Management of the Curb Space Allocation in Urban Transportation System

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Abstract

Curb space management and traffic flow are two important elements of the transportation system that interact with each other and affect the overall system performance. With the growth of new mobility operators and goods delivery, the demand for access to curb space is increasing rapidly. Thus, the traditional use of curb space solely for parking is challenged and it becomes important to manage curb space effectively. Our study investigates the allocation of curb space for various uses (i.e., parking, pick-up/drop-off, and loading/unloading) so that overall transportation system performance can be enhanced. We simulate the transportation system and analyze the interactions between traffic flow and curb space usage by investigating the impact on traffic congestion of the allocation of curb spaces for different uses. We build an optimization model to determine dynamic curb space allocation decisions that ensure a smooth traffic flow. Our objective is to maximize the cities' profit from curb space allocation decisions and minimization of traffic delay. We further evaluate the value of dynamic curb space allocation policies over fixed allocation policies and find that the dynamic policies can result in improvements in traffic delay and total distance driven.

Keywords: Curb space management, resource allocation, mobility solutions, urban transportation

1. Introduction

Curb spaces have evolved very rapidly with the arrival of new mobility services and increased needs for goods delivery. Currently, curb spaces are not only used for parking but also used for the pick-up/dropoff zone of ride-sharing services, bike share or scooter parking racks, delivery zones for online shopping companies, etc. Although due to the growing demand in ride-sharing services (i.e., Uber, Lyft, Chariot), the need for curb-side parking has decreased, the need for other uses of the curb spaces (i.e., pick-up, wait, drop-off) has risen. Further, the increasing demand in online shopping, which was supposed to reduce traffic jams by reducing individual trips to stores (19), has resulted in an explosion in the trips made by delivery trucks (i.e., UPS, FedEx) and their use of the curb spaces. Hence, concerns

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about traffic congestion has arisen, and it has been found that cruising for parking spaces alone contributes to around 30% of the total traffic congestion in business areas during the rush hour (31). Similarly, it has been found that the total traffic delay from pick-up and delivery activities ranks third among all activities, indicating that the magnitude of this traffic delay is more severe than the expectation (18). Moreover, illegal parking in cities (i.e., blockages of bus lanes, bicycle facilities, and crosswalks by double-parked vehicles) has escalated, and it is reported that the delivery vans of companies such as FedEx and UPS received millions of dollars in parking tickets due to illegal parking in 2018 (22). Thus, the inefficient use of curb spaces can cause a potential safety hazard for people, traffic delays, and loss of city profits (35), and it is very crucial for cities to utilize curb spaces efficiently.

To make the transportation system more reliable, cities across North America are shifting curb spaces from solely parking lanes to flexible zones, where the use of the curb zones can vary dynamically during the day. For example, these flexible zones could shrink, grow, or be assigned to other purposes by considering varying demands for different usages. Some cities have adopted policies that define the use of curb spaces. For example, the city of Seattle uses flexible zones and assigns the curb spaces to different uses according to some predefined priorities. However, no standard methodology exists for cities to assess the potential for dynamic curb space allocation and the subsequent impacts of those changes. Also, despite the importance of the curb space planning, the consideration of dynamic use of the curb spaces during the day limits its large-scale adoption. In this paper, we study the dynamic allocation of curb spaces by cities for different uses. We consider three possible uses of the curb spaces (i.e., parking, pick-up/dropoff, and loading/unloading). We address the benefits of dynamic curb space allocation by considering the interaction between the traffic and the curb spaces, and we develop answers to the following operational questions:

1. Given the number of existing on-street parking spots inside a transportation network, what is the optimal dynamic curb space allocation policy that considers the flexible assignment of curb spaces for different uses (i.e., parking, pick-up/drop-off, and loading/unloading)?
2. What is the value of the dynamic curb space allocation policy in terms of vehicle traffic delay and vehicle driven distance?

To address these questions, we first build a macroscopic simulation model to capture the interaction between the transportation system and the curb space allocation policy. The macroscopic simulation model allows us to analyze several curb space allocation scenarios for different uses and observe the impacts of the model parameters (i.e., vehicle free speed, traffic demand, etc.) on the overall traffic flow. Second, we build an integer programming (IP) model using the outputs of the macroscopic simulation model to determine the optimal dynamic curb space allocation policy among different uses of the curb space (i.e., parking, pick-up/drop-off, and loading/unloading). The objectives of the integer programming are: (i) to maximize the cities' profit from parking, and (ii) to minimize the traffic delay. Since the simulation model should be run for each possible curb space allocation, it becomes intractable to solve the IP model. Hence, we propose a curb space allocation heuristic to solve efficiently the dynamic curb space allocation model. Finally, in our study, we consider both the fixed curb space allocation policy, in which the use of curb spaces is fixed over time, and the dynamic allocation policy, in which the use of curb spaces can vary over time. We further compare both policies to analyze the value of the dynamic curb space allocation

implementation. We show that the dynamic allocation of curb spaces can yield a decrease in both the traffic delay and the total distance driven within the network.

The remainder of this paper is structured as follows: In Section 2, we review the relevant literature. In Section 3, we describe our simulation model, while we present the capacity allocation model in Section 4. In Section 5, we propose a curb space allocation heuristic to efficiently solve the integer programming model. In Section 6, we perform numerical analysis to present our results from both macroscopic simulation and optimization models. Finally, our conclusions are outlined in Section 7.

2. Literature Review

In recent years, with the rapid growth of mobility services, the need for the effective use of curb space has attracted several researchers. Some of them study how cities manage their curb spaces and the existing approaches that are used for curb space management (10; 26; 35). Some propose new policies to find solutions for mitigating traffic congestion. Among studies that focus on policy development in curb space management (31) points out that the congestion within a network is mostly caused by parked vehicles and that the parking rate can be adjusted to decrease the traffic demand entering the network and to better control the traffic delay. In another similar study, (15) proposes policies to mitigate the traffic congestion, such as greatly expanding road capacity, using intelligent transportation system devices to speed traffic flow, and greatly expanding public transit capacity. These policies would be helpful if cities can afford the huge cost and time for the changes, for example, new urban planning to expand road capacity and include high-occupancy vehicle lanes. However, most cities prefer a lower-cost strategy that takes a shorter time to see the effect. Thus, it is more practical and efficient to provide solutions by using the current resources and allocating them efficiently for possible different uses. Researchers have also investigated drivers' parking and cruising behaviors and provided solutions related to parking fees and duration to mitigate the congestion and traffic delay (7; 10; 23). However, these studies focus on high-level policies that are not necessarily based on any methodological framework/model.

Another stream of literature that is relevant to our study is on economics and traffic assignment. In this stream, studies investigate the interaction between parking and the traffic system and analyze the equilibrium of curbside parking (4; 2; 3; 5). Different from these studies, we consider the dynamics of the traffic system (i.e., time-varying conditions). We further build an optimization model to effectively allocate the cities' curb spaces. The studies related to the curb space management are almost solely about parking use, and there are few studies that investigate other uses, such as pick-up/drop-off and loading/unloading. How cities manage curb spaces for major uses (e.g., parking, loading/unloading) is studied by (35). They propose three models related to curb space management planning, price regulation, and community strategies to help cities in curb space management policies and decision-making processes. Although the effect of the existing curb space management policies and the use of curb space for loading are discussed in the paper, no simulation models or optimization models are presented to further validate the efficiency of the proposed curb space management policies.

