# A recentering approach for interpreting interaction effects from logit, probit, and other nonlinear models 

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#### Abstract

Research Summary: Strategic management has seen numerous studies analyzing interaction terms in nonlinear models since Hoetker's (Strat Mgmt J., 2007, 28(4), 331-343) best-practice recommendations and Zelner's (Strat Mgmt J., 2009, 30(12), 1335-1348) simulation-based approach. We suggest an alternative recentering approach to assess the statistical and economic importance of interaction terms in nonlinear models. Our approach does not rely on making assumptions about the values of the control variables; it takes the existing model and data as is and requires fewer computational steps. The recentering approach not only provides a consistent answer about statistical meaningfulness of the interaction term at a given point of interest, but also helps to assess the effect size using the template that we offer in this study. We demonstrate how to implement our approach and discuss the implications for strategy researchers.


Managerial Summary: In industry settings, the relationship between multiple corporate strategy-related inputs and corporate performance is often nonlinear in nature. Furthermore, such relationships tend to vary for different types of firms represented within the broader population of firms in a given industry. It is thus imperative for managers to know how to take nonlinear relationships between related business factors into account
when they make strategic decisions. We suggest a simple and easily implementable way of assessing and interpreting interactions in a nonlinear setting, which we term a recentering approach. We demonstrate how to apply our approach to a strategic management setting.

## KEYWORDS

effect size, interaction effects, nonlinear models, odds ratio, recentering

## 1 | INTRODUCTION

Interaction terms are frequently modeled in strategic management research in order to evaluate the effect of one explanatory variable on the response variable given the magnitude of another explanatory variable (e.g., the relationship between corporate strategy-related inputs and management performance outcomes varies depending on the internal and external business environments). Assessing and interpreting interaction terms becomes more complicated when models are nonlinear. Unlike linear models where the effect of a one-unit change in a covariate on the outcome variable (i.e., marginal or partial effect) is constant over the whole range of the covariate given the level of the other covariates in the model, the same effect in nonlinear models relies on the values of all other covariates in the model (Ai \& Norton, 2003; Norton, Wang, \& Ai, 2004). Given the frequency with which strategic management researchers have encountered interaction terms in nonlinear models (see, e.g., Hoetker, 2007; Shook, Ketchen, Cycyota, \& Crockett, 2003), we will argue and show by way of mathematical proof and empirical analysis that there is room for another methodological option for achieving simplicity and consistency of interpretation of those interaction terms.

In strategic management research, Hoetker (2007) recommended a set of best practices for the use of logit and probit models, including interpreting interaction terms. To further improve the assessment of statistical meaningfulness and interpretation of logit and probit results, Zelner (2009, p. 1336) suggested "a simulation-based technique developed by King, Tomz, and Wittenberg (2000)" ${ }^{1}$ and argued for the benefits of this technique over the conventional calculus-based method known as the delta method (Zelner, 2009, pp. 1341-1,342) ${ }^{2}$ proposed by Dorfman (1938). In particular, Zelner proposed (a) calculating and interpreting a difference in predicted probabilities associated with discrete changes in key predictor values (known as the cross-partial derivative or cross-difference, which measures how the marginal effect of one variable changes when the other variable in the interaction term changes) and (b) testing whether the difference in predicted probabilities is different from zero by constructing a confidence interval (CI) around the estimated quantity and finding out if the interval

[^0]contains zero. ${ }^{3}$ This simulation-based technique argued for by Zelner (2009) requires user-written Stata commands "CLARIFY" and "intgph" (Tomz, Wittenberg, \& King, 2003; Zelner, 2009).

This simulation approach, however, must by inherent definition include the researcher picking assumed values for all the control variables in the model in order to generate output about whether an interaction effect is statistically meaningful. To address this concern, we propose and recommend a recentering approach, which focuses on the main independent variable at a point of theory-motivated interest. The recentering approach does not require assumed values for any of the control variables, takes the data and model as is, is computationally simpler and is easier to implement with one simple mathematical transformation as seen below. Last but not least, our approach enables one to assess, with the help of the template we provide in this study, the effect size of the interaction term in a nonlinear model. Overall, the recentering approach we propose gives researchers an additional option to consider when assessing the interaction effect in nonlinear models.

Our recentering approach is based on a recentered regression where one or both variables involved in the interaction term are centered at a value of interest-whether it be at the sample mean, sample median, sample 75th percentile value, sample 25th percentile value, or any other theory-driven value. That is, every value of the variable being interacted in the data set is deducted by the same value of the researcher's interest. For ease of explication and comparison, we begin our discussion below by showing our simulation results from three logit model examples used in prior research. We then illustrate our recentering approach where we first show the link to generalized linear models and discuss the benefits of using the log of odds ratio in assessing and interpreting interaction effects in nonlinear models. Next, we present our mathematical proof that concisely illustrates why the recentering method provides a simple and consistent identification process. We then show how the recentering approach can help researchers assess the effect size of the interaction term in a nonlinear setting beyond its statistical meaningfulness. There, we provide a table that researchers can easily consult to evaluate the relationship between the odds ratio and Cohen's $d$ (Cohen, 1988), a widely accepted measure for assessing the effect size in the field of statistics and in the behavioral and health sciences. Lastly, we demonstrate the steps to implement the recentering approach. We conclude by discussing the benefits of using the recentering approach in comparison to the simulation-based approach.

## 2 | THE SIMULATION-BASED APPROACH

For ease of explication and comparison across studies, we utilize three well-specified logit models (Models IV A, VI and V) and the data $(N=469)$ used in Leiblein and Miller (2003) as our examples. The three logit models specified in Equation (2) below take the form of the following population logistic model of the binary outcome variable Y with the vector of independent variables $X \equiv\left(X_{1}, \ldots, X_{i}\right)$ :

$$
\begin{align*}
\operatorname{Pr}(Y=1 \mid X) & =F\left(\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{1} X_{2}+\cdots+\beta_{i} X_{i}\right) \\
& =\frac{1}{1+e^{-\left(\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{1} X_{2}+\cdots+\beta_{i} X_{i}\right)}} \tag{1}
\end{align*}
$$

[^1]where $F$ is the cumulative standard logistic distribution function, $X_{1}$ denotes a continuous variable, $X_{2}$ denotes a dummy variable, and $X_{1} X_{2}$ denotes an interaction term whose effect is the change in the predicted probability that $Y=1$ for a change in both $X_{1}$ and $X_{2} .{ }^{4}$ Given Equation (1), Leiblein and Miller's (2003) two logit models specify, for a given firm in a given year:

Vertical integration $=\operatorname{Pr}($ make $=1)$

$$
\begin{equation*}
=F\binom{\beta_{0}+\beta_{1} \text { Demand uncertainty }+\beta_{2} \text { Asset specificity }}{+\beta_{3}(\text { Demand uncertainty•Asset specificity })+\sum \beta_{i} \text { Controls }_{i}} \tag{2}
\end{equation*}
$$

where the outcome variable, vertical integration takes a value of one for a "make" decision and zero for a "buy" decision, and the explanatory variables include the two interacted variables (a continuous measure of "demand uncertainty" and a binary measure of "asset specificity" that takes a value of one when asset specificity is present and zero otherwise) and the interaction term that is the product of the two variables. A set of control variables that are theoretically believed to influence firms' vertical integration are controlled for in each of the three models. In particular, Model IV A adjusts for Fabrication Experience, Sourcing Experience, Ex ante Small Numbers, Small Numbers Squared, Firm Size, Firm Tenure, US Firm, Japanese Firm, and Other Asian Firm. Model V replaces Fabrication Experience and Sourcing Experience with Fabrication Experience Hat and Sourcing Experience Hat in Model VI A and adds Diversification Strategy and Diversification Squared to Model IV A. Model VI adds Diversification Strategy, Diversification Squared, and year fixed effects to Model IV A.

