Essays in International Macroeconomics

by

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To Verónica Olivares Larenas and Marco Rojas San Martín

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TABLE OF CONTENTS

LIST OF FIGURES

Figure

LIST OF TABLES

Table

LIST OF APPENDICES

Appendix

ABSTRACT

This dissertation contains three essays in international macroeconomics and macroeconomics, more broadly defined. Chapter I answers the question about how does the zero lower bound (ZLB) on the international interest rate affect monetary policy in small open economies (SOE)? When the Fed's rate was at the ZLB (2008-2015), data for sixteen SOE shows a significantly lower correlation between interest rates and inflation, which is at odds with the empirical regularity. This is explained in a model where the distribution of shocks that affect SOE changes when the international interest rate hits the ZLB. Two opposing channels affect the exchange rate. At the international ZLB, the depreciating channel is amplified, while the appreciating channel is attenuated. Then, the SOE currency depreciates more than in a scenario without international ZLB. This passes through to inflation, which affects SOE's ability to stabilize the economy as it cannot lower its interest rate as much. In an estimated model, this mechanism can explain 26 percent of the lower correlation observed in the data.

Chapter II studies the relevance of taking into account the switching in the currency of invoicing in international trade. First, using a highly detailed dataset of customs in Chile , I present novel descriptive statistics on the invoicing dynamics of exporters. For instance, 9% of total export value in a given month comes from firms that switch their currency of invoicing. Second, I derive a model to understand how firm-specific time-varying variables can explain the switching among currencies. In particular, I find that changes in the currency composition of imports in the firm can explain switching *to* the U.S. dollar as a vehicle currency, but not the switching away *from* the US dollar. Third, exploiting switches in invoicing for narrowly define trade links, one can better measure the relation between exchange rate pass-through and the currency of invoicing, as the firm-effect can be disentangled from the currency-effect. Under this framework, I identify switches where the pass-through is no longer associated to the currency used.

Finally, in Chapter III, which is work co-authored with Javier Cravino and Andrei Levchenko, we quantify the role of population aging in the structural transformation process. Household-level data from the U.S. show that the fraction of expenditures devoted to services increases with household age. We use a shift-share decomposition and a quantitative model

to show that U.S. population aging accounted for about a fifth of the observed increase in the service share in consumption between 1982 and 2016. The contribution of population aging to the rise in the service share is about the same size as the contribution of real income growth, and about half as large as that of changes in relative prices.

CHAPTER I

Monetary Policy in Small Open Economies and the International Zero Lower Bound

1.1 Introduction

During the Great Recession – and ongoing Covid Crisis – the Fed's rate hit the zero lower bound (ZLB), which had several and known implications for the US economy (Eggertsson and Woodford, 2003; Gust et al., 2017). In addition, because this interest rate can be interpreted as the international interest rate, the ZLB may also affect other countries. Recent papers have studied how the ZLB in one economy can influence another economy in reaching it too (Caballero et al., 2020). Others have looked at how unconventional policies in the US during this period affect emerging and small open economies (SOE) (Alpanda and Kabaca, 2020). However, little is known about how the ZLB restriction itself on the international interest rate may affect SOE.

This paper contributes to this question by studying monetary policy in SOE when the international interest rate is at the ZLB. The first part documents a novel fact in a key relationship for monetary policy. The correlation between domestic interest rates and inflation falls significantly for several SOE during the Fed's rate ZLB episode that took place amid, and in the aftermath, of the Great Recession. Second, it presents a simple model to rationalize this finding. At the international ZLB, there is additional depreciation in SOE that generates an increase in inflation at a moment when domestic interest rates are falling, which may break the usual positive correlation between these two variables. Next, this mechanism is included in a larger quantitative model that can explain part of the lower correlation observed in the data. Finally, this framework is used to understand the impact that the ZLB restriction on the international interest rate can have SOE.

Using sixteen small open economies with inflation-targeting central banks, I find that the correlation between core CPI inflation, which excludes energy and food prices, and policy rates is significantly lower during the international ZLB period when compared to normal times. On average, the coefficient goes from 0.75 to 0.31.

This finding is understood through the lens of a standard SOE model (Galí and Monacelli, 2005), where the large economy or Rest of the World (ROW), is affected by discount rate or preference shocks. I study the whole distribution of shocks that end up impacting the SOE, rather than an individual shock by its own (e.g., foreign demand), and how this distribution may vary with the monetary policy characteristics in ROW. For instance, whether there is a zero bound restriction or not.

In the model, when ROW – interpreted as the US – enters a recession, two channels affect a SOE. First, a large negative discount rate shock impacting US households may cause a recession. This triggers a policy response that lowers the international interest rate. Ceteris paribus, this depreciates the US dollar and appreciates the SOE currency. Second, US output drops, which lowers the foreign demand that the SOE faces. Ceteris paribus, this depreciates the SOE currency. Therefore, total depreciation depends on which channel dominates. In this context, the model evaluates what is the differential effect on SOE from two scenarios. One where the international interest rate can move freely (No ZLB*) and one with a zero bound restriction (ZLB*). In the ZLB* scenario, the first effect is smaller and the second is larger compared to a No ZLB* scenario. This generates added depreciation that may pass through overall inflation. At the same time, the SOE is trying to lower its interest rate due to the external crisis. However, because of this higher than otherwise inflation, the SOE cannot lower its rate as much or keep it low for long. This produces a weaker relationship between interest rate and inflation, and affects the ability the SOE has to combat the recession.

To better illustrate the channels and how they change in both scenarios, the model focuses on a simple case with complete markets and unitary intertemporal and intratemporal elasticities, which allow for clean analytical expressions. The first channel stays constant in either scenario, while the second channel becomes relatively more relevant in the ZLB* scenario. To understand this consider the following. Note that the risk-sharing condition in this context states that the value of marginal utilities in both economies must be equal to each other when measured in the same currency. And, that lowering the international interest rate affects the extent to which the second channel is absent or not. If there is no restriction, the shock is fully accommodated and there is no drop in the foreign demand faced by the small economy.

In the No ZLB* scenario, there is no change in US output, therefore the discount rate shock in the US lowers the value of marginal utility of US households. Due to complete markets, the value of marginal utility of SOE households must be lowered too. This can be happen in two ways. Either by increasing contemporaneous consumption or by appreciating the currency of SOE. In equilibrium, both happen. The first is achieved by lowering the interest rate in SOE, and the second implies that imported inflation falls. Together they pin down a positive correlation between interest rates and CPI inflation.

In the ZLB* scenario, the international interest rate cannot fully accommodate the shock and US output now drops which lowers the foreign demand faced by the SOE. The first channel is still present and in the same magnitude, so the only difference is larger depreciation, which makes CPI inflation to increase with respect to the previous scenario. Depending on the magnitude of the latter, this second channel may lower, cancel or outweigh the appreciation coming from the first channel. Because the interest rate falls by the same amount as in the No ZLB* scenario, the relationship with inflation is weaker, null or positive, respectively, in the ZLB* scenario. Thus, the simple model provides a rationale for why we observe a drop in the correlation between interest rate and inflation during the international ZLB.

Then, I embed this mechanism in a quantitative SOE model that builds on Justiniano and Preston (2010a,b), and adds local currency pricing for domestic firms when exporting (Gopinath et al., 2010a) and forward guidance in the international interest rate, i^*. The purpose of this is to have a model that can match better the data. The model has incomplete markets, habit formation, and sticky wages and prices. In the SOE the law of one price does not necessarily hold for both imports and exports. There are retail firms that import at the competitive price, but have monopolistic power when setting their prices internally. Domestic producing firms set their prices in ROW currency when exporting. In addition to the discount rate shocks in ROW that explain the ZLB on the international interest rate, the model considers discount rate shocks in SOE too. Also, productivity, cost-push, monetary policy and risk-premium shocks are included. Finally, monetary policy in ROW contemplates forward guidance as it can characterize better what happened to the Fed's rate amid and in the aftermath of the Great Recession. For this, the model follows Del Negro et al. (2013) which proposes a Taylor rule that reacts to not only contemporaneous, but also past inflation.

To solve the model I focus on Australia as the SOE and US as ROW. Most of the parameters come from Justiniano and Preston (2010a) and related literature, and some others are calibrated such that they match their average data counterparts (e.g. discount factor and average interest rate). To estimate the remaining parameters, I use the simulated method of moments. One key moment is the share of quarters i^* is at the ZLB, which relies heavily on the parametrization of the discount rate shocks to ROW households. Because of this non-linearity, the model cannot be solved using traditional perturbation methods, so instead it follows the approach in Guerrieri and Iacoviello (2015) that provides piecewise linear solutions.

To evaluate how the model performs, I quantify the ability the model has to explaint the lower correlation between interest rate and core inflation observed in the data. I simulate the fully estimated model and compute the equivalent correlations to those of the data. The model can explain 26 percent of the drop in the correlation that happens when comparing periods where the international rate is not at the ZLB and periods where it is.

Finally, the quantitative model can be used to understand the implications on a SOE that a restricted international interest rate can have. I do this by studying impulse response functions from large discount rate shocks to US households under two scenarios. The baseline or ZLB* scenario, and an alternative one where the international interest rate can be adjusted freely, or No ZLB* scenario. For instance, under large shocks, it could become negative. When comparing these scenarios, the main result is verified: there is larger depreciation in SOE when the international interest rate is at the ZLB. This gets passed to imported and overall inflation, which together with a higher international interest rate, results in a higher domestic interest rate compared to a No ZLB* scenario. The ability to lower the interest rate further allows for output in the SOE to fall by less when facing the external recession.

This exercise illustrates how the monetary policy structure in the US can affect small economies by producing abnormal exchange rate movements due to the mismatch between the structural shock, the policy response and the effects on activity.

This paper contributes to several strands of the literature. First, those that study the international spillover effects in SOE and emerging economies from monetary policy in the US or other large economies (e.g. Eurozone). Some papers have examined the impacts of conventional monetary policy.[1] For example, they have used identified shocks to Fed's rate movements to study effects on the exchange rates. Other have estimated the effects of unconventional monetary policy.[2] For instance, they have implemented event study techniques to study the effects of large-scale asset purchases done by the Fed on international bond yields. I contribute to this by studying the effect of a particular feature of Fed's monetary policy, which is that its main instrument cannot fall below zero, on monetary policy itself in SOE. My paper provides descriptive evidence on monetary policy in SOE during this period, and lends a theoretical rationale of why we observe a break in the relationship between two key variables that characterize monetary policy.

Second, the paper builds on the literature at the intersection of international economics and the ZLB on interest rates, either understood as a consequence of secular stagnation or as a transitory shock, which is produced, for example, because of a discount rate shock as in

[1]See, for example, Kalemli-Özcan (2019); Iacoviello and Navarro (2019); Albagli et al. (2019); Buch et al. (2019); Vicondoa (2019); Lakdawala et al. (2020); Miranda-Agrippino and Rey (2020).

[2]See, for example, Neely (2015); Curcuru et al. (2018); Gajewski et al. (2019).

this paper.[3] When the ZLB is the result of long-term trends, Eggertsson et al. (2016) and Caballero et al. (2020) propose different models to study two symmetric economies, and what occurs when one enters secular stagnation. They predict that, under certain conditions, the ZLB in one country generates the other economy to reach it too. With a world economy structure like in this paper, Corsetti et al. (2019) challenge that prediction by studying a SOE affected by secular stagnation in ROW, and show that the SOE can isolate itself from it.

When the ZLB takes place as a transitory shock, Cook and Devereux (2013) study how the zero restriction generates odd exchange rate variations. This is in a model with symmetric economies and where the country of interest is the one initially affected by the ZLB. In this context, this paper fills in the gap in the literature by studying a SOE when the ZLB is foreign (as in Corsetti et al., 2019), but in the presence of a transitory shock (as in Cook and Devereux, 2013).

Finally, it relates to research on the impact of the ZLB on the economy, and the associated literature on the effects of negative interest rates. Gust et al. (2017) study how the ZLB affected the US during the Great Recession and restricted its ability to overcome the recession. They do this by using an alternative model where the Fed rate can be negative. Ulate (2021) and Lopez et al. (2020) examine the effects of negative interest rate and their impact to commercial banks. Sims and Wu (2021) study negative policy rates as a tool of unconventional monetary policy. I further this understanding by studying how the ZLB in one country spills over to other economies. For this, the paper compares the baseline scenario against an economy where the international interest rate can be negative.

The rest of the paper is organized as follows. Section 1.2 provides the data and descriptive evidence on what happens to monetary policy in several SOE during the international ZLB. Section 1.3 presents a simple model that delivers the main mechanism, which is then included in a quantitative model described in Section 1.4. Section 1.5 presents the parametrization of the model, together with the estimation of certain parameters and the solution method. Section 1.6 evaluates the performance of the model and carries out impulse response exercises. Finally, Section 2.6 concludes.

1.2 Inflation and interest rates in small open economies

This section studies whether the relationship between interest rates and inflation, in small open economies, changes during the period when the Fed's rate was at its zero lower bound.

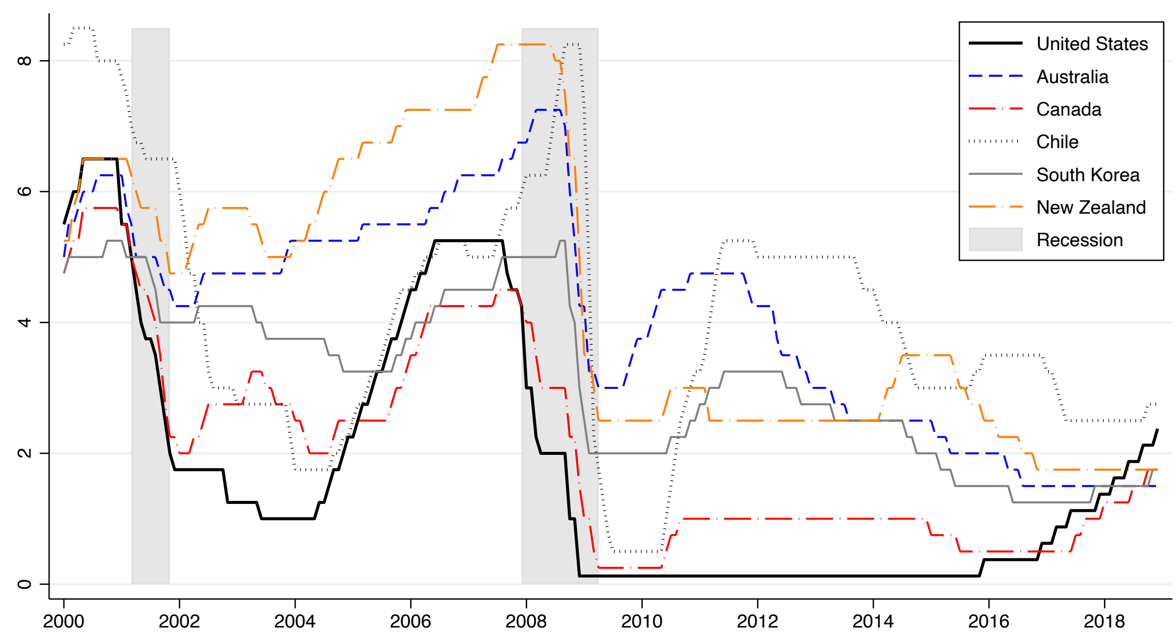
[3] In addition, important contributions have been made in this intersection. For instance when ZLB occurs within currency unions. See, for example, Gomes et al. (2015); Farhi and Werning (2016); Hettig and Müller (2018); Cook and Devereux (2019).

1.2.1 Data

The main sources of data are the BIS statistics for country's policy rates, and OECD statistics for core CPI inflation indexes. The baseline uses CPI of all non-food non-energy items. I include all small open economies in the BIS dataset with at least 20 years of data between 1990 and 2019. SOE are defined as countries integrated with world markets, and whose policies do not affect world prices.[4] In particular, they take the international interest rate as given and cannot affect it. This leaves 16 countries at quarterly frequency, which are listed in Appendix Table A.3. All these countries have inflation-targeting central banks (Hammond, 2012).[5] More details are provided in Appendix A.1.

To provide context, Figure 1.1 shows the Fed's rate together with the policy rate of five SOE. The Fed's rate drops during the Dot-Com bubble and Great Recession (gray areas), which is accompanied by drops in the other policy rates too. This paper investigates the potential different mechanism taking place during – and after – the last recession when the Fed's rate hit – and stayed – at the ZLB, and how that may have affected SOE. If the Fed's rate is considered to be at the ZLB when it is below 0.25%, then the international ZLB period takes place between 2008Q4 and 2015Q4.

Figure 1.1: Interest rate in US and selected economies, 2000-2019



[4]See the definition in Deardoff's glossary of international economics.

[5]Switzerland is not included in that study, but it has an inflation-targeting central bank (See https://www.snb.ch/en/iabout/snb/id/snb_tasks).

1.2.2 Inflation and interest rates during the international ZLB

One of the many empirical regularities in macroeconomies is the positive correlation between interest rates and different measures of inflation. When these are equilibrium rates in the financial markets, for instance in the Treasury bond market, this relationship is sometimes referred to as the Fisher relationship (Fisher, 1930). Despite many ways that monetary policy can be understood (e.g., through Taylor rules), studying their correlation is a simple model-free approach that can inform whether the international ZLB may affect the relationship between key variables for monetary policy.

Figure 1.2 plots the correlation between the average policy rate and year-ended core CPI inflation in a given quarter. This is done for two distinct periods. The correlation during periods when the international interest rate is not bound by zero is displayed in the y-axis. And the correlation during the international ZLB in the x-axis. For all countries, we observe a drop during the international ZLB as they all are above the 45° line. For some economies, this drop is small (see Sweden), but for most of them is a sizable drop (see Australia, Canada or Israel). The solid diamond shows that, on average, the correlation coefficient goes from 0.75 during normal times to 0.31during the international ZLB.

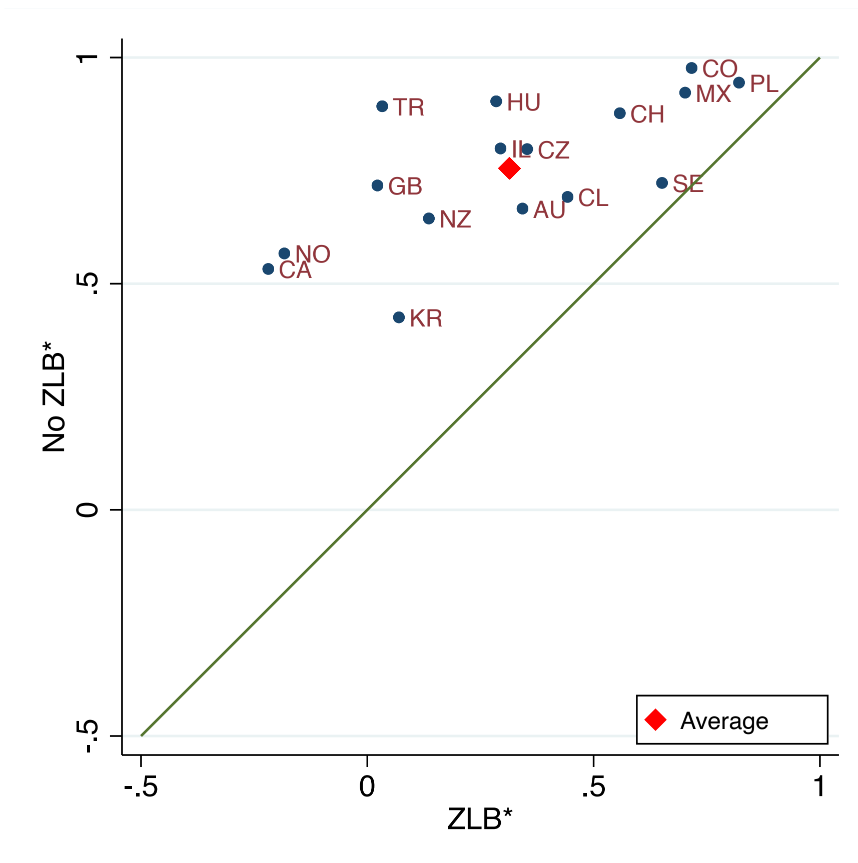
Table 1.1 accompanies the correlation coefficients with their standard deviation. This verifies that correlation coefficients during the international ZLB are statistically different to those during normal times (significance level of 5%), with the exception of Sweden. This finding is robust to using quarter-to-quarter inflation, using different measures for core CPI inflation and using headline instead of core CPI inflation. Appendix A.1 performs these robustness checks.

Discussion. The correlation coefficient can hardly tell something about causality between inflation and policy rates. A positive relationship can be viewed as a reaction of the policy rate to current inflation or expected inflation. Given the inflation targeting scheme, higher (lower) inflation requires a rise (drop) in the interest rate to keep inflation under control. And because it acts with a lag, we can still observe a positive relationship within a quarter. Alternatively, this positive correlation is also consistent with the Neo-Fisherian view that reverses this causality. Because agents in the economy care about real interest rates, the theory goes, a higher (lower) nominal interest rate will only have an effect through a higher (lower) inflation. Nominal interest rate equals inflation plus real interest rate, which in the long-run is unaffected by nominal variables.

Thus, the drop in this correlation can be viewed as one of these hypothesis becoming less strong during the international ZLB period. For instance, policy rates may also respond to output gap, to output growth, and – especially in open economies – to the exchange rate.

If the international ZLB changes the distribution of shocks affecting SOE, such that the relative proportion of variables the policy rate responds to is different compared to normal times, then we can expect the relationship of interest rates to each of those variables to also change. Alternatively, nominal interest rates have an impact not only through demand in the short-run, but also through the supply side of the economy (Baqaee et al., 2021), which can end up affecting real variables in the long-run, and thus the one-to-one relationship between nominal variables. If the international ZLB exacerbates nominal rigidities, then we can expect the relationship between interest rates and other nominal variables to change too. The following section shows why small economies observe a weaker relationship between these two variables during the international ZLB period.

Figure 1.2: Correlation of interest rate and year-ended core CPI inflation



Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Table 1.1: Correlation of interest rate and year-ended core inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.67	0.34	IL - Israel	0.80	0.29
	(0.06)	(0.16)		(0.05)	(0.16)
CA - Canada	0.53	-0.22	KR - South Korea	0.43	0.07
	(0.07)	(0.21)		(0.11)	(0.19)
CH - Switzerland	0.88	0.56	MX - Mexico	0.92	0.70
	(0.04)	(0.13)		(0.04)	(0.11)
CL - Chile	0.69	0.44	NO - Norway	0.57	-0.18
	(0.08)	(0.14)		(0.07)	(0.21)
CO - Colombia	0.98	0.72	NZ - New Zealand	0.64	0.14
	(0.02)	(0.10)		(0.06)	(0.18)
CZ - Czechia	0.80	0.35	PL - Poland	0.94	0.82
	(0.06)	(0.15)		(0.03)	(0.08)
GB - Great Britain	0.72	0.02	SE - Sweden	0.72	0.65
	(0.06)	(0.19)		(0.06)	(0.11)
HU - Hungary	0.90	0.28	TR - Turkey	0.89	0.03
	(0.03)	(0.16)		(0.05)	(0.19)

Notes: This table reports sample correlations between core CPI inflation measures and interest rates for two periods at quarterly frequency. The ZLB* period is given by 2008Q4 to 2015Q4. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

1.3 Simple model

This section presents a simple model which is used to illustrate the main mechanism, and make sense of the data presented above. It is a simplified version of what is outlined in Section 1.4, and follows closely Galí and Monacelli (2005). However, instead of focusing on the effect of a given shock, the analysis looks at the entire distribution of shocks that, stemming from ROW, affect a SOE. And in particular, how that distribution may change with monetary policy features of ROW.

First, the world economy is presented with separate ROW and SOE blocks. Both contain households, firms and a central bank/government. Second, the log linearized equilibrium is derived. Finally, the international ZLB is analyzed by studying what happens after a one-time large negative discount rate shock hits ROW households.

1.3.1 World Economy

Time is indexed by t. The world economy is made of a large economy (or ROW) of size 1 and a SOE of size 0, indexed by R and S respectively. Given their relative sizes, ROW is in practice a closed economy. There is a unit mass of firms in each economy that can set their prices à la Calvo in their own currency, i.e., producer currency pricing. There are complete financial markets. International trade is frictionless and the law of one price (LOP) holds for individual goods. Because households have home bias, LOP fails to hold for consumption price indexes. ROW values are denoted with *.

ROW. There is a unit mass of households in ROW with the following utility function:

$$U^* = E_0 \sum_{t=0}^{\infty} \beta^t \exp(\nu_t^*) \left[\log C_t^* - \frac{N_t^{*1+\varphi}}{1+\varphi} \right]$$

where φ is the inverse of the Frisch elasticity and ν_t^* is a discount rate or preference shock that changes the relative weight given to marginal utility in period t with respect to $t+1$. This is the key shock that drives the mechanism and provokes a recession by making households extremely patient.[6] N_t^* is the labor supplied by the household. C_t^* is the consumption index that aggregates varieties produced by ROW firms: $C_t^* \equiv C_{R,t}^* \equiv \left(\int_0^1 C_{R,t}^*(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$, where $\epsilon > 1$ is the elasticity of substitution among differentiated goods, and $P_{R,t}^* = \left(\int_0^1 P_{R,t}^*(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ the price. Note that because of ROW's relative size, this is in practice a closed economy, which means $P_t^* = P_{R,t}^*$.

ROW households have access to a complete set of fully contingent claims. B_t^* is the nominal payoff in period $t+1$ of the portfolio of such claims held by ROW households at the end of period t. $\mathbb{Q}_{t,t+1}^*$ is the stochastic discount factor for a nominal payoffs in $t+1$ from the perspective in t. Then, budget constraint is:

$$P_t^* C_t^* + E_t \mathbb{Q}_{t,t+1}^* B_{t+1}^* = W_t^* N_t^* + B_t^* + T_t^* + \Gamma_t^*.$$

[6]This shock to characterize recessions that may lead interest rates to hit the zero lower bound have been used widely in the literature. See for example Gust et al. (2017); Christiano et al. (2015); Nakata (2016). Alternatively, a shock that drives agents away from risky assets into safe assets (à la Krishnamurthy and Vissing-Jorgensen (2012)) can also generate similar results in terms of bringing down output and prices simultaneously. In order to keep tractability and analytical expressions, this paper opts for a one-asset model in ROW. Further research is needed to understand what additional implications would this have on SOE.

where T_t^* are taxes/transfers and Γ_t^* are firms profits. Households maximize their utility subject to the budget constraints:

$$C_t^* N_t^{*\varphi} = \frac{W_t^*}{P_t^*} \qquad \text{and} \qquad \beta E_t \exp(\Delta\nu_{t+1}^*)(1+i_t^*)\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-1}\frac{P_t^*}{P_{t+1}^*} = 1, \tag{1.1}$$

where $1 + i_t^* = (E_t \mathbb{Q}_{t,t+1}^*)^{-1}$.

There is a continuum $[0,1]$ of firms, where firm j produces with production function, $Y_t^*(j) = N_t^*(j)$. Firms enjoy monopolistic power, so there is a wage subsidy such that they charge marginal costs in steady state. If $\tau^* = \frac{1}{\epsilon}$, then $P_t^* = (1-\tau^*)\frac{\epsilon}{\epsilon-1}W_t^* = W_t^*$. This is financed with lump-sum tax to households T_t^*.

In each period, a share θ of firms cannot adjust their price, so for them $P_t^*(j) = P_{t-1}^*(j)$. The remaining $(1-\theta)$ share set $\tilde{P}_t^*(j)$ to solve the following problem:

$$\max_{\tilde{P}_t^*(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \mathbb{Q}_{t,t+k}^* \left[\tilde{P}_t^*(j) Y_{t+k|t}^* - (1-\tau^*)\Psi^*\left(Y_{t+k|t}^*\right)\right]\right\}. \tag{1.2}$$

where $\mathbb{Q}_{t,t+k}^* \equiv \beta^k (C_{t+k}^*/C_t^*)^{-1}(P_t^*/P_{t+k}^*)$, Ψ^* is the cost function, $Y_{t+k|t}^* = (\tilde{P}_t^*(j)/P_{t+k}^*)^{-\epsilon} Y_{t+k}^*$, and τ^* is the labor subsidy. Because of the relative size of ROW, $Y_t^* = C_t^*$.

Finally, the central bank at ROW has a stabilization objective of strict inflation target, $\bar{\Pi}_t^* = 1$ (Eggertsson and Woodford, 2003). The only tool available to attain such target is the nominal interest rate i_t^*. In this context, the simple model assesses two different scenarios:

$$\begin{aligned} &\text{(a) No ZLB}^* : i_t^* \in \mathbb{R} \\ &\text{(b) ZLB}^* : i_t^* \geq 0. \end{aligned} \tag{1.3}$$

SOE. In describing the SOE block, I omit the details that mirror those of the ROW, and just point out relevant differences.

There is a unit mass of households in SOE with the following utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right].$$

C_t is a consumption basket made of SOE and ROW goods: $C_t \equiv C_{S,t}^{1-\alpha} C_{R,t}^{\alpha}$, where $1-\alpha$ is the home bias.[7] In turn, $C_{S,t}$ and $C_{R,t}$ are indexes for the differentiated goods coming from

[7]This aggregation is assuming that the elasticity of substitution between domestic and foreign goods is equal to one.

SOE itself and ROW, respectively,

$$C_{S,t} \equiv \left(\int_0^1 C_{S,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \quad , \quad C_{R,t} \equiv \left(\int_0^1 C_{R,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} .$$

The prices corresponding to C_t, $C_{R,t}$, and $C_{S,t}$ are $P_t \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} P_{S,t}^{1-\alpha} P_{R,t}^{\alpha}$, $P_{S,t} \equiv \left(\int_0^1 P_{S,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ and $P_{R,t} \equiv \left(\int_0^1 P_{R,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$, respectively. Budget constraints faced by SOE households are:

$$P_t C_t + E_t \mathbb{Q}_{t,t+1} B_{t+1} = W_t N_t + B_t + T_t + \Gamma_t.$$

SOE households maximize their utility subject to the budget constraints:

$$C_t N_t^{\varphi} = \frac{W_t}{P_t} \quad \text{and} \quad \beta E_t (1+i_t) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{P_t}{P_{t+1}} = 1,$$

where $1 + i_t = (E_t \mathbb{Q}_{t,t+1})^{-1}$.

There is a continuum $[0,1]$ of firms, where firm j produces with production function, $Y_t(j) = N_t(j)$. Firms enjoy monopolistic power, so there is a wage subsidy such that they charge optimal marginal costs in steady state: $\tau = \frac{1}{1-\alpha}$ (Galí and Monacelli, 2005). This is financed with a lump-sum tax to households T_t.

For a given differentiated product, the LOP holds, then

$$P_{S,t}(j) = \mathcal{E}_t P_{S,t}^*(j) \quad \text{and} \quad P_{R,t}(j) = \mathcal{E}_t P_{R,t}^*(j).$$

The nominal exchange rate is denoted by $\mathcal{E}_t$ and is defined as the price of one unit of ROW's currency in terms of SOE's currency (e.g. Chilean pesos per US dollar). Then, an increase in $\mathcal{E}_t$ is a depreciation of SOE's currency. Given the preferences and the parity holding for individual goods prices, $P_{S,t} = \mathcal{E}_t P_{S,t}^*$ and $P_{R,t} = \mathcal{E}_t P_{R,t}^*$. However, due to home bias $P_t \neq \mathcal{E}_t P_t^*$.

Using households' preferences, the total demand for SOE goods is given by,

$$Y_t = \left(\frac{P_{S,t}}{P_t} \right)^{-1} \left[(1-\alpha) C_t + \alpha Q_t C_t^* \right]. \tag{1.4}$$

where $Q_t \equiv \dfrac{\mathcal{E}_t P_t^*}{P_t}$ is the real exchange rate (e.g. Chilean consumption baskets per US basket).

An increase (decrease) in Q_t is a real depreciation (appreciation) in the small economy.

In each period, a share θ of firms cannot adjust their price, so for them $P_{S,t}(j) = P_{S,t-1}(j)$. The remaining $(1-\theta)$ share set $\tilde{P}_{S,t}(j)$ to solve the following problem:

$$\max_{\tilde{P}_{S,t}(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \mathbb{Q}_{t+k|t} \left[\tilde{P}_{S,t}(j) Y_{t+k|t} - (1-\tau)\Psi\left(Y_{t+k|t}\right) \right] \right\}, \tag{1.5}$$

where $Y_{t+k|t} = (\tilde{P}_{S,t}(j)/P_{S,t+k})^{-\epsilon} Y_{t+k}$.

Due to complete markets, the value of marginal utilities of households in SOE and ROW equal each other when priced in the same currency:

$$Q_t \cdot C_t^{-1} = \exp(\nu_t^*) C_t^{*-1}. \tag{1.6}$$

This expression is key to understand the mechanism described below.[8] Under constant prices (i.e., Q_t constant), when ROW households become patient (i.e., $\nu_t^* < 0$), even if ROW consumption does not fall, consumption in SOE increases. A drop in the value of marginal utility in ROW households requires an equally sized drop in SOE. Otherwise, given complete markets, gains from trade arise. Then, either a drop in marginal utility itself (through consumption), a drop in its price (through real appreciation) or both must occur. Due to home bias, this is effectively a rise in demand for SOE goods, which then rises domestic consumption.

Finally, the central bank at SOE maximizes households' utility subject to equilibrium conditions (1.4) (1.5) and (1.6). More details on its derivation are provided in Appendix A.2.

1.3.2 Log-linearized system

Next, the paper proceeds to present log-linear approximations to ROW and SOE blocks. Further equations, details and derivations appear in Appendix A.2. Lowercases denote percent deviations with respect to steady state ($x_t \equiv \log X_t - \log X$), with the exception of interest rates, i_t and i_t^*, that already correspond to percentages.

[8]It is however not necessary for the results, but it helps to obtains clean analytical solutions. The quantitative model of Section 1.4 gets rid off the complete markets assumption.

In ROW, the market clearing condition plus equations (1.1) and (1.2) are written as:

$$\begin{aligned} \pi_t^* &= \kappa^* y_t^* + \beta E_t \pi_{t+1}^* \\ c_t^* &= E_t c_{t+1}^* - \left(i_t^* - E_t \pi_{t+1}^* - r_t^{n*}\right), \\ y_t^* &= c_t^* \end{aligned} \tag{1.7}$$

where $r_t^{n*} = \rho - E_t \Delta\nu_{t+1}^*$, $\kappa^* \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\epsilon\varphi)}(1+\varphi)$, and $\rho \equiv 1/\beta - 1$. The next Lemma summarizes how the central bank in ROW implements its objective and determines equilibrium, depending on the size of the shock and the restriction (or lack of thereof) on its nominal interest rate.

Lemma I.1 (ROW Implementation). *Under a central bank in ROW with inflation target of zero percent, $\overline{\pi}^* = 0$, and equilibrium conditions in system* (1.7)*:*

1. *If the nominal interest rate can take any value, $i_t^* \in \mathbb{R}$. Then, the target is implemented with $i_t^* = r_t^{n*}$, and output is stabilized, $y_t^* = 0$ for all t.*

2.(a) *If the nominal interest rate is bound by zero, $i_t^* \geq 0$, and $\rho \geq E_t \Delta\nu_{t+1}^*$. Then, the target is implemented with $i_t^* = r_t^{n*}$, and output is stabilized, $y_t^* = 0$ for all t.*

2.(b) *If the nominal interest rate is bound by zero, $i_t^* \geq 0$, and $\rho < E_t \Delta\nu_{t+1}^*$. Then, the target cannot be implemented, so $i_t^* = 0$, and $y_t^* < 0$.*

Next, in SOE, equations (1.4), (1.5) and (1.6) are written as:

$$\begin{aligned} \pi_{S,t} &= \kappa \left(c_t + \varphi y_t + \frac{\alpha}{1-\alpha} q_t\right) + \beta E_t \pi_{S,t+1}, \\ c_t &= y_t^* + q_t - \nu_t^*, \\ y_t &= (1-\alpha) c_t + \alpha y_t^* + \tilde{\alpha} q_t, \end{aligned} \tag{1.8}$$

where $\tilde{\alpha} \equiv \alpha(2-\alpha)/(1-\alpha)$ and $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\epsilon\varphi)}$.[9] The following Lemma summarizes how the central bank at SOE implements its objective taking into consideration SOE's and ROW's equilibrium conditions.

Lemma I.2 (SOE Implementation). *Under a central bank in SOE that maximizes (a second-order approximation to) households' utility subject to the system in* (1.8)*, the desired equilibrium is attained with the following optimal rule:*

$$\pi_{S,t} + \frac{1}{\epsilon} \Delta y_t = 0. \tag{1.9}$$

[9] SOE equilibrium conditions already consider $y_t^* = c_t^*$.

Proof. See Appendix A.2. □

Note that given standard values for the elasticity of substitution across differentiated goods (e.g., $\epsilon = 8$), the optimal rule implies strong domestic price stabilization. It stays close to fully stable at the expense of movements in output

Finally, because the focus is on CPI inflation and interest rate, these can be written as:

$$\begin{aligned} \pi_t &= \pi_{S,t} + \frac{\alpha}{1-\alpha}(q_t - q_{t-1}), \\ i_t &= \rho + E_t \Delta c_{t+1} + E_t \pi_{t+1}. \end{aligned} \tag{1.10}$$

The first expression comes from the Euler equation and the second from the definition of P_t and Q_t.

Definition I.3 (World equilibrium)**.** The world economy is in equilibrium if, for a given series of discount rate shocks $\{\nu_t^*\}_{t=0}^{\infty}$, the following holds:

(i) ROW variables $\{y_t^*, \pi_t^*\}$ and i_t^* satisfy the system in (1.7) and one of the scenarios in Lemma I.1. And,

(ii) SOE variables $\{c_t, y_t, q_t, \pi_{S,t}, \pi_t\}$ and i_t satisfy the system in (1.8) and (1.10), and rule (1.9).

1.3.3 International ZLB

First, the model proceeds to derive the equilibria in ROW under each scenario. Then, it analyzes how each scenario affects the SOE differently. For that it begins by deriving supply and demand functions for SOE output.

To study the effect of the zero bound in the international interest rate, consider the following sequence of discount rate shocks $\{\nu_0^*, 0, 0, ...\}$. This series is known in ROW and SOE at the beginning of $t = 0$. In order to make the comparison among scenarios relevant, from Lemma I.1 the shock must satisfy: $\rho > -\nu_0^*$. This ensures $r_0^{n*} < 0$. Given the monetary policies in both economies, it can be shown that this one-time shock in $t = 0$ only generates deviations from steady state in that period. Therefore, $E_0 x_{t+1} = E_0 x_{t+1}^* = 0$ and $E_0 i_{t+1} = E_0 i_{t+1}^* = \rho$ for all $t \geq 0$, where x_t is any variable in ROW or SOE.[10]

In scenario (a), i_0^* can take any value (No ZLB*), so the shock is fully absorbed such that no recession takes place, and the policy objective is attained. The equilibrium at ROW in

[10]The dynamic system in the ROW block is not determinate, however, the proposed solution is indeed a *possible* equilibrium. In order to show that this also holds for a determinate model. Appendix A.2 the same equilibria in a model with money-in-the utility and exogenous money supply.

this context is denoted with N and given by:

$$\pi^*_{N,0} = 0 \quad , \quad y^*_{N,0} = 0 \quad , \quad i^*_{N,0} = r^{n*}_0 = \rho + \nu^*_0. \tag{1.11}$$

In scenario (b), i^*_0 has a zero lower bound (ZLB*), so the response is halted for large enough shocks. The equilibrium at ROW in this context is denoted with Z and given by:

$$\pi^*_{Z,0} = \kappa^* r^{n*}_0 \quad , \quad y^*_{Z,0} = r^{n*}_0 \quad , \quad i^*_{Z,0} = 0. \tag{1.12}$$

To understand the differential effect on SOE, the following demand and supply curves for the output produced in SOE are derived from the system in (1.8):

$$-q_0 = (1-\alpha)\big(y^*_0 - y_0 - (1-\alpha)\nu^*_0\big) \tag{DD}$$

$$-q_0 = (1-\alpha)\left(\varphi' y_0 + y^*_0 - \nu^*_0\right) \tag{SS}$$

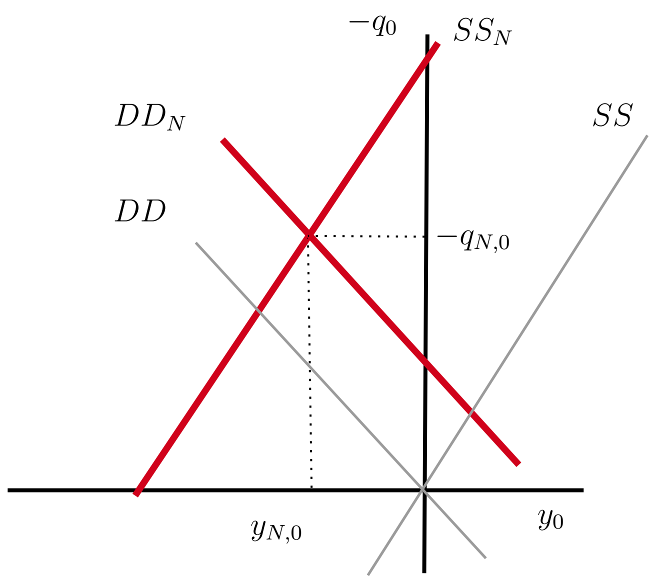
They are conveniently written to be drawn in the axis $(y_0, -q_0)$, so that (DD) and (SS) have standard negative and positive slopes, respectively. Note that the domestic price of SOE output is negatively correlated to a real exchange rate appreciation. A rise in $p_{S,0}$ rises the overall price in the economy, p_0, which by the definition of the real exchange, lowers q_0. Gray curves in Figure 1.3 show both curves when $\nu^*_0 = y^*_0 = 0$.

Note that in the No ZLB* scenario, only the structural shock affects the SOE. In the ZLB* scenario, it is also present, but there is an additional shock coming from a drop in y^*_0. Now, the model proceeds to assess how these two scenarios affect the SOE differently.

Figure 1.3 displays what happens in the No ZLB* scenario. Only the discount rate shock affects the SOE. From (1.6) the structural shock, under constant prices, generates upward pressure on domestic consumption. On one hand, this increases marginal costs by rising workers' opportunity cost, which pushes (SS) in. It shifts curve SS to SS_N. For a given level of q_0, SOE's production falls. For a given level of y_0, SOE's competitiveness falls. On the other hand, it increases demand which pushes (DD) out. It shifts curve DD to DD_N. The overall drop in output occurs as the increase in marginal costs is larger than the increase in demand, which by (1.10) means a drop in interest rate. CPI inflation depends on output and real exchange rate, but is determined by the latter.[11] Thus, the appreciation means a drop in inflation. This pins down a positive relationship between interest rate and inflation.

[11] It can be shown that due to (1.9), variations in domestic inflation are very small, and given $\epsilon > 1$, the sign of CPI inflation is always determined by that of the real exchange rate.

Figure 1.3: Negative ROW discount rate shock under No ZLB* scenario



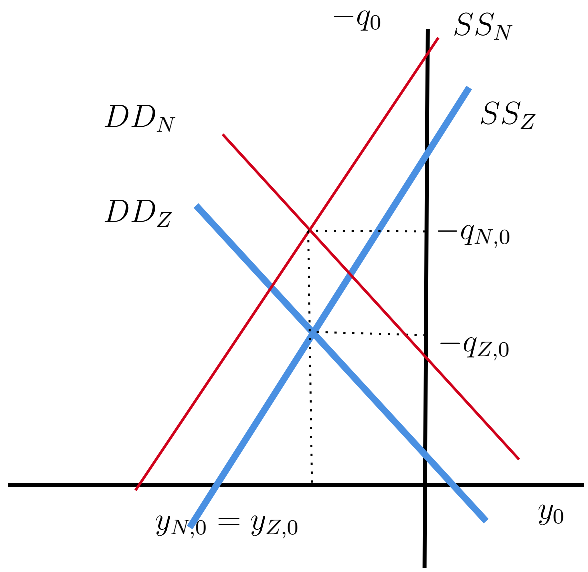
Intuitively, the structural shock generates a rise in the *relative demand* for SOE goods. This increases its relative price, so the real exchange rate appreciates. Because of this, imports into the SOE are cheaper which lowers overall inflation. In addition, the increase in consumption can only be achieved if nominal interest rates fall.[12] Given that this occurs despite the response of ROW's central bank, it is a *direct* channel that affects the SOE.

Figure 1.4 displays what happens in the ZLB* scenario. The effect of ν_0^* is still present, but now there is another channel given by the drop in y_0^*. From (1.4) it generates downward pressure on consumption. On one hand, this decreases marginal costs which pushes (SS) out compared to the previous scenario. It pushes curve SS_N to SS_Z. On the other hand, it decreases demand which pushes (DD) in compared to the previous scenario. It pushes curve DD_N to DD_Z. Given the simplified model, the drop in output in this scenario is the same as before, so is the drop in the domestic interest rate.[13] The difference comes from what happens to the real exchange rate which appreciates less than before, $q_Z > q_N$. This occurs because of the lower depreciation produced by $y_0^* < 0$, which turns into an increase in CPI inflation.

[12] Alternatively, one could think in nominal terms. By the UIP, the drop in i_0^* goes partly to a drop in i_0 and partly to an expected depreciation, which requires current appreciation. So the drop in domestic interest rates is accompanied by a drop in inflation.

[13] Specifically, a simplified model means complete markets, unitary elasticity of intertemporal substitution, unitary elasticity of substitution between domestic and foreign goods, and optimal mark-up in SOE.

Figure 1.4: Negative ROW discount rate shock under ZLB* scenario



This new channel goes in the opposite direction, with respect to what happens from ν_0^*. Plus, there is no change in the interest rate. Then, in this scenario the positive relationship between inflation and interest rate in the SOE weakens. And it can potentially be zero or negative if, for example, y_0^* drops too severely. Figure A.6 and A.7 in the Appendix show the cases for zero and negative relationships, respectively.

Discussion. The comparison between both scenarios informs about how domestic variables behave depending on the restriction that the international interest rate may have or not. International recessions characterized by no restriction, either because it does not exist or because the recession is of smaller magnitude, delivers a positive relationship between interest rate and inflation. If those variables are positively correlated due to other domestic shocks, then no difference should arise in the data and the empirical regularity holds. Conversely, an international recession characterized by the zero bound restriction delivers a weaker, null or negative relationship compared to the previous scenario. If the structural shock driving the recession is large enough, then a noticeable difference in the data may arise. This overall result is not dependent on the simplification of the model. Appendix A.2 shows that the findings hold using a Taylor rule instead.

Section 1.2 presented descriptive evidence that is explained by a mechanism presented here. Next, I include that mechanism into a quantitative model to quantify the ability it has in explaining the drop in the correlation.

