Sowing the Seeds: Radicalization as a Political Tool

Supplemental Materials

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A Appendix

Proof of Remark 1: The expected wage in the economic sector is

$$\mathbb{E}_z\left[\int_{w^*(s_j;z)}^{D} wdF(w)\right] = \pi(x)\int_0^D wdF(w) + (1-\pi(x))\int_B^D wdF(w). \tag{A.1}$$

Recall that the material return among citizens who participate in dissidence is B, and observe that

$$\mathbb{E}_z\left[\int_0^{w^*(s_j;z)} BdF(w)\right] = \pi(x)\int_0^0 BdF(w) + (1-\pi(x))\int_0^B BdF(w). \tag{A.2}$$

That (A.1) is strictly larger than (A.2) follows by inspection. ■

Proof of Proposition 1: The level of dissidence follows by combining the expression for dissidence activities with Lemma 1, which leads to

$$Y_z(\rho; w(0;1)) = \rho\int_0^{\overline{w}} dF(w) + (1-\rho)\int_0^{z\cdot w(0;1)} dF(w),$$

which, after carrying through the integrals, gives the expression in the text.

That $Y_z(\rho)$ is strictly increasing in z follows by inspection of (2). By differentiating with respect to ρ:

$$\frac{dY_z(\rho)}{d\rho} = 1 - zF(B) > 0,$$

establishing the second part. For the third part, consider a first-order stochastic increase from F_1 to F_2, which implies that

$$Y_z(\rho; F_1) = \rho + z(1-\rho)F_1(B) > \rho + z(1-\rho)F_2(B) = Y_z(\rho; F_2).$$

For the last part, let $z' > z$, then

$$Y_{z'}(\rho) - Y_z(\rho) = (1-\rho)F(B)(z'-z).$$

Differentiating this expression with respect to ρ,

$$-F(B)(z' - z),$$

which is strictly negative, thus establishing that dissidence exhibits strict decreasing differences in ρ and z. ■

Proof of Proposition 2: The government's problem is

$$\max_{x} \; -\pi(x)Y_0(\rho) - (1 - \pi(x))Y_1(\rho) - c(x).$$

Let $\theta = \pi(x)$ and denote the induced cost function $c(\pi^{-1}(\theta)) = \kappa(\theta)$. Then, using this change of variables, we write the government's problem as

$$\max_{\theta \in [0,1]} \; -\theta Y_0(\rho) - (1 - \theta)Y_1(\rho) - \kappa(\theta).$$

Notice that κ is strictly convex, and by the Inverse Function Theorem, κ is smooth and strictly increasing. By Proposition 1, the government's problem can be written as

$$\max_{\theta \in [0,1]} \; -\theta\rho - (1 - \theta)(\rho + (1 - \rho)F(B)) - \kappa(\theta),$$

and after combining terms,

$$\max_{\theta \in [0,1]} \; -\rho - (1 - \theta)((1 - \rho)F(B)) - \kappa(\theta),$$

which can be further simplified to

$$\max_{\theta \in [0,1]} \theta[(1 - \rho)F(B)] - \kappa(\theta). \tag{A.3}$$

That the best-response level of repression is unique follows because the objective function is strictly concave in θ and this relates to the level of repression through $x^* = \pi^{-1}(\theta^*)$.

The first-order condition characterizing an interior solution θ^* is

$$(1-\rho)F(B) = \kappa'(\theta).$$

$\blacksquare$

Proof of Proposition 3: Since κ is strictly convex, κ' is strictly increasing. Then, the first two parts follow because $(1-\rho)F(B)$ increases in B and decreases in ρ. Next, consider a first-order stochastic increase from F_1 to F_2. Then,

$$\kappa'(x^*(\rho; F_1)) = (1-\rho)F_1(B) > (1-\rho)F_2(B) = \kappa'(x^*(\rho; F_2)),$$

and hence, $x^*(\rho; F_1) > x^*(\rho; F_2)$. $\blacksquare$

Proof of Proposition 4: Sequential rationality implies that the opposition's choice e must be held fixed, and consequently, the choices for citizens and the government follow by Lemma 1 and Proposition 2 by replacing ρ with $\rho(e)$ and B with $B(e)$.

Moving backward to the first stage, the opposition's problem is

$$\max_e \, (1-\theta^*(e))(1-\rho(e))F(B(e)),$$

Consider the change of variables, $\gamma = \theta^*(e) = (\kappa')^{-1}(((1-\rho(e))F(B(e)))$, then the opposition's problem becomes

$$\max_\gamma \, (1-\gamma)\kappa'(\gamma).$$

That a solution exists follows by the Extreme Value Theorem. The first-order condition in terms of γ is then

$$-\kappa'(\gamma) + (1-\gamma)\kappa''(\gamma) = 0, \tag{A.4}$$

and an optimal γ^* corresponds to any γ that solves

$$\kappa''(\gamma) = \frac{\kappa'(\gamma)}{1-\gamma}. \tag{A.5}$$

An equilibrium radicalization effort, e^*, is then characterized implicitly by the condition

$$\kappa'(\gamma^*) = (1-\rho(e^*))F(B(e^*)). \tag{A.6}$$

Since the right-hand side is strictly decreasing in e, while the left-hand side is constant, the equilibrium level of radicalization effort, e^*, is uniquely determined by γ^*. ■