In order to assess interaction effects in nonlinear models, Zelner (2009) proposed looking at the difference in predicted probabilities associated with a discrete change in key predictor values and testing whether such difference is statistically different from zero by constructing a CI around the estimated quantity. If the CI includes zero, then it is concluded that there is no statistically meaningful interaction effect. For this hypothesis testing, Zelner (2009) proposed computing the CIs using King et al.'s (2000) simulation-based approach that implements "CLARIFY". This Stata user-written program uses Monte Carlo simulation which relies on asymptotic theory (Cameron \& Trivedi, 2005; Wooldridge, 2010). ${ }^{5}$

[^2]To illustrate this approach, Zelner (2009) generated 10 sets of simulated coefficients from Model V of Leiblein and Miller (2003) using "CLARIFY" (by default the program draws $M=1,000$ sets of simulated parameters) and showed the results with the $80 \%$ (two-tailed) CI for each simulated coefficient in the logit model to assess its statistical meaningfulness. Following this approach, we also run our three logit model examples and simulate the coefficients using "CLARIFY." Although the overall simulation process is the same, ours differs from Zelner (2009) in three ways. First, we analyze Leiblein and Miller (2003)'s three logit models (Models IV A, V, and VI) whereas Zelner (2009) did one (Model V). ${ }^{6}$ Second, in consideration of Cameron and Trivedi (2010) who imposed a caveat on running only 1,000 simulations for reported results given "considerable simulation noise, especially for estimates of test size (and power)" (Cameron \& Trivedi, 2010, p. 140), we run both 1,000 and 10,000 simulations. Third, we report the $95 \%$ (two-tailed) CIs using the percentiles of the simulated results as Zelner (2009) did (e.g., the $95 \%$ two-tailed CI for each coefficient in the case of 1,000 simulated results is bounded by the 25 th-lowest and 975 th-highest simulated values for the coefficient). ${ }^{7}$

We report the results from Leiblein and Miller (2003)'s three logit models in Tables 1-3, respectively. In Table 1 (Model IV A), note that the original coefficients of asset specificity $(-1.158)$ and firm size ( 0.214 ) variables are moderately meaningful in a statistical sense with the $p$ values equal to .053 and .056 respectively. In contrast, the simulation-based approach tells that neither is statistically meaningful, regardless of the number of simulations.

## 3 | THE RECENTERING APPROACH

The recentering approach we propose is in the branch of statistics called Generalized Linear Models (GLM). GLM, first invented in 1972 (Nelder \& Wedderburn, 1972) and widely considered to be one of the pioneering achievements in the last 50 years of the field of statistics, exists to unify linear and nonlinear models in the spirit of greater analyzability. In GLM, the nonlinear representation of the dependent variable appears on the left-hand side and the linear representation of the independent variables including any interaction term(s) appears on the right-hand side. The left-hand side can be nonlinear while the right-hand side is linear because of an invertible linearizing "link function" on the left-hand side, which transforms the expectation of the dependent variable such that it can be equal to a linear function of the independent variables. To express this point in proper notation in the classic linear model, the equation can be written in the following form:

$$
\begin{equation*}
Y=X \beta+\varepsilon \tag{3}
\end{equation*}
$$

[^3]TABLE 1 Results from Zelner (2009)'s simulation-based approach for Leiblein and Miller's (2003) Model IV A

| Independent variables | Logit Model IV A |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coefficient estimates | $\boldsymbol{p}>\|\mathbf{z}\|$ | Simulated coefficients Mean $_{1,000}$ | 95\% CI |  | Simulated coefficients Mean $_{10,000}$ | 95\% CI |  |
|  |  |  |  | Lower $_{1,000}$ | Upper $_{1,000}$ |  | Lower $_{10,000}$ | Upper $_{10,000}$ |
| Demand uncertainty | -16.795 | . 007 | -16.990 | -29.574 | -4.565 | -16.803 | -28.989 | -4.228 |
| Asset specificity | -1.158 | . 053 | -1.167 | -2.345 | 0.020 | -1.157 | -2.335 | 0.016 |
| Asset specificity $\times$ Demand uncertainty | 30.886 | . 002 | 31.427 | 10.917 | 51.871 | 30.847 | 11.408 | 49.838 |
| Firm size | 0.214 | . 056 | 0.217 | -0.013 | 0.444 | 0.214 | -0.001 | 0.430 |
| Firm tenure | 0.076 | . 000 | 0.077 | 0.038 | 0.117 | 0.076 | 0.037 | 0.114 |
| U.S. firm | -0.367 | . 447 | -0.342 | -1.282 | 0.571 | -0.368 | -1.336 | 0.583 |
| Japanese firm | -0.092 | . 884 | -0.090 | -1.415 | 1.173 | -0.090 | -1.324 | 1.151 |
| Other Asian firm | 0.219 | . 760 | 0.213 | -1.188 | 1.515 | 0.214 | -1.191 | 1.648 |
| Ex Ante small numbers | -0.342 | . 009 | -0.349 | -0.590 | -0.104 | -0.343 | -0.601 | -0.089 |
| Small numbers squared | 0.012 | . 002 | 0.012 | 0.005 | 0.019 | 0.012 | 0.004 | 0.019 |
| Fabrication experience | 0.187 | . 000 | 0.184 | 0.113 | 0.260 | 0.187 | 0.113 | 0.264 |
| Sourcing experience | -0.269 | . 000 | -0.271 | -0.386 | -0.150 | -0.269 | -0.381 | -0.155 |
| Intercept | 0.924 | . 483 | 0.944 | -1.502 | 3.375 | 0.932 | -1.652 | 3.539 |
| DD $\hat{\pi}_{.194, ~ .015 ~}^{\text {en }}$ | 1.003 |  | 0.809 | 0.202 | 1.222 | 0.800 | 0.190 | 1.220 |

[^4]TABLE 2 Results from Zelner (2009)'s simulation-based approach for Leiblein and Miller's (2003) Model V

| Independent variables | Logit Model V |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coefficient estimates | $p>\|z\|$ | Simulated coefficients Mean $_{1,000}$ | 95\% CI |  | Simulated coefficients Mean $_{10,000}$ | 95\% CI |  |
|  |  |  |  | Lower $_{1,000}$ | Upper $_{1,000}$ |  | Lower ${ }_{\mathbf{1 0 , 0 0 0}}$ | Upper $_{10,000}$ |
| Demand uncertainty | -21.940 | . 001 | -21.808 | -35.571 | -8.153 | -21.954 | -34.842 | -9.009 |
| Asset specificity | -1.662 | . 012 | -1.628 | -2.969 | -0.356 | -1.667 | -2.963 | -0.370 |
| Asset specificity $\times$ Demand uncertainty | 34.931 | . 001 | 34.831 | 13.647 | 55.916 | 34.978 | 13.063 | 56.609 |
| Firm size | 0.390 | . 007 | 0.387 | 0.097 | 0.684 | 0.391 | 0.110 | 0.668 |
| Firm tenure | 0.069 | . 001 | 0.069 | 0.029 | 0.108 | 0.069 | 0.029 | 0.110 |
| U.S. firm | -0.366 | . 462 | -0.382 | -1.344 | 0.558 | -0.371 | -1.360 | 0.598 |
| Japanese firm | 0.148 | . 818 | 0.134 | -1.069 | 1.300 | 0.146 | -1.095 | 1.413 |
| Other Asian firm | -0.582 | . 470 | -0.601 | -2.255 | 0.998 | -0.583 | -2.154 | 1.000 |
| Ex ante small numbers | -0.374 | . 028 | -0.378 | -0.709 | -0.053 | -0.374 | -0.703 | -0.043 |
| Small numbers squared | 0.011 | . 026 | 0.011 | 0.001 | 0.021 | 0.011 | 0.001 | 0.020 |
| Fabrication experience hat | 0.305 | . 000 | 0.301 | 0.212 | 0.394 | 0.306 | 0.210 | 0.404 |
| Sourcing experience hat | -0.611 | . 000 | -0.605 | -0.927 | -0.278 | -0.609 | -0.921 | -0.300 |
| Diversification strategy | 1.885 | . 000 | 1.886 | 0.863 | 2.969 | 1.880 | 0.847 | 2.894 |
| Diversification squared | -0.312 | . 000 | -0.310 | -0.463 | -0.166 | -0.311 | -0.454 | -0.163 |
| Intercept | -0.217 | . 893 | -0.188 | -3.355 | 2.869 | -0.202 | -3.314 | 2.916 |
| DD $\hat{\pi}_{\text {.194, }} .015$ | 0.854 |  | 0.713 | 0.040 | 1.326 | 0.717 | 0.039 | 1.330 |

