# DESIGN REVIEW 3

April 22, 2022
Team 12: Design of a Benard Cell Demonstrator
ME 450 - 004 WN22

Stephen Mundy, Neil Wagner, Daria Bizyaeva, Dalton Wawro

# ABSTRACT

Our project is to design and develop a functional model that demonstrates the Benard convection cells for use in Professor Boehman's heat transfer class. He desires the model to be highly visible and visually interesting to all viewers in the class and those who are viewing it online. From our concept generation phase, we developed an open top setup that relies on reaching a critical Rayleigh number in order to generate cellular motion. We have conducted successful tests that demonstrated Benard cells. We have further verified and validated our final design. This design was presented to our sponsor with a user manual for his use in ME 335 lectures.

# EXECUTIVE SUMMARY

The goal of this project is to design a lecture demonstration of Benard cells to allow students to easily visualize natural convection. Through research and discussion with our stakeholders, we found our main requirements that the demonstration must be visible within 40 minutes of lecture start; portable; visually interesting; easily viewable to all students in the classroom and online; and easily set up and broken down by the professor within an appropriate time. The main challenge we faced when designing this demonstration is figuring out the correct proportions, hot plate temperature, and quantities of the materials used. The theory of Benard cells is based on two main equations using parameters from the system. First, the critical Rayleigh number determines whether or not Benard cells will form from a thermal gradient present in a working fluid. Secondly, the critical Marangoni number determines whether or not Benard cells will form due to surface tension. For Benard cells to form in our demonstration, one of these constraints must be met. We assured that we chose viable parameters that can reach the critical Rayleigh number. We generated potential designs based on initial brainstorming and research into previous setups. From this, we selected potential designs while considering the requirements and specifications conveyed to us by our stakeholders. Our final design is an open-top design without cooling, as we found a lid might obscure the view of the cells and a cooling mechanism might increase set-up time. Consequently, we are focusing on reaching a critical Rayleigh number. As previously implied, the majority of our information has come from research papers and other demonstrations that we have found online. We have combined this information with what we learned from textbooks and heat transfer professors at the University of Michigan to perform preliminary analysis on the temperatures required to attain cellular motion. We had a soft deadline of March 14th to finish our project in order to meet Professor Boehman's goal of using the demonstration in his natural convection lecture this semester. However, this goal has not been stressed as an essential requirement and the demonstration completion date could be pushed back so that the project could have been used for the first time in the Fall 2022 semester. We have validated and verified our design and engineering analysis to assure that our design meets the requirements and criteria of the project. The visually interesting demonstration displays Benard cells with an easy setup and disposal process. We have presented our design to Professor Boehman with a user manual. The demonstration was recently successfully shown in lecture.

# Table of Contents

# BACKGROUND AND INTRODUCTION

Undergraduate mechanical engineering students at the University of Michigan are required to take a class in heat transfer, MECHENG 335 (also known as ME 335 or just 335 among mechanical engineering students) [1]. This class covers the three modes of heat transfer- conduction, convection, and radiation- and goes further into exploring some more specific types of heat transfer. Among these, convection is arguably the most complex to understand yet the most versatile in application. Convective heat transfer occurs in fluids due to fluid motion [2]. Interactions between fluid motion and heat transfer create a variety of interesting features. Ranging from the flow of air over a microcontroller heat sink to turbulence in stellar nebulas, applications of convection and heat transfer play an important role across multiple fields of science.

Professor Andre Boehman hopes to introduce natural convection to his ME 335 class this semester via a demonstration of Benard cell convection (shown in figure 1 below). Inspired by this image from the Album of Fluid Motion, Professor Boehman asked this team to create a live demonstration of this setup for the lecture hall and purchased materials to reproduce the effect. The goal is to show this particular Benard cell convection, where the conditions are such that they form even, consistent cells for the students to see in action. Though there are other kinds of visually interesting convection-driven heat transfer systems, this one is simplest to explain. Explaining it in its most basic form, a thin layer of fluid is heated from the bottom, which causes these convection cells to form due to temperature differences between the top and bottom.

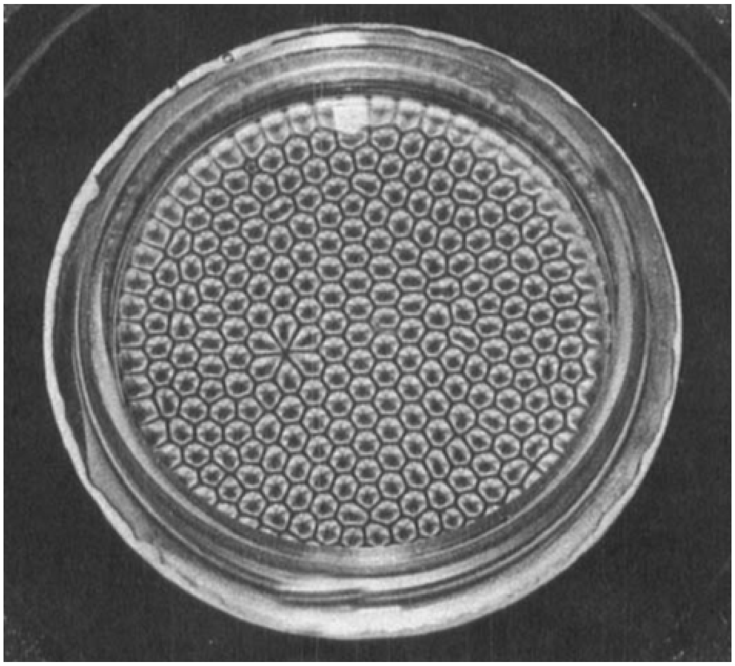


**Fig. 1**. Benard convection cells with rigid-free boundary, formed using silicone oil and aluminum powder [4, 5]

## Scientific Background

### *Fluid Flow within Cells*

Thermobuoyant flow is driven by pressure and/or temperature gradient in a fluid under the influence of gravity. The interaction between buoyancy and fluid density gives rise to Rayleigh-Benard convection, which can cause Benard cells to form [2, 3]. Hotter fluid rises at the center of the cells and colder fluid falls around the edges. The formation of these neat hexagonal structures can depend on both the Rayleigh number and the Marangoni number, where the former is more convection-driven (rigid-rigid boundary) and the latter more connected to surface tension (rigid-free boundary) [3, 4].

The main assumption in studies of Benard cells is that the fluid follows the Boussinesq approximation, where all fluid parameters are independent of temperature except for fluid density [3, 6]. Nondimensionalizing the Navier-Stokes equation with the continuity equations, we get the Rayleigh number. Without surface tension, the main result becomes that there is a critical Rayleigh number. The Rayleigh number is found from equation 1 below:

$$Ra = \frac{g\beta}{\alpha\nu}\left(T_H - T_C\right)L^3 \tag{1}$$

Where g is the acceleration due to gravity (m/s$^2$), β is the thermal expansion coefficient, ν is the kinematic viscosity, α is the thermal diffusivity, L is the height of the fluid layer (m), $T_H$ is the temperature at the controlled-temperature surface (K), and $T_C$ is the temperature of the fluid on top (K) [2, 3]. With rigid-rigid boundary conditions (the top and bottom surfaces are enclosed by a solid boundary), the critical Rayleigh number to form hexagonal Benard cells is about 1700 [4, 7]; rigid-free boundary conditions have a critical Rayleigh number of about 1100 [7]. Below that number, there is not enough buoyancy for formation while above, it starts to become turbulent convection.

Temperature differences between the top and bottom layers of a fluid can also form gradients in surface tension. This creates an imbalance which- with sufficient forces to overcome the frictional viscous forces- causes convection. While this effect is not defined as thermobuoyant, a similar process (with a different equation of state) to the one earlier was used to derive another dimensionless quantity called the Marangoni number:

$$M = -\frac{\delta S}{\delta T}\frac{L\left(T_H - T_C\right)}{\alpha\nu} \tag{2}$$

Where $\frac{\delta S}{\delta T}$ is the change in surface tension with respect to temperature, ν is the kinematic viscosity, α is the thermal diffusivity, L is the height of the fluid layer, $T_H$ is the temperature at the controlled-temperature surface, and $T_C$ is the temperature of the fluid on top [3]. The critical Marangoni number to form hexagonal Benard cells is about 80 [4, 8].

For any of these two cases to work, experiments must also consider the aspect ratio (ratio of the length of the container to the depth of the fluid). To eliminate boundary effects (conductive transfer) from the sides of an enclosing container, the aspect ratio needs to be large enough, as infinity is assumed in theoretical calculations but would be impractical in reality [2, 4, 6]. Fluid layer thickness is especially important in the case of surface-tension-driven convection. If the fluid layer is too thick, Benard cells may become unfavorable and rolls will form instead [9]. However, in very thin layers, surface tension effects dominate in such a way that the Marangoni number can be subcritical for the formation of hexagonal cells [10].

### *Heat Transfer from Cells*

For a given fluid, the fluid thickness and temperature difference are the two things that can be varied. However, in the case of an open surface as in the surface-tension-driven convection, the top surface temperature must be carefully considered. This temperature is often difficult to control and maintain compared to a rigid-rigid boundary, where cooling can more readily be applied to the top surface. The temperature difference between the air (at $T_{amb}$) and the top surface (at $T_C$) can still be estimated. Assuming the fluid is a semi-infinite, solid surface with constant heat flux [2]:

$$T_C - T_{amb} = \frac{Q \times d}{A \times Nu \times k} \tag{3}$$

Where Q is the heat transferred through the top (W), d is the container diameter (m), A is the heat transfer area ($m^2$), Nu is Nusselt's number, and k is the thermal conductivity (W/(m·K)). The Nusselt number, which gives the ratio of surface conductance to convectance, is given in this case by the following equation:

$$Nu = \left( \left( \frac{1.4}{ln\left(1+1.4/\left(0.835\times\frac{4}{3}\times\frac{0.503}{\left(1+(0.492/Pr)^{9/16}\right)^{4/19}}\times Ra^{0.25}\right)\right)} \right)^{10} + \left(0.14Ra^{1/3}\right)^{10} \right)^{0.1} \qquad (4)\ [2]$$

Where Ra is the Rayleigh number and Pr is the Prandlt number (ν/α).

## Previous Setups

There were some clearly described demonstrations from which we drew inspiration and additional recommendations.

The setup that yields clear results (similar to figure 1) comes from various studies by Koschmieder, who used this successful setup throughout multiple papers [4, 9]. A copper plate heated by a wire evenly distributed the heat to a glass dish, which was filled with a thin layer of silicone oil with aluminum powder. The walls of the container were insulated from the rest of the environment to minimize heat transfer through the walls. A lid on top was kept at a constant temperature via a constant flow of water being pumped in and out. There was an air gap, so the boundary was free on top. A thermocouple measured the temperature at the bottom of the plate and top of the fluid.

Undergraduate students from Oral Roberts University designed a Benard cell testing set up as part of a course project [7]. Their system included both an enclosed fluid (rigid-rigid boundary) and open-top (rigid-free boundary). Similar to what we intended, they used silicone oil. Their open system gave our intended result at a Rayleigh number of about 2120, and though they never mentioned the Marangoni number, they did state that this pattern of Benard convection was driven by surface tension effects.

The Indian Institute of Technology in Kharagpur also used silicone oil but with the intention of demonstrating surface-tension-driven Benard cells [8]. There was some discussion given in regards to the Benard cell experiment, explaining the critical number and stating the effects of thermal diffusion and viscosity, along with specifying that the fluid layer must be thin to neglect buoyancy.

# DESIGN PROCESS

From the very first lecture, it was clear to us that carefully choosing a design process would be essential to promoting the efficacy of our time and the quality of our product. Thus, shortly after receiving our project assignment, we began investigating the best possible fit for an excellent design process to guide our semester's work. We started from the Wynn and Clarkson article [17] and began a process of analysis to determine our strategy. At the highest level, our task was to promote students' understanding of free convection in ME335. Because of the large number of open questions with this project (e.g., how to make the demonstration visible on Zoom), we leaned toward a problem-oriented approach. We felt that such an approach would allow us to focus fully on the needs of the stakeholders, and in particular on the needs of the students, which would give us the freedom to tailor our design directly to them, rather than proposing a design as in the solution-oriented approach. Next, we noted that our challenge was very concrete; at its most basic, we were to construct a physical device. Thus, we found that a procedural approach would be more useful than an abstract one. At this point, we considered our timeline. Due to the short timeline of the class, in that our whole project must be completed in a single semester, we felt that we did not have time to perform an analysis of the processes used by other designers of heat transfer demonstrators. Thus, we selected a prescriptive approach to our design process.

Now that we understood what we wanted from our design process, we began reviewing resources available to us, looking for a problem-oriented, procedural, prescriptive design process. The logical first step was to review the suggested ME450 design process constructed by Skerlos et.al [21] in the lecture slides. We also considered French's process [18] but decided that we wanted final detailing to be included in the feedback loop due to the fine nature of the project (thermodynamic instabilities tend to be very sensitive to environmental conditions). Simpler processes such as Ehrlenspiel's [19] were very attractive due to their compactness and useability, but ultimately, we selected the ME450 suggested process due to its inclusion of the five full-process ribbons. We felt that incorporating these overarching themes, such as stakeholder engagement, engineering principles, and inclusivity, to name a few, throughout the entire process and constantly returning to them at each stage would lead us to a much higher quality of design. Since we were looking for a prescriptive approach, we wanted the best fit of the process model so that we could use it without modification. And thus, we decided that this setup was the best fit.

At this point, implementing the process was all that remained. We performed thorough need identification, ensuring that we gathered relevant information from our stakeholders, applied our prior knowledge and engineering principles, explored the need space rigorously, and thought through context assessment. While keeping our stakeholders engaged, we developed a clear, concise problem definition, working through iteration and reflection to determine the best possible definition. Finally, we arrived at concept generation and selection. We ran calculations and tests through the selected designs to narrow it down to one design, on which further iterations were run. This is described in more detail later in this report.

# DESIGN CONTEXT

The most important place to begin when considering the context in which a design takes its form is with the humans involved. Thus, our design begins with stakeholders. We identified three primary groups of stakeholders: ME335 professors, students, and GSI's. As we worked with our stakeholders, we learned that our project exceeds the interest of the sponsor in an interesting lecture-demonstration - our project is driven by a desire to make engineering education as a whole more accessible. Education plays a critical role in combating inequity in the engineering community.

Due to the open nature of the project, it is inherently collaborative and public domain; thus, intellectual property plays a minimal role. There are no intellectual property protections applicable to our project, as one of our goals is to make our design as open to interested parties as possible. A stakeholder even mentioned that it should be intentionally designed to be replicated by curious students. Thus, since no intellectual property is created in this project, there will be no owners of it.

Ideally, all of our stakeholders will be positively affected by our project. We aim first of all to ensure that the students benefit from it most directly. However, we do expect that the impact on the GSI's will be minimal, as they have little direct involvement in the classroom time. There is a chance that the professors may be affected negatively by the use of the demonstration; one of the professors we interviewed expressed concern about the safety features of the device. Further, since an aluminum powder suspension in silicone oil must be disposed of as hazardous waste, our design has the potential for negative environmental impact, which could affect all of our stakeholders. Additionally, there may be implicit factors in the extraction of some of our materials, such as human labor, land use, and/or chemical processes that may be used to produce aluminum powder.

In the context of our project, we believe that education and student impact is of the highest priority to our sponsor, while the environment is a secondary concern. This order of priorities may influence us to select a design that is more visible and more easily usable over one that is recyclable or reusable. Emphasis on education makes our project more likely to yield positive results. The small scope of the project will make the environmental impact small. Given the demonstration is not novel in the field of research, there will not be any spillover effects either- use is limited to those who really would like to see the demonstration and have the resources to do so.

The least sustainable aspects of our design are the fluid-powder suspension in which the Benard cells will form and the consumption of energy needed to form them. The material is difficult to replace due to the strict constraints needed to form the instability. However, it could be made more sustainable through the use of a sealed design, such that the fluid can be reused every time the demonstration is run. This change would require a small monetary investment for parts to seal the device but would have the larger impact of potentially making the cells more difficult to see. The device could also be adapted to use a more sustainable energy source, such as solar energy for heat. Due to inconsistent sunlight and power supply, especially during the winter term, such a change could make the device unreliable. Finally, sealing the design would result in the disposal of the entire device at its end-of-life rather than simply changing the fluid, which could also have negative environmental effects. Our device- if properly implemented- will not emit pollutants; thus, energy consumption is the only pollution-generating process associated with its use. However, its disposal does have the potential to generate pollution.

In the design of our project, we expect to face several ethical dilemmas, mostly related to selecting priorities in our design. Do we prioritize the needs of the students first, and potentially introduce a dangerous system that could injure a professor? Do we choose to prioritize the sealed design to reduce pollutant production, and risk increasing energy consumption? We will manage such ethical dilemmas through lifecycle analysis, realistic quantification of the harms, and ultimately, through stakeholder group discussion and team-based decision-making.

Our team's personal ethics reflect those of our University very well. We all value academic honesty and hard work, environmental stewardship, and inclusivity. We recognize that something as complex as the design of a lecture-demonstration has the potential to cause much unintended harm, and we take responsibility to minimize this potential. We recognize that potential future employers may prioritize profits over social impacts, so we do not expect that these ethics will necessarily be reflected by our employers.

Between us and our sponsor, there is the inherent power dynamic of professor-student relations. Some of our team members are in classes taught by our sponsor, which adds a significant aspect to the power dynamic. However, our sponsor is conscious of these power dynamics and so far has worked to ensure they do not affect our design. As designers working with faculty, there is a level of respect that the students in ME335 offer to us, which creates a very slight imbalance of power. However, due to the proximity of the class to our own academic progressions, this power dynamic is extremely minimal and has not had an effect. Among our team, we have worked to ensure that all team members have a say in team decisions, minimizing the impact of any existing power dynamics.

# REQUIREMENTS AND SPECIFICATIONS

Through communication with our stakeholders and sponsors, and research into similar demonstration rigs designed by other universities for educational purposes, we determined the requirements needed for our project. The model must develop the convection cells during class, must be visually interesting for the students and professor, must be visible to all in the lecture hall and online, must use the materials provided to us via Professor Boehman, must be transportable between the professor's office and the lecture hall, and preferably will be functionally prototyped by the natural convection unit in the heat transfer class this semester. To form clear specifications, we identified quantifiable measurements to characterize each requirement. Thus, the formation of cells must occur within 40 minutes to keep it within the first half of the class period. At least half of the cells formed must be hexagonal unit cells to keep the model visually interesting for students. The individual cells must be visible to students in the back of a large lecture hall when projected to the class. For this specification, we will use a student of 20/40 vision as the baseline and check against a Snellen chart to determine if it is visible. We will use the Snellen equation for 20/40 vision:

$$w = 2d\,tan(\frac{\theta}{2}) \qquad (5)\ [22]$$

Where *w* is the projected size of the cell, *d* is the distance from the back of the room to the projected image, and $\theta$ is the subtended angle (10 arcminutes for 20/40) [22]. 20/40 vision was selected because 95% of the population has 20/40 vision or better when wearing their corrective lenses [23]. The cells must be visible to students online and on the lecture recording. The model must use the resources given which are silicone oil, alumina/aluminum powder, Petri dishes, and a hot plate. The model must fit on a 24-inch by 36-inch area on a cart that is used for transport, as well as be able to be delivered to the lecture hall within 15 minutes of leaving Dr. Boehman's office. Lastly, the complete setup will be functionally prototyped by March 14th, on which the natural convection lecture occurs for the heat transfer class. We have also determined that the formation of the cells within time, the cells being visually interesting, and the cells being visible are of the highest priority for the project, and the rest of the requirements are of lower importance to achieve. For safety reasons, we must also make sure that nothing within the model is flammable or is at risk of igniting for the duration of the model working. The information above is presented in table 1 below.