Proof of Proposition 5: Denote the equilibrium level of radicalization efforts, depending on F, by e^*_F, and consider a first-order stochastic increase from F_1 to F_2. Then,

$$\kappa'(\gamma^*) = (1-\rho(e^*_{F_1}))F_1(B(e^*_{F_1})) > (1-\rho(e^*_{F_1}))F_2(B(e^*_{F_1})).$$

Since $(1-\rho(e))F(B(e))$ is strictly decreasing in e, to restore equality, $e^*_{F_2} < e^*_{F_1}$.

By substitution, equilibrium dissidence can be written as

$$Y^* = \rho(e^*) + (1-\gamma^*)\kappa'(\gamma^*). \tag{A.7}$$

Since γ^* is constant, a first-order stochastic increase in F has no effect on γ^*, but decreases e^*, and since ρ is an increasing function, decreases Y^*. ■

Proof of Proposition 6: Denote the equilibrium level of radicalization efforts, depending on ρ, by e^*_ρ, and consider a pointwise increase from ρ_1 to ρ_2. Then,

$$\kappa'(\gamma^*) = (1-\rho_1(e^*_{\rho_1}))F(B(e^*_{\rho_1})) > (1-\rho_2(e^*_{\rho_1}))F(B(e^*_{\rho_1})).$$

Since $(1-\rho(e))F(B(e))$ is strictly decreasing in e, to restore equality, $e^*_{\rho_1} > e^*_{\rho_2}$. Non-monotonicity of Y^* follows from the argument in the text. ■

B Direct Benefit to Radicalization

In this section we consider an extension of the model where the opposition's objective function depends directly on the level of radicalization, not just the level of organized dissent it mobilizes. In this extension we let $\pi(x) = x$, since this is without loss of generality (see the proof of Proposition 2).

Suppose that the opposition receives a direct benefit from decentralized extremist dissent, given by the strictly increasing and smooth function $V(\rho)$, which depends on the level of radicalization. The opposition's objective is

$$z(Y_1 - Y_0) + V(\rho). \tag{B.1}$$

Since the second and third stages of this extended game are the same as in the main text, by sequential rationality, Lemma 1 and Proposition 2 characterize the sequential best responses for that part of the game by replacing ρ with $\rho(e)$ and B with $B(e)$.

A direct benefit experienced by the opposition is only relevant for the opposition's effort decision. We can write the opposition's problem in the initial stage in terms of e as

$$\max_{e} \, (1 - x^*(e))(1 - \rho(e))F(B(e)) + V(\rho(e)). \tag{B.2}$$

That an equilibrium exists follows by identical arguments to that in the proof of Proposition 4 in the main text.

For completeness, we compare this extension with the main model.

Proposition B.1 *The level of radicalization effort, e^* is strictly higher in the model in which the opposition receives a direct benefit from the level of radicalization among the population.*

Proof: We write the objective function in (B.2) as

$$(1 - x^*(e))(1 - \rho(e))F(B(e)) + t \cdot V(\rho(e)), \tag{B.3}$$

where $t \in \{0,1\}$, with the main model following when $t = 0$ and the extension when $t = 1$. Noticing that (B.3) has strict increasing differences in e and t, by the Strict Monotone Selection Theorem of Edlin and Shannon (1998), $e^*(t)$ is strictly increasing in t, establishing the result. ■

We next consider the case where the opposition enjoys the total level of dissent, a special case of when the opposition receives direct benefits from radicalization, where opposition's payoff is zY_1.

Corollary 1 *In the model where the opposition's payoff is zY_1, then the level of radicalization effort, e^*, is strictly higher than in the main model.*

Proof: The result follows by writing Opposition's problem as

$$\max_e \, (1 - x^*(e))(Y_1(e) - Y_0(e)) + (1 - x^*(e))Y_0(e).$$

Since $Y_0(e) = \rho(e)$, which is strictly increasing in e, and because $x^*(e)$ is strictly decreasing in e, we have that $(1-x^*(e))Y_0(e)$ is a strictly increasing function of e. The result follows by applying Proposition B.1 by setting $V(\rho(e)) = (1 - x^*(e))Y_0(e)$. ■

We next address the empirical implications from the main text, focusing solely on interior solutions.