[^5]TABLE 3 Results from Zelner (2009)'s simulation-based approach for Leiblein and Miller's (2003) Model VI

| Independent variables | Logit Model VI |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coefficient estimates | $\boldsymbol{p}>\|\mathrm{z}\|$ | Simulated coefficients Mean $_{1,000}$ | 95\% CI |  | Simulated coefficients Mean $_{10,000}$ | 95\% CI |  |
|  |  |  |  | Lower $_{1,000}$ | Upper $_{1,000}$ |  | Lower $_{10,000}$ | Upper $_{10,000}$ |
| Demand uncertainty | -19.869 | . 011 | -20.097 | -35.408 | -4.941 | -19.888 | -35.132 | -4.314 |
| Asset specificity | -1.198 | . 116 | -1.206 | -2.652 | 0.340 | -1.196 | -2.699 | 0.295 |
| Asset specificity $\times$ Demand uncertainty | 35.693 | . 006 | 36.304 | 9.778 | 63.901 | 35.660 | 9.925 | 61.202 |
| Firm size | 0.174 | . 300 | 0.177 | -0.167 | 0.519 | 0.174 | -0.149 | 0.498 |
| Firm tenure | 0.034 | . 209 | 0.035 | -0.019 | 0.090 | 0.034 | -0.019 | 0.086 |
| U.S. firm | -1.533 | . 020 | -1.505 | -2.796 | -0.266 | -1.533 | -2.857 | -0.224 |
| Japanese firm | 0.440 | . 592 | 0.446 | -1.227 | 2.035 | 0.442 | -1.153 | 2.050 |
| Other Asian firm | 0.295 | . 750 | 0.276 | -1.508 | 2.015 | 0.288 | -1.520 | 2.114 |
| Ex ante small numbers | -0.511 | . 003 | -0.517 | -0.831 | -0.194 | -0.512 | -0.848 | -0.187 |
| Small numbers squared | 0.014 | . 004 | 0.014 | 0.005 | 0.023 | 0.014 | 0.005 | 0.024 |
| Fabrication experience | 0.184 | . 000 | 0.179 | 0.082 | 0.273 | 0.184 | 0.089 | 0.283 |
| Sourcing experience | -0.223 | . 003 | -0.226 | -0.370 | -0.078 | -0.223 | -0.370 | -0.077 |
| Diversification strategy | 2.886 | . 000 | 2.891 | 1.560 | 4.143 | 2.877 | 1.612 | 4.132 |
| Diversification squared | -0.406 | . 000 | -0.406 | -0.582 | -0.219 | -0.405 | -0.584 | -0.227 |
| Intercept | 3.847 | . 052 | 3.845 | 0.155 | 7.550 | 3.862 | -0.030 | 7.783 |
| DD $\hat{\pi}_{.194, .015}$ | 0.524 |  | 0.510 | 0.010 | 1.127 | 0.496 | 0.010 | 1.121 |

Note: This model includes year fixed effects (1988-1996), but we do not report the results for brevity. CI stands for confidence interval. Subscripts 1,000 and 10,000 denote the number of simulation sets using CLARIFY (Tomz et al., 2003). The last row of this table ( $\Delta \Delta \hat{\pi} .194, .015$ ) shows the original and simulated double differences (DD) in predicted probabilities ( $\Delta \hat{\pi} .{ }_{.194}-\Delta \hat{\pi} .015$ ) associated with an increase in Asset specificity from zero to one when Demand uncertainty is set to a value of 0.194 ( $\Delta \hat{\pi} .194$ ) and set to a value of $0.015\left(\Delta \hat{\pi}_{.015}\right)$ and all other variables in the model set to a value of zero, respectively.
where $Y$ is a response variable, $X$ is a set of explanatory variables, $\beta$ is a set of estimated coefficients, and $\varepsilon$ is a column vector of disturbances. The linear model follows a set of Gaussian assumptions, including but not limited to the facts that the relationship between each explanatory variable and the response variable is approximately linear, and that the residuals are independent and identically distributed (i.i.d.) normal with mean zero and constant variance. These last two restrictions are eliminated in a GLM, which in turn provides a way to learn the effect of the explanatory variables that closely resembles the process of analyzing independent variables in the classic linear model.

The key to a GLM is the specification of a so-called link function, which links the systematic component of the linear model $X \beta$ with a wider class of nonlinear representations of the response variable. The link function " $g$ " can be written in the following form:

$$
\begin{equation*}
E(Y)=\mu=g^{-1}(X \beta) \tag{4}
\end{equation*}
$$

where $E(Y)$ is the expected value of the response variable $Y, \mu$ is the mean of $Y, X \beta$ is the linear predictor, a linear combination of the unknown parameters $\beta$, and $g$ is the link function. The link function can be a logit, probit, poisson, negative binomial, or any other nonlinear transformation of the response variable $Y$ such that the right-hand side can be a linear representation of the independent variables. To emphasize, the recentering method that we will show below will work not just for logit, but also for poisson, negative binominal, or any other nonlinear transformation through a known link function. Going from logit link to probit link, for example, only changes the left-hand side of the equation, while recentering happens on the righthand side of the equation. So the changing of the link function will not impact the math result on the right-hand side of the equation. In the following example, we will take the logit link function:

$$
\begin{equation*}
\log \frac{p}{1-p}=\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{1} X_{2}+\beta_{4} X_{3}+\ldots \tag{5}
\end{equation*}
$$

where $p$ denotes probability and $\log (p / 1-p)$ is the logit function, which is the logarithm of the odds ratio ${ }^{8}$ of the "make-or-buy" decision in Equation (2). One of the reasons for transforming probability (ranging from 0 to 1 ) to log odds (ranging from negative infinity to positive infinity) is because it is hard to statistically model a variable which has a restricted range like probability. One way to circumvent such a restricted range issue is this transformation. Also, the log of odds is one of the easiest to understand and interpret, among all of the limitless options for transformation. ${ }^{9}$ There is a one-step conversion for that: (a given odds ratio)/(1+ that same given odds ratio) $=$ probability. Also when one is looking at the incremental effect on baseline probability, one needs to start the research project in any case (whether one is utilizing the

[^6]simulation method or the recentering approach) with knowledge of what is the baseline probability of an event occurring in one's sample.

Second, in a nonlinear world, the conversion between odds ratio and probability is monotonic but not intended to be symmetric. It is the case that at a starting odds ratio of 1:1 (equal to a starting probability of 0.5 ), the relationship between odds ratio and probability is symmetric. Starting at the original odds ratio of $1: 1$, that is the same as a probability of ( $1 / 1$ )/ $(2 / 1)=1 / 2=0.5$. When that original odds ratio of $1: 1$ goes up by 5 , that is the same as a probability of $(5 / 1) /(6 / 1)=5 / 6=0.833$. When that original odds ratio of $1: 1$ gets divided by 5 , that is the same as a probability of $(1 / 5) /(6 / 5)=1 / 6=0.167$. Thus, at a baseline odds ratio of $1: 1$, the absolute value of the positive impact of multiplying the odds ratio by 5 is the same as (thus is "symmetric to") the absolute value of the negative impact of dividing the odds ratio by 5 .

It is also true that the same coefficient expressed in log odds can have a smaller nominal effect in changing the probability for a group with small baseline odds than a group with larger baseline odds (Hoetker, 2007, p. 334). It is thus important for the researcher to explicitly depict whether and how a given change in log odds ratio means a different change in probability for various groups in the population.

Having the regression run efficiently in GLM and getting the log odds ratio out of it means that one does not have to test out all combinations of the control variables in a simulation. Let's say that one has 10 control variables, each of which takes on 10 different possible values in one's data set. At most, one runs 10 different recentered regressions. The reason for why the recentering method is efficient and provides consistent results in nonlinear models is because, through a simple mathematical transformation that we will see next, we are able to subtract out the effects of the control variables. We are thus able to arrive at the answer that is unbiased and consistent regardless of the number of and all combinations of the control variables in the data set.

In summary, the log of odds ratio is particularly helpful for studying interaction effects in nonlinear models for several reasons. First, the $\log$ of odds ratio lends itself to broadly applicable statistical analysis (because it can be used through the recentering method to tell us the statistical and economic meaningfulness of an interaction term that is true and consistent no matter what the values of the control variables are). Second, the log of odds ratio, while not previously held to be intuitive, can be readily converted into a probability that is more easily understood using any odds-probability online converter tool or a simple calculation. ${ }^{10}$ Third, the $\log$ of odds ratio provides a clear benchmark for assessing the economic meaningfulness/ effect size of an interaction term.

