FRAMING, NORMATIVITY, AND SERVICEABILITY IN TEACHERS' DECISION MAKING DURING LESSONS

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To operationalize the notion that lessons are the knowledge base of the teaching profession and enable the study of teacher decision making during lessons, we provide a conceptualization of lesson as part of a multiverse of mathematics teaching. Using the case of problem-based instruction and a particular example in teaching high school geometry, we characterize framing, normativity, and serviceability as three systems of choice within the practical rationality of mathematics teaching which are useful for a teacher in managing decision making in problem-based lessons.

Notions of *lesson* are common across different genres of research on mathematics teaching. Lessons are sometimes seen as instances, say of a teaching practice. Lessons are also seen as unique events, where a teacher and their students work on specific tasks, in specific settings. But the Lesson Study literature has also brought attention to a notion of lesson that treats them as specifically dependent on tasks and conceptions, yet sufficiently general that they could be taught in different settings and by different people (Fernández, 2002). Different research purposes, have called for different notions of lesson. The latter notion of lesson is particularly helpful in accounting for teaching knowledge and teacher decision-making. Indeed, Hiebert, Gallimore, and Stigler (2002), building on what has been observed in Japanese lesson study, have described lessons as containers of the knowledge of the teaching profession. In this latter set of works, we observe the possibility to talk about lessons in ways that defy the objectivism assumed by those who see lessons as expressions of latent constructs (e.g., mathematical quality of instruction, teaching signature) and the subjectivism required of researchers to capture when teaching episodes are considered as unique events, characterized by individuals' adaptation to each other and their context. A third position takes inspiration from Bourdieu (1990) and considers teaching a lesson as a practice within the larger practice of mathematics teaching (Lampert, 2010). This position is supported by a metaphor of lesson as a region in the multiverse that teachers navigate while teaching. Along with the multiverse metaphor, they can help explain how lessons can be the knowledge base of the teaching profession and help describe how teachers use this knowledge.

The goal of this paper, taking up the aspiration of Hiebert et al. (2002) for the role of lessons as containers of professional knowledge, is to elaborate a multiversal concept of lesson that can support describing the knowledge of the teaching profession. Intuitively, we want to describe the phenomenon whereby different practitioners may teach "the same lesson" multiple times, even if they may make different choices within each enactment. This intuition is familiar to any secondary school teacher who teaches several sections of the same class: enactments of a lesson may differ in their contingencies (e.g., different students may bring up different ideas for the same problem) but those differences, inasmuch as they can be anticipated, are elements of knowledge that inform a teacher's decisions. Some lessons are connected across practitioners inasmuch as they rely on similar resources (e.g., lesson plans or other documents; see Guedet & Trouche, 2009; Morris & Hiebert, 2011) and use them in similar contexts (e.g., in 9th grade geometry courses across different states). And while individuals

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may interpret those lesson plans in idiosyncratic ways when they teach, a shared sense of the contingencies a teacher in similar circumstances may encounter when teaching that lesson (e.g., similar student errors or other contingencies) supports the expectation that knowledge of a lesson is in part collective even if tacit (Collins, 2010). In this paper we are interested in identifying and illustrating some systems of choice that arguably are part of the practical rationality of teaching in that they help name and organize some of the choices that teachers can make as they teach a lesson.

THE PRACTICE OF MATHEMATICS TEACHING

We come to this theoretical research problem from a perspective we've been building over the years under *practical rationality*. This perspective is informed by Bourdieu's (1990) approach to a theory of practice, whereby a practice is accounted for by spelling out relationships among three meta-theoretical constructs: field, habitus, and capital. The *field* of mathematics teaching alludes initially to objective elements of the practice, including positions (teacher, students), institutions (schools, departments) and their policies and rules (schedules, prescribed curriculum), or material resources available to practitioners (e.g., lesson plans). *Capital* includes teachers' social and intellectual capital that might provide individuals with leverage for handling elements of field. This includes knowledge of topics, resources, tools, courses, programs, and students, and also, importantly, their knowledge of lessons. The *habitus* or practical rationality of mathematics teaching alludes to practitioners' dispositions toward elements in the field, including their recognition of those elements and their sense of what are appropriate ways of handling those: It includes practitioners' recognition of instructional norms and professional obligations to the stakeholders of school mathematics (Herbst & Chazan, 2012). Crucially, the dispositions of habitus bridge between the individual resources of capital and the objective positions of field: Bourdieu (1990) describes them as "structured structures predisposed to function as structuring structures" (p. 53), thus capable of generating recurrence in ways of handling events and socialization of newcomers to practices. The systemic linguistic notion of system of choice is, we argue, an example of how the dispositions of habitus are structured structures that function as structuring structures: Individuals make meaning by choosing language uses from systems of choice that organize the language tokens and texts individuals know. Analogously, we wish to speak of the practical rationality containing systems of choice that organize individual's knowledge of a lesson into resources that enable different enactments of a lesson. These systems of choice along with the totality of lesson knowledge held by individuals and the objective resources for a lesson available in the field can be used to account for the professional knowledge associated with a lesson.

