QUANTITATIVE ASSESSMENT OF RISK SENSITIVITY FOR CONSTRUCTION ESTIMATING

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ABSTRACT :
Realistic construction estimating requires optimal decisions that not only minimize construction time and cost, but also address other important factors such as project risks. Probabilistic estimating is generally used to incorporate risk into estimated costs, the most common measure of which is the expected monetary value (EMV). Yet, EMV does not capture the way most people make decisions under uncertainty because it cannot reflect the decision maker's risk sensitivity. This paper presents a probabilistic estimating model that can quantify and incorporate the contractor's risk aversion. A highway tunneling project is presented to illustrate the application of the proposed model. Tunnel cost estimating is formulated as a risk-sensitive Markov decision process (MDP) where the contractor's risk sensitivity is modeled by an exponential utility function. The contractor's decisions depend on his risk aversion coefficient and the variability in tunneling costs, which in turn depend on geologic uncertainty. The risk-sensitive MDP is solved by maximizing the expected utility value (EUV) of tunneling costs. The final results include the optimal excavation and support sequence, and the risk-adjusted tunneling costs for the project, as functions of the contractor's risk sensitivity.

KEYWORDS : CONSTRUCTION ESTIMATING, CONSTRUCTION RISKS, RISK SENSITIVITY, TUNNELING

1. Introduction

Construction estimating should account for future construction risks during the planning (bidding) phase. Realistic estimating involves optimal decisions that not only minimize construction cost and time, but also address other important factors such as potential risks associated with the project. Probabilistic estimating is commonly used to incorporate risk into estimated costs. However, its most common measure, expected monetary value (EMV), does not reflect the way most people make decisions under uncertainty because it does not consider the decision maker's risk sensitivity.

In this paper, we present a probabilistic estimating model that can quantify and incorporate the contractor's risk sensitivity. The proposed model is applied to tunnel estimating to illustrate its power to systematically integrate the contractor's risk sensitivity into construction estimating.

2. Decision Making in Construction Estimating

During the construction planning (bidding) phase, contractors use information obtained from a variety of sources (e.g., the bidding documents developed by the architect and engineer) for establishing a construction plan and estimating its associated costs. To develop an appropriate construction plan, a chief estimator needs to make several major decisions based on available information and personal experience, including site layout, construction methods, major construction equipment, and material and labor management. These decisions are usually left to the contractor because he has a better understanding of construction practices than the architect and engineer. Allowing the contractor to use construction plans that are most compatible with his expertise and available resources can significantly enhance the overall project economy.

Often, there are many feasible construction alternatives for a particular project. To be competitive, contractors must be able to measure and compare the performance of these alternatives. The most important attributes for appraising construction performance are construction cost and time, which can be determined by construction estimating.

Decision making in construction estimating is characterized by a variety of uncertainties, depending upon the nature of project. For example, risks in tunnel construction stem from three major factors: geologic uncertainty, geologic variability, and uncertainty in the productivity of tunneling operations. The probabilistic estimating approach can be used to quantify and incorporate these risks into construction estimating, as presented in detail in [1].

3. Risk Sensitivity in Construction Estimating

Decision making under uncertainty (e.g., construction estimating) can be described by an uncertain proposition, termed *lottery*. When a person participates in a lottery, he will receive one of a specified set of outcomes (e.g., construction costs). Each outcome has the associated probability that the decision maker will receive that outcome. Let us consider a lottery of a coin tossing, as illustrated in Figure 1. Suppose that the participant in this lottery will receive nothing if the coin falls heads, but he will receive $100 if the coin falls tails. Assume that the coin is fair; that is, P(Heads) = P(Tails) = 0.5. A common measure of the lottery for a risk-neutral decision maker is the *expected monetary value* (EMV), which is computed by multiplying the amount of each outcome by its corresponding probability and summing over all outcomes. Thus, the EMV of this coin-tossing lottery equals to 0.5($0) + 0.5($100) = $50.

