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Game theory based bidding strategy for prosumers in a distribution system with a retail electricity market

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Abstract: Distributed energy resources (DERs) are deployed vastly to reduce carbon emission, improve power quality and maintain the reliability of distribution systems. With the introduction of new players, such as prosumers, which are constructed with DERs, distribution system operators (DSOs) are facing changes in the retail electricity market. Prosumers need a well-defined strategic bidding mechanism to maximize their operation revenue, while DSOs need a new market clearing mechanism for the changed retail electricity market. Thus, an innovative game-theoretic market framework for a prosumer-centric retail electricity market is proposed. A bilevel algorithm is adopted to model new features of DSOs, utility companies and prosumers. The supply function equilibrium model, Nikaido–Isoda functions, and relaxation algorithms are applied to analyse the competition among key participants in a retail electricity market. Extensive simulation results are employed to illustrate and validate the effectiveness of the proposed framework for bidding strategies of prosumers with a retail electricity market. Specifically, the strategy with dumping-bid or abnormal-bid from a prosumer is suppressed by the market operator in the model. Moreover, the sensitivity analysis shows that the proposed framework can handle various numbers of prosumers in the retail electricity market with reasonable computational time and convergence rate.

Nomenclature

Indices and sets

i	index of prosumer
k	index of the bus node
K	set of buses located downstream of the bus node k
N_i	number of prosumers
N_k	number of bus nodes
m	number of lower level constraints
n	number of iterations
$\vec{X}$	set of bidding strategies

Parameters

a_i	generation cost coefficients of ith prosumer ($/MW^{2}h)
b_i	generation cost coefficients of ith prosumer ($/MWh)
c_i	generation cost coefficients of ith prosumer ($)
M_{service_i}	'subscription fee' paid by the ith electricity prosumer to the utility company ($)
R_i	revenue of the ith prosumer ($)
C_i	cost of the ith prosumer ($)
$P_{i,\min}$	lower bound of the ith prosumer's energy capacity (MW)
$P_{i,\max}$	upper bound of the ith prosumer's energy capacity (MW)
P_{di}	active power demand of the ith prosumer (MW)
α_i	electricity price coefficient of the ith prosumer
β_i	electricity price coefficient of the ith prosumer
P_{base}	base load of the system except prosumers (MW)
P_{utility}	power supplied by the utility company to the system (MW)
c_{retail_u}	market clearing price of the utility company ($/MWh)
$\boldsymbol{A}$	symmetric positive semi-definite matrix
$\boldsymbol{x} \in \mathbb{R}$	auxiliary matrix
$\boldsymbol{y} \in \mathbb{R}$	auxiliary matrix
r_k	resistance of the distribution line connecting node k and node $k+1$ (Ω)
x_k	inductance of the distribution line connecting node k and node $k+1$ (Ω)
v_0	bus voltage magnitude at the slack bus (kV)
$V_{\min}$	lower bound of bus voltage magnitude (kV)
$V_{\max}$	upper bound of bus voltage magnitude (kV)
$\theta_{\min}$	lower bound of weighting term
$\theta_{\max}$	upper bound of weighting term
ε	small enough value

Variables

P_{gi}	active power generation from CCHP units of the ith prosumer (MW)
C_{gi}	generation cost of the ith prosumer ($)
lmp_i	locational marginal price of the ith prosumer ($/MWh)
k_i	bidding strategy of the ith prosumer
ρ_i	bidding price of the ith prosumer ($/MWh)
P_i	total active power generation of the ith prosumer (MW)
P_{ri}	renewable power generation of the ith prosumer (MW)
c_{retail_i}	market clearing price of the ith prosumer ($/MWh)
P_{loss}	total active power loss (MW)
U	payoff function of the utility company ($)
P_k	active power flowing from node k to node $k+1$ (MW)
Q_k	reactive power flowing from node k to node $k+1$ (MW)
p_k	net active power consumption for consumers on the node k (MW)
q_k	net reactive power consumption for consumers on the node k (MVAR)
v_k	bus voltage magnitude at node k (kV)
P_{kk}	active power consumption/generation downstream of the node k (MW)
Q_{kk}	reactive power consumption/generation downstream of the node k (MVAR)
P_{loss_k}	active power loss on the distribution line between buses k and $k+1$ (MW)
φ_i	payoff function of the ith prosumer ($)
x^0	feasible initial estimation of the solutions
x^*	Nash equilibrium point
$\Psi(x, y)$	Nikaido–Isoda function
$Z(x)$	optimal strategy maximises the payoff function
θ_n	weighting term at the nth iteration

λ_i	Lagrange multiplier

1 Introduction

Electricity has become an integral part of the world making it hard to imagine a life without power. According to the Electricity Consumers Resource Council (ELCON) report [1], the 2003 North America Blackout made >50 million people suffer from a power outage and resulted in a national economy loss of seven to ten billion dollars. The increasing dependency on fossil fuel-based power plants has led to a potential energy crisis. Environmental problems may include air pollution, global warming, land desertification and so on. As a result, the green energy generated from renewable energy sources (RESs) has become a promising solution. However, due to the nature of RES units, large solar power plants that require huge areas to generate electricity and wind farms that produce loud noises along with the power generation process are often placed in suburbs, far away from consumers [2]. Besides, the aggregated large solar power plants and wind farms are connected to the transmission system through several substations, which are hard to guarantee the efficiency of when delivering electricity to demand side consumers. Moreover, the deployment of RES units has been suffering from a very expensive and inefficient transmission system expansion and maintenance. Thus, the integration of distributed energy resources (DERs) into distribution systems is becoming a solution with great promise to restructure the current power system infrastructure and ensure the stability of the electricity supply.

