

**Ordinary Brilliance: Understanding Black Children's Conceptions of Smartness  
and How Teachers Communicate Smartness Through their Practice**

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## **Dedication**

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## **Abstract**

This dissertation is an effort to better understand Black children's conceptions of smartness and the ways that teachers communicate smartness through their practice. Here, I reimagine smartness as a verb rather than a noun—that is, smartness is about what one does that is smart. I develop a conceptual framework that attends to race, mathematics, and teacher practice that disrupts a traditional, white supremacist, and antiBlack mathematics education. Key elements of my conceptual frame incorporate tenets of critical race theory (Ladson-Billings, 1999) to attend to race. I drew on the mathematical task framework (Stein, Grover, Henningson, 1996) to appraise and analyze the mathematics problems used in the class, and the concept of normative identity (Cobb, Gresalfi, & Hodge, 2009) was a key analytic tool in identifying the obligations and messages communicated by the teacher.

I used my conceptual framework to conduct a multi-case study that explores the conceptions of smartness of five Black learners (three girls and two boys) during a summer mathematics program. Additionally, I identified five episodes of instruction that included critical moments that student-participants highlighted as related to their conceptions of smartness. I also then analyzed these episodes to understand what messages about smartness the teacher seemed to be communicating during the program. The data comprise three interviews with each student, pre- and post-surveys, the students' notebooks from the program, and video recordings of classroom

instruction. I used these data to answer the research questions: *(1) How do these Black students describe what it means to be smart in a summer mathematics program?* And *(2) How does a teacher communicate smartness during a summer mathematics program?*

The first part of my findings highlights the number and types of conceptions students have about smartness. The conceptions that I identified are complex and offer three key takeaways: conceptions among students are alike superficially, but different substantively, their conceptions of smartness are malleable, and their conceptions can be seen as strategies that pushback against antiBlackness.

The second part of my findings highlights the messages that the teacher communicated about smartness and the methods she used to communicate those messages during the summer mathematics program. The teacher's messages about smartness defined smartness to include characteristics such as *listening to and learning from others' thinking, revising your own thinking, and explaining your thinking to convince peers*. I also identified six methods the teacher used to communicate these messages: *interrupting normalized patterns of classroom interaction, scaffolding students to explain their thinking and to orient their explanations to the rest of the class, encourage students to revise their thinking, using the routines of "notes to self" and end-of-class checks as opportunities to reflect and focus on metacognitive development, strategically and intentionally acknowledge competence, designing and using tasks focused on key ideas and practices that challenge and surface multiple ways of thinking.*

Together these findings raise several important implications for future research and practice. Future research should focus on discovering and unpacking the conceptions that Black learners bring into classrooms and how that connects to their enactment of smartness in different contexts, focusing in particular more explicitly on the interpersonal and intrapersonal dynamics between learners and environment. Needed are methods that capture the complexity and dynamism of students' conceptions of smartness, which would allow for understanding the relationship between students' conceptions and their enactments of smartness. From a practice perspective, focusing on the messages about smartness that teachers communicate as well as how they communicate them also seems particularly productive.

## Chapter 1 Introduction

Dr. Abdelkrim Brania was my professor for Real Analysis II at Morehouse College. Professor Brania was from Algeria. His classes were always challenging. I had only had him for one class prior to Real Analysis II, and in that class, I had earned a C. In Real Analysis II, I earned a B-. At least as important as the grade I received were my interactions with Professor Brania. I remember visiting him during office hours frequently to get help with the homework problem sets he would assign, which seemed impossible to me. I remember I would write the problems on the board and attempt to solve them. I would often ask him questions to get a sense of whether I was heading in the right direction for the solution. He would usually refuse to answer my questions directly; instead he would answer my question with a question, and make gestures about what I wrote or what I said. I often entered his office feeling hopeful and left feeling frustrated and demoralized. While in his office, he would rant about his own experiences learning math in Algeria, where, according to him, all he had was a book and how he taught himself. He would also critique me as a student, stating that I only focused on the solution and not the process or telling me that I wanted him to do the work for me.

At some point during my many trips to his office hours, his rants, and my rants about him to my classmates, I realized what he was trying to do. Professor Brania wanted me and his other students to understand mathematics for ourselves. He wanted us to be empowered to do mathematics, and he wanted us to focus on learning. After finally understanding “the method to his madness,” I realized that I wanted the same thing he

wanted. However, the means by which I could get there were different from what he was offering. For instance, as a tenured professor at Morehouse College, the only all-male Historically Black College in the country, I expected he would have gained knowledge of the students he taught, especially how they might have been prepared, or not, in K-12 education. I thought he would know different pedagogical techniques that could be helpful or that he would have reflected on his experiences with the students he taught and realized that at an HBCU the goal is to do everything one can to set students up to be successful. Over the course of Dr. Brania's tenure at Morehouse College, I thought he would have adjusted his pedagogy and instruction to support the needs of his students. I thought he should have recognized that all of the students he taught might not have had the opportunities to develop the skills and beliefs he thought they should have. However, instead of identifying and understanding the skills and beliefs that students had and using them to adapt his instruction, it seemed he was satisfied doing things the way he always did.

My experience with Dr. Brania illuminated three important points. One, he made me consider what it actually means to be good at or smart in mathematics. Prior to taking his classes, I had always thought I was good at mathematics: I did well in my math classes in K-12, I did well on the math parts of standardized tests, and I did well in the math classes I took at the community college. However, I never felt smart in Dr. Brania's class. The processes that worked for me to be successful previously did not work in his class. Second, I felt Dr. Brania did not actually support and develop me as a doer of mathematics. He had his way of teaching, and I had my beliefs. And it seemed as though the two would never align unless I just morphed myself into the person, he wanted me to



be—that is, to become a critical thinker, teach myself the content, and depend less on others for help. Finally, I wondered what it would take for Dr. Brania to help me bridge the gap from the beliefs I had developed as a mathematics learner to his conception of what it means to do mathematics. It is in this story, and what I took from this experience, that I situate my study.

### **1.1 This Experience and Me**

The experience with Dr. Brania was not unique for me. As a Black man, once a Black boy, I have had both positive and negative experiences in mathematics. During my primary and secondary education, I was considered by my teachers to be *smart* and *gifted*. These labels were attached to me because of several factors, including skipping the fourth grade, graduating high school at the age of 16, and graduating college at the age of 20. Some of these labels were not just because of these achievements, but also because of who I am and where I come from: I am a Black male who was raised by a single-parent Black mother in a mostly Black neighborhood and who attended public schools in K-12. I include these details because they typically contribute to a master narrative that Black boys under these circumstances do not succeed. However, my experience serves as a counter-story. I loved math because I thought it was logical, and I was good at computing. I often did what the model student in traditional mathematics classrooms is expected to do: I sat at the front of the class; wrote down verbatim the notes that teachers wrote on the board; completed the problems that were assigned for homework from the textbook; checked my answers at the back of the textbook; and, if I could not complete the homework, I would ask my teacher for help.

It wasn't until I got to college that I realized that my approach up to this time might not be enough. At Morehouse, I learned that mathematics was not just about computation but also involved seeing structures and patterns, writing proofs, and constructing arguments—just to name a few things to which I had not been exposed to before then. I also noticed that some professors were not as helpful or held expectations of their students with which I was unfamiliar. For example, the quality of instruction from professor to professor was inconsistent. Some professors did not cover the necessary mathematical content and often did not engage us in the mathematical practices of the content that we needed for future courses. Other professors, like Dr. Brania, brought their own philosophies, which were closely connected to their cultures and how they were trained in mathematics. In other words, what they understood to be learning mathematics was inconsistent with my own experiences with the teaching and learning of mathematics. Despite these experiences I was able to survive—although not thrive—and earn a B.S. in mathematics. However, these experiences left me unsure about my mathematical competence and ability to pursue graduate level mathematics.

My uncertainty was confirmed in graduate school when I began to pursue a MA in mathematics. The first, and only, graduate mathematics course I took was Real Analysis II. My class was taught by a Latinx postdoc. My classmates were mostly Asian students, a few White students, and just two Black students (including me). Despite taking Real Analysis II at Morehouse College, I struggled. Although I attempted to get help from my professor during his office hours, he often spoke to me dismissively and told me I should already know the content related to the questions I asked. Although I tried to work with

my classmates, that also was not helpful; my peers did not want to collaborate with me, choosing instead to work on their own or with other students in our class.

To say my experience was traumatizing is an understatement. Walking to class three days a week in the mathematics department as a Black male made me realize and internalize how anti-Black the space was. My experience in graduate school contrasted the pro-Black experience I had at Morehouse College. There, my teachers and peers looked like me, were invested in me, and were willing to give the necessary time and effort to ensure my success. However, my graduate school experience made me question myself, my ability, and everything I had accomplished to this point. Given how far I had come, how could I feel this way? In the end, I withdrew from the course and abandoned my pursuit of a graduate degree in mathematics. These experiences serve as the foundation and motivation for my research.

## **1.2 The Problem**

My personal experiences are not unique. For Black children, experiences like mine pervade U.S. schooling (Martin, 2012). It is not unusual that a Black boy like me would doubt his own ability and that he might feel marginalized in his classes. It is not rare that a Black boy could see himself as “good at math” growing up, only to discover that what he was “good at” in his K-12 schooling was seen as limited once he entered the university. In the first case, Black boys’ identities are shaped by broader racial narratives that assume they are intellectually deficient (Martin, 2012; Nasir & Shah, 2011). In the second, Black boys who are affirmed as “bright” are often not supported when placed in environments that are mismatched with their preparation, causing them to question their own ability instead of the environment and system (Sander & Taylor, 2012). In neither

case are Black boys seen as “smart.” In neither case are their identities or their capabilities affirmed.

These patterns have been corroborated in numerous studies. Nasir and Shah (2011), for example, describe the racial narratives that position Black students as inferior in mathematics and Asian students as smart. Through Black boys’ voices, the authors illuminate how they negotiate those narratives in math classes. McGee and Martin (2011) found that Black college students in STEM had developed strategies to manage the stereotype threats that permeate their classes. These findings are instances of what Gholson and Wilkes (2017) argue is a history of education refusing to see Black children as mathematicians, as scientists, or as scholars; instead, Black children's identities are (mis)taken as troublemakers, criminals, and terrorists which renders them as inferior and unteachable.

One important way that Black children are constructed as "not smart" in mathematics has to do with teachers' subjective assessments of Black children. Teachers' subjectivity is connected to the discretion that they exercise in their classrooms and in the decisions, they make about children (Ball, 2018). It is in the subjectivity that we see Black children's capacity repeatedly questioned and undermined. For example, although Black children comprise 16.7% of the school age population, only 9.8% of children identified for gifted programs are Black (Grissom & Redding, 2016). In their analysis, Grissom and Redding found that this disproportionality was principally the product of teachers' subjective judgments, not differences in children's capability. Further, in her study of teacher's perceptions of children, Wickstrom (2015) found that a teacher receiving professional development in growth mindset still did not see the competence

and quality of the work of some of her students. For example, for similar work done by a White boy and a Black girl, the teacher thought that how the mathematical task was phrased didn't allow the White boy to show all he knew. Whereas for the Black girl, the teacher interpreted her work as reflective of inflexibility in her understanding of a concept, inability to use knowledge in context, and inability to visualize concepts in ways that would allow her to generate different solutions.

Teachers' tendency to construct Black children as "not smart" is connected to messages about who is smart and what it means to be smart that occur in society writ large. These messages are communicated and embedded throughout popular culture, such as in movies like *A Beautiful Mind* (2001) that are often centered around White men and describe a master narrative about mathematicians that includes being innately gifted in mathematics, working independently, and being social introverts. This should come as no surprise, as Hottinger (2016) poignantly argues that mathematics is a White, Eurocentric, male-dominated field. These messages communicated through media platforms make their way into classrooms. We see this in studies that find that children often depict White men in laboratory jackets when asked to draw a scientist (Rock & Shaw, 2000; Picker & Berry 2000), or those that explore children's descriptions of the negative messages that they have heard or received about being a Black learner in mathematics (Gholson & Robinson, 2018).

Across all of this, a major theme stands out: Black children are unlikely to see themselves as smart in math. Their conceptions of what it means to "be smart in math" are the product of the messages they hear and the experiences they have, which are rooted in a history of presuming Black intellectual inferiority (Darby & Rury, 2018).

Moreover, the conceptions that students develop about seeing or not seeing themselves as smart, and what it means to be smart in mathematics, are connected to how students engage and participate in mathematics classrooms. In this study, I investigate what conceptions of smartness Black students have and the messages a teacher communicates through her practice.

### **1.3 Overview of Study, RQs**

To understand more about how Black children describe what “smartness” in mathematics entails, I conducted a multi-case study of five fifth-grade Black children during a summer mathematics program. The following research questions guide my study:

- 1) How do Black students describe what it means to be smart in a summer mathematics program?
- 2) How does the teacher communicate what it means to be smart during a summer mathematics program?

The first question attends both to Black students and smartness. Often research either focuses on Black students or smartness. Connecting the specificity of Black learners’ identities to their descriptions of smartness provides an opportunity to see a critical aspect of mathematics instruction through students. The second question is grounded in the assumption that teachers signal to students what it means to be smart during instruction—in other words, teachers communicate smartness in ways that shape learners’ experiences. Regardless of teachers’ intent, what they say and do is the focus of this question because the ways a teacher communicates smartness implicitly and

explicitly are important for understanding how students come to construct and perform smartness in their classes.

#### **1.4 Outline of Dissertation Proposal Chapters**

I now outline each subsequent chapter in this dissertation proposal. In the second chapter I discuss relevant literature, beginning with literature that attends to Black learners' constructions of smartness in mathematics classrooms. Next, I turn to literature that investigates smartness for all students with a focus on broadening what it means to be smart in mathematics as well as ideas about who can be smart in mathematics. Then, I focus on literature that aims to understand how competence is constructed in mathematics classrooms. I follow this section by examining literature that focuses on the role of the teacher in constructing smartness in mathematics classrooms. I close the chapter by arguing for the need to understand how Black learners construct smartness in environments designed to broaden what it means to be smart and who can be smart, with a focus on describing how teachers communicate smartness.

In the third chapter, I describe the methods that I use for my study. I describe the context of my study and argue why that context is appropriate. I provide an overview of the summer mathematics program curriculum along with focal mathematical tasks, describe how the program recruits and admits students, describe the process I used to select participants for my study, and provide a profile of each participant. Next, I describe the data I collected and the purpose for each type of data. Then, I discuss my methodological approach for each research question in my study. I close the chapter with potential implications of the study.

In the fourth chapter, I address the first research question. I analyze student-created artifacts and describe the different conceptions of smartness they have. I close the chapter by discussing three themes: *superficial similarities, but substantive differences malleability*, and *conceptions as strategies to push back against antiBlackness*. The first theme describes the similarity and differences among focal students' conceptions of smartness. The second describes how focal students' conceptions potentially shift during the summer mathematics program. The third describe how students' conceptions of smartness and ways of being could serve as strategies to push back against antiBlackness.

The fifth chapter addresses the second research question. In this chapter, I analyze the transcripts of five critical moments that occurred during the summer mathematics program. In these moments I name the different moves the teacher made and the signals she communicated about smartness. I close the chapter by discussing signaling methods. In total I describe six signaling methods that the teacher used to communicate messages about smartnes.

The sixth and final chapter I make connections between chapter 4 and chapter 5. I emphasize the role of the mathematical tasks as key for the conceptions students described, and the messages the teacher could communicate. I also, describe some of the limitations of this study and what that means for the field, as well as steps for future researcher. I close the dissertation restating what the goal was, my commitment to this work, and the importance, as well as my rationale for including students' names images, and words in the dissertation.



## **Chapter 2 Literature Review and Conceptual Framework**

### **2.1 Literature Review**

I turn now to discuss areas of prior research that inform my study. Central to my research questions is how conceptions of “mathematical smartness” are constructed by Black learners and inside mathematics classrooms. My aim is to capture the voices and experiences of Black students as they describe smartness in mathematics—namely, what they think it means to be smart, whether they think they are smart, why they believe what they believe about smartness, and how classrooms contribute to their understandings of smartness.

In the first section, I examine studies that focus on Black learners and how notions of “smartness in math” or “mathematical ability” shape their experiences and senses of identity in different ways. Due to the relative paucity of studies that address Black learners’ constructions of what it means to be “smart” in mathematics and who is seen as smart, I turn in the second section to literature that attends to broader notions of smartness or competence in mathematics. I specifically selected literature to identify how traditional views of smartness in mathematics have been described. I also selected studies that attend to different ways of thinking about smartness in mathematics, as well as scholarship that unpacks the benefits of broadening what it means to be smart in mathematics for students.

In the third section, I review the work of scholars who investigate how smartness is constructed in classrooms. Although there is a master narrative about what it means to be smart generally, that narrative is enacted in particular ways in classrooms. Investigating this empirically is important because it opens possibilities to reconstruct or redefine smartness in classrooms by identifying features that might be most salient. These features could include the tasks the students work on, how students participate, and the teacher's moves during instruction. This literature argues that smartness is locally and socially constructed and highlights the ways that smartness is defined differently in different contexts. Knowing that smartness is both locally and socially constructed offers a useful perspective for examining who contributes to the construction of smartness and how in a given context.

In the fourth section, I examine literature that focuses on teaching and how teachers frame and enact conceptions of "smartness." Teachers are one of several factors that influence how students understand what it means to be smart and who is smart. Seeing what teachers say during instruction, what they describe when talking about students, and which media they use to communicate smartness provides examples of how teachers construct smartness and opens space for considering how they might construct smartness differently. We see in these studies that there are examples of teachers doing this work well, as well as examples that show how even well-intentioned teachers frame smartness in ways that reinforce traditional views of what it is to be "good at math" that limit students' opportunities to learn.

I close the chapter by arguing for the need to extend the existing research by investigating Black learners' constructions and performances of smartness in

mathematics classrooms, especially in environments designed to broaden what it means to be smart and by examining how teachers can work to reframe smartness to support student learning.

### **2.1.1 *Black Learners' Constructions of Mathematical Competence***

An increasing number of studies have focused on Black learners in mathematics education over the last two decades (Martin, 2000; Martin 2009, Martin & Leonard, 2013; Joseph, 2017; McGee, 2020). These studies have been important as they have increased knowledge about the experiences of Black learners, offered critiques of current policies, and described the teaching and learning of Black learners. I mention this to acknowledge the valuable work of many scholars and to specify my rationale for the four sets of studies I discuss in this section. I begin by focusing on studies that are most closely aligned to the focus of my dissertation. My criteria yielded research conducted by (1) Danny Martin, (2) Kara Jackson, (3) Na'ilah Suad Nasir with Niral Shah, Cyndy Snyder, kihana ross, and (4) Teresa Dunleavy. Although some scholars have investigated and theorized about Black learners' constructions of competence in mathematics classrooms, these scholars focus specifically on Black learners' ideas around their "smartness," competence, and/or mathematical ability.

Martin (2009) argues that race has been undertheorized in mathematics education and describes how this under-theorization has historical and political roots that position and accept non-White racial groups as inferior based on racial achievement gaps. He unpacks what Ernst (1991) calls the racial hierarchy of mathematical ability which views White and Asian learners as having the most mathematical ability while Black, Latinx, and Native American learners are assumed to have the least mathematical ability. Martin's

work advances a critical analysis of the historical and contemporary contexts of Black learners and notions of smartness in mathematics and shows the pervasiveness of views of Black people as intellectually inferior.

Whereas Martin's analysis addresses the macro-historical and social levels, Jackson (2009) provides an empirical investigation of Black learners' constructions of mathematical ability inside of classrooms. Although Jackson does not focus specifically on smartness, her study is relevant because she investigates how Black youth in elementary mathematics are constructed both inside and outside of the classroom. She found that two students, Nikki (a Black girl) and Timothy (a Black boy), were "socialized into a construction of mathematics that emphasized speed and accuracy over process and involved completing procedures that were absent of mathematical meaning" (p. 195). She argues that this construction of Black youth in mathematics is a result of the institutional discourses in school which viewed students as academically deficient and lacking basic skills. Jackson's analysis suggests that the construction of Black learners may be connected to the ways teachers enact school-level discourse in classrooms to reinforce traditional notions of smartness.

A third program of research investigates racial storylines and narratives in mathematics education (Nasir & Shah, 2011; Nasir, Snyder, Shah, & Ross, 2012; Shah, 2017). Racial narratives as defined by Shah are "stories that circulate in society about the supposed traits and behaviors of particular racial groups, and also about whether and how race matters in a given context" (2017, p. 8). This research has described the types of racial narratives and storylines that learners have access to and the tensions for learners in countering those narratives. For Black learners, common narratives include

that Black learners are not good at school or mathematics, as well as that Asian learners are good at school and math.

For example, Nasir and colleagues (2017) investigated Black learners' constructions of smartness using a combination of surveys and interviews. They found that elementary and middle school students in urban schools are aware of and hold school-related racial stereotypes about who can be good at school; they also found that these stereotypes get stronger in middle school. In addition, they found that Black and Latinx students in urban schools experience greater divergence between their views of their group's abilities in school (endorsement), and their beliefs about how others view their group (stereotype). Said another way, Black and Latinx students believed they were smart, however they believed society viewed them as less smart. Further, the authors found that students' recognition and endorsement of stereotypes were complex, highlighting the nuance of students' constructions of smartness in mathematics classrooms.

Dunleavy (2018) investigated how three Black girls defined smartness in an Algebra I class. At the beginning of the school year the girls described having negative perceptions of their own smartness in mathematics. Over the course of the Algebra I class The author found that the girls' perceptions of their smartness in mathematics shifted and they defined smartness in four ways: (1) consistently and unapologetically affording time and space to value multiple solution strategies; (2) belief in mathematical justification and explanation as the goal for demonstrating mastery of mathematical content; (3) valuing mathematically valid ideas from all class members; and (4) valuing collaborative problem solving as a way to help group members, distribute mathematical knowledge, and orient

students toward learning with one another. These beliefs about smartness run counter to traditional views of smartness that are associated with grades, test scores, speed, and correctness.

Although the literature on Black learners' constructions of mathematical smartness is sparse, these offer three complementary conceptualizations of competence, or smartness, in mathematics with respect to Black learners. Martin (2009) examines how smartness is constructed in mathematics theoretically, while Jackson (2009) takes an emic perspective (insider view) to examine how smartness is constructed in mathematics empirically. These authors' findings demonstrate how Black learners are positioned as intellectually inferior through pervasive normative rhetoric at the macro-level (e.g., mathematics education research), as well as at the micro-level (e.g., schools and classrooms). From an etic perspective, Nasir and colleagues (2017) and Dunleavy (2018) leverage data about student experiences and perspectives to investigate how students construct smartness. These studies' findings suggest that students have their own constructions of smartness, which can be quite distinct from the traditional views of smartness that schools offer. Dunleavy (2018) looks at how Black students themselves construct smartness in mathematics. Thus, although these studies make important contributions, as a group they also highlight how few studies in mathematics education research attend explicitly to Black learners' constructions of smartness in mathematics classrooms. I turn next to studies that do not focus on Black learners specifically, but that do investigate learners' constructions of smartness. In the studies that follow, some of the studies do not name the racial identities of the focal students, and the ones that do, do not take up race in any meaningful way thereby taking a colorblind perspective on

learners' construction of smartness. By colorblind perspective I mean that students' race was not taken into consideration in understanding students' constructions of smartness.

### **2.1.2 All Learners' Constructions of Smartness in Mathematics**

A broader group of studies have explored the idea of smartness in mathematics, although, problematically, without considering learners' identities. These studies describe how smartness had traditionally been constructed in mathematics and how particular environments can shape smartness in more productive ways. In their investigation of learners' ways of knowing mathematics, Boaler and Greeno (2000) acknowledged that mathematicians deem creativity, connectivity, and intuition as important to develop. However, mathematicians also argue for a pedagogy that emphasizes teaching “abstract mathematical procedures through repeated practice of the procedures.” This pedagogical approach reinforces the idea that smartness in mathematics means that students know and can apply mathematical procedures quickly and accurately. The authors found that typical K-12 pedagogical practices that emphasize repeated procedural practice undermine children's development of creativity, connectivity, and intuition, impeding the development of the skills they will need in post-secondary mathematics. The authors highlight how K-12 mathematics instruction contributes to traditional views of smartness through emphases on skills such as memorizing procedures, quickly answering questions, and focusing mainly on accuracy at the expense of other characteristics that are important for mathematics.