Measuring a lottery by using EMV does not capture the way most people make decisions under uncertainty because it cannot reflect the decision maker's risk sensitivity. Individual valuation of benefits and costs for decisions involving risk is often nonlinear. That is, decisions made regarding risky situations are not based on the maximization of EMV. When making decisions under uncertainty a decision maker is commonly sensitive to risk, either risk averse or risk preferring. An individual's risk sensitivity is influenced by a variety of factors, especially his current financial position. Typically, as a person's net asset position increases, the less risk averse his behavior toward the same risk. Risk sensitivity is also influenced by the magnitude, range, and likelihood of the outcomes. For example, a fair coin lottery with monetary outcomes equal to $49 and $51 also has EMV=$50 but has much less variability and thus presents much less risk.

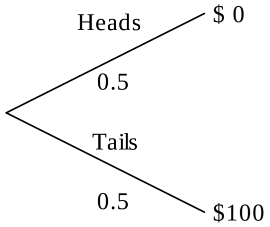

Figure 1 Coin-Tossing Lottery

In construction, a contractor's decisions under uncertainty depend on his risk sensitivity, the range of monetary outcomes, and the associated probabilities. Thus, the factors influencing his decisions include:

- The contractor's current financial status;
- The identity of the owner and the engineer;
- The type and number of available projects;
- Project conditions (e.g., size, location, and complexity);
- The contractor's state of uncertainty about the work, which is based primarily on available information; and
- The amount of risk assumed by each party to the contract.

A contractor's risk aversion and the degree of risk exposure can have a major influence on his construction decisions and the necessary amount of risks premium (contingency) embedded in his price in order to undertake the work. A more risk-averse contractor adopts a more conservative construction plan and includes a higher allowance as contingencies in his bid than a less risk-averse contractor does [2].

A measure of a lottery that can reflect the decision maker's risk sensitivity is the certain equivalent (CE). The CE of a lottery is defined as the minimum selling price of the lottery (i.e., the minimum amount of money a person would accept to give up a lottery he already owns). The CE of a particular lottery is different for different decision makers because of different personal risk attitudes. Suppose that the minimum amount of money a person wants to be paid to give up the coin-tossing lottery in Figure 1 is $40. The CE of this lottery for this person is therefore $40. That is, this person is indifferent between playing this lottery and receiving the amount of money equal to the CE ($40) with certainty. Figure 2 illustrates the meaning of CE graphically. The symbol ~ means that the decision maker is indifferent between this coin-tossing lottery and another lottery that pays $40 (i.e., CE) with probability one.

If a lottery is described by a random variable X, where larger values of X are preferred, the difference between the EMV and the CE of the lottery is defined as the *risk premium*:

$$X_p = E[X] - \tilde{X} \tag{1}$$

where X_p is the risk premium of the lottery X, $E[X]$ is the EMV of the lottery, and $\tilde{X}$ is its certain equivalent.

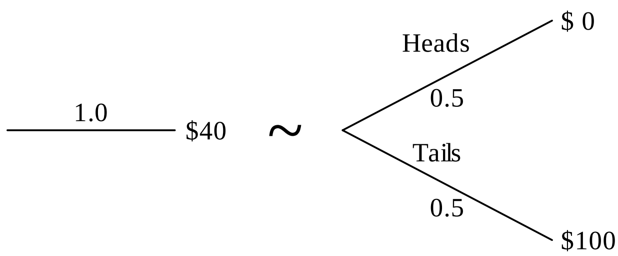

Figure 2 Certain Equivalent (CE) of the Coin-Tossing Lottery

The risk premium can be considered the algebraic amount of money a person is willing to forgo to avoid the risk of the lottery. For example, it is the amount that a contractor would be willing to pay a subcontractor to do the work (above the expected cost) and thus avoid risk. If the risk premium is zero, the person is risk neutral. If the risk premium is not zero, the person is risk sensitive. For risk-averse persons, the risk premium is positive, whereas for risk preferring persons (gamblers) the risk premium is negative. A positive risk premium means that a risk-averse decision maker would accept the lower but certain amount of money to give up the uncertain lottery with the higher EMV. In contrast, the gambler would give up his lottery only if he is offered with the higher amount of certain money than the EMV of the lottery. Thus, the risk premium for the above coin-tossing lottery is \$50 – \$40 = \$10, so the person is risk averse.