DERs, including distributed generation (DG) units, distributed storage (DS) units and controllable loads, are deployed in a distribution system to replace traditional units such as coal-fired power plants. The increasing deployment of DERs in distribution systems is driven by several benefits [3]: (i) DERs are located close to the demand side, which can improve power quality and system reliability; (ii) RES units can significantly reduce carbon emissions; and (iii) combined cooling, heat and power (CCHP), also known as trigeneration, can locally utilise waste heat in the electricity generation process to provide useful cooling and heat to improve overall efficiency in a distribution system [4]. Moreover, some of the customers in distribution systems are not only consumers but also producers, since they proactively participate in the electricity market with their DERs. Thus, the concept of the prosumer is proposed.

In the proposed prosumer-centric distribution system with a retail electricity market, prosumers have the ability to sell surplus electricity. However, they may have a partial or total conflict of interest with others. Moreover, the conventional electricity market is not designed for a prosumer-based clearing mechanism. Hence, setting up a well-defined strategic bidding mechanism to maximise the operation revenue for prosumers in the proposed distribution system becomes a critical issue. Game theory is gaining increasing attention as an important analysis tool for future power system design. It involves several mathematical tools to study the complex interactions between independent and rational participants that can seize the new features of the proposed distribution system in the following aspects: (i) communication and control of the prosumers on different nodes in the distribution system; (ii) the heterogeneous nature of the distribution system with its multi-objective prosumers; and (iii) using low-complexity distributed algorithms to represent the competitive (non-cooperative game) scenarios between the prosumers in the distribution system [5]. Thus, game theory methodology and algorithms are used together to compare relationships between prosumers in the proposed distribution system.

There are several kinds of literature investigating the feasibility of applying game theory to a distribution system, where the latest prior works are summarised in Table 1. In the table, we also listed the major algorithms that are adopted in prior works. Moreover, prior works considering prosumers in the distribution system are compared to demonstrate the unique features of our work. The authors in [23] propose a game-theoretic approach to study the dynamic interactions between different prosumers in a distribution system with AC power flow constraints. However, our unique bidding strategy for prosumers participating in the retail electricity market is not considered. In [24], a game-theory-based electricity market clearing mechanism for a distribution system is studied, but power quality and system reliability issues are not included. The authors in [25] demonstrate a game-theoretic framework for the economic dispatch of future distribution systems with multiple prosumers. Again, the new features of a retail electricity market with multiple prosumers are not discussed. To the best of our knowledge, none of the works mentioned above have fully considered all the unique features of prosumers in a distribution system with a retail electricity market using combined game theory and bilevel algorithms.

Table 1 Comparison of game theory method in the distribution system

Game theory method	Algorithms and literature
cooperative game	Lagrangian relaxation (LR) [6]
	benders decomposition (BD) [7, 8]
	Shapley value (SV) [9, 10]
	bilateral SV (BSV) [11]
	bottom-up modelling [12]
non-cooperative game	Nash equilibrium [13–17]
	iterative synchronous best response algorithm (ISBRA) and Nash equilibrium [18]
	Nikaido–Isoda function [19]
	reinforcement learning (RL) and Nash equilibrium [20]
	Lagrange dual decomposition [21]
	Epsilon–Nash equilibria [22]

In this paper, an innovative game-theoretic retail electricity market bidding mechanism for a prosumer-centric distribution system is proposed. The new features of the prosumers in this distribution system and the retail electricity market are modelled with a bilevel algorithm. Novel game-theoretic methodologies and algorithms are applied to analyse the competition among the prosumers when bidding in the retail electricity market. Extensive simulation results are employed to illustrate and validate the proposed framework. The main contributions of this paper are summarised as follows:

- New features of the retail electricity market and prosumers in the distribution system with bilevel algorithms are identified and modelled.
- Novel game-theoretic methodologies are applied to analyse the competition among the prosumers when bidding in the retail electricity market.
- An optimal bidding strategy is investigated to maximise the operation revenues of the prosumers while maintaining the reliability of the distribution system.
- Extensive simulation results are employed to illustrate and validate the proposed framework.

The remaining of the paper is organised as follows. In Section 2, a well-defined price mechanism related to the proposed system for future prosumers is introduced. In Section 3, game theory is introduced to solve the problem described in Section 2, and the applied algorithms are also investigated. In Section 4, the case study simulation results are demonstrated. The conclusion draws in Section 5.

2 System modelling

2.1 Prosumer-centric distributed grid framework

Fig. 1 presents an overview of the future prosumer-centric distribution system, which consists of three main components: (i) prosumers, (ii) utility companies, and (iii) a distribution system operator (DSO). The difference between the proposed framework

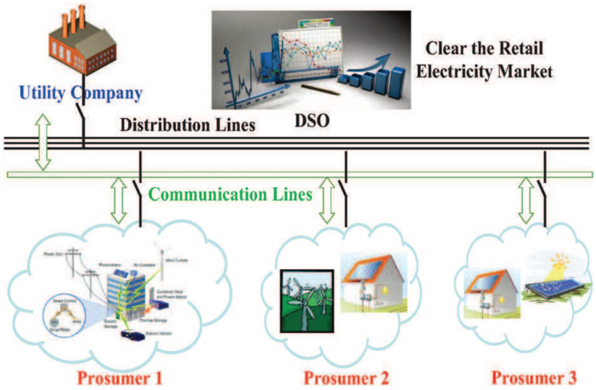

Fig. 1 *Framework of a prosumer-centric distribution system*

and the conventional one is that energy can be generated inside the distribution system instead of the transmission system.

