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How does an economy's capital respond to aggregate productivity shocks when firms make lumpy investments? We show that capital's transitional dynamics are structurally linked to two steady-state moments: the dispersion of capital to productivity ratios—an indicator of capital misallocation—and the covariance of capital to productivity ratios with the time elapsed since their last adjustment—an indicator of asymmetric costs of upsizing and downsizing the capital stock. We compute these two sufficient statistics using data on the size and frequency of investment of Chilean plants. The empirical values indicate significant effects of aggregate productivity shocks and favor investment models with a strong downsizing rigidity and random opportunities for free adjustments.

Keywords: inaction, lumpiness, transitional dynamics, sufficient statistics, non-convex adjustment costs, investment, state-dependence, time-dependence.

1. INTRODUCTION

Economies are exposed to productivity, monetary, and many other aggregate shocks. In a frictionless world, agents immediately respond to these shocks and bring the economy back to normal without delay. In contrast, in the presence of microeconomic adjustment frictions, agents gradually respond to these shocks slowing the economy's transition.

Lumpiness—periods of inaction followed by bursts of activity—is one of the most pervasive manifestations of microeconomic adjustment frictions. Capital investment, price and wage setting, labor hiring and firing, inventory management, consumption of durable goods, portfolio choice, and many other economic decisions made by firms and households exhibit lumpy adjustment. How large are the effects of aggregate shocks in lumpy environments? Understanding the role of lumpy adjustment for the propagation of aggregate shocks is crucial for the design and implementation of policies aimed at stabilizing the business cycle and promoting growth.

We propose a new sufficient statistics approach to quantitatively assessing the role of lumpiness for aggregate transitional dynamics. The approach consists of two steps. First,

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we represent the speed of convergence of aggregate variables after an aggregate shock as a function of two steady-state cross-sectional moments. The premise is that observing agents' responses to idiosyncratic shocks in steady state conveys information about their responses to an aggregate shock. Second, we recover these steady-state cross-sectional moments using microdata on adjustments. The premise is that the size and timing of the actions taken by adjusters inform us about the behavior of non-adjusters during their period of inaction.

We apply the sufficient statistics approach to investigate the propagation of productivity shocks when firms make lumpy investments. In the first step, we link the speed of convergence of average capital following an aggregate productivity shock to (i) the steady-state variance of log capital-to-productivity ratios, and (ii) the covariance of log capital-to-productivity ratios with the time elapsed since their last adjustment. These sufficient statistics have a meaningful economic interpretation. The variance of log capital-to-productivity ratios reflects the degree of capital misallocation. In turn, the covariance of log capital-to-productivity ratios with the time elapsed since their last adjustment reflects firms' response to depreciation and the relative costs of shrinking and expanding the capital stock. Thus, our theory indicates that matching these two steady state moments is critical for understanding the transitional dynamics of aggregate capital.[1]

In the second step, we recover these sufficient statistics using data on the size and timing of investment from manufacturing plants in Chile. We discover that the empirical values of these two steady-state moments imply that micro frictions in investment significantly slow down the propagation of productivity shocks. Because different types of adjustment frictions give rise to different values for these moments, the sufficient statistics also serve as model discrimination devices. As a case in point, we find that the investment data discriminates in favor of lumpy models with random fixed costs. Within this subclass, models that feature higher costs to downsizing than upsizing the capital stock.

In summary, when applying our methodology to the study of lumpy investment, we establish structural links between the transitional dynamics of aggregate capital that fol-

[1] Lanteri, Medina and Tan (2019) make a similar point by showing that the transitional dynamics of domestic production following an import-competition shock depend on the size of frictions in capital reallocation; and Moll (2014) shows in a model with financial frictions that the speed of transitions and the steady-state level of capital misallocation jointly depend on the persistence of idiosyncratic productivity.

low an aggregate productivity shock, steady-state moments such as capital misallocation, and the nature of capital adjustment costs. More generally, our sufficient statistics capture the economic forces that shape aggregate dynamics, serve as model discrimination devices, provide researchers with a unique set of moments to be targeted by lumpy models, and guide empirical efforts to collect the most informative statistics for the theory. Next, we explain the theory in more detail and provide economic intuitions for the results.

Sufficient statistics for aggregate dynamics. Consider the following economic environment. There is a continuum of agents. Each agent's uncontrolled state x follows a diffusion, $\mathrm{d}x_t = -\nu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t$, where the trend is common and the Brownian shocks are idiosyncratic. Payoffs depend on the state x. To control their state, agents pay an adjustment cost. The adjustment cost is different for upward and downward adjustments, and there are random opportunities for free adjustments that arrive at a constant rate. The decision rule consists of (i) a constant reset point x^*, to which agents set their state when they decide to adjust, and (ii) the timing of adjustments, which occur when the state reaches one of two thresholds $\{x^-, x^+\}$ or a free opportunity to adjust arrives. The economy features a steady-state distribution of idiosyncratic states $F(x)$. We conceptualize aggregate variables as functions of cross-sectional moments of the state (e.g., the mean, the variance, or other higher-order moments).

In this environment, we characterize analytically the transitional dynamics of the cross-sectional distribution after a common exogenous disturbance. Consider the following hypothetical experiment. Initially, the economy is at its steady-state distribution $F(x)$. At time zero, an aggregate shock hits—a small, identical, and once-and-for-all change in agents' states—displacing the distribution away from the stationary one. As agents gradually respond to the aggregate shock by actively changing their state, the distribution follows a deterministic transition to its steady state. Assuming that agents follow their steady-state decision rules $\{x^-, x^*, x^+\}$ along the transition, that is, neglecting any feedback from the distribution to policies, what can we say about the speed of convergence to steady state?

As a first step, following Álvarez, Le Bihan and Lippi (2016), we define our notion of the speed of convergence as the area under the impulse-response function of any moment of

x relative to its steady-state value. We label this object the cumulative impulse response (CIR). The CIR is a useful metric of convergence: It summarizes in one scalar both the impact and persistence of the economy's response, eases comparison across models, and represents a multiplier of aggregate shocks. In the frictionless benchmark, instantaneous adjustment to the aggregate shock implies a CIR of zero. With adjustment frictions, the larger the CIR, the longer it takes firms to respond to the aggregate shock, and the slower the transitional dynamics.[2]

Our first theoretical result proves that the CIR can be expressed, up to first order, as a linear combination of two steady-state cross-sectional moments. In particular, the CIR of the average of the distribution depends on (i) the steady-state variance of the state, $\mathbb{V}ar[x]$, and (ii) the covariance of the state with its age a, $\mathbb{C}ov[x,a]$, where age is the time elapsed since the last adjustment.

A major challenge to applying our sufficient statistics approach arises if $F(x)$ is unobservable, as in the majority of applications. Thus the steady-state moments cannot be computed directly from the data. As economists, however, we have available detailed panel data $\Omega = \{\Delta x, \tau\}$ with information on the size of discrete adjustments Δx and the duration of completed inaction spells τ. Our second theoretical result provides analytic mappings from the data Ω to moments of the invariant distribution $F(x)$ and the stochastic process parameters (ν, σ^2, x^*). To obtain these mappings, we exploit, exclusively, the properties of Markov processes and the constant reset state x^*.

Taken together, our theoretical results provide researchers with the sufficient statistics that characterize the transitional dynamics of aggregate variables in lumpy environments, as well as with mappings to infer the sufficient statistics and parameters using microdata.

Capital dynamics with lumpy investment. To investigate the propagation of aggregate productivity shocks, we set up a parsimonious partial equilibrium investment model with adjustment costs, in the spirit of Caballero and Engel (1999) and the related literature.[3] Firms produce output with capital. They are subject to depreciation, technological growth,

[2] Álvarez, Le Bihan and Lippi (2016), Baley and Blanco (2019), and Álvarez, Lippi and Oskolkov (2020) use the CIR to compare the effects of monetary shocks across different price-setting models.

[3] Similar environments have been studied by Dixit and Pindyck (1994); Bertola and Caballero (1994); Caballero, Engel and Haltiwanger (1995); Cooper and Haltiwanger (2006); and others.

and idiosyncratic productivity shocks. To change their capital, firms pay a fixed cost that scales with firms' size and could be different for upward and downward adjustments. Also, firms face random opportunities for free adjustments. Defining the state x as the log capital-to-productivity ratio, the model falls into the basic environment described above.

How does the economy's capital respond to a permanent change in aggregate productivity? What do the data tell us about the role of micro lumpiness for capital dynamics? And which type of investment rules best match the data?

Using plant-level investment data from Chile, we recover the two sufficient statistics that characterize the propagation of aggregate productivity shocks: The steady-state variance of log capital-to-productivity ratios, $\mathbb{V}ar[x]$, and the covariance of log capital-to-productivity ratios with their age, $\mathbb{C}ov[x, a]$. Concretely, we recover these sufficient statistics using the following empirical moments: the average and dispersion in duration of inaction, the dispersion and skewness of adjustment size, and the covariance between duration of inaction and adjustment size.

The sufficient statistics inferred from the data imply significant effects of aggregate shocks. We obtain a CIR of 3.7: A 1% decrease in aggregate productivity generates a total deviation in capital-to-productivity ratios of 3.7% above steady state along the transition path. The implied half-life of the aggregate capital response (assuming exponential decay) is 2.5 years. To put these numbers in context, a symmetric fixed adjustment cost model that matches the average frequency of inaction produces a CIR of 0.4 and a half-life of 0.3 years; these numbers are ten times smaller than what the data suggest.

Our analysis reveals that (i) capital adjustment frictions at the micro-level significantly slow down the propagation of aggregate shocks; and that (ii) allowing for asymmetric policies—through large downsizing costs—and randomness in adjustment—through infrequent opportunities for free adjustments—is key for correctly matching the sufficient statistics that shape aggregate capital dynamics.

Contributions to the literature. We highlight three contributions to previous work. First, we provide sufficient statistics that capture the role of micro lumpiness for aggregate dynamics. Álvarez, Le Bihan and Lippi (2016) provide the first step in this direction by studying the transitions of the first moment of the distribution in economies with zero drift and symmetric policies. They show that in a large class of price-setting models, the CIR of real output following a monetary shock is proportional to the kurtosis of price changes times the average price duration. Their theoretical strategy links the CIR directly to the observables in the data. Our strategy is different because we split this challenging problem into two simpler subproblems: From the CIR to steady-state moments and from steady-state moments to the data. Our approach has various advantages. It improves our understanding of the economic forces behind these links. It eases the analysis of sufficient statistics in richer economic environments than previously studied, including drift and asymmetric policies. And finally, it allows us to characterize the transitional dynamics of higher-order moments beyond the average.[4]

Second, we strengthen the bridge between two branches of the literature that study lumpy economies with different objectives and methodologies. The first aims to understand the role of lumpiness for the propagation of aggregate shocks; see Caplin and Spulber (1987); Caplin and Leahy (1991, 1997); and Caballero and Engel (1991) for early work. The second aims to quantify the role of lumpiness for productivity losses in steady state. For example, Álvarez, Beraja, Gonzalez-Rozada and Neumeyer (2018) and Blanco (2020) examine inefficient price dispersion and Asker, Collard-Wexler and De Loecker (2014) examine capital misallocation. To our best knowledge, we are the first to show theoretically the structural links that exist between transitional dynamics of higher-order moments and the steady-state distribution of agents in lumpy economies with drift and asymmetries. We believe our approach may engage researchers in exploiting the connections between these two dimensions of the same environment.

Third, our work speaks to the debate about the nature of capital adjustment frictions. The response of aggregate capital to productivity shocks consists of a direct channel (changes in the marginal product of capital) and an indirect channel (changes in the user cost of capital). The quantitative investment literature has jointly analyzed both channels in models calibrated to match moments that appear ex ante to be sensible choices, but that sometimes lead to opposite conclusions.[5] Instead, our approach focuses exclusively

[4]In a frictionless environment, Gabaix, Lasry, Lions and Moll (2016) studies the dynamics of inequality and provide a lower bound for the speed of convergence by the dominant eigenvalue.

