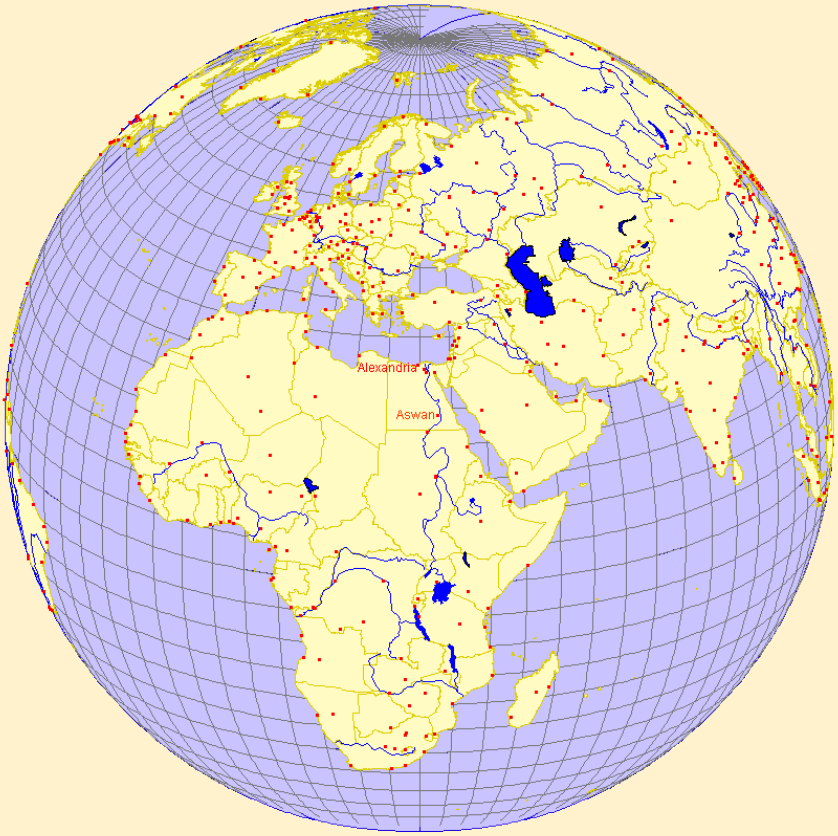
ISSN: 1059-5325

Solstice: An Electronic Journal of Geography and Mathematics

Volume XXXII Number 2
December 21, 2021



Founded: 1990
Persistent archive: Deep Blue, University of Michigan
Mathematical Geography (Institute of);
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SOLSTICE:

An Electronic Journal of Geography and Mathematics

32 YEARS OF PUBLICATION!

Persistent URL: **http://deepblue.lib.umich.edu/handle/2027.42/58219**

Cite articles as: Author name(s), Year. Title of article, *Solstice: An Electronic Journal of Geography and Mathematics*, Vol. YY, No. ZZ. Ann Arbor: Institute of Mathematical Geography.

Virtual reality of The University of Michigan "Diag", below. Note the observatory dome atop Angell Hall, in the foreground.

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Frontmatter

Solstice was born digital in 1990, before the advent of the Internet. Early volumes were typeset using the digital typesetting language, TeX. The digital files were sent to subscribers via email and the receiver printed out the TeX code, if desired, to produce a typeset-quality journal, on-demand. Selected monographs in the IMaGe Monograph series contain typeset versions of *Solstice*, printed from the code transmitted as the original version of that issue of *Solstice*. Later, when the Internet became available, *Solstice* switched to the Internet as the platform for transmission, writing documents in html rather than TeX.

Early in *Solstice's* production history, some authors worried that their electronic files could be maliciously altered by random readers and uploaded to replace their own writings. Of course, that could not have happened (because everything was passworded). However, as reassurance to prospective authors not yet familiar with the mechanics of servers and such, early documents were edited to introduce deliberate errors in spacing, inserted by hand, that a random word-processed document would fail to automatically duplicate. Hence, a bogus copy could be detected simply by overlaying a 'new' printout on the 'old' printout on a light table. The hand-insertion of erroneous spaces motivated the oriental rug motif, photographed from a Bokhara rug from the 1964 New York City World's Fair; that symbol is carried forward (although the practice itself is not) in *Solstice* today, as a subtle reminder of one element of the journal's history.

Over the years, *Solstice* has gained media attention from a variety of sectors: from *Science* (AAAS) and *Science News* early on. A bit later with interaction with a museum, the Exploratorium (San Francisco), and the TV show, *Nova*. For all these notices, as well as for those in more conventional academic arenas, our primary thanks go to our contributors, volunteers, and readers who have been with us for so many years. Best wishes to all!

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Transient URLs that point to the persistent URL: imagenet.org; mathematicalgeography.org; mathematicalgeography.com; instituteofmathematicalgeography.org; instituteofmathematicalgeography.com

AUTHORS OF ORIGINAL MATERIALS APPEARING IN SOLSTICE, 1990-present:
Ainslie, V. | Albert, D. | Ard, K. | Arlinghaus, D. E. | Arlinghaus, S. L. | Arlinghaus, W. C. | Arlinghaus, W. E. | Austin, R. F. | Baird, N. D. | Bandyopadhyay, G. | Barmore, F. E. | Barr, D. | Batty, M. | Beal, F. J. | Beier, K.-P. | Bhatia, S. S. | Blake, B. | Borgers, A. W. J. | Burkhalter, B. R. | Chattopadhyay, S. | Chaudhuri, S. | Childers, P. E. | Chirapiwat, T. | Crane, J. | Crawford, T. C. | De la Sierra, R. | Derudder, B. | Dison, A. C. | Drake, W. D. | Eagle, K. A. | Earl, E. | Emas, T. | Favro, R. | Frank, A. I. | Gober, C. | Glunz, B. | Goodman, F. L. | Gorstein, J. | Griffith, D. A. | Haidar, S. | Hall, B. | Hall, D.| Hamilton, R. C. | Han, S.-H. | Harary, F. | Haug, R. | Haynes, D. | Hoffman, L. L. | Hogh, G. | Iyer, S. D. | Jacobs, D. A. | Kaur, K. | Kelbaugh, D. S. | Kerski, J. | Kim, E-Y. | Koenig, R. | Kos, S. | Kwon, T. | Lach, A. | Larimore, A. E. | Laug, A. | Lazzaro, A. A. | Lee, H. | Licate, J. A. | Lindemann, W. | Maeda, J. | Martin, P. | McCloskey, J. C. | Metzler, D. | Mokray, G. | Moore, K. | Muhilal | Naud, M. | Noguchi, M. | Nystuen, J. A. | Nystuen, J. D. | Onstott, B. | Oppenheim, P. J. | Oswalt, K. S. | Pak, S. | Phillips, L. R. | Pogany, T. | Ponce de Leon, J. S. | Pothukuchi, K. | Purvis, E. M. | Ram, B. | Rayle, R. | Robinson, Craig | Rosenblum, A. | Ross, S. | Rushing, D. | Ryznar, R. | Sammataro, D. | Schlossberg, M. | Schumann, L. | Stern, H. L. | Stuckman, P. | Tarwotjo, I. | Tilden, R. | Timmermans, H. J. P. | Tobler, W. R. | Van Acker, V. | Wagner, T. | Wallace, R. | Walton, A. | Ward, M. | Wilson, M. L. | Wilson, S. | Witlox, F. J. A. | Woldenberg, M. J. | Zander, R. H.

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Update on Activities of the Editor…

During the course of the Pandemic of 2020/2021, I have had time to develop ideas and strategies for various projects, but almost no time or other resources to implement them (due to shutdowns of various sorts and subsequent economic, travel, and other limitations). The virtual networking, represented by *Solstice*, became all the more important!

- Working with co-authors Joseph Kerski and William C. Arlinghaus on a book (book contract with CRC Press, signed in November 2021) entitled *Teaching Mathematics Using Interactive Mapping.* (See related article later in this issue of *Solstice*.)
- I have much enjoyed reconnecting with friends and colleagues during the difficult times we have all experienced over the last year and a half or so. Of particular interest is an ongoing conversation with Jonathan D. Mayer (Professor Emeritus, Epidemiology, Medicine, and Geography, University of Washington) involving Erdös Numbers. Stay tuned…!
- Category theory is an interesting way of organizing mathematical objects. How might it be interesting in the real world? I continue to search for associations to bring a loose abstract parallel down to Earth. Thus, I am collecting 'commutative diagrams' when I see them in the real world. In a general way, a commutative diagram is to category theory as an equation is to algebra. A commutative diagram is a diagram in which all directed paths, with the same starting and ending points, lead to the same result.

  For example, if I am sitting in the living room with Bill, I can tell him that 'today, the high temperature is forecast to be 44 degrees. Or I can send him that information via email. If I use email, the message goes from me to a server at my university. Then the server at my university sends it to a server at his university. Then, the latter server sends to Bill. Either way, Bill receives the same information. The diagram thereby traced out, commutes. Both paths, direct or through two servers, yield the same result. Should one of the servers fail, causing the diagram not to commute, I will be notified and so will seek alternate means of delivering my message. I will be confident that the system is reliable.