Future prosumers will have local generators like rooftop solar panels, wind turbines, and energy storage (ES) devices such as batteries to generate and store their own power. In addition, their loads are likely to be controllable. These prosumers can be considered larger than a city or smaller than a community with several commercial buildings. Moreover, they are all self-sufficient and have the ability to sell their surplus power to others. The prosumers are rational such that they will only maximise their own payoff, which is common sense for all prosumers. This characteristic provides prosumers with significant economic incentives for self-installation of DERs.

A utility company in the future prosumer-centric distribution system is mainly a provider of ancillary services and maintains the stability of the power system, which is quite distinct from its role in the conventional one. In other words, they will provide the distribution system infrastructure, as well as sell power to those consumers who cannot generate enough power to fulfil their own demand. As a result, utility companies make a profit through a 'renting out' fee, along with power sales to base load consumers.

Meanwhile, a DSO mainly clears the retail electricity market after collecting the participants' bidding strategies. Specifically, the DSO uses a security-constrained economic dispatch method to minimise the cost of supplying load, as well as customers' payments, for the purpose of market clearing.

The functionalities of all the roles listed above are subject to the AC power flow constraints and physical capabilities constraints of DERs. It always holds true that the total generation equals the total consumption.

2.1.1 Electricity prosumers: Electricity prosumers who are able to self-sufficiently meet their basic needs they can sell surplus power to others. The ith electricity prosumer who has CCHP's generation cost function can be modelled in a second-order manner

$$C_{gi} = a_i P_{gi}^2 + b_i P_{gi} + c_i + M_{\text{service}_i}, \tag{1}$$

where P_{gi} is the active power generation of the ith electricity prosumer from CCHP units. a_i, b_i, c_i are the generation cost coefficients, and M_{service_i} is the constant 'subscription fee' paid by the ith electricity prosumer to the utility company.

Then the locational marginal price of the ith electricity prosumer is calculated as in [23]

$$\text{lmp}_i = 2a_i P_{gi} + b_i\,. \tag{2}$$

The supply function equilibrium (SFE) model is adopted to simulate the prosumer-centric distribution system since SFE enables electricity prosumers to connect the bidding price to the bidding quantity of its output [26]. It offers capabilities beyond the traditional Cournot framework [27] and other alternative models, such as multi-unit auction models [28] and agent-based simulations [29], which are only related to the bidding prices of the prosumers or bidding quantities of the surplus electricity. Since the SFE model requires specification of the dependence of demand on bidding, however, it is not immune to the problem of sensitivity to the specification of the market demand. The SFE model also provides the possibility of investigating the behaviour of different players. One more advantage of the SFE model is that it can explicitly demonstrate a long-time unchanged bidding strategy.

In one word, the SFE model provides a more practical point of view about the retail electricity market. As a result, the electricity bidding function of prosumers can be represented as a quadratic function about P_{gi}, similar to the cost function. For ease of control and observation, the electricity prosumers are assumed to bid with a linear supply curve, thus the bidding function of the prosumers can be represented as

$$\rho_i = k_i \text{lmp}_i = k_i(2a_i P_{gi} + b_i), \tag{3}$$

where ρ_i is the bidding price and k_i is the bidding strategy of the ith prosumer. These bidding strategies will be submitted to the DSO, while the DSO will clear the retail electricity market based on the gathered bidding strategy.

Therefore, the payoff function of the ith electricity prosumer is expressed as

$$\begin{aligned} &\max\ R_i = c_{\text{retail}_i} \sum_{i=1}^{N_i} (P_i - P_{di}) - \sum_{i=1}^{n} C_i \\ &\qquad P_i = P_{gi} + P_{ri} \\ &\text{s.t.}\quad P_{i,\min} \le P_i \le P_{i,\max}, \end{aligned} \tag{4}$$

where N_i indicates the total number of electricity prosumers; P_i, P_{ri} are total active power generation and renewable power generation, respectively, from the ith electricity prosumer; P_{di} is the active power demand of the ith electricity prosumer; C_i is the cost and $P_{i,\min}$, $P_{i,\max}$ are the lower and upper bounds of the ith prosumer's energy capacity, respectively.

As the SFE model is used in this paper, the demand curve can be represented as a linear function. Then $c_{\text{retail}_i} = \beta_i - \alpha_i(\sum_{i=1}^{N_i} P_i - P_{\text{loss}})$, where α_i and β_i are coefficients.

2.1.2 Utility companies: In the proposed electricity market, the function of the future utility company is different from that of the current utility company. The future utility company is primarily responsible for guaranteeing power system reliability by complementing power shortage areas and providing a day-ahead reference to avoid congestion. In some cases, it is possible that the power generated by the electricity prosumers does not allow for self-sufficiency, meaning utility companies would sell electricity to those electricity prosumers. The utility company still owns the distributed system infrastructure, so it can earn profit from leasing the infrastructure. Electricity prosumers that participate in the selling of electricity to others need to pay for these leasing expenses. Thus, the payoff function of the utility company can be expressed as

$$U = c_{\text{retail}_u} P_{\text{utility}} + \sum_{i=1}^{N_i} M_{\text{service}_i}, \tag{5}$$

where P_{utility} is the power supplied by the utility company; c_{retail_u} denotes the market clearing price of the utility company.

