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ABSTRACT

Research Summary: Strategic management has seen numerous studies analyzing interaction terms in nonlinear models since Hoetker's (2007) best-practice recommendations and Zelner's (2009) simulation-based approach. We suggest an alternative recentering approach to assess the statistical and economic importance of interaction terms in nonlinear models. Our approach does not rely on making assumptions about the values of the control variables; it takes the existing model and data as is and requires fewer computational steps. The recentering approach not only provides a consistent answer about statistical meaningfulness of the interaction term at a given point of interest, but also helps to assess the effect size using the template that we offer in this study. We demonstrate how to implement our approach and discuss the implications for strategy researchers.

Managerial Summary: In industry settings, the relationship between multiple corporate strategy-related inputs and corporate performance is often nonlinear in nature. Furthermore, such relationships tend to vary for different types of firms represented within the broader population of firms in a given industry. It is thus imperative for managers to know how to take nonlinear relationships between related business factors into account when they make strategic decisions. We suggest a simple and easily implementable way of assessing and interpreting interactions in a nonlinear setting, which we term a recentering approach. We demonstrate how to apply our approach to a strategic management setting.

INTRODUCTION

Interaction terms are frequently modeled in strategic management research in order to evaluate the effect of one explanatory variable on the response variable given the magnitude of another explanatory variable (e.g., the relationship between corporate strategy-related inputs and management performance outcomes varies depending on the internal and external business environments). Assessing and interpreting interaction terms becomes more complicated when models are nonlinear. Unlike linear models where the effect of a one-unit change in a covariate on the outcome variable (i.e., marginal or partial effect) is constant over the whole range of the covariate given the level of the other covariates in the model, the same effect in nonlinear models relies on the values of all other covariates in the model (Ai & Norton, 2003; Norton, Wang, & Ai, 2004). Given the frequency with which strategic management researchers have encountered interaction terms in nonlinear models (see, e.g., Shook, Ketchen, Cycyota, & Crockett, 2003; Hoetker, 2007), we will argue and show by way of mathematical proof and empirical analysis that there is room for another methodological option for achieving simplicity and consistency of interpretation of those interaction terms.

In strategic management research, Hoetker (2007) recommended a set of best practices for the use of logit and probit models, including interpreting interaction terms. To further improve the assessment of statistical meaningfulness and interpretation of logit and probit results, Zelner (2009, p. 1336) suggested "a simulation-based technique developed by King, Tomz, and Wittenberg (2000)"[1] and argued for the benefits of this technique over the conventional calculus-based method known as

[1] For an introduction to the simulation method, see Krinsky and Robb (1986, 1990, 1991). See Greene (2018, pp. 647-648, p. 752) for an instructive discussion on the simulation-based method and the specific method of Krinsky and Robb.

the delta method (Zelner, 2009, pp. 1341-1342)[2] proposed by Dorfman (1938). In particular, Zelner proposed (i) calculating and interpreting a difference in predicted probabilities associated with discrete changes in key predictor values (known as the *cross-partial derivative* or *cross-difference*, which measures how the marginal effect of one variable changes when the other variable in the interaction term changes) and (ii) testing whether the difference in predicted probabilities is different from zero by constructing a confidence interval (C.I.) around the estimated quantity and finding out if the interval contains zero.[3] This simulation-based technique argued for by Zelner (2009) requires user-written Stata commands 'CLARIFY' and 'intgph' (Tomz, Wittenberg, & King, 2003; Zelner, 2009).

This simulation approach, however, must by inherent definition include the researcher picking assumed values for all the control variables in the model in order to generate output about whether an interaction effect is statistically meaningful. To address this concern, we propose and recommend a recentering approach, which focuses on the main independent variable at a point of theory-motivated interest. The recentering approach does not require assumed values for any of the control variables, takes the data and model as is, is computationally simpler and is easier to implement with one simple mathematical transformation as seen below. Last but not least, our approach enables one to assess, with the help of the template we provide in this study, the effect size of the interaction term

[2] For an analysis comparing the delta method and the simulation method, see Krinsky and Robb (1990). For an additional description of the delta method, see Rothenberg (1984) and Horowitz (2001). For a separate way to implement the simulation method, see the NLOGIT software package and its WALD command.

[3] Greene (2010) notably raised issues with this common practice of computing the cross-partial derivative and testing the interaction effect in nonlinear models. He pointed out the difficulty of interpreting the interaction effect measured by the cross-partial derivative in nonlinear models given the relationships among the variables.

in a nonlinear model. Overall, the recentering approach we propose gives researchers an additional option to consider when assessing the interaction effect in nonlinear models.

Our recentering approach is based on a recentered regression where one or both variables involved in the interaction term are centered at a value of interest—whether it be at the sample mean, sample median, sample 75th percentile value, sample 25th percentile value, or any other theory-driven value. That is, every value of the variable being interacted in the data set is deducted by the same value of the researcher's interest. For ease of explication and comparison, we begin our discussion below by showing our simulation results from three logit model examples used in prior research. We then illustrate our recentering approach where we first show the link to generalized linear models and discuss the benefits of using the log of odds ratio in assessing and interpreting interaction effects in nonlinear models. Next, we present our mathematical proof that concisely illustrates why the recentering method provides a simple and consistent identification process. We then show how the recentering approach can help researchers assess the effect size of the interaction term in a nonlinear setting beyond its statistical meaningfulness. There, we provide a table that researchers can easily consult to evaluate the relationship between the odds ratio and Cohen's *d* (Cohen, 1988), a widely accepted measure for assessing the effect size in the field of statistics and in the behavioral and health sciences. Lastly, we demonstrate the steps to implement the recentering approach. We conclude by discussing the benefits of using the recentering approach in comparison to the simulation-based approach.

THE SIMULATION-BASED APPROACH

For ease of explication and comparison across studies, we utilize three well-specified logit models (Models IV A, VI and V) and the data (N=469) used in Leiblein and Miller (2003) as our examples. The three logit models specified in equation (2) below take the form of the following population logistic model of the binary outcome variable Y with the vector of independent variables $X \equiv (\mathrm{x}_1, \ldots, \mathrm{x}_i)$:

$$\begin{aligned}\Pr(Y=1 \mid X) &= F(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \cdots + \beta_i X_i) \\ &= \frac{1}{1 + e^{-(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \cdots + \beta_i X_i)}}\end{aligned} \tag{1}$$

where F is the cumulative standard logistic distribution function, X_1 denotes a continuous variable, X_2 denotes a dummy variable, and X_1X_2 denotes an interaction term whose effect is the change in the predicted probability that Y=1 for a change in both X_1 and X_2.[4] Given equation (1), Leiblein and Miller's (2003) two logit models specify, for a given firm in a given year:

$$\begin{aligned}&\text{Vertical Integration} \\ &= \Pr(\text{make} = 1) = F\left(\begin{aligned}&\beta_0 + \beta_1 \text{Demand Uncertainty} + \beta_2 \text{Asset Specificity} \\ &+ \beta_3 (\text{Demand Uncertainty} \cdot \text{Asset Specificity}) + \sum \beta_i \text{Controls}_i\end{aligned}\right)\end{aligned} \tag{2}$$

[4] For the logit model from equation (1), the interaction effect where a continuous covariate (X_1) and a dummy covariate (X_2) are interacted is the discrete difference (with respect to X_2) of the single derivative (with respect to X_1), which is:

$$\frac{\Delta \dfrac{\partial F(\cdot)}{\partial X_1}}{\Delta X_2} = (\beta_1 + \beta_3)\begin{pmatrix} F\{(\beta_1 + \beta_3)X_1 + \beta_2 + X\beta\} \\ \times (1 - F\{(\beta_1 + \beta_3)X_1 + \beta_2 + X\beta\}) \end{pmatrix} - \beta_1 \left[F(\beta_1 X_1 + X\beta)\{1 - F(\beta_1 X_1 + X\beta)\}\right]$$

where $F(\cdot)$ denotes $F(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + X\beta)$ and $X\beta$ denotes the vectors of covariates and the associated parameters. For further technical computation of the interaction effects in logit and probit models when the two interacted variables are both continuous or both binary, see Norton et al. (2004, pp. 155-159).

where the outcome variable, vertical integration takes a value of one for a 'make' decision and zero for a 'buy' decision, and the explanatory variables include the two interacted variables (a continuous measure of 'demand uncertainty' and a binary measure of 'asset specificity' that takes a value of one when asset specificity is present and zero otherwise) and the interaction term that is the product of the two variables. A set of control variables that are theoretically believed to influence firms' vertical integration are controlled for in each of the three models. In particular, Model IV A adjusts for Fabrication Experience, Sourcing Experience, Ex ante Small Numbers, Small Numbers Squared, Firm Size, Firm Tenure, US Firm, Japanese Firm, and Other Asian Firm. Model V replaces Fabrication Experience and Sourcing Experience with Fabrication Experience Hat and Sourcing Experience Hat in Model VI A and adds Diversification Strategy and Diversification Squared to Model IV A. Model VI adds Diversification Strategy, Diversification Squared, and year fixed effects to Model IV A.

In order to assess interaction effects in nonlinear models, Zelner (2009) proposed looking at the difference in predicted probabilities associated with a discrete change in key predictor values and testing whether such difference is statistically different from zero by constructing a C.I. around the estimated quantity. If the C.I. includes zero, then it is concluded that there is no statistically meaningful interaction effect. For this hypothesis testing, Zelner (2009) proposed computing the C.I.s using King et al.'s (2000) simulation-based approach that implements 'CLARIFY'. This Stata user-written program uses Monte Carlo simulation which relies on asymptotic theory (Cameron & Trivedi, 2005; Wooldridge, 2010).[5]

[5] As Wooldridge (2010, pp. 437-438) points out, "it is important not to rely too much on Monte Carlo simulations. Many estimation methods have asymptotic properties which do not rely on underlying distributions. In the nonlinear regression model, the nonlinear least squares estimator is asymptotically normal, and the usual asymptotic variance

To illustrate this approach, Zelner (2009) generated 10 sets of simulated coefficients from Model V of Leiblein and Miller (2003) using 'CLARIFY' (by default the program draws M = 1,000 sets of simulated parameters) and showed the results with the 80% (two-tailed) C.I. for each simulated coefficient in the logit model to assess its statistical meaningfulness. Following this approach, we also run our three logit model examples and simulate the coefficients using 'CLARIFY'. Although the overall simulation process is the same, ours differs from Zelner (2009) in three ways. First, we analyze Leiblein and Miller (2003)'s three logit models (Models IV A, V, and VI) whereas Zelner (2009) did one (Model V).[6] Second, in consideration of Cameron and Trivedi (2010) who imposed a caveat on running only 1,000 simulations for reported results given "considerable simulation noise, especially for estimates of test size (and power)" (Cameron & Trivedi, 2010, p. 140), we run both 1,000 and 10,000 simulations. Third, we report the 95% (two-tailed) C.I.s using the percentiles of the simulated results as Zelner (2009) did (e.g., the 95% two-tailed C.I. for each coefficient in the case of 1,000 simulated results is bounded by the 25th-lowest and 975th-highest simulated values for the coefficient).[7]

matrix is valid under a set of assumptions. However, in a typical Monte Carlo simulation, the implied error (u) is assumed to be independent of x, and the distribution of u must be specified. The Monte Carlo results then pertain to this distribution, and it can be misleading to extrapolate to different settings. Furthermore, one can never try more than just a small part of the parameter space. Because one never knows the true population value, one can never be sure how well one's Monte Carlo study describes the underlying population."