Featherstone and colleagues (2011) argue for broader notions of smartness in mathematics teaching. They define “math smarts” not as abilities with which a person is born but as abilities that are built through use and developed over time by engaging with

challenging math problems. Taking this perspective, the goal of instruction is to appreciate smarts that children already have and nurture new ones. Their view runs counter to the idea that ability is fixed and innate. They operationalize this view in at least two ways. First, they argue for the use of complex instruction, which is a form of instruction in which students work in small groups on tasks that require students to take on different roles (e.g., recorder and presenter) and work together to solve problems that they would not be able to solve individually (Cohen, Lotan, Scarloss & Arellano, 1999). Second, they advance the teaching practice of assigning competence in which teachers publicly acknowledge the competence of students who have a lower status in the classroom to highlight the value of their contributions to peers (Featherstone, Crespo, Jilk, Oslund, Parks, & Wood, 2011). Through their work, these authors argue that broader notions of smartness can be developed through teaching practices such as complex instruction and assigning competence and that these efforts have potential to make mathematics learning more equitable, especially for students who may not typically be perceived as competent in mathematics.

The work of Boaler and Staples (2008) and Horn (2008) provides empirical evidence of the benefits of broadening competence. Boaler and Staples (2008) conducted a mixed methods longitudinal study of the practices at three different high schools and how they impact student learning and promote equity. They found that students learned to “appreciate the different ways that students saw mathematics problems and learned to value the contribution of different methods, perspectives, representations, partial ideas and even incorrect ideas as they worked to solve problems” (p. 22). Additionally, they found that students in their study scored higher on assessments in comparison to



students at other schools that adhered to traditional views of mathematics instruction. Although these researchers do not specifically investigate students' conceptions of smartness, the study makes three important points with respect to conceptions of smartness: first, broadening conceptions of smartness beyond speed and accuracy (traditional views) has a positive impact on students' scores on assessments; second, broadening conceptions of smartness encourages students to redefine what it means to be smart at math; and, third, broadening conceptions of smartness supports students to develop more complete, accurate, and holistic views of what it means to do mathematics.

In her investigation of “turnaround students,” Horn (2008) found that teachers framed students' struggles in advancing in mathematics not as inability to advance in mathematics, but rather as students having not yet decided to succeed. Teachers decided to focus on students' maturity, determination, readiness, organization, and skills—broader notions of mathematical competence, instead of innate ability or lack of home support. Horn argues that a teacher's commitment to framing students' struggles in more productive ways and highlighting competence in broader ways supports students to develop positive mathematical identities and to continue taking mathematics courses (i.e., staying in the mathematics pipeline). As a result, the author argues that teachers play an intricate role in developing positive mathematical identities and keeping them intact, as well as ensuring that students remain in the mathematics pipeline.

In these studies, we see examples of researchers arguing for broader conceptions of smartness, defining what some of those broader conceptions can look like, and identifying the benefits those broader conceptions have for student learning. However, what is missing from these studies is the role of social identities, namely race. Although

these studies make important contributions, not attending to the role of social identities, including, race assumes that social identities and race are not important for the teaching and learning of mathematics generally, and for Black learners specifically.

### **2.1.3 *Smartness as Locally and Socially Constructed in Classrooms***

Critical to investigating smartness is understanding how it gets constructed in classrooms. In other words, how do students and teachers develop shared meanings around what it means to be smart? What do teachers communicate about what and who is valued, and what do students take up? Whereas the previous section focused on students' conceptions of smartness, this section attends to the role of classroom contextual features (e.g., participation patterns; explicit and implicit behavioral expectations; the nature and quality of content instruction; etc.) in constructing smartness. Some work has been done to uncover the ways that smartness is *locally* (i.e., within specific classroom contexts) and *socially* (i.e., by groups of individuals developing shared definitions) constructed in classrooms.

In their investigation of how competence is constructed in mathematics classrooms, Gresalfi, Martin, Hand, and Greeno (2009) define competence as “a collection of skills or abilities that are attributed to individuals apart from the specific contexts in which they participate” and “what students need to know or do in order to be considered successful by the teacher and other students in the classroom” (p. 50). The authors operationalize this definition of competence as a system to argue that

competence gets constructed as students and the teacher negotiate: (1) the kind of mathematical agency that the [mathematical] task and the participation structure afford, (2) what the students are supposed to be accountable *for* doing, and (3)

whom they need to be accountable *to* in order to participate successfully in the classroom activity system. (p. 52)

Using their system of competence to analyze two classrooms, they found that competence was constructed differently in each classroom. In the sixth-grade classroom competence was constructed as working hard and fixing one's mistakes. In the eighth-grade classroom, competence was constructed as replicating and repeating procedures the teacher showed and practicing. Specifically, the authors found in the sixth-grade classroom that competence was constructed through having an open-ended task, providing students with opportunities to exercise conceptual agency, and having students be accountable to the mathematical content and each other, which was demonstrated and negotiated through interactions among the teacher and students.

Although Hatt (2012) doesn't specifically focus on mathematics, during her ethnographic study of a kindergarten classroom, she found that smartness was constructed through artifacts such as the stoplight. Each student's name was inscribed on a car and put on a large stoplight for everyone to see. Each day students were put on green and based on their behavior stayed on green or moved to yellow and/or red, with each color after green indicating the student's behavior was worse or inappropriate. Based on this artifact, students came to understand that smartness meant following rules and behaving in ways expected by the teachers, (i.e., compliance). In practice, compliance looked like "[being] conservative in speech (e.g., speak only when called on), in movement (e.g., hands to self, legs crossed when sitting), and in communication with authority (e.g., 'yes, ma'am')" p. 447. The teacher constructed smartness implicitly and explicitly by telling students they were smart if they "listened to authority figures, knew

curricular material and behavioral expectations before being taught” (p. 456). Through artifacts such as the stoplight and interactions with students, the teacher constructed smartness based on the teacher’s biases around race and class and thus reinforcing broader societal narratives. This problematic construction signaled to students how to understand their own smartness and value as well as how to understand the competence and value of their peers.

These studies highlight that, although broader societal systems structure who and what is seen as smart, and what counts as mathematical smartness, smartness is also locally and socially constructed. This point is important because it highlights relevant features of classrooms and the individuals within them that to construct smartness. Knowing that the mathematical tasks and the opportunities students have to exercise their agency and accountability contributes to constructions of smartness and that different classrooms construct smartness differently provides additional insight for understanding students’ constructions of smartness. Also, knowing that artifacts within classrooms, along with interactions among and between teachers and students, offer examples of both subtle and non-subtle ways smartness is constructed and shape students’ experiences. Together these studies make important contributions, namely that smartness is locally and socially structured, which highlights the need to understand how smartness, with a focus on teacher practice and students’ perspectives is constructed locally and socially within different contexts and what that means for teachers and students.

## **2.1.4 Teachers' Roles in Constructing and Enacting Smartness in Mathematics Classrooms**

Several factors influence how smartness gets constructed in classrooms; however, teachers also play a specific and important role. During instruction, teachers teach content knowledge as well as ideas about what it means to be good in mathematics. In some cases, teachers' constructions of smartness are more productive (Lampert, 1990; Lampert, 2001; Boaler & Staples 2008; Horn 2008). By more productive, I mean some teachers' constructions of smartness are more equitable and more aligned with practices that are authentic to the discipline of mathematics and equitable. In other cases, despite their efforts, some teachers' constructions of smartness are less productive (Hand 2010; Wickstrom, 2015; Louie, 2017). In this section I provide examples of both.

In her seminal work, Lampert (1990) explored whether students could come to know mathematics in ways that resembled mathematicians' practice. Drawing on the work of Polya (1954) and Lakatos (1976), Lampert wondered whether it was possible to foster students' intellectual courage, intellectual honesty, and wise restraint. By selecting particular mathematical tasks for students to work on and focusing on student participation structures, she was able to show that students could do mathematics in the ways she envisioned. She argued that

the content of the lesson is the arguments that support or reject solution strategies rather than the findings of answers. Students' strategies yield answers to teachers' questions, but the solution is more than the answer, just as the problem is more than the question. Generating a strategy and arguing its legitimacy indicates what the students knows about mathematics. p. 40.

Lampert's research focused on using her own teaching, in which she sought to engage students in authentic experiences doing mathematics in ways that aligned with disciplinary mathematics, which often contrasts with the ways in which school mathematics is learned and taught in K-12. Despite not having student data such as students' perspectives and outcomes, particularly with respect to smartness, her analysis includes student voices, work, and performance in class. This study provides an example of how a teacher's practice can shift the meaning of knowing, learning, and doing mathematics.

Two studies highlight how even well-intentioned teachers frame smartness in problematic ways. Wickstrom (2015) found that teachers' practice and beliefs may be contradictory. She found that despite aspects of the teacher's practice that encouraged students to be resilient and view their learning in an incremental way, the teacher still had fixed ideas of smartness that influenced how she viewed students. Specifically, the teacher held static views of students based on their beginning of the year test scores, which impacted positively how she saw students with high scores and negatively how she viewed students with low scores. The teacher's lens limited her ability to see the competence of all students in the class.

Similarly, Louie (2017) adds to the literature that highlights the difficulty for teachers attempting to shift toward more equitable practice. She observed the practice of four geometry teachers and categorized their practice as exclusive or inclusive of students based upon the teachers' framing of what it means to do mathematics (the nature of mathematical activity) and who can do mathematics (nature of mathematical ability). She found that despite teachers' aim "to move toward more equitable and

inclusive forms of instruction, the teachers in this study created and enacted instructional strategies that were rooted in exclusionary framings of mathematical activity and ability” (p. 512). One explanation for her findings is that the dominant culture of mathematics education excludes learners from being good at math if “they do not know a lot of formulas, always answer questions quickly and correctly and are White or Asian males from economically privileged backgrounds” (p. 514).

### **Building Upon and Extending the Literature**

Looking across all of the studies discussed, I have identified four themes. First, few studies have focused specifically on Black learners and how they construct smartness in mathematics, and of those that do, only one that I found takes up how Black learners define smartness for themselves.

Second, findings suggest that broadening notions of what it means to be smart has benefits for learners' success in mathematics in secondary school. This takeaway is important because it demonstrates that there are examples of the benefits of broadening conceptions of smartness at the secondary level while raising the question of whether these same benefits accrue at the elementary level.

Third, the constructions of smartness described in these studies show how these notions and identities are locally and socially constructed. That is, what it means to be smart and who is smart was not the same in each class. Several factors including the teacher, mathematical tasks, and the ways in which students could participate during instruction impacted how smartness was constructed. This points to the significance of contexts in shaping how students come to know and do mathematics. This takeaway is important because it considers the features and actors in classrooms that are relevant to

the construction of smartness. Further, highlighting the value of understanding the social dynamics in classrooms contributes to imagining the ways in which smartness might be redefined in different contexts, as well as potential challenges that might make it difficult to do so.

The fourth takeaway is that teachers contribute to conceptions of smartness that uphold, disrupt, or reflect some problematic combination of views about what it means to be smart and who is smart in mathematics classrooms. This highlights the point that there is still work to be done to examine and understand the ways in which teachers communicate smartness.

My examination of the literature highlights the few studies that have focused on smartness or the mathematical competence of students in K-12 with respect to how smartness is conceptualized, how it develops, and how it shifts. A number of these studies also take a raceless perspective. That is, they do not study or acknowledge the race of the students in their respective studies and how that might shape their data and analyses. This point is critical because taking a raceless or colorblind perspective hides how these phenomena are racialized, and not neutral, thus reinforcing dominant lenses that deny the role that race, and racism play in students' experiences and development. Additionally, there are even fewer studies that focus on Black learners specifically. The lack of studies that focus on Black learners indicates a critical and important perspective missing in the literature.

I seek to build on and extend this existing literature by focusing specifically on Black learners and their conceptions of smartness. Focusing on Black learners extends beyond the often deficit-based constructed narratives and stereotypes that are associated



with Black learners in general and mathematics specifically (see Chapter 1). Fundamentally the perspectives of Black learners' matter, are valuable, and too often have not been heard, which limits what is known and what can be done with and for Black learners. Additionally, I observe a classroom environment with a teacher who focuses on broadening students' views of smartness. I examine the teacher's practice and interpret the messages about smartness that she communicates during classroom instruction. In the next section, I highlight the different theories and frameworks that I draw on to frame my study.

## **2.2 Conceptual Framework**

In this section I identify the theoretical lenses I assembled to ask, investigate, and analyze my research questions. I use the analyses that these lenses afford to make an argument for how educators need to redefine smartness in mathematics with Black learners in mind. First, I briefly discuss Critical Race Theory (CRT) and Martin's (2013) argument that the brilliance of Black children is axiomatic. Here I use this work, to not only show the reader how I focus on Black learners in my work, but also how I use this work to disrupt traditional notions of Black learners as deficient and reposition them as capable and worthy of challenging mathematics instruction—that is, as smart in ways that educators and researchers systematically fail to acknowledge and lift up into view. I then describe smartness both generally and in the context of mathematics. I argue that current theories and literature are not sufficient for understanding smartness as they do not define what one *does* to enact smartness. I propose a new definition of smartness that does not focus on evaluative assessments (e.g., grades and test scores) of students, instead focusing on what students *do* during instruction. In the final section, I discuss the

instructional triangle (Cohen, Raudenbush, & Ball, 2003; Ball, 2018), a normative identity analytic framework (Cobb, Gresalfi, & Hodge, 2009), assigning competence (Featherstone et. al, 2011), and the mathematical task framework (Stein, Grover, and Henningson, 1996) as lenses to focus on particular aspects of classroom instruction in order to attend to what and how smartness gets constructed in classrooms.

### **2.2.1 Critical Race Theory**

My dissertation foregrounds Black learners and conceptualizes race as socially constructed. That is, I argue that race is not biological but a construct that is negotiated in society both explicitly and implicitly. I draw upon Critical Race Theory (CRT) as a theoretical frame to unpack the ways in which race is constructed, as well as to describe the ways in which race is present in my design and analysis. CRT was born out of Critical Legal Studies (CLS), which through doctrinal and policy analysis shows how class structures in society are legitimized and are oppressive (Crenshaw, 1988). The contributions of CLS, though important, did not address race, leading subsequent scholars to develop CRT to account for how racism is embedded in society (Bonilla-Silva, 2006; Delgado, 1995). The central tenets of CRT that I apply to this study are the permanence of racism in the U.S., an understanding of Whiteness as property (Harris, 1993), and the necessity of centering experiential knowledge and counter-storytelling to disrupt dominant narratives (Solórzano & Yosso, 2002).

The CRT tenet concerning the permanence of racism in the U.S. asserts that racism is embedded in the fabric of U.S. history, highlighting the ways that even when race and racism are not explicitly discussed they are present in U.S. social and legal relations. This is particularly useful as I think about the Black learners in my study.

Although race does not come up explicitly in what students say and do, that does not mean that race and racism are not present. For example, one way that racism is enacted in classrooms is through punishment practices. Black learners are punished more frequently and more severely for subjective infractions (e.g., what they wear, facial expressions, tone of voice, etc.) as compared to their white counterparts (Morris, 2007; Epstein, Black, Gonzalez, 2017). This is just one way that racism could occur without explicit mention of race or racism in classrooms.

Whiteness as property connects to the material and physical idea of how whiteness is enacted in the world. Whiteness as property is the manifestation of White Supremacist ideology that privileges White people's interests through physical, legal, and theoretical property. Framing whiteness as property comes with "...the right to transfer, the right of use and enjoyment, and the right of exclusion..." (Dixson, 2004, p. 28). I argue that, in classrooms, the construct of smartness embodies whiteness. Smartness is constructed from white interests, beliefs, and practices, and only particular people—white people—and ways of being—white ways of being—are able to be considered smart. I aim to disrupt this traditional construction by identifying the ways students' views of smartness and the messages they receive about what constitutes smartness may or may not embody traditional, white supremacist definitions. Focusing on Black learners is particularly important as historically and presently these have been the students who have arguably been impacted the most yet have been heard the least (Howard, 2001; Howard, 2008).

I use the tenet of counter-storytelling to draw upon the voices of the Black students with whom I am most concerned. Although there are different narratives about Black learners' conceptions of smartness and Black learners' own perceptions of smartness, as

discussed in earlier sections of this chapter, few studies have specifically examined how Black learners themselves conceive of smartness. One of my goals in this study is to capture and listen to the voices of Black learners in order to elevate their counter-narratives about what they think it means to be smart in mathematics.

Further, Ladson-Billings (1999) argues that, in education, CRT enables us to critically interrogate how racism is enacted through curriculum, instruction, assessment, school funding, and desegregation. For example, using a CRT lens reveals that generic “best practices” instructional strategies, which are promoted as working for “all” students, typically presume a deficit orientation when used with Black learners. Such approaches encourage teachers to focus on employing the “right” strategy or technique during instruction as a mechanism to control Black learners (see for example, *Teach Like a Champion* (Lemov, 2010)). When Black learners do not respond to these techniques, especially when they are unsuccessful based upon evaluative measures valued by schools, Black learners are viewed as deficient. With this in mind, it is important to understand Black learners' constructions of smartness given that their experiences are often grounded in educational racism. Therefore, investigating points of intersection or overlap between students' experiences and their constructions of smartness is relevant to understand the relationship between Black learners' perspectives and the educational racism they experience provides nuance for Black learners. Next, it is important to understand teachers' practice, particularly with respect to what gets communicated to learners, in this case Black learners, as curriculum and instruction are key sites for educational racism. This makes it important to understand how teachers can work intentionally to disrupt patterns of racism. Understanding this is relevant for my study as

it provides a lens to interpret the ways in which Black learners discuss their conceptions about smartness that are grounded in their broader experiences in mathematics, as well as the teacher's practice and communication.

### **2.2.2 *Brilliance of Black Children as Axiomatic***

An essential part of my framing and conceptualization of Black learners is the axiomatic principle that Black children are brilliant (Martin, 2013). Treating the brilliance of Black children as axiomatic means that their brilliance is something that doesn't need to be proved. Leonard and Martin (2013) first described Black children's brilliance as a counter to the prevalence of studies, especially within mathematics education, that assume Black learners need to be fixed and/or are naturally inferior. Leonard and Martin focus on successful Black learners in mathematics and argue for a redefinition of what it means for us to be successful by focusing on Black learners' resilience and persistence. Reframing success in this way allows for Black learners to be seen as competent.

In this dissertation, I take the brilliance of Black children as axiomatic. This comes up in my design. For example, the questions I ask in surveys and interviews are not centered on learners proving they are smart. Instead, the questions I pose were generated to elicit students' understandings of what it means to be smart in math along with how they make sense of smartness given their experiences. Also, this axiom influences my analysis. In Chapter 4 it guides how I make sense of what students are saying and how I identified relevant moments, through the perspective of my participants, that took place in the summer mathematics program. In Chapter 5, this axiom allowed me to see and interpret both the teacher's practice and the students' participation. For example, in a key episode, the teacher reframes an incorrect answer from La'Rayne by

focusing on identifying what question she is answering. The teacher's move here highlights La'Rayne as someone whose thinking makes sense by focusing on what she knows, instead of what she does not.

Overall, this axiom guides how I choose to represent students including how I write about the students, what I attend to and what I do not attend to, the images of use of them, as well as their work that I include. I focus on and trust their representations, explanations, and ideas, and analyze them for what we can learn about Black students' interpretations of their encounters with mathematics and their constructions of themselves as doers of mathematics.

### **2.2.3 *Smartness***

This study is grounded in a conception of mathematical competence, confidence, and skill that I term "smartness." I focus on smartness not as a single trait, but instead as a collection of practices and enacted orientations to oneself and the context. From this perspective, I focus on how learners do things that are smart, not whether a person "is smart" as a character trait. I take this perspective because treating smartness as a single character trait reinforces a static notion of smartness in which you either are or are not smart. Privileging enactment as the primary marker of smartness, on the other hand, allows for flexibility, multiple modes of expression, and the ability to adapt and respond to local contexts.

Traditionally, "smartness" is connected to the notion of intelligence, which has a long history in education in the United States (Murray & Hernstein, 1996). Although intelligence is often conceived as an objective measure of an individual's ability to know information and think analytically, it has historically been used to privilege some groups

of people and marginalize others. In the US, the same ideologies that stemmed from and were used to justify the institution of slavery have also served as the foundation on which the idea of intelligence has been socially constructed (Darby & Rury, 2018). Darby and Rury define the "color of mind" ideology, which holds that Black people are inferior to White people with respect to character, content, and behavior. To justify the enslavement of Black people, White people developed a set of beliefs about us that insists we lack morals (character), knowledge (content), and work ethic (behavior). Further, supposedly objective methods such as examining the skulls of humans and giving individuals the intelligence quotient (IQ) test were designed to provide "empirical" evidence of Black people's assumed inferiority (Gould, 1981). These findings were then used to justify and reinforce the ongoing oppression of Black people. Collectively this research has created a narrative that Black learners are ineducable. The enactment of this narrative through policy and practice has created a context in which Black learners become unteachable.

Two key assumptions underlying the racialized conception of intelligence I've just described are that intelligence is innate and that it is relatively stable over time. For over a century, researchers have debated whether this is, in fact, the case (Dweck, 2002). Recently, researchers such as Carol Dweck have argued for the malleability of intelligence by framing it in terms of "mindsets." Dweck and others have argued that individuals either have a fixed mindset or a growth mindset. Having a fixed mindset means individuals are more likely to attribute success to genes or talent, whereas individuals who have a growth mindset are likely to attribute success to effort (Dweck, 2008).

Framing intelligence as a product of a growth mindset is useful in that it refutes the idea of intelligence as something you are born with and reconceives it as something that

can change through effort and work ethic. However, framing intelligence in this way has its shortcomings, particularly for Black learners. In the context of classroom education, growth mindset does not help teachers consider why Black learners might have developed particular mindsets; it does not interrupt assumptions teachers might hold that Black learners don't have a growth mindset; and it allows teachers to conclude that Black learners are less successful because they do not work as hard as their peers. In these ways, though a growth mindset may be seen as disrupting the assumption that Black people are naturally intellectually inferior, it can also be used to reinforce the assumption that Black people have an inferior work ethic (Darby & Rury, 2018). Although effort and work ethic are relevant, framing intelligence in this way fails to either consider or address the systematic ways Black children are oppressed in school.

One's mindset does not account for what it means to be successful or smart at a particular task or in a particular domain. Instead, it is essential to consider what are the things that one has to *do* to be considered successful or smart? For example, in mathematics having a growth mindset means one is resilient or demonstrates perseverance, but what does one *do* to persevere? The growth mindset conception of intelligence lacks a focus on the enactment of smartness, on the things that are smart to *do*. And what one *does* to enact smartness is contextual—that is, related to specific domains and settings of human activity and learning—which growth mindset fails to account for. I turn next to consider, more specifically, the practice of smartness in the domain of mathematics.



#### **2.2.4 Smartness in Mathematics**

Conceptions of smartness in mathematics are shaped by particular views of the discipline. For example, my experiences in school shaped my early conceptions of math as a domain centered on speed, accuracy, and right answers, which were reflected in good grades and high-test scores (see Chapter 1). As a young learner, I came to believe that I was smart in math and that to be smart one had to work hard, ask questions, study, and seek help. Later I encountered other views of what it means to be smart in mathematics and struggled to adjust to the view of mathematical competence communicated by my professors, which included teaching yourself content, being logical, having intuition, and being able to prove. My experiences provide an example of the disconnect between how K-12 mathematics education frames mathematical smartness and how the discipline of mathematics defines itself.

In this dissertation, I conceptualize smartness in mathematics as engaging in reasoning, explaining, experimenting, and representing, as well as the robust use of mathematical concepts and ideas. From this perspective, a child who offers a partially developed solution or approach is doing something smart mathematically, as is one who asks a well-pointed question about the approach. This conception of mathematical smartness does not privilege speed or memorized procedures; instead, it foregrounds thinking, connecting, and building ideas. Conceptualizing smartness in this way does still allow for attention to solutions and accuracy, but it also makes room for mathematical imagining and thinking.