2.1.3 Distribution system operator: After collecting the submitted bids of the electricity prosumers, the DSO, which is similar to a market clearing centre, minimises the market price to benefit the prosumers without generating equipment, subject to bids and power flow constraints. The locational marginal price is defined as

$$\min \sum_{i=1}^{N_i} \rho_i P_i, \quad \forall i \in [1, N_i]\,. \tag{6}$$

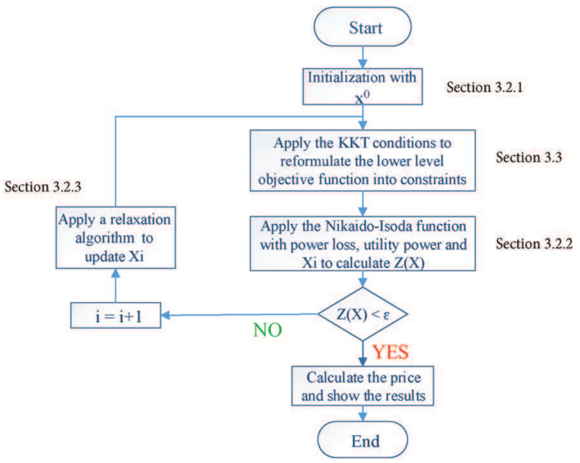

Fig. 2 *Flowchart of the combined game theory and bilevel algorithm*

The objective of DSO is to maintain the stability of the distribution system and to minimise the cost of supplying the load, while the objective of prosumers is to maximise their own revenues. Therefore, a bilevel problem is adopted to handle this multi-objective problem. The upper level is for prosumers with constrained capabilities, while the lower level represents the calculation process of the DSO. For the simplicity of demonstration, the objective function for the prosumers (4) in the upper level can be represented as $F(x, y)$, subject to the constraints related to the prosumers (1)–(4), $G(x, y) \le 0$; meanwhile, the lower level can be defined as $f(x, y)$ and $g(x, y) \le 0$, which represent the objective function for the utility company (6) and constraints associated with AC power flow and power quality issues (details are shown in the following sections), respectively.

As aforementioned, this is the lower level problem. After applying (3)–(6), the formulation is obviously a quadratic function, which means it is convex and regular. In other words, the proposed bilevel algorithm can be used in the proposed problem. The proof of the convexity of the quadratic function is as follows.

Assume there is a quadratic function $F(x) = x^T Ax + b^T x + c$, which is similar to the proposed lower level objective function. $\boldsymbol{A}$ is a symmetric positive semi-definite matrix, as in (6).

By the definition of convex, for $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}$:

$$f\left(\frac{\boldsymbol{x}+\boldsymbol{y}}{2}\right) \ge \frac{1}{2}(f(x) + f(y)). \tag{7}$$

Hence, it can be proven that

$$\begin{aligned} \frac{1}{2}(x+y)^T \boldsymbol{A}(\boldsymbol{x}+\boldsymbol{y}) &\ge x^T \boldsymbol{A}\boldsymbol{x} + y^T \boldsymbol{A}\boldsymbol{y} \\ x^T \boldsymbol{A}\boldsymbol{y} + y^T \boldsymbol{A}\boldsymbol{x} &\ge x^T \boldsymbol{A}\boldsymbol{x} + y^T \boldsymbol{A}\boldsymbol{y}. \end{aligned} \tag{8}$$

That is to say

$$(x-y)^T \boldsymbol{A}(\boldsymbol{x}-\boldsymbol{y}) \ge 0, \tag{9}$$

which is directly followed by a positive semi-definite matrix.

2.1.4 Constraints: The mathematical formulations in the previous sections are constrained to the power flow of the distribution system. In this paper, the DistFlow method is applied as the simplified AC power flow [30].

The system of DistFlow equations for active power, reactive power, and voltage is as follows:

$$\begin{aligned} P_{k+1} - P_k &= p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2}, \\ Q_{k+1} - Q_k &= q_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2}, \\ v_{k+1}^2 - v_k^2 &= -2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2)\frac{P_k^2 + Q_k^2}{v_k^2}, \end{aligned} \tag{10}$$

where $k = 1, \ldots, N_k$ denotes the node of the feeder. P_k and Q_k represent the active power and reactive power, respectively, flowing from node k to node $k+1$. v_k is the bus voltage magnitude at the node k. p_k and q_k show the net active power and reactive power consumptions, respectively, for consumers. r_k and x_k demonstrate the resistance and inductance, respectively, of the line connecting node k and node $k+1$.

It needs to be noted that for a radial distribution network, P_k is equal to the sum of active power consumption/generation downstream of the node k and Q_k is equal to the sum of reactive power consumption/generation downstream of node k:

$$P_k = \sum_{kk \in K} P_{kk}, \quad Q_k = \sum_{kk \in K} Q_{kk}, \tag{11}$$

where K is a set of buses that are downstream of node k.

The quadratic terms in (10) are very small compared with the branch power. Therefore, the quadratic terms can be dropped to simplify the equation.

With assumptions of $(v_k - v_0)^2 \simeq 0$ and $v_0^2 + 2v_0(v_k - v_0) \simeq v_k^2$ [31], the AC power flow equations can be re-write as

$$\begin{aligned} P_{k+1} - P_k &= p_k, \quad Q_{k+1} - Q_k = q_k, \\ v_{k+1} - v_k &= -\frac{(r_k P_k + x_k Q_k)}{v_0}. \end{aligned} \tag{12}$$

For the stability and security of the distribution system, the voltage should be in the following range:

$$V_{\min} \le v_k \le V_{\max}. \tag{13}$$

As a result, the loss on the distribution line between buses k and $k+1$ is expressed as

$$P_{\text{loss}_k} = r_k \frac{P_k^2 + Q_k^2}{v_k^2} \simeq r_k[P_k^2 + Q_k^2]. \tag{14}$$

Hence, the total active power loss can be calculated as

$$P_{\text{loss}} = \sum_{k=1}^{N_k} P_{\text{loss}_k} = \sum_{k=1}^{N_k} r_k \frac{P_k^2 + Q_k^2}{v_k^2} \simeq \sum_{k=1}^{N_k} r_k[P_k^2 + Q_k^2], \tag{15}$$

where P_{loss} denotes the total active power loss.