**Table 1**. This table shows the requirements and specifications that our model must meet to be considered successful as well as the level of priority.

| Requirements | Specifications | Priority |
|---|---|---|
| Cells must form at least halfway through the lecture | - Formation of cells within 40 minutes | High |
| The model must be visually interesting | - At least half of the Petri dish area is filled with hexagonal Benard cells | High |
| Must be visible to all viewers in the lecture hall | - Individual cells must be visible to students in the very back of a large lecture hall (seen by students with 20/40 vision)<br>- Individual cells must also be visible to students on Zoom | High |
| Must use provided materials | - silicone oil [6], aluminum/alumina powder [8, 11], Petri dishes [7], and hot plate [11] provided must be used | Low |
| Must be easily transported between locations of the lecture | - Materials must fit on an available 24"x36" cart<br>- 15 minutes from Auto Lab to GGB lecture hall | Medium |

| | | |
|---|---|---|
| Preferable completion by the lecture on natural convection | - Complete setup and instruction manual by the week of March 14 | Low |
| The model must not be prone to be flammable | -Ensure all materials will not ignite at any temperature produced by the hot plate | High |

Another limiting factor in our design is the ethical sourcing and disposal of materials used in the demonstration. Since the oil and visual aids used in the demonstration may need to be replaced in between uses, we need to assure that those materials are sourced and disposed of ethically. When researching the sources of the initial materials list given to us by our sponsor, we found no unethical sourcing of materials or unnecessary/excessive damage to the environment or other social groups. When disposing of the used materials, we have been following University of Michigan disposal guidelines for lab chemicals and materials. This includes labeling of used materials and assuring that they are not disposed with any other goods that may be reactive with one another.

# CONCEPT GENERATION

### Methodology

When approaching concept generation, we first focused on heating generation as it is the most important portion of our design. Without proper heating, the working fluid will not turn over and the cells will not form. After generating our heating concepts, we began to discuss potential additions that we could make to the design in order to improve its utility. These changes will not affect the overall design or procedure of the demonstration. Instead, they solely serve to improve how our design meets our specifications and requirements.

### Broad Concept Generation and Idea Formation

We developed four heating concepts for our design in order to reach the necessary temperature gradients to have cellular formation. These concepts vary in the presence or method of cooling of the top plate (one design does not have the top plate at all). The heating concepts were compared to our requirements and specifications from design review 1 and also were assessed on their ease of use, manufacturing, and development. The heating concepts can be seen in figures 2-5:

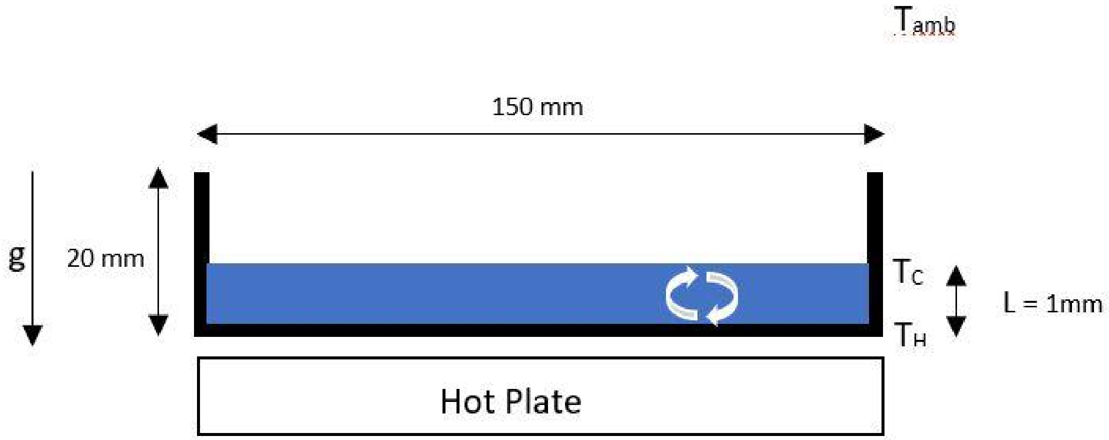


**Fig. 2**. Heating concept generation 1. The design has an open top with a fluid layer of 1 mm. The Marangoni number must be larger than 80 in order for cellular motion to occur.

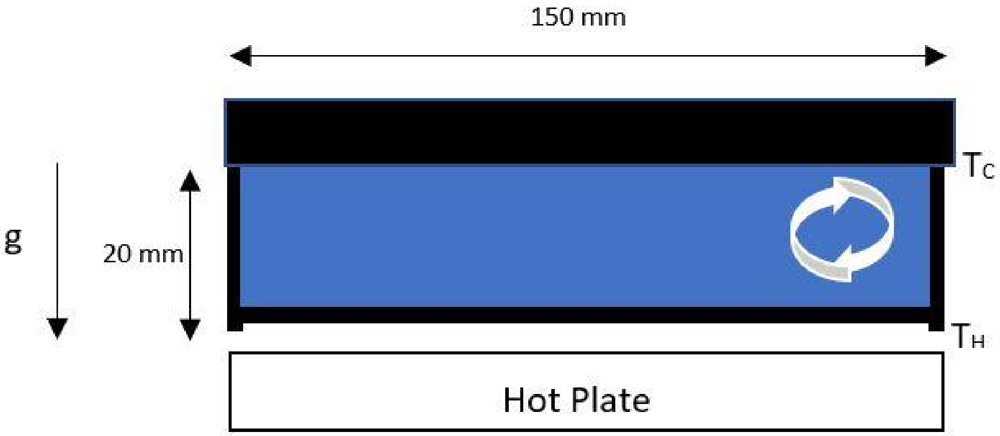


**Fig. 3**. Heating concept generation 2. The design has a closed top with a fluid layer of 20 mm. The Raleigh number must be larger than 1700 in order for cellular motion to occur.

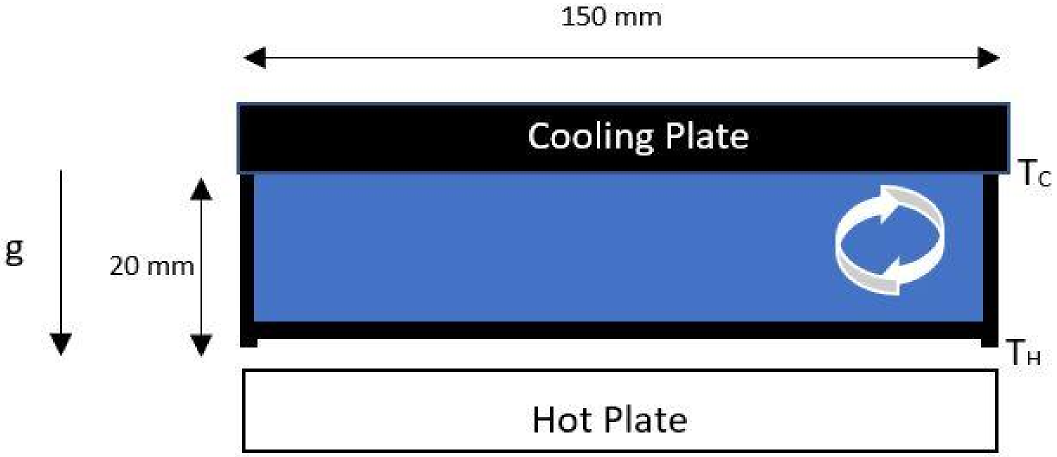


**Fig. 4**. Heating concept generation 3. The design has a closed top with a cooling plate. The fluid layer has a height of 20 mm. The Raleigh number must be larger than 1700 in order for cellular motion to occur.

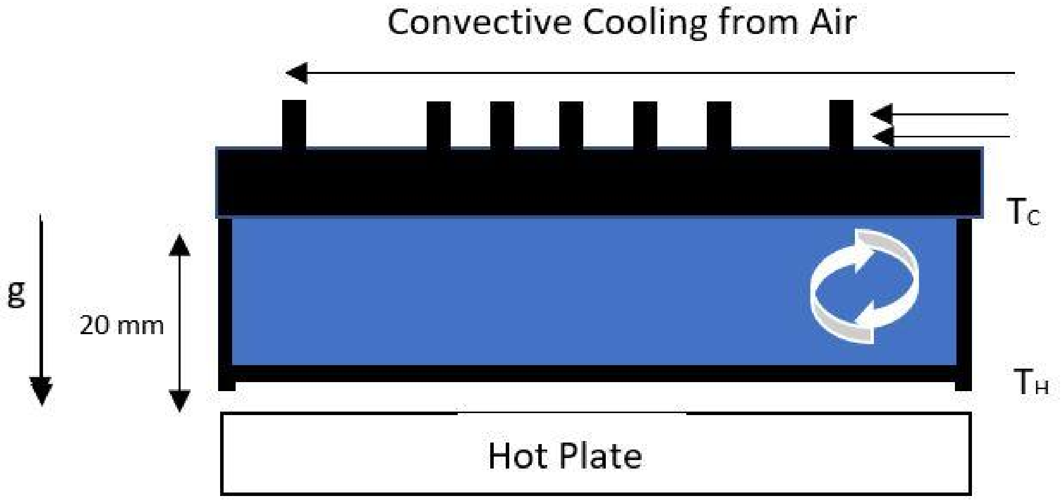


**Fig. 5**. Heating concept generation 4. The design has a closed top with fins on top for convective cooling. The fluid layer has a height of 20 mm. The Raleigh number must be larger than 1700 in order for cellular motion to occur.

The four heating concepts give a wide array of methods of heating and cooling in order to reach the required temperature. Concept 1 is the only design that does not have an upper plate and also is the only fluid layer less than 20 mm. Concept 2 has a closed top with no additional heating aspect. Concepts 3 and 4 have a cooling plate and convective fins respectively. Concept 1 relies upon a critical Marangoni number to create cellular motion while concepts 2, 3, and 4 all rely upon a critical Rayleigh number.

**Potential Additions/Subcomponents**

The potential additions and the subcomponents of the design have the sole intention of improving the design's ability to meet the requirements and specifications. The proposed potential additions to the design are:

1. Different powders/visual aids - aluminum or alumina powder
2. Larger Petri dish - alter the aspect ratio of the cells
3. Black material underneath the dish - improve visibility
4. Thermocouples - improve qualitative data

**Rejected Design Choices**

Designs were rejected based on their inability to meet the requirements and specifications created by our group and our sponsor. Our aim was to create a design that meets all the requirements and specifications set out by our team. This would create a demonstration that best meets Professor Boehman's needs and would help his students understand natural convection to the best of their ability. The closed top solutions were not chosen for our final design because the top decreases the visibility of the demonstration. For closed top solutions, the fluid layer would have to completely fill the Petri dish until it reaches the lid. For our given Petri dishes, this height is 2 cm. We ruled out these closed top solutions because we worried that the relatively large fluid height would cause a large aspect ratio and cell size. The size of cells are determined by the ratio between the height and width of the fluid. Since the fluid height is very high, this may cause the cell size to be abnormally large. If the cells become too large, they may be difficult to see or not visually interesting to students in the classroom or connecting over zoom. The convective cooling design and the cooling plate design were not chosen as they require outside materials that were not given to us at the start of the project. The larger Petri dish similarly requires outside materials, however we believe that if the aspect ratio needs to be changed in the future, this option is still available. Lastly, the black material was deemed unnecessary as the powder mixture used in the design created enough visual contrast in the demonstration. After removing rejected designs, we were left with our final selected design. This design has an open top setup and relies on reaching a critical Rayleigh number to form Benard cells. This design had the best ability to meet all of our requirements and specifications compared to the other generated concepts.

# SELECTED DESIGN (ALPHA)

The chosen design is centered around heating concept 1. This design has an open top and relies on a Rayleigh number greater than 1100. The fluid layer in this design is initially set at 1 mm in height. The additional components added to this design are a mixture of aluminum and alumina powder as well as thermocouples to increase the quantitative data. A CAD drawing of the design can be seen below in figure 6:

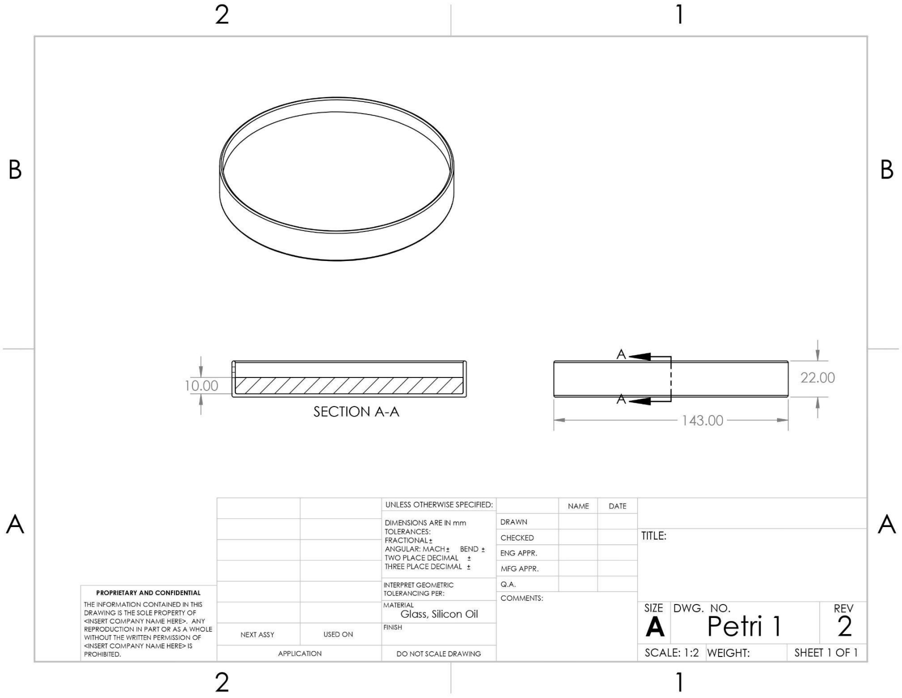


**Fig. 6**. CAD drawing of the final selected design

This design was heavily influenced by the given materials from our sponsor. Since utilizing the given materials is a requirement and specification set by our group, we tried to create a design which stuck to that specification as closely as possible. From the designs that met that criterium, we then based our decision off of engineering analysis and research papers of similar designs. We used engineering analysis in order to determine if the heating concepts were feasible and we used the research papers to determine the ease of use of certain setups. The values determined from our analysis were based upon concepts learned in the ME 335 (Heat Transfer) course and are true estimates of what we believe the temperature gradient will resemble. Using the thermocouples in our design, we will be able to verify that our design creates a temperature gradient that is close to our estimates from our analysis.

# ENGINEERING ANALYSIS

Because the demonstration requires developing a very specific phenomenon, it is critical that we develop a thorough understanding of the conditions occurring in the device. Once we understand well the conditions and how to manipulate them, we can create the needed conditions such that the Benard cells form on demand, reliably, with a reasonable margin of safety such that natural environmental variations will not disrupt our device.

In order to most rapidly and effectively develop this understanding, we elected to perform a combination of empirical testing and computational modeling. This combination allowed us to develop and verify an accurate model rapidly. For the computational model, we considered FEA heat transfer software, but found that an accurate FEA model would require significant time to develop and significant computational power to execute. We noted that our design relies heavily on idealized conditions, and is in fact propelled by simple heat transfer principles. Therefore, it was ideally suited to developing an analytical model based on a series of simplifying assumptions. Thus, we developed code in MatLab (see appendix A) to allow us to vary the critical parameters, particularly fluid thickness and target dimensionless quantity, to optimize our design. Verifying this model with testing is an important part of our engineering analysis, as it gives us confidence in the results and allows us to refine our model and assumptions.

## Assumptions

To keep the model simple, we selected a series of assumptions. First, we assumed that the bottom of the Petri dish was at a constant temperature. This assumption is accurate because the Petri dish rests flat on the hot plate, which extends past the edges of the Petri dish and is at constant temperature, so any surface temperature variations across the dish will be small. Next, we assumed that negligible heat transfer occurs through the sides of the Petri dish. The thinness of the fluid layer relative to its surface area on the top and bottom ensures that this is justified, as thermal transfer through the sides will be small relative to the top and bottom. We also assumed that negligible thermal contact resistance exists between the various items. This assumption is justified due to the fact that the fluid establishes excellent surface contact with the Petri dish. Additionally, we assumed that the air above the liquid is not forced to move, and is at a constant 25°C and 1 atm. These conditions approximate what we will encounter in the lab and in the classroom, and thus provide a reasonable assumption for the ambient conditions for the demonstration. We assumed negligible heat transfer occurs through radiation, which is justified due to the significance of the heat transfer through conduction and convection, which will render the transfer through radiation minimal. In particular, the hot plate transfers heat through conduction, not radiation, and it is our main source of heat. We further assumed that the system is at steady state; while the system will indeed be transitory during the entire formation period, assuming steady state allows us to calculate operating parameters that will guarantee the cells will achieve steady state, and thus stay visible throughout the entire duration of the lecture. These assumptions were easily justified, and allowed us to begin setting up our calculations.

Since there is no analytical method to model the convection cells themselves, as they are a stochastic process, we assumed that the liquid is at rest (no convection cells having formed) during our calculations. This assumption relies on the fact that we are calculating the minimum parameters that will achieve the critical Rayleigh and Marangoni numbers, so we are assuming that we reach the critical value just prior to cells forming. While this is unlikely to be exactly the case in testing, careful control of the conditions as we heat the dish should make this assumption reasonable. Next, we noticed that all of the properties of the liquid were temperature-dependent. However, setting up a calculation that calculated the gradient of each property through the liquid would be massively computationally expensive, so to keep the model simple we assumed that the properties of the liquid were constant throughout it. This assumption is justified due to the layer being thin enough that the temperature gradient (and thus the change in the properties) would be relatively small. Finally, we began searching for numerical values of the properties to use. We found that the spec sheet provided by the oil manufacturer listed properties only at a single temperature. Thus, we assumed that these values applied at all temperatures. We justify this assumption by ensuring that we are within the rated operating temperature range of the fluid, which should ensure that these properties are close to the actual properties of the fluid.

## Problem Setup

Literature review led us to the Rayleigh and Marangoni numbers as the dimensionless quantities of interest, as described earlier in the report. Using our assumptions, we model the process with thermal resistances.

