Received: 21 November 2021 | Revised: 27 February 2022 | Accepted: 2 March 2022
DOI: 10.1111/ssm.12522

RESEARCH PAPER – MATHEMATICS EDUCATION

Investigating preservice teachers' assessment skills: Relating aspects of teacher noticing and content knowledge for assessing student thinking in written work

Nama Namakshi[1] | **Hiroko K. Warshauer**[2] | **Sharon Strickland**[2] | **Lauren McMahon**[3]

[1]Department of Mathematics, University of Arkansas, Fayetteville, Arkansas, USA
[2]Department of Mathematics, Texas State University, San Marcos, Texas, USA
[3]School of Education, University of Michigan, Ann Arbor, Michigan, USA

Correspondence
Hiroko K. Warshauer, Department of Mathematics, Texas State University, 601 University Drive, San Marcos, TX 78666, USA.
Email: hw02@txstate.edu

Abstract
This mixed methods study explored the development of elementary and middle school preservice teachers' (PSTs) learning to assess student thinking through analysis of students' written work. Six sections of the first mathematics content course for teaching implemented two writing assignments focused on analysis of student work aligned to the mathematics topics covered in the PSTs' classes. Using quantitative methodology, we examined the PSTs' ($n = 99$) change in their noticing and assessment skills of attending to and interpreting students' thinking. We also documented their mathematical content knowledge (MKT) as measured by the Learning for Mathematics Teaching in the domains of specialized content knowledge and knowledge of content and students. Findings show that the PSTs increased in their assessment skills and in their MKT. Using a qualitative methodology on interview data of a subset of eight PSTs, we found they acknowledged that having a firm grasp of mathematical content is essential to understanding and explaining student thinking. Findings indicate that engaging PSTs in the real work of teaching such as noticing and assessing students' thinking through written work at the early stages of their teacher preparation may motivate PSTs to learn mathematics more deeply with an eye towards strengthening their assessment skills.

KEYWORDS
Teacher education, Teachers and teaching, Teacher knowledge, Student assessment, Students and learning, Math/math education

The purpose of assessment is to help educators in the decision-making process to support students' learning, summarize and monitor student achievements at a certain time, and aid research and development in education (Harlen, 2005). Formative assessment (assessment for learning) is generally defined as the process of collecting and interpreting relevant evidence of student learning and understanding with the goal of facilitating learning and improving instruction (Harlen, 2005; Lee & Lim, 2020). The evidence of student learning may be collected through various artifacts such as written work on in-class activities, assignments, or content specific classroom discussions. Perhaps the most important characteristic of formative assessment is that it can be a catalyst for change when productive change can still occur (Dingman et al., 2019). The practice of formative assessment has been identified as critical for improving teacher effectiveness and student learning outcomes (Black & Wiliam, 2009). However, both prospective and practicing teachers need support in developing such an important pedagogical skill which involves interpreting students' mathematical thinking (Boerst et al., 2019) and providing effective feedback to move learners forward (Lee & Lim, 2020). In our present study we focus on the former aspect of formative assessment—attending to and interpreting students' mathematical thinking.

It is imperative that mathematics teacher preparation programs provide experiences that will deepen preservice teachers' (PSTs) conceptual understanding of the discipline while also engaging them in key activities of teaching mathematics such as assessing students' mathematical thinking (AMTE, 2017; Ball et al., 2008). There has been some research on the importance of formative assessment to improve student learning (Allsopp et al., 2008; Collins, 2012; Schoenfeld, 2015) and several related studies in the field of professional noticing of children's mathematical thinking that often involve observing student–teacher interactions in a classroom setting (Jacobs et al., 2010; Sherin et al., 2011). While the use of student work focuses on mathematical detail, there is a scarcity of research on developing mathematics teachers' assessment reasoning through analysis of students' written work (Fernandez et al., 2013; Lee & Lim, 2020) and hardly any research that examines the development of assessment reasoning among PSTs in the context of Mathematical Knowledge for Teaching (MKT) and professional noticing of children's mathematical thinking, which we henceforth refer to as teacher noticing, as it relates to analysis of students' written responses to formative assessment (Thomas et al., 2017). Therefore, our interest is in investigating and understanding the nature and development of PSTs' assessment thinking, and how it can be studied using the Teacher Noticing framework developed by Jacobs et al. (2010).

Based on the notion that it is beneficial for PSTs to learn "about children's mathematical thinking concurrently while learning mathematics." (Philipp, 2008, p. 8), we hypothesized that a mathematics content course for PSTs is an ideal setting in which to begin the development of PSTs' assessment thinking in conjunction with strengthening their content knowledge. In our study, the content course sections in which our PSTs enrolled intentionally embedded opportunities that would support their MKT. This was done through various activities and classroom discussions analyzing students' mathematical thinking. Classroom discussions focused not only on the content but also on making sense of others' thinking. A study by Fernandez et al. (2013) suggests that collaborative discussions serve to increase PSTs' level of noticing. Thus, our overall goal was to provoke and develop PSTs' assessment skills through focusing on the noticing skills of attending and interpreting during the course, while maintaining ties to teaching practices advocated by researchers such as Ball and Cohen (1999) and national documents (AMTE, 2017; NCTM, 2014).

Our study was guided by the following research questions:

1. What are the changes in the PSTs' assessment of student thinking and their Mathematical Knowledge for Teaching (MKT) in the content course that supported PSTs' MKT development and analysis of student work?
2. How did a subset of the PSTs describe their experiences engaging with an activity that focused on analyzing student work?