There are several studies that build multi-agent traffic simulation models to investigate the dynamics of the traffic system (6; 11; 27; 12; 20). Although multi-agent traffic simulation models allow inclusion of personal preferences, driver behaviors, etc., they require detailed data for all specific conditions, and thus their results cannot be easily generalized. Also, the integration of the

multi-agent simulation models with the optimization models would require more computational effort.

In the area of macroscopic simulation models, (34) build a macroscopic framework to investigate the allocation of a certain portion of road space to on-street parking for vacant ride-sourcing vehicles, and they analyze the trade-off between reduction in cruising and capacity loss. More relevant to our study, (8) build a macroscopic simulation model that analyzes the interaction between urban parking and the urban traffic systems and shows their effects on urban congestion. In a follow-up study, (9) present a case study of an area within the city of Zurich, Switzerland, using their macroscopic simulation model and analyze the traffic performance measures (i.e., traffic delay, total distance) within the network. Different from them, we consider other uses of the curb space (i.e., pick-up/drop-off and loading/unloading) in addition to parking-only use and investigate the optimal curb space allocation by building an optimization model on the top of the macroscopic simulation model.

3. Simulation Model

In this section, we build on the study of (9) and develop a macroscopic simulation model to investigate the interaction between the transportation system and curb space allocation. Different from (9), we introduce additional system states by introducing new curb space uses (i.e., pick-up/drop-off, loading/unloading). We consider a relatively small urban area where all existing on-street public parking spaces[1] are randomly distributed. Also, we assume that all existing curb spaces are uniformly distributed such that the drivers do not have a preference. We use P, PD, and LU to denote the cases of parking, pick-up/drop-off, and loading/unloading, respectively. A vehicle's trip starts when the vehicle enters the urban network area and ends when the vehicle leaves the urban area. We assume that trips are uniformly distributed after the vehicles enter the network.

When a vehicle enters the network, the following cases can occur: (i) the vehicle can go through traffic, (ii) the vehicle can search for a parking (P) spot, (iii) the vehicle can search for a pick-up/drop-off (PD) spot, or (iv) the vehicle can search for a loading/unloading (LU) spot. We assume that only a proportion of traffic entering the network will look for a curb space; the other traffic will go through the network after driving for a certain distance. Also, vehicles that look for a $P/PD/LU$ spot may leave the network without accessing any curb space after cruising for a certain time. More specifically, as illustrated in Figure 1, we consider three scenarios that can occur after a vehicle enters the network:

- Scenario 1: Vehicles that look for a $P/PD/LU$ spot enter the network and successfully access a curb space (Figure 1(a)).
- Scenario 2: Vehicles that look for $P/PD/LU$ spot enter the network and then leave the network after cruising for more than a certain time without accessing a curb space (Figure 1(b)).
- Scenario 3: Vehicles that do not look for a curb space enter the network and go through the network (Figure 1(b)).

Let $z \in \{P,PD,LU\}$ denote the different types of curb space usage and J be the set of system states. We use the following system states to simulate the vehicle movement:
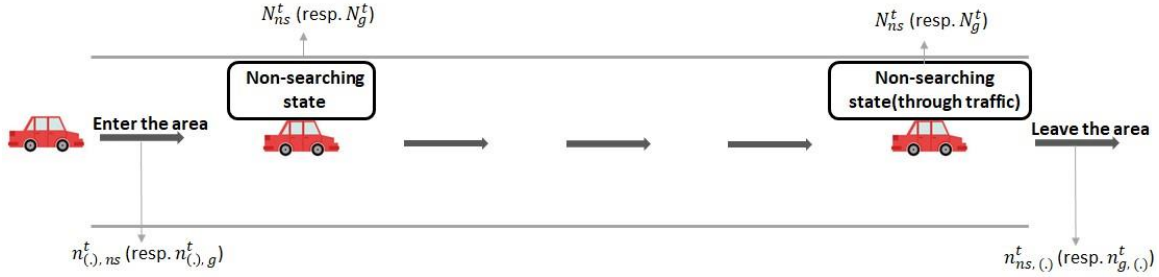
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[1] Off-street parking spaces and private parking spaces are not considered

1. *Non-searching (ns)*: This state includes vehicles that are not searching for any spot. The vehicles may have either just entered the network or just departed from the curb space.
2. *Searching (s_z)*: The vehicles in this state are cruising to find a curb spot $z \in \{P, PD, LU\}$.
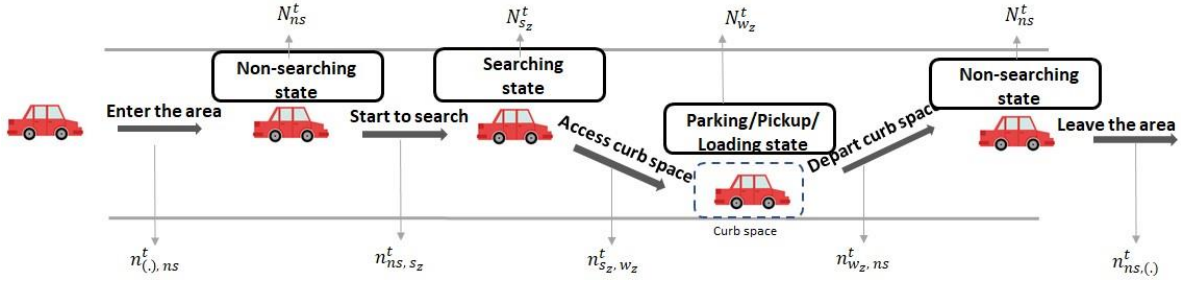
(a) Scenario 1

(b) Scenario 2 (resp. Scenario 3)

Fig. 1. Illustration of all state events and transition events for each scenario

3. *Stationary (w_z)*: This state involves vehicles that have accessed a curb spot $z \in \{P, PD, LU\}$.
4. *Going through traffic (g)*: In this state vehicles do not enter the searching state and simply go through the network.

During the simulation, we assume that there are $t \in T$ time periods. In order to capture the changes in the number of vehicles, we define N_j^t to represent the number of vehicles in each system state $j \in J = \{ns, s_z, w_z, g\}$ in time period t, and we define $n_{j,j'}^t$ to represent the number of vehicles transitioning from system state $j \in J$ to system state $j' \in J$ in time period t.

$$N_{ns}^t = N_{ns}^{t-1} + n_{(.),ns}^{t-1} + n_{(.),g}^{t-1}$$

$$+ \sum_{z \in \{P,PD,LU\}} \left[n_{w_z,ns}^{t-1} - n_{ns,s_z}^{t-1} \right] - n_{ns,(.)}^{t-1} - n_{g,(.)}^{t-1} \qquad \forall t \in \mathcal{T} \qquad (1)$$

$$N_{s_z}^t = N_{s_z}^{t-1} + n_{ns,s_z}^{t-1} - n_{s_z,w_z}^{t-1} \qquad \forall z \in \{P,PD,LU\}, t \in \mathcal{T} \qquad (2)$$

$$N_{w_z}^t = N_{w_z}^{t-1} + n_{s_z,w_z}^{t-1} - n_{w_z,ns}^{t-1} \qquad \forall z \in \{P,PD,LU\}, t \in \mathcal{T} \qquad (3)$$

$$N_g^t = n_{(.),g}^{t-1} - n_{g,(.)}^{t-1} \qquad \forall t \in \mathcal{T} \qquad (4)$$

Then, we use the following equations to calculate the number of vehicles in each system state $j \in J$:

We note that $n_{(.),ns}^{t-1}$ denotes the number of vehicles entering the network and transitioning into the nonsearching state, while $n_{(.),g}^{t-1}$ denotes the number of vehicles entering the network and going through traffic without entering the searching state. Similarly, $n_{j,(.)}^{t-1}$ denotes the number of vehicles leaving the network in period $t - 1$. Equations (1)-(4) define the number of vehicles in the states of non-searching, searching (i.e., for parking, for picking-up/dropping-off, for loading/unloading), stationary (i.e., P, PD, LU), and going through traffic, respectively, in time period t.