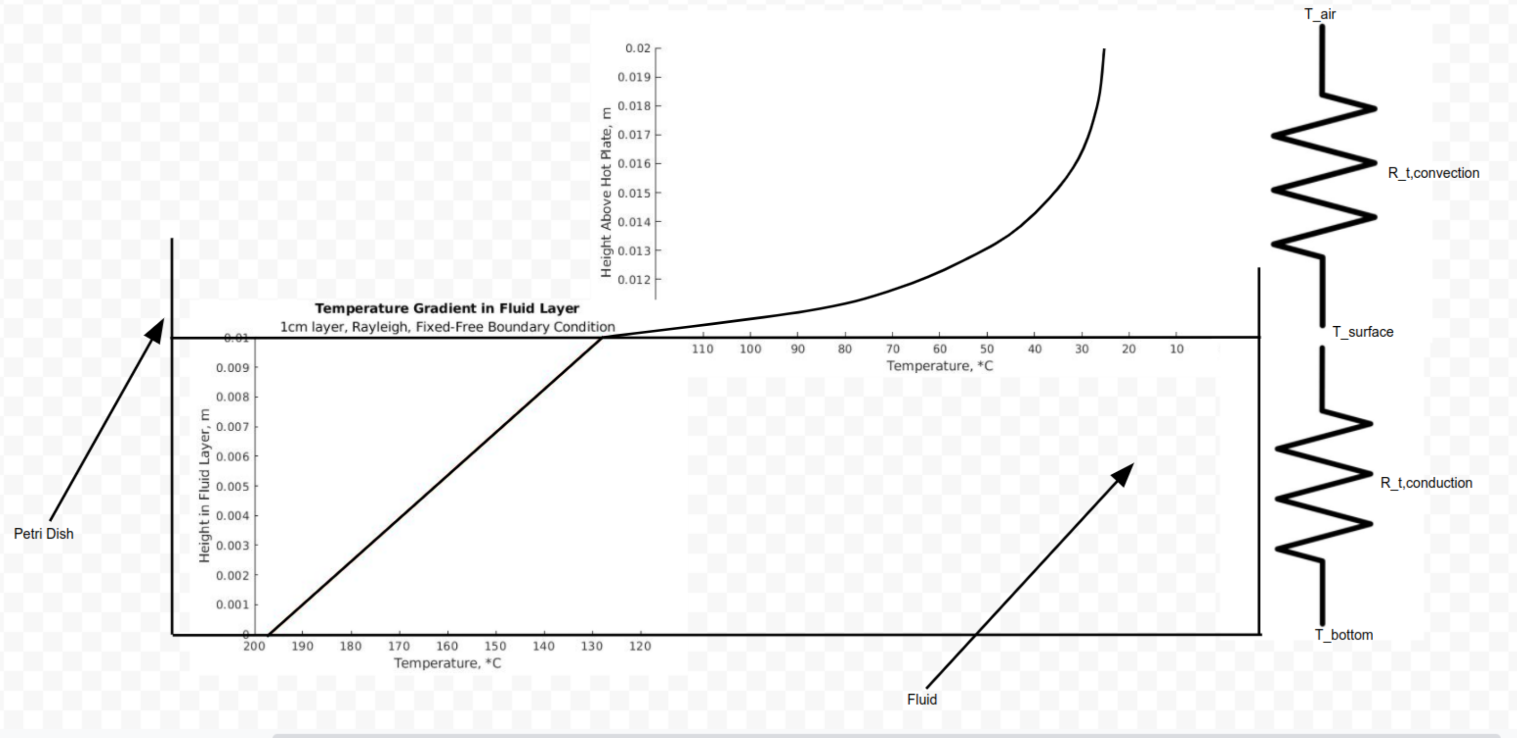


**Fig. 7.** Thermal Circuit Model with Temperature Gradients. Note the temperature gradient in the air is an estimate, and the drawing is not to scale except where noted.

## Calculations

Using this thermal resistance model, we determined the following heat transfer equations would be useful. Note that our critical Rayleigh number (1100) falls between $10^3$ and $10^7$:

$$L_c = \frac{A}{P} \tag{6}$$

$L_c$ is the characteristic length of the air, A is the area of the air in contact with the fluid, and P is the perimeter of the area of air in contact with the fluid.

$$\overline{h} = \frac{k}{L_c} \bullet 0.54\, Ra^{\left(\frac{1}{4}\right)} \tag{7}$$

h_bar is the convection coefficient and k is the thermal conductivity of the air.

$$R_{t,\, Convection} = \frac{1}{\overline{h} \bullet A} \tag{8}$$

$$R_{t,\, Conduction} = \frac{L}{k_f A} \tag{9}$$

$R_t$ is the thermal resistance, L is the thickness of the fluid, and $k_f$ is the thermal conductivity of the fluid.

$$q = \frac{T_{base} - T_{air}}{R_{t,\, Convection} + R_{t,\, Conduction}} \tag{10}$$

T is temperature and q is heat flux.

$$T_{surface} = T_{base} - (q \bullet R_{t,\, Conduction}) \tag{11}$$

Coupling these equations with equations 1 and 2, we can calculate all the needed properties from known parameters. In order to streamline our analysis, and allow for the most rapid iteration of various parameters, we constructed a MatLab code using these equations. The code is available in Appx. A.

## Results

We divided our engineering analysis results into two sections, based on the dimensionless quantity being analyzed. We began with Marangoni analysis, and concluded with Rayleigh analysis when Marangoni analysis proved unsatisfactory.

### Marangoni Analysis

As indicated in equation 2, the surface tension-temperature gradient is critical to the Marangoni number. Using data from Super Lube, the supplier of the silicone oil, we generated a plot of surface tension vs. temperature:

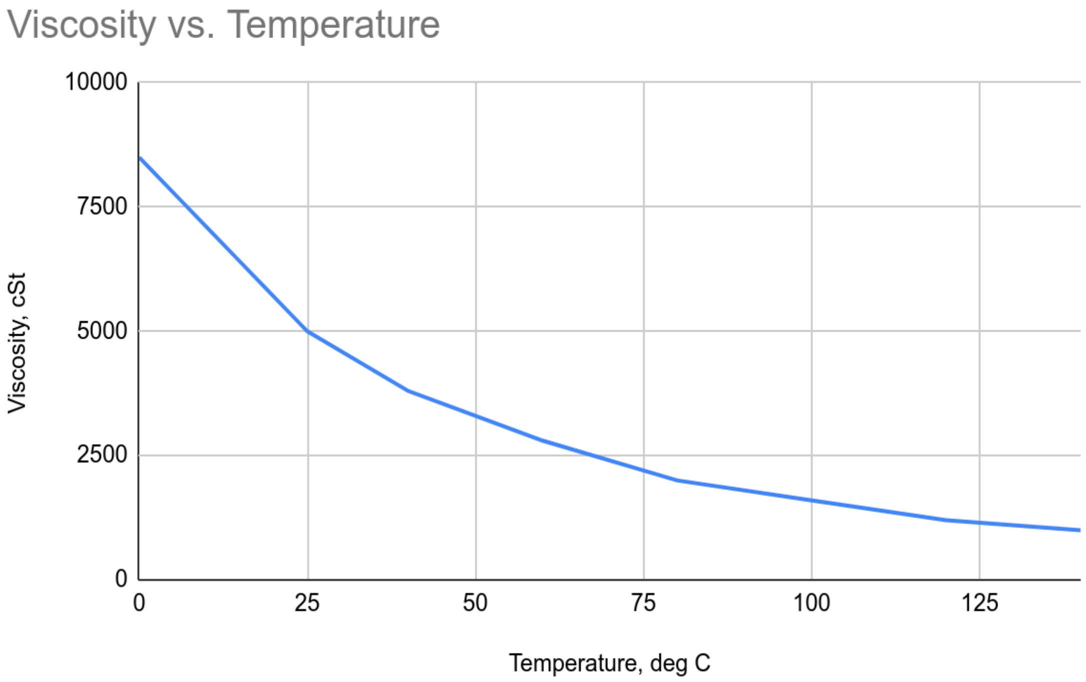


**Fig. 8.** Viscosity vs. Temperature for the silicone oil.

From this diagram, we extrapolated out to the maximum rated temperature of our fluid (200°C). We then took the derivative of the best-fit equation for this data to obtain a working estimate of $\frac{\delta S}{\delta T}$. Using equation 2, we constructed a MatLab model.

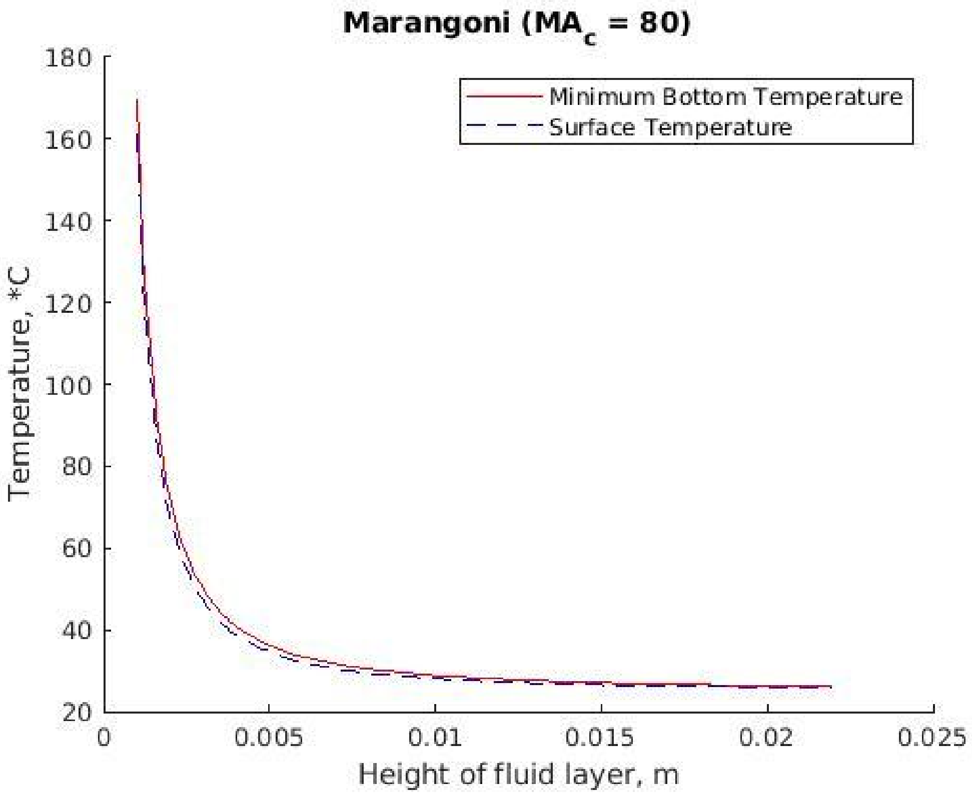


**Fig. 9.** Base and Surface Temperature as a function of fluid layer height for the critical Marangoni Number of 80.

This graph shows the temperature of the base of the fluid (in red) and the temperature at the surface of the fluid (in blue, dashed) that are needed to cause cell formation across a variety of fluid thicknesses. We see that as the fluid thickness is increased, the temperatures required reduce, and the temperature gradient across the fluid becomes smaller. Examining Fig. 9, it seems that the largest temperature gradient occurs at the smallest fluid layer. Fig. 9 seems to imply that the cells will nearly spontaneously form at room temperature if the fluid layer is 2cm thick; this result seems on face value to be incorrect, and does not agree with our intuition or testing. Thus, in order to maximize the possible margin of safety in our design parameters, and to minimize the material waste associated with testing a model that we know is inaccurate, we selected a 1mm thick layer for Marangoni analysis.

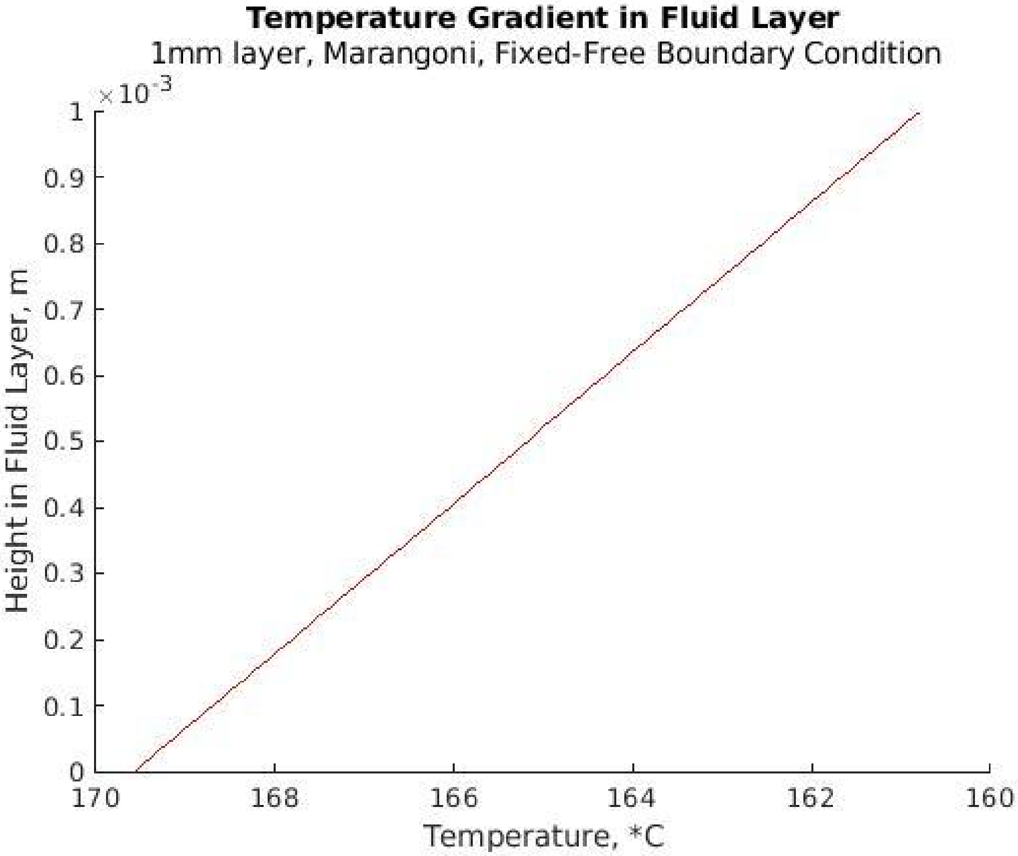


**Fig. 10.** Temperature Gradient through fluid layer, for a 1mm thick fluid layer, with a critical Marangoni number of 80.

With a fluid thickness selected and a temperature gradient modeled, all that remained was to test our assumptions and model.

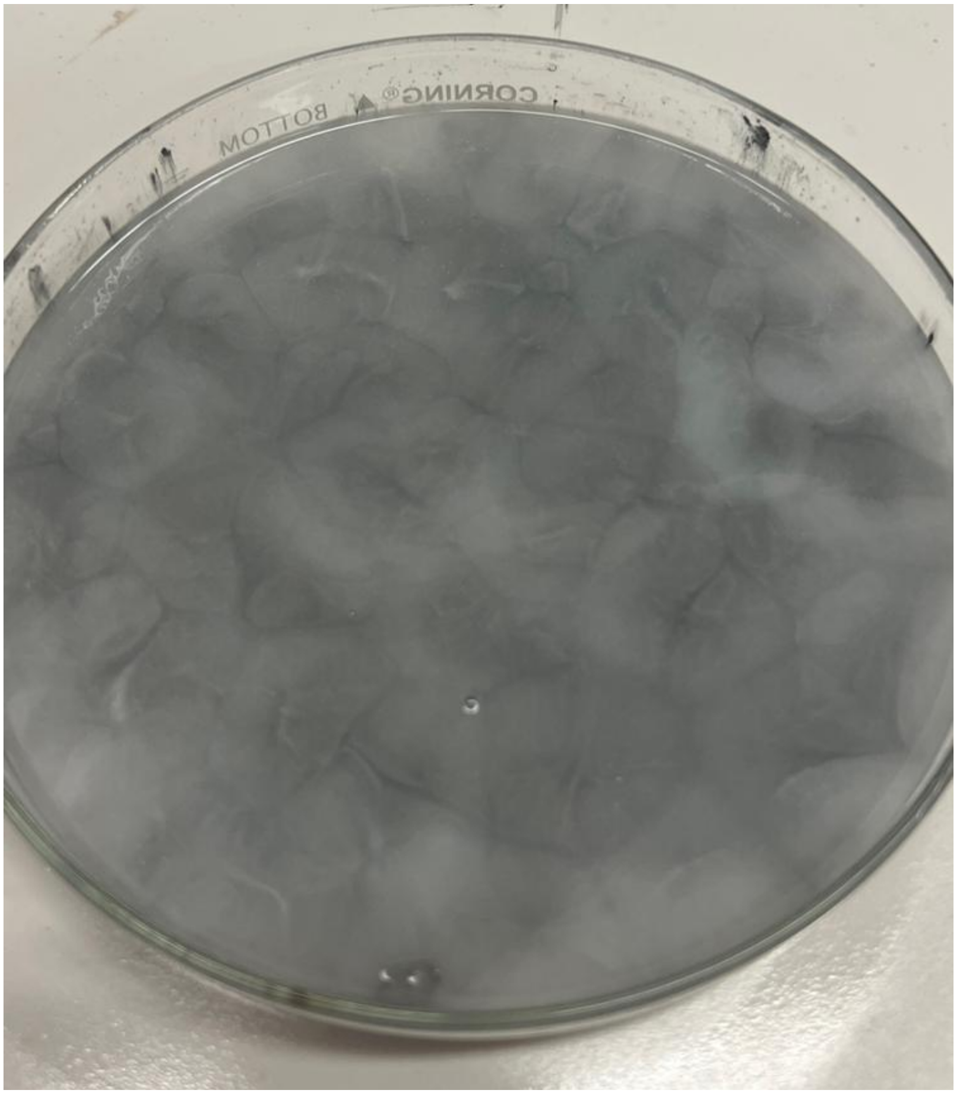


**Fig. 11.** 1mm thick fluid layer test.

Unfortunately, as expected empirical testing did not confirm Marangoni analysis. While we can clearly see convection cells in the fluid in Fig. 11, they are not at all regular. These cells formed significantly above the predicted base temperature, and as a result the small bubbles near the bottom of the image indicate that the maximum rated temperature of the fluid has been exceeded, and some boiling is beginning to occur. This test reached a temperature of 230°C, at which point we deemed it unsafe to continue and ended the experiment.

With our Marangoni model contradicted by our testing, it is clear that our assumptions for this analysis are incorrect. Thus, we turned our focus to Rayleigh analysis. Note that the Rayleigh number is heavily dependent on fluid thickness, as it is proportional to the layer height to the third power. Coupled with the constant-properties assumption, it is noteworthy that the Rayleigh number will not accurately model small fluid thicknesses.

## Rayleigh Analysis

Limiting our model to temperatures under 200°C, the rated maximum temperature of the fluid, will ensure that we meet our non-flammability requirement. Thus, with that constraint, we modeled the heat transfer with a critical Rayleigh number of 1100, which was drawn from the fixed-free boundary conditions assumed present at the bottom and top of the fluid, respectively.

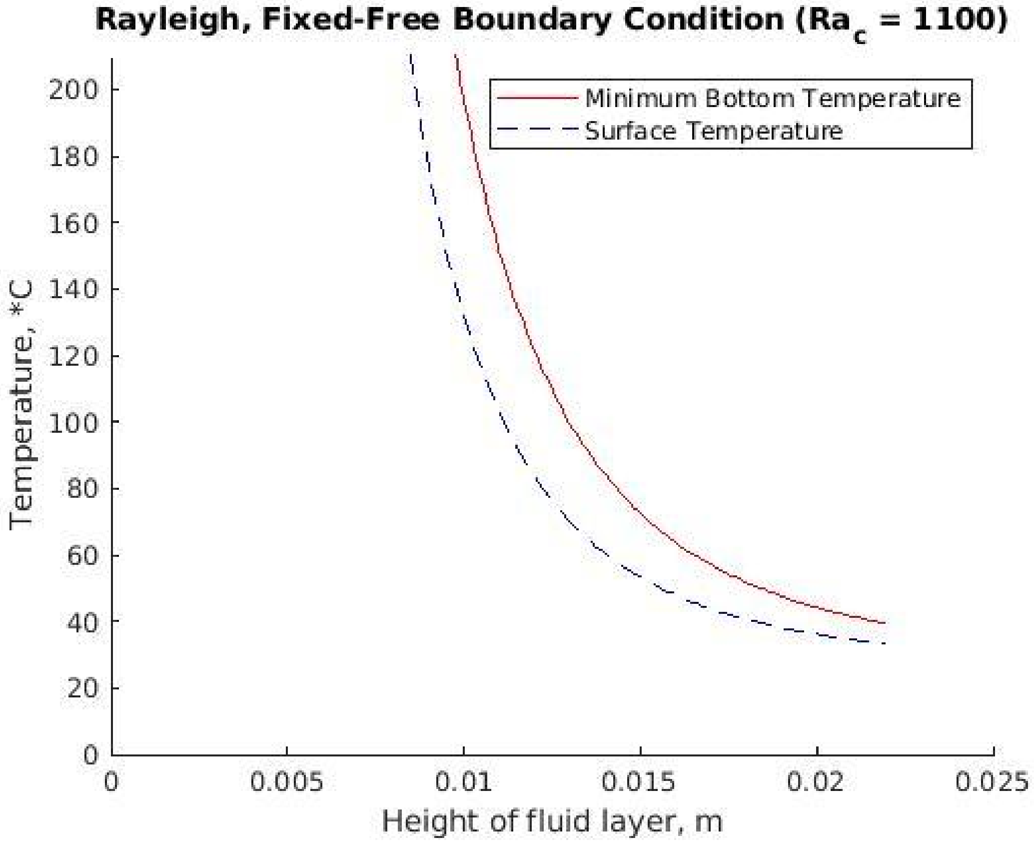


**Fig. 12.** Temperature of the bottom of the fluid (in red) and the surface of the fluid (blue, dashed) as a function of fluid layer height for the critical Rayleigh number of 1100.