For the remainder we will make use of the following:

Lemma B.1 *Let $f(z,y) : \mathbb{R}^2 \to \mathbb{R}$ and $g(z,y) : \mathbb{R}^2 \to \mathbb{R}$ be smooth and define the set of selections by*

$$z^*(y) = \max_z \, f(z,y) + g(z,y).$$

Then, at an interior solution

$$\frac{dz^*(y)}{dy} = -\frac{\frac{\partial^2 f(z,y)}{\partial z \partial y} + \frac{\partial^2 g(z,y)}{\partial z \partial y}}{\frac{\partial^2 f(z,y)}{\partial z^2} + \frac{\partial^2 g(z,y)}{\partial z^2}}.$$

Proof: An interior solution is characterized by the first-order condition

$$\frac{\partial f(z,y)}{\partial z} + \frac{\partial g(z,y)}{\partial z} = 0.$$

Totally differentiating

$$dz\left[\frac{\partial^2 f(z,y)}{\partial z^2} + \frac{\partial^2 g(z,y)}{\partial z^2}\right] + dy\left[\frac{\partial^2 f(z,y)}{\partial z \partial y} + \frac{\partial^2 g(z,y)}{\partial z \partial y}\right] = 0,$$

which, after distributing and rearranging, yields

$$\frac{dz^*(y)}{dy} = -\frac{\frac{\partial^2 f(z,y)}{\partial z \partial y} + \frac{\partial^2 g(z,y)}{\partial z \partial y}}{\frac{\partial^2 f(z,y)}{\partial z^2} + \frac{\partial^2 g(z,y)}{\partial z^2}}.$$

■

To apply calculus techniques to our main comparisons, thereby using Lemma B.1, create a linearly ordered parameter set $(\mathcal{T}, \succeq)$, where $\succeq$ orders elements of $\mathcal{T}$ according to first-order stochastic shifts of $F(\cdot;\tau)$, i.e. $F(\cdot;\tau')$ strictly first-order stochastically dominates $F(\cdot;\tau)$ if and only if $\tau' \succ \tau$. Since everything in the model is smooth, standard techniques (see, e.g., Guillemin and Pollack (2010)) can establish that $\mathcal{T}$ is a manifold.

We begin with the parallel result associated with Proposition 5.

Proposition B.2 *When the opposition receives a direct benefit from radicalization, radicalization effort e^* is strictly decreasing in first-order stochastic increases of F, and the equilibrium level of dissent Y^* is strictly decreasing in economic conditions.*

Proof: Follows by applying Lemma B.1, observing that $\frac{\partial^2 V(\rho(e))}{\partial e \partial \tau} = 0$. ■

Naturally, the analogy to Proposition 6 depends on the interaction of the functions ρ and V. Using the same technique to apply calculus techniques, create a linearly ordered parameter set $(\mathcal{T}, \succeq)$, where $\succeq$ orders elements of $\mathcal{T}$ according to pointwise increases of $\rho(\cdot;\tau)$, i.e. $\rho(\cdot;\tau')$ pointwise increases from $\rho(\cdot;\tau)$ if and only if $\tau' \succ \tau$.

Proposition B.3 *When the opposition receives a direct benefit from radicalization, there* *exists a finite $M > 0$ such that if*

$$\left.\frac{\partial^2 V(\rho(e;\tau))}{\partial e \partial \tau}\right|_{e=e^*} \leq M,$$

then radicalization effort e^* is strictly decreasing in pointwise increases of ρ, and the equilibrium level of dissent Y^* is nonmonotone in pointwise increases of ρ.

Proof: Denote $\psi(e,\tau) = (1 - x^*(e))(1 - \rho(e))F(B(e))$, then applying Lemma B.1, $\frac{de^*(\tau)}{d\tau}$ has the same sign as

$$\frac{\partial^2 \psi(e,\tau)}{\partial e \partial \tau} + \frac{\partial^2 V(\rho(e;\tau))}{\partial e \partial \tau}.$$

The first term is negative by Proposition 6, hence, $\frac{de^*(\tau)}{d\tau} < 0$ if the second term does not exceed a finite bound around the equilibrium value, $e^*(\tau)$. ■

The analysis of the case where the opposition's payoff is zY_1 employs similar techniques and Lemma B.1.

C Macroeconomic Effects of Repression

In this section we focus on macroeconomic factors and consider two different ways of conceptualizing of economic channels which might impact the relationship between radicalization and targeted repression. There are a number of similarities between these two conceptualizations, and so we begin this section with some general results, where we represent a general economic channel by a parameter t, that influences the distribution function $F(w \mid t)$, and may potentially effect the government's cost function $c(x,t)$.

Since an individual's decision (radicalized or unradicalized) depends on macroeconomic factors only through F, Lemma 1 characterizes citizen decisions. Moving on to dissidence, we present an analogue to Proposition 1.

Proposition C.1 *Dissidence is given by*

$$Y_z(\rho;t) = \rho + z(1-\rho)F(B \mid t),$$

which is strictly increasing in z, strictly increasing in B, and strictly increasing in ρ. Moreover, the level of dissidence has strict decreasing differences between the opposition's survival, z, and the level of radicalization, ρ.

Proof: Dissidence follows by combining the expression

$$Y_z(\rho; t) = \rho \int_{0)}^{w(1;z)} dF(w \mid t) + (1-\rho) \int_0^{z \cdot w(0;1)} dF(w \mid t),$$

with Lemma 1, and evaluating the integrals, giving the expression in the text.