[5]See Thomas (2002); Veracierto (2002); Gourio and Kashyap (2007); Khan and Thomas (2008); Bachmann, Caballero and Engel (2013); House (2014); and Winberry (2021) for different conclusions about

on the direct channel. This permits us to identify precisely the empirical moments that lumpy models must target that capture the role of lumpiness for transitional dynamics and gauge the strength of the partial equilibrium response to aggregate shocks.

2. A PARSIMONIOUS MODEL OF LUMPY INVESTMENT

How does an economy's capital respond to aggregate productivity shocks when firms face capital adjustment frictions? We present a parsimonious partial equilibrium model of lumpy investment to derive sufficient statistics that characterize the role of micro lumpiness for aggregate dynamics. We first study the problem of an individual firm and characterize its optimal investment policy in terms of capital-to-productivity ratios. Then we consider the steady state of an economy with a continuum of ex ante identical firms and perturb it with an aggregate productivity shock. Finally, we define the cumulative impulse response (CIR) of aggregate capital, which summarizes transitional dynamics.

2.1. *The problem of an individual firm*

Time is continuous and extends forever. Consider a firm that produces output using capital. It faces capital adjustment frictions and a constant real interest rate r.

Technology and shocks. The firm produces output y_t using capital k_t according to a production function with decreasing returns to scale

$$(1) \qquad y_t = (z_t e_t)^{1-\alpha} k_t^{\alpha}, \quad \alpha < 1.$$

The firm's total productivity is driven by aggregate z_t and idiosyncratic e_t components. Aggregate productivity z_t grows deterministically at a rate $\mu_z > 0$,

$$(2) \qquad \mathrm{d}\log(z_t) = \mu_z \, \mathrm{d}t.$$

Idiosyncratic productivity shocks e_t follow a geometric Brownian motion with zero drift (w.l.o.g) and volatility σ,

$$(3) \qquad \mathrm{d}\log(e_t) = \sigma \, \mathrm{d}W_t, \quad W_t \sim Wiener.$$

the role of lumpiness when GE effects are present.

The capital stock, if uncontrolled, depreciates at a rate $\zeta > 0$.

The firm can control its capital stock through purchasing or selling capital. For every change in its capital stock (investment) $i_t \equiv \Delta k_t$, the firm must pay an adjustment cost θ_t that is proportional to its total productivity.[6] The adjustment cost is different for positive and negative investments, and there exist random opportunities for free adjustments. Concretely, the adjustment cost takes the form

$$\theta_t \;\equiv\; \Theta\left(i_t, \Delta N_t\right) z_t e_t, \tag{4}$$

where N_t is a Poisson counter with arrival rate λ. The function $\Theta\left(i_t, \Delta N_t\right)$ takes the following values:

$$\Theta\left(i_t, \Delta N_t\right) \;\equiv\; \begin{cases} 0 & \text{if } i_t = 0 \;\text{ or }\; \Delta N_t = 1, \\ \theta^- & \text{if } i_t > 0 \text{ and } \Delta N_t = 0, \\ \theta^+ & \text{if } i_t < 0 \text{ and } \Delta N_t = 0. \end{cases} \tag{5}$$

We label this type of adjustment friction—i.e., asymmetric fixed costs with random free adjustments—Bernoulli fixed costs. We consider different costs for downsizing and upsizing the capital stock to reflect, in a parsimonious way, several asymmetric frictions in capital adjustment. In turn, we consider random free adjustments as a proxy for frictions that contain a stochastic element, e.g., information or search frictions.[7] Our analysis shows that both frictions are relevant to match the data.

An advantage of this formulation is that it nests two benchmark cases of strict state- and time-dependence within a more general framework. Setting $\lambda = 0$ shuts down the random free adjustments and collapses the model into a standard state-dependent fixed cost problem, whereas in the limiting case of infinite fixed costs, i.e., $\{\theta^-, \theta^+\} \to \{\infty, \infty\}$, the model collapses into a standard time-dependent problem that allows adjustment only at random dates that arrive at a rate $\lambda > 0$.[8]

[6]For any stochastic process q_t, we use the notation $\Delta q_t = q_t - q_{t^-}$, where $q_{t^-} \equiv \lim_{s\uparrow t} q_s$ denotes the limit from the left.

[7]Investment with asymmetric adjustment frictions, e.g., partial irreversibility, is studied by Abel and Eberly (1996); Bertola and Caballero (1994); Dixit and Pindyck (1994); and Lanteri (2018); investment with information frictions is studied by Verona (2014); and investment with search frictions is studied by Kurmann and Petrosky-Nadeau (2007) and Ottonello (2018).

[8]The Bernoulli fixed-cost formulation originated in the pricing literature to match the empirical distribution of price changes. See Nakamura and Steinsson (2010) and Álvarez and Lippi (2014).

Investment problem. Let $V(k,z,e)$ be the value of the firm. Given the initial conditions (k_0, z_0, e_0), the firm chooses a sequence of capital adjustment dates $\{T_h\}_{h=1}^{\infty}$ and investments $\{i_{T_h}\}_{h=1}^{\infty}$, where h counts the number of adjustments, to maximize its expected discounted stream of profits. The sequential problem of the firm is described by

$$(6) \qquad V(k_0, z_0, e_0) \;=\; \max_{\{T_h, i_{T_h}\}_{h=1}^{\infty}} \mathbb{E}\left[\int_0^{\infty} e^{-rt} y_t \,\mathrm{d}t \;-\; \sum_{h=1}^{\infty} e^{-rT_h}\left(\theta_{T_h} + i_{T_h}\right)\right],$$

subject to the production function (1), aggregate productivity (2), idiosyncratic productivity (3), adjustment costs (4 and 5), and the law of motion for its capital stock

$$(7) \qquad \log(k_t) \;=\; \log(k_0) \;-\; \zeta t \;+\; \sum_{h:\, T_h \le t} \log\left(1 + \frac{i_{T_h}}{k_{T_h^-}}\right),$$

which describes a period's capital stock as a function of the firm's initial stock k_0, the depreciation rate ζ, and the sum of the investments made at prior adjustment dates.

2.2. *Optimal Policy*

We solve the sequential problem in (6) recursively as a stopping-time problem using the Principle of Optimality. The resulting investment policy is characterized by an asymmetric inaction region

$$(8) \qquad \mathcal{R} \equiv \left\{(k,z,e) : k^-(z,e) \le k \le k^+(z,e)\right\},$$

where $k^-(z,e)$ and $k^+(z,e)$ are the lower and upper inaction thresholds, together with a reset value $k^*(z,e)$ to which capital is set upon every adjustment. Given these three functions, $\{k^-, k^*, k^+\}$, adjustment happens at every date T_h when the capital stock falls outside the inaction region $\mathcal{R}$ or there is an opportunity of free adjustment:

$$(9) \qquad T_h \;=\; \inf\left\{t \ge T_{h-1} :\; (k_t, z_t, e_t) \notin \mathcal{R} \;\text{ or }\; \Delta N_t = 1\right\}.$$

Investment i_{T_h} is the difference between the reset value and the capital immediately before adjustment:

$$(10) \qquad i_{T_h} \;=\; k^*(z,e) - k_{T_h^-}.$$

Given the optimal adjustment dates in (9), we define two useful notions of duration of inaction: the duration of completed spells, denoted by τ, equal to the difference of two consecutive adjustment dates

$$\tau_h \;\equiv\; T_h - T_{h-1}, \quad \text{with} \quad T_0 = 0, \tag{11}$$

and the duration of uncompleted spells or capital age, denoted by a, equal to the time elapsed since the last adjustment

$$a_t \;\equiv\; t - \max\{T_h : T_h \leq t\}. \tag{12}$$

After each adjustment, the capital age is reset to zero, i.e., $a_{T_h} = 0$.

Log capital-to-productivity ratio. To characterize the policy, it is convenient to reduce the state space and recast the problem in terms of a new variable, the log of the capital-to-productivity ratio:

$$\hat{x}_t \;\equiv\; \log\left(\frac{k_t}{z_t e_t}\right). \tag{13}$$

The problem admits the reformulation because of the homothetic production function and the adjustment costs proportional to productivity.

Lemma 1 characterizes the firm value and the optimal investment policy in terms of the log capital-to-productivity ratio through the standard sufficient optimality conditions. The firm value and the policy must satisfy: (i) the Hamilton-Jacobi-Bellman equation, which describes the evolution of the firm's value during periods of inaction, (ii) the value-matching conditions, which set the value of adjusting equal to the value of not adjusting at the borders of the inaction region, and (iii) the smooth-pasting and optimality conditions, which ensure differentiability at the borders of inaction and the reset point. To simplify notation, we define $\nu \equiv \zeta + \mu_z$, which reflects the drift affecting the uncontrolled $\hat{x}$'s, and $\rho \equiv r + \lambda - \mu_z - \sigma^2/2$. All proofs appear in Appendices A and B; each proof begins with an outline of the proof's strategy and an intuitive explanation.

LEMMA 1 *Let $\mathcal{V}(\hat{x}) : \mathbb{R} \to \mathbb{R}$ be a function of the log capital-to-productivity ratio. If $\mathcal{V}(\hat{x})$ and the values $\{\hat{x}^-, \hat{x}^*, \hat{x}^+\}$ satisfy the following three conditions, then the optimal policy is $\{k^-, k^*, k^+\} = ze \times \{\exp(\hat{x}^-), \exp(\hat{x}^*), \exp(\hat{x}^+)\}$ and $\mathcal{V}(\hat{x}) = V(ze\exp(\hat{x}), z, e)/(ze)$.*

(i) In the interior of the inaction region, $\mathcal{V}(\hat{x})$ solves the HJB equation:

$$\rho\mathcal{V}(\hat{x}) = \exp(\alpha\hat{x}) - \nu\frac{\mathrm{d}\mathcal{V}(\hat{x})}{\mathrm{d}\hat{x}} + \frac{\sigma^2}{2}\frac{\mathrm{d}^2\mathcal{V}(\hat{x})}{\mathrm{d}\hat{x}^2} + \lambda\left[\mathcal{V}(\hat{x}^*) - (\exp(\hat{x}^*) - \exp(\hat{x}))\right], \qquad \forall\hat{x} \in \left(\hat{x}^-, \hat{x}^+\right). \tag{14}$$

(ii) At the borders of the inaction region, $\mathcal{V}(\hat{x})$ satisfies the value-matching conditions:

$$\mathcal{V}(\hat{x}^-) = \mathcal{V}(\hat{x}^*) - \theta^- - \left(\exp(\hat{x}^*) - \exp(\hat{x}^-)\right), \tag{15}$$

$$\mathcal{V}(\hat{x}^+) = \mathcal{V}(\hat{x}^*) - \theta^+ - \left(\exp(\hat{x}^*) - \exp(\hat{x}^+)\right). \tag{16}$$

(iii) At the borders of the inaction region and the reset state, $\mathcal{V}(\hat{x})$ satisfies the smooth-pasting and the optimality conditions:

$$\mathcal{V}'(\hat{x}) = \exp(\hat{x}), \quad \forall\hat{x} \in \{\hat{x}^-, \hat{x}^*, \hat{x}^+\}. \tag{17}$$

Notice that when expressed in terms of log capital-to-productivity ratios, the inaction region and the reset state are constant and thus memoryless. The constant policy implies that each adjustment completely erases the history of idiosyncratic shocks. Also notice that adjustment dates in (9), duration of inaction in (11), and age in (12) can be written as functions of log capital-to-productivity ratios (just exchanging k for $\hat{x}$ in their expressions) and their distributions remain unchanged. In the case of investment, the continuity of the productivity process allows us to recover the investment rate in (7) from the change in the log capital-to-productivity ratio as follows:

$$1 + \frac{i_{T_h}}{k_{T_h^-}} = \frac{k^*(z_{T_h}, e_{T_h})}{k_{T_h^-}} = \frac{k^*(z_{T_h}, e_{T_h})/(z_{T_h}e_{T_h})}{k_{T_h^-}/(z_{T_h^-}e_{T_h^-})} = \exp(\Delta\hat{x}_{T_h}). \tag{18}$$

In the first equality we apply the definition of investment. In the second equality, we multiply and divide by total productivity at the moment of adjustment and use the continuity of the stochastic process in the denominator to exchange (z_{T_h}, e_{T_h}) for $(z_{T_h^-}, e_{T_h^-})$. In the third equality, we substitute the definition of capital-to-productivity ratios.

