Persistence of playing school: Examining an immersive 90-day semester-program for shaping students' mathematical perceptions and practices

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How can a semester-program that organizes students' mathematical learning in the posing and investigation of natural mathematical questions shape students' mathematical perspectives and practices? Every semester, hundreds of young adults across the US leave their schools to partake in unique educational experiences offered in one of dozens of semester programs. While the experiences students have in such programs can be thought of as quite rare and distinct from their learning in more typical schooling environments, we argue these programs offer an important window into a genre of reform emerging in local schools called *place-based learning* (Grunewald, 2003; Smith, 2002). These programs offer a laboratory environment for studying what might be possible in schools committed to comprehensive place-based learning.

In this paper, we investigate the outcomes of a 90-day immersive semester-program located on an island in the western Atlantic Ocean that utilizes an immersive place-based curriculum for shaping students' learning. Program faculty aim to leverage the surrounding ocean, local culture, and real constraints that come with residing on an island to immerse students in posing meaningful questions in order to make tangible connections between their disciplinary learning and the broader world. Along with learning to live on a green campus (powered by renewable energy), students have weekly opportunities to engage in field-based research projects of local and regional significance. In these projects, students work collaboratively with resident mentors from one of two sister institutions—one focused on environmental engineering for sustainable living, the other focused on studying environmental issues facing our oceans. The program's core classes maintain a place-based focus —with the mathematics class focused on equipping students with the fundamentals of statistical and mathematical analysis useful posing and answering questions related to improving the campus' sustainability and supporting students' engagement in their research projects.

Theoretical Framework

While place-based efforts in mathematics education are still emerging (see Showalter, 2013), the foundations of such work are not new. Research on *ethnomathematics* (D'Ambrosio, 1985) demonstrated the role of context in student learning—sensitizing scholars to the differences between school mathematics and street mathematics (e.g., Carraher, Carraher, & Schliemann, 1985). Similarly, research on *funds of knowledge* (e.g., Civil, 2007) demonstrated how school mathematics tends to privilege the kind of knowledge held by white, middle-class students. The *suspension of sensemaking* literature (e.g., Silver et al., 1993; Schoenfeld, 1991) demonstrated ways that school mathematics in general, and the story problem genre in particular is woefully inadequate for eliciting students' realistic considerations. In this work, researchers illustrated children's tendencies to answer a question like, *A captain owns 26 sheep and 10 goats. How old is the captain?*, with nonsensical solutions like 36 (obtained by adding 26 and 10, see Baruk, 1985).

Other bodies of work have sought ways to remedy such problems in school mathematics. Work organized under the banner of *problems-based learning* (e.g., Hmelo-Silver et al., 2007) has attempted to disrupt students' typical ways of doing mathematics by starting with a novel task that invites students to pose and grapple with interesting mathematical problems. Similarly, *Realistic Mathematics Education* (RME) (Van den Heuvel-Panhuizen & Drijvers, 2020) has sought to shift the story problem genre offered in schools by starting with problem contexts that: (1) students find meaningful and (2) provide students with opportunities to engage with the complexities of mathematics as contextualized in real life as opposed to the sanitized version of the “real-world” offered in typical story problems. While some of this work has demonstrated how classroom-based interventions can lead to positive changes in students’ disposition towards mathematical sense-making (e.g., Verschaffel & De Corte, 1997), other work has highlighted how the institutionalized contexts of schools create challenges not easily overcome by teachers interested in engaging in such instruction (Silver & Stein, 1996).

The supplementary nature of semester programs may enable them to escape the kinds of normal constraints that come with trying to reform instruction in schools—making semester programs fertile ground for exploring what might be possible in a fully-operational model of place-based mathematics education. To that end, we share findings from a study conducted at the above described school in which we had the following three research questions: (1) How do students describe their experiences in a mathematics class embedded in a 90-day semester program centered on place-based learning?, (2) What changes (if any) can be observed in students’ attitudes towards mathematics at the beginning and end of their participation in such a program?, (3) What evidence of mathematical sensemaking can be observed in students’ responses to story problems during their participation in such a program?