I build, in part, on the conception of “mathematical proficiency” in Kilpatrick, Swafford, & Findell (2001), which comprises conceptual understanding, procedural

fluency, strategic competence, and adaptive reasoning. In addition, Kilpatrick et al. include a trait that they call “productive disposition,” which they define as the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 5). These elements are useful in broadening conventional views of mathematical skill and knowledge but remain somewhat static and rooted in a view of mathematics as something external and produced by others. The conception of “smartness” on which my study is based includes developing a sense of oneself as a mathematical knower and doer (see Lampert, 1990, 2001) as well as developing the disposition to try things about which one is not sure and to propose alternatives. Although this has a resemblance to productive disposition, I focus on this as an *enacted orientation*, rather than as an innate quality or trait.

The *Common Core State Standards in Mathematics* (2010) builds on earlier standards documents produced by the National Council of Teachers of Mathematics (1989, 2000) by emphasizing the centrality of mathematical practices, forwarding a view of mathematics as something one does and makes. The Standards for Mathematical Practice (SMP) in the Common Core Standards attempt to convey this perspective by focusing on practices which include making sense of problems and persevering in them (SMP 1), constructing and critiquing arguments (SMP 3), and attending to precision (SMP 5). The SMPs represent expectations of what students in K-12 should be proficient in throughout their matriculation. This view of mathematics as practice is fundamental to the lens, I take on mathematical smartness and how children do smart things as they engage in mathematical problem solving, reasoning, knowing, and learning.

My definition of smartness in mathematics informed my dissertation study from design to execution to analysis. I intentionally focused on what students do and their practices during mathematics instruction by developing surveys and interviews that centered students' own accounts. Similarly, I analyzed student artifacts to identify their conceptions of what it means to be smart as well as examples in their work of how they enacted smartness. Lastly, when interrogating the ways a teacher communicates smartness through her practice, I paid special attention to the messages she communicated to students around how they might enact smartness in her classroom.

### ***2.2.5 Mathematical Environment: Communicating Smartness Through Teaching Practice and Mathematical Tasks***

Mathematics instruction is central to the teaching and learning of mathematics. Although teaching and learning often refer to mathematical content, other content is taught and learned in classrooms, including ideas about smartness. Ideas about smartness are communicated in various ways during instruction through interactions among children, the teacher, and the mathematical content. Communication occurs in the form of what the teacher says and does, as well as the types of mathematical tasks students work on. Throughout instruction, messages are signaled to students about what it means to be smart and who is smart. Although these messages are communicated, I do not assume that students receive the messages or take them up. However, that does not mean that they do not exist or are not important.

#### **2.2.5.1 Instruction.**

I leverage the work of Ball (2018) to name the different components of instruction and the components that are most relevant for my study. In Ball (2018) she introduces a

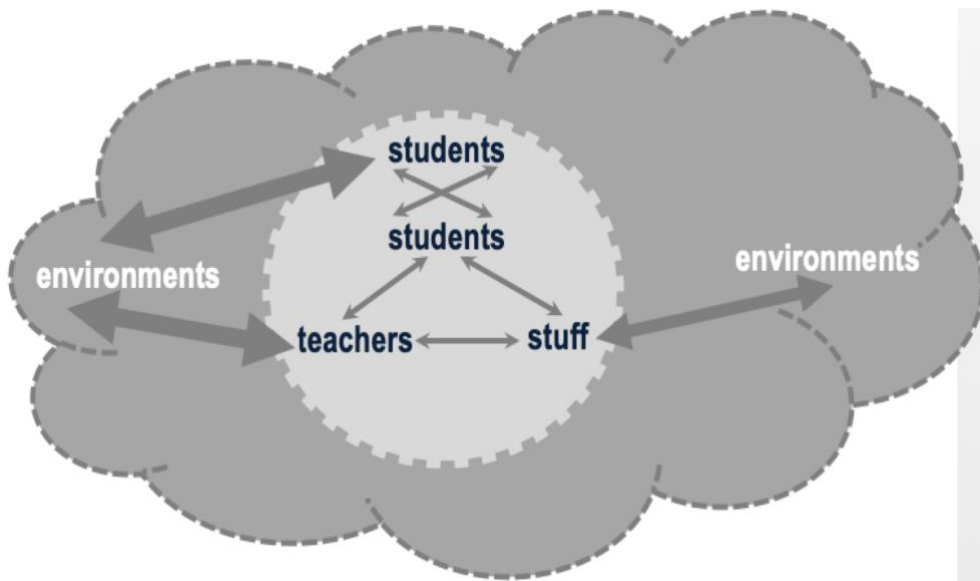
revised version of the instructional triangle, an explication of the original instructional triangle (Cohen, Raudenbush, & Ball, 2003), that elaborates the components of instruction to include the teacher, *stuff*, and students, situated in an environment. *Stuff* refers to the content that is taught during instruction which can be both disciplinary (e.g., mathematics) or non-disciplinary (e.g., what does it mean to be a doer of mathematics). It can also include what is taught and learned in school more broadly than subject matter. These components are interdependent on one another and situated within classrooms, which are situated, within schools, which are situated within school districts, which are situated within communities, and so on and so forth. This point is important because the larger environment includes the different histories, systems, and structures that take place at a macro-level and that impact the interdependent relationships that take place in the classroom (Ball, 2018). These histories include racism, sexism, and ableism. Ball argues that these *isms* impact every level (macro, meso, and micro). For example, with respect to teachers, this impacts their expectations, beliefs, and overall practice. It is here that I connect the work of Cobb and colleagues (2009) who argue that teachers' expectations along with other characteristics create a normative identity. The authors define normative identity as:

... [comprising] both the general and the [specific] mathematical obligations that delineate the role of an effective student in a particular classroom. A student would have to identify with these obligations in order to develop an affiliation with classroom mathematical activity and thus with the role of an effective doer of mathematics as they are constituted in the classroom. Normative identity is a collective or communication note rather than an individualistic notion. (p. 4)

The idea of a normative identity highlights how a teacher within a classroom community constructs what it means to be a successful or effective doer of mathematics, including through the ways the teacher's practice communicates mathematical obligations, norms, and expectations. I leverage this perspective to consider the types of agency students have, who they are accountable to, and what they are accountable for.

**Figure 2.1**

*Revised instructional triangle*



### **2.2.5.2 Assigning Competence.**

One way that teachers communicate smartness is through the practice of assigning competence (Cohen & Lotan, 1988). Assigning competence as outlined by Featherstone and her colleagues (2011) consists of three parts: (1) the teacher recognizes children's intellectual contributions, (2) selects a student or students who have low status in the classroom, and (3) publicly acknowledges that student's or students'

contribution to the class. Assigning competence is a teacher technique that was designed to intervene on inequities that result from status differentials in classrooms. I use assigning competence as an analytic tool to better understand the type of intellectual contributions that are acknowledged in classrooms, the ways in which the teacher acknowledges intellectual contributions, and whose intellectual contributions the teacher attends to. My interpretation of assigning competence takes as a given that students come into classrooms with knowledge and skills, and it is the teacher's responsibility to recognize and acknowledge their competence. With this in mind, it is important to consider what the teacher considers intellectual contributions, as that bounds what it means to engage in the discipline of mathematics. For example, acknowledging the intellectual contribution of a student seeing patterns across tasks is different from recognizing the contribution of giving a correct answer. Leveraging assigning competence is important and relevant because students' descriptions and performance does not occur in a vacuum. The ways in which the teacher defines smartness and who is smart plays a significant role in how students understand what it means to be smart and who is smart.

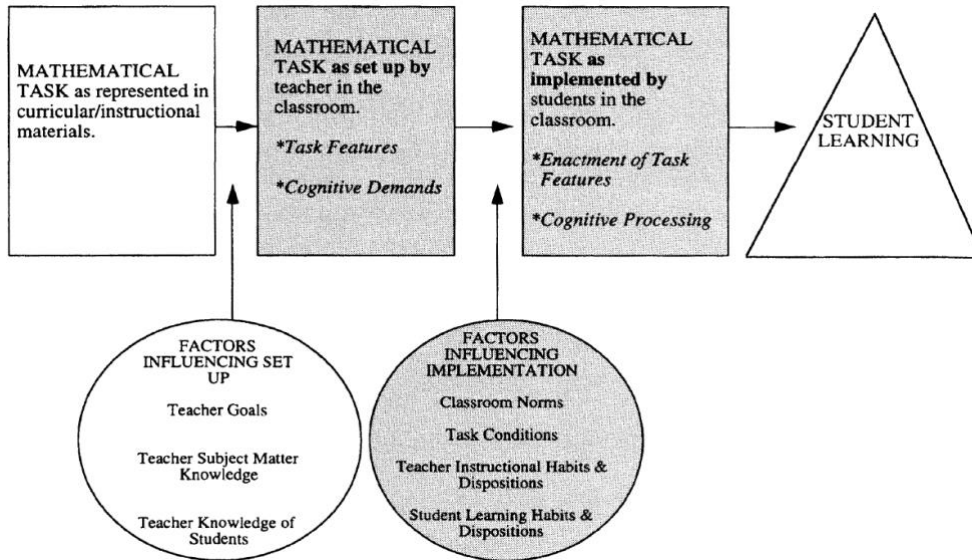
#### **2.2.5.3 Mathematical Tasks.**

Teaching practice is one way that smartness is communicated. The nature of mathematical tasks is another way smartness is communicated to students. According to Stein and colleagues (1996), mathematical tasks contribute to how students come to know and do math. The authors label features of mathematical tasks based upon their cognitive demand (e.g., low or high). Low cognitive demand tasks emphasize procedural fluency, whereas high cognitive demand tasks emphasize conceptual understanding. The level of cognitive demand is important because emphasizing procedural fluency requires

different skills than emphasizing conceptual understanding. It is here through the different opportunities to *do* mathematics that messages are communicated to students. For example, mathematical tasks that only focus on finding one answer in one particular way provides limited opportunities to do mathematics. That is, the task itself is limited to promoting certain skills which are likely to reinforce common ways of doing mathematics such as working independently, memorizing, and getting answers quickly. By contrast, tasks that have multiple solutions and strategies or even no solution and multiple strategies provide students with more opportunities to *do* mathematics. In this case, the task emphasizes more skills such as collaborating with peers, conjecturing, and prioritizing understanding. As the mathematical task frameworks highlights, there are multiple variables and relationships to consider. However here I argue that mathematical tasks which students work on are critical for the potential opportunities they could provide, which communicates messages about smartness.

**Figure 2.2**

*Mathematical Task Framework with Variables and Relationships*



## Conclusion

In this chapter, I described my assumptions about Black learners and the ways in which race appears in my study. I defined and operationalized smartness and its historical context. Although there are myriad ways and environmental features that shape and communicate messages about smartness, I focus here on a teacher’s communication of smartness and how students notice and hear what is communicated, refracted through their own experience. I briefly described what I mean by “communication”, which I will also elaborate on in Chapter 5, and provide frames that offer evidence of when and how messages are communicated particularly during mathematics instruction. I also included the mathematical task framework, which highlights how mathematical tasks themselves can communicate messages about smartness.

Smartness is the doing of mathematics which places an emphasis on reasoning, critiquing, and representing mathematics and other practices that often are not foregrounded or prioritized in mathematics classrooms. Smartness is communicated in classrooms in a variety of ways. I focus on the mathematical tasks that students work on



which afford opportunities to do math that is aligned with the discipline; I also focus on teacher practice, which includes what teachers emphasize, who they acknowledge, how they emphasize students *doing math* and acknowledge students, and when they do this work. My conceptualization allows me to (1) capture the ways students construct smartness, (2) identify critical moments during instruction that may be particularly relevant for communicating messages about smartness, and (3) identify teacher practices that signal smartness.

## Chapter 3 Methods

I take the brilliance of Black children as axiomatic, meaning it does not have to be proven.<sup>1</sup> However, as I discussed in Chapter 2, typical mathematics education most often constructs Black children—and teaches them to construct themselves—as "not smart." This dissertation is a multi-case case study focused on how five Black children constructed and/or reconstructed their understanding of smartness while participating in a summer mathematics program. The following questions guide my study:

- 1) How do Black students describe what it means to be smart in a summer mathematics program?
- 2) How does the teacher communicate what it means to be smart during a summer mathematics program?

In this chapter, I describe the design and methods of my study as well as the dataset on which the study is focused. First, I describe the context in which my study takes place, as well as explain why the context is relevant. Next, I describe the participants and my process for selecting them. I then turn to detail the types of data I collected and my rationale for collecting these. Finally, I unpack my approach to data analysis and how each part addresses my research questions.

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<sup>1</sup> Danny Martin presented "Proofs and Refutations: The Truth About Black Children and Mathematics at the iMathination 2013 Conference in St. Charles, Illinois on January 26, 2013. In the presentation, he unpacks the axiom that Black children are brilliant.

### **3.1 Context for the Study**

My study took place in the context of a two-week (10-day) summer mathematics program that has taken place at a Midwestern university since 2007. Approximately 30 rising fifth-grade students attend each year. The program has two major components: each morning, the students attend mathematics class for two hours; then, in the afternoons, students participate in activities such as tutoring, visiting the art museum on campus, or participating in other extra-curricular activities. One objective of the summer mathematics program is to ensure that the children have opportunities to engage in both foundational mathematical content (e.g., fractions) and content that extends beyond fifth grade (e.g., integers), as well as opportunities to engage in mathematical practices that foster both conceptual understanding and procedural fluency. The program does not emphasize grades, tests, speed, or correctness. Instead, it is designed to challenge students to solve cognitively demanding tasks and provide learners with opportunities to participate in class through explaining their own thinking and attending closely to and building on their peers' thinking.

A second objective of the program is to nurture students' development of positive mathematical identities. This is evident in the designers' intentionality about the goals and structures created, which expand definitions of what it means to do well in mathematics as well as broaden participants' ideas about who can do well in mathematics. Within the program, "doing well in mathematics" extends beyond speed and accuracy to focus on students learning from one another, asking questions, and explaining their thinking. This framing of what "doing well in mathematics" is designed to reinforce and support learners to see themselves as doers of mathematics.

Most of the children who attend the summer mathematics program are Black, some are Latinx, and a few each year are White. The children are recruited from a local school district based on teacher recommendations, and an equal number of boys and girls are admitted. The teacher is a White woman with over 25 years of experience teaching culturally, linguistically, and racially diverse groups of learners. The teacher sees the children as sensemakers of the world and actively seeks to position learners as smart. The work the teacher does toward this goal includes intentional planning that attends to specific students as well as to the collective. During instruction, the teacher pays close attention to the ways she wields power to intervene on moments she perceives as inequitable or as reinforcing of societal stigmas (e.g., Black children are not good at math, boys are better at math, policing and punishing Black bodies for subjective infractions is okay). Moreover, the teacher is a White woman who thinks about her race and gender in relation to the learners in the classroom, as well as in relation to how they are situated in the larger societal context. With these things in mind, the teacher has a heightened awareness around how practice can be used to disrupt or intervene on common patterns of inequity.

### **3.2 Relevance of the Context**

Historically, Black children have been positioned as intellectually inferior in mathematics classrooms (Martin, 2009). In addition to these deficit frames, mathematics classrooms across the country also tend to promote very narrow conceptions of what it means to be smart in mathematics (Leonard & Martin, 2013). Smartness in math is often equated to being fast, getting right answers, and earning high scores on assessments (Dunleavy, 2018). Because research suggests Black students' identities

are often shaped in part through the experiences, they have in mathematics classrooms (Cobb, Gresalfi, Hodge, 2009; Martin, 2000; Nasir, 2002), it is valuable to study an environment like the summer mathematics program that is intentionally designed to diverge from the norm. By grounding my study in an environment that challenges the experiences that Black children have likely had related to mathematics and smartness, I am able to interrogate what students think it means to be smart and how they might respond to signals of what it means to be smart that may be unfamiliar or uncommon. I am also able to examine the messages about smartness or about being a doer of math that such a context offers and analyze the degree to which they might deviate from or challenge normative patterns in classrooms.

Further, the summer mathematics program is part of a larger project that is designed to investigate the work of teaching. Specifically, the project explores the practice, knowledge, and tensions that are not always visible in teaching but that teachers are always negotiating in their practice, sometimes as a function of habit (Ball, 2018; Ball & Forzani, 2009; Noel, 2018). Because of this, detailed data are collected to capture different aspects of the program, some of which I used for my study. My data sources comprise high-quality video recordings of classroom instruction from the summer of 2015; scans of all student work; and classroom artifacts. These data enable me to address my research questions by allowing me to describe key features of the environment and interactions involving the students and teacher.

### **3.3 Overview of Program Curriculum, Recruitment and Admission Process, and Participant Sample**

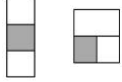
In this section I provide more detail about the program's curriculum, the program's recruitment and admission process, and the participants I selected for this study. First, I describe the curriculum, including the mathematics problems students worked. Second, I explain how the program recruits and admits students, as well as implications for the program's structure. I close the section by providing a rationale for the students I selected and briefly describing the students from my observations of their classroom engagement and artifacts.

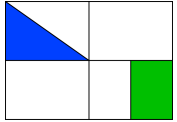
#### **3.3.1 *Program Curriculum***

I collected data during the summer of 2015. That year, the program curriculum focused on fractions, integers, and combinatorics. Tasks were designed for collective work—that is, students often collaborated with one another to solve problems instead of working independently. Table 3.1 provides a synoptic overview of the problems and their key features related to the doing of mathematics.

**Table 3.1**

*Mathematical Problems*

Day launched	Number of days worked on	Problem name	Problem	Materials	Key mathematical concepts	Key mathematical practices and mindsets	Orientation: Individual/collective
1	1	Cuisenaire Rod Riddles	Which rod is three times a light green?	Cuisenaire rods	<ul style="list-style-type: none"> <li>proportional relationships</li> <li>“three times”</li> </ul>	<ul style="list-style-type: none"> <li>interpreting language</li> <li>geometric reasoning</li> <li>explaining to others</li> <li>justifying</li> </ul>	Collective: to explore different interpretations, develop shared meaning of the question, and agreement on the solution
2	2	Magic Triangle		Diagram	<ul style="list-style-type: none"> <li>addition</li> <li>“exactly once”</li> <li>“sum”</li> </ul>	<ul style="list-style-type: none"> <li>persevering</li> <li>justifying</li> </ul>	Individual: finding solutions that work
2	1	Grey Rectangle	What fraction of each rectangle below is shaded gray? 	Diagram Black line sticky	<ul style="list-style-type: none"> <li>fractions, area models</li> <li>equal areas in areas models</li> </ul>	<ul style="list-style-type: none"> <li>manipulating a diagram to reason</li> <li>explaining with a diagram</li> <li>using language</li> <li>convincing others</li> </ul>	Collective: provocative representation and determining how to reason about and name the fraction in the second figure
2	2	Train Problem Part 1		Cuisenaire rods	<ul style="list-style-type: none"> <li>addition</li> <li>length models of addition</li> </ul>	<ul style="list-style-type: none"> <li>making sense of a problem</li> <li>identifying boundaries of a solution set</li> <li>keeping track</li> <li>justifying</li> <li>flexibility</li> <li>convincing others</li> </ul>	Collective: developing a solution for all the numbers together

Day launched	Number of days worked on	Problem name	Problem	Materials	Key mathematical concepts	Key mathematical practices and mindsets	Orientation: Individual/collective
3	2	Blue-Green Rectangle	 <p>What fraction of the big rectangle is shaded blue?</p> <p>What fraction of the big rectangle is shaded green?</p>	Diagram Scissors Blue triangles and green rectangle stickies	<ul style="list-style-type: none"> <li>fractions</li> <li>equal areas need not be congruent</li> <li>defining the whole in an area model</li> </ul>	<ul style="list-style-type: none"> <li>proving</li> <li>persevering</li> <li>manipulating a diagram to reason</li> <li>explaining with a diagram</li> <li>convincing others</li> </ul>	Collective: provocative representation and determining how to reason about the blue triangle and the green rectangle
4	7	Train Problem Part 2		Cuisenaire rods	<ul style="list-style-type: none"> <li>addition</li> <li>length models of addition</li> <li>combinations</li> <li>impossibility, problems might not have solutions</li> </ul>	<ul style="list-style-type: none"> <li>keeping track</li> <li>justifying</li> <li>flexibility</li> <li>convincing others</li> </ul>	Collective: solving the problem; convincing a protagonist








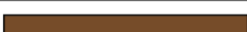




### 3.3.1.1 Cuisenaire Rod Riddles.

Cuisenaire rods are concrete materials that enable students to reason about proportions. A set of Cuisenaire rods comprises ten rectangular rods of different colors and lengths (See Figure 3.1). The rods are in proportional relationship to one another—e.g., the red rod is two times the length of the white, the black rod is six times the length of the white and three times the length of the red, etc. (See Figure 3.2).

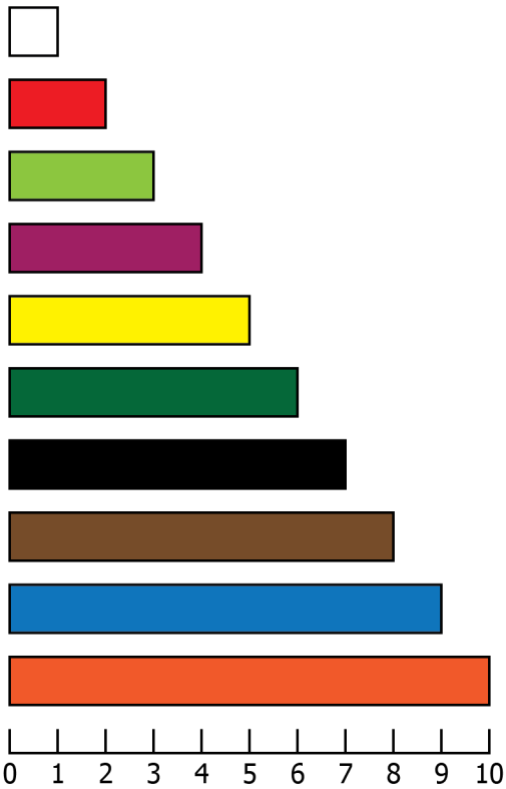
**Figure 3.1**

*Cuisenaire Rods Information*

Number	Cuisenaire rod color	Cuisenaire rod
1	white (w)	
2	red (r)	
3	green (g)	
4	purple (p)	
5	yellow (y)	
6	dark green (d)	
7	black (k)	
8	brown (n)	
9	blue (e)	
10	orange (ø)	

**Figure 3.2**

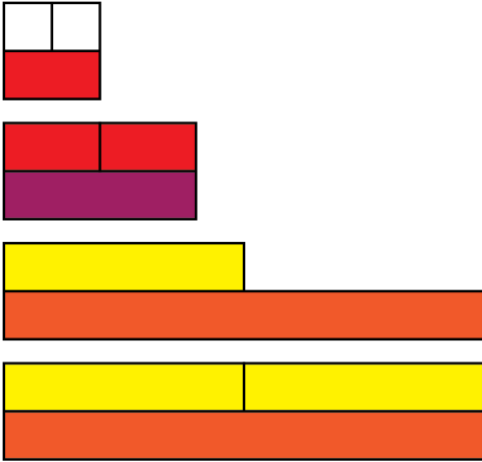
*Cuisenaire Rods Proportional Relationship*



Each Cuisenaire rod can be used to represent a number geometrically. Because they are physical objects, they allow students to test ideas in ways that can be challenging otherwise. Some of the initial questions that the teacher posed during the program get at the relationships between and among the Cuisenaire rod problems. Mathematically, Cuisenaire rods offer a rich context for reasoning and problem solving.

**Figure 3.3**

*Initial Problems*



### **3.3.1.1.1 What Is the Main Mathematical Point?**

The task “Which rod is three times as long as a light green?” is a challenging one for students. The rods are an environment for students to work on proportional reasoning for both whole and rational numbers. In the context of this problem as it appears in the program curriculum, students are beginning to work on fractions. For example, the question, “Which rod is three times a light green?” could be rephrased as “One light green rod is one-third of which rod? The formulation of the problem allows for students to work on mathematical language and notation, centered on the mathematical topic of proportional reasoning.

### **3.3.1.1.2 Key mathematical concepts and practices.**

This problem asks students to compare three light green rods, arranged end to end, and to determine which other rod has the same length. One challenging part of this problem is being able to make sense of the question being asked, which has to do with how students interpret the phrasing. They might interpret the question as “Which rod requires three to make a light green?” or as “Which rod is made with three light

greens?” depending on how they understand the phrase “three times as long as.” In the former case, the answer is the white rod—it takes three white rods to be the same length as a light green rod; in the latter, the answer is the blue rod—three light greens are the same length as a blue rod. As such, this problem engages students in work on interpreting mathematical language, geometric reasoning, explaining to others, and justifying.