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Now, suppose I replace the inanimate intermediate servers with intermediate people. Will the reliability of the network remain? Will the direct path and the path with intermediaries consistently produce the same results? Probably not. Primary source contact, direct contact, is superior to secondary sources, with intervening humans. Diagrams that commute give a reliable reporting system; those that do not commute, do not. They create 'gossip.' Is there a way to test reliability in communications, when secondary sources are involved, using commutative diagrams? A number of ideas come to mind, but implementation might be a challenge. When conceptually rich abstract structure becomes embedded in a technological revolution, what sorts of associated culture shifts might arise?

- Transformations. Another collection of real-world observations, as have been made by a number of scholars in the past (D'Arcy Thompson, Waldo Tobler, and more). A simple phone call is a voice transmission from one person to another; it is one-to-one. A simple phone call from one person to a group of people, through a speaker phone or other means, is one-to-many. In parallel, a FaceTime call is like a simple phone call, one-to-one, and its extension to become one-to-many is via a Zoom call.
- An article in USA Today found that Meridian was the only US city in a set of 50 cities that had no PFAs ("forever chemicals") in its water supply. Presumably, that means that there were none in a point sample of our aquifer. Of course, we would wish to keep it that way—not only at that single point, but also throughout the aquifer. Therefore, we need to encourage businesses to remove PFAs from other nearby water resources: streams, lakes, etc. as well as good plans for surface runoff and onsite retention. All so that our region can be kept clean—how might this goal be achieved?
    - Beds of activated charcoal can adsorb PFAs. Study the North Carolina model.
    - We have lots of coal in the Meridian area (lignite, not good for much else).
    - Trap PFAs in carbon. Then to dispose of them, subject the beds of carbon to high temperatures (as with refreshing activated charcoal bags using sunlight).

- Might the remnants of the former Kemper county coal plant might serve as a regional (or more) center for PFA removal? Presumably, they have/had access to carbon beds and to the technology to subject them to high temperatures?
- Approach utility companies or environmental groups or others to work with regional political leaders?

- Continuing work on Meridian Renaissance Project:
  - Implement 'History Garden' pilot project for plant identification of local species, including varieties of kudzu, including as a possible food source. Plant the kudzu or other species in a manner consistent with general goals of urban sequoias, but at a very reduced scale. Use the kudzu to shield vertical concrete or other hard surfaces with plant materials that absorb carbon. Integrate with existing ideas associated with kudzu, archived in previous issues of *Solstice*.
  - Crime Mapping Project. Mapping, in the style of "Urban Graffiti as Territorial Markers" (Ley and Cybriwski, 1974, *Annals*, Association of American Geographers) donated to Meridian municipal authorities in 2021.
  - Training of elderly in enhanced computer capability might use culinary matters and Green Tables issues and content to promote interest.
- Projects with culinary association.
  - Increase membership in Ann Arbor Chapter of Les Dames d'Escoffier. Established a satellite of that Chapter in Meridian—model for involving small clusters of interest into larger groups, clearly appropriate given current interest in not needing to be in one geographical locale in order to work.
  - Develop a series of Zoom Tours and Projects. Initial focus in Meridian. Make a digital Library/Archive of Zoom Tours and Project. The first one appears later in this issue of *Solstice*.
  - Book donation project.

- Possible publications. *LDEI Quarterly*: Green tables update on kudzu project in Mississippi; culinary families in LDE Chapters—Lach/Arlinghaus, Glunz/Donovan, others. Include related archives and such.
- Continuing Board, Review, and Similar Work
  - 2021. Member Culinary Historians of Chicago.
  - 2019 to present. Director of Outreach for the Meridian, MS, Bonus Use Group in association with the Ann Arbor Chapter of Les Dames d'Escoffier,
  - 2019 to present**.** Committee Member, Les Dames d'Escoffier International Collection at the John and Bonnie Boyd Hospitality and Culinary Library, 1609 Oretha Castle Haley Boulevard, New Orleans, LA 70113. The committee's goal is to assemble a repository at the Boyd Library of all the volumes about food and beverages authored by current and past members in good standing.
  - 2016-present. Continuing work as founding member of the Ann Arbor Chapter of Les Dames d'Escoffier International, a leading haute cuisine organization. Some of that work involves attempting to develop connections between the academic and culinary worlds. Also a member of the Chicago Chapter of LDEI. 2019--: Director of Outreach and Acting Vice-President, Ann Arbor Chapter. http://www.ldei.org
  - 2015-present. Continuing association with the Special Collections Research Center of the Joseph Regenstein Library of the University of Chicago, Alma Lach Culinary and Book special collections.

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Predomination, or, Which Comes First: The Concept or the Activity?

Sandra L. Arlinghaus* and Joseph J. Kerski

Introduction

A persistent issue in communicating and understanding ideas and the real world involves the balance between 'concepts' and 'activities'. Different people think differently and that is a good thing, lest we all become clones of a single mind. Some of us want to understand the conceptual picture first with a full-blown (or at least fairly complete) mental image of an abstract structure. Then, we place particular activities within the broad context: we start with the 'whole' and insert the 'parts'. Others of us like to have mental hooks, perhaps in the form of a real-world activity, on which to hang concepts, as a practical handle to understanding them. We start with the parts and assemble the whole from them. Which is better? The answer is…Neither! Each has merits and each has drawbacks. We illustrate how even very different approaches can lead back to the same place—with difference in approaches depending on whether the 'concepts' or the 'activities' predominate (consider the comments about 'commutative diagrams' in the Update section of this issue…)..

Activity Predominates

Ask a student to measure the height of a flagpole on a sunny day. Assume the student has necessary conceptual knowledge, including a basic knowledge of plane geometry and elementary right-triangle trigonometry. Also assume the student has access to appropriate physical tools: a tape measure, a protractor, and needed office supplies.

On a sunny day, the flagpole casts a shadow. Figure 1 illustrates the process for determining the height of a flagpole.

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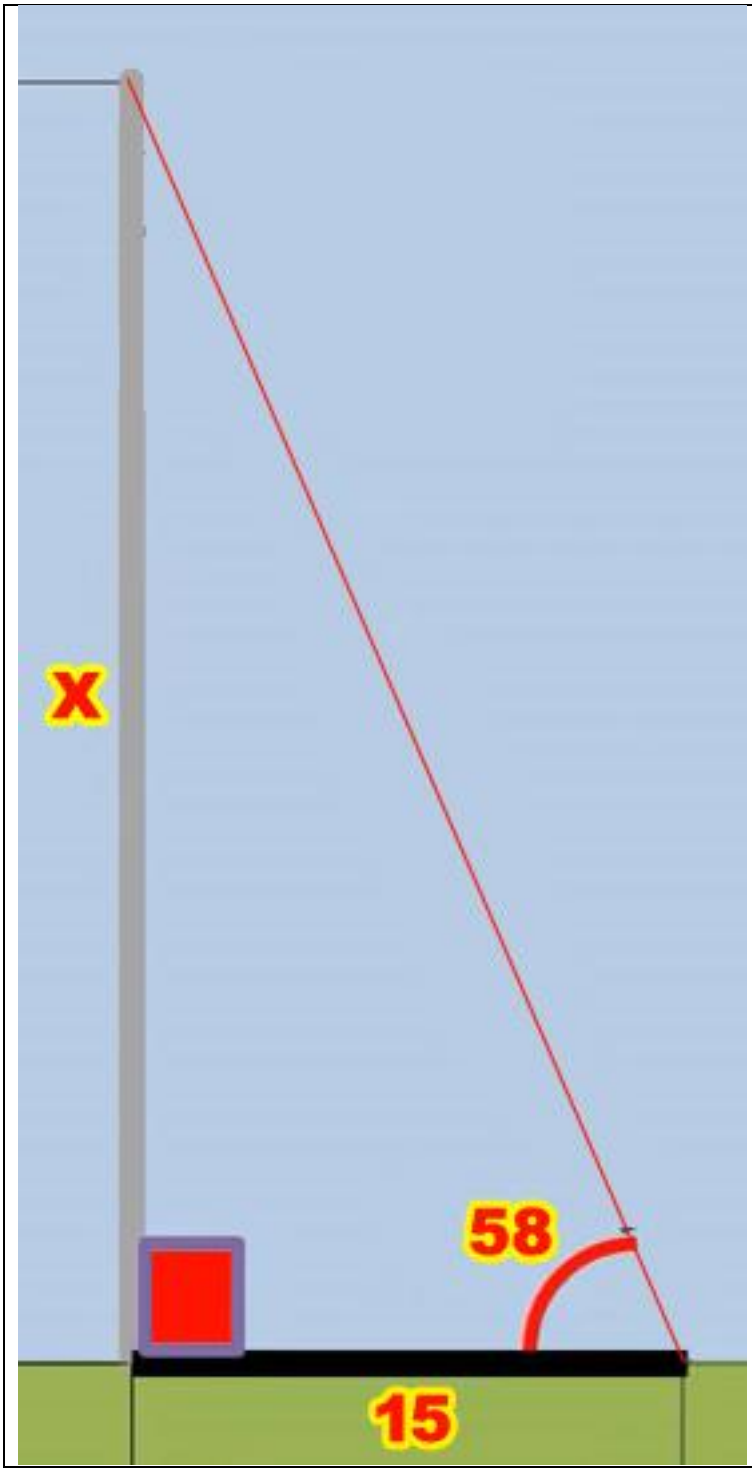

Measure the length of the shadow using a tape measure. The length is 15 feet.

