

Four-Dimensional Relativistic Scattering of Electromagnetic Waves from an Arbitrary Collection of Moving Lossy Dielectric Spheres

Hamed Radpour, Ali Pourziad, Kamal Sarabandi

Abstract— In this paper, four-dimensional (4D) relativistic scattering of electromagnetic waves from an arbitrary collection of uniformly translational moving lossy dielectric spheres is discussed. Two reference frames, four 4D coordinate systems and Lorentz transformation are used to obtain the scattered electromagnetic fields. The direct scattering of the spheres and their interactions are considered with a novel approach. The introduced method is straightforward and the analytical relations for the fields are achieved. To check the validity of the proposed method, different examples for both stationary and moving scatterers are investigated. The effect of the key parameters such as the size, material, velocity, number, position of the spheres and also the frequency of the incident wave are discussed. The derived scattered fields are valid for low, medium and high velocities but according to practical applications low and moderate velocities are highlighted in numerical results.

I. INTRODUCTION

Very interesting subject in electromagnetics is related to the scattering of electromagnetic (EM) waves from moving objects which has been investigated by researchers over the last century. From the standpoint of applications, for low and moderate speed cases, it can be used to calculate the attenuation and transmission of EM waves for rainy, snowy and dusty mediums, which is very important in meteorology, satellite communications, environmental issues, radar applications and remotely sensed data. Furthermore, for the high-speed objects profile, it has applications in the understanding of scattering by relativistically moving interstellar dust grains [1], moving plasma columns [2-4] and mass flows in pneumatic pipes [5].

The important properties of objects such as shape, material, and velocity could be obtained by processing the scattered fields which is known as the inverse scattering. Furthermore, through post-processing, other significant practical characteristics of a collection of objects such as scattering, extinction and absorption cross sections could be derived. The main challenge in the random and multiple scattering is obtaining the scattered fields from a moving medium or targets. In this case, four 4D coordinate systems for the rest and moving frames and Special Theory of Relativity (STR) should be considered which causes mathematical difficulties.

Electromagnetic scattering of a translational moving body with STR, which has been introduced firstly by Einstein in 1905 [6], has been used for a moving perfectly reflecting mirror [7],[8]. By applying STR, EM scattered waves for different moving shapes such as dielectric medium [9, 10] cylinder [11-15], conducting sphere [16], electrically small circular sphere [17], small particle [18], perfectly conducting flat plate [19], rough surface [20], arbitrary obstacle [21-27], wedge [28, 29], and half-plane [30-32] have been derived. Also, scattering characteristics (scattering cross section, extinction, and absorption) for a uniformly moving object [33] and a moving concentrically layered sphere [34] are discussed. The

backscattered signal by a uniformly moving sphere considering incident wave to be a pulsed plane wave, is investigated [35]. All foregoing works discuss only one moving object whereas practical applications mostly deal with a collection of many moving random objects. In that case, not only investigation on relativistic translational motion of the individual object is required, the mutual interactions of the moving objects also have significant effects, which make the solution more complicated.

In this paper, time-domain scattered fields from an arbitrary collection of uniformly translational moving lossy-dielectric spheres are calculated in the far field region. The size, material, velocity, number, position of the spheres, and the frequency of the incident wave can be selected, arbitrarily which make studying of effective parameters possible. By considering the intrinsic inaccuracies of using numerical techniques, such as the finite-difference-time-domain method (FDTD) and Lorentz precise integration time-domain method (Lorentz-PITD), for a moving object [36-42], here the STR and Mie theory [43-46] are employed. Also other kinds of motions for an individual object such as rotational [47-50] and vibrational [52-54] have been investigated, but, here the translational motion is of interest.

II. FORMULATION OF THE PROBLEM

A collection of uniformly translational moving spheres of radius a , complex refractive index of $n = n' + jn''$ and constant velocity of $\vec{v} = v\hat{z}$ moving along z-direction is considered. Four 4D coordinate systems (three dimensions are associated with the position and one with the time) are considered regarding the rest and moving frames which are denoted with K and K' , respectively, as shown in Fig.1. In the rest frame, the spheres appear to be moving and in the moving frame the spheres seem to be stationary.

In order to synchronize times, K and K' are considered to coincide at the time $t = t' = 0$. Each frame has two spatial coordinate systems. In order to characterize and solve the whole

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problem, Cartesian (x, y, z) and spherical (r, θ, ϕ) coordinate systems are considered for the rest frame (K) and similarly, the prime forms (x', y', z') and (r', θ', ϕ') are chosen to indicate the quantities in the moving frame (K'). Considering that all spheres are moving in the rest frame, it would be more appropriate to transform the problem to the moving frame. In other words, the problem is solved in K' and then the resulting scattered fields transformed back into K . The incident wave and the observation point are given in the K frame. According to symmetry characteristics of spheres, without losing generality, an incident plane wave is considered to propagate in the negative \hat{x} direction¹ and has a polarization in the y direction which can be expressed in the rest frame by:

$$\vec{E}^i = E^i e^{-jkx} \hat{y}, \quad \vec{H}^i = -\frac{1}{\eta} \hat{x} \times \vec{E}^i \quad (1)$$

where η is the intrinsic impedance of the free space and k is the wave number in the rest frame.

A. Electromagnetic scattering from a moving sphere:

At the time $t=0$, the coordinates of the i th moving sphere is represented by (x_{i0}, y_{i0}, z_{i0}) . Moreover, τ is the time when the i th sphere scatters a spherical wavefront and r_i is the instantaneous distance between the i th moving sphere and the observation point (x_p, y_p, z_p) , as shown in Fig.2. So the position of this sphere in the rest frame can be expressed by $(x_i, y_i, z_i, t) = (x_{i0}, y_{i0}, z_{i0} + v\tau, \tau)$ and the angle θ_i is defined as:

$$\cos \theta_i = \frac{z_p - z_{i0} - v\tau}{r_i} \quad (2)$$

$$r_i = [(x_p - x_i)^2 + (y_p - y_i)^2 + (z_p - z_{i0} - v\tau)^2]^{\frac{1}{2}} \quad (3)$$

In order to associate K and K' using the Lorentz transformation [7, 8]

$$x'_i = x_i; y'_i = y_i; z'_i = \frac{z_i - v\tau}{\sqrt{1-\beta^2}}; t' = \frac{t - (\frac{v}{c^2})z_i}{\sqrt{1-\beta^2}} \quad (4)$$