## 3.1 | Why the recentering method offers a useful option for interpreting interaction effects

Through simple mathematical steps, we next illustrate why the recentering method provides a simple and consistent identification process. Recalling Equation (5), consider we seek to

[^7]determine the effect of $X_{2}$ given $X_{1}$ being equal to a defined point, a. $X_{2}$ can be either a continuous or a dummy variable. From Equation (5), note what happens when $X_{2}$ shifts from the value b to the value $(\mathrm{b}+1)$. The following is the effect of $X_{2}$ given $X_{1}=a$ as $X_{2}$ shifts from b to $(\mathrm{b}+1)$ :
\[

$$
\begin{gather*}
X_{2}=b \rightarrow \log \left(\frac{p_{1}}{1-p_{1}}\right)=\beta_{0}+\beta_{1} a+\beta_{2} b+\beta_{3} a b+\beta_{4} X_{3}+\ldots  \tag{6}\\
X_{2}=b+1 \rightarrow \log \left(\frac{p_{2}}{1-p_{2}}\right)=\beta_{0}+\beta_{1} a+\beta_{2}(b+1)+\beta_{3} a(b+1)+\beta_{4} X_{3}+\ldots \tag{7}
\end{gather*}
$$
\]

To find the effect of $X_{2}$ as it goes from $b$ to $(b+1)$, one can examine the $\log$ of odds ratio by subtracting Equation (6) from Equation (7). The outcome will then be:

$$
\begin{equation*}
\log \left(\frac{p_{2}}{1-p_{2}}\right)-\log \left(\frac{p_{1}}{1-p_{1}}\right)=\log \left(\frac{p_{2} / 1-p_{2}}{p_{1} / 1-p_{1}}\right)=\beta_{2}+\beta_{3} a \tag{8}
\end{equation*}
$$

Note that as a result of this subtraction, all the terms for the control variables are removed. In contrast, the methods for examining interaction terms in strategic management (e.g., Wiersema \& Bowen, 2009) are focused on the direct change in probabilities, where from the odds $(p / 1-p)$ one can derive the probability p as shown below:

$$
\begin{align*}
& \left(\frac{p}{1-p}\right)=e^{\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{1} X_{2}+\beta_{4} X_{3}+\ldots}  \tag{9}\\
& p=\frac{e^{\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{1} X_{2}+\beta_{4} X_{3}+\ldots}}{1+e^{\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{1} X_{2}+\beta_{4} X_{3}+\ldots}} \tag{10}
\end{align*}
$$

Examining the effect of $X_{2}$ given $X_{1}=\mathrm{a}$ in this approach entails a relatively more complicated math problem in which the control variables do not disappear:

$$
\begin{gather*}
X_{2}=b \rightarrow P_{1}=\frac{e^{\beta_{0}+\beta_{1} a+\beta_{2} b+\beta_{3} a b+\beta_{4} X_{3}}}{1+e^{\beta_{0}+\beta_{1} a+\beta_{2} b+\beta_{3} a b+\beta_{4} X_{3}}}  \tag{11}\\
X_{2}=b+1 \rightarrow P_{2}=\frac{e^{\beta_{0}+\beta_{1} a+\beta_{2}(b+1)+\beta_{3} a(b+1)+\beta_{4} X_{3}}}{1+e^{\beta_{0}+\beta_{1} a+\beta_{2}(b+1)+\beta_{3} a(b+1)+\beta_{4} X_{3}}} \tag{12}
\end{gather*}
$$

In other words, to examine the direct change in probability as the effect of the change in $X_{2}$ from $b$ to $(b+1)$, one can attempt to subtract Equation (11) from Equation (12), but the other control variables will not be removed in this case. A notable takeaway from this demonstration is that one can never assess the effect of a change in probability when looking at the world this way unless one plugs in assumed values for each and every control variable. In contrast, the recentering method enables one to subtract away all control variables through a simple mathematical transformation, and as a result of that simple mathematical transformation, easily and consistently assess the statistical and economic meaningfulness of the interaction term at hand in nonlinear models.

To achieve that goal, we present below what is learned when operating in the GLM world using the link function in which the control variables are subtracted away, and
where one gets the consistent answer no matter what the values of the control variables are. Using the example of Leiblein and Miller (2003), we start with the logit link function on the left-hand side and the matching linear representation of the independent variables on the right-hand side. Here we know in advance that in a GLM framework, all of the control variables will be subtracted away; thus, we can focus fully on the main variables of interest: Demand Uncertainty ( $D U$ ), which is a continuous variable and Asset Specificity $(A S)$, which in the real world is ultimately a continuous variable but in Leiblein and Miller (2003) is measured as a binary dummy variable. Recalling Equations (2) and (5), let $D U, A S$, and ( $D U \cdot A S$ ) in Equation (2) be $X_{1}, X_{2}$, and $X_{1} X_{2}$ in Equation (5) respectively. We will then get:

$$
\begin{equation*}
\log \left(\frac{p}{1-p}\right)=\beta_{0}+\beta_{1} D U+\beta_{2} A S+\beta_{3}(D U \cdot A S)+\ldots \tag{13}
\end{equation*}
$$

In Equation (13), we know in advance, as shown in Equation (8), the effect of $A S$ on the make-or-buy decision (the response variable) given $D U=$ a as the value of $A S$ goes from 0 to 1 is $\beta_{2}+\beta_{3}(a)$. Thus, one can see that the effect of $A S$ critically depends on the value of $D U$ set at a. Therefore, the effect of $A S$ on the dependent variable in this interaction context can only be told by the value for $\beta_{2}$ alone if $D U=0$. Our interest here is in how to easily and consistently assess the effect of $A S$ on the dependent variable given a specific value of $D U$ in this nonlinear context. Based on theory from strategic management research, one does typically have an interest in learning about the effect of the interaction for a chosen region of $D U$. The efficient way to identify this interaction effect with consistency is to perform a simple mathematical transformation that makes $D U$ become zero so that the $D U$ term is canceled out by algebra. How does this work? In Equation (13), we simultaneously add and subtract the value of $D U$ that would make $D U=0$ at the $D U$ point of interest in the $D U$ data distribution, which we call $\overline{D U}$ below:

$$
\begin{equation*}
\log \left(\frac{p}{1-p}\right)=\beta_{0}+\beta_{1}(D U-\overline{D U}+\overline{D U})+\beta_{2} A S+\beta_{3}[(D U-\overline{D U}+\overline{D U}) \cdot A S]+\ldots \tag{14}
\end{equation*}
$$

By reorganizing the terms on the right-hand side in Equation (14), we get:

$$
\begin{equation*}
=\beta_{0}+\beta_{1} \overline{D U}+\beta_{1}(D U-\overline{D U})+\beta_{2} A S+\beta_{3} \overline{D U} \cdot A S+\beta_{3}(D U-\overline{D U}) \cdot A S+\ldots \tag{15}
\end{equation*}
$$

which is the same as:

$$
\begin{equation*}
=\left(\beta_{0}+\beta_{1} \overline{D U}\right)+\beta_{1}(D U-\overline{D U})+\left(\beta_{2}+\beta_{3} \overline{D U}\right) \cdot A S+\beta_{3}(D U-\overline{D U}) \cdot A S+\ldots \tag{16}
\end{equation*}
$$

The power of the above mathematical transformation is that all of the positive $\overline{D U}$ terms get subtracted out and we are left with only the negative $\overline{D U}$ terms. For the sake of simplicity, we then group together terms in Equation (16) using the alternative notation of $\delta_{i}$ :

$$
\begin{equation*}
=\delta_{0}+\delta_{1}(D U-\overline{D U})+\delta_{2} A S+\delta_{3}(D U-\overline{D U}) \cdot A S+\ldots \tag{17}
\end{equation*}
$$

where $\delta_{0}=\beta_{0}+\beta_{1} \overline{D U}, \delta_{1}=\beta_{1}, \delta_{2}=\beta_{2}+\beta_{3} \overline{D U}$, and $\delta_{3}=\beta_{3}$.