3.1. Intermediate variables

In order to capture the change in the number of vehicles between transitioning system states during a given time period, we introduce some intermediate variables, as shown in Table 1[2].

Table 1

Intermediate variables

Notation	Definition
A_z^t	The number of available P/PD/LU spots at the beginning of time period t
k^t	Average traffic density in time period t
v^t	Average travel speed in time period t
d^t	Maximum drive distance of a vehicle in time period t
s^t	Spacing between vehicles that are searching for P/PD/LU at the beginning of time period t
m^t	Maximum number of vehicles that can pass by the same spot on the network during time period t

The number of curb spots available of type z in period t (i.e., A_z^t) equals the total capacity of curb spots in type z (A_z) minus the number of spots that are occupied in type z in period t. We define this relation by using Equation (5):

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[2] We summarize all notation that are used in Appendix A with Tables 9 and 10.

$$A_z^t = A_z - N_{w_z}^t \qquad\qquad \forall z \in \{P,PD,LU\}, t \in T \qquad\qquad (5)$$

where $A_z^t \leq A_z$. Let L be the length of the traffic network. In Equation (6), we define the average traffic density in period t (k^t) as the division of the total number of vehicles on the road at the beginning of time period t by the length of the network.

$$k^t = \frac{N_{ns}^t + N_g^t + \sum_{z \in \{P,PD,LU\}} N_{s_z}^t}{L} \qquad\qquad \forall t \in T \qquad\qquad (6)$$

In Equation (7), v^t denotes the average vehicle speed during time period t, and we calculate it based on a triangular fundamental diagram (FD) (14). To this end, we use k_c and k_j to denote the critical and the jam traffic density, respectively. We define Q_{max} as the maximum traffic flow rate that can be adopted on the network. We consider that congestion occurs if the traffic density for a given period is greater than the critical traffic density. To calculate the average vehicle speed, we compare the current traffic density with the jam traffic density. For example, if k^t is not greater than k_j (i.e., traffic density in time period t is not greater than the jam density), we assume a free speed in the network, and we will use the FD methodology to update the travel speed during the time period t. Otherwise (i.e., if traffic density in time period t is greater than the jam density), we assume all vehicles are not able to move any farther in the network, indicating zero vehicle speed. During a given time period t, we assume all vehicles drive at the same speed such that no overtaking is allowed in the network, and a curb space is always occupied by the first vehicle that passes by.

$$v^t = \begin{cases} \frac{Q_{max}}{k_c - k_j} \cdot \left(1 - \frac{k_j}{k^t}\right), & k^t \leq k_j \\ 0, & k^t > k_j \end{cases} \qquad\qquad \forall t \in T \qquad\qquad (7)$$

The maximum driven distance of a vehicle in time period t (d^t) is the multiplication of the vehicle speed in time period t (v^t) by the length of the time period (t_l), and we define this relation through Equation (8).

$$d^t = v^t \cdot t_l \qquad\qquad \forall t \in T \qquad\qquad (8)$$

To calculate the distance between two consecutive vehicles in time period t (s^t), we divide the length of the network by the number of vehicles searching for $P/PD/LU$ spots as shown in Equation (9).

$$s^t = \frac{L}{\sum_{z \in \{P,PD,LU\}} N_{s_z}^t} \qquad\qquad \forall t \in T \qquad\qquad (9)$$

In Equation (10), we describe m^t, which is the maximum number of vehicles that can pass by the same curb space in the network during time period t. We formulate m^t by using the maximum distance a vehicle can drive and the space between two consecutive vehicles in period t. We note that all curb spaces on the network could potentially be visited by $m^t - 1$ vehicles.

$$m^t = \left\lceil \frac{d^t}{s^t} \right\rceil \qquad\qquad\qquad \forall t \in \mathrm{T} \qquad\qquad\qquad (10)$$

3.2. Definition of the number of transitioning vehicles

In this section, we discuss the details of how we define the equations to update the changes of the number of vehicles in each transition event.

3.2.1. Entering the network

We first define the number of vehicles that enter the network (i.e., $n_{(.),ns}^t$ and $n_{(.),g}^t$). We consider that there is a probabilistic traffic demand that enters the network. Among those vehicles, we assume that a δ percentage of the vehicles will go through the traffic and leave the network directly after driving a distance of l_g^t during the time period t. The remainder of the vehicles (i.e., $(1 - \delta)$ percentage of the vehicles) will search for a spot. More specifically, vehicles will search for a *P*, *PD*, or *LU* spot with a percentage of α, β, and γ, respectively, where $\alpha + \beta + \gamma = 1$. However, we assume that if the vehicles cruise more than a certain time before entering the searching state, they will leave the area instead. We consider that δ, α, β, and γ values are fixed throughout the simulation.

3.2.2. Searching for the curb space

After vehicles enter the network, they start to search for a *P/PD/LU* spot after driving a distance l_s^t during time period t [3]. However, some vehicles leave the network without entering the searching state after they cruise for a certain time, and they leave the network after driving for a distance of l_{ns}^t during time period t. We denote vehicle cruising time as CT in the following equations. We formulate the number of vehicles that cannot enter the searching state in time period t after cruising for a certain time through Equations (11) and (12), where $\phi_{ns,ns}'$ defines the binary variables indicating whether these vehicles can drive the required distance to start searching.

$$n_{ns,ns}^t = \sum_{t'=1}^{t-CT} n_{(.),ns}^{t'} \cdot \phi_{ns,ns}' \ , \ d^{t'} < l_s^{t'} \qquad\qquad \forall t \in \mathrm{T} \qquad\qquad (11)$$

where

$$\phi_{ns,ns}' = \begin{cases} 1, & l_s^t > \sum_{j=t-CT}^{j=t} d^j \\ 0, & \text{otherwise} \end{cases} \qquad\qquad (12)$$

In Equation (11), $n_{(.),ns}^{t'}$ consists of vehicles that entered the area in any time period between 1 and $t-CT$. In time period $t' \in [1, t-CT]$, among the vehicles which have not started searching in previous periods, the vehicles that do not drive the required distance l_s^t cannot enter the searching state. The statement $d^{t'} < l_s^{t'}$ ensures that the driven distance of a vehicle is less than the required distance in each time period. We define these conditions through the above equations. We further formulate the number of vehicles that start searching for *P/PD/LU* spots during time period t after driving a certain distance l_s^t with Equations (13) and (14).

