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Supporting Information for

Conditions of loss cone filling by scattering on the curved field lines for 30 keV protons during geomagnetic storm as inferred from numerical trajectory tracing

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Introduction

The main paper is based on the results of the numerically tracing of the particle trajectories in the magnetic field of the magnetospheric model. The particle trajectories are integrated using the method with an adaptive time-step. The accuracy of tracing should be such that to provide the final error in the particle pitch angle within the desired fraction of the atmospheric loss cone size. In the main paper, we present the equation for the calculation of the time step size as a function of the allowed pitch angle error at the final point of the particle trajectory, local magnetic field, and the magnetic field gradient at the final point of the particle trajectory (Equation 7). In S1, we present a derivation of this equation.

Text S1.

In our study, we need to know the location of the particle in the velocity space with respect to the edge of the loss cone. In other words, we need to know accurately enough the local particle pitch angle and the size of the loss cone. The numerical error accumulated during the trajectory integration has two components: the error in the velocity vector and the error in the spatial coordinate. The error in the velocity vector obviously results in the pitch angle error. The error in the spatial coordinate results in the model magnetic field vector being calculated at wrong location. This means that both magnetic field direction and magnitude are incorrect. The former results again in the pitch angle error and latter results in the loss cone size error.

1.1 Pitch-angle error due to inaccuracy of the final velocity estimation

It has been found that the relative error in particle velocity after one time-step can be approximated by following equation:

$$\frac{\delta V}{V} = C \cdot \Delta t^5 \cdot B^4$$

and the absolute error:

$$\delta V = C \cdot V \cdot \Delta t^5 \cdot B^4$$

Here, C is the constant ($C = 10^{-4}\ s^{-5}\ nT^{-4}$), V is the full particle velocity, h is the time-step size, B is the magnetic field magnitude.

We assume that the particle parallel velocity can be approximated as V (small pitch-angles). Then, the time (T) during which particle passes distance dS along a field line can be estimated as:

$$T = \frac{dS}{V},$$

and we need to do dN time-steps to trace particle along this distance:

$$dN = \frac{T}{\Delta t} = \frac{dS}{V \Delta t}$$

The velocity error accumulated on the element of field line dS can be estimated as:

$$d\delta V = dN \cdot \delta V = dN \cdot C \cdot V \cdot \Delta t^5 \cdot B^4 = dS \cdot C \cdot \Delta t^4 \cdot B^4$$

Then total accumulated error is:

$$\Delta V = C \cdot \int \Delta t^4 \cdot B^4 dS$$

It is convenient to define h so that $h^4 B^4 = g$, where g is a constant, then

$$\Delta V = C \cdot g \cdot \int dS = C \cdot g \cdot L$$

Here L is the length of the field line. Then the accumulated error in pitch angle related to numerical error in velocity vector is

$$\Delta \alpha_V = \frac{\Delta V}{V} = \frac{C \cdot g \cdot L}{V}$$

1.2 Pitch-angle error resulting from the spatial coordinate error

1.2.1 Estimation of the spatial coordinate error.

We found that the relative error during the one time-step can be approximated as:

$$\frac{\delta r}{r} = A \cdot \Delta t^4 \cdot B^4$$

Here, A is the constant ($A=10^{-4}\ s^{-4}\ nT^{-4}$) and r is the displacement for one time-step which can be approximated as:

$$r = V \cdot \Delta t$$

Then, the absolute error is:

$$\delta r = A \cdot V \cdot \Delta t^5 \cdot B^4$$

We need to do *dN* time-steps to trace particle along *dS* distance.

$$dN = \frac{T}{\Delta t} = \frac{dS}{V\Delta t}$$

Thus, the error in the spatial coordinate accumulated on the element of the field line *dS* is:

$$d\delta r = A \cdot \Delta t^4 \cdot B^4 \cdot dS$$

The total error accumulated during particle trajectory tracing is:

$$\Delta r = \int A \cdot \Delta t^4 \cdot B^4 \cdot dS = A \cdot g \cdot L$$

Here, we again use the notation $g = h^4 B^4$

1.2.2 Pitch angle error resulting from the incorrect direction of magnetic field vector.

An estimate of the angular difference of the magnetic field at the true and estimated locations can be expressed as:

$$\Delta_{r1}\alpha = \frac{1}{B} \cdot \frac{\partial B}{\partial r} \Delta r$$

It can be assumed that it leads to the same error in the particle pitch angle.