Where $\beta = v/c$ and c is the speed of light in free space. The components of the spherical coordinate system in K' can be written as [7, 8]:

$$r'_i = \frac{1 - \beta \cos \theta_i}{\sqrt{1 - \beta^2}} r_i \quad (5)$$

$$\cos \theta'_i = \frac{\cos \theta_i - \beta}{1 - \beta \cos \theta_i}; \quad \sin \theta'_i = \frac{\sin \theta_i \sqrt{1 - \beta^2}}{1 - \beta \cos \theta_i} \quad (6)$$

$$\cos \phi'_i = \frac{x'_p - x'_i}{r'_i \sin \theta'_i}; \quad \sin \phi'_i = \frac{y'_p - y'_i}{r'_i \sin \theta'_i}; \quad \phi'_i = \phi_i \quad (7)$$

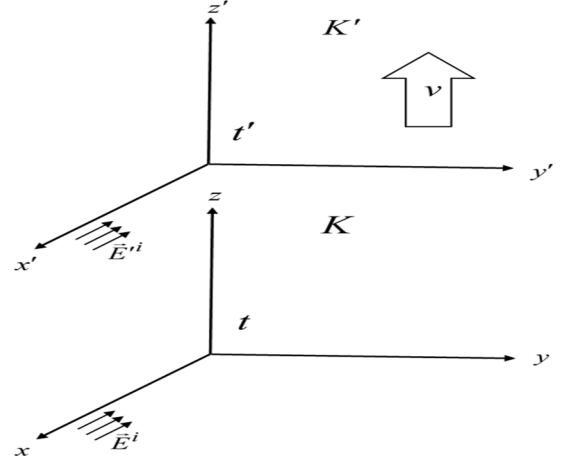


Fig.1. Two reference frames for relative motion.

The incident plane wave is transformed into the moving frame [7, 16]:

$$\vec{E}^{\prime i} = \frac{1}{\sqrt{1-\beta^2}} E^i e^{-jkx'} \hat{y}' \quad (8)$$

$$k' = \frac{k}{\sqrt{1-\beta^2}} \quad (9)$$

By applying Mie theory to the incident field² for the far-field region ($k'r' \gg 1$) in the moving frame, the direct scattered field can be derived as

$$\vec{E}'_{\theta ds} = \frac{1}{\sqrt{1-\beta^2}} [S_1(\delta') \sin \phi'_i \cos \theta'_i + 0.25 S_2(\delta') \sin 2\theta'_i \sin 2\phi'_i] \frac{j e^{jk'r'_i}}{k'r'_i X'^2} \quad (10)$$

$$\vec{E}'_{\phi ds} = \frac{1}{\sqrt{1-\beta^2}} [S_1(\delta') \cos^2 \theta'_i \cos \phi'_i - S_2(\delta') \sin^2 \theta'_i \sin \phi'_i] \frac{j e^{jk'r'_i}}{k'r'_i X'^2} \quad (11)$$

with $X' = (1 - \sin^2 \theta'_i \cos^2 \phi'_i)^{\frac{1}{2}}$ and δ' is the angle between incident direction (\hat{k}'_i) and scattering direction (\hat{k}'_s). The scattering amplitude matrix coefficients for Mie theory [45] can be stated as

$$S_1(\delta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \pi_n(\cos \delta) + b_n \tau_n(\cos \delta)] \quad (12a)$$

$$S_2(\delta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \tau_n(\cos \delta) + b_n \pi_n(\cos \delta)]$$

¹ Throughout this paper $e^{-j\alpha}$ used to transform to the time-harmonic fields.

² For the convenience amplitude of the incident field is considered to be 1V/m.

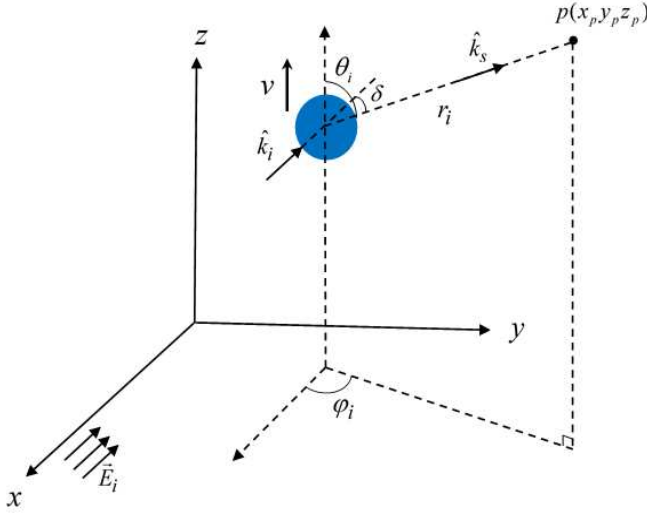


Fig.2. Scattering configuration and angles for a sphere moving with velocity v along z -direction.

where

$$\pi_n(\cos \delta) = -\frac{P_n^1(\cos \delta)}{\sin \delta} \quad (12b)$$

$$\tau_n(\cos \delta) = -\frac{dP_n^1(\cos \delta)}{d\delta}$$

$$a_n = \frac{k_p^2 a^2 j_n(k_p a) [k a j_n(k a)]' - k^2 a^2 j_n(k a) [k_p a j_n(k_p a)]'}{k_p^2 a^2 j_n(k_p a) [k a h_n(k a)]' - k^2 a^2 h_n(k a) [k_p a j_n(k_p a)]'}$$

$$b_n = \frac{j_n(k_p a) [k a j_n(k a)]' - j_n(k a) [k_p a j_n(k_p a)]'}{j_n(k_p a) [k a h_n(k a)]' - h_n(k a) [k_p a j_n(k_p a)]'}$$

(12c)

Where a is the radius of the sphere, a_n , b_n are the Mie scattering coefficients, j_n , h_n are the spherical Bessel and Hankel functions of the first kind, respectively, P_n is the associated Legendre function and prime is the notation for derivation.

To transform back to the stationary frame, the following relations are used [7]

$$E_\theta^{ds} = \frac{\sqrt{1-\beta^2}}{1-\beta \cos \theta_i} E_\theta'^{ds} ; \quad E_\phi^{ds} = \frac{\sqrt{1-\beta^2}}{1-\beta \cos \theta_i} E_\phi'^{ds} \quad (13)$$

Hence, the time-harmonic expressions³ for E_θ^{ds} and E_ϕ^{ds} can be written according to (10), (11) and (13) as

$$E_\theta^{ds} = \frac{1}{1-\beta \cos \theta_i} \left[\frac{1}{X'^2} (S_1(\delta') \sin \phi'_i \cos \theta'_i + 0.25 S_2(\delta') \sin 2\theta'_i \sin 2\phi'_i) \right] \frac{j e^{jk'(r'_i - ct')}}{k' r'_i} \quad (14)$$

$$E_\phi^{ds} = \frac{1}{1-\beta \cos \theta_i} \left[\frac{1}{X'^2} (S_1(\delta') \cos^2 \theta'_i \cos \phi'_i - S_2(\delta') \sin^2 \phi'_i \sin \theta'_i) \right] \frac{j e^{jk'(r'_i - ct')}}{k' r'_i} \quad (15)$$

