

Supporting Information. Clay, P.A., M.H. Cortez, and M.A. Duffy. 2021. Dose relationships can exacerbate, mute, or reverse the impact of heterospecific host density on infection prevalence. *Ecology*.

Appendix S2: Parameter Values and Derivations

Section S1: Parameter Values

Table S1: Model parameter values

| Parameter | Value |
|-----------------------|---------------------------------|
| r_1 | 1 |
| r_2 | 0-2 |
| α_{11} | 1 |
| α_{22} | 1 |
| α_{12} | 0.5 |
| α_{21} | 0.5 |
| f | 1 |
| k | 0.5, 1, 1.5 |
| m | 0.4 |
| x_1 (scenario 1) | 100 |
| x_2 (scenario 1) | $0, \frac{x_1}{2}, x_1, 1.5x_1$ |
| x_1 (scenario 2) | 0 |
| x_2 (scenario 2) | 100 |
| μ | 0.1 |
| γ (Figure 3,5) | -3,0,0.5 |

| | |
|---------------------|-------------|
| ρ (Figure 4,5) | 0.5, 1, 1.5 |
|---------------------|-------------|

Section S2: Calculating β

For each model parameterization, we calculated the per propagule infectivity constant, β , which would yield $I_1 = S_1 > 0$, and $I_2 + S_2 = N_2 = 0$ when the system of equations was at equilibrium. When we set $I_1 = S_1$ and $N_2 = 0$, eq. 3 becomes

$$2r_1 - \alpha_{11}4 S_1 = \beta_1(f_1P)^{k_1} \quad (\text{S1})$$

With the same assumptions, eq. 4 becomes

$$\beta_1(f_1P)^{k_1} = m_1 \quad (\text{S2})$$

Combining eq. S1 and S2, we get

$$2r_1 - \alpha_{11}4 S_1 = m_1 \quad (\text{S3})$$

Which we can solve for S_1 :

$$S_1 = \frac{2r_1 - m_1}{4\alpha_{11}} \quad (\text{S4})$$

We then solve eq. 7 for P , replace I_1 and S_1 with eq. S4, and get

$$P = \frac{x_1 \frac{2r_1 - m_1}{4\alpha_{11}}}{\mu + f_1 \frac{2r_1 - m_1}{2\alpha_{11}}} \quad (\text{S5})$$

Subbing eq. S5 into eq. S2 and solving for β_1 , we get

$$\beta_1 = \frac{m_1}{\left(f_1 \frac{x_1 \frac{2r_1 - m_1}{4\alpha_{11}}}{\mu + f_1 \frac{2r_1 - m_1}{2\alpha_{11}}} \right)^{k_1}} \quad (\text{S6})$$

Unless otherwise stated, we assume that $\beta_2 = \beta_1$.

For Scenario 2 (Host cannot maintain parasite transmission, Appendix S3: Section S10),

$x_1 = 0$ and thus β_1 becomes infinite according to eq. S6. Thus, for scenario 2, we calculated β_1 according to eq. S6 as though $x_1 = 100$. Therefore β_1 is the same for both scenario 1 and scenario 2 for a given value of k_1 . As in scenario 1, we assume that $\beta_2 = \beta_1$.