1.4 Quantitative model

In this section, I incorporate the previous mechanism into a DSGE model that builds on Justiniano and Preston (2010a). The main departures from that model are the following. First, ROW's Taylor rule allows for forward guidance in the interest rate. This does not come as an insight from the simple model, but rather as a way to better fit the data as the original model does not contemplate the ZLB on i^*. Second, SOE firms invoice their exports in the currency of the ROW, which makes the LOP to not hold for goods produced domestically. In addition, the model no longer assumes a unitary elasticity of intertemporal substitution or unitary elasticity between domestic and foreign goods.

As in the simple model, the main source of the recession is due to a structural shock to the discount rate of ROW households (ν_t^* and ν_t). However, because the model attempts to match other moments in the data, it also includes risk-premium shocks ($\xi_{rp,t}$), productivity shocks ($\xi_{a,t}$), cost-push shocks to domestic and importing firms ($\xi_{cpS,t}$ and $\xi_{cpR,t}$) and monetary policy shocks ($\xi_{i,t}$). The model is presented from SOE's perspective, but differences with the ROW's counterpart are pointed out.

1.4.1 Households

There is a unit mass of households with the following utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \exp(\nu_t) \big[u(C_t, C_{t-1}) - v(N_t)\big]$$

where ν_t is the discount rate shock to SOE households. $u(C_t, C_{t-1}) = \dfrac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma}$ and $v(N_t) = \dfrac{N_t^{1+\varphi}}{1+\varphi}$, where $h \in (0,1)$ is an external habit coefficient. C_t is a consumption index,

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{S,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{R,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where η is the elasticity of substitution between SOE and ROW goods, and $(1-\alpha)$ is the home bias. C_t in Section 1.3 assumed $\eta = 1$. The corresponding price is $P_t = \big[(1-\alpha)P_{S,t}^{1-\eta} + \alpha P_{R,t}^{1-\eta}\big]^{\frac{1}{1-\eta}}$. Also, $C_{S,t}$ and $C_{R,t}$ are consumption indexes defined, together with corresponding prices, as in Section 1.3.

Households have access to bonds in SOE currency, D_t, and in ROW currency, B_t. Then, the budget constraint is

$$P_tC_t + D_t + \mathcal{E}_tB_t = (1 + i_{t-1})D_{t-1} + \mathcal{E}_tB_{t-1}(1 + i^*_{t-1})\phi_t(A_t) + \Gamma_{S,t} + \Gamma_{R,t} + W_tN_t,$$

where the function $\phi_t(\cdot)$ is interpretable as a debt elastic interest rate premium given by:

$$\phi_t = \exp[-\chi A_t + \xi_{p,t}], \qquad \text{with } A_t \equiv \frac{\mathcal{E}_{t-1}B_{t-1}}{P_{t-1}},$$

where χ is the elasticity and $\xi_{rp,t}$ is a risk-premium shock. All households are assumed to start with the same initial wealth of 0, i.e. $A_{-1} = B_{-1} = 0$. The international interest rate is given by i^*_t and set by the central bank in ROW as it is explained below. $\Gamma_{S,t}$ and $\Gamma_{R,t}$ are profits from domestic and importing firms respectively.

The demand functions for each category are:

$$C_{S,t}(i) = \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\epsilon} C_{S,t} \quad \text{and} \quad C_{R,t}(i) = \left(\frac{P_{R,t}(i)}{P_{R,t}}\right)^{-\epsilon} C_{R,t}$$

Optimal allocation of expenditure across domestic and foreign goods implies demand functions:

$$C_{S,t} = (1-\alpha)\left(\frac{P_{S,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{R,t} = \alpha\left(\frac{P_{R,t}}{P_t}\right)^{-\eta} C_t$$

ROW households face a similar problem to the one outlined above, but with a few differences. As in Section 1.3, C^*_t aggregates differentiated goods instead of other composite goods. Because SOE is of negligible size, ROW own's debt is in zero net supply, so is SOE own's debt too. However, SOE can still access bonds denominates in ROW currency.

As in Justiniano and Preston (2010a), this model allows the intertemporal elasticity of substitution and habit formation coefficient to be different for ROW households, $\sigma^* \neq \sigma$ and $h^* \neq h$.

1.4.2 Optimal labor supply

Each domestic firm produces good j with technology $Y_t(j) = \exp(\xi_{a,t})f(N_t(j))$, where $f(\cdot)$ is the identity. The labor input used in production of j comes from:

$$N_t(j) = \left[\int_0^1 N_t(k)^{\frac{\epsilon_W - 1}{\epsilon_W}} dk\right]^{\frac{\epsilon_W}{\epsilon_W - 1}} \quad \text{and} \quad W_t = \left[\int_0^1 W_t(k)^{1-\epsilon_W} dk\right]^{\frac{1}{1-\epsilon_W}},$$

where ϵ_W is the elasticity of substitution. Then, firm j's demand for each type of labor k is given by:

$$N_t^d(k) = N_t(j)\left(\frac{W_t(k)}{W_t}\right)^{-\epsilon_W}.$$

Households supply labor under monopolistic competition. A fraction $(1-\theta_W)$ of households set wages optimally, while a fraction θ_W adjusts according to the following rule:

$$W_t(k) = W_{t-1}(k)\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_W},$$

where $\gamma_W \in (0,1)$ is a degree of indexation to the previous period CPI inflation. Households solve the following problem when setting their wage $W_t(k)$:

$$\max_{\tilde{W}_t(k)} E_t \sum_{T\geq t} (\theta_W \beta)^{T-t}\left[\frac{\tilde{W}_t(k)}{P_T} N_{T|t}(k)\left(\frac{P_{T-1}}{P_{t-1}}\right)^{\gamma_W} \nu_T u_{1,T} - v_{1,T}(N_{T|t}(k))\right],$$

where $N_{T|t}(k) = N_T\left(\frac{\tilde{W}_t(k)}{W_T}\right)^{-\epsilon_W}$, $u_{1,T} \equiv \frac{\partial u}{\partial C_T}(C_T, C_{T-1})$, $v_T \equiv v(N_T)$ and $v_{1,T} \equiv \frac{\partial v}{\partial N_T}(N_T)$. ROW households solve a similar problem with parameters γ_W^* and θ_W^* instead.

1.4.3 Domestic producers

There is a continuum $[0,1]$ of monopolistically competitive domestic firms producing differentiated goods. They sell in the SOE where they set prices in their own currency ($P_{S,t}$), and sell in ROW where they set prices in the ROW currency ($P_{S,t}^*$). Namely, local or destination currency pricing for exports.

Each period a fraction θ_S of firms cannot adjust both prices optimally, and only adjust them according to the following rules:

$$P_{S,t}(j) = P_{S,t-1}(j)\left(\frac{P_{S,t-1}}{P_{S,t-2}}\right)^{\gamma_S} \quad \text{and} \quad P_{S,t}^*(j) = P_{S,t-1}^*(j)\left(\frac{P_{S,t-1}^*}{P_{S,t-2}^*}\right)^{\gamma_S^*},$$

where $\gamma_S \in (0,1)$ and $\gamma_S^* \in (0,1)$ are the degree of indexation to relevant inflation in the previous period. The other fraction $(1-\theta_S)$ of firms set prices optimally. They choose $\tilde{P}_{S,t}(j)$ for domestic sales and $\tilde{P}_{S,t}^*(j)$ for foreign sales to maximize the present discounted value of their nominal profits:

$$\max_{\tilde{P}_{S,t}(j),\tilde{P}^*_{S,t}(j)} E_t\sum_{T=t}^{\infty}\theta_S^{T-t}\mathbb{Q}_{t,T}\left[\tilde{P}_{S,t}(j)\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{\gamma_S}y_{S,T|t}(j)+\mathcal{E}_T\tilde{P}^*_{S,t}(j)\left(\frac{P^*_{S,T-1}}{P^*_{S,t-1}}\right)^{\gamma^*_S}y^*_{S,T|t}(j)\right.$$
$$\left.-W_Tf^{-1}\left(\frac{y_{S,T|t}(j)+y^*_{S,T|t}(j)}{\xi_{a,t}}\right)\right],$$

where $y_{S,T|t}(j)=\left(\frac{\tilde{P}_{S,t}(j)}{P_{S,T}}\right)^{-\epsilon}\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{-\epsilon\gamma_S}C_{S,T}$ and $y^*_{S,T|t}(j)=\left(\frac{\tilde{P}^*_{S,t}(j)}{P^*_{S,T}}\right)^{-\epsilon}\left(\frac{P^*_{S,T-1}}{P^*_{S,t-1}}\right)^{-\epsilon\gamma^*_S}C^*_{S,T}$ are the demands faced in T when setting prices in t, and $\mathbb{Q}_{t,T}\equiv\beta^{T-t}\frac{\nu_T}{\nu_t}\left(\frac{C_T}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_T}\right)$ is the stochastic discount factor. In addition, these firms are subject to cost-push shocks, $\xi_{cpS,t}$.

Local currency pricing breaks the LOP of the S goods exported to ROW. It means that $P_{S,t}(j)\neq\mathcal{E}_tP^*_{S,t}(j)$. To keep track of this we define the deviations in the LOP for S goods, $\Psi_t\equiv\frac{\mathcal{E}_tP^*_{S,t}}{P_{S,t}}$, which is always equal to one in the simple model. It is important to note that terms of trade in this context are defined as $S_t\equiv\frac{P_{R,t}}{\mathcal{E}_tP^*_{S,t}}$.

ROW firms solve a similar problem with parameters γ^* and θ^*, but only selling in their own market from their perspective. And because it acts like as a closed economy, $P^*_{R,t}=P^*_t$. In addition, ROW demand for SOE good – though negligible from ROW's perspective – is given by

$$C^*_{S,t}=\left(\frac{P^*_{S,t}}{P^*_t}\right)^{-\lambda^*}C^*_t,$$

where λ^* is ROW's elasticity between ROW and SOE goods. These firms are also subject to cost-push shocks, $\xi^*_{cp,t}$.

1.4.4 Retail firms

Retail or importing firms are only present in the SOE. There is a continuum [0,1] of importing firms that buy ROW goods and sell them in SOE. The law of one price holds at the docks, however, in firms setting the price in terms of SOE currency they are monopolistically competitive. This pricing power leads to a violation of LOP, so $P_{R,t}(j)\neq P^*_{R,t}(j)\mathcal{E}_t$. To keep track of this we define the deviation in the LOP for R goods, $\Psi^*_t\equiv\frac{P_{R,t}}{\mathcal{E}_tP^*_t}$, which is always equal to one in the simple model.

A fraction $(1-\theta_R)$ of firms set prices optimally, while a fraction θ_R adjusts according to the following rule:

$$P_{R,t}(j) = P_{R,t-1}(j) \left(\frac{P_{R,t-1}}{P_{R,t-2}} \right)^{\gamma_R},$$

where $\gamma_R \in (0,1)$ is a degree of indexation to imported inflation in the previous period. The firm's price setting problem in t is to maximize their expected present discounted value of profits:

$$\max_{\tilde{P}_{R,t}(j)} \; E_t \sum_{T=t}^{\infty} \theta_R^{T-t} \mathbb{Q}_{t,T} \left[\tilde{P}_{R,t}(j) \left(\frac{P_{R,T-1}}{P_{R,t-1}} \right)^{\gamma_R} - \mathcal{E}_T P^*_{R,T}(j) \right] y_{R,T|t}(j),$$

where $y_{R,T|t}(j) = \left(\frac{\tilde{P}_{R,t}(j)}{P_{R,T}} \right)^{-\epsilon} \left(\frac{P_{R,T-1}}{P_{R,t-1}} \right)^{-\epsilon\gamma_R} C_{R,t}$. In addition, these firms are subject to cost-push shocks, $\xi_{cpR,t}$.

1.4.5 International risk sharing and prices

Optimality conditions of SOE households lead to the following equations to determine domestic and foreign bond allocations:

$$\exp(\nu_t)(C_t - hC_{t-1})^{-\sigma} \frac{1}{P_t} = \beta(1+i_t) E_t \left[\exp(\nu_{t+1})(C_{t+1} - hC_t)^{-\sigma} \frac{1}{P_{t+1}} \right],$$

$$\exp(\nu_t)(C_t - hC_{t-1})^{-\sigma} \frac{\mathcal{E}_t}{P_t} = \beta(1+i^*_t) E_t \left[\exp(\nu_{t+1})(C_{t+1} - hC_t)^{-\sigma} \phi_{t+1} \frac{\mathcal{E}_{t+1}}{P_{t+1}} \right].$$

Similarly for ROW households:

$$\exp(\nu^*_t)(C^*_t - h^* C^*_{t-1})^{-\sigma^*} \frac{1}{P^*_t} = \beta(1+i^*_t) E_t \left[\exp(\nu^*_{t+1})(C^*_{t+1} - h^* C^*_t)^{-\sigma^*} \frac{1}{P^*_{t+1}} \right].$$

Combining optimality conditions for ROW bonds, we arrive to the following incomplete risk-sharing condition,

$$\frac{\Phi_t}{\Phi^*_t} Q_t = \frac{E_t \Phi_{t+1} \mathcal{E}_{t+1} / P_{t+1} \Phi_{t+1}}{E_t \Phi^*_{t+1} / P^*_{t+1}},$$

where $\Phi_t \equiv \exp(\nu_t)(C_t - hC_{t-1})^{-\sigma}$. This expression reflects a similar idea to that in (1.6). Under constant prices, a discount rate shock to ROW households that lowers the relative value of current marginal utility (and that is uncorrelated to SOE households, i.e. $\nu_t = 0$) lowers the relative current marginal utility of SOE households too. This provokes a rise in the relative demand for SOE, which may require a contemporaneous increase in C_t. Note that it occurs even if consumption at the ROW does not change.

This expression can also be written as the uncovered interest parity condition (UIP):

$$(1+i_t) = (1+i_t^*)E_t\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\phi_t.$$

1.4.6 Monetary policy

ROW. The central bank follows a Taylor rule that has a zero lower bound and that can also capture forward guidance in its interest rate. For this, the model uses Del Negro et al. (2013) and defines $\tilde{i}_t^*$ as the shadow rate:

$$\begin{aligned} (1+\tilde{i}_t^*) &= \left(1+\tilde{i}_{t-1}^*\right)^{\psi_i^*}\left(\left(\frac{P_t^*}{P_{t-4}^*}\right)^{\psi_\pi^*}\left(\frac{Y_t^*}{Y_{t-4}^*}\right)^{\psi_y^*}\right)^{1-\psi_i^*}\exp(\xi_{i,t}^*), \\ i_t^* &= \max\left\{0,\tilde{i}_t^*\right\}, \end{aligned} \tag{1.13}$$

where $\xi_{i,t}^*$ is a monetary policy shock. This rule captures forward guidance as a shock that happened in the past, say $t-3$, and that generated low prices and low output, is pushing interest rates down in t, even if the shock is no longer present.

SOE. The central bank in SOE sets the nominal interest rate i_t according to:

$$(1+i_t) = (1+i_{t-1})^{\psi_i}\left(\Pi_t^{\psi_\pi}Y_t^{\psi_y}\left(\frac{Y_t}{Y_{t-1}}\right)^{\psi_{\Delta y}}\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\psi_{\Delta e}}\right)^{1-\psi_i}\exp(\xi_{i,t}),$$

where $\xi_{i,t}$ is a monetary policy shock. This is a rule that responds to CPI inflation, output, output growth and nominal depreciation. This is a good approximation to what monetary policy can be characterized in countries like Australia.

1.4.7 World equilibrium

Goods markets in SOE and in ROW clear:

$$Y_t = C_{S,t} + C_{S,t}^* \quad , \quad Y_{S,t} = C_{S,t} \quad , \quad Y_{R,t} = C_{R,t} \quad , \quad Y_t^* = C_t^*.$$

Asset market for bonds denominated in SOE currency clear:

$$D_t = 0.$$

Households in SOE are homogenous and they all start with the same wealth. And, given SOE's size, they do not trade with ROW.

Definition I.4 (World equilibrium)**.** An equilibrium is a set of

(i) Prices: $\{P_{S,t}, P^*_{S,t}, P_{R,t}, P_t, \mathcal{E}_t, Q_t, W_t, P^*_t, W^*_t\}$

(ii) Quantities: $\{Y_t, Y_{S,t}(j), Y^*_{S,t}(j), Y_{R,t}(j), C_t, C_{S,t}, C_{R,t}, N_t, N^d_t, B_t, D_t, Y^*_t, Y^*_t(j), C^*_t, C^*_{S,t}, N^*_t, N^{d^*}_t, B^*_t\}$

(iii) Price decisions: $\{\tilde{P}_{S,t}, \tilde{P}^*_{S,t}, \tilde{P}_{R,t}, \tilde{W}_t, \tilde{P}^*_t, \tilde{W}^*_t, \}$

(iv) Interest rates: $\{i_t, i^*_t, \tilde{i}^*_t\}$

such that,

1. Given prices, $\{C_t, C_{S,t}, C_{R,t}, N_t, B_t, D_t\}$ solve SOE households problem.
2. Given prices, $\{C^*_t, N^*_t, B^*_t\}$ solve ROW households problem.
3. Given prices and demand for labor, $\{\tilde{W}_t\}$ and $\{\tilde{W}^*_t\}$ solve SOE and ROW household wage-setting problems, respectively.
4. Given prices, $\{Y_{S,t}, Y_{S,t}(j), Y^*_{S,t}, Y^*_{S,t}(j), Y_{R,t}, Y_{R,t}(j), Y_t, N^d_t\}$ solve problem of domestic firms and of importing firms in SOE.
5. Given prices, $\{Y^*_t, Y^*_t(j), N^{d^*}_t\}$ solve problem of firms in ROW.
6. Given prices and demand for goods, $\{\tilde{P}_{S,t}, \tilde{P}^*_{S,t}\}$ and $\{\tilde{P}_{R,t}\}$ solve domestic and importing firms price-setting problems in SOE, respectively.
7. Given prices and demand for goods, $\{\tilde{P}^*_t\}$ solves firms price-setting problem in ROW.
8. Nominal interest rates in SOE $\{i_t\}$ and in ROW $\{i^*_t, \tilde{i}^*_t\}$ satisfy their corresponding Taylor rules.
9. Labor, goods and asset markets clear.

1.5 Solution method and parametrization

The purpose of this section is to supply a solution method that takes into consideration the non-linearity in the model, and to provide a parametrization for the quantitative model.

Solution method. Given the occasionally binding constraint on i_t^*, the model is no longer linear. Thus traditional perturbation methods that rely on the model being always linear cannot be implemented. Therefore, I use the package (*OccBin*) and approach provided by Guerrieri and Iacoviello (2015) that uses a piecewise linear perturbation method that can accommodate non-linearities in the model. In particular, for each variable it delivers a perturbation based solution for when i_t^* binds and when it does not. This is also the solution method used for the estimation results described below.

Now that the model has been presented and that a solution method has been provided, the model proceeds to focus on one small open economy and the US as the ROW for the parametrization. The SOE is Australia as it has had the same monetary policy framework since 1990, and was not affected as much by similar structural shock as the US was.[14]

Parametrization. Following Justiniano and Preston (2010b), I assume all shocks x are AR(1) with persistence parameter ρ_x and standard deviation σ_x:

$$\log \xi_{x,t} = \rho_x \log \xi_{x,t-1} + \sigma_x u_t, \tag{1.14}$$

where $u_t \sim (0,1)$ and i.i.d. The exceptions are each country's own cost-push shocks $(\xi_{cpS,t}, \xi^*_{cp,t})$ and monetary policy shocks $(\xi_{i,t}, \xi^*_{i,t})$, which are assumed to be i.i.d. $(\rho_x = 0)$.

The parameters of the model are divided into two groups. The first group comes from the related literature and the second one is estimated using the simulated method of moments or calibrated using data.

For the first group of parameters, and in order for the parametrization to not guide the results, I set several parameters in SOE and ROW equal to each other. The intertemporal elasticity of substitution, Frisch elasticity, and habit formation coefficient follow Gust et al. (2017). This means $\sigma = \sigma^* = 1$, $\varphi = \varphi^* = 2$ and $h = h^* = 0.70$. In their same vein, I set $\epsilon = \epsilon_W = \epsilon^* = \epsilon^*_W = 6$. The rest of common parameters come from the ROW block in Justiniano and Preston (2010a), which estimates a related model with the US as ROW. This includes the parameters governing firms' cost-push and productivity shocks, the indexation parameters, and the probability of resetting prices for firms and wages for households. It is also assumed that domestic firms in SOE have the same indexation coefficient when selling at home or abroad. The details are found in Table 1.2.

The rest of the parameters for the small economy are taken from Justiniano and Preston (2010b), which estimates the parameters for Australia as a SOE. This is done as the SOE

[14] Canada was another candidate, but it is more likely to have been affected by similar shocks than the US was, then not allowing a proper assessment of how the Great Recession in the US affects a SOE via the zero restriction the international interest rate has.

block of their model resembles most of the model presented above. It is worth noting that here we also follow the assumption that the elasticity of substitution between domestic and foreign goods is the same from both SOE's and ROW's perspective (i.e., $\eta = \lambda^*$). Also, the parameters governing monetary policy and discount rate shocks are not equal to those in ROW, because these are estimated to match the US data as is explained below.

The home bias parameter and discount factors are calibrated such that they correspond to their data average counterpart. I set $\alpha = 0.2$ as this is the average import to GDP ratio in Australia during this period. β and β^* are set such that annual steady state interest rates are 4% and 3%, respectively.

Finally, there are key parameters which are estimated to match relevant moments in the US data. In particular, moments that were not intended to be relevant in Justiniano and Preston (2010a) (e.g., share of quarters that i^* binds at zero) or that may be affected by the use of a different Taylor rule (e.g., correlation between i^* and y^*). Given the discussion of Section 1.3, the characteristics of the discount rate shock determine the ZLB in the international interest rate, which in the quantitative model is informed by parameters ρ_ν^* and σ_ν^*. And due to the inclusion of a different Taylor rule for ROW, its parameters are also estimated (ψ_i^*, ψ_π^*, ψ_y^* and σ_i^*).

Parameter estimation. To estimate the parameters, I use the simulated method of moments (SMM) as analytical expressions for the moments are not available given the non linearities of the model. Given the structure of the model, where ROW is in practice a closed economy, the set of parameters can be divided into the ones pertaining to ROW and to SOE separately, $\Theta = (\Theta^R, \Theta^S)$. This lowers the computational burden as only the ROW block is now solved for when estimating a subset of Θ^R.

In particular, I estimate $\bar{\Theta} \equiv (\sigma_\nu^*, \rho_\nu^*, \psi_i^*, \psi_\pi^*, \psi_y^*, \sigma_i^*) \subset \Theta^R$ by solving the following distance problem:

$$\widehat{\bar{\Theta}} = \arg\min_{\bar{\Theta}} \left[\mu(x_t) - \frac{1}{\mathcal{S}}\sum_{s=1}^{\mathcal{S}} \mu\left(x\left(\boldsymbol{\xi}_t^s, \bar{\Theta}\right)\right)\right] \widehat{W}^{-1} \left[\mu(x_t) - \frac{1}{\mathcal{S}}\sum_{s=1}^{\mathcal{S}} \mu\left(x\left(\boldsymbol{\xi}_t^s, \bar{\Theta}\right)\right)\right]'. \tag{1.15}$$

x_t is the observed data and $\mu(x_t)$ is a function that computes the moments that appear in column 'Data' of Table 1.3. $\boldsymbol{\xi}_t^s$ is a vector draw of random shocks for simulation s and $\mathcal{S}$ is the total number of simulations, which considers all shocks affecting ROW. The length of the shocks is the same as that of the data. $x\left(\boldsymbol{\xi}_t^s, \bar{\Theta}\right)$ is the simulated data, which is the piecewise-linear solution obtained from the model under shocks $\boldsymbol{\xi}_t^s$, parameters $\bar{\Theta}$ and the *OccBin* approach. $\widehat{W}$ is an estimate of the optimal weighting matrix (Ruge-Murcia, 2012). The data used in this estimation is described in Appendix A.4.

Discussion of estimation results. Panel B of Table 1.2 displays the results of the estimation procedure, and Table 1.3 shows the model-based moments obtained under the results.

The Taylor rule coefficients are in line with standard estimates for them, with the exception of the smoothing parameter (ψ_i^*) which is slightly higher. Justiniano and Preston (2010a) use data until 2007 and obtain $\widehat{\psi}_i^* = 0.85$. Given the mechanical persistence of i^* during the Great Recession, it is not surprising our estimate is higher. This is compounded by the forward guidance structure in (1.13).

The estimated persistence of the discount rate shocks of ROW pairs to those found in the literature. The standard deviation, though, is considerably lower (by around 5 times) when compared to the one obtained in Justiniano and Preston (2010a). This difference is expected as the mentioned paper does not take into consideration the existence of a lower bound and uses data up to 2007. Compared to studies that match moments in the US and share of ZLB periods, our finding of $\sigma_\nu^* = 0.55$ is near to that of Nakata (2016) that finds a range between 0 and 0.40. Furthermore, our higher estimate is consistent with our estimated moment for the share of periods at the ZLB, which is of 16 percent compared to 6 in Nakata (2016).

By looking at Table 1.3, we can verify that the estimated parameters discussed above match the relevant moments reasonably well. One exception is the autocorrelation of inflation which is considerably higher in the model.[15] It is worth noting the close match to the mean share of quarters the international interest rate is at the ZLB. This moment has very high variance which affects its ability to be matched, thus in general we do not expect it to be as close.

[15] A potential remedy for this is to include γ^* into the parameters to be estimated too.

Table 1.2: Fixed and estimated parameter values

Panel A: Small open economy			
Coeff.	Description	Value	Source
β	Discount factor	0.99	4% interest rate
α	Openness	0.20	Average import/GDP
η	Elasticity of SOE demand	0.58	Justiniano and Preston (2010b)
ψ_i	Taylor rule, smoothing	0.84	" " " "
ψ_π	Taylor rule, inflation	1.83	" " " "
ψ_y	Taylor rule, output	0.09	" " " "
$\psi_{\Delta y}$	Taylor rule, output growth	0.74	" " " "
$\psi_{\Delta e}$	Taylor rule, nominal depreciation	0.14	" " " "
ρ_{rp}	Risk-premium, persistence	0.94	" " " "
σ_{rp}	Risk-premium, std. deviation	0.35	" " " "
ρ_ν	Preferences, persistence	0.93	" " " "
σ_ν	Preferences, std. deviation	0.16	" " " "
σ_{cpR}	Cost-push imports, std. deviation	1.58	" " " "
σ_i	Monetary policy, std. deviation	0.26	" " " "
θ_R	Calvo import prices	0.55	" " " "
γ_R	Index. import. prices	0.07	" " " "
χ	Elasticity of risk premium to debt	0.01	" " " "
ρ_a	Technology, persistence	0.93	Justiniano and Preston (2010a)
σ_a	Technology, std. deviation	0.47	" " " "
σ_{cpS}	Cost-push domestic, std. deviation	0.22	" " " "
γ_S	Index. dom. prices in SOE	0.58	" " " "
γ_S^*	Index. dom. prices in ROW	0.58	" " " "
γ_W	Index. wages	0.29	" " " "
θ_S	Calvo domestic prices	0.75	" " " "
θ_W	Calvo wages	0.75	" " " "
Panel B: Rest of the World			
Coeff.	Description	Value	Source
β^*	Discount factor	0.9925	3% interest rate
ψ_i^*	Taylor rule, smoothing	0.94	SMM
ψ_π^*	Taylor rule, inflation	1.38	SMM
ψ_y^*	Taylor rule, output	0.99	SMM
ρ_ν^*	Preferences, persistence	0.88	SMM
σ_ν^*	Preferences, std. deviation	0.55	SMM
σ_i^*	Monetary policy, std. deviation	0.00	SMM
ρ_a^*	Technology, persistence	0.93	Justiniano and Preston (2010a)
σ_a^*	Technology, std. deviation	0.47	" " " "
σ_{cp}^*	Cost-push, std. deviation	0.22	" " " "
θ^*	Calvo prices	0.75	" " " "
θ_W^*	Calvo wages	0.75	" " " "
γ^*	Index. prices	0.58	" " " "
γ_W^*	Index. wages	0.29	" " " "
λ^*	Elasticity ROW demand	0.58	" " " "

Table 1.3: Key moments: Data and Model

	Standard deviation				Autocorrelation				Corr w/ output			
	Data	Model	[2,	98]	Data	Model	[2,	98]	Data	Model	[2,	98]
Output (y^*)	1.22	1.56	1.05	2.29	0.89	0.87	0.80	0.92	–	–	–	–
Inflation (π^*)	0.23	0.27	0.22	0.33	0.12	0.62	0.45	0.74	0.45	0.52	0.25	0.73
Interest rate (i^*)	0.32	0.29	0.19	0.39	0.79	0.92	0.88	0.95	0.59	0.51	0.26	0.71
ZLB* (mean)	18.24	15.66	0.00	41.51	–	–	–	–	–	–	–	–

Notes: This table reports the moments used to solve (1.15), and the simulated moments under $\widehat{\Theta}^R$. It employs quarterly US data between 1980-2019. The total number of simulations is $\mathcal{S} = 1,000$ where each one is of the same length as the data ($T = 160$) with a burning period of 100 quarters. Column headings $[2, 98]$ denote the confidence intervals. The last row is the share of periods that i^* in the data (or simulated model) is at the ZLB.

1.6 Results

This section quantifies the model's ability to explain the lower correlation between interest rate and inflation in Australia during the international ZLB. In addition, it illustrates the implications of the quantitative model through impulse response functions after a large discount rate shock.

1.6.1 Correlations in the quantitative model

In order to know how the model performs in explaining what happens in SOE during the international ZLB, I simulate the model and compute the same correlations as in Table 1.1. To do so, I consider all shocks affecting both the ROW and SOE under the parametrization and structure given in Section 1.5. For each simulated economy I separate the periods between those when i^* is at the ZLB and those when i^* is not. Then, I calculate the contemporaneous correlation between interest rate and CPI inflation for the SOE, and with imported inflation too. Table 1.4 presents the comparison between the data and model.

Table 1.4: Correlation of interest rate and inflation: Data and Model

		Data	Model
$\rho(i, \pi)$	No ZLB*	0.6660	0.7288
	ZLB*	0.3429	0.6454
	% explained	25.82	

Notes: Column 1 of this table reports sample correlations between core year-ended CPI inflation and interest rates for Australia for two periods. The ZLB* period in the data is given by 2008Q4 to 2015Q4. Column 2 reports the correlation for the same elements in the quantitative. The ZLB* period in the model are all the quarters when (1.13) binds. The number of simulation is 1,000.

The model matches the drop in the correlation between ZLB* and No ZLB*. In particular, it explains around 26% of the drop observed in the data. Given the mechanism I exploit in this model and the extend of the international ZLB, it is reasonable to expect other shocks affecting Australia between 2008 and 2015 that may also play a role in explaining a lower correlation. For instance the flattening of the Phillips Curve in Australia (Ruberl et al., 2021). In addition, Appendix Table A.10 repeats this exercise for the correlation with quarterly inflation.

It is worth mentioning that the parametrization does not intent to match any moments for the SOE. Therefore, the share explained is likely a lower bound of how much this mechanism can explain. If parameters in the SOE were to be estimated such that they can match, on average, moments for Australia, then we would expect this percentage explained to be higher. For instance, the correlation between Australia's interest rate and CPI inflation.

The parameter estimation together with the model's ability to explain a significant part of the drop in the correlation point to the validity of this framework to understand what happens in a SOE when there is an external recession characterized by a binding international interest rate.

1.6.2 Impulse response functions

To understand the implications the international ZLB have on a SOE, I study impulse response functions stemming from discount rate shocks in two different scenarios. First, the baseline scenario where monetary policy follows (1.13). Second, an alternative scenario where i_t^* can potentially become negative:

$$(1+\tilde{i}_t^*) = \left(1+i_{t-1}^*\right)^{\psi_i^*} \left(\left(\frac{P_t^*}{P_{t-4}^*}\right)^{\psi_\pi^*} \left(\frac{Y_t^*}{Y_{t-4}^*}\right)^{\psi_y^*} \right)^{1-\psi_i^*} \exp(\xi_{i,t}^*),$$

$$i_t^* = \tilde{i}_t^*.$$

This resembles scenario (a) in (1.3) in the simple model of Section 1.3.

To illustrate the differences that arise between both scenarios, this exercise considers an international interest rate that is bound at zero for 15 quarters. This is attained by a one-time disturbance to the discount rate shock of around 30%.[16]

Figure 1.5 shows impulse responses after a large enough discount rate disturbance. Unsurprisingly, we can observe how the international interest rate goes into negative territory in the No ZLB* scenario. When this occurs, foreign output fall significantly less. With respect to the real exchange rate, we corroborate that the prediction made in Section 1.3 holds for the quantitative model as well. There is larger depreciation in the No ZLB* scenario with respect to the ZLB* scenario, as the solid line is above the dashed line.[17]

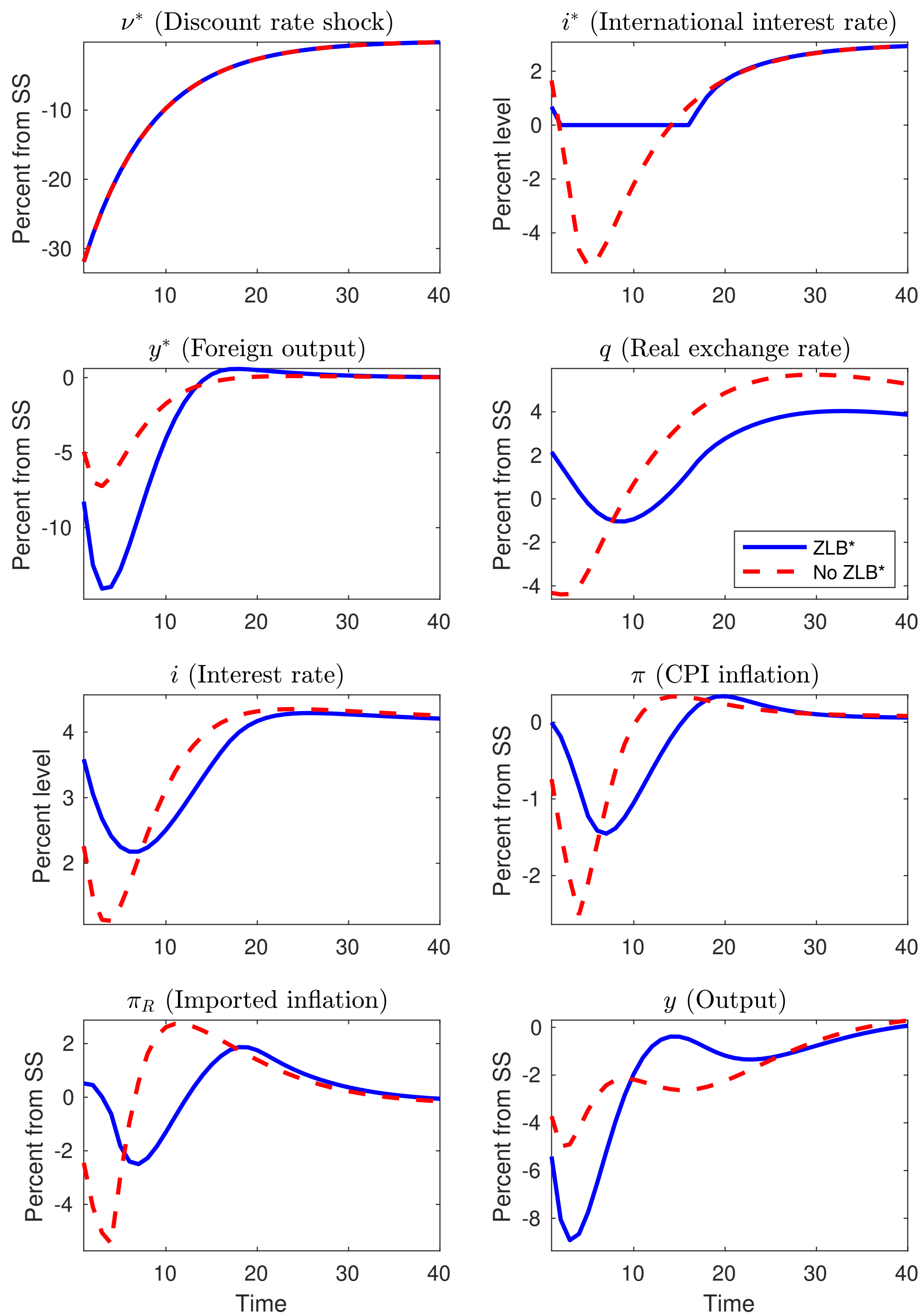
We can see that the domestic interest is able to fall by more under the No ZLB* scenario. In this particular case is by almost 100bps. This is partially explained by a lower international interest rate, but also by lower CPI inflation, which in turn is explained by lower imported inflation. As the SOE currency loses more of its value under the No ZLB* scenario, the price of imports increases. Finally, output falls on impact one percentage point less when the international interest rate can move freely. The responses of additional variables, such as domestic inflation, are shown in Appendix Figure A.8.

This exercise highlights how the zero bound restriction in the international interest rate affects the ability the SOE has in combating the external recession. It generates less appreciation by a contained drop in international interest rate and produces more depreciation by a larger drop in foreign output.

[16]There is nothing special about 15 quarters. Similar insights are obtained from shorter or longer international ZLB periods.

[17]In addition, there is *depreciation* under ZLB* and *appreciation* under No ZLB*, which is not an implication of the mechanism necessarily. The result from the simple model is that there is *more depreciation* under ZLB*, so their relative orders and not their levels.

Figure 1.5: Impulse responses to a foreign discount rate shock under ZLB* and No ZLB*



1.7 Conclusions

This paper studies what happens to small open economies in a context where the international interest rate is bound by zero. In particular, when this occurs as a result of a strong recession in the large economy. To understand what happens and guide the analysis, I study the only recent period when this has occurred. Namely, the period during and after the Great Recession where the Fed's rate, and therefore the international interest rate, was at the ZLB.

Using several SOE and different sources of data, I find that the usual positive relationship between interest rates and inflation weakens, breaks or flips during this period. This is explained by a model where the small economy is affected by two forces that have opposite effects on the exchange rate, which in turn can pass-through inflation. At the international ZLB, the relative size of these forces changes in such a way that breaks the usual positive relationship between inflation and interest rate.

This mechanism is embedded in a medium-size model for Australia and the US. Once the model is estimated and parametrized, it is used to quantitatively measure that 26 percent of the drop in the correlation is explained by this mechanism. Further research is needed to understand other aspects of SOE when the international interest rate is at the ZLB, such as their fiscal response, potential exchange rate interventions, or capital controls that may alleviate the added effect the international ZLB brings about.

CHAPTER II

Switching in the Currency of Invoicing

2.1 Introduction

The currency used for invoicing international trade is a key element to understand how exchange rate movements and other shocks in the world economy are transmitted into the local economy via, mostly, export and import prices. In general, international trade can be done using producer currency pricing (PCP) when is invoiced in the exporter's currency, local currency pricing (LCP) when is invoiced in the importer's currency, or vehicle currency pricing (VCP) when is invoiced in a third currency. Because this last one is usually the US dollar (USD), it is also known as dollar currency pricing (DCP).

The literature has studied, among other things, two aspects with regards to the currency of invoicing. The first one has to do with what are the determinants of invoicing in one currency versus another one. The second aspect is with respect to the implications of invoicing in different currencies, specifically how the exchange rate pass-through (ERPT) into import or export prices differs among currencies. This paper identifies issues with the current approaches to these two aspects and – by focusing on switchers in the currency of invoicing – proposes ways to improve and better answer those questions. In particular, it focuses on within-firm time-varying variables when studying currency choices, and its approach separates the firm-effect from the currency-effect when studying how invoicing affects the ERPT.

First, the study of why firms decide to use one specific currency to carry out their international transactions has been a topic in International Economics that dates back a few decades starting by Krugman (1980) and Friberg (1998). However, it was not until the work by Goldberg and Tille (2008) that this was taken to the data.[1] They use aggregate data

[1]Around the same time, Friberg and Wilander (2008) collect survey data for Swedish firms on many aspects in the currency of invoicing. Also, Gopinath and Itskhoki (2010) and Gopinath and Rigobon (2008) show novel data on import prices and their currencies for the US.

from many countries and try to understand how country and product differentiation can explain invoicing in a particular currency. This is connected to a model where firms try to minimize their price differences with respect to that of their competitors'. Then, Goldberg and Tille (2016) use millions of invoice level data from Canadian imports to try to understand which macro (e.g. exchange rate volatility), micro (e.g. exporting country market share) or transaction level (e.g. relative size of transaction) variables determine whether to invoice using LCP, PCP or some VCP. They find macro variables as the most important, and also evidence about hedging as exporters use currencies that co-move with their productions costs. More recently, Amiti et al. (2020) use Belgian export data to understand how the import intensity in USD may explain their invoicing decision between Euro (PCP) or non-Euro currencies (LCP or DCP). This is very important as is the first paper that links the currency of invoicing of both imports and exports at the firm level.[2]

Despite the contribution that all of these papers have had, none of them has been able to measure how time-varying firm-specific characteristics affect the currency of invoicing, such as the currency intensities in a firm's imports. Amiti et al. (2020) get close, however their import data are for only two years, so they extrapolate this to the other six years in their data. This is both a strong assumption and does not allow to measure the effects of potentially changing intensities of their currencies used in imports. Therefore, their analysis is a cross-sectional one.

In order to remedy this, this paper propose to focus on switchers. The massive prevalence of the USD documented by several authors (Gopinath, 2015; Gopinath et al., 2020) together with the idea of a historical dependence (Krugman, 1980; Mukhin, 2021) may lead firms to not really *choose*, but rather follow what other firms are doing. As it becomes clear below, the preponderance of the USD is also observed in the Chilean export data used in this paper. However, by focusing on firms that switch, we are able to observe variation within firm, which helps to better test whether the fundamentals in the models for currency choice can explain why firms decide (or potentially do not decide) the currency of their international transactions.

Following Amiti et al. (2019) we derive an equation which aims at explaining how changes in a firm's imports currency composition affect the invoicing decision for the firm's products and destination markets. When analyzing switching between LCP and DCP in the Chilean exports between 2011 and 2017, we find that indeed a higher share of imports invoiced in US dollars is associated with a higher probability of switching from LCP to DCP as the model suggests. We discuss the connection of this finding with the results obtained when studying

[2] This idea was first proposed in Gopinath et al. (2010b) in a model of incomplete pass-through where this depends on how marginal costs and markup vary with the exchange rate.

the implications of switching below.

The second issue tackled in this paper deals with our measures of ERPT and its relation to the currency of invoicing. By studying the exchange rate pass-through into import prices and its relation to the currency of invoicing, we can assess the role that the invoicing currency plays in the transmission of shocks. A simple theoretical prediction is that PCP will have a full pass-through in the short run, LCP a zero pass-through and VCP an ERPT somewhere in between.[3] Many authors have studied this. Gopinath et al. (2010b) uses import prices in the US and separates them among the ones invoiced in USD (i.e. LCP) and non-USD (mostly PCP). For the former, it finds an ERPT of zero in the short-run that grows slightly in a 24-month period. For the latter, it finds an ERPT of close to one in the short- and long-run. This is in line with the theoretical prediction. Auer et al. (2018) find something similar when studying the effect of the Swiss Franc appreciation in 2015 in import prices in Switzerland. Amiti et al. (2020) use Belgian exports data between 2012 and 2017 and compute the ERPT for products invoiced in Euro (i.e. PCP), in LCP and in DCP. Their results are again as expected.

Despite the fact that all the results in these papers are in line with the theory, they all suffer from a selection problem. The firms invoicing in one currency or another may not necessarily be the same and thus they may react very differently to exchange rate movements independent of the currency that they use.[4] This problem deepens when we consider the prevalence of the USD as a vehicle currency in international trade (Gopinath, 2015; Gopinath et al., 2020) as firms may potentially not have any other alternative than using DCP. So far the literature has not been able to separate the firm effect from the currency effect, and hence it has confounded both.

In order to correct for this selection problem, we focus on switchers as defined by the firm-product-destination links that change the currency of invoicing across time. By doing this, we can observe the same firm exporting the same good to the same destination, but invoicing in two different currencies in different moments in time. This allows to isolate the currency effect and thus better measure the ERPT for different currencies. Using Chilean exports data at the transaction level and concentrating on switches between LCP and DCP, we compute the ERPT between these currencies when switching and when not switching. We find that when invoicing in DCP and switching to LCP, the ERPT is in line with the theoretical prediction. However, when invoicing in LCP and then switching to DCP, the ERPT stays at zero in the short-run, which is at odds with the theoretical prediction.[5]

[3]See Engel (2006) for an earlier treatment of this.

[4]Instead of thinking of merely firms, one could think of firm-product-destionation combinations differing on how they invoice. This potentially intensifies the problem.

[5]A potential explanation is beyond the scope of this paper. However, one option is to separate the price

The rest of the paper is organized as follows. Section 2.2 describes the data used for both analysis summarized above and for Section 2.3 which shows the importance and preponderance of switching behavior in the data. Section 2.4 explores how the currency composition of a firm's imports affects their invoicing decision. Then, Section 2.5 tackles how the ERPT differs by currency. There we focus on switchers which are invoicing in different currencies at different moments in time. Finally, Section 2.6 concludes and proposes venues to continue this study.

2.2 Data

This paper uses a unique datasets that allows to measure many aspects in relation to the currency of invoicing that earlier papers had not been able to. The data are all the transactions carried out by exporters in Chile between 2011 and 2017. When firms in Chile export they need to fill in a form called DUS[6] which is administered by the Customs Office. In the form, the exporters need to declare, among other things, the product code of the export using an HS 8-digit code, the FOB and CIF value, quantity, destination, and the currency used for settling the price. We have access to all that information, together with a unique firm identifier.

With the data we can study the invoicing behavior of a firm across time, and also the currency used for the different products and destinations that a given firm has. Considering the state of the literature, firms choose their currencies of invoicing at the product-destination level for a given time period. Thus, we can define *trade-link* as the firm-destination-product tuple, which is also the unit of analysis. The data have 334,682 of such links which are observed anything between once and throughout the whole sample period. The trade links come from 20,045 individual firms. A total of 128,406 trade links (38% of all) are observed only one time, but represent less than 1% of total export value.

Additionally, we use import data that are then merged with exporters. These data also come from the Customs Office, and in particular from the form called DIN.[7] In the form, the firms need to report, among other things, the product code as in the DUS form, the CIF and FOB value, quantity, origin, and the currency used for settling the price. Because the firm identifier is the same in the exports and imports data, these can be merged.

The information cointained in the DUS and DIN are readily available at `http://datos.gob.cl/` and do not require any type of agreement or disclosure for their use.

decision from that of the currency decision. For instance, the Calvo assumption in this context ties these two decision together.

[6]Acronym in Spanish for Export Unique Document.

[7]Acronym in Spanish for Import Unique Document.

For the analysis below we use GDP per capita and Producer Price Index in Chile. The data comes from the World Development Indicators in the World Bank Data Bank and from the Central Bank of Chile, respectively.