The comparative static implications follow by identical arguments to the proof of Proposition 1. ■

The important part of this result shows that in a more complex model, which explicitly incorporates additional macroeconomic factors, the key feature of our main model remains, i.e. where dissidence exhibits strict decreasing differences in z and ρ.

C.1 Economic Instruments

In this supplement we focus on economic instruments, denoted by $t = h$, that the government can use to impact economic returns, represented by the smooth distribution function $F(w \mid h)$, where h is from a compact subset of $\mathbb{R}$.[1]

Proposition C.2 *There is an equilibrium characterized by the pair $(w^*(s_j; z), x^*(\rho), h^*(\rho))$.*

Proof: The government's problem is

$$\max_{x,h} \; -\pi(x)Y_0(\rho; h) - (1-\pi(x))Y_1(\rho; h) - c(x, h). \tag{C.1}$$

Existence follows by the Extreme Value Theorem. ■

In what follows we assume an interior solution, thus implying that the second-order condition associated with (C.1) is satisfied.

[1]It is straightforward to extend to multidimensional economic instruments.

Proposition C.3 *Targeted repression, $x^*(\rho)$, is strictly decreasing in ρ, and economic instruments, $h^*(\rho)$, are increasing if $\frac{\partial F(w|h)}{\partial h} > 0$ and decreasing if $\frac{\partial F(w|h)}{\partial h} < 0$.*

Proof: The government's problem can be simplified to

$$\max_{x,h} \; -(1-\pi(x))(1-\rho)F(B \mid h) - c(x,h).$$

The first-order conditions that characterize an interior solution are

$$\pi'(x)(1-\rho)F(B \mid h) - \frac{\partial c(x,h)}{\partial x} = 0; \tag{C.2}$$

and

$$-(1-\pi(x))(1-\rho)\frac{\partial F(B \mid h)}{\partial h} - \frac{\partial c(x,h)}{\partial h} = 0. \tag{C.3}$$

Next, taking the implicitly defined $h^*(x)$ from (C.3) and plugging it into (C.2), and totally differentiating, we have

$$\frac{dx^*(\rho)}{d\rho} = -\frac{-\pi'(x)F(B \mid h^*(x))}{\pi''(x)(1-\rho)F(B \mid h^*(x)) + (\pi'(x)(1-\rho)\frac{\partial F(B|h^*(x))}{\partial h} - \frac{\partial c(x,h^*(x))}{\partial h \partial x}) \cdot \frac{dh^*(x)}{dx} - \frac{\partial^2 c(x,h^*(x))}{\partial x^2}}.$$

Since the denominator is negative by the second-order condition, the sign is determined solely by the sign of the numerator. Thus, $\frac{dx^*(\rho)}{d\rho}$ is negative since $\pi' > 0$.

Last, taking the implicitly defined $x^*(h)$ from (C.2) and plugging it into (C.3), and totally differentiating, we have

$$\frac{dh^*(\rho)}{d\rho} = -\frac{(1-\pi(x^*(h)))\frac{\partial F(B|h)}{\partial h}}{-(1-\pi(x^*(h)))(1-\rho)\frac{\partial^2 F(B|h)}{\partial h^2} + (\pi'(x^*(h))(1-\rho)\frac{\partial F(B|h)}{\partial h} - \frac{\partial^2 c(x^*(h),h)}{\partial h \partial x}) \cdot \frac{dx^*(h)}{dh} - \frac{\partial^2 c(x^*(h),h)}{\partial h^2}}$$

Since the denominator is negative by the second-order condition, the sign is determined solely by the sign of the numerator, whose sign is the same as the sign of $\frac{\partial F(B|h)}{\partial h}$. ■

Introducing macroeconomic instruments means that the government can manipulate participation in dissidence by nonradicalized citizens in two ways. First, through the channel in the main manuscript, the government can eliminate the operational capacity of the opposition, thus preventing them from mobilizing dissidence. Second, in this extension the government can also directly manipulate the economy through various economic or redistributive policies. Proposition C.3 shows that the level of radicalization affects the government's choice of economic instruments, depending on how economic instruments affect the economy, i.e. depending on the sign of $\frac{\partial F(w|h)}{\partial h}$, which is determined by the nature of macroeconomic instruments available to the government.

C.2 Economic Influence of Repression

In this supplement we consider when the government's targeted repression, like freezing financial assets or eliminating opposition leaders, have macroeconomic effects that alter the state of the economy, captured in our model by the distribution $F(w \mid x)$. We make use of the general results from the beginning of this section substituting $t = x$.

Proposition C.4 *There is an equilibrium characterized by the pair* $(w^*(s_j; z), x^*(\rho))$.

Proof: The government's problem is

$$\max_x \; -\pi(x)Y_0(\rho; x) - (1 - \pi(x))Y_1(\rho; x) - c(x). \tag{C.4}$$

Existence follows by the Extreme Value Theorem. ■

In what follows we assume an interior solution, thus implying that the second-order condition associated with (C.4) is satisfied.