In order to represent the phase factor ($e^{jk'(r'_i - ct')}$) of the obtained scattered fields in an unprimed form and to relate it with τ it is noted that

$$r'_i = c(t' - \tau') \quad \text{and} \quad jk'(r'_i - ct') = -jk'c\tau \sqrt{1-\beta^2} \quad (16)$$

By applying (16) and (9) to (14) and (15), the time-harmonic direct scattered fields in the rest frame could be expressed by:

$$E_\theta^{ds} = \frac{1-\beta^2}{(1-\beta \cos \theta_i)} \left[\frac{1}{X'^2} (S_1(\delta') \sin \phi'_i \cos \theta'_i + 0.25 S_2(\delta') \sin 2\theta'_i \sin 2\phi'_i) \right] \frac{j e^{-jk c \tau}}{k r_i} \quad (17)$$

$$E_\phi^{ds} = \frac{1-\beta^2}{1-\beta \cos \theta_i} \left[\frac{1}{X'^2} (S_1(\delta') \cos^2 \theta'_i \cos \phi'_i - S_2(\delta') \sin^2 \phi'_i \sin \theta'_i) \right] \frac{j e^{-jk c \tau}}{k r_i} \quad (18)$$

$$\vec{H}^{ds} = \frac{1}{\eta} \hat{r} \times (E_\theta^{ds} \hat{\theta} + E_\phi^{ds} \hat{\phi}) \quad (19)$$

It is important to state that although these direct scattered fields components are functions of τ , they represent the scattered fields in the observation point at the time $t = \tau + \frac{r_i}{c}$.

B. Secondary electromagnetic scattering fields from two spheres configurations

In this section, two moving spheres are considered and the problem is to evaluate the secondary scattered fields. The secondary scattered fields are the fields scattered from a moving sphere, when illuminated by a primary scattered field from another moving sphere. Assuming that i th and j th spheres are moving along the z -direction, as represented in Fig.3, their positions in the K frame can be expressed by

$$(x_i, y_i, z_i, t) = (x_{i0}, y_{i0}, z_{i0} + v\tau, \tau) \quad (20)$$

$$(x_j, y_j, z_j, t) = (x_{j0}, y_{j0}, z_{j0} + v\tau, \tau)$$

The incident field in (1) is upon the i th sphere and this sphere scatters a field which illuminates the j th sphere, then the j th sphere scatters a field which would be calculated. Since these two spheres move with an equal velocity and have an identical direction of motion, they appear stationary to each other in the moving frame; therefore, the secondary scattered fields can be called coupling fields. It is assumed that the j th sphere is in the far-field region of the i th sphere. If the distance between the two

³ For the sake of simplicity the $\text{Real}\{\cdot\}$ operation is not represented.

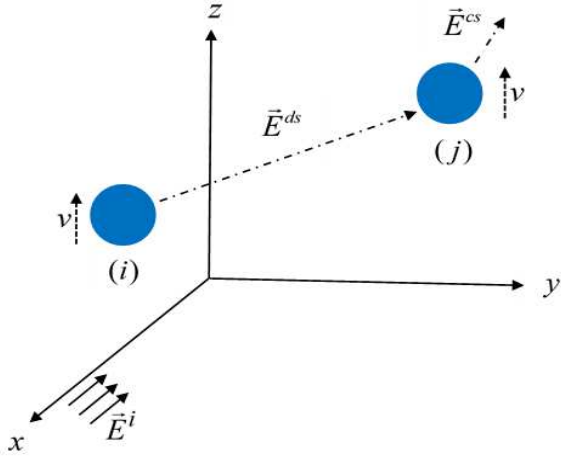


Fig.3. Two moving spheres with the direct and coupling scattered fields.

spheres represented by d_{ij} and the radius of the i th sphere denoted by a_i then the far-field condition mathematically can be stated as

$$d_{ij} \geq \frac{8a_i^2}{\lambda} \quad (21)$$

According to the length-contraction property of STR, it can be written that

$$d'_{ij} = \frac{1 - \beta \cos \theta_{ij}}{\sqrt{1 - \beta^2}} d_{ij}; \quad r'_j = \frac{1 - \beta \cos \theta_j}{\sqrt{1 - \beta^2}} r_j \quad (22)$$

where r_j is considered as the instantaneous distance between sphere number j and the observation point (x_p, y_p, z_p) in the K frame.

The parameters $(\theta_{ij}, \varphi_{ij})$ and (θ_j, φ_j) are defined as

$$\cos \theta_{ij} = \frac{1}{d_{ij}} (z_j - z_i) \quad (23)$$

$$\cos \varphi_{ij} = \frac{1}{d_{ij} \sin \theta_{ij}} (x_j - x_i); \quad \sin \varphi_{ij} = \frac{1}{d_{ij} \sin \theta_{ij}} (y_j - y_i) \quad (24)$$

$$\cos \theta_j = \frac{1}{r_j} (z_p - z_{j0} - v \tau) \quad (25)$$

$$\cos \varphi_j = \frac{1}{r_j \sin \theta_j} (x_p - x_j); \quad \sin \varphi_j = \frac{1}{r_j \sin \theta_j} (y_p - y_j) \quad (26)$$

Prime forms of $(\theta_{ij}, \varphi_{ij})$ and (θ_j, φ_j) in moving frame can be obtained in a similar way (6-7).

The coupling fields would be calculated by applying Mie theory in two levels and transformations between stationary and moving frames would be employed.

The parameters S_{1i}, S_{2i}, S_{1j} and S_{2j} construct the scattering amplitude matrices that are used to calculate the primary and secondary scattered fields. The parameters S_{1i}, S_{2i} which are

used for evaluation of the scattered fields from the i th sphere could be obtained with direct replacement of δ'_i in (12a) and (12b) and S_{1j}, S_{2j} are related to the j th sphere that could be achieved by replacing δ'_j in (12a) and (12b).

$$\cos(\delta'_i) = -\sin \theta'_{ij} \cos \varphi'_{ij} \quad (27a)$$

$$\cos(\delta'_{ij}) = \frac{1}{d'_{ij} r'_j} [(x'_{pj} (x'_j - x'_i) + y'_{pj} (y'_j - y'_i) + z'_{pj} (z'_j - z'_i))] \quad (27b)$$