These three constructs, field, capital, and habitus, can help us consider the teaching of a lesson as one of many practices included in the practice of mathematics teaching more generally (Lampert, 2010). In talking about teaching a lesson as a practice, we counter a gaze on lesson as an instance of a general construct (e.g., teaching signature) described by an objective observer, as well as a gaze on lesson as a unique event that must be captured through the participants' perspective. In speaking of teaching a lesson as a practice, we allude to how elements of field such as problem statements, instructional goals, lesson representations (e.g., plans, scripts, storyboards), or contingencies (e.g., students' actions), and elements of individual teacher knowledge (e.g., knowledge of the mathematics at stake in an instructional goal or of the conceptions that explain students actions) can be managed with elements of practical rationality to generate a manifold of possible enactments of a lesson. In this paper we focus on characterizing three systems of choice that are part of this practical rationality. We zoom into a particular type of lesson, a problem-based lesson that starts from the posing of a problem and eventuates in the introduction of a theorem that helps solve the problem. Specifically, we use The Tangent Circle lesson taught by Ms. A and Ms. K, in two high schools in the American Midwest.

Beginning with a metaphor: The multiverse of lessons as the orbit of teaching practice

Drawing on the phenomenon of photon interference, quantum physics envisions parallel universes (or multiverse) as composed of shadow as well as tangible particles (Deutsch, 1998, pp. 43-49). We find the multiverse as a useful metaphor for thinking about the space of all mathematics lessons, where possible events in a lesson correspond to a particle (tangible if observable, shadow if not). Mathematics teaching is the navigation of this multiverse, and teachers' decision making as choosing particular actions rather than others which exist along with other possibilities in which that choice is not made.

A lesson can be described as a region of the multiverse, anchored in some elements of field, such as a lesson plan. Different sequences of events that might respond to the same lesson plan can be considered enactments of the same lesson--allowing for the notion of teaching the lesson as practice, with elements of capital and habitus, to organize a multitude of events from field (particularly actions taken by individual teachers and students), as choices against the background of a multiversal web of possible actions. The metaphor of the multiverse helps conceive the total space covered by the practice of teaching anchored in a set of lesson resources. The notion of orbit of a group serves as a metaphor to link teaching practice and lesson: If one thinks of teaching practice (emergent from field, capital, and habitus) as acting on those lesson resources, the set of outcomes of that action is the multiversal lesson.

Thinking about the set of all enactments of a lesson as a region of the multiverse, particular choices and subsequent decisions and actions might, as in the cases studied by physicists, lead to discernible patterns of sequences of actions such as the emergence of particular responses from students in some enactments of the lesson but not in others. Just as the quantum conception of causality speaks of the likelihood with which a phenomenon will follow another, a possible characteristics of a lesson might be conceived of as a consequence of particular choices on the part of the teacher in a statistical sense: Of the many possible enactments of the lesson in which students' realization of a particular mathematical relationship could be observed, there might be many more in which the teacher makes the choice to pose a task in a particular way. While the multiverse of mathematics teaching contains all such sequences of actions in all lessons, a particular lesson is a region of the multiverse, containing all the potential enactments of a plan. The multiversal notion of lesson helps build lesson-specific models of professional knowledge, instantiating Bourdieu's constructs of field, capital, and habitus to represent the rationality used navigating a lesson. The concepts of framing, normativity, and serviceability are introduced as systems of choice that can help understand what elements of this rationality support the navigation of a lesson. We do this in the context of The Tangent Circle lesson.

The Tangent Circle lesson

Consider field resources for a lesson, such as the problem statement "what would you have to do to find a circle tangent to two intersecting lines at two given points of tangency" and the purported goal of introducing the tangent segments theorem.[2] Ms. A and Ms. K were each teaching high school

[2] The tangent segments theorem says that there exists a circle tangent to two intersecting lines if and only if the points of tangency are equidistant to the point of intersection of the two lines.

geometry when they taught this lesson: Ms. A in her 3 accelerated geometry classes of mostly 9th graders (aged 14-15) in Eastside HS; Ms. K in her 3 regular geometry classes (mostly 10th and 11th graders) in Midwestern HS. All classes had included experiences with constructions and proofs.