The key insight here is that, when we are interested in the effect of $A S$ on the dependent variable at any particular part of the actual $D U$ data distribution, all we need to do is to subtract from every value of $D U$ the data point of interest $(\overline{D U})$. As a result, $\delta_{1}$ and $\delta_{3}$ in Equation (17) disappear because ( $D U-\overline{D U}$ ) becomes zero. The estimated coefficient $\left(\delta_{2}\right)$ of $A S$ in terms of the log of odds ratio then tells us whether the interaction term ( $D U \cdot A S$ ) is statistically meaningful, given the $p$ value associated with that coefficient, at the $D U$ point of interest $(\overline{D U})$ when the value of $A S$ moves from one value to another (here, zero to one in our example). In essence, after recentering, the coefficient ( $\delta_{2}$ ) represents the log-odds ratio effect of a one-unit change in $A S$ holding $D U$ constant at the point of interest and holding all other control variables constant. Also note that in contrast to the simulation approach, this recentering approach also helps us assess the effect size or the economic meaningfulness of the interaction term $(D U \cdot A S)$ at the $D U$ point of interest $(\overline{D U})$ with the template that we will discuss and provide below.

Up to now we used the coefficient of $A S$ to learn the change in the log of odds ratio at a specific $D U$ point of interest. Researchers, however, can also learn the covariate-invariant effect of the other main variable among the two interacted variables in the same regression. In our example above, this means that if we instead wanted to know the effect of $D U$ at the $A S$ point of interest (e.g., 1), we just recenter the $A S$ variable at 1 by subtracting 1 from all values of $A S$ ( 0,1 in our prior example) which makes the $A S$ at that point of interest equal to zero. Because of this subtracting process of an interacted variable, which essentially makes that variable to be equal to zero at a specific value of that variable, we call this technique a recentered regression. This recentered regression makes it possible to assess the statistical and economic meaningfulness of the interaction effect in a nonlinear setting, that is consistent regardless of the values of all other control variables.

It is noteworthy that the recentering approach can also help when there are two sets of interaction terms in one nonlinear model. For example, consider four main variables, $A, B, C$, and $D$ and two interaction terms $(A \cdot B)$ and $(C \cdot D)$ in the same nonlinear model. In this case, the recentering approach can diagnose more than just one interaction term at a time or both at the same time (like if one were to recenter " $B$ " at the $B$ point of interest and " $D$ " at the $D$ point of interest simultaneously). In the latter case, the recentering approach can help identify the interaction term $(A \cdot B)$ at the $B$ point of interest $(\bar{B})$ and $D$ point of interest $(\bar{D})$ simultaneously. To understand how the recentering approach works here, suppose a nonlinear model specified as $\log (\mathrm{p} /[1-\mathrm{p}])=\alpha_{0}+\alpha_{1} A+\alpha_{2} B+\alpha_{3}(A \cdot B)$ $+\alpha_{4} C+\alpha_{5} D+\alpha_{6}(C \cdot D)+\alpha_{i} \Sigma$ (Other Covariates ${ }_{i}$ ). Then via centering $B$ and $D$ at $\bar{B}$ and $\bar{D}$ respectively, the coefficient of $A\left(\alpha_{1}\right)$ would tell us the effect of $A$ when $B$ is at $\bar{B}$, no matter what $C, D$ or the other control variable values are (i.e., "covariate-invariant"). Likewise, the coefficient of $C\left(\alpha_{4}\right)$ would tell us the covariate-invariant effect of $C$ when $D$ is at $\bar{D}$. If the nonlinear model were specified as $\log (p /[1-p])=\pi_{0}+\pi_{1} A+\pi_{2} B+\pi_{3} C+\pi_{4}(A \cdot B)+\pi_{5}(A \cdot C)+$ $\pi_{k} \Sigma$ (Other Covariates ${ }_{k}$ ), then via centering $B$ and $C$ at $\bar{B}$ and $\bar{C}$ respectively, the coefficient of $A\left(\pi_{1}\right)$ would tell us the effect of $A$ when $B$ is at $\bar{B}$ and $C$ is at $\bar{C}$, no matter what the other covariate values are. Similarly via centering $A$ at $\bar{A}$, the coefficient of $B\left(\pi_{2}\right)$ would tell us the covariate-invariant effect of $B$ when $A$ is at $\bar{A}$ and the coefficient of $C\left(\pi_{3}\right)$ would tell us the covariate-invariant effect of $C$ when $A$ is at $\bar{A}$. Here all the "effect" discussed above refers to the $\log$ of odds ratio.

## 3.2 | The recentering approach helps assess the effect size in a nonlinear model.

The recentering approach also helps researchers assess the effect size of the interaction term in a nonlinear model beyond its statistical meaningfulness. From prior literature in epidemiology (Chen et al., 2010), there is a precisely defined template for interpreting the size of the effect expressed in terms of the log of odds ratio, which we introduce in Table 4. Specifically, the table evaluates the effect size in direct mathematical comparison to Cohen's $d^{11}$ (Cohen, 1988), a widely accepted measure for assessing effect size in the field of statistics and in the behavioral and health sciences. Recall that the coefficient of the explanatory variable in a logistic regression corresponds to the log of odds ratio of the outcome variable per unit increase in the explanatory variable. The first two columns of Table 4 show odds ratio and $\log$ of odds ratio, respectively. The rest of the columns show the equivalent Cohen's $d$ given $P_{0}$ in the second row, which is the baseline rate of outcome of interest in the group of subjects. In our example above, it is the rate of vertical integration in the group of firms where asset specificity $=0$, holding demand uncertainty and all other control variables constant. Because $P_{0}$ could vary for different values of demand uncertainty, it makes sense to interpret the effect size, or lack thereof, based on the answer being true regardless of the exact value of $P_{0}$ (in epidemiology it is the rate of contracting a disease of interest in the non-exposed group, i.e., the group not exposed to a particular harm, for example, and is estimated from the general population, and in our present empirical context, it would be the rate at which firms engage in vertical integration). Our main interest in Table 4 is whether Cohen's $d$ equivalent to log odds ratio is clearly indicating an economically large or economically small effect. Values in bold with shading in Table 4 indicate Cohen's $d<0.20$ or $>0.80$. Values of Cohen's $d$ less than 0.20 suggest that the effect sizes are small for all plausible values of $P_{0}$, and values of Cohen's $d$ greater than 0.80 suggest that the effect sizes are large for all plausible values of $P_{0}$ (Cohen, 1988).

As shown in Table 4, the log of odds ratio $<0.26$ always corresponds to Cohen's $d<0.20$ and the $\log$ of odds ratio $>1.95$ always corresponds to Cohen's $d>0.80$. Cohen's $d<0.20$ reflects the fact that the effect size of the interaction effect is small. Cohen's $d>0.80$ reflects the fact that the effect size of the interaction effect is large. Thus, if we see a log of odds ratio less than or equal to 0.26 , we know that the size of the interaction effect is economically small no matter what the $P_{0}$ is. Similarly, if we see a log of odds ratio greater than or equal to 1.95 , we know that the size of the interaction effect is large no matter what the $P_{0}$ is. Therefore, when the log of odds ratio identified from the recentering method is less than or equal to 0.26 , we can say that the size of the interaction effect is small (clearly mapping onto Cohen's $d<0.20$ ) regardless of the values of $P_{0}$. Similarly, when the log of odds ratio identified from the recentering method is greater than or equal to 1.95 , we can say that the size of the interaction effect is large (clearly mapping onto Cohen's $d>0.80$ ) no matter what the $P_{0}$ is. Of course, one might know from prior studies that $P_{0}$ is likely smaller than say 0.20 , for example. In our present context, $P_{0}$ might be such that the rate of vertical integration with asset specificity equal to zero and all other control variables held constant is 0.05 . This is the same thing as saying that vertical integration is occurring $5 \%$ of the time in the general population