4. Modeling Risk Sensitivity

The risk sensitivity of a decision maker can be encoded by a unique utility function [3]. A utility function $u(v)$ assigns a real number u in an ordinal scale to each of the possible lottery outcomes v. The utility function has two important properties:

- The utility of any lottery is the expected utility value (EUV) of its outcomes.
- If the decision maker prefers one lottery to another, it must have the higher utility.

Accordingly, the decision maker's preference in ranking alternative lotteries with uncertain outcomes can be quantified by the EUV of each lottery [4]. If the larger value of v is preferred, then $u(v)$ is a monotonically increasing function of v. When the decision maker has to choose between several lotteries, the one with the largest EUV is the most desirable choice.

The certain equivalent (CE) of a lottery $\tilde{v}$ is defined as the value of the outcome that has the same utility as the EUV of the lottery.

$$u(\tilde{v}) = E\left[u(v)\right] \tag{2}$$

Thus, the decision maker is indifferent between facing the uncertain outcomes of a lottery and receiving the CE with certainty. It should be noted that the CE of a lottery must be interpreted algebraically. For example, the CE (selling price) of a lottery involving monetary loss (e.g., construction costs) is negative, which represents the amount of money the decision maker is willing to "pay" (e.g., subcontract) in order to sell the risk of that lottery to other parties (e.g., subcontractors) [5].

A utility function can be expressed by a utility curve, which assigns a utility number to every value of the lottery outcome. An individual's utility curve can be developed by an assessment procedure [3]. The utility function of risk-neutral decision makers is linear, that of risk-averse persons is concave, and that of gamblers is convex, as shown in Figure 3.

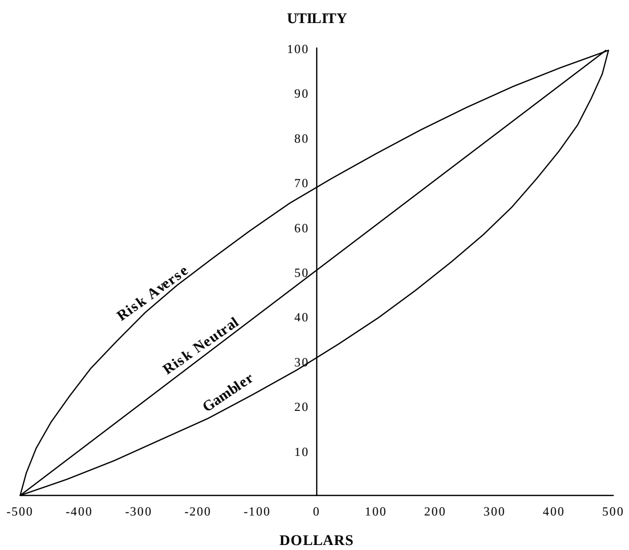

Figure 3 Utility Curves of Three Types of Risk Preference

5. Risk-Sensitive Tunnel Cost Model

In this paper, we illustrate the procedure for quantifying and incorporating an estimator's (e.g., a contractor's) risk sensitivity into construction estimating through the risk-sensitive tunnel cost model.

Tunneling operations can be described as a risk-sensitive Markov decision process (MDP) where the risk-sensitive tunnel cost model is formulated by integrating utility theory with stochastic dynamic programming.

Modeling this risk-sensitive tunnel cost model also requires an additional assumption concerning risk preference known as the delta property [3]. An important implication of this assumption is that a multistage decision problem can be broken down into single-stage decision problems that are easier to solve.

Since the utility function of a decision maker who accepts the delta property is restricted to be either linear or exponential [3], the exponential utility function, which is the general case, is used to construct the tunneling cost function:

$$u(v) = -(\text{sign } \gamma)e^{-\gamma v} \tag{3}$$

where $u(v)$ is the utility of tunneling costs v. The parameter γ is the risk aversion coefficient, and (sign γ) is the sign of γ. A positive γ means the decision maker is risk averse. A negative γ means the decision maker prefers risk. A detailed discussion about the formulation of this model can be found in [1].

6. Application

As an example, we applied the proposed model to determine tunneling costs and plans for the Hanging Lake Tunnel Project in Colorado, USA. We focused on the part of the westbound tunnel with the total length of 1.1 km (3,609 ft) excavated by the multiple-drift drill and blast method.