Proposition C.5 *Targeted repression,* $x^*(\rho)$*, is strictly decreasing in* ρ *if*

$$\frac{\pi'(x)}{1 - \pi(x)} > \frac{\frac{\partial F(B|x)}{\partial x}}{F(B \mid x)}, \tag{C.5}$$

and is nondecreasing otherwise.

Proof: The government's problem can be simplified to

$$\max_x \; -(1 - \pi(x))(1 - \rho)F(B \mid x) - c(x).$$

The first-order condition that characterizes an interior solution is

$$(1-\rho)\left[\pi'(x)F(B \mid x) - (1-\pi(x))\frac{\partial F(B \mid x)}{\partial x}\right] - c'(x) = 0.$$

Totally differentiating this condition yields

$$\frac{dx^*(\rho)}{d\rho} = -\frac{-[\pi'(x)F(B \mid x) - (1-\pi(x))\frac{\partial F(B|x)}{\partial x}]}{(1-\rho)[\pi''(x)F(B \mid x) + 2\pi'(x)\frac{\partial F(B|x)}{\partial x} - (1-\pi(x))\frac{\partial^2 F(B|x)}{\partial x^2}] - c''(x)}.$$

Since the denominator is negative by the second-order condition, the sign is determined solely by the sign of the numerator. Thus, $\frac{dx^*(\rho)}{d\rho}$ is negative whenever

$$\pi'(x)F(B \mid x) - (1-\pi(x))\frac{\partial F(B \mid x)}{\partial x} > 0,$$

which rearranges to the expression above. ■

Introducing macroeconomic effects of targeted repression introduces another concern for the government since this additional channel implies that targeted repression also increases participation by nonradicalized citizens. Because targeted repression now works through two different channels, one by potentially eliminating the opposition's operational capacity and the other through macroeconomic effects of repression, the government's response to the level of radicalization depends on the relative elasticities of each channel. Condition (C.5) shows that when the elasticity of the opposition-eliminating effect is higher, then targeted repression is strictly decreasing in the level of radicalization. It also shows that when the elasticity of the macroeconomic effect of targeted repression is larger, then targeted repression is primarily an economic instrument (as above), and is strictly increasing in the level of radicalization.

D Countering Radicalization

In this supplement we consider when the government can manipulate the intensity of radicalization, through things like propaganda, public education, and policies aimed at winning hearts and minds. To capture this possibility we further extend our extended model by allowing the government to manipulate the intensity of self-motivation experienced by radicalized individuals. In the main text we assume that $D > \overline{w}$, but this extension requires us to extend our framework to allow for lower levels of D. It is important to emphasize that the results of this section depend on other aspects of the model, in particular, players' payoff functions.

In the initial stage, when the opposition chooses the level of effort toward radicalizing citizens, e, the government simultaneously chooses the magnitude of the duty term, $D \in [0, \overline{D}]$, where $\overline{D} > \overline{w}$. The cost of counter-radicalization measures is captured by the smooth, strictly decreasing, and strictly convex function $\psi(D)$, which is strictly decreasing to reflect that lower D is more difficult to achieve than higher D. To simplify the presentation, suppose that counter-radicalization is sufficiently costly so that $\lim_{D \to 0} \psi(D) = \infty$. Finally, in this extension we let $\pi(x) = x$, since this is without loss of generality (see the proof of Proposition 2).

Recall that citizen j dissents if and only if

$$w_j \leq s_j D + z(1 - s_j)B(e).$$

This leads to the following analogue of Lemma 1.

Lemma D.1 *There exists a unique cutoff, $w^*(s_j; z) : \{0, 1\}^2 \to [0, D]$, such that citizen j dissents if $w_j \leq w^*(s_j; z)$, and joins the economic sector if $w_j > w^*(s_j; z)$. Specifically, citizens follow the decision rule:*

$$y^*(w_j, s_j, z) = \begin{cases} 1 & \text{if } w_j \leq w^*(s_j; z) \\ 0 & \text{otherwise} \end{cases}$$

where

$$0 = w^*(0;0) < w^*(0;1) = B(e) \quad and \quad w^*(1;0) = w^*(1;1) = D.$$

Proof: Recall from (1) that $w^*(s_j; z) = s_j D + z(1 - s_j)B(e)$. The last part follows by substitution. ■

Next, we focus on the level of dissidence, presenting an analogue of Proposition 1.

Proposition D.1 *Dissidence is given by*

$$Y_z(\rho; D) = \rho F(D) + z(1 - \rho)F(B(e)),$$

which is strictly increasing in z, strictly decreasing in first-order stochastic increases in F, and strictly increasing in ρ if $D > B(e)$ and nonincreasing otherwise. Moreover, dissidence has strict decreasing differences between z and ρ.

Proof: Dissidence follows by combining the expression

$$Y_z(\rho; D) = \rho \int_{0)}^{w(1;z)} dF(w) + (1 - \rho) \int_{0}^{z \cdot w(0;1)} dF(w),$$

with Lemma D.1, and evaluating the integrals, giving the expression in the text.