Another constraint is the supply and demand balance, P_{base} represents the base load of the system except prosumers, the equation below needs to be satisfied all the time

$$\sum_{i=1}^{N_i} P_{di} + P_{\text{loss}} + P_{\text{base}} = \sum_{i=1}^{N_i} P_i + P_{\text{utility}}. \tag{16}$$

3 Solution methodology

3.1 Solution method

The process of clearing the retail electricity market with the combined bilevel and game-theoretic algorithms is shown in Fig. 2, where the Nikaido–Isoda function and a relaxation algorithm are utilised together to determine the Nash equilibrium point.

The detailed combined algorithms are described below and illustrated in the following subsections:

1. Let null vector $\boldsymbol{x}^0$ be a feasible initial estimation of the solutions.
2. Apply the Karush–Kuhn–Tucker (KKT) conditions to reformulate the DSO's objective function as each prosumer's constraints with $\boldsymbol{x}$.
3. Use the combination of relaxation algorithm with the Nikaido–Isoda function and P_{loss}, P_{utility} and $\boldsymbol{x}$ to get the Nash equilibrium point x^*. Then solve the optimisation problem (21) to get the argument of the maximum $Z(x)$.
4. If $Z(x) < \varepsilon$, where ε is a small tolerance value, then stop. Otherwise, update $i \leftarrow i + 1$ and go back to Step 2.

3.2 Game theory

Game theory has become a useful tool to solve real-world problems. There are four key components of game theory: players, payoffs, consequences, and rules.

3.2.1 Concept of game theory: Note that N_i players participate in one game, where vector $\boldsymbol{x}_i$ denotes the action taken by the ith player, $i = 1, 2, 3, \ldots, N_i$. Therefore, a collective action set $\boldsymbol{x} = x_1, x_2, \ldots, x_{N_i}$ is formed when all the players act at the same time. Moreover, φ_i is adopted as the ith player's payoff, which represents the ith player's profit, earned from setting its own strategy when others strategies are settled [25]. For our proposed framework, utility companies and energy prosumers are considered players. Hence, the payoff function φ_i is the difference between sale revenue and the cost of electricity production and facilities, as

$$\varphi_i = R_i - C_i, \tag{17}$$

where R_i represents the revenue of the ith participants and C_i denotes the cost functions for the ith participants. In addition, $\boldsymbol{x}_i$ is the electricity sold by the ith participant. Moreover, as mentioned before, $(y_i | x)$ represents the ith participant's action set, i.e. $(x_1, \ldots, x_{(i-1)}, y_i, x_{(i+1)}, \ldots, x_{N_i})$, which indicates that the ith participant takes the action of y_i while the other participants take the actions set of $(x_1, \ldots, x_{(i-1)}, x_{(i+1)}, \ldots, x_{N_i})$.

When all the participants are competing with each other, at the final stage, there should be an equilibrium point that can achieve a balance among all the participants. This consequence (balanced state) is defined as the Nash equilibrium point

$$\begin{aligned} x^* &= (x_1^*, \ldots, x_{N_i}^*), \quad \forall i \\ \varphi_i(x^*) &= \max_{(x_i | x^*) \in \vec{X}} \varphi_i(x_i | x). \end{aligned} \tag{18}$$

Here (18) shows the rules that all participants act at the same time and stop competing with each other until the equilibrium is reached [24].

3.2.2 Problem reformulation: In order to reformulate the Nash equilibrium searching problem as an optimisation problem that can be solved by commercial solvers, the Nikaido–Isoda function is adopted [32]

$$\Psi(x, y) = \sum_{i=1}^{N_i} [\varphi_i(y_i | x) - \varphi_i(x)] \tag{19}$$

On the right-hand side of the Nikaido–Isoda function, i.e. (19), $[\varphi_i(y_i | x) - \varphi_i(x)]$ shows the payoff amount difference when the ith participant is changing its bidding strategy from x_i to y_i while other participants stick to strategy set $\boldsymbol{x}$. Therefore, the total amount of changes in the payoff functions when making different choices of bidding strategies can be summarised by adding all the differences together. Thus, the Nash normalised equilibrium point x^* can be achieved when the following criteria are met:

$$\max_{(x^*, y) \in \vec{X}} \Psi(x^*, y) = 0 \tag{20}$$

When (20) reaches zero, none of the participants can adjust their payoff amounts by unilaterally changing their own bidding strategy while the other participants still follow the same bidding strategy. In addition, a Nash normalised equilibrium point can be the Nash equilibrium point when certain concavity conditions are satisfied. For our proposed framework, the convex concave function describes the payoff function for all the prosumers and utility companies. Thus, the Nash equilibrium problem can be solved by reformulating the optimisation problem as follows:

$$Z(x) = \arg\max_{y \in \boldsymbol{X}} \arg\max \Psi(x, y)x, \quad Z(x) \in \boldsymbol{X}, \tag{21}$$

where argmax denotes the argument of the maximum.