The two teachers had differential personal resources going into the teaching of this lesson--Ms. K was more experienced and she had also participated in a project in which she and other experienced geometry teachers had watched and commented on an animation (https://youtu.be/xtHXVa8bkGA) showing a lesson aimed at the same goal and based on a very similar problem. In the animation, the students are given a sheet with two intersecting lines and two points plotted on them (Figure 1), the teacher in the animation describes the points as being on the lines making no comment on their spatiographical properties. In the animation, students are asked to draw a circle tangent to the two lines at the two points. Teachers' individual knowledge of mathematics, supported by the explicit statement of the tangent segments' theorem as the goal of the lesson, would likely enable them both to see that if a diagram were given with two marked points, a construction that took the spatiographical location as specifying each point would either produce a circle tangent (if the points were equidistant) or not (if otherwise). Additionally, Ms. K knew that for the points displayed in the animation's given diagram, if those were taken as given locations, there would be no circle tangent to both lines at those two points simultaneously (see Figure 1). Though unclear that Ms. A and Ms. K possessed this knowledge, the statement of the original problem (to find) without a provided diagram created an opportunity for students to draw their own points and think of the marked points initially as only representing the theoretical properties ascribed to them--being on the lines. The problem could have been understood as one that required finding additional constraints that enable the sought construction (see Figure 2). Teachers' knowledge of content and students might also suggest to them that students would unlikely see the diagram as only representing theoretical properties, but likely locate the points of tangency spatiographically.[3] Teachers' knowledge of how students relate to spatiographical properties would predict low probability that students would use an analytic method (suppose the circle was drawn, deduce what are the properties such a figure should have and see whether any provides leverage to construct the circle; Nagel, 1939) to find that the tangent segments have to be congruent.

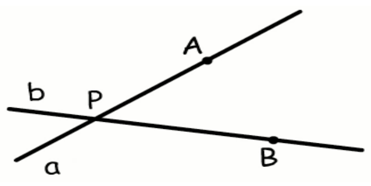

Figure 1. Diagram given in the tangent circle animation

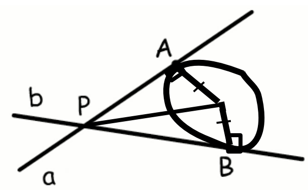

Figure 2. Assuming the circle has been constructed, those triangles must be congruent

Facing the prospect of teaching the Tangent Circle lesson, Ms. A and Ms. K had ways of relating to teaching the tangent segments theorem and to the problem to find a circle tangent to two intersecting lines at two given points of tangency. These were dispositions of the habitus, according to which it is customary for teachers to introduce new theorems through mini-lectures in which they provide official statements followed by diagrams that represent and exemplify the theorem stated. Likewise, problems

[3] See Laborde (2005) for the distinction between spatiographical and theoretical properties of figures.

"to find" are common in geometry only in the context of geometric calculations (Boileau & Herbst, 2015) where students must find measures of given objects rather than the objects themselves. In contrast, students usually participate in coming up with geometric objects in situations of construction, where they are asked "to construct" a given figure (e.g., an angle bisector) using some primitives (viz., a given angle). Their habitus as teachers of geometry would likely dispose Ms. A and Ms. K to recognize that discovering the tangent segments theorem through work on a problem would be a departure from the norm for how new knowledge is usually encountered in mathematics classes and that a problem that stated the need to find a geometric object would be seen as novel rather than familiar. Teaching this lesson would present an experienced geometry teacher with choices.

FRAMING A LESSON AND A PROBLEM

The notion of *framing* was introduced by Bateson (1954/2000) to identify metacommunicative acts that provide a context within which to interpret words and actions transacted. We argue that as Ms. A and Ms. K prepared to teach the tangent circle lesson, they had available two systems of choice that involved framing. The first one was whether or not to frame the lesson as one that would require a different division of labor vis-à-vis knowledge than what they were used to. Different didactical contracts could provide the alternatives from which the teachers could have chosen to frame this lesson (see Figure 3): The lesson could be framed as a problem-based lesson, many characteristics could be included simultaneously to frame the lesson as special and hence distinguish it from usual lessons that students might participate in. For example, Ms. K framed the lesson as one where they would have to "justify your ideas using prior knowledge" and "ask your other group members questions to understand their thinking." The lesson could also not be framed as special. Within the special lesson choice they would have further choices as to whether to allude to its special nature in regard to the contractual norm of how new knowledge is introduced. We conjecture that a teacher's decision to frame the lesson or not is connected not only to how the lesson deviates from what is normative in their classroom but also with the differential likelihood teachers attribute to events possible to happen in the lesson. We use the conventions of systemic functional linguistics to denote inclusive and exclusive choices in systems, with curly brackets for the former and square brackets for the latter (see Figure 3).