Methods

During Fall 2018, we engaged in a study of the semester program described above where a total of 51 students were enrolled. To investigate RQ2, the Attitudes Towards Mathematics Instrument (ATMI, Tapia & March, 2004) was deployed with all students using paper-and-pencil surveys administered at the beginning and end of the semester program. In the first administration, students were directed to reflect on their most recent mathematics class back at their sending school. In the second administration, students were directed to reflect on their mathematics class at the semester program. The ATMI isa 40-question survey with 5-point likert scale. It has been shown to have sound psychometric properties—with a well-established 4-factor model for measuring high school and college students' attitudes towards mathematics as made up of: *mathematical value* (e.g., Mathematics is a very worthwhile and necessary subject), *mathematical motivation* (e.g., I want to develop my mathematical skills), *mathematical confidence* (e.g., Mathematics makes me feel uncomfortable), and *mathematical enjoyment* (e.g., I have usually enjoyed studying mathematics in school).

For RQ1 & RQ3, we engaged 17 students in a 30-minute interview. Students were selected with help from the program's mathematics teachers. Each teacher identified four students from each of their classes—two that were thriving and two that were not—and those who assented and whose parents consented were interviewed. In the first half of the interview, we asked students about the nature of their mathematical activity within their mathematics coursework in the semester program (e.g. "Can you describe what a regular day in your mathematics class looks like?"). In the latter portion, we asked students to solve three story problems, drawn from the suspension of sense-making literature, sharing their thinking as they solved. Our final selection criteria for problems was guided by the following: The problem (1) required mathematics we could be reasonably assured that all participants would have prior experience with, (2) resembled the kinds of problems that could appear in high school curricula, and (3) had prior results reported for similarly-aged students, without intervention—useful for informing our expectations regarding the proportion of students likely to demonstrate realistic reactions (see Figure 1 for our final problem selection and data from prior studies).

Figure 1. Percentage of students demonstrating realistic reactions across 4 studies

Problem	Rope Problem	Runner Problem	Bruce & Alice Problem
Problem Text	A man wants to have a rope long enough to stretch between two poles 12 meters apart, but only has pieces of rope 1.5 meters long. How many of these would he need to tie together to stretch between the poles?	John's best time to run 100 meters is 17 seconds. How long will it take him to run 1 kilometer?	Bruce and Alice go to the same school. Bruce lives at a distance of 17 km from the school and Alice lives at 8 km. How far do Bruce and Alice live from each other?
Expected Answer Realistic Answer	8 pieces More than 8 pieces	170 seconds More than 170 seconds	9 km or 25 km Between 9 and 25 km
Study 1 (n=100)	12%	6%	N/A
Study 2 (n=67)	6%	3%	N/A
Study 3(n=75)	0%	3%	3%
Study 4 (n=45)	2%	7%	2%

Study 1: Greer (1993); Study 2: Reusser & Stebler (1995);
Study 3: Verschaffel et al (1994); Study 4: Yoshida et al (1997).

Analysis and Results

Students Descriptions of their Mathematics Class at the Semester Program—To analyze interview data about students' descriptions of their mathematics classes, we engaged in rounds of descriptive, analytical, and thematic coding with sets of interview data containing students' responses to questions about the quality of their experiences of mathematics classes and their perceptions of typical and atypical mathematics class activities and interactions in the semester program. We employed constant comparative analysis by coding and analyzing across various sub-groups of respondents (e.g. respondents whose teachers identified them as thriving and respondents whose teachers identified them as not-thriving). Such analysis allowed us to test our analytical findings by demonstrating confirmation or disconfirmation between sub-groups of students.

As part of the interview, students were asked to describe both an "ordinary" and "out of the ordinary" day in their semester math class. Responses generally suggested that students' "ordinary" semester program experiences were, in fact, quite out of the ordinary when compared to a conventional mathematics classroom. For example, when asked to describe an "out of the ordinary" day, one student initially responded by saying "There weren't. No, there aren't, really", but added immediately, "There was one day we went outside and measured solar panels...and we went and visited one of the cisterns…[we] did mini field trips." These field trips to collect and analyze data from elements of the campus' sustainability infrastructure were commonly recounted by students as standard parts of their mathematical learning. A common sentiment shared by students was that such activities were clear and salient applications of the mathematical concepts they were learning in class. For example, one student said that, when doing such activities, he's "getting more of a physical look" at class concepts. Every interviewee said that they saw their coursework as being related to the "real world," a result that suggests the power of place-based learning for mediating students' grounded engagement with mathematics.