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[3] We assume that this distance is the same for all searching states (i.e., P, PD, and LU)

$$n_{ns,s_z}^t = \sum_{t''=t-CT-1}^{t-1} n_{(.),ns}^{t''} \cdot \phi_{ns,s_z}'' \ , \ d^{t''} < l_s^{t''} \qquad\qquad \forall z \in \{P,PD,LU\}, t \in T \qquad (13)$$

where

$$\phi_{ns,s_z}'' = \begin{cases} 1, & l_s^t \le \sum_{j=t''}^{j=t-1} d^j \text{ and } \sum_{j=t''}^{j=t-1} d^j \le l_s^t + d^{t-1} \\ 0, & \text{otherwise} \end{cases} \qquad (14) \qquad \begin{array}{l} \forall z \in \{P,PD,LU\}, t \in T \end{array}$$

In Equation (13), n_{ns,s_z}^t consists of vehicles that entered the area in any time period between $t{-}CT{-}1$ and $t-1$. We do not consider the vehicles that cruise more than the cruising time CT. In time period $t'' \in [t - CT - 1, t-1]$, $n_{(.),s_z}^{t''}$ vehicles will search for $P/PD/LU$ spots after entering the area. Two conditions must be satisfied: (i) the vehicles should drive a certain distance to start searching, and
(ii) they have not started searching in previous periods. In Equation (14), ϕ_{ns,s_z}'' indicates whether $n_{(.),ns}^{t''}$ vehicles can drive the distance l_s^t within the cruising time CT and transit from non-searching state to the searching state (i.e., searching for parking, picking-up/dropping off, or loading/unloading).

3.3. Accessing curb space

Once vehicles drive enough distance to enter the searching state, they are able to access any curb space as long as there is a vacancy. However, we keep track of only the number of vehicles that can access curb space and not which vehicles. More specifically, we do not model the exact location of each vehicle and each curb space. Our goal is to observe how the curb space allocation decisions impact the overall traffic. Thus, we model the number of vehicles that access curb space and the number of spots that are occupied at time period t.

At the beginning of each time period, the number of vehicles searching for $P/PD/LU$ spots and the number of available curb spaces are calculated in Equation (2) and Equation (5) respectively. We use the following two assumptions in the model: First, the locations of the available curb space are random at the beginning of each time period. Second, the locations of searching vehicles are uniformly distributed on the network at the beginning of each time period. The first assumption ensures stochasticity of the parking availability. The second assumption guarantees that the demand is homogeneously generated. The second assumption is necessary because if vehicles are located mostly within a few streets, the other available curb spaces will not be occupied even if they are vacant. Also, the model can provide an average amount of curb space being taken, and this average value is meaningful only when all searching vehicles are uniformly distributed in the network.

We use x to denote the curb space location. Assume a $P/PD/LU$ spot is located at location x_z, and the remaining P, PD, and LU spots are located at location x_μ, for $\mu \in \{1,2,3,\dots,A_z^t - 1\}$ (i.e., there remain $A_z^t - 1$ spots for each curb space use z). We consider that the searching vehicles' initial positions are at location x_c, for $c \in \{1,2,3,.,,,.N_{s_z}^t\}$. Then, we consider three different cases based on the relations between d^t, s^t, and L to calculate the number of searching vehicles that access a curb space for parking, picking-up/dropping-off, or loading/unloading.

- *Case 1: If $d^t \in [0, s^t]$*: Under this case, the maximum driving distance of a vehicle (d^t) is shorter than the spacing between two consecutive vehicles (s^t). Therefore, no two vehicles' trajectories

will ever overlap during a single time period. As a result, a curb space can be visited at most by one vehicle. Then, the following two conditions should be satisfied to guarantee that a *P/PD/LU* spot at location x_z is occupied during time period t:

- *Condition 1:* The available spot at location x_z must be within the reach of a vehicle. $x_z \in [x_c, x_c + d^t]$ for any $c \in [1, N_{s_z}^t]$. The probability for this condition is as follows: $\sum_{c=1}^{N_{s_z}^t} \int_{x_c}^{x_c+d^t} \frac{1}{L} d x_z$

- *Condition 2:* There should not be any other curb spaces between x_c and x_z. The probability for this condition is stated as follows: $\prod_{x_\mu=1}^{A_z^t-1}(1 - \int_{x_c}^{x_z} \frac{1}{L} dx_\mu)$.

Thus, the probability of a random *P/PD/LU* spot being taken during the time period t is the product of the two probabilities defined under Condition 1 and Condition 2. Then, the average number of *P/PD/LU* spots that are occupied during the time period t equals the multiplication of the number of available spots in each use (i.e., A_z^t) by the product of these two probabilities. We define this expression through Equation (15).

$$
n_{s_z,w_z}^t = A_z^t \cdot \sum_{c=1}^{N_{s_z}^t} \int_{x_c}^{x_c+d^t} \frac{1}{L} dx_z \cdot \prod_{x_\mu=1}^{A_z^t-1} \left(1 - \int_{x_c}^{x_z} \frac{1}{L} dx_\mu \right)
$$

$$\forall z \in \{P, PD, LU\}, t \in \mathcal{T} \tag{15}$$

- *Case 2: If $d^t \in [s^t, L]$:* In this case, vehicles' trajectories can overlap and a curb space can be visited by more than one vehicle (although it accommodates only the first one). We define the probability of a spot at location x_z being occupied during time period t through three sub-cases (i.e., $m^t > A_z^t$, $m^t = A_z^t$, and $m^t < A_z^t$). We investigate the number of vehicles that transit from the searching state to the stationary state for each curb use type z (i.e., $n_{s_z,w_z}^t = A_z^t$) for each sub-case and include the steps and details in Appendix B.

- *Case 3: if $d^t \in [L, \infty]$:* In this case, each vehicle can drive around the whole network at least once, so all vehicles will access curb spaces if there are enough curb spaces. Otherwise, all available curb spaces will be taken. Then, n_{s_z,w_z}^t can be written as follows:

$$
n_{s_z,w_z}^t = \min \left\{ A_z^t, N_{s_z}^t \right\}
$$

$$\forall z \in \{P, PD, LU\}, t \in \mathrm{T} \tag{16}$$

3.4. Departing the curb space

In this section, we define the number of vehicles that transit from one of the stationary states to the nonsearching state. We use the probability distribution function of the parking, picking-up/dropping-off, and loading/unloading durations. Equation (17) shows the number of vehicles that depart from stationary state z in time period t.

$$
n_{w_z,ns}^t = \sum_{t'=1}^{t-1} n_{s_z,w_z}^{t'} \cdot \int_{(t-t') \cdot t_l}^{(t+1-t') \cdot t_l} f(\tau_z) dt_z
$$

$$\forall z \in \{P, PD, LU\}, t \in \mathrm{T} \tag{17}$$

In Equation (17), $n_{w_z,ns}^t$ consists of vehicles that access curb spaces in any time period between 1 and $t-1$, and $f(\tau_z)dt_z$ represents the probability distribution of each curb space use duration. We note that the vehicles that access curb spaces during time period t are not included, as they already experience one transition event during this time period.