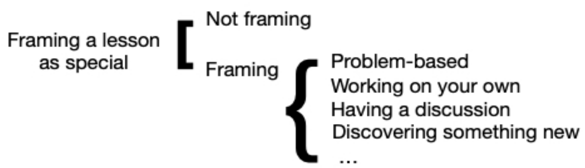

Figure 3. Framing lesson as special

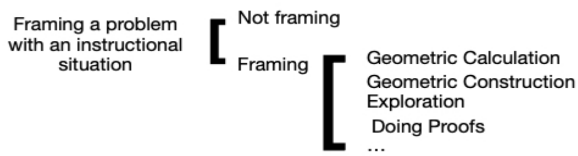

Figure 4. Framing a problem with a situation

Regardless of whether the teacher framed the lesson as special and problem-based, the lesson actually had those characteristics. The characteristics of the problem included in the lesson plan might predispose the teachers to either be attentive to some distress on the part of students as they received the problem as stated or to do some work with them at the onset to help them calibrate the effort they would need to put into their work. This we describe as *framing the problem with an instructional situation*. Another system of choices was available, that including ways of framing the problem as one students had seen before. *Instructional situation* refers to the distinct types of problems used in a course of studies insofar as they appeal to particular norms for mathematical work. In high school geometry, we have described calculation, construction, exploration, and proof as different instructional situations (Herbst, Boileau, & Gürsel, 2018). In the tangent circle lesson, teachers could choose to frame the problem as one that would fit an existing instructional situation or not. Those choices would require the teacher to amend how the problem had been stated in the plan. Figure 4 shows this framing system.

The teacher in the animation Ms. K had seen had framed the problem as construction. Following the norms of a construction situation, the teacher provided a diagram (Figure 1) that included some givens. The statement was also changed into "draw" as opposed to "find," and teachers could expect students would act as if the figure should exist. Teachers' experience with situations of construction and proof in high school geometry, and their recognition of what is normative in those situations (Herbst, et al., 2018), might have suggested to them that students would not consider making a construction in response to a question that asks them if there exists a tangent circle, and would therefore be disposed to frame the problem as a construction, giving the two lines and two points. In doing that, they might have been disposed to notice something of a dilemma in regard to what points to provide as points of tangency: If the given points of tangency were given to be close to equidistant from the intersection of the given lines, students would likely not doubt the task to be possible and produce a circle that looked tangent without becoming aware that the points had to be equidistant; and, if the two given points were given not equidistant, they might feel deceived by having been asked to construct something that could not be constructed--this would be a breach of contract. Overall, our point here is that Ms. A and Ms. K's habitus would include dispositions to consider a variety of possibilities that flesh out alternatives for how the lesson could proceed. The versions of the lesson we recorded document those alternatives.

As we think of Ms. A and Ms. K as individuals, it is clear that some possibilities for how the Tangent Circle lesson could unfold might be more visible to Ms. K than to Ms. A on account of her differential knowledge. At the same time, Ms. A's knowledge of her students, based partly on their higher academic accomplishments, reassured her that her students could engage with this problem without much distress. The anticipated and realized instances of using the Tangent Circle problem to teach the Tangent Segments' theorem lesson help envision a network of possible paths as an aggregate background. Each teacher's instance of using the problem to teach the theorem can take meaning against that aggregate background: Individual teacher moves to flesh out the lesson have meaning in the sense of being choices made against the possibility of other choices present in the aggregate. As the quantum conception of causality speaks of the likelihood with which a phenomenon will follow another, a possible characteristic of a lesson (e.g., students not realizing that the existence of the circle tangent depends on the location of the points of tangency) might be conceived as a consequence of particular choices on the part of the teacher in a statistical sense: Of the many possible universes in which such lack of realization could be observed, there are many more in which the teacher makes the choice not to provide a diagram or to provide a diagram with equidistant-seeming points of tangency than those in which the diagram contains points that seem not equidistant.