Measure the angle to the top of the flag pole, from the tip of the shadow, by sighting on a 'sextant' made from a protractor. That angle is 58 degrees.

Find, X, the height of the flagpole.

tan 58 = X/15; X = 15 * tan 58.

tan 58 is approximately 1.600 so

X=15 * 1.600 = 24.

The height of the flagpole is approximately 24 feet.

Figure 1. Measuring the height of a flagpole on a sunny day.

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What are possible error factors causing inaccuracy in the result from this activity? Can the user control them?

- Are you measuring the size of the acute angle from the ground or from a few feet off the ground. You can control that error. How?
- Is the flagpole actually perpendicular to the Earth's surface? Can you control that?
- Is the flagpole, itself, actually straight and not warped? Can you control that?
- Is the 15 foot measurement along the Earth's surface affected by the curvature of the Earth?

The latter comment, involving the curvature of the Earth, is one that is richer in conceptual content than the pragmatic comments that lead up to it. It thus opens the door to more conceptual questions derived from this single activity.

Derivative conceptual ideas.

- The tangent function was used here. If we had different values for the right triangle, would different trigonometric functions become involved? Design activities to suggest how to achieve this strategy.
- Why is the tangent function called 'tangent'? I don't see any tangent line to a circle?
- Do we need to use trigonometry? Is it possible to design an activity based only on plane geometry?
- What is the general history of the concept of 'measurement'?
- The protractor was used to create a 'sextant'. What is a 'sextant"? What else is it used for? Read a podcast by Kerski on the topic. Do research on the history of navigation, and once again we return to the concept of the Earth as a whole.

In this activity, we were led from a specific task to think a bit more broadly, both in terms of error functions and derivative concepts; each sequence led us back to the idea of some measurement issues of the Earth as a whole. The activity predominated; the concepts were in the background.

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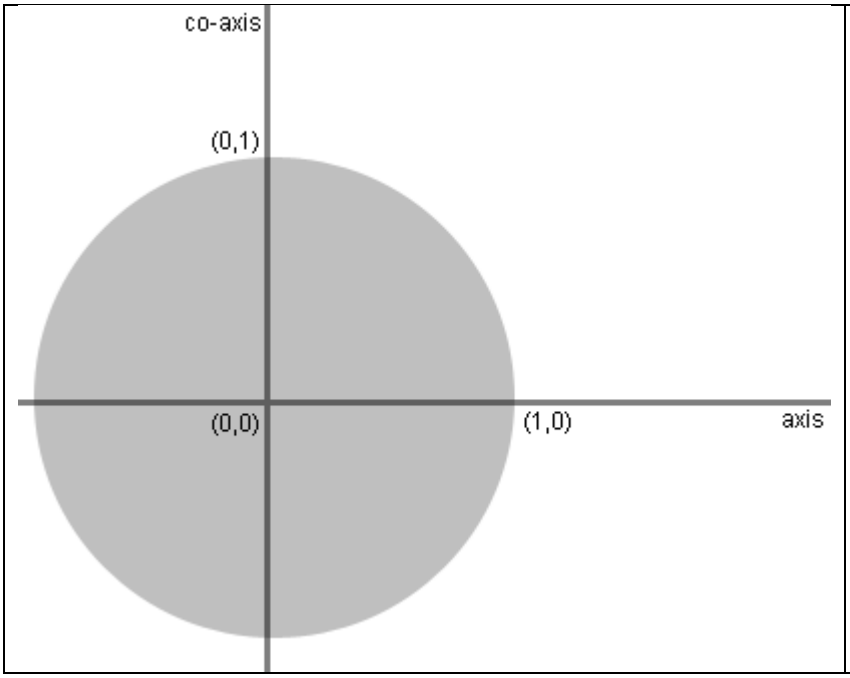
Concept Predominates

Here we make the same assumptions: the student has knowledge of basic plane geometry and elementary right triangle trigonometry. Often students with only very basic knowledge of right triangle trigonometry become perplexed by the names of the various functions, especially ones like tangent and secant that they associate visually with circles. Derivative of such confusion, there often arises difficulty in figuring out trigonometric identities and other more advanced skills that often bleed into difficulties in trigonometric substitutions in calculus and elsewhere. Thus, we focus here on the broad structure of trigonometric functions with a visual trigonometry review.



The basic setup: a unit circle with axes labeled as 'axis' and 'co-axis': the motivation is to make plausible names like co-sine, co-tangent, and co-secant. The ***co***mplementary (orthogonal) axis is designated as the ***co***-axis. Orthogonal axes are assumed and are necessary.

Figure 2. The basic setup.

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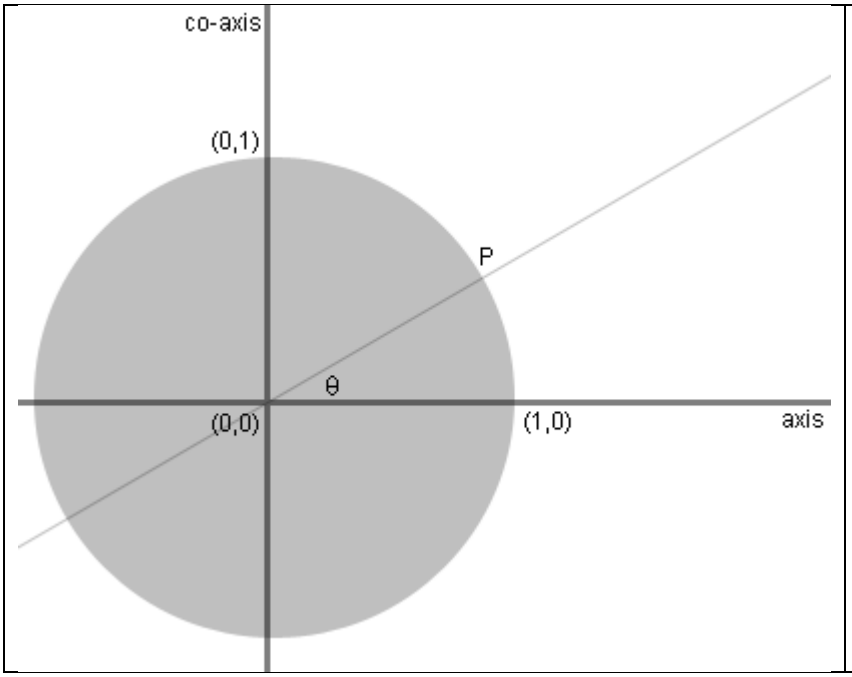
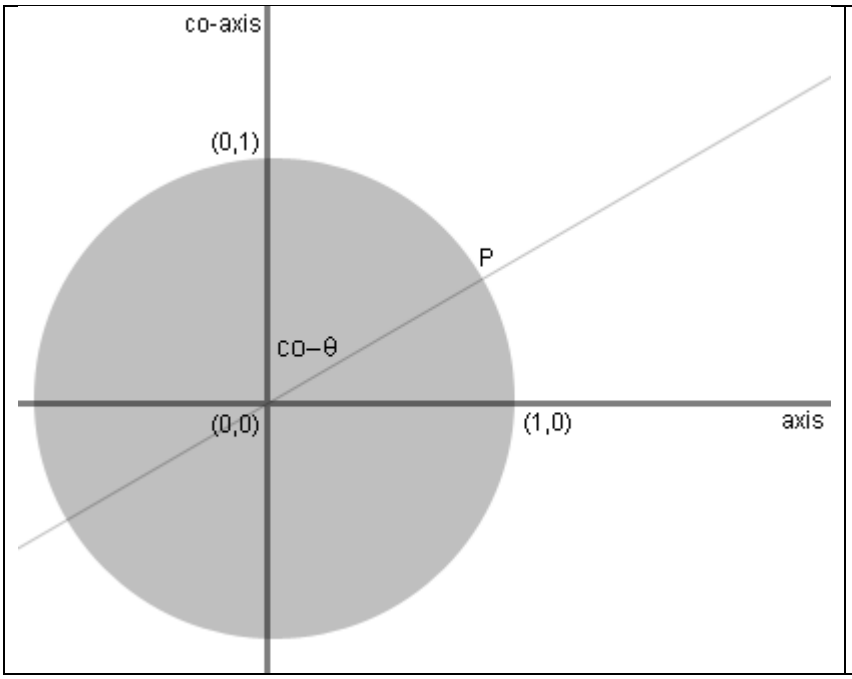


A secant line, is a geometric object. This one cuts through the center of the unit circle at an angle of theta to the horizontal axis. This secant line intersects the unit circle at point *P*.