Before going into the switching behavior in the currency of invoicing is important to know about the invoicing features in the economy itself. Table 2.1 shows the share of export transactions and share of export value for the 10 most important currencies during our sample period. The USD is by far the most used currency, with the Euro (EUR) in second place. Table 2.2 also shows the share of transactions and export value, by type of currency. 66% of transactions are carried out using the USD as a vehicle currency (or VCP USD). This happens when the United States (or another dollarized economy) is not the destination. 32% of transactions use the currency of the destination country (i.e. LCP). The Chilean Peso (i.e. PCP) is used extremely rarely.[8] In the Section below we will show that in spite of this distribution, there is significant switching between currencies.

Table 2.1: Currency use in sample

	% of transactions	% of export value
USD	90.07	95.52
EUR	6.01	3.09
UKP	1.78	0.42
CAD	0.89	0.12
YEN	0.59	0.43
CLP	0.19	0.14
MEX	0.15	0.09
REA	0.13	0.05
SWE	0.09	0.03
NOR	0.05	0.01
Other	0.05	0.11

[8] There is no trend in any of these aspects through the 7 years we study this, thus we omit year-by-year tables.

Table 2.2: Currency type use in sample

	% of transactions	% of export value
PCP	0.19	0.14
LCP	31.58	16.38
VCP USD	66.33	82.93
VCP EUR	1.77	0.42
VCP UKP	0.07	0.01
VCP CAD	0.01	0.00
VCP YEN	0.01	0.00
Other	0.06	0.11

2.3 Switching Behavior

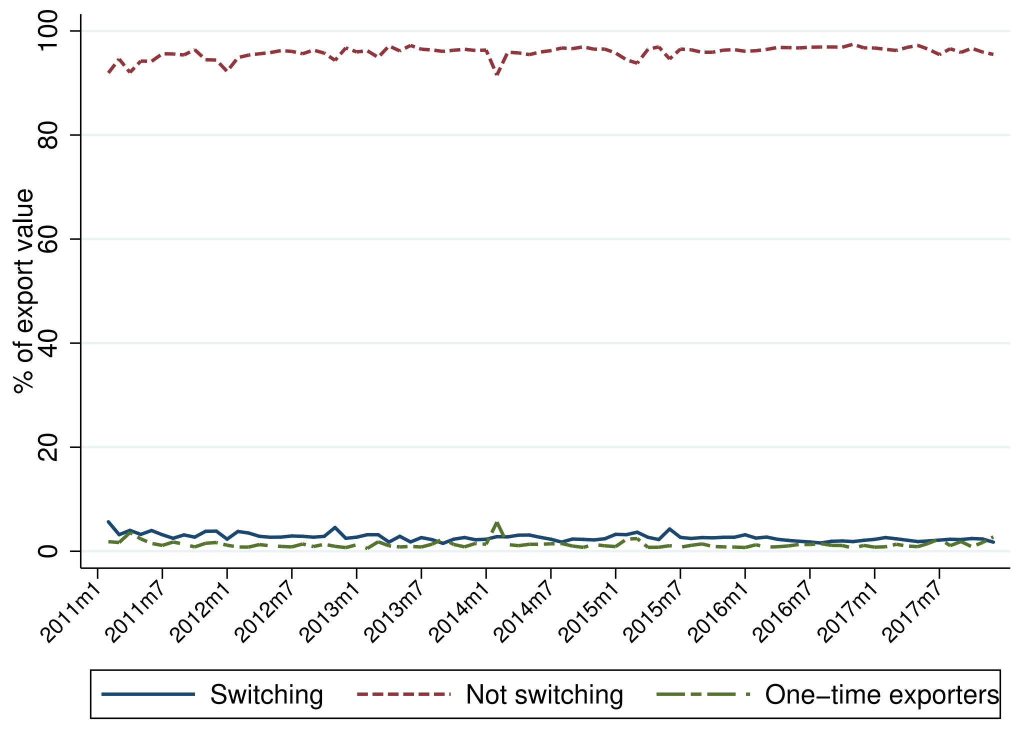
In order to account for the switching behavior the paper documents how relevant this is in the data, and to what extent these switches can be explained by something we already know from the literature. This section seeks to answer this. We investigate the size of the switching behavior, how frequent the switches occur, their duration and the currency composition of these switches. We find that a sizable share of firms enact switching in the currency of invoicing, that the duration of the switch depends on the currency composition of the switch itself, and that the Chilean Peso (CLP) is relatively more prevalent among switchers than in the entire sample.

To study the switching behavior with more detail, together with defining a unit of analysis (i.e. firm-product-destination), we need to define a time period to study when the switch occurs. To exploit as much as possible the granularity of the data, we use month as the relevant time period. With this, we can define a *switch* as a trade link's monthly transaction that uses a set of currencies in month t which differs to the set of currencies used in month $t-1$.

A couple of observations with respect to this definition. First, to do this we collapse the raw data (at the transaction level) to the firm-product-destination-month level, which can potentially generate observations with more than one currency. We keep track of this and is accounted for when in the definition above we refer to the *set* of currencies. Second, the definition is extended to consider switches also the instances when a trade links does not appear every month. In this case, a switch can still occur (unless it is the first month *ever* observed in the sample) when, for example, a trade links appears in month 1 using one currency, then does not export in month 2, and then in month 3 appears but using a different currency. Given the fact that we are using firm-product-destination level data, it is reasonable that this trade link will not show up every month (e.g. seasonal products to specific regions such as fruits to the Northern Hemisphere in the opposite season).

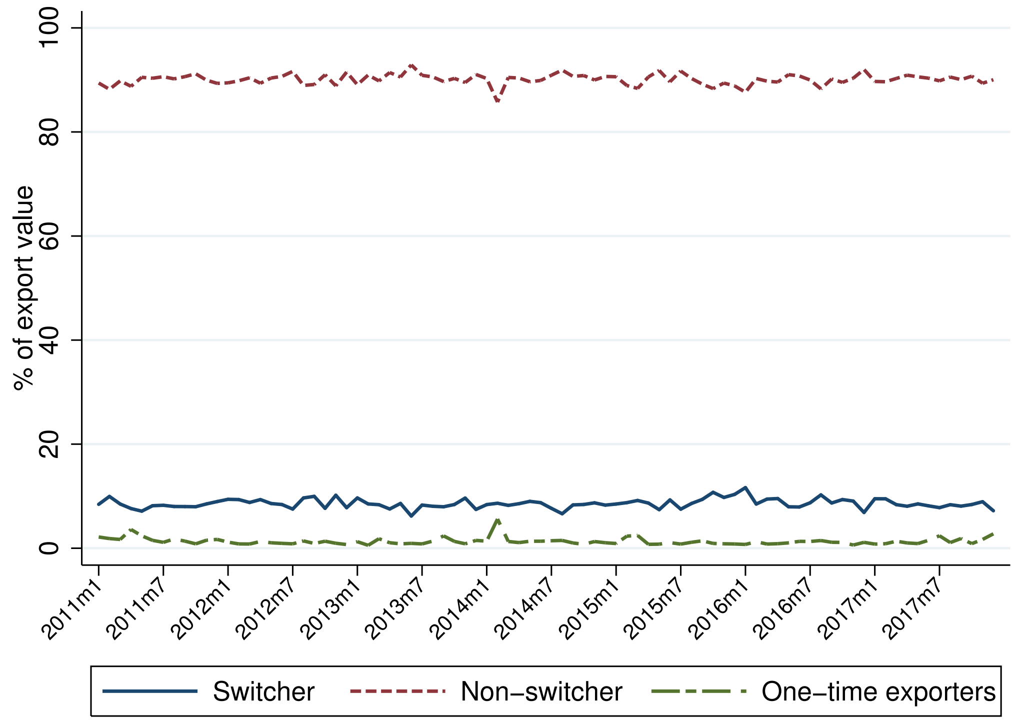
Figure 2.1 shows the export value share by type of transaction in a given month. Specifically, whether the transaction is a switch, not a switch, or a one-time export, in the sense that the tuple firm-product-destination is only observed once. It can be noted that around 3% of *all* export value at any month comes from transactions that switch the currency of invoicing. Similarly, this represents 2% of all transactions in a given month.

Figure 2.1: Export value by type of transaction



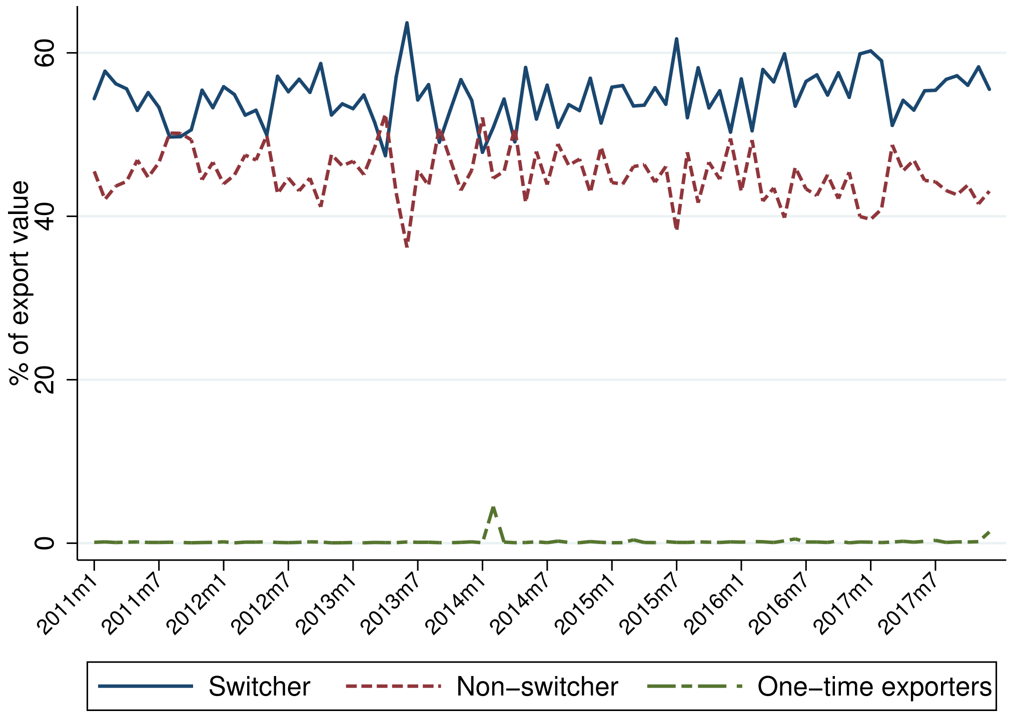
Due to potential costs of switching, we cannot expect firms or trade links to be switching every month. Thus, we can define a *switcher* as a trade link that enacts at least one switch during the sample period we study. This is an important element as it is the relevant group of study in terms of switching behavior even if they are not switching every period. Figure 2.2 shows the export value share in a given month by type of trade links. It can be noted that around 9% of all the export value in a given month comes from trade links that at some point during the sample switch their currency of invoicing. It is worth pointing that is not a negligible share of trade, and that is stable across time.

Figure 2.2: Export value by type of trade link



One could also define a switcher over firms rather than doing it over the firm-product-destination combination (i.e. trade link). When doing so we unsurprisingly get a share that is considerably larger. Specifically, Figure 2.3 shows that around 55% of all the export value in a given month comes from firms that at some point during the sample switch the currency for at least one of their product-destination pairs. Similarly, when looking at the share of firms, 6% of them switch at some point. This figure goes up to 14% if we do not consider firms that only export once in our sample.

Figure 2.3: Export value by type of firm



During the rest of the paper, a *switcher* is referred to the trade link that enacts switching, rather than the firm. Considering this, we can assess how the distribution of currencies is for this group (switchers) compared to the whole sample, which was displayed in Section 2.2. Tables 2.3 and 2.4 show the distribution by currency and type of pricing for switchers, respectively. Two aspects are worth noting. First, the CLP is used ten times more for this group compared to the whole sample (compare row 1 in Table 2.4 with row 1 in Table 2.2). This can be an indication of CLP either used as a "safeguard" from other currencies or an "unstable" currency such that exporters that use CLP quickly move to another currency. Second, VCP-USD is the most important pricing type for the whole sample. In contrast, for this subsample, this is disputed with LCP which covers almost 2/3 of all switcher's transactions.

Table 2.3: Currency use for switchers in sample

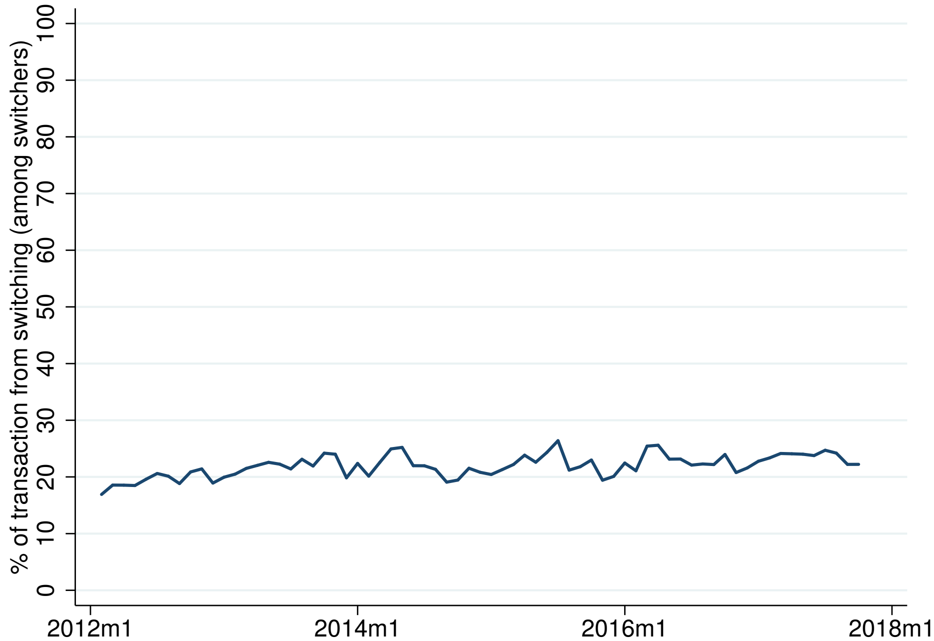
	% of transactions	% of export value
USD	58.58	60.47
EUR	20.36	25.30
UKP	9.61	4.22
CAD	4.46	1.02
YEN	3.47	4.54
CLP	1.08	1.42
MEX	0.98	1.08
REA	0.89	0.62
SWE	0.32	0.16
NOR	0.01	0.01
Other	0.23	1.16

Table 2.4: Currency type use for switchers in sample

	% of transactions	% of export value
PCP	1.08	1.42
LCP	64.96	44.18
VCP USD	27.53	50.29
VCP EUR	5.74	2.81
VCP UKP	0.37	0.10
VCP CAD	0.03	0.01
VCP YEN	0.02	0.02
Other	0.27	1.17

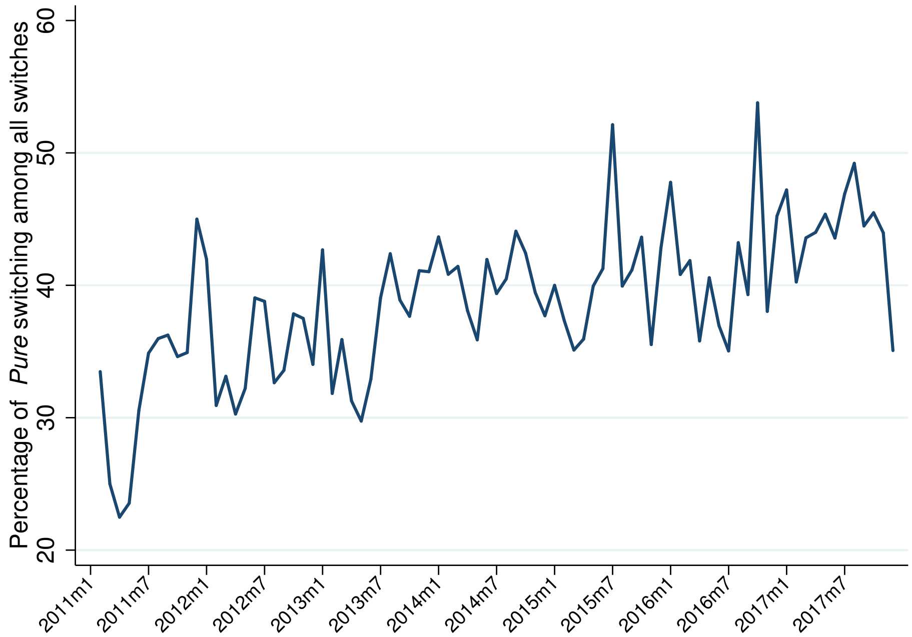
We can also assess how frequent switching occurs at the trade link level. Figure 2.4 displays that around 22% of transactions of switchers are switches at any given month. Similarly, we would like to know the duration of these switches.

Figure 2.4: Share of switching transactions among switchers



Before diving into that, we can differentiate switches based on their currency composition. Two groups naturally appear: *Pure* and *Mixed* switches. In the former group, the sets of currencies before and after the switch are singleton. For example, EUR to USD or LCP to PCP. In the latter, at least one of the sets of currencies before and/or after contains more than one currency. For example, 50% EUR and 50% UKP to 100% USD, or 25% LCP and 75% VCP-USD to 50% LCP and 50% VCP-USD. In Figure 2.5 we can see that around 39% (61%) of switching transactions are Pure (Mixed) switches. Now we can take this into account when studying the duration of switches. Pure switches last 6.80 months on average (with a median of 3 months), whereas mixed switches last only 2.91 months (with a median of 1 month). In general, switches last 4.03 months (with a median of 1 month).

Figure 2.5: Export value by type of switching



To gain further understanding one can study transition matrices for different types of switches (Pure or Mixed) and for how currencies are classified (currency itself and pricing). This is key to get a first glance on how switching occurs. Table 2.5 shows this for pricing type in the case of pure switches and Table 2.6 in the case of mixed switches. In this last case, given that a mixed switch is composed of more than one currency either before or after, we associate a trade-link's monthly transaction to a currency if more than 75% of its transactions are invoiced that way. Hence, the diagonal in Table 2.6 is not necessarily zero as a trade link can change its composition, but still invoice mostly in one given currency. Also, the category "Other" makes reference to cases where either it is mostly invoiced in a currency not listed in the table or where no currency is used in more than 75% of monthly transactions. In the Appendix B.1 we show the transition matrices for currencies themselves. In these Tables below we can observe that, with the exception of VCP UKP, all currencies are mostly likely to switch to VCP USD. And from VCP USD and VCP UKP, the most probable currency to switch is LCP. This is clearer in pure switches than in mixed switches where mostly invoicing in the current currency is the most likely switch. From now on we will refer to VCP USD as DCP (dollar currency pricing)

Given the relationship between DCP and LCP, together with being the currencies used the most, we focus the rest of the paper on these two currencies. In order to make the most of the data, we pull together pure and mixed switches and define to be invoiced in LCP or DCP when transactions more than 75% of monthly transaction are in either currency, i.e. it includes pure switches. A total of 3,536 trade-links switch between DCP and LCP according to this definition and correspond to 46,320 monthly observations. Additionally, it is important to remark that because trade-links are not necessarily observed every month, we define switches as the change in currency compared to the last time that trade link took place.

In the following sections we try to understand switching from two perspectives. First, why do trade links switch their currency, and in particular what is the role played by their own currency composition when importing. And second, what is the implication of these switches. Specifically, how the exchange rate pass-through and its relation to the currency of invoicing is affected by these changes.

Table 2.5: Transition matrix between Pricing types for Pure Switches

		Currency type in t							
		PCP	LCP	V. USD	V. EUR	V. UKP	V. CAD	V. YEN	Other
Type in $t-1$	PCP	0.0	10.2	88.3	0.3	0.0	0.0	0.0	1.2
	LCP	2.6	0.0	88.3	2.9	5.6	0.4	0.0	0.3
	VCP USD	7.6	62.9	0.0	23.9	1.7	0.4	0.3	3.3
	VCP EUR	0.3	7.8	89.4	0.0	2.0	0.0	0.0	0.5
	VCP UKP	0.0	66.4	25.8	7.9	0.0	0.0	0.0	0.0
	VCP CAD	0.0	31.2	68.8	0.0	0.0	0.0	0.0	0.0
	VCP YEN	0.0	7.1	92.9	0.0	0.0	0.0	0.0	0.0
	Other	0.9	7.0	88.7	3.3	0.0	0.0	0.0	0.0

Table 2.6: Transition matrix between Pricing types for Mixed Switches

		Currency type in t							
		PCP	LCP	V. USD	V. EUR	V. UKP	V. CAD	V. YEN	Other
Type in $t-1$	PCP	28.3	4.4	21.2	0.9	0.9	1.8	3.5	38.9
	LCP	0.2	52.8	3.0	3.2	1.9	0.5	0.2	38.2
	VCP USD	0.9	5.0	35.1	3.8	1.3	0.3	0.3	53.3
	VCP EUR	0.0	19.5	14.7	20.3	1.2	0.0	0.1	44.1
	VCP UKP	0.0	37.8	16.4	3.3	2.7	1.3	1.0	37.5
	VCP CAD	1.5	46.2	18.5	3.1	3.1	0.0	3.1	24.6
	VCP YEN	10.3	33.3	20.5	2.6	2.6	7.7	0.0	23.1
	Other	0.9	36.4	26.0	7.7	1.6	0.2	0.1	27.1

2.4 Why switching?

In this section, we focus on how the changes in the currency composition of a firm's imports can explain the switches observed. First, we propose a simple model of currency switching which delivers an estimating equation. Second, inspired by this we give some firm-level evidence on the synchrony between the importing and exporting currencies. Finally, we provide the results from the model which indicate that changes in the importing currency is indeed associated with switching.

2.4.1 Model of Currency Switching

The optimal price of firm f selling product p to destination d in period t is given by $p_{fpd,t}$. For the ease of notation we define $i := f \times p \times d$. Thus, we can write the optimal price in the producer's currency as:

$$\tilde{p}_{it} = \tilde{\mu}_{it} + mc_{it}$$

where $\tilde{\mu}_{it}$ is the desired markup[9] and mc_{it} is the marginal cost. Following Amiti et al. (2019) we can totally differentiate the equation for the desired price and arrive to:

$$\mathrm{d}\tilde{p}_{it} \quad = \quad \frac{1}{1+\Gamma_{it}}\mathrm{d}mc_{it} + \frac{\Gamma_{it}}{1+\Gamma_{it}}\mathrm{d}\left(p^{*}_{-it} + e_t\right) + \varepsilon_{it} \tag{2.1}$$

[9]The markup $\tilde{\mu}_{it} = \mathcal{M}(p_{it}, p_{-it}; \boldsymbol{\xi}_t)$ is a function of its own price, the price of the competition p_{-it} and demand shifters $\boldsymbol{\xi}_t$.

where $\Gamma_{it} \equiv -\frac{\partial \mu_{it}}{\partial p_{it}}$ or the elasticity of the desired markup with respect to price, p^*_{-it} is the average price in the local currency of product p at destination d charged by all the competition of firm f, and ε_{it} is a residual.[10] It is worth mentioning that the elasticity of the desired markup with respect to price is an increasing function of the size of i, thus we can write $\Gamma_{it} = \Gamma(S_{it})$.

Equipped with this, we can obtain an expression for the optimal short-run ERPT:

$$\tilde{\psi}_{it} \equiv \frac{\mathrm{d}\tilde{p}_{it}}{\mathrm{d}e} = \frac{1}{1+\Gamma_{it}}\varphi_{it} + \frac{\Gamma_{it}}{1+\Gamma_{it}}\left(1 - \Psi^*_{pd,t}\right) \tag{2.2}$$

where $\varphi_{it} = \dfrac{\mathrm{d}mc_{it}}{\mathrm{d}e_t}$, $\Psi^*_{pd,t} = -\dfrac{\mathrm{d}p^*_{it}}{\mathrm{d}e_t}$ and we are implicitly assuming that $\dfrac{\mathrm{d}\varepsilon_{it}}{\mathrm{d}e_t} = 0$. If firms could choose between LCP, PCP and DCP we should observe the following values for $\tilde{\psi}_{it}$:

$$\tilde{\psi}_{it} = \begin{cases} 0 & \text{then LCP} \\ 1 & \text{then PCP} \\ (0,1) & \text{then DCP} \end{cases}$$

However, in our setup and given the discussion above we will focus only on firms that switch between LCP and DCP, thus we define $\tilde{\psi}^*_{it}$:

$$\tilde{\psi}^*_{it} = \begin{cases} 0 & \text{then LCP} \\ 1 & \text{then DCP} \end{cases}$$

Then, following Amiti et al. (2020) we do a first-order approximation to (2.2) which leads to:

$$\Pr\left(\psi^*_{fpd,t} = 1\right) = \alpha_{pd} + \beta\varphi_{i,t} + \gamma S_{i,t} \tag{2.3}$$

where α_{pd} is a product-destination fixed effect. Here we are assuming that the short run aggregate response to the competition's price is constant across time.

Finally, we are interested in switches between currencies, then we can define the following elements:

$$\Delta\psi^{DCP}_{fpd,t} \equiv \begin{cases} 1 & \text{if } \mathrm{LCP}_{fpd,t-1} \rightarrow \mathrm{DCP}_{fpd,t} \\ 0 & \text{if } \mathrm{LCP}_{fpd,t-1} \rightarrow \mathrm{LCP}_{fpd,t} \end{cases} \quad \text{and} \quad \Delta\psi^{LCP}_{fpd,t} \equiv \begin{cases} 1 & \text{if } \mathrm{DCP}_{fpd,t-1} \rightarrow \mathrm{LCP}_{fpd,t} \\ 0 & \text{if } \mathrm{DCP}_{fpd,t-1} \rightarrow \mathrm{DCP}_{fpd,t} \end{cases}$$

So combining this with (2.3) we derive the following linear probability models which will

[10]The residual corresponds to $\varepsilon_{it} = \frac{1}{1+\Gamma_{it}} \sum_{j \in \mathcal{F}_{pd,t}} \frac{\partial \mathcal{M}_{it}(p_{it}, p_{-it}; \boldsymbol{\xi}_t)}{\partial \xi_{j,t}} \mathrm{d}\xi_{j,t}$ which is i's demand shock at t, and where $\mathcal{F}_{pd,t}$ is all the firms exporting p at d.

be our main estimating equations:

$$\Pr\left(\Delta\psi_{fpd,t}^{DCP}=1\right) = \beta^{DCP}\Delta\varphi_{fpd,t}+\gamma^{DCP}\Delta S_{fpd,t} \tag{2.4}$$
$$\Pr\left(\Delta\psi_{fpd,t}^{LCP}=1\right) = \beta^{LCP}\Delta\varphi_{fpd,t}+\gamma^{LCP}\Delta S_{fpd,t} \tag{2.5}$$

Given the data we have we can measure size $S_{fpd,t}$ as:

$$S_{fpd,t}=\frac{\text{Export value}_{fdp,t}}{\sum_{j\in\mathcal{F}_{pd,t}}\text{Export value}_{jdp,t}}$$

However, we cannot measure $\varphi_{fpd,t}$ directly which is defined as follows:

$$\varphi_{fpd,t}=\frac{\text{Total USD import value}_{fpd,t}}{\text{Total variables costs}_{fpd,t}}$$

This is because we do not observe total variables costs and we cannot observe which inputs are used for which goods, or to pin down a particular cost to a particular destination. To overcome this issue, we make the following two assumptions. First, costs are constant across destinations and products, i.e. $\varphi_{fpd,t}=\varphi_{f,t}$. Second, the ratio of imports to total variable costs is constant. Then, we can write $\varphi_{f,t}$ as:

$$\varphi_{f,t}=\frac{\text{Total imports}_{f,t}}{\text{Total var. costs}_{f,t}}\cdot\frac{\text{Tot. USD imp. value}_{f,t}}{\text{Total imports}_{f,t}}=\kappa\cdot\frac{\text{Tot. USD imp. value}_{f,t}}{\text{Total imports}_{f,t}}$$

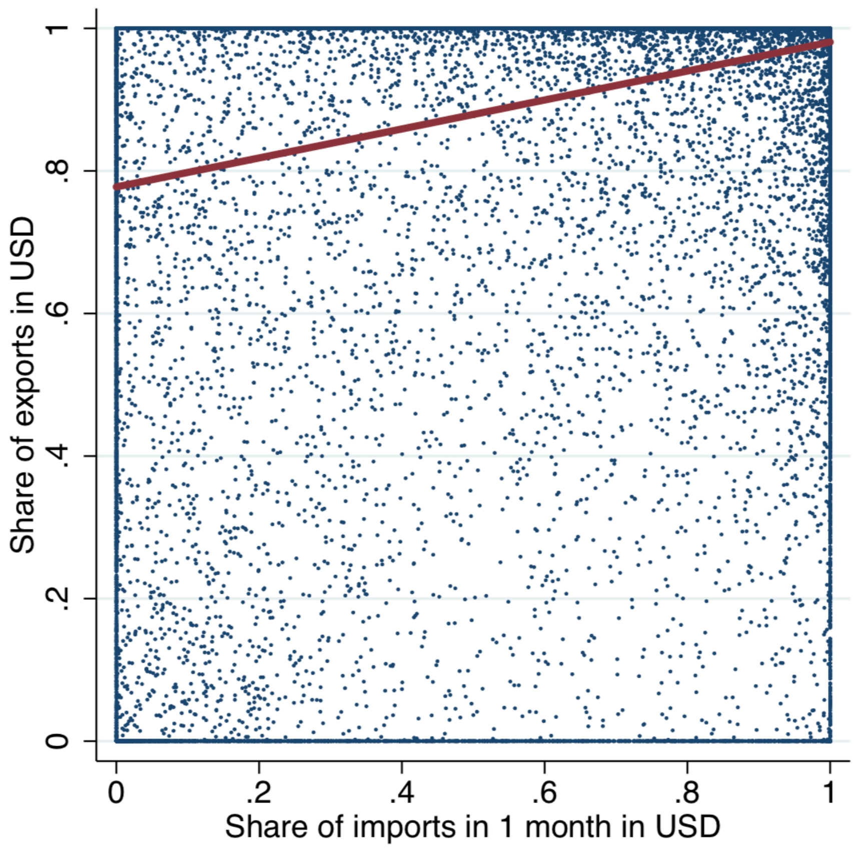
The first assumption is reasonable as long as firms do not produce many different goods that imply different production functions. The second assumption is valid as long as the country's international stance does not change in the period studied.

2.4.2 Cross-section evidence on currency synchronization

Before we proceed to the results of estimating (2.4) and (2.5), we can study the cross sectional evidence on how the share of imports in certain currency correlates with the currency of invoicing of exports. To do so we can focus on the two mostly used currencies, the US dollar and the Euro. First, we plot the shares of exports and imports at the firm level for different currencies. Then, we run simple reduced-form regressions to better control for the correlations.

Figures 2.6 and 2.7 show the share of monthly exports invoiced in USD and EUR by a firm against its share of monthly imports invoiced in USD and EUR, respectively. The solid line is the fitted line of each corresponding scatter plot. In both figures we can clearly see a positive correlation and also a very similar slope. In Figure 2.6 we can also see that the

Figure 2.6: Share of exports and imports invoice in USD in a firm

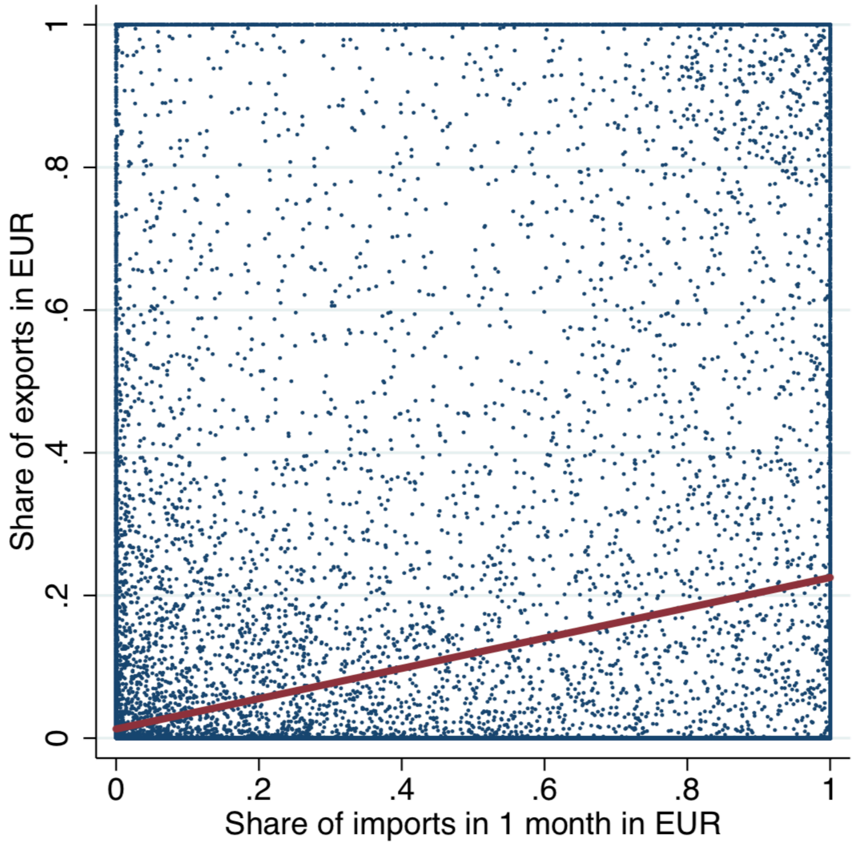


“constant” implied by the solid line drawn is around 0.8 which is very high and in line with the idea that USD invoicing might not be easily explained by firm-specific fundamentals. In Appendix B.2 we describe and run different regressions that confirm that the currency composition of imports is highly associated to the currency of invoicing of exports.

2.4.3 Results

Here we present the results from estimating (2.4) and (2.5). First, we proceed to present the baseline results using one-month import currency composition as it is implied by the model above. Table 2.7 presents the results from this and adds a trade-link fixed effect. In columns (2) and (3) we can see that adding this fixed effect marginally affect the point estimate of $\Delta\varphi^{DCP}_{f,t}$, however it does change the significance on $\Delta S_{fpd,t}$. These columns also confirm that positive changes in the US dollar intensity of a firm’s imports is in fact associated with a higher probability of switching from LCP to DCP. In particular, a 50% increase in the USD import intensity increases in 1% the probability of switching.

Figure 2.7: Share of exports and imports invoice in EUR in a firm



Depending on whether this is a pure or a mixed switcher,[11] the relative magnitude of this probability varies. Table B.3 in Appendix B.1 shows the transition matrix for pure switchers. The probability of going from LCP to DCP is 15%, so an increase of 1 percent is relevant, but only 1/15 of the overall probability. However, for Mixed Switchers (Table B.4, Appendix B.1) this probability is of 6%. Then, an increase of 1 percent represents 1/6 of the overall probability, which is relatively more substantial.

Going back to Table 2.7, we can see in columns (5) and (6) that changes in the USD import intensity do not have an effect on a trade link switching from DCP to LCP. One could have also expected this to have a negative sign as more USD in imports should lead to less switching aways from DCP. Inspired by this one could modify equations (2.4) and (2.5) and replace $\Delta\varphi^{DCP}_{f,t}$ by $\Delta\varphi^{LCP}_{f,t}$ and potentially expect the opposite results as the ones displayed in Table 2.7. Table 2.8 displays this using changes in LCP intensity rather than

[11]We did not define this earlier, but their definition is very intuitive. A *Pure* switcher is a trade link that only has pure switches. A *Mixed* switcher is a trade link that has at least one mixed switch.

DCP.[12] As expected, we can observe in column (3) that higher shares of LCP generate less switching away from LCP into DCP. However, that it also contributes to less switching from DCP into LCP, which is counterintuitive. This is observed in columns (4)-(6). In order to see how both play a role, we finally present results of equations (2.4) and (2.5) but using both changes in import intensity. Column (3) in Table 2.9 shows that only changes in the DCP import intensity is associated with more switching from LCP to DCP when exporting. On the other hand, column (6) shows that neither changes in the DCP import intensity nor in the LCP are associated with the switches from DCP to LCP.

Table 2.7: Results for Switching Currency using DCP import intensity

	(1) $\Delta\psi^{DCP}_{fpd,t}$	(2) $\Delta\psi^{DCP}_{fpd,t}$	(3) $\Delta\psi^{DCP}_{fpd,t}$	(4) $\Delta\psi^{LCP}_{fpd,t}$	(5) $\Delta\psi^{LCP}_{fpd,t}$	(6) $\Delta\psi^{LCP}_{fpd,t}$
$\Delta\varphi^{DCP}_{f,t}$	0.0178*	0.0181*	0.0231***	0.0219	0.0215	0.0131
	(0.0101)	(0.0101)	(0.00888)	(0.0154)	(0.0154)	(0.0124)
$\Delta S_{fpd,t}$		-0.128	-0.261**		0.104	0.0928
		(0.121)	(0.121)		(0.105)	(0.125)
Trade-link FE	No	No	Yes	No	No	Yes
Currency	LCP→LCP	LCP→LCP	LCP→LCP	DCP→DCP	DCP→DCP	DCP→DCP
	LCP→DCP	LCP→DCP	LCP→DCP	DCP→LCP	DCP→LCP	DCP→LCP
N	9,478	9,478	9,478	7,089	7,089	7,089
R^2	0.000	0.001	0.465	0.000	0.001	0.553

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

[12]Because we do not keep track of all currencies when studying currency composition of imports, we focus on Euro countries, Canada, Japan and the UK.

Table 2.8: Results for Switching Currency using LCP import intensity

	(1)	(2)	(3)	(4)	(5)	(6)
	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$
$\Delta\varphi^{LCP}_{f,t}$	-0.0178	-0.0183	-0.0207**	-0.0386**	-0.0381**	-0.0218
	(0.0116)	(0.0116)	(0.0104)	(0.0180)	(0.0180)	(0.0149)
$\Delta S_{fpd,t}$		-0.138	-0.286**		0.0882	0.125
		(0.126)	(0.129)		(0.110)	(0.137)
Trade-link FE	No	No	Yes	No	No	Yes
Currency	LCP→LCP	LCP→LCP	LCP→LCP	DCP→DCP	DCP→DCP	DCP→DCP
	LCP→DCP	LCP→DCP	LCP→DCP	DCP→LCP	DCP→LCP	DCP→LCP
N	8,596	8,596	8,596	6,592	6,592	6,592
R^2	0.000	0.001	0.464	0.001	0.001	0.564

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2.9: Results for Switching Currency using DCP and LCP import intensity

	(1)	(2)	(3)	(4)	(5)	(6)
	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$
$\Delta\varphi^{DCP}_{f,t}$	0.0221	0.0221	0.0254*	-0.0220	-0.0219	-0.00295
	(0.0155)	(0.0155)	(0.0139)	(0.0228)	(0.0228)	(0.0179)
$\Delta\varphi^{LCP}_{f,t}$	0.00162	0.00107	0.00157	-0.0574**	-0.0569**	-0.0243
	(0.0177)	(0.0177)	(0.0161)	(0.0262)	(0.0261)	(0.0212)
$\Delta S_{fpd,t}$		-0.138	-0.285**		0.0880	0.125
		(0.126)	(0.129)		(0.110)	(0.137)
Trade-link FE	No	No	Yes	No	No	Yes
Currency	LCP→LCP	LCP→LCP	LCP→LCP	DCP→DCP	DCP→DCP	DCP→DCP
	LCP→DCP	LCP→DCP	LCP→DCP	DCP→LCP	DCP→LCP	DCP→LCP
N	8,596	8,596	8,596	6,592	6,592	6,592
R^2	0.000	0.001	0.464	0.001	0.001	0.564

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.5 in Appendix B.1 shows how the reported effects vanish when instead of using changes in the import intensity in 1 month, we use the changes in three months. This informs that switching between currencies is only explained by changes in the currency composition of imports as long as these changes happen contemporaneously.

2.5 Implications of Currency Switching

In this section we study how switching can affect the short-run exchange rate pass-through. Specifically, whether the relation between the currency of invoicing and the magnitude of the ERPT is affected when we focus on switchers. By focusing on switchers we can separate the firm effect from the actual effect of invoicing in a particular currency. Previous research such as Gopinath et al. (2010b) compute ERPT for imports in the United States using LCP and non-LCP, i.e. USD or non-USD. However, the specific firms, products and origins of those transactions where not necessarily the same when using LCP and when using something else. Thus, there is a selection problem of which are the firms that choose to invoice in one currency and not the other.

2.5.1 Conceptual framework

It is important to mention that we can only study short-run ERPT and not medium- or long-run ERPT. The main reason for this is having very few observations that satisfy the following requirements. First, a trade-link needs to be observed invoiced in both currencies for sufficiently extended periods of time (e.g. 12 months), such that the comparison between currencies works. However, as it was explained above switching is not a persistent behavior. Firms (or trade-links) when they switch they only stay in the new currency for 4.3 months on average and with 1 month as the median. Second, trade-links have to occur every month, so no gaps in time, which is not what happens in the data. Namely, the same firm is not exporting the same product (at the 8-digit code) to the same destination *every* month, which does not allow for ERPT regressions for longer horizons.

One way to remedy any of these issues would be aggregating at a lower frequency (i.e. quarters instead of months) or less digits (4-digit code instead of 8). However, this stills leads to few observations as the switching might not occur as we have defined it.[13]

So, we estimate a modified short-run ERPT regression which is standard in the literature (Burstein and Gopinath, 2014):

$$\Delta p_{fpd,t} = \alpha_{fpd} + \beta_0 \Delta e_{d,t} + \beta_1 \Delta e_{d,t} \times Z_{fpd,t} + \beta_2 Z_{fpd,t} + \gamma X_t + \delta W_{d,t} + \varepsilon_{fpd,t} \tag{2.6}$$

[13]In order to classify a transaction in LCP or DCP, we require the firm-product-destination to have at least 75% of their transactions in the relevant period. Thus, when aggregating at the quarterly level from the monthly level, we could have, for example, a firm that was switching from LCP (month 1) to DCP (month 4), but now at the quarterly level the firm has 50%-50% in both currencies.

where $p_{fpd,t}$ is the price in the local (or destination) currency of product p from firm f to destination d in month t, α_{fpd} is the firm fixed effect, $e_{d,t}$ is the bilateral exchange rate between the Chilean peso and the currency at d,[14] X_t is a supply-side variable which is measured by the Producer Price Index, and $W_{d,t}$ is a demand-side variable which is measured by the GDP per capita at destination d.

We estimate (2.6) for two set of observations, which each intend to measure what is the effect of switching from LCP to DCP, and then from DCP to LCP. The first set is comprised of trade-links that mostly invoice in DCP, which in turn defines $Z_{fpd,t}$ taking value 0 when DCP and 1 when LCP. The second set is made of trade-links that mostly invoice in LCP, which defines $Z_{fpd,t}$ to take value 0 when LCP and 1 when DCP.

2.5.2 Results

Table 2.8 displays the baseline results of estimating (2.6). The columns differ on which controls and fixed effects are being used, but most importantly on which currency they are switching from and to. Odd columns show the results of switching from LCP to DCP and even columns when switching from DCP to LCP. Our preferred specifications correspond to columns 7 and 8, however the results are maintained for other specifications too. First, we can note that when there is no switching, both LCP and DCP deliver the expected result of invoicing in those currencies, i.e. LCP has an ERPT of zero and DCP an ERPT of 3.47% when there is a 10% depreciation of the destination's currency with respect to the Chilean peso. These results are qualitatively very similar to estimating a version of (2.6) for non-switchers.[15] Second, in column 8 we can see that when switching from DCP to LCP the ERPT is not different from zero, which is expected of invoicing in the local currency. However, when the switch occurs in the opposite direction the ERPT is positive, but not significantly different from zero. This is at odds with DCP, which is associated with a larger pass-through.

[14] It is measured as units of destination's currency per Chilean peso, so an increase in e means a depreciation of the destination's currency with respect to the Chilean peso.

[15] See Table B.6 in Appendix B.1.

Figure 2.8: Baseline results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$
$\Delta e_{d,t}$ (LCP)	0.0204	-0.213	0.00216	-0.211	0.0156	-0.0784	-0.000851	-0.0744
	(0.115)	(0.376)	(0.118)	(0.386)	(0.117)	(0.386)	(0.119)	(0.396)
$\Delta e_{d,t}$ (DCP)	0.169	0.335*	0.200	0.332*	0.0551	0.344*	0.0903	0.347*
	(0.326)	(0.178)	(0.335)	(0.182)	(0.337)	(0.182)	(0.346)	(0.186)
Most of invoicing	LCP	DCP	LCP	DCP	LCP	DCP	LCP	DCP
Controls	No	No	Yes	Yes	No	No	Yes	Yes
HS8-Dest. FE	Yes	Yes	Yes	Yes	No	No	No	No
Trade-link FE	No	No	No	No	Yes	Yes	Yes	Yes
N	24,046	17,267	24,046	17,267	22,194	16,187	22,194	16,187

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

There are many reasons that may explain this results. One is that we do not observe prices, but rather unit price values. The variable $p_{fpd,t}$ is computed by dividing the total revenue from firm f exporting p to d in month t by the total quantity associated with this. This is a measurement error problem in the dependent variable, which should not bias the results as long as it is independent from our explanatory variables, which is a reasonable assumption.

Another idea is to departure from the paradigm that because pricing by firms is à la Calvo (Gopinath et al., 2010b; Cravino, 2012; Mukhin, 2021), when firms can change their prices they can also simultaneously change the currency used for invoicing. Instead, keep the Calvo assumption for prices, but allow currency switches to occur independently of prices. Then, we can claim that when firms switch from LCP to DCP they cannot necessarily change their prices too, but that they can do it when switching from DCP to LCP. We do not have any evidence to back this claim, however, from our results in Section 2.4 we can speculate based on the idea that switching from LCP to DCP was actually explained by changes in the currency composition of imports, whereas the switching from DCP to LCP was more idiosyncratic, i.e. our data and model could not explain the switch. Hence, one can claim that switching from LCP to DCP occurs as a response from the changing currency composition of imports which would not allow the firm to change the price too necessarily. A potential model with these features and its implications can be a way to extend the work in this paper.

In order to be sure about these results one could, instead of running (2.6) for two sets of trade-links, run it for two set of observations. One of those that satisfy the structure $\{\text{DCP}_t, \text{DCP}_{t-1}\}$ and $\{\text{LCP}_t, \text{DCP}_{t-1}\}$ in order to measure the effect of switching to LCP. And another one for those that satisfy $\{\text{LCP}_t, \text{LCP}_{t-1}\}$ and $\{\text{DCP}_t, \text{LCP}_{t-1}\}$ in order to measure the effect of switching to DCP. Tables 2.10 and 2.11 do this controlling in the same way as in the previous table. We can see that the conclusions drawn from Table 2.8 hold here too. Namely, DCP and LCP when not switching resemble qualitatively the same magnitudes as before. Then, when switching from LCP to DCP the effect gets very noise, but ultimately the switch is not associated with the ERPT of invoicing in the dollar as a vehicle currency. And, when switching from DCP to LCP the effect, the ERPT in fact becomes zero.