Note that due to the heavy dependence of the Rayleigh number on fluid thickness, it predicts that fluid depths less than 1cm will require temperatures that exceed the rated temperature of the fluid. Examining Fig. 12, we find that the temperature gradient across the fluid (the vertical distance between the two lines) increases with decreasing fluid thickness. Thus, we selected a fluid depth of 1cm, which the model indicates is the minimum thickness we can achieve without exceeding the rated fluid temperature. This selection is made to minimize the materials needed (minimizing disposal costs and environmental impact), as well maximizing the temperature gradient to provide as large a factor of safety as possible (since the model does not account for the loss in temperature that will accompany cell propagation). Finally, selecting the smallest possible fluid layer should reduce the time needed to heat the fluid, helping us to meet the 40 minute maximum to cell formation specification.

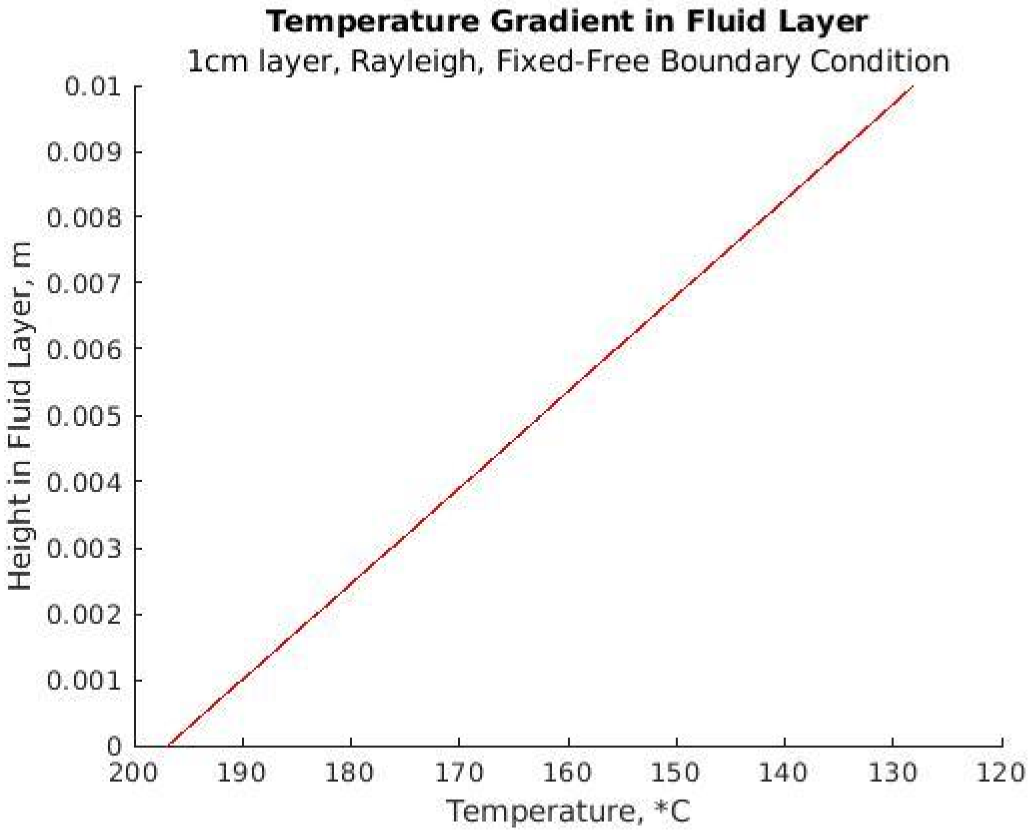


**Fig. 13.** Temperature gradient within the fluid layer for a 1cm layer, with the critical Rayleigh number of 1100.

Note in figure 13 that, due to our assumption of fluid at rest, the temperature gradient is modeled as perfectly linear. In testing, this is impossible to achieve exactly, but slow heating has shown much better results than rapid heating. Thus, since slow heating tends to create a more linear temperature gradient, this model is in agreement with our testing.

Finally, the model was employed to calculate the heat flux through the system:

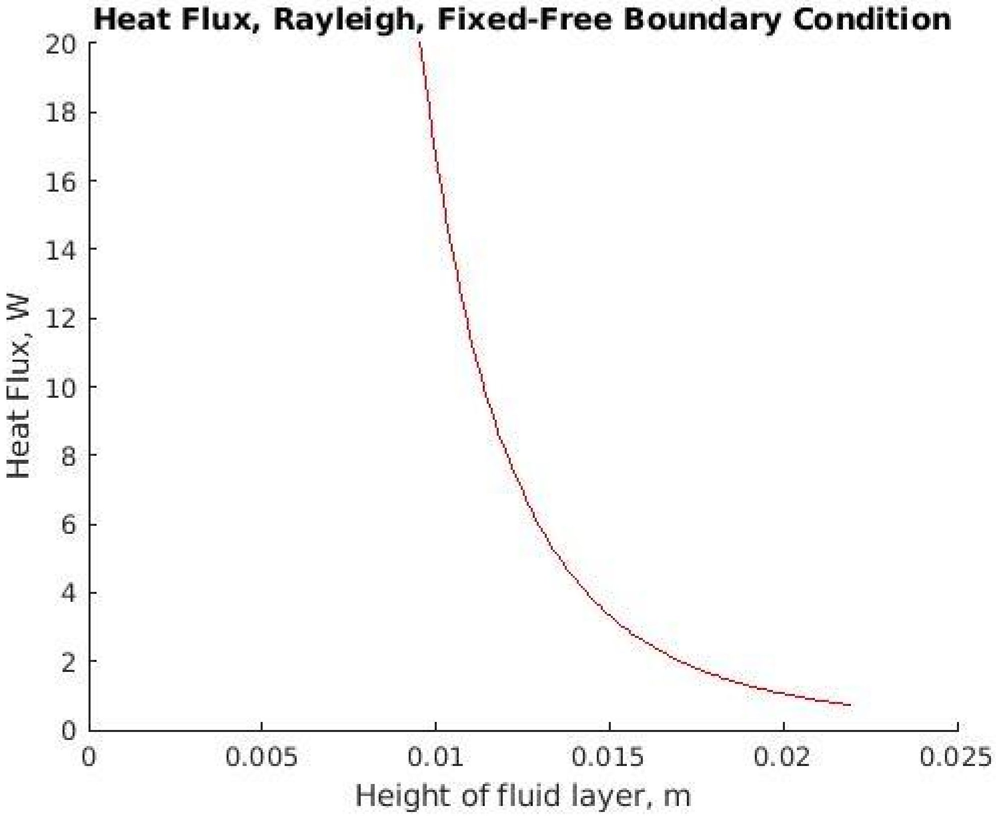


**Fig. 14.** Heat Flux through the system as a function of fluid layer thickness, for the critical Rayleigh number of 1100.

Note in figure 14 that we can once again observe the asymptotic behavior with fluid layers thinner than 1cm. Since the rated total power output of our hot plate is 628W [16], and some of that power will be expended heating the air due to the exposed surface area of the hot plate, our choice of fluid layer height at 1cm is confirmed. Since this keeps the power consumption of the demonstration below 50W, we have an excellent factor of safety built in to ensure our hot plate is powerful enough to achieve the necessary heating within the first half of the class period.

Thus, the model indicates that the temperature of the base of the Petri dish should be 197°C. At this temperature, the modeled heat flux into the 1cm layer of fluid is 16.7 W.

Finally, knowing that the Petri dish has a bottom thickness of 2mm and a thermal conductivity of 1.15 W/mK [2], we can calculate the hot plate temperature using a simple thermal resistance model and assuming zero thermal contact resistance between the hot plate and the Petri dish. Employing equations 9 and 11:

$$R_t = \frac{2mm}{1.15\frac{W}{m^*K}{}^*0.0152m^2} = 0.115\,\frac{K}{W}$$

$$T_{Dish} = (16.7W * 0.115\frac{K}{W}) + 197°C = 199°C$$

Thus, the hot plate should be at about 200°C to achieve Benard convection cells with a 1cm thick layer of fluid.

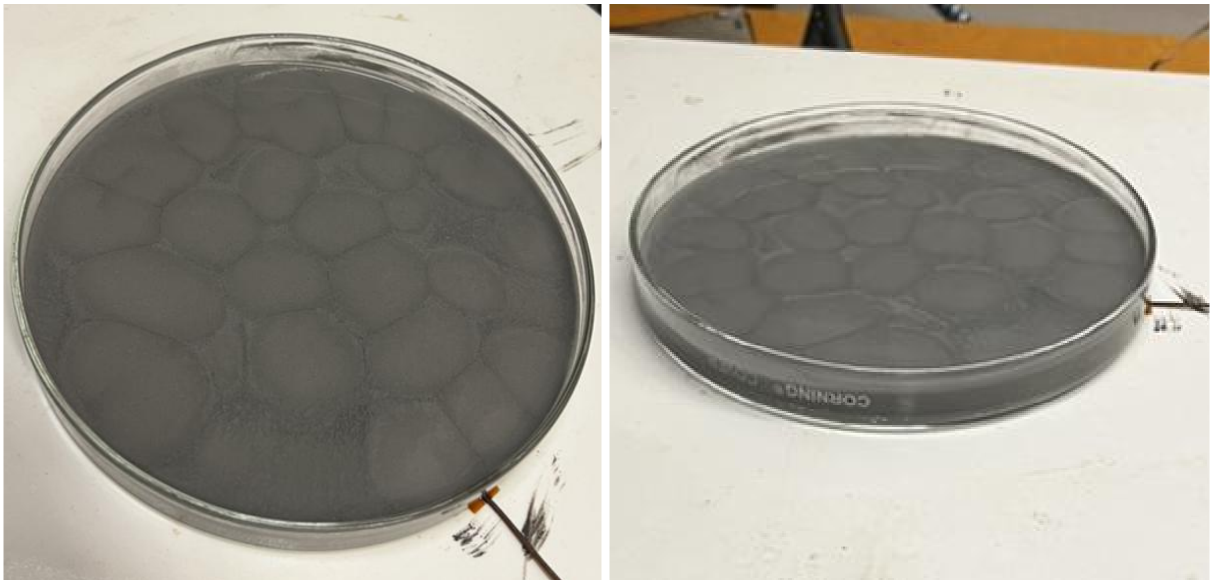


**Fig. 15.** Cell formation in the 1cm layer.

Fig. 15 was taken with the hot plate between 200°C and 210°C, showing excellent agreement with the fluid base temperature predicted by our model. The lack of bubbles in the images above shows that the fluid has not exceeded its maximum rated temperature, so it seems that our model is indeed accurate.

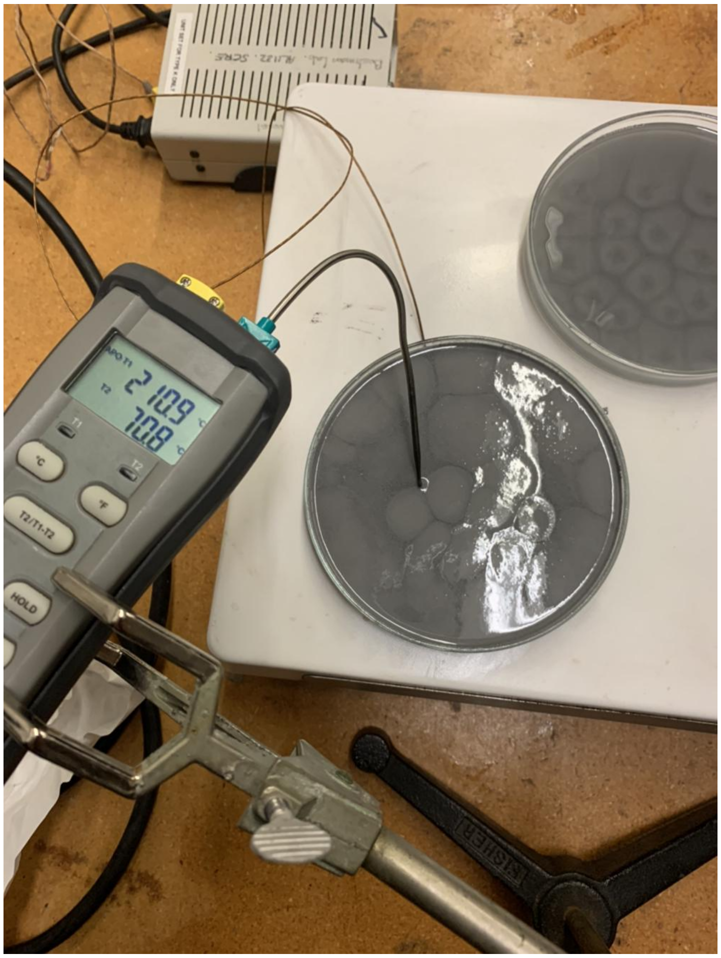


**Fig. 16.** Cell formation in the 1cm layer with Thermocouple data.

Note that in Fig. 16 you can see the low-contrast setup in the upper right-hand corner; this setup uses less of the powders, and is a better visualization to the naked eye, but does not appear clearly on the document camera. The high-contrast setup (with the thermocouples attached) provides a better visualization when viewed through the camera.

We can see here that the hot plate is at 210°C, and the surface of the fluid is at 70°C. This base temperature agrees well with our Rayleigh model. The surface temperature is lower than expected, which makes sense given that the Benard cells will increase the heat transfer rate as compared to the linear, at-rest fluid assumed for the model. Further, note that the surface temperature probe is at the border between two cells; the cells are hottest in the middle, and coldest at the borders.

We then tested two additional fluid thicknesses, 1.5cm and 0.5cm, to further investigate our model's accuracy.

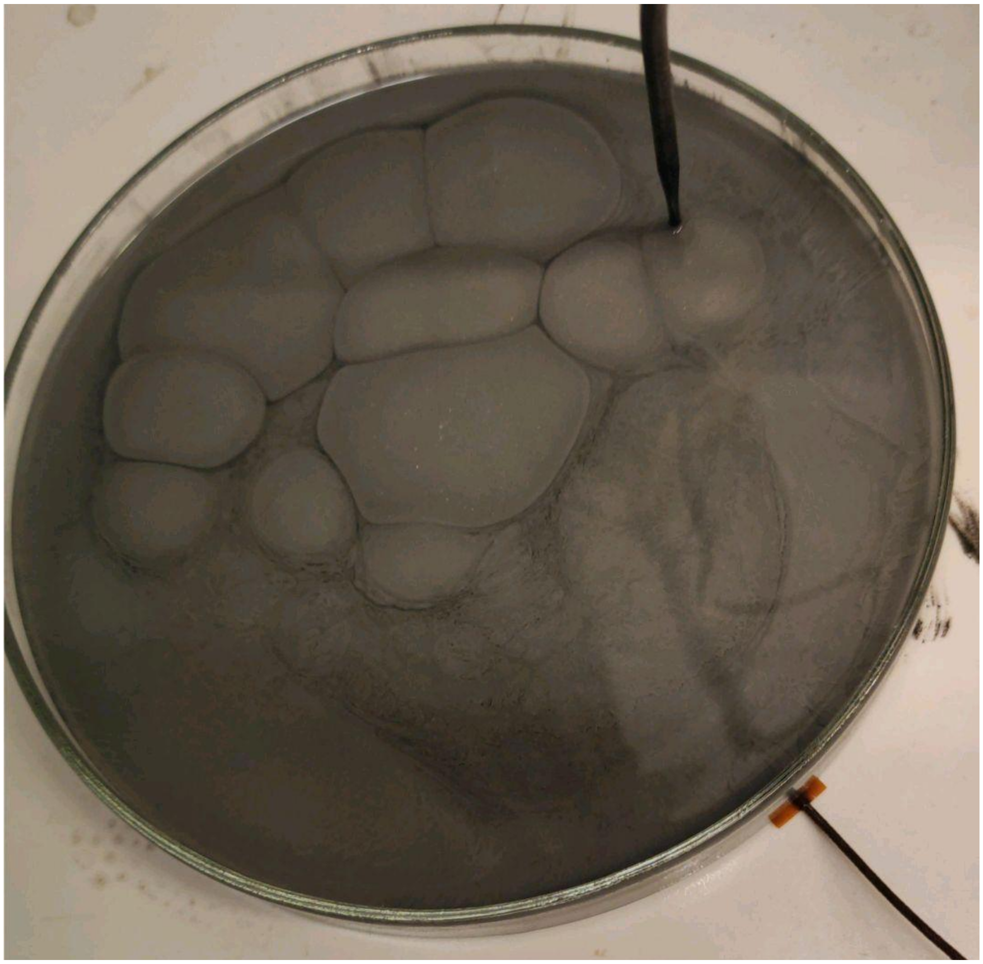


**Fig. 17.** 1.5cm layer cell formation.

Figure 17 shows cells forming in a 1.5cm thick layer. Unfortunately, at this point our base temperature thermocouple failed so we were unable to get an accurate reading of the base temperature. As discussed above, the surface thermocouple is a rough estimate due to the additional heat transfer from the cells. What we can conclude from this particular test is that these cells are too large to be useful; the cells formed into random blob-like shapes with large borders instead of the regular geometric cells we are hoping to generate.

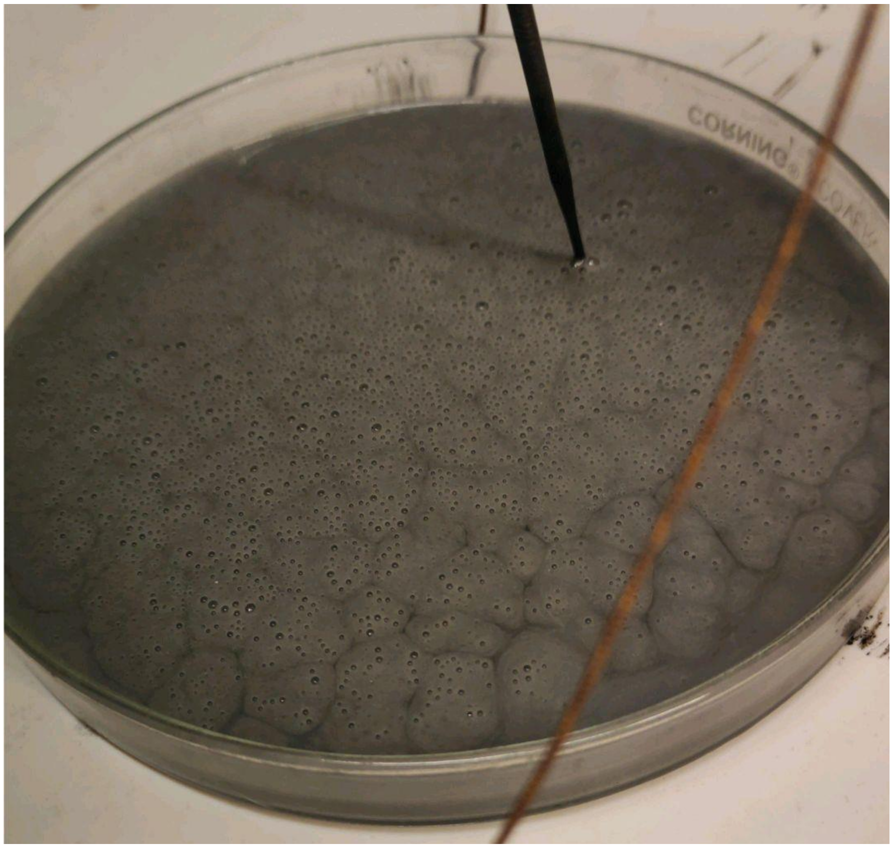


**Fig. 18.** 0.5cm layer cell formation.

In Figure 18, we see the small, regular geometric cells forming, but we also see something else - the many small bubbles indicating that our fluid is beginning to boil. This result nicely verifies our model, which indicates that thicknesses less than 1cm will require temperatures exceeding the rated temperature of the fluid to form Benard cells.