The parameters \hat{k}_i and \hat{k}_s are incident and scattered fields propagation directions, respectively, associated with the j th sphere. Therefore $(\hat{1}'_i, \hat{2}'_i, \hat{k}'_i)$ and $(\hat{1}'_s, \hat{2}'_s, \hat{k}'_s)$ are the orthonormal unit systems [45] to characterize scattering by the j th sphere which can be defined as

$$\begin{aligned} x'_{pj} &= x'_p - x'_j; y'_{pj} = y'_p - y'_j; z'_{pj} = z'_p - z'_j \\ \hat{k}'_i &= \frac{1}{d'_{ij}} [x'_{pj} \hat{x}' + y'_{pj} \hat{y}' + z'_{pj} \hat{z}'], \quad \hat{k}'_s = \hat{r}'_j \\ \hat{1}'_i &= \hat{1}'_s = \frac{\hat{k}'_i \times \hat{k}'_s}{|\hat{k}'_i \times \hat{k}'_s|} = \frac{1}{N'} [\hat{\theta} L'_4 + \hat{\phi} L'_6] \end{aligned} \quad (28)$$

$$\hat{2}'_i = \hat{k}'_i \times \hat{1}'_i = -\frac{1}{d'_{ij} N'} [\hat{\theta} L'_5 + \hat{\phi} L'_6]$$

$$\hat{2}'_s = \hat{k}'_s \times \hat{1}'_s = \frac{1}{N'} [-\hat{\theta} L'_4 + \hat{\phi} L'_3]$$

where

$$\begin{aligned} u'_1 &= x'_{pj} (y'_j - y'_i) - y'_{pj} (x'_j - x'_i) \\ u'_2 &= y'_{pj} (z'_j - z'_i) - z'_{pj} (y'_j - y'_i) \\ u'_3 &= z'_{pj} (x'_j - x'_i) - x'_{pj} (z'_j - z'_i) \\ u'_4 &= u'_1 (y'_j - y'_i) - u'_3 (z'_j - z'_i) \\ u'_5 &= u'_2 (z'_j - z'_i) - u'_1 (x'_j - x'_i) \\ u'_6 &= u'_3 (x'_j - x'_i) - u'_2 (y'_j - y'_i) \\ N' &= (u_1'^2 + u_2'^2 + u_3'^2)^{\frac{1}{2}} \end{aligned} \quad (29)$$

$$\begin{aligned} L'_1 &= S_{1i} \cos \theta'_{ij} \sin \varphi'_{ij} + 0.25 S_{2i} \sin 2\theta'_{ij} \sin 2\varphi'_{ij} \\ L'_2 &= S_{1i} \cos^2 \theta'_{ij} \cos \varphi'_s - S_{2i} \sin \theta'_{ij} \sin^2 \varphi'_s \\ L'_3 &= u'_2 \cos \theta'_j \cos \varphi'_j + u'_3 \cos \theta'_j \sin \varphi'_j - u'_1 \sin \theta'_j \\ L'_4 &= -u'_2 \sin \varphi'_j + u'_3 \cos \varphi'_j \\ L'_5 &= u'_4 \cos \theta'_j \cos \varphi'_j + u'_5 \cos \theta'_j \sin \varphi'_j - u'_6 \sin \theta'_j \\ L'_6 &= -u'_4 \sin \varphi'_j + u'_5 \cos \varphi'_j \end{aligned} \quad (30)$$

$$X' = (1 - \sin^2 \theta'_{ij} \cos^2 \phi'_{ij})^{\frac{1}{2}} \quad (31)$$

Then the secondary scattered fields components in the moving frame can be written as

$$E_{\theta}^{'cs} = -\left[\frac{1}{X'^2 N'^2} (S_{1j} L'_3 (L'_1 L'_3 + L'_2 L'_4)) + \frac{S_{2j}}{d'_{ij}} L'_4 (L'_1 L'_5 + L'_2 L'_6) \right] \frac{e^{jk'(d'_{ij} + r'_j)}}{k'^2 d'_{ij} r'_j} \quad (32)$$

$$E_{\phi}^{'cs} = \left[\frac{1}{X'^2 N'^2} (-S_{1j} L'_4 (L'_1 L'_3 + L'_2 L'_4)) + \frac{S_{2j}}{d'_{ij}} L'_3 (L'_1 L'_5 + L'_2 L'_6) \right] \frac{e^{jk'(d'_{ij} + r'_j)}}{k'^2 d'_{ij} r'_j} \quad (33)$$

Referring (13) for transforming scattered fields back into the stationary frame, the time-harmonic field components can be expressed as

$$\bar{E}_{\theta}^{cs} = -\frac{\sqrt{1-\beta^2}}{1-\beta \cos \theta_j} \left[\frac{1}{X'^2 N'^2} (S_{1j} L'_3 (L'_1 L'_3 + L'_2 L'_4)) + \frac{S_{2j}}{d'_{ij}} L'_4 (L'_1 L'_5 + L'_2 L'_6) \right] \frac{e^{jk'(d'_{ij} + r'_j - ct')}}{k'^2 d'_{ij} r'_j} \quad (34)$$

$$\bar{E}_{\phi}^{cs} = -\frac{\sqrt{1-\beta^2}}{1-\beta \cos \theta_j} \left[\frac{1}{X'^2 N'^2} (S_{1j} L'_4 (L'_1 L'_3 + L'_2 L'_4)) - \frac{S_{2j}}{d'_{ij}} L'_3 (L'_1 L'_5 + L'_2 L'_6) \right] \frac{e^{jk'(d'_{ij} + r'_j - ct')}}{k'^2 d'_{ij} r'_j} \quad (35)$$

Considering t'_{ij} and t'_j the corresponding elapsed times for d'_{ij} and r'_j respectively; it can be written that

$$t' = \tau + t'_{ij} + t'_j \quad (36)$$

$$jk'(d'_{ij} + r'_j - ct') = -jk'c\tau \sqrt{1-\beta^2} \quad (37)$$

Applying (9), (24) and (37) to (34-35) leads to the time-harmonic secondary scattered fields in the stationary frame

$$\bar{E}_{\theta}^{cs} = -\frac{(1-\beta^2)^{\frac{5}{2}}}{(1-\beta \cos \theta_j)(1-\beta \cos \theta_j)^2} \left[\frac{1}{X'^2 N'^2} (S_{1j} L'_3 (L'_1 L'_3 + L'_2 L'_4)) + \frac{S_{2j}}{d'_{ij}} L'_4 (L'_1 L'_5 + L'_2 L'_6) \right] \frac{e^{-jk'c\tau}}{k'^2 d'_{ij} r'_j} \quad (38)$$

$$\bar{E}_{\phi}^{cs} = \frac{(1-\beta^2)^{\frac{5}{2}}}{(1-\beta \cos \theta_j)(1-\beta \cos \theta_j)^2} \left[\frac{1}{X'^2 N'^2} (S_{1j} L'_4 (L'_1 L'_3 + L'_2 L'_4)) - \frac{S_{2j}}{d'_{ij}} L'_3 (L'_1 L'_5 + L'_2 L'_6) \right] \frac{e^{-jk'c\tau}}{k'^2 d'_{ij} r'_j} \quad (39)$$

$$\bar{H}^{cs} = \frac{1}{\eta} \hat{r}_j \times (E_{\theta}^{cs} \hat{\theta} + E_{\phi}^{cs} \hat{\phi}) \quad (40)$$

It is important to notice that components of these fields represent secondary scattered fields in the observation point at the time $t = \tau + \frac{d'_{ij} + r'_j}{c}$.