3.5. Leaving the network

Vehicles that do not access the curb space (i.e., $n_{(.),g}^t$ and $n_{ns,ns}^t$), or that access and leave the curb space ($n_{w_z,ns}^t$), will leave the network after driving a certain distance. We use l_g^t and l_{ns}^t to denote the required distances that the vehicles need to drive to leave the network for different system states. Then we define the number of vehicles leaving the network at time period t with Equations 18-20.

$$
n_{ns,(.)}^t = \sum_{t'=1}^{t-1} \left(n_{(.),g}^{t'} \cdot \phi_{g,(.)}' + \sum_{z \in \{P,PD,LU\}} \left(n_{w_z,ns}^{t'} \cdot \phi_{ns,(.)}' \right) \right.
$$
$$
\left. + n_{ns,ns}^{t'} \cdot \phi_{ns,(.)}' \right)
$$
$$
\forall t \in T \qquad (18)
$$

where

$$
\phi_{g,(.)}' = \begin{cases} 1, & l_g^t \leq \sum_{j=t'}^{j=t-1} d^j \text{ and } \sum_{j=t'}^{j=t-1} d^j \leq l_g^t + d^{t-1} \\ 0, & \text{otherwise} \end{cases}
$$
$$(19)$$

$$
\phi_{ns,(.)}' = \begin{cases} 1, & l_{ns}^t \leq \sum_{j=t'}^{j=t-1} d^j \text{ and } \sum_{j=t'}^{j=t-1} d^j \leq l_{ns}^t + d^{t-1} \\ 0, & \text{otherwise} \end{cases}
$$
$$(20)$$

As shown in Equation (18), $n_{ns,(.)}^t$ consists of the vehicles that go through the traffic and leave the network, the vehicles that access the curb space and leave the network, and the vehicles that leave the network without accessing a curb space due to the congestion. $\phi_{g,(.)}'$ and $\phi_{ns,(.)}'$ are binary variables indicating whether these vehicles can drive the required distance to leave the network at time period t.

4. Curb Space Allocation Model

In this section, we build a curb space allocation model by integrating the outputs of the simulation model. We develop an optimization model to allocate the curb space optimally among three different uses (i.e., P, PD, and LU). Given the total number of existing curb spaces, our goal is to maximize the total profit of an urban network by allocating the available spaces for P, PD, and LU uses over time. First, we consider a static use of curb space by assigning a fixed allocation for parking, picking-up/dropping-off and loading/unloading. In practice, the curb space allocation strategies of cities, where the use of the curb spaces are fixed, are mostly static. Indeed, most of the curb spaces are currently used solely for parking.

To this end, we define ρ_p as the unit profit obtained from the parked vehicles and c_d as the unit cost of traffic delay. We further use v^t to denote the total traffic delay in time period t due to the congestion. We note that we calculate the traffic delay through the simulation model and that the traffic delay varies as the curb space allocation for different uses changes. We define the traffic delay v^t as a function of $n_{j,j'}^t$ the number of vehicles transitioning from system state $j \in J$ to system state $j' \in J$ in period t. For example, the delay time to find a curb space is the difference between the cumulative number of vehicles that start searching for parking and the cumulative number of vehicles that access parking over time t. Let $n_{j,j'}'^t$ represent the cumulative number of vehicles transitioning from system state $j \in J$ to system state $j' \in J$ in period t. Then $n_{j,j'}'^t$ can be defined as follows:

$$n_{j,j'}'^t = n_{j,j'}'^{t-1} + n_{j,j'}^t \tag{21}$$

Hence the delay time to find a curb space for each stationary state w_z at time t is defined as follows: $v_{w_z}^t = \int_0^t n_{ns,s_z}'^t \, dt - \int_0^t n_{s_z,w_z}'^t \, dt$. Similarly, the delay time until the vehicles start searching at period t is defined as follows: $v_{ns}^t = \int_0^t n_{(.),ns}'^t \, dt - \int_0^t n_{ns,s_z}'^t \, dt$. Then, the total delay v^t is defined as: $v^t = v_{ns}^t + \sum_{z \in \{P, PD, LU\}} v_{w_z}^t$.

In our optimization model, we use A_z, which is the fixed number of curb spots allocated for curb use type z, as the decision variable of the model. Let M_A be the total curb space available. Then, the optimization model for the static curb use can be defined as follows:

$$\max \quad F(A_z) = \sum_{t=1}^{|T|} N_p^t \cdot \rho_p - v^t \cdot c_d \tag{22}$$

$$s.t. \sum_{z \in \{p, pd, lu\}} A_z = M_A \tag{23}$$

Constraints (1)-(20)

$$A_z \geq 0 \qquad\qquad\qquad \forall z \in \{P, PD, LU\} \tag{24}$$

In the above model, Equation (22) represents the objective function, which is the profit obtained from the curb space allocation decisions over all periods. The first term represents the return obtained from the parked vehicles over all periods, while the second term is the total cost due to the traffic delay. Constraint (23) states that the total allocated spots for different uses should be equal the total available curb space spots. The model is also subject to constraints (1)-(20). Finally, constraint (24) defines the non-negativity constraints.

We further consider that the allocated curb spaces can be flexible and can change during the day by considering the demand for different curb uses. Hence, we define $h \in H$ to represent the number of epochs where the number of allocated curb spaces for different types of uses can change in each epoch h. We redefine the time as follows: $t \in \{1, 2, \dots, \frac{|T|}{|H|}, \frac{|T|}{|H|} + 1, \dots, \frac{2 \cdot |T|}{|H|}, \dots, |T|\}$. Then, our dynamic curb space allocation model can be defined as follows:

$$\max \quad F(A_z^h) = \sum_{h=0}^{|H|-1} \sum_{t=1+h\frac{|T|}{|H|}}^{(h+1)\frac{|T|}{|H|}} N_p^t \cdot \rho_p - v^t \cdot c_d \tag{25}$$

$$s.t. \sum_{z \in \{p, pd, lu\}} A_z^h = M_A \qquad\qquad \forall h \in H \tag{26}$$

Constraints (1)-(20)

$$A_z^h \geq 0 \qquad\qquad \forall\, z \in \{p, pd, lu\}, h \in H \qquad (27)$$

Where A_z^h represents the number of allocated curb spaces for the curb use z in epoch h. The dynamic curb space allocation model is similar to the static model. More specifically, equation (25) is used to define the profit function. Constraint (26) states that the allocated curb spots in each epoch h equal the total available capacity. The model is also subject to constraints (1)-(20). Finally, constraint (27) defines the non-negativity. The dynamic model allows that the curb space allocation policy can change over time. This flexibility can ensure that the traffic delay within a specific time interval can be minimized as well as that the curb space can be utilized to the greatest extent.

5. Heuristic Policy

The above curb space allocation model is difficult to solve, as it requires the traffic delay output of the simulation model for all different curb space use configurations (A_z) to find the optimal configuration. In our model, as the numbers of time periods and available curb spots increase, it becomes intractable to compute the optimal objective function and find the optimal allocation policy. In this section, to address computational and practical challenges, we describe a simplistic curb space allocation heuristic
(CSAH). To this end, we consider $\omega \in \Omega$ iterations. Let $A_z^\omega, z \in \{p, pd, lu\}$ be the capacity of curb use z at iteration ω. We further define $\Delta(F)$ to represent the change in the objective function as follows:

$$\Delta\left(F\left(A_p, A_{z'}, A_z\right)\right) = F\left(A_p - 1, A_{z'} + 1, A_z\right) - F\left(A_p, A_{z'}, A_z\right) \qquad \forall\, z \in \{pd, lu\}, z' \in \{pd, lu\}\backslash z$$
(28)

Then, we define the curb space allocation heuristic through Algorithm 1.