#### ***3.3.1.1.3 What About the Problem Requires/Benefits from Collective Work?***

One of the affordances of the Cuisenaire Rod Riddles problem is the opportunity it offers for students to do collective work. Cuisenaire rod problems are accessible and allow for students to bring a range of conceptual understandings of the focal mathematics. However, because these problems are also not straightforward, students benefit from thinking and talking together about them. In doing so, they have opportunities to continue to develop their conceptual understanding of proportional reasoning, using one another’s ideas and interpretations, and explaining their thinking to one another. Arriving at the solution to this problem depends on students listening to and seeing one another’s proposals.

#### **3.3.1.2 The Grey Rectangle Problem.**

The Grey Rectangle Problem is a problem that uses area models to help students understand key steps in naming a fraction which include naming the whole, making sure the parts are partitioned equally, counting the total number of parts, counting the numbers shaded, and finally naming the fraction. The Grey Rectangle Problem comprises two questions (see Figure 3.4). Embedded within these tasks are the concepts of unit fraction, numerator and denominator, and notation for fractions.

One physical material that is used for the Grey Rectangle Problem is a black sticky line that supports students' understanding and explanations for equally partitioning the whole to accurately identify and name fractions.

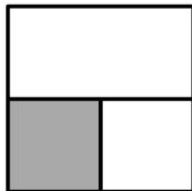
**Figure 3.4**

*Grey Rectangle Problem*

What fraction of the rectangle below is shaded gray? \_\_\_\_\_



What fraction of the rectangle below is shaded gray? \_\_\_\_\_



**3.3.1.2.1 What Is the Main Mathematical Point?**

The first task is relatively simple. Students are to determine the name of the fraction represented by an equally partitioned rectangle. Students are usually able to readily identify the fraction as  $\frac{1}{3}$ . In the second task, students are confronted with another area model, but this one is unequally partitioned. Again, one region of the model is shaded gray and two are not. Therefore, if a student's understanding and

procedure for identifying and naming fractions is counting the shaded and unshaded parts, they may think  $\frac{1}{3}$  is the answer again. However, the problem brings that understanding and procedure into question for some students because the parts are not the same size as they were in the first task (i.e., they are unequally partitioned). Thus, the mathematical point of the Grey Rectangle Problem is to ensure that students understand that in order to name a fraction all of the parts have to be equal.

#### ***3.3.1.2.2 Key Mathematical Concepts and Practices.***

The Grey Rectangle Problem provides students with opportunities to apply their conceptions of and procedures for reasoning about fractions within the same problem context. When this problem is typically presented in the summer mathematics program, most students understand how to name a fraction by counting the total pieces (denominator) and pieces shaded (numerator). However, the second task in The Grey Rectangle Problem requires a more nuanced understanding of how to identify the whole and of the necessity of ensuring that the parts are equally partitioned in order to name a fraction. By engaging with these conceptual elements of this problem, students can develop a more complex and complete understanding of fractions.

#### ***3.3.1.2.3 What About the Problem Requires/Benefits from Collective Work?***

An important affordance of the Grey Rectangle Problem is collective work. Although most students quickly agree that the answer to the first task is  $\frac{1}{3}$ , responses to the second question often include a range of answers such as  $\frac{1}{3}$ ,  $\frac{1}{2}$  or  $1\frac{1}{2}$ . All of these answers have apparently justifiable explanations. Having different answers and justifiable explanations for each, forces collective discussion among students to come to an agreement on an answer and explanation. Specifically working collectively on the

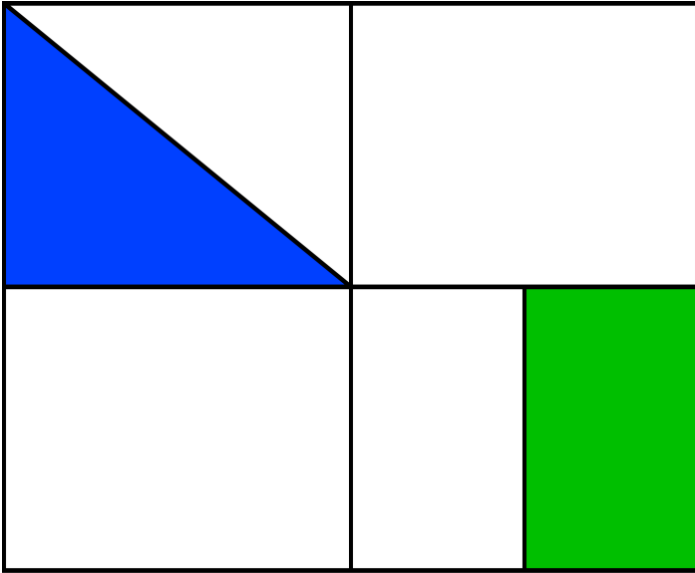
Grey Rectangle Problem allows for students to construct a shared understanding of how to name a fraction when using an area model.

### **3.3.1.3 The Blue-Green Rectangle Problem.**

The Blue-Green Rectangle Problem is challenging for several reasons. First, like the Grey Rectangle Problem, the Blue-Green Rectangle Problem asks students to name fractions based on an area model representation. However, in the Blue-Green Rectangle Problem the whole is partitioned into three different types of parts: four equal white rectangles, one blue triangle, and one green rectangle. The problem asks students to figure out what fraction of the whole is shaded blue and what fraction is shaded green. Because the blue triangle and the green rectangle are different shapes, students typically do not intuit that they each equal  $\frac{1}{8}$  of the whole. Naming the fractions of the different-shaped partitions of the whole forces students to develop a more flexible understanding of how to identify fractions based on area models.

### **Figure 3.5**

*Blue-Green Rectangle Problem*



What fraction of the big rectangle is shaded blue?

What fraction of the big rectangle is shaded green?

### ***3.3.1.3.1 What Is the Main Mathematical Point?***

Mathematically, this problem extends students' work on the Grey Rectangle Problem. The Blue-Green Rectangle Problem allows for students to learn and practice several skills. First, they continue to develop their understanding of fractions with respect to using area models. To solve the problem, they must apply their understanding that the whole must be divided into equal parts to name a fraction. Doing this necessarily requires identifying what the whole is. Second, they must equally partition the same whole in two different ways (i.e., into equal-sized triangles and into equal-sized rectangles). Third they must recognize and figure out how to prove that, despite those two shapes being different, they have the same area and represent the same fraction of the whole. Collectively, these skills and topics are what make this task challenging.



### **3.3.1.3.2 Key Mathematical Concepts and Practices.**

The opportunities that the Blue-Green Rectangle Problem offers students to work on key concepts and practices are centered on the different ways of thinking and interpreting that the problem elicits. For instance, students might struggle to identify the whole and thus misidentify the fraction that one or both shapes represent. Students might also conclude that one shape—either the blue triangle or the green rectangle—is larger than the other one. This problem thus challenges students' conceptions about how to identify and partition a whole which supports their development of a deeper understanding of fractions.

### **3.3.1.3.3 What About the Problem Requires/Benefits from Collective Work?**

The nature of the problem creates the opportunity for students to learn from one another. Also, the problem is scaffolded to support skills of proving and representing. In addition to students having opportunities to prove, collective discussion around different students' proofs allows for students to explore both their peers and own reasoning. In doing this work, students have the opportunity to deepen their conceptual understanding, which highlights the benefit of the collective work within the context of the Blue-Green Rectangle Problem.

### **3.3.1.4 The Train Problem.**

The train problem is decomposed into two parts (see Figure 3.6). The Train Problem Part 1 allows students to become familiar with the context and conditions of the problem, work together to make sense of the problem, and practice keeping track of their thinking as they work on the problem over the course of two days. Students use a subset of the Cuisenaire rods to represent the train cars and how many passengers

they hold: white holds 1 passenger, red holds two passengers, green holds three passengers, purple holds four passengers, and yellow holds five passengers. The question in the first part is to find all the ways a train could be made for 1-15 passengers, given the conditions of the problem. More importantly, students are challenged to determine *if* a train can be made for each number of passengers from 1-15. The second part builds on the first by asking students whether it's possible to create smaller trains from the complete train that hold each number of passengers from 1-15 without changing the order of the train cars in the whole train. What makes the problem is challenging is there is no solution to the problem, which students do not often experience in elementary school. In addition to the problem not having a solution, the problem requires skills such as being organized, keeping track of your thinking, and remembering the conditions of the problem. Failure to enact any of the previous practices could make the problem more challenging and potentially frustrating.

**Figure 3.6**

*The Train Problem (Part 1 and Part 2)*

### The Train Problem

The EML Train Company has five different-sized train cars: a 1-passenger car, a 2-passenger car, a 3-passenger car, a 4-passenger car, and a 5-passenger car. These cars can be connected to form trains that hold different numbers of people.

- 1-passenger car
- 2-passenger car
- 3-passenger car
- 4-passenger car
- 5-passenger car

#### Part 1

Try to build some trains. You can use only these five types of cars to build trains, and you can use at most one of each type of car in each train.

What are the different numbers of people that the EML Train Company can build trains to hold?

#### Part 2

Ms. McDuff wants to order a special 5-car train that uses exactly one of each of the different-sized cars. Ms. McDuff wants to be able to break apart the 5-car train to form smaller trains that hold exactly each number of people from 1 to 15. Ms. McDuff wants to be able to build these smaller trains using cars that are next to each other in the 5-car train.



















Can the EML Train Company build Ms. McDuff's order? Explain how you know.

### 3.3.1.4.1 What Is the Main Mathematical Point?

The Train Problem is a mathematically rich context in which students can practice foundational skills like addition as well as explore combinatorics. The Train Problem Part 1 focuses on the combinations of ways trains can be made to hold specific numbers of passengers, given the condition that you can only use one of each train car. In completing this part of the problem, students discover that you can make some trains in more than one way (see Figure 3.7).

#### Figure 3.7

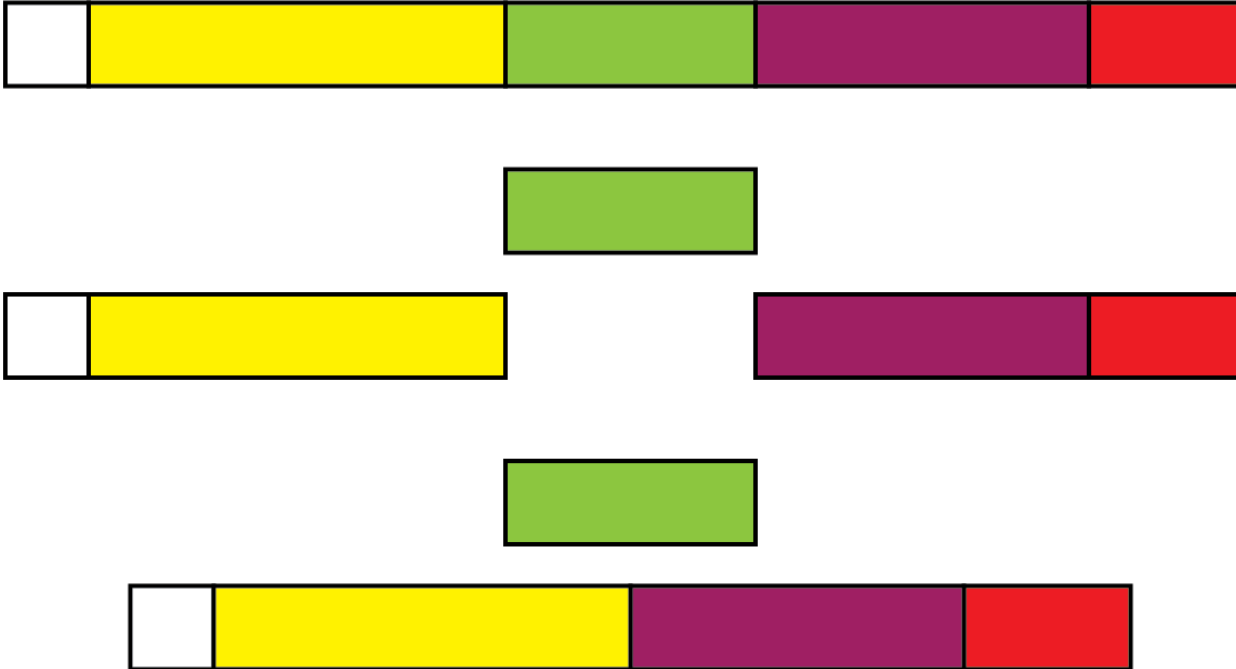
*Expressions and Representations of Trains*

Number of passengers	Expressions		Cuisenaire rods
1	1	w	
2	1 + 1	w + w	
	2	r	
3	1 + 1 + 1	w + w + w	
	2 + 1	r + w	
	3	g	
4	1 + 1 + 1 + 1	w + w + w + w	
	2 + 1 + 1	r + w + w	
	2 + 2	r + r	
	3 + 1	g + w	
	4	p	
5	1 + 1 + 1 + 1 + 1	w + w + w + w + w	
	2 + 1 + 1 + 1	r + w + w + w	
	2 + 2 + 1	r + r + w	
	3 + 1 + 1	g + w + w	
	3 + 2	g + r	
	4 + 1	p + w	
	5	y	

The second part of the problem focuses on permutations. In this part, the order of the train cars matters. Students must find a train that can be composed of the five available cars in an order that will allow it to be decomposed into smaller trains for each value 1-15 without changing the order of the cars (see Figure 3.8). As students experiment with different permutations, they discover that, unlike the first part of the problem, the conditions of the second part mean that there is no solution.

**Figure 3.8**

*Example of Train*



**Note:** In this picture the train is wygpr. However, to make certain trains sometime students move the trains. A train of yellow, green, and purple makes a train that holds 12 passengers. However, it would be incorrect to remove the green and make the train “yellow and purple” because in the original train is” wygpr”. Sometimes students rearrange trains because they cannot make trains that hold a certain number of passengers. In this case this train can make every number except 10 and 11.

### **3.3.1.4.2 Key Mathematical Concepts and Practices.**

The Train Problem offers students multiple opportunities for mathematical development. First, both parts of the problem allow for students to work on a problem that they cannot solve right away, thus supporting them to further develop their perseverance in the context of mathematics. Second, both parts of the problem require students to practice organizing and tracking their own work. Third, the problem challenges students to develop their skills with mathematical proof. This is especially true for the second part of the problem. There are 120 possible trains that can be tried

for the second part (i.e.,  $5!$  Or  $5 \times 4 \times 3 \times 2 \times 1 = 120$ ). Students could conceivably try every single possibility. However, being able to reason without trying all possibilities is an important mathematical skill and is something that students typically do not have a chance to practice in elementary school.

#### **3.3.1.4.3 What About the Problem Requires/Benefits from Collective Work?**

The features of The Train Problem create a context in which collective work is required and benefits both individuals and the collective. As alluded to in Table 3.1 The Train Problem takes nine days to complete. During this time, students work together to understand the conditions of the problem and reduce the problem in ways that make it solvable. Given the complexity of the problem, it is unlikely that any one person will solve it by themselves.

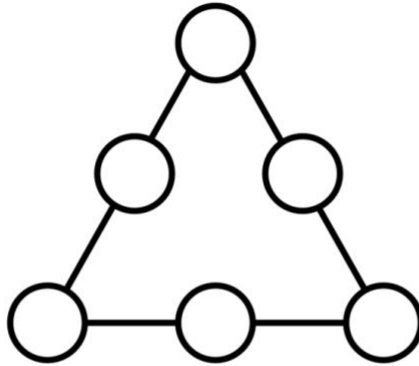
#### **3.3.1.5 The Triangle Problem.**

The Triangle Problem first asks students to find the combination of numbers such that the sum of each side of the triangle is nine (See Figure 3.9). Students might try to solve the problem by thinking about the different combinations of expressions that equal nine (e.g.,  $3 + 1 + 5$  and  $4 + 3 + 2$  equals 9) or by plugging numbers into the vertices. After solving for nine, students are challenged to determine what other sums are possible if all three sides of the triangle are to remain equal.

#### **Figure 3.9**

*The Triangle Problem*

### The Triangle Problem

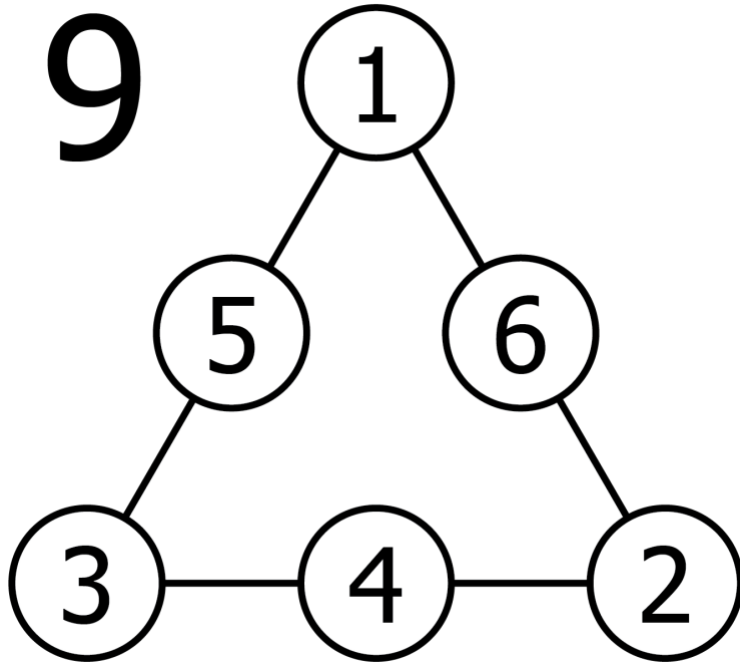


Place each of the numbers 1, 2, 3, 4, 5, and 6 in a different circle on the triangle so that the sum of the numbers on each side of the triangle is equal to 9.

The Triangle Problem has a finite set of solutions. That is, the only numbers that fit the conditions are 9, 10, 11, and 12. (See figure 3.10 for a solution for 9 and Appendix A for solutions for 10, 11, and 12). Unlike some of the other tasks, The Triangle Problem does not have physical materials to support with solving the problem. Despite this, the task is accessible in the sense that anyone can get started working on the problem.

#### **Figure 3.10**

*Solution of 9*



#### ***3.3.1.5.1 What Is the Main Mathematical Point?***

There are several features of the Triangle Problem that make it rich from a mathematical perspective. The students do some foundational work building number sense as they solve. They also work on the mathematical practice of making sense of problems by examining the constraints of the problem. These include the conditions that a number can only be placed in the triangle once (e.g.,  $4+4+1$  is not a valid response to the problem), only the numbers 1-6 are allowed, and the three groupings of numbers that are used must have the same sum. The problem also connects to geometry because students are finding sums such that the sums make an equilateral triangle. In secondary mathematics, these types of problems fall into the category of geometric algebraic tasks.



### ***3.3.1.5.2 Key Mathematical Concepts and Practices.***

Although the task is accessible to a range of learners, it is also challenging. Many students do not immediately find the right combinations of numbers to make all the different sums possible. In fact, students often begin to solve by using the trial by error method. That is, they may just plug in numbers to get the sum for one side and then move on to the next. However, thinking about the relationships among the numbers and the triangle could allow students to make connections with the constraints of the problem. For instance, there are three vertices that two sides (i.e., the vertex at the top and the two vertices in the corners). Knowing that allows students to strategically put numbers in the vertices such that that have the maximal utility. Within this problem students practice making sense of problems.

### ***3.3.1.5.3 What About the Problem Requires/Benefits from Collective Work?***

The Triangle Problem is the only task discussed here that was not a whole group discussion problem. It was first introduced as a warm-up problem. Typically, warm-up problems are designed to be completed individually. However, the teacher extended the Triangle Problem beyond the warm-up to include opportunities for mini discussions about the constraints of the problem and the reasoning students were using to try to solve the problem

## ***3.3.2 Student Recruitment and Admission Process***

To participate in the summer mathematics program, students are recruited during the previous school year. Program staff ask educators (e.g., superintendent, principals, teachers, etc.) from a neighboring school district to communicate about the summer mathematics program to students and families and to help identify potential participants.

A program flyer is distributed to educators along with a note suggesting they send the flyer to students who fit at least one of the following criteria:

- Lack of confidence or motivation in mathematics
- Score well on tests but have other difficulties such as completing homework or explaining their work and reasoning
- Have not done well on mathematics tests
- Are multilingual and/or need support with linguistic demands of mathematics learning

Care is taken to make clear that the program is open to all students and that it is not remedial.

Upon application, the only requirements for students are that they will: (1) attend the fifth grade the following fall, (2) participate in the program every day, and (3) commit to the daily components of the program (e.g., completing homework). If there are more applications than the number of openings, students are randomly selected to enter the program. In addition to students who are selected from the neighboring district, a few additional students are sometimes included from other school districts. Regardless of the number of students selected, the program organizers attempt to ensure that there is an equal number of boys and girls.

A distinctive feature of the admissions process is that the teacher does not examine students' prior academic and behavioral records, including assignment to special education. This feature could be considered problematic as some may argue that to meet students' individual needs, the teacher's access to students' background information is critical to preparing for classroom instruction and interactions with

students (Perouse-Harvey, 2020). A counterargument is that, given the purpose of the program, the teacher might want to avoid being influenced by others' past perceptions of students. This could not only help the teacher build authentic rapport with them, but also open space for the children to foreground or try out a different self (Marcus & Nurius, 1986). For example, students often end up with labels (e.g., smart, slow, lazy, etc.) that follow them throughout their educational experiences, shaping how others see them and how they come to see themselves. By refusing to engage with others' past perceptions of the students in the summer mathematics program, the teacher is intentionally attempting to use her power and responsibility to open space for them to take on positive and affirming identities that may have been denied to them in their prior classroom experiences.

### **3.3.3 Participant Sample**

For my dissertation study, I collected data for 28 students (14 boys, 14 girls) enrolled in the program during the summer of 2015. Because I did not have information about how students self-identify in terms of race and gender, references to students' racial and gender identities are based on my interpretation of students' self-presentations.

I selected a subset of five Black children (three girls and two boys) to interview and follow throughout the program. My participants were Arianna, Chandler, and La'Rayne (girls) and Jeremiah and Kasim (boys). I selected five children for two reasons. First, five students seemed feasible to interview and observe given how the students' days were structured. Each day included the following: breakfast, math class, lunch, museum, recess, tutoring, and preparation for students to go home. Due to this

structure, I often interviewed students in the morning during breakfast, during recess, and as they transitioned from tutoring to get prepared to go home. Second, I was interested in conducting multiple interviews with the same students to build rapport and to follow their experiences throughout the program. Five interviews were a reasonable number for me to be able to accomplish this goal.

I employed purposeful selection (Maxwell, 2013) to identify students who had both similar and different constructions of smartness. To determine how students constructed smartness at the beginning of the program, I created and administered a pre-survey on the first day of the program. In it, I asked students to provide examples of times they had felt smart in math and examined their responses to learn more about their perceptions and ideas about smartness (see Table 3.2).

**Table 3.2**

*Pre-survey questions connection to smartness*

<b>Pre-Survey Question</b>	<b>Connection to Smartness</b>
1. What do you think it means to be smart in math class? Please give at least two examples to explain your answer.	It's important to get a sense of what students think it means to be smart in math class and if possible, provide some examples of what that looks like.
2. Can you describe a time during the school year when you felt smart in math class? Please give an example.	Not only is it important to see what a student thinks it means to be smart, but it is also important for them to see themselves as such.
3. Can you tell me about someone in your school that you think is smart in math class? What does this person do that	Describing other people you know are smart and why you think they are smart is another dimension in understanding students' construction of smartness.