The developed approach for deriving direct and coupling scattered fields can be generalized to a collection of arbitrary number of spheres by the employment of an iterative procedure to achieve the scattered fields up to the second order. This is a good approximation regarding that the spheres are in the farfield of each other then the amplitudes of scattered fields more than second order are negligible.

III. NUMERICAL RESULTS

Theoretical results achieved in the last section for a collection of both stationary and moving spheres are simulated to have a deeper physical insight of the problem. The incident field is considered to propagate in the negative \hat{x} direction ($\vec{E}^i = e^{-jkx} \hat{y}$) and the maximum value used for n in the Mie theory is set to be 100 to calculate the numerical results. The azimuth and elevation angles are angular measurements in the spherical coordinate system and have the intervals of $[0, 2\pi]$ and $[0, \pi]$ respectively.

A. Fields for Stationary Scatterers

In this section the refractive index of the spheres is set to be $n = 3.2 + j0.32$. An individual stationary sphere with a radius of $a = 1\text{cm}$ and size parameter of $ka = 10$ ($f = 47.75\text{GHz}$) which is located at $(x, y, z) = (0, -5\text{m}, 0)$ is considered. Fig.4.a represents the 3D scattered field pattern at a distance of 10m from the origin of the coordinate system (radius $r = 10\text{m}$) and Fig.4.b illustrates the azimuth pattern for a 90° elevation angle. According to the position of the sphere, it is expected for the elevation pattern to be symmetric about the elevation angle of 90° which is confirmed by Fig. 4.c.

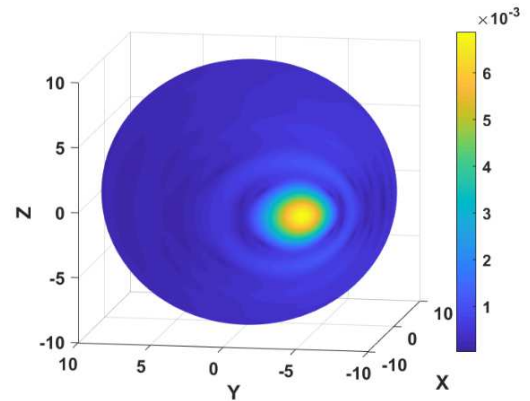


Fig.4. a. Electric scattered field pattern for a stationary sphere at $r = 10\text{m}$.

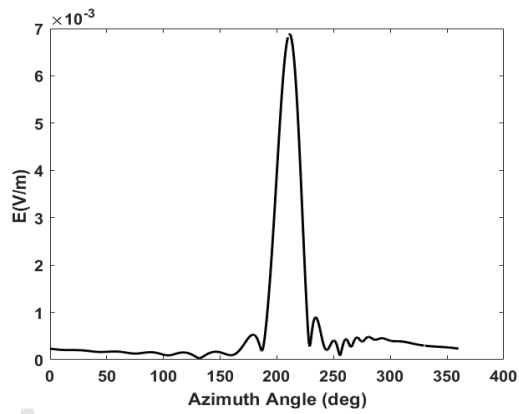


Fig.4. b. Bistatic scattered field amplitude for an elevation angle of $\frac{\pi}{2}$.

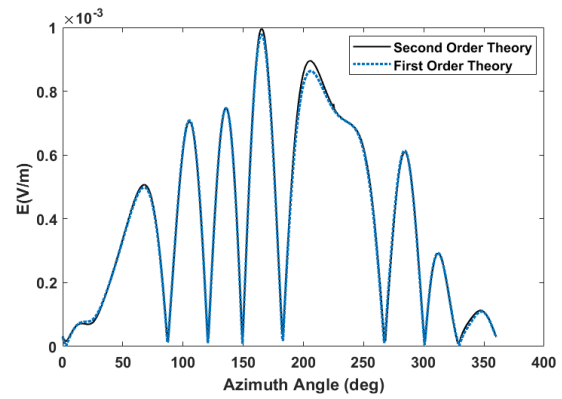


Fig.5. b. Bistatic scattered field amplitude for an elevation angle of $\frac{\pi}{2}$.

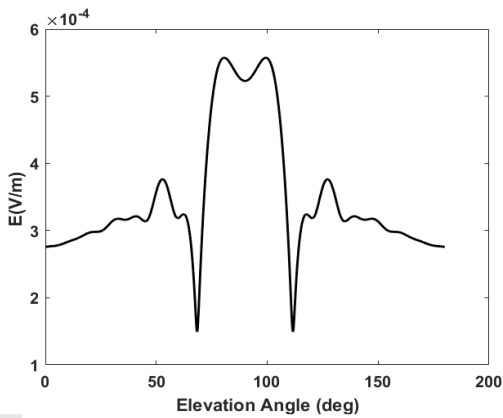


Fig.4. c. Bistatic scattered field amplitude for an azimuth angle of π .

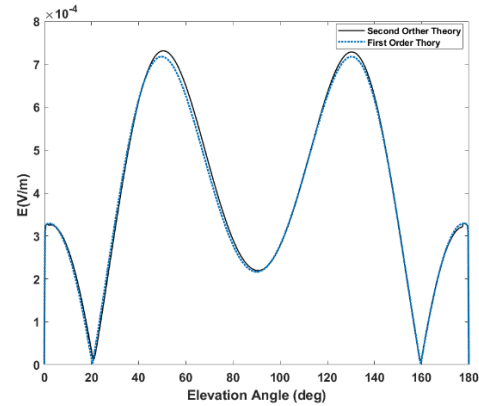


Fig.5. c. Bistatic scattered field amplitude for an azimuth angle of π .

In the following, scattered fields are calculated for two similar spheres with $a = 5\text{mm}$ and $ka = 1.5$ ($f = 14.3\text{GHz}$) which are located at $(1.5\text{cm}, 1.5\text{cm}, 0)$ and $(-1.5\text{cm}, -1.5\text{cm}, 0)$.

Fig.5.a shows the field pattern at $r = 10\text{m}$ and the 2D patterns in the azimuth and elevation planes are illustrated in Figs 5.b and 5.c, respectively. Fig.5.b demonstrates that the maximum level for coupling fields are approximately around the azimuth angle of 225° which is expected regarding the position of the spheres and direction of the incident field. Fig.5.c again has the property of symmetry due to the position of spheres.