Algorithm 1 Curb Space Allocation Heuristic (CSAH)

$\quad \omega = 0,\, A_p \leftarrow M_A,\, A_{pd} \leftarrow 0,\, A_{lu} \leftarrow 0$
$\quad$ **while** $\sum_{z \in \{p, pd, lu\}} A_z^\omega \leq M_A$ **do**
$\qquad$ Calculate $\Delta\left(F(A_p^\omega, A_{z'}^\omega, A_z^\omega)\right) \quad \forall z \in \{pd, lu\}, z' \in \{pd, lu\}\backslash z$
$\qquad$ **if** $\Delta\left(F(A_p^\omega, A_{z'}^\omega, A_z^\omega)\right) \leq 0, \quad \forall z \in \{pd, lu\}, z' \in \{pd, lu\}\backslash z$ **then**
$\qquad\quad$ **break**
$\qquad$ **end if**
$\qquad z^* = argmax_{z'}\, \Delta\left(F(A_p^\omega, A_{z'}^\omega, A_z^\omega)\right), \forall z \in \{pd, lu\}, z' \in \{pd, lu\}\backslash z$
$\qquad A_p^{\omega+1} \leftarrow A_p^\omega - 1,\, A_{z^*}^{\omega+1} \leftarrow A_{z^*}^\omega + 1,$
$\qquad$ **while** $\Delta\left(F(A_p^\omega, A_{z^*}^\omega, A_z^\omega)\right) \leq \Delta\left(F(A_p^\omega - 1, A_{z^*}^\omega + 1, A_z^\omega)\right) \forall z \in \{pd, lu\}\backslash z^*$ **do**
$\qquad\quad A_p^{\omega+1} \leftarrow A_p^\omega - 1,\, A_{z^*}^{\omega+1} \leftarrow A_{z^*}^\omega + 1$
$\qquad$ **end while**
$\qquad \omega \leftarrow \omega + 1$
$\quad$ **end while**
$\quad A_z \leftarrow A_z^\omega$

In the curb space allocation heuristic, we calculate $\Delta\left(F\left(A_p, A_{z'}, A_z\right)\right)$ at each step and find the value of increasing the capacity of curb use type $z' \in \{pd, lu\}$ by one. We increase the allocated capacity of curb use type with the highest gain. We continue increasing the capacity by one for the same curb use until the increase does not provide a sufficient gain (i.e., $\Delta\left(F\left(A_p^\omega, A_{z^*}^\omega, A_z^\omega\right)\right) \leq \Delta\left(F\left(A_p^\omega - 1, A_{z^*}^\omega + 1, A_z^\omega\right)\right)$). The algorithm stops when the allocated capacity reaches the available capacity or when adding one more capacity for all appointments yields a negative profit gain. We note that the above heuristic can be applied to the dynamic case as well by running the heuristic for each epoch separately.

Our heuristic has similarities with the coordinate search type methods (16; 33). More specifically, similar to the coordinate search type methods, our heuristic is an iterative method, and at each iteration, the search continues in one direction while the other components are fixed. Different from the coordinate search type methods our step size (newly allocated curb space) is fixed, integer, and equals to one as at each step we are checking to allocate one new curb space for different uses. Also as a stopping rule, we check the change in the objective function while the coordinate search type methods use the step size as a stopping rule.

6. Numerical Analysis

This section comprises three main parts. First, we describe our parameter settings. Second, we present the results of the simulation model for varying scenario settings. Third, we explore how curb spaces can be allocated to maximize the profit for different scenarios.

6.1. Parameter settings

In this section, we consider an urban network located in downtown Detroit and conduct numerical experiments to validate the efficiency of the proposed simulation model and optimization model. We select a network in the downtown Detroit area with a radius of 300 meters. In total, this network consists of 260 on-street curb spaces for public use (25). First, we calculate the length of all streets inside this network that provide curb spaces for public use by using data provided from the website (25). We further calculate the curb space width by using the "Parking Area Design Report" (32). Figure 2 displays the layout of the selected urban network. Basically, this network contains 12 streets with a total length of 5.32 kilometers (calculated using the Google Distance API). We assume that each street has two directions and one lane per direction on average. Then, the total length of the network is 5.32 * 2 = 11.7 kilometers. Additionally, we study the rush-hour traffic in the downtown area, and we assume that the critical traffic density is k_c = 25 veh/km/lane and jam density is k_j=55 veh/km/lane (8).

We use the Regional Traffic Counts Database (29) to estimate the approximate number of vehicles that enter the network within a given time period. This database provides the daily traffic of each street so that we can estimate the proportionate traffic demand of the streets that are in the selected network. The average vehicle speed in Detroit is about 40 KPH (kilometers per hour) without traffic, based on a Detroit city speed report (21). We use an average speed of 30 KPH by considering the traffic in the downtown area during rush hours. We further perform sensitivity analysis on speed

by using a speed range between 20 KPH and 40 KPH. Since all the existing on-street parking spots in the selected network are metered parking, we consider the metered parking duration for our setting. Based on the studies in the literature (1; 17; 30), we use gamma distribution to model the duration for parking, pick-up/drop-off, and loading/unloading. We use similar parameter values to those of the study of (1), where the authors study metered on-street parking behavior by using both historical data and survey
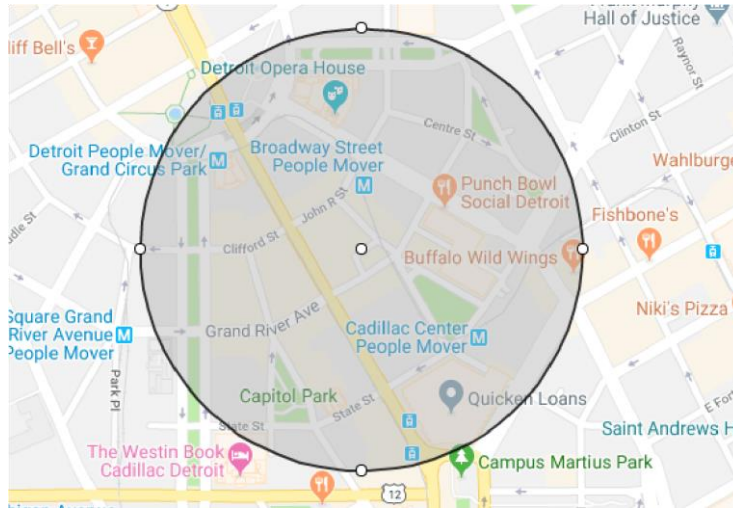

Fig. 2. Selected urban network in downtown Detroit area

data from downtown Ann Arbor, Michigan. We include the figure of the probability distribution function of the parking duration in Figure 4 in Appendix C.

We estimate parameters of the LU duration distribution based on a survey conducted in a study about commercial vehicles' parking duration in New York City and its implications for planning (28). The duration of picking-up/dropping-off is expected to be shorter than loading/unloading goods, in general. Thus, we assume a shorter PD duration and estimate our parameters accordingly.