With the problem of an individual firm fully characterized, we examine an economy with a continuum of firms.

2.3. *Economy with a continuum of firms*

Consider a continuum of ex ante identical firms that face the problem described in the previous section. The stochastic processes of idiosyncratic productivity W_t and the arrival of free adjustments N_t are independent across firms. The economy features a steady-state distribution $G(\hat{x})$, with density $g(\hat{x})$, that solves the following Kolmogorov forward equation with its boundary conditions:

$$(19)\qquad \nu \frac{\mathrm{d}g(\hat{x})}{\mathrm{d}\hat{x}} + \frac{\sigma^2}{2}\frac{\mathrm{d}^2 g(\hat{x})}{\mathrm{d}\hat{x}^2} - \lambda g(\hat{x}) = 0 \ \forall \hat{x} \neq \hat{x}^*,$$
$$\int_{\hat{x}^-}^{\hat{x}^+} g(\hat{x})\,\mathrm{d}\hat{x} = 1, \quad g(\hat{x}^-) = g(\hat{x}^+) = 0.$$

We denote by $\mathbb{E}_g[\cdot]$ the expectations computed with the steady-state density g.

Capital gaps. Using the steady-state distribution, for every firm we define the capital gap as its log capital-to-productivity ratio relative to the steady-state average:

$$(20)\qquad x_t \equiv \hat{x}_t - \mathbb{E}_g[\hat{x}],$$

where $\mathbb{E}_g[\hat{x}] \equiv \int_{\hat{x}^-}^{\hat{x}^+} \hat{x}\,\mathrm{d}g(\hat{x})\,\mathrm{d}\hat{x}$. Notice that in the absence of adjustment frictions, capital gaps would always be equal to zero. Similarly, we redefine the investment policy by centering the borders of the inaction region and the reset state around the average:

$$(21)\qquad (x^-,\ x^*,\ x^+) = (\hat{x}^- - \mathbb{E}_g[\hat{x}],\ \hat{x}^* - \mathbb{E}_g[\hat{x}],\ \hat{x}^+ - \mathbb{E}_g[\hat{x}]).$$

From now on, we will work with capital gaps x. We use $F(x)$ and $f(x)$ to denote the distribution and density of capital gaps. We will also denote by $\mathbb{E}[\cdot]$ the expectations computed with their steady-state distribution F. Given the centralization, the reset gap x^* is understood as the gap of adjusters relative to the average gap in the cross-section.

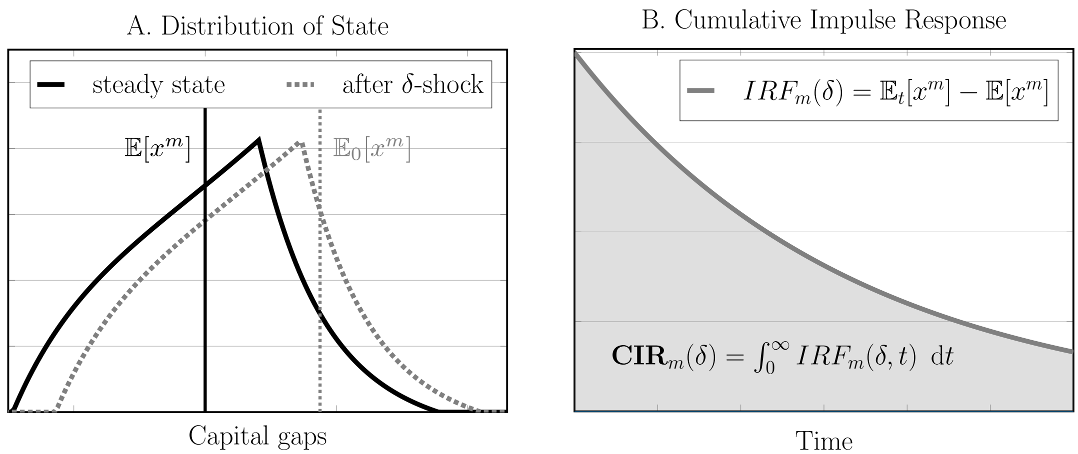
2.4. *Aggregate productivity shock*

How does aggregate capital respond to an aggregate productivity shock? Starting from the steady state, we introduce a small and unanticipated decrease in the (log) level of aggregate productivity of size $\delta > 0$, which we label as δ-perturbation. We normalize its arrival date to $t = 0$, so aggregate productivity is $\ln z_0 = \ln z_{0^-} - \delta$. The negative aggregate productivity shock generates a homogeneous increase in the capital gap of all firms, as they now have too much capital relative to their productivity: $x_0 = x_{0^-} + \delta$.

Let $F_t(x)$ be the distribution t periods after the aggregate shock and $\mathbb{E}_t[\cdot]$ denote expectations computed with F_t. The distribution of capital gaps displaces horizontally to the right relative to the steady-state distribution, i.e., the gap distribution immediately after the aggregate shock is $F_0(x) = F(x-\delta)$.[9] After the initial displacement, the gap distribution evolves according to the firms' policies, and eventually, it converges back to its steady state. By assuming a constant interest rate, investment policies do not respond to changes in the distribution and they are fixed along the transition path.

Panel A in Figure 1 plots the steady-state density $f(x)$ and the initial density $f_0(x)$ following the δ-perturbation; it also shows an arbitrary m-th cross-sectional moment before and after the shock. Our exercise consists of tracking these moments as they make their way back to their steady-state value.

FIGURE 1.— Distributional Dynamics and Cumulative Impulse Response



Notes: Panel A shows the steady-state distribution of the idiosyncratic state $f(x)$ and an initial distribution $f_0(x) = f(x-\delta)$ following the δ-shock. It also illustrates an arbitrary m-th cross-sectional moment to be tracked from its initial value $\mathbb{E}_0[x^m]$ toward its steady-state value $\mathbb{E}[x^m]$. Panel B shows the transitional dynamics of the m-th moment: the IRF (solid line) and the CIR (area under the IRF).

Aggregate deviations from steady state. We are interested in characterizing the effects of the aggregate productivity shock on the average capital-to-productivity ratio, $\hat{K}_t \equiv \mathbb{E}_{g_t}[\exp(\hat{x})]$, expressed as percent deviations from its steady-state value $\hat{K} \equiv \mathbb{E}_g[\exp(\hat{x})]$. This deviation, up to a first-order approximation, can be expressed as the average gap:

$$\frac{\hat{K}_t - \hat{K}}{\hat{K}} = \frac{\mathbb{E}_{g_t}[\exp(\hat{x})]}{\mathbb{E}_g[\exp(\hat{x})]} - 1 = \frac{\mathbb{E}_t[\exp(x)]}{\mathbb{E}[\exp(x)]} - 1 \approx \mathbb{E}_t[x]. \tag{22}$$

[9]The analysis of infinitesimal shocks that displace the cross-sectional distribution away from steady-state is closely related to the marginal response function in Borovička, Hansen and Scheinkman (2014).

To obtain expression (22), the first equality applies the definition of aggregate capital-to-productivity ratios during the transition and in steady state. The second equality is obtained by multiplying and dividing by $\exp(\mathbb{E}_g[\hat{x}])$ and writing in terms of capital gaps. The third step uses a first-order approximation of the exponential function, i.e., $e^x \approx 1+x$, and applies the definition of capital gaps. In this way, we connect the deviation in the average capital-to-productivity ratio to the average capital gap.

While the aggregate capital deviations are not exactly equal to the average capital gap, the approximation is quite helpful for exposition. To exactly compute the deviation, one needs all of the moments of the capital gap distribution, as the full expansion is $\mathbb{E}_t[\exp(x)] = \sum_{n=0}^{\infty} (\mathbb{E}_t[x^m]/m!)$. Later in the paper, we characterize the transitions of all moments of x.

2.5. *Cumulative Impulse Response (CIR)*

To analyze transitional dynamics, we consider the impulse response function of the m-th moment of capital gaps following the δ-perturbation. It is denoted by $\mathrm{IRF}_m(\delta, t)$, it is a function of time, and it is defined as the difference between the moment's value at time t and its steady-state value:

$$\mathrm{IRF}_m(\delta, t) \;\equiv\; \mathbb{E}_t\left[x^m\right] - \mathbb{E}[x^m]. \tag{23}$$

Following Álvarez, Le Bihan and Lippi (2016), we define the cumulative impulse response, denoted by $\mathrm{CIR}_m(\delta)$, as the area under the $\mathrm{IRF}_m(\delta, t)$ curve across all dates $t \in (0, \infty)$:

$$\mathrm{CIR}_m(\delta) \;\equiv\; \int_0^{\infty} \mathrm{IRF}_m(\delta, t)\;\mathrm{d}t. \tag{24}$$

Panel B in Figure 1 plots these two objects. The solid line represents the IRF, and the area underneath it is the CIR. The CIR is a useful metric: It summarizes both the impact and the persistence of the response in one scalar, eases the comparison of models, and represents a multiplier of aggregate shocks. It is illustrative to compare the CIR with and without adjustment frictions. Without frictions, individual gaps are always equal to zero. When the aggregate shock hits the economy, all firms respond instantly to keep their gap at zero. The impulse response is a jump with zero area underneath, i.e., $\mathrm{CIR}_m = 0$ for all m. With frictions, the larger the CIR, the longer it takes firms to respond to the aggregate shock through investment.

Remarks on the definition of gaps. Our definition of capital gaps centers log capital-to-productivity ratios around their steady-state average. In contrast, the standard approach defines gaps using a micro target, which is usually the frictionless optimal capital choice (Caballero, Engel and Haltiwanger, 1997; Caballero and Engel, 1993; Cooper and Willis, 2004). We base our approach on the fact that specifying a micro target is irrelevant for the study of impulse responses centered around steady-state: The micro target cancels out as it enters the impulse response and the steady state symmetrically. Since the micro target does not affect the investment distribution either, we only specify the relative position of a firm's capital-to-productivity ratio in the distribution and not its absolute level.

3. PROPAGATION AND STEADY-STATE MOMENTS

Next, we establish the theoretical relationships between the transitional dynamics of the average capital-to-productivity ratio following an aggregate productivity shock and two steady-state moments of the capital gap distribution.

3.1. *Characterizing the CIR*

As the first step, Lemma 2 expresses the cumulative impulse response of moment m—the CIR_m defined in (24)—as the solution to a collection of stopping-time problems indexed by the initial capital gap. It establishes that it is only necessary to keep track of firms from the arrival of the aggregate shock at $t = 0$ until their first adjustment at $t = \tau$, correcting by the average behavior in steady state. This result is extremely convenient, since it allows us to characterize the propagation of the aggregate shocks without tracking the evolution of the whole distribution of capital gaps.

LEMMA 2 *Given the steady-state policies (x^-, x^*, x^+) and distribution F, the CIR_m can be written as*

$$CIR_m(\delta) = \int_{x^-}^{x^+} v_m(x)\, \mathrm{d}(F(x-\delta) - F(x)), \tag{25}$$

where the function $v_m(x)$ measures the cumulative deviations of the m-th moment from *its steady-state value for a firm with initial capital gap x:*

$$(26) \qquad v_m(x) \equiv \mathbb{E}\left[\int_0^\tau (x_t^m - \mathbb{E}[x^m])\,\mathrm{d}t \middle| x_0 = x\right].$$

An analogous result was first shown by Álvarez, Le Bihan and Lippi (2016) in a driftless and symmetric environment for $m = 1$, noting that after the first adjustment, a firm's expected contribution to the average gap is zero, since positive and negative contributions are equally likely. Thus, in their environment, the average gap conditional on adjustment is equal to zero at every date.