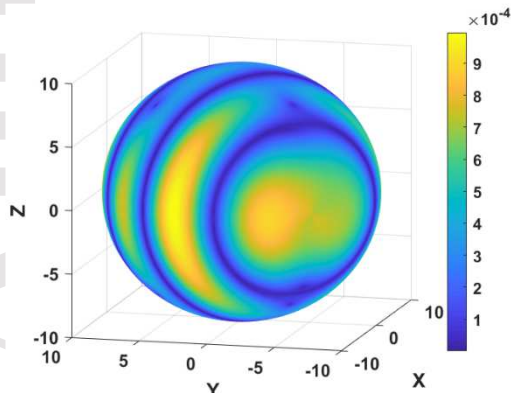


Fig.5. a. Electric scattered field pattern for two stationary spheres at $r = 10\text{m}$.

Ten similar spheres with $a = 5\text{mm}$ and $ka = 3$ ($f = 28.6\text{GHz}$) with locations shown in Table.1 are considered and the bistatic scattered field pattern at $r = 20\text{m}$ is illustrated in Fig.6.a. Maximum amplitude in both Figs 6.a and 6.b occurs around azimuth angle of 180° which states the effect of the addition of direct scattered fields. Maximum deviations of the first and second-order fields, which happen at about $45^\circ, 225^\circ$ as highlighted, are due to the coupling interactions regarding the locations of spheres. These deviations show the importance of the coupling fields when the number of spheres increases. Fig.6.c is also symmetric about the elevation angle of 90° .

Table.1. Configurations of two stationary spheres.

NO	x(cm)	y(cm)	z(cm)
1	1.5	1.5	0
2	4.5	4.5	0
3	7.5	7.5	0
4	10.5	10.5	0
5	13.5	13.5	0
6	-1.5	-1.5	0
7	-4.5	-4.5	0
8	-7.5	-7.5	0
9	-10.5	-10.5	0
10	-13.5	-13.5	0

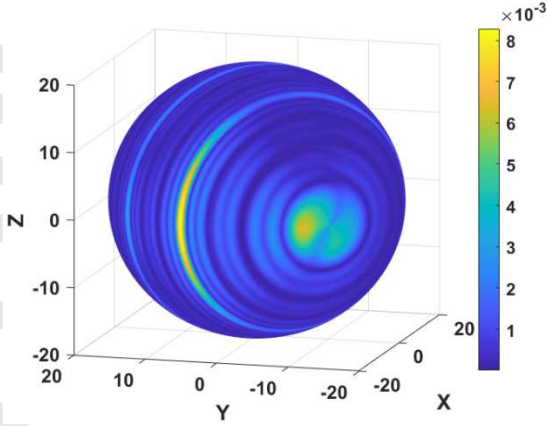


Fig.6. a. Electric scattered field pattern for ten stationary spheres at $r = 20\text{m}$.

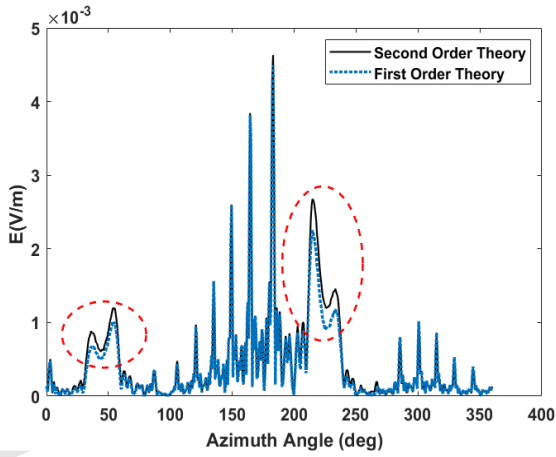


Fig.6. b. Bistatic scattered field amplitude for an elevation angle of $\frac{\pi}{2}$.

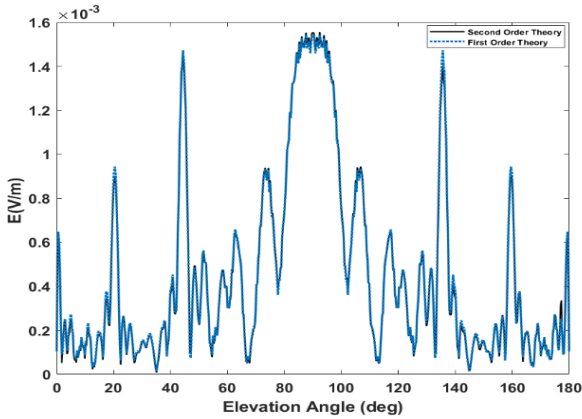


Fig.6. c. Bistatic scattered field amplitude for an azimuth angle of π .

A. Fields for Moving Scatterers

In this section, the amplitude of the scattered electric field is illustrated in a determined observation point with respect to the time. Influence of the effective parameters such as the size, material, number, velocity, position of the spheres and the frequency of incident wave on the scattered fields of the moving spheres has been demonstrated in this section. According to the quickly time varying phase of the both primary and secondary

scattered fields, it is expected that the field patterns appear with a slowly varying amplitude envelop with a rapidly varying carrier.

Firstly, one sphere with $a = 1\text{mm}$, $ka = 10$ ($f = 477\text{GHz}$), $n = 3.2 + j0.32$ and initial position in the origin of the coordinate system, moving with the velocity of $v = 0.5\text{m/s}$ is considered. The scattered field amplitude in the observation point of $(x_p, y_p, z_p) = (-5\text{m}, 0, 5\text{m})$ is illustrated in Fig.7. According to the speed of the sphere and height of the observation point, it is expected that the maximum level of field amplitude to occur at about $t = 10\text{s}$ regarding that the forward scattering is dominant for this ka value. Since the position of the sphere is symmetric about the observation point, scattered field amplitude must be either symmetric which is in full agreement with Fig.7.

In the next step, the conditions are as the same as the previous sphere except that the velocity is set to $v = 2 \times 10^8\text{m/s}$. As it is seen in Fig.8, amplitudes before the peak moment (t_{peak}) are larger than their symmetric corresponding moments (after t_{peak}) which is because of the effect of aberration in propagation direction phenomenon. Also, the peak amount of the amplitude is decreased compared to the previous condition.

For two spheres scenario, a reference mode is considered and only one parameter would be changed in each following mode

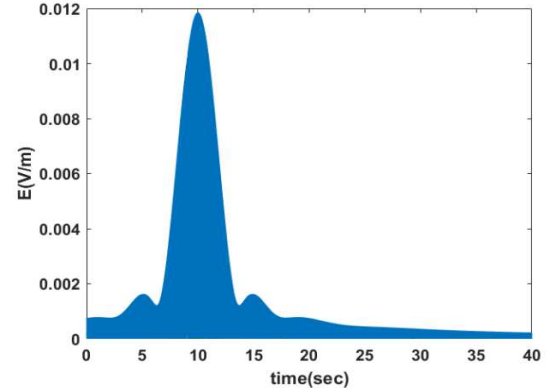


Fig.7. Time-Domain electric scattered field amplitude for a moving sphere.