Next, we define the additional parameters used in the optimization model. On-street parking fees vary depending on the region. For example, the on-street parking fee in Detroit ranges between \$1/h and \$2/h (24). However, the parking fee in the selected network is the same, and it is \$2/h. Hence, we use a fixed parking rate (i.e., \$0.025/min) in the optimization model. We further use \$0.217/min as a delay cost, which is defined and described in detail in the "INRIX Global Traffic Scorecard" (13).

6.2. Change in the traffic flow for the fixed curb allocation

In this section, we conduct our numerical experiments to observe the change in the traffic flow, traffic delay, and occupancy of curb space for different uses by considering several scenarios. We assume that the curb space allocation for different uses is given, and we investigate the optimal curb space allocation decisions in the next section. In this section, we assume that the allocated curb spaces are proportional to the average demand ratio considered in the model. To this end, we consider that the percentages of the allocated curb spaces for parking, picking-up/dropping-off, and

loading/unloading are 70%, 20%, and 10% respectively. Current parking policy in Detroit is static in this selected area, which means that the use of the curb space is fixed over time. Thus, we consider only a static curb space allocation policy in this section. We simulate the traffic system in Detroit for six hours (i.e., between 6:00 a.m. and 12:00 noon). We summarize the parameters used in the numerical analysis in Table 2. We note that $\delta, \alpha, \beta, \gamma$ are the percentage of vehicles that will search for a P, PD, or LU, and we change their values through our scenarios. For example for δ, we consider that it can take three possible values; 0.5, 0.6 and 0.7. Since the sum of the demand proportions of P, PD, and LU should be equal to 1, we consider 18 combinations composed by $\delta, \alpha, \beta, \gamma$. For the traffic demand, vehicle speed, and cruising time we consider three possible values. Hence, in total, we analyze $18 * 3^3 = 486$ instances.

Table 2
Parameter setting for the simulation study

Notation	Definition	Unit	Values
Demand	Traffic demand entering the network.	veh	3500; 4500; 6000
δ	Traffic demand entering the network and go through the network.	veh	0.5; 0.6; 0.7
α	Traffic demand entering the network and headed to P.	veh	0.6; 0.7; 0.8
δ	Traffic demand entering the network and headed to PD.	veh	0.1; 0.2; 0.3
γ	Traffic demand entering the network and headed to LU.	veh	0.1; 0.2; 0.3
v	The free flow speed of network (with traffic flow)	km/h	20; 30; 40
CT	Cruising time	min	5; 10; 15
l	Distance to drive to leave the network for through traffic.	km	Uniform[0, 0.5]
l_s	Distance to drive to transition from non-searching to searching.	km	Uniform[0, 0.5]
$l_{ns,(.)}$	Distance to drive to leave the network after cruising for a certain time.	km	Uniform[0, 0.5]

In Figure 3, we illustrate the change in the cumulative number of vehicles that transit between states over time for both the parking and the picking-up/dropping-off cases[4]. Figure 3 illustrates the total number of vehicles that enter the network (i.e., the line "enter the area"), that start searching for a curb space (i.e., the line "start searching for parking"), that leave the area after cruising for a certain time before entering the searching state (i.e., the line "leave the area without parking (resp. pick-up/dropoff)"), that access the curb space (i.e., the line "access parking (resp. pick-up/drop-off)"), that depart the curb space after parking (resp. pick-up/drop-off) (i.e., the line "depart parking"), and that leave the network after parking (resp. pick-up/drop-off) (i.e., the line "leave the area"). Through Figure 3, we can calculate the average traffic delay and average driven distance. For example, to calculate the searching time to find a curb space, we find the difference between the areas under the

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[4] We note that we include the change in the cumulative number of vehicles that transit between states over time for loading/unloading in Figure 5 in Appendix C.

lines "start searching for parking (resp. pick-up/drop-off)" and "access parking (resp. pick-up/drop-off)".

In Table 3, we present the minimum, average, and maximum traffic delay for different states among all scenarios. We note that we let all vehicles leave the network even after the simulation ends. As shown, the total traffic delay per vehicle ranges from 70 to 371 minutes. The average delay time of searching for a curb space ranges between 69 and 296 minutes, while the delay time in the non-searching state ranges from 1 to 75 minutes. This varying range shows that it is important to have an efficient and dynamic curb space allocation policy that can change over time as a response to varying demand. We can also observe that most of the delay time is caused by vehicles searching for parking, although the allocated curb spaces for parking are greater compared to the other uses.
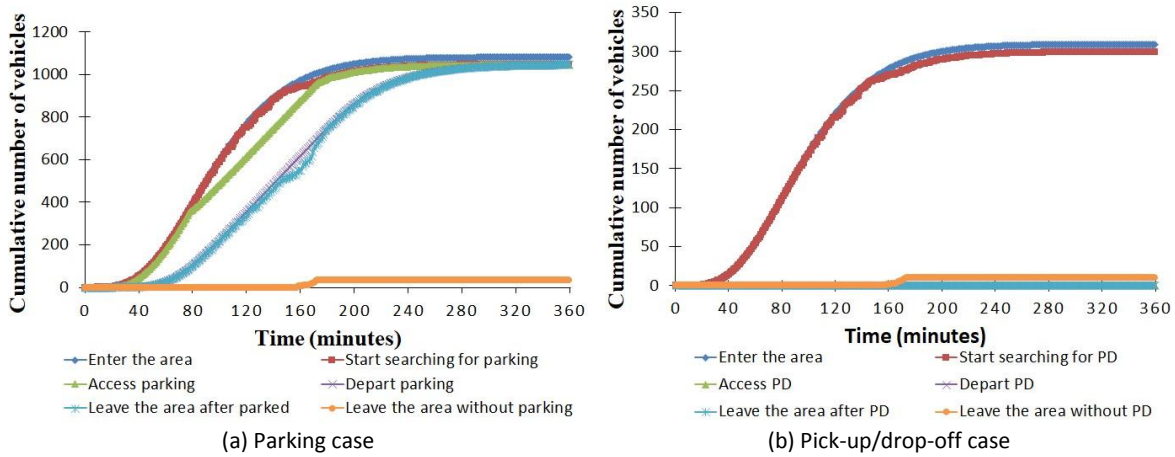

(a) Parking case   (b) Pick-up/drop-off case

Fig. 3. Cumulative number of vehicles transitioning between states over time

Table 3
Average traffic delay when $\alpha =0.7, \beta =0.2, \gamma =0.1$

	Average Delay Time Per Vehicle (minutes)				
Scenario	Non-searching	Searching for Parking	Searching for Pick-up/ Drop-off	Searching for Loading/ Unloading	Total
Minimum	0.88	42.48	10.57	16.59	70.52
Average	29.79	90.63	26.76	30.04	177.24

Maximum	74.97	121.14	61.96	112.83	370.91

Table 4

Average vehicle driving distance when α =0.7,β =0.2,γ =0.1

Scenario	Non-searching	Searching for Parking	Searching for Pick-up/ Drop-off	Searching for Loading/ Unloading	Total
Minimum	0.11	1.13	3.4	4.23	8.88
Average	0.21	7.8	37.47	71.78	117.27
Maximum	0.28	21.92	175.38	350.72	548.3

In Table 4, we present the minimum, average, and maximum values of average driven distance over all scenarios. As shown in the table, we observe that among 486 scenarios, the minimum average driven distance is 8.88 kilometers and the maximum average driven distance 548.3 kilometers. The delay time and the driven distance are highly related to the allocated curb space for different uses. Hence, as a next step, we investigate efficient ways of allocating the curb space for different uses.