1 | CONCEPTUAL FRAMEWORK

We frame our study with a social-constructivist perspective that learning occurs in a social context. The meaning and knowledge about teaching is built and made more explicit when set among and enacted with others (Vygotsky, 1978). This project is a merging of three large bodies of research in teacher education literature: formative assessment, teacher noticing, and mathematical knowledge for teaching. In our research we were interested in studying the assessment skills of PSTs through the Teacher Noticing framework (Jacobs et al., 2010) in the context of a content course that was designed to support PSTs' MKT. In addition, we investigated the PSTs' perceptions of the tasks intended to develop their assessment skills. In the following section we discuss each construct and how it contributes to the area of study that integrates these important ideas.

1.1 | Formative assessment and teacher noticing

AMTE (2017) and NCTM (2014) emphasize that the practice of assessing students is integral to teaching. Teacher's assessment practices help establish learning intentions and criteria for success. These practices support effective classroom discussions that elicit evidence of student understanding and provide timely feedback thereby promoting a productive cycle of teaching and learning in the classroom (Hattie & Timperley, 2007; Lee & Lim, 2020). Assessing student thinking occurs through a variety of avenues such as analyzing students' written work, observing students as they work with peers or independently, and interacting with students by asking questions and carefully listening to their responses and reasoning (Black & Wiliam, 2009). We are guided by studies that have examined ways in which PSTs' expertise in formative assessment can be supported and developed in both science and mathematics settings (Casey & Amidon, 2020; Sleep & Boerst, 2012; Talanquer et al., 2015). Sleep and Boerst (2012) suggest that as we prepare future teachers, "...learning experiences grounded in formative assessment could support beginners' work on two crucial elements of mathematics teaching: unpacking mathematics and attending to student thinking" (p. 1039).

The effectiveness of formative assessment to foster student learning depends on how well teachers can analyze students' mathematical thinking, "notice critical issues and generate productive interpretations about the major challenges that students face to grasp targeted concepts" (Talanquer et al., 2015, p. 587). In this way, formative assessment may be considered an aspect of teacher noticing. Therefore, in order to study the development of PSTs' assessment skills, we adopted the noticing framework proposed by Jacobs et al. (2010), which focuses on student thinking and is conceptualized as a set of three interrelated skills: Attending to children's strategies, interpreting children's understandings, and deciding how to respond based on the children's understandings. Although analyzing the data on the deciding component may be useful for a study that focuses on PSTs' skills in providing feedback or exploring the broader construct of teacher noticing, our present study did not include this focus. Therefore, as mentioned earlier, we only used the attending and interpreting aspects of the framework for analyzing our data because our present study investigated the PSTs' assessment skills in interpreting and understanding students' mathematical thinking (Boerst et al., 2019). This focus on student's mathematical thinking provides a way for teachers to connect their own understanding of mathematics for teaching with the thinking of students (Fennema et al., 1993; Jacobs et al., 2010).

The majority of existing literature about teacher noticing (e.g. Sherin & van Es, 2009; Star et al., 2011) relates to analyzing videos or observations of classroom situations which often occur in dynamic settings where teaching and learning interact in complex ways. There are few studies which have examined PSTs' development of assessment skills through analyzing students' work. Crespo (2000), Fernandez et al. (2013), and Dick (2013), are examples of studies that focused on PSTs' experiences with noticing students' thinking through analysis of student work. In Crespo's study, PSTs had an opportunity to "explore and investigate students' ways of thinking and communicating in mathematics" (p. 158) and in the process began to focus their interpretation of their students' thinking towards meaning making rather than correctness. Fernandez et al. (2013) studied 39 PSTs at the final stages of their teacher preparation program to examine and characterize their developing skills in noticing students' problem solving specific to attending and interpreting students' work in multiplicative and additive situations. Dick's work (2013) examined how student teachers learned to notice students' mathematical thinking through analysis of their students' multi-digit addition and subtraction work samples. Her study suggests the importance of providing PSTs the opportunities to practice analyzing students' written work. Informed by these studies in mathematics teaching, we looked towards engaging PSTs with the notion of assessment early on in their teacher preparation program as an integral part of what teachers do in practice.

1.2 | Mathematical knowledge for teaching and analyzing students' mathematical thinking on formative assessment tasks

The Mathematical Knowledge for Teaching (MKT) framework is a way for us to examine the domains of subject matter knowledge and pedagogical content knowledge for teaching mathematics (Ball et al., 2008). A subdomain of subject matter knowledge includes specialized content knowledge (CK) and a subdomain of pedagogical content knowledge includes knowledge of content and students (KCS) (Ball et al., 2008). KCS focuses on anticipating what students might think or are likely to do, and interpreting students' thinking as students express themselves (Ball et al., 2008).

In our classes, we wove in opportunities for PSTs to learn mathematics content for teaching with a focus on different ways people come to think about, understand, approach, and solve mathematical tasks. Our team met weekly to discuss our research as well as plan for classes. For example, we consistently made use of materials in the textbook (Billstein et al., 2013) regarding student work to promote discussions about student thinking in our classes. As stated earlier, our course activities also included reflecting on and understanding non-standard ways of solving problems, video analysis of children's mathematical thinking, and classroom discussions to support PSTs' MKT (Ball et al., 2008; Thames & Ball, 2010). In this way our MKT focused content course provided a context within which we investigated PSTs' assessment thinking skills.

FIGURE 1 Sample released item from LMT instrument

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35	35	35
x 25	x25	x 25
125	175	25
+75	+700	150
875	875	100
		+600
		875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

The Learning Mathematics for Teaching (LMT) (Hill et al., 2004) are instruments that assess the MKT domains and include assessments for the two subdomains CK and KCS. We used the LMT instruments in Elementary Number Concepts and Operations which aligned with the content of our course to measure the PSTs' level of MKT at two points during the semester. Figure 1 is an example of a released item from the LMT (Ball & Hill, 2008) related to interpreting student work.