<p>makes you think that they are smart in math class?</p>	
<p>4. Do you think you are smart in math class? Explain why or why not.</p>	<p>It is important to see if students see themselves as smart and why. This question allows for students to make sense of it on their own terms.</p>
<p>5. What do you think a good math teacher does?</p>	<p>Teachers play an important role because often what and who they deem as good or smart directly informs students' interpretations of what it means to be smart and if they are smart. Although I didn't ask students specifically about what a teacher does to make a student smarter, this question gets at students' expectations of what a good teacher does which may have implications for how students make sense of the mathematics classroom.</p>

In addition to looking at students' responses to the pre-surveys, I observed their actions during the first two days of the summer mathematics program. I was especially interested in including students who participated in class in a variety of ways. For example, some students volunteered to go to the board, others asked questions of the teacher and/or their peers, still others participated by doing their work in their notebooks, etc. By considering both students' pre-survey responses and my initial observations of them, I was able to select a group of focal students who would allow me to explore potential similarities and differences in the construction and performance of smartness during the summer mathematics program.

Next, I describe each student and offer my observations of them, as well as the type of case each represents. My goal here is, first, to humanize them. Although they are participants in my study, they are people first, and I value each one of them. In

addition, it is important to contextualize the students to better understand who they are and why I chose them, which informs my analysis and has implications for my study.

### **3.3.3.1 Arianna.**

Arianna is a light-skinned Black girl who has long brown hair. She was quiet during instruction, answering some questions, but rarely asking the teacher for anything. During my initial observations, she appeared to prefer to either work by herself or with her seatmate, Helen. In her pre-survey, she said that being smart in math means knowing a lot of math, and she thought she was smart because she does well on tests. Despite this apparently traditional view of smartness in mathematics, Arianna mentioned in different artifacts the value of listening to others' ideas and learning from others. Arianna is an example of someone who may be perceived as smart based upon traditional ideas, but who does other things that could be seen as smart through a non-traditional lens.

### **3.3.3.2 Chandler.**

Chandler is a brown-skinned Black girl who wore glasses and long braids. Chandler came from a different school district than most of the students in the summer mathematics program and did not know any of the other children in the program. In class, Chandler was very quiet, although she answered some questions during whole-group discussions. She often wrote detailed notes in her notebook. In her pre-survey, when describing smartness, Chandler mentioned working hard, paying attention, and raising her hand. I selected Chandler as a participant because she seemed to think about smartness in ways that the other students did not associate with being smart in math class. That is, she highlighted working hard, studying hard, and paying attention in class.

### **3.3.3.3 La'Rayne.**

La'Rayne is a brown-skinned Black girl who often wore two braids. In class, La'Rayne participated frequently during whole-group discussions. In fact, she was one of the first students to go to the board to provide a solution to a mathematical task. She not only offered solutions, but she also listened to others' thinking, which was apparent when she voiced her peers' thinking during discussions. After I observed La'Rayne in class and reviewed her pre-survey, I was surprised to learn that she actually did not see herself as smart. I included her in my sample because she embodied, from my perspective, the student who does not perceive herself as smart even though others do.

### **3.3.3.4 Jeremiah.**

Jeremiah is a dark-skinned Black boy who wore a box fade haircut. Although he did answer some questions, he tended to work independently and was generally quiet during the first two days of the summer mathematics program. I found his pre-survey interesting because, when asked the question "Name someone who you think is smart," he wrote "IDK." He also wrote that he believed he was smart, despite getting answers incorrect sometimes. When I reviewed his notebook, I saw that he mentioned fractions were hard and when asked whether he had learned anything from anyone during class he wrote "IDK" again. I selected Jeremiah because of what he wrote and what he did during math class seemed to contradict each other. His actions in class seemed to project that he was confident but reserved, while his written responses seemed to indicate that he was unsure of his or others' smartness.

### **3.3.3.5 Kasim.**

Kasim is a light-skinned Black boy who wore an Afro. Kasim was very confident in his mathematical ability and showed it during the first two days. He often presented ideas at the board and wrote interesting notes about his work in his notebook. Even if he did not answer a question correctly or did not understand something, he was confident he would find the answer. Despite Kasim not completing the entire pre-survey, the information he did provide seemed typical of a student who does well in math. For example, he mentioned getting a particular number of answers right, getting good grades, and earning high scores. I selected Kasim because he seemed like a student who was accustomed to being perceived as smart and believing he is smart.

## **3.4 Data Sources**

In this study, I use a range of qualitative data sources to track the experiences of these five fifth-grade children during the summer mathematics program. Following Yin (2017), my study is a multiple-case study in which each case, or unit of analysis, is one fifth-grader's experiences. Given my research questions, I examined the following sources of data for this study: student pre- and post-surveys, student interviews, student artifacts (i.e., notebooks), video recordings of classroom instruction, and classroom artifacts (e.g., classroom charts). Given my interest in exploring Black learners' conceptions of smartness, a multiple-case study was most appropriate.

### **3.4.1 Student Data**

Student data refers to the data I collected to capture student's thinking. I designed some of these data sources (i.e., the pre- and post-surveys; student interviews), while others were collected as part of the larger research project. The student data used for this



study comprises pre- and post-surveys, interviews, and notebooks (see Table 3.3). In the subsections that follow I describe each data type, provide an example of each, and describe its relevance for the study.

**Table 3.3**

*Student data collected*

<b>Data type</b>	<b>Arianna</b>	<b>Chandler</b>	<b>La'Rayne</b>	<b>Jeremiah</b>	<b>Kasim</b>
Notebook	x	x	x	x	x
Pre-survey	x	x	x	x	x
Post-Survey	x	x	x	x	x
Interview 1	x	x	x	x	x
Interview 2	x	x	x	x	x
Interview 3	x	x	x	x	x

### **3.4.1.1 Surveys.**

I created two paper-and-pencil surveys designed to capture students' ideas about what they thought it means to be smart in mathematics, who they thought was smart, and if they thought they were smart in mathematics. I administered the pre-survey on the first day of the summer mathematics program before students went to class. The purpose of the pre-survey was to understand the background knowledge learners were bringing into

the summer mathematics program, which could come from a variety of experiences they had prior to attending. I administered the post-survey on the last day of the summer mathematics program. It asked similar questions to the pre-survey but focused specifically on the summer mathematics context. For example, the first question on the pre-survey asked students “What does it mean to be smart in math class?”, whereas the first question on the post-survey asked, “What does it mean to be smart in our summer math class?” The purpose of the post-survey was to capture what learners noticed about smartness during the summer mathematics program.

#### **3.4.1.2 Interviews.**

I conducted 15 interviews total comprising one interview with each learner at the beginning, middle, and end of the program. During the first interview, I followed up on students’ responses to the pre-survey as well as tried to get a sense of each learner’s mathematics experiences. In the second interview, I focused on further probing what students said in prior interviews, what they wrote in their notebooks, and what they did in class to understand how they were making sense of the summer mathematics program. In the third and final interview, I focused on what students learned from the summer mathematics program.

#### **3.4.1.3 Notebooks.**

As a standard feature of the program, each student was given a notebook to write in each day. The notebooks contained warm-up problems, tasks that learners worked on during instruction, and end-of-class checks (see Figure 3.11 for a sample notebook page). The notebook also included students’ responses to a routine called *Note to Self*. *Note to Self* was a writing prompt that the teacher often deployed before transitioning to another

segment of instruction. The prompt was used to capture students' thinking in-the-moment, to record if they were stuck on a problem or unclear of their own or other students' ideas, or as an opportunity to reflect. These were not shared publicly, although the teacher had access to them in the students' notebooks (except in cases where they covered them with post-its to signal private writing). In addition to students' work on problems, the notebooks also contained writings and drawings that students wrote on their own. The notebooks are rich in that they contain students' thinking throughout the summer mathematics program, including those things they may have written that they would not say during classroom instruction.

**Figure 3.11**

*Chandler Day NTS*

Add three different numbers together to equal 9.  
 The numbers you can choose from are 1, 2, 3, 4, 5, or 6.  
 How many different three-number equations can you write like this?

Conditions

$1, 6 + 3 = 9$   
 $2, 5 + 4 = 9$

~~1, 2, 3, 4, 5, 6~~  
~~2, 3, 4, 5, 6~~

6 5  
 3 3  
 2 3 6 4 3 1

I remember that these numbers are the same numbers as the other page.

equation

note → I think I did well but it is really hard to do but it takes practice practice makes perfect!

center

### **3.4.2 Classroom Data**

Classroom data refers to data that was collected as part of the larger research project, as well as artifacts that were created for the purposes of teaching and learning in the classroom. These data sources comprise video recordings of classroom instruction, lesson plans, classroom artifacts, and the teachers' comments to students in their notebooks. In the subsections that follow, I describe each data source and its relevance for my study.

#### **3.4.2.1 Video Recordings of Classroom Instruction.**

The video recordings of classroom instruction are high-quality and include two camera angles (one angle that always focuses on the teacher and another angle that focuses on students) and professional sound recording. The teacher wore a lavalier microphone clipped to her lapel, and there were table microphones at the students' seats to capture as much dialogue as clearly as possible during class. The recordings capture all the instruction each day from before the first students enter the classroom in the morning until the last students leave the classroom after completing the end-of-class check.

I used the video recordings to analyze how students participated in the mathematics classroom, for triangulation with information captured in the interviews and surveys, and to develop probes for future interviews. Additionally, I paired video recordings with student artifacts to investigate how students took up the mathematical content and their ideas about the summer mathematics program in-the-moment, as well as to triangulate their actions with comments they made in the interviews or during math

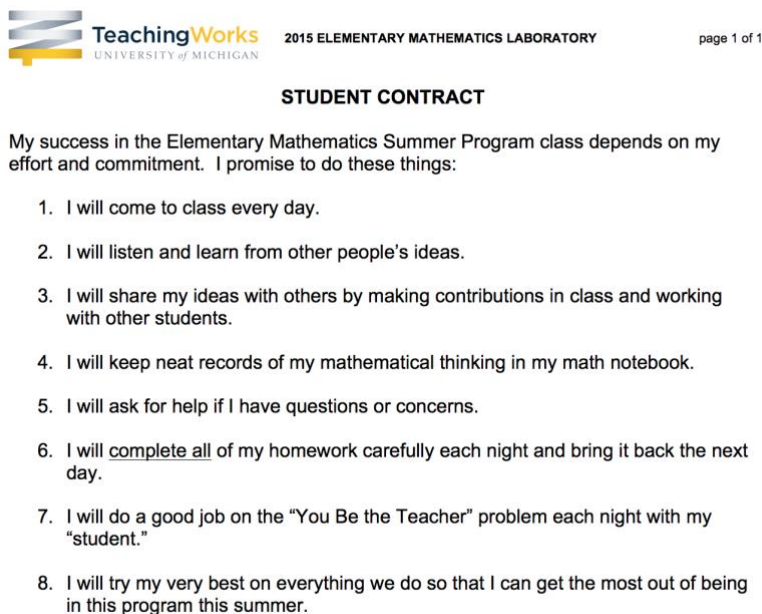
class. I also used the videos to analyze the teacher's practice during classroom instruction.

### 3.4.2.2 Classroom Artifacts.

Classroom artifacts include the teacher and student contracts, as well as posters of students' work. Student contracts were designed to create a culture within the summer mathematics program that included expectations that students could hold themselves and each other accountable to. For example, students were expected to come to class every day and listen to other people's ideas. (See Figure 3.12).

**Figure 3.12**

*Student Contract*



Classroom artifacts such as the student contract are important because they signal messages about what it means to be smart and to be a doer of mathematics in

the summer mathematics program. Additionally, they help to illuminate ideas that were taken up during whole group discussions including whose ideas were taken up.

### **3.5 Data Analysis**

I used open coding for each research question to analyze my data. In this section, I describe the data analysis for each research question.

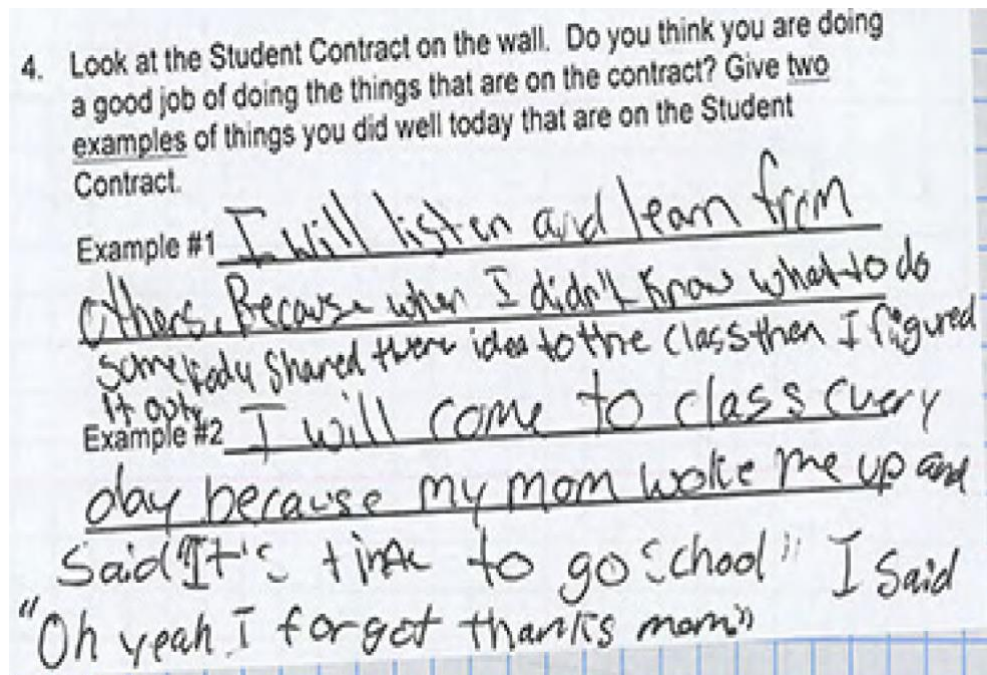
#### **3.5.1 Research Question #1**

The first research question is: How do Black students describe what it means to be smart in a summer mathematics program? To answer this question, I analyzed students' notebooks, surveys, and interviews. For each data source I took an open coding approach to analyze the data (Corbin & Strauss, 2007). Open coding is an approach to data analysis that allows one to start with the data to attempt to understand a particular phenomenon. My goal was to understand the students' conceptions of smartness by identifying relationships and patterns in their responses.

I began by coding each student's notebook for each day of the summer mathematics program. I focused on the End-of-Class (EOC) Checks and how students responded to prompts (e.g., Note-to-Self (NTS)) the teacher gave them. For the end-of-class checks, I examined students' responses to questions such as: "What are smart things you did today?" "What were your goals for today?" and "Who do you think is smart in our class?" These responses helped me to understand how students think about smartness. I also examined how students responded to the teacher's prompts as well as their own writings and drawings to understand what students identified as most salient. For example, on Day 2 of the end-of-class checks Arianna responded to a question about how she was doing on her goals (see Figure 3.13).

**Figure 3.13**

*Arianna's Day 2 End-of-class Check*



After coding each notebook, I coded both surveys and the interviews in the same manner. I went through two rounds of coding in this way to get clarity on the codes and further develop the codebook. After all the coding was completed, I created a case for each student by looking across the coded artifacts for each. I used NVivo qualitative coding software to run different queries, including frequency, matrices, and crosstabs to identify important themes across student cases. I then examined the common themes for each student individually and wrote a memo for each student detailing how each theme was specific to each student. I considered the codes across students first to get a sense of the similarities and differences in how students were thinking about and constructing smartness in general. I then examined each student specifically to provide

more nuance and context. Often examining individual students led me to review the artifacts again to better understand and make sense of what was coming up in the data.

After completing a general analysis, I focused in on the “smart” code. For each student, I looked at the related memos to determine what conceptions of smartness it appeared each student had. I then reviewed the artifacts for evidence to substantiate, refine, or alter my analysis of their conceptions. I also analyzed each conception to interpret and explain what it meant for the student. Finally, I again looked across cases to determine if students had the same conceptions or different ones and used this analysis to further refine my analysis.

### **3.5.2 Research Question #2**

The second research question is: How does the teacher communicate what it means to be smart during a summer mathematics program. To address this question, I focused specifically on the teacher’s moves during classroom instruction and what they seemed to convey about what it means to be smart in the summer mathematics program and who is smart. The purpose of this question is not to make causal claims about what the teacher is doing in relation to what the students wrote and did, or to claim that the students heard or read these messages. Rather, focusing on the teacher makes it possible to analyze how she enacts practice as well as what ideas about smartness might be being signaled to students. To address this question, I analyzed video recordings of classroom instruction and the corresponding transcripts.

In the first phase of analysis, I watched all the videos from beginning to end of the mathematics instruction. This provided a holistic picture of what was going on in the classroom which was important to understanding the signals the teacher employed to



communicate various messages about smartness. Because instruction lasted for 2 hours and 15 minutes, the teacher provided a 10-minute break for students approximately midway through instruction each day. Therefore, each video was divided into two parts, before break and after break. As I watched each video, I segmented it based upon the whole-group discussions that took place and took observation notes. I focused on whole group discussion for two reasons. First, whole-group discussions are where the teacher would be likely to signal messages about smartness and what it means to be doers of mathematics to the class. Second, although messages about smartness could be communicated in small group and one-on-one interactions, the quality of data available for whole-group discussions was much more consistent than in these smaller scale interactions.

In the second phase of analysis, I re-watched the whole-group discussion segments only. While re-watching the segments I kept records of the following: the day the clip took place, whether the clip took place before or after the break, the time stamps of the clips, the mathematics problem discussed, a summary of the discussion captured in the clip, which students spoke during the clip, and initial takeaways about what messages were being communicated about what it means to be smart and who is smart as well as the teacher's moves to signal these messages. If the clip was longer than nine minutes, I sub-segmented the video into five-minute intervals. I selected five-minute segments to reduce the size of the clip, while still allowing me to accurately describe what was going on. Table 3.4 below summarizes the number of whole-group discussions that took place each day and during the entire summer mathematics program.

**Table 3.4**

*Total and daily number of whole group discussions during the summer mathematics program*

Day	# of Whole-group discussions
1	3
2	5
3	5
4	6
5	3
6	4
7	3
8	3
9	3
10	2
<b>Total</b>	<b>38</b>

Once I had segmented the whole-group discussions, I drew upon the student data and the work of Maher and Martino (1996) to identify episodes and critical moments during discussions. Maher and Martino defined critical events as events that serve as a key point in students' trajectory with relation to their development around proofs. I adapted that idea to identify what I call critical moments, which are moments that came up in students' artifacts that seemed to indicate moments in the class that might communicate ideas about what it means to be smart or a doer of mathematics.

These critical moments took place during larger episodes that capture the full activity in a segment of interest (e.g., discussion of mathematical problem).

I selected three episodes based upon critical moments that were identified by the students. One critical moment was identified by Arianna, who mentioned in our first interview that she thinks it's important to listen to others and named the mathematical task "three times a light green," which took place on the first day of the summer mathematics program, as a relevant moment that embodied this idea. Another moment came from Kasim, who highlighted that he learned from Michio on an end-of-class check. Kasim's mention of learning from Michio stood out because he had earlier asserted that he did not learn from his peers in his fourth-grade class, while also describing himself as the "smartest" and "top star" student. The third moment came from Chandler who, on an end-of-class check, named revising her thinking as one thing she did that was smart. Each of these excerpts from the focal students' artifacts hinted that something may have been communicated about smartness in the related episodes.

In addition to the moments identified by students, I identified two moments that were relevant based on which students were acknowledged, what the teacher acknowledged about the students, and the messages that the teacher might have communicated. Twice during the summer mathematics program, the teacher asked students to come up to the document camera to show the work that had done in their notebooks. These moments served as opportunities to highlight the competence of students who normally did not share during whole group. I selected one of these episodes to analyze as an opportunity to see what messages the teacher communicated. The final moment I selected was the only time an episode of this type

occurred during the summer mathematics program. After a couple of days of students not working effectively together during whole group, the teacher called students over to the rug. During this episode the teacher had a discussion with students to get their suggestions on what they thought the class could do to work together more effectively. This episode provided multiple opportunities to analyze how the teacher signaled what it means to be smart, as well as to hear student voices and perspectives on their own roles as mathematics learners. Collectively these five episodes, moments within them, provide an idea of the type of messages and signals the teacher communicated during the summer mathematics program.

## Chapter 4 Findings

### 4.1 Introduction

To begin this chapter, I reiterate, for reasons provided in chapters 1 and 2, that my project here is to represent and probe Black learners' perspectives and what they see with respect to smartness, not to prove that they are smart. Before I present and analyze students' conceptions of smartness, I highlight two key points.

First, students' conceptions of smartness are representative of their ordinary brilliance. Ordinary brilliance counters racialized patterns that position Black brilliance as exceptional (Gholson & Martin, 2019). It also resists the tendency to focus only on Black students' successes or failures. Instead, ordinary brilliance includes the high, the lows, and everything in between — in other words, the messiness of Black children's brilliance. The second point is the summer mathematics program that is the setting for this study serves as a stage with opportunities to see students. Seeing the summer mathematics as a stage, emphasizes that students come into the program with beliefs, experiences, and skills. Their beliefs, experiences, and skills are demonstrated in varied ways through their agency and the opportunities that are and are not provided.

Together, these two points— about ordinary brilliance and about the notion of a stage on which that brilliance can be seen — underlie the findings in this chapter. I argue that we can see the Black children in two intersecting lights: one where students are describing and demonstrating on this stage and the other, that other factors (e.g., beliefs, experiences, and skills) combine with the context of the stage to contribute to

what students are describing and demonstrating. With this in mind, I provide descriptions of these Black children and their views about themselves and about math, about what it means to be smart, and about the different things it seems they are pulling from their environments to describe how they see it. Through this process, the reader will see the complex picture that is painted across six children.

In this chapter, I address my first research question, How do Black students describe what it means to be smart in a summer mathematics program? To answer this question, I focus on five Black learners, Arianna, Chandler, Jeremiah, Kasim, and La’Rayne. I analyze their artifacts which comprise interviews, surveys, and their notebooks to identify their conceptions of smartness. I take conceptions to mean students’ beliefs and descriptions about smartness in mathematics. In each section of the chapter, I identify and unpack conceptions for each student using the data. Through my analysis I found that each student had at least one conception of smartness and at most five conceptions of smartness (see table 4.1). After identifying and unpacking the conceptions students have, in the last section I describe three themes across the five students: 1) Superficial Similarities, Substantive Differences, 2) Conceptions as Coping Strategies and 3) Malleability.

**Table 4.1**

*Students’ Conceptions of Smartness*

Names	Conceptions
Arianna	(1) knowing a lot of math  (2) trying your best and working hard, and

	(3) taking your time on tasks
Chandler	(1) being smart is about knowing a lot of math; (2) being smart is about studying and working hard; (3) being smart is about participating in class (listening, paying attention, and raising your hand); (4) being smart is about revising your thinking; and (5) being smart is about helping and receiving help from others
La'Rayne	(1) getting correct answers and (2) keep trying
Kasim	(1) getting correct answers, and (2) sharing ideas publicly
Jeremiah	(1) knowing math

## 4.2 Arianna

**Figure 4.1**

*Arianna in class*





The very first time I met Arianna; she was like an open book. She was happy that I interviewed her and would often tell me stories while in transit before and after the interviews, like the time she learned karate. She also was very excited about the summer mathematics program and even in the last interview asked if she could come back next year and the year after.

Arianna talked about smartness in traditional ways. For example, her primary definition of smartness in mathematics was that smartness meant knowing a lot of math. She told me that she thought she was smart because she knew multiplication, addition, subtraction, and fractions. Despite voicing these more traditional views of smartness, she also talked about how much she enjoyed collaborating with other students, especially with her seatmate Helen:

**Charles:** Okay. So, in this math class with Dr. Ball, who do you work with the most in class?

**Arianna:** Uh...

**Charles:** Like, which classmate?

**Arianna:** Helen.

**Charles:** Okay.

**Arianna:** She's very nice and she helps me a lot. We help each other out.

**Charles:** Okay.

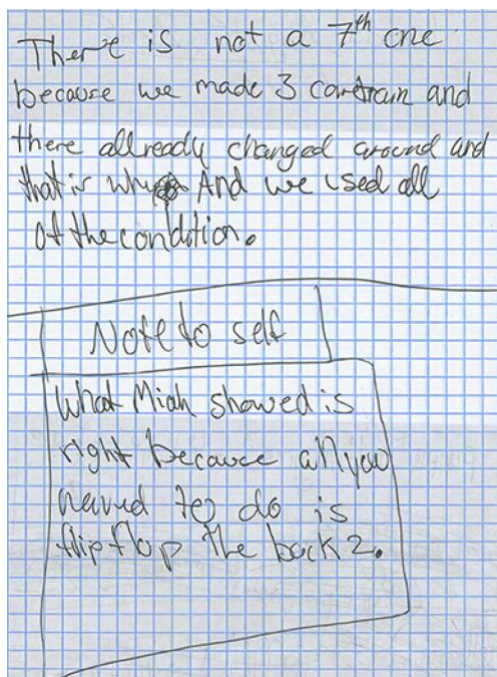
**Arianna:** Cause one day it was like, we would help each other about the, on like, the train problem. I'm like, "Oh, Helen. I give up." And she's like, "Aw, don't give up. Don't give up. You know, it's just- We're, we'll get this. We'll get this." And after that, I said, "Okay."