6.3. Comparison of the proposed algorithm results with the optimal solution

In this section, we investigate how scarce curb space spots should be allocated among different uses. Considering the scale of the considered urban network, it is not tractable to solve the large-scale setting optimally for varying settings. Hence, in order to examine the efficiency of the proposed algorithm according to the optimal solution, we first consider a small setting. In the small setting, we consider a small urban network with a total network length of 1 km and total simulation time of three hours. Since the network is small, we further adjust the demand in the network and consider two different values for the demand (i.e., 600 and 800). We summarize the parameters that are different from the large case setting in Table 5.

Table 5

Parameters used in the optimization model for small case setting

Notation	Definition	Unit	Value
L	Total network length	km	1
T	Total time length	hour	3

Demand	Traffic demand entering the network.	veh	600; 800
CT	Cruising time	min	5

We first solve the optimal curb space allocation for all instances of the small case using a nonlinear solver (Scipy in Python) and the proposed algorithm. We note that we also enumerate the potential solutions to find the optimal solution. Since the nonlinear solver gets the optimal solution faster than the enumeration, we used its results in the comparison. In Table 6, we present the average process time of the algorithms and the average percent objective gap. We calculate the percent objective difference between different algorithms by using the following formula:

$$Percent\ Objective\ Gap = \frac{Objective\ Value\ of\ the\ Optimal\ Solution - Objective\ Value\ of\ the\ CSAH}{Objective\ Value\ of\ the\ Optimal\ Solution} \tag{29}$$

In Table 6, it is observed that the run time of the CSAH is ten times faster than that of the nonlinear solver and that the percent objective gap is less than 0.6%.

As a next step, we consider the large-scale setting defined in Section 6.1. More specifically we consider the parameters defined in Table 2 and analyze 486 instances in total. For all instances defined, we compare the solution of the nonlinear solver (NLS) with the proposed algorithm solution. We note that we limit the run time of the nonlinear solver to two hours for each instance and report the best results obtained within two hours. The comparison results for the large-scale setting are shown in Table 7. To calculate the percent objective gap, we use a similar formula as presented in Equation (29).

Table 6

Comparison of the proposed and the optimal solutions for the small case

	Optimal	CSAH
Average process time (min.)	38.86	3.33
Average % objective gap	-	0.59%

According to our results, the proposed algorithm is six times faster than the NLS solution. The percent objective gap between the NLS solution and the CSAH is around -2.2%, which indicates that the CSAH has good performance for large-scale settings as well. The CSAH can reach a better solution more time efficiently compared to the NLS. We note that both Table 6 and Table 7 illustrate the results for the static curb space allocation model, and the results are similar to the dynamic curb space allocation model.

Table 7

Comparison of the proposed and the optimal solutions for the large-scale setting

	NLS	CSAH
Average process time (min.)	120	18.99

Average % objective gap	-	-2.21%

We further investigate the average traffic delay that is obtained by using the NLS and CSAH for the large-scale setting. In Table 8, we compare the average vehicle delay time in different system states for the NLS, the CSAH, and a fixed-allocation policy (FAP), which is discussed in Section 6.2 (i.e., 70% for parking, 20% for pick-up/drop-off, and 10% for loading/unloading). For the NLS and CSAH, we consider both the static and the dynamic curb space allocation policies in comparison. We calculate the percent change in the total vehicle delay with respect to FAP and use the following equation for calculation:

$$Percent\ Objective\ Gap = \frac{Vehicle\ Delay\ of\ the\ Proposed\ Policy - Vehicle\ Delay\ of\ the\ FAP}{Vehicle\ Delay\ of\ the\ FAP} \tag{30}$$

Table 8
Comparison of average vehicle delay time

Policies	Non-searching state (min.)	Searching for P state (min.)	Searching for PD state (min.)	Searching for LU state (min.)	Total (min.)	Percent gap from FAP
FAP	29.79	90.63	26.76	30.04	177.24	-
NLS-Static	16.36	66.38	41.59	16.27	140.63	-20.66%
NLS-Dynamic	13.54	68.31	14.79	14.65	111.31	-37.19%
CSAH-Static	15.97	66.43	39.19	15.91	137.52	-22.41%
CSAH-Dynamic	13.20	67.54	14.43	14.77	109.97	-37.95%

As shown in Table 8, the total average delay time per vehicle in the FAP is greater than both the ones in NLS and CSAH. In addition, we see that the dynamic curb space allocation policy yields lower traffic delay per vehicle than the static curb space allocation policy. When the dynamic allocation policy is applied, both NLS and CSAH yield lower traffic delay in all system states compared to the FAP, whereas when the static allocation policy is applied, both NLS and CSAH yield lower traffic delay in all system states than the FAP except the searching for picking-up/dropping-off state. To analyze the benefit of the dynamic allocation policy with respect to the static allocation policy, we also compare the average objective function values over all instances. We find that on average the dynamic allocation policy yields higher profit than the static policy by around 20% for both the NLS and the CSAH.

7. Conclusion

As cities become larger and more complex, it becomes more significant to address mobility challenges. In many cities, curb space is an increasingly contested piece of urban real estate, and the importance of effective curb space allocation is increasing rapidly. Curb space management for different uses is essential for smooth traffic, especially during rush hours in urban areas. In this study,

we provide insights on the interplay between traffic flow and different configurations of curb space usages to assist cities in their curb space allocation decision making.

   More specifically, we build a transportation system simulation model in this study to analyze the interaction between traffic flow and curb space usage. We observe that traffic delay and vehicle driven distance are highly dependent on the usage of curb spaces. Hence, we further analyze how curb spaces can be allocated to mitigate the traffic delay and maximize the city's earnings. To provide computationally tractable policies, we propose a curb space allocation heuristic and compare its performance with that of the optimal policy. We find that the proposed algorithm is a more practical procedure, with a shorter run time, that outperforms the existing nonlinear solver. Also, for large-scale settings, the proposed heuristic can reach a better solution in ten minutes than the non-linear solver can reach within two hours. Through our analysis, we show that the curb space allocation policies should be adapted by considering different demand and network structures and that having a fixed policy does not yield effective performance measures. By having dynamic curb space allocation policies, the needs can be addressed more efficiently so that the traffic delay can be mitigated and the profits of the cities can be increased. Our model is a general parametric model and it can be applied to any urban setting. On the other hand, our numerical experiments and our suggestions are based on the data obtained for the city of Detroit. Hence, the results that we obtain from our model could be applied to the mid-sized cities that have similar characteristics to the city of Detroit.

   As part of future research, the following extensions can be considered. First, in this study, we do not specifically consider which exact spot should be assigned for which use. We provide an overview of how to allocate efficiently the curb space for different uses and consider only a proportional split. Thus, the model can be extended to include the determination of which spots should be assigned for which use in real-world implementations. Second, we consider that the number of curb spaces is given and fixed in the model, but cities can consider redesigning of the curb space. Hence, the optimal number of initial curb spaces can also be investigated as part of a future study. Third, currently we use a fixed hourly parking rate, as this is the current implementation in many cities. As a third extension, dynamic pricing policies can be also investigated.

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