While the LMT instrument was designed with in-service teachers in mind (Hill et al., 2004), researchers have used the LMT to measure PSTs' MKT as well (e.g. Flake, 2014; Swars et al., 2007). For example, Flake (2014) investigated noticing and MKT in a methods class using the LMT as measure of specialized content knowledge. Similarly, Swars et al. (2007) used the LMT in a methods class to examine specialized content knowledge and beliefs among PSTs. Our study thus hopes to add to the literature by using LMT to measure PSTs' MKT in a content course setting.

2 | METHODOLOGY

We implemented a mixed-methods design to enhance the experimental study with a qualitative method. By using the explanatory sequential design (Creswell & Plano Clark, 2018), we measured the variables in our data on PSTs' assessment skills and MKT then gave voice to the interviewed PSTs to capture their perceived value and the challenges of engaging in activities that focused on analyzing student work. The data sources for our study include PSTs' written analyses of a sample of real elementary/middle school students' responses in each of two formative assessment tasks, LMT instruments (Hill et al., 2004): CK and KCS in the domain of Number Concepts and Operations, and interview transcripts of eight PSTs. A detailed description of our data sources will follow in the data collection section.

2.1 | Setting and participants

This study was conducted during the academic year 2014–2015 in a 14-week spring semester mathematics content course for PSTs at a large Hispanic-Serving public university in the southern United States. The course is the first of a two-course mathematics sequence for PSTs in preparation for elementary and middle school teaching; the prerequisite is successful completion of College Algebra. The course focused on the development of the number system with associated operations and properties, problem solving, algebra, and patterning. Emphasis is placed on examining multiple strategies for solving problems and the mathematical basis for a variety of standard and non-standard algorithms. Students typically enroll in this course before official acceptance into the teacher preparation program, so it is one of their first experiences with ideas related to teaching. We assumed no prior knowledge of methods. These course sections were intended to provide a strong foundation in the PSTs' content knowledge with a focus on the specialized mathematical content needed for teaching. The PST participants in our study were taking their first mathematics content course and had little prior experience with assessing student work.

Participants consisted of PSTs in six sections of the mathematics content course ($n = 108$) taught by four instructors and took a demographic survey at the beginning of the semester. The demographics of the participants was 98% female, 2% male; 62% White, 28% Hispanic, 7% Black/African-American, 1% Asian, and 2% Other. The instructors were two mathematics education faculty and two advanced mathematics education doctoral students.

2.2 | Data sources and implementation

The results of this study are based on three sources of data described below:

2.2.1 | Participants' written assessment of student work

The findings of the first research question are based on the PSTs' written analysis and evaluation of a sample of an elementary/middle school student's response in each of the two formative assessment tasks focused on the concepts of problem-solving through pattern recognition. Out of the 108 participants, 99 participants submitted their written evaluations on both tasks. Therefore, when analyzing data we only included the 99 participants for which the data set was complete. We used tasks and student work found in Schoenfeld's (1999) *Balanced Assessment* because the tasks were open-ended, had potential for multiple solution strategies, incorporated a range of cognitive demands, and were sufficiently rich to promote discussion. The first task, entitled "Table Tennis - round robin tournament", was administered at the beginning of the semester. The concepts in this task related to general problem-solving strategies in counting methods and computations with conversion between units of measure involving time. The second task entitled "Consecutive Addends," administered midway through the semester, focused on patterning and generalizing by systematically investigating numbers that can be expressed as sums of consecutive addends. Our goals for the PSTs were to notice the misconceptions or well-formed conceptions in student solutions and to interpret student work in light of the strategies used and the mathematical thinking that would lead to such a solution.

The general process of implementing both formative assessment activities was for the PSTs to work on the task in class then engage in small group discussions about their process and solutions to the questions on the task. Each instructor then coordinated a whole class conversation where each group had an opportunity to share their work. Discussions included the similarities and differences across the PSTs' solutions and methods as well as any connections to content from class. This 90-min class period was intended to support PSTs in making sense of their peers' mathematical thinking and diverse ways of problem-solving. After the discussions on how to solve the task had concluded, the instructor passed out the packets containing a sample child's work on the same task which the PSTs had just worked on and discussed. The PSTs were then instructed to read through the packet and write an analysis of the child's mathematical thinking when assessing their work as a take-home assignment. These formative assessment (FA) assignments 1 and 2 consisted of a particular sample, entitled Student B, because the student's work exhibited a mix of successful and unsuccessful strategies on the task; included a fair amount of written description of his/her process; some correct and some incorrect calculations; and a table or diagram. This allowed opportunities for PSTs to notice and assess a range of student performance, modes of communication (symbolic, pictorial/diagram, and written), and strategies across the parts of the task. The sample of Student B's work on the first task and sample writing prompts/instructions are included in the Appendix A.

2.2.2 | LMT instruments

Between FA1 and FA2, we engaged PSTs in a variety of activities to promote their assessment skills and analysis of students' mathematical thinking. For example, the class session following the return of FA1 to the PSTs included an in-depth discussion of major themes the instructor identified across their written assessments. The goal of the feedback was to help orient PSTs in improving their subsequent written reports and their noticing and assessment skills. Our course activities included reflecting on and understanding non-standard ways of solving problems included in the course textbook (such as dividing integers, making sense of fractions, looking for patterns etc.), video analysis of children's mathematical thinking, and classroom discussions to support PSTs' MKT related to the content being covered in class. This way of instruction was maintained throughout the duration of the course.