What is surprising is that this property still holds in the presence of drift and asymmetric policies. A firm's investment fully responds to the aggregate shock with its first adjustment. Any subsequent deviations are purely driven by idiosyncratic shocks and are unrelated to the response to the aggregate shock. However, in contrast to the symmetric and driftless case, a firm's expected contribution to the average gap is not necessarily equal to zero, and it depends on the stage of its inaction spell. Completed inaction spells can be ignored because they are equal to the steady-state moment when averaged across all agents; but uncompleted spells cannot be ignored. For this reason, the term $-\int_{x^-}^{x^+} v_m(x)\,\mathrm{d}F(x)$ appears in the expression for the CIR in (25) to correct for the uncompleted idiosyncratic–driven deviations.[10]

Lemma 2 hinges exclusively on properties of Markov processes. It does not need to assume a specific stochastic process for x, the source of the rigidity, the moment we wish to track, or the type of initial perturbation. The crucial assumption is that an adjustment erases the history of shocks—a property embedded in the constant reset state.

3.2. *Sufficient statistics for aggregate transitional dynamics*

Now we proceed to characterize the CIR as a function of steady-state moments. For expositional purposes, we consider the joint steady-state distribution of capital gaps and age, denoted by $F(x, a)$, and for any two numbers $k, l \in \mathbb{N}$, we define the joint steady-state

[10] We are in debt with Andrey Alexandrov for pointing out to us that we missed the correction term $\int_{x^-}^{x^+} v_m(x)\,\mathrm{d}F(x)$ in expression (25) that arises from the asymptotic behavior; see Alexandrov (2020) for further details. Appendix C verifies numerically Lemma 2.

moments of capital gap and age as

$$(27) \qquad \mathbb{E}[x^k a^l] \equiv \int_x \int_a x^k a^l \; \mathrm{d}F(x,a), \quad \forall k,l \in \mathbb{N}.$$

For the Bernoulli fixed-cost model described in Section 2, Proposition 1 characterizes the $\mathrm{CIR}_m(\delta)$. It considers the first-order Taylor expansion $\mathrm{CIR}_m(\delta) = \mathrm{CIR}_m(0) + \delta \mathrm{CIR}'_m(0) + o(\delta^2)$, where $\mathrm{CIR}_m(0) = 0$ and the term $\mathrm{CIR}'_m(0)$ is expressed as a linear combination of two steady-state moments of the distribution $F(x,a)$. Appendix C verifies numerically that the first-order approximation is accurate for small δ-perturbations.

Proposition 1 *Up to first order, the CIR_m with Bernoulli fixed costs is*

$$(28) \qquad \frac{CIR_m(\delta)}{\delta} = \frac{\mathbb{E}\left[x^{m+1}\right] + \nu \mathbb{C}ov\left[x^m, a\right]}{\sigma^2} + o(\delta).$$

Equation (28) shows that up to first order, the transitional dynamics of the m–th moment of capital gaps are structurally linked to the $m+1$ steady-state moment plus a covariance term that corrects for the presence of drift. To better understand why these two moments are sufficient statistics for the propagation of aggregate shocks, let us focus on the case $m = 1$, stated in the following Corollary.

Corollary 1 *Up to first order, the CIR_1 with Bernoulli fixed costs is*

$$(29) \qquad \frac{CIR_1(\delta)}{\delta} = \frac{\mathbb{V}ar\left[x\right] + \nu \mathbb{C}ov\left[x, a\right]}{\sigma^2} + o(\delta).$$

Equation (29) presents the CIR of the mean of the cross-sectional distribution as a linear combination of the steady-state variance of capital gaps, $\mathbb{V}ar\left[x\right]$, and the steady-state covariance between capital gaps and their age, $\mathbb{C}ov\left[x,a\right]$. Since aggregate shocks z_t and idiosyncratic shocks e_t enter symmetrically into a firms' capital gaps, firms' responsiveness to idiosyncratic shocks (encoded by steady-state moments) is informative about their responsiveness to aggregate shocks (measured by the CIR).

Insensitivity to idiosyncratic shocks. To explain heuristically the link between the two sides of Equation (29), we propose the notion of insensitivity to idiosyncratic shocks. Let $\tilde{W}_t \equiv (W_t - W_{t-a_t})/\sigma$ be the sum of all shocks received by a firm since its last adjustment, normalized by their volatility. We define the economy's insensitivity to idiosyncratic shocks as the covariance of capital gaps with $\tilde{W}_t$ in the population of firms, i.e., $\mathbb{C}ov[x_t, \tilde{W}_t]$. Intuitively, if firms sluggishly incorporate changes in their productivity into their capital stock, then the cross-section features a strong relationship between capital gaps and idiosyncratic shocks. In that case, the covariance is large. In the opposite case without adjustment frictions, firms are extremely sensitive to idiosyncratic shocks, so they continuously adjust their gaps to keep them at zero, yielding a zero covariance.

Let us link our definition of insensitivity to idiosyncratic shocks to the CIR_1. The capital gap of any firm at time t can be written as $x_t = x^* - \nu a_t + \sigma^2 \tilde{W}_t$. Multiplying both sides by x_t, taking the cross-sectional average, and using $\mathbb{E}[x] = 0$, we obtain $\mathbb{V}ar[x] = -\nu \mathbb{C}ov[x, a] + \sigma^2 \mathbb{C}ov[x, \tilde{W}]$. Rearranging, it yields $\mathbb{C}ov[x, \tilde{W}] = (\mathbb{V}ar[x] + \nu \mathbb{C}ov[x, a])/\sigma^2$, which is exactly the expression for the $\text{CIR}_1'(0)$ in (29).

This analysis reveals two novel insights. First, when the drift is zero, the propagation of aggregate productivity shocks is proportional to the steady-state variance of capital gaps, normalized by idiosyncratic volatility, i.e., $\mathbb{V}ar[x]/\sigma^2$. This ratio of ex post to ex ante dispersions is a sufficient statistic for aggregate capital's insensitivity to productivity shocks. A large ratio signals insensitivity and slow convergence of average capital gaps.

Second, when the drift is different from zero, the propagation of aggregate productivity shocks is also affected by the covariance of capital gaps and their age, $\mathbb{C}ov[x, a]$. The covariance term corrects for the additional dispersion generated by the drift to identify the insensitivity to productivity shocks. This result implies that capital depreciation ζ or technological progress μ_z—the two components of the drift—directly affect the propagation of aggregate productivity shocks.

Extreme sensitivity. Besides the frictionless case, an environment with a large drift parameter ($\nu \to \infty$) also features extreme sensitivity to idiosyncratic shocks. In this limiting case, the joint steady-state distribution of gaps and age $F(x, a)$ weakly converges to the distribution of an economy without idiosyncratic shocks $\sigma^2 = \lambda = 0$. Capital gaps in the limit are generated by $dx_t = -\nu \, \mathrm{d}t$, so that they become an affine function of age, i.e., $x_t = x^* - \nu a_t$. Multiplying both sides by x_t and taking expectations, we obtain

$\mathbb{V}ar[x] = -\nu\mathbb{C}ov[x,a]$ and thus $\text{CIR}_1'(0) = 0$. Corollary 2 formalizes this argument.[11]

COROLLARY 2 *With Bernoulli fixed costs, when the drift goes to infinity, then*

$$\lim_{\nu\to\infty,\sigma^2>0} \frac{\mathbb{V}ar[x]}{\nu\mathbb{C}ov[x,a]} = -1, \tag{30}$$

which implies that $\lim_{\nu\to\infty,\sigma^2>0} CIR_1(\delta)/\delta = o(\delta)$.

Corollary 2 speaks to a classic result in the pricing literature developed by Caplin and Spulber (1987).[12] That paper considers an environment with nonzero drift (inflation) and zero idiosyncratic risk. The authors show that money shocks do not affect real output: Money is neutral. In our jargon, the $\text{CIR}_1'(0)$ equals zero when $\lim_{\sigma^2\to 0,\nu>0}(\nu/\sigma^2) = \infty$. We replicate an analogous result by taking an equivalent limit: $\lim_{\nu\to\infty,\sigma^2>0}(\nu/\sigma^2) = \infty$. In the investment context, the neutrality result says that aggregate productivity shocks are immediately absorbed by investment when the drift to volatility ratio goes to infinity.

In summary, the variance of capital gaps $\mathbb{V}ar[x]$ and the covariance between capital gaps and their age $\mathbb{C}ov[x,a]$ encode firms' insensitivity to the idiosyncratic shocks, acting as a sufficient statistic for the speed at which the mean of the cross-sectional distribution converges back to the steady state following an aggregate productivity shock.

The CIR of higher-order moments. The macro literature mainly focuses on the dynamics of cross-sectional averages—e.g., capital, output, and inflation. There is increasing interest, however, in the dynamics of higher-order moments. The sufficient statistics for the CIR_m in Proposition 1, for $m > 1$, could in principle be helpful to researchers interested in measuring and characterizing higher-order dynamics using models of lumpy adjustment. For example, the CIR_2—which measures the dynamics of the second moment—relates to the steady-state third moment. This relationship could be used to connect the cyclical fluctuations in the dispersion of investment rates (Bachmann, Caballero and Engel, 2013;

[11]Clearly, as the drift goes to infinity, the duration of inaction goes to zero as well, $\overline{\mathbb{E}}[\tau] \to 0$. This mechanically makes $\text{CIR}_1'(0)$ equal to zero, as firms are constantly adjusting. However, our proof shows a stronger result that the one stated in Corollary 2: Even if we rescale the fixed costs such that for every drift level expected duration $\overline{\mathbb{E}}[\tau]$ is constant, $\text{CIR}_1'(0)$ also goes to zero as the drift goes to infinity. We verify this result numerically in Appendix D.

[12]We thank Fernando Álvarez for suggesting to explore the connection to this limiting case.

Bachmann and Bayer, 2014), marginal products of capital (Oberfield, 2013), or prices (Vavra, 2014; Nakamura, Steinsson, Sun and Villar, 2018) to the steady-state skewness of those distributions. In turn, the CIR_3—which measures the dynamics of the third moment—relates to the steady-state fourth moment. This relationship could be used to connect the cyclical fluctuations in the skewness of sales' growth (Salgado, Guvenen and Bloom, 2019) to their steady-state kurtosis.

3.3. *Sufficient statistics as model discrimination devices*

Our analysis shows that the CIR_1 with Bernoulli fixed costs is structurally linked to two steady-state cross-sectional moments of the capital-to-productivity distribution. Any configuration of this model that generates the empirical values of the sufficient statistics is relevant to the study of aggregate dynamics. Nevertheless, the theory imposes restrictions that can systematically rule in some model configurations and rule out others. We use two benchmark adjustment cost structures nested in our framework to illustrate the power of sufficient statistics in distinguishing across model configurations.

Fully state-dependent adjustments. The Bernoulli fixed-cost model nests the widely used state-dependent model of investment (Caballero and Engel, 1999) by shutting down free adjustments ($\lambda = 0$). This model does not impose any restriction on the sign of the covariance $\mathbb{C}ov[x, a]$. On the one hand, the drift reduces capital gaps as they get older, which pushes the covariance to be negative. On the other hand, the combination of idiosyncratic shocks with asymmetric barriers can generate either a positive or negative covariance. In particular, if downward adjustments are more expensive than upward adjustments, then capital gaps increase as they get older, which pushes the covariance to be positive. Therefore, the relative strength of these two opposing forces determines the value of the covariance of gaps and age.[13] This configuration, however, generates a variance of gaps $\mathbb{V}ar[x]$ close to zero because the distribution of adjustment size concentrates at the borders of the inaction region (in Section 4.2, equations (39) and (41) explicitly show the positive relationship between the variance of gaps and the variance of adjustment sizes). Therefore, if the empirical variance of capital gaps is larger than what a fully state-

[13]In a state-dependent model without drift and with symmetric barriers, $\mathbb{C}ov[x, a] = 0$.

dependent model would predict, this configuration would fall short of explaining the data.

Fully time-dependent adjustments. Another model nested in our framework consists of fully time-dependent adjustments (Calvo, 1983), and it is obtained when both fixed costs go to infinity $\lim\{\theta^-, \theta^+\} \to \{\infty, \infty\}$. In the limit, the inaction region disappears and adjustments occur at a constant rate λ—i.e., they are exponentially distributed. In contrast to the fully state-dependent model, this model generates a larger variance of gaps thanks to the free adjustments (for a given expected duration). This configuration, however, restricts the covariance between gaps and age to be negative $\mathbb{C}ov[x, a] < 0$. Since firms cannot decide when to adjust, the drift renders capital gaps negative as time goes by.[14] Therefore, if the empirical covariance is positive, this configuration would fall short of explaining the data.