Table 2.10: Additional results defining switching based on observations

	(1)	(2)	(3)	(4)
	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$
Δe_t (LCP)	0.100	-0.734	0.0513	-0.686
	(0.129)	(0.479)	(0.131)	(0.503)
Δe_t (DCP)	-0.393	0.494**	-0.510	0.472**
	(0.384)	(0.197)	(0.401)	(0.200)
Invoicing	LCP→DCP	DCP→LCP	LCP→DCP	DCP→LCP
	LCP→LCP	DCP→DCP	LCP→LCP	DCP→DCP
Controls	No	No	Yes	Yes
HS8-Dest. FE	Yes	Yes	Yes	Yes
Trade link FE	No	No	No	No
N	20,830	15,663	19,299	14,530

Robust standard errors in parentheses   * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2.11: Additional results defining switching based on observations

	(1)	(2)	(3)	(4)
	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$
Δe_t (LCP)	0.101	-0.862	0.0545	-0.780
	(0.134)	(0.588)	(0.137)	(0.611)
Δe_t (DCP)	-0.0905	0.474**	-0.285	0.459**
	(0.428)	(0.209)	(0.446)	(0.212)
Invoicing	LCP→DCP	DCP→LCP	LCP→DCP	DCP→LCP
	LCP→LCP	DCP→DCP	LCP→LCP	DCP→DCP
Controls	No	No	Yes	Yes
HS8-Dest. FE	No	No	No	No
Trade-link FE	Yes	Yes	Yes	Yes
N	20,830	15,663	19,299	14,530

Robust standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

2.6 Conclusion and future work

Earlier approaches on studying the determinants and implications of the currency of invoicing in international trade have usually assumed or treated this as a static element. This has been due to a direct treatment in the models or the available data. This paper studies switching in the currency of invoicing for a narrowly defined unit of analysis. It reports some stylized facts for the switching behavior, together with using this to better understand why firms decide to use a particular currency and what is the effect in the exchange rate pass-through into prices when firms switch their pricing.

By using Chilean exports data at the transaction level we are able to find that switching is sizable, with almost half of the firms enacting switching at some point in the sample period studied, and around 2% of all export share every month coming from transactions that changed their currency of invoicing with respect to the previous month. We also find that the currency composition of imports can in fact be associated with switching between currencies. Specifically, a larger share of imports denominated in US dollars does explain switching from LCP to DCP. However, the changes in import composition does not explain the switches from DCP to LCP. Finally, we compute the ERPT around the switching to find that the link between currency of invoicing and pass-through is kept when the switching is from DCP to LCP, but not when it is from LCP to DCP. Hence, in this last case the ERPT is not different from zero, which is at odds with earlier literature.

In order to reconcile these aspects one would need to depart from the Calvo pricing that simultaneously allow to change prices and currency. This would allow firms to change their currencies as a response to changes in their fundamentals, but not necessarily allow price changes. Including an explicit model with these mechanisms is not explored in this paper, but it is a path to continue this work.

CHAPTER III

Population Aging and Structural Transformation

with Javier Cravino and Andrei Levchenko

3.1 Introduction

Economic growth is accompanied by large reallocations of economic activity across broad sectors, a phenomenon known as structural transformation (Kuznets, 1957). In advanced economies, the structural transformation process is associated with a decline in the relative size of the Agriculture and Manufacturing sectors and a corresponding rise in the Service sector. Traditional theories that attempt to rationalize this process have relied on non-homothetic preferences with a high income elasticity for services (e.g. Kongsamut et al., 2001), or on a technology-driven increase in the relative price of services coupled with a low elasticity of substitution across sectors (Baumol, 1967; Ngai and Pissarides, 2007).

This paper documents and quantifies the role of population aging in the structural transformation process. Older individuals devote a larger share of their expenditures to services, thus the relative size of the service sector grows as the population ages. We show that, across a large sample of countries, increases in population age are accompanied by a rise in the relative size of the service sector. Using household-level data for the US, we document large differences in sectoral expenditure shares across households of different ages, with older households spending relatively more on services. We then use a shift-share decomposition and a quantitative model of structural change to quantify how much of the rise in the relative size of the service sector in the US over the period 1982-2016 can be accounted for by changes in population age.

To document how structural transformation is related to population aging across countries and time, we use multiple data sources following the Handbook chapter by Herrendorf et al. (2014). Across many countries and years, and several datasets, the service shares of employment, value added, and consumption expenditures are positively related to population aging. Importantly, this empirical regularity persists when controlling for the (possibly nonlinear) relationship between the service shares and income per capita that has been emphasized in the previous literature. After controlling for income, a 1 percentage point increase in the fraction of population that is over 65 is associated with a 1.3-1.5 p.p. increase in the service shares of value-added and employment, and a 0.7 p.p. increase in the service share of consumption expenditures.

We then use household-level data from the US Consumer Expenditure Survey (CES) to document large differences in sectoral expenditure shares across households of different ages. Our data cover the 1982-2016 period and have been widely used to study how service expenditures vary with household income. Older households spend significantly more on services, a pattern monotonic in household age throughout the age distribution. Compared to households in their early 30s, the service expenditure shares of households in their early 60s (resp. over 80) are 8 (resp. 27) percentage points higher. These differences are stable over the sample period, and are equally large when controlling flexibly for household income. The largest differences in expenditure patterns arise in Health, Utilities, and Domestic Services, which are intensively consumed by the old, and in Vehicle Purchases, Leasing, and Gasoline and Motor Oil, which are intensively consumed by the young.[1]

We quantify the contribution of population aging to structural change in the US in two complementary ways. First, we perform a simple within-between decomposition of the change in the service expenditure share between 1982 and 2016 (the sample period available in the CES). We write the change in the aggregate service expenditure share as a sum of two terms, one capturing changes in the service share of expenditures within each household-age group, and another capturing changes in the relative aggregate expenditure of the age groups. This decomposition shows that changes in the age-structure of the population accounted for 20% of the observed change in the service expenditure share over this period.

We then use our data along with a structural model to evaluate the relative contributions of changes in relative prices, real income, and the age distribution to the structural change process. We use a two-sector model with heterogenous households whose preferences over goods and services take the Price-Independent Generalized Linear (PIGL) form, augmented with age-specific taste shifters. These preferences were introduced by Muellbauer (1975, 1976), and recently applied to the analysis of structural change by Boppart (2014). In the model, the household-specific expenditure share on goods depends on the relative price of goods vs. services, the household real expenditures, and the household taste shifter. An

[1] It is well-known that the CES only contains health expenditures paid directly by households (i.e., it excludes payments made by Medicare, Medicaid, or private insurance). According to the National Health Expenditure Survey (NHES), out-of-pocket health expenses represent a similar fraction of total health expenses across the age distribution, so the differences in health expenditures persist after adding non-out-of-pocket expenditures. Online Appendix C.2.3 repeats our analysis after rescaling household-specific expenditure shares in the CES to match the aggregate expenditures reported in National Accounts data.

advantage of the PIGL preferences is that household-level expenditures can be easily aggregated, so that the aggregate expenditure shares are a function of relative prices, aggregate income per capita, and a weighted average of the taste shifters, with weights that correspond to the relative importance of each age group in total expenditures.

The relative strengths of the mechanisms that determine structural change in the model depend on the elasticity of substitution across sectors, the income elasticity of each sector, and the relative size of the age-specific taste shifters. Following Boppart (2014), we use the model's structural equations for the household-specific expenditure shares and cross-sectional household data to estimate the sectoral income elasticities, and use the same methodology to estimate age-specific taste shifters. We then use the structural equation for the aggregate expenditure shares and aggregate data on expenditures and prices to estimate the parameter governing the elasticity of substitution between goods and services.

Having estimated the preference parameters allows us to decompose the log change in the services share additively into the components driven by aging, technology, real income growth, and a residual which can be interpreted as arising from age- and income-neutral changes in preferences over time. We find that population aging played a significant role in the increase in the expenditure share of services during this period, accounting for about 20 percent of the total. The increase in the relative price of services accounted for about 40% of the overall change, the rise in the real incomes another 20%, and residual taste changes the remaining 20%. Finally, we combine our estimates of age-specific taste shifters for services with population estimates to project that the US service expenditure share will increase by a further 0.1 log points between today and 2050. The impact of aging on structural transformation is set to become stronger in the future compared to its past role.

Our paper contributes to a large literature that attempts to rationalize the structural transformation process (see the recent survey by Herrendorf et al., 2014). Most theories focus on the non-homotheticity of the relative demand for services with respect to income (e.g. Kongsamut et al., 2001), or on changes in relative prices driven by differential long-growth rates of productivity (e.g. Ngai and Pissarides, 2007) or capital deepening and factor intensity differences across sectors (Acemoglu and Guerrieri, 2008). Alternative recent theories for the structural transformation process have also emphasized the roles of international trade (Matsuyama, 2009; Uy et al., 2013; Cravino and Sotelo, 2019), home production (Buera and Kaboski, 2012), and changes in the labor supply driven by changes in schooling (Porzio et al., 2020), or by cohort-specific occupational choices (Cociuba and MacGee, 2018; Hobijn et al., 2018). We contribute to this literature by quantifying a complementary demand-side mechanism for the structural transformation process. Closest to our cross-country empirical results is Siliverstovs et al. (2011), who relate employment shares in 9 goods and service sectors to aging in a panel of countries. Brembilla (2018) argues that aging slows down the process of structural transformation because the price elasticity of demand for services is lower for the old compared to the young. In contrast, our analysis indicates that aging speeds up the structural transformation process since older households consume more services.

Our analysis is also related to the quantitative literature that combines the mechanisms listed above to evaluate their relative importance. Herrendorf et al. (2013) show that the relative strength of the income and substitution forces depend on whether expenditures and prices are measured using expenditure or value-added data. Boppart (2014) and Comin et al. (2021) introduce the PIGL and Generalized CES preferences, respectively, and re-evaluate these mechanisms allowing for non-vanishing long-run income effects. Swiecki (2017) uses a framework that allows for international trade across countries and shows that substitution effects are most important in developed countries, while income effects are more important in accounting for the shift out of agriculture during the early stages of the development process. We contribute to this body of work by showing that expenditure patterns differ across the age distribution, and thus an important portion of the structural change process may be driven by the population composition changes.

Finally, our paper builds on the literature documenting the differences in consumption patterns across the age distribution. Hobijn and Lagakos (2005) show that differences in spending patterns by age lead to differences in CPI inflation across age groups. Like us, they find that the largest disparities are in health care expenditures (disproportionally consumed by the elderly) and gasoline prices (disproportionally consumed by the young). Aguiar and Hurst (2013) analyze consumption expenditures on non-durable goods, and find large differences in consumption patterns of young vs. old households in food, nondurable transportation, and clothing and personal care. Hall and Jones (2007) and Reinhardt (2003) explore, theoretically and empirically, the role of aging in health expenditures. We contribute to this literature by quantifying how age-related differences in consumption patterns affect the structural transformation process.

The rest of the paper is organized as follows. Section 3.2 describes the relationship between population age and the share of services in the economy across countries, US households, and time. Section 3.3 quantifies the contribution of the observed population aging to structural change, and Section 3.4 concludes. The Online Appendix collects additional exercises and robustness results.

3.2 Population aging and structural transformation: Facts

This section presents empirical evidence documenting that population aging is systematically related to a shift in economic activity from Agriculture and Manufacturing sectors towards Service sectors. We organize our evidence in two sections, one showing how structural transformation relates to population aging across countries and time using aggregate data, and another showing how sectoral expenditure shares vary with household age using microdata for the US.

3.2.1 Cross-country evidence

We start by describing how population aging is related to structural transformation across space and time. The empirical analysis follows the approach in the Handbook chapter by Herrendorf et al. (2014), who document how economic activity reallocates across Agriculture, Manufacturing, and Services as income per capita rises. We use the same data sources and empirical strategy to document how this reallocation is related to population aging. We document the relation between population aging and the share of services in (i) employment; (ii) value added, and (iii) consumption, after controlling for changes in income.

With this in mind, we take sectoral value added and employment shares for a broad set of developed countries from EU KLEMS, which is compiled by the Groningen Growth and Development Center. The database reports hours worked and value added by sector for a sample of 20 developed countries over the 1970-2007 period. Consumption shares come from the OECD Statistics. Consumption shares can differ from value added and employment shares since they do not include investment nor exports, and they do include imports. OECD statistics report consumption for 11 countries in 16 expenditure categories for the 1970-2007 period. We follow Herrendorf et al. (2014) and classify Food Consumption as Agriculture, Semi-, Durable-, and Non-Durable Goods minus Food Consumption as Manufacturing, and the remaining categories as Services. The aging indicator is the share of the population that is 65 or older, taken from the World Development Indicators.

We evaluate the relation between population aging and the structural transformation process by estimating the following regressions:

$$\omega_{i,t}^{j} \;=\; \alpha_i^j + \beta^j Age_{i,t} + \gamma_1^j gdp\_pc_{i,t} + \gamma_2^j gdp\_pc_{i,t}^2 + \varepsilon_{i,t}^j. \tag{3.1}$$

Here, $\omega_{i,t}^{j}$ is the share of employment, value-added, or consumption in sector j in country i in year t, α_i^j is a country fixed effect, $gdp\_pc_{i,t}$ is the log of GDP per capita in country i year t, and $Age_{i,t}$ is population age in country i in year t, measured either by the share of population that is over 65 or by the average age in the country. We cluster standard errors by country.

Table 3.1: Population aging and the sectoral shares of employment and value added

	(1) $\omega_{i,t}^{Agr}$	(2) $\omega_{i,t}^{Agr}$	(3) $\omega_{i,t}^{Man}$	(4) $\omega_{i,t}^{Man}$	(5) $\omega_{i,t}^{Ser}$	(6) $\omega_{i,t}^{Ser}$
Employment Share						
Share of pop 65+	-1.980***	-0.653**	-1.351***	-0.877**	3.330***	1.530***
	(0.440)	(0.285)	(0.323)	(0.381)	(0.586)	(0.490)
Log GDP per capita		-1.240***		1.243***		-0.00306
		(0.0802)		(0.234)		(0.229)
(Log GDP per capita)^2		0.0590***		-0.0677***		0.00874
		(0.00456)		(0.0133)		(0.0126)
R^2	0.802	0.951	0.487	0.681	0.825	0.924
Value Added Share						
Share of pop 65+	-1.012***	-0.0575	-1.533***	-1.252***	2.545***	1.309***
	(0.261)	(0.105)	(0.297)	(0.381)	(0.353)	(0.352)
Log GDP per capita		-0.705***		1.528***		-0.823***
		(0.0402)		(0.166)		(0.138)
(Log GDP per capita)^2		0.0326***		-0.0818***		0.0492***
		(0.00234)		(0.00990)		(0.00812)
Observations	707	707	707	707	707	707
R^2	0.700	0.953	0.579	0.760	0.772	0.874

Notes: This table reports the results of estimating equation (3.1). The outcome variables are employment shares (top panel) and value added shares (bottom panel) in agriculture (Agr), manufacturing (Man) and services (Ser). Population age is proxied by the share of population over 65. All specifications include country fixed effects. Standard errors clustered at the country level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Table 3.2: Population aging and the sectoral consumption share, OECD

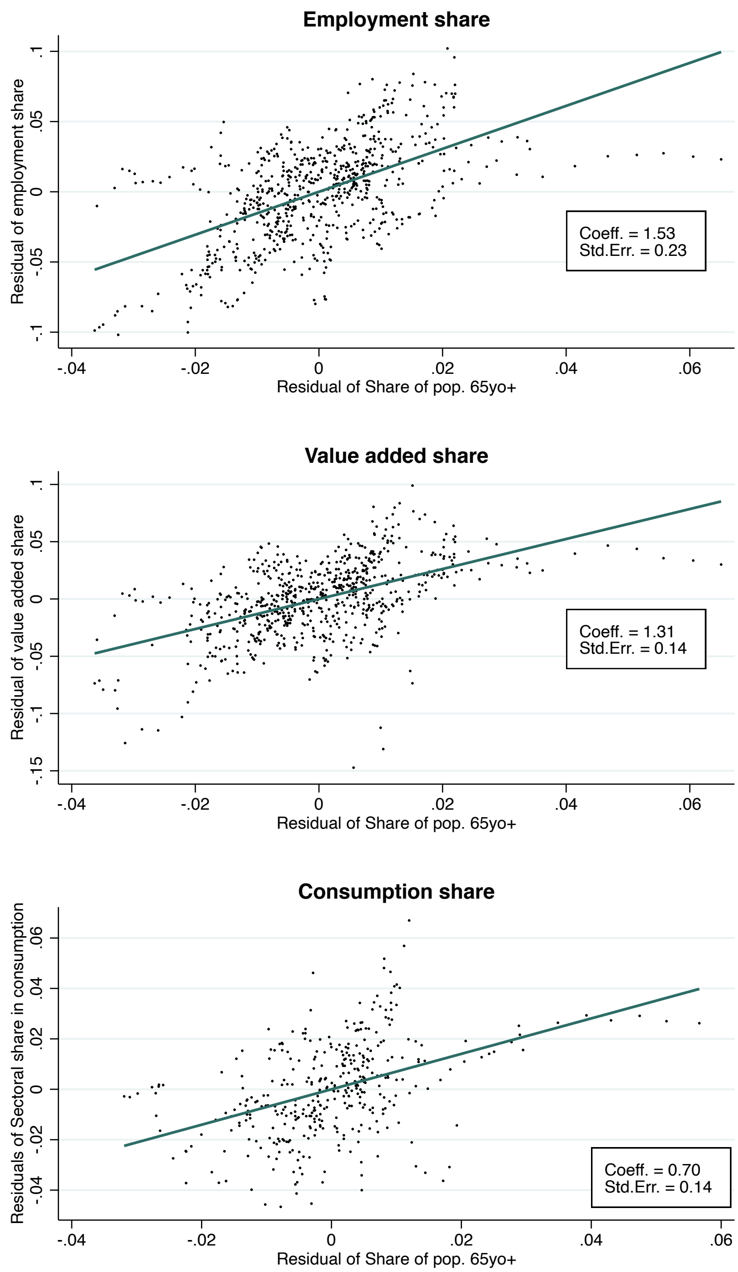
	(1) $\omega_{i,t}^{Agr}$	(2) $\omega_{i,t}^{Agr}$	(3) $\omega_{i,t}^{Man}$	(4) $\omega_{i,t}^{Man}$	(5) $\omega_{i,t}^{Ser}$	(6) $\omega_{i,t}^{Ser}$
Share of pop 65+	-1.702**	-0.498*	-0.793**	-0.205	2.496***	0.703***
	(0.560)	(0.261)	(0.293)	(0.271)	(0.614)	(0.219)
Log GDP pc		-0.455***		0.714***		-0.259
		(0.136)		(0.170)		(0.145)
(Log GDP pc)^2		0.0181**		-0.0406***		0.0225**
		(0.00780)		(0.00987)		(0.00852)
Observations	377	377	377	377	377	377
R^2	0.767	0.957	0.803	0.860	0.789	0.948

Notes: This table reports the results of estimating equation (3.1). The outcome variables are consumption expenditure shares (bottom panel) in agriculture (Agr), manufacturing (Man) and services (Ser). Population age is proxied by the share of population over 65. All specifications include country fixed effects. Standard errors clustered at the country level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Table 3.1 reports the results of separately estimating equation (3.1) for each sector. Both the shares of hours worked and of value added are decreasing in income per capita in the Agriculture and Manufacturing sectors, but increasing in the Service sector, in line with the evidence surveyed by Herrendorf et al. (2014). The coefficient of interest β^j is negative for Agriculture and Manufacturing, and positive for Services, indicating that indeed aging is associated with a reallocation of economic activity towards services, even after controlling for changes in income. These findings are robust to measuring shares both in terms of value-added or employment, and to using either of our two measures of population age. Table 3.2 reports the analogous results for consumption. The coefficients are economically significant. The 0.7 coefficient in column 6 of Table 3.2 implies that, other things constant, if the US had the age structure of Japan in 2007, its share of services in consumption would be 5 percentage points higher.

Figure 3.1 plots employment, value added, and consumption shares, residualized with respect to the log of GDP per capita, the log of GDP per capita squared, and country fixed effects against population age. For all three outcome variables, there is a pronounced positive relationship between population aging and the share of services.

Figure 3.1: Residualized service sector shares



Notes: Each dot represents a country-year. On the y-axis is the residual of a regression of the share of services in hours worked (top panel), value added (middle panel), or consumption (bottom panel) on log GDP per capita, log GDP per capita squared, and country fixed effects. On the x-axis is the residual of a regression of the share of the population that is 65+ on GDP per capita, GDP per capita squared, and country fixed effects. The boxes report the slope coefficient and the standard errors clustered at the country level.

The Online Appendix presents a comprehensive analysis of the cross-country data. In particular, it: (i) shows unconditional relationships; (ii) uses average age as an alternative aging variable; (iii) adds further controls, such as trade openness, government size, and relative price trends, and (iv) uses sectoral shares data from other datasets (WDI and the UN), that cover a much broader range of countries.

Cross-country data let us establish macro-level correlations between aging and sectoral expenditure shares across time and space. The downside of the macro approach is that it is difficult to distinguish between the effects of aging *per se* and other confounding factors, such as other country characteristics or long-run trends. The following Section overcomes these limitations by using instead household-level microdata for the US.

3.2.2 Household-level evidence

Our household-level data come from the US Consumer Expenditure Survey (CES) and cover the 1982-2016 period. We use the Interview Module of the CES, which surveys about 12,000 households per year. The Interview Module collects households' responses about their purchases across 350 distinct expenditure categories, as well as other demographic information at the household level. We consider urban households of all ages and drop household/quarter observations if either service expenditures, goods expenditures, or pre-tax income are zero.

We weight the households using household weights in the *FMLI.dta* files of the CES. The CES interviews households up to four times about their expenditures in the preceding quarter to capture annual expenditures. Since our analysis is at the calendar quarter level, we follow Cravino et al. (2020) and multiply the raw CES weights by the fraction of months from each interview corresponding to each given calendar quarter. The sum of our modified weights in each calendar quarter approximately adds up to the number of US urban households. We use the average age of household members as the measure of age for our baseline analysis. Online Appendix C.2.1 shows that our results are robust to using the reference person's age, i.e. age of the household head.

We aggregate expenditures into goods and services following Aguiar and Bils (2015).[2] For our baseline results, we focus on how the share of non-housing service expenditures

[2]See Appendix Table C.4 for the breakdown. Relative to Aguiar and Bils (2015), we disaggregate two sectors considered goods in that paper – "Personal Care" and "Other vehicle expenses" – into their service and goods sub-components. For instance, instead of counting all "Personal Care" expenditures as goods, we take advantage of the fact that the CES disaggregates this category into "Personal Care Goods" and "Personal Care Services," and apportion those to goods and services accordingly. None of the quantitative or qualitative conclusions change if we use the exact Aguiar and Bils (2015) classification without this refinement (results available upon request). We classify 87 percent of the non-housing expenditures in the CES as either goods or services (following Aguiar and Bils, 2015, we do not classify "Pensions" and "Personal Insurance").

to the overall non-housing expenditures changes with household age. We do not include housing in expenditure because in the CES the rental value of owner-occupied housing is self-reported and thus may not be directly comparable to rent payments for renters. Since home ownership rates change substantially over the life cycle, the switches between owner-occupied implicit rent value and actual paid rent may complicate the comparison across age groups. Online Appendix C.2.1 reports results including housing in the analysis and shows that the treatment of housing does not alter the main conclusions.

The top panel of Figure 3.2 plots the expenditure share on services across households of different ages. Each color of the dots represents a different period. There is a clear positive monotonic relationship between the service expenditure share and the average age of the household members. The differences are large: service expenditure shares of households in their 60s are about 25 percent larger than for the households in their 30s (0.5 vs. 0.4). Households in their 80s have expenditure shares in services that are almost 70% higher than those in their 30s (0.68 vs 0.40). These patterns are stable over time. While later periods tend to feature higher service expenditure shares overall, the cross-age differences are pronounced in all time periods.

Controlling for income The unconditional patterns in Figure 3.2 may arise from income differences across age groups. This section shows that this is not the case. To control flexibly for income, we estimate a regression that projects the service expenditure shares on age dummies, while controlling for income decile dummies and other household characteristics and region-time fixed effects:

$$\omega_t^{s,h} \quad = \quad \delta^a + \delta^{inc} + \gamma X_{r,t}^h + \delta_{r,t} + \varepsilon_t^{s,h}. \tag{3.2}$$

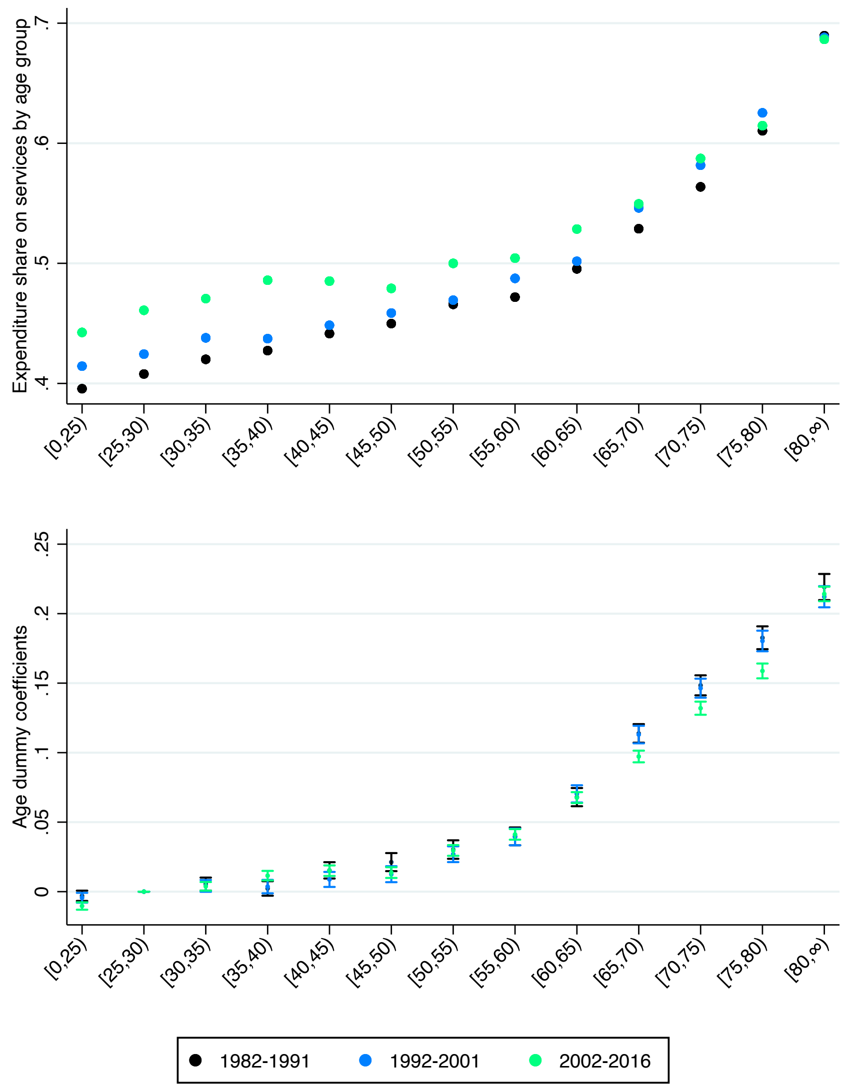
Here $\omega_t^{s,h}$ is the service expenditure share of household h at time t, δ^a are household age group dummies and δ^{inc} are income decile dummies (the income decile dummies are provided by the CES). X_{rt}^h are demographics dummies for the number of household members (2, 3-4, 5+) and dummies for the number of household earners (1, 2+). These are typically used in the literature (e.g. Aguiar and Bils, 2015). Following Comin et al. (2021) we also control for differences in household-specific prices by including region-time dummies, $\delta_{r,t}$. The implicit assumption behind this control is that households within a region face the same prices.

We estimate equation (3.2) separately for each decade for which the CES data are available. The bottom panel of Figure 3.2 plots the age group dummies, which measure differences in service expenditure shares of the age group relative to age group 25-30. The 95% confidence bands based on standard errors clustered by household are depicted around the point estimates. There are large differences in service expenditures across households of different ages, even conditioning on income and prices. These conditional differences are nearly as large as the unconditional ones reported in Figure 3.2. As in the raw data, households in their 60s have service expenditure shares 10-12 percentage points higher than households in their 30s, and households in their 80s' service expenditure shares are more than 20 points higher. The age dummies are precisely estimated, and quite stable over time.

Online Appendix C.2.1 presents additional descriptive statistics regarding service consumption patterns in the CES. Appendix Figure C.12 reproduces Figure 3.2 using the age of the reference person (i.e. household head) instead of average age in the household, and shows that the results are virtually unchanged. Additionally, Appendix Figure C.13 replicates Figure 3.2 adding housing as part of the overall consumption and services. Appendix Figure C.14 adds age-group-specific price indices as controls in equation (3.2).

Decomposing consumption differences Table 3.3 shows the differences in expenditure shares across young and old households for the main consumption categories. It reports the difference in expenditure shares for each category for the 25-30 vs. the age groups starting at 60-65. Unsurprisingly, the largest disparity arises in health expenditures, where the consumption expenditure share of the 60-65 (80+) age group is 5.6 (15.3) percentage points larger than that of the 25-30 age group. The table shows that the elderly also spend relatively more on Cash Contributions, Domestic Services, and Utilities. In contrast, for Vehicle Purchasing and Leasing, the expenditure share of the 60-65 (80+) age group is 3.8 (11.24) percentage points smaller than that of the 25-30 age group.

Figure 3.2: Service consumption by average age of household members



Notes: The top panel displays the average household-level expenditure shares on services in the CES by age group (x-axis), for 3 time periods. The bottom panel displays the age dummies resulting from estimating equation (3.2). Each dot represents the point estimate of the age dummies for a particular decade in the CES data. The omitted dummy is that of age group 25-30. The bands report the 95% confidence intervals based on standard errors clustered at the household level.

It is worth noting that the differences in consumption patterns across age groups are not mainly driven by retirement. The CES contains an indicator for whether the reference person in the household is retired. We can include this indicator when estimating equation (3.2).

When controlling for age dummies, the retirement dummy has at most a modest positive effect on the service expenditure share. In contrast, the age dummies are mainly unchanged after including retirement as a control (results available upon request).

Table 3.3: Expenditure shares by consumption category relative to age group 25-30

	Age groups				
	60-65	65-70	70-75	75-80	80+
Health	5.62	7.90	10.17	12.42	15.25
Cash contributions	3.41	4.44	5.59	6.45	9.48
Domestic services (excl. childcare)	1.45	1.77	2.15	2.85	6.05
Utilities	1.06	1.23	1.88	2.57	3.41
Personal care services	0.13	0.20	0.30	0.36	0.44
Food at home	-0.89	-0.57	0.03	0.51	0.45
Personal care goods	-0.01	-0.01	-0.01	-0.02	-0.01
Public transport	0.37	0.36	0.25	0.18	-0.41
Tobacco	0.03	-0.17	-0.38	-0.58	-0.77
Childcare	-0.85	-0.85	-0.86	-0.87	-0.80
Shoes and other apparel	-0.37	-0.47	-0.58	-0.79	-0.85
Children's clothing	-0.76	-0.77	-0.88	-0.94	-1.03
Entertainment fees, adm., reading	-0.08	-0.14	-0.32	-0.61	-1.04
Alcoholic beverages	-0.33	-0.46	-0.64	-0.84	-1.04
Furnitures and Fixtures	-0.17	-0.30	-0.62	-0.83	-1.21
Appliances	0.14	-0.20	-0.49	-0.74	-1.36
Men's and women's clothing	-0.32	-0.57	-0.73	-1.13	-1.69
Car maintenance, repairs, insurance	-0.31	-0.55	-0.71	-0.78	-1.84
Food away from home	-0.55	-0.77	-1.17	-1.64	-2.26
Entertainment equipment	-0.20	-0.83	-1.78	-2.23	-2.80
Education	-2.63	-2.86	-2.90	-2.80	-2.99
Gas	-0.98	-1.41	-1.89	-2.48	-3.70
Vehicle purchasing, leasing	-3.75	-4.98	-6.41	-8.04	-11.24
Services	7.61	10.73	14.37	18.12	25.26

Notes: This table reports the differences in expenditure shares across the major consumption categories between age groups starting at 60-65 and households aged 25-30 in the CES.

Accounting for differences between CES and National Accounts data It is well known that the aggregated expenditure shares in the CES do not match those in the Personal Consumption Expenditure module of the National Income and Product Accounts compiled by the BEA. One reason for this discrepancy is that the CES only reports out-of-pocket expenses by private households, which may differ from economy-wide aggregate consumption and misrepresent expenditure differences across households. This may be especially salient in healthcare, since the CES data do not include spending by Medicaid, Medicare, and private insurance for services rendered to the household. Appendix Table C.6 reports the shares of out-of-pocket expenditures in total health expenditures in National Health Expenditure Survey (NHES) in the first and last year available in that survey, by broad age groups. Out-of-pocket expenditures represent a similar fraction of the total health expenditures across the age distribution. Thus, the relative health expenditure differences across the age distribution would persist after adding the non-out-of-pocket expenses.

With this in mind, we map our analysis to the National Accounts data, by augmenting the CES data with data from the Personal Consumption Expenditures (PCE) from the BEA. In particular, we rescale the expenditures in each consumption category to match aggregate consumption expenditures by category in the National Accounts (PCE BEA) data. These rescaled data reproduce the aggregate sectoral expenditure shares in the BEA, while preserving the heterogeneity across households present in the CES. Online Appendix C.2.3 details this procedure and replicates the results in this section and Section 3.3 using the rescaled dataset, and shows that the results are similar to the baseline.[3]

3.3 Accounting for structural change in the US

This section quantifies the contribution of observed changes in the age distribution to the observed changes in sectoral consumption shares in the US between 1982 and 2016. We conduct this exercise using two alternative methodologies. The first is a shift-share decomposition of the increase in the share of services in total consumption into the part that arises from reallocation of expenditures between age groups vs. changes in expenditures within age groups. The second is a quantitative model of structural transformation that allows us to compare the contribution of population aging to the contributions of the income and price effects that have been the focus of most of the structural transformation literature.

3.3.1 Within-between decomposition

We start with a decomposition of the observed rise in the share of services in total consumption in the CES between 1982 and 2016. We can write the share of services in

[3] Rescaling the CES data using NHES is challenging because the expenditure categories in the CES do not map readily into those in NHES, as the former presents the expenses from the perspective of the household, whereas the latter records the sources of revenue of the healthcare provider. In addition, NHES by age group only goes back to 2002.

aggregate consumption as:

$$\Omega_t^s = \frac{\sum_a e_t^{s,a}}{\sum_a \sum_j e_t^{j,a}} = \sum_a \omega_t^{s,a} \times s_t^a, \tag{3.3}$$

where $e_t^{j,a}$ are total consumption expenditures by age group a in consumption sector j, $\omega_t^{s,a} \equiv \frac{e_t^{s,a}}{\sum_j e_t^{j,a}}$ is the share of services in total expenditures by age group a, and $s_t^a \equiv \frac{\sum_j e_t^{j,a}}{\sum_a \sum_j e_t^{j,a}}$ is the share of age group a in aggregate expenditures. Letting $\Delta x \equiv x_1 - x_0$ and $\overline{x} \equiv [x_1 + x_0]/2$ denote the change and the average of a variable across periods $t = 1$ and $t = 0$ we can write:

$$\Delta\Omega^s = \underbrace{\sum_a \Delta\omega^{s,a} \cdot \overline{s}^a}_{\text{Within}} + \underbrace{\sum_a \overline{\omega}^{s,a} \cdot \Delta s^a}_{\text{Between}}. \tag{3.4}$$

Equation (3.4) expresses the change in the service share of expenditures as the sum of two terms. The term labeled 'Within' captures changes in the age-specific expenditure shares, $\Delta\omega^{s,a}$, while the term labeled 'Between' captures changes in the share of age group a in aggregate expenditures, Δs^a.

Table 3.4: Population aging and changes in the services share

Panel A: Expenditure shares across the age distribution

	Pop $_{1982}$	s^a_{1982}	$\omega^{s,a}_{1982}$	Pop $_{2016}$	s^a_{2016}	$\omega^{s,a}_{2016}$
0-25	31.8	31.2	38.8	20.4	19.8	47.2
25-30	13.5	16.1	39.9	11.4	11.8	47.6
30-35	9.4	11.2	42.1	9.4	10.8	50.3
35-40	6.2	7.6	43.0	7.1	7.9	49.5
40-45	4.6	5.4	45.4	5.9	6.5	53.4
45-50	3.6	4.0	45.6	5.2	5.5	51.4
50-55	3.8	4.0	45.7	6.1	6.1	51.4
55-60	5.1	4.9	47.4	6.7	6.9	51.9
60-65	5.7	5.2	50.6	7.5	7.8	58.1
65-70	5.9	4.5	53.0	6.8	6.3	56.7
70-75	4.3	2.9	58.7	5.1	4.6	57.4
75-80	3.3	1.8	59.5	3.4	2.8	60.8
80+	2.9	1.2	67.5	5.0	3.4	69.6

Panel B: Within-between decomposition

	Average		Reference	
	Value	%	Value	%
Within	0.0663	78.2	0.0675	79.7
Between	0.0185	21.8	0.0172	20.3
Total	0.0848	100	0.0848	100

Notes: In Panel A, ‘Pop’ reports the share of the population in each age group, and s^a_t and ω^a_t are defined as in Equation (3.4). Panel B reports the results of the decomposition in equation (3.4). ’Average’ uses the average age across all household member as the age of the household. ’Reference’ uses the age of the head in the household.

We take equation (3.4) to the data by breaking the US population into the 13 age groups as in Section 3.2.2, measuring age both by the average age of all household members and by the age of the household head. Panel A of Table 3.4 reports the terms $\omega^{s,a}_t$ and s^a_t in equation (3.3) for each age group in 1982 vs. 2016. As already documented in Figure 3.2, older households allocate a significantly larger fraction of their expenditures towards services than younger ones: both in 1982 and 2016, the share of expenditure in services is more than 50% higher for households over 80 than for those aged 25-30. In addition, the table shows a large increase in the share of expenditures that is accounted for by older households: households 65 and older accounted for 10.4 percent of total expenditures in 1982, and 17.1 percent in 2016, a 64% increase. The share of expenditures that goes to households 80 and older nearly tripled, going from 1.2 to 3.4 percent. The counterpart of this increase is the decline in the share of expenditures that goes to households 30 and younger, from 47.3 to 31.6 percent.

Panel B of Table 3.4 reports the results of the decomposition in equation (3.4). The share of services in total expenditures increased by 8.5 percentage points during the 1982-2016 period. The table shows that 1.85 percentage points, about a fifth of the increase, are attributed to between age group changes in expenditures. The remainder is attributed to changes in expenditure shares within groups. The table shows that the numbers are similar if we instead measure household age by the age of the household head. Appendix Table C.5 shows that the results are somewhat smaller though still economically significant if we count housing as part of service expenditures. The decomposition in (3.4) is implemented using age-specific total expenditure shares s^a, which change both due to demographics and the age distribution of income. Appendix C.2.4 shows that most of the Between effect documented in this section is due to demographics rather than the changing age distribution of income.

3.3.2 Structural model

This section sets up a model to quantify the contribution of changes in population age, income, and relative prices to the structural transformation process in the US. We study an economy populated by N_t households indexed by h that are heterogeneous in their preferences and their expenditure levels e_t^h. Households consume goods (g) and services (s). The indirect utility of household h takes the form:

$$\mathcal{V}^h\left(P_t^s, P_t^g, e_t^h\right) = \frac{1}{\epsilon}\left[\frac{e_t^h}{P_t^s}\right]^{\epsilon} - \frac{\nu_t^h}{\gamma}\left[\frac{P_t^g}{P_t^s}\right]^{\gamma} - \frac{1}{\epsilon} + \frac{\nu_t^h}{\gamma}, \tag{3.5}$$

where P_t^s and P_t^g are the prices of goods and services, and the parameters satisfy $0 \leq \epsilon \leq \gamma \leq 1$ and $\nu_t^h \geq 0$. This utility function belongs to a subclass of Price Independent Generalized Linearity (PIGL) preferences (Muellbauer, 1975, 1976; Boppart, 2014), with household-specific taste shifters ν_t^h. Using Roy's identity, we can show that expenditure shares are given by:

$$\omega_t^{g,h} \equiv \frac{e_t^{g,h}}{e_t^h} = \nu_t^h \left[\frac{P_t^s}{e_t^h}\right]^{\epsilon}\left[\frac{P_t^g}{P_t^s}\right]^{\gamma}, \tag{3.6}$$

where $e_t^{j,h}$ is the expenditure by h on sector j, and $\omega_t^{s,h} \equiv \frac{e_t^{s,h}}{e_t^h} = 1 - \omega_t^{g,h}$. The aggregate expenditure share on goods is:

$$\Omega_t^g \equiv \frac{\sum_h e_t^{g,h}}{\sum_h e_t^h} = \left[\frac{P_t^s}{e_t}\right]^\epsilon \left[\frac{P_t^g}{P_t^s}\right]^\gamma \frac{1}{N_t} \sum_h \nu_t^h \left[\frac{e_t^h}{e_t}\right]^{1-\epsilon},$$

where $e_t \equiv \frac{1}{N_t} \sum_h e_t^h$ denotes average expenditures per household. Aggregate shares depend on real per capita expenditures in units of services, $\frac{e_t}{P_t^s}$, the relative price of goods vs. services, $\frac{P_t^g}{P_t^s}$, the extent of income inequality, $\frac{e_t^h}{e_t}$, and the taste shifters, ν_t^h.

In what follows we assume that households can be grouped according to their age, and denote the number of households of age a by N_t^a, with $\sum_a N_t^a = N_t$. We further assume that the taste shifters take the form $\nu_t^h = \nu_t \mu^a \mu_t^h$, with $\frac{1}{N_t} \sum_h \mu_t^h = 1$. This implies that the household-specific taste shifter has an aggregate component ν_t, an age-specific component μ^a, and an idiosyncratic component μ_t^h. The aggregate expenditure share can then be written as:

$$\Omega_t^g = \left[\frac{P_t^s}{e_t}\right]^\epsilon \left[\frac{P_t^g}{P_t^s}\right]^\gamma \bar{\mu}_t \phi_t \nu_t. \tag{3.7}$$

Here, $\bar{\mu}_t \equiv \sum_a s_t^a \mu^a$ is the weighted average of the age-specific taste shifters, with weights given by expenditure shares $s_t^a = \frac{e_t^a N_t^a}{e_t N_t}$. The composite $\phi_t \equiv \frac{1}{N_t} \sum_h^{N_t} \frac{\mu^a}{\bar{\mu}_t} \left[\frac{e_t^h}{e_t}\right]^{1-\epsilon}$ is a measure of the inequality in the economy, weighted by household preferences.[4]

Parameterization We are interested in decomposing changes in expenditure shares into the components due to changes in real income per capita $\frac{e_t}{P_t}$, relative prices $\frac{P_t^g}{P_t^s}$, and changes in the share of expenditures that correspond to the different age groups in the population, $s_t^a \equiv \frac{e_t^a N_t^a}{e_t N_t}$. To conduct this exercise we need to parameterize the income and substitution effects governed by ϵ and γ, as well as the age effects captured by $\bar{\mu}_t$.

We follow Boppart (2014) and proceed in two steps. First we use the cross-section of households from the CES and estimate equation (3.6) in logs. The estimating equation is:

$$\ln \omega_t^{g,h} = \beta_0 + \beta_1 \ln e_t^h + D^a + \delta_{r,t} + \varepsilon_t^h, \tag{3.8}$$

where $\beta_0 + \delta_{r,t} = \ln (P_t^s)^{\epsilon-\gamma} (P_t^g)^\gamma$, $\beta_1 = -\epsilon$, and $\varepsilon_t^h = \ln \mu_t^h$. $D_a = \ln \mu^a$ is an age dummy that captures the taste shifter of the age group relative to an omitted age group. Without loss of generality we normalize $\mu^a = 1$ for age group [25,30). Using these estimates for ϵ and μ^a, we can construct the time series of $\bar{\mu}_t$ and ϕ_t. We can then obtain the price elasticity γ

[4]This assumes that within age groups income and idiosyncratic preferences are uncorrelated.

from a regression of equation (3.7) in logs:

$$\ln \Omega_t^g = b_1 \ln P_t^g + b_2 \ln P_t^s + b_3 X_t + \ln \nu_t, \tag{3.9}$$

where $X_t \equiv \ln\left(e_t^{-\epsilon}\bar{\mu}_t\phi_t\right)$, $b_1 = \gamma$, and the other coefficients satisfy the restrictions $b_3 = 1$, and $b_2 = \epsilon - b_1$.

Columns 1 and 2 in Table 3.5 report the results of estimating (3.8) with OLS. To address measurement error in the CES expenditure data, Columns 3 and 4 report the results of IV estimation with expenditure instrumented by income, as is customary in the literature (see, e.g. Boppart, 2014; Aguiar and Bils, 2015).[5] Table 3.5 yields an estimate of ϵ of 0.12, which is somewhat smaller than the $\epsilon = 0.2$ found by Boppart (2014).[6] Appendix Table C.7 displays the estimates for the age dummies, and shows that our results are robust to using the age of the reference person. Appendix Table C.8 shows that the results for ϵ are only slightly different if we consider housing as part of service consumption. The age dummies are relatively large and statistically different from zero, and decrease monotonically with age, indicating that older households spend relatively less on goods after controlling for real income.

Table 3.6 reports the estimation results for equation (3.9). To implement it, we construct P_t^g and P_t^s by chain-weighting category-specific price series from NIPA Table 2.4.4, using the expenditure shares for each category within either goods or services.[7] Our estimate for γ is 0.15. Both γ and ϵ are precisely estimated and significantly different from zero, and satisfy the restriction $\gamma > \epsilon > 0$.

[5] We use pre-tax income inclusive of transfers and pension income.

[6] Roughly half of the difference with the Boppart (2014) value is due to the different controls used in that paper vs. ours (our regression includes age decile dummies, which are key for our exercise). The remaining half is mainly due to differences in the classification of CES categories into goods and services (see Table C.4).

[7] Since the price data are not required to estimate equation (3.8), our estimates of ϵ and the taste shifters μ_t^a are not affected by potential biases in these data.