The four instructors administered the LMT instruments (Hill et al., 2004) on the first day of the course as a pre-test and again on the day of the final examination as a post-test. The pre and post-test scores were analyzed and compared to obtain empirical evidence of any change in PSTs' MKT in the subdomains of CK and KCS during the course.

2.2.3 | PST interviews

We used interviews of a subset of eight PSTs to study the second research question. Out of the PSTs who had consented to being interviewed and were invited, eight PSTs participated in the interviews. These eight PSTs represented a mix of varied levels of assessment skills on their FA1 of student work ranging from novice (3 PSTs), intermediate (3 PSTs), to advanced (2 PSTs) levels.

The individual interviews of the eight PSTs were conducted by the fourth author shortly before final examinations were administered. These interviews were semi-structured and asked the participants to share their experiences in the course and with the two FA activities. The questions focused on the processes the PSTs engaged in as they completed the written analysis of student work, particularly how these processes changed from the first

TABLE 1 Modified coding scales

Modified scales	Description
Attend score 0	Attended to none or just one instance of student's mathematical thinking with an inaccurate description of the strategy used by the student OR only attended to what the student did correctly/incorrectly, without a description of the strategy.
Attend score 1	Attended to at least two instances of student's mathematical thinking and included an accurate description of the strategy used by the student.
Interpret score 0	PST's interpretation of student thinking is not present, or the interpretations are not evidence-based or mathematically correct.
Interpret score 1	Evidence-based, but limited mathematical interpretation of student thinking - *PST explains only part of the student work and process with minor lack of clarity.*
Interpret score 2	Robust mathematical interpretation of student thinking - *PST explanations are evidence-based and clearly explains what the work shown by the student means in terms of how they got those numbers or/and why they did that process.*

task to the next, if at all, and their overall experience with engaging in activities that focused on analyzing student work and the course. We recognize the possible effects of being interviewed by one's own instructor such as candor, criticism, or undue influence (Corbin & Strauss, 2008; Weiss, 1994). However, of the eight PSTs, only two were from the interviewer's class. We also triangulated the interview data among all eight PSTs in order to report corroboration of responses.

2.3 | Data analysis

To analyze the data from the PSTs' written assessments of student work on FA1 and FA2, we used a modified form of coding scales described by Jacobs et al. (2010). In their Noticing Framework, Jacobs et al. (2010) describe a system for distinguishing between participants' attending to student thinking, interpreting student thinking, and using those skills to make decisions about students' learning needs. Recall, we only used the PSTs' attending and interpreting analysis for this study. Table 1 below illustrates the modified coding scales.

Three of the authors first met to establish the modified codes described above and coded one written assessment on FA1 together to get a sense of our coding scheme. We then separately coded 15 written assessments on FA1 from the PSTs in a section where none of the three had taught. After coding, we met to reconcile any differences and came to an agreement. We equally divided the remaining written assessments for coding. Next, we each randomly selected 10% of the written assessments from our assigned cluster and shared them with the other two coders for inter-rater reliability checks. We maintained 90% or better agreement throughout the process, after the initial alignment. We followed a similar cycle of coding with the written assessments on FA2 and again maintained 90% reliability or better. In summary, each PST was scored for both their FA1 and FA2 assessments of student-work for attending and for interpreting according to the described scale above. The growth in PSTs' MKT during the course was analyzed according to the MKT analysis manual for the CK and KCS sub-scales, respectively. We made no distinction among PSTs seeking elementary or middle school certification as most (90%) were pursuing elementary certification. We present PST scores as scaled z-scores. We ran matched pair t-tests to determine if there were any improvements in the PSTs' scores from pre to post.

We open coded the semi-structured interview data from the eight PSTs (Corbin & Strauss, 2008) to answer research question 2. In particular, we examined how the PSTs described their experiences engaging with an activity focused on analyzing student work. Three of the researchers coded one interview transcript together related to the PST descriptions and discussed the emerging codes e.g. Aspects of FAs in relation to noticing; Aspects of classroom teaching in relation to noticing; or PST growth evidence. We repeated this inductive coding and discussed the remaining interview transcripts. Two of the researchers then separately analyzed for common themes about the experiences across the eight participants (Miles et al., 2014). We met and clustered these themes according to the literature on content knowledge and noticing (e.g. Hill et al., 2008; Jacobs et al., 2010). We found two categories emerged from the PST descriptions: (a) value in the FA with subcategories of seeing multiple strategies and knowing the mathematical content and (b) challenges in the FA with subcategories of difficulty in examining student thinking, lack of confidence in one's own content knowledge, and novelty of the assignment. We now report our findings.

3 | FINDINGS

3.1 | Research question 1

Our first research question asked whether there was any development in the PSTs' assessment of student thinking as measured by their analysis of student work from FA1 to FA2 and their MKT as measured by the LMT instrument.

TABLE 2 FA-activity (written evaluations) outcomes

FA-activity (n = 99)	Attend		Interpret	
	M	SD	*M*	SD
FA1	0.3535	0.481	0.1616	0.445
FA2	0.6465	0.481	0.3535	0.628
Difference	0.2930***	0.593	0.1919*	0.680

*$p < 0.01$. ***$p < 0.001$.

The mean "attend" score for PSTs' written evaluations on FA1 more than doubled between the two assignments by 0.2923 and was statistically significant at $p < 0.001$. Similarly, we also saw nearly doubled gains in the interpret scores with a difference of 0.191 and statistically significant at $p < 0.01$. The results are reported in Table 2.