3.2.3 Relaxation algorithm: The reformulated optimisation problem can be solved iteratively through a relaxation algorithm until it converges to the Nash equilibrium point [33]. Firstly, an initial guess x^0 is provided in order to begin the iteration process. Note that x^0 is a null vector. After that, the relaxation algorithm can be implemented on the reformulated optimisation problem as follows:

$$x^{n+1} = (1 - \theta_n)x^n + \theta_n Z(x^n), \quad 0 \le \theta_n \le 1, \tag{22}$$

where n is the iteration number, θ_n denotes the weighting term at the nth iteration, and x^n is the participant's strategy at the nth iteration. Note that the value of θ_n is fixed as 0.5 for the simplicity of demonstration. Moreover, the stopping criteria of the solution process are defined as follows:

$$\max_{(x^n, y) \in \vec{X}} \Psi(x^n, y) \le \varepsilon, \tag{23}$$

where ε is a small enough value that it can be used to control the convergence rate.

3.3 Bilevel algorithm

Bilevel programming has been successfully applied in various areas, including economics, management science, engineering and so on, since the concept of a bilevel algorithm was first proposed by Candler and Norton [34].

In a bilevel problem, there are two sets of constraints and two objective functions. The two levels are regarded as the upper level and the lower level. In the upper level, the objective function and constraints contain every variable, while in the lower level, only part of the variables are included in the objective function and constraints. This does not mean that the rest of the variables disappear from the lower level; instead of variables, they are regarded as some constant values in the lower level problem. This is an important concept in a bilevel algorithm [34].

For the simplicity of demonstration, the objective function for the prosumers (4) in the upper level can be represented as $F(x, y)$, subject to the constraints related to the prosumers (1)–(4), $G(x, y) \le 0$; meanwhile, the lower level can be defined as $f(x, y)$ and $g(x, y) \le 0$, which represent the objective function for the utility company (6) and constraints associated with AC power flow and power quality issues (5) and (10)–(16), respectively.

Then the proposed bilevel problem can be defined as follows:

$$\begin{aligned} \text{Upper level:} \quad & \min F_{\text{upper}}(x, y) \\ \text{s.t.} \quad & G_{\text{upper}}(x, y) \le 0 \\ \text{Lower level:} \quad & \min f_{\text{lower}}(x, y) \\ \text{s.t.} \quad & g_{\text{lower}}(x, y) \le 0. \end{aligned} \tag{24}$$

A Lagrange multiplier is adopted in our proposed bilevel algorithm, which is a classic method used to solve the extremum

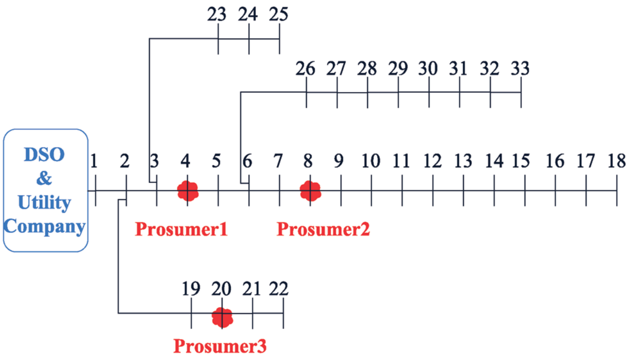

Fig. 3 *Modified IEEE 33-bus test feeder*

problem [35]. Note that when minimising or maximising the function $F(x, y)$ without limitations and boundaries, $\partial F(x, y)/\partial x = 0$ and $\partial F(x, y)/\partial y = 0$ can be used to attain the optimal solutions. Taking the constraints into consideration, however, the problem can still be reformulated into equations without constraints and solved in a similar way. For the proposed bilevel problem, when the lower level objective function is regular and convex, it can be reformulated as similar equations by using its KKT conditions. Thus, the reformulated single-level problem is presented as follows:

$$\begin{aligned} &\min_{x \in \vec{X}, y} \ F(x, y) \\ &\text{s.t.} \\ &\quad G(x, y) \le 0 \\ &\quad g(x, y) \le 0 \\ &\quad \lambda_i \ge 0, \quad \forall i \in [1, m] \\ &\quad \lambda_i g_i(x, y) = 0, \quad \forall i \in [1, m] \\ &\quad \nabla_y L(x, y, \lambda) = 0 \\ &\text{where } L(x, y, \lambda) = f(x, y) + \sum_{i=1}^{m} \lambda_i g_i(x, y), \end{aligned} \tag{25}$$

where m is the number of lower level constraints.

It is still hard to solve the single level problem due to some non-convexities that occur in the complementarities and Lagrange constraints though. Branch-and-bound is mainly to traverse the tree and seek all the roots of the tree. However, not all of the roots are able to reach the optimal solution, or even a solution. As a result, λ is used as an index constraint for branch-and-bound to deal with the non-convexities.

If the ith index λ_i is greater than 0, then the ith constraint condition $g_i(x, y) = 0$; conversely, if $\lambda_i = 0$, then $g_i(x, y)$ will work as is, which means it is beyond the bound. With the help of the definitions mentioned above, the optimal solutions of the proposed bilevel problem are valid for the corresponding sub-tree. After applying the bilevel algorithm, the bilevel problem can be reformulated as a single level conventional multiple participants game theory problem. In this paper, a novel game theoretic algorithm is proposed to transform the equilibrium problem into an optimisation problem and find the optimal solutions. All these algorithms are processed in Matlab on an Intel Core i7-2450M CPU 2.50 GHz computer with 8 GB RAM. The convergence tolerance for the proposed algorithm is set to $\varepsilon = 10^{-3}$.