The comparative static implications follow by identical arguments as the proof of Proposition 1, with the exception of ρ. Differentiating with respect to ρ yields

$$F(D) - zF(B(e)),$$

which is strictly positive if $D > B(e)$, and nonpositive if $D \leq B(e)$. ■

The only difference between Proposition 1 and Proposition D.1 is that whether dissidence is increasing in the level of radicalization depends on which citizens, radicalized or un-radicalized, participate more, which depends on whether ideological incentives, captured by D, are larger than material incentives, captured by $B(e)$.

Proposition D.2 *There is a unique sequential best response to the subgame starting at the government's targeted repression decision, characterized by the pair* $(w^*(s_j; z), x^*(\rho; F))$.

Proof: The government's problem in this stage is

$$\max_{x\in[0,1]} \; - xY_0(\rho; D) - (1-x)Y_1(\rho; D) - c(x),$$

which by substitution, and after collecting terms, becomes

$$\max_{x\in[0,1]} \; - \rho F(D) - (1-x)(1-\rho)F(B(e)) - c(x),$$

and can be further simplified to

$$\max_{x\in[0,1]} x[(1-\rho)F(B(e))] - c(x). \tag{D.1}$$

That the best-response level of repression is unique follows because the objective function is strictly concave in x. ■

It is important to emphasize that the condition characterizing the government's sequential best-response level of repression, (D.1), is identical to that in the main model, (A.3). Consequently, we have the following Corollary:

Corollary 2 *The sequential best-response level of repression,* $x^*(\rho; F)$, *is strictly increasing in* $B(e)$, *strictly decreasing in first-order stochastic increases in* F, *and strictly decreasing in* ρ.

Proof: Follows since the government's decision problem is identical to that in the main model, and following the arguments from the proof of Proposition 3. ■

We are now prepared to analyze the initial stage, where the opposition chooses effort e to devote toward radicalizing citizens, and the government simultaneously chooses D, reflecting counter-radicalization measures achieved through propaganda, public education, and hearts and minds policies. In this extension, the opposition's problem is

$$\max_e \,(1 - x^*(e, D))[Y_1(\rho(e)) - Y_0(\rho(e))],$$

Using the same change of variables as in the main model, we have that a best-response effort level, $e^*(D)$, is characterized by any e that satisfies

$$c(\gamma^*) = (1 - \rho(e))F(B(e)). \tag{D.2}$$

This condition has two noteworthy features. First, it is identical to the condition characterizing the opposition's effort choice in the main model, (A.6). Second, and importantly, it does not directly depend on the government's choice of D.

Corollary 3 *Opposition's best-response radicalization effort, e^*, is independent of the magnitude of the duty term, i.e. $\frac{de^*}{dD} = 0$.*

Proof: Follows by inspection of (D.2). ■

This result establishes that D does not directly affect e^*, and hence, exogenous changes in the environment that affect D^* do not have a strategic influence on e^*.

Moving on, we now consider the government's choice of D. The government's initial stage decision problem is

$$\max_D \; - x^*(e, D)Y_0(\rho(e), D) - (1 - x^*(e, D))Y_1(\rho(e), D) - \psi(D) - c(x^*(e, D)). \tag{D.3}$$

By substitution, this can be written as

$$\max_D -x^*(e, D)\rho(e)F(D) - (1 - x^*(e, D))(\rho(e)F(D) + (1 - \rho(e))F(B(e))) - \psi(D) - c(x^*(e, D)),$$

which can be simplified to

$$\max_D \; - \rho(e)F(D) - (1 - x^*(e, D))(1 - \rho(e))F(B(e)) - \psi(D) - c(x^*(e, D)).$$

Using the envelope theorem, the first-order condition for the government's initial stage problem is

$$-\rho(e)f(D) = \psi'(D). \tag{D.4}$$

Define an interior solution to (D.4) by $\hat{D}^*(e)$, which exists whenever two conditions are met. First, the second-order condition implies that at any interior solution,

$$\Phi(D) \equiv -\rho(e)f'(D) - \psi''(D) < 0.$$

Second, Since any $D \geq \overline{w}$ implies that every radicalized citizen dissents, any interior optimal choice must fall below $\overline{w}$. Thus, we can characterize the government's decision by[2]

$$D^*(e) = \begin{cases} \overline{D} & \text{if } \hat{D}^*(e) \geq \overline{w} \text{ or } \Phi(D) > 0 \\ \hat{D}^*(e) & \text{otherwise} \end{cases} \tag{D.5}$$

Combining the above analysis we can present the analogue to Proposition 4.

Proposition D.3 *There exists an equilibrium, characterized by the triple* $(e^*, D^* = D^*(e^*), x^*(e,g), w^*(s_j; z))$, *where* $w^*(s_j; z)$ *follows from Lemma D.1,* $x^*(e, D)$ *follows from Proposition D.2 where* $\rho = \rho(e^*)$, $D = D^*$, *and* (e^*, D^*), *where* e^* *solves* (D.2) *and* D^* *solves* (D.5) *evaluated at* e^*.