Figure 3. Geometric secant line determines the angle theta.



The same secant line also determines the ***co***mplementary angle to theta (another geometric object), denoted ***co***-theta.

Figure 4. The same secant line as in Figure 3 determines the ***co***mplementary angle to theta, ***co***-theta.

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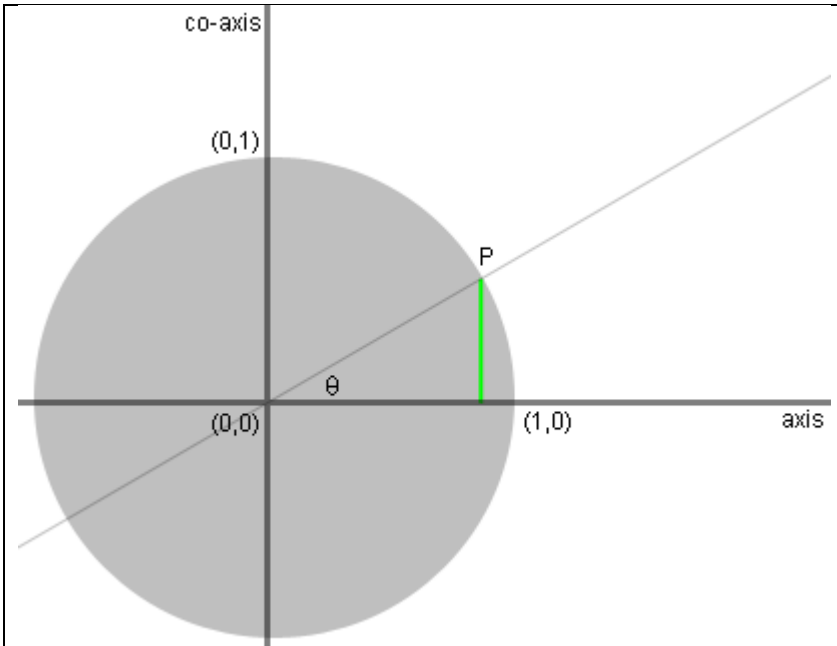
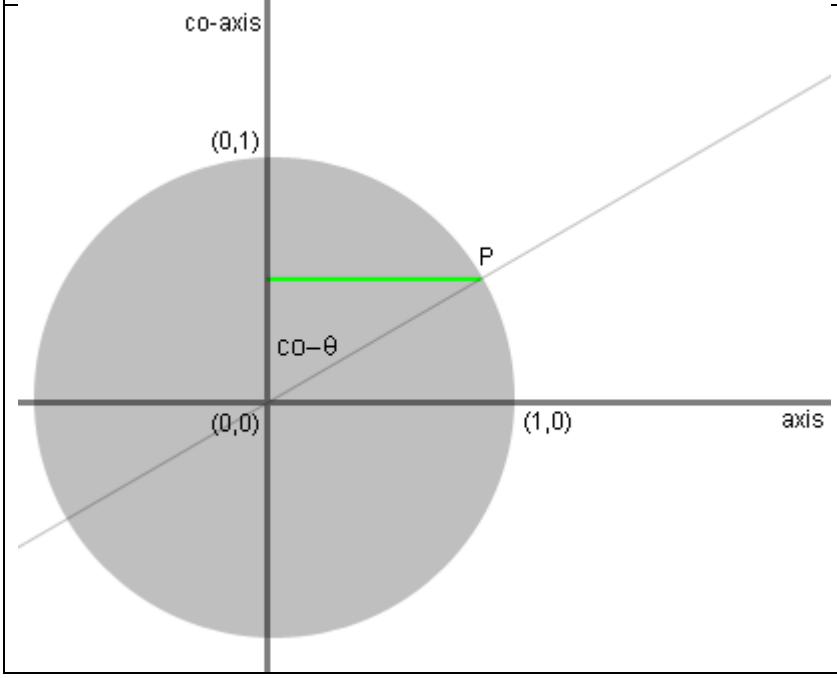


The length of the green line, dropping from *P* to the axis, measures the sine of theta: opposite side of a right triangle over hypotenuse of the unit circle.



The length of the green line, dropping from *P* to the ***co***-axis, measures the sine of *co*-theta: opposite side of a right triangle over a hypotenuse of the unit circle. Hence, ***co***sine of theta.

Figures 5a and 5b. Sine and cosine. Familiar right triangle construction. Linguistic derivation visualized.

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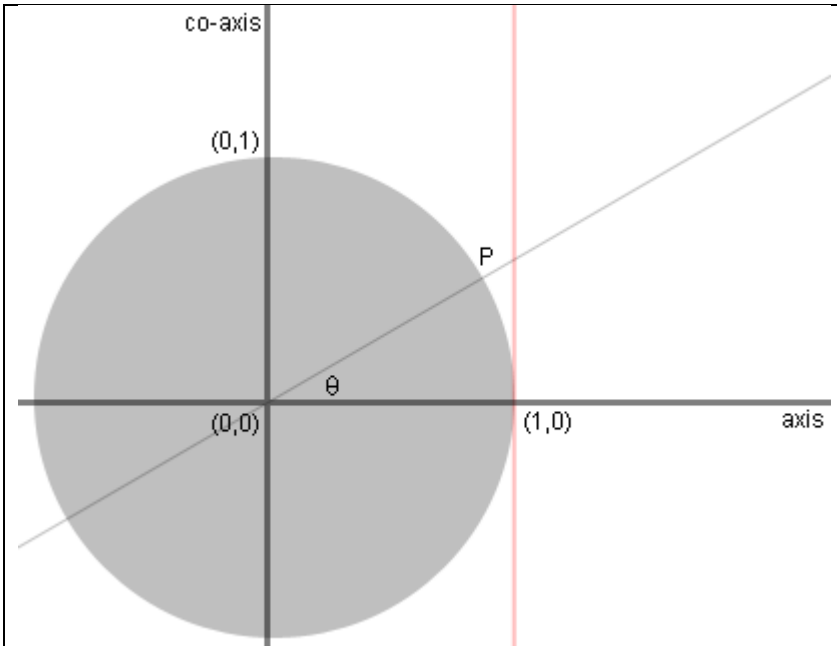
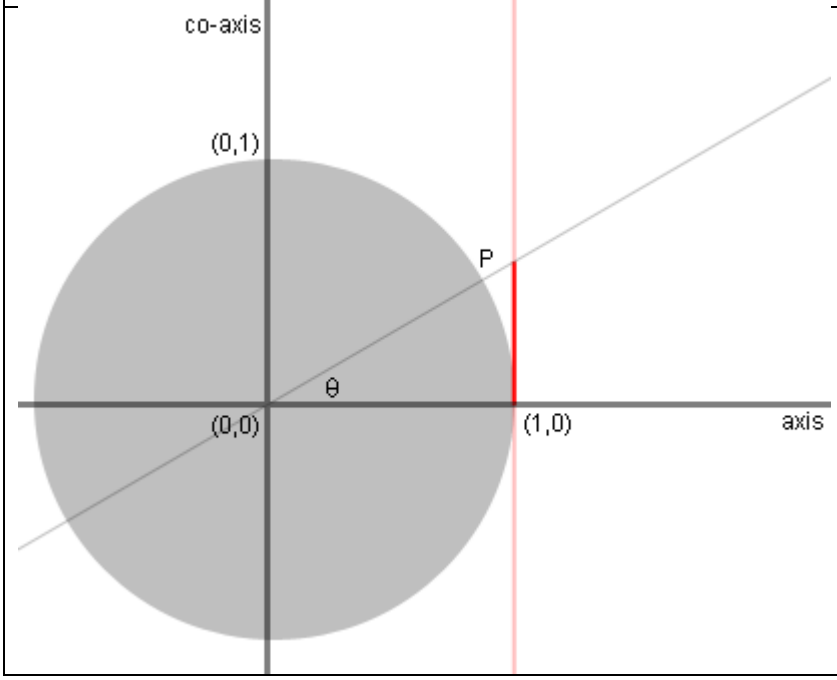


The pink line is a geometric line tangent to the unit circle at (1,0) on the horizontal axis.



The length of the red line, intercepted by the secant line along the tangent line, measures the tangent of theta.

Figures 6a and 6b. Tangent, geometric and trigonometric terms unified.