Taken together, a positive covariance of gaps an age, $\mathbb{C}ov\,[x, a]$, is a tell-tale sign of state dependence, while a large variance of gaps, $\mathbb{V}ar[x]$, is a tell-tale sign of time dependence. We use these facts when applying the theory to the data in Section 5.

Economic forces behind sufficient statistics. Between the two extreme configurations of fully state-dependent and fully time-dependent adjustments analyzed above, there exists a myriad of parametrizations that bring the model closer to one or the other specification. To better understand the economic forces at play, Appendix D presents comparative statics for the five fundamental parameters governing the process of idiosyncratic shocks and the structure of adjustment costs $(\nu, \sigma, \lambda, \theta^+, \theta^-)$ and discusses how they shape the sufficient statistics and the CIR.

4. STEADY-STATE MOMENTS AND MICRODATA

We have established a structural link between the CIR_m and the steady-state moments of capital gaps. The challenge ahead lies in computing these moments, as capital gaps are difficult to observe. The actions of adjusters, however, are readily available in the microdata. Let $\Omega \equiv (\Delta x, \tau)$ denote a panel of observations of discrete capital gap changes

[14] The covariance is $\mathbb{C}ov[x, a] = -\nu\overline{\mathbb{E}}[\tau]^2 < 0$. See Appendix E.1 for the proof. Additionally, Appendix E.2 shows that, for any drift $\nu \in \mathbb{R}$, the sufficient statistic for the CIR_1 in fully time-dependent models is the average age of the capital gaps $\mathbb{E}[a]$.

(or adjustment size) and the duration of completed inaction spells, and let $R(\Omega)$ denote their distribution. This section shows how to use the distribution of observable actions $R(\Omega)$ to infer the behavior of non-adjusters, and reverse engineer the steady-state cross-sectional moments of $F(x,a)$ and the parameters of the stochastic process.[15]

To establish the inverse mapping from data Ω to steady-state moments and parameters, we need two inputs: A parametric stochastic process for the uncontrolled gaps that is Markovian and has continuous paths (in our case, a Brownian motion with drift ν and volatility σ), and a constant reset gap x^*. These inputs are enough to pin down these mappings. We emphasize that we do not need to assume a specific model of inaction as long as it delivers a constant reset state. For this reason, in this part of the theory, we consider the reset state x^* a parameter.

Notation. To express our results succinctly, we use the following notation. We denote with bars the cross-sectional moments computed with the distribution of adjusters $R(\Omega)$ (e.g., $\overline{\mathbb{E}}[\cdot]$ and $\overline{\mathbb{C}ov}[\cdot,\cdot]$). We denote with tildes the variables that are expressed relative to their mean (e.g., $\tilde{\tau} \equiv \tau/\overline{\mathbb{E}}[\tau]$). Lastly, for any random variable $y \in \mathbb{R}$ and $\psi > 0$, we define the generalized coefficient of variation as $\overline{\mathbb{CV}}^{\psi}[y] \equiv (\overline{\mathbb{E}}[y^{\psi}] - \overline{\mathbb{E}}[y]^{\psi})/\overline{\mathbb{E}}[y]^{\psi}$.[16]

4.1. *Recovering parameters from microdata*

Proposition 2 provides mappings that allow an economist with observables Ω to make inferences about the parameters (ν, σ^2, x^*).

PROPOSITION 2 *Let $\Omega \equiv (\Delta x, \tau)$ be a panel of observations of adjustment size and duration of inaction. Then the drift ν and volatility σ^2 of the stochastic process for capital gaps and the reset capital gap x^* are recovered from the data through the following system:*

$$\nu = \frac{\overline{\mathbb{E}}[\Delta x]}{\overline{\mathbb{E}}[\tau]}, \tag{31}$$

$$\sigma^2 = \frac{\overline{\mathbb{E}}[\Delta x^2]}{\overline{\mathbb{E}}[\tau]} - 2\nu x^*, \tag{32}$$

$$x^* = \nu(\overline{\mathbb{E}}[\tau] - \mathbb{E}[a]) + \overline{\mathbb{C}ov}\,[\tilde{\tau}, \Delta x]\,. \tag{33}$$

[15]Clearly, the age of gaps can be directly measured, but not the joint distribution of (x,a).

[16]For $\psi = 2$, we obtain the standard definition: $\overline{\mathbb{CV}}^{2}[y] \equiv \overline{\mathbb{V}ar}[y]/\overline{\mathbb{E}}[y]^2$.

Drift and volatility. Expressions in (31) and (32) provide a mapping to infer the parameters of the stochastic process. The first expression shows that in a stationary environment, the average adjustment size $\overline{\mathbb{E}}[\Delta x]$ must compensate for the average drift between two adjustments $\nu\overline{\mathbb{E}}[\tau]$. Similarly, the dispersion in adjustment size $\overline{\mathbb{E}}[\Delta x^2]$ reflects the cumulative shocks received during the inaction period: A high dispersion in adjustment size must mean that either idiosyncratic volatility is high or the time between two adjustments is high. Álvarez, Le Bihan and Lippi (2016) obtain an analogous expression for idiosyncratic volatility σ^2 in the symmetric and driftless environment, i.e. $x^* = \nu = 0$, given by $\sigma^2 = \overline{\mathbb{E}}[\Delta x^2]/\overline{\mathbb{E}}[\tau]$. Our mapping includes a new term $-2\nu x^*$ to correctly identify idiosyncratic volatility from the ratio of these two statistics.

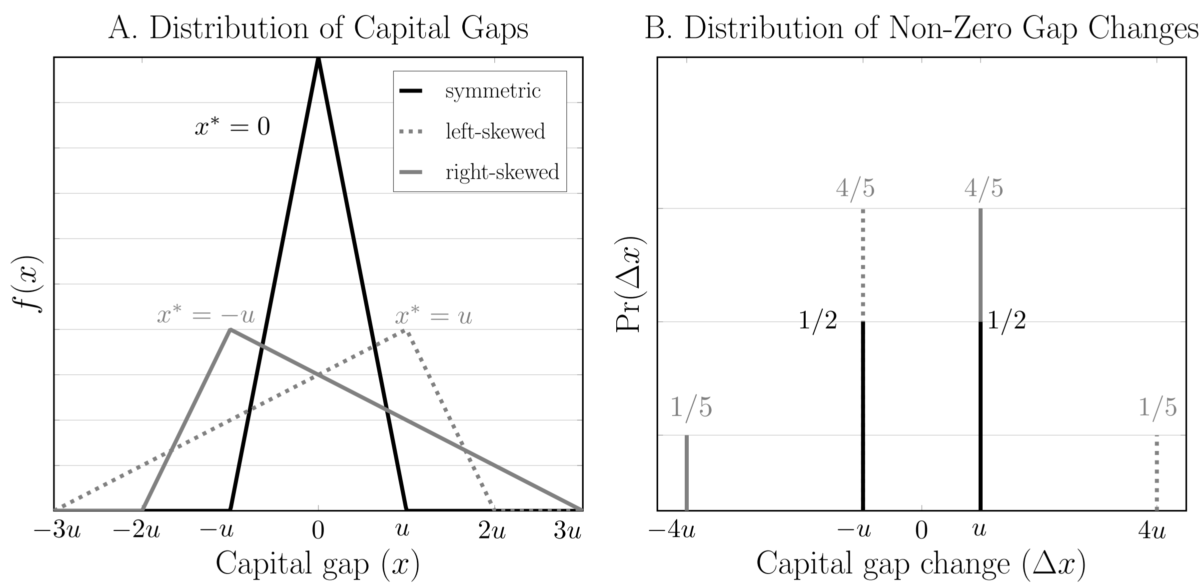
Reset capital gap. Equation (33) shows how to recover the reset gap x^* from the microdata. This object carries information about optimal behavior in environments with nonzero drift and asymmetric policies. Its value is derived from the restriction that capital gaps have a zero mean. Thus x^* must compensate for the average deviations that arise by inactivity and ensure that the mean of the stationary distribution remains at zero.

A nonzero drift and an asymmetric policy may push average deviations in opposite directions. For instance, a negative drift pushes average deviations down, while an asymmetric policy arising from relatively more costly downward than upward adjustment pushes average deviations up (i.e, if $\theta^+ >> \theta^-$, units that have positive gaps tend to get stuck and do not want to adjust down). The reset state summarizes how firms optimally balance these opposing forces. We examine three special cases to showcase each force separately.

Role of policy asymmetry. We start by discussing how the reset state reflects policy asymmetry. We assume away the drift and free adjustments, i.e., $\nu = \lambda = 0$. In this case, the reset gap is a weighted average of gap changes, with weights equal to relative durations of inaction, i.e., $x^* = \overline{\mathbb{E}}[\tilde{\tau}\Delta x]$. The analysis makes use of Figure 2. Panel A plots three distributions of capital gaps (symmetric, left-skewed, and right-skewed) that correspond to alternative assumptions about fixed costs, which generate different policies. Panel B plots the corresponding distributions of non-zero capital gap changes. The idea is to use Panel B (observables) to infer Panel A (unobservables).

If the distribution of gaps is symmetric around zero (black solid line), positive and negative adjustments are equality likely ($\Pr(\Delta x > 0) = \Pr(\Delta x < 0) = 1/2$), implying a zero reset gap $x^* = \overline{\mathbb{E}}[\Delta x] = 0$. If the distribution of gaps is right-skewed (gray solid line), the upper border is further from x^* than the lower border and we observe few negative adjustments ($\Pr(\Delta x > 0) = 4/5 > \Pr(\Delta x < 0) = 1/5$). The negative adjustments get weighted by their longer relative duration, generating a negative reset gap $x^* = \overline{\mathbb{E}}[\tilde{\tau}\Delta x] = -u < 0$. Thus a negative reset gap suggests a right-skewed distribution generated by relatively more costly downward adjustments ($\theta^+ > \theta^-$). An analogous argument applies for positive reset gaps that suggest a left-skewed distribution. Overall, the sign of x^* indicates the shape of the gap distribution and the adjustment costs that generate it.

FIGURE 2.— Reset State, Capital Gaps, and Capital Gap Changes ($\nu = \lambda = 0$)



Notes: Panel A plots three distributions of capital gaps x in the Bernoulli fixed-cost model without drift and free adjustments. The symmetric distribution (black solid line) is generated by the policy $(x^-, x^*, x^+) = (-u, 0, u)$; the left-skewed distribution (gray dotted line) is generated by $(x^-, x^*, x^+) = (-3u, u, 2u)$; and the right-skewed distribution (gray solid line) is generated by $(x^-, x^*, x^+) = (-2u, -u, 3u)$. Panel B shows the corresponding three distributions of non-zero capital gap changes.

One may naively conjecture that the simple average gap change $\overline{\mathbb{E}}[\Delta x]$ provides information about asymmetries. However, this intuition is flawed: Larger adjustments happen with lower occurrence, so that $\overline{\mathbb{E}}[\Delta x] = 0$ regardless of the policy, eliminating any possibility to learn from the average gap change (see Panel B of Figure 2). Our analysis shows that appropriately reweighing the distribution of gap changes using relative durations $\tilde{\tau}$ circumvents this identification challenge.[17]

Role of the drift. Now we discuss the role of the drift by considering two polar cases of the Bernoulli fixed-cost model with symmetric policies. In the limiting case without idiosyncratic shocks ($\sigma^2 \to 0$), the duration of all inaction spells is identical for all firms—say $\overline{\mathbb{E}}[\tau] = \mathcal{T}$—and evidently the covariance between duration and size in (33) disappears. The surviving term, $\nu(\overline{\mathbb{E}}[\tau] - \mathbb{E}[a])$, reflects how the reset gap compensates for the expected erosion caused by the drift that accumulates between adjustments. The expected erosion is proportional to the average length of completed spells ($\overline{\mathbb{E}}[\tau]$, adjusting firms) minus the average length of uncompleted spells ($\mathbb{E}[a]$, inactive firms). The reset gap becomes $x^* = \nu\mathcal{T} - \nu\mathbb{E}[a] = \nu\mathcal{T}/2 > 0$, where $\mathbb{E}[a] = \mathcal{T}/2$ is the average age of uncompleted spells.