[^8]TABLE 4 Odds ratio (OR) and the equivalent Cohen's $d$ (Chen et al., 2010)

| Odds ratio | Log of odds ratio | $P_{0}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
| 1.1 | 0.10 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 |
| 1.2 | 0.18 | 0.07 | 0.07 | 0.08 | 0.08 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 | 0.09 |
| 1.3 | 0.26 | 0.10 | 0.11 | 0.11 | 0.12 | 0.12 | 0.13 | 0.13 | 0.13 | 0.13 | 0.14 | 0.15 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.15 | 0.13 |
| 1.4 | 0.34 | 0.13 | 0.14 | 0.15 | 0.15 | 0.16 | 0.16 | 0.17 | 0.17 | 0.17 | 0.18 | 0.20 | 0.21 | 0.21 | 0.21 | 0.21 | 0.20 | 0.19 | 0.17 |
| 1.5 | 0.41 | 0.15 | 0.17 | 0.18 | 0.19 | 0.19 | 0.20 | 0.20 | 0.21 | 0.21 | 0.21 | 0.24 | 0.25 | 0.25 | 0.25 | 0.25 | 0.24 | 0.23 | 0.20 |
| 1.6 | 0.47 | 0.18 | 0.20 | 0.21 | 0.22 | 0.22 | 0.23 | 0.24 | 0.24 | 0.25 | 0.25 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.28 | 0.26 | 0.23 |
| 1.7 | 0.53 | 0.20 | 0.22 | 0.24 | 0.25 | 0.25 | 0.26 | 0.27 | 0.27 | 0.28 | 0.28 | 0.31 | 0.33 | 0.33 | 0.33 | 0.32 | 0.31 | 0.29 | 0.26 |
| 1.8 | 0.59 | 0.23 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.30 | 0.31 | 0.31 | 0.35 | 0.36 | 0.37 | 0.37 | 0.36 | 0.35 | 0.32 | 0.29 |
| 1.9 | 0.64 | 0.25 | 0.27 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.33 | 0.34 | 0.34 | 0.38 | 0.40 | 0.40 | 0.40 | 0.39 | 0.38 | 0.35 | 0.31 |
| 2 | 0.69 | 0.27 | 0.29 | 0.31 | 0.32 | 0.34 | 0.35 | 0.35 | 0.36 | 0.37 | 0.37 | 0.41 | 0.43 | 0.43 | 0.43 | 0.42 | 0.40 | 0.38 | 0.34 |
| 3 | 1.10 | 0.44 | 0.48 | 0.51 | 0.53 | 0.55 | 0.56 | 0.58 | 0.59 | 0.60 | 0.61 | 0.66 | 0.68 | 0.68 | 0.67 | 0.66 | 0.63 | 0.58 | 0.52 |
| 4 | 1.39 | 0.56 | 0.62 | 0.65 | 0.68 | 0.71 | 0.73 | 0.74 | 0.76 | 0.77 | 0.78 | 0.84 | 0.86 | 0.86 | 0.84 | 0.81 | 0.78 | 0.72 | 0.64 |
| 5 | 1.61 | 0.66 | 0.73 | 0.77 | 0.81 | 0.83 | 0.85 | 0.87 | 0.89 | 0.90 | 0.92 | 0.98 | 1.00 | 0.99 | 0.97 | 0.93 | 0.89 | 0.83 | 0.74 |
| 6 | 1.79 | 0.75 | 0.82 | 0.87 | 0.91 | 0.94 | 0.96 | 0.98 | 1.00 | 1.02 | 1.03 | 1.09 | 1.11 | 1.09 | 1.07 | 1.03 | 0.98 | 0.91 | 0.81 |
| 7 | 1.95 | 0.82 | 0.90 | 0.96 | 1.00 | 1.03 | 1.06 | 1.08 | 1.10 | 1.11 | 1.12 | 1.19 | 1.20 | 1.18 | 1.15 | 1.11 | 1.05 | 0.98 | 0.87 |
| 8 | 2.08 | 0.89 | 0.98 | 1.03 | 1.08 | 1.11 | 1.14 | 1.16 | 1.18 | 1.19 | 1.21 | 1.27 | 1.28 | 1.26 | 1.22 | 1.17 | 1.11 | 1.03 | 0.92 |
| 9 | 2.20 | 0.94 | 1.04 | 1.10 | 1.15 | 1.18 | 1.21 | 1.23 | 1.25 | 1.27 | 1.28 | 1.34 | 1.35 | 1.32 | 1.28 | 1.23 | 1.17 | 1.08 | 0.97 |
| 10 | 2.30 | 1.00 | 1.10 | 1.16 | 1.21 | 1.25 | 1.27 | 1.30 | 1.32 | 1.33 | 1.35 | 1.41 | 1.41 | 1.38 | 1.34 | 1.28 | 1.21 | 1.13 | 1.01 |
| 15 | 2.71 | 1.21 | 1.33 | 1.40 | 1.46 | 1.50 | 1.53 | 1.55 | 1.57 | 1.59 | 1.60 | 1.65 | 1.63 | 1.59 | 1.53 | 1.47 | 1.39 | 1.29 | 1.16 |
| 20 | 3.00 | 1.36 | 1.50 | 1.58 | 1.64 | 1.68 | 1.71 | 1.73 | 1.75 | 1.76 | 1.78 | 1.81 | 1.78 | 1.73 | 1.67 | 1.60 | 1.51 | 1.40 | 1.26 |
| 25 | 3.22 | 1.49 | 1.64 | 1.72 | 1.78 | 1.82 | 1.85 | 1.87 | 1.89 | 1.90 | 1.91 | 1.93 | 1.89 | 1.84 | 1.77 | 1.69 | 1.60 | 1.49 | 1.34 |
| 30 | 3.40 | 1.60 | 1.75 | 1.83 | 1.89 | 1.93 | 1.96 | 1.98 | 2.00 | 2.01 | 2.02 | 2.03 | 1.98 | 1.92 | 1.85 | 1.77 | 1.67 | 1.56 | 1.40 |

Note: $P_{0}$ : rate of outcome of interest in the group with no event. In the case of our example, it is the rate of vertical integration in the group of firms where asset specificity $=0$ holding demand uncertainty and all the other covariates constant. $\mathrm{OR}=P\left(1-P_{0}\right) / P_{0}(1-P)$ where $P$ is the rate of outcome of interest in the group with an event. The rows are the values of odds ratio (and equivalent log of odds ratio for each odds ratio). For each row, one sees the equivalent values of Cohen's $d$ for a given log of odds ratio and given $P_{0}$. Values in bold indicate Cohen's $d<0.20$ or $>0.80$, where Cohen's $d<0.20$ means that the economic significance of the log of odds ratio result would be considered economically small, and where Cohen's $d>0.80$ means that the economic significance of the log of odds ratio would be considered economically large. Anything in between 0.20 and 0.80 should be regarded as a continuous line in which those values of Cohen's $d$ closest to 0.20 are quite small, those in the middle are of approximately medium economic significance, and those closest to 0.80 are close to have large economic significance. Cohen's $d=\mathrm{Z}_{0}-\mathrm{Z}$ (standardized mean difference) where $\mathrm{Z}_{0}$ is the standard normal deviation for $P_{0}$ and Z is the standard normal deviation for $P$.
of interest where asset specificity equals zero, holding all other covariates constant. Under that case ( $P_{0}=0.05$ ), the $\log$ of odds ratio less than 0.41 (equivalent Cohen's $d=0.19$ ) would be considered economically small, and the log of odds ratio greater than 1.61 (equivalent Cohen's $d=0.83$ ) would be considered economically large, according to Table 4. It is also straightforward to convert the predicted $\log$ of odds ratio to a predicted probability. That is because there is a $1: 1$ monotonic relationship between log of odds ratio and probability.

Specific to our vertical integration example, the coefficient of demand uncertainty corresponds to the log of odds ratio for a "make" decision when asset specificity is fixed at a specific value ( 0 or 1 ). Given uncertainty over $P_{0}$ in prior make-or-buy literature, a coefficient of demand uncertainty less than 0.26 , according to Table 4 , indicates that the effect size of demand uncertainty on vertical integration is clearly small, given fixed asset specificity equal to 1 no matter what the $P_{0}$ is. Similarly, a coefficient of demand uncertainty greater than 1.95, according to Table 4, indicates that the effect size of demand uncertainty on vertical integration is clearly large, given fixed asset specificity equal to 1 no matter what the $P_{0}$ is.