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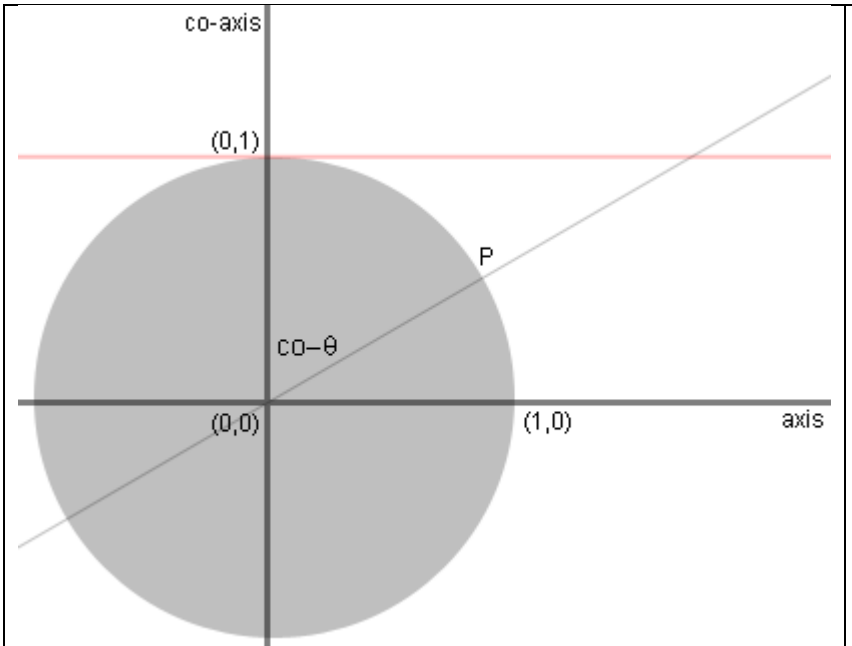
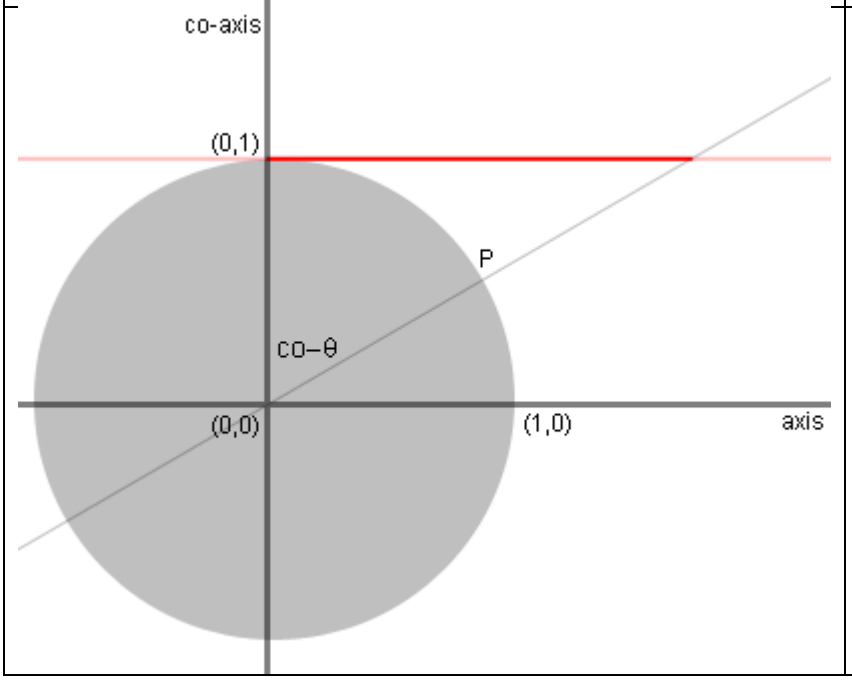


The pink line is a geometric line tangent to the unit circle at (1,0) on the co-axis.



The length of the red line, intercepted by the secant line along the tangent line, measures the tangent of ***co***-theta. Hence, ***co***tangent of theta

Figures 7a and 7b. ***Co***tangent, linguistic derivation visualized along tangent line with respect to ***co***mplementary angle, ***co***-theta.

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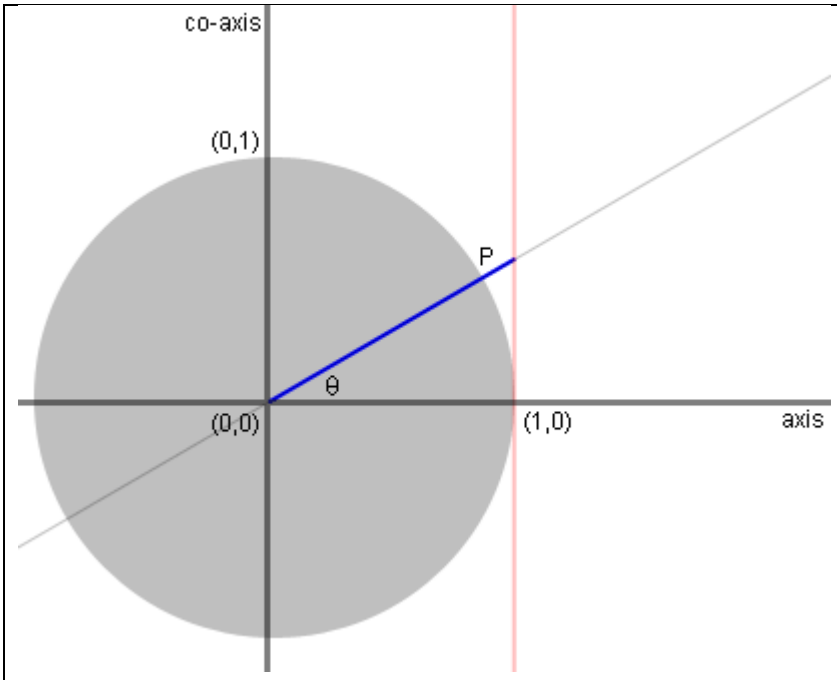
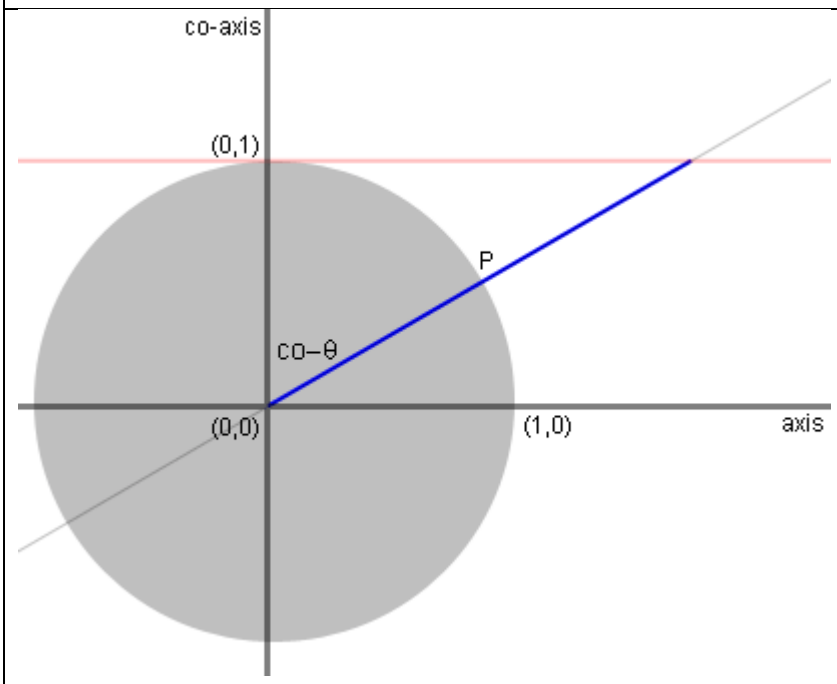
	The length of the blue line, intercepted by the tangent line along the secant line, measures the secant of theta.
	The length of the blue line, intercepted by the tangent line along the secant line, measures the secant of ***co***-theta. Hence, ***co***secant of theta.

Figures 8a and 8b. Secant and ***co***secant.