Proof: Follows from the preceding discussion. ■

We next consider analogues to Propositions 5 and Proposition 6, which we break up into two parts. First, we consider the opposition's choice of radicalization effort.

Corollary 4 *Equilibrium radicalization effort* e^* *is strictly decreasing in first-order stochastic increases of* F *and strictly decreasing in pointwise increases of* ρ.

Proof: Follows by noticing that the characterization of the opposition's choice is identical to that in the main model, and independent of D, combined with identical arguments as

[2]Note, $D \leq \overline{w}$ could be introduced as a constraint and Langrangian techniques applied.

Propositions 5 and 6. ■

This result establishes that opposition effort devoted to radicalization between our main model and this extension is identical.

We conclude our extension by considering equilibrium dissidence, which in this extension is

$$Y^* = \rho(e^*)F(D^*) + (1 - \pi(x^*(e^*)))(1 - \rho(e^*))F(B(e^*)).$$

We present the results as a series of corollaries.

Corollary 5 *If $\hat{D}^*(e) \geq \overline{w}$ or $\Phi(D) > 0$, then equilibrium dissidence Y^* is strictly decreasing in first-order stochastic increases of F, and is nonmonotone in pointwise increases of ρ.*

Proof: Follows by observing that when $\hat{D}^*(e) \geq \overline{w}$ or $\Phi(D) > 0$ the model reduces to the extended model in main text where $D = \overline{D}$. ■

We can now focus on the case when there is an interior D^*, which is characterized by (D.4). In this case, by total differentiation

$$\frac{dD^*(e)}{de} = -\frac{-\rho'(e)f(D)}{-\rho(e)f'(D) - \psi''(D)}. \tag{D.6}$$

The denominator of (D.6) is strictly negative by the second-order condition, hence the sign is determined by the numerator, which is strictly negative, meaning that e and D are strategic substitutes.

Corollary 6 *Suppose that D^* is interior, then equilibrium dissidence Y^* is nonmonotone in both first-order stochastic increases of F and pointwise increases of ρ.*

Proof: Starting with the second part, nonmonotonicity with respect to ρ follows from an identical argument as Proposition 6. For first-order stochastic changes to F, combining the argument from Proposition 5 with the fact that dissidence is increasing in e^* and D^* establishes the result. ■

E Public Finance Considerations for Opposition

In this section we expand on the public finance apparatus of opposition groups.

E.1 Rival Political Goods

Here we consider opposition-provided benefits that exhibit rivalry, i.e. goods where consumption by one individual prevents simultaneous consumption by another individual. Rivalry in opposition-provided benefits implies that such benefits need to decrease with the level of dissidence, Y, represented formally by the smooth and strictly decreasing function $B(Y)$.

Recall that citizen j dissents when

$$w_j \leq s_j D + z(1 - s_j)B(Y),$$

which, following an identical argument as in the main text, yields $w^*(s_j; z, Y) = s_j D + z(1 - s_j)B(Y)$. Since $B(Y)$ is strictly decreasing in Y, inspection shows that $w^*(s_j; z, Y)$ is nondecreasing in Y.

Proposition E.1 *There exists a unique sequential best-response level of dissidence, $Y^\dagger$, which is the unique solution to*

$$Y_z^\dagger = \rho + z(1 - \rho)F(B(Y_z^\dagger)). \tag{E.1}$$

The sequential best-response level of dissidence, $Y^\dagger$, is strictly increasing in ρ and z; strictly decreasing in first-order stochastic increases in F; and has strict decreasing differences in ρ and z.

Proof: The sequential best-response level of dissidence must satisfy

$$Y = \rho \int_0^{\overline{w}} dF(w) + (1 - \rho) \int_0^{z \cdot w^*(s_j; z, Y)} dF(w). \tag{E.2}$$

The left-hand side is strictly increasing in Y and ranges from 0 to 1, and the right-hand side is weakly decreasing in Y and also bounded between 0 and 1. Since both the right-hand and left-hand sides of (E.2) are continuous in Y, by the Intermediate Value Theorem there is a unique $Y^\dagger$ characterized by

$$Y_z^\dagger = \rho \int_0^{\overline{w}} dF(w) + (1-\rho) \int_0^{zB(Y_z^\dagger)} dF(w), \tag{E.3}$$

where the expression above follows by integrating.