APPRECIATING STUDENTS' WORK AS NORMATIVE AND AS SERVICEABLE

As Brousseau (1997) notes, in creating and modifying the activities for students to work on, the teacher contextualizes, personalizes, and temporalizes the knowledge at stake, while the students' learning of that knowledge will require them to eventually decontextualize, depersonalize, and detemporalize it. A problem-based lesson thus confronts the teacher with contingencies in students' responses to the activities in which they are involved and requires the teacher to make decisions that use students' responses as the class navigates toward laying claim on the instructional goal. Those decisions are neither determined by the context nor plainly willed by the teacher, but they can be understood as a response to the complexity informed by individual resources and the social resources of the context. In particular, the instructional context with which the teacher makes decisions is mathematically-specific, not only because of the instructional goal and the problem being used, but also by the instructional situations the teacher may have used to frame the problem for students to work.

The theory of practical rationality (Herbst & Chazan, 2012) provides concepts that can be used to produce such understandings of the decisions a teacher makes and may eventually help assess the value(s) of the series of decisions a teacher makes in the enactment of a lesson. The framing move described above brings in the norms of a situation of construction, which might scaffold the students' work on the problem. But this does not necessarily mean their work will move the class toward the instructional goal. When a problem has been framed within an instructional situation, the concepts of normativity and serviceability support our understanding of what characteristics of a student's work a teacher may notice and appreciate, which may predict how they interpret and respond to that work. Inasmuch as an instructional situation has norms that describe what students are expected to do, *normativity* describes a possible quality the teacher might see in a student's work: whether the work abides by the norms of the situation used to frame the work. In the Tangent Circle lesson, the response offered by a student who drew a shape freehand could be described as non-normative, given that students are expected to construct diagrams using straightedge and compass. Separately, inasmuch as the problem is an instrument to get the class to move toward a planned instructional goal, *serviceability* describes the extent to which students' work creates material that supports the class's advancement toward such goal. In the Tangent Circle lesson, one student claims that the task is impossible unless one moves the points. Though non-normative, as student are expected to complete the tasks that their teachers assign them, this claim could be seen as serviceable, as it raises the question of where one would have to move the point, which could lead to the students concluding that the points of tangency of the two tangent lines must be equidistant from their point of intersection.

Normativity and serviceability are distinct dimensions for appraising students' work. Serviceability is a dimension of student work in problem-based lessons. Normativity depends on framing inasmuch as a lesson framed after an instructional situation not only enables more student work but also compels the teacher to appreciate student work that responds to this framing, even if this work may not be serviceable. The notion of framing (as construction) and the notions of normativity and serviceability have proved useful to us in explaining teachers' decisions to bring particular students' work to the attention of the whole class and their decisions how to respond to particular students' work.

CONCLUSION: A LESSON IN THE MULTIVERSE OF MATHEMATICS TEACHING

The aggregate background of possible enactments of a plan is what we'd like to call a *lesson*, and for the case being discussed, The Tangent Circle lesson: a region of the multiverse of mathematics teaching anchored in some field resources (e.g., a plan). Given an instructional goal and activities planned to accomplish that goal, we define a lesson as the set of all possible enactments that could result from the negotiation of contingencies associated with the enactment of those activities and the achievement of the instructional goals. As activities depend on contingent, codependent actions by students and teacher, the enactment that takes place in any one classroom is one possibility among many that could happen. The encounter of an individual teacher, a particular class, and a lesson produces an instance of that lesson in which some possible choices are made and others are forgone. This lesson embodies systems of choice, categories of perception and appreciation, that contain what practitioners would find sensible to do as well as ways of expressing differences among what different practitioners might find sensible to do. In that sense, a lesson can contain the knowledge of the profession: Someone who did not belong in the profession would unlikely see the variety of alternatives that present to them as they embark a particular class on the Tangent Circle problem. In providing this interpretation of how a lesson contains the knowledge of the profession, we are drawing heavily on a conceptualization of knowing-in-practice: Not only individual content knowledge for teaching associated with the lesson but also the aggregate set of dispositions (recognitions of instructional situations available to frame work on the lesson and the norms of those situations, recognition of opportunities to attend to obligations to the stakeholders of mathematics teaching) that are part of the habitus of mathematics teachers and which are activated as they work with specific elements of field. Along with Hiebert et al. (2002), we argue that lessons so conceptualized are containers of professional knowledge. Our main contention is that we need concepts to study and represent lessons in ways that make them knowable, learnable, and teachable. Such concepts should assist us as we compare lesson enactments. In addition to the framing systems we identified as available for the teacher to launch students' work on the problem, serviceability and normativity are two systems can be proposed as helping teachers appreciate students' work particularly in problem-based lessons that aim to use students' work on a problem to introduce a new idea.

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