4 Simulation results

4.1 Numerical settings

The proposed bidding strategy for multiple prosumers in a retail electricity market is tested on a modified IEEE 33-bus test feeder, as shown in Fig. 3. Note that all the transformers, switches, and voltage regulators are ignored [36]. The modified IEEE 33-bus test feeder is assumed to be a single-phase balanced distribution system. The reactance and resistance data for each bus in the modified IEEE 33-bus test feeder are listed in Table 2. The active and reactive power demand for each bus in the modified IEEE 33-bus test feeder are from [37]. For the simplicity of demonstration, three prosumers are assumed on buses 4, 8 and 20 that are able to provide self-generated electricity using RES units, CCHP, and distributed ES systems. The capacity of each aggregated generator varies largely, which means every prosumer may have a different cost function. The coefficients of the prosumers' cost functions, a_i and b_i, are listed in the first two lines of Table 3. In order to reduce the computational cost, let the subscription fee be a multiple of the amount of quantity of the power output, so that M_{service_i} can be regarded as contained in b_i, also, assume that every c_i is zero \$/h. The generation capabilities of all the prosumers are <20 MW, as shown in Tables 3 and 4. The minimum generation of all prosumers is set to 0 MW. The bidding strategies and demand curve coefficients of the three prosumers are also listed in Table 3.

Table 2 IEEE 33-bus test feeder data

Begin node	End node	Resistance, Ω	Reactance, Ω
1	2	0.0922	0.0470
2	3	0.4930	0.2511
3	4	0.3660	0.1864
4	5	0.3811	0.1941
5	6	0.8190	0.7070
6	7	0.1872	0.6188
7	8	0.7114	0.2351
8	9	1.0300	0.7400
9	10	1.0440	0.7400
10	11	0.1966	0.0650
11	12	0.3744	0.1238
12	13	1.4680	1.1550
13	14	0.5416	0.7129
14	15	0.5910	0.5260
15	16	0.7463	0.5450
16	17	1.2890	1.7210
17	18	0.7320	0.5740
2	19	0.1640	0.1565
19	20	1.5042	1.3554
20	21	0.4095	0.4784
21	22	0.7089	0.9373
3	23	0.4512	0.3083
23	24	0.8980	0.7091
24	25	0.8960	0.7011
6	26	0.2030	0.1034
26	27	0.2842	0.1447
27	28	1.0590	0.9337
28	29	0.8042	0.7006
29	30	0.5075	0.2585
30	31	0.9744	0.9630
31	32	0.3105	0.3619
32	33	0.3410	0.5302

Table 3 Data of three prosumers

Parameters	Prosumer 1	Prosumer 2	Prosumer 3
a_i, \$/MW^{2}h	10.8508	6.5455	0.0455
b_i, \$/MWh	14.6738	37.258	26.1739
k_i	2.1	2.1	1.3
α_i	8	9	10
β_i	320	330	300
capacity, MW	10.5	9.4	7.3

Table 4 Data of the fourth prosumer

Parameters	Prosumer 4
a_i, \$/MW^{2}h	0.00677
b_i, \$/MWh	0.1739
k_i	1.1
α_i	8
β_i	250
capacity, MW	3.8

Table 5 Expected payoff of three prosumers

Variables	Prosumer 1	Prosumer 2	Prosumer 3
P_i, MW	6.9338	7.1945	6.6493
revenue, \$/h	1167.9133	1121.5742	461.7081

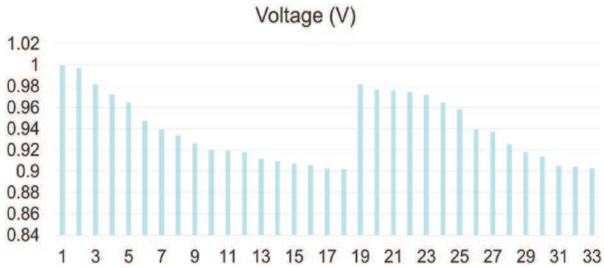

Fig. 4 *Voltage magnitude of each node on the IEEE 33-bus test feeder*

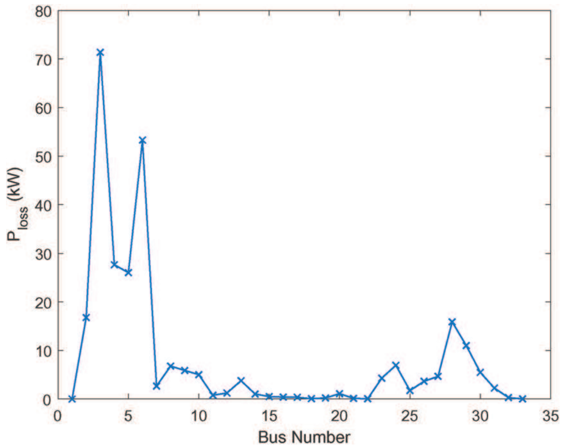

Fig. 5 *Detailed active power loss on the distribution line between each bus*

4.2 Case study

4.2.1 System stability: As shown in Table 5, when the bidding strategies and demand curve are similar and regular, every prosumer will generate a similar quantity of power. The difference between the revenue of the third prosumer and the other two prosumers is caused by the payoff coefficient. Moreover, voltage magnitude represents the stability level of the power system, especially for distribution systems. Fig. 4 shows that the voltages of all nodes are between 0.9 and 1.1, which means the system is stable. There is a huge difference between node 18 and node 19, which is caused by the system configuration, as node 19 is connected to node 2. In addition, the detailed active power losses on the distribution line between bus k to bus $k+1$ is shown in Fig. 5. The total active power loss of the whole distribution system is 0.247 MW after 14 iterations with three prosumers, which is only 1.2% compared with the total active power delivered in the distribution system. Therefore, the proposed model is proved to be stable and effective.