First, totally differentiating (E.1),

$$\frac{dY_z^\dagger}{d\rho} = -\frac{1 - zF(B(Y_z^\dagger))}{z(1-\rho)f(B(Y_z^\dagger))B'(Y_z^\dagger) - 1} > 0. \tag{E.4}$$

For the second part notice that $Y_1^\dagger - Y_0^\dagger = (1-\rho)F(B(Y_1^\dagger)) > 0$. Third, a first-order stochastic change from F_1 to F_2 implies

$$0 = \rho + z(1-\rho)F_1(B((Y_z^\dagger)_1)) - (Y_z^\dagger)_1 > \rho + z(1-\rho)F_2(B((Y_z^\dagger)_1)) - (Y_z^\dagger)_1,$$

so to restore equality $Y_z^\dagger$ must decrease. Fourth, by (E.4) and the quotient rule,

$$\frac{d^2Y_z^\dagger}{d\rho dz} = \frac{F(B(Y_z^\dagger))(z(1-\rho)f(B(Y_z^\dagger))B'(Y_z^\dagger) - 1) + (1 - zF(B(Y_z^\dagger)))((1-\rho)f(B(Y_z^\dagger))B'(Y_z^\dagger))}{[z(1-\rho)f(B(Y_z^\dagger))B'(Y_z^\dagger) - 1]^2},$$

which is negative since $B'(Y_z^\dagger) < 0$. ■

This shows that rivalry in opposition-provided benefits introduces an additional condition to equilibrium, similar to a market-clearing constraint in a general equilibrium model, as the "market" for opposition-provided goods must clear in equilibrium. All the results from the main model follow by the addition of (E.1) to statements of equilibrium and replacing Proposition 1 with Proposition E.1.

E.2 Explicit Budget Constraint

Suppose that M is the total level of resources the opposition controls, obtained by drug trade, natural resource extraction, or illicit financial networks (among other potential sources). Suppose further that the individual citizen receives benefits B from participating in dissidence through the provision of public (or club) goods or services. Denoting the price of good i by p_i, the opposition's budget must satisfy

$$p_B B + p_e \cdot e \leq M.$$

Radicalization and dissidence are normal goods, and hence, the opposition group's budget constraint binds. Putting these together we have that

$$B(e) = \frac{M - p_e e}{p_B}.$$

Inspection shows that opposition-provided benefits, $B(e)$, is strictly decreasing in e.

F Responsive Repression

To capture responsive repression we add an additional last stage of the game where the government can repress following dissidence. Suppose that responsive repression, r, comes from a compact subset of $\mathbb{R}$. We represent the government's utility over responsive repression by the smooth and strictly concave function $u(Y, r)$ which is strictly decreasing in the level of dissidence, Y, and strictly increasing in the level of responsive repression, r. The government's payoff function is thus $u(Y, r) - Y_z - c(x)$ where the first term captures responsive incentives, those that matter after the dissent has actualized, and the second captures preventative incentives, i.e. those preceding the mobilization of dissent.[3] We assume that the government's incentive to repress responsively gets larger as dissidence

[3] It is straightforward to extend this model to have multidimensional repression, where repression vectors are ordered according to the product order.

gets larger, which is captured by assuming that u has strict increasing differences. Notice that the envelope theorem allows us to separate responsive from preventative decisions, and that sequential rationality requires we consider the government's problem for any level of dissidence, Y.

Proposition F.1 *There exists an equilibrium to this extended model where the characterization of e^*, $x^*(e)$, and $w^*(s_j; z)$ follow from Proposition 4, and responsive repression $r^*(Y)$, which is determined by the unique solution to*

$$\frac{\partial u(Y, r)}{\partial r} = 0.$$

Moreover, responsive repression is strictly increasing in the level of dissidence.

Proof: We can focus on the government's final stage problem

$$\max_r u(Y, r).$$

Since r is chosen from a compact subset and u is smooth and strictly concave, we can focus on the unique solution to the first-order condition

$$\frac{\partial u(Y, r)}{\partial r} = 0.$$

By total differentiation we have that

$$\frac{dr^*}{dY} = -\frac{\partial^2 u(Y, r) / \partial r \partial Y}{\partial^2 u(Y, r) / \partial r^2}.$$

Since u is strictly concave the denominator is negative, and because u has strict increasing differences the numerator is positive. ■

G Nonmonotonic Targeted Repression

In our model, government repression has one channel of influence: it increases the probability that the opposition group's operational capacity or leadership is eliminated. In this section we consider the possibility that such government efforts are counterproductive, meaning that government repression designed to eliminate the operational capacity or leadership instead improves the opposition's operational capacity or the leadership's survival. This is captured formally by assuming that for some levels of repression, x, the probability the opposition is eliminated is strictly decreasing. Although this is not problematic for our results, we cannot exploit the same change of variables used above.

Recall the government's problem:

$$\max_{x} \; -\pi(x)Y_0(\rho) - (1-\pi(x)Y_1(\rho) - c(x),$$

which gives the first-order condition

$$\pi'(x)(Y_1(\rho) - Y_0(\rho)) = c'(x).$$

After substitution for dissidence levels, this rearranges to

$$(1-\rho)F(B) = \frac{c'(x)}{\pi'(x)}. \tag{G.1}$$

This condition characterizes the unique interior solution. Note that because the left-hand side of (G.1) is strictly positive, and since c' is strictly positive, there cannot be a best-response x^* whenever $\pi'(x) < 0$. Hence, any best-response x^* must take place when $\pi'(x^*) > 0$, as in our model where we have assumed that π' is strictly positive globally. Since our comparative static analysis is local, this implies our results are unchanged.

References

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