## 3.3 | An example of the recentering approach

For illustrative purposes, we rewrite Equation (2) in the following basic form which represents Leiblein and Miller's (2003) original logit models that include the interaction term of ( $D U \cdot A S$ ) and recenter the $D U$ variable at 0.015 as an example where one seeks to assess the statistical and economic meaningfulness of the interaction term:

$$
\begin{align*}
& \log \left(\frac{p}{1-p}\right) \text { where } p=\operatorname{Pr}(\text { Vertical integration })  \tag{18}\\
& =\delta_{0}+\delta_{1}(D U-0.015)+\delta_{2} A S+\delta_{3}[(D U-0.015) \cdot A S]+\sum \delta_{i} \text { Controls }_{i}+\varepsilon
\end{align*}
$$

To implement this recentered regression, one can follow the steps below:
1 Before running any of the logit models that include the interaction term ( $D U \cdot A S$ ), set the entire range of sample values of $D U$ recentered at the specific point of $D U$ of interest (here 0.015 ) by subtracting the value of interest ( 0.015 ) from each sample value of $D U$, so that the $D U$ value of interest becomes zero (0.015-0.015) in a newly recentered $D U$ variable.
2 Replace the original $D U$ variable with the newly recentered $D U$ variable ( $D U-0.015$ ) in the model. Do this for both the main effect variable and the interacted variable of the interaction term as shown in Equation (18). Perform the recentered logit regression.
3 Look for the $p$ value associated with the parameter estimate $\left(\delta_{2}\right)$ for $A S$ from the recentered logit model as it indicates the statistical meaningfulness of the $\log$ of odds ratio for $A S=1$ versus $A S=0$ when $D U=0.015$.
4 Run the logit model recentered at a different $D U$ point of interest, if desired by theory, by repeating steps 1-3 above.
5 For graphical illustrations, if desired, plot the predicted probabilities (make $=1$ ) against the range of sample values of $D U$ using the Stata command "marginsplot" after logit, as shown in Figure 1.
6 In order to conduct a substantive analysis of the theory-driven effect size or the economic meaningfulness which measures the strength of the association between the event (here $A S=1$ ) when $D U=0.015$ and the outcome (here make $=1$ ), one can refer to Table 4 where we provide a statistical framework for assessing the effect size of the log of odds ratio in


FIGURE 1 Statistical meaningfulness of interaction effects at specific demand uncertainty levels for the "mean values" firm based on recentering approach using Leiblein and Miller (2003)'s three Models IV A, V and VI with cluster-robust standard errors (S.E.s) and with all other non-binary variables in each model set to their estimating sample means and all other binary variables set to their estimating sample modes [Color figure can be viewed at wileyonlinelibrary.com]
terms of Cohen's $d$ for its easier interpretation based on a widely accepted method in statistics and health sciences (Chen et al., 2010).

In Table 5, we show the results of our logit model examples (Models IV A, V, and VI) from the recentering approach using the steps above. The baseline log of odds ratio (baseline ratio) in Table 5 denotes the estimated coefficient of $A S$ from each logit model. Given the nested data structure at the firm level in Leiblein and Miller (2003)'s data, we report the results from the recentering approach using more appropriate cluster-robust standard errors (SE) (Moulton, 1990) in addition to the original SEs that Leiblein and Miller (2003) used. $\mathrm{P}>|\mathrm{z}|$ denotes two-tailed $p$ values for z statistics. As expected, the results with cluster-robust SEs for each model in Table 5 are less statistically meaningful. For example, in Model IV A, the baseline ratio ( -1.158 ) without cluster-robust SEs is marginally meaningful in a statistical sense with the $p$ value equal to .053 , whereas the same ratio with cluster-robust SEs is no longer statistically meaningful. Similarly, when demand uncertainty is recentered at 0.074 , the associated log of odds ratio for each model (1.127 for Model IV A and 1.443 for Model VI) becomes no longer statistically meaningful with the $p$ value equal to .106 and .129 respectively when using cluster-robust SEs. The same holds true in Models V and VI: when demand uncertainty is recentered at 0.106 , the associated log of odds ratio ( 2.041 and 2.586 for Models
TABLE 5 Results from the recentering approach for Leiblein and Miller's (2003) models

| The recentering approach with different standard errors (SEs) using Leiblein and Miller (2003)'s logistic regression models | Logit Model IV A ${ }^{\text {b }}$ |  |  |  |  | Logit Model $\mathbf{V}^{\text {c }}$ |  |  |  |  | Logit Model VI ${ }^{\text {d }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. estimate (size of the ratio) | Original SE | $\boldsymbol{p}>\|\mathrm{z}\|$ | Cluster robust SE | $\boldsymbol{p}>\|\mathrm{z}\|$ | Coeff. estimate (size of the ratio) | Original SE | $\boldsymbol{p}>\|\mathrm{z}\|$ | Cluster robust SE | $\boldsymbol{p}>\|\mathrm{z}\|$ | Coeff. estimate (size of the ratio) | Original SE | $\boldsymbol{p}>\|\mathrm{z}\|$ | Cluster robust SE | $\boldsymbol{p}>\|\mathrm{z}\|$ |
| Baseline ratio ${ }^{\text {a }}$ | -1.158 | 0.599 | . 053 | 0.712 | . 104 | -1.662 | 0.663 | . 012 | 0.714 | . 020 | -1.198 | 0.761 | . 116 | 0.898 | . 182 |
| Ratio with demand uncertainty recentered at 0.015 | -0.695 | 0.502 | . 166 | 0.585 | . 235 | -1.138 | 0.554 | . 040 | 0.559 | . 042 | -0.663 | 0.633 | . 295 | 0.728 | . 363 |
| Ratio with demand uncertainty recentered at 0.025 | -0.386 | 0.452 | . 393 | 0.527 | . 463 | -0.789 | 0.499 | . 114 | 0.493 | . 109 | -0.306 | 0.569 | . 591 | 0.656 | . 641 |
| Ratio with demand uncertainty recentered at 0.026 | -0.355 | 0.448 | . 428 | 0.523 | . 496 | -0.754 | 0.495 | . 128 | 0.488 | . 123 | -0.270 | 0.564 | . 632 | 0.651 | . 678 |
| Ratio with demand uncertainty recentered at 0.027 | $-0.325$ | 0.445 | . 465 | 0.519 | . 531 | -0.719 | 0.491 | . 143 | 0.484 | . 138 | -0.234 | 0.559 | . 675 | 0.647 | . 717 |
| Ratio with demand uncertainty recentered at 0.074 | 1.127 | 0.503 | . 025 | 0.696 | . 106 | 0.923 | 0.559 | . 099 | 0.778 | . 236 | 1.443 | 0.666 | . 030 | 0.951 | . 129 |
| Ratio with demand uncertainty recentered at 0.106 | 2.115 | 0.731 | . 004 | 1.044 | . 043 | 2.041 | 0.816 | . 012 | 1.215 | . 093 | 2.586 | 0.982 | . 008 | 1.442 | . 073 |
| Ratio with demand uncertainty recentered at 0.194 | 4.833 | 1.525 | . 002 | 2.162 | . 025 | 5.115 | 1.705 | . 003 | 2.539 | . 044 | 5.727 | 2.051 | . 005 | 2.972 | . 054 |

[^9]V and VI respectively) with cluster robust SEs becomes barely statistically meaningful ( $p$ value $=.093$ and .073 respectively).

Lastly, as introduced above, one can assess the effect size of the interaction term using Table 4 after running the recentered regression. For example, in Model IV A of Table 5, the size of the ratio with demand uncertainty recentered at 0.194 is estimated as 4.833 which is statistically meaningful. Since the ratio of 4.833 is greater than 1.61 (which corresponds to Cohen's $d>0.8$ ), according to Table 4, one can say that there is a large interaction effect for a one-unit change in asset specificity when demand uncertainty is equal to 0.194 .

## 4 | CONCLUSION

Our recentering approach is intended to provide strategic management researchers with an additional option when it comes to assessing interaction effects in nonlinear models. Instead of requiring the researcher to know how to make the assumptions about each control variable and to enter in assumed values for implementation, the recentering approach, through a simple mathematical transformation, takes the data and model as is and tells the researcher what the statistical and economic meaningfulness of the interaction effect is at that chosen point in the data distribution. What it takes to implement the recentering approach is to recenter the regression and implement it. If the independent variable of interest has 10 different values, one might have to run the recentered regression 10 times. Even that number of 10 regressions would be fewer if one's theory, as seen in Jeong and Siegel (2018), were explicitly focused on the upper, middle, or lower part of the distribution of a variable of interest.