Thus, with a verified analytical model based on Rayleigh analysis, we can optimize the design. Thus, we calculate the operating parameters of our setup (detailed in the table below).

**Table 2.** Calculated Operating Parameters.

| **Critical Rayleigh Number** | **Height of fluid layer, mm** | **Temperature at Base of Fluid, °C** | **Temperature at Surface of fluid, °C** | **Hot Plate temperature, °C** | **Heat Flux, W** |
|---|---|---|---|---|---|
| 1100 | 10 | 197 | 132 | 200 | 16.7 |

It is important to understand the limitations of our analysis. It does not accurately model the surface temperature of the fluid, since the motion of the fluid increases the heat transfer rate. It does not take into account other forms of convection cells, which form at different conditions, as we only modeled the critical Rayleigh number for Benard convection. Testing has shown irregular convection cells forming as low as 130°C, but Benard cells don't form until close to the expected temperature. The model is only as good as the assumptions; careful control of the conditions leads to higher accuracy of the predictive model and more reliable results. Small changes, such as the hot plate not being level, invalidate our assumptions and lead to failure to meet requirements (such as half or more of the dish being filled with cells).

# FINAL DESCRIPTION

### Final Design

Upon completing the engineering analysis, we were able to arrive at a final, optimized design. This design employs a single Petri dish with an open top, filled with a 1cm deep layer of fluid, and the appropriate amounts of aluminum and alumina powder to create sufficient contrast. For viewing with the naked eye, testing indicates this is 0.75g aluminum powder and 0.25g alumina powder. For viewing with the document camera, this quantity should be increased, but no more than 7.5g aluminum and 2.5g alumina should be used. 150ml of the 5000 cst Silicone oil is needed. The details of this design can be found in the CAD drawing.

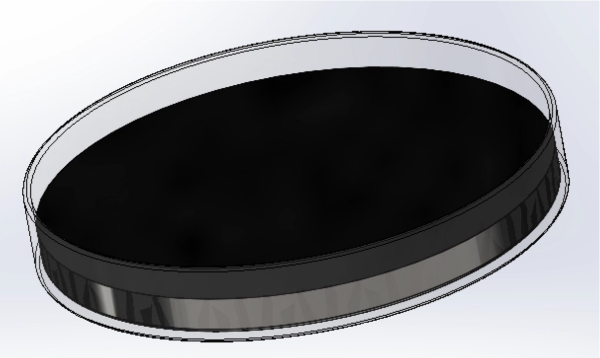


**Fig. 19.** Perspective view of the CAD drawing of the Petri dish filled with 1cm of fluid mixed with aluminum and alumina powder.

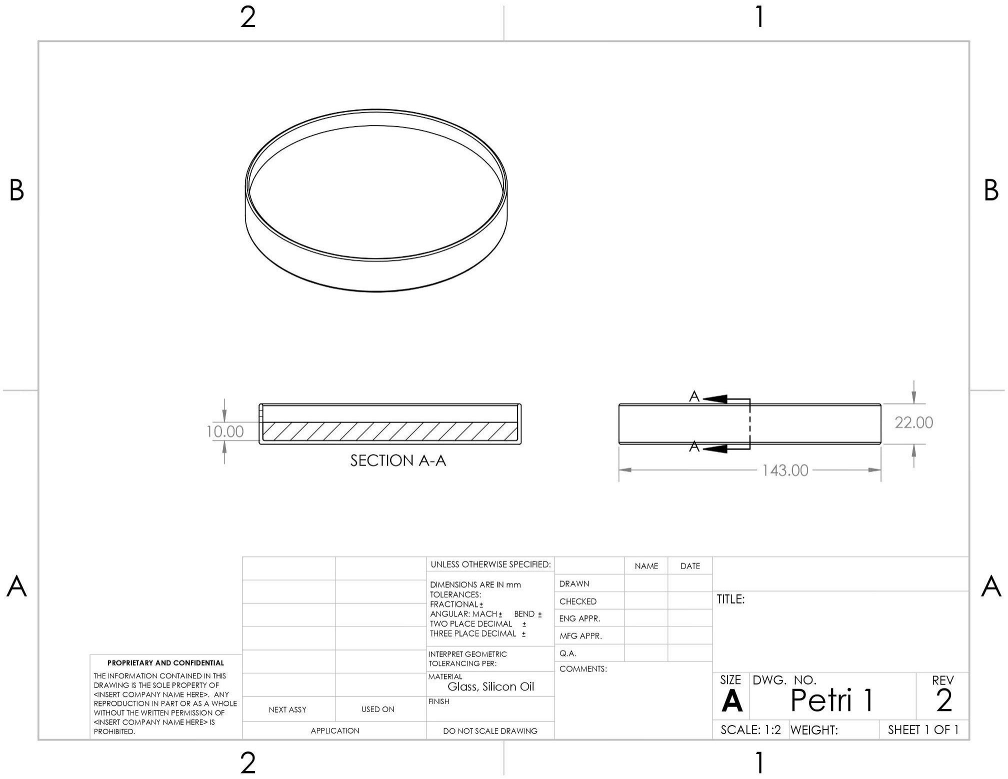


**Fig. 20.** Detail drawing showing the dimensions of the device, in mm.

Our device was fully prototyped for testing.

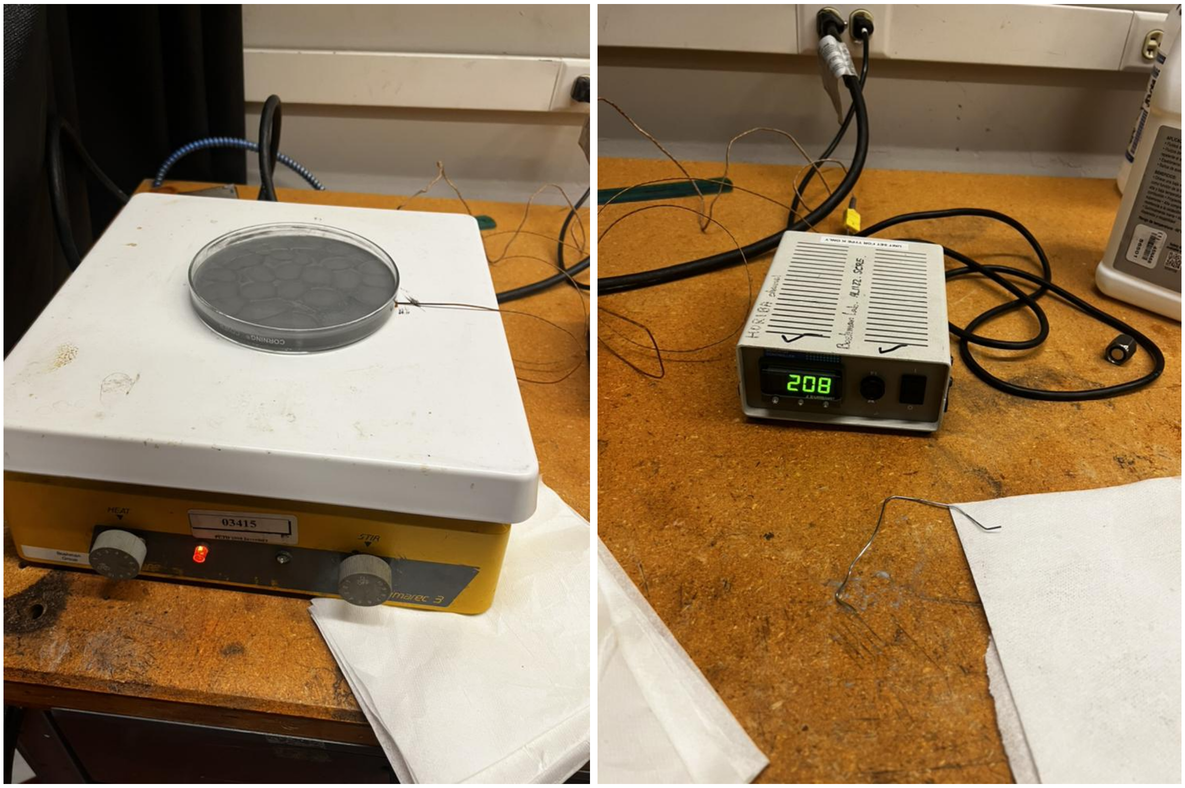


**Fig. 21.** Prototype device. Note the hot plate differs from the spec hot plate, the thermocouple reader only has a single channel, and there is only a single thermocouple in use.

The final design differs from the prototype in that a different hot plate will be used and two thermocouples and a dual-channel thermocouple reader will be used to show both plate and surface temperatures simultaneously.

Our testing verifies that this is the optimal solution in several ways. First, testing the various thicknesses verified our Rayleigh-based model, providing significant evidence that the 1cm thick fluid layer is the optimized design solution. Second, testing in the classroom showed that our dual-powder solution does indeed provide enough contrast for viewing both the cells themselves and the motion of the fluid (when zoomed in) over the in-classroom projector when viewed with the in-classroom document camera. Third, extensive testing across multiple runs with the 1cm fluid layer shows consistent cell formation around the predicted 200°C base temperature within the first 15 minutes of powering on the hot plate. Finally, endurance testing the prototypes at the Design Expo verified the usefulness of our steady-state assumption approach; once the cells had formed, the demonstration was able to run for at least 40 minutes before the fluid temperature began to rise to the point where the critical Rayleigh number was exceeded and the cells were disrupted. Thus, our extensive modeling and testing does indeed verify that the 1cm, open-top, Petri dish on hot plate, combined-powder design is optimal, and will work well to form visible, visually interesting Benard convection cells.

None of this would have been possible without meeting failure. Testing of the 1mm layer showed that there were serious shortcomings in our Marangoni model, but did allow us to determine that we could safely reach a temperature of 230°C and boil the fluid without encountering an ignition hazard. This gives us good confidence in our margins of safety to prevent and fire or burn risk. Additionally, as we had initially been in favor of using Marangoni analysis, this failure showed us the importance of validating our models before drawing conclusions from them. We also tested a closed-top version, in which we placed another, smaller petri dish upside-down on top of the fluid to close the dish. This version proved difficult to control, and resulted in undesirable fluid behaviors when heat was applied (fluid expansion under heating caused the fluid to climb the walls of the smaller petri dish). This test cemented the ease-of-use of the open-top design, which is desirable for making the demo as easy to integrate into lecture as possible in order to best serve the professors of ME 335. A final unexpected, although not necessarily unsuccessful, result is that under certain ambient conditions, different convection cell modes form at significantly lower temperatures. This came as a surprise the first time we encountered this result, and highlighted the importance of understanding the limits of our model - specifically, that our model only describes Benard convection, so other modes may occur that are not modeled. Thus, we recommend that the verified model be used to build a well-understood, visually-interesting demonstration, as described above.

### Build Description

Our build design follows our final design without any modifications. The prototype used a different hot plate than the production model requires, and relied on a single-channel thermocouple reader rather than the dual-channel one specified for the production version. For later tests, a dual-channel thermocouple reader was borrowed, enabling tests more similar to the final production version. We will give the assembly plan assuming the dual-channel thermocouple reader is available. As seen from figures 16 and 17, the Petri dish is filled with a 1cm thick layer of oil and powder mixture. We use a 3:1 ratio of aluminum to alumina powder, consistent with our final design. Small changes in the total powder amount depend on the intended use of the demonstration. Smaller powder amounts (i.e. 1 g total) are good for viewing cells and details in person. Meanwhile, for our intended final usage in the classroom, a higher powder concentration (no more than 10 g per 150 mL oil) helps in creating more contrast for the cell edges, which makes them clearer to see with the document camera. A detailed Bill of Materials, including manufacturer and cost analysis, is available in Appx. C.

An important aspect of our build design is making sure that it is visible in the lecture hall. For this, we need to set up the document camera. Camera setup takes about 5 minutes, at most. It can be set up at any time in heating or cell formation, depending on whether we want to see the formation or the resulting cells. The IT support available from the facilities crews is very helpful if issues are encountered with document camera setup.

Since all of the components required for our demonstration are off-the-shelf, no manufacturing is needed. Thus, our assembly instructions become the following:

1. Measure appropriate quantity of silicone oil in a graduated cylinder, and pour into Petri dish.
2. Measure appropriate quantity of alumina powder on a scale, and pour into Petri dish.
3. Measure appropriate quantity of aluminum powder on a scale, and pour into Petri dish.
4. Mix thoroughly.
5. Attach fluid surface thermocouple to thermocouple reader.
6. Attach thermocouple reader to test stand such that the tip of the hard fluid surface thermocouple is just in contact with the surface of the fluid.
7. Place Petri dish onto hot plate. Turn hot plate to lowest setting. Wait approximately 3 minutes.
8. Turn hot plate up in 1-setting increments approximately 3 minutes apart to keep heating gradual.
9. Continue turning up hot plate until plate temperature reaches 200°C (about 20 minutes).
10. Turn on camera and adjust lighting, focus, magnification, and brightness.

**Fig. 22.** Build design setup running with the document camera (above, not shown) running. Note the broken base thermocouple has been removed.

Because the differences between the build and the final design are trivial (hot plate type and thermocouple reader channels), this build inherently confirms the critical elements of our final design. In particular, because the petri dish, fluid type and height, and powder amounts are identical to our final design, critical temperatures, cell formation speed, and visual interest will be identical as well. This allows us to examine the performance and usability of our final design directly, and allows us to use the build for validation testing directly - which we did both at the design expo and during an in-class demonstration of the device in Prof. Boehman's final class session of ME 335 this semester. Because the critical parameters of the build are determined directly from the engineering performed by our team, and because the build consistently develops Benard cells at temperatures very similar to the modeled base temperatures, the build directly demonstrates the quality and value of the engineering that we performed.

# VERIFICATION AND VALIDATION PLANS

## Verification of Requirement Satisfaction

To verify each of the requirements we need to test each one individually and assess the performance of the model based on the specifications we have set.

To test the model on its ability to produce cells within half of the class period, we will take the time it takes to form cells and compare if it can reliably form within 40 minutes which is half of the class period. In preliminary testing, we have achieved the reliable formation of cells within 40 minutes with the average being about 30 minutes from the start of heating. This fully satisfies the requirements as it is within the desired time period by a sizable margin of time.

To test the requirement of the model being visually interesting we require at least half of the Petri dish to be filled with hexagonal Benard convection cells. From testing, we have consistently run the model to have the Petri dish be completely full of the hexagonal cells as shown in figure 15. With the dish being completely full of cells, we believe the demonstration to be visibly interesting for the students within the heat transfer class.

For our team to test the model on visibility and being visible to all viewers in the lecture hall we tested the picture produced from the document camera that is used in all classrooms from our model. Our results showed a very clear picture of the cells and the movement within them and were consistently produced. We believe that the document camera will produce a clear picture for everyone within the classroom and anyone attending class on zoom to see clearly and provide a useful demonstration of natural convection. As seen in figure 23, the setup successfully shows the formation of the cells. Furthermore, we could zoom in and see some cellular motion and even some individual particle motion at close proximity. We understand that some individuals may miss the demonstration or may have trouble seeing the model which would not be covered via our tests so we will also have a media file containing the model working and several in-depth photos of the movement of the fluid in the cells.

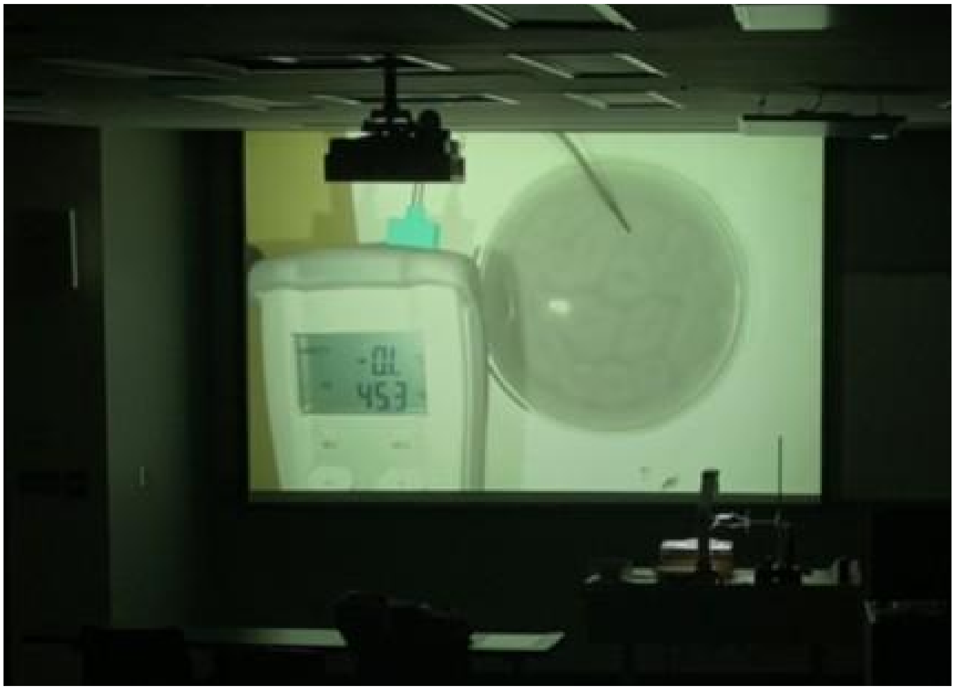


**Fig. 23.** Demonstration implemented in an empty classroom, showing cells and fluid surface temperature measured by thermocouple. Note this was taken with a phone camera and looks better in person.

Our model was built, tested, and experimented with only the materials that were provided to us by Professor Boehman which satisfied a requirement that had been set and helped save money for the Professor as well as expedited the process as we did not have to order materials.
To test the model on its portability we must prove that the model's components can all fit on the Professor's cart which is 24" by 36". The hot plate is within 15 inches by 15 inches and the filled Petri dish is within 5" by 5", so all of the materials that are required will be able to fit within the cart and will be mobile for the Professor to take between his office and the classroom. This satisfies the portability requirement of the model. We have also tested to make sure that it won't take long to transport the materials. It took about 5 minutes to get from our base- the Auto Lab- to the EECS lecture hall without hurrying. The full demonstration setup was placed on the cart for this, shown in figure 24 on the next page.

