur prior to an activity/instruction where they will be identifying events as certain, likely, or impossible. As a result, there may be a review of the key words used to identify events (i.e., certain, likely, impossible).

6.7.4. Predicting the Outcome of a Probability Experiment (Behavior) poc

Probability>Predicting the Outcome of a Probability Experiment should be coded when students are involved in activities/instruction related to predicting the outcome of a probability experiment. For example, before conducting a probability experiment (i.e., a coin flip) the teacher asks the students to make their predictions as to the outcome of the experiment. Also, students working in a small group are asked as part of their activity/instruction to make a prediction for the outcome of their probability experiment (i.e., spinning a spinner to either "blue" or "red").

6.7.5. Conducting a Probability Experiment (Behavior) pex

Probability>Conducting a Probability Experiment should be coded when students are involved in activities/instruction related to conducting a probability experiment. For example, the teacher conducts the experiment of flipping a coin to determine if the coin will land on heads or tails. The "coin flip" experiment is repeated several times in order to better understand the nature of probability. Also, students use a spinner to spin and determine if it lands on the blue color or red color. Again the experiment is conducted repeatedly to better understand the nature of probability.

6.10. Multi-Component General

6.10.1. Multi-component (Behavior) mul

Multi-Component General>Multi-Component should be coded when a variety of combined, yet different *math-related* behaviors occur within at least the 15 second instruction minimum and/or for a longer duration. This code is used when a variety of different behaviors (i.e., addition, geometry, measurement, data analysis, etc.) are occurring together and are not better coded using a more *specific* multi-component code (i.e., multi-component addition, multi-component measurement, etc.). Multi-Component General>Multi-Component should also be used as a *default* code when a specific math-related instruction code cannot be determined (i.e., math instruction is occurring, but it does not fit into any math-related behavior codes provided in the mathematics coding manual). *In this case* a *brief description of the activity should be noted in the comment field.*

APPENDIX C

Primary Caregiver Questionnaire

Start of Block: Child Information

Q1 What is your child's name?

____

Q2 What is your child's gender?

- ○ Male (1)
- ○ Female (2)

Q3 What is your child's race/ethnicity?

____

Q4 What is your child's native language?

- ○ English (1)
- ○ Other (specify) (2) ____

Q5 What is your child's date of birth?

____

Q6 Who is completing this questionnaire?

- ◯ Biological Mother (1)
- ◯ Biological Father (2)
- ◯ Other (specify) (3) ____

End of Block: Child Information

Start of Block: Family Information

Q8 What is your Relationship Status?

- ◯ Married (1)
- ◯ Single (2)
- ◯ Divorced (3)
- ◯ Other (Specify) (4) ____

Q9 On average, how many hours/day are you responsible for your child (i.e., in a normal 9am-5pm job, you would be responsible for your child 16 hours/day)?

▼ 1 (1) ... 24 (24)

End of Block: Family Information

Start of Block: Parent #1

Q12 Parent #1 name:

____

Q11 Relation to Child:

____

Q13 Age:

____

Q14 Native Language:

- ◯ English (1)
- ◯ Other (specify) (2) ____

Q15 Ethnicity/Race:

____

Q16 What is your occupation? (Please be as specific as possible)

____

Q17 Are you currently employed?

- ◯ Yes (1)
- ◯ No (2)

Display This Question:

If Q17 = Yes

Q18 If “Yes” do you work part-time or full-time?

- ○ Part-time (1)
- ○ Full-time (2)

Display This Question:

If Q18 = Part-time

Q19 If part-time, please specify how many hours per week:

____

Q20 From all sources of income, please tell me your total family income before taxes in 2017.

____

Q21 IF LEFT BLANK, what would be your best guess?

- $5000-$10,000 (1)
- $10,000-$15,000 (2)
- $15,000-$20,000 (3)
- $20,000-$25,000 (4)
- $25,000-$30,000 (5)
- $30,000-$35,000 (6)
- $35,000-$40,000 (7)
- $40,000-$45,000 (8)
- $45,000-$50,000 (9)
- $50,000-$55,000 (10)
- $55,000-$60,000 (11)
- $60,000-$65,000 (12)
- $65,000-$70,000 (13)
- $70,000-$75,000 (14)
- $75,000-$80,000 (15)
- $80,000-$85,000 (16)
- $85,000-$90,000 (17)
- $90,000-$95,000 (18)
- $95,000-$100,000 (19)
- $100,000+ (20)

Q22 Birthdate:

____

Q23 What is the highest educational level you have attained?