Hereinafter we will use following estimation for the magnetic field gradient:

$$\frac{\partial B}{\partial r} = \sqrt{\sum_{k,i} \left| \frac{\partial B_i}{\partial x_k} \right|^2},$$

where $k,i=\{x,y,z\}$. The partial derivatives are estimated at the final point of the particle trajectory.

1.2.3 The loss cone size error resulting from the incorrect magnitude of magnetic field vector

Since we need to estimate the particle pitch-angle with respect to the angular size of the loss cone at the final point of the traced trajectory, the incorrect estimation of the loss cone size also leads to the undesired error. The loss cone size as a function of the local magnetic field magnitude (*B*) is expressed as:

$$\alpha_{LC}(B) = \arcsin\left(\sqrt{\frac{B}{B_{atmos}}}\right),$$

where B_{atmos} is the magnetic field magnitude at the altitude where the particles are lost due to collision with the atmosphere.

The loss cone size error can be estimated as:

$$\Delta\alpha_{r2} \approx \frac{\partial \alpha_{LC}}{\partial B} \cdot \frac{\partial B}{\partial r} \Delta r = \frac{1}{2} \cdot \frac{1}{\sqrt{B_{atmos} B \left(1 - \frac{B}{B_{atmos}}\right)}} \cdot \frac{\partial B}{\partial r} \Delta r \approx \frac{1}{2} \cdot \frac{1}{\sqrt{B_{atmos} B}} \cdot \frac{\partial B}{\partial r} \Delta r$$

Here, we assume that B_{atmos} > 20000 nT >> *B* which is reasonable because our trajectory tracing is stopped at the distance r = 3.5 R_E.

It can be shown that for the ratio of the two errors is:

$$\Delta_{r1}\alpha / \Delta_{r2}\alpha = 2 \cdot \sqrt{\frac{B_{atmos}}{B}} \geq 15$$

and we can neglect the latter error.

1.3 Final error estimation:

The total error can be computed as a sum of two aforementioned errors, that is:

$$\Delta_V \alpha + \Delta_{r1}\alpha = \frac{C \cdot g \cdot L}{V} + \frac{1}{B}\frac{\partial B}{\partial r} \cdot A \cdot g \cdot L = \left(\frac{C}{V} + \frac{A}{B}\frac{\partial B}{\partial r} \right) \cdot g \cdot L$$

For the allowed error in pitch-angle $\Delta\alpha_{allowed}$, we can estimate g parameter:

$$g = \frac{\Delta \alpha_{allowed}}{\left(\frac{C}{V} + \frac{A}{B}\frac{\partial B}{\partial r} \right) \cdot L}$$

$\partial B/\partial r$ and B are evaluated at the final point of particle trajectory.

Finally, recalling the definition of g, one can calculate the time-step size as:

$$\Delta t = \frac{\sqrt[4]{g}}{B}$$

Here, B is the local magnetic field magnitude.

Text S2.

Figure S1 shows the absolute error of the K parameter estimation by the analytical approximation of the MHD field versus time and MLT. Note for the majority of configurations median error is well within ΔK=1.