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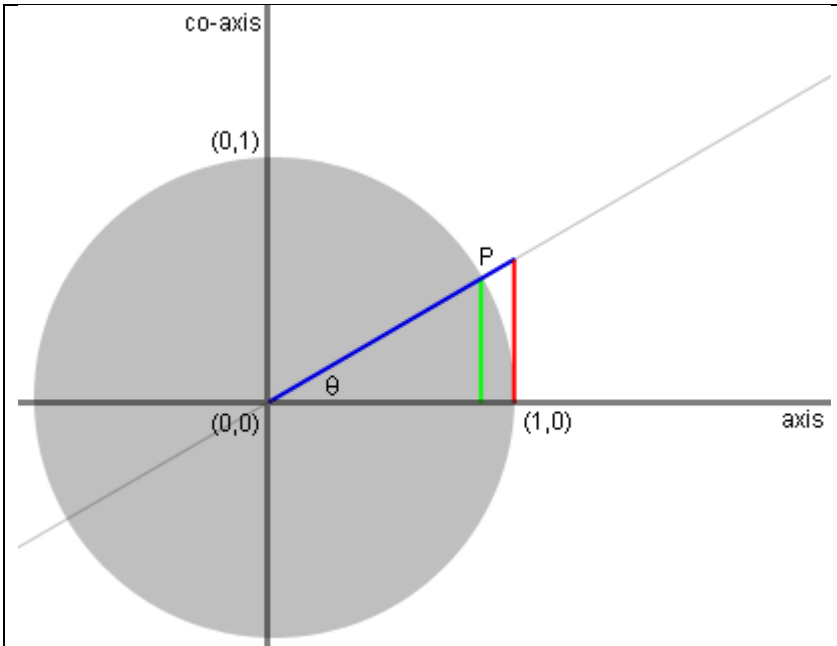
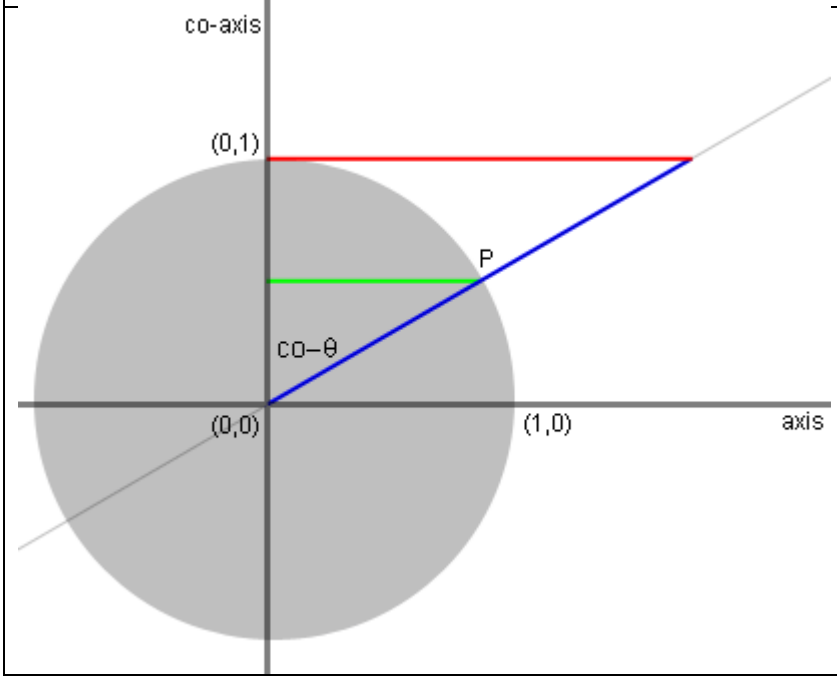
	The three functions of theta measuring sine, tangent, and secant shown together.
	The three functions of co-theta measuring sine of co-theta (cosine of theta), tangent of co-theta (cotangent of theta), and secant of co-theta (cosecant of theta).

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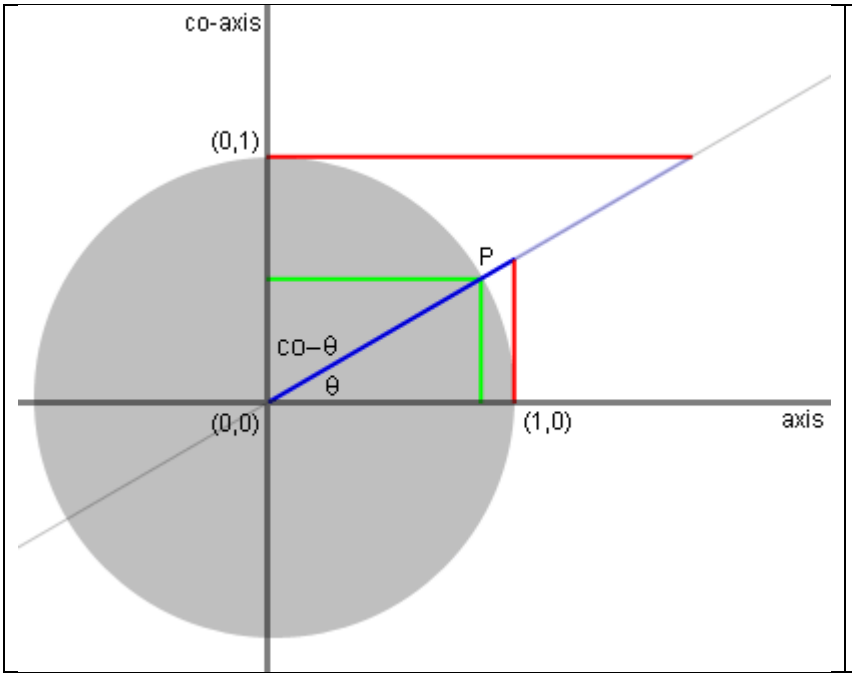
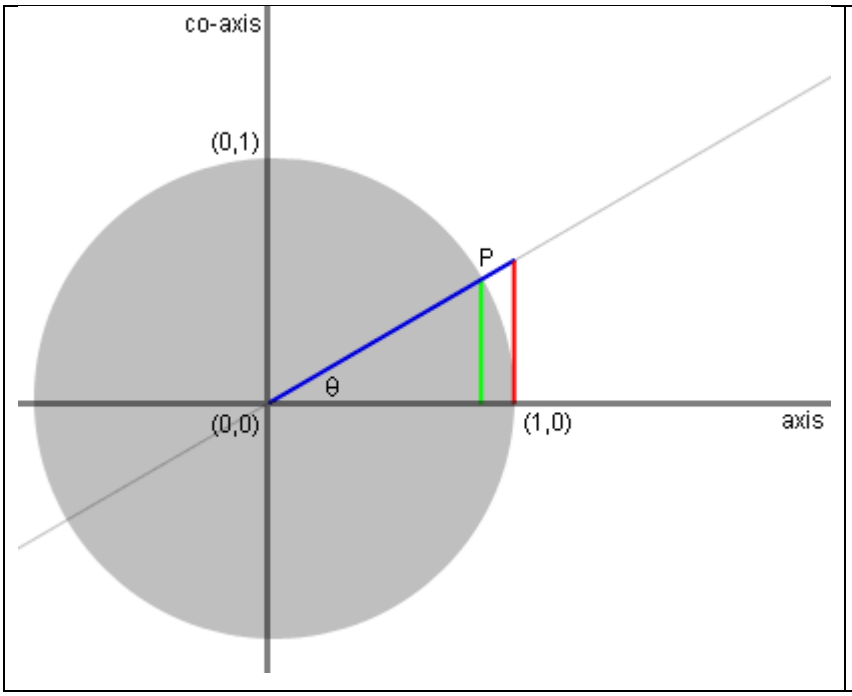


All six functions of theta are shown in this image. The representations for secant lies on top of that for cosecant of theta. The latter is thus shaded a lighter shade.

Figures 9a, 9b, and 9c. Trigonometric functions of theta, co-theta, and all six, visualized together.



An animation of the ideas shown as in Figure 9c. If the animation does not play in your browser, the separate animated file is also housed in Deep Blue on the same page as this document. Or view it in the Esri Cascading Story Map (Classic) style, by linking here.

Figure 10. All visualized trigonometric functions unified in a single animation.

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A number of trigonometric identities are evident from this visual approach.

From the Pythagorean Theorem, it follows that:

$\sin^2 \theta + \cos^2 \theta = 1$, the radius of the unit circle measured along the secant line;
$\sec^2 \theta = \tan^2 \theta + 1$, the radius of the unit circle measured along the horizontal axis;
$\csc^2 \theta = \cot^2 \theta + 1$, the radius of the unit circle measured along the co-axis.

What others do you note?

What are the possible sources or error that can result from these concepts? Can the user control them?

- There are assumptions about the underlying geometry that produce a rectilinear world. But is the real world rectilinear? When applying these concepts, the user needs to keep that in mind and control for any lack of fit in modeling by being clear about where concepts do not fit reality, rather than trying to 'bend' the analysis to fit a preordained agenda.
- Are there, instead, other geometries that might be considered, and what sorts of trigonometric functions do they lead to and do these yield useful models of real-world activity?
- If one considers geometry/trigonometry in higher dimensions, what sorts of error might be introduced in associated real-world interpretations and associated activities?
- In particular, if we consider the three-dimensional measurement of the Earth, in various contexts, what are the difficulties? Is the Earth a sphere? What problems arise if it is not and can the user compensate for them.

The latter comment, involving the curvature of the Earth, is one that is richer in activity content than the more abstract comments that lead up to it. It thus opens the door to more activity questions derived from this conceptual consideration.

Derivative activity ideas.

- What is the actual shape of the Earth, as a geometric solid?

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- When considering the Earth as a whole, should we also include mountain ranges, gorges, and other topographic features as well? What sorts of scale problems are there with including more detail?
- How do we measure topography? How are topographic maps made?
- Indeed, how do we measure the whole Earth?

In this sequence of concepts, we were led from a specific conceptual visualization to think a bit more broadly, both in terms of error functions and derivative concepts; each sequence led us back to the idea of some measurement issues of the Earth as a whole. The concepts predominated; the activities were in the background.