[6] We were able to analyze the data that were generously shared with us. The first data shared with us enabled us to analyze Models IV A and VI. Later we were sent separate data to run Model V. Using those separate Model V data sent to us in a second batch, we get substantively identical findings (but with somewhat different coefficients) from the original article. We have shown our log file to the original authors, and they have confirmed via email communication on 3 October 2019 that we have run the same model that they did.

[7] Note that constructing C.I.s using the percentiles of the simulated results can work if the sample of, for example, 1,000 replicates generates a sample from the sampling distribution of some estimator. If this is a maximum simulated likelihood (MSL) estimation of a random parameter, the method does not work. We are grateful to William H. Greene for his clarifying advice on when this percentile method can work.

We report the results from Leiblein and Miller (2003)'s three logit models in Tables 1a, 1b, and 1c, respectively. In Table 1a (Model IV A), note that the original coefficients of asset specificity (-1.158) and firm size (0.214) variables are moderately meaningful in a statistical sense with the p-values equal to 0.053 and 0.056 respectively. In contrast, the simulation-based approach tells that neither is statistically meaningful, regardless of the number of simulations.

[Insert Tables 1a, 1b, and 1c near here]

THE RECENTERING APPROACH

The recentering approach we propose is in the branch of statistics called Generalized Linear Models (GLM). GLM, first invented in 1972 (Nelder & Wedderburn, 1972) and widely considered to be one of the pioneering achievements in the last 50 years of the field of statistics, exists to unify linear and nonlinear models in the spirit of greater analyzability. In GLM, the nonlinear representation of the dependent variable appears on the left-hand side and the linear representation of the independent variables including any interaction term(s) appears on the right-hand side. The left-hand side can be nonlinear while the right-hand side is linear because of an invertible linearizing "link function" on the left-hand side, which transforms the expectation of the dependent variable such that it can be equal to a linear function of the independent variables. To express this point in proper notation in the classic linear model, the equation can be written in the following form:

$$Y = X\beta + \varepsilon \tag{3}$$

where Y is a response variable, X is a set of explanatory variables, β is a set of estimated coefficients, and ε is a column vector of disturbances. The linear model follows a set of Gaussian assumptions, including but not limited to the facts that the relationship between each explanatory variable and the response variable is approximately linear, and that the residuals are independent and identically distributed (i.i.d.) normal with mean zero and constant variance. These last two restrictions are eliminated in a GLM, which in turn provides a way to learn the effect of the explanatory variables that closely resembles the process of analyzing independent variables in the classic linear model.

The key to a GLM is the specification of a so-called link function, which links the systematic component of the linear model $X\beta$ with a wider class of nonlinear representations of the response variable. The link function 'g' can be written in the following form:

$$E(Y) = \mu = g^{-1}(X\beta) \tag{4}$$

where E(Y) is the expected value of the response variable Y, μ is the mean of Y, $X\beta$ is the linear predictor, a linear combination of the unknown parameters β, and g is the link function. The link function can be a logit, probit, poisson, negative binomial, or any other nonlinear transformation of the response variable Y such that the right-hand side can be a linear representation of the independent variables. To emphasize, the recentering method that we will show below will work not just for logit, but also for poisson, negative binominal, or any other nonlinear transformation through a known link function. Going from logit link to probit link, for example, only changes the left-hand side of the equation, while recentering happens on the right-hand side of the equation. So the changing of the link function will not impact the math result on the right-hand side of the equation. In the following example, we will take the logit link function:

$$\log\frac{p}{1-p} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_3 + \ldots \tag{5}$$

where p denotes probability and log (p/1–p) is the logit function, which is the logarithm of the odds ratio[8] of the 'make-or-buy' decision in equation (2). One of the reasons for transforming probability (ranging from 0 to 1) to log odds (ranging from negative infinity to positive infinity) is because it is hard to statistically model a variable which has a restricted range like probability. One way to circumvent such a restricted range issue is this transformation. Also, the log of odds is one of the easiest to understand and interpret, among all of the limitless options for transformation.[9] There is a one-step conversion for that: (a given odds ratio)/(1 + that same given odds ratio)=probability. Also when one is looking at the incremental effect on baseline probability, one needs to start the research project in any case (whether one is utilizing the simulation method or the recentering approach) with knowledge of what is the baseline probability of an event occurring in one's sample.

Second, in a nonlinear world, the conversion between odds ratio and probability is monotonic but not intended to be symmetric. It is the case that at a starting odds ratio of 1:1 (equal to a starting probability of 0.5), the relationship between odds ratio and probability is symmetric. Starting at the original odds ratio of 1:1, that is the same as a probability of (1/1)/(2/1)=1/2=0.5. When that original odds ratio of 1:1 goes up by 5, that is the same as a probability of (5/1)/(6/1)=5/6=0.833. When that original odds ratio of 1:1 gets divided by 5, that is the same as a probability of (1/5)/(6/5)=1/6=0.167.

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[8] An odds ratio, a measure of association between an event and an outcome, was originally proposed to decide whether the probability of an event is the same or differs between two groups, usually one with the event and the other without the event. The range of odds ratios is from 0 to infinity where a value of one indicates that the event is equally likely in the two groups with and without the event, suggesting no effect of the event on the odds of outcome. As the value of odds ratio rises or drops from a value of one, the association between the event and the odds of outcome becomes much stronger positively or negatively (Chen, Cohen, & Chen, 2010, p. 861).

[9] UCLA: Statistical Consulting Group. "FAQ: How Do I Interpret Odds Ratios in Logistic Regression?" from https://stats.idre.ucla.edu/other/mult-pkg/faq/general/faq-how-do-i-interpret-odds-ratios-in-logistic-regression/ (accessed December 2019).

Thus, at a baseline odds ratio of 1:1, the absolute value of the positive impact of multiplying the odds ratio by 5 is the same as (thus is "symmetric to") the absolute value of the negative impact of dividing the odds ratio by 5.

It is also true that the same coefficient expressed in log odds can have a smaller nominal effect in changing the probability for a group with small baseline odds than a group with larger baseline odds (Hoetker, 2007, p. 334). It is thus important for the researcher to explicitly depict whether and how a given change in log odds ratio means a different change in probability for various groups in the population.

Having the regression run efficiently in GLM and getting the log odds ratio out of it means that one does not have to test out all combinations of the control variables in a simulation. Let's say that one has 10 control variables, each of which takes on 10 different possible values in one's data set. At most, one runs 10 different recentered regressions. The reason for why the recentering method is efficient and provides consistent results in nonlinear models is because, through a simple mathematical transformation that we will see next, we are able to subtract out the effects of the control variables. We are thus able to arrive at the answer that is unbiased and consistent regardless of the number of and all combinations of the control variables in the data set.

In summary, the log of odds ratio is particularly helpful for studying interaction effects in nonlinear models for several reasons. First, the log of odds ratio lends itself to broadly applicable statistical analysis (because it can be used through the recentering method to tell us the statistical and economic meaningfulness of an interaction term that is true and consistent no matter what the values of the control variables are). Second, the log of odds ratio, while not previously held to be intuitive, can be readily converted into a probability that is more easily understood using any odds-probability online converter tool or a simple calculation.[10] Third, the log of odds ratio provides a clear benchmark for assessing the economic meaningfulness/effect size of an interaction term.

Why the recentering method offers a useful option for interpreting interaction effects

Through simple mathematical steps, we next illustrate why the recentering method provides a simple and consistent identification process. Recalling equation (5), consider we seek to determine the effect of X_2 given X_1 being equal to a defined point, a. X_2 can be either a continuous or a dummy variable. From equation (5), note what happens when X_2 shifts from the value b to the value (b+1). The following is the effect of X_2 given $X_1 = a$ as X_2 shifts from b to (b+1):

$$X_2 = b \quad \rightarrow \quad \log\left(\frac{p_1}{1-p_1}\right) = \beta_0 + \beta_1 a + \beta_2 b + \beta_3 ab + \beta_4 X_3 + \ldots \tag{6}$$

$$X_2 = b+1 \quad \rightarrow \quad \log\left(\frac{p_2}{1-p_2}\right) = \beta_0 + \beta_1 a + \beta_2 \left(b+1\right) + \beta_3 a\left(b+1\right) + \beta_4 X_3 + \ldots \tag{7}$$

To find the effect of X_2 as it goes from b to (b+1), one can examine the log of odds ratio by subtracting equation (6) from equation (7). The outcome will then be:

$$\log\left(\frac{p_2}{1-p_2}\right) - \log\left(\frac{p_1}{1-p_1}\right) = \log\left(\frac{p_2/1-p_2}{p_1/1-p_1}\right) = \beta_2 + \beta_3 a \tag{8}$$

[10] For example, see https://www.calculatorsoup.com/calculators/games/odds.php. Or one can simply convert from log of odds ratio to odds by exponentiating the log of odds ratio (i.e., odds ratio = exp(log of odds ratio)). To convert from an odds ratio to a probability, one can divide the odds by one plus the odds (e.g., to convert odds of 1/9 to a probability, divide 1/9 by 10/9 to obtain the probability of 0.10).