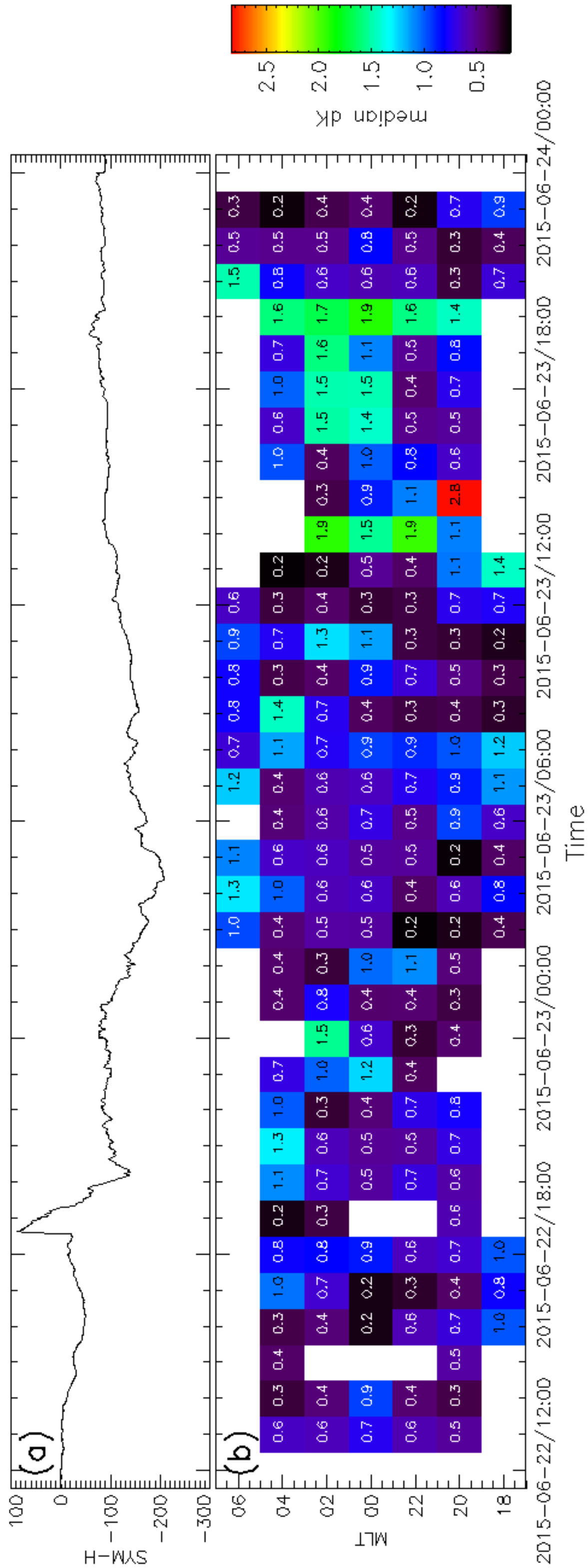

Figure S1. (a) SYM-H index. (b) Color shows the median error of the K parameter estimation by analytical approximation versus time and MLT. The numbers in the bins duplicate the color scale.

Text S3.

In Section 5 of the paper, we introduced criterion for the IB formation by FLC scattering mechanism. Since our criterion is adapted to the NOAA/POES detector characteristics, it may look artificial. To give the reader an idea about its relation to more general criterion, we calculated the percentage of the loss cone area filled when the central 1/3 zone of the loss cone becomes fully filled, that is, for K_{cr}^{low} value (black histogram in Figure S2). The red histogram shows the percentage of the loss cone area filled for K_{cr}^{up} value.

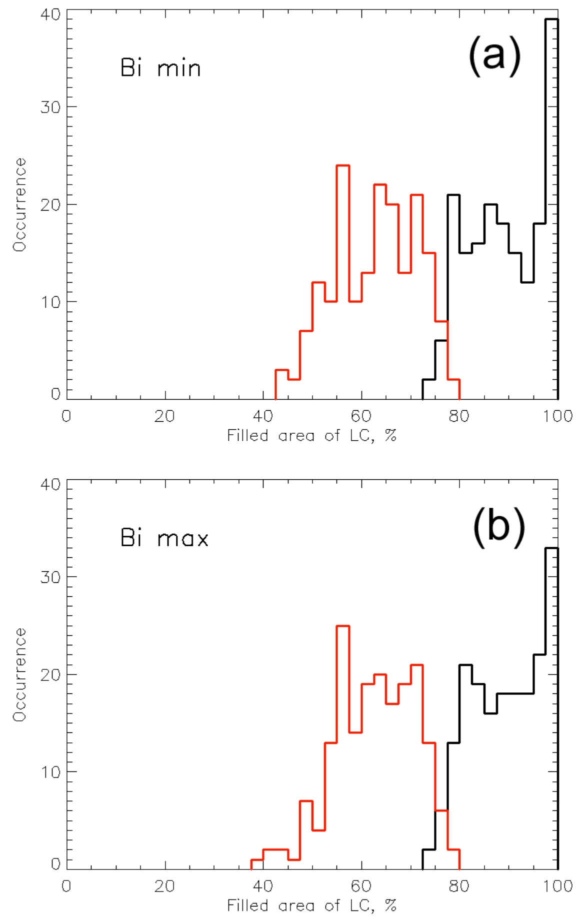

Figure S2. The figure shows the percentage of the loss-cone area filled corresponding to the K_{cr}^{low} and K_{cr}^{up} values. Panels (a) and (b) show the results obtained using minimum and maximum ionospheric field values.