4.2.2 Anti-dumping: In order to examine the global optimal solution, the bids of the third prosumer are decreased, as shown in

Table 6 Decreasing the bids of prosumer three

Bidding strategy	Prosumer 1	Prosumer 2	Prosumer 3
k_i	2.1	2.1	0.3

Table 7 Expected payoff of three prosumers based on updated bidding strategy

Variables	Prosumer 1	Prosumer 2	Prosumer 3
P_i, MW	8.0311	8.7321	1.1445
revenue, \$/h	1566.8391	1652.2364	3.1996

Table 8 Increasing the bids of prosumer three

Parameters	Prosumer 1	Prosumer 2	Prosumer 3
k_i	2.1	2.1	300
β_i	320	330	50

Table 9 Expected payoff of three prosumers based on updated bidding strategy and demand parameters

Variables	Prosumer 1	Prosumer 2	Prosumer 3
P_i, MW	2.1101	2.3394	0.4812
revenue, \$/h	915.2671	999.5875	67.458

Table 6, which is at node 4, while keeping the same bidding strategy for the other two prosumers. The expected payoff for the updated bidding strategy of the three prosumers is listed in Table 7. Dumping is often referred to selling at less than 'normal value' on similar quality goods in the ordinary course of trade, so as to sell more goods. In our case study, an extreme scenario is picked, in which prosumer 3's bidding strategy is 0.3. This means the prosumer is bidding much lower than cost, at thirty percent of the cost, which is less than the normal value, so the DSO will suppress its production to keep the market stable as an anti-dumping action. As shown in Table 7, the power production of prosumer 3 is only 1.1445 MW, much less than that of the other two prosumers.

4.2.3 Macro-control: Macro-control means macroeconomic regulation and control, which is operated by an independent agency, such as to centrally-plan an economy or remedy market failure. In our system, the DSO is such an independent agency. Due to the computing complexity and convergence rate selection, the proposed method cannot guarantee the global solution but may locate the global solution only if the initial bidding strategies are selected properly. To test the sensitivity of the retail electricity market and the DSO to the bidding strategies of our system, the bidding strategies and demand curve coefficients of the third prosumer are increased to an abnormal value, as shown in Table 8. The expected payoff of the updated bidding strategy and demand curve coefficients of the three prosumers is listed in Table 9. In our model, if one participant prices himself out of the market, the model will also suppress its revenue by limiting its power production. In this scenario, prosumer 3 prices himself out of the market at 300 times its cost, so the DSO will also suppress its revenue by limiting its power production, to about 0.48 MW, as the table shows.

4.2.4 Sensitivity analysis: In order to show the effectiveness of our proposed bidding strategy for multiple prosumers in the retail electricity market, a sensitivity analysis is performed on changing the number of prosumers from two to four. Note that the case studies for the proposed framework with three prosumers have been presented above. All environmental settings are the same as in the previous case studies. As shown in Table 10, the revenue and active power injected into the distribution system of prosumer 1 and prosumer 2 have increased from that in previous case studies. This is reasonable because, in this scenario, prosumer 3 is eliminated, thus prosumer 1 and prosumer 2 have more opportunity to compete in the retail electricity market and sell their surplus

Table 10 Expected payoff of two prosumers

Variables	Prosumer 1	Prosumer 2
P_i, MW	8.2628	9.0541
revenue, $/h	1656.9289	1775.0173

Table 11 Bidding strategy of four prosumers

Parameters	Prosumer 1	Prosumer 2	Prosumer 3	Prosumer 4
k_i	2.1	2.1	1.5	1.7
β_i	320	330	300	250

Table 12 Expected payoff of four prosumers

Variables	Prosumer 1	Prosumer 2	Prosumer 3	Prosumer 4
P_i, MW	4.8864	5.1281	4.8812	2.5307
revenue, $/h	1152.5424	1101.5898	569.5199	181.3805

power. Besides, the convergence rate of this two prosumers' scenario is much faster than that of three prosumers', which is 5 iterations compared with 14 iterations.

In addition, we test our proposed framework in another scenario when adding one more prosumer on a random bus. In this scenario, prosumer 4 is added on bus 25. The data of prosumer 4 is listed in Table 4 (Table 11). Table 12 shows the revenue and active power injected into the distribution system of the four prosumers. The differences between the revenue and the active power injected into the distribution system between the fourth prosumer and the other three prosumers are caused by the payoff coefficient. Moreover, the convergence rate of this four prosumers' scenario is much slower than that of three prosumers', which is 37 iterations compared with 14 iterations.

5 Conclusion

An innovative game-theoretic market framework has been proposed for a prosumer-centric retail electricity market. A bilevel algorithm has been adopted to model the new features of DSOs, utility companies and prosumers in a distribution system. An SFE model, Nikaido–Isoda functions and relaxation algorithms have been applied to analyse the competition among the key participants in a retail electricity market. Extensive simulation results have been employed to illustrate and validate the effectiveness of the proposed framework with following major findings: (i) prosumers' bidding strategies are tested with various numbers of participants in the retail electricity market; (ii) the combination of bilevel algorithm and relaxation algorithm can guarantee the convergence of the proposed game-theoretic market framework; and (iii) dumping-bidding and abnormal bidding from prosumers have been suppressed by the market operator in the model.

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