Table 3.5: Estimates of equation (3.8)

	(1)	(2)	(3)	(4)
Dep. var.: $\ln \omega_t^{g,n}$				
$\ln e_t^n$	-0.0476***	-0.0478***	-0.116***	-0.117***
	(0.000642)	(0.000643)	(0.00178)	(0.00179)
Type	OLS	OLS	IV	IV
Time FE	Yes	No	Yes	No
Region-Time FE	No	Yes	No	Yes
Observations	1,324,874	1,319,092	1,226,096	1,220,472
R^2	0.122	0.125	0.099	0.100

Notes: This table reports the results of estimating equation (3.8). The outcome variable is household expenditure share on goods. Standard errors clustered at the household level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Table 3.6: Estimates of equation (3.9)

	(1)	(2)
Dep. var.: $\ln \Omega_t^g$		
$b_1 = \gamma$	0.143***	0.153***
	(0.0105)	(0.0105)
Age variable	Average	Reference
Observations	35	35
R^2	0.846	0.862

Notes: This table reports the results of estimating equation (3.9). The outcome variable is aggregate expenditure share on goods. Standard errors in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Quantitative results Taking logs in equation (3.7) and rewriting everything in terms of share of consumption on services, we obtain:[8]

$$\hat{\Omega}_t^s \approx -\frac{\Omega_{82}^g}{\Omega_{82}^s}\left\{\underbrace{\epsilon\left[\hat{P}_t - \hat{e}_t\right]}_{\text{Income}} + \underbrace{\left[\gamma - \epsilon\Omega_t^g\right]\left[\hat{P}_t^g - \hat{P}_t^s\right]}_{\text{Substitution}} + \underbrace{\hat{\mu}_t}_{\text{Aging}} + \underbrace{\hat{\phi}_t + \hat{\nu}_t}_{\text{Residual}}\right\}, \tag{3.10}$$

where we used the notation $\hat{x}_t \equiv \ln x_t - \ln x_{82}$ to denote the cumulative log change of a variable between the first year in our sample and time t, and $\hat{P}_t \equiv \left[1 - \Omega_t^g\right] \hat{P}_t^s + \Omega_t^g \hat{P}_t^g$

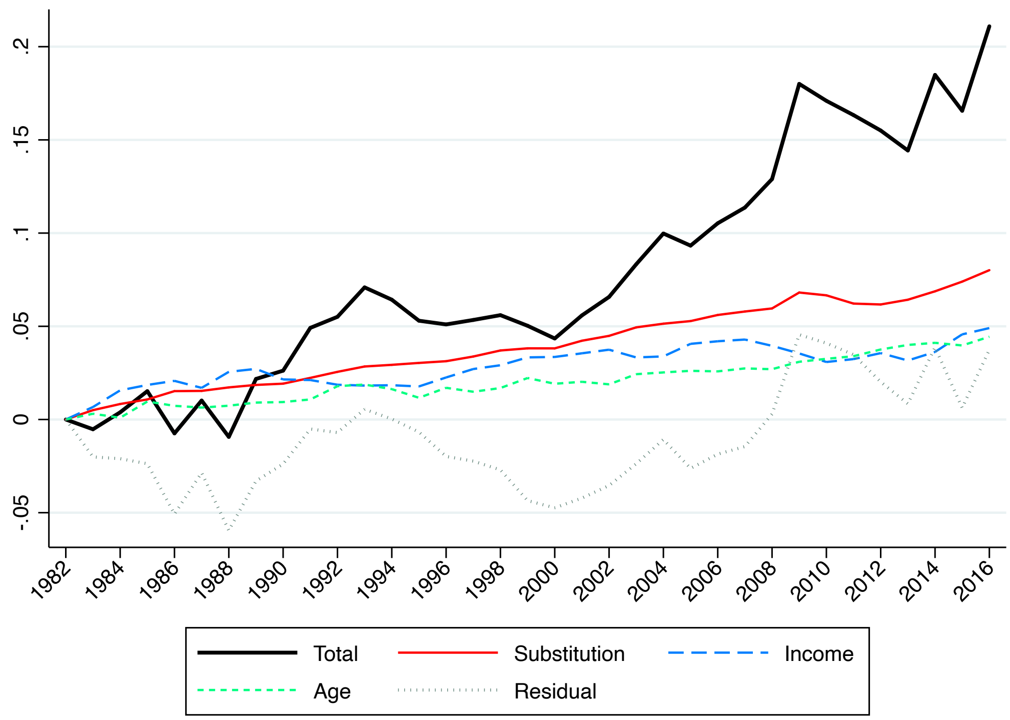
[8]See the Online Appendix C.2.5 for the derivation. The elasticity of the expenditure share on goods with respect to the relative price of goods to services, $\gamma - \epsilon\Omega_t^g$, ranges from -0.08 to -0.09 depending on year given our estimates of γ and ϵ and the goods expenditure share Ω_t^g in the data. The income elasticity of the goods expenditure share is simply ϵ.

to denote the log change in the aggregate price index. Equation (3.10) shows that log-changes in the aggregate expenditure share of goods are additively separable into the effects of changes in 'Income', 'Substitution', 'Aging', and a residual.[9] This decomposition is plotted in Figure 3.3. The expenditure share in services grew by about 0.2 log points between 1982 and 2016 in the CES data. The contribution of population aging $\hat{\bar{\mu}}_t$ was nearly 0.05 log points, about a fifth of the total change. About 40% of the total was due to the rise in the relative price of services (labeled 'Substitution'), and another 20% due to the income effect.[10] The residual accounted for remaining roughly 0.05 log points. Appendix Figure C.15 shows that the results are unchanged when using the age of the reference person as the household age variable. Appendix Figure C.16 shows that the absolute contribution of aging stays unchanged when considering housing as part of service consumption. Appendix C.2.4 implements an alternative decomposition that isolates purely demographic change, and shows that most of the Aging effect documented in this section is due to demographics rather than the changing age distribution of income.

[9]The residual includes both the change in the inequality measure $\hat{\phi}_t$ and the unexplained shifts in taste $\hat{\nu}_t$. In the data, the changes in the inequality term have a negligible effect on the aggregate service share the throughout the period.

[10]Ignoring the impact of aging on the service expenditure share increases the size of the substitution effect, from 0.08 to 0.13 log points. This is because abstracting from aging increases the estimate of γ by about 20%. This is intuitive: γ is estimated by relating the change in the aggregate service share to the change in prices (equation 3.9). Our procedure nets out the impact of aging on the service share (X_t), and thus the relative prices have a smaller change in expenditure shares to explain. Thus, if we ignore the impact of aging, a higher γ is needed for the relative price changes to account for the change in expenditure shares. A higher γ, in turn, increases the size of the implied substitution effect.

Figure 3.3: Accounting for structural change in the US



Notes: This figure displays the decomposition (3.10) for the US from 1982 to 2016.

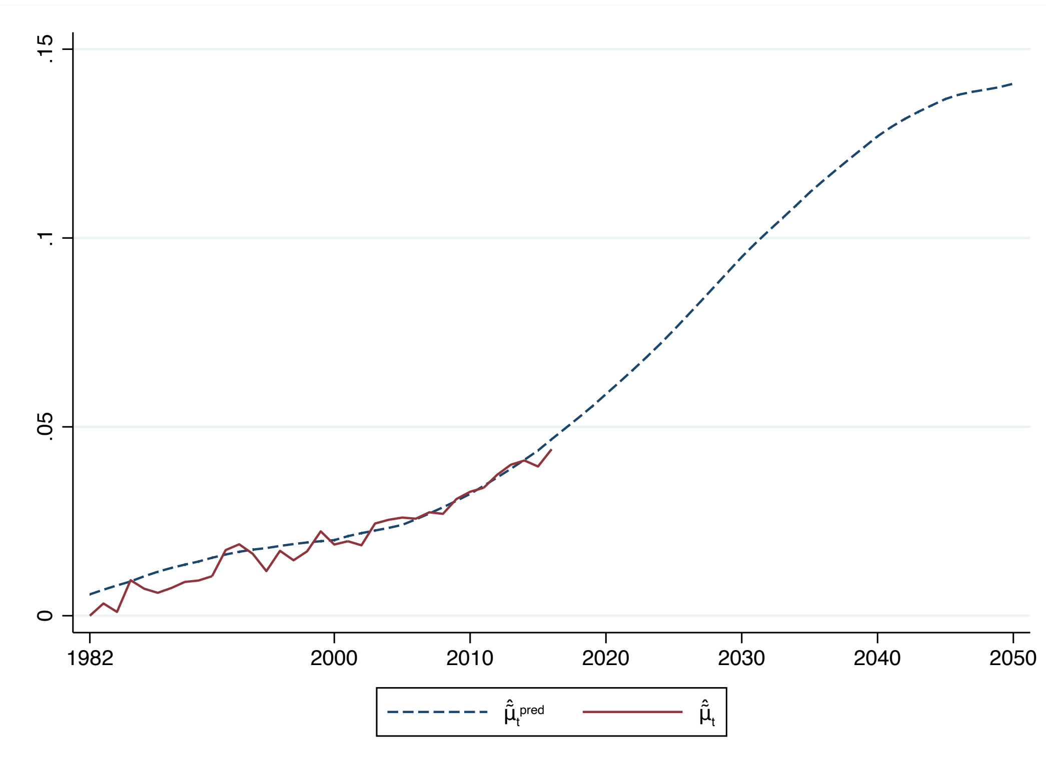
Projected changes in expenditure shares To further illustrate the potential strength of aging as a driver of structural transformation, we compute the contribution of the projected changes in population composition to structural transformation in the future. To do this, we use the US population projections to the year 2050 from the World Bank's "Population estimates and projections" database. Because our estimates of the age taste shifters μ^a are at the household level, while the population projections are for population shares by age group, we must convert trends in population into trends in numbers of households. We do this by means of fitting the following regression to map population shares ($PopSh_t^a$) into household age shares:

$$\frac{N_t^a}{N_t} = \beta_1 PopSh_t^a + \beta_2 (PopSh_t^a)^2 + \varepsilon_t \qquad \text{for } t = 1982, ..., 2016. \tag{3.11}$$

We use a squared term because this specification fits the in-sample data better. Then, for future years we construct $s_t^{a,pred}$ putting together the prediction for the share of age a households among all households $\frac{N_t^a}{N_t} = \widehat{\beta}_1 PopSh_t^a + \widehat{\beta}_2 (PopSh_t^a)^2$ for $t = 2017, ..., 2050$

and $\frac{\bar{e}^a_{2011-16}}{\bar{e}_{2011-16}}$ computed using data for 2011-16. We then construct $\bar{\mu}_t^{pred} = \sum_a s_t^{a,pred} \mu^a$ for t =2017,...,2050. Note that this exercise captures only the contribution of projected population aging on the service share, as it assumes the relative incomes of the different age groups stay constant.

Figure 3.4: Projected change in the service share due to aging in the US



Notes: This figure displays the estimated $\hat{\bar{\mu}}_t$ from 1982 to 2016, the estimated $\hat{\bar{\mu}}_t$ from 1982 to 2016 based on the quadratic projection of household numbers on population shares (3.11), and the projected $\hat{\bar{\mu}}_t$ for 2017-2050 for the US.

Figure 3.4 reports the results. It turns out that the contribution of aging to structural change over the past 35 years is relatively modest compared to its projected future contribution. The service expenditure share will increase by a further 0.1 log points under the current population aging projections to 2050, even with price of services and real income held constant at today's values. This implies that the service expenditure share in the CES will go from 0.52 in 2016 to 0.57 in 2050. The pace of the increase in the service share accelerates modestly from current rates, before leveling off. This is driven by the faster projected pace of aging between now and the mid-2030s. To evaluate the fit of the population-to-household projection (3.11), the figure also plots the "prediction" for the structural change over the period for which we do have household data, 1982-2016. The projection fits quite well.

3.4 Conclusion

This paper documented and quantified the role of population aging in the structural transformation process. Older individuals devote a larger share of their expenditures to services, so the relative size of the service sector grows as the population ages. We document large differences in sectoral expenditure shares across households of different ages in the US CES data, with older households spending relatively more on services. We then use a shift-share decomposition and a quantitative model to show that changes in the US population age accounted for about a fifth of the increase in the consumption share of service expenditures observed between 1982 and 2016. In our quantitative model, the contribution of population aging to the observed structural change in the US during this period is similar to the contribution of real income growth. Projections for the changes in the service expenditure share due to aging in the US suggest that the future impact of aging on structural transformation will be, if anything, larger than its role to date.

APPENDICES

APPENDIX A

Appendix to Chapter I

A.1 Additional results from Section 1.2

A.1.1 Data

For the baseline analysis, I use the following data sources:

Table A.1: Data (variables, description, source, period), several countries, 1990-2019

Variable	Description	Source	Period
BIS_CBPOL	Policy rate	BIS	1990M1:2019M12
CPGRLE01	CPI: All items non-food non-energy	OECD	1990Q1:2019Q4
CPALTT01	CPI: All items	OECD	1990Q1:2019Q4

Alternatively, I also use inflation data coming from each country's central bank:

Table A.2: Core CPI inflation measures, several countries, 1990-2019

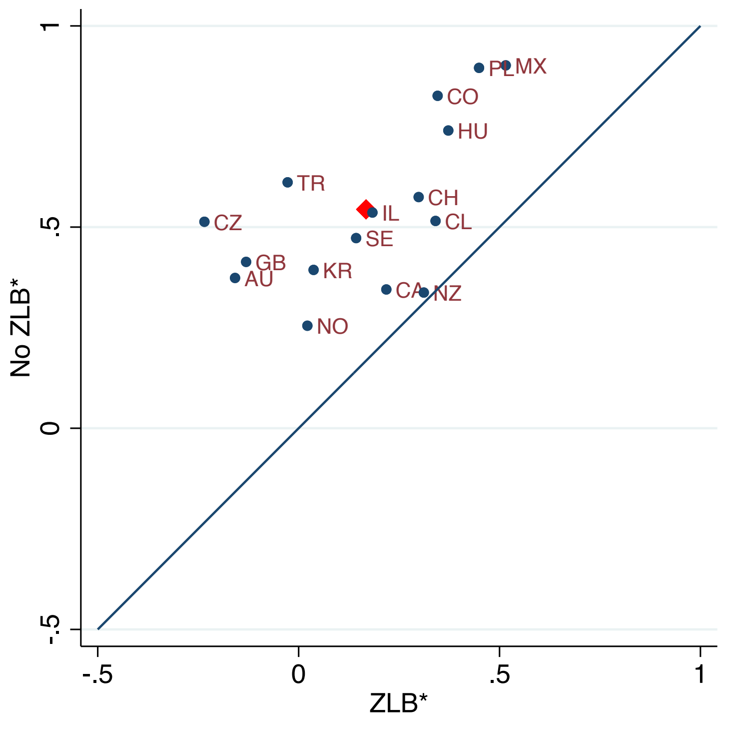
Variable/Table	Country	Source	Period
Table G1	AU - Australia	RBA	1990Q1:2019Q12
Table 18-10-0256-01	CA - Canada	Statistics Canada	1990Q1:2019Q4
TM15	CH - Switzerland	SNB	1990Q1:2019Q4
IPCSAE	CL - Chile	BCCh	1999Q1:2019Q4
Table 2.3.2	CO - Colombia	BanRep	1996Q1:2019Q4
IND9	CZ - Czechia	CNB	1996Q1:2019Q4
Table 7.4.2	KR - South Korea	BoK - ECOS	1990Q1:2019Q4
Table CP151	MX - Mexico	Banxico	1990Q1:2019Q4
Table HM1	NZ - New Zealand	RBNZ	1990Q1:2019Q4

Table A.3: Small open economies

Country	Time	No obs. (max)
AU - Australia	1990Q1:2019Q12	120
CA - Canada	1990Q1:2019Q4	120
CH - Switzerland	1990Q1:2019Q4	120
CL - Chile	1999Q1:2019Q4	84
CO - Colombia	1996Q1:2019Q4	96
CZ - Czechia	1996Q1:2019Q4	96
GB - Great Britain	1990Q1:2019Q4	120
HU - Hungary	1990Q1:2019Q4	120
IL - Israel	1990Q1:2019Q4	120
KR - South Korea	1990Q1:2019Q4	120
MX - Mexico	1990Q1:2019Q4	120
NO - Norway	1990Q1:2019Q4	120
NZ - New Zealand	1990Q1:2019Q4	120
PL - Poland	1996Q1:2019Q4	96
SE - Sweden	1990Q1:2019Q4	120
TR - Turkey	1995Q1:2019Q4	100

A.1.2 Additional figures and tables

Figure A.1: Correlation of interest rate and quarter-to-quarter core CPI inflation



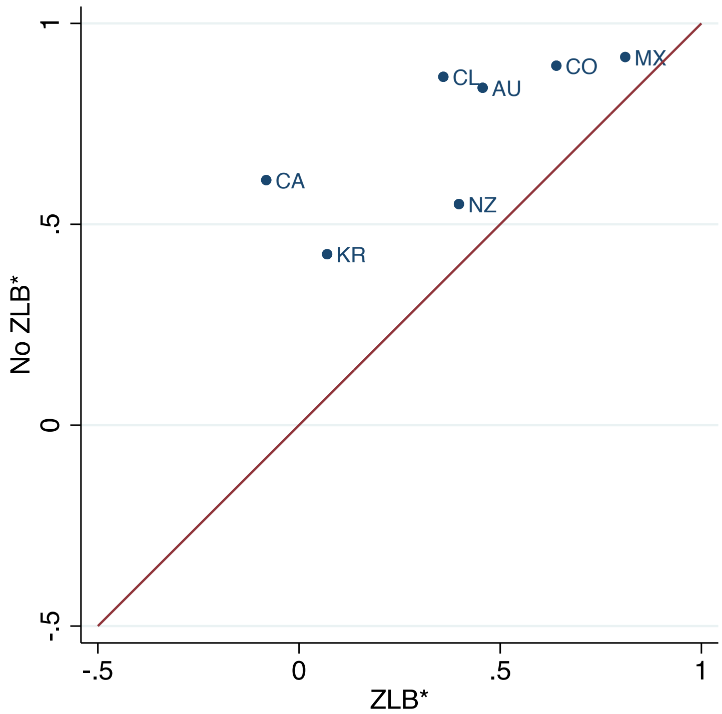
Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Table A.4: Correlation of interest rate and quarter-to-quarter core CPI inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.37	-0.16	IL - Israel	0.54	0.18
	(0.08)	(0.21)		(0.08)	(0.17)
CA - Canada	0.34	0.22	KR - South Korea	0.39	0.04
	(0.09)	(0.17)		(0.11)	(0.19)
CH - Switzerland	0.57	0.30	MX - Mexico	0.90	0.52
	(0.07)	(0.16)		(0.04)	(0.13)
CL - Chile	0.52	0.34	NO - Norway	0.25	0.02
	(0.10)	(0.16)		(0.09)	(0.19)
CO - Colombia	0.83	0.35	NZ - New Zealand	0.34	0.31
	(0.05)	(0.16)		(0.09)	(0.16)
CZ - Czechia	0.51	-0.23	PL - Poland	0.90	0.45
	(0.09)	(0.21)		(0.04)	(0.14)
GB - Great Britain	0.41	-0.13	SE - Sweden	0.47	0.14
	(0.08)	(0.20)		(0.08)	(0.18)
HU - Hungary	0.74	0.37	TR - Turkey	0.61	-0.03
	(0.05)	(0.15)		(0.10)	(0.20)

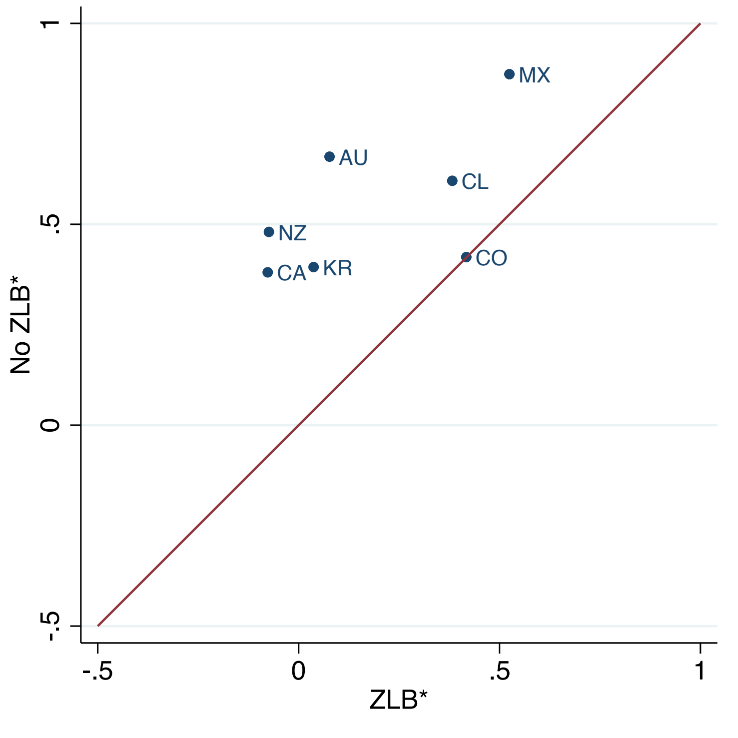
Notes: This figure reports correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1-2019Q4. ZLB*: 2008Q4-2015Q4. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Figure A.2: Correlation of interest rate and year-ended core CPI inflation, Central bank data



Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Figure A.3: Correlation of interest rate and quarter-to-quarter core CPI inflation, Central bank data



Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Table A.5: Correlation of interest rate and core CPI inflation, Central bank data

Panel A: Year-ended inflation

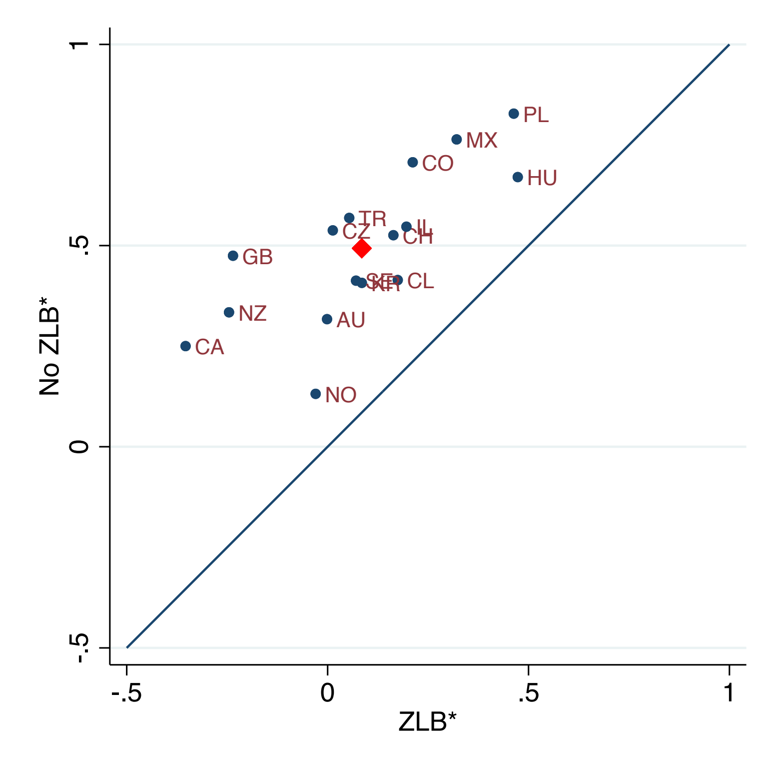
	No ZLB*	ZLB*
AU - Australia	0.84	0.46
	(0.04)	(0.14)
CA - Canada	0.61	-0.08
	(0.07)	(0.20)
CL - Chile	0.87	0.36
	(0.05)	(0.15)
CO - Colombia	0.89	0.64
	(0.04)	(0.12)
KR - South Korea	0.43	0.07
	(0.11)	(0.19)
MX - Mexico	0.92	0.81
	(0.04)	(0.08)
NZ - New Zealand	0.55	0.40
	(0.08)	(0.15)

Panel B: Quarter-to-quarter inflation

	No ZLB*	ZLB*
AU - Australia	0.67	0.08
	(0.06)	(0.18)
CA - Canada	0.38	-0.08
	(0.08)	(0.20)
CL - Chile	0.61	0.38
	(0.08)	(0.15)
CO - Colombia	0.42	0.42
	(0.11)	(0.15)
KR - South Korea	0.39	0.04
	(0.11)	(0.19)
MX - Mexico	0.87	0.52
	(0.05)	(0.13)
NZ - New Zealand	0.48	-0.07
	(0.11)	(0.20)

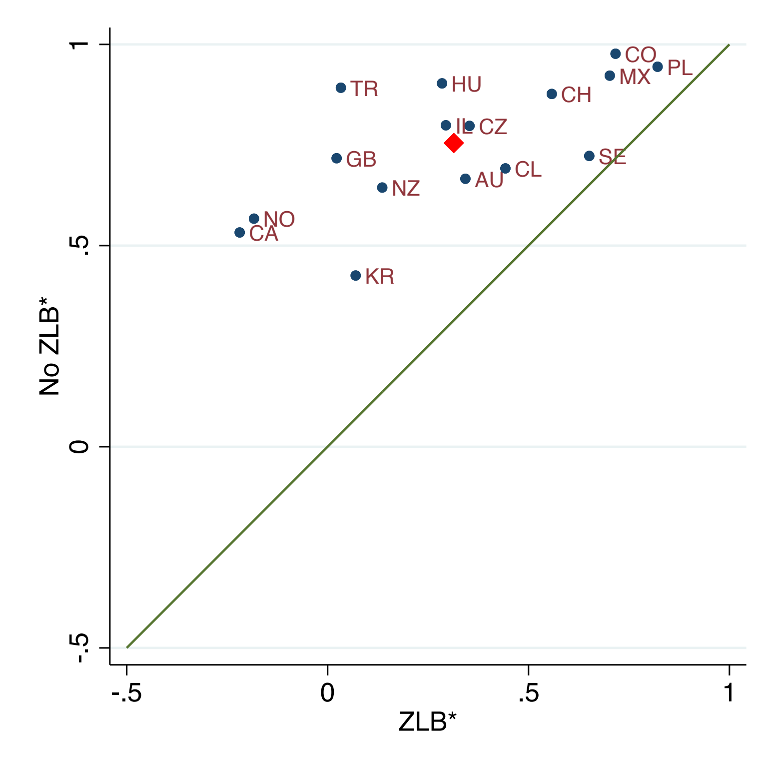
Notes: This table reports correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Figure A.4: Correlation of interest rate and year-ended headline CPI inflation



Notes: This figure plots correlations between headline CPI inflation and interest rates for two periodsat quarterly frequency during 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Figure A.5: Correlation of interest rate and quarter-to-quarter headline CPI inflation



Notes: This figure plots correlations between headline CPI inflation and interest rates for two periodsat quarterly frequency during 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Table A.6: Correlation of interest rate and headline CPI inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.65	0.62	IL - Israel	0.82	0.64
	(0.06)	(0.12)		(0.05)	(0.12)
CA - Canada	0.53	0.48	KR - South Korea	0.58	0.59
	(0.07)	(0.14)		(0.09)	(0.12)
CH - Switzerland	0.85	0.74	MX - Mexico	0.92	0.77
	(0.04)	(0.10)		(0.04)	(0.09)
CL - Chile	0.70	0.65	NO - Norway	0.32	0.26
	(0.07)	(0.11)		(0.09)	(0.17)
CO - Colombia	0.97	0.89	NZ - New Zealand	0.63	0.10
	(0.02)	(0.06)		(0.06)	(0.18)
CZ - Czech Republic	0.87	0.49	PL - Poland	0.94	0.91
	(0.04)	(0.14)		(0.03)	(0.06)
GB - Great Britain	0.77	0.29	SE - Sweden	0.62	0.72
	(0.05)	(0.16)		(0.07)	(0.10)
HU - Hungary	0.90	0.77	TR - Turkey	0.87	0.30
	(0.03)	(0.09)		(0.06)	(0.16)

Notes: The tables report correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1-2019Q4. ZLB*: 2008Q4-2015Q4. Core inflation comes from country's central bank or statistical agency. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Table A.7: Correlation of interest rate and headline CPI inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.32	-0.00	IL - Israel	0.55	0.20
	(0.09)	(0.19)		(0.08)	(0.17)
CA - Canada	0.25	-0.35	KR - South Korea	0.41	0.09
	(0.09)	(0.22)		(0.11)	(0.18)
CH - Switzerland	0.53	0.16	MX - Mexico	0.76	0.32
	(0.07)	(0.18)		(0.07)	(0.16)
CL - Chile	0.41	0.17	NO - Norway	0.13	-0.03
	(0.10)	(0.17)		(0.10)	(0.20)
CO - Colombia	0.71	0.21	NZ - New Zealand	0.33	-0.25
	(0.07)	(0.17)		(0.09)	(0.21)
CZ - Czech Republic	0.54	0.01	PL - Poland	0.83	0.46
	(0.08)	(0.19)		(0.05)	(0.14)
GB - Great Britain	0.47	-0.24	SE - Sweden	0.41	0.07
	(0.08)	(0.21)		(0.08)	(0.19)
HU - Hungary	0.67	0.47	TR - Turkey	0.57	0.05
	(0.06)	(0.14)		(0.10)	(0.19)

Notes: The tables report correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1-2019Q4. ZLB*: 2008Q4-2015Q4. Core inflation comes from country's central bank or statistical agency. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Table A.8: Policy rate movements, selected countries

	No ZLB*		ZLB*	
	No.	%	No.	%
AU - Australia				
Drop	35	13.6	13	16.6
No change	202	78.6	58	74.4
Hike	20	7.8	7	9.0
CA - Canada				
Drop	21	21.7	6	10.5
No change	56	57.7	48	84.2
Hike	20	20.6	3	5.3
CL - Chile				
Drop	34	20.6	16	18.8
No change	102	61.8	56	65.9
Hike	29	17.6	13	15.29
KR - South Korea				
Drop	15	10.3	10	11.8
No change	117	80.7	70	82.4
Hike	13	9.0	5	5.9
NZ - New Zealand				
Drop	16	15.1	9	15.8
No change	68	64.2	42	73.7
Hike	22	20.1	6	10.5

Notes: This table reports the frequency, and corresponding shares, of policy rate movements during monetary policy meetings. ZLB*: 2008Q4–2015Q4.

A.2 Additional results from Section 1.3

A.2.1 Model under Taylor rule

Here I present a modified version of the two-period SOE model in Section 1.3. Instead of optimal monetary policy in SOE, here I assume that SOE central bank follows a Taylor rule:

$$i_t = \rho + \psi_\pi \pi_t + \psi_y y_t,$$

Then, the equations that define the equilibrium are given by:

$$
\begin{aligned}
y_0 &= (1-\alpha)c_0 + \alpha y_0^* + \tilde{\alpha} q_0, \\
c_0 &= y_0^* + q_0 - \nu_0^*, \\
\pi_{S,0} &= \kappa\left(c_0 + \varphi y_0 + \frac{\alpha}{1-\alpha} q_0\right), \\
\pi_0 &= \pi_{S,0} + \frac{\alpha}{1-\alpha} q_0, \\
i_0 &= \rho + \psi_y y_0 + \psi_\pi \pi_0, \\
c_0 &= -(i_0 - E_0 \pi_1 - \rho).
\end{aligned}
$$

The corresponding demand and supply functions are now:

$$
q_0 = (1-\alpha)(y_0 - y_0^* + (1-\alpha)\nu_0^*) \tag{DD'}
$$

$$
q_0 = -\frac{1-\alpha}{A}\left[B y_0 + (1+\kappa\psi_\pi)(y_0^* - \nu_0^*)\right] \tag{SS'}
$$

where $A \equiv (\kappa+\alpha)\psi_\pi + 1 > 1$ and $B \equiv \psi_y + \kappa\varphi\psi_\pi \geq 0$. Curve (DD') is the same as curve (DD), but now the supply curve (SS') depends on how the Taylor rule responds to inflation and output. Note that if we let $\psi_\pi = 0$ and $\psi_y = \varphi'$, we are back at the baseline scenario.

These curves can be used to find the equilibrium as function of foreign shocks, y_0^* and ν_0^*:

$$
\begin{aligned}
y_0 &= \frac{\alpha}{A+B}(A-\psi_\pi)\nu_0^* + \frac{\alpha\psi_\pi}{A+B} y_0^* \\
q_0 &= (1-\alpha)\left[\frac{\kappa\psi_\pi + 1 + B(1-\alpha)}{A+B}\nu_0^* - \frac{\kappa\psi_\pi + 1 + B}{A+B} y_0^*\right].
\end{aligned} \tag{A.1}
$$

Compared to the baseline, now the response on output from external shocks is ambiguous and depends on the parameters. And, when going from a No ZLB* to a ZLB* scenario, output falls.

In contrast to what happens to output, real exchange rate reacts in the same way as it did in the baseline when we compare a No ZLB* to a ZLB* scenario. From (A.1), we can observe that a depreciation takes place for the whole parameter space. In fact,

$$
q_{Z,0} - q_{N,0} = -\frac{\kappa\psi_\pi + 1 + B}{A+B}\underbrace{(y_{Z,0}^* - y_{N,0}^*)}_{<0} > 0.
$$

In addition, we can compute domestic inflation, CPI inflation and interest rate:

$$\begin{aligned}
\pi_{S,0} &= \kappa\left[\frac{\alpha\psi_\pi(1+\varphi)}{A+B}y_0^* + \frac{\alpha(\varphi A - B) - \alpha\psi_\pi(1+\varphi)}{A+B}\nu_0^*\right] \\
\pi_0 &= -\alpha\frac{\psi_y+1}{A+B}y_0^* + \alpha\frac{\kappa(\varphi A - B) + \psi_y + 1 - \alpha B}{A+B}\nu_0^* \\
i_0 &= \rho + \frac{\alpha}{A+B}\left[(\psi_\pi + B)\nu_0^* - \psi_\pi y_0^*\right]
\end{aligned}$$

As predicted in the baseline model, CPI inflation increases from a No ZLB* to a ZLB* scenario. Now, interest rate increases even if the drop in output is stronger in the No ZLB* scenario. However, because CPI inflation is rising, while output is falling, the increase in interest rate is substantially smaller when faced with negative foreign output shocks, $y_0^* < 0$. We can compare:

$$\frac{\partial i_0}{\partial y_0^*} = -\alpha\frac{\psi_\pi}{A+B} \quad > \quad \frac{\partial \pi_0}{\partial y_0^*} = -\alpha\frac{\psi_\pi + 1}{A+B}$$

A.2.2 Derivation of optimal target rule in SOE

The second-order approximation to the utility function of SOE households in the two-period model corresponds to:

$$U_0 = E_0\sum_{t=0}^{\infty}\beta^t\left[c_t - \frac{y_t}{\mu} - \frac{1}{2}\frac{1+\varphi}{\mu}y_t^2 - \frac{1}{2}\frac{\epsilon}{\kappa\mu}\pi_{S,t}^2\right] + t.i.p. + \mathcal{O}(\|\zeta\|^3).$$

The problem that the monetary authority at SOE solves is:

$$\max_{c_t, y_t, q_t, \pi_{S,t}} U_0 \quad \text{subject to system in (1.8)}$$

The FOC are:

$$\begin{aligned}
1 - \kappa\lambda_{1,t} + \lambda_{2,t} - (1-\alpha)\lambda_{3,t} &= 0 \\
\lambda_{1,t}\kappa\frac{\alpha}{1-\alpha} + \lambda_{2,t} + \tilde{\alpha}\lambda_{3,t} &= 0 \\
-\frac{1}{\mu} - \frac{1+\varphi}{\mu}y_t - \kappa\varphi\lambda_{1,t} + \lambda_{3,t} &= 0 \\
-\frac{\epsilon}{\kappa\mu}\pi_{S,t} + \lambda_{1,t} - \lambda_{1,t-1} &= 0
\end{aligned}$$

Solving this system and imposing $\mu = \frac{1}{1-\alpha}$, we arrive to (1.9) in the main text.

A.2.3 Model with money-in-the-utility

Consider the following utility function,

$$U(C_t^*, M_t^*/P_t^*, N_t^*) = \log C_t^* + \log\left(\frac{M_t^*}{P_t^*}\right) - \frac{N_t^{*1+\varphi}}{1+\varphi}$$

where M_t^* are money holdings. The budget constraint in this context is:

$$P_t^* C_t^* + \mathbb{Q}_t^* B_t^* + M_t^* = W_t^* N_t^* + B_{t-1}^* + M_{t-1}^* - T_t^* + \Gamma_t^*.$$

From the first-order conditions we can derive the following demand function:

$$\frac{M_t^*}{P_t^*} = \frac{Y_t^*}{1-\mathbb{Q}_t^*}.$$

Doing a first-order log approximation we can arrive to:

$$l_t^* \equiv m_t^* - p_t^* = y_t^* - \frac{1}{\rho} i_t^*.$$

Then, by combining the Euler equation and Phillips curve in (1.8), together with the demand for money and noting that, $l_{t-1}^* = l_t^* + \pi_t^* - \Delta m_t^*$, we can derive the following system:

$$\underbrace{\begin{pmatrix} 1+1/\rho & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}}_{A_0} \begin{pmatrix} y_t^* \\ \pi_t^* \\ l_{t-1}^* \end{pmatrix} = \underbrace{\begin{pmatrix} 1/\rho & 1/\rho & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{A_1} E_t \begin{pmatrix} y_{t+1}^* \\ \pi_{t+1}^* \\ l_t^* \end{pmatrix} + \underbrace{\begin{pmatrix} 1/\rho & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}}_{B} \begin{pmatrix} \rho - E_t \Delta \nu_{t+1}^* \\ \Delta m_t^* \end{pmatrix}$$

It can be shown that, for any relevant parametrization, $A_0^{-1} A_1$ has two eigenvalues inside the unit circle and one outside. This means that there is a unique and stationary solution in the system.

Under initial conditions, $l_{-1}^* = m_{-1} = 0$, the ZLB* equilibrium in (1.11) is achieved by setting: $m_{N,0}^* = -(\rho + \nu_0^*)/\rho$. The No ZLB* equilibrium in (1.12) is achieved by setting: $m_{Z,0}^* = (\kappa + 1)(\rho + \nu_0^*)$.

A.2.4 Derivation of (DD) and (SS)

To solve for all the endogenous variables $(c_0, q_0, y_0, \pi_{S,0})$ we proceed as follows. First, we replace the optimal rule into the NKPC:

$$
\begin{aligned}
-\frac{1}{\epsilon}y_0 &= \kappa\left(c_0 + \varphi y_0 + \frac{\alpha}{1-\alpha}q_0\right) \\
0 &= \left(\kappa\frac{\alpha}{1-\alpha}\right)q_0 + \left(\kappa\varphi + \frac{1}{\epsilon}\right)y_0 + \kappa c_0
\end{aligned}
$$

Now we replace the risk-sharing condition into the expression above and solve for q_0:

$$
\begin{aligned}
0 &= \left(\kappa\frac{\alpha}{1-\alpha}\right)q_0 + \left(\kappa\varphi + \frac{1}{\epsilon}\right)y_0 + \kappa\left(y_0^* + q_0 - \nu_0^*\right) \\
0 &= \frac{1}{1-\alpha}q_0 + \varphi' y_0 + \left(y_0^* - \nu_0^*\right) \\
q_0 &= -(1-\alpha)\left(\varphi' y_0 + y_0^* - \nu_0^*\right) \qquad \text{(SS)}
\end{aligned}
$$

where $\varphi' \equiv \varphi + \frac{1}{\epsilon\kappa}$. Now, the risk sharing condition into the demand function:

$$
\begin{aligned}
y_0 &= (1-\alpha)(y_0^* + q_0 - \nu_0^*) + \alpha y_0^* + \tilde{\alpha} q_0 \\
\left[(1-\alpha) + \tilde{\alpha}\right] q_0 &= y_0 - y_0^* + (1-\alpha)\nu_0^* \\
q_0 &= (1-\alpha)\big(y_0 - y_0^* + (1-\alpha)\nu_0^*\big) \qquad \text{(DD)}
\end{aligned}
$$

This leads to,

$$
y_0 = \frac{\alpha}{1+\varphi'}\nu_0^* \quad \text{and} \quad q_0 = (1-\alpha)\frac{1+\varphi'(1-\alpha)}{1+\varphi'}\nu_0^* - (1-\alpha)y_0^* \quad \text{and} \quad \pi_{S,0} = -\frac{1}{\epsilon}\frac{\alpha}{1+\varphi'}\nu_0^*
$$

A.2.5 Additional figures

Figure A.6: Negative ROW discount rate shock under ZLB* scenario and $y_0^* << 0$

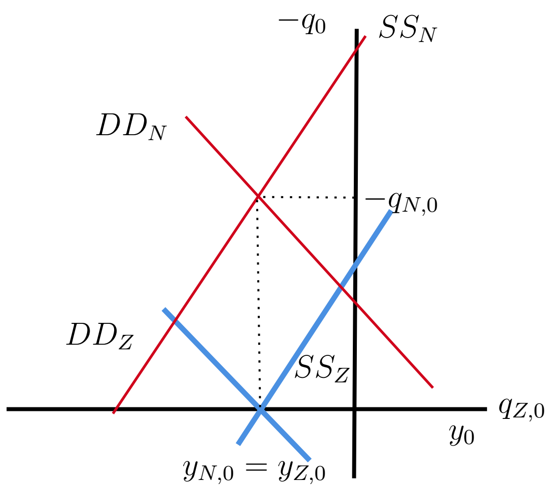
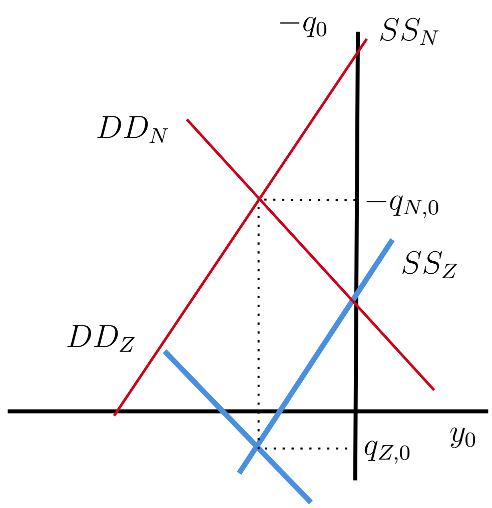


Figure A.7: Negative ROW discount rate shock under ZLB* scenario and $y_0^* <<< 0$



A.3 Additional results from Section 1.4

A.3.1 Derivation of first-order approximations

A.3.1.1 Domestic firms selling in SOE

Taking the first-order condition with respect to $\tilde{P}_{S,t}$ leads to:

$$E_t \sum_{T=t}^{\infty} \theta_S^{T-t} \mathbb{Q}_{t,T} y_{S,T|t}(i) \left[\left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S} - \frac{\epsilon}{\epsilon-1} \frac{W_T}{\xi_{a,T}} \frac{1}{\tilde{P}_{S,t}} \right] = 0$$

$$\tilde{P}_{S,t} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \frac{1}{P_T} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S} y_{S,T|t}(i) = \frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \overline{MC}_T y_{S,T|t}(i)$$

$$\frac{\tilde{P}_{S,t}}{P_{S,t}} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \frac{P_{S,t}}{P_T} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S(1-\epsilon)} {P_{S,T}}^{\epsilon} C_{S,T} = \dots$$

$$\frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \overline{MC}_T \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{-\gamma_S \epsilon} {P_{S,T}}^{\epsilon} C_{S,T}$$

$$\frac{\tilde{P}_{S,t}}{P_{S,t}} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \frac{\nu_T}{C_T^{\sigma}} Q_T \overline{P}_{S,T} \left(\frac{P_{S,T}}{P_{S,t}} \right)^{\epsilon-1} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S(1-\epsilon)} C_{S,T} = \dots$$

$$\frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \frac{\nu_T}{C_T^{\sigma}} \overline{MC}_T \left(\frac{P_{S,T}}{P_{S,t}} \right)^{\epsilon} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{-\gamma_S \epsilon} C_{S,T}$$

where $\overline{MC}_T = \frac{1}{\xi_{a,T}} \frac{W_T}{P_T}$ and $\overline{P}_{S,T} = \frac{P_{S,T}}{P_T}$.

Doing a first-order approximation:

$$\frac{1}{1-\beta\theta_S} (\tilde{p}^*_{S,t} - p^*_{S,t}) = E_t \sum_{T \geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}_{S,T} + (p_{S,T} - p_{S,t}) - \gamma_S (p_{S,T-1} - p_{S,t-1}) \right)$$

$$= E_t \sum_{T \geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}_{S,T} \right) + \frac{\beta\theta_S}{1-\beta\theta_S} E_t \sum_{T \geq t} (\beta\theta_S)^{T-t} (\pi_{S,T+1} - \gamma_S \pi_{S,T})$$

The price index for exported goods in ROW:

$${P_{S,t}}^{1-\epsilon} = \left[(1-\theta_S) {\tilde{P}_{S,t}}^{1-\epsilon} + \theta_S \left(P^*_{S,t-1} \left(\frac{P_{S,t-1}}{P_{S,t-2}} \right)^{\gamma_S} \right)^{1-\epsilon} \right]$$

$$\frac{\tilde{P}_{S,t}}{P^*_{S,t}} = \left[\frac{1 - \theta_S {\Pi_{S,t}}^{\epsilon-1} \Pi_{S,t-1}^{\gamma_S(1-\epsilon)}}{1-\theta_S} \right]^{\frac{1}{1-\epsilon}}$$

Doing a first-order approximation:

$$\tilde{p}_{S,t} - p_{S,t} = \frac{\theta_S}{1-\theta_S} (\pi_{S,t} - \gamma_S \pi_{S,t-1})$$

Putting both first-order approximations together:

$$\frac{\theta_S}{(1-\beta\theta_S)(1-\theta_S)}(\pi_{S,t}-\gamma_S\pi_{S,t-1}) = \overline{mc}_t - \overline{p}_{S,t} - q_t + \frac{\beta\theta_S}{1-\beta\theta_S}E_t(\pi_{S,t+1}-\gamma_S\pi_{S,t}) + E_t\sum_{T\geq t+1}(\beta\theta_S)^{T-t}\big(\overline{mc}_T - \overline{p}_{S,T} - q_T\big) + \frac{\beta\theta_S}{1-\beta\theta_S}E_t\sum_{T\geq t+1}(\beta\theta_S)^{T-t}(\pi_{S,T+1}-\gamma_S\pi_{S,T})$$

Doing it in $t+1$, multiplied by $\beta\theta$ and then replaced back into the original equation:

$$\pi_{S,t}-\gamma_S\pi_{S,t-1} = \kappa_S(\overline{mc}_t - \overline{p}_{S,t}) + \beta E_t(\pi_{S,t+1}-\gamma_S\pi_{S,t})$$

And, by looking at the price index,

$$\begin{aligned} P_t^{1-\eta} &= (1-\alpha)P_{S,t}^{1-\alpha} + \alpha P_{R,t}^{1-\alpha} \\ 1 &= (1-\alpha)\left(\frac{P_{S,t}}{P_t}\right)^{1-\eta} + \alpha\left(\frac{P_{R,t}}{P_t}\right)^{1-\eta} \\ \Rightarrow \bar{p}_{S,t} = p_{S,t} - p_t &= -\frac{\alpha}{1-\alpha}(q_t + \psi_t^*) \end{aligned}$$

A.3.1.2 Domestic firms selling in ROW

Taking the first-order condition with respect to $\tilde{P}^*_{S,t}$ leads to:

$$E_t\sum_{T=t}^{\infty}\theta_S^{T-t}\mathbb{Q}_{t,T}y^*_{S,T|t}(i)\left[\mathcal{E}_t\left(\frac{P^*_{S,T-1}}{P^*_{S,t-1}}\right)^{\gamma_S^*} - \frac{\epsilon}{\epsilon-1}\frac{W_T}{\xi_{a,T}}\frac{1}{\tilde{P}^*_{S,t}}\right] = 0$$

$$\tilde{P}^*_{S,t}E_t\sum_{T=t}^{\infty}(\beta\theta_S)^{T-t}\nu_T C_T^{-\sigma}\frac{\mathcal{E}_T}{P_T}\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{\gamma_S^*}y^*_{S,T|t}(i) = \frac{\epsilon}{\epsilon-1}E_t\sum_{T=t}^{\infty}(\beta\theta_S)^{T-t}\nu_T C_T^{-\sigma}\overline{MC}_T y^*_{S,T|t}(i)$$

$$\frac{\tilde{P}^*_{S,t}}{P^*_{S,t}}E_t\sum_{T=t}^{\infty}(\beta\theta_S)^{T-t}\nu_T C_T^{-\sigma}Q_T\frac{P^*_{S,t}}{P^*_T}\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{\gamma_S^*(1-\epsilon)}{P^*_{S,T}}^{\epsilon}C^*_{S,T} = ...$$

$$\frac{\epsilon}{\epsilon-1}E_t\sum_{T=t}^{\infty}(\beta\theta_S)^{T-t}\nu_T C_T^{-\sigma}\overline{MC}_T\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{-\gamma_S^*\epsilon}{P^*_{S,T}}^{\epsilon}C^*_{S,T}$$

$$\frac{\tilde{P}^*_{S,t}}{P^*_{S,t}}E_t\sum_{T=t}^{\infty}(\beta\theta_S)^{T-t}\frac{\nu_T}{C_T^{\sigma}}Q_T\overline{P}^*_{S,T}\left(\frac{P^*_{S,T}}{P^*_{S,t}}\right)^{\epsilon-1}\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{\gamma_S^*(1-\epsilon)}C^*_{S,T} = ...$$

$$\frac{\epsilon}{\epsilon-1}E_t\sum_{T=t}^{\infty}(\beta\theta_S)^{T-t}\frac{\nu_T}{C_T^{\sigma}}\overline{MC}_T\left(\frac{P^*_{S,T}}{P^*_{S,t}}\right)^{\epsilon}\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{-\gamma_S^*\epsilon}C^*_{S,T}$$

where $\overline{MC}_T = \frac{1}{\xi_{a,T}}\frac{W_T}{P_T}$ and $\overline{P}^*_{S,T} = \frac{P^*_{S,T}}{P^*_T}$.