During the summer mathematics program, Arianna didn't always share her thinking with the class, especially in the first week. When asked why she didn't like to share, she told me that in previous classes her peers would laugh at her if she had the wrong answer. She said that at one point she even had to have a parent-teacher conference about it.

However, when Arianna did share her ideas, she made important contributions to the group's discussions. She was especially adept at revoicing other students' thinking to make ideas clearer. For example, during the second week she revoiced Miah's proof, which was an important idea that helped the class make progress on the train problem part 2.

**Figure 4.2**

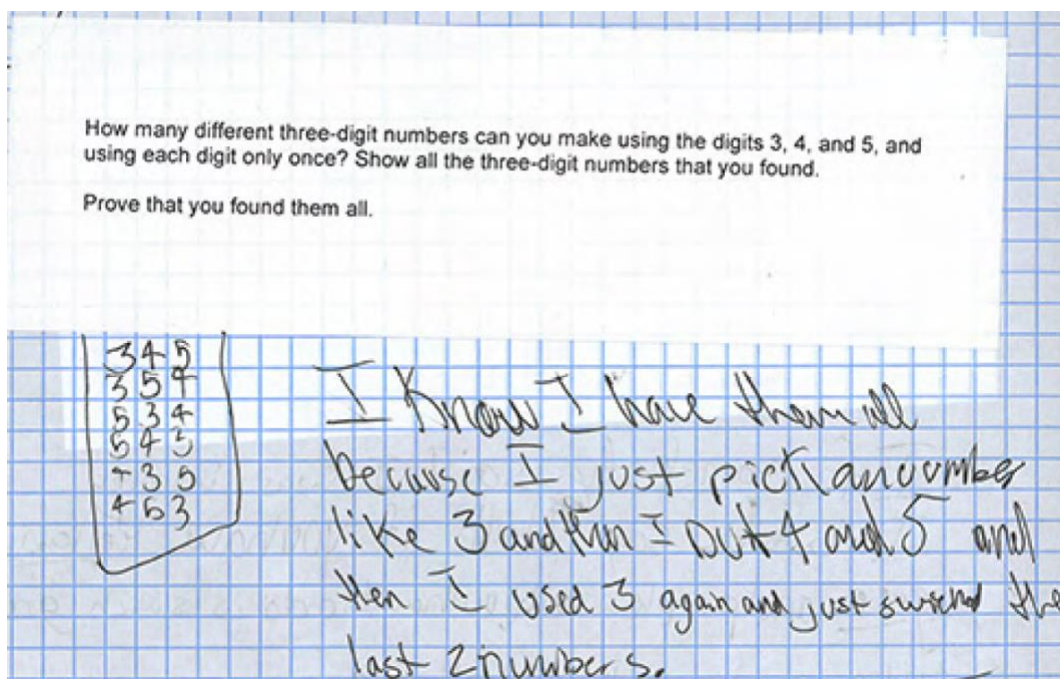
*Arianna NTS*



She also became more comfortable sharing her own ideas as the program progressed. An example of this came the day after Miah's proof, when Arianna explained her answer to the 3-4-5 Warm-up Problem, showing that there were 6 ways to arrange 3 digits and connected it to Miah's proof.

**Figure 4.3**

*Arianna 3-4-5 Warm Up*



Additionally, during one whole group discussion, the teacher told the class that Arianna has ideas that others could learn from.

#### **4.2.1 Arianna's Explicit Definitions of Smartness**

Across the surveys and interviews, Arianna named three conceptions of what it means to be smart in mathematics: (1) knowing a lot of math, (2) trying your best and

working hard, and (3) taking your time on tasks. I unpack each of these conceptions in the sections below.

#### **4.2.1.1 Knowing a lot of math.**

For Arianna, knowing a lot of math was connected to doing well on tests, doing difficult math, and learning things that she didn't know before. In her pre-survey responses, Arianna exclusively described being smart in math as knowing a lot of math. In fact, in response to the question "What does it mean to be smart?" she wrote, "That you know a lot of math." When asked if she was smart, she responded, "Yes because we would do [addition, subtraction, multiplication, and division] tests to see how good we were." In addition, when asked if there was someone in her class, she thought was smart in math and why, she talked about a student named John. She said he was smart because he had high scores. When asked about what a good teacher does, she said, "Teach me hard math."

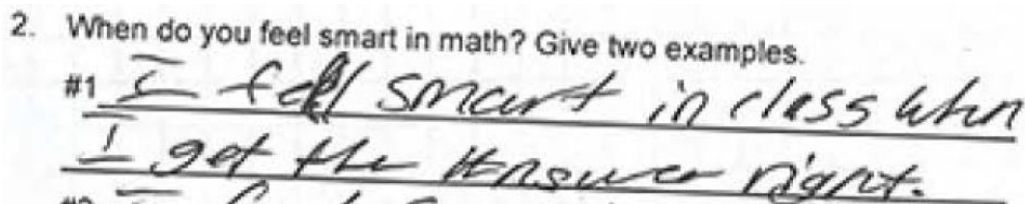
Each of Arianna's responses to the pre-survey questions were connected to knowing math in ways that were justified through test scores. Later in the pre-survey, when she identified the teacher's responsibility to teach hard math, she meant that a good teacher teaches math that is above grade level. The points Arianna raises are consistent with traditional views of being smart at mathematics. That is knowing a lot of math is connected to traditional values that students get answers right and get high scores on tests. The idea that you learn or know math that is "beyond" or "above" where you currently are is a part of the culture that to be smart means to know things others don't or things you're not supposed to learn. Arianna's identification of what a "good" teacher should do reinforces the idea of knowing a lot generally, as well as the

assumption that students who learn things that are “beyond” grade level are exceptional or gifted.

Arianna’s conception of knowing a lot of math only came up twice outside of her pre-survey, both times during the second week of the summer mathematics program. The first instance occurred on one of the end-of-class checks that asked about smartness in math. She wrote, “I felt smart in math when I got the answer right.”

**Figure 4.4**

*Arianna EOC Check*



The second instance occurred on the post-survey. Then, when asked about the teacher, she wrote that the teacher made her smarter because she helped the students learn things that they didn’t already know. In both of these cases, Arianna’s responses revert back to the idea of smartness as knowing a lot of mathematical content. Further, she again reiterates that knowing a lot of math is about getting answers right. However, her response to the question about the teacher also leaves room for the possibility that there is math that smart students don’t already know, and it’s the teacher’s job to further contribute to each student’s knowledge base. As such, Arianna’s response here affirms that the teacher plays an intricate role in smart students knowing a lot of math.

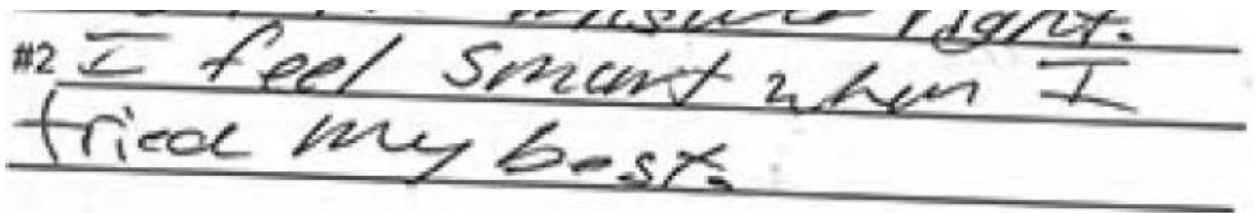
#### 4.2.1.2 Trying Your Best and Working Hard.

Arianna described trying her best and working hard as another conception of what it means to be smart in math. Working hard and trying your best are separate from grades, speed, or test scores. In this conception, the emphasis is on *doing*. In other words, knowing a lot of math doesn't necessarily mean that one works hard or tries their best; nor does trying your best and working hard necessarily mean one knows a lot of math; but regardless of how much math one knows, working hard and trying your best is a way for students to *enact* smartness

For example, in our first interview I asked Arianna if there were other reasons, she thought she was smart in math, beyond what she'd written about in her pre-survey. She added that she thought she was smart in math because "I work hard." She continued to describe trying her best as a reflection of smartness throughout the program. For instance, on the end-of-class check during the second week, when asked about smartness, she wrote, "I feel smart when I tried my best."

**Figure 4.5**

*Arianna EOC Check*



#2 I feel smart when I tried my best.

When asked if she had ever not felt smart, she responded she always felt smart because she always tries her best. These responses stand out because not only is

working hard something that a student does, but it is also something that she can control (e.g., as opposed to students' inability to control whether the teacher teaches them difficult math). Stating that she works hard as a reason for why she is smart highlights agency as a component of her understanding of smartness. Thus, "I work hard" and "I always try my best" can be understood as "I am smart because I exercise control over how I do math."

She again affirmed this conception of working hard and trying her best in her post-survey. In particular, when asked what she thinks it means to be smart in the summer mathematics program, she specifically named "... to try your best" as a key indicator of smartness. Additionally, when asked about specific times she felt smart during the summer mathematics program she responded, "I feel smart when I tried my best on the triangle problem because I almost got it!!!!!" and "When I almost got the train problem." When asked to name someone who is smart in the summer mathematics program she wrote, "Everybody because they try their hardest!!!!!"

**Figure 4.6**

*Arianna Post-survey*

1. What do you think it means to be **smart** in our summer math class? Using complete sentences give two examples to explain your answer.

#1 To persevere!

#2 To try your best!

2. Can you describe a time during summer math class when you felt **smart**? Using complete sentences give two examples.

#1 I feel smart when I tried my best on the triangle problem because I almost got it!!!

#2 When I almost got the train problem.

3. Can you tell me someone in our summer math class that you think is **smart**? Using complete sentences explain what does this person do that makes you think that they are **smart** in math class?

Everybody because they try their hardest!!!

In each of these examples, Arianna named trying your best as a key indicator of smartness even in cases where she didn't actually get the right answer (i.e., she "almost" got the triangle and train problems). Here, her acknowledgment that working hard and trying your best as justifications of smartness de-emphasizes the importance of right or wrong answers, grades, or tests. Instead, her responses reflect the idea that students exercising agency over their learning reflects smartness. Working hard and trying your best are things students can control and, in these responses, they are just as important to defining smartness as more traditional classroom markers.

#### 4.2.1.3 Taking Your Time on Tasks.

The third and final conception of smartness that Arianna surfaced is the importance of taking your time on tasks. Like working hard and trying your best, taking your time on tasks as a conception of smartness emphasizes what students can *do* to

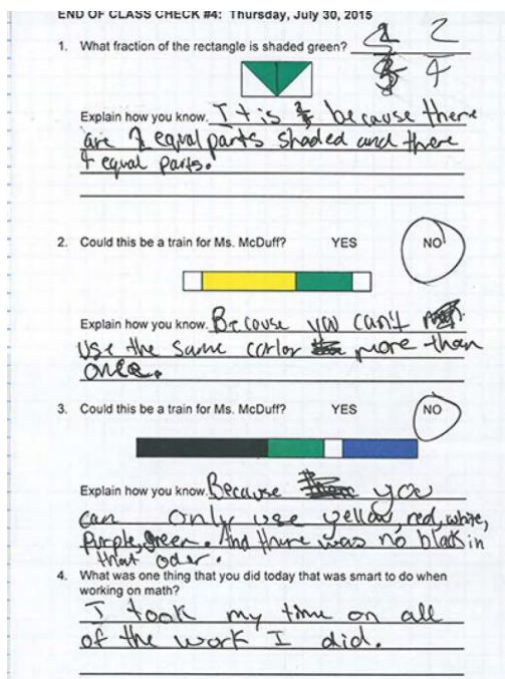


help themselves learn more math. In addition, taking your time runs contrary to the evaluation of speed, which is often emphasized in math.

On one of the first week's end-of-class checks, Arianna said, "I took my time on all of the work I did" when asked to name one thing she had done that was smart.

**Figure 4.7**

*Arianna Day 4 EOC Check*



Further, in my second interview with Arianna, I asked if there was anything that she did that she thought was smart but hadn't already named. Although at first, she said she didn't know, she later said, "Cause, I take my time."

Taking one's time is important in mathematics but often runs counter to the more dominant classroom idea of quickly getting answers right as a marker of smartness. Normally, the need to take your time only arises in classrooms as a result of students

making minor mistakes as they are solving problems. This leads to teachers emphasizing the need to slow down and take your time in order to ensure accuracy. However, this more typical insistence on taking one's time as a corrective for careless errors may lead students who work more slowly to think of themselves as slower, less capable thinkers who need the time to understand and convey their thoughts.

Arianna doesn't talk about taking your time in this way, though. Instead, she seems to equate taking her time with a more positive conception of her own smartness. Taking her time could allow her to be meticulous in her work, for example, ensuring that she could really focus on learning and being able to convey her thoughts. Further, her conception of taking your time on tasks is a practice that, like working hard and trying your best, students have agency over. The conception of "taking your time on tasks" demonstrates Arianna's agency as well as highlight an internal characteristic of smartness, which contrasts the conception of "knowing a lot of math"

#### **4.2.1.4 Summary.**

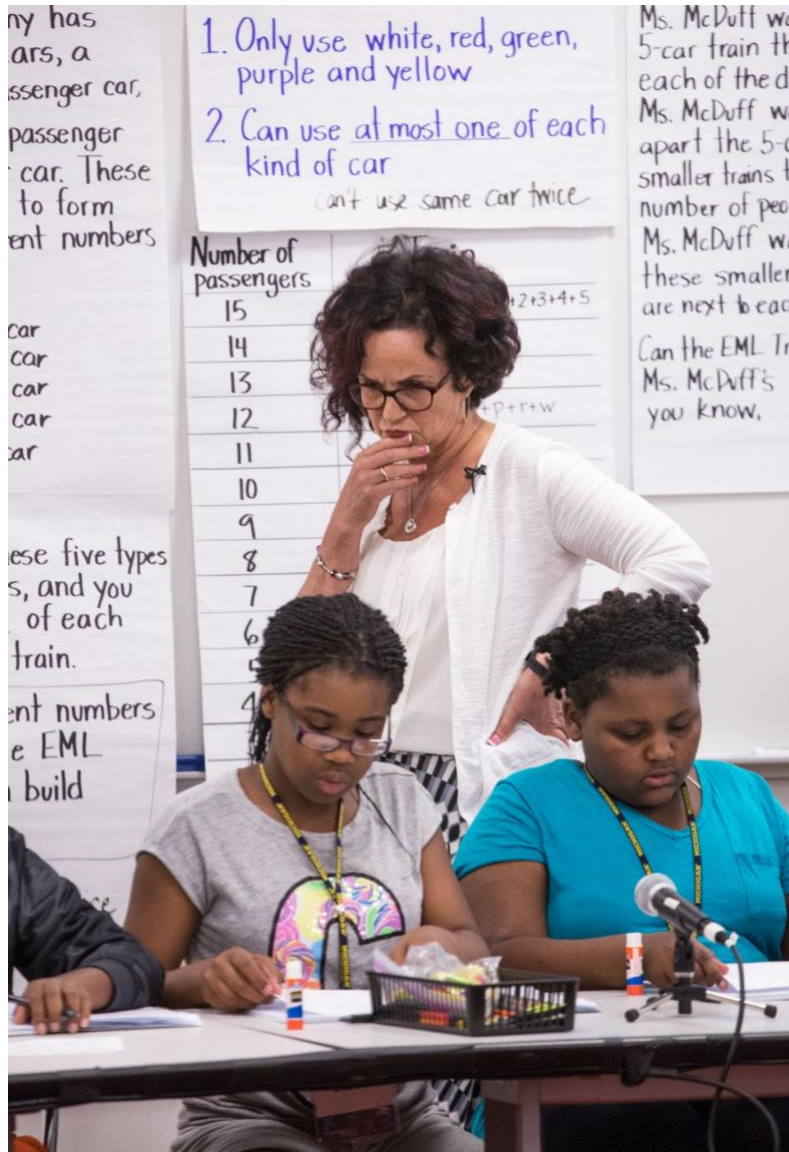
Although Arianna's initial conception of smartness focused squarely on knowing a lot of math, her responses to the end-of-class checks and interview questions during the course of the summer mathematics program indicated a more expansive definition of smartness. In particular, Arianna identified working hard and trying her best as well as taking her time as important aspects of smartness. This more expansive definition of smartness shifts the focus away from an exclusive reliance on test scores and grades as measures of smartness. Instead, her definition included the process of learning and knowing a lot of math. In other words, she began to foreground ways of *doing* smartness.



### 4.3 Chandler

Figure 4.8

*Chandler in class*



Chandler appeared to exemplify the student who is compliant, no-nonsense, and does everything “right” (i.e., follows adults’ instructions and doesn’t get into trouble). For instance, during the second week of the program when the teacher took suggestions from students about how the class could work better together, Chandler’s suggestion was to take away breaks from the students who wouldn’t follow instructions. Chandler’s response is indicative of her view that in math classrooms that learning is the primary purpose and anyone that disrupts learning should be removed so that students that want to learn can.

Although Chandler described some traditional ways of defining smartness, such as using grades, her responses and actions during class often revealed alternative ways of thinking about smartness. For example, she talked about working hard, paying attention, and revising her thinking as key aspects of smartness that she valued. In line with this thinking in interviews these ideas came across consistently. One characteristic that consistently came out for Chandler is listening and learning from others. For example, when asked about the importance of learning from others and why she responded:

Yes, cause your answer might not be wrong, but their answer might be a little bit better...I would listen to them because we might have the same answer, or ours might be different from each other and we all, we can learn from other people.

Chandler continued to elaborate when talking about listening to her peers that there are not “right and wrong answers for everything” and that “you can learn from mistakes”. The previous transcripts highlight the orientation that Chandler has about mathematics classrooms.

Despite Chandler not going to the board often, instead preferring to stay at her seat and explain or revoice the thinking of others she was very attuned to what was going on in class. Although quiet, I noticed from Chandler’s notebook and conversations with Chandler had a confidence, maturity, and humility that she carried that stood out from the other students in the program.

#### **4.3.1 Chandler’s Explicit Definitions of Smartness**

Chandler shared five explicit conceptions of smartness across her interviews, surveys, and notebook entries: (1) being smart is about knowing a lot of math; (2) being smart is about studying and working hard; (3) being smart is about participating in class (listening, paying attention, and raising your hand); (4) being smart is about revising your thinking; and (5) being smart is about helping and receiving help from others.

##### **4.3.1.1 Knowing a lot of math.**

One conception of smartness Chandler shared with other students was that smartness means knowing a lot of math. In this case, as with Arianna, knowing a lot of math refers specifically to knowing mathematical content like multiplication facts, fractions, etc. A part of knowing a lot of math for Chandler also included being able to figure out problems on your own. She made this point in her responses to both the pre- and the post-survey when asked “What do you think it means to be smart in math class/our summer math class?” (See Figures 4.9 and 4.10).

## Figure 4.9

### Chandler Pre-survey Question #1

1. What do you think it means to be **smart** in math class? Please give at least two examples to explain your answer.

To be smart means to know a lot of things and figure out things on your own. ex.  $11 \times 4 = 44$  I figured  $11 \times 9$  on my own. To be smart mean to mean that u got this, example if I say I can't do it then I realize I can do it because im smart.

## Figure 4.10

### Chandler Post-survey

1. What do you think it means to be **smart** in our summer math class? Using complete sentences give two examples to explain your answer.

#1 I think it means that you can do your work and know a lot of stuff.

Both of these responses highlight the idea that knowing a lot of math is a critical component of mathematical smartness. Chandler's mention of knowing a lot of "stuff" refers to knowing mathematical content such as multiplication facts, as well as numbers and operations. They also illustrate the importance of being able to do work on your own. By putting these two conceptions together, it seems that Chandler understands "knowing a lot of math" as something that individuals do independently. The idea of

working independently or by yourself reinforces the traditional views of mathematics that to be smart in math is an individualistic process. One may be able to work collaboratively, but one is really smart when they can learn or do math by themselves.

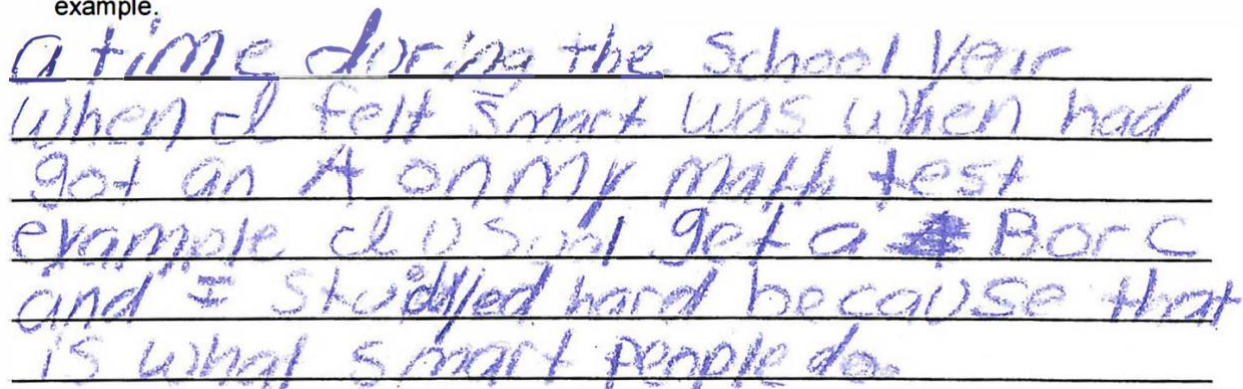
#### 4.3.1.2 Studying and Working Hard.

Chandler repeatedly brought up the ideas of studying and working hard as markers of smartness. Unlike some of her other conceptions of smartness, studying and working hard highlight the role of effort in both defining and embodying smartness. She first mentioned the idea of studying by acknowledging that she got an A on a test because she studied hard. She concluded, “that is what smart people do” (see Figure 4.11).

**Figure 4.11**

*Chandler Pre-survey Question #2*

2. Can you describe a time during the school year when you felt **smart** in math class? Please give an example.



A time during the school year when I felt smart was when had got an A on my math test example I usual get a B or C and I studied hard because that is what smart people do.

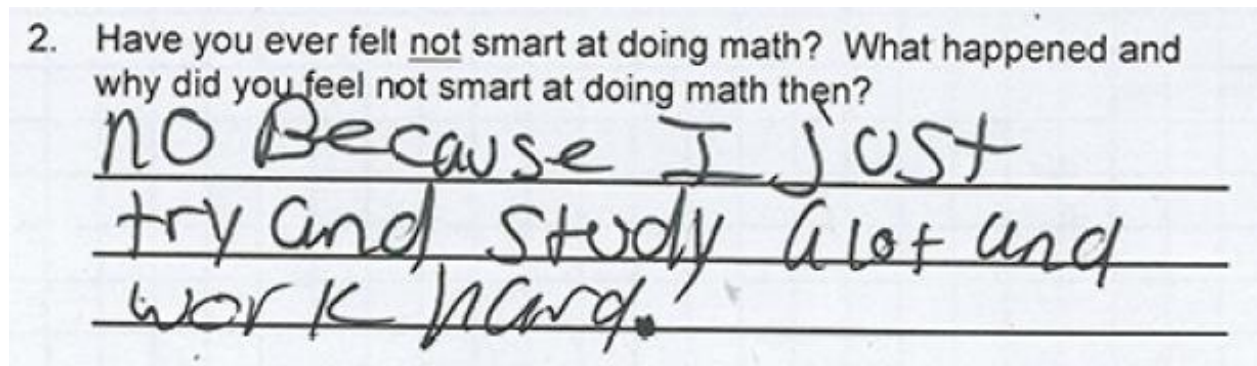
*Note.* Chandler wrote: “a time during the school year when I felt smart was when had got an A on my math test example I usual get a B or C and I studied hard because that is what smart people do”



Later, on the Day 9 end-of-class check when she was asked if she ever felt not smart at math, she responded that she never feels that way because she studies and works hard (see Figure 4.12).