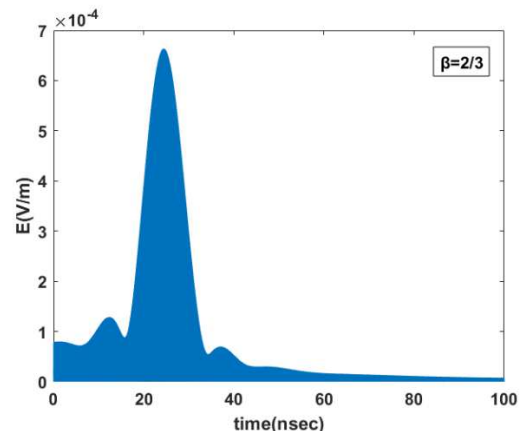


Fig.8. Time-Domain electric scattered field amplitude for a moving sphere ($\beta = 2/3$)

to have a better understanding of the intended parameter. In the reference mode, two spheres are assumed to have $a = 1\text{mm}$, $v = 1\text{m/s}$ and $n = 3.2 + j0.32$. The size parameter is set to $ka = 10$ ($f = 477\text{GHz}$) and coordinates of observation point and primary location of spheres are given in Table.2. The maximum amplitude at $t = 5\text{s}$ and symmetry in Fig.9 could be predicted by physical interpretation. The peak at $t = 5\text{s}$ is about two times of the peak of Fig.7 which indicates that the direct scattered field of each sphere has been added constructively.

This time, the radius of the spheres is changed to $a = 1\mu\text{m}$ and Fig.10 shows that the amplitude reduction of the scattered field (about 1000 times) is proportional to the reduction of the radiuses. Fig.11 relates to the mode that $ka = 5$. Thus comparing this with the results of the reference mode reveals that the lower ka causes the wider beam which is in agreement with the general fact that moving from optical through Mie and Rayleigh scattering regions makes scattering pattern more homogeneous then forward scattered pattern becomes wider.

In the next mode to represent the effect of the **dielectric material**, the extinction coefficient is omitted and the refractive index is set to be $n = 3.2$, as shown in Fig.12.

Table.2. Configurations of Reference Mode for two moving spheres.
units in meter

Observation point			Sphere 1			Sphere 2		
x_p	y_p	z_p	x_1	y_1	z_1	x_2	y_2	z_2
-5	0	5	0	-0.01	0	0	0.01	0

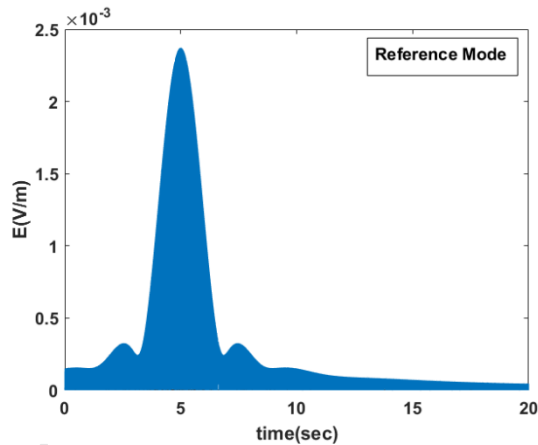


Fig.9. Time-Domain electric scattered field amplitude for a moving sphere. (Reference Mode)

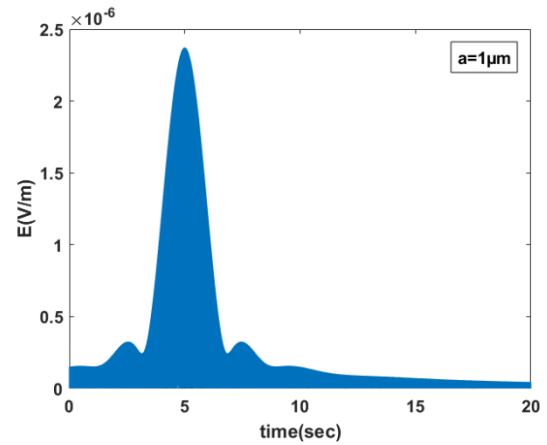


Fig.10. Time-Domain electric scattered field amplitude for two moving spheres. ($a = 1\mu\text{m}$)

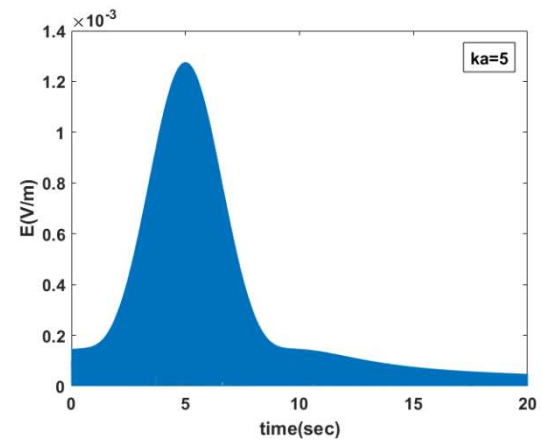


Fig.11. Time-Domain electric scattered field amplitude for two moving spheres. ($ka = 5$)

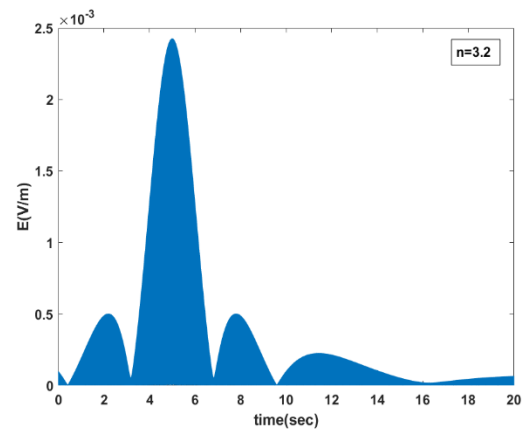


Fig.12. Time-Domain electric scattered field amplitude for two moving spheres. ($n = 3.2$)

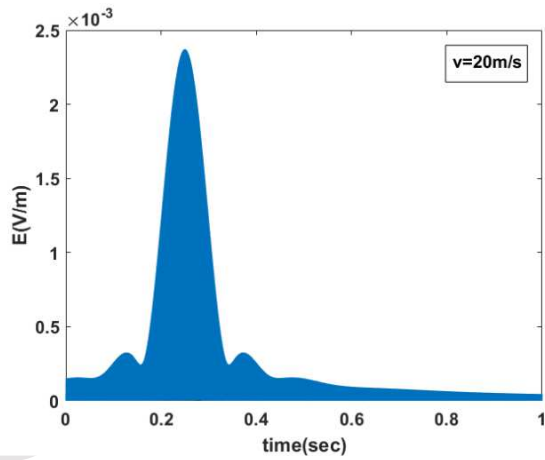


Fig.13. Time-Domain electric scattered field amplitude for two moving spheres. ($v = 20\text{m/s}$)

Fig.13 is for a condition that spheres move with the velocity of $v = 20\text{m/s}$ which states that the pattern has been scaled. Since the velocity of spheres is negligible when compared to the velocity of light, the electric scattered field amplitude is similar to the reference mode.