Doing a first-order approximation:

$$\frac{1}{1-\beta\theta_S}(\tilde{p}^*_{S,t} - p^*_{S,t}) = E_t \sum_{T\geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}^*_{S,T} - q_T + (p^*_{S,T} - p^*_{S,t}) - \gamma^*_S(p^*_{S,T-1} - p^*_{S,t-1})\right)$$

$$= E_t \sum_{T\geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}^*_{S,T} - q_T\right) + \frac{\beta\theta_S}{1-\beta\theta_S} E_t \sum_{T\geq t} (\beta\theta_S)^{T-t} (\pi^*_{S,T+1} - \gamma^*_S \pi^*_{S,T})$$

The price index for exported goods in ROW:

$$P^{*\,1-\epsilon}_{S,t} = \left[(1-\theta_S)\tilde{P}^{*\,1-\epsilon}_{S,t} + \theta_S \left(P^*_{S,t-1}\left(\frac{P^*_{S,t-1}}{P^*_{S,t-2}}\right)^{\gamma^*_S}\right)^{1-\epsilon}\right]$$

$$\frac{\tilde{P}^*_{S,t}}{P^*_{S,t}} = \left[\frac{1-\theta_S \Pi^{*\,\epsilon-1}_{S,t} \Pi^{\gamma^*_S(1-\epsilon)}_{S,t-1}}{1-\theta_S}\right]^{\frac{1}{1-\epsilon}}$$

Doing a first-order approximation:

$$\tilde{p}^*_{S,t} - p^*_{S,t} = \frac{\theta_S}{1-\theta_S}(\pi^*_{S,t} - \gamma^*_S \pi^*_{S,t-1})$$

Putting both first-order approximations together:

$$\frac{\theta_S}{(1-\beta\theta_S)(1-\theta_S)}(\pi^*_{S,t} - \gamma^*_S\pi^*_{S,t-1}) = \overline{mc}_t - \overline{p}^*_{S,t} - q_t + \frac{\beta\theta_S}{1-\beta\theta_S} E_t(\pi^*_{S,t+1} - \gamma^*_S \pi^*_{S,t}) + E_t \sum_{T\geq t+1} (\beta\theta_S)^{T-t} \big(\overline{mc}_T$$
$$- \overline{p}^*_{S,T} - q_T\big) + \frac{\beta\theta_S}{1-\beta\theta_S} E_t \sum_{T\geq t+1} (\beta\theta_S)^{T-t} (\pi^*_{S,T+1} - \gamma^*_S \pi^*_{S,T})$$

Doing it in $t+1$, multiplied by $\beta\theta$ and then replaced back into the original equation leads to:

$$\pi^*_{S,t} - \gamma^*_S \pi^*_{S,t-1} = \kappa_S(\overline{mc}_t - \overline{p}^*_{S,t} - q_t) + \beta E_t(\pi^*_{S,t+1} - \gamma^*_S \pi^*_{S,t})$$

where

$$\overline{p}^*_{S,t} + q_t = \psi_t + (p_{S,t} - p_t)$$
$$= \psi_t - \frac{\alpha}{1-\alpha}(q_t + \psi^*_t).$$

A.3.1.3 Importing firms

Taking the first-order condition with respect to $\tilde{P}_{R,t}$ leads to:

$$E_t \sum_{T=t}^{\infty} \theta_R^{T-t} \mathbb{Q}_{t,T} y_{R,T|t}(i) \left[\left(\frac{P_{R,T-1}}{P_{R,t-1}} \right)^{\gamma_R} - \frac{\epsilon}{\epsilon - 1} \mathcal{E}_t \frac{P^*_{R,t}(i)}{\tilde{P}_{R,t}} \right] = 0$$

$$\tilde{P}_{R,t} E_t \sum_{T=t}^{\infty} (\beta\theta_R)^{T-t} \nu_T C_T^{-\sigma} \frac{1}{P_T} \left(\frac{P_{R,T-1}}{P_{R,t-1}} \right)^{\gamma_R} y_{R,T|t}(i) = \frac{\epsilon}{\epsilon - 1} E_t \sum_{T=t}^{\infty} (\beta\theta_R)^{T-t} \nu_T C_T^{-\sigma} \mathcal{E}_t \frac{P_t^*}{P_t} y_{R,T|t}(i)$$

$$\frac{\tilde{P}_{R,t}}{P_{R,t}} E_t \sum_{T=t}^{\infty} (\beta\theta_R)^{T-t} \nu_T C_T^{-\sigma} \frac{P_{R,t}}{P_{R,T}} \overline{P}_{R,T} \left(\frac{P_{R,T-1}}{P_{R,t-1}} \right)^{\gamma_R} y_{R,T|t}(i) = \frac{\epsilon}{\epsilon - 1} E_t \sum_{T=t}^{\infty} (\beta\theta_R)^{T-t} \nu_T C_T^{-\sigma} \frac{\overline{P}_{R,T}}{\Psi_t^*} y_{R,T|t}(i)$$

where $\overline{P}^*_{S,T} = \frac{P^*_{S,T}}{P^*_T}$. A first-order approximation:

$$\begin{aligned} \frac{1}{1-\beta\theta_R}(\tilde{p}_{R,t} - p_{R,t}) &= E_t \sum_{T \geq t} (\beta\theta_R)^{T-t} \left[(p_{R,T} - p_{R,t}) - \gamma_R(p_{R,T-1} - p_{R,t-1}) - \psi_t^* \right] \\ &= E_t \sum_{T \geq t} (\beta\theta_R)^{T-t} (-\psi_t^*) + \frac{\beta\theta_R}{1-\beta\theta_R} E_t \sum_{T \geq t} (\beta\theta_R)^{T-t} (\pi_{R,T+1} - \gamma_R \pi_{R,T}) \end{aligned}$$

Following similar steps to the ones above lead to the NKPC for importing goods.

A.3.1.4 Optimal labor supply

Taking the first-order condition with respect to $\tilde{W}_t$ leads to:

$$\begin{aligned} E_t \sum_{T=t} (\theta_W \beta)^{T-t} N_{T|t}(k) \left[\frac{\tilde{W}_t}{P_T} \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\gamma_W} \nu_t u_{1,T} - \frac{\epsilon_W}{\epsilon_W - 1} v_{1,T}(N_{T|t}(k)) \right] &= 0 \\ E_t \sum_{T=t} (\theta_W \beta)^{T-t} N_{T|t}(k) \left[\frac{\tilde{W}_t}{W_t} \frac{W_t}{P_T} \Pi^{\gamma_W}_{T-1,t-1} \nu_t u_{1,T} - \frac{\epsilon_W}{\epsilon_W - 1} v_{1,T}(N_{T|t}(k)) \right] &= 0 \end{aligned}$$

Doing a first-order approximation:

$$\frac{1}{1-\theta_W\beta}(\tilde{w}_t - w_t) + E_t \sum_{T=t}^{\infty} (\theta_W\beta)^{T-t} \left[\zeta_T + \gamma_W \pi_{T-1,t-1} + \nu_t + \hat{u}_{1,T} - \hat{v}_{1,T|t} - \sum_{k=1}^{T-t} \pi_{W,t+k} \right]$$

where

$$\begin{aligned} v_{1,T}(N_{T|t}(k)) &= N_{T|t}(k)^{\varphi} = \left[\left(\frac{\tilde{W}_t}{W_t} \cdot \frac{W_t}{W_T} \right)^{-\theta_W} N_T \right]^{\varphi} \\ \Rightarrow \hat{v}_{1,T|t} &= -\theta_W \varphi \left[(\tilde{w}_t - w_t) + (w_t - w_T) \right] + \varphi n_T \\ &= -\theta_W \varphi \left[(\tilde{w}_t - w_t) - \sum_{k=1}^{T-t} \pi_{W,t+k} \right] + \varphi (y_T - \xi_{a,T}) \end{aligned}$$

Back into the first order approximation:

$$\frac{1+\theta_W\varphi}{1-\theta_W\beta}(\tilde{w}_t - w_t) + E_t\sum_{T=t}^{\infty}(\theta_W\beta)^{T-t}\left[\zeta_T + \gamma_W\pi_{T-1,t-1} + \nu_t + \hat{u}_{1,T} - \varphi n_T - -(1+\theta_W\varphi)\sum_{k=1}^{T-t}\pi_{W,t+k}\right] = 0$$

$$\frac{\theta_W}{(1-\theta_W\beta)(1-\theta_W)}(\pi_{W,t} - \gamma_W\pi_{t-1}) + \frac{1}{1+\theta_W\varphi}E_t\sum_{T=t}^{\infty}(\theta_W\beta)^{T-t}(\zeta_T + \nu_t + \hat{u}_{1,T} - \varphi n_T -)$$
$$-E_t\sum_{T=t}^{\infty}(\theta_W\beta)^{T-t}\sum_{k=1}^{T-t}\pi_{W,t+k} + \gamma_W E_t\sum_{T=t}^{\infty}(\theta_W\beta)^{T-t}\pi_{T-1,t-1} = 0$$

Then, in $t+1$:

$$\frac{\theta_W}{(1-\theta_W\beta)(1-\theta_W)}(\pi_{W,t+1} - \gamma_W\pi_t) + \frac{1}{1+\theta_W\varphi}E_{t+1}\sum_{T=t+1}^{\infty}(\theta_W\beta)^{T-t-1}(\zeta_T + \nu_t + \hat{u}_{1,T} - \varphi n_T -)$$
$$-E_{t+1}\sum_{T=t+1}^{\infty}(\theta_W\beta)^{T-t-1}\sum_{k=1}^{T-t-1}\pi_{W,t+1+k} + \gamma_W E_{t+1}\sum_{T=t+1}^{\infty}(\theta_W\beta)^{T-t-1}\pi_{T-1,t} = 0$$
$$\frac{\theta_W^2\beta}{(1-\theta_W\beta)(1-\theta_W)}(E_t\pi_{W,t+1} - \gamma_W\pi_t) + \frac{1}{1+\theta_W\varphi}E_t\sum_{T=t+1}^{\infty}(\theta_W\beta)^{T-t-1}(\zeta_T + \nu_t + \hat{u}_{1,T} - \varphi n_T)$$
$$-E_t\sum_{T=t}^{\infty}(\theta_W\beta)^{T-t}\sum_{k=1}^{T-t}\pi_{W,t+k} + \frac{\theta_W\beta}{1-\theta_W\beta}E_t\pi_{W,t+1} + \gamma_W E_t\sum_{T=t}^{\infty}(\theta_W\beta)^{T-t}\pi_{T-1,t-1} - \frac{\gamma_W\theta_W\beta}{1-\theta_W\beta} = 0$$

Subtracting the last terms of expressions in t and $t+1$:

$$\frac{\theta_W}{(1-\theta_W\beta)(1-\theta_W)}\left[(\pi_{W,t} - \gamma_W\pi_{t-1}) - \theta_W\beta(E_t\pi_{W,t+1} - \gamma_W\pi_t)\right] - \frac{\theta_W\beta}{1-\theta_W\beta}E_t\pi_{W,t+1}$$
$$+\frac{1}{1+\theta_W\varphi}(\zeta_t + \nu_t + \hat{u}_{1,t} - \varphi n_t -) + \frac{\gamma_W\theta_W\beta}{1-\theta_W\beta}\pi_t = 0$$
$$\Rightarrow \pi_{W,t} - \gamma_w\pi_{t-1} = \beta(E_t\pi_{W,t+1} - \gamma_W\pi_t) + \kappa_W\left[\varphi y_t - \varphi\xi_{a,t} + \nu_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) - \zeta_t\right]$$

where $\kappa_W = \frac{(1-\theta_W\beta)(1-\theta_W)}{\theta_W(1+\epsilon_W\varphi)}$

A.3.2 Log-linearized equilibrium

The ROW is characterized by the following set of equations:

1. Euler equation

$$c_t^* - h^*c_{t-1}^* = E_t(c_{t+1}^* - h^*c_t^*) - \sigma^{*-1}(1-h^*)(i_t^* - E_t\pi_{t+1}^* + E_t\Delta\nu_{t+1}^*)$$

2. Firms price setting

$$\pi_t^* - \gamma^*\pi_{t-1}^* = \beta^*E_t(\pi_{t+1}^* - \gamma^*\pi_t^*) + \kappa^*(\zeta_t^* - \xi_{a,t}^*) + \xi_{cp,t}^*$$

where $\kappa^* = \frac{(1-\theta^*)(1-\beta^*\theta^*)}{\theta^*}$

3. Household price setting

$$\pi^*_{W,t} - \gamma^*_W \pi^*_{t-1} = \beta^* E_t(\pi^*_{W,t+1} - \gamma^*_W \pi^*_t) + \kappa^*_W \left[\varphi^* y^*_t - \varphi^* \xi^*_{a,t} + \frac{\sigma^*}{1-h^*}(c^*_t - h^* c^*_{t-1}) - \zeta^*_t\right]$$

where $\kappa^*_W = \frac{(1-\theta^*)(1-\beta^*\theta^*)}{\theta^*}\frac{1}{1+\epsilon^*_W\varphi^*}$

4. Real wages law of motion

$$\zeta^*_t = \zeta^*_{t-1} + \pi^*_{W,t} - \pi^*_t$$

5. Monetary policy

$$\tilde{i}^*_t = \psi^*_i i^*_{t-1} + (1-\psi^*_i)\left[\psi^*_\pi(\pi^*_t + \pi^*_{t-1} + \pi^*_{t-2} + \pi^*_{t-3}) + \psi^*_y(y^*_t - y^*_{t-4})\right] + \xi^*_{i,t}$$
$$i^*_t = \max\{0, \tilde{i}^*_t\}$$

The SOE is characterized by the following set of equations:

1. Euler equation:

$$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \sigma^{-1}(1-h)(i_t - E_t\pi_{t+1} - E_t\Delta\nu_{t+1})$$

2. Market clearing:

$$y_t = (1-\alpha)c_t + \alpha y^*_t + \eta\alpha(q_t + \psi^*_t) - \lambda\alpha(\psi^*_t - s_t)$$

3. Households' wage setting

$$\pi_{W,t} - \gamma_W \pi_{t-1} = \beta E_t(\pi_{W,t+1} - \gamma_W \pi_t) + \kappa_W \left[\varphi y_t - \varphi \xi_{a,t} + \frac{\sigma}{1-h}(c_t - hc_{t-1}) - \zeta_t\right]$$

where $\kappa_W = \frac{(1-\theta_W)(1-\beta\theta_W)}{\theta_W}\frac{1}{1+\epsilon_W\varphi}$

4. Domestic firms price setting at SOE

$$\pi_{S,t} - \gamma_S \pi_{S,t-1} = \beta E_t(\pi_{S,t+1} - \gamma_S \pi_{S,t}) + \kappa_S \left(\zeta_t - \xi_{a,t} + \frac{\alpha}{1-\alpha}(q_t + \psi^*_t)\right) + \xi_{cpS,t}$$

where $\kappa_S = \frac{(1-\theta_S)(1-\beta\theta_S)}{\theta_S}$

5. Domestic firms price setting at ROW

$$\pi^*_{S,t} - \gamma^*_S \pi^*_{S,t-1} = \beta E_t(\pi^*_{S,t+1} - \gamma^*_S \pi^*_{S,t}) + \kappa_S \left(\zeta_t - \xi_{a,t} + \frac{\alpha}{1-\alpha}(q_t + \psi^*_t) - \psi_t \right) + \xi_{cpS,t}$$

6. Retail firms price setting

$$\pi_{R,t} - \gamma_R \pi_{R,t-1} = \beta E_t(\pi_{R,t+1} - \gamma_R \pi_{R,t}) + \kappa_R(-\psi^*_t) + \xi_{cpR,t}$$

where $\kappa_R = \frac{(1-\theta_R)(1-\beta\theta_R)}{\theta_R}$

7. Terms of trade

$$s_t = p_{R,t} - e_t - p^*_{S,t}$$

8. Domestic inflation

$$\pi_{S,t} = p_{S,t} - p_{S,t-1}$$

9. Imported inflation

$$\pi_{R,t} = p_{R,t} - p_{R,t-1}$$

10. CPI Inflation

$$\pi_t = \pi_{S,t} + \alpha(s_t - s_{t-1}) + \alpha(\psi_t - \psi_{t-1})$$

11. Risk sharing

$$\begin{aligned} \frac{\sigma}{1-h}\left(E_t \Delta c_{t+1} - h\Delta c_t\right) = & \frac{\sigma^*}{1-h^*}\left(E_t \Delta y^*_{t+1} - h\Delta y^*_t\right) \\ & + E_t\left[\Delta q_{t+1} - \Delta \nu^*_{t+1} + \Delta \nu_{t+1} - \chi a_t - \xi_{rp,t}\right] \end{aligned}$$

12. Real exchange rate

$$q_t = e_t + p^*_t - p_t$$

13. LOP gap for exports

$$\psi_t = e_t + p^*_{S,t} - p_{S,t}$$

14. LOP gap for imports

$$\psi^*_t = p_{R,t} - e_t - p^*_t$$

15. Real wages law of motion

$$\zeta_t = \zeta_{t-1} + \pi_{W,t} - \pi_t$$

16. Budget constraint

$$c_t + a_t = \beta^{-1} a_{t-1} - \frac{\alpha}{1-\alpha}(q_t + \psi^*_t) + \psi_t + y_t$$

17. Taylor rule

$$i_t = \psi_i i_{t-1}(1-\psi_i)(\psi_\pi \pi_t + \psi_y y_t + \psi_{\Delta y}\Delta y_t + \psi_{\Delta e}\Delta e_t) + \xi_{i,t}$$

A.4 Additional results from Section 1.5 and Section 1.6

A.4.1 Data

Here I provide details on the macroeconomic data for Australia and the US, including sources and period.

Table A.9: Data (variables, description, source, period), US and Australia, 1990-2019

Variable	Description	Source	Period
Australia			
AUSGDPRQDSMEI	Constant Price Gross Domestic Product	FRED	1990Q1:2019Q4
NAEXKP02AUQ189S	Constant Price Final Private Consumption	FRED	1990Q1:2019Q4
POPTOTAUA647NWDB	Population	FRED	1990:2019
GCPIXVIQP	Core CPI quarterly inflation (Table G.1)	RBA	1990Q1:2019Q4
GCPIXVIYP	Core CPI yearly inflation (Table G.1)	RBA	1990Q1:2019Q4
A2298279F	Imported consumption goods price index (Tables 4, 5 and 6)	ABS	1990Q1:2019Q4
BIS_CBPOL	Policy rate	BIS	1990M1:2019M12
DEXUSAL	Nominal exchange rate US$/AU$	FRED	1990Q1:2019Q4
United States			
A939RX0Q048SBEA	Real gross domestic product per capita	FRED	1990Q1:2019Q4
FEDFUNDS	Effective Federal Funds Rate	FRED	1990Q1:2019Q4
CPILFESL	Core CPI	FRED	1990Q1:2019Q4

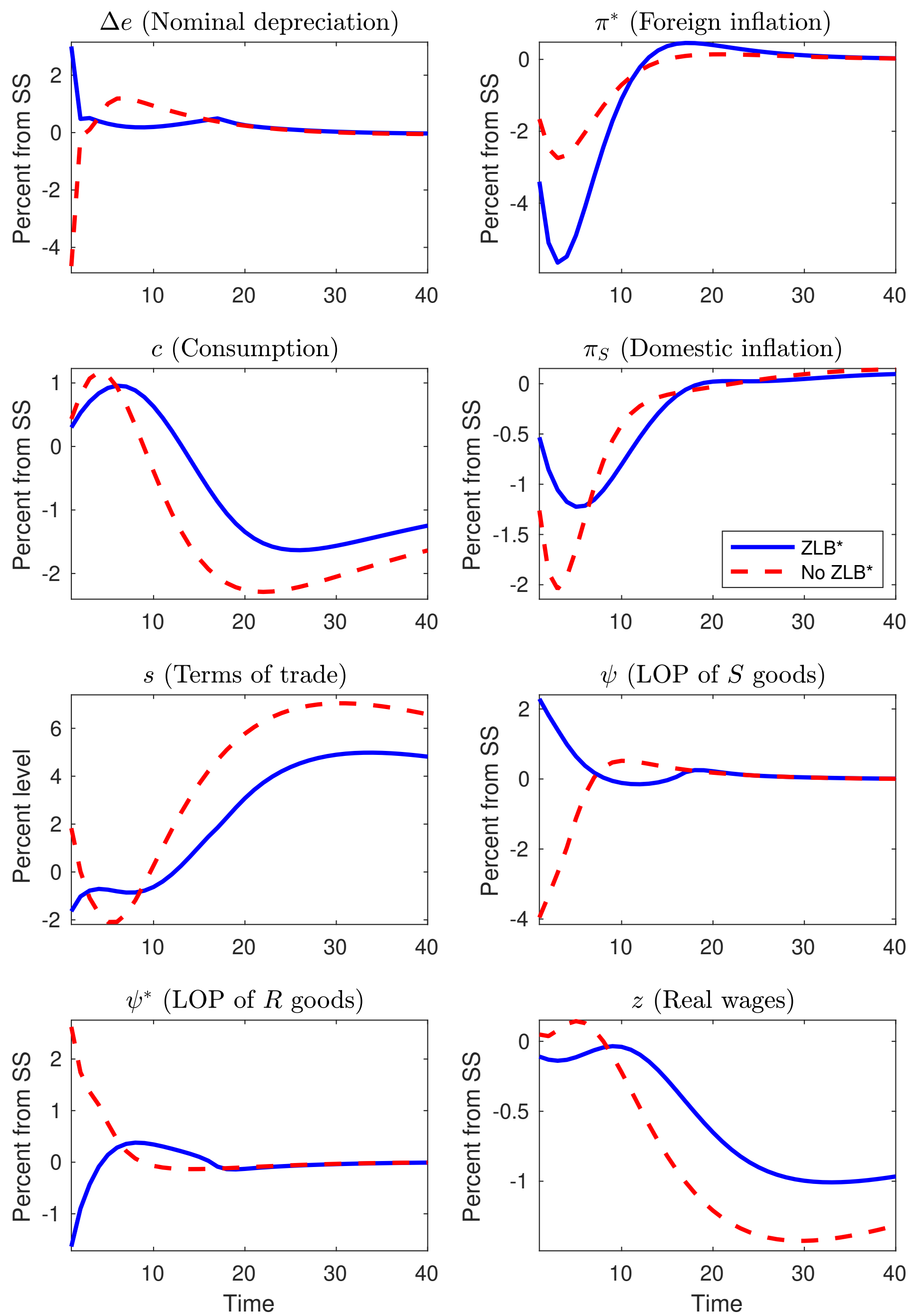
A.4.2 Additional figures and tables

Table A.10: Correlation of interest rate and inflation: Data and Model

		Data	Model
$\rho(i, \pi)$	No ZLB*	0.3735	0.5473
	ZLB*	-0.1583	0.4647
	% explained	15.52	

Notes: Column 1 of this table reports sample correlations between core quarterly CPI inflation and interest rates for Australia for two periods. The ZLB* period in the data is given by 2008Q4 to 2015Q4. Column 2 reports the correlation for the same elements in the quantitative. The ZLB* period in the model are all the quarters when (1.13) binds. The number of simulation is 1,000.

Figure A.8: Impulse responses to a foreign discount rate shock under ZLB* and No ZLB*



APPENDIX B

Appendix to Chapter II

B.1 Additional Tables

Table B.1: Transition matrix between Currencies for Pure Switches

		Currency in t										
		USD	EUR	UKP	CAD	YEN	CLP	MEX	REA	SWE	NOR	Other
Currency in $t-1$	USD	0.0	71.7	6.5	7.2	1.5	8.0	1.0	0.7	0.2	0.1	3.1
	EUR	90.6	0.0	7.0	0.4	0.0	1.5	0.0	0.0	0.1	0.1	0.4
	UKP	57.0	42.7	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.1
	CAD	96.4	3.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	YEN	92.0	3.0	0.0	0.0	0.0	5.0	0.0	0.0	0.0	0.0	0.0
	CLP	94.2	4.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	1.2
	MEX	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	REA	100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SWE	93.5	6.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	NOR	66.7	33.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Other	87.6	11.2	0.0	0.0	0.0	1.2	0.0	0.0	0.0	0.0	0.0

Table B.2: Transition matrix between Currencies for Mixed Switches

		Currency in t										
		Not main	USD	EUR	UKP	CAD	YEN	CLP	MEX	REA	SWE	Other
Currency in $t-1$	Not main	0.8	38.4	36.6	12.6	7.6	0.7	0.7	1.1	0.3	0.4	0.8
	USD	13.6	61.8	16.6	3.6	1.7	1.0	0.5	0.4	0.3	0.0	0.5
	EUR	13.1	16.7	66.9	2.7	0.1	0.0	0.4	0.0	0.0	0.1	0.0
	UKP	11.8	9.1	7.2	71.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	CAD	18.2	11.2	0.7	0.0	69.9	0.0	0.0	0.0	0.0	0.0	0.0
	YEN	5.0	15.8	0.0	0.0	0.0	79.1	0.0	0.0	0.0	0.0	0.0
	CLP	6.2	20.2	12.4	0.0	0.0	0.0	61.2	0.0	0.0	0.0	0.0
	MEX	8.3	9.1	0.0	0.0	0.0	0.0	0.0	82.5	0.0	0.0	0.0
	REA	4.7	11.2	0.0	0.0	0.0	0.0	0.0	0.0	84.0	0.0	0.0
	SWE	18.0	0.0	10.0	0.0	0.0	0.0	0.0	0.0	0.0	72.0	0.0
	Other	12.2	30.5	2.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	54.9

Table B.3: Transition matrix between Pricing types for Pure Switchers

		Currency type in t							
		PCP	LCP	V. USD	V. EUR	V. UKP	V. CAD	V. YEN	Other
Type in $t-1$	PCP	69.3	2.3	28.1	0.1	0.0	0.0	0.0	0.3
	LCP	0.6	83.4	14.9	0.5	0.5	0.1	0.0	0.0
	VCP USD	2.0	10.9	81.0	5.0	0.2	0.1	0.1	0.8
	VCP EUR	0.1	1.3	21.8	76.7	0.1	0.0	0.0	0.1
	VCP UKP	0.0	26.1	9.8	1.6	62.4	0.0	0.0	0.0
	VCP CAD	0.0	23.5	64.7	0.0	0.0	11.8	0.0	0.0
	VCP YEN	0.0	4.0	44.0	0.0	0.0	0.0	52.0	0.0
	Other	0.2	2.0	39.4	0.7	0.0	0.0	0.0	57.6

Note: This includes *all* transactions done by trade-links that only have pure switches

Table B.4: Transition matrix between Pricing types for Mixed Switchers

		Currency type in t							
		PCP	LCP	V. USD	V. EUR	V. UKP	V. CAD	V. YEN	Other
Type in $t-1$	PCP	78.9	3.1	8.9	0.2	0.2	0.3	0.7	7.7
	LCP	0.1	88.9	3.8	0.6	0.6	0.1	0.0	5.9
	VCP USD	0.2	5.3	85.6	1.7	0.4	0.1	0.0	6.7
	VCP EUR	0.0	3.8	7.2	81.7	0.5	0.0	0.0	6.8
	VCP UKP	0.0	28.4	11.7	3.5	43.0	0.5	0.3	12.6
	VCP CAD	1.4	44.4	16.7	2.8	2.8	6.9	2.8	22.2
	VCP YEN	9.5	31.0	23.8	2.4	2.4	7.1	2.4	21.4
	Other	0.8	33.7	24.5	7.2	1.5	0.2	0.1	32.0

Note: This includes *all* transactions done by trade-links that have at least one mixed switch

Table B.5: Results for Switching Currency using 3-month changes

	(1)	(2)	(3)	(4)	(5)	(6)
	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{DCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$	$\Delta\psi^{LCP}_{fpd,t}$
$\Delta\varphi^{DCP}_{f,\overline{t-3}}$	0.00863	0.00887	0.000314	0.0224	0.0226	0.0563***
	(0.0119)	(0.0119)	(0.0104)	(0.0192)	(0.0192)	(0.0160)
$\Delta S_{fpd,\overline{t-3}}$		-0.154	-0.184		0.121	0.110
		(0.118)	(0.116)		(0.101)	(0.120)
Trade-link FE	No	No	Yes	No	No	Yes
Currency	LCP→LCP	LCP→LCP	LCP→LCP	DCP→DCP	DCP→DCP	DCP→DCP
	LCP→DCP	LCP→DCP	LCP→DCP	DCP→LCP	DCP→LCP	DCP→LCP
N	13,048	13,048	13,048	9,859	9,859	9,859
R^2	0.000	0.001	0.453	0.000	0.001	0.557

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.6: Results for of ERPT by currency for Non Switchers

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$	$\Delta p_{fpd,t}$
$\Delta e_{d,t}$	0.108*	0.540***	0.0337	0.585***	0.0978	0.536***	0.0192	0.578***
	(0.0527)	(0.0248)	(0.0558)	(0.0266)	(0.0550)	(0.0254)	(0.0575)	(0.0270)
$W_{d,t}$			-0.00328	0.00441			0.0168	-0.0126
			(0.0434)	(0.0283)			(0.0456)	(0.0320)
X_t			0.00579	-0.0140*			0.00783	-0.00822
			(0.0157)	(0.00687)			(0.0168)	(0.00763)
Currency	LCP	DCP	LCP	DCP	LCP	DCP	LCP	DCP
Controls	No	No	Yes	Yes	No	No	Yes	Yes
HS8-Dest. FE	Yes	Yes	Yes	Yes	No	No	No	No
Trade-link FE	No	No	No	No	Yes	Yes	Yes	Yes
Observations	212,006	999,807	186,222	863,083	212,006	999,807	186,222	863,083
R^2	0.018	0.029	0.016	0.029	0.068	0.075	0.066	0.072

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

B.2 Firm-level evidence on synchrony of currencies in international trade

We proceed to estimate the following reduced-form regression:

$$y^i_{ft} = \alpha + X'_{ft}\boldsymbol{\beta} + \varepsilon_{ft} \tag{B.1}$$

where y^i_{ft} is the share of exports of firm f in month t invoiced in currency i, where $i \in \{USD, EUR\}$. X_{ft} has the share of imports in USD and EUR, x^{USD}_{ft} and x^{EUR}_{ft}. Tables B.7 and B.8 display the results of the equation above using different fixed effects.

Table B.7: Firm-level regression on the synchrony of USD invoicing and imports

	(1)	(2)	(3)	(4)	(5)	(6)
	y_{ft}^{USD}	y_{ft}^{USD}	y_{ft}^{USD}	y_{ft}^{USD}	y_{ft}^{USD}	y_{ft}^{USD}
x_{ft}^{USD}	0.203***	0.203***	0.0123***	0.0749***	0.0748***	0.0173**
	(0.0128)	(0.0128)	(0.00367)	(0.0172)	(0.0172)	(0.00725)
x_{ft}^{EUR}				-0.152***	-0.152***	0.00629
				(0.0220)	(0.0220)	(0.00821)
Constant	0.778***	0.778***	0.923***	0.906***	0.906***	0.918***
	(0.0123)	(0.0124)	(0.00280)	(0.0172)	(0.0172)	(0.00699)
Time FE	No	Yes	No	No	Yes	No
Firm FE	No	No	Yes	No	No	Yes
Observations	110,939	110,939	110,939	110,939	110,939	110,939
R^2	0.105	0.106	0.623	0.115	0.116	0.623

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.8: Firm-level regression on the synchrony of EUR invoicing and imports

	(1)	(2)	(3)	(4)	(5)	(6)
	y_{ft}^{EUR}	y_{ft}^{EUR}	y_{ft}^{EUR}	y_{ft}^{EUR}	y_{ft}^{EUR}	y_{ft}^{EUR}
x_{ft}^{EUR}	0.212***	0.212***	0.00952**	0.200***	0.200***	0.00545
	(0.0142)	(0.0142)	(0.00393)	(0.0158)	(0.0158)	(0.00682)
x_{ft}^{USD}				-0.0132*	-0.0131*	-0.00440
				(0.00784)	(0.00784)	(0.00557)
Constant	0.0129***	0.0129***	0.0537***	0.0255***	0.0254***	0.0578***
	(0.00145)	(0.00145)	(0.000790)	(0.00772)	(0.00771)	(0.00542)
Time FE	No	Yes	No	No	Yes	No
Firm FE	No	No	Yes	No	No	Yes
Observations	110,939	110,939	110,939	110,939	110,939	110,939
R^2	0.121	0.122	0.629	0.121	0.122	0.629

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

APPENDIX C

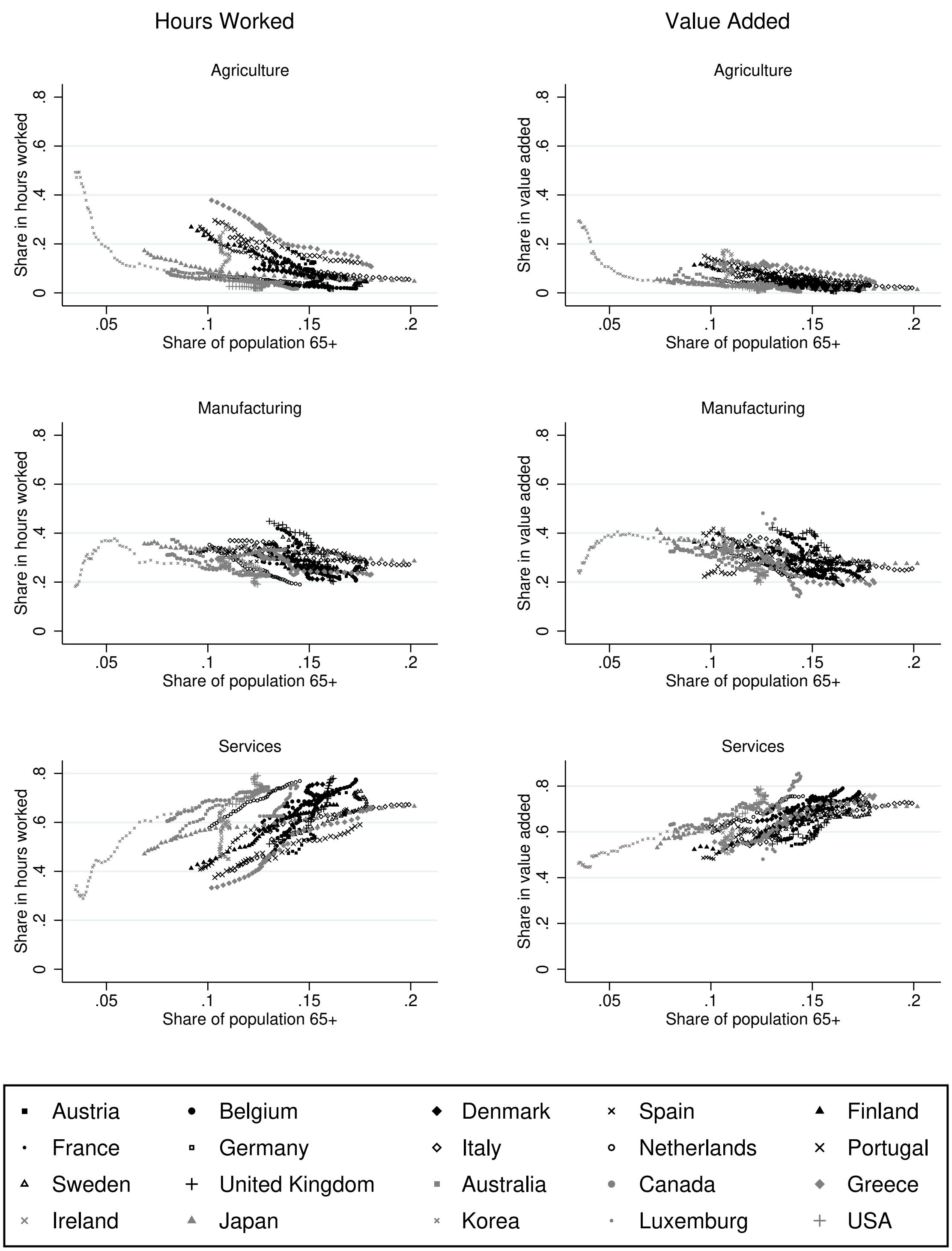
Appendix to Chapter III

C.1 Additional cross-country results

C.1.1 Unconditional patterns

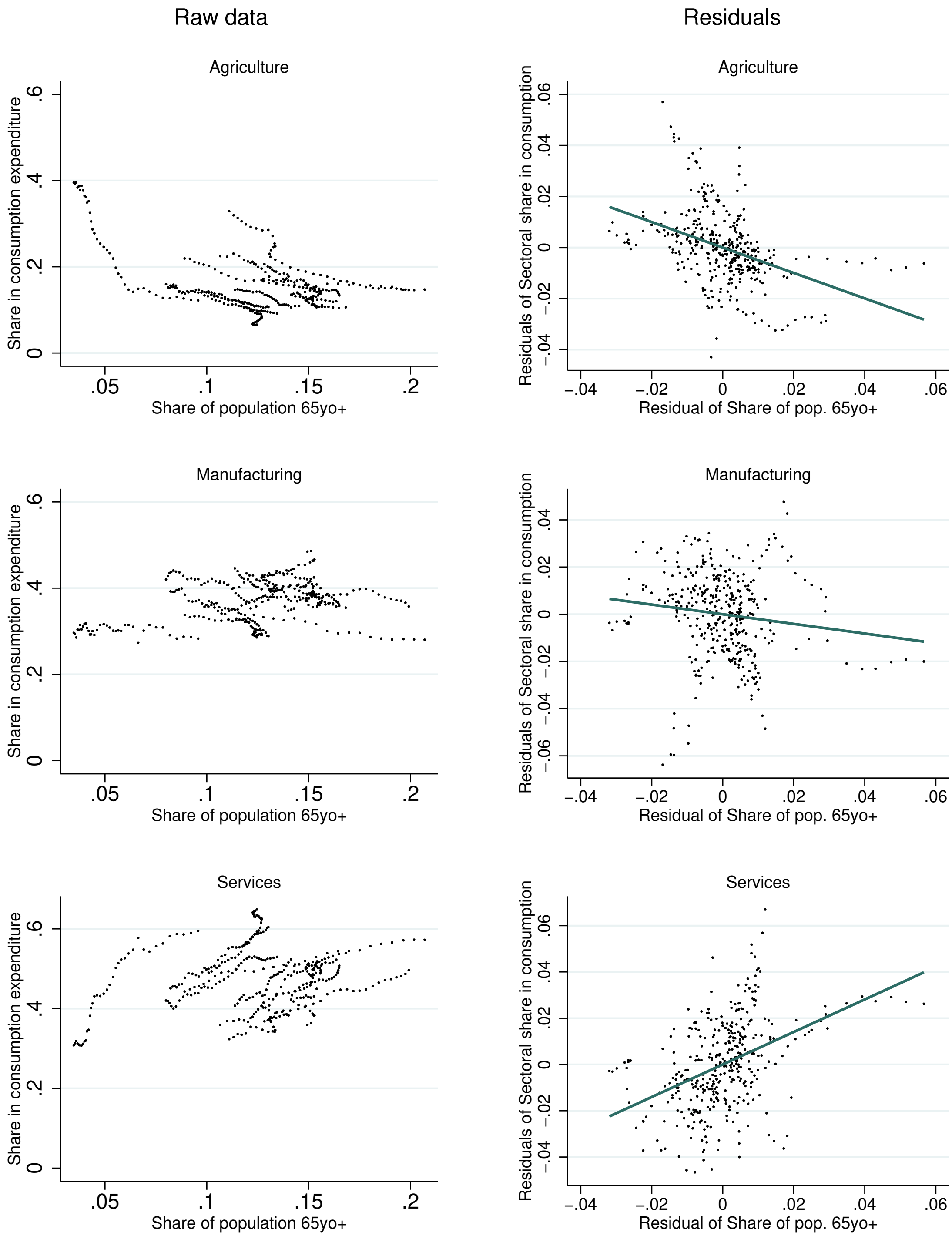
Figure C.1 reports the unconditional sectoral shares of hours worked and the share of population over 65, for each country-year in EU KLEMS. The share of hours in Agriculture decreases as population ages, while the share of hours in Services increases. The employment share in Manufacturing is somewhat hump-shaped. The right panel in the figure shows that the same pattern emerges if we use sectoral value added instead of sectoral hours worked shares. The left panel in Figure C.2 plots the unconditional sectoral consumption shares against the share of population over 65 for each country-year pair in our sample. These figures indicate that even in raw data, economic activity reallocates towards the service sector as the population ages.

Figure C.1: Sectoral shares of employment and value added



Notes: Each dot represents a country-year. The x-axis reports the share of the population that is 65 and over (source: WDI). The y-axis reports the sectoral share in hours worked (left panel) and the sectoral shares in value added (right panel) using data from EU KLEMS.

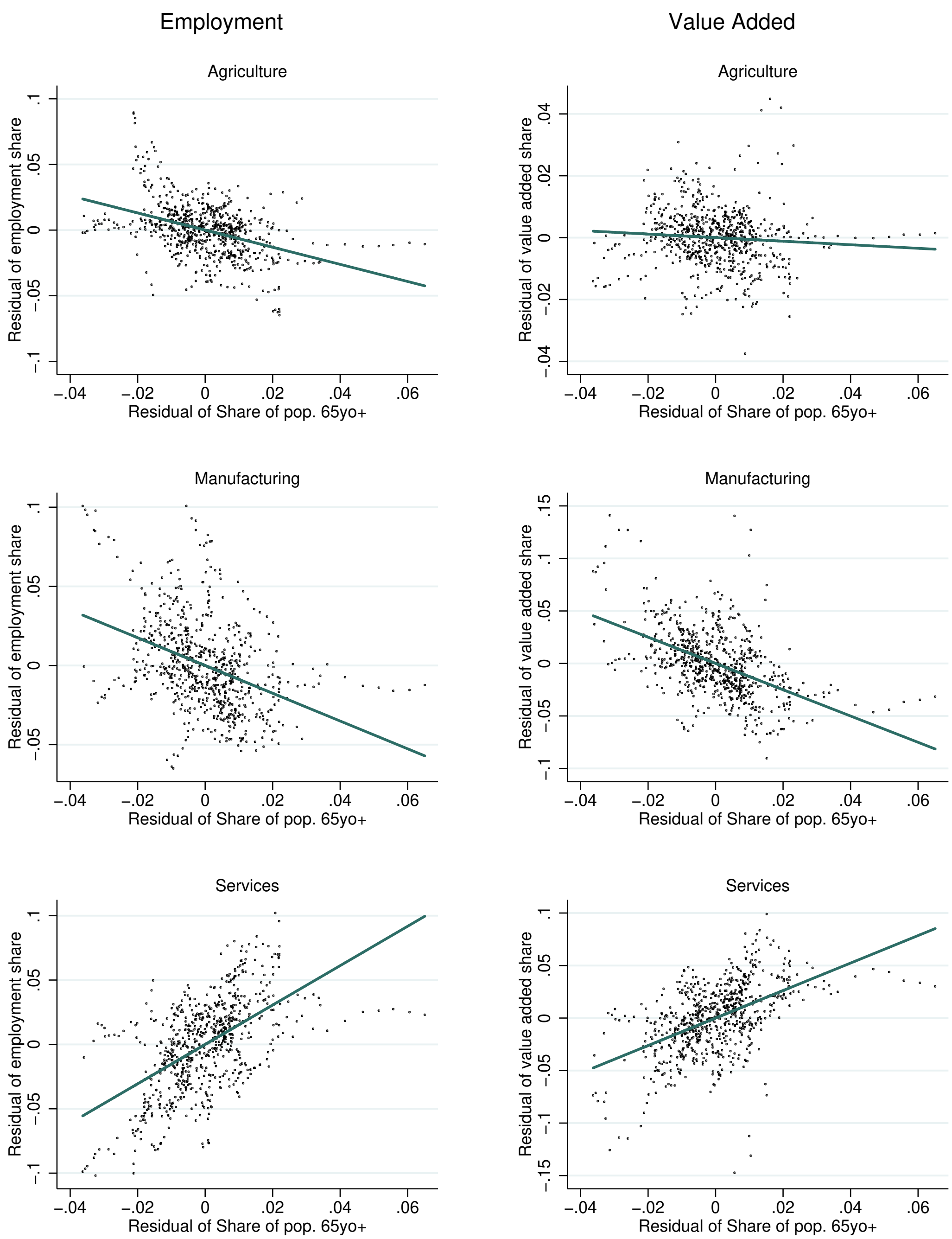
Figure C.2: Sectoral consumption shares



Notes: Each dot represents a country-year. The x-axis reports the actual (left panel) and the residualized (right panel) share of the population that is 65 and over. The y-axis reports the sectoral share in actual (left panel) and the residualized (right panel) sectoral shares in consumption using data from OECD.