The anticipated tunnel geology was classified into three ground classes: *GC*1 (best), *GC*2 (medium), and *GC*3 (worst). The engineer specified three excavation methods (*EM*1, *EM*2, and *EM*3) and three primary support systems (*SS*1, *SS*2, and *SS*3) in accordance with the three ground classes. For example, *EM*3 and *SS*3 represent the most conservative and the most expensive construction method for this project. There are nine possible combinations (called *tunneling alternatives*) of different excavation and support methods and the geologic conditions to which they may be applied in this example (i.e., 3 excavation and support methods × 3 possible ground classes). For example, the tunneling alternative (*EM*1,*GC*3) represents the decision to apply *EM*1 in a particular round and the actual ground class of that round after blasting to be *GC*3 (leading to excessive overbreak). The distributions of cost associated with different alternatives are different and are determined separately.

Based on available project information (e.g., [6]), the equipment, material, and labor costs for each alternative were organized and calculated by using a computer spreadsheet. These costs were then categorized into fixed costs and variable costs, as shown in Table 1. The calculated tunneling variable costs were then used as inputs for the probabilistic scheduling analysis of tunneling operations by Monte Carlo simulation, as discussed in [7]. The simulation results provided distributions of tunneling unit costs ($/m) for all nine alternatives, which can be approximated very well by normal distributions with parameters shown in Table 1.

As can be seen, the tunneling unit costs for applying an excavation method in a particular round depend upon the prevailing ground class after blasting. If the selected method is appropriate for the revealed geologic conditions, this decision will lead to the lowest unit cost for the geologic conditions in that round [e.g., (*EM*1,*GC*1), (*EM*2,*GC*2), and (*EM*3,*GC*3)]. In contrast, if the selected method is structurally inadequate [e.g., (*EM*1,*GC*2)], or overly conservative [e.g., (*EM*3,*GC*1)] for the actual ground conditions, the tunneling unit costs will be higher than those of the right decision cases.

Another input for the risk-sensitive tunnel cost model is the predicted geologic conditions of the project in the form of ground class transition probability matrices along the tunnel profile. Detailed discussion of this can be found in [1] and [8]. An example of the ground class transition probability matrix between locations 746.1 m (2,448 ft) and 749.8 m (2,460 ft) is:

$$\mathbf{P}^{GC}(746.1, 749.8) = \begin{bmatrix} 0.44522 & 0.46793 & 0.08685 \\ 0.11387 & 0.76330 & 0.12283 \\ 0.14624 & 0.63995 & 0.21380 \end{bmatrix}$$

For example, given that the tunnel geology is ground class 2 at location 746.1 m, the probabilities that it will make a transition to ground class 1, ground class 2 (remain the same), and ground class 3 at location 749.8 m are 11.39, 76.33, and 12.28 percent, respectively (i.e., the second row of the matrix).

The parameters of the tunneling unit cost normal distributions and the ground class transition probability matrices determined above provided inputs for the risk-sensitive tunnel cost model. This model was then solved by using stochastic dynamic programming to determine the risk-adjusted tunneling costs (certain equivalents) and the optimal excavation and support sequence (optimal policy) for different degrees of risk sensitivity (i.e., different risk aversion coefficients γ).

Figure 4 shows the CE of tunneling costs for different degrees of risk sensitivity of the contractor. As can be seen, the EMV of tunneling costs for this project ($\gamma = 0$) is approximately $30.3M. As the risk aversion coefficient γ increases (i.e., a contractor becomes more risk averse), the risk-adjusted tunneling cost (CE) increases almost linearly. For example, the CE of the tunneling cost for a risk-averse contractor with $\gamma = 25$ is approximately $35.6M (i.e., the risk premium is about 17%). In contrast, as the risk aversion coefficient γ decreases (i.e., a contractor is more risk preferring), the risk-adjusted tunneling cost decreases almost linearly. For example, the risk-adjusted cost for a risk-preferring contractor with $\gamma = -25$ is $26.9M or about 11% below the EMV of tunneling cost.