In the next situation, the observation point is approached to $(x_p, y_p, z_p) = (-1\text{m}, 0, 5\text{m})$, as shown in Fig.14, which means that the closer distances result in narrower scattered beam-widths.

Next, the velocity is set to $v = 2 \times 10^8\text{m/s}$. As it is depicted in Fig.15, the scattered electric field amplitude is going to be more asymmetric by increasing the velocity to the relativistic speeds due the aberration in propagation direction phenomenon. Also, the peak amplitude decreases compared to the previous and reference mode.

In the following, scattered field amplitude for a collection of ten moving spheres with $a = 1\text{mm}$ and $v = 1\text{m/s}$ is calculated. The size parameter and the refractive index are set to be $ka = 10$ ($f = 477\text{GHz}$) and $n = 3.2 + j0.32$ respectively. Table.3 specifies the configuration of the spheres collection and the observation point is considered to be $(x_p, y_p, z_p) = (-1\text{m}, 0, 20\text{m})$. Fig.16 demonstrates the resulting scattered field amplitude.

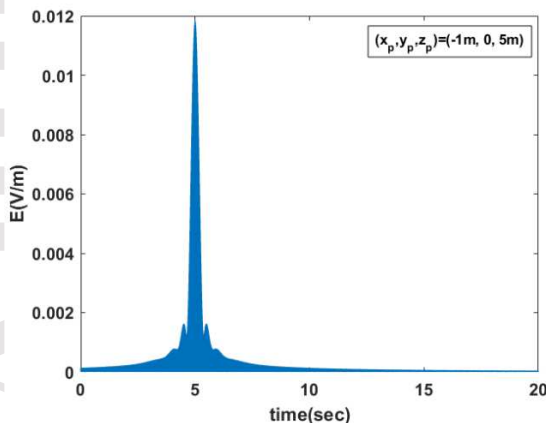


Fig.14. Time-Domain electric scattered field amplitude for two moving spheres with $(x_p, y_p, z_p) = (-1\text{m}, 0, 5\text{m})$

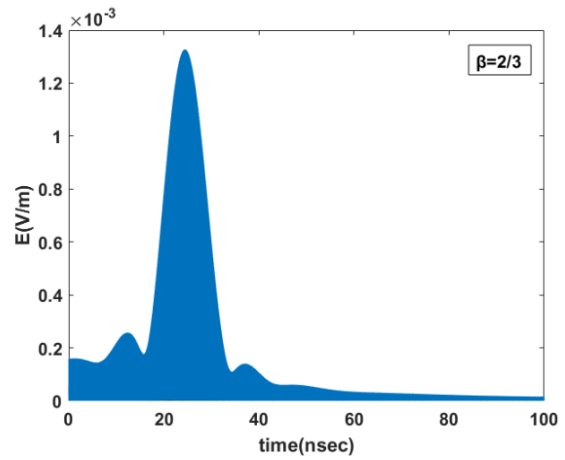


Fig.15. Time-Domain electric scattered field amplitude for two moving sphere ($\beta = 2/3$)

Table.3. Configurations of ten moving spheres.

NO	$x(\text{cm})$	$y(\text{cm})$	$z(\text{m})$
1	1.5	1.5	2
2	4.5	4.5	4
3	7.5	7.5	6
4	10.5	10.5	8
5	13.5	13.5	10
6	-1.5	-1.5	-2
7	-4.5	-4.5	-4
8	-7.5	-7.5	-6
9	-10.5	-10.5	-8
10	-13.5	-13.5	-10

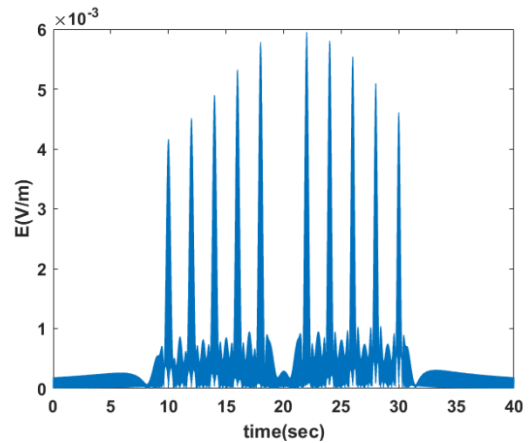


Fig.16. Time-Domain electric scattered field amplitude for ten moving spheres.

Finally, time-domain electric scattered field amplitude for a collection of ten spheres moving in relativistic speed ($\beta = 2/3$) is represented in Fig.17. The remaining parameters are the same as in the previous case.

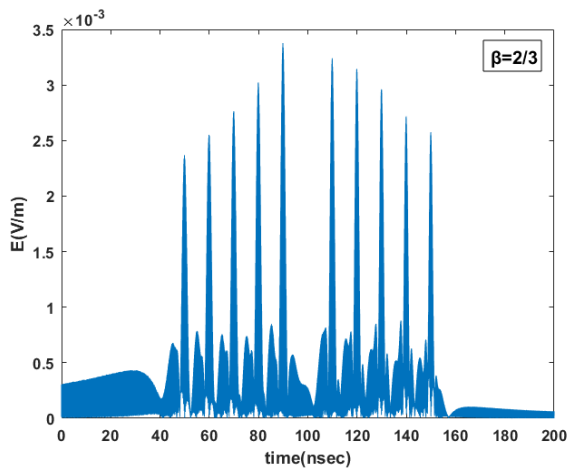


Fig.17. Time-Domain electric scattered field amplitude for a moving sphere ($\beta = 2/3$)

IV. CONCLUSION

In this work, the Frame-Hopping Method (FHM) which is based on the Special Theory of Relativity (STR) is used to obtain time-domain relativistic scattered fields up to the second-order from an arbitrary collection of uniformly translational moving lossy-dielectric spheres. To gain a deeper physical insight of the problem, scattered fields for a collection of both stationary and moving spheres have been simulated. The influence of effective parameters such as the size, material, velocity, number, position of the spheres and also the frequency of the incident field on the scattered fields of a collection of moving spheres has been investigated and the obtained numerical results are in good agreement with physical concepts. Also, a wide variety of objects, such as raindrops, snowflakes, and dust particles, could be approximated by spheres and the study of scattered fields from a collection of moving spheres has a substantial significance for many practical applications. The procedure applied in this work may be the basis for the study of multiple and random scattering from other collections of moving objects considering their mutual interactions.

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