One End Result: Two Different Approaches

By allowing either a sequence of activities, or a sequence of concepts, to predominate, we were led to the same eventual end: a quest for measurement at the global scale. Considering the interaction of these two alternative views, from the conceptual and from the activities vantage points, captured a great problem: Eratosthenes's measurement of the circumference of the Earth

Eratosthenes of Alexandria (appointed Director of the Great Library at Alexandria in 236 B.C.) was an innovator in measurement. Not only did he create a prime number sieve, but also he figured out how to measure the circumference of the Earth. To do so, he used Euclidean Geometry and simple measuring tools. The following link shows some of the detail of the underlying assumptions and manipulations. http://www-personal.umich.edu/~copyrght/image/books/Spatial%20Synthesis/Eratosthenes/index.html

Here, we mention just a few of the underlying assumptions.

- The Earth is a sphere.
- The circumference of the sphere is measured along a great circle on the sphere.
- Finding the circumference of the Earth by finding the length of intercepted arc of a small central angle.

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- Finding two places on the surface of the Earth that lie on the same meridian (or close to it): meridians are halves of great circles.
- Eratosthenes choosing Alexandria and Syene, near contemporary Aswan. Figure 11 shows that they are close to lying on the same meridian, but do not.
- Assuming that the rays of the Sun are parallel to each other.
- Assuming the Sun's rays are directly overhead, on the Summer Solstice (c. June 21), at 23.5 degrees N. Latitude.
- Assuming Syene is located at about 23.5 degrees N. Latitude. Hence, on the Summer Solstice, the reflection of the sun will appear in a narrow well (and it will not on other days). Alexandria is north of Syene. Thus, on June 21, objects at Alexandria will cast shadows whereas those at Syene will not.
- Focusing on an obelisk or post located in an open area. He measured the shadow that the obelisk cast, and then measured the height of the obelisk (perhaps using a string anchored to the tip of the obelisk). **Think back to our first activity of measuring the height of the flagpole!**

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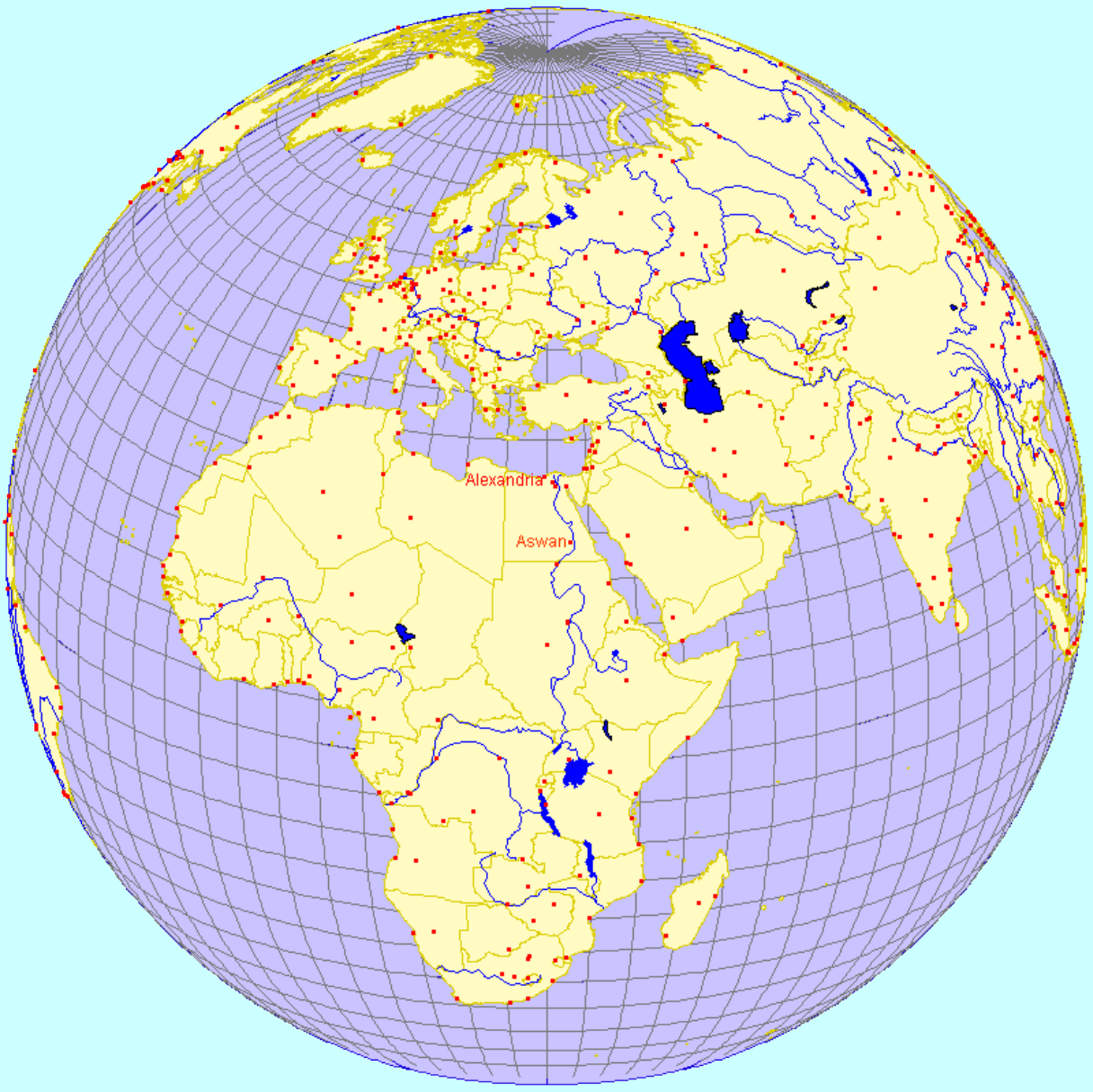

Figure 11. Relative location of Alexandria and Aswan. They are close to lying on the same meridian (half of a great circle)—but they are not quite on the same meridian.

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Many of the assumptions made by Eratosthenes were not accurate; apparently, however, underfit and overfit of error balanced out to produce a good result. For example, Syene and Alexandria are not on the same meridian; Syene is not at exactly 23.5 degrees N. Latitude, and so forth.

Predomination can yield different focal points on related topics but in the end, it is the great ideas that transcend either approach and, in fact, unify them. History is filled with examples of the general idea of such interaction, although methods of implementation vary greatly depending on available technology of the times. It is our goal to attempt to employ this sort of interaction in our forthcoming work entitled *Teaching Mathematics Using Interactive Mapping* (Arlinghaus, Kerski, and Arlinghaus, 2022/23).

Earlier work of R. H. Atkin (1974), which found application in data science and other aspects of the digital world, developed a theory of connectivity of spaces based on a pair of related sets in which one serves as a static backcloth for the other. Albert Einstein (1905) created a 'special relativity theory' with hyperbolic geometry as its conceptual backdrop. Indeed, hyperbolic is one arm of non-Euclidean geometry, a geometry in which Euclid's Parallel Postulate (parallel lines never meet) does not hold (Coxeter, 1955).

More generally, projective geometry has served as an interesting world from which to draw inspiration not only in 20th century physics and the art of M.C. Escher (2013 description, EscherMath, use of hyperbolic geometry in Circle Limit Series), but also in the art of Leonardo Da Vinci (employing a perspective vanishing point, behind Christ's eye, for the ceiling beams in his Last Supper) (2002, described by Green and McKie). The elegant underlying feature of projective geometry is the Principle of Duality. The projective world exhibits perfect symmetry. There is a dictionary of dual terms. Thus, 'Two lines meet in a point" has as its dual "Two points join in a line". Some dual terms are: line/point; meet/join. That duality is not present in Euclidean geometry for parallel lines do not meet in a point.

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Broadly viewed, one might see the activity/concept predomination pair described here as dual terms. When concepts act as a backdrop against which to view activities—activities predominate. When activities act as a backdrop against which to view concepts—concepts predominate. The challenge remains to merge interactive approaches in order to ask, and solve, great problems of global significance.

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*Some of the materials in this article were first published in *Spatial Synthesis, An eBook*, Volume I. By S. L. Arlinghaus and W. C. Arlinghaus. Used here with permission. Archived in Institute of Mathematical Geography, http://www.imagenet.org.

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Culinary Essays

Sandra Lach Arlinghaus, Toria Emas, Barbara Glunz

With input from Alma Lach (deceased); William E. Arlinghaus, and Donna K. Tope

Introduction

Benchmarks help us to tie the real world to the mapped world. They offer recognition points from which we can move from one aspect of one world, to another, employing a different tool—navigation through the real world using a paper map. Similarly, timelines with key events highlighted such as marriage, birth of children, moving to a new home, and so forth help us fix our locations in time-space. Combined, these offer an extra way to nail down our progress in the world, perhaps as a 'GEOMAT' (2019, Arlinghaus, Kerski, Larimore, and Naud) linking maps, archives, and timelines. In broad contextual characterizations of one's path through life or the world, what happens may be captured abstractly. Here, we offer some captures of one view of the culinary, and associated cultural, world through photo essays, tributes, and more.