Note that as a result of this subtraction, all the terms for the control variables are removed. In contrast, the methods for examining interaction terms in strategic management (e.g., Wiersema & Bowen, 2009) are focused on the direct change in probabilities, where from the odds (p/1−p) one can derive the probability p as shown below:

$$\left(\frac{p}{1-p}\right) = e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_3 + \ldots} \tag{9}$$

$$p = \frac{e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_3 + \ldots}}{1+e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_3 + \ldots}} \tag{10}$$

Examining the effect of X_2 given X_1=a in this approach entails a relatively more complicated math problem in which the control variables do not disappear:

$$X_2 = b \;\rightarrow\; P_1 = \frac{e^{\beta_0 + \beta_1 a + \beta_2 b + \beta_3 ab + \beta_4 X_3}}{1+e^{\beta_0 + \beta_1 a + \beta_2 b + \beta_3 ab + \beta_4 X_3}} \tag{11}$$

$$X_2 = b+1 \;\rightarrow\; P_2 = \frac{e^{\beta_0 + \beta_1 a + \beta_2 (b+1) + \beta_3 a(b+1) + \beta_4 X_3}}{1+e^{\beta_0 + \beta_1 a + \beta_2 (b+1) + \beta_3 a(b+1) + \beta_4 X_3}} \tag{12}$$

In other words, to examine the direct change in probability as the effect of the change in X_2 from b to (b+1), one can attempt to subtract equation (11) from equation (12), but the other control variables will not be removed in this case. A notable takeaway from this demonstration is that one can never assess the effect of a change in probability when looking at the world this way unless one plugs in assumed values for each and every control variable. In contrast, the recentering method enables one to subtract away all control variables through a simple mathematical transformation, and as a result of that simple mathematical transformation, easily and consistently assess the statistical and economic meaningfulness of the interaction term at hand in nonlinear models.

To achieve that goal, we present below what is learned when operating in the GLM world using the link function in which the control variables are subtracted away, and where one gets the consistent answer no matter what the values of the control variables are. Using the example of Leiblein and Miller (2003), we start with the logit link function on the left-hand side and the matching linear representation of the independent variables on the right-hand side. Here we know in advance that in a GLM framework, all of the control variables will be subtracted away; thus, we can focus fully on the main variables of interest: Demand Uncertainty (DU), which is a continuous variable and Asset Specificity (AS), which in the real world is ultimately a continuous variable but in Leiblein and Miller (2003) is measured as a binary dummy variable. Recalling equations (2) and (5), let DU, AS, and (DU · AS) in equation (2) be X_1, X_2, and X_1X_2 in equation (5) respectively. We will then get:

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 DU + \beta_2 AS + \beta_3 (DU \cdot AS) + \ldots \tag{13}$$

In equation (13), we know in advance, as shown in equation (8), the effect of AS on the make-or-buy decision (the response variable) given DU=a as the value of AS goes from 0 to 1 is $\beta_2 + \beta_3(a)$. Thus, one can see that the effect of AS critically depends on the value of DU set at a. Therefore, the effect of AS on the dependent variable in this interaction context can only be told by the value for β_2 alone if DU=0. Our interest here is in how to easily and consistently assess the effect of AS on the dependent variable given a specific value of DU in this nonlinear context. Based on theory from strategic management research, one does typically have an interest in learning about the effect of the interaction for a chosen region of DU. The efficient way to identify this interaction effect with consistency is to perform a simple mathematical transformation that makes DU become zero so that the DU term is canceled out by algebra. How does this work? In equation (13), we simultaneously add and subtract the value of DU that would make DU=0 at the DU point of interest in the DU data distribution, which we call $\overline{DU}$ below:

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1\left(DU - \overline{DU} + \overline{DU}\right) + \beta_2 AS + \beta_3\left[\left(DU - \overline{DU} + \overline{DU}\right) \cdot AS\right] + \ldots \quad (14)$$

By reorganizing the terms on the right-hand side in equation (14), we get:

$$= \beta_0 + \beta_1 \overline{DU} + \beta_1\left(DU - \overline{DU}\right) + \beta_2 AS + \beta_3 \overline{DU} \cdot AS + \beta_3\left(DU - \overline{DU}\right) \cdot AS + \ldots \quad (15)$$

which is the same as:

$$= \left(\beta_0 + \beta_1 \overline{DU}\right) + \beta_1\left(DU - \overline{DU}\right) + \left(\beta_2 + \beta_3 \overline{DU}\right) \cdot AS + \beta_3\left(DU - \overline{DU}\right) \cdot AS + \ldots \quad (16)$$

The power of the above mathematical transformation is that all of the positive $\overline{DU}$ terms get subtracted out and we are left with only the negative $\overline{DU}$ terms. For the sake of simplicity, we then group together terms in equation (16) using the alternative notation of δ_i:

$$= \delta_0 + \delta_1\left(DU - \overline{DU}\right) + \delta_2 AS + \delta_3\left(DU - \overline{DU}\right) \cdot AS + \ldots \quad (17)$$

where $\delta_0 = \beta_0 + \beta_1 \overline{DU}$, $\delta_1 = \beta_1$, $\delta_2 = \beta_2 + \beta_3 \overline{DU}$, and $\delta_3 = \beta_3$.

The key insight here is that, when we are interested in the effect of AS on the dependent variable at any particular part of the actual DU data distribution, all we need to do is to subtract from every value of DU the data point of interest ($\overline{DU}$). As a result, δ_1 and δ_3 in equation (17) disappear because ($DU - \overline{DU}$) becomes zero. The estimated coefficient (δ_2) of AS in terms of the log of odds ratio then tells us whether the interaction term ($DU \cdot AS$) is statistically meaningful, given the p-value associated with that coefficient, at the DU point of interest ($\overline{DU}$) when the value of AS moves from one value to another (here, zero to one in our example). In essence, after recentering, the coefficient (δ_2) represents the log-odds ratio effect of a one-unit change in AS holding DU constant at the point of interest and holding all other control variables constant. Also note that in contrast to the simulation approach, this recentering approach also helps us assess the effect size or the economic meaningfulness of the interaction term (DU · AS) at the DU point of interest ($\overline{\text{DU}}$) with the template that we will discuss and provide below.

Up to now we used the coefficient of AS to learn the change in the log of odds ratio at a specific DU point of interest. Researchers, however, can also learn the covariate-invariant effect of the other main variable among the two interacted variables in the same regression. In our example above, this means that if we instead wanted to know the effect of DU at the AS point of interest (e.g., 1), we just recenter the AS variable at 1 by subtracting 1 from all values of AS (0, 1 in our prior example) which makes the AS at that point of interest equal to zero. Because of this subtracting process of an interacted variable, which essentially makes that variable to be equal to zero at a specific value of that variable, we call this technique a *recentered regression*. This recentered regression makes it possible to assess the statistical and economic meaningfulness of the interaction effect in a nonlinear setting, that is consistent regardless of the values of all other control variables.

It is noteworthy that the recentering approach can also help when there are two sets of interaction terms in one nonlinear model. For example, consider four main variables, A, B, C, and D and two interaction terms (A · B) and (C · D) in the same nonlinear model. In this case, the recentering approach can diagnose more than just one interaction term at a time or both at the same time (like if one were to recenter "B" at the B point of interest and "D" at the D point of interest simultaneously).

In the latter case, the recentering approach can help identify the interaction term ($A \cdot B$) at the B point of interest ($\overline{B}$) and D point of interest ($\overline{D}$) simultaneously. To understand how the recentering approach works here, suppose a nonlinear model specified as $\log(p/(1-p)) = \alpha_0 + \alpha_1 A + \alpha_2 B + \alpha_3 (A \cdot B) + \alpha_4 C + \alpha_5 D + \alpha_6 (C \cdot D) + \alpha_i \Sigma(\text{Other Covariates}_i)$. Then via centering B and D at $\overline{B}$ and $\overline{D}$ respectively, the coefficient of A (α_1) would tell us the effect of A when B is at $\overline{B}$, no matter what C, D or the other control variable values are (i.e., "covariate-invariant"). Likewise, the coefficient of C (α_4) would tell us the covariate-invariant effect of C when D is at $\overline{D}$. If the nonlinear model were specified as $\log(p/(1-p)) = \pi_0 + \pi_1 A + \pi_2 B + \pi_3 C + \pi_4 (A \cdot B) + \pi_5 (A \cdot C) + \pi_k \Sigma(\text{Other Covariates}_k)$, then via centering B and C at $\overline{B}$ and $\overline{C}$ respectively, the coefficient of A (π_1) would tell us the effect of A when B is at $\overline{B}$ and C is at $\overline{C}$, no matter what the other covariate values are. Similarly via centering A at $\overline{A}$, the coefficient of B (π_2) would tell us the covariate-invariant effect of B when A is at $\overline{A}$ and the coefficient of C (π_3) would tell us the covariate-invariant effect of C when A is at $\overline{A}$. Here all the "effect" discussed above refers to the log of odds ratio.

The recentering approach helps assess the effect size in a nonlinear model

The recentering approach also helps researchers assess the effect size of the interaction term in a nonlinear model beyond its statistical meaningfulness. From prior literature in epidemiology (Chen et al., 2010), there is a precisely defined template for interpreting the size of the effect expressed in terms of the log of odds ratio, which we introduce in Table 2. Specifically, the table evaluates the effect size in direct mathematical comparison to Cohen's *d*[11] (Cohen, 1988), a widely accepted measure for assessing effect size in the field of statistics and in the behavioral and health sciences. Recall that the coefficient of the explanatory variable in a logistic regression corresponds to the log of odds ratio of the outcome variable per unit increase in the explanatory variable. The first two columns of Table 2 show odds ratio and log of odds ratio, respectively. The rest of the columns show the equivalent Cohen's *d* given P_0 in the second row, which is the baseline rate of outcome of interest in the group of subjects. In our example above, it is the rate of vertical integration in the group of firms where asset specificity=0, holding demand uncertainty and all other control variables constant. Because P_0 could vary for different values of demand uncertainty, it makes sense to interpret the effect size, or lack thereof, based on the answer being true regardless of the exact value of P_0 (in epidemiology it is the rate of contracting a disease of interest in the non-exposed group, i.e., the group not exposed to a particular harm for example and is estimated from the general population, and in our present empirical context, it would be the rate at which firms engage in vertical integration). Our main interest in Table 2 is whether Cohen's *d* equivalent to log odds ratio is clearly indicating an economically large or economically small effect. Values in bold with shading in Table 2 indicate Cohen's $d < 0.20$ or > 0.80. Values of Cohen's *d* less than 0.20 suggest that the effect sizes are small for all plausible values of P_0,

[11] "A measure of effect size, the most familiar form being the difference between two means (M1 and M2) expressed in units of standard deviations: the formula is $d = (M_1 - M_2)/\sigma$, where σ is the pooled standard deviation of the scores in both groups. [Named after the US psychologist Jacob (Jack) Cohen (1923–98) who devised it and popularized it in his book *Statistical Power Analysis for the Behavioral Sciences* (1969, 1988)]" (Cohen's *d* - Oxford Reference at http://www.oxfordreference.com/view/10.1093/oi/authority.20110803095622509).

and values of Cohen's *d* greater than 0.80 suggest that the effect sizes are large for all plausible values of P_0 (Cohen, 1988).