Figure 5 illustrates the optimal tunneling policies for the west tunnel segment given that the contractor is risk averse with $\gamma = 5$. Nine bars in the figure correspond to the nine possible combinations (states) of ground classes and excavation methods during construction. For example, given that the tunnel geology encountered at location 40.2 m (132 ft) is *GC*1, and *EM*1 was used in the previous round, the optimal policy for the risk-averse contractor with $\gamma = 5$ is to use the same method (i.e., the first bar). However, if the geologic conditions at the same location are *GC*2, and *EM*1 was used in the previous round, the contractor should switch to *EM*2 for the next round (i.e., the fourth bar).

Figure 6 illustrates the optimal tunneling policies given that the contractor is risk preferring with $\gamma = -5$. As can be seen, the optimal policies at several tunnel locations for this risk-preferring case are different from those in the previous risk-averse case such as for state (*GC*3,*EM*1), the seventh bar, at location 304.8 m (1000 ft).

7. Conclusion

The risk-sensitive tunnel cost model presented in this paper demonstrates the procedure to quantify and incorporate a contractor's risk sensitivity into tunnel cost estimating in a systematic manner. The results show the influence of the contractor's degree of risk aversion on the certain equivalent of tunneling cost and the optimal construction policies. These optimal decisions can be used not only for planning and estimating project prior to construction but also for choosing optimal construction methods based on actual construction conditions (e.g., prevailing ground conditions and current excavation method). The proposed model can be modified and applied to other types of construction projects such as buildings and highways as well.

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Table 1 Summary of Tunneling Costs and Parameters of Tunneling Unit Cost Normal Distributions

No	Tunneling Alternative	Consequence	Material Unit Cost per Length ($/m)	Hourly cost ($/hr)	Parameters of Tunneling Unit Cost Normal Distribution ($/m)	
					Mean	SD
1	(*EM*1,*GC*1)	Right Decision	6,896	8,720	19,626	509
2	(*EM*1,*GC*2)	Exc. Overbreak	8,304	9,065	34,895	722
3	(*EM*1,*GC*3)	Exc. Overbreak	8,855	9,065	60,236	1,532
4	(*EM*2,*GC*1)	Underbreak	6,962	8,720	30,315	679
5	(*EM*2,*GC*2)	Right Decision	7,648	9,065	26,306	492
6	(*EM*2,*GC*3)	Exc. Overbreak	8,622	9,065	47,041	1,096
7	(*EM*3,*GC*1)	Underbreak	7,087	8,720	42,329	1,342
8	(*EM*3,*GC*2)	Underbreak	7,766	9,065	44,580	1,050
9	(*EM*3,*GC*3)	Right Decision	8,022	9,065	31,706	653

Note: The total fixed cost is $874,347. If the entire project is in *GC*1, this cost could decrease to $867,137.

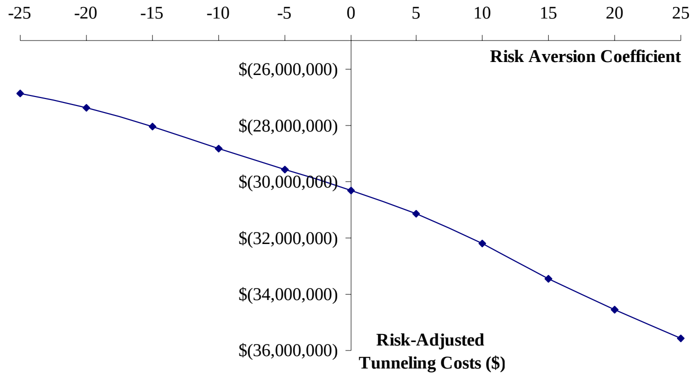

Figure 4 Relation between Risk-Adjusted Tunneling Costs (Certain Equivalents) and Risk Aversion Coefficients (γ)