▼ Some High School (1) ... Postgraduate or Professional Degree (e.g. NA, MS, PhD, JD, MD) (7)

Q24 Field of Study/Major

____

Display This Question:

If Q23 = Postgraduate or Professional Degree (e.g. NA, MS, PhD, JD, MD)

Q25 Graduate School

▼ MA (1) ... JD (7)

Q26 Name of the last school attended and/or received a degree:

____

End of Block: Parent #1

Start of Block: Parent #2

Q59 Parent #2 name:

____

Q60 Relation to Child:

____

Q61 Age:

____

Q62 Native Language:

- ◯ English (1)
- ◯ Other (specify): (2) ____

Q63 Ethnicity/Race:

____

Q64 What is your occupation? (Please be as specific as possible)

____

Q65 Is Parent #2 currently employed?

- ◯ Yes (1)
- ◯ No (2)

Display This Question:

If Q65 = Yes

Q66 If “Yes” does he/she work part-time or full-time?

○ Part-time (1)

○ Full-time (2)

Display This Question:

If Q66 = Part-time

Q67 If part-time, please specify how many hours per week:

____

Q70 Birthdate:

____

Q71 What is the highest educational level he/she has attained?

▼ Some High School (1) ... Postgraduate or Professional Degree (e.g. NA, MS, PhD, JD, MD) (7)

Q72 Field of Study/Major

____

Q73 Graduate School

▼ MA (1) ... MBA (6)

Q74 Name of the last school attended and/or received a degree:

____

End of Block: Parent #2

Start of Block: Preschool/Child Care History

Q43 Please list all forms of childcare and/or preschool experiences your child has had since birth:

○ Type (e.g. small group home, relative, day care, preschool, etc.) (1) ____

○ Dates attended (mm/yr) (2) ____

○ Hours per week (3) ____

Q45

○ Type (e.g. small group home, relative, day care, preschool, etc.) (1) ____

○ Dates attended (mm/yr) (2) ____

○ Hours per week (3) ____

Q46

○ Type (e.g. small group home, relative, day care, preschool, etc.) (1) ____

○ Dates attended (mm/yr) (2) ____

○ Hours per week (3) ____

Q48 Please answer how often you participate in these activities with your child:

	Almost Never (1)	Every so often (2)	1 to 3 times a week (3)	4 to 6 times per week (4)	Daily (5)
How often do you do math activities (i.e., workbooks or math problems) with your child? (1)	○	○	○	○	○
How often do you play number games such as “This Old Man” or “1, 2, Buckle My Shoe” with your child? (2)	○	○	○	○	○
How often do you do math-related activities, such as “connect the number” pictures, mazes, and puzzles with your child? (3)	○	○	○	○	○

How often do you play card games with your child (i.e., war, skip-bo, Uno)? (4)					
How often do you count objects with your child? (5)					
How often do you sort things by size, color, or shape with your child? (6)					
How often do you talk about money with your child (i.e., when shopping saying "which costs more?")? (7)					
How often do you measure ingredients with your child when cooking? (8)					

How often do you use calendars with your child? (9)	○	○	○	○	○
How often does your child wear a watch? (10)	○	○	○	○	○
How often do you engage with your child in identifying written numbers? (11)	○	○	○	○	○
How often do you engage with your child in printing numbers? (12)	○	○	○	○	○
How often do you use analog clocks with your child? (13)	○	○	○	○	○

Q49 Please answer how much you identify with these statements:

	Not at all like me (1)	Slightly like me (2)	Somewhat like me (3)	A lot like me (4)	Very much like me (5)
I find math activities enjoyable. (1)	○	○	○	○	○
I believe that literacy activities are more important than numeracy activities for young children. (2)	○	○	○	○	○
I believe that it is important for caregivers to focus on math skills in young children. (3)	○	○	○	○	○
I believe it is as much my responsibility as the school's to help my child learn. (4)	○	○	○	○	○

End of Block: Math Specific Questions

Start of Block: Math Specific Questions (cont.)

Q50 How many puzzels do you have in the house?

____

Q51 How many analog clocks do you have in the house?

____

Q52 How many board games do you have in the house?

____

Q56 How many decks of cards do you have in the house?

____

End of Block: Math Specific Questions (cont.)

Start of Block: Block 7

Q74 Please rank these from 1 (being most important) to 5 (least important) what you think will have the largest impact on your child's math performance?
______ Your attitude toward math (1)
______ The amount of time spent doing math at home (2)
______ Genetics (3)
______ How early they were exposed to math concepts (4)
______ My child's teacher's attitudes towards math (5)

End of Block: Block 7

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