Now consider the limiting case with infinite adjustment costs, $\lim(\theta^-, \theta^+) \to (\infty, \infty)$, so that investments occur at a constant rate $\lambda > 0$. The *iid* nature of adjustment dates makes the expected duration for adjusting and non adjusting firms identical $\overline{\mathbb{E}}[\tau] = \mathbb{E}[a]$. The first term in equation (33) disappears and the reset gap is identified by the second term: $x^* = \overline{\mathbb{C}ov}[\tilde{\tau}, \Delta x]$. It is easy to show that $\overline{\mathbb{C}ov}[\tilde{\tau}, \Delta x] = \nu\overline{\mathbb{E}}[\tau]$, corroborating that this covariance correctly identifies the effect of the drift on the average capital gap.[18]

We have discussed at length the forces that shape the reset state for various reasons. It enters into the formulas for volatility and steady-state moments. It indicates the shape of distribution of x. But most importantly, the forces that shape the reset point—and the way they manifest in the data—are also responsible for determining $\mathbb{C}ov[x, a]$.

4.2. *Recovering steady-state moments from microdata*

Proposition 3 provides mappings from observables Ω to steady-state moments of gaps.

PROPOSITION 3 *Let $\Omega \equiv (\Delta x, \tau)$ be a panel of observations of adjustment size and duration of inaction. Construct the gaps immediately before adjustment as $x_\tau = x^* - \Delta x$. Then the following relationships hold for any $m \geq 1$:*

[17] Appendix F presents an instructive example that computes explicitly the steady-state moments in a driftless environment with a right-skewed distribution, as presented in Figure 2.

[18] See Appendix E.3 for the proof.

(i) Average age relates to the average and the dispersion of duration of inaction as

$$(34) \qquad \mathbb{E}[a] \;=\; \frac{\overline{\mathbb{E}}[\tau]}{2}\left(1+\overline{\mathbb{CV}}^2[\tau]\right).$$

(ii) With zero drift ($\nu = 0$), the steady-state moments are given by

$$(35) \qquad \mathbb{E}[x^m] \;=\; \frac{2}{(m+1)(m+2)}\left(\frac{\overline{\mathbb{E}}\left[x_\tau^{m+2}\right]-x^{*m+2}}{\overline{\mathbb{E}}\left[\Delta x^2\right]}\right),$$

$$(36) \qquad \mathbb{E}[x^m a] \;=\; \frac{2\overline{\mathbb{E}}[\tau]}{(m+1)(m+2)}\left(\frac{\overline{\mathbb{E}}\left[\tilde{\tau}x_\tau^{m+2}\right]-\mathbb{E}[x^{m+2}]}{\overline{\mathbb{E}}[\Delta x^2]}\right).$$

(iii) With nonzero drift ($\nu \neq 0$), the steady-state moments are given by

$$(37) \qquad \mathbb{E}[x^m] \;=\; \frac{1}{(m+1)}\left(\frac{x^{*m+1}-\overline{\mathbb{E}}\left[x_\tau^{m+1}\right]}{\overline{\mathbb{E}}\left[\Delta x\right]}\right)+\frac{m\sigma^2}{2\nu}\mathbb{E}[x^{m-1}],$$

$$(38) \qquad \mathbb{E}[x^m a] \;=\; \frac{\overline{\mathbb{E}}[\tau]}{(m+1)}\left(\frac{\mathbb{E}[x^{m+1}]-\overline{\mathbb{E}}\left[\tilde{\tau}x_\tau^{m+1}\right]}{\overline{\mathbb{E}}[\Delta x]}\right)+\frac{m\sigma^2}{2\nu}\mathbb{E}[x^{m-1}a].$$

Average age. Equation (34) relates the average age (the average length of uncompleted spells in the whole population) to the average and the dispersion in duration of inaction (the length of completed spells by adjusters), where the dispersion is measured using the coefficient of variation squared. The relationship between average age and average duration is straightforward: If adjusters take longer to adjust on average, then the average capital gap in the cross-section will be older. Why does the dispersion in duration also increase age? The reason is the fundamental renewal property: The probability that a random firm has an expected duration of inaction of τ is increasing in τ—i.e., many inaction spells are short, but the average spell is attributable to firms with long duration. Dispersion in duration implies that some firms take a longer time to adjust, and those firms are more representative of the economy, raising the average age.

To analyze the relationships between steady-state moments and the microdata, the following Corollary presents simplified expressions for the case $m = 1$ and zero reset gap.

COROLLARY 3 *Assume the reset point is zero, i.e., $x^* = 0$, so that $x_\tau = -\Delta x$. Then, we recover the steady-state moments $\mathbb{V}ar[x]$ and $\mathbb{C}ov[x,a]$ as follows:*

(i) With zero drift ($\nu = 0$)*:*

$$(39) \qquad \mathbb{V}ar[x] = \frac{\overline{\mathbb{E}}[\Delta x^2]}{6}\left(1 + \overline{\mathbb{CV}}^2\left[\Delta x^2\right]\right),$$

$$(40) \qquad \mathbb{C}ov[x,a] = \frac{\overline{\mathbb{E}}[\tau]}{3}\left(\frac{\overline{\mathbb{E}}\left[\tilde{\tau}x_\tau^3\right] - \mathbb{E}[x^3]}{\overline{\mathbb{E}}[\Delta x^2]}\right).$$

(ii) With nonzero drift ($\nu \neq 0$)*:*

$$(41) \qquad \mathbb{V}ar[x] = \frac{\overline{\mathbb{E}}[\Delta x]^2}{3}\left(1 + \overline{\mathbb{CV}}^3\left[\Delta x\right]\right),$$

$$(42) \qquad \mathbb{C}ov[x,a] = \frac{\overline{\mathbb{E}}[\tau]}{2}\left(\frac{\mathbb{E}[x^2] - \overline{\mathbb{E}}\left[\tilde{\tau}x_\tau^2\right]}{\overline{\mathbb{E}}[\Delta x]}\right) + \frac{\sigma^2}{2\nu}\mathbb{E}[a].$$

Variance of capital gaps. The drivers behind the cross-sectional variance of capital gaps $\mathbb{V}ar[x]$ are described in equations (39) and (41) for the cases without and with drift. The first term in these expressions relates to the average adjustment size (measured by squared gap changes or gap changes squared), and the second term relates to the dispersion of adjustment size (measured by generalized coefficients of variation).[19] Clearly, large average adjustments signal more dispersed gaps. But what is the connection between the dispersion in adjustment size and the dispersion of capital gaps? It is the fundamental renewal property again: The average behavior in the economy is attributable to firms with longer periods of inaction, which coincidentally are firms that make larger adjustments. Accordingly, higher dispersion in x_τ^2 (squared gaps of adjusters) increases $\mathbb{E}[x^2]$ (squared gaps of non-adjusters or $\mathbb{V}ar[x]$).

Covariance of capital gaps with their age. The drivers behind the covariance between capital gaps and their age $\mathbb{C}ov[x,a]$ are described in (40) and (42). As with the reset gap, this covariance can be positive or negative, depending on the relative importance of the drift and policy asymmetry.

When the drift is equal to zero, the covariance in (40) is proportional to the excess asymmetry in the capital gaps of adjusters relative to non-adjusters—namely, the difference in the third moments of their respective distributions, i.e., $\overline{\mathbb{E}}\left[\tilde{\tau}x_\tau^3\right] - \mathbb{E}[x^3]$. A positive

[19]With zero drift, the dispersion in adjustment size is measured by the generalized coefficient of variation of Δx^2 with $\psi = 2$; with nonzero drift, it is measured by the generalized coefficient of variation of Δx with $\psi = 3$ (this is close to the skewness, as the presence of the drift alters the notion of dispersion).

difference reflects a right-skewed distribution of gaps and vice versa. Note that the distribution of adjusters is weighted by the relative duration $\tilde{\tau}$, as with the reset state. The two additional terms in (40) are rescaling factors: the denominator $\overline{\mathbb{E}}[\Delta x^2]$ ensures that the covariance is of order 1 (canceling the cubic powers in the numerator) and $\overline{\mathbb{E}}[\tau]$ accounts for age's dependence on σ^2.

When the drift is different from zero, the covariance in (42) is obtained from different moments, but the economic interpretation is the same. In this case, the excess asymmetry of adjusters relative to non-adjusters is measured by the second moment of the respective distributions, i.e., $\mathbb{E}[x^2] - \overline{\mathbb{E}}\left[\tilde{\tau} x_\tau^2\right]$. Again, the distribution of adjusters is reweighed by relative duration $\tilde{\tau}$ and there is a rescaling factor, $\overline{\mathbb{E}}[\Delta x]$, to ensure that the covariance remains linear. Lastly, the term $\sigma^2 \mathbb{E}[a]/(2\nu)$ compensates for the direct effect of idiosyncratic volatility on second moments (see equation 32).

4.3. *The CIR in terms of microdata*

Now we combine our main theory results—the mapping from steady-state moments to CIR_1 in Corollary 1 and the mappings from microdata to parameters in Proposition 2 and to steady-state moments in Proposition 3—to express concisely the propagation of an aggregate productivity shock as a function of microdata Ω.

COROLLARY 4 *Assume the reset point is zero, i.e., $x^* = 0$, so that $x_\tau = -\Delta x$. Then the CIR_1 can be computed using microdata moments as follows:*

(i) With zero drift ($\nu = 0$):

$$(43) \qquad \frac{CIR_1(\delta)}{\delta} \;=\; \frac{\overline{\mathbb{E}}[\tau]}{2} \, \frac{\overline{\mathbb{K}ur}[\Delta x]}{3} \;+\; o(\delta).$$

(ii) With nonzero drift ($\nu \neq 0$):

$$(44) \qquad \frac{CIR_1(\delta)}{\delta} \;=\; \frac{\overline{\mathbb{E}}[\tau]}{2} \left(\frac{\overline{\mathbb{CV}}^2[\tau] - 1}{2} + \frac{\overline{\mathbb{E}}\left[\widetilde{\Delta x}^3\right]}{\overline{\mathbb{E}}\left[\widetilde{\Delta x}^2\right]} - \overline{\mathbb{C}ov}[\tilde{\tau}, \widetilde{\Delta x}^2] \right) + o(\delta).$$

The previous expressions summarize many economic forces that shape the propagation of aggregate shocks in lumpy economies and show how these forces are reflected in the data. We use these expressions to organize a short literature review.

Zero drift and symmetric policy. Álvarez, Le Bihan and Lippi (2016) characterize the CIR_1 for zero drift and a symmetric policy ($\nu = x^* = 0$), and obtain their well-known kurtosis formula, as presented in (43). In the price-setting context of their paper, x represents the markup gap and Δx represents the price change, and the formula expresses the response of real output to a one-time monetary policy shock as the product of the kurtosis of price changes and the expected time between adjustments.[20] The kurtosis of gap changes—which measures the dispersion in adjustment sizes—has proven to be extremely useful sufficient statistic in evaluating the empirical relevance of various models of price adjustment, as long as the drift (inflation) is not too large.

Three amplification channels with nonzero drift and asymmetric policies. Average duration of inaction matters for propagation because it reflects the average speed at which agents adjust to the aggregate shock. According to expression (44), three additional channels shape aggregate dynamics: (i) dispersion in duration of inaction, (ii) dispersion in adjustment size, and (iii) the covariance between duration of inaction and adjustment size. The first two statistics have been analyzed in the price-setting context, so we refer to this literature for a discussion. The third statistic is a novel contribution of our analysis. We will discuss each in turn.

Dispersion in the duration of inaction amplifies the CIR_1 because it reflects the coexistence of fast and slow adjusters; the latter slow the response to the shock and they are more representative of the economy by the renewal property. This insight is formalized by Carvalho and Schwartzman (2015) and Alvarez, Lippi and Paciello (2016) in fully time-dependent models with zero drift. Our formula formally demonstrates that this insight extends beyond fully time-dependent models and also applies to environments with nonzero drift and asymmetric policies.