**Figure 4.12**

*Chandler EOC Check*



From her responses to these questions, it seems that studying and working hard are key behaviors that Chandler uses to define smartness. These behaviors are things that she can do and that she is in control of. Within these responses Chandler also moves between a conception that focuses on grades (*external*) to one that focuses on *doing (internal)*, highlighting the complexity she has with her conceptions of smartness. Having a conception of smartness that is different from traditional reviews is useful as it provides another important measure for Chandler to identify smartness.

**4.3.1.3 Participating in Class.**

Chandler talked about participation as listening to others, paying attention, and raising your hand. She described these as things that smart students do to learn. For

instance, in her pre-survey, she described a student named Sydnee as smart because she pays attention and raises her hand in class (see Figure 4.13).

**Figure 4.13**

*Chandler Pre-survey Question #3*

3. Can you tell me about someone in your school that you think is **smart** in math class? What does this person do that makes you think that they are **smart** in math class?

This girl name Sydnee is smart  
I think because this person always  
pay attention in class and raises  
her hand up.

Later in the same survey, Chandler wrote that she believes she is smart because she listens, raises her hand, and pays attention (see Figure 4.14).

**Figure 4.14**

*Chandler Pre-Survey \* Question*

\*Do you think you are **smart** in math class? Explain why or why not.

I think I am smart because I  
always listen and pay attention in class.  
also raise my hand sometimes.

Although brief, the fact that Chandler identifies these behaviors as reasons for concluding one is smart highlights their importance to her definition of smartness.

Chandler’s definition of participation is important and consistent with her other conceptions identified that attend to agency and the *doing* of smartness. Chandler's inclusion of listening to others, paying attention, and raising your hand are all examples of what some define as learning practices, but Chandler also sees them as characteristics one does, *enacting* smartness. Her view of participation is a view that highlights characteristics that are often not foregrounded through a traditional view of what it means to be smart in mathematics. This conception along with others described provide understanding how Chandler defines smartness in math.

#### **4.3.1.4 Revising Your Thinking.**

Another conception of smartness that Chandler shared was revising your thinking. At several points throughout the program, she talked about how once you learn something that you didn’t previously know you should change your thinking. She consistently classified this as smart behavior. For example, on one of the end-of-class checks during the first week, students were asked to name one thing that they did that was smart. She wrote that she changed her thinking and “made it correct the second time” (see Figure 4.15).

#### **Figure 4.15**

*Chandler EOC Check*

4. What was one thing that you did today that was smart to do when working on math?

~~I kept~~ I changed my answer and made it correct the second time.

In my second interview with her, I asked her to say more about this idea. She responded:

“Because um, if you know it's wrong, you want to change it to make it correct um, and you can remember it more and smart people know that um, if it is wrong, you can change your answer to make it different.”

Here, Chandler not only identifies revising thinking as smart, but also provides a justification for why it is important to do (i.e., “... you can remember it more...”). Further, her matter-of-fact tone during this exchange indicated that not only is it okay to revise your thinking, but it's an expected part of doing and learning math.

#### **4.3.1.5 Helping and Receiving Help from Others.**

Chandler described her ideas about helping and receiving help as an indicator of smartness throughout the program. In fact, in the second interview to provide examples of things she had done that was smart, she responded “...listen to other people's ideas

and uh, help them ... My um, partner.” After that she elaborated about her collaboration with Alex during class. When I asked her who she worked with most in the summer mathematics program, we had the following exchange:

**Charles:** Okay. So, who do you work with the most in class?

**Chandler:** Alexandria.

**Charles:** And do you like working with her?

**Chandler:** Yeah.

**Charles:** What, why do you like working with Alex?

**Chandler:** Because if she needs something, I help her. And if I need something, she'll help me.

**Charles:** Okay. Does she help you with a lot of different things?

**Chandler:** Mm-hmm (affirmative).

In the transcripts above Chandler mentions that she and Alex help each other in class. Although she doesn't explicitly mention what they helped each other with, we see that there are a lot of different things they help each other with and that she values their relationship.

Then on the post-survey, Chandler explicitly connected helping and receiving help to smartness when she wrote that she thought her classmate Alex, whom she sat next to during the summer mathematics program, was smart because “she helped me” (see Figure 4.16).

## Figure 4.16

### Chandler Post-survey Question #3

3. Can you tell me someone in our summer math class that you think is **smart**? Using complete sentences explain what does this person do that makes you think that they are **smart** in math class?

I think Alex is smart because she helped me,

The idea of helping one another is important especially given the collaborative nature of mathematics in general and during the summer mathematics program. Connecting both the second interview and post-survey it seems that Chandler values the idea of helping others and receiving help from others, with Alex being an example of that conception. This conception is similar to some of the previous perspectives described. That is, this way of thinking about what it means to be “smart” centers on doing or enacting smartness opposed to a static property or characteristic. This particular conception “helping and receiving help from others” also challenges traditional individualistic views of smartness. Instead, Chandler describes the value of collaboration, collective engagement, and its connection to smartness.

#### **4.3.1.5.1 Summary.**

Chandler holds several conceptions of smartness at once. Although some of her ideas reflect more traditional and limited classroom notions of smartness (e.g., being smart is doing well on tests), together her multiple conceptions create a more nuanced definition of smartness centered around things that she does. One thing that sticks out

about Chandler is the different conceptions of smartness she has. On the one hand she has a conception of knowing a lot of math, which within this conception she emphasizes “figuring it out on your own” when working on math. However, later she describes another conception of smartness “helping and receiving help from others”, in which she acknowledges her seatmate Alex as smart because she helps her. Whereas the first conception is more aligned with traditional views of smartness in mathematics the second seems less traditional and counters the first conception. One could imagine these conceptions in a binary way; however, these conceptions represent a more nuanced and fluid understanding of smartness that she has and uses. Chandler’s conceptions in totality paints an interesting picture that provided more understanding of her as the traditional compliant successful student and the student who has more expansive definitions of smartness.

#### 4.4 La'Rayne

Figure 4.17

*La'Rayne in class*





From the very first time I met La'Rayne, she spoke her mind. She was always honest in our interviews; she would express how she preferred reading to math and that she was only in the summer mathematics program because of her dad. She would also describe how she felt about the math problems the class worked on during the summer mathematics program. For example, on the triangle problem, she said it was confusing and frustrating because she had to put the numbers in a circle, instead of writing them straight which she was accustomed to. She also spoke openly about her disdain for previous mathematics experiences, which included not liking her 4th grade math teacher because the teacher always "screamed" at her and that she always had a problem with that teacher. In this case by screamed La'Rayne could be referring to the teacher yelling at her or raising her tone. The fact that La'Rayne mentioned it, indicates that it was important to her in her mathematical trajectory.

La'Rayne's conceptions of smartness were very complex. She described being smart at math as answering big questions and problems. By this La'Rayne meant getting correct answers to big questions and problems, which came up early in the program when she described not being smart because she didn't know answers. This view was just one part of her understanding of smartness as she described another conception that moved beyond answering big questions and problems.

Although she didn't see herself as smart in math, La'Rayne was very attuned to the classroom. She always participated and would speak her mind just like she did in the interviews, whether prompted by the teacher or not. Examples of this included shouting out answers, revoicing her classmates' ideas, and even calling out the teacher

for only calling on students on one side of the room. Regardless of how she felt, she always participated in some observable way. For example, on the first day of class on one of the first mathematical tasks students worked on, she raised her hand confidently and went to the board to explain her answer. Despite her answer being wrong (although the teacher didn't communicate her answer that way—see Chapter 5) she continued to share her thinking throughout the summer mathematics program.

#### **4.4.1 *La'Rayne's Explicit Definitions of Smartness***

Across La'Rayne's data I identified two conceptions of smartness that she expressed: (1) getting correct answers and (2) keep trying. Despite this, La'Rayne's understanding of mathematical smartness was quite complex. Below I unpack each conception.

##### **4.4.1.1 Getting Correct Answers.**

La'Rayne repeatedly spoke and wrote about the importance of getting answers right as a sign of smartness in math. In the pre-survey, when asked what it means to be smart, she wrote, "I think that to be smart means to be able to answer very Big questions and Problems" (see Figure 4.17).

#### **Figure 4.18**

*La'Rayne Pre-survey Question #1*

1. What do you think it means to be **smart** in math class? Please give at least two examples to explain your answer.

I THINK THAT TO BE SMART MEANS  
TO BE ABLE TO ANSWER VERY  
BIG QUESTIONS AND PROBLEMS

Additionally, the example she gave of when she felt smart in math class was a time, she was working on a division problem and correctly guessed what the answer was (see Figure 4.19).

**Figure 4.19**

*Chandler Pre-survey Question #2*

2. Can you describe a time during the school year when you felt **smart** in math class? Please give an example.

When I was a division  
problem in school and  
I had guessed what the  
answer was and I got  
it right.

In response to the question of whether she thinks she's smart, she responded, "Yes No, because I don't know a lot of the problems" (see Figure 4.20).

**Figure 4.20**

*La'Rayne Pre-survey \* Question*

\*Do you think you are **smart** in math class? Explain why or why not.

*Yes I do. I can do all the problems.*

Across these responses, La'Rayne emphasizes getting correct answers as an important sign of smartness, even when she has to guess, as well as answering "Big questions and problems". These views shape a very narrow conception of what it means to be smart. Grounding her definition in only right and wrong responses limits other ways of being in classrooms that are just as important if not more. We see this in her responses as nothing speaks to other characteristics such as working hard or trying her best, as well as paying attention as other students mentioned. Therefore, for her to be smart in math is grounded in one thing and if one does not do that one thing, they are not smart.

During our first interview, I further probed her responses from the pre-survey. I asked whether she thought she was smart. She responded:

**La'Rayne:** Uh, not really.

**Charles:** Why not?

**La'Rayne:** Cause sometimes when they did the triangle problem, I think that was confusing.

**Charles:** Mm-hmm (affirmative).

**La'Rayne:** Because when- It was frustrating me because I had to put the numbers in order, or whatever. I had to put them in the triangle-

**Charles:** Okay.

**La'Rayne:** And it was confusing cause it was shaped like a triangle.

**Charles:** Okay.

**La'Rayne:** And I'm used to numbers just being straight, or like that, or just normal. I can't do stuff like that; it's confusing.

Realizing that she interpreted my question as only about the context of the summer mathematics program, I followed up with a question that focused on prior experiences:

**Charles:** Okay. What about like, in your fourth-grade class? Do you think you are smart there in math class?

**La'Rayne:** [nods head]

**Charles:** Okay. So, why do you think you're smart there?

**La'Rayne:** Cause it was easy. [laughter 01:40]

**Charles:** Oh, okay. So then, you don't think you're smart here because why?

**La'Rayne:** The math is harder.

**Charles:** The math is harder? So, do you typically like, get the problems quickly here?  
Are you-

**La'Rayne:** No.

**Charles:** Okay.

**La'Rayne:** Not all the time.

**Charles:** Not all the time? How's it make you feel?

**La'Rayne:** Mm, frustrated sometimes. [02:00] Sometimes, I just don't care.

**Charles:** Okay. In your fourth grade class, did you have this feeling a lot? Like, were you frustrated with problems?

**La'Rayne:** Mm, no. Not all the time.

**Charles:** Okay. All right, thank you for sharing that. I know it's kind of hard when you get stuff and stuff comes easy, and then things are difficult.

These exchanges are interesting because her assessment of her own smartness seems to depend on how difficult she perceives the mathematical content to be. She believed she was smart in fourth grade because the content was easy. However, when she found the work in the summer mathematics program to be more challenging and confusing, she felt frustrated and not smart in this context. Whereas students often readily consider getting right answers as an indication of smartness, La'Rayne also takes into account the difficulty of the content. This adds a layer of nuance to her conception that considers the relationship between getting right answers and the mathematical content. That is, for her getting answers right when the content is easy is different from getting answers right when the content is hard. For La'Rayne this

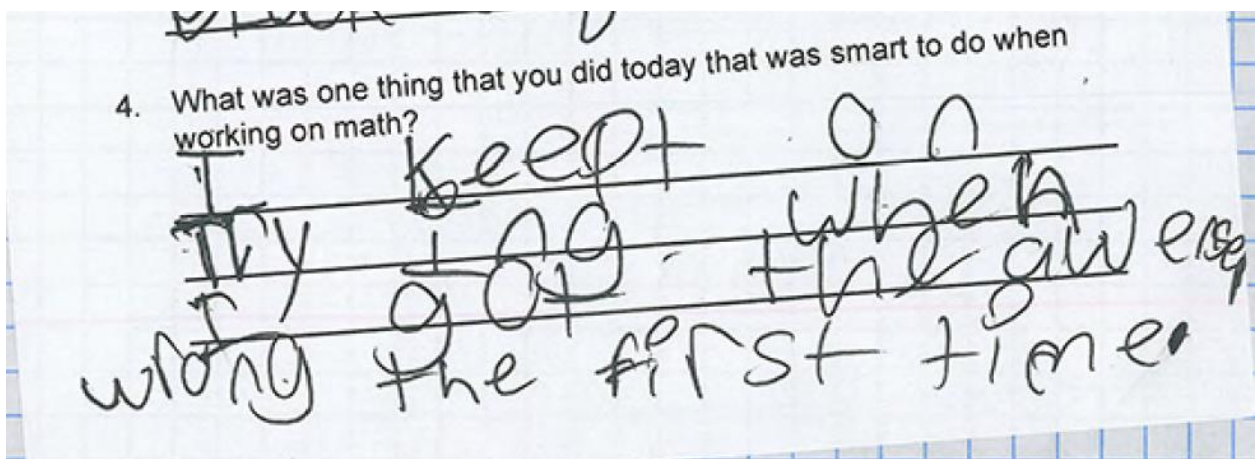
understanding impacts how she defines smartness and therefore sees herself as smart or not. Therefore, it's not necessarily sufficient getting right answers to "easy" math as justification for being smart. Instead, getting right answers to "hard" math holds more weight or is a better justification for being smart. This observation illuminates the negotiation that La'Rayne does in defining smartness and how she herself fits within that definition.

#### 4.4.1.2 Keep Trying.

After the first couple of days in the first week, La'Rayne's descriptions included more layers of complexity to her conception of smartness. An example of this is in one of the end-of-class checks. In response to the prompt to name one thing she did that was smart, she wrote that "I kept on trying after I got an answer wrong the first time" (see Figure 4.21).

**Figure 4.21**

*La'Rayne EOC Check*



This response is a sharp contrast with what she named in the pre-survey and first interview. Here, rather than focusing on a right/wrong binary, La'Rayne seems to be expanding her idea of smartness to include both persisting when the work is difficult and revising your thinking when necessary. This is important because it broadens the possibilities that she has to name and identify her smartness. La'Rayne's conception "keep trying" is an example of a conception that broadens possibilities for students to see non-traditional ways of smartness that are often not emphasized or encouraged. Her response highlights the value that if you get an answer wrong the first time, it is not only important to keep trying, but it is smart to do so.

In the second interview, I followed up with her about some of the things I had noticed about how her ideas seemed to contradict:

**Charles:** When do you ... When do you feel smart in math? Give two examples.

**La'Rayne:** Train problem, square problem.

**Charles:** Okay, what about the train problem and the square problem make you feel smart?

**La'Rayne:** Easy.

**Charles:** They're easy?

**La'Rayne:** Yes.

**Charles:** So if a problem is easy, that makes you feel smart?

**La'Rayne:** Well, I mean, the train problem's not that easy, but the square problem is.

**Charles:** Okay. The square problem is easy. What makes you feel smart when you do the train problem?



**La'Rayne:** Cause it's hard... [inaudible]... and sometimes I can't.

**Charles:** Wait a minute. Repeat that. I couldn't hear what you said.

**La'Rayne:** I said some, cause sometimes I can answer it and sometimes I can't.

**Charles:** What you mean sometimes you can, it's just sometimes you can't?

**La'Rayne:** Cause I'm doing it over and over again.

In the first interview, La'Rayne had talked about being smart in her fourth-grade class because the math was easy. Although she mentions this point again here with the square problem, she also states that the train problem made her feel smart because it was difficult (for more information about the Train Problem see chapter 3). La'Rayne's comments highlight her tension with respect to defining smartness. Her assertion that she felt smart about doing the square problem because it was easy highlights her conception of getting answers right. At the same time, her comments about the train problem offer a more expansive and nuanced take on smartness. She implies that she kept trying on the problem even when she didn't get the answer and also there were moments when she could see her progress on the task (i.e., sometimes I get it and sometimes I don't). That is with the train problem she was able to try trains, however she was not able to find the train that solved the problem. Being able to broaden what it means to be smart to other things beyond getting answers right is valuable because there are other characteristics that are just as important if not more important that learners can do that aligns with what it means to do mathematics. It seems that as the summer mathematics program went on, La'Rayne used other features to characterize smartness.

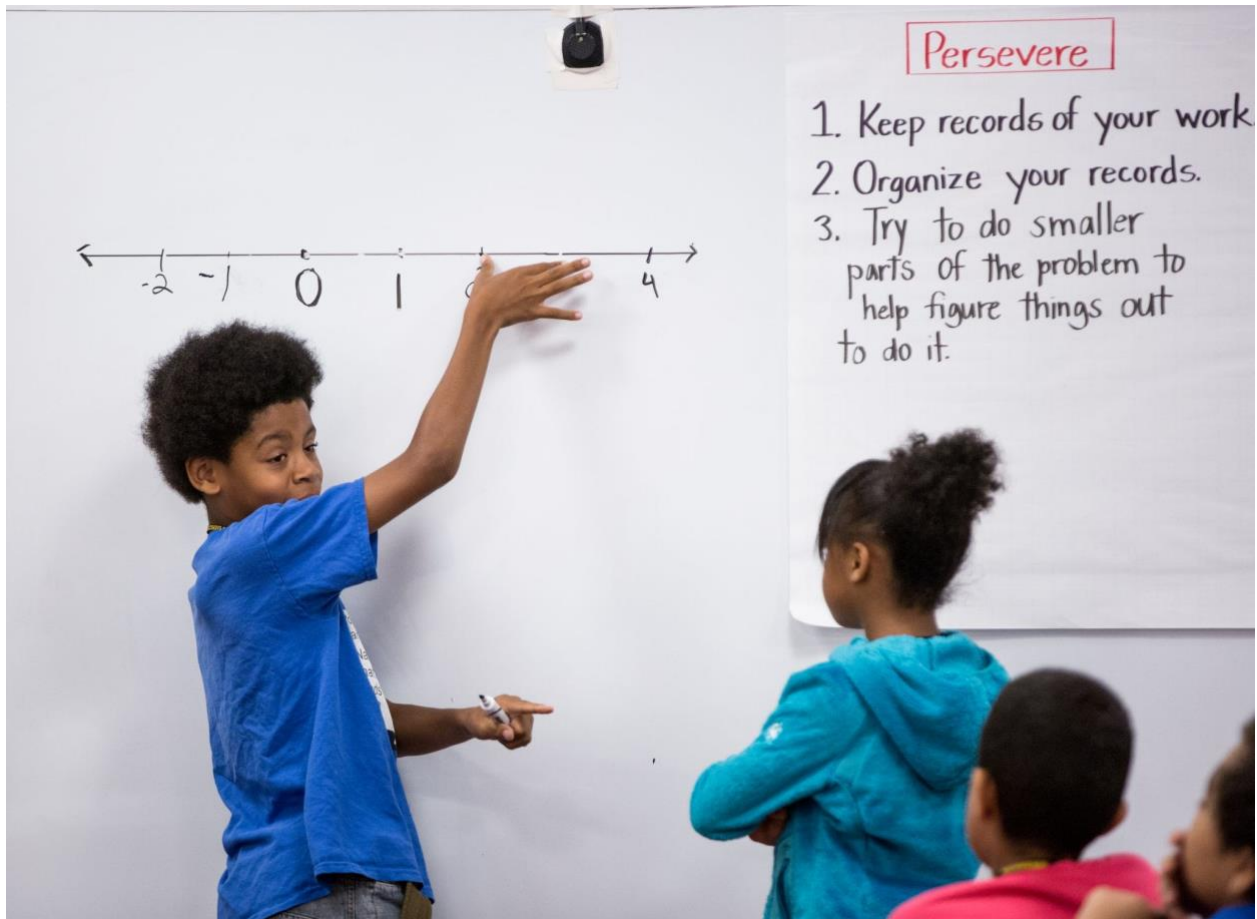
#### **4.4.1.3 Summary.**

Overall, across the summer mathematics program La'Rayne seemed to be navigating important tensions around how she conceptualizes smartness. Her initial conception of smartness as getting correct answers occurred frequently, especially early in the program. For example, problems she could solve right away were "easy", which reinforce the conception that getting answers correct is justification for smartness. However, her conception that smartness is related to perseverance (i.e., keep trying), which means you may not solve problems right away, emerged as the summer mathematics program unfolded. Additionally, she began to talk about working on difficult content as a sign of smartness. Her initial conception focused on external factors, such as getting correct answers. However, across the program, she came to also attend to aspects of smartness that are related to what one does (e.g., try).

## 4.5 Kasim

Figure 4.22

*Kasim in class*



Kasim was *that* student. Of the five focal students in this study, he was the one who was very vocal about his prior experiences in mathematics. He was very confident in his ability. His confidence came through in interviews in several ways. He mentioned going to the board “32 times” or “a lot” in his previous classes. He also mentioned being viewed as a role model for his peers and being viewed as the “top star” in his class. Despite not always getting answers right (although he did get them right most of the time) or getting stuck on problems he still had confidence which is represented in his response to a question that asked why he keeps working on hard problems: “

Cause I know I can do it. Can't nobody tell me I can't. And then if they do, I'll be like, "Where's your paper at? Did you do the problem yet?"

Kasim was also that student who adults seemed to love. He was all personality. For example, at one point during the summer mathematics program I had worn a Morehouse College shirt that read “You can tell a Morehouse man. But you can't tell him much.” Then during our second interview I asked him whether he learned a lot from his classmates in fourth grade, to which he responded, “I was like, I was like, the smartest kid in my classroom, so like ... [a] Morehouse man. Like, they couldn't tell me much.” His response was indicative of his personality. He also told me that his teachers told him they wished there were more of him in class. It was easy to understand why: Kasim pushed himself hard to succeed. For example, on the triangle problem he wrote, “I will solve this problem” to motivate himself.

Kasim was very proud of his work and his capabilities. During the summer mathematics program, Kasim was a confident presenter and often got the correct answer when he went to the board. For instance, after he presented at the board on the first day, the teacher asked the class to name something Kasim did well. Kasim acknowledged that he spoke loudly and clearly when he gave his explanation. But there were also moments when his pride and confidence seemed to limit his engagement with mathematical content. This was evident when the class worked on the grey white rectangle problem (see Chapter 3). While discussing this problem with the class, Kasim used the fact that he studied fractions in school as justification for why he thought another student's answer was right. The teacher explained that Kasim's reasoning was not sufficient. Kasim was so confident that he knew the right answer that he might have stopped working on the problem if the teacher hadn't pressed the class to keep going.

#### **4.5.1 Kasim's Explicit Definitions of Smartness.**

Across the data for Kasim, I identified two conceptions of smartness: (1) getting correct answers, and (2) sharing ideas publicly.

##### **4.5.1.1 Getting Correct Answers.**

Kasim repeatedly connected smartness to the idea of getting answers correct. For instance, in his pre-survey, when asked, "What do you think it means to be smart in math class?" he said that you're smart if you get the correct answer "five times in a row."

#### **Figure 4.23**

*Kasim Pre-survey Question #1*

1. What do you think it means to be **smart** in math class? Please give at least two examples to explain your answer.

When you get the ~~answer~~ answer right at least five times in a row.

When asked for an example of his own smartness, he wrote, "When I got a math problem right when nobody else did."

**Figure 4.24**

*Kasim Pre-survey Question #2*

2. Can you describe a time during the school year when you felt **smart** in math class? Please give an example.

When I got a math problem right when nobody else did.

These responses highlight both the importance of getting answers correct consistently and being the only one to get the answer correct, which are examples of traditional views of what it means to be smart in math class.