We also include sample responses from our PSTs to provide examples of what the written assessments looked like for different levels of scores (advanced, intermediate, novice) reported above. These examples are included in Table 3.

In our findings related to the growth in PSTs' MKT during the course, we found that the PSTs entered the course with limited knowledge of mathematics related to teaching. This was not surprising as it was the PSTs' first course related to mathematics with a teaching focus. Table 4 summarizes the findings from our analysis of the PSTs' CK and KCS scores.

The mean PST scaled score on the pre-CK measure was $\underline{x} = -0.93$, indicating that the PSTs, on average, scored nearly one standard deviation below the mean of the norming groups' scaled score. Because the LMT measures were developed with and for use with practicing teachers, this is not surprising. It suggests that our PSTs enter with little experience with this kind of mathematics. The KCS scaled score pre-test mean was $\underline{x} = -1.74$. This indicates that at the beginning of the course, the PSTs demonstrated a lack of knowledge about students' mathematical thinking, even more so than the lack of content knowledge as demonstrated on the CK measure at the start of the semester. From the post-test means we can see that the PST mean gain on CK was slightly above 1 standard deviation and on KCS was nearly 2 standard deviations. On both measures PSTs' knowledge grew despite their limited experiences prior to the course. This suggests that it is reasonable to assume that the course helped improve their content knowledge and their knowledge of students' mathematical thinking.

3.2 | Research question 2

We now report our findings for our second research question in which a subset of the PSTs describe their experiences with the FA-activity and analysis of student work. The consensus of the eight PSTs was to note the benefits that they gained when they did the task in class and discussed the solutions. They expressed difficulty when analyzing the students' work and writing their analysis report. We present our findings from the interviews first characterizing the value that the PSTs ascribed to the activity and then the challenges that they faced.

3.2.1 | Value of the Formative Assessment (FA) Activity

The PSTs identified the following aspects of the FA activity that were useful in analyzing the student work: (a) Doing the problems in class and sharing their solutions revealed multiple strategies in solving problems and (b) Knowledge of the mathematical content gave insight into how students solve problems. In addition, PSTs reflected on their overall FA activity experience and the course. We provide some student responses as examples of these characterizations.

Seeing multiple strategies

All eight PSTs shared a similar perspective on the in-class FA activity portion. They stated that doing the task in class, sharing their own strategies, seeing different approaches to solving the problem, and seeing similar possible mistakes in their group and whole-class discussions helped to focus their eyes towards the thinking of others. According to the PSTs, the in-class portion helped them to anticipate what the students might think or were likely to do. For example, in her interview, Stacey remarked, "one of the big things I learned in class was that there are so many different ways to look at something." Skye felt that, "we probably came up with some of the same mistakes at first that the students came up with, so we would be able to know that and like, "oh they did this" cuz they thought this." Meg tied her class experience to the student work analysis by sharing, "… everyone in my class had different ideas than me, but we were all right. So that taught me there are multiple ways to do a problem and so I could understand where the students were coming from." Katherine more specifically referred to the class work in relation to its usefulness in assessing student thinking as she stated, "it kind of helped point out things I might notice or might not notice. …they [students] might look at the problem differently…it helped me to be open-minded to what students were doing."

Knowing the mathematical content

The PSTs expressed value in knowing the mathematics related to the FA activity problems and the ways the students carried out the work. Meg reported that, "I would go

TABLE 3 Examples of levels of assessment skills among the PSTs

Levels of assessment	Explanation	Excerpts from PSTs' written evaluations
Novice	PSTs who primarily attended to correctness of answers and/or whose interpretations were not evidence-based.	"I noticed that he took the time to figure out the first answer which was correct…[0n] #2 I'm not quite understanding his work, he might have been on the right track and just didn't complete it."
Intermediate	PSTs who noticed more than correctness of answers and whose interpretations were more evidence-based, but who did not consistently, throughout their analysis, sustain these levels of *assessments*.	"[student's] plan was to draw out all …[the] lines and match them up by drawing lines to each opponent and they got the right answer. [On #2] the reason they multiplied 11 by .5 is because 11 represents how many matches are played at each table and .5 is ½ an hour. They were not able to come to an answer."
Advanced	PSTs who throughout attended to more than correctness of answers, and whose interpretations were evidence-based	"on part 1, Student B writes 45 matches and drew 10 lines to each person they had to play, and I take this as evidence that the student has the concept of solving how many matches will be played…They were right about there being 11 matches with one left over because 4 can go into 45 a total of 11.25 times…they knew how to figure out the number of total matches by dividing 45 by 4. Their next step was to separate the 11 and the .25 and to multiply the 11 by .5 because each match takes half an hour. They were confused after they multiplied the 11 by .5 though because they added the extra .25 that was left over, but they messed up the decimal point which made the final answer incorrect."

TABLE 4 PSTs scaled scores on the CK and KCS measures

$n = 108$	preCK	postCK	CK gains	preKCS	postKCS	KCS gains
PST mean scaled scores	−0.93	0.27	1.20* Effect Size 1.89	−1.74	0.16	1.90* Effect Size 2.82

*$p < 0.0001$.

through and figure out why he picked to do like this division first…I guess I would kind of try to break it down and see….[the FA activity] makes you explain and think about why it works." Mary found it essential to understand the concepts before analyzing student work. So, she reread the book, looked at her notes, and even went online as she states "to understand the concept more and I would do some of the problems [found online]… read the definitions and look at different examples." To explain why she went to great lengths to do the background work, Mary said, "So I understood the problems and so it was easier for me to understand the student's work. I really understood the concept is what I'm trying to say." Jennifer stated "I talked to [a classmate] a lot. We kind of went over what [classmate] did and I did." Skye also reported talking to her peers about the math content helped her with analyzing students' work "I think working in groups; when we did them in groups that helped too…People like to popcorn off of each other. If you're thinking that and they have something similar to that they can just add on to the discussion so that was helpful." Dora echoed Mary's sentiment, though for Dora, it was more about having a clear example worked out that helped her, "[instructor] went over with us, and we went over in groups to see what it really was and how it was really supposed to be solved. And just having that with me and looking back at that…and comparing it to the student's helped."