[Insert Table 2 near here]

As shown in Table 2, the log of odds ratio < 0.26 always corresponds to Cohen's $d < 0.20$ and the log of odds ratio > 1.95 always corresponds to Cohen's $d > 0.80$. Cohen's $d < 0.20$ reflects the fact that the effect size of the interaction effect is small. Cohen's $d > 0.80$ reflects the fact that the effect size of the interaction effect is large. Thus, if we see a log of odds ratio less than or equal to 0.26, we know that the size of the interaction effect is economically small no matter what the P_0 is. Similarly, if we see a log of odds ratio greater than or equal to 1.95, we know that the size of the interaction effect is large no matter what the P_0 is. Therefore, when the log of odds ratio identified from the recentering method is less than or equal to 0.26, we can say that the size of the interaction effect is small (clearly mapping onto Cohen's $d < 0.20$) regardless of the values of P_0. Similarly, when the log of odds ratio identified from the recentering method is greater than or equal to 1.95, we can say that the size of the interaction effect is large (clearly mapping onto Cohen's $d > 0.80$) no matter what the P_0 is. Of course, one might know from prior studies that P_0 is likely smaller than say 0.20, for example. In our present context, P_0 might be such that the rate of vertical integration with asset specificity equal to zero and all other control variables held constant is 0.05. This is the same thing as saying that vertical integration is occurring 5% of the time in the general population of interest where asset specificity equals zero, holding all other covariates constant. Under that case ($P_0 = 0.05$), the log of odds ratio less than 0.41 (equivalent Cohen's $d = 0.19$) would be considered economically small, and the log of odds ratio greater than 1.61 (equivalent Cohen's $d = 0.83$) would be considered economically large, according to Table 2. It is also straightforward to convert the predicted log of odds ratio to a predicted probability. That is because there is a 1:1 monotonic relationship between log of odds ratio and probability.

Specific to our vertical integration example, the coefficient of demand uncertainty corresponds to the log of odds ratio for a "make" decision when asset specificity is fixed at a specific value (0 or 1). Given uncertainty over P_0 in prior make-or-buy literature, a coefficient of demand uncertainty less than 0.26, according to Table 2, indicates that the effect size of demand uncertainty on vertical integration is clearly small, given fixed asset specificity equal to 1 no matter what the P_0 is. Similarly, a coefficient of demand uncertainty greater than 1.95, according to Table 2, indicates that the effect size of demand uncertainty on vertical integration is clearly large, given fixed asset specificity equal to 1 no matter what the P_0 is.

An example of the recentering approach

For illustrative purposes, we rewrite equation (2) in the following basic form which represents Leiblein and Miller's (2003) original logit models that include the interaction term of (DU · AS) and recenter the DU variable at 0.015 as an example where one seeks to assess the statistical and economic meaningfulness of the interaction term:

$$\begin{aligned} &\log\left(\frac{\mathrm{p}}{1-\mathrm{p}}\right) \text{ where } \mathrm{p} = \Pr\left(\text{Vertical Integration}\right) \\ &= \delta_0 + \delta_1\left(\mathrm{DU}-0.015\right) + \delta_2\mathrm{AS} + \delta_3\left[\left(\mathrm{DU}-0.015\right)\cdot\mathrm{AS}\right] + \sum\delta_i\mathrm{Controls}_i + \varepsilon \end{aligned} \tag{18}$$

To implement this recentered regression, one can follow the steps below:

1. Before running any of the logit models that include the interaction term (DU · AS), set the entire range of sample values of DU recentered at the specific point of DU of interest (here 0.015) by subtracting the value of interest (0.015) from each sample value of DU, so that the DU value of interest becomes zero (0.015−0.015) in a newly recentered DU variable.
2. Replace the original DU variable with the newly recentered DU variable (DU−0.015) in the model. Do this for both the main effect variable and the interacted variable of the interaction term as shown in equation (18). Perform the recentered logit regression.
3. Look for the p-value associated with the parameter estimate (δ_2) for AS from the recentered logit model as it indicates the statistical meaningfulness of the log of odds ratio for AS=1 vs. AS=0 when DU=0.015.
4. Run the logit model recentered at a different DU point of interest, if desired by theory, by repeating steps 1–3 above.
5. For graphical illustrations, if desired, plot the predicted probabilities (make=1) against the range of sample values of DU using the Stata command 'marginsplot' after logit, as shown in Figure 1.

[Insert Figure 1 near here]

6. In order to conduct a substantive analysis of the theory-driven effect size or the economic meaningfulness which measures the strength of the association between the event (here AS=1) when DU=0.015 and the outcome (here make =1), one can refer to Table 2 where we provide a statistical framework for assessing the effect size of the log of odds ratio in terms of Cohen's *d* for its easier interpretation based on a widely-accepted method in statistics and health sciences (Chen et al., 2010).

In Table 3, we show the results of our logit model examples (Models IV A, V, and VI) from the recentering approach using the steps above. The baseline log of odds ratio (baseline ratio) in Table 3 denotes the estimated coefficient of AS from each logit model. Given the nested data structure at the firm level in Leiblein and Miller (2003)'s data, we report the results from the recentering approach using more appropriate cluster-robust standard errors (S.E.) (Moulton, 1990) in addition to the original S.E.s that Leiblein and Miller (2003) used. $P > |z|$ denotes two-tailed p-values for z statistics. As expected, the results with cluster-robust S.E.s for each model in Table 3 are less statistically meaningful. For example, in Model IV A, the baseline ratio (-1.158) without cluster-robust S.E.s is marginally meaningful in a statistical sense with the p-value equal to 0.053, whereas the same ratio with cluster-robust S.E.s is no longer statistically meaningful. Similarly, when demand uncertainty is recentered at 0.074, the associated log of odds ratio for each model (1.127 for Model IV A and 1.443 for Model VI) becomes no longer statistically meaningful with the p-value equal to 0.106 and 0.129 respectively when using cluster-robust S.E.s. The same holds true in Models V and VI: when demand uncertainty is recentered at 0.106, the associated log of odds ratio (2.041 and 2.586 for Models V and VI respectively) with cluster robust S.E.s becomes barely statistically meaningful (p-value = 0.093 and 0.073 respectively).

[Insert Table 3 near here]

Lastly, as introduced above, one can assess the effect size of the interaction term using Table 2 after running the recentered regression. For example, in Model IV A of Table 3, the size of the ratio with demand uncertainty recentered at 0.194 is estimated as 4.833 which is statistically meaningful. Since the ratio of 4.833 is greater than 1.61 (which corresponds to Cohen's $d > 0.8$), according to

Table 2, one can say that there is a large interaction effect for a one-unit change in asset specificity when demand uncertainty is equal to 0.194.

CONCLUSION

Our recentering approach is intended to provide strategic management researchers with an additional option when it comes to assessing interaction effects in nonlinear models. Instead of requiring the researcher to know how to make the assumptions about each control variable and to enter in assumed values for implementation, the recentering approach, through a simple mathematical transformation, takes the data and model as is and tells the researcher what the statistical and economic meaningfulness of the interaction effect is at that chosen point in the data distribution. What it takes to implement the recentering approach is to recenter the regression and implement it. If the independent variable of interest has 10 different values, one might have to run the recentered regression 10 times. Even that number of 10 regressions would be fewer if one's theory, as seen in Jeong and Siegel (2018), were explicitly focused on the upper, middle, or lower part of the distribution of a variable of interest.

In conclusion, the recentering method provides researchers a consistent answer through an efficient way. With the recentering approach, one can assess not just statistical meaningfulness of the interaction effect at each and every point along the spectrum in a nonlinear model, but also economic meaningfulness/effect size of the interaction term. The recentering method also can be easily applied to a situation where a nonlinear model specifies more than one interaction term. We recommend the recentering method to strategy researchers for its consistency in results, its ability to assess both statistical and economic meaningfulness, its methodological efficiency, and its relative simplicity in implementation.

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Table 1a. Results from Zelner (2009)'s simulation-based approach for Leiblein and Miller's (2003) model IV A

Independent variables	Logit Model IV A							
	Coefficient estimates	P > |z|	Simulated coefficients $Mean_{1,000}$	95% C.I.		Simulated coefficients $Mean_{10,000}$	95% C.I.	
				$Lower_{1,000}$	$Upper_{1,000}$		$Lower_{10,000}$	$Upper_{10,000}$
DEMAND UNCERTAINTY	-16.795	0.007	-16.990	-29.574	-4.565	-16.803	-28.989	-4.228
ASSET SPECIFICITY	-1.158	0.053	-1.167	-2.345	0.020	-1.157	-2.335	0.016
ASSET SPECIFICITY * UNCERTAINTY	30.886	0.002	31.427	10.917	51.871	30.847	11.408	49.838
FIRM SIZE	0.214	0.056	0.217	-0.013	0.444	0.214	-0.001	0.430
FIRM TENURE	0.076	0.000	0.077	0.038	0.117	0.076	0.037	0.114
US FIRM	-0.367	0.447	-0.342	-1.282	0.571	-0.368	-1.336	0.583
JAPANESE FIRM	-0.092	0.884	-0.090	-1.415	1.173	-0.090	-1.324	1.151
OTHER ASIAN FIRM	0.219	0.760	0.213	-1.188	1.515	0.214	-1.191	1.648
EX ANTE SMALL NUMBERS	-0.342	0.009	-0.349	-0.590	-0.104	-0.343	-0.601	-0.089
SMALL NUMBERS SQUARED	0.012	0.002	0.012	0.005	0.019	0.012	0.004	0.019
FABRICATION EXPERIENCE	0.187	0.000	0.184	0.113	0.260	0.187	0.113	0.264
SOURCING EXPERIENCE	-0.269	0.000	-0.271	-0.386	-0.150	-0.269	-0.381	-0.155
INTERCEPT	0.924	0.483	0.944	-1.502	3.375	0.932	-1.652	3.539

DD $\hat{\pi}_{.194,\,.015}$	1.003	0.809	0.202	1.222	0.800	0.190	1.220

Notes. Subscripts 1,000 and 10,000 denote the number of simulation sets using CLARIFY (Tomz et al., 2003). C.I. stands for confidence interval. The last row of this table ($\Delta\Delta\hat{\pi}_{.194,\,.015}$) shows the original and simulated double differences (DD) in predicted probabilities ($\Delta\hat{\pi}_{.194} - \Delta\hat{\pi}_{.015}$) associated with an increase in ASSET SPECIFICITY from zero to one when DEMAND UNCERTAINTY is set to a value of 0.194 ($\Delta\hat{\pi}_{.194}$) and set to a value of 0.015 ($\Delta\hat{\pi}_{.015}$) and all other variables in the model set to a value of zero, respectively.