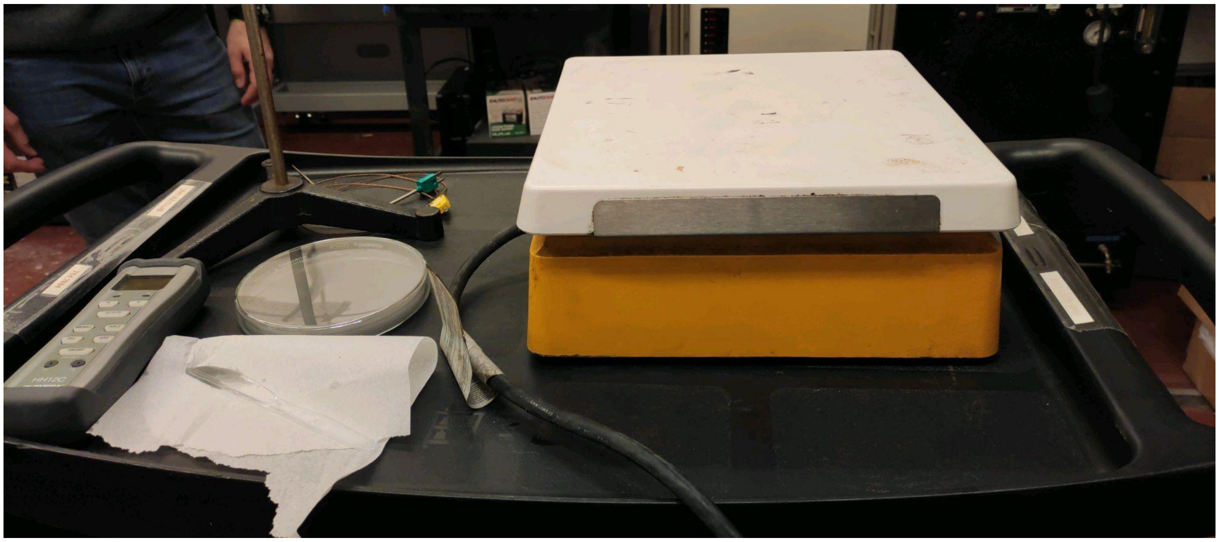


**Fig. 24.** Complete setup of demonstration on cart with the prepared mixture in the Petri dish

The requirement of being completed by the natural convection lesson was not achieved due to the prototype of the model not being completed by March 14th. This was unfortunate as the current students will not be able to see the model in action to complement their learning of the relevant content but they were able to see it at the end of the semester, as per the Professor's plan. This does mean that the requirement was not met; however, this particular requirement was low priority and the completion of the model is much more important for our sponsors and stakeholders.

Finally, the model had to be tested for flammability as a safety protocol. Our group accomplished this by ensuring the operating temperatures were within the acceptable ranges for our working fluid and we also completed tests that heated the temperature above the ranges set by the manufacturer and confirmed that the fluid wouldn't ignite during any phase of operation and was safe for the students and professor to operate without the model igniting.

### Testing Limitations

Due to funding and time constraints, our ability to fully test and refine our model accordingly has been limited. This largely unaffected our final model as the testing that was completed was rigorous enough to produce a consistent working model that the sponsor is pleased with. We had to make assumptions within the heating profiles to ensure our results were accurate but later confirmed them using engineering analysis and verification of heat transfer principles. We also did not have the time or funds to accurately and consistently read all relevant temperatures within the model and test all desirable variables. This leads to uncertainties in the heating rate and the resulting size and color of the cells; however, the consistent processes that were used to form the cells have to lead to reliable results which can be expected while in use in the classroom.

**Validation of Model**
To properly validate the model and its ability to meet the requirements set for us, we will meet with our stakeholders and determine if the model is successful based on their responses. This includes meeting with Professor Boehman, the heat transfer graduate student instructor, and several past students that have previously taken heat transfer. Their interviews will provide us with insight into whether or not we have reached our goals and if this model is beneficial to the students and staff that justifies the time and materials put into this project by our group and sponsor over the course of the semester. From the response of Professor Boehman, we feel confident in our model and believe that the resources used in the model are well worth the outcome.

# PROBLEM ANALYSIS

Our design in its current state requires an operation manual for the user as to get the results consistently, it needs to be used correctly. Our model is not as simple as turn on and wait, but rather careful watching of the temperature and the heating rate so that the cells can have the opportunity to form properly. The misuse of our model will make it fail so following the manual is imperative to its operation. Fortunately, the liquid and other components won't be rendered unusable if used improperly unless the system is taken above 250 degrees celsius which is warned in the manual. We believe this to be our greatest challenge moving forward. We plan to submit a comprehensive report on the manual so that no misuse while using it will occur in future usage.

Another problem we may have going into the final stages of the project is the inconsistency of heating results for our final testing when making our user manual and tutorial media for future users. Our current hot plate is operated based on the power output according to the setting, rather than a set temperature like the newer hotplate that will be used in the future and that belief may be wrong or inconsistent as the current hot plate is over 30 years old and may malfunction due to wear and tear. We plan to avoid this by receiving the newer hot plate as soon as possible on which we can control the plate temperature directly and avoid any unwanted heating.

# LESSONS AND REFLECTIONS FROM PREVIOUS DESIGNS

**Failed Designs and Components**
From early testing of our preliminary designs and early versions of our components we have learned several flaws in designs and what the limitations are with our current materials.

While using a closed top design which would use the critical Rayleigh number of 1700, we have learned that the design greatly decreases the visibility of the Benard convection cells which would make our demonstrator useless. It would also prove to need higher temperatures according to our primary engineering analysis of our model, which would result in higher lifetime power consumption and thus greater environmental impact.

While testing the alumina powder we have gathered that the powder is too light and doesn't produce a high enough contrast to see the motion of the convection. We had much greater success using the aluminum powder or a mix of the two powders as the aluminum powder is darker and tends to mix with the silicone oil much better than the alumina powder producing a much more visible effect in the convection. We also saw the alumina tend to sink and not be reintroduced into the system after sitting for a decent amount of time. This means that the fluid would have to be remade or sufficiently mixed to create good suspension.

When testing the temperature ranges of the silicone oil as the working fluid for the convection cells, we saw the fluid start to smoke at around 200 degrees celsius and began to boil. Our model indicates that the cells should form prior to 200 celsius, so our fluid should work. However, if the temperature of the top surface rises too much the temperature gradient will not be enough and the fluid won't be able to handle the heat required. Thus, further testing will indicate if we may have to select a new working fluid that can meet higher temperature requirements.

To control our top surface temperature we have experimented with several different types of cooling. While using water to try and take away the heat from the top plate, it proved too great of a temperature change for the Petri dish and ended up shattering the top plate. Thus, using water to cool the top plate is not feasible for our design as our testing has indicated that it could present a safety hazard.

When utilizing the new thermocouples in the design, the thermocouples often had trouble adhering properly to the Petri dish and therefore provided inaccurate and inconsistent results in early testing. To fix this problem we had to use the phone stand that our sponsor donated to us for taking videos and repurpose it as a thermocouple reader holder to stabilize the thermocouples and provide much more accurate results.

### Stakeholder Recommendations

When talking with our main stakeholder Professor Boehman, he has provided us with several recommendations that we will test in the future. He first recommended the use of thermocouples to detect the temperature at the top and bottom of the plate to find the temperature gradient between the two and therefore we can alter the heating element accordingly. He also recommended the implementation of a new hot plate for the heating element that can adjust the temperature to known values; we can then bypass the thermocouple on the bottom element and directly adjust the gradient to our desired temperature differences. Lastly, he recommended using the phone stand provided to get a high detail timelapse of the cells forming and moving in the fluid.

# DISCUSSION

**Problem Definition**

With our project we were constrained to only this semester to complete our objectives which made our design choices and testing ability limited. If our project wasn't limited to the semester then our team would've like to explore our problem further, especially in the realm of testing. We wanted to further test the fluid properties of our working fluid and how it performs under different heating conditions such as faster heating or more even fluid heating from the hot plate. We also wanted to test new working fluids with different properties in which we could change the design to get smaller hexagonal cells within the demonstration. Lastly, we would have liked to test different powders to see how we can change the contrast and visibility of the design. Specifically, one of our early concepts was to use a thermochromic powder to show the change in temperature and the transfer of heat within the cells. Each of these different factors could have pushed our design further and enhanced it to be truly unique and useful for the heat transfer class.

**Design Critique**

Our design was successful in addressing the needs of our sponsor. We fulfilled all our requirements and managed to contribute to a heat transfer lecture this semester, though we were not able to finish on time for the originally-intended convection lecture. The design was visible in the lecture hall and online, with the classroom recording also showing the cells and even some of the fluid motion via the specs of white alumina powder in contrast with the darker aluminum powder. The cells formed rapidly with a high stability and without breaching safety. It was easy to transport between the locations. Overall, it was also cheaper than most of the other demonstrations we have found in our research.

There are some elements of the design that could have been improved. While the classroom document camera was sufficient to provide a basic demonstration of the motion, it would have been better if we could increase the size and contrast of the motion. We could not manually focus the camera to focus on fine details, and attempts to work with zooming in automatically have failed due to glare from the fluid surface. For improvements, we could have acquired a small, controllable camera to mount on a stand. This might help not only in zooming in but also in reducing glare by slightly angling the camera to prevent it. In line with visibility improvements, our restrictions led us to form cells that were not perfectly hexagonal. However, pictures of the more uniform cells as seen from studies (such as those from Koschmieder [3]) were derived under a more rigorous setup with the intentions of thoroughly examining Benard cells under a scientific, analytical lense. We are merely trying to create the simplest setup for Professor

Boehman, his students, and others who might be interested in a quick and relatively accessible demonstration. Had our intentions been to create a research-scale demonstration, we would have done more research into the setups used previously by researchers. More complex systems tend to include a mechanism for even temperature distribution across the hot plate and petri dish, a different medium for holding the fluid, a different fluid, a top-cooling mechanism, and more.

# REFLECTION

**Public Health, Safety, and Welfare**

With our project, we are more concerned with public safety. We initially considered making sure nothing is flammable within our possible operation range. As we began looking more into what we had to work with, our temperatures were set to a maximum by the petri dish and silicone oil. At the temperature range of around 200°C, we were no longer concerned as much about igniting the aluminum powder. Our concerns were reaching the upper limits of the continuous Petri dish temperature and boiling of the silicone oil. The hot plate would remain the greatest source of safety concerns for us. After heating the mixture, we also started to consider how long it might take to cool before it could be safely handled without protective hand wear (ie. oven mitts). Silicone oil would spread more easily on surfaces after heating up, but this wasn't a concern in the realm of health. Our silicone oil was food grade, so its ingestion in miniscule amounts is harmless (caution: please do not, under any circumstances, drink large quantities of any kind of oil). Our greatest concern with that was the spreading of this oil outside the Petri dish, on the hot plate, which occasionally made our Petri dish slide on the surface. However, we are not moving the Petri dish extensively so the safety concerns there are minimal.

**Global, Societal, and Economic Context**

Our demonstration has a minimal global impact. Our intention was not to make something novel but to make something simple and accessible while still being useful in the academic setting. Our materials also have minimal impact. Sourcing for the vendors is unknown. Perhaps there is some harm in the sourcing of some materials, such as mining for the aluminum. However, we are not aware of this. Due to designing our demonstration to use minimal quantities of each material, we do not expect it to produce a wide impact, so there will not be any foreseeable societal or economic impact from its production, use, or disposal.. The demonstration individually does not require much itself, so even our requirements- had we needed to purchase additional supplies- would not have an impact on the supply chain of the suppliers. Our stakeholder map showed that our potential for negative societal impacts is very small, but the potential to make engineering education more accessible through our demonstration is high. In that way, our stakeholder map showed that our design has a very positive societal impact.

**Inclusion and Equity**

Our demonstration does not, to the best of our knowledge, explicitly or intentionally exclude anyone. In fact, it is designed to promote the accessibility of complex heat transfer topics, which may increase the inclusivity of engineering education. Our stakeholders included people from the student population. However, we did exclude those who have not taken heat transfer. There may have been some useful information there, but we determined that it would have been less useful to include people who did not know what the course was about and have not experienced the difficulties with learning convection in this specific course at this university.

Given the frequency with which professors have taught this material and the experiences and direct feedback on the course from former students, we were more biased towards the professors' opinions. We did not exclude considerations and suggestions from the student stakeholders but we tended to emphasize the professors' feedback. Professors had more to say in terms of what the demonstration might entail. Our stakeholder meetings with students were more focused on making sure Benard cells were understood and could be explained properly within a short time frame. Professor Boehman, who teaches this course, had a strong influence on which way the demonstration should go. This is expected as he is the sponsor of our project. It is worth noting that Prof. Boehman was very aware of the power dynamic he held as the sponsor of our project, and worked hard to make himself available and approachable to minimize any negative impact that such a power dynamic can have. Indeed, Prof. Boehman, Prof. Kaviany, and Prof. Adera - all professors who potentially could have benefited from the power dynamics of an academic setting - instead took time to give us sound technical advice, resulting in the power dynamics of the class proving to be a strong benefit to our final design.

We did not have any cultural considerations come into play, in either consideration or implementation. Each team member worked to respect the cultures and identities of fellow team members. We worked well together with little to no concerns between team members regarding timely and good-quality work completion. Differences of opinion were dealt fairly, with discussion and input from all team members. We consistently felt as though the stylistic and personal differences between each team member made our design and reports more interesting, and thus, our differences in perspective became a source of strength rather and not a source of conflict.

### Ethics

We did not face any considerable ethical dilemmas in our work on this design. The greatest potential dilemma was perhaps more internal in that we had to remind ourselves that the intention of our project was a simple setup, not a complicated array with near-perfect results.

If our project was to enter the marketplace as a widely used demonstration (which we anticipate is highly unlikely), the greatest ethical dilemma would be in material sourcing. Mining remains an arduous field of labor despite definite improvements in the past century with modernization. Furthermore, the environmental impacts of mining are considerable, requiring stripping of land and disturbing the soil to obtain the desired resources. Thus, we would face the ethical dilemma of a tradeoff between expense and sustainability. A less expensive product would be more accessible to underprivileged and underfunded educational institutions serving the underserved, while a more expensive product would inherently be less accessible. The less expensive product would rely on virgin materials, which would come from mining, and would be thrown away at the end of their useful lifecycle - both of which would have serious environmental consequences and impacts on the health and wellbeing of those affected by the mining and garbage industries. The more expensive product would rely on recycled materials, and would itself be recycled at the end of its useful life. The dilemma of accessibility vs. sustainability is a great dilemma indeed.

We found overall that our personal ethics with regard to engineering design were very similar to those we were expected to uphold by the University. Indeed, the engineering honor code captures well our feelings in terms of engineering ethics, and we do believe that potential future employers will expect similar professional ethics.

# RECOMMENDATIONS

When using this demonstration in class, we highly recommend following the user manual for best results. This user manual is given in Appendix D and has also been given to our sponsor. The user manual outlines all materials needed and the steps that should be taken to maximize the utility of the demonstration in class. It outlines all steps from initial setup to storage for the next time the demonstration is used. In the user manual, we recommend using 7.5 grams of aluminum powder. This quantity gives the best contrast when the setup is displayed using the document camera. However, it should be noted that a lesser amount of aluminum powder can be used. When using roughly 1 gram of aluminum powder, dark spots can be seen in the center of the Benard cells. This creates a different effect and may be more visually interesting to the naked eye. However, this amount of powder should only be used if the setup is being utilized in a small group setting and not displayed on a projector or document camera. In the future, we recommend potentially investigating the effects of using different working fluids. We used 5000 cSt Silicone oil for the duration of our project, but did not investigate other working fluids. A different fluid might be able to demonstrate the Benard cells at a lower temperature or faster rate than the silicon oil. This would be an interesting investigation for future students. When utilizing the demonstration, it is important to keep the fluid level in the range of 190 °C - 220 °C. If the fluid is at too high of a temperature the fluid may boil off and become unstable. If it is at too low of a temperature the cells may not form. Keeping the working fluid within this ideal range will create best results when utilizing this demonstration in a classroom setting. Always keep safety practices in mind when using this demonstration. The silicon oil, Petri dish, and hot plate can remain at an extremely high temperature even after turning off the hot plate. Always be careful when handling these components and use heat resistant gloves whenever it is necessary to touch high temperature components of the demonstration. Portions of the demonstration can be easily replaced with similar counterparts when necessary. A replacement Petri dish or hot plate might be necessary for many reasons. When replacing any part of this setup, make sure to test the component with the rest of the demonstration before using it in an instructional setting. This will allow for ease of use and best safety practices. If there are any issues or questions about the demonstration or design in the future, please do not hesitate to contact a member of our team for insight and suggestions. We have really enjoyed working on this project throughout the semester and conducted many tests on the design. We have faced many challenges throughout our time working on the design and would be happy to help with any portion of the demonstration if possible.

# CONCLUSION

We have designed, reiterated, redesigned and created our demonstration to present Benard cells to ME 335 students. Our team generated concepts and potential subcomponents in order to ensure that our design best meets the requirements and specifications set out by our group in design review 1. Throughout the testing process, we have reiterated the design in order to troubleshoot and guarantee that it best meets the needs of our sponsors. The final design is an open-top setup with a 1cm fluid layer which relies upon reaching a critical Rayleigh number of 1100 in order to generate cellular motion. This design and our engineering analysis has been verified and validated in order to assure that our simulation model is correct and that we meet the requirements and specifications created for this project. The successful design has been presented to our sponsor for his use in ME 335 lectures. On Monday, April 17th, the design was used in Professor Boehman's class. Our team and Professor Boehman were very happy with the results of the demonstration; the successful formation of Benard cells helped his class to better visualize natural convection.

# ACKNOWLEDGEMENTS

Special thanks to our sponsor, Prof. Andre Boehman. His support and technical expertise made our project possible. We would like to thank our instructor, Prof. Massoud Kaviany, who provided consistent guidance throughout our work and kept our project on track. We are especially appreciative of our stakeholders, Prof. Solomon Adera, Courtney Videchak, Ekim Koca, and Kayra Ilkbahar, whose perspectives and input enabled us to design this system to better serve the students and instructors of ME 335. And finally, many thanks to Anna DeMarco and Kevin Wall from SuperLube, who provided us with data and technical support that proved invaluable in characterizing the silicone oil to enable accurate engineering analysis of our system. Our success on this project throughout this semester would not have been possible without the help of our sponsors, instructor, stakeholder, and product suppliers.

# REFERENCES

1. "ME Courses- Mechanical Engineering," ME Courses, University of Michigan, 2021, https://me.engin.umich.edu/academics/courses/.
2. Kaviany, M., 2011, Essentials of Heat Transfer. Cambridge University Press, New York.
3. Koschmieder, E.L., 1993, Bénard Cells and Taylor Vortices. Cambridge University Press, New York. https://books.google.com/books?id=arpUkoBD8uYC&pg=PA54&source=gbs_toc_r&cad=4#v=onepage&q&f=false
4. Koschmieder, E. L., 1974, "Bénard Convection," Advance in Chemical Physics, 26, pp. 177-212. https://onlinelibrary-wiley-com.proxy.lib.umich.edu/doi/pdfdirect/10.1002/9780470143780.ch4
5. Van Dyke, M., 1982, An album of fluid motion. Parabolic Press. Stanford, CA.
6. Bodenschatz, E., Pesch, W., and Ahlers, G., 2000, "Recent Developments in Rayleigh-Benard Convection," Annual Review of Fluid Mechanics, 32, pp. 709–778, https://courses.physics.ucsd.edu/2018/Winter/physics116_216/Convection-Recent%20Results-Bodenschatz_W17.pdf
7. Mattson, J., 2008. "A student project on Rayleigh-Benard convection," ASEE Annual Conference and Exposition, Pittsburgh, PA, June 22-25, 2008, pp. 1 - 8, DOI 10.18260/1-2--3591, https://peer.asee.org/3591.
8. Kalelkar, C., 2018, "The Inveterate Tinkerer 11. Marangoni Effect," Resonance, 23, pp. 225–233. https://doi.org/10.1007/s12045-018-0607-1
9. Bragard, J. and Velarde, M. G., 1998, "Benard–Marangoni convection: planforms and related theoretical predictions," J. Fluid Mech., 368, pp. 165-194, https://jeanbragard.github.io/publi_ref_files/jfm368.pdf.
10. Koschmieder, E. L. and Biggerstaff, M. I., 1986, "Onset of surface-tension-driven Benard convection," J. Fluid Mech., 167, pp. 49-64, doi:10.1017/S0022112086002720.
11. "SUPER LUBE® SILICONE OIL - Technical Data Sheet," super-lube.com/silicone-oil, Mar-2021. [Online]. Available: https://www.super-lube.com/silicone-oil. [Accessed: Feb-2022].
12. "Pyrex® 150x20 mm petri dish with cover," Corning. [Online]. Available: https://ecatalog.corning.com/life-sciences/b2c/US/en/General-Labware/Dishes/Dishes%2C-Glass/PYREX%C2%AE-Petri-Dishes-with-Cover/p/3160-152.
13. "Fisher Scientific," Aluminium oxide, 99.99%, (trace metal basis), extra pure, Thermo Scientific | Fisher Scientific. [Online]. Available: https://www.fishersci.com/shop/products/aluminium-oxide-99-99-trace-metal-basis-extra-pure-thermo-scientific/AC212250250. [Accessed: Feb-2022].
14. "Fisher Scientific," Fisher Science Education Aluminum, Powder | Fisher Scientific. [Online]. Available:

https://www.fishersci.com/shop/products/aluminum-powder-31/S25144. [Accessed: Feb-2022].