PSTs felt that understanding the underlying mathematics improved their ability to make sense of student strategies and the work that appeared on the students' papers. In addition, PSTs came to value various aspects of their content course such as learning about different mathematical models such as "the chip model or the pumpkin Pascal's Triangles", practice showing and justifying their work, and activities like the FA activity which related the math content to students. Meg offered, "I thought this class is really helpful because it makes you explain and think about why it works…[our instructor] would take the time to explain how everything works and why. So, that was helpful for me because I'm going to have to tell my students why someday and I don't want just [to] be the teacher that like "do this, like circle this and this…I think it will help me definitely teach math way better." Similar sentiments were expressed by other PSTs as well who found the course and the FA activity challenging at first but going through the process came to appreciate both. Below we describe some of the challenges that the PSTs encountered in their assignments.

3.2.2 | Challenges encountered when completing the FA activity

The PSTs identified the following challenges in analyzing the student work: (a) Examining student thinking is difficult, (b) Lack of confidence in one's own content knowledge, and (c) Novelty of the assignment. We provide some student responses as examples of these characterizations of the challenges in assessing students' work.

Examining student thinking is difficult

All eight PSTs reported that examining student thinking was the most challenging part of the FA activity. Six out of eight PSTs reported that lack of work shown by the student attributed to the difficulty of examining student thinking. For example, Skye stated that the "... because their work was incomplete, so it was like harder, cuz there wasn't much information to go by. I had to make a lot of assumptions like guesses on my own of what they were thinking." Abby echoed a similar sentiment "Seeing more of the student's work helps make sense of student's thinking." As PSTs reported on this challenge, they also expressed the realization they came to about the importance of showing work. For example, Jennifer stated, "Growing up it is like "show your work" and like "why?" Becoming a teacher now makes me appreciate it." While most PSTs cited the lack of student work as a reason for the difficulty with examining student thinking, two of the eight PSTs also reported that assessing incorrect work of students was more challenging than assessing the work of students that was correct. For example, Jennifer stated "[when] they got everything right…So, it is easy to say… oh they know the concept…but when they are not getting it correct it is harder to figure out what they are doing and thinking." Similarly, Meg described that making sense of the somewhat vague work shown by the student was challenging "some of the answers [were] right but I wouldn't understand their work. I didn't really know how to explain because whatever they did, it worked."

Lack of confidence in one's own content knowledge

Six out of the eight PSTs reported that not knowing the mathematical content themselves posed a challenge in analyzing students' work. For example, Abby stated that when she started struggling with the math content, analyzing students' approaches became increasingly difficult. As Abby became uncertain of her mathematical understanding, she attributed that to her difficulty in understanding the student's work, "…when like the student would do something different than I did, if I didn't have like a reference sheet, I didn't feel like I had a well enough understanding…to assess and see what they did or if something was wrong…which is just my own poor mathematical ability." Similarly, Skye stated "I struggled with that so when they [students] struggled with that it was hard for me to like, "they were thinking this"…The last one was more challenging because it was dealing with [what] students just struggle with in general…[I] felt the content [of the task] posed another challenge." Therefore, it was important for the PSTs to understand the problems themselves before they could make sense of the students' work. PSTs came to this realization as they navigated through the challenges of completing the written assessments. The value of future teachers understanding the mathematical content at a deeper level was expressed by all 8 PSTs.

Novelty of the assignment

Two out of the eight PSTs explicitly reported that the FA activity was challenging at first because they had never done anything like this before where they were asked to figure out what the actual school children did and why. For example, Katherine stated "I have never had to actually sit down and figure out what the kids did wrong, and I think that was the toughest part like because I have never actually [had] to do anything like this… I think the first one was tough because I hadn't done anything like it before." Meg echoed similar sentiments "the first one I was pretty stressed out cuz like oh my gosh, I've never done this kind of thing before…But I also didn't have experience like grading other people's work." Four other PSTs, though not explicitly stating this reason, implied that lack of experience with FA-style activity and not knowing their teacher's expectations contributed to the challenge. However, after the first FA activity, PSTs reported getting more comfortable with analyzing student work. For Mary it was just getting more experience with the assignment that helped her "we got better …just [with] the experience [of] doing it."

The eight PSTs described various challenges they encountered while completing the FA activity. They all recognized that examining student thinking is difficult and discovered various reasons that contribute to this challenge such as lack of student work shown or making sense of incorrect student work. Not knowing the mathematical content, themselves and lack of experience with evaluating someone else's work were also reported as challenges when analyzing student work. However, as the PSTs encountered these challenges, they also identified what helped them navigate the challenges - importance of understanding mathematical content at a deeper level, feedback from their teachers, being open-minded to diverse ways of solving a problem, and just getting more experience.

4 | DISCUSSION AND CONCLUSION

The primary goal of our study was to examine whether elementary PSTs in the early stages of their teacher preparation improve their ability to assess students' mathematical work on a task after experiencing small group and whole class discussion of the same task in a content course that embedded opportunities to support their MKT. A secondary goal of the study was to describe the experiences of our PSTs as they engaged with these activities in this course that afforded opportunities to assess students' mathematical thinking.