Table 1b. Results from Zelner (2009)'s simulation-based approach for Leiblein and Miller's (2003) model V

	Logit Model V							
Independent variables	Coefficient estimates	P > |z|	Simulated coefficients Mean$_{1,000}$	95% C.I. Lower$_{1,000}$	95% C.I. Upper$_{1,000}$	Simulated coefficients Mean$_{10,000}$	95% C.I. Lower$_{10,000}$	95% C.I. Upper$_{10,000}$
DEMAND UNCERTAINTY	-21.940	0.001	-21.808	-35.571	-8.153	-21.954	-34.842	-9.009
ASSET SPECIFICITY	-1.662	0.012	-1.628	-2.969	-0.356	-1.667	-2.963	-0.370
ASSET SPECIFICITY * UNCERTAINTY	34.931	0.001	34.831	13.647	55.916	34.978	13.063	56.609
FIRM SIZE	0.390	0.007	0.387	0.097	0.684	0.391	0.110	0.668
FIRM TENURE	0.069	0.001	0.069	0.029	0.108	0.069	0.029	0.110
US FIRM	-0.366	0.462	-0.382	-1.344	0.558	-0.371	-1.360	0.598
JAPANESE FIRM	0.148	0.818	0.134	-1.069	1.300	0.146	-1.095	1.413
OTHER ASIAN FIRM	-0.582	0.470	-0.601	-2.255	0.998	-0.583	-2.154	1.000
EX ANTE SMALL NUMBERS	-0.374	0.028	-0.378	-0.709	-0.053	-0.374	-0.703	-0.043
SMALL NUMBERS SQUARED	0.011	0.026	0.011	0.001	0.021	0.011	0.001	0.020

FABRICATION EXPERIENCE HAT	0.305	0.000	0.301	0.212	0.394	0.306	0.210	0.404
SOURCING EXPERIENCE HAT	-0.611	0.000	-0.605	-0.927	-0.278	-0.609	-0.921	-0.300
DIVERSIFICATION STRATEGY	1.885	0.000	1.886	0.863	2.969	1.880	0.847	2.894
DIVERSIFICATION SQUARED	-0.312	0.000	-0.310	-0.463	-0.166	-0.311	-0.454	-0.163
INTERCEPT	-0.217	0.893	-0.188	-3.355	2.869	-0.202	-3.314	2.916
DD $\hat{\pi}_{.194,\,.015}$	0.854		0.713	0.040	1.326	0.717	0.039	1.330

Notes. Subscripts 1,000 and 10,000 denote the number of simulation sets using CLARIFY (Tomz et al., 2003). C.I. stands for confidence interval. The last row of this table ($\Delta\Delta\hat{\pi}_{.194,\,.015}$) shows the original and simulated double differences (DD) in predicted probabilities ($\Delta\hat{\pi}_{.194} - \Delta\hat{\pi}_{.015}$) associated with an increase in ASSET SPECIFICITY from zero to one when DEMAND UNCERTAINTY is set to a value of 0.194 ($\Delta\hat{\pi}_{.194}$) and set to a value of 0.015 ($\Delta\hat{\pi}_{.015}$) and all other variables in the model set to a value of zero, respectively.

Table 1c. Results from Zelner (2009)'s simulation-based approach for Leiblein and Miller's (2003) model VI

Independent variables	Logit Model VI							
	Coefficient estimates	P > |z|	Simulated coefficients	95% C.I.		Simulated coefficients	95% C.I.	
			$\text{Mean}_{1,000}$	$\text{Lower}_{1,000}$	$\text{Upper}_{1,000}$	$\text{Mean}_{10,000}$	$\text{Lower}_{10,000}$	$\text{Upper}_{10,000}$
DEMAND UNCERTAINTY	-19.869	0.011	-20.097	-35.408	-4.941	-19.888	-35.132	-4.314
ASSET SPECIFICITY	-1.198	0.116	-1.206	-2.652	0.340	-1.196	-2.699	0.295
ASSET SPECIFICITY * UNCERTAINTY	35.693	0.006	36.304	9.778	63.901	35.660	9.925	61.202
FIRM SIZE	0.174	0.300	0.177	-0.167	0.519	0.174	-0.149	0.498
FIRM TENURE	0.034	0.209	0.035	-0.019	0.090	0.034	-0.019	0.086
US FIRM	-1.533	0.020	-1.505	-2.796	-0.266	-1.533	-2.857	-0.224

JAPANESE FIRM	0.440	0.592	0.446	-1.227	2.035	0.442	-1.153	2.050
OTHER ASIAN FIRM	0.295	0.750	0.276	-1.508	2.015	0.288	-1.520	2.114
EX ANTE SMALL NUMBERS	-0.511	0.003	-0.517	-0.831	-0.194	-0.512	-0.848	-0.187
SMALL NUMBERS SQUARED	0.014	0.004	0.014	0.005	0.023	0.014	0.005	0.024
FABRICATION EXPERIENCE	0.184	0.000	0.179	0.082	0.273	0.184	0.089	0.283
SOURCING EXPERIENCE	-0.223	0.003	-0.226	-0.370	-0.078	-0.223	-0.370	-0.077
DIVERSIFICATION STRATEGY	2.886	0.000	2.891	1.560	4.143	2.877	1.612	4.132
DIVERSIFICATION SQUARED	-0.406	0.000	-0.406	-0.582	-0.219	-0.405	-0.584	-0.227
INTERCEPT	3.847	0.052	3.845	0.155	7.550	3.862	-0.030	7.783
DD $\hat{\pi}_{.194,\ .015}$	0.524		0.510	0.010	1.127	0.496	0.010	1.121

Notes. This model includes year fixed effects (1988-1996), but we do not report the results for brevity. C.I. stands for confidence interval. Subscripts 1,000 and 10,000 denote the number of simulation sets using CLARIFY (Tomz et al., 2003). The last row of this table ($\Delta\Delta\hat{\pi}_{.194,\ .015}$) shows the original and simulated double differences (DD) in predicted probabilities ($\Delta\hat{\pi}_{.194} - \Delta\hat{\pi}_{.015}$) associated with an increase in ASSET SPECIFICITY from zero to one when DEMAND UNCERTAINTY is set to a value of 0.194 ($\Delta\hat{\pi}_{.194}$) and set to a value of 0.015 ($\Delta\hat{\pi}_{.015}$) and all other variables in the model set to a value of zero, respectively.

Table 2. Odds ratio (OR) and the equivalent Cohen's *d* (Chen et al., 2010)

Odds ratio	Log of odds ratio	P_0																	
		0.01	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*	*0.10*	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*	*0.70*	*0.80*	*0.90*
1.1	0.10	**0.04**	**0.04**	**0.04**	**0.04**	**0.04**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**	**0.06**	**0.06**	**0.06**	**0.06**	**0.06**	**0.05**	**0.05**
1.2	0.18	**0.07**	**0.07**	**0.08**	**0.08**	**0.09**	**0.09**	**0.09**	**0.09**	**0.09**	**0.09**	**0.11**	**0.11**	**0.11**	**0.11**	**0.11**	**0.11**	**0.10**	**0.09**
1.3	0.26	**0.10**	**0.11**	**0.11**	**0.12**	**0.12**	**0.13**	**0.13**	**0.13**	**0.13**	**0.14**	**0.15**	**0.16**	**0.16**	**0.16**	**0.16**	**0.16**	**0.15**	**0.13**
1.4	0.34	**0.13**	**0.14**	**0.15**	**0.15**	**0.16**	**0.16**	**0.17**	**0.17**	**0.17**	**0.18**	0.20	0.21	0.21	0.21	0.21	0.20	**0.19**	**0.17**