Dispersion in adjustment sizes amplifies the CIR_1 because it reflects a weak selection effect—namely, that adjusting firms are not necessarily those with the largest need for adjustment. A weak selection effect arises if the measure of firms whose gaps lie in the

[20] In a model with monopolistic price-setters, the average markup gap is equal, up to a first order, to aggregate real output. Therefore, the CIR_1 tracks the deviation of real output relative to steady state following a monetary shock δ.

neighborhood of the adjustment thresholds is small; in that case, most firms are dispersed away from their adjustment threshold. Hence, the distribution of the adjustment sizes exhibits large dispersion. With zero drift, the dispersion in adjustment sizes is measured by the kurtosis of gap changes; with nonzero drift, it is measured by the skewness of gap changes. Quantitatively, this connection is explored in Midrigan (2011) and Luo and Villar (2020). We formally demonstrate that accounting for the dispersion of adjustment size (either through kurtosis or skewness) is key to studying transitional dynamics in lumpy economies more generally, beyond price-setting models.

Lastly, the covariance between duration of inaction and adjustment size also shapes the CIR_1, as it reflects the presence of asymmetric policies. Identifying and quantifying this channel is one of the key payoffs from our theory. A naive approach is to identify an asymmetric policy through an asymmetric distribution of adjustments. We have already shown, however, that this approach is incorrect because time-dependent models—which are inherently symmetric—generate asymmetric adjustments in the presence of drift. Our analysis shows that the correct way to identify asymmetric policies in the presence of drift is through the excess asymmetry of adjusters relative to non-adjusters, as measured by $\overline{\mathbb{C}ov}[\tilde{\tau}, \widetilde{\Delta x}^2]$. This statistic complements alternative methodologies that aim to diagnose whether frictions in capital allocation mainly affect upsizing firms or downsizing firms, as the ones put forward by Caballero and Engel (2007) and Lanteri, Medina and Tan (2019).

5. EMPIRICAL APPLICATION

We apply the theoretical results obtained in Sections 3 and 4 using establishment-level data from Chile. First, we construct the distributions of capital gap changes Δx and duration of inaction τ from the data. Second, we use these empirical distributions as inputs into our formulas and obtain parameters, sufficient statistics, and the CIR_1 as outputs. Lastly, we use the sufficient statistics to discriminate across configurations of the Bernoulli cost model and settle on the best calibration to explain the data.

5.1. *Data description*

Sources. We use yearly data on manufacturing plants in Chile from the Annual National Manufacturing Survey (*Encuesta Nacional Industrial Anual*) for the period 1979 to 2011.

Chilean National Accounts and Penn World Tables provide information on the depreciation rates and price deflators used to construct the capital series. We examine the total capital stock and structures, a capital category that represents 30% of all investment in the manufacturing sector and features the strongest lumpy behavior. We consider plants that appear in the sample for at least 10 years (more than 60% of the sample) and have more than 10 workers. The Data Appendix describes the sample selection, the variable construction, and the analysis for vehicles, machinery, and equipment.

Capital stock and investment rates. We construct the capital stock series using the perpetual inventory method. Given an initial K_0, a plant's capital stock in year t is

$$K_t \;=\; (1-\zeta)K_{t-1} \;+\; I_t/D_t, \tag{45}$$

where ζ is the depreciation rate, D_t is the gross fixed capital formation deflator, and initial capital K_0 is a plant's self-reported nominal capital stock at current prices for the first year in which it is nonnegative. Gross nominal investment I_t is based on information on purchases, reforms, improvements, and sales of fixed assets. We define the investment rate ι_t as the ratio of real gross investment to the capital stock [21]

$$\iota_t \;\equiv\; \frac{I_t/D_t}{K_{t-1}}. \tag{46}$$

5.2. *Construction of capital gap changes and duration of inaction*

To apply the theory, for each plant and each inaction spell h, we record the capital gap change upon action Δx_h and the spell's duration τ_h. We construct capital gap changes with investment rates from (46):

$$\Delta x_h \;=\; \begin{cases} \log\left(1+\iota_h\right) & \text{if} \quad |\iota_h| > \underline{\iota}, \\ 0 & \text{if} \quad |\iota_h| < \underline{\iota}. \end{cases} \tag{47}$$

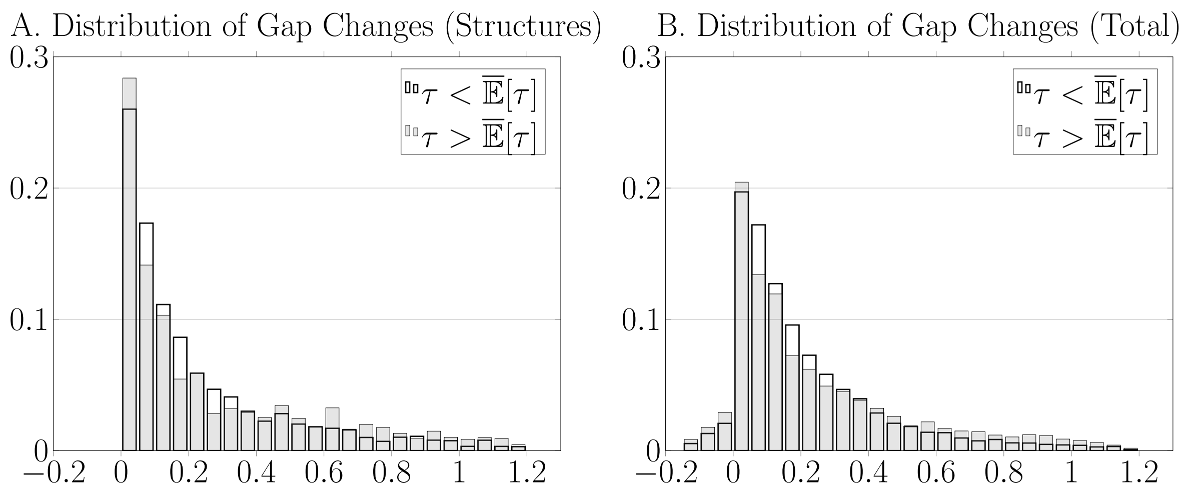
The threshold $\underline{\iota} > 0$ reflects the idea that small maintenance investments should be excluded. Following Cooper and Haltiwanger (2006), we set $\underline{\iota} = 0.01$, such that all investment

[21]Note that the investment rate equals the ratio in the last term of equation (7): $\iota_{T_h} \equiv i_{T_h}/k_{T_h^-} = (k_{T_h} - k_{T_h^-})/k_{T_h^-}$, where $k_{T_h^-} = \lim_{t\uparrow T_h} k_t$. In contrast to the continuous-time model, in which investment is computed as the difference in the capital stock between two consecutive instants, in the data we compute it as the difference between two consecutive years.

rates below 1% in absolute value are considered to be part of an inaction spell. Given the capital gap changes, we define an adjustment date T_h from $\Delta x_{T_h} \neq 0$ and compute a spell's duration as the difference between two adjacent adjustment dates: $\tau_h = T_h - T_{h-1}$. Finally, we truncate the distribution at the 2nd and 98th percentiles of the investment distribution to eliminate outliers.[22]

Figure 3 plots the resulting cross-sectional distribution of non-zero capital gap changes for structures (Panel A) and total capital (Panel B). Both histograms show sizable asymmetry and positive skewness. In each figure, we plot the distribution for two subsamples: observations with duration of inaction above the average duration (gray bars) and below the average duration (white bars). Notice that capital gap changes in both subsamples lie on top of each other, which is a sign of lack of covariance between adjustment size and duration of inaction. Below, we interpret this fact through the lens of the theory.

FIGURE 3.— Empirical Distribution of Non-Zero Capital Gap Changes



Notes: Own calculations using establishment-level data from Chile. Sample: Firms with at least 10 years of data, truncation at 2nd and 98th percentiles, and an inaction threshold of $\underline{\iota} = 0.01$. Panel A plots the distribution of non-zero capital gap changes Δx for structures and Panel B for total capital. Solid bars = inaction spells with duration below average; white bars = inaction spells with duration above average.

[22]Table I in the Data Appendix presents descriptive statistics on investment rates. In particular, the inaction rate ($|\iota| < 0.01$) equals 77.3% for structures and 40.1% for total capital. For comparison, the table includes numbers reported by Cooper and Haltiwanger (2006) for US manufacturing plants and by Zwick and Mahon (2017) for US firms from tax records.

5.3. *Putting the theory to work*

We put the theory to work by computing the cross-sectional statistics of capital gap changes and duration of inaction to infer the parameters and sufficient statistics related to the CIR_1. We apply the formulas in Propositions 2 and 3. Table I summarizes the results. The left side of the table shows the inputs from the data: cross-sectional statistics of duration and capital gap changes. The right side shows the outputs from the theory: parameters (ν, σ^2, x^*), sufficient statistics $(\mathbb{V}ar[x], \mathbb{C}ov[x,a])$, and the CIR_1.

TABLE I

INPUTS FROM MICRO DATA AND OUTPUTS FROM THE THEORY

Inputs from Data			**Outputs from Theory**		
	Structures	**Total**		**Structures**	**Total**
Duration			**Parameters**		
$\overline{\mathbb{E}}[\tau]$	2.510	1.749	ν	0.095	0.119
$\overline{\mathbb{CV}}^2[\tau]$	1.107	0.872	σ^2	0.049	0.049
			x^*	0.006	0.028
Gap Changes			**Sufficient Statistics**		
$\overline{\mathbb{E}}[\Delta x]$	0.239	0.207	$\mathbb{V}ar[x]$	0.124	0.092
$\overline{\mathbb{E}}[\Delta x^2]$	0.126	0.098	$\mathbb{C}ov[a,x]$	0.592	0.293
$\overline{\mathbb{E}}[x_\tau^3]$	-0.089	-0.057	$\mathbb{E}[a]$	2.644	1.637
$\overline{\mathbb{K}ur}[\Delta x]$	4.635	5.683			
Covariances			**CIR_1**		
$\overline{\mathbb{C}ov}[\tilde{\tau}, \Delta x]$	0.019	0.015	Drift + Asymmetric	3.661	2.562
$\overline{\mathbb{E}}[\tilde{\tau} x_\tau^2]$	0.141	0.103	Driftless + Symmetric	1.939	1.657

Notes: Own calculations using establishment-level data from Chile. Sample: Firms with at least 10 years of data, truncation at 2nd and 98th percentiles, and inaction threshold of $\underline{i} = 0.01$. Gap of adjusters: $x_\tau = x^* - \Delta x$. Normalized duration: $\tilde{\tau} \equiv \tau / \overline{\mathbb{E}}[\tau]$.

Inputs from microdata. We focus our discussion on the values obtained for structures. The duration of inaction has an average of $\overline{\mathbb{E}}[\tau] = 2.51$ years and a coefficient of variation squared of $\overline{\mathbb{CV}}^2[\tau] = 1.11$, suggesting substantial heterogeneity in the adjustment frequency. Adjustment size has an average of $\overline{\mathbb{E}}[\Delta x] = 0.24$; a second moment of

$\overline{\mathbb{E}}[\Delta x^2] = 0.13$; a third moment of $\overline{\mathbb{E}}[x_\tau^3] = -0.09$ (the distribution is right-skewed); and a kurtosis of $\overline{\mathbb{K}ur}[\Delta x] = 4.64$ (the distribution is leptokurtic). The covariance between adjustment size and relative duration is almost zero $\overline{\mathbb{C}ov}[\tilde{\tau}, \Delta x] = 0.02$, as suggested by Figure 3. The duration-weighted second moment of gap changes is $\overline{\mathbb{E}}[\tilde{\tau} x_\tau^2] = 0.14$.

We compute average age in two ways: directly from the data and using the formula in (34), which connects it to the duration of inaction $\mathbb{E}[a] = \overline{\mathbb{E}}[\tau](1 + \overline{\mathbb{CV}}^2[\tau])/2 = 2.64$ (for this reason, we show it in the second column with other outputs from the theory). We obtain similar numbers using both methods, confirming the validity of the mapping.[23] Next, we input these statistics into the formulas derived in the previous section.