We saw significant growth in the PSTs' attend and interpret scores on their written evaluations of the student work. This might be unsurprising as we offered feedback on their first written report that encouraged them to think about students more explicitly. We believe that such feedback as well as activities directed at student thinking helped focus the PSTs' analysis on the subsequent FA activity task (Hattie & Timperley, 2007). The interviews revealed that PSTs considered the in-class discussions of multiple strategies helpful for their own recognition of others' thinking (Ball et al., 2008). Despite growth in their assessment thinking, we see from Table 2 that this growth was small (with a difference of 0.2930 in the mean scores for Attend and a difference of 0.1919 in the mean scores for Interpret between the two FA activities). Again, this seems reasonable given that such a task was difficult for them (per the interviews) especially for interpreting student thinking. These results align with the research study by Jacobs et al. (2010) where they found less than half of the PSTs in their study provided evidence of interpreting children's understanding. The authors provided one possible explanation for their finding; the mathematical concepts may have been too challenging, and in order to interpret the students' understanding they had to understand the mathematical concepts behind the students' work. While we designed the FA activities to align with the course topic and sequence, the added cognitive load of examining students' work may have been very high. In addition, we found that similar to the Jacobs et al. (2010) findings, our PSTs often attended to children's affect (e.g. being "lazy") with little evidence and failed to focus on children's strategies and the mathematics children employed. These course sections were intended to not only provide a strong foundation in the PSTs' content knowledge but also focus on the specialized mathematical content needed for teaching. In this way our MKT focused content course provided a context within which we investigated PSTs' assessment skills. Therefore, we were curious to see if there was any growth in PSTs' MKT during this course. While the LMT instruments used to measure the PSTs' MKT were developed to analyze practicing teachers' MKT (Hill et al., 2004), the PSTs improvement over the course of a semester in the two measures, CK and KCS, helped affirm for us as instructors that the objectives of the content course supports the PSTs' CK with the added bonus of KCS. It is promising to note that the PSTs' changes in these measures were slightly over a standard deviation for CK and nearly two standard deviations for KCS. Interestingly, our work contrasts with findings from the study conducted by Flake (2014) with PSTs in methods courses. Unlike our findings, Flake found little change in the PSTs' content knowledge as measured by the LMT instruments. Our PSTs were just beginning their teacher preparation trajectory with relative lack of experience in the practices and mathematics entailed in teaching. We suspect there was a lot of room for growth and the PSTs were motivated by the activities embedded into the course which focused on examining non-standard ways to solve problems and different ways of thinking as proxy for their interest in and caring about teaching children (Philipp, 2008).

In our analysis of the interviews, we found that PSTs were giving voice to the quantitative findings which positively correlated the change in the FA scores with the change in CK and KCS scores. All 8 PSTs acknowledged that a firm grasp of mathematical content is essential for understanding and explaining student thinking. Several PSTs commented that course material which related content to students made mathematics more interesting and was helpful in preparing them for the teaching profession (Philipp, 2008). The PSTs noted in the FA activity intervention that teaching with models such as the chip model or Pascal's Triangle were particularly beneficial indicating an appreciation for the specialized content knowledge and knowledge of content and students to support their teaching. Hill et al. (2008) note in their work that CK and KCS are widely believed to be important components of teacher knowledge by the research and practitioner community. Our findings widen this circle of acceptance by including the perspective of the PSTs. In the voices of our PSTs, we heard their appreciation for MKT as a result of working on activities such as the FA activity intervention designed to develop their assessment skills. Findings show that PSTs appreciate higher order thinking in mathematics if it is made relevant to them in interventions such as the FA activity. This may be one way to address issues raised by McDiarmid (1990) that PSTs "…do not see the relevance of much that they are taught. Without immediate need for the knowledge, they do not attend to it closely" (p. 12). Philipp (2008) underscores this issue that PSTs "…do not know what mathematics they need to know to teach effectively, and many are not open to approaching the content anew in a deeper and more conceptual way…" (p. 8). However, our study contributes to the literature on developing assessment thinking among PSTs especially in the context of MKT and Teacher Noticing through the analysis of students' written responses (Thomas et al., 2017). Furthermore, the voices of the PSTs give us insight into the challenges and the value of activities designed to develop assessment thinking from the perspective of the participants themselves. The PSTs clearly articulated that they felt the FA activities focused them into paying attention to student strategies and that doing so was a new and somewhat anxiety-provoking experience at times. They also linked their own knowledge of the content related to the tasks they were asked to analyze and their perception of their ability to analyze student work on that task. When they felt knowledgeable of the content, they felt more confident about their analysis; and when they felt less knowledgeable, they felt less capable or more unsure. Discussions in class helped them but so did some of their own help-seeking actions such as looking up content online, revisiting notes, and having conversations with classmates. Our hope is that this program will be valuable to our PSTs as they continue to strengthen their assessment skills and content knowledge for teaching and that our intervention was a jumpstart on their emerging skills.

4.1 | Limitations of our study and future recommendations

We understand that more layers of differentiation in scoring of Attend and Interpret dimensions could be added to build upon the Teacher Noticing Framework of Jacobs et al. (2010) which could have captured a more detailed portrayal of PSTs' assessment capabilities. Furthermore, while we were able to capture the development of the PSTs' MKT and their assessment reasoning, we were unable to explore any connections between these two constructs. We, therefore, recognize the limitations of our study.