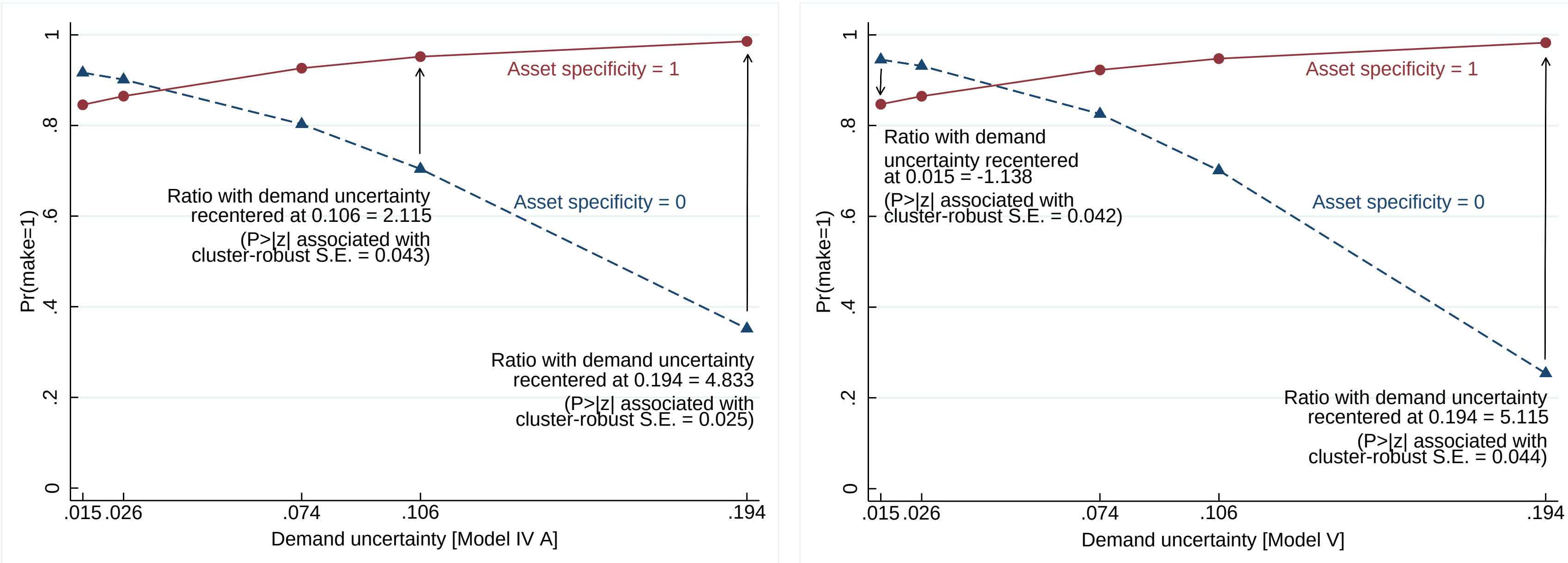
1.5	0.41	**0.15**	**0.17**	**0.18**	**0.19**	**0.19**	0.20	0.20	0.21	0.21	0.21	0.24	0.25	0.25	0.25	0.25	0.24	0.23	0.20
1.6	0.47	**0.18**	0.20	0.21	0.22	0.22	0.23	0.24	0.24	0.25	0.25	0.28	0.29	0.29	0.29	0.29	0.28	0.26	0.23
1.7	0.53	0.20	0.22	0.24	0.25	0.25	0.26	0.27	0.27	0.28	0.28	0.31	0.33	0.33	0.33	0.32	0.31	0.29	0.26
1.8	0.59	0.23	0.25	0.26	0.27	0.28	0.29	0.30	0.30	0.31	0.31	0.35	0.36	0.37	0.37	0.36	0.35	0.32	0.29
1.9	0.64	0.25	0.27	0.29	0.30	0.31	0.32	0.33	0.33	0.34	0.34	0.38	0.40	0.40	0.40	0.39	0.38	0.35	0.31
2	0.69	0.27	0.29	0.31	0.32	0.34	0.35	0.35	0.36	0.37	0.37	0.41	0.43	0.43	0.43	0.42	0.40	0.38	0.34
3	1.10	0.44	0.48	0.51	0.53	0.55	0.56	0.58	0.59	0.60	0.61	0.66	0.68	0.68	0.67	0.66	0.63	0.58	0.52
4	1.39	0.56	0.62	0.65	0.68	0.71	0.73	0.74	0.76	0.77	0.78	**0.84**	**0.86**	**0.86**	**0.84**	**0.81**	0.78	0.72	0.64
5	1.61	0.66	0.73	0.77	**0.81**	**0.83**	**0.85**	**0.87**	**0.89**	**0.90**	**0.92**	**0.98**	**1.00**	**0.99**	**0.97**	**0.93**	**0.89**	**0.83**	0.74
6	1.79	0.75	**0.82**	**0.87**	**0.91**	**0.94**	**0.96**	**0.98**	**1.00**	**1.02**	**1.03**	**1.09**	**1.11**	**1.09**	**1.07**	**1.03**	**0.98**	**0.91**	**0.81**
7	1.95	**0.82**	**0.90**	**0.96**	**1.00**	**1.03**	**1.06**	**1.08**	**1.10**	**1.11**	**1.12**	**1.19**	**1.20**	**1.18**	**1.15**	**1.11**	**1.05**	**0.98**	**0.87**
8	2.08	**0.89**	**0.98**	**1.03**	**1.08**	**1.11**	**1.14**	**1.16**	**1.18**	**1.19**	**1.21**	**1.27**	**1.28**	**1.26**	**1.22**	**1.17**	**1.11**	**1.03**	**0.92**
9	2.20	**0.94**	**1.04**	**1.10**	**1.15**	**1.18**	**1.21**	**1.23**	**1.25**	**1.27**	**1.28**	**1.34**	**1.35**	**1.32**	**1.28**	**1.23**	**1.17**	**1.08**	**0.97**
10	2.30	**1.00**	**1.10**	**1.16**	**1.21**	**1.25**	**1.27**	**1.30**	**1.32**	**1.33**	**1.35**	**1.41**	**1.41**	**1.38**	**1.34**	**1.28**	**1.21**	**1.13**	**1.01**
15	2.71	**1.21**	**1.33**	**1.40**	**1.46**	**1.50**	**1.53**	**1.55**	**1.57**	**1.59**	**1.60**	**1.65**	**1.63**	**1.59**	**1.53**	**1.47**	**1.39**	**1.29**	**1.16**
20	3.00	**1.36**	**1.50**	**1.58**	**1.64**	**1.68**	**1.71**	**1.73**	**1.75**	**1.76**	**1.78**	**1.81**	**1.78**	**1.73**	**1.67**	**1.60**	**1.51**	**1.40**	**1.26**
25	3.22	**1.49**	**1.64**	**1.72**	**1.78**	**1.82**	**1.85**	**1.87**	**1.89**	**1.90**	**1.91**	**1.93**	**1.89**	**1.84**	**1.77**	**1.69**	**1.60**	**1.49**	**1.34**
30	3.40	**1.60**	**1.75**	**1.83**	**1.89**	**1.93**	**1.96**	**1.98**	**2.00**	**2.01**	**2.02**	**2.03**	**1.98**	**1.92**	**1.85**	**1.77**	**1.67**	**1.56**	**1.40**

Notes. P_0: rate of outcome of interest in the group with no event. In the case of our example, it is the rate of vertical integration in the group of firms where asset specificity=0 holding demand uncertainty and all the other covariates constant. OR = $P(1-P_0)/P_0(1-P)$ where P is the rate of outcome of interest in the group with an event. The rows are the values of odds ratio (and equivalent log of odds ratio for each odds ratio). For each row, one sees the equivalent values of Cohen's d for a given log of odds ratio and given P_0. Values in bold indicate Cohen's $d < 0.20$ or > 0.80, where Cohen's $d < 0.20$ means that the economic significance of the log of odds ratio result would be considered economically small, and where Cohen's $d > 0.80$ means that the economic significance of the log of odds ratio would be considered economically large. Anything in between 0.20 and 0.80 should be regarded as a continuous line in which those values of Cohen's d closest to 0.20 are quite small, those in the middle are of approximately medium economic significance, and those closest to 0.80 are close to have large economic significance. Cohen's $d = Z_0 - Z$ (standardized mean difference) where Z_0 is the standard normal deviation for P_0 and Z is the standard normal deviation for P.

Table 3. Results from the recentering approach for Leiblein and Miller's (2003) models

The Recentering approach with different standard errors (S.E.) using Leiblein and Miller (2003)'s logistic regression models	Logit Model IV A[b]					Logit Model V[c]					Logit Model VI[d]				
	Coeff. estimate (size of the ratio)	Original S.E.	P > |z| (S.E.)	Cluster robust S.E.	P > |z| (cluster-robust S.E.)	Coeff. estimate (size of the ratio)	Original S.E.	P > |z| (S.E.)	Cluster robust S.E.	P > |z| (cluster-robust S.E.)	Coeff. estimate (size of the ratio)	Original S.E.	P > |z| (S.E.)	Cluster robust S.E.	P > |z| (cluster-robust S.E.)
Baseline ratio[a]	-1.158	0.599	0.053	0.712	0.104	-1.662	0.663	0.012	0.714	0.020	-1.198	0.761	0.116	0.898	0.182
Ratio with demand uncertainty recentered at 0.015	-0.695	0.502	0.166	0.585	0.235	-1.138	0.554	0.040	0.559	0.042	-0.663	0.633	0.295	0.728	0.363
Ratio with demand uncertainty recentered at 0.025	-0.386	0.452	0.393	0.527	0.463	-0.789	0.499	0.114	0.493	0.109	-0.306	0.569	0.591	0.656	0.641
Ratio with demand uncertainty recentered at 0.026	-0.355	0.448	0.428	0.523	0.496	-0.754	0.495	0.128	0.488	0.123	-0.270	0.564	0.632	0.651	0.678
Ratio with demand uncertainty recentered at 0.027	-0.325	0.445	0.465	0.519	0.531	-0.719	0.491	0.143	0.484	0.138	-0.234	0.559	0.675	0.647	0.717
Ratio with demand uncertainty recentered at 0.074	1.127	0.503	0.025	0.696	0.106	0.923	0.559	0.099	0.778	0.236	1.443	0.666	0.030	0.951	0.129
Ratio with demand uncertainty recentered at 0.106	2.115	0.731	0.004	1.044	0.043	2.041	0.816	0.012	1.215	0.093	2.586	0.982	0.008	1.442	0.073
Ratio with demand uncertainty recentered at 0.194	4.833	1.525	0.002	2.162	0.025	5.115	1.705	0.003	2.539	0.044	5.727	2.051	0.005	2.972	0.054

Notes. Vertical integration is the binary outcome variable of all three models. [a] Baseline ratio denotes the estimated coefficient of Asset Specificity from each model used here. [b] Model IV A includes Asset Specificity, Demand Uncertainty, Asset Specificity * Demand uncertainty, Fabrication Experience, Sourcing Experience, and other control variables. [c] Model V uses Fabrication Experience Hat and Sourcing Experience Hat (instead of Fabrication Experience and Sourcing Experience) and adds Diversification Strategy and Diversification Squared to Model IV A. [d] Model VI adds Diversification Strategy, Diversification Squared, and year fixed effects to Model IV A. Because the data used for Leiblein and Miller (2003) were clustered at the firm level, we use cluster robust S.E.s (Moulton, 1990) that adjust for clustering at the firm level. P > |z| denotes two-tailed p-values for z statistics.