Changes in Students' Attitudes Towards Mathematics—For the analysis of students' survey responses, we organized the data according to the four constructs and used simple averages to compute scores for each student that completed every item within a given construct (after reverse coding 11 items according to their polarity). Descriptive statistics for participants' scores on each of these constructs are reported in Table 1. Using a two-tailed paired-samples t-test, we found a significant increase from pre to post survey in students' scores related to *mathematical value*, $t(29) = 0.98$, $p<0.05$, and *mathematical enjoyment*, $t(29) = 0.96$, $p<0.05$.

Table 1: Descriptive statistics for participants scores on 4 constructs of the ATMI

		Pre-Survey		Post-Survey	
Construct	n	$\bar{x}$	σ	$\bar{x}$	σ
Mathematical Value	30	3.68	0.69	4.12	0.63
Mathematical Enjoyment	30	3.41	0.63	3.79	0.73
Mathematical Self-Confidence	30	3.73	0.70	3.83	0.80
Mathematical Motivation	27	3.44	0.74	3.42	0.90

No statistically significant differences existed for students' *mathematical motivation*, $t(26)=0.35$, $p>0.94$ or *mathematical self-confidence*, $t(29)=0.53$, $p>0.63$. These results suggest some aspects of students' mathematical attitudes towards mathematics, namely their perception of mathematics as valuable and enjoyable, may improve during engagement in such interventions, while others may not.

Evidence of Students' Tendencies Towards Sense-Making—We coded students' answers to the story problems (using their written and verbal response) using four of the five[1] categories outlined by Verschaffel et al (1994): Expected Answer (EA), Technical Error (TE), Realistic Answer (RA), No Answer (NA) . We also examined the transcript/video containing explanations students provided for indications of sense-making (hesitations, criticizing the problem, qualifiers) and augmented the five categories with a "+" if any such indications were found, and with a "-" if not. For example, in the runner problem, an EA- was used to code student responses

[1] Other Answer (OA) did not present in our data

that simply multiplied 17 by 100 to get 1,700 seconds; while EA+ was used if such responses were accompanied by reasoning like the following:

> *But then again, you probably can't run that kilometer because if it's his best time to run 100 meters then you probably can't maintain that time for a full kilometer. But I feel like the number is 170, so I'm going to go with 170.*

Table 2 provides an overview of students' reactions to the three problems with the final row providing a summary of all the Realistic Reactions (RR)—combining categories with a "+".

Table 2. Number (percentage) of students' (n=17) according to various reactions to the problems

Answer Category	Rope	Runner	Bruce & Alice
EA+	0 (0%)	1 (6%)	7 (41%)
EA-	13 (76%)	13 (76%)	9 (53%)
TE+	0 (0%)	0 (0%)	0 (0%)
TE-	1(6%)	1 (6%)	0 (0%)
NA+	0 (0%)	0 (0%)	0 (0%)
NA-	0 (0%)	1 (6%)	0 (0%)
RA+	3 (18%)	1 (6%)	1 (6%)
RA-	0 (0%)	0 (0%)	0 (0%)
RR	3 (18%)	2 (12%)	8 (47%)

The percentage of students with realistic reactions is somewhat underwhelming for the first two problems, given that similar percentages have been reported in other studies of similarly-aged students without intervention (see Figure 1). We are less certain how to make sense of the larger percentage of RR responses in the final problem. One possibility is students' exposure to the triangle inequality in high school geometry gives them routine ways for thinking about the problem. Another is the back-to-back administration of problematic items may have sensitized students to the need to pay attention to context. Yet, we find it perplexing that such a large percentage of students engaged in this kinds of mathematical experiences gave EA+ type responses—unwilling to assert a realistic answer saying things like, "I'm just going to say it's nine kilometers," even after showing some signs of sense-making with questions like, "Do they live on the same side of the school?"