15. Doering, Charles R., 5 May 2020 “Turning up the Heat in Turbulent Thermal Convection.” *PNAS*, National Academy of Sciences, https://www.pnas.org/content/117/18/9671.
16. “Corning® 5 x 7 Inch Top PC-400D Hot Plate with Digital Display, 120V/60Hz”, Corning. [Online]. Available: https://ecatalog.corning.com/life-sciences/b2b/us/en/Equipment/Constant-Temperature-Equipment/Digital-Hot-Plates/Corning%C2%AE-Digital-Hot-Plates/p/6795-400D. [Accessed: Feb-2022].
17. Wynn, D., and Clarkson, J., 2005, “Models of Designing,” Design Process Improvement, pp. 34-59. https://link.springer.com/book/10.1007/978-1-84628-061-0.
18. French, M. J., 1999, Conceptual design for engineers. Springer.
19. Ehrlenspiel, K., 1995, Integrierte Produktentwicklung: Methoden für die Prozessorganisation, Produkterstellung und Konstruktion. Hanser.
20. “Labor Force Statistics from the Current Population Survey”, Bureau of Labor Statistics. [Online]. Available: https://www.bls.gov/cps/cpsaat11.htm. [Accessed: Feb-2022].
21. “450 course intro WN22”, University of Michigan, https://umich.instructure.com/courses/508936/files/23785236/download, [Accessed: Feb-2022]
22. “Size of Letters Required for Visibility as a Function of Viewing Distance and Observer Visual Acuity”, National Institute of Standards and Technology. [Online]. Available: https://www.govinfo.gov/content/pkg/GOVPUB-C13-ff8dc22d75e66f29ebdb2bb2085ee683/pdf/GOVPUB-C13-ff8dc22d75e66f29ebdb2bb2085ee683.pdf.
23. “Binocular Visual Acuity of Adults”, Center for Disease Control. [Online]. Available: https://www.cdc.gov/nchs/data/series/sr_11/sr11_003.pdf.

# APPENDICES

## Appendix A: Matlab Code

```
global d_pl h_pl h_fl T_air debug;
d_pl = 0.139; % m
h_pl = 0.022; % m
h_fl = 0.001; % m
T_air = 298.15; % K

TMa_80_const = FindTbaseMa(80)-273.15;
Tsurf_Ma80_const = FindTsurf(TMa_80_const)-273.15;
q_const_Ma = Findq(TMa_80_const, Tsurf_Ma80_const);

% TRa_1100_const = FindTbaseRa(1100)-273.15;
% Tsurf_Ra1100_const = FindTsurf(TRa_1100_const)-273.15;
% q_const_Ra1100 = Findq(TRa_1100_const, Tsurf_Ra1100_const);


for i = 0:0.00001:0.001
    index = round(i*100000+1);
    TgradMa(1,index) = i;
    TgradMa(2,index) = FindTatLevel(i, TMa_80_const, Tsurf_Ma80_const)-273.15;
end

for i = 0:0.00001:0.01
    index = round(i*100000+1);
    TgradRa1100(1,index) = i;
    TgradRa1100(2,index) = FindTatLevel(i, TRa_1100_const,
Tsurf_Ra1100_const)-273.15;
end

% for i = 0.001:0.0001:0.022
%     h_fl = i;
%     index = round(i*10000-9);
%     TMa_80(1,index) = h_fl;
%     TMa_80(2,index) = FindTbaseMa(80)-273.15;
%     TMa_80(3,index) = FindTsurf(TMa_80(2,index))-273.15;
%     q_Ma(1, index) = h_fl;
%     q_Ma(2, index) = Findq(TMa_80(2,index), TMa_80(3,index));
% end

for i = 0.004:0.0001:0.022
    h_fl = i;
```

```
    index = round(i*10000-39);
    TRa_1100(1,index) = h_fl;
    TRa_1100(2,index) = FindTbaseRa(1100)-273.15;
    TRa_1100(3,index) = FindTsurf(TRa_1100(2,index))-273.15;
    q_Ra1100(1, index) = h_fl;
    q_Ra1100(2, index) = Findq(TRa_1100(2,index), TRa_1100(3,index));
end

% for i = 0.005:0.0001:0.022
%     h_fl = i;
%     index = round(i*10000-49)
%     TRa_1700(1,index) = h_fl;
%     TRa_1700(2,index) = FindTbaseRa(1700) - 273.15;
%     TRa_1700(3,index) = FindTsurf(TRa_1700(2,index))-273.15;
% end


% figure(1)
% hold on
% plot(TMa_80(1,:),TMa_80(2,:), '-r')
% plot(TMa_80(1,:),TMa_80(3,:), '--b')
% legend({'Minimum Bottom Temperature','Surface
Temperature'},'Location','northeast')
% title('Marangoni (MA_c = 80)')
% xlabel('Height of fluid layer, m')
% ylabel('Temperature, *C')
% %axis([0 0.025 0 100])
% hold off

figure(2)
hold on
plot(TRa_1100(1,:),TRa_1100(2,:), '-r')
plot(TRa_1100(1,:),TRa_1100(3,:), '--b')
legend({'Minimum Bottom Temperature','Surface Temperature'},'Location','northeast')
title('Rayleigh, Fixed-Free Boundary Condition (Ra_c = 1100)')
xlabel('Height of fluid layer, m')
ylabel('Temperature, *C')
axis([0 0.025 0 210])
hold off

% figure(3)
% hold on
% plot(TRa_1700(1,:),TRa_1700(2,:), '-r')
% plot(TRa_1700(1,:),TRa_1700(3,:), '--b')
% legend({'Minimum Bottom Temperature','Surface
Temperature'},'Location','northeast')
% title('Rayleigh, Fixed-Fixed Boundary Condition (Ra_c = 1700)')
```

```
% xlabel('Height of fluid layer, m')
% ylabel('Temperature, *C')
% axis([0 0.025 0 2000])
% hold off


% figure(4)
% hold on
% plot(q_Ma(1,:),q_Ma(2,:), '-r')
% title('Heat Flux, Marangoni, Fixed-Free Boundary Condition')
% xlabel('Height of fluid layer, m')
% ylabel('Heat Flux, W')
% %axis([0 0.025 0 2000])
% hold off

figure(5)
hold on
plot(TgradMa(2,:),TgradMa(1,:), '-r')
title('Temperature Gradient in Fluid Layer')
subtitle('1mm layer, Marangoni, Fixed-Free Boundary Condition')
xlabel('Temperature, *C')
ylabel('Height in Fluid Layer, m')
set(gca, 'XDir', 'reverse')
axis([160 170 0 0.001])
hold off

figure(6)
hold on
plot(q_Ra1100(1,:),q_Ra1100(2,:), '-r')
title('Heat Flux, Rayleigh, Fixed-Free Boundary Condition')
xlabel('Height of fluid layer, m')
ylabel('Heat Flux, W')
%axis([0 0.025 0 2000])
hold off

figure(7)
hold on
plot(TgradRa1100(2,:),TgradRa1100(1,:), '-r')
title('Temperature Gradient in Fluid Layer')
subtitle('1cm layer, Rayleigh, Fixed-Free Boundary Condition')
xlabel('Temperature, *C')
ylabel('Height in Fluid Layer, m')
%axis([0 0.025 0 2000])
set(gca, 'XDir', 'reverse')
hold off

% figure(8)
```

```
% hold on
% plot(TgradRa1100(2,:),TgradRa1100(1,:), '-r')
% %title('Temperature Gradient above Fluid Layer')
% %subtitle('1cm layer, Rayleigh, Fixed-Free Boundary Condition')
% xlabel('Temperature, *C')
% ylabel('Height Above Hot Plate, m')
% axis([0 120 0.01 0.02])
% xticks(0:10:120)
% set(gca, 'XDir', 'reverse')
% hold off

function Tsurf = FindTsurf(Tbase)
    global d_pl h_pl h_fl T_air debug;
    g = 9.8; % gravity, m/s^2

    Tbase = Tbase + 273.15;

    Tsguess = 0;
    Tsactual = 273.15;
    Afl = (d_pl/2)^2*(pi);
    Pfl = (pi)*(d_pl);
    Lair = Afl/Pfl;

    while ( Tsguess < Tbase && abs(Tsactual-Tsguess)>0.1)
        Tsguess = Tsactual;
        Tavg = (Tsguess+T_air)/2;

        vair = (0.000063245*(Tavg)^2+0.06194*(Tavg)+-8.486)*10^(-6); % m^2/s
        alphair = (0.0001343*(Tavg)^2+0.0561*(Tavg)+-6.1952)*10^(-6); % m^2/s
        b_air = (987.239*(Tavg^(-0.9965)))*10^(-3); % 1/K
        k_air = (-0.00001631*(Tavg^2)+0.081885*(Tavg)+2.95)*10^(-3); % W/m*K

        k_fl = 0.16; % W/m*C

        Ra_L = (g*b_air*(abs(Tsguess-T_air))*Lair^3)/(vair*alphair);
        hbarair = (k_air/Lair)*0.54*Ra_L^(1/4);

        Rt_cnv  = 1/(hbarair*Afl);
        Rt_cnd  = h_fl/(k_fl*Afl);

        Rtot = Rt_cnv + Rt_cnd;

        q = (Tbase-T_air)/Rtot;

        Tsactual = -(q*Rt_cnd)+Tbase;
    end
```

```
    Tsurf = Tsactual;

end

function Tb = FindTbaseRa(Ra_c)
    global d_pl h_pl h_fl T_air debug;
    g = 9.8; % gravity, m/s^2

    Tbguess = 250;

    Tbactual = 273.15;

    debug = 0;

    while (abs(Tbactual-Tbguess)>0.1)
        Tbguess = Tbactual;
        Tbc = Tbguess-273.15;
        Tu = FindTsurf(Tbc);
        Tavgfl = (Tbguess+Tu)/2;
        k_fl = 0.16; % W/m*K
        v_fl = 0.005; % m^2/s
        b_fl = 0.000945; % 1/K
        rho_fl = 973; % kg/m^3
        Cp_fl = 1500; % J/kg*K
        alphfl = 0.00000010963; % m^2/s
        dsdt = -0.00005; % N/m*K

        Tbactual = Ra_c * ((alphfl*v_fl)/(g*b_fl)) * (1/(h_fl^3))+Tu;
        debug = debug +1;
    end

    Tb = Tbactual;

end

function Tb = FindTbaseMa(Ma_c)
    global d_pl h_pl h_fl T_air;
    g = 9.8; % gravity, m/s^2

    Tbguess = 250;

    Tbactual = 273.15;

    while (abs(Tbactual-Tbguess)>0.1)
        Tbguess = Tbactual;
        Tbc = Tbguess-273.15;
        Tu = FindTsurf(Tbc);
```

```
        Tavgfl = (Tbguess+Tu)/2;
        k_fl = 0.16; % W/m*K
        v_fl = 0.005; % m^2/s
        b_fl = 0.000945; % 1/K
        rho_fl = 973; % kg/m^3
        Cp_fl = 1500; % J/kg*K
        alphfl = 0.00000010963; % m^2/s
        dsdt = -0.000005; % N/m*K
        %dsdt =
(1.409904*10^(-10))*(Tavgfl^3)+(-1.582703136*10^(-7))*(Tavgfl^2)+(5.9182582*10^(-5)
)*(Tavgfl)+(-0.0074311525);
        %dsdt =(-1.60429885*10^11)*Tavgfl^(-6.16769997247372);


        Tbactual = Ma_c*(1/(-dsdt))*(alphfl*v_fl)/(h_fl)+Tu;

    end

    Tb = Tbactual;

end



function Tl = FindTatLevel(height, Tbasein, Tsurfin)
    global d_pl h_pl h_fl T_air debug;
    g = 9.8; % gravity, m/s^2

    Tbase = Tbasein+273.15;
    Tsurf = Tsurfin+273.15;

    Tavgfl = (Tbase+Tsurf)/2;
    Tavg = (Tbase + T_air)/2;
    k_fl = 0.16; % W/m*K
    v_fl = 0.005; % m^2/s
    b_fl = 0.000945; % 1/K
    rho_fl = 973; % kg/m^3
    Cp_fl = 1500; % J/kg*K
    alphfl = 0.00000010963; % m^2/s
    dsdt = -0.00005; % N/m*K

    vair = (0.000063245*(Tavg)^2+0.06194*(Tavg)+-8.486)*10^(-6); % m^2/s
    alphair = (0.0001343*(Tavg)^2+0.0561*(Tavg)+-6.1952)*10^(-6); % m^2/s
    b_air = (987.239*(Tavg^(-0.9965)))*10^(-3); % 1/K
    k_air = (-0.00001631*(Tavg^2)+0.081885*(Tavg)+2.95)*10^(-3); % W/m*K
```

```
    Afl = (d_pl/2)^2*(pi);
    Pfl = (pi)*(d_pl);
    Lair = Afl/Pfl;

    Ra_L = (g*b_air*(abs(Tbase-T_air))*Lair^3)/(vair*alphair);
    hbarair = (k_air/Lair)*0.54*Ra_L^(1/4);

    Rt_cnv  = 1/(hbarair*Afl);
    Rt_cnd  = h_fl/(k_fl*Afl);
    Rt_lvl = height/(k_fl*Afl);

    Rtot = Rt_cnv + Rt_cnd;

    q = (Tbase-T_air)/Rtot;

    Tl = Tbase-q*Rt_lvl;
end

function qout = Findq(Tbasein, Tsurfin)
    global d_pl h_pl h_fl T_air debug;
    g = 9.8; % gravity, m/s^2

    Tbase = Tbasein+273.15;
    Tsurf = Tsurfin+273.15;

    Tavgfl = (Tbase+Tsurf)/2;
    Tavg = (Tbase + T_air)/2;
    k_fl = 0.16; % W/m*K
    v_fl = 0.005; % m^2/s
    b_fl = 0.000945; % 1/K
    rho_fl = 973; % kg/m^3
    Cp_fl = 1500; % J/kg*K
    alphfl = 0.00000010963; % m^2/s
    dsdt = -0.00005; % N/m*K

    vair = (48.00006*(Tavg)^2+0.06194*(Tavg)+-8.486)*10^(-6); % m^2/s
    alphair = (0.0001343*(Tavg)^2+0.0561*(Tavg)+-6.1952)*10^(-6); % m^2/s
    b_air = (987.239*(Tavg^(-0.9965)))*10^(-3); % 1/K
    k_air = (-0.00001631*(Tavg^2)+0.081885*(Tavg)+2.95)*10^(-3); % W/m*K


    Afl = (d_pl/2)^2*(pi);
    Pfl = (pi)*(d_pl);
    Lair = Afl/Pfl;

    Ra_L = (g*b_air*(abs(Tbase-T_air))*Lair^3)/(vair*alphair);
    hbarair = (k_air/Lair)*0.54*Ra_L^(1/4);
```

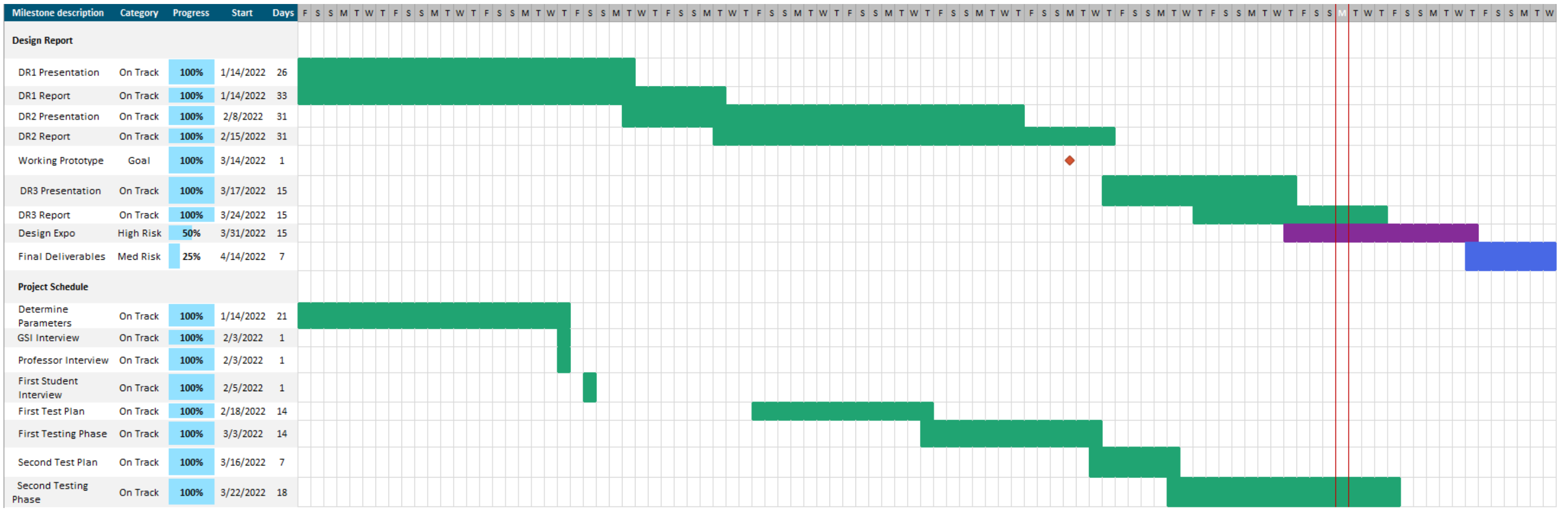
```
    Rt_cnv  = 1/(hbarair*Afl);
    Rt_cnd  = h_fl/(k_fl*Afl);

    Rtot = Rt_cnv + Rt_cnd;

    q = (Tbase-T_air)/Rtot;

    qout = q;
end
```

# Appendix B: Gantt Chart



| Milestone description | Category | Progress | Start | Days |
|---|---|---|---|---|
| **Design Report** | | | | |
| DR1 Presentation | On Track | 100% | 1/14/2022 | 26 |
| DR1 Report | On Track | 100% | 1/14/2022 | 33 |
| DR2 Presentation | On Track | 100% | 2/8/2022 | 31 |
| DR2 Report | On Track | 100% | 2/15/2022 | 31 |
| Working Prototype | Goal | 100% | 3/14/2022 | 1 |
| DR3 Presentation | On Track | 100% | 3/17/2022 | 15 |
| DR3 Report | On Track | 100% | 3/24/2022 | 15 |
| Design Expo | High Risk | 50% | 3/31/2022 | 15 |
| Final Deliverables | Med Risk | 25% | 4/14/2022 | 7 |
| **Project Schedule** | | | | |
| Determine Parameters | On Track | 100% | 1/14/2022 | 21 |
| GSI Interview | On Track | 100% | 2/3/2022 | 1 |
| Professor Interview | On Track | 100% | 2/3/2022 | 1 |
| First Student Interview | On Track | 100% | 2/5/2022 | 1 |
| First Test Plan | On Track | 100% | 2/18/2022 | 14 |
| First Testing Phase | On Track | 100% | 3/3/2022 | 14 |
| Second Test Plan | On Track | 100% | 3/16/2022 | 7 |
| Second Testing Phase | On Track | 100% | 3/22/2022 | 18 |

## Appendix C: Bill of Materials

| *Item* | *Quantity* | *Source* | *Catalog No.* | *Cost* | *Contact* | *Notes* |
|---|---|---|---|---|---|---|
| Hot Plate | 1 | Corning | PC-400D | $319.20 | corning.com | |
| Petri Dish | 1 | Pyrex | 3160-152 | $1.77 | corning.com | |
| Silicone Oil | 150 ml | SUPER LUBE | 56501 | $5.03 | super-lube.com | |
| Aluminum Powder | 7.5g | Fischer Scientific | S25144 | $1.18 | fishersci.com | |
| Alumina Powder | 2.5g | Fischer Scientific | AC212250250 | $8.83 | fishersci.com | |
| Rolling Cart | 1 | N/A | N/A | N/A | Auto Lab Staff | Provided by facility |
| Document Camera | 1 | N/A | N/A | N/A | UM Facilities and Ops | Provided in classroom |
| Projector | 1 | N/A | N/A | N/A | UM Facilities and Ops | Provided in classroom |
| Thermocouple | 2 | N/A | N/A | ~$30 | Prof. Boehman | Borrowed from decommissioned lab |
| Thermocouple Reader | 1 | N/A | N/A | ~$100 | Prof. Kaviany | Borrowed |
| | | | **Total:** | $466.01 | | |

Table 3 also shows the cost of materials. Note that the listed cost is for the quantity indicated in the table, which may be less than the minimum order quantity. Also note that some items do not include details, such as the document camera; these items are either provided by facilities or are present in the classroom and exact specs can vary. While we are not concerned with their cost (as they are provided to us), we want to ensure that we can gauge the price of this setup and experiment. This will be especially helpful for future demonstrations or testing where one would start without the materials.