Controlling for income: The patterns that underlie Tables 3.1-3.2 can be visualized in Figure C.3 and the right panel of Figure C.2. The y-axis plots the residuals of the regressions of the employment and value added shares on the log of GDP per capita, log of GDP per capita squared and country fixed effects. The x-axis shows the residuals of the share of population that is over 65 on those same variables. The changes in sectoral shares that are orthogonal to the changes in income per capita are strongly correlated to the changes in population age that are orthogonal to income per capita. The figures show that consumption in Agriculture and Manufacturing products decline with population age, while the share of Service consumption increases with population age, after controlling for income and country effects.

Figure C.3: Residualized sectoral shares of employment and value added

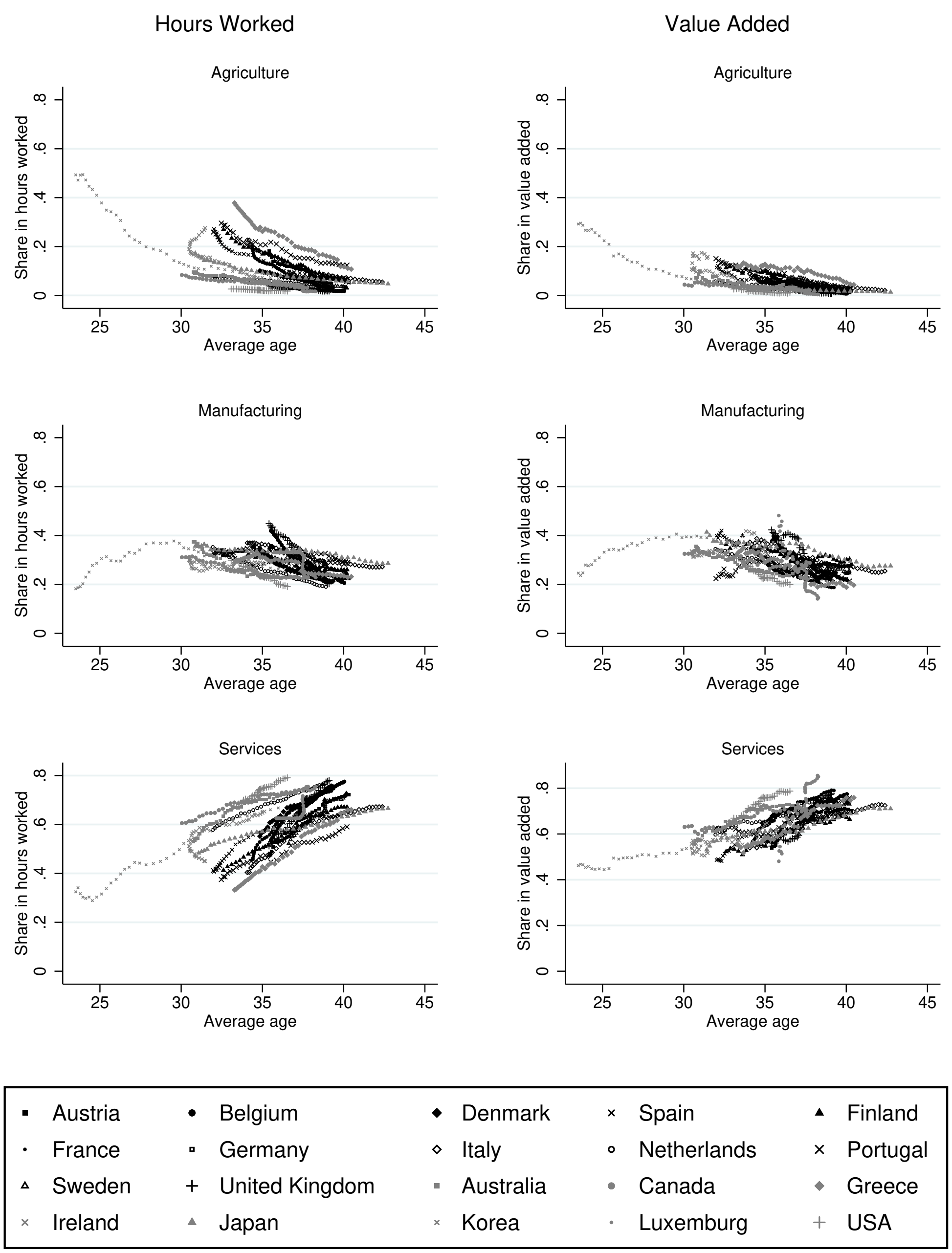


Notes: Each dot represents a country-year. The x-axis reports the residual of a regression of the share of the population that is 65 and over on GDP per capita, GDP per capita squared, and country fixed effects. The y-axis reports the residual of a regression of the sectoral share in hours worked (left panel) and the sectoral shares in value added (right panel) on GDP per capita, GDP per capita squared, and country fixed effects. Data sources are the same as in Figure C.1.

C.1.2 Using average age

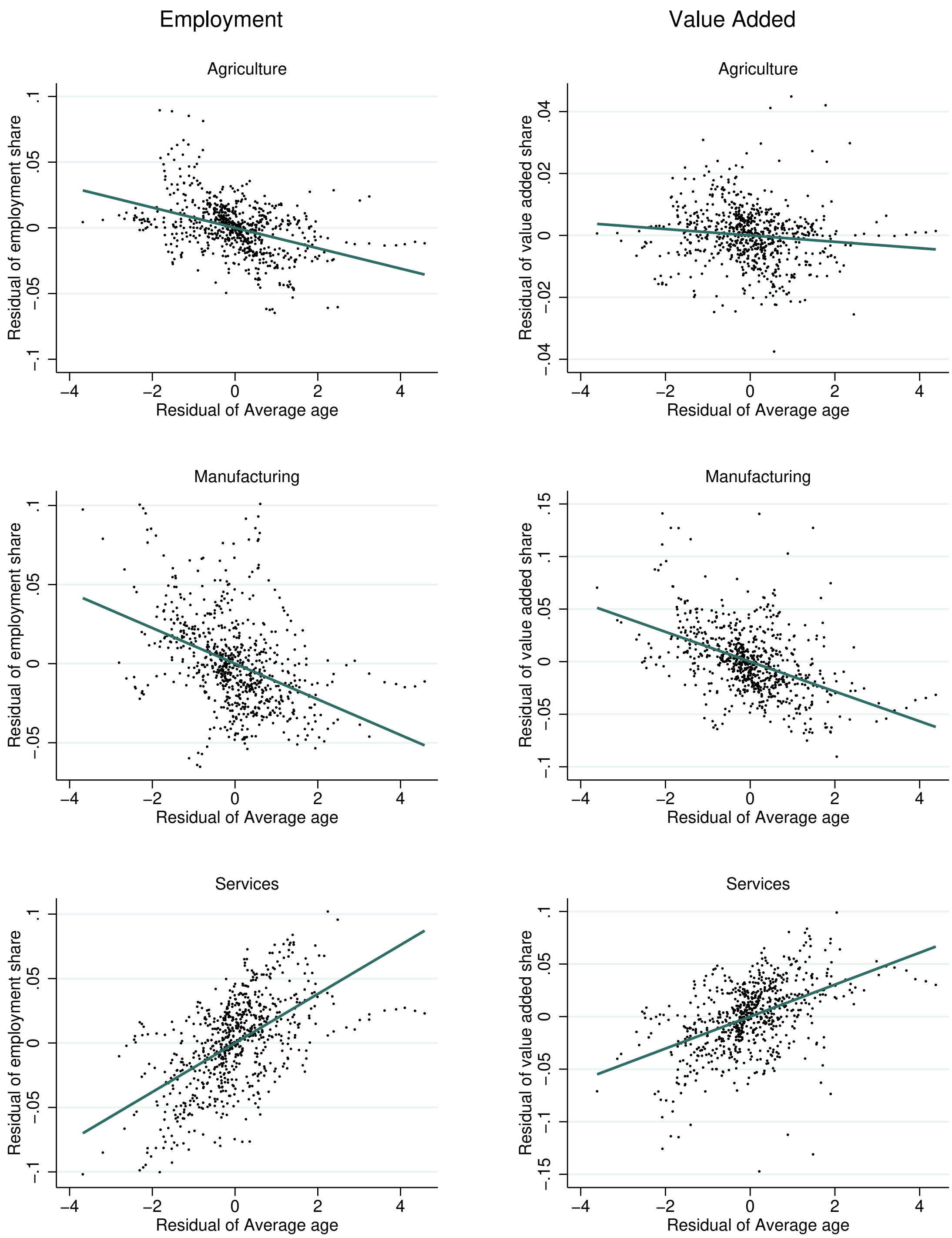
As an alternative measure of aging, we use the average age in the country, computed from the World Bank's "Population estimates and projection" database. This database divides a country's population into 5-year age brackets. To compute the average age, we multiply the midpoint of each bracket (e.g. 2 in the 0-4 years old bracket) times its population, then add across age groups, and finally divide this by the total population. Appendix Figures C.4, C.5, and C.6 show that the patterns documented in the main text and in this Appendix persist if we use the average age in the population instead of the share of population over 65 as our age measure.

Figure C.4: Sectoral shares of employment and value added



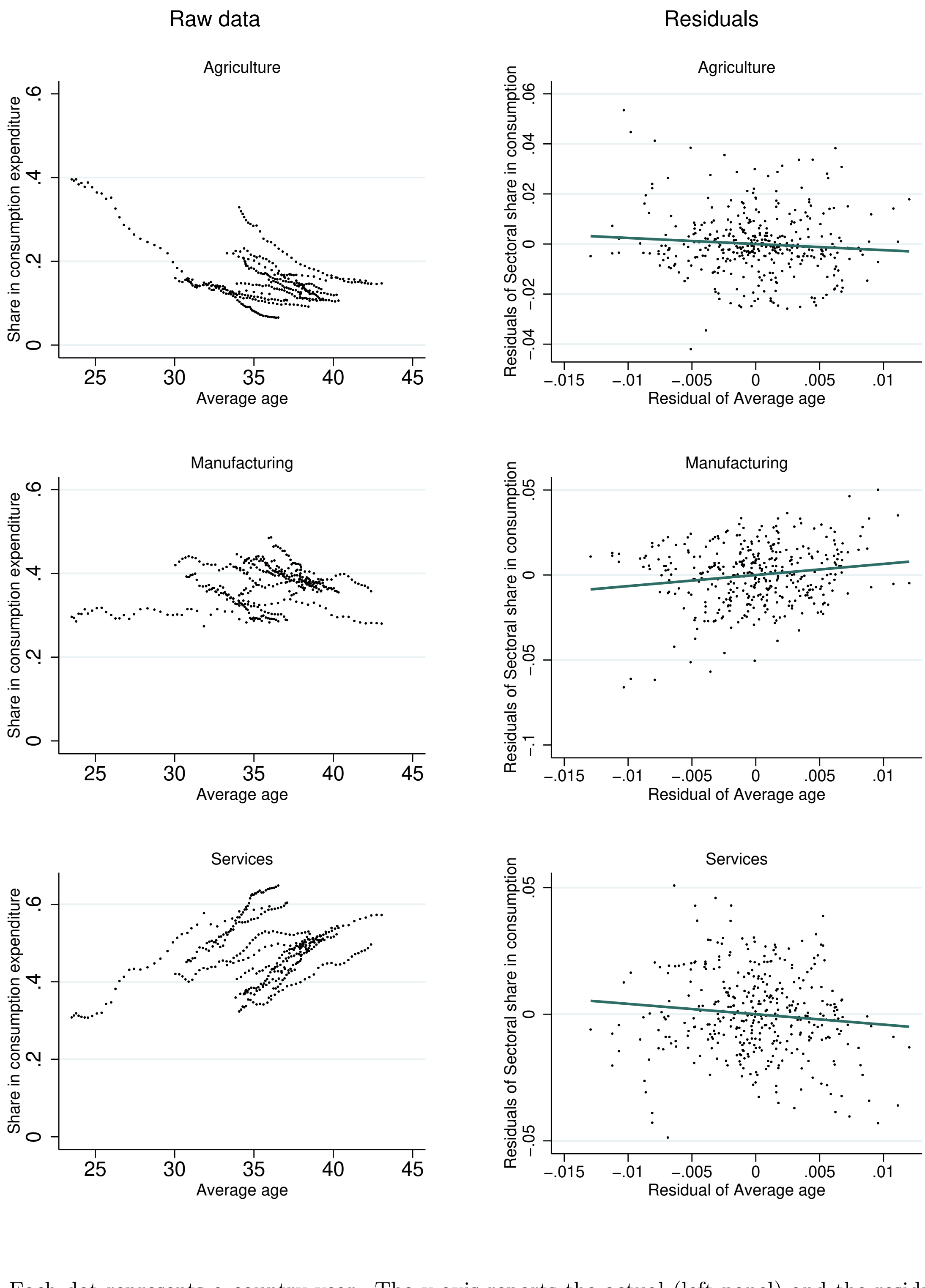
Notes: Each dot represents a country-year. The x-axis reports the average age in the population (source: WDI). The y-axis reports the sectoral share in hours worked (left panel) and the sectoral shares in value added (right panel) using data from EU KLEMS.

Figure C.5: Residualized sectoral shares of employment and value added



Notes: Each dot represents a country-year. The x-axis reports the residual of a regression of the average age in the population on GDP per capita, GDP per capita squared, and country fixed-effects. The y-axis reports the residual of a regression of the sectoral share in hours worked (left panel) and the sectoral shares in value added (right panel) on GDP per capita, GDP per capita squared, and country fixed-effects. Data sources are the same as in Figure C.1.

Figure C.6: Sectoral consumption shares



Notes: Each dot represents a country-year. The x-axis reports the actual (left panel) and the residualized (right panel) average age in the population. The y-axis reports the sectoral share in actual (left panel) and the residualized (right panel) sectoral shares in consumption using data from OECD.

C.1.3 Additional controls

Table C.1 presents the main results for each of the three main outcome variables, controlling for (i) trade openness, (ii) investment/GDP ratio; (iii) government expenditures as a share of GDP, and (iv) the relative price of services. We take the controls (i)-(iii) from the World Development Indicators. The relative price of services was computed by aggregating sectorial price indexes from EU KLEMS. Sectors 15 to 37 in KLEMS were aggregated into Goods, and sectors G, H, 60 to 64, J, 70 to 74, L, M, N, O, P, Q were aggregated into Services. Following Herrendorf et al. (2013) and Bonadio et al. (2021), the indexes were aggregated using a cyclical expansion procedure. In particular, let Y_{it}, Q_{it}, and P_{it} denote the nominal output, the quantity index, and the price index for a sub-sector i at time t provided by KLEMS. Aggregate quantity indexes for Goods and for Services were computed as:

$$Q_t^j \equiv \sqrt{\frac{\sum_{i\in j} P_{it} Q_{it-1}}{\sum_{i\in j} Y_{it-1}} \frac{\sum_{i\in j} Y_{it}}{\sum_{i\in j} P_{it-1} Q_{it}}},$$

and the corresponding price indexes were computed as $P_t^j \equiv \sum_{i\in j} Y_{it}/Q_t^j$. We note that, since our regressions include country fixed effects, the price indexes are sufficient for the purposes of controlling for the within-country changes in the relative price of services over time. The coefficients on the age variable in these alternative specifications are similar to our baseline and statistically significant.

C.1.4 Evidence from the WDI and the UN Statistics Division

This section complements the evidence from Section 3.2.1 using employment data from the WDI and value-added data from the UN. Relative to the data presented in the main text, these sources cover a much broader sample of both developed and developing countries. On the other hand, unlike the EU-KLEMS data, the WDI only reports number of employed persons as opposed to number of hours worked, and the value-added data from the UN are obtained from country-specific sources that are not necessarily harmonized. The WDI yields an unbalanced sample of 157 countries covering 1980-2007, while the UN data cover 188 countries over 1970-2007.

We replicate the fact reported in Section 3.2.1 using these alternative data. Table C.2 and Figure C.7 summarize the results from a regression analogous to Equation (3.1) that is estimated on the WDI data. They show that, after controlling for income, there is a clear negative relation between population age and the employment shares in Agriculture and Manufacturing, and a strong positive relation between population age and the share of employment in the Service sector. These relations are observed for each of our population

Table C.1: Population aging and the services share in hours worked, value added and consumption

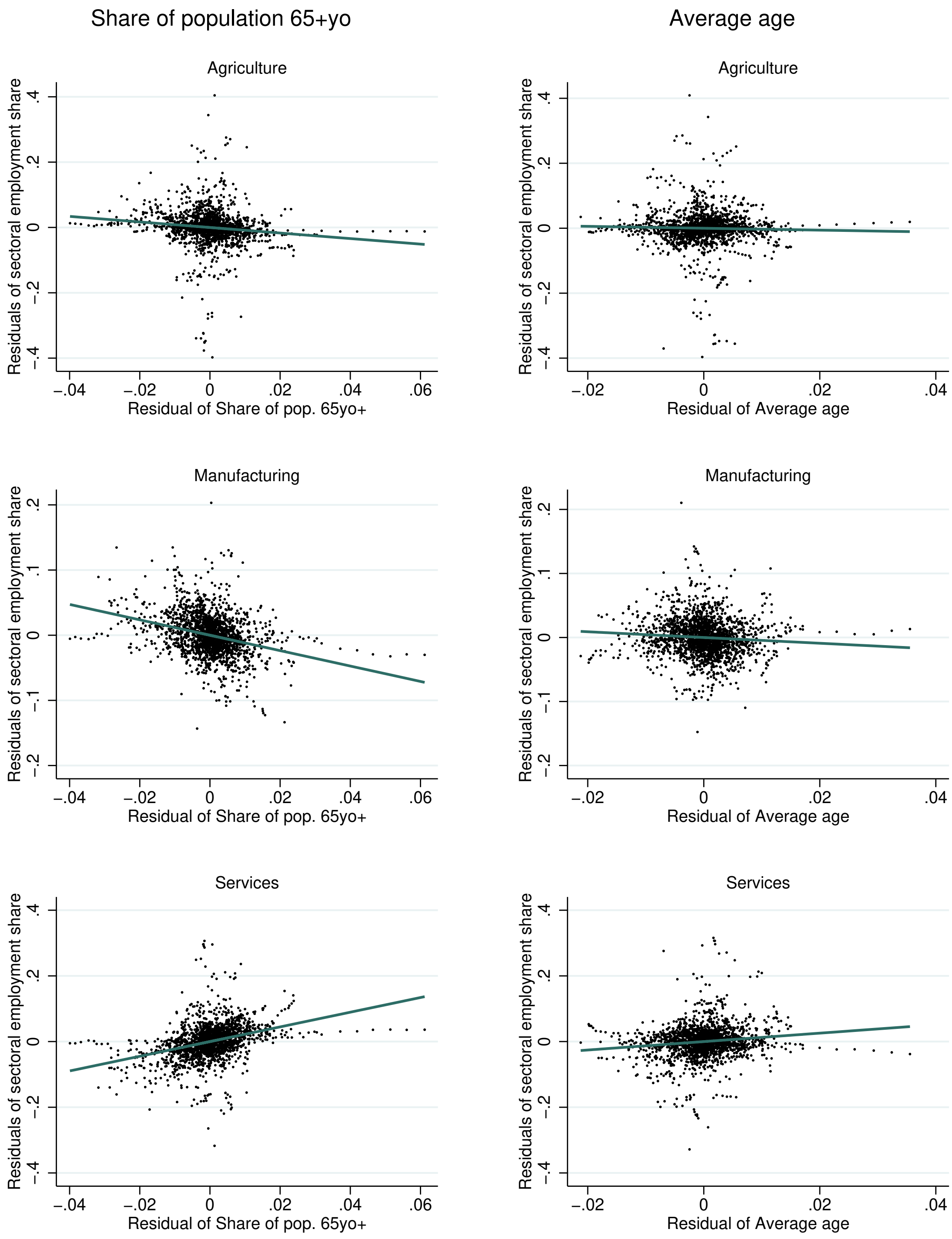
	Hours worked				Value added				Consumption			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$	$\omega_{i,t}^{Ser}$
Share of pop 65+	1.520***	0.828*	1.024*	1.547***	1.278***	0.791**	0.692**	1.385***	0.690***	0.239	0.511*	0.509**
	(0.479)	(0.434)	(0.511)	(0.501)	(0.348)	(0.364)	(0.294)	(0.380)	(0.204)	(0.169)	(0.230)	(0.206)
Log GDP p.c.	-0.023	0.247	-0.067	0.057	-0.890***	-0.638***	-0.901***	-0.555***	-0.200	-0.091	-0.328**	-0.297**
	(0.286)	(0.171)	(0.259)	(0.232)	(0.221)	(0.140)	(0.127)	(0.152)	(0.148)	(0.113)	(0.143)	(0.129)
(Log GDP p.c.)^2	0.010	-0.004	0.012	0.005	0.053***	0.040***	0.054***	0.031***	0.019*	0.014*	0.026**	0.022**
	(0.016)	(0.009)	(0.014)	(0.013)	(0.013)	(0.008)	(0.007)	(0.009)	(0.009)	(0.007)	(0.008)	(0.007)
Trade/GDP	-0.000				-0.000				0.000			
	(0.000)				(0.000)				(0.000)			
Investment/GDP		-0.006***				-0.005***				-0.005***		
		(0.001)				(0.001)				(0.001)		
Goverment/GDP			0.007**				0.009***				0.004*	
			(0.003)				(0.003)				(0.002)	
P_t^s/P_t^g				0.017				0.078***				0.074**
				(0.016)				(0.014)				(0.027)
Observations	707	707	707	707	707	707	707	707	377	377	377	369
R^2	0.924	0.953	0.934	0.924	0.874	0.902	0.901	0.893	0.949	0.964	0.952	0.959

Notes: This table reports the results of estimating equation (3.1) with additional controls. The outcome variables are hours worked, value added and consumption shares in services (*Ser*). Population age is proxied by the share of population 65 years or older. Additional controls Trade/GDP, Investment/GDP and Government/GDP come from WDI. Trade/GDP is the sum of imports and exports as a share of GPD. Control variable $P_t^s/P_t^g$is the ratio of the price of services to manufacturing goods in EU-KLEMS. All specifications include country fixed effects. Standard errors clustered at the country level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

age variables.

Figure C.8 and Table C.3 corroborate that the same patterns described in Section 3.2.1 are also present in the value-added data from the UN. After controlling for income, there is a clear negative relation between population age and the employment shares in Agriculture and Manufacturing, and a strong positive relation between population age and the share of employment in the service sector.

Figure C.7: Residualized sectoral employment shares: WDI data



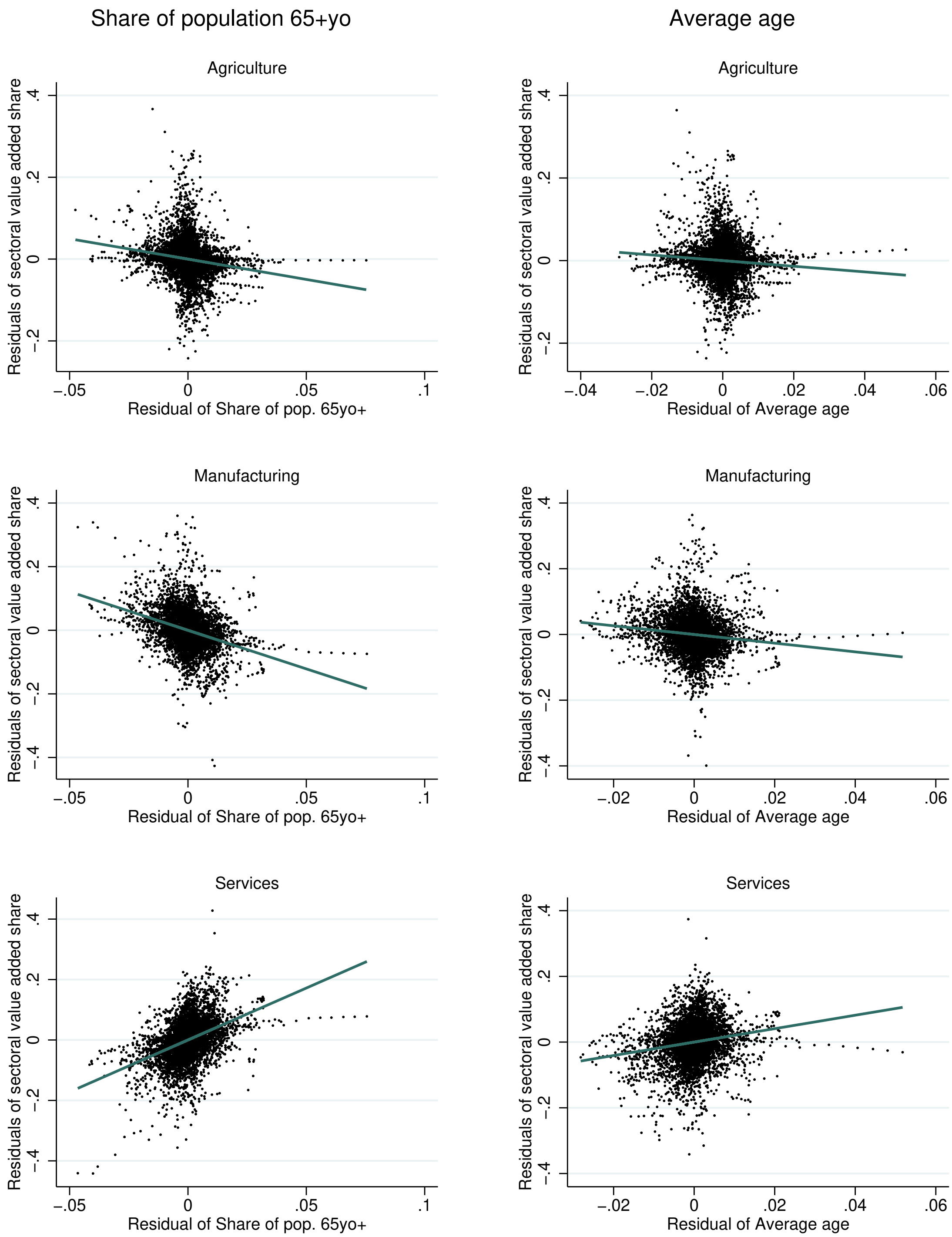
Notes: Each dot represents a country-year. The x-axis reports the residual of a regression of the share of the population that is 65 and over (left panel) or the average age of the population (right panel) on GDP per capita, GDP per capita squared, and country fixed effects. The y-axis reports the residual of a regression of the sectoral share in employment on GDP per capita, GDP per capita squared, and country fixed effects.

Table C.2: Population aging and the services share in employment: WDI data

	(1) $\omega_{i,t}^{Agr}$	(2) $\omega_{i,t}^{Agr}$	(3) $\omega_{i,t}^{Man}$	(4) $\omega_{i,t}^{Man}$	(5) $\omega_{i,t}^{Ser}$	(6) $\omega_{i,t}^{Ser}$
Average age	-0.0136***	-0.00807**	-0.0103***	-0.0110***	0.0249***	0.0189***
	(0.0023)	(0.0035)	(0.0024)	(0.0028)	(0.0024)	(0.0037)
Log GDP per capita		-0.404**		0.771***		-0.304**
		(0.155)		(0.167)		(0.153)
(Log GDP per capita)^2		0.0189**		-0.0416***		0.0194**
		(0.0083)		(0.0093)		(0.0084)
Observations	2206	2029	2214	2037	2214	2037
R^2	0.921	0.919	0.805	0.854	0.904	0.898

Notes: This table reports the results of estimating equation (3.1). The outcome variables are employment shares in agriculture (Agr), manufacturing (Man) and services (Ser). Population age is proxied by the average age. All specifications include country fixed effects. Standard errors clustered at the country level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Figure C.8: Residualized sectoral value-added shares: UN data



Notes: Each dot represents a country-year. The x-axis reports the residual of a regression of the share of the population that is 65 and over (left panel) or the average age of the population (right panel) on GDP per capita and country fixed effects. The y-axis reports the residual of a regression of the sectoral share in value added (second panel) on GDP per capita and country fixed effects.

Table C.3: Population aging and the services share in value-added: UN data

	(1) $\omega_{i,t}^{Agr}$	(2) $\omega_{i,t}^{Agr}$	(3) $\omega_{i,t}^{Man}$	(4) $\omega_{i,t}^{Man}$	(5) $\omega_{i,t}^{Ser}$	(6) $\omega_{i,t}^{Ser}$
Average age	-0.0117***	-0.00570***	-0.00648***	-0.0153***	0.0180***	0.0210***
	(0.00136)	(0.00143)	(0.00166)	(0.00267)	(0.00163)	(0.00282)
Log GDP pc		-0.380***		0.276***		0.113
		(0.0642)		(0.0783)		(0.0910)
(Log GDP pc)^2		0.0181***		-0.0105**		-0.00822
		(0.00360)		(0.00514)		(0.00563)
Observations	6509	6156	6547	6194	6547	6194
R^2	0.880	0.908	0.778	0.822	0.829	0.826

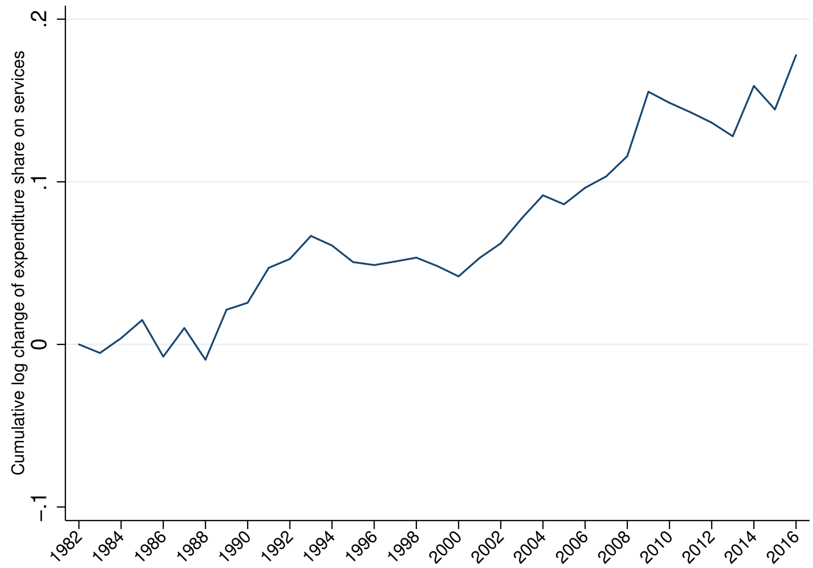
Notes: This table reports the results of estimating equation (3.1). The outcome variables are value added shares in agriculture (Agr), manufacturing (Man) and services (Ser). Population age is proxied by the average age. All specifications include country fixed effects. Standard errors clustered at the country level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

C.2 Additional results, household-level data and model

C.2.1 Additional tables and figures, CES

Figure C.9 plots the cumulative change in the aggregate expenditure share on services in the CES data. Consistent with the aggregate evidence on structural transformation, the service expenditure share rises in the CES, by about 0.18 log points over this period. Appendix Table C.4 reports the trends in broad service expenditure categories. The rise in the healthcare is the main, but not the only, driver of the upward trend in the service expenditure. Other categories showing substantial proportional increases are Cash Contributions and Education.

Figure C.9: Service consumption in the CES



Notes: This figure displays the cumulative log change in the aggregate expenditure share on services in the CES.

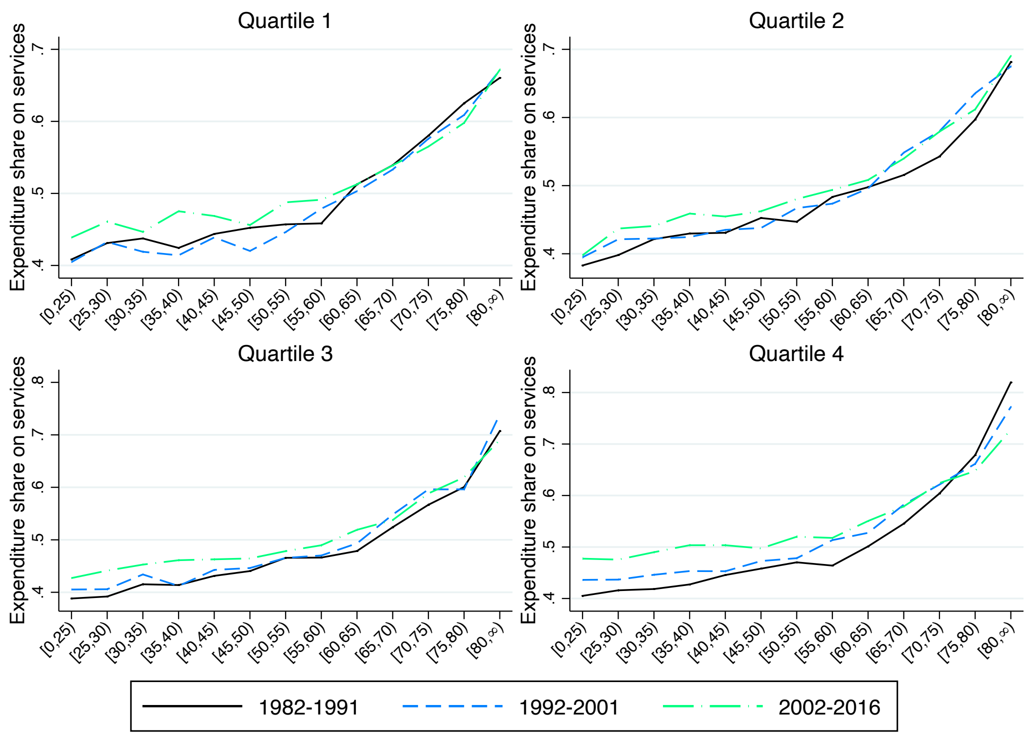
Table C.4: Expenditure shares on goods and services

	Baseline			Baseline w/ housing			All exp. in CES		
	82-91	92-01	02-16	82-91	92-01	02-16	82-91	92-01	02-16
Goods	*51.0*	*49.8*	*47.7*	*40.5*	*38.1*	*35.4*	*37.0*	*34.6*	*31.6*
Food at home	15.6	15.1	14.7	12.4	11.5	10.9	11.4	10.6	9.8
Vehicle purchasing, leasing	12.0	13.6	12.0	9.6	10.4	8.9	8.7	9.4	7.9
Gas	5.4	4.3	6.3	4.3	3.2	4.7	3.9	2.9	4.2
Entertainment equipment	4.1	4.7	5.3	3.2	3.6	3.9	2.9	3.3	3.5
Appliances	2.7	2.8	2.3	2.1	2.2	1.7	2.0	2.0	1.5
Men's and women's clothing	3.9	3.1	1.9	3.1	2.4	1.4	2.8	2.2	1.3
Furnitures and Fixtures	2.4	2.0	1.7	1.9	1.5	1.3	1.8	1.4	1.1
Alcoholic beverages	1.5	1.2	1.2	1.2	0.9	0.9	1.1	0.8	0.8
Shoes and other apparel	1.5	1.2	0.9	1.2	0.9	0.7	1.1	0.9	0.6
Tobacco	1.3	1.1	1.0	1.0	0.8	0.7	0.9	0.7	0.6
Children's clothing	0.6	0.6	0.4	0.5	0.5	0.3	0.4	0.4	0.3
Personal care goods	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Services	*49.0*	*50.2*	*52.3*	*59.5*	*61.9*	*64.6*	*63.0*	*65.4*	*68.4*
Health	9.1	10.1	12.1	7.2	7.7	9.0	6.8	7.2	8.2
Utilities	11.0	10.7	11.6	8.8	8.2	8.6	8.1	7.5	7.8
Cash contributions	4.9	5.1	5.7	3.9	3.9	4.3	3.7	3.7	3.9
Car maint, repairs	5.4	5.9	5.2	4.3	4.5	3.9	3.9	4.1	3.5
Food away from home	6.4	5.9	5.0	5.1	4.5	3.7	4.6	4.1	3.3
Domestic services	4.2	4.1	4.2	3.3	3.2	3.1	3.1	2.9	2.8
Education	1.4	1.7	2.7	1.1	1.3	2.0	1.0	1.2	1.8
Entertainment fees/adm., read.	2.9	3.0	2.5	2.3	2.3	1.8	2.1	2.1	1.6
Public transport	1.9	2.0	1.8	1.5	1.5	1.3	1.4	1.4	1.2
Personal care services	1.4	1.3	1.0	1.1	1.0	0.8	1.0	0.9	0.7
Childcare	0.3	0.4	0.5	0.3	0.3	0.4	0.2	0.3	0.3
Housing	.	.	.	20.6	23.5	25.7	18.9	21.5	23.0
Personal insurance	.	.	.	.	.	.	1.4	1.3	0.8
Pensions	.	.	.	.	.	.	6.7	7.3	9.4

Notes: This table reports the aggregate expenditure shares on broad categories of goods and services, in the three decades separately, in the baseline using the CES, including housing and using the entire Interview dataset in the CES.

Figure C.10 plots the age-service expenditure share relationships separately for each quartile of the income distribution. It is clear that the relationship is about equally strong within broad income groups.

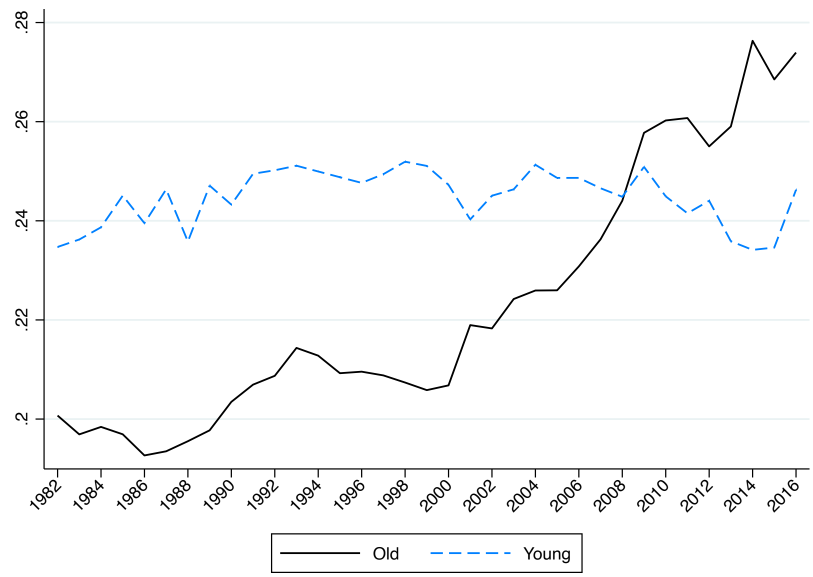
Figure C.10: Service consumption by average age of household members and income



Notes: This figure displays the average household-level expenditure shares on services in the CES by age group (x-axis), for 3 time periods, and each income quartile.

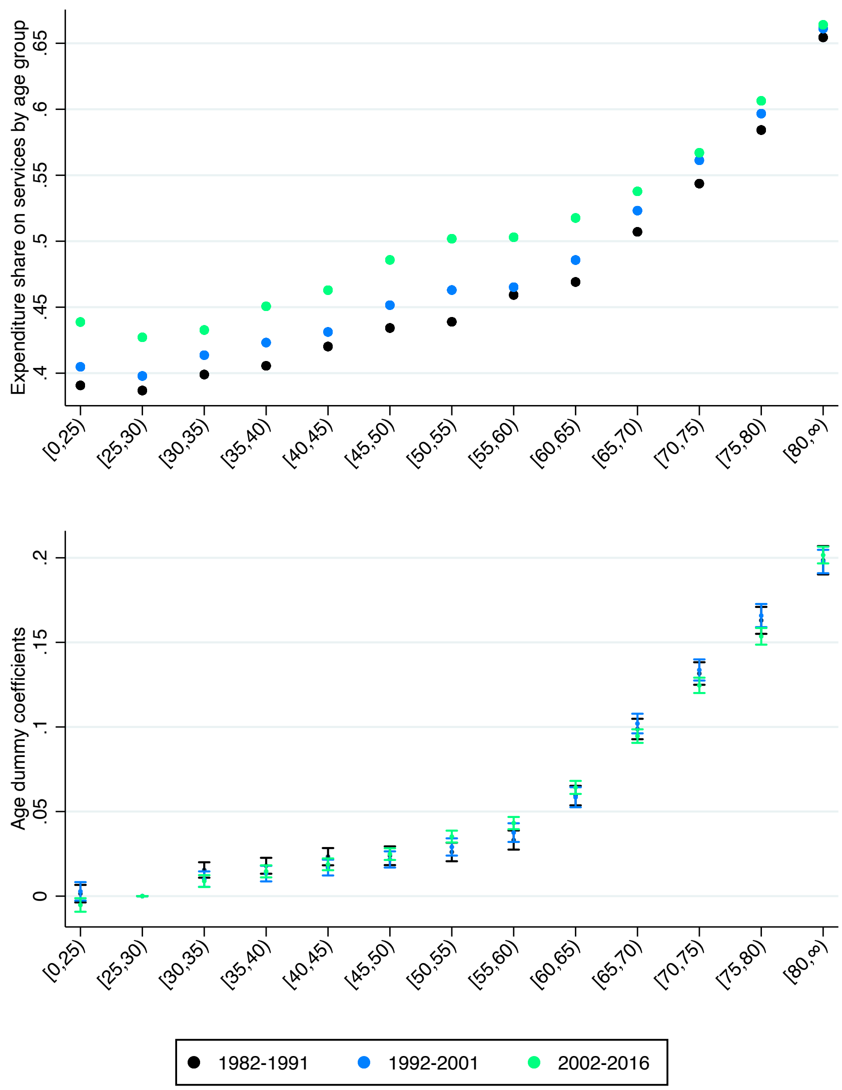
Structural change within the service sector The rise in service expenditures has been concentrated in categories that are disproportionally consumed by older households. Figure C.11 divides service categories into two groups: one for the categories that are disproportionally consumed by the old (Health, Utilities, and Domestic Services), and one for the remaining categories. The figure shows a dramatic increase in the aggregate expenditure share for Health, Utilities, and Domestic Services, the combined expenditure share in these categories goes from 21 to over 28 percent over our period. In contrast, there is no change in the expenditure share in the remaining service categories. Figure C.20 shows that a similar pattern emerges in the Personal Consumption Expenditure data from the BEA: the increase in service consumption is concentrated among those categories that are disproportionally consumed by the old.

Figure C.11: Evolution of expenditure shares on service categories in the CES



Notes: 'Old' displays the aggregate expenditure share in the CES on categories that are disproportionally consumed by the old: Health, Utilities, and Domestic Services (excluding Childcare). 'Young' displays the expenditure share on the remaining service categories.

Figure C.12: Service consumption by age of the reference person



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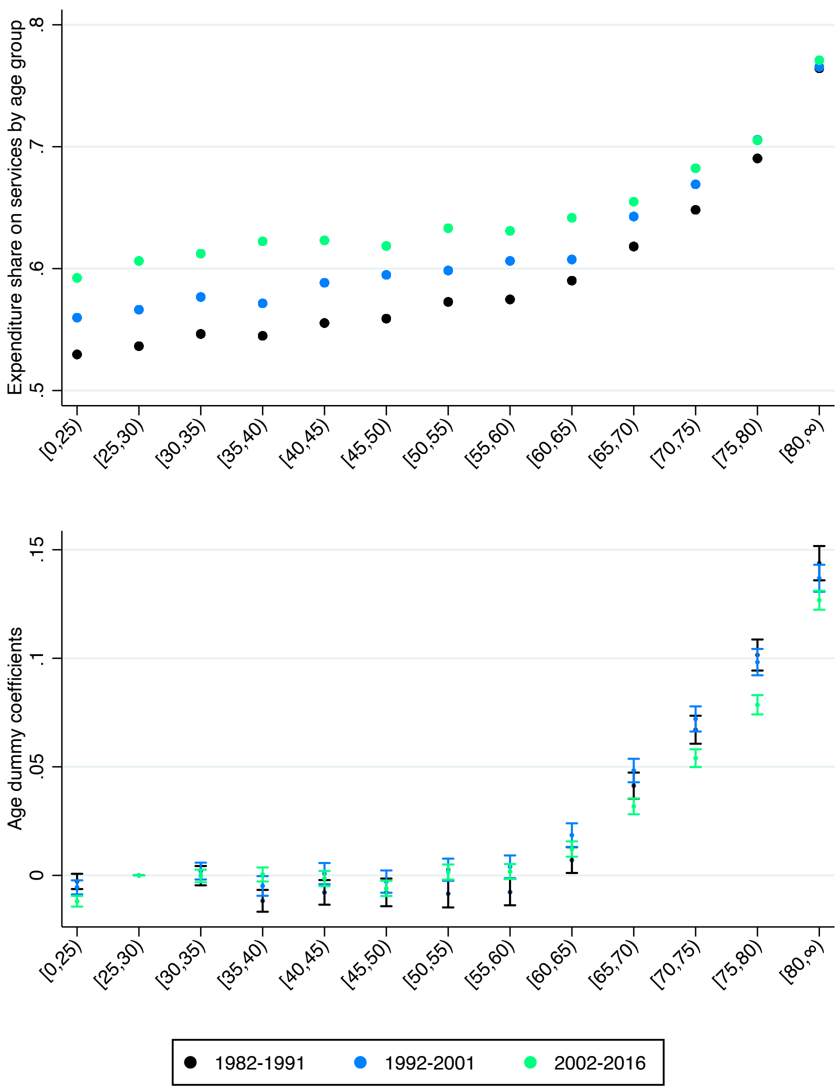
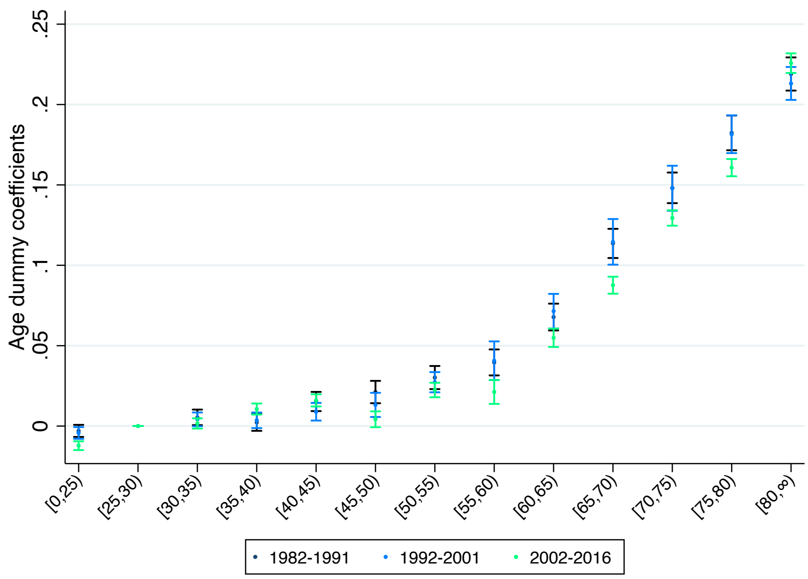
Notes: The top panel displays the average household-level expenditure shares on services in the CES by age group according to the age of the reference person (x-axis), for 3 time periods. The bottom panel displays the age dummies resulting from estimating equation (2). Each dot represents the point estimate of the age dummies for a particular decade in the CES data. The omitted dummy is that of age group 25-30. The bands report the 95% confidence intervals based on standard errors clustered at the household level.