Pr(make=1)
0
.2
.4
.6
.8
1
Asset specificity = 1
Asset specificity = 0
Ratio with demand uncertainty recentered at 0.106 = 2.115
(P>|z| associated with cluster-robust S.E. = 0.043)
Ratio with demand uncertainty recentered at 0.194 = 4.833
(P>|z| associated with cluster-robust S.E. = 0.025)
.015 .026
.074
.106
.194
Demand uncertainty [Model IV A]
Pr(make=1)
0
.2
.4
.6
.8
1
Asset specificity = 1
Asset specificity = 0
Ratio with demand uncertainty recentered at 0.015 = -1.138
(P>|z| associated with cluster-robust S.E. = 0.042)
Ratio with demand uncertainty recentered at 0.194 = 5.115
(P>|z| associated with cluster-robust S.E. = 0.044)
.015 .026
.074
.106
.194
Demand uncertainty [Model V]

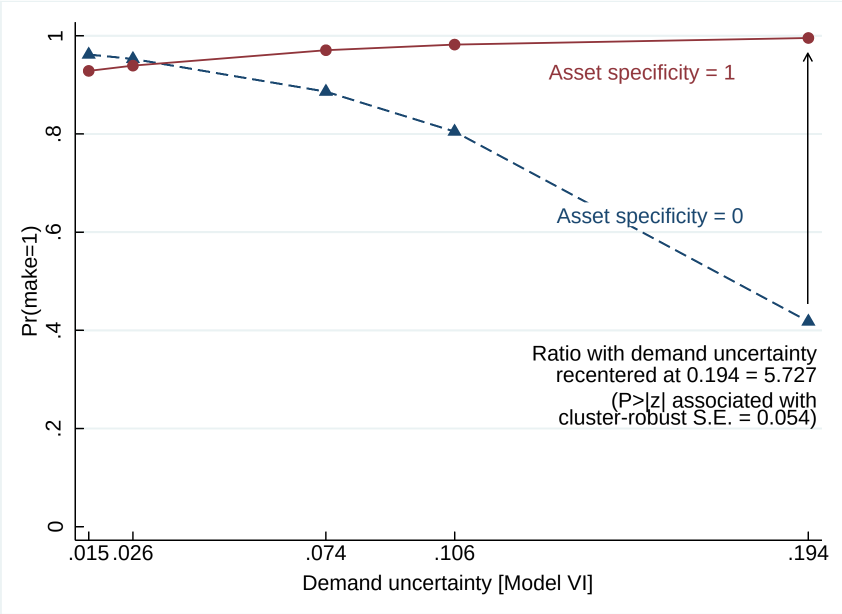


Figure 1. Statistical meaningfulness of interaction effects at specific demand uncertainty levels for the 'mean values' firm based on recentering approach using Leiblein and Miller (2003)'s three Models IV A, V and VI with cluster-robust S.E.s and with all other non-binary variables in each model set to their estimating sample means and all other binary variables set to their estimating sample modes

A RECENTERING APPROACH FOR INTERPRETING INTERACTION EFFECTS FROM LOGIT, PROBIT, AND OTHER NONLINEAR MODELS

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Running head: A Recentering Approach for Nonlinear Interaction Effects

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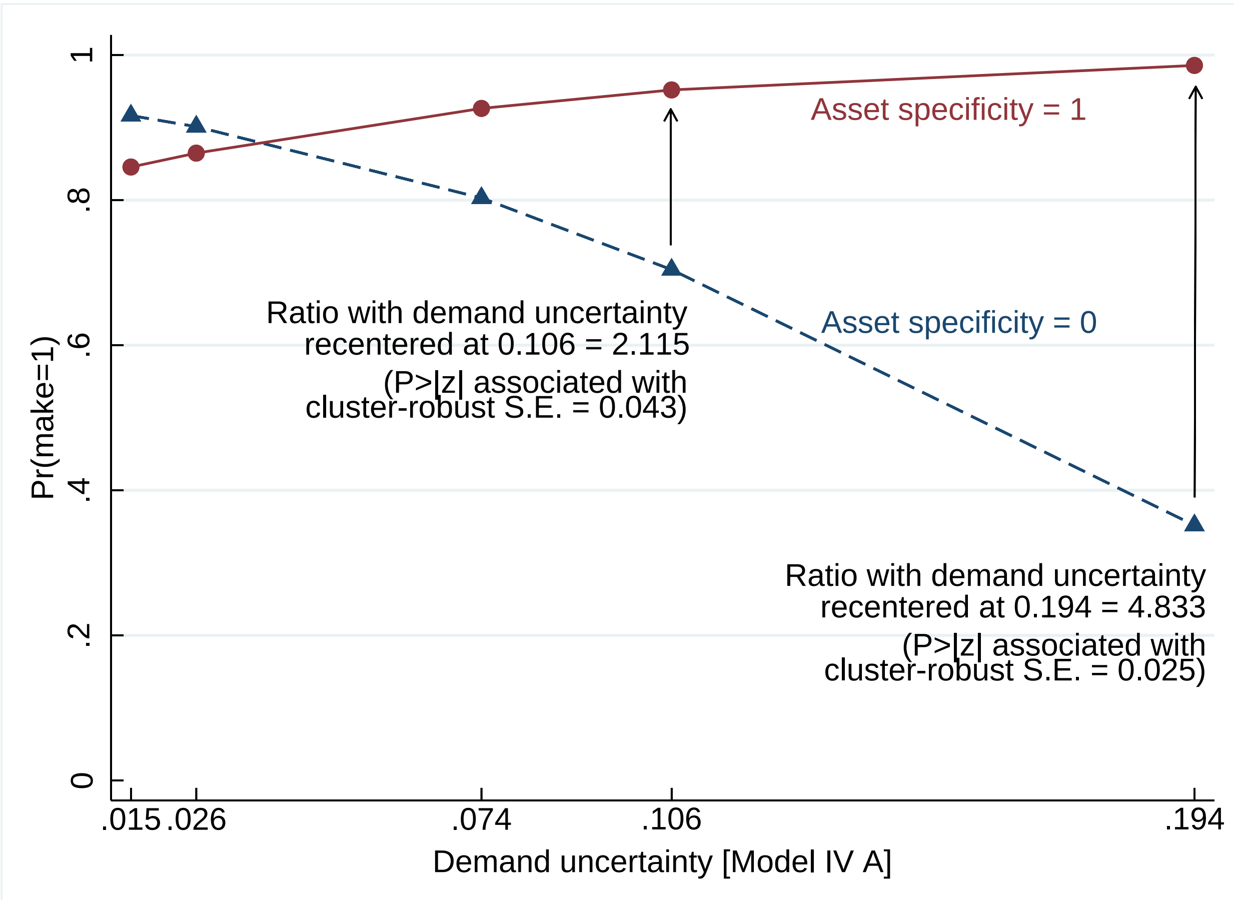
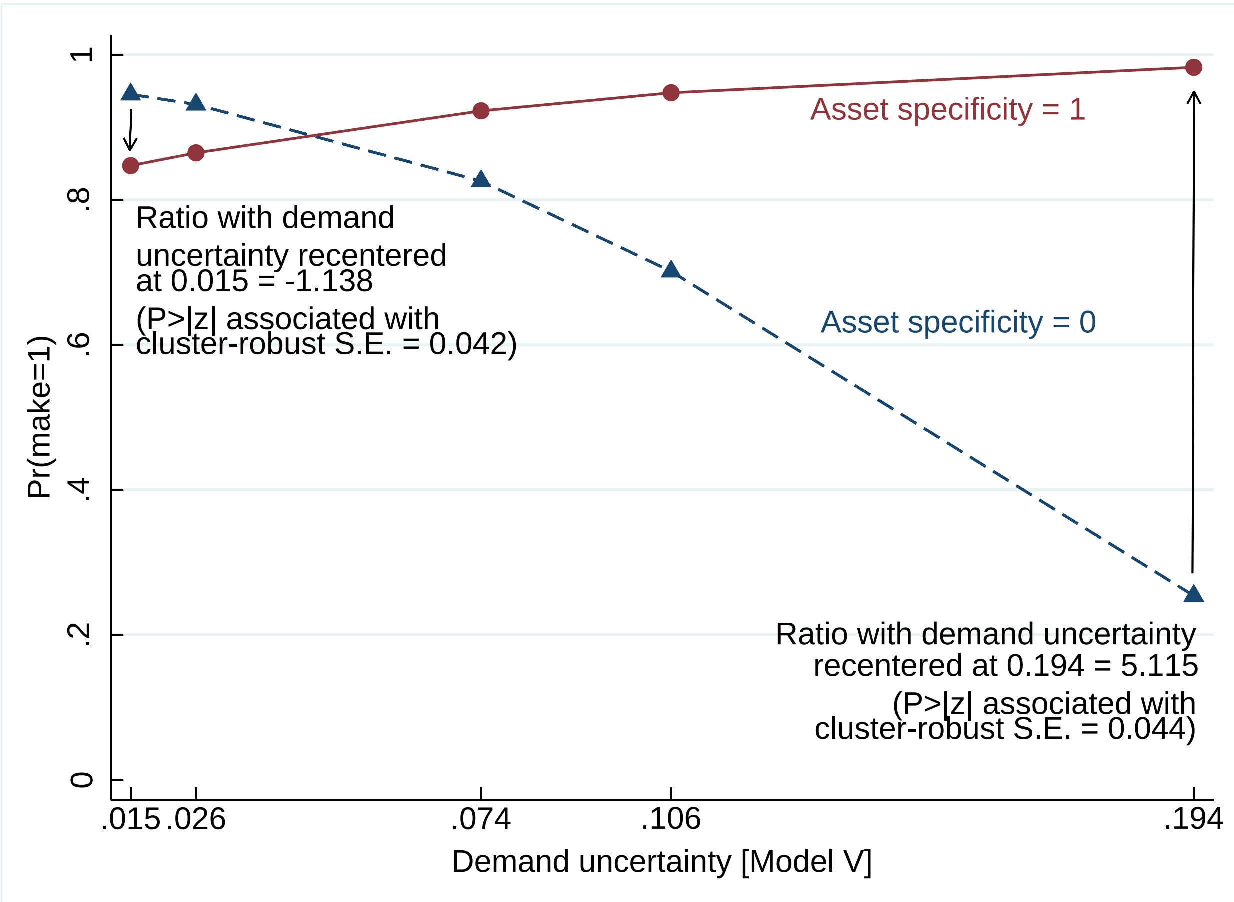
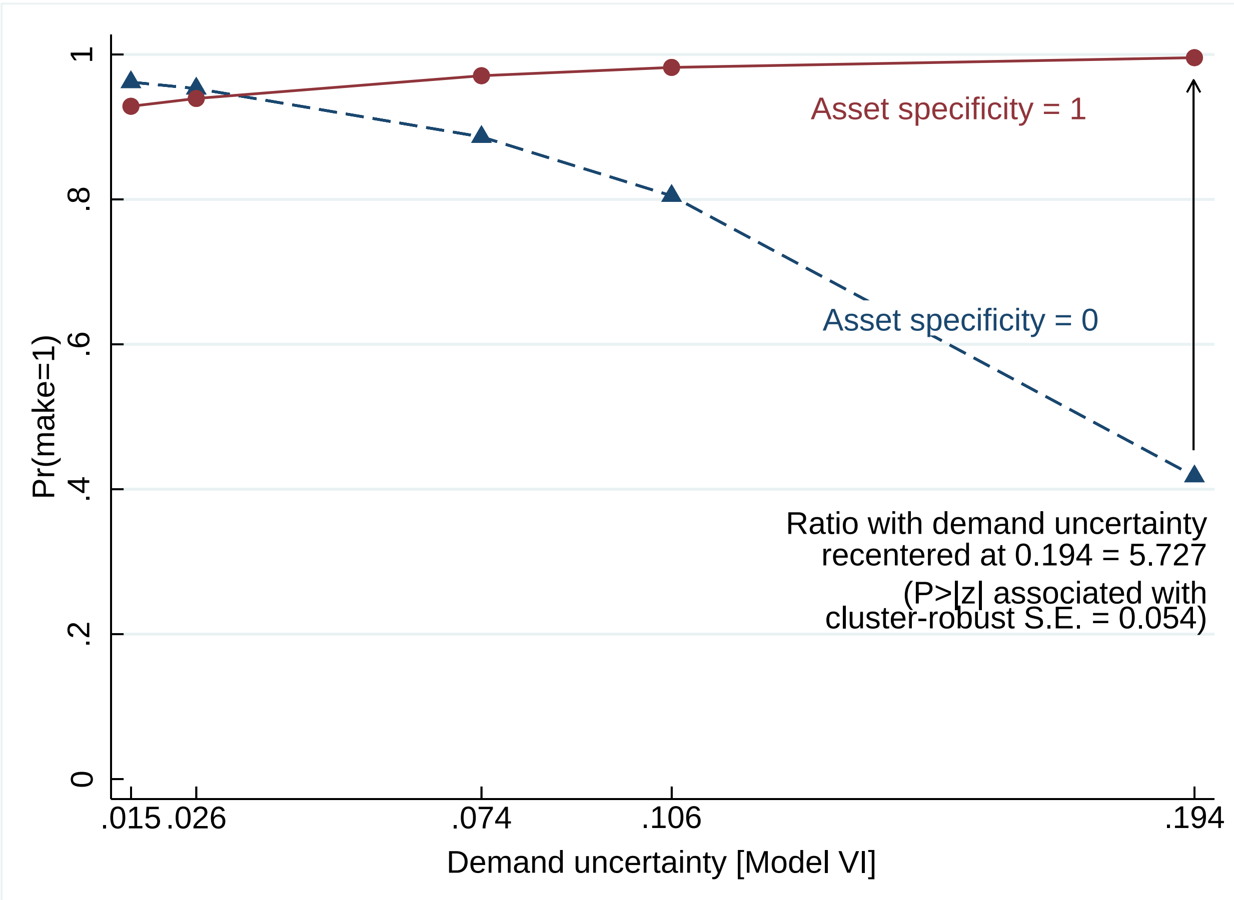