Links to Culinary Essays: Emphasis on Connection to Geography

- Photo Essay. Sandra Lach Arlinghaus: Wine Label Booklet; also stored in Deep Blue on same page as this issue of *Solstice*.
- Photo Essay. Sandra Lach Arlinghaus: Alma Lach's Kitchen and Pantry; also stored in Deep Blue on same page as this issue of *Solstice*.
- Tribute: Toria Emas. Banquet Speech, Les Dames d'Escoffier, Chicago Chapter, 2007. Also stored in Deep Blue on same page as this issue of *Solstice*.
- Tribute: Barbara Glunz: Banquet Speech on receipt of Les Dames d'Escoffier, Chicago Chapter, Dame of Distinction Award, 2021. Also stored in Deep Blue on same page as this issue of *Solstice*.

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Culinary Photo Essay: Emphasis on Connection to Geometry

(S. L. Arlinghaus with input from D. K. Tope).

Antique French Potato Steamer, from the Alma Lach Copper Collection, is shown in this photo essay with accompanying commentary.

The copper potato steamer has a lid, two opposing brass handles, and a base. The curved base is welded to the slanted upper part. Alma Lach purchased it in the 1950s in France. She used it occasionally at dinner parties she gave in Chicago. Take note of the circular cross section where welding has taken place.

Figure 1. Copper potato steamer.

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Dimensions: about 8.5 inches from the table to the circular edge of the truncated cone; 6.25 inches diameter across the top.

Manufacturer, perhaps Mauviel.

Figure 2. Remove the lid. What remains is a single solid piece.

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Notice the circular, perforated plate inside the steamer. The plate is loose and can be removed. It is used to separate the potatoes from the water below it.

Figure 3. Look inside.

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Fill the volume below the circular plate with water. Do not allow the water to come up over where the plate was: the circular cross section where welding is.

Figure 4. Remove the circular plate.

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Now, insert some potatoes and other vegetables (carrots…) in the steamer above the circular plate. Some vegetables might be 'carved' to enhance space-packing while leaving airspace. Minimize the contact of the vegetables with each other. Do not allow the potatoes to lean on the slanted sides of the container. The idea is to keep plenty of steam surrounding all vegetables.

Figure 5. Replace the circular plate. Make certain it does not touch the water under it.

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Turn on the heat; boil the water, and let the potatoes and other vegetables steam until they are fully cooked. Cook in it; then serve from it at the table (with a hot pad under it).

Figure 6. Replace the lid. Using the handles on opposing sides of the vessel, pick up the filled container and place it on the stove.

- The water in the base boils quickly (there is not much of it).
- Steam rises around the vegetables and condenses on the inside of the copper lid.
- The shape of the lid guides the steam to the lid's edge and down the slanted sides, back into the base.

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Geometry makes it work!

The steamer is a closed system that cooks moist, but not soggy, potatoes.

Water never touches the potato.

Clever. Function follows form!

Acknowledgments for the photo essay.

Ann Arbor Chapter Les Dames d'Escoffier, Alma Lach (deceased), Sandra Lach Arlinghaus, William E. Arlinghaus (Affiliate Member) Antoinette Benjamin, Linda Powell, and Donna K. Tope.

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  Exhibition on view from Sept. 19, 2016-Jan. 6, 2017 – The Hanna Holborn Gray Special Collections Research Center. The University of Chicago Library. https://www.lib.uchicago.edu/collex/exhibits/almalach/

  Special document based on the exhibition: https://deepblue.lib.umich.edu/bitstream/handle/2027.42/60304/LachExhibitionUCJuly31.pdf?sequence=44&isAllowed=y

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Some Chicago Chapter members of Les Dames d'Escoffier visit the Exhibition, photo below:

- Lach, Alma. Alma Lach Papers. https://www.lib.uchicago.edu/collex/exhibits/almalach/le-cordon-bleu/
- Lach, Alma. Alma Lach Papers https://www.lib.uchicago.edu/collex/exhibits/almalach/honors-and-accolades/
- Lach, Alma. Links to Pixellist Art displays shown at the Tavern Club of Chicago (zipped files). Archived at http://www.imagenet.org/ (search the collection using search term 'Alma Lach'):
    - 1999: https://deepblue.lib.umich.edu/bitstream/handle/2027.42/60304/1999TavernClubDisplay.pdf?sequence=8&isAllowed=y
    - 2000: https://deepblue.lib.umich.edu/bitstream/handle/2027.42/60304/2000TavernClubDisplay.pdf?sequence=9&isAllowed=y
    - 2002: https://deepblue.lib.umich.edu/bitstream/handle/2027.42/60304/TavernClubSummer2002.zip?sequence=32&isAllowed=y

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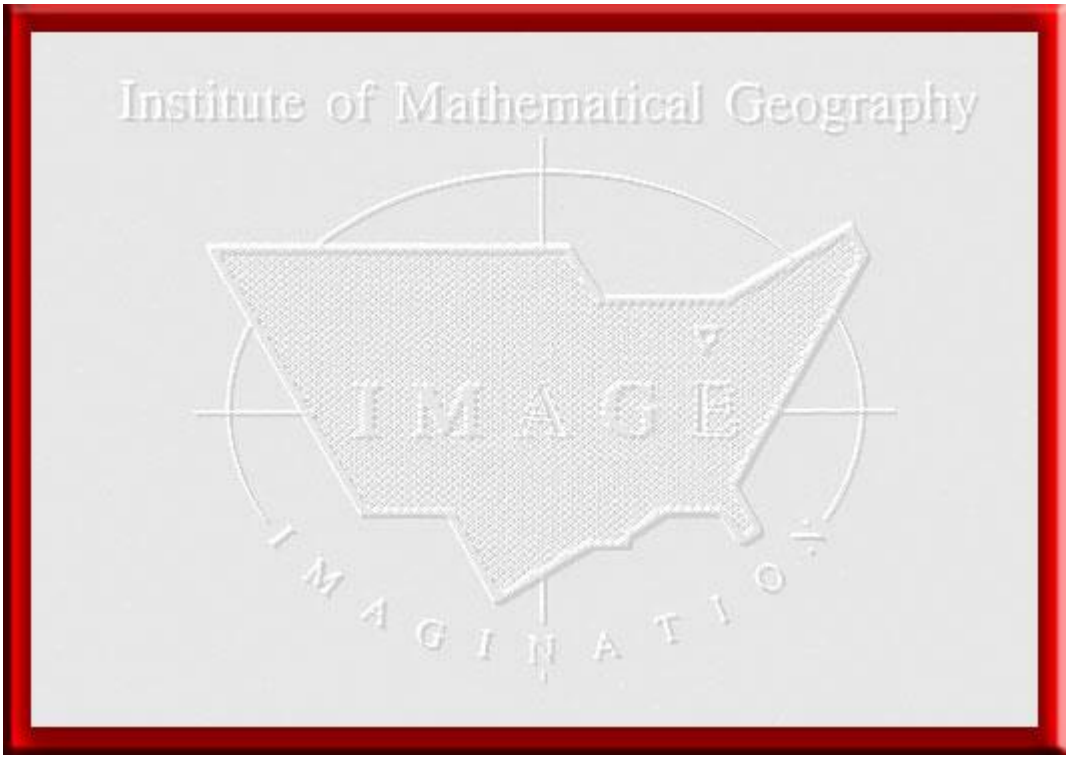
Endmatter



Original logo designed by Sandra L. Arlinghaus and stylized and redrafted by Allen K. Philbrick.

AWARDS AND SELECTED COMMENTS

- ***Solstice* page translated into Belorussian, April, 2016; many thanks to Valerie Bastiaan.**
- ***Solstice* cover materials translated into Ukranian, August 25, 2011; many thanks to Galina Miklosic.**
- ***Solstice* was a Pirelli INTERNETional Award Semi-Finalist, 2001 (top 80 out of over 1000 entries worldwide)**
- **One article in Solstice was a Pirelli INTERNETional Award Semi-Finalist, 2003 (Spatial Synthesis Sampler).**
- ***American Mathematical Monthly*, September 1992, in Telegraphic Reviews section notes *Solstice* as "one of the world's first electronic journals using TeX." L. A. Steen.**
- ***Science News*, 25 January, 1992. Article about *Solstice*.**
- ***Science*, AAAS, 29 November, 1991. Article about *Solstice*.**

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LISTINGS IN DIRECTORIES AND DATABASES

- **IMaGe is listed as a "Collection" in the persistent online archive, DeepBlue, of The University of Michigan library. It is listed under "Mathematical Geography" on the Collections link.**
- ***Solstice* has been listed in the Directory of Open Access Journals (for its first 28 years) maintained by the University of Lund.**
- ***Solstice* is listed on the journals section of the website of the American Mathematical Society, http://www.ams.org/**
- ***Solstice* has been listed in the EBSCO database.**
- **IMaGe has been listed on the website of the Numerical Cartography Lab of The Ohio State University, with thanks to Harold Moellering.**
- ***Solstice* was listed in Geoscience e-Journals, with thanks to Bruno Granier.**

Sandra L. Arlinghaus, celebrated over 30 full years of archived *Solstice* publication in 2020.