**Appendix D: Demonstration User Manual**

# *Introduction*

*Benard cells are an interesting state of thermobuyant convection cells, driven by a density gradient (Rayleigh-Taylor instability). Heating a fluid from the bottom causes the hotter, less dense fluid to rise while driving down the colder, denser fluid. These cells form under specific conditions where the instabilities are not turbulent but provide enough energy to drive cellular motion.*

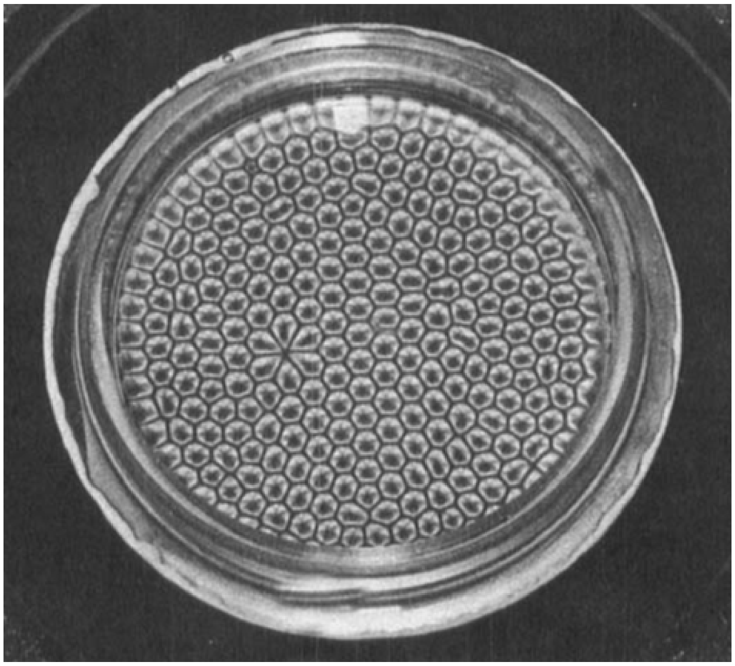


***Figure 1**. Benard cells formed in a layer of silicone oil with aluminum powder [2]*

## *Objective*

*This document will provide instructions for a simple, in-class demonstration for Benard cells. The objective of this demonstration is to provide students with a more concrete, visual representation of convection and/or heat transfer. We want the demonstration to engage students and potentially provide them with a way to experiment with this setup themselves.*

## *Scientific Background*

*Thermobuoyant flow is driven by pressure and/or temperature gradient in a fluid under the influence of gravity. The interaction between buoyancy and fluid density gives rise to Rayleigh-Benard convection, which can cause Benard cells to form [1, 2]. Hotter fluid rises at the center of the cells and colder fluid falls around the edges. The formation of these neat hexagonal structures can depend on both the Rayleigh number* and the Marangoni number, where the former is more convection-driven (rigid-rigid boundary) and the latter more connected to surface tension (rigid-free boundary) [2, 3].

The main assumption in studies of Benard cells is that the fluid follows the Boussinesq approximation, where all fluid parameters are independent of temperature except for fluid density [2, 4]. Nondimensionalizing the Navier-Stokes equation with the continuity equations, we get the Rayleigh number. Without surface tension, the main result becomes that there is a critical Rayleigh number. The Rayleigh number is found from equation 1 below:

$$Ra = \frac{g\beta}{\alpha\nu}\left(T_H - T_C\right)L^3 \qquad (1)$$

Where g is the acceleration due to gravity ($m/s^2$), $\beta$ is the thermal expansion coefficient, $\nu$ is the kinematic viscosity, $\alpha$ is the thermal diffusivity, L is the height of the fluid layer (m), $T_H$ is the temperature at the controlled-temperature surface (K), and $T_C$ is the temperature of the fluid on top (K) [1, 2]. With rigid-rigid boundary conditions (the top and bottom surfaces are enclosed by a solid boundary), the critical Rayleigh number to form hexagonal Benard cells is about 1700 [3, 5]; rigid-free boundary conditions have a critical Rayleigh number of about 1100 [5]. Below that number, there is not enough buoyancy for formation while above, it starts to become turbulent convection.

Temperature differences between the top and bottom layers of a fluid can also form gradients in surface tension. This creates an imbalance which- with sufficient forces to overcome the frictional viscous forces- causes convection. While this effect is not defined as thermobuoyant, a similar process (with a different equation of state) to the one earlier was used to derive another dimensionless quantity called the Marangoni number:

$$M = -\frac{\delta S}{\delta T}\frac{L\left(T_H - T_C\right)}{\alpha\nu} \qquad (2)$$

Where $\frac{\delta S}{\delta T}$ is the change in surface tension with respect to temperature, $\nu$ is the kinematic viscosity, $\alpha$ is the thermal diffusivity, L is the height of the fluid layer, $T_H$ is the temperature at the controlled-temperature surface, and $T_C$ is the temperature of the fluid on top [2]. The critical Marangoni number to form hexagonal Benard cells is about 80 [3, 6].

For any of these two cases to work, experiments must also consider the aspect ratio (ratio of the length of the container to the depth of the fluid). To eliminate boundary effects (conductive transfer) from the sides of an enclosing container, the aspect ratio needs to be large enough, as infinity is assumed in theoretical calculations but would be impractical in reality [1, 3, 4]. Fluid layer thickness is especially important in the *case of surface-tension-driven convection. If the fluid layer is too thick, Benard cells may become unfavorable and rolls will form instead [7]. However, in very thin layers, surface tension effects dominate in such a way that the Marangoni number can be subcritical for the formation of hexagonal cells [8].*

## ***Materials***

| *Item* | *Quantity* | *Manufacturer* | *Part No.* | *Cost* |
|---|---|---|---|---|
| *Hot Plate* | *1* | *Corning* | *PC-400D* | *$319.20* |
| *Petri Dish* | *1* | *Pyrex* | *3160-152* | *$1.77* |
| *Silicone Oil* | *150 ml* | *SUPER LUBE* | *56501* | *$5.03* |
| *Aluminum Powder* | *7.5g* | *Fischer Scientific* | *S25144* | *$1.18* |
| *Alumina Powder* | *2.5g* | *Fischer Scientific* | *AC212250250* | *$8.83* |
| *Rolling Cart* | *1* | *N/A* | *N/A* | *N/A* |
| *Document Camera* | *1* | *N/A* | *N/A* | *N/A* |
| *Projector* | *1* | *N/A* | *N/A* | *N/A* |
| *Thermocouple* | *2* | *N/A* | *N/A* | *~$30* |
| *Thermocouple Reader* | *1* | *N/A* | *N/A* | *~$100* |
|  |  |  | ***Total:*** | *$466.01* |

*Exact equipment and prices may vary. Specifically, instructions were written for the prototype version hot plate (not listed), while the final production version will employ the hot plate listed above, which includes a built-in plate temperature thermocouple. This document will include instructions on the target temperatures of the top and bottom fluid layers, though the instructions will still focus on our prototype hot plate. Note that variation in the thermocouples, the thermocouple reader, and the ambient conditions in the classroom will cause variation in the exact temperatures at which the demonstration functions. While a temperature range is provided, it is not unexpected* *for the demonstration to function below the specified temperatures. However, if the demonstration has exceeded the specified temperature range without functioning, please discontinue use and contact ME450 Team 12, as damage to the fluid and a possible fire risk can occur. If the device must be used in excessively warm ambient conditions (such as outdoors or in the Auto Lab during the summer), 10mL additional fluid should be added to compensate.*

## *Setting Up and Running the Experiment*

1. *Pour the silicone oil into the petri dish*
2. *Mix in the aluminum and alumina powders until mixture is homogeneous*
3. *Attach plate surface thermocouple to plate with Kapton tape*
4. *Place the petri dish onto the hot plate, on top of the plate surface thermocouple*
5. *Attach the surface thermocouple and the plate thermocouple to the thermocouple reader*
6. *Attach the thermocouple reader to the stand, and position such that the surface thermocouple is just barely in contact with the surface of the fluid*
7. *Heat the fluid*
    a. *10 minutes*
        i. *Turn hot plate to setting 6*
        ii. *Cells should begin to form once the hot plate temperature reaches 190-210 °C*
    b. *15 minutes*
        i. *Turn hot plate to setting 5*
        ii. *Cells should begin to form once the hot plate temperature reaches 190-210 °C*
    c. *20 minutes*
        i. *Turn hot plate to setting 4*
        ii. *Cells should begin to form once the hot plate temperature reaches 190-210 °C*
    d. *Note: These times of formation and heating settings are only true for the hot plate we used throughout testing. If it is switched, these settings and times may not be correct*

8. *Maintain the temperature at 200°C within ±5°C for best results. Some variance is acceptable, but note that exceeding the rated temperature of the fluid may damage the fluid and reduce demonstration lifetime*

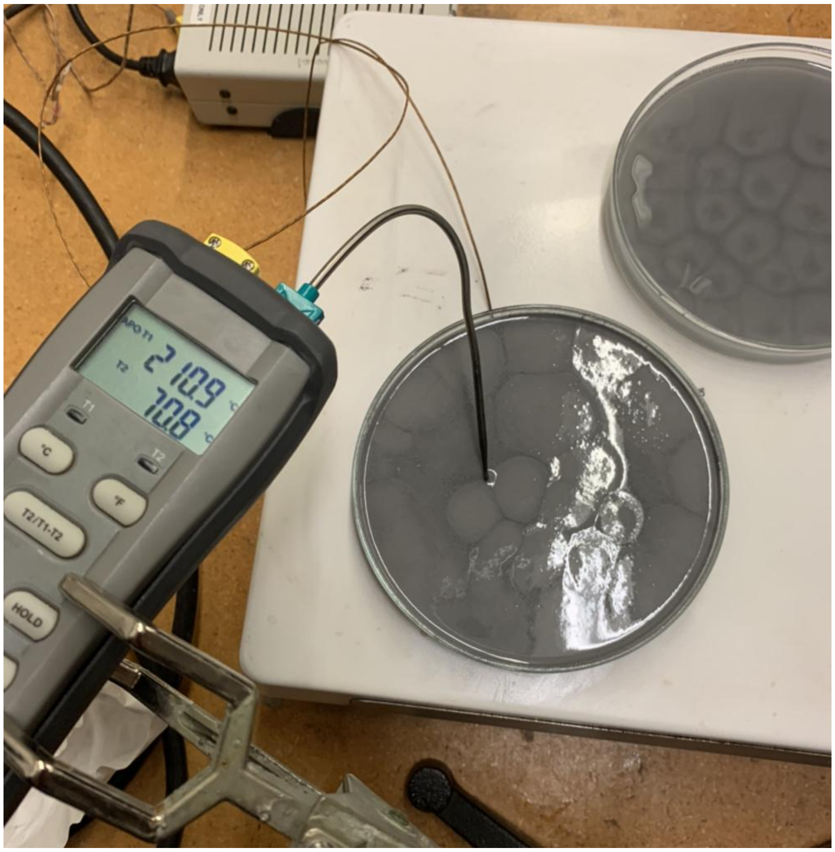


***Figure 2**. Benard cells formed in a 1cm fluid layer thickness, with a bottom temperature around 210°C and a top temperature around 71°C with ambient temperature of around 25°C*

## *Storage and Disposal*

*For storage of the demonstration, secure the lid on top of the Petri dish. Store the dish in a level and safe location where it will not spill. If it is knocked over, some of the silicon oil will likely spill from the dish. When reusing a stored setup, mix the fluid and powders back to a fully mixed state. Make sure that you scrape the bottom of the dish with your mixing tool as some of the powder can get stuck to the bottom of the dish. This will decrease the visibility of the demonstration when it is reused. After mixing the powder and fluid, begin from step 3 above and continue with demonstration setup.*

*When disposing of the used materials, follow University of Michigan disposal guidelines for lab chemicals and materials. This includes labeling of used materials and assuring that they are not disposed with any other goods that may be reactive with one another. The aluminum powder can be fairly reactive with some other chemicals, so be sure to follow typical laboratory guidelines and procedures when disposing of this material.*

# *Acknowledgements*

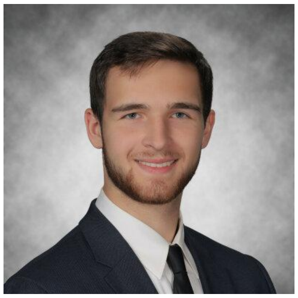
*Special thanks to our sponsor, Prof. Andre Boehman. His support and technical expertise made our project possible. We would like to thank our instructor, Prof. Massoud Kaviany, who provided consistent guidance throughout our work and kept our project on track. We are especially appreciative of our stakeholders, Prof. Solomon Adera, Courtney Videchak, Ekim Koca, and Kayra Ilkbahar, whose perspectives and input enabled us to design this system to better serve the students and instructors of ME 335. And finally, many thanks to Anna DeMarco and Kevin Wall from SuperLube, who provided us with data and technical support that proved invaluable in characterizing the silicone oil to enable accurate engineering analysis of our system.*

# *References*

1. *Kaviany, M., 2011, Essentials of Heat Transfer. Cambridge University Press, New York.*
2. *Koschmieder, E.L., 1993, Bénard Cells and Taylor Vortices. Cambridge University Press, New York. https://books.google.com/books?id=arpUkoBD8uYC&pg=PA54&source=gbs_toc_r&cad=4#v=onepage&q&f=false*
3. *Koschmieder, E. L., 1974, "Bénard Convection," Advance in Chemical Physics, 26, pp. 177-212. https://onlinelibrary-wiley-com.proxy.lib.umich.edu/doi/pdfdirect/10.1002/9780470143780.ch4*
4. *Bodenschatz, E., Pesch, W., and Ahlers, G., 2000, "Recent Developments in Rayleigh-Benard Convection," Annual Review of Fluid Mechanics, 32, pp. 709–778, https://courses.physics.ucsd.edu/2018/Winter/physics116_216/Convection-Recent%20Results-Bodenschatz_W17.pdf*
5. *Mattson, J., 2008. "A student project on Rayleigh-Benard convection," ASEE Annual Conference and Exposition, Pittsburgh, PA, June 22-25, 2008, pp. 1 - 8, DOI 10.18260/1-2--3591, https://peer.asee.org/3591.*
6. *Kalelkar, C., 2018, "The Inveterate Tinkerer 11. Marangoni Effect," Resonance, 23, pp. 225–233. https://doi.org/10.1007/s12045-018-0607-1*
7. *Bragard, J. and Velarde, M. G., 1998, "Benard–Marangoni convection: planforms and related theoretical predictions," J. Fluid Mech., 368, pp. 165-194, https://jeanbragard.github.io/publi_ref_files/jfm368.pdf.*
8. *Koschmieder, E. L. and Biggerstaff, M. I., 1986, "Onset of surface-tension-driven Benard convection," J. Fluid Mech., 167, pp. 49-64, doi:10.1017/S0022112086002720.*

# TEAM

Neil Wagner



I'm from Bloomfield Hills, Michigan, and the reason I chose mechanical engineering is that I come from a long line of mechanical engineers and I grew up watching them work and design things which sparked my interest. During my childhood, I always played with Legos or Erector sets in which I could see basic engineering principles at work. I also have an interest in robotics and mechatronics which is where my passion lies when it comes to engineering. In the future, I plan to go straight to graduate school as soon as I finish my undergraduate work here at Michigan. I then plan to work at a company that works on mechatronics and my dream job is designing the mechatronics on robots that are used in incredibly unique situations like satellites or rovers on different planets. Some fun facts about me are that I'm an eagle scout, I play two sports year-round and I am an avid gamer in my free time.

Dalton Wawro

I currently live in Petoskey, Michigan but originally am from Birmingham, Michigan. I chose Mechanical Engineering because of my love for Physics and the Physics courses that I took in high school. I recently added a Physics minor and I plan to graduate this semester. I am still undecided on post-graduation plans and am deciding between attending graduate school and beginning to work. I would love to work in design engineering or a thermal analysis position in the future. My dream job is to be a design engineer at a golf club company such as Taylormade, Titleist, or Callaway. My hobbies include chess, golf, cooking, and reading. A fun fact about myself is that I have lived in 4 different states and 5 different cities in Michigan.

Stephen Mundy

I hail from Pleasant Hill, California, where I was born and raised. I chose mechanical engineering because of a lifelong curiosity about how things work and a love of cars. Like Neil, I grew up playing with Legos - a toy which is due no small credit in my choice to pursue engineering. In the future, I plan to go to grad school, where I will pursue my doctorate doing research on internal combustion engines. In particular, I hope to research improved diesel engines and fuel injection systems. After obtaining my doctorate, my dream job would be to work as a consultant to OEM's or to launch a startup producing inexpensive diesel engines for cars. Some fun facts about me: in High School I was a competitive Bagpiper, I have completed multiple triathlons, and I am an avid Ballroom Dancer.

Daria Bizyaeva

I've moved around a bit and now live in New Mexico but I originally come from Moscow, Russia. My interest in mechanical engineering was born out of the desire to apply scientific knowledge, specifically to the case of renewable energy. Since I started, I found that I also had an interest in materials and heat transfer, and am currently cultivating my interest in atomic physics. Given my ever-evolving interests, I hope to go into industry after graduation and eventually go back into academia to pursue research, gaining something to focus on. My favorite things to do include reading, consuming tea, and watching old shows and movies. I am currently learning Mandarin and continuing to watch Star Trek.