Figure C.13: Service consumption with housing by average age of household members



Notes: The top panel displays the average household-level expenditure shares on services in the CES by age group (x-axis), for 3 time periods. The bottom panel displays the age dummies resulting from estimating equation (2). Each dot represents the point estimate of the age dummies for a particular decade in the CES data. The omitted dummy is that of age group 25-30. The bands report the 95% confidence intervals based on standard errors clustered at the household level. Housing is included in expenditures.

Figure C.14: Age dummies (controlling for income decile), including age-specific price indices



Notes: Each dot represents the point estimate of the age dummies in modified Equation (3.2) for a particular decade in the CES data. The modified equation includes age-specific price indices as controls. The omitted dummy is that of age group 25-30. The bands report the 95% confidence intervals based on standard errors clustered at the household level.

Table C.5: Population aging and changes in the services share, including housing

Panel A: Expenditure shares across the age distribution						
	Pop $_{1982}$	s^a_{1982}	$\omega^{s,a}_{1982}$	Pop $_{2016}$	s^a_{2016}	$\omega^{s,a}_{2016}$
0-25	31.8	31.6	51.8	20.4	20.0	61.2
25-30	13.5	16.1	52.1	11.4	12.0	61.9
30-35	9.4	11.3	54.3	9.4	10.8	63.4
35-40	6.2	7.5	53.9	7.1	7.9	62.7
40-45	4.6	5.3	55.2	5.9	6.5	65.5
45-50	3.6	3.9	55.6	5.2	5.5	63.7
50-55	3.8	3.9	56.0	6.1	6.1	63.7
55-60	5.1	4.8	57.2	6.7	6.8	63.6
60-65	5.7	5.2	60.1	7.5	7.5	67.7
65-70	5.9	4.5	62.1	6.8	6.2	67.4
70-75	4.3	2.8	66.9	5.1	4.4	67.4
75-80	3.3	1.8	68.0	3.4	2.7	70.2
80+	2.9	1.3	76.5	5.0	3.5	78.6

Panel B: Within-between decomposition

	Average		Reference	
	Value	%	Value	%
Within	0.0811	86.3	0.0834	88.7
Between	0.0129	13.7	0.0107	11.3
Total	0.0940	100	0.0940	100

Notes: In Panel A, 'Pop' reports the share of the population in each age group, and s^a_t and ω^a_t are defined as in Equation (3.4). Panel B reports the results of the decomposition in equation (3.4). 'Average' uses the average age across all household member as the age of the household. 'Reference' uses the age of the head in the household. Housing is included in expenditures.

Table C.6: Share of out-of-pocket expenses in total personal healthcare expenses, NHES

Age group	2002	2014
0-44	0.144	0.112
45-64	0.164	0.121
65+	0.173	0.153

Notes: This table reports the ratios of out-of-pocket to total personal healthcare expenditures by broad age group from the National Health Expenditure Survey.

C.2.2 Additional tables and figures for Section 3.3.2

Table C.7: Estimates of equation (3.8) for different age measures

	(1)	(2)	(3)	(4)
Dep. var.: $\ln \omega_t^{g,n}$				
$\ln e_t^n$	-0.116***	-0.117***	-0.118***	-0.119***
	(0.00178)	(0.00179)	(0.00191)	(0.00191)
$D^{[0,25)}$	0.0139***	0.0141***	-0.0557***	-0.0555***
	(0.00206)	(0.00206)	(0.00330)	(0.00331)
$D^{[30,35)}$	-0.0150***	-0.0155***	0.000930	0.000256
	(0.00254)	(0.00254)	(0.00275)	(0.00275)
$D^{[35,40)}$	-0.0258***	-0.0266***	0.00153	0.000858
	(0.00283)	(0.00283)	(0.00278)	(0.00279)
$D^{[40,45)}$	-0.0454***	-0.0461***	-0.00562**	-0.00629**
	(0.00313)	(0.00314)	(0.00286)	(0.00286)
$D^{[45,50)}$	-0.0562***	-0.0575***	-0.0264***	-0.0270***
	(0.00325)	(0.00326)	(0.00292)	(0.00293)
$D^{[50,55)}$	-0.0932***	-0.0930***	-0.0594***	-0.0597***
	(0.00332)	(0.00333)	(0.00302)	(0.00302)
$D^{[55,60)}$	-0.118***	-0.118***	-0.0879***	-0.0888***
	(0.00326)	(0.00326)	(0.00316)	(0.00317)
$D^{[60,65)}$	-0.172***	-0.173***	-0.142***	-0.142***
	(0.00338)	(0.00338)	(0.00335)	(0.00336)
$D^{[65,70)}$	-0.255***	-0.255***	-0.224***	-0.225***
	(0.00360)	(0.00360)	(0.00349)	(0.00349)
$D^{[70,75)}$	-0.340***	-0.341***	-0.309***	-0.310***
	(0.00402)	(0.00403)	(0.00397)	(0.00397)
$D^{[75,80)}$	-0.435***	-0.436***	-0.406***	-0.407***
	(0.00483)	(0.00482)	(0.00462)	(0.00462)
$D^{[80,\infty)}$	-0.592***	-0.592***	-0.551***	-0.552***
	(0.00548)	(0.00548)	(0.00508)	(0.00508)
Age variable	Average	Average	Reference	Reference
Time FE	Yes	No	Yes	No
Region-Time FE	No	Yes	No	Yes
Observations	1,226,096	1,220,472	1,226,096	1,220,472
R^2	0.099	0.100	0.085	0.087

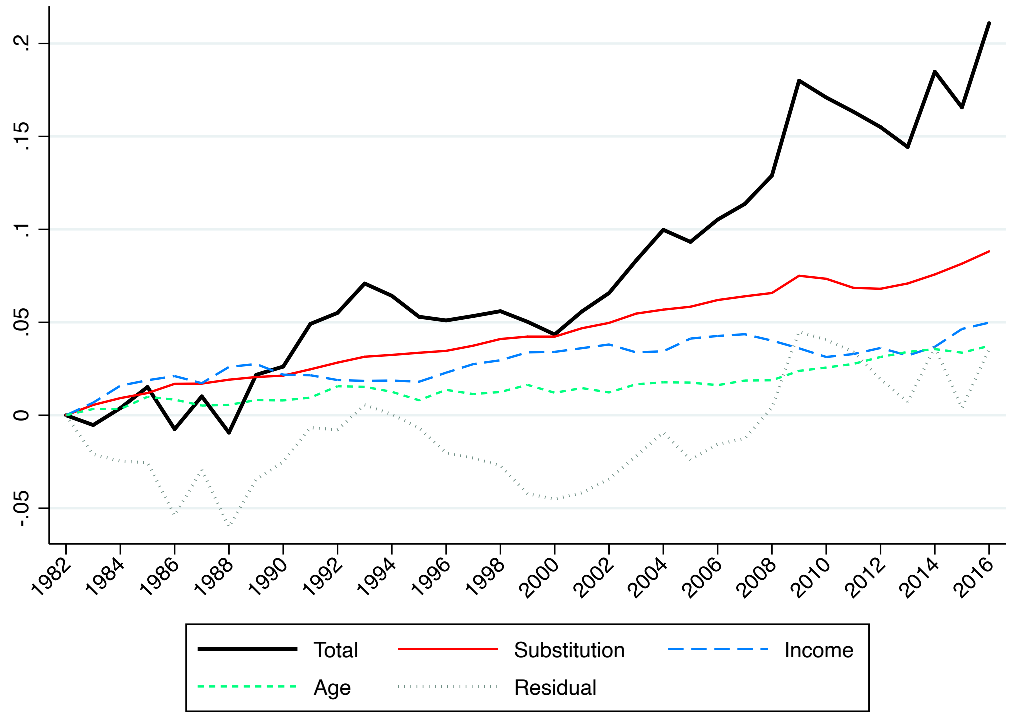
Notes: This table reports the results of estimating equation (3.8). The outcome variable is household expenditure share on goods. Standard errors clustered at the household level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Table C.8: Estimates of equation (3.8) with housing

	(1)	(2)	(3)	(4)
Dep. var.: $\ln \omega_t^{g,n}$				
$\ln e_t^n$	-0.0906***	-0.0869***	-0.0893***	-0.0847***
	(0.00218)	(0.00219)	(0.00238)	(0.00239)
$D^{[0,25)}$	0.0344***	0.0345***	-0.00426	-0.000668
	(0.00254)	(0.00253)	(0.00397)	(0.00396)
$D^{[30,35)}$	-0.0152***	-0.0151***	-0.000360	-0.00143
	(0.00320)	(0.00318)	(0.00343)	(0.00341)
$D^{[35,40)}$	-0.0155***	-0.0157***	0.00620*	0.00525
	(0.00358)	(0.00355)	(0.00348)	(0.00347)
$D^{[40,45)}$	-0.0370***	-0.0366***	0.0139***	0.0126***
	(0.00394)	(0.00393)	(0.00354)	(0.00352)
$D^{[45,50)}$	-0.0360***	-0.0372***	0.00962***	0.00802**
	(0.00404)	(0.00404)	(0.00361)	(0.00360)
$D^{[50,55)}$	-0.0684***	-0.0692***	-0.0132***	-0.0143***
	(0.00408)	(0.00408)	(0.00371)	(0.00370)
$D^{[55,60)}$	-0.0723***	-0.0734***	-0.0263***	-0.0283***
	(0.00397)	(0.00396)	(0.00384)	(0.00383)
$D^{[60,65)}$	-0.106***	-0.108***	-0.0617***	-0.0630***
	(0.00401)	(0.00400)	(0.00399)	(0.00398)
$D^{[65,70)}$	-0.178***	-0.178***	-0.128***	-0.128***
	(0.00414)	(0.00414)	(0.00408)	(0.00408)
$D^{[70,75)}$	-0.251***	-0.252***	-0.202***	-0.203***
	(0.00453)	(0.00455)	(0.00452)	(0.00452)
$D^{[75,80)}$	-0.351***	-0.351***	-0.299***	-0.299***
	(0.00531)	(0.00532)	(0.00512)	(0.00513)
$D^{[80,\infty)}$	-0.560***	-0.558***	-0.487***	-0.484***
	(0.00657)	(0.00659)	(0.00612)	(0.00614)
Age variable	Average	Average	Reference	Reference
Time FE	Yes	No	Yes	No
Region-Time FE	No	Yes	No	Yes
Observations	1,226,096	1,220,472	1,226,096	1,220,472
R^2	0.078	0.084	0.064	0.070

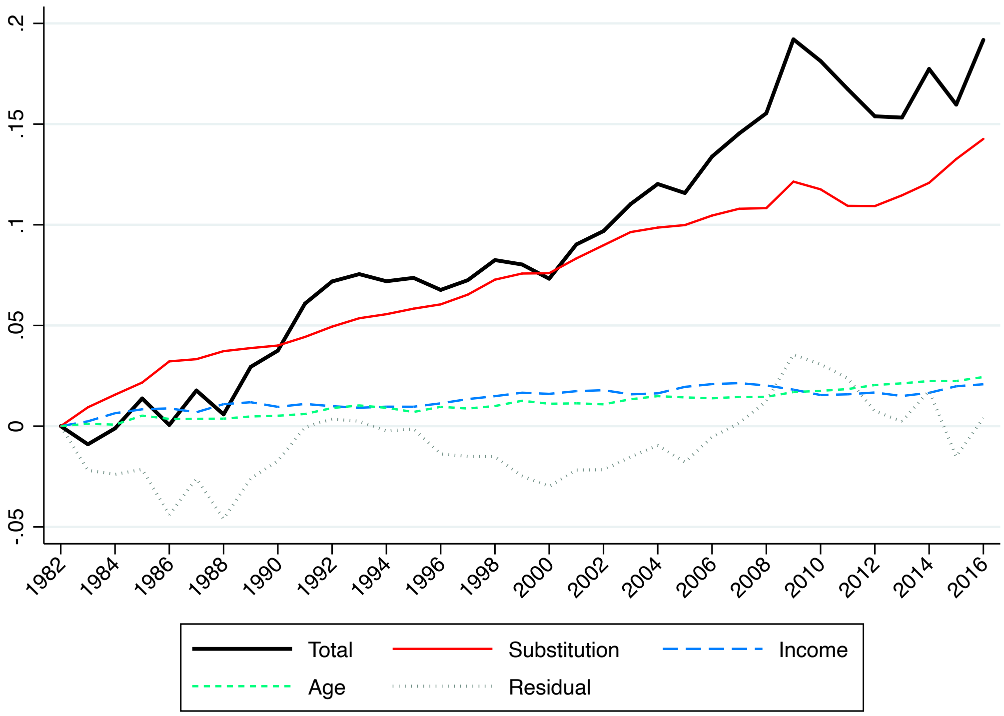
Notes: This table reports the results of estimating equation (3.8). The outcome variable is household expenditure share on goods including housing. Standard errors clustered at the household level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%. Housing is included in expenditures.

Figure C.15: Accounting for structural change in the US, using reference person's age



Notes: This figure displays the decomposition (3.10) for the US from 1982 to 2016, using the age of the reference person as the age variable.

Figure C.16: Accounting for structural change in the US, using housing as service



Notes: This figure displays the decomposition (3.10) for the US from 1982 to 2016, using the average age of members as the age variable and including housing as part of service consumption.

C.2.3 Rescaling CES expenditure data to aggregate data

Rescaling procedure This section rescales the expenditure data in the Consumption Expenditure Survey to match the aggregate Personal Consumption Expenditure (PCE) shares reported by the BEA. In principle, these data need not coincide, since they are collected from different sources that use very different methodologies.[1] After concording the expenditure categories in the CES to PCE items in the BEA data, we compute total expenditures in the CES, $e_t^{j,CES}$, for each category j and year t. We then create the scaling factor for each category that reflects the discrepancy in the aggregate expenditure between the CES and the BEA: $X_t^j = e_t^{j,BEA}/e_t^{j,CES}$. Then, we rescale the consumption expenditure of each household by this factor: $e_t^{j,h} = e_t^{j,h,CES} \times X_t^j$. In this way, the aggregate expenditure on each category in each year in the CES in the rescaled data match the BEA aggregates in every category and year.

Using the rescaled expenditures, we compute the expenditure shares $\omega_t^{j,h} \equiv e_t^{j,h} / \sum_j e_t^{j,h}$,

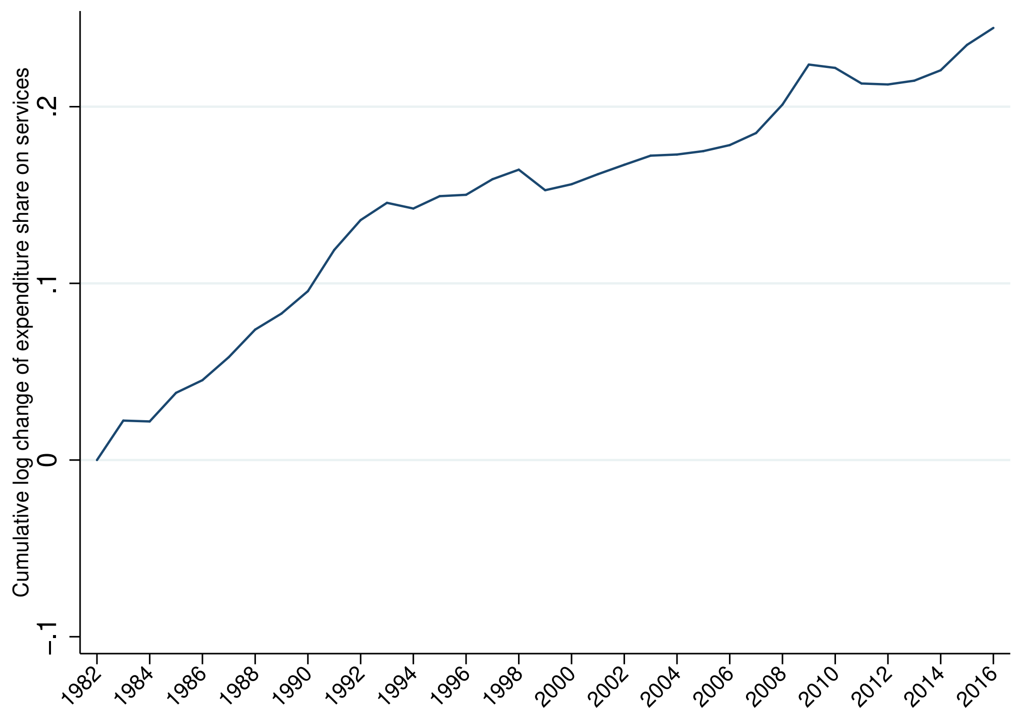
[1]The CES collects expenditures from households surveys, while the BEA final sales made by businesses in a way that is consistent with the National Income and Product Accounts.

and the total expenditures by household: $e_t^h \equiv \sum_j e_t^{j,h}$. From this, we compute the new e_t^h / e_t. These steps give us all the elements of a new dataset, on which we repeat the household-level estimation in Section 3.2.2 and the quantitative analysis of Section 3.3. This approach relies on the assumption that the micro variation across households in the CES is an accurate reflection of the differences in spending patterns by age group. In the main text, we argued based on evidence from another survey that this is likely to be the case with healthcare, where the ratio to out-of-pocket to total expenditure is stable across age groups. Unfortunately, similar data on other categories of public expenditures by age group are not readily available. A particularly concerning category is education, which is a service consumed disproportionally by the young where public expenditures are large. We construct a lower bound for the effect of aging on the service share of consumption by adopting the extreme assumption that all of the public education expenditure goes to the younger (below 65) households.[2] The age profile of service consumption is quite similar to the baseline reported below.

Replication of main results using rescaled data Figure C.17 plots the cumulative log change in the aggregate expenditure share on services in the BEA PCE data. These data show a somewhat larger change than the CES, with the expenditure share of services rising by 0.24 log points. Figure C.18 shows the service expenditure shares for households of different ages, and the three time periods. It also displays the age dummies controlling for income, as in equation (3.2). The magnitudes of the differences across households are similar to the baseline analysis. Figure C.19 breaks down by income quartile. The results are quite similar to the baseline.

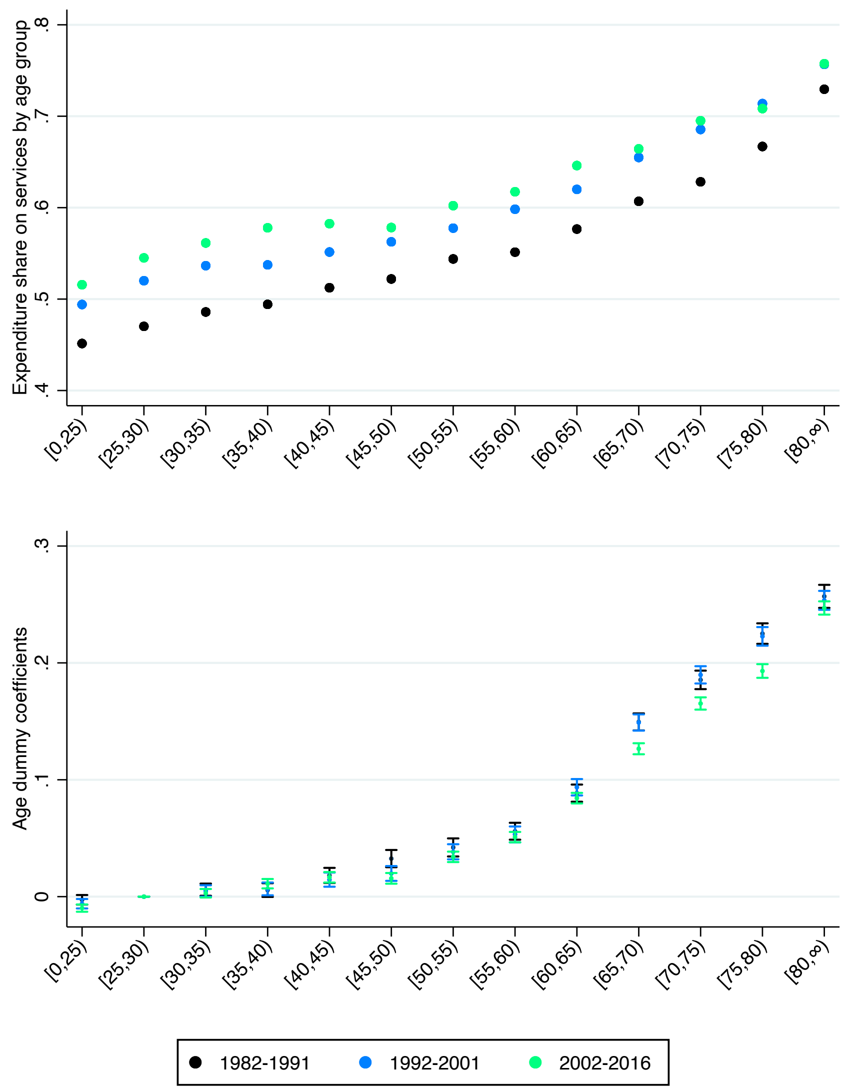
[2]That is, we rescale the CES data to match the BEA aggregates, assuming that the over-65s receive zero public education expenditure. This gives us an lower bound on the impact of aging on the service share, since education is a service and we are in effect increasing the service expenditure share of the young by more than the old.

Figure C.17: Service consumption share, BEA



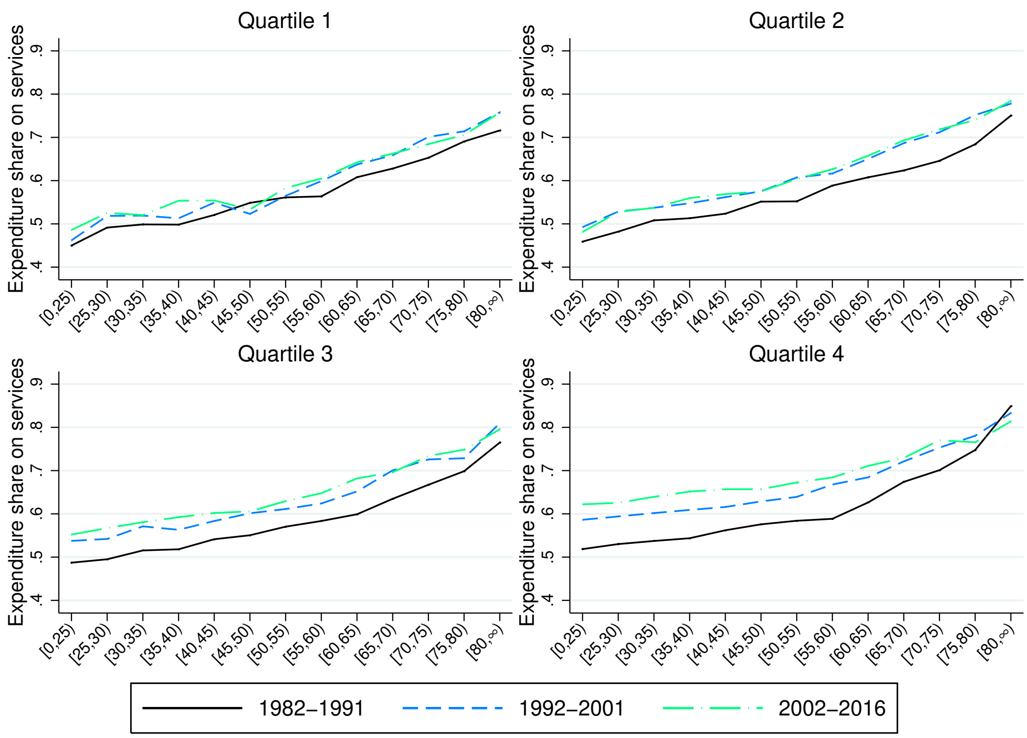
Notes: This figure displays the cumulative log change in the aggregate expenditure share on services in the BEA. Housing is excluded from expenditures.

Figure C.18: Service consumption by average age of household members, rescaled to BEA



Notes: The top panel displays the average household-level expenditure shares on services in the CES, rescaled to BEA, by age group (x-axis), for 3 time periods. The bottom panel displays the age dummies resulting from estimating equation (2). Each dot represents the point estimate of the age dummies for a particular decade in the CES data. The omitted dummy is that of age group 25-30. The bands report the 95% confidence intervals based on standard errors clustered at the household level.

Figure C.19: Service consumption by average age of household members and income, rescaled to BEA

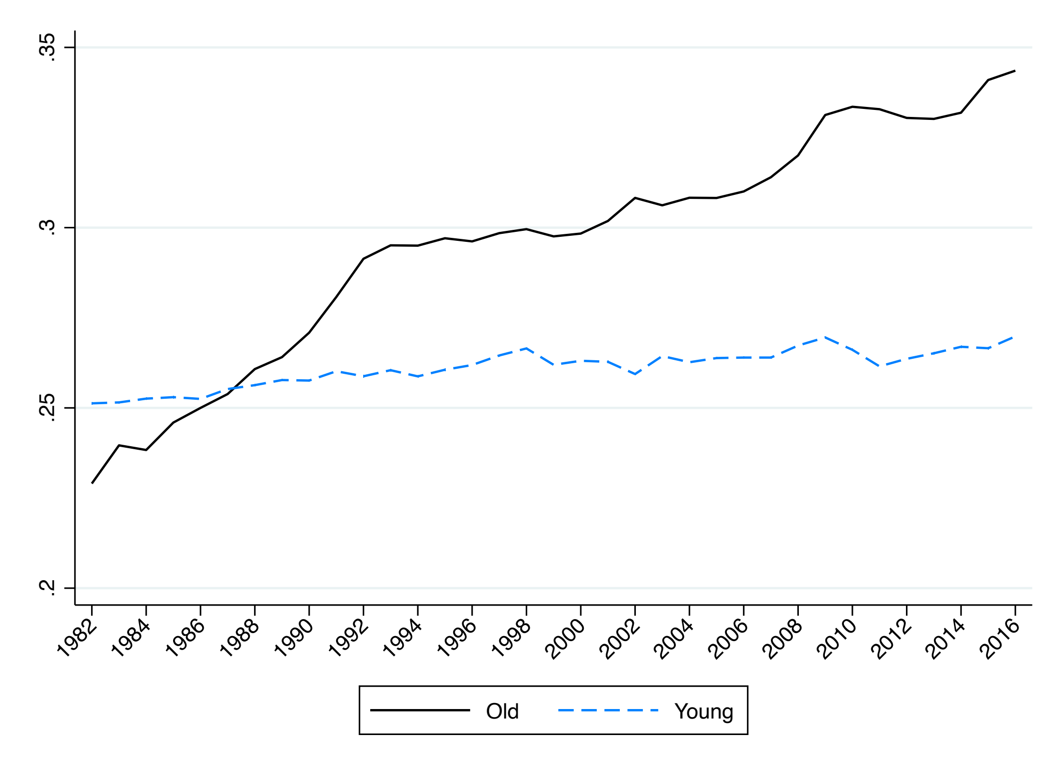


Notes: This figure displays the average household-level expenditure shares on services in the rescaled CES by age group (x-axis), for 3 time periods, and each income quartile.

Table C.9 reports the differences in consumption expenditures by category for older households, expressed as a difference relative to the households aged 25-30. While the ranking of categories according to young-old expenditure share differences is similar, the BEA-rescaled data show larger absolute differences in Healthcare.

Moving on to the replication of the results in Section 3.3, Table C.10 reports the changes in the services expenditure shares and income shares, and the within-between decomposition. In the BEA-rescaled data, the absolute size of the between effect due to population aging is slightly larger than in the baseline. However, because the change in the aggregate service expenditure share is also larger in the BEA, the between effect represents 14.3% of the total rise in the service expenditure share.

Figure C.20: Evolution of expenditure share on selected service categories using CES and re-scaling to BEA



Notes: 'Old' displays the aggregate expenditure share in the BEA on categories that are disproportionally consumed by the old: Health, Utilities, and Domestic Services and Childcare. 'Young' displays the expenditure share on the remaining service categories.

Table C.9: Differences in expenditures by consumption category: 25-30 vs 60-65, 65-70, 70-75, 75-80 and 80+, rescaled to BEA

	Age groups				
	60-65	65-70	70-75	75-80	80+
Health	9.65	13.70	17.53	21.63	25.75
Cash contributions	2.31	3.04	3.79	4.34	6.27
Domestic services	0.11	0.30	0.50	0.99	3.18
Utilities	-0.02	-0.04	0.12	0.31	0.45
Personal care services	-0.01	0.03	0.09	0.11	0.14
Personal care goods	-0.02	-0.01	-0.02	-0.02	-0.02
Public transport	0.07	0.05	-0.03	-0.07	-0.38
Tobacco	-0.11	-0.34	-0.56	-0.78	-0.97
Shoes and other apparel	-0.52	-0.63	-0.75	-0.95	-1.01
Children's clothing	-0.84	-0.86	-0.96	-1.01	-1.09
Alcoholic beverages	-0.55	-0.71	-0.91	-1.13	-1.34
Car maintenance, repairs	-0.53	-0.70	-0.83	-0.88	-1.45
Furnitures and Fixtures	-0.47	-0.64	-1.01	-1.24	-1.65
Personal Insurance	3.36	1.97	0.89	-0.90	-1.67
Appliances	-0.22	-0.62	-0.96	-1.24	-1.91
Men's and women's cloth.	-0.66	-0.94	-1.12	-1.50	-2.03
Entertainment fees, ...	-0.70	-0.90	-1.17	-1.63	-2.26
Entertainment equipment	-0.60	-1.05	-1.72	-2.01	-2.40
Education	-2.31	-2.48	-2.52	-2.45	-2.59
Food at home	-2.92	-2.88	-2.53	-2.18	-2.73
Gas	-1.05	-1.37	-1.70	-2.06	-2.82
Food away from home	-1.71	-2.10	-2.70	-3.30	-4.16
Vehicle purchasing, leasing	-2.26	-2.81	-3.42	-4.06	-5.30
Services	10.22	12.87	15.67	18.16	23.28

Notes: This Table reports the differences in expenditure shares across the major consumption categories between households aged 60-65 (first panel) or 80+ (second panel) and households aged 25-30. Source: authors' calculations based on the CES, rescaled to BEA.

Table C.10: Population aging and the services share, rescaled to BEA

Panel A: Expenditure shares across the age distribution

	Pop $_{1982}$	s^a_{1982}	$\omega^{s,a}_{1982}$	Pop $_{2016}$	s^a_{2016}	$\omega^{s,a}_{2016}$
0-25	31.8	30.3	42.3	20.4	18.5	54.4
25-30	13.5	15.6	44.7	11.4	11.5	56.7
30-35	9.4	11.1	46.4	9.4	10.5	59.2
35-40	6.2	7.5	48.0	7.1	7.7	58.7
40-45	4.6	5.4	49.7	5.9	6.6	62.4
45-50	3.6	4.0	52.2	5.2	5.6	61.3
50-55	3.8	4.0	49.9	6.1	6.0	60.8
55-60	5.1	5.1	52.2	6.7	7.2	64.2
60-65	5.7	5.6	55.3	7.5	8.1	67.0
65-70	5.9	4.9	58.5	6.8	6.9	67.2
70-75	4.3	3.1	61.8	5.1	5.0	68.9
75-80	3.3	1.9	61.8	3.4	2.9	69.6
80+	2.9	1.4	69.3	5.0	3.6	75.4

Panel B: Within-between decomposition

	Average		Reference	
	Value	%	Value	%
Within	0.1181	88.8	0.1036	77.8
Between	0.0150	11.2	0.0295	22.2
Total	0.1331	100.0	0.1331	100.0

Notes: In Panel A, 'Pop' reports the share of the population in each age group, and s^a_t and ω^a_t are defined as in Equation (3.4). Panel B reports the results of the decomposition in equation (3.4). 'Average' uses the average age across all household member as the age of the household. 'Reference' uses the age of the head in the household.

Tables C.11-C.12 re-estimate the model parameters on the BEA-rescaled data, while Figure C.21 reports the decomposition of the US structural change. The income effect plays a higher role compared to the baseline results, but none of the substantive conclusions change when using these data. Population aging still contributes about 0.05 log points to the change in the service share since 1982, same as in the baseline. This absolute contribution is smaller as a proportion of the total, since the aggregate service share rises by more in the BEA than the CES.

Table C.11: Estimates of equation (3.8), rescaled to BEA

	(1)	(2)	(3)	(4)
Dep. var.: $\ln \omega_t^{g,n}$				
$\log e_t^n$	-0.138***	-0.138***	-0.225***	-0.226***
	(0.000722)	(0.000720)	(0.00194)	(0.00195)
Type	OLS	OLS	IV	IV
Time FE	Yes	No	Yes	No
Region-Time FE	No	Yes	No	Yes
Observations	1,325,402	1,319,609	1,226,453	1,220,823
R^2	0.198	0.202	0.170	0.173

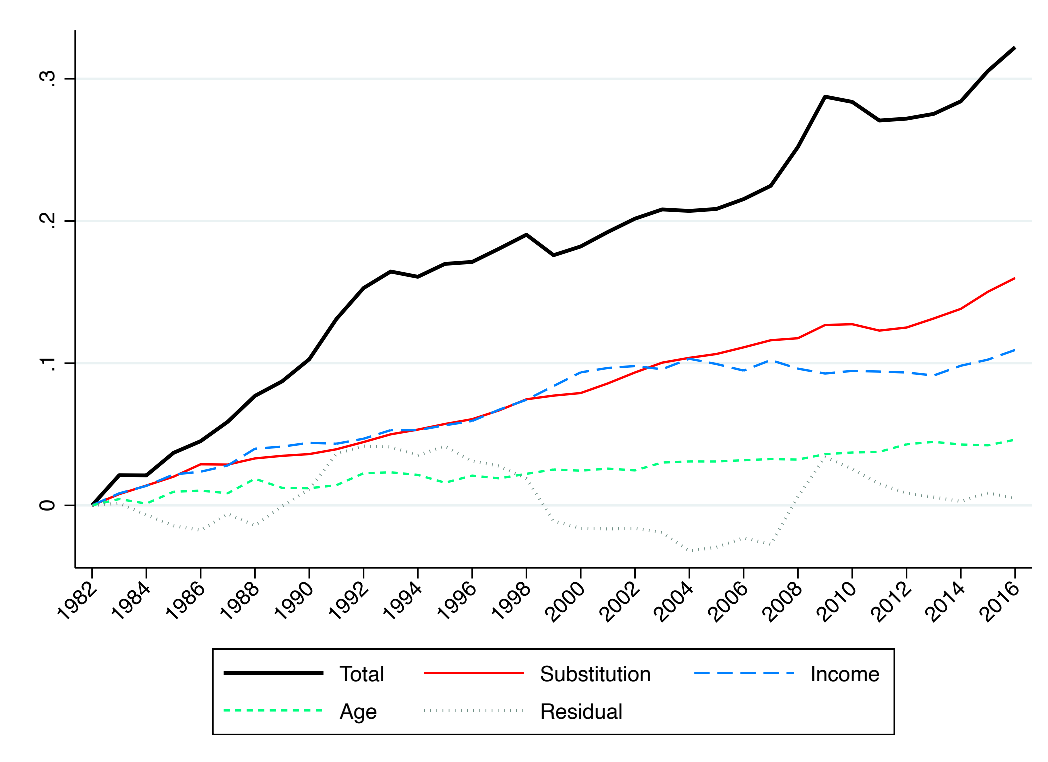
Notes: This table reports the results of estimating equation (3.8). The outcome variable is household expenditure share on goods. Standard errors clustered at the household level in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Table C.12: Estimates of equation (3.9), rescaled to BEA

	(1)	(2)
Dep. var.: $\ln \Omega_t^g$		
$b_1 = \gamma$	0.296***	0.309***
	(0.00933)	(0.00925)
Age variable	Average	Reference
Observations	35	35
R^2	0.967	0.970

Notes: This table reports the results of estimating equation (3.9). The outcome variable is aggregate expenditure share on goods. Standard errors in parentheses. *: significant at 10%; **: significant at 5%; ***: significant at 1%.

Figure C.21: Accounting for structural change in the US, rescaled to BEA.



Notes: This figure displays the decomposition (3.10) for the US from 1982 to 2016, using data rescaled to BEA.

C.2.4 Changes in relative number of households vs. relative income

The results in Section 3.3 arise from changes in the share of each age group in total expenditures across time. The share of age group a in aggregate expenditures can be written as:

$$s_t^a \equiv \frac{\sum_j e_t^{j,a}}{\sum_a \sum_j e_t^{j,a}} \quad = \quad n_t^a \times \tilde{e}_t^a,$$

where $n_t^a \equiv N_t^a / \sum_a N_t^a$ is the share of households that are in age group a, and $\tilde{e}_t^a \equiv \frac{\sum_j e_t^{j,a}/N_t^a}{\sum_a \sum_j e_t^{j,a} / \sum_a N_t^a}$ are the expenditures per household of age group a relative to expenditures per household in the economy. This appendix explores how large is the contribution of aging to structural change if we instead focus solely on the shares of households component of changing expenditure shares, n_t^a.

C.2.4.1 Within-between decomposition

To focus on the role of changes in the share of households that are in age group a, we perform a within-between decomposition on the *average* service expenditure share across household age groups, rather than on the *aggregate* service expenditure share in the economy. The average expenditure share in services across age groups is defined as

$$\omega_t^s \equiv \sum_a n_t^a \omega_t^{s,a},$$

and can be decomposed into

$$\Delta\omega^s = \underbrace{\sum_a \Delta\omega^{s,a} \cdot \overline{n}^a}_{\text{Within}} + \underbrace{\sum_a \overline{\omega}^{s,a} \cdot \Delta n^a}_{\text{Between}}, \tag{C.1}$$

where ω^s is the cross-age group average share of services expenditure. The average ω_t^s and aggregate Ω_t^s shares are very similar, and thus experienced very similar changes over this period (ω^s went from 0.447 in 1982 to 0.524 in 2016, whereas Ω^s went from 0.435 to 0.520). So the decomposition of the average (C.1) should still be informative, while at the same time focusing purely on the population changes Δn^a rather than expenditure share changes Δs^a. Table C.13 below presents the results of the decomposition (C.1). The results are quite similar to the baseline. The contribution of the Between effect is still about 20% of the total.

Table C.13: Within-between decomposition

	Average		Reference	
	Value	%	Value	%
Within	0.0636	81.8	0.0660	83.7
Between	0.0141	18.2	0.0128	16.3
Total	0.0777	100	0.0789	100

Notes: The table reports the results from the decomposition in equation (C.1). 'Average' uses the average age across all household member as the age of the household. 'Reference' uses the age of the reference person in the household.

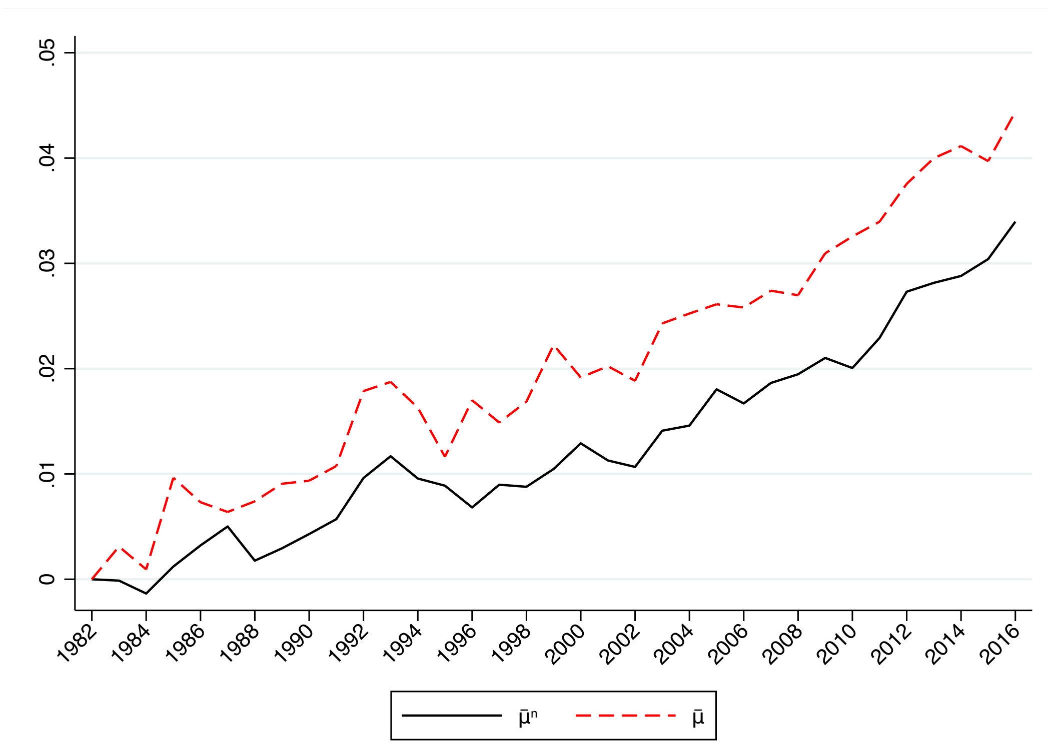
C.2.4.2 Structural model

To focus purely on changes in household numbers by age group, we implement an alternative version of equation (3.7):

$$\Omega_t^g = \left[\frac{P_t^s}{e_t}\right]^{\epsilon} \left[\frac{P_t^g}{P_t^s}\right]^{\gamma} \bar{\mu}_t^n \phi_t^n \nu_t,$$

with $\bar{\mu}_t^n \equiv \sum_a n_t^a \mu^a$ and $\phi_t^n \equiv \frac{1}{N_t} \sum_h^{N_t} \frac{\mu^a}{\bar{\mu}_t^n} \left[\frac{e_t^h}{e_t}\right]^{1-\epsilon}$. Note that this alternative simply redefines the aggregate aging term $\bar{\mu}_t$ to sum over number of households shares n_t^a instead of expenditure shares s_t^a. While this affects the inequality term ϕ_t, it leaves the rest of the decomposition unchanged, and thus the Income and Substitution terms in (3.10) are the same as in the Baseline. Figure C.22 plots the original Aging component of (3.10), $\hat{\bar{\mu}}_t$, alongside the alternative $\hat{\bar{\mu}}_t^n$. The two are quantitatively similar, though the latter has a somewhat smaller contribution.

Figure C.22: Measures $\bar{\mu}_t^n$ and $\bar{\mu}_t$



Notes: This figure displays the changes across time of two different aging measures from the structural model. $\bar{\mu}_t \equiv \sum_a s_t^a \mu^a$ and $\bar{\mu}_t^n \equiv \sum_a n_t^a \mu^a$.

C.2.5 Derivation of equation (3.10)

We are interested in computing the elasticity of the expenditure share on goods with respect to the relative price of goods $\frac{P_t^g}{P_t^s}$. To compute this elasticity, solve for e_t^h to obtain the expenditure function associated with the utility level $\mathcal{V}^h$:

$$\begin{aligned}
\frac{1}{\epsilon}\left[\frac{e_t^h}{P_t^s}\right]^{\epsilon} &= \mathcal{V}^h + \frac{\nu_t^h}{\gamma}\left[\frac{P_t^g}{P_t^s}\right]^{\gamma} + \frac{1}{\epsilon} - \frac{\nu_t^h}{\gamma} \\
e_t^h &= P_t^s\left\{\epsilon\left[\mathcal{V}^h + \frac{\nu_t^h}{\gamma}\left(\frac{P_t^g}{P_t^s}\right)^{\gamma} + \frac{1}{\epsilon} - \frac{\nu_t^h}{\gamma}\right]\right\}^{\frac{1}{\epsilon}}.
\end{aligned}$$

By Roy's identity, the demand for goods is:

$$c_t^{g,h} = \frac{\nu_t^h\left[\frac{P_t^g}{P_t^s}\right]^{\gamma}\frac{1}{P_t^g}}{\left[\frac{e_t^h}{P_t^s}\right]^{\epsilon-1}\frac{1}{P_t^s}} = \frac{\nu_t^h\left[\frac{P_t^g}{P_t^s}\right]^{\gamma}\frac{e_t^h}{P_t^g}}{\left[\frac{e_t^h}{P_t^s}\right]^{\epsilon}},$$

and therefore the goods spending share is:

$$\omega_t^{g,h} = \frac{\nu_t^h\left(\frac{P_t^g}{P_t^s}\right)^{\gamma}}{\epsilon\left[\mathcal{V}^h + \frac{\nu_t^h}{\gamma}\left(\frac{P_t^g}{P_t^s}\right)^{\gamma} + \frac{1}{\epsilon} - \frac{\nu_t^h}{\gamma}\right]}.$$

The elasticity of this share with respect to $\frac{P_t^g}{P_t^s}$ is:

$$\gamma - \epsilon\omega_t^{g,h}.$$

Then at the household level, the substitution effect is defined as

$$\left(\gamma - \epsilon\omega_t^{g,h}\right)\left[\hat{P}_t^g - \hat{P}_t^s\right].$$

As Muellbauer (1975, 1976) shows, this economy admits a representative agent, defined as the household that exhibits the aggregate expenditure shares. In our framework, this is the household with income $e_t^{rep} \equiv e_t\left(\bar{\mu}_t\phi_t\nu_t\right)^{-\frac{1}{\epsilon}}$. This allows us to define the aggregate substitution effect as just the substitution effect of the representative consumer, or:

$$\left(\gamma - \epsilon\Omega_t^g\right)\left[\hat{P}_t^g - \hat{P}_t^s\right]. \tag{C.2}$$

The log change in the aggregate expenditure share (3.7) is:

$$\hat{\Omega}_t^s \approx -\frac{\Omega_{82}^g}{\Omega_{82}^s} \left\{ \epsilon \left[\hat{P}_t^s - \hat{e}_t \right] + \gamma \left[\hat{P}_t^g - \hat{P}_t^s \right] + \hat{\bar{\mu}}_t + \hat{\phi}_t + \hat{\nu}_t \right\}. \tag{C.3}$$

The first two terms, $\epsilon \left[\hat{P}_t^s - \hat{e}_t \right] + \gamma \left[\hat{P}_t^g - \hat{P}_t^s \right]$ can be thought of as capturing the sum total of the income and substitution effects. They can be combined with (C.2) to isolate the two effects separately, leading to (3.10).

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