Output from theory: Parameters. From (31), we infer a drift of $\nu = 0.095$, which reflects the depreciation rate ζ, productivity growth μ_z, and changes in relative prices between consumption and capital goods (ignored in the model). We apply (32) to estimate the volatility of idiosyncratic shocks as $\sigma^2 = 0.05$. The volatility estimate is in line with the value used by Khan and Thomas (2008). Lastly, using (33), we estimate a tiny reset capital gap of $x^* = 0.006$. Although small, the positive reset capital gaps suggests that the distribution of capital gaps is right-skewed and that the drift effect is overcome by policy asymmetry (recall the discussion in Section 4.1). In particular, we infer that downward adjustment is more costly than upward adjustment.

Outputs from theory: Sufficient statistics. Using (37), we infer a steady-state variance of capital gaps of $\mathbb{V}ar[x] = 0.12$.[24] The estimated variance of gaps is large (when compared to a purely state-dependent model) and suggests a significant role for free adjustments.[25]

Using (38), we infer a covariance between capital gaps and age of $\mathbb{C}ov[x, a] = 0.60$. The positive covariance between capital gaps and age means that plants that have not adjusted for a long time—their capital is old—have larger capital-to-productivity ratios than those

[23]We thank Francesco Lippi for suggesting this robustness exercise.

[24]Appendix G connects $\mathbb{V}ar[x]$ to the notion of capital misallocation, defined as the cross-sectional dispersion in log marginal revenue products of capital as $Std[\log \text{MPK}] = (\alpha - 1)\mathbb{V}ar[x]^{1/2}$. We compare our estimation strategy that uses exclusively investment data with the standard approach that requires additional data on value added or sales.

[25]A state-dependent model ($\lambda = 0$) that matches the drift, idiosyncratic volatility, and average duration implies a variance of $\mathbb{V}ar[x] = 0.072$, 40% lower than what is observed in the data. See Table II.

that have recently adjusted. This observation is at odds with a pure time-dependent model, which reinforces our assessment that it is more costly for firms to downsize in response to negative productivity shocks than to upsize in response to positive ones.[26]

To summarize, the large variance of gaps $\mathbb{V}ar[x]$ and the positive covariance with age $\mathbb{C}ov[x,a]$ strongly suggest that firms follow a hybrid investment policy with both time- and state-dependent components. Time-dependent adjustments increase the dispersion of gaps, while state-dependent asymmetric adjustments generate a positive covariance. Since the Bernoulli fixed-cost model nests these two alternatives, it serves as an adequate laboratory to study the relative importance of these components. In the following section, we search for a configuration of the Bernoulli model that best explains the data.

Output from theory: CIR_1. Assuming that the Bernoulli fixed-cost model is a good description of the data, the sufficient statistics imply that $\text{CIR}_1(\delta)/\delta = 3.66$. This number says that a negative aggregate productivity shock of 1% generates a cumulative deviation of 3.66% in the average capital-to-productivity ratio (and in aggregate capital, up to first order) above its steady-state value along the transition path. In other words, aggregate productivity shocks have a long-run multiplier effect of approximately 3.7. As firms gradually scale down to accommodate the fall in aggregate productivity, capital is adjusted downward by selling it or letting it depreciate. We approximate the half-life of the response assuming exponential decay, obtaining 2.5 years ($\ln(2) \times \text{CIR}_1$).[27]

Notice that naively applying the kurtosis formula in (43), which is invalid for environments with nonzero drift and policy asymmetry, implies $\text{CIR}_1(\delta)/\delta = 1.94$. This underestimates the effects of an aggregate productivity shock by about 50%.

5.4. *Parametrization of the Bernoulli fixed-cost model*

With the estimated parameters of the stochastic process and the steady-state moments at hand, a natural question arises: Which configuration of the Bernoulli fixed-cost model

[26]A time-dependent model (with infinite fixed costs) that matches the drift, idiosyncratic volatility, and average duration implies a covariance of $\mathbb{C}ov[x,a] = -0.602$, which has the opposite sign of the one observed in the data. See Table II.

[27]Assuming exponential decay at rate ρ, the half-life $= \ln(2)/\rho$. Applying the definition of the CIR, we have $\text{CIR}_1 = \int_0^\infty e^{-\rho t}\, \mathrm{d}t = 1/\rho$. Together, half-life $= \ln(2) \times \text{CIR}_1 = 0.69 \times 3.66 = 2.54$.

generates the Chilean data? How important are the fixed costs relative to free adjustment opportunities?

To answer these questions, we explore the benchmark configurations nested within the Bernoulli fixed cost model to assess their ability to generate the data. These special cases illustrate the relationship between the structure of adjustment frictions and the sufficient statistics. In all exercises, we take as given the estimated parameters of the stochastic process and match the average duration of inaction spells. Given the parameters, matching average duration imposes additional constraints—e.g., the average adjustment size is also matched by equation (31). Table II summarizes the calibrated parameters.

Column I considers a purely state-dependent model by shutting down the free adjustments, $\lambda = 0$. Anticipating that this configuration generates a tiny variance of gaps, and to give it the largest possibility of matching the positive covariance of capital gaps and age, we set an inaction threshold for negative investments of $\theta^+ = \infty$ (this is effectively a one-sided inaction region). To match average duration, the inaction threshold for positive investments is $\theta^- = 0.043$. The physical adjustment costs represent 0.1% of yearly revenue.[28] The implied sufficient statistics are $\mathbb{V}ar[x] = 0.073$ and $\mathbb{C}ov[x, a] = 0.661$. The CIR_1 equals 2.734, which is 25% below the data.

Column II considers the limiting case with infinite fixed costs, $\{\theta^-, \theta^+\} \to \{\infty, \infty\}$, which produces a purely time-dependent model. We calibrate the arrival rate of free adjustments $\lambda = 0.397$ to match average duration $\overline{\mathbb{E}}[\tau]$. As expected, this model produces a significant variance of gaps of $\mathbb{V}ar[x] = 0.182$, larger than in the data. However, it produces a negative covariance with age of $\mathbb{C}ov[x, a] = -0.601$, which we do not observe in the data. Surprisingly, this configuration implies a CIR_1 of 2.512, which is similar to the one obtained in the state-dependent model of Column (1).

This analysis illustrates how two extreme calibrations can generate the same CIR_1 by matching one of the two sufficient statistics. The state-dependent model correctly captures the covariance of gaps with age—but misses the variance of gaps—whereas the time-dependent model does the opposite. Calibrations that lie between these two extremes

[28]Assuming an output to capital elasticity of $\alpha = 0.6$ (adjusted by the absence of labor in the model), the average yearly payment of adjustment costs relative to yearly revenue is equal to $(\theta^- \Pr[x_\tau = x^-] + \theta^+ \Pr[x_\tau = x^+])/(\overline{\mathbb{E}}[\tau]\mathbb{E}_g[\exp(\alpha\hat{x})]) = (0.043 \times 1)/(2.519 \times 17) = 0.001$.

TABLE II

CONFIGURATIONS OF THE BERNOULLI FIXED-COST MODEL

Parameters	**Data**	**(I) Bernoulli**	**(II) Bernoulli**	**(III) Extended**
		$\lambda = 0$	$\{\theta^-, \theta^+\} \to \{\infty, \infty\}$	$\lambda^- \neq \lambda^+$
θ^- (for $i_t > 0$)	—	0.043	∞	0.945
θ^+ (for $i_t < 0$)	—	∞	∞	∞
λ^- (for $i_t > 0$)	—	0	0.397	0.800
λ^+ (for $i_t < 0$)	—	0	0.397	0
Moments				
$\overline{\mathbb{E}}[\tau]$	2.519	2.519*	2.519*	2.519*
$\mathbb{V}ar[x]$	0.124	0.073	0.182	0.107
$\mathbb{C}ov[x, a]$	0.600	0.661	−0.601	0.564
x^*	0.005	−0.140	0.239	−0.037
CIR_1	3.663	2.734	2.512	3.237

Notes: Data from Chilean plants. Configurations: (I) State-dependent Bernoulli with $\lambda = \lambda^+ = \lambda^- = 0$. (II) Time-dependent Bernoulli with $\lim\{\theta^-, \theta^+\} \to \{\infty, \infty\}$ and $\lambda = \lambda^+ = \lambda^-$. (III) Extended Bernoulli with $\lambda^+ \neq \lambda^-$. Parameters for the stochastic process: $\nu = 0.095$ and $\sigma^2 = 0.050$. * = targeted moment.

only decrease the CIR_1. We conclude that the Bernoulli fixed-cost model falls short of generating the two sufficient statistics for propagation of aggregate shocks in the data.

Extended Bernoulli fixed-cost model. Can a simple modification of the Bernoulli model enable it to explain the data? The answer is yes. The extension considers different rates of free adjustments for positive and negative investments, λ^- and λ^+. We verify numerically that the sufficient statistics for the CIR_1 remain valid under this extension.[29]

Column III shows the calibration. The extended model breaks the trade-off between asymmetry and randomness embedded in the original model, and it does an excellent job of matching the data. The best match has fixed costs of $(\theta^-, \theta^+) = (0.945, \infty)$ and arrival rates of free adjustments of $(\lambda^-, \lambda^+) = (0.800, 0)$. The average physical adjustment costs represent 0.1% of yearly revenue. The implied sufficient statistics are $\mathbb{V}ar[x] = 0.107$ and $\mathbb{C}ov[x, a] = 0.564$, and the CIR_1 equals 3.237, which is almost 90% of its empirical value.

As in the pure state-dependent case, we obtain a one-sided inaction region that matches

[29] Appendix H verifies numerically that the CIR_1 in the extended Bernoulli model is well approximated by the sufficient statistics in expression (29) for small δ shocks.

the positive covariance of gaps and age. Additionally, the free adjustments introduce a random element in the policy that increases the variance of gaps, but it applies exclusively to upward adjustments. Finally, notice that the reset state x^* implied by the extended Bernoulli model is closer to the data than in the other two configurations.

Heterogeneity and robustness checks. Our theory assumes that one can exploit the cross-section to learn about the behavior of individual firms over time. In practice, fixed heterogeneity may affect the computation and interpretation of the cross-sectional statistics (Blanco and Cravino, 2020). In Appendix I we present a multisector extension that allows for fixed heterogeneity (e.g., across sectors or plant size) and we show how to aggregate sectorial statistics. Additionally, we conduct a series of robustness checks in the Data Appendix. Throughout these checks, we consistently obtain similar sufficient statistics.

6. CONCLUSION

We develop a parsimonious framework to study the propagation of aggregate productivity shocks when firms make lumpy investments. Through a sufficient statistics approach, we discover that the transitional dynamics of capital are structurally linked to the degree of steady-state capital misallocation and the relative costs of upsizing and downsizing. Our results indicate that policies that impact the lumpiness of investment—e.g., investment tax credits (Chen, Jiang, Liu, Suárez Serrato and Xu, 2019)—directly affect the propagation of aggregate shocks.

Looking forward, we foresee four avenues for developments that would extend the scope of our theory. First, we focus on a one-dimensional state. Extending the theory to a multidimensional state would facilitate studying transitional dynamics with multiplant firms (Kehrig and Vincent, 2019), several production inputs (Hawkins, Michaels and Oh, 2015), or the interaction of lumpy investment and price-setting (Sveen and Weinke, 2007).

Second, we assume full adjustment upon action. Extending the theory to accommodate partial adjustments would allow for interactions of lumpiness with convex adjustment costs, time-to-build, learning, or other features that may generate a correlation between adjustments. Our work in Baley and Blanco (2019) makes progress in this direction by providing bounds for the CIR in environments with learning by carrying the aggregate forecast error as an additional state.

Third, we characterize the CIR, but not the complete profile of the impulse-response function; moreover, we only consider marginal perturbations around the steady state. Extending the theory to characterize the full IRF and general perturbations is key to discuss non linearities and different types of aggregate shocks. Contemporaneous work makes progress in these directions: Álvarez and Lippi (2019) characterizes the complete impulse response function using eigenvalue-eigenfunction decompositions, and Alexandrov (2020) studies the effect of non-marginal shocks in the presence of drift.

Finally, our theory assumes constant prices along the transition path. Appendix J relaxes this assumption and presents a general equilibrium model that delivers constant prices as an equilibrium outcome. However, incorporating complex feedback from the distribution to individual policies, e.g., strategic complementaries, is likely the most important extension ahead, but also the most challenging.

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