1
.8
.6
.4
.2
0
Pr(make=1)
Asset specificity = 1
Asset specificity = 0
Ratio with demand uncertainty recentered at 0.015 = -1.138
(P>|z| associated with cluster-robust S.E. = 0.042)
Ratio with demand uncertainty recentered at 0.194 = 5.115
(P>|z| associated with cluster-robust S.E. = 0.044)
.015
.026
.074
.106
.194
Demand uncertainty [Model V]


Pr(make=1)
0
.2
.4
.6
.8
1
.015
.026
.074
.106
.194
Demand uncertainty [Model VI]
Asset specificity = 1
Asset specificity = 0
Ratio with demand uncertainty
recentered at 0.194 = 5.727
(P>|z| associated with
cluster-robust S.E. = 0.054)

ONLINE APPENDIX

Why the 'margins' Approach in Stata Does Not Sufficiently Solve the Weakness of the Simulation Method

Soon after Zelner (2009) was written, Stata introduced in July 2009 an updated version (version 11) of its software which superseded the simulation approach.[1] Previously, Stata did not have the functionality to automate both the calculation of interaction terms and their marginal effects. This means that one had to do the hundreds of lines of Stata programming that went behind the simulation method. Starting with July 2009, Stata began to offer such functionality with its 'margins' command.

The 'margins' command just takes the logistic regression and transforms it using the delta method (Wooldridge, 2010, p. 47) to get the change in probability. The simulation approach is supposed to give the same answer as what the simple 'margins' command produces. If there should be any difference in the results between the simulation method and the 'margins' command, it will be due to the randomness of the simulation approach and its simulated data. Yet this randomness should be controlled to be a very small randomness that does not affect the final output. With Stata's 'margins' command, one no longer needs to use user-written commands such as 'CLARIFY' and 'intgph' (Tomz, Wittenberg, & King., 2003; Zelner, 2009) for assessing interaction terms in nonlinear models anymore. This enables one to avoid many steps in the simulation that can go wrong because one has to make numerous assumptions for any simulation model behind the scenes.

Although the 'margins' command has an advantage over user-written commands for the simulation approach for its substantially improved simplicity, it should be noted that the 'margins' approach also needs to make highly consequential assumptions about the values of the covariates just like the simulation method does. In particular, what the 'margins' command does is to produce mean predicted probabilities, calculated across the observations in the estimation sample and subject to the

[1] https://www.stata.com/support/faqs/resources/history-of-stata/

'at' option in the 'margins' command where one can fix certain covariates at a chosen value or a set of values, and subject to integrating across all the other control variables (which will be explained further below). At this point, the 'margins' approach could have theoretically been designed to do either of two things: to assume the covariate values at their means or to integrate over them. The problem with assuming covariate values at their means is that often one has a series of categorical variables as controls. Yet it would be meaningless to have those categorical variables take on a value of 0.5, for example. In the case of an individual-level sample controlling for the often-important categorical variable of gender, for example, it would be simply erroneous to treat the entire sample as being the new entity of half-male/half-female. The problem with assuming categorical variables at their mean is that there is no such thing as half-and-half and thus the sample does not represent anyone. As a result, the output in this case would not generalize to the general population.

If one were to instead pursue the alternative of calculating margins by integrating over the control variables, which is the default for the 'margins' command, this also does not help get interaction results that can provide inference about the larger population of interest. Integrating means taking the original sample data and taking a group within the data like gender, which has a binary distribution, and assuming that the distribution in the sample in terms of, for example, gender is the same as the population distribution of interest. So if one looks at the whole population of interest, what the 'margins' command will produce will be the change in probability given the proportion of male and female staying the same in the general population as was in the sample. But as soon as the proportion of male and female in the general population turns out to be different, the results from the 'margins' command cannot be used to make inference to the general population. In fact, the potential for problems is yet more severe when we consider that the 'margins' command assumes that the distribution for every single continuous as well as categorical variable in the sample is the same as in the general population of interest. Note that even when just one assumption for one control variable is wrong, there can be erroneous inference about the general population of interest. Yet quite often in applied research, a researcher does not know for sure whether the distribution of every covariate is the same as the distribution in the population of interest. And all it takes is for at least one assumption to be wrong for the study's conclusion to be perhaps wrong.

In the field of statistics, there is a commonly shared desire to use methods that deliver answers that are robust to alternative assumptions—or that do not depend on one perhaps questionable assumption. When focusing on the log odds ratio, one does not require any assumption about the proportion of male and female in the general population. In contrast, the 'margins' command, like the simulation method, requires one to have the correct assumption about the distribution of every subgroup in the data for the output to be generalizable. One would prefer a method that does not require this assumption about the distribution of every subgroup in the data, and we discuss below why this is the case using two illustrative examples.

In numerous real-world instances, there is something unknown about the true distribution proportions in the population of interest. One good illustrative example was the 2016 U.S. presidential election results. Nearly all the experts doing fine-grained statistical analysis predicting the election results on the day of the election based on polling and prior distributions of groups in the electorate were surprised that Trump got elected as president, but in statistical terms (a less than 80,000 vote difference in the three tipping states of Wisconsin, Michigan, and Pennsylvania), it was because of a small difference in the distribution between polled samples and actual voter population. There was very little bias in terms of the sample compared to the population. Yet the small difference in distribution results in a meaningful difference in terms of the final predicted outcome. But if you do a logistic regression and look at only the log odds ratio, then this small difference in proportions will not affect the log odds ratio answer and its generalizability.

To take another illustrative example, in the total population, one conjectures that the number of U.S. children exposed to lead would be much smaller than that of U.S. children not exposed to lead. To study the effect of exposure to lead on child development by gender, consider that one collected a sample whose ratio of lead exposure to non-exposure was 1:2. If in the whole true population the same ratio were 1:1,000, then the researcher using the 'margins' command on gender in a nonlinear setting would see a biased result (in terms of the difference in probability) that cannot be generalized to the true population. One may then wonder if the 'margins' command would generate an unbiased result should one collect a sample whose ratio of lead exposure to non-exposure is 1:1000, exactly the same as the true population. Theoretically one could, but practically it would not be feasible to collect a far greater number of samples to achieve the same testing power of the sampling design of 1:2. It would not be viable particularly when the study has a limited budget and acquiring a sample is costly. In contrast, examining the effect of exposure to lead on child development by gender using the log odds ratio will provide a consistent and robust answer. In fact, using the log odds ratio will provide a consistent and robust answer regardless of what the proportion turns out to be in the general population for the number of children exposed to lead relative to those not exposed to lead. In summary, these illustrative examples help show why one would prefer a method that does not rely on knowing every subgroup proportion in the general population to a method that critically relies on having the correct assumptions about every subgroup proportion.

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