In conclusion, the recentering method provides researchers a consistent answer through an efficient way. With the recentering approach, one can assess not just statistical meaningfulness of the interaction effect at each and every point along the spectrum in a nonlinear model, but also economic meaningfulness/effect size of the interaction term. The recentering method also can be easily applied to a situation where a nonlinear model specifies more than one interaction term. We recommend the recentering method to strategy researchers for its consistency in results, its ability to assess both statistical and economic meaningfulness, its methodological efficiency, and its relative simplicity in implementation.

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## SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of this article.

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[^0]:    ${ }^{1}$ For an introduction to the simulation method, see Krinsky and Robb (1986, 1990, 1991). See Greene (2018, pp. 647-648, 752) for an instructive discussion on the simulation-based method and the specific method of Krinsky and Robb.
    ${ }^{2}$ For an analysis comparing the delta method and the simulation method, see Krinsky and Robb (1990). For an additional description of the delta method, see Rothenberg (1984) and Horowitz (2001). For a separate way to implement the simulation method, see the NLOGIT software package and its WALD command.

[^1]:    ${ }^{3}$ Greene (2010) notably raised issues with this common practice of computing the cross-partial derivative and testing the interaction effect in nonlinear models. He pointed out the difficulty of interpreting the interaction effect measured by the cross-partial derivative in nonlinear models given the relationships among the variables.

[^2]:    ${ }^{4}$ For the logit model from Equation (1), the interaction effect where a continuous covariate $\left(X_{1}\right)$ and a dummy covariate $\left(X_{2}\right)$ are interacted is the discrete difference (with respect to $X_{2}$ ) of the single derivative (with respect to $X_{1}$ ), which is:

    $$
    \frac{\Delta \frac{\partial F(\cdot)}{\partial X_{1}}}{\Delta X_{2}}=\left(\beta_{1}+\beta_{3}\right)\binom{F\left\{\left(\beta_{1}+\beta_{3}\right) X_{1}+\beta_{2}+X \beta\right\}}{\times\left(1-F\left\{\left(\beta_{1}+\beta_{3}\right) X_{1}+\beta_{2}+X \beta\right\}\right)}-\beta_{1}\left[F\left(\beta_{1} X_{1}+X \beta\right)\left\{1-F\left(\beta_{1} X_{1}+X \beta\right)\right\}\right]
    $$

    where $F(\cdot)$ denotes $F\left(\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{1} X_{2}+X \beta\right)$ and $X \beta$ denotes the vectors of covariates and the associated parameters. For further technical computation of the interaction effects in logit and probit models when the two interacted variables are both continuous or both binary, see Norton et al. (2004, pp. 155-159).
    ${ }^{5}$ As Wooldridge (2010, pp. 437-438) points out, "it is important not to rely too much on Monte Carlo simulations. Many estimation methods have asymptotic properties which do not rely on underlying distributions. In the nonlinear regression model, the nonlinear least squares estimator is asymptotically normal, and the usual asymptotic variance matrix is valid under a set of assumptions. However, in a typical Monte Carlo simulation, the implied error ( $u$ ) is assumed to be independent of $x$, and the distribution of $u$ must be specified. The Monte Carlo results then pertain to this distribution, and it can be misleading to extrapolate to different settings. Furthermore, one can never try more than just a small part of the parameter space. Because one never knows the true population value, one can never be sure how well one's Monte Carlo study describes the underlying population."

[^3]:    ${ }^{6}$ We were able to analyze the data that were generously shared with us. The first data shared with us enabled us to analyze Models IV A and VI. Later we were sent separate data to run Model V. Using those separate Model V data sent to us in a second batch, we get substantively identical findings (but with somewhat different coefficients) from the original article. We have shown our log file to the original authors, and they have confirmed via email communication on October 3, 2019 that we have run the same model that they did.
    ${ }^{7}$ Note that constructing CIs using the percentiles of the simulated results can work if the sample of, for example, 1,000 replicates generates a sample from the sampling distribution of some estimator. If this is a maximum simulated likelihood (MSL) estimation of a random parameter, the method does not work. We are grateful to William H. Greene for his clarifying advice on when this percentile method can work.

[^4]:    Note: Subscripts 1,000 and 10,000 denote the number of simulation sets using CLARIFY (Tomz et al., 2003). CI stands for confidence interval. The last row of this table ( $\Delta \Delta \hat{\pi} .194, .015$ ) shows the original and simulated double differences (DD) in predicted probabilities ( $\Delta \hat{\pi} .194-\Delta \hat{\pi} .015$ ) associated with an increase in Asset specificity from zero to one when Demand uncertainty is set to a value of $0.194\left(\Delta \hat{\pi}_{.194}\right)$ and set to a value of $0.015\left(\Delta \hat{\pi}_{.015}\right)$ and all other variables in the model set to a value of zero, respectively.

[^5]:    Note: Subscripts 1,000 and 10,000 denote the number of simulation sets using CLARIFY (Tomz et al., 2003). CI stands for confidence interval. The last row of this table ( $\Delta \Delta \hat{\pi} .194, .015$ ) shows the original and simulated double differences (DD) in predicted probabilities ( $\Delta \hat{\pi} .194-\Delta \hat{\pi} .015$ ) associated with an increase in Asset specificity from zero to one when Demand uncertainty is set to a value of $0.194\left(\Delta \hat{\pi}_{.194}\right)$ and set to a value of $0.015(\Delta \hat{\pi} .015)$ and all other variables in the model set to a value of zero, respectively.

[^6]:    ${ }^{8}$ An odds ratio, a measure of association between an event and an outcome, was originally proposed to decide whether the probability of an event is the same or differs between two groups, usually one with the event and the other without the event. The range of odds ratios is from 0 to infinity where a value of one indicates that the event is equally likely in the two groups with and without the event, suggesting no effect of the event on the odds of outcome. As the value of odds ratio rises or drops from a value of one, the association between the event and the odds of outcome becomes much stronger positively or negatively (Chen, Cohen, \& Chen, 2010, p. 861).
    ${ }^{9}$ UCLA: Statistical Consulting Group. "FAQ: How Do I Interpret Odds Ratios in Logistic Regression?" from https://stats. idre.ucla.edu/other/mult-pkg/faq/general/faq-how-do-i-interpret-odds-ratios-in-logistic-regression/ (accessed December 2019).

[^7]:    ${ }^{10}$ For example, see https://www.calculatorsoup.com/calculators/games/odds.php. Or one can simply convert from log of odds ratio to odds by exponentiating the $\log$ of odds ratio (i.e., odds ratio $=\exp (\log$ of odds ratio)). To convert from an odds ratio to a probability, one can divide the odds by one plus the odds (e.g., to convert odds of $1 / 9$ to a probability, divide $1 / 9$ by $10 / 9$ to obtain the probability of 0.10 ).

[^8]:    11"A measure of effect size, the most familiar form being the difference between two means (M1 and M2) expressed in units of standard deviations: the formula is $d=\left(\mathrm{M}_{1}-\mathrm{M}_{2}\right) / \sigma$, where $\sigma$ is the pooled standard deviation of the scores in both groups. [Named after the US psychologist Jacob (Jack) Cohen (1923-98) who devised it and popularized it in his book Statistical Power Analysis for the Behavioral Sciences (1969, 1988)]" (Cohen's $d$-Oxford Reference at http://www. oxfordreference.com/view/10.1093/oi/authority.20110803095622509).

[^9]:    Note: Vertical integration is the binary outcome variable of all three models. Because the data used for Leiblein and Miller (2003) were clustered at the firm level, we use cluster-robust SEs (Moulton, 1990) that adjust for clustering at the firm level. $p>|\mathrm{z}|$ denotes two-tailed $p$ values for z statistics.
    ${ }^{\text {a }}$ Baseline ratio denotes the estimated coefficient of asset specificity from each model used here.
    ${ }^{5}$ Model IV A includes asset specificity, demand uncertainty, asset specificity $\times$ demand uncertainty, fabrication experience, sourcing experience, and other control variables. Model V uses fabrication experience hat and sourcing experience hat (instead of fabrication experience and sourcing experience) and adds diversification strategy and diversification squared to Model IV A.
    ${ }^{\mathrm{d}}$ Model VI adds diversification strategy, diversification squared, and year fixed effects to Model IV A.