Discussion & Conclusions

Results from RQ1 and RQ2 paint a picture of a somewhat idyllic context in which students are engaged in authentic mathematical activity that seems to have played a role in improving their value for and enjoyment of mathematics. Results from RQ3, however, are more challenging to interpret. One interpretation casts doubt on the impact of mathematical activities like those afforded at this semester program on students' propensities to make sense of problem situations. Another interpretation casts doubt on this genre of items for gauging students' propensity to make sense, a concern expressed by others (e.g., Gerofsky, 1996, 2010). We suggest a third interpretation: this genre of items may not work well for gauging students' propensity to make sense in this kind of supplementary program. This interpretation draws on two premises: (1) the radically different organization of students' mathematical activity in such a program could be seen by students as something wholly different from the kinds of things valued/expected in school mathematics, and (2) the representation of the context in these problems evokes something closer to the norms of school and in this way may be at odds with students' experiences in the program. (1) may suggest to students that the kind of mathematical practices developed and used in such a program are not applicable for the kinds of mathematical work expected of them back to their sending school. (2) may suggest to students that the story problem presented to them in the context of this interview "looks" like school, rather than like the work they have been doing in the program, and respond accordingly.

Authentic problem posing and inquiry often leaves its users with more questions than answers. This is no less true in research as it is in mathematics classrooms. The ease with which students in this semester program reverted to the kinds of persistent, well-rehearsed routines of playing school when primed with a school-like story problem surprised us—suggesting norms of school may be quite challenging for students to unlearn. We are interested in investigating ways to support students in developing the kind of awareness and agency that would enable them to both question and challenge norms of schooling that may be unproductive for their learning. Such work would have important implications for supporting students to leverage the knowledge and experiences they gain in supplementary, out-of-school programs.

References

Baruk, S. (1985). L'age du capitaine. *De l'erreur en mathématiques.* Paris: Seuil.

Carraher, T.N., Carraher, D.W., & Schliemann, A.D. (1985). Mathematics in the street and in schools, *British Journal of Developmental Psychology 3*, 21–29.

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. Improving access to mathematics: Diversity and equity in the classroom, 105-117.

D'Ambrosio, U. (1985), Ethnomathematics and its place in the history and pedagogy of mathematics, *For The Learning of Mathematics 5*(1), 44–48.

Gerofsky, S. (1996). A linguistic and narrative view of word problems in mathematics education. *For the Learning of Mathematics, 16*(2), 36-45.

Gerofsky, S. (2010). The impossibility of 'real-life' word problems (according to Bakhtin, Lacan, Zizek and Baudrillard). *Discourse: Studies in the Cultural Politics of Education*, *31*(1), 61-73.

Gruenewald, D. A. (2003). Foundations of place: A multidisciplinary framework for place-conscious education. *American Educational Research Journal*, *40*(3), 619-654.

Greer, B. (1993). The modeling perspective on wor(l)d problems. *Journal of Mathematical Behavior, 12,* 239 - 250.

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: a response to Kirschner, Sweller, and. *Educational psychologist*, *42*(2), 99-107.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution: The suspension of reality and sense-making in the culture of school mathematics. Learning and Instruction, 7(4), 309-327.

Schoenfeld, A.H. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J.F. Voss, D.N. Perkins & J.W. Segal (Eds.), *Informal reasoning and education* (pp. 3.11-343). Hillsdale, NJ: Erlbaum.

Showalter, D. A. (2013). Place-based mathematics education: A conflated pedagogy. *Journal of Research in Rural Education*, *28*(6), 1-13.

Silver, E. A., Shapiro, L. J., & Deutsch, A. (1993). Sense making and the solution of division problems involving remainders: An examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education*, *24*(2), 117-135.

Silver, E. A., & Stein, M. K. (1996). The QUASAR project: The" revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education*, *30*(4), 476-521.

Smith, G. A. (2002). Place-based education: Learning to be where we are. *Phi Delta Kappan, 83*(8), 584-594.

Treffers, A., & de Moor, E. (1990). *Proeve van een nationaal programma voor het reken-wiskundeonderwijsop de baissschool. Deel 2. Basisvaardigheden en cijferen. [Towards a national mathematics curriculum for the elementary school. Part 2. Basic skills and written computation].* Tiburg: Zwijssen.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic mathematics education. In S. Lerman (ed.), *Encyclopedia of Mathematics Education*, (pp. 713-717) Springer Nature, Switzerland.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction, 4*(4), 273-294.

Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modeling in the elementary school: A teaching experiment with fifth graders. *Journal for Research in Mathematics Education, 28*(5), 577-601.

Yoshida, H., Verschaffel, L., & De Corte, E. (1997). Realistic considerations in solving problematic word problems: Do Japanese and Belgian children have the same difficulties?. *Learning and Instruction, 7*(4), 329-338.