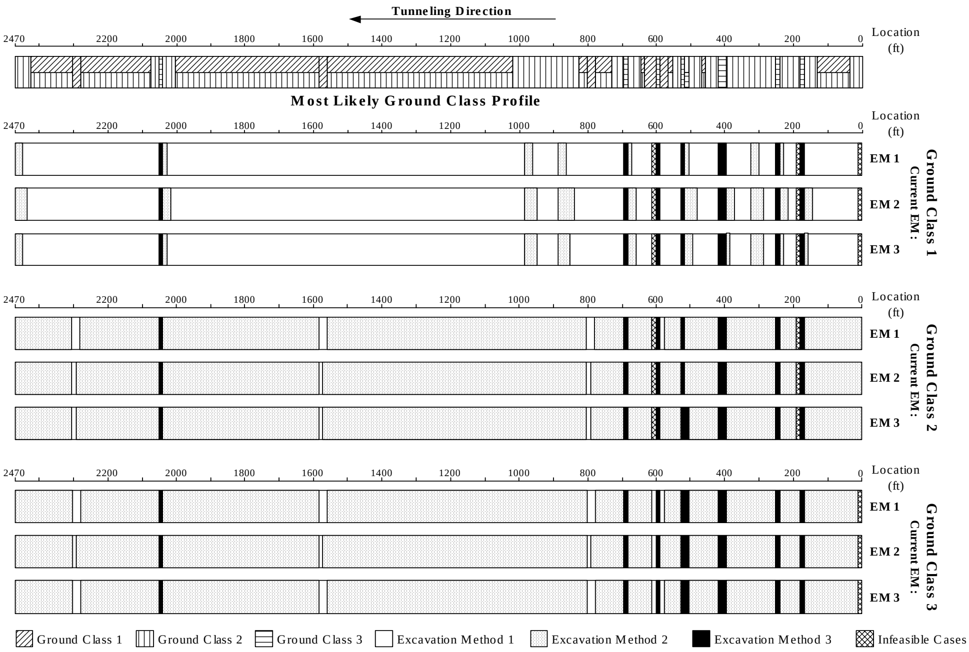

Figure 5 Optimal Tunneling Policies for $\gamma = 5$ (Risk-Averse Contractors)

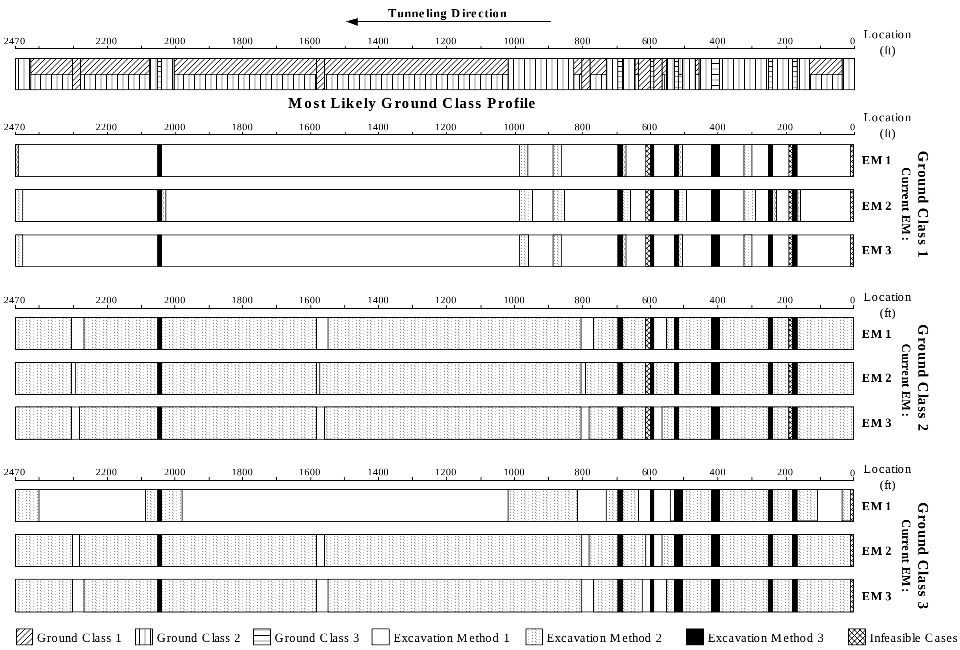

Figure 6 Optimal Tunneling Policies for $\gamma = -5$ (Risk-Preferring Contractors)