As a result of our study, we recognize a need to differentiate PSTs' assessment reasoning that can better capture the subtle differences in the PSTs' assessment reasoning skills particularly in interpreting students' thinking (Boerst et al., 2019). Further investigation may also give insight into what connections exist, if any, between PSTs' MKT, specifically in the CK and KCS subdomains, and the development of their assessment reasoning. We also recommend that it would be beneficial for researchers to explore the connections between MKT and the broader construct of Professional Teacher Noticing of Children's mathematical thinking by including the decisional component.

Finally, our study suggests the following recommendations: (1) incorporate aspects of teacher noticing, assessment reasoning, and MKT explicitly into content courses for PSTs through formative assessment activities such as analysis of student work, (2) provide detailed feedback to PSTs to build assessment skills, and (3) teacher educators should regularly ask their PSTs about their experiences of course content and activities. These recommendations can inform the teacher educators' teaching practices in better supporting the PSTs.

ORCID

Hiroko K. Warshauer https://orcid.org/0000-0002-7338-8996

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How to cite this article: Namakshi, N., Warshauer, H. K., Strickland, S., & McMahon, L. (2022). Investigating preservice teachers' assessment skills: Relating aspects of teacher noticing and content knowledge for assessing student thinking in written work. *School Science and Mathematics*, *122*(3), 142–154. https://doi.org/10.1111/ssm.12522

APPENDIX A

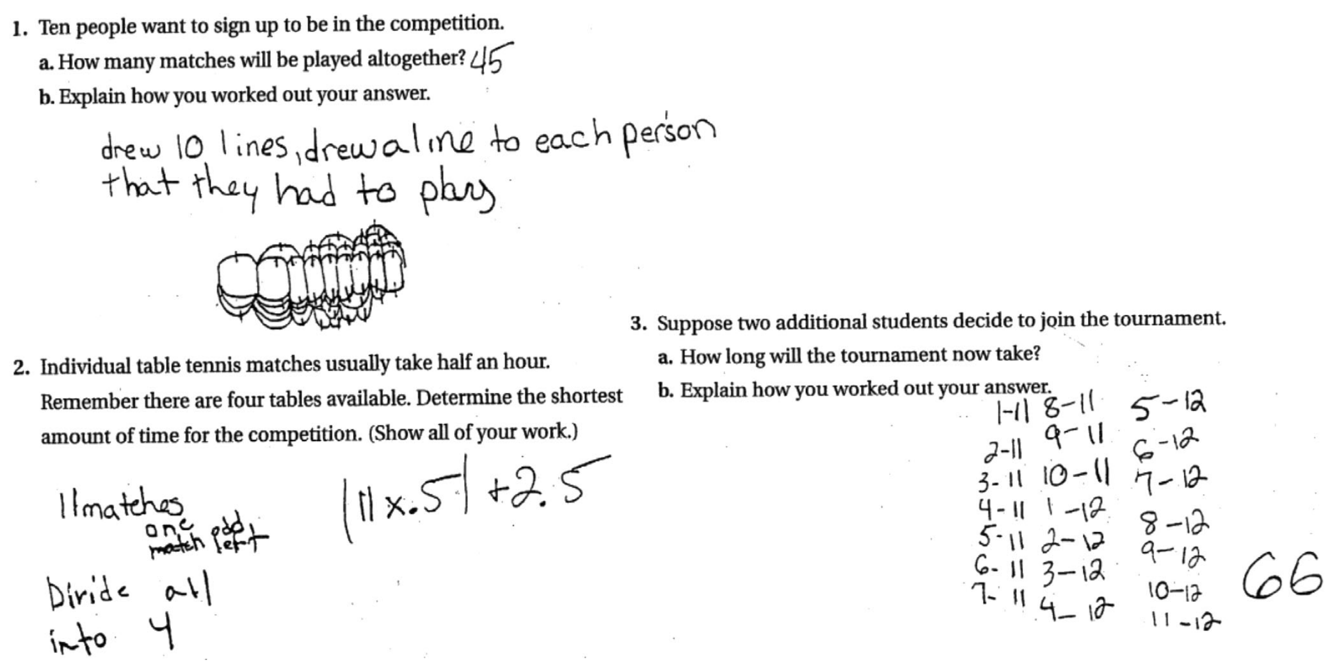
Student B Work on FA Activity Task

Sample Instructions and Writing Prompts (focused on Attend and Interpret aspects only)

For this assignment you will be given a packet that includes a task we will have done together in class as well as sample student's work/solutions to that task. Your job is to carefully analyze the student's solutions, and write a report documenting your assessment of their work and progress.

A. *Skills/concepts and/or thinking and reasoning*: write about what the student's work reveals about their thinking, strategies, and understanding of skills/concepts. *Attend to* what the student has done correctly or incorrectly and then *Interpret* their work to make conjectures about their thought process. Write about what you think the student may have been thinking, evidenced by what they wrote and why it would be mathematically reasonable to interpret their work in that way.

B. *Evidence*: include examples from student's work as evidence to support your assessment or claims that you made above in part A. Note every claim you make when attending and interpreting student responses should be supported by evidence from student work.



1. Ten people want to sign up to be in the competition.
   a. How many matches will be played altogether? 45
   b. Explain how you worked out your answer.
   drew 10 lines, drew a line to each person that they had to play

2. Individual table tennis matches usually take half an hour. Remember there are four tables available. Determine the shortest amount of time for the competition. (Show all of your work.)
   11 matches one match left
   (11 x .5) + 2.5
   Divide all into 4

3. Suppose two additional students decide to join the tournament.
   a. How long will the tournament now take?
   b. Explain how you worked out your answer.
   1-11 2-11 3-11 4-11 5-11 6-11 7-11 8-11 9-11 10-11 1-12 2-12 3-12 4-12 5-12 6-12 7-12 8-12 9-12 10-12 11-12
   66