During the second interview, when asked to describe things that show that he's smart, he replied:

**Kasim:** Like, when, let's say um ... Uh ... Like ... If you go through like, a page of homework or a page of work that you gotta do, in about five minutes.

**Charles:** Okay.

**Kasim:** Not five minutes, but one minute. Under a page of, of work or un- If you do a full packet under five minutes, that make me feel really smart.

**Charles:** Okay.

**Kasim:** Like I did the triangle problem in under 10 minutes.

In addition to getting answers correct consistently, the transcripts above add two dimensions, volume and speed, to Kasim's conceptions of smartness. In his transcripts Kasim defines feeling "really smart" when he's able to do work, a lot of work, in the least amount of time. Part of the idea of doing work is implicit in his transcripts in the inclusion of the triangle problem, which Kasim described as challenging. However, including the triangle problem and associating it with time highlights both his connection with correctness to speed and challenging problems.

#### **Figure 4.25**

*Kasim Work on The Triangle Problem*

① Persevere

I will listen to others ideas so I can add on to it and make my own to share.

The Triangle Problem

Place each of the numbers ~~1, 2, 3, 4, 5~~ and 6 in a different circle on the triangle so that the sum of the numbers on each side of the triangle is equal to 9.

I feel like stuck but I think I can ~~do it~~ WILL DO THIS PROBLEM

Kasim did not answer several questions on the pre-survey, so I followed up with him during our first interview. I began by asking him to name someone he thought was smart. He named Jaden, a student who also went to the same school as Kasim. Kasim described Jaden as a “math all-star” because he could just “look at the problem and get the right answer.” When I asked Kasim if he thought he was smart in math, he responded, “... I always used to get um, As and Bs.” He then went on, unprompted, to provide vivid descriptions of moments in his mathematical experiences that went as far back as second grade when he knew he was smart. He described that he was the only student to get what he called “The greatest problem I solved” which led to him getting three pieces of candy instead of one.

These responses highlight Kasim’s orientation to getting correct answers, but also



provide nuances about getting correct answers. For Jaden, the description of “math all-star” and “look at the problem and get the right answer” hints that Kasim sees values that reinforces innate ability as well as speed. Additionally, Kasim uses grades as justification for why he is smart, which reinforces a traditional view of what it means to be smart in math. Further Kasim mentioning a problem that he did in second grade that only he got right is another example of getting correct answers, however this particular example also highlights the value in being the “only one” to get the answer correct. This idea of being the only person is part of the notion that to be smart in math means one is an outlier or anomaly. Collectively Kasim’s responses complicates Kasim’s conception of “getting correct answers”, as well as highlights traditional views about smartness in math that Kasim has.

In my last interview with Kasim, he offered a somewhat different view of smartness with respect to getting answers correct. For Kasim, getting answers right is one of his conceptions, however he also connected getting answers right with frequency (e.g., do you get answers right most of the time) and speed (e.g., how long does it take you to get the answer right). Below we see Kasim include speed as he describes smartness as in getting answers right.

**Charles:** Okay. Is there anything else you think might show like, how you're smart?

**Kasim:** Like, when, let's say um ... Uh ... Like ... If you go through like, a page of homework or a page of work that you gotta do, in about five minutes.

**Charles:** Okay.

**Kasim:** Not five minutes, but one minute. Under a page of, of work or un- If you do a full packet under five minutes, that make me feel really smart.

**Charles:** Okay.

**Kasim:** Like I did the triangle problem in under 10 minutes.

**Charles:** Okay, so you think it's about- What if, what if it take longer than five or ten minutes?

**Kasim:** Well...

**Charles:** Will you still say like, that's smart?

**Kasim:** If you still do it. Yeah, if you still do it, you doing pretty good. Like, cause other people would kind of give up.

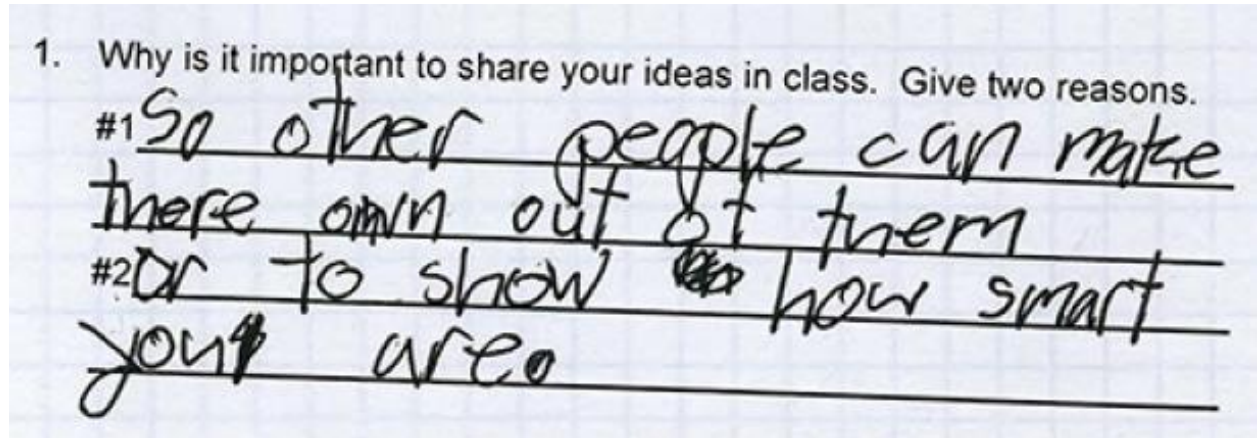
Kasim's initial descriptions about getting correct answers quickly aligned closely with traditional ways of conceiving smartness in classrooms. However, in our later interactions he seemed to also focus on not giving up. In these exchanges above, Kasim's definition of smartness includes *eventually* getting the answer right. Additionally, he indicates that not giving up is an important feature of smartness.

#### **4.5.1.2 Sharing Ideas Publicly.**

Kasim also seemed to connect smartness to particular ways of performing in math class. In particular, he connected the act of sharing ideas publicly with smartness. During the second week, when asked on an end-of-class check why it's important to share your ideas in math class, he wrote in part, "So other people can make their own out of them" and "To show how smart you are."

**Figure 4.26**

*Kasim EOC Check*

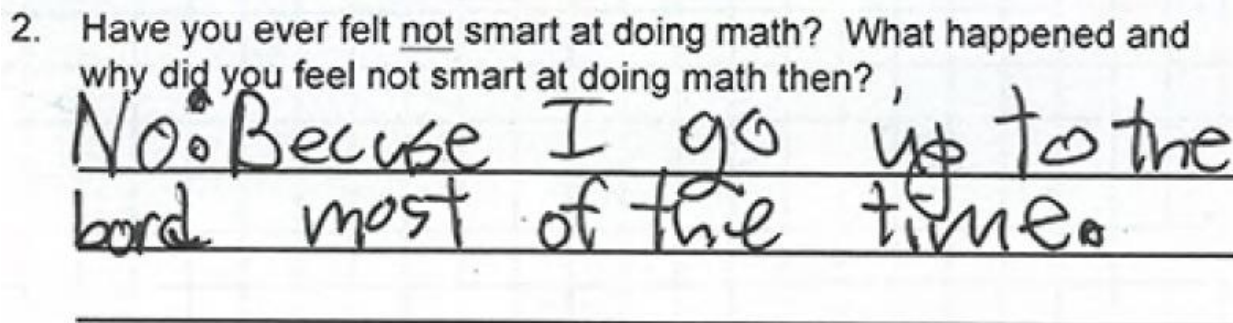


Both of Kasim's responses speak to Kasim's confidence and pride in his own ability. However, they both represent different aspects of his conception of sharing ideas publicly. The first response highlights Kasim's attention to sharing his ideas and learning. Although he doesn't mention that he too learns from others' ideas here, he does recognize that others can learn from him when he shares his ideas. His second response speaks to the performance aspect that Kasim has about sharing his ideas. He sees sharing his ideas as an opportunity to demonstrate his smartness to the teacher and his peers. This view is embedded in a traditional view of smartness that relies upon external validation as affirmation for one's legitimacy in this case as smart in math. Sharing ideas publicly often came up for Kasim in the context of going to the board. On the next question in that same end-of-class check, when asked when he feels smart in math, he wrote, "... when you go up to the board." He reiterates the importance of going

to the board on the Day 9 end-of-class check which asked if he ever felt not smart. He responded, “No Because I go to the board most of the time” (see Figure 4.27).

**Figure 4.27**

*Kasim EOC Check*



Similarly, in the post-survey when asked about what it means to be smart in the summer mathematics program, he responded, “To go up to the board a lot” (see Figure 4.28).

**Figure 4.28**

*Kasim Post-survey Question #1*

1. What do you think it means to be **smart** in our summer math class? Using complete sentences give two examples to explain your answer.

#1 Two get to go up to the board a lot.

For Kasim, going to the board is a high-status act in his math classroom. He repeatedly associates going to the board--or, more generally, sharing his ideas publicly--with smartness, especially with having the opportunity to show off his smartness to others. This aligns with mainstream conceptions of smartness in mathematics which

affirms that the student that talks the most and goes to the board frequently are the students who are validated as smart.

By the end of the summer mathematics program, he was incorporating more nuanced criteria into his definition of smartness. This was especially clear in how he talked about sharing ideas publicly. In my final interview with him, he described in more detail what sharing ideas publicly meant to him:

**Charles:** So, I'mma ask you this question. Do you think ... That the only way to be smart in a math class is to do good on tests?

**Kasim:** No.

**Charles:** Okay.

**Kasim:** Like, I like going up to the board. It make me feel smart a lot. It make me feel like [10:00] I know a lot.

**Charles:** Okay.

**Kasim:** It make me- Yeah, it make me feel like I know a lot when I go up to the board and explain. And then other people be like, "Oh, so that's what that means."

**Charles:** Okay.

**Kasim:** Stuff like that.

Here, Kasim emphasizes that going to the board makes him feel smart and shows that he knows a lot, which he had said repeatedly throughout the program. However, he also identifies sharing publicly as an opportunity for others to understand math through him. This is connected to his first reason for sharing his ideas in class.

Although Kasim highlights that sharing his thinking publicly affirms his own smartness there is some nuance to that conception in that it also connects to others learning not just for his own benefit. It is important that Kasim values and sees sharing his ideas as an opportunity to help or support his peers, but it raises questions around can his peers do for him what he perceives that he does for them? Also, does his idea of sharing his thinking publicly only occur when he is sharing the correct answers or is the only one that shares the correct answers?

#### **4.5.1.3 Summary.**

Kasim offered two conceptions of smartness: getting correct answers and sharing ideas publicly. Initially, these conceptions seemed to be grounded in traditional ways to think about smartness. For example, Kasim described getting correct answers with both how frequently and how quickly one gets correct answers. However, by the end of the program Kasim had expanded his ideas about accuracy to include not giving up. Although subtle, Kasim's acknowledgement of the importance of perseverance is an important and contrast traditional definitions of smartness. His second conception focuses on sharing ideas publicly. Normally, the context in which Kasim does this is going to the board. For Kasim, this seems to be an opportunity for him to both show off his smartness and justify to himself that he is smart. Although this conception highlights a traditional view of mathematics, by the end of the program Kasim also adds an additional rationale to his conception when he mentions that one important reason for going to the board is to be able to help his classmates. This is an important addition to the idea of sharing ideas publicly because it seems to signal to some degree Kasim

holding to ideas: 1) sharing his smartness as an individual and self-serving act, and 2) sharing his smartness as supporting the learning of others.

## 4.6 Jeremiah

Figure 4.29

*Jeremiah in Class*





Jeremiah was one of the hardest students for me to read. When I asked questions during interviews, he often responded with “Uh huhs” or head nods. Even when prompted to use “yes” or “no”, he often reverted back to “Uh huhs” or head nods. When he did respond in more detail, he spoke concisely and voiced supreme confidence about his mathematical ability. For example, in the first interview I followed up with Jeremiah, about his comment during class that the math was easy. He responded that all the math was easy in the summer mathematics program and that he could do it quickly. In addition, Jeremiah mentioned how easy mathematics was in general. His confidence also showed up when he said he did not learn from anyone in the class, pointing out the things that others had done that he could also have done. He even wrote on the post-survey, “I haven’t seen anyone as smart as me.”

In some ways, Jeremiah seemed to just want attention. This desire showed up in how he interacted with the teacher, often seeking her out for help if he had questions and asking to sit at the front of the class nearest to her and to the board. Before Jeremiah sat at the front of the room he sat in between Langston and Madison. While seated there, Jeremiah didn't share his thinking, worked alone, and sometimes had issues with Langston and Madison. However, after changing seats Jeremiah seemed to share his thinking more during whole-group discussions, including sharing his thinking at the board.

Jeremiah’s views of smartness were grounded in traditional conceptions such as having mathematical skills and understanding mathematical concepts, getting answers right, and getting good test scores. At the same time, Jeremiah named other characteristics of smartness such as the importance of sharing ideas to help others and

of persevering in math. Although Jeremiah voiced very common ways of thinking about smartness, there was more than met the eye with him.

#### **4.6.1 Jeremiah's Explicit Definitions of Smartness.**

For Jeremiah, the primary conception of smartness that he shared across the data was that being smart is knowing math. Below I unpack this conception, highlighting where Jeremiah's beliefs around knowing a lot of math seemed to be grounded in external as well as internal justification.

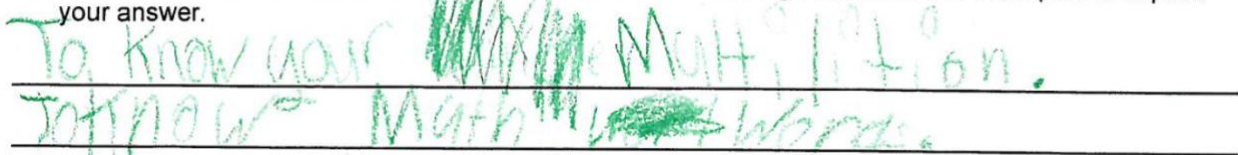
##### **4.6.1.1 Knowing Math.**

Jeremiah's primary conception of smartness was knowing math, which he seemed to define as mathematical skills and concepts. For instance, when asked on the pre-survey what it means to be smart in mathematics, he wrote, "To know your multiplication. To know your math words" (see Figure 4.30).

**Figure 4.30**

*Jeremiah Pre-survey Question #1*

1. What do you think it means to be **smart** in math class? Please give at least two examples to explain your answer.



To know your multiplication.  
To know math words.

On the next question, when asked about a time during the school year when he felt he was smart, he wrote that he felt smart when he knew the answer on a test and scored a 100% (see Figure 4.29). This justification is similar to those of Chandler,

Arianna, and Kasim, whose conceptions of smartness included doing well on tests and getting answers correct as evidence of knowing a lot.

**Figure 4.31**

*Jeremiah Pre-survey #2*

2. Can you describe a time during the school year when you felt **smart** in math class? Please give an example.

IN Math when I ~~know~~  
identified on a test and got a 100% on it.

Further, during the first interview, when I asked Jeremiah to reflect on his pre-survey response about whether he feels smart in math, he said:

**Jeremiah:** I put yes because some of the times I get hundreds.

**Charles:** Okay.

**Jeremiah:** On tests.

**Charles:** Are there any other reasons why you think you're smart?

**Jeremiah:** Mm ... 'Cause I know math...

Jeremiah's responses to these questions emphasize "knowing math," which includes knowing math facts and vocabulary and doing well on math tests. These are common conceptions of being smart in math that reflect the traditional focus on memorization and accuracy in classroom mathematics instruction. In these responses,


the idea of knowing a lot for Jeremiah is grounded in the external justification of getting the right answers and earning high test scores.

Additional evidence of this conception of smartness surfaces in his responses to an interview that I conducted with him about his answers to an end-of-class check from the end of the first week.

**Figure 4.32**


*Jeremiah EOC Check Questions #2-4*

2. Could this be a train for Ms. McDuff? YES  NO



Explain how you know. because she is use a  
white.

3. Could this be a train for Ms. McDuff? ~~YES~~  NO




Explain how you know. ~~because she is use a~~  
no because useing  
aback.

4. What was one thing that you did today that was smart to do when working on math?  
I think I was smart on everything  
today

Figure 4.33

Jeremiah EOC Check Question #3

3. Could this be a train for Ms. McDuff? ~~YES~~ NO



Explain how you know. ~~Yes because she is using a black.~~ No because she is using a black.

Figure 4.34

Jeremiah EOC Check Question #4

4. What was one thing that you did today that was smart to do when working on math?

I think I was smart to do everything today

In question 2 Jeremiah identified in his explanation that one of the conditions, you can't have more than one of any car, was violated which meant the train could not be made. Again, in question 3, Jeremiah's explanation identified one of the conditions, you can't use a black Cuisenaire rod was violated indicating that the train could not be made. In question 4, Jeremiah's explanation demonstrates his evaluation on the mathematical tasks, which was that everything he did was smart. Collectively, Jeremiah's responses to questions 2, 3, and 4 were consistent with the earlier data that being smart at math means knowing a lot and, therefore, being able to get correct answers. When I followed up about the second and third questions, Jeremiah was able to explain each answer he had written and further justify his answers.

**Charles:** ... What about here? [Points at Question 2, Figure 4.30.] ...

**Jeremiah:** I said you can't make that because you're using two whites and, and in our thingy it said you couldn't use two whites. Like, two of the same colors.

**Charles:** Okay. Well then, what about the last thing [points at Question 3 (see Figure 4.31)]? 'Cause you said everything you did was smart.

**Jeremiah:** Everything I did was smart.

**Charles:** Well what's smart about this response right here?

**Jeremiah:** Everything.

**Charles:** Can you explain that a little bit more?

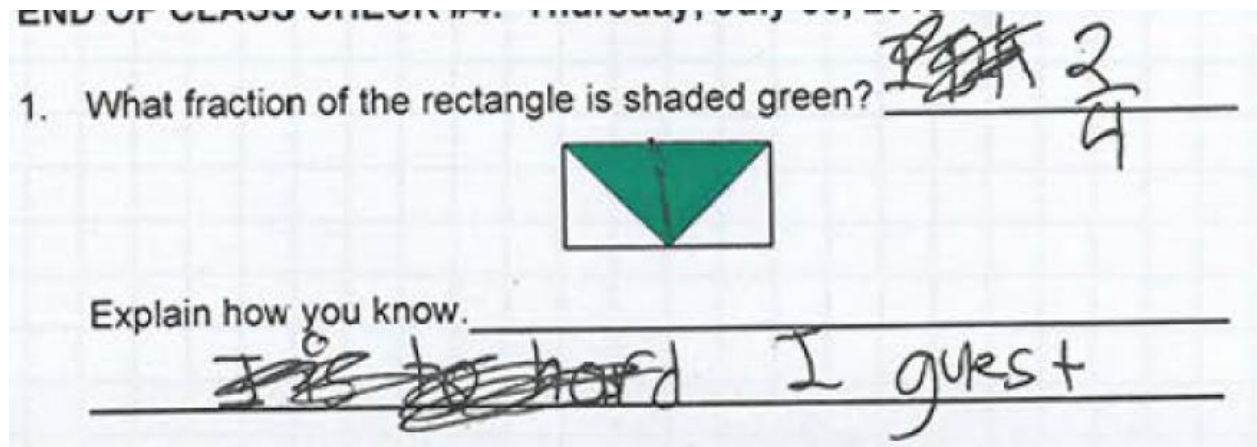
**Jeremiah:** 'Cause they used the black rod and you, they didn't like, you're not supposed to be using black rods. We only supposed to be using yellow, white, and green, and red, and white.

**Charles:** Okay. All right. Thank you for explaining that.

However, on the first question of that same end-of-class check, Jeremiah had written "I guest [sic]" as a justification for his answer (see Figure 4.33). This response is interesting because it seems to reveal a lack of confidence in his (correct) answer before he goes on to write that everything, he did was smart.

**Figure 4.35**

Jeremiah EOC Check Question #1



I followed up with him about this in the interview:

**Charles:** Okay. At the bottom, you said that you think you did everything smart. Can you like, explain what you were trying to say?

**Jeremiah:** I know I'm smart. I know I'm smart, and I could do anything smart.

**Charles:** But on this page when you said it, you said "everything today," so what were things up here that you think you did that were smart?

**Jeremiah:** Everything.

**Charles:** What's- Give me an example.

**Jeremiah:** Well, I know everything-

**Charles:** You know-

**Jeremiah:** Just for a minute.

**Charles:** You knew everything?

**Jeremiah:** Yep.

**Charles:** You said you guessed that answer, though. Did you know that answer?

[Referring to Question 1, Figure 4.33]

**Jeremiah:** Nope.

**Charles:** So ... Would this be an example of what you said like, you was smart in everything? That you guessed this answer?

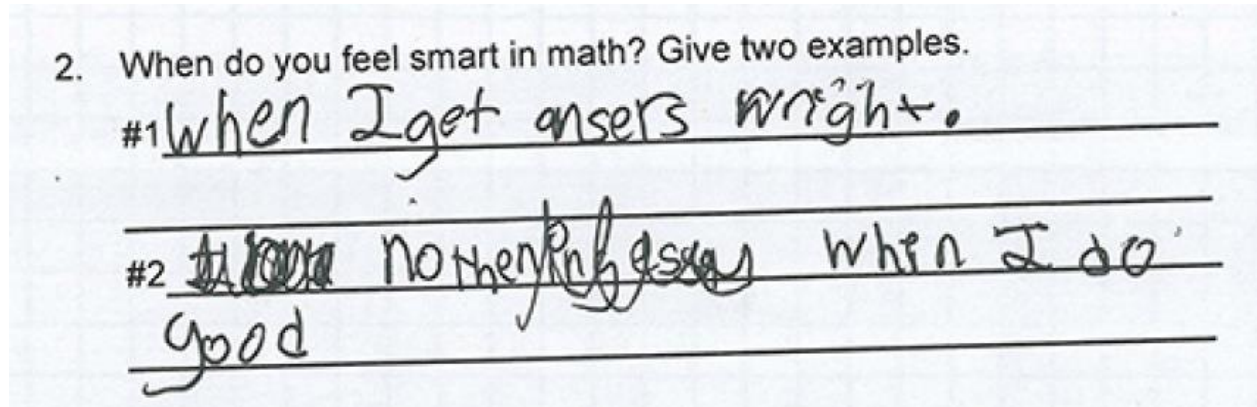
**Jeremiah:** Oh. Yeah, I still am smart.

Jeremiah's description of smartness in this moment made me revert back to what he said at another point in the summer mathematics program. In the figure Jeremiah mentions getting the "right answers" and when he "do good" as examples of when he feels smart (See figure 4.36). His responses seemed to run counter to what he said above, which he describes that he is still smart although he guessed. While getting correct answers seem to be aligned with his conception of knowing a lot of math, it is not clear what "doing good" means.



**Figure 4.36**

*Jeremiah EOC Check*



Below I probe Jeremiah to better understand the complexity of how he defines smartness.

**Charles:** In one of the end-of-class checks, you said you feel smart when you get the right answers and when you do good. What is, what does it mean to do good?

Particularly we're talking about in math class.

**Jeremiah:** To get the right answers and ... Like, to do good work.

**Charles:** Do you think that right answers and good work are the only ways that show that you are smart?

**Jeremiah:** Nope.

**Charles:** Well, what are some other things that you think show that you're smart?

**Jeremiah:** You could get wrong answers, but you still could be smart.

**Charles:** Okay. Any other things that you think you could do and still show that you're smart?

**Jeremiah:** No. Not in my head right now.

Jeremiah's response here hints at a conception of smartness that moves beyond the right answers. Whereas his conception of knowing a lot of math seems to rely on common external factors that serve as justifications of smartness (e.g., getting answers right, doing well on tests), his assertion that "you can guess" and "you could get wrong answers, but you still could be smart" could point towards an internal justification around his own self-concept with respect to smartness in math (e.g., perseverance is smart). However, as he did not elaborate on what he meant, it is difficult to draw firm conclusions about this aspect of his conception of smartness.

#### **4.6.1.2 Summary.**

Overall, Jeremiah was difficult to understand and to read. His responses over the course of the summer program seemed to vary from very traditional, relatively narrow conceptions of smartness as knowing to potentially broader and more nuanced conceptions of smartness as doing. However, his views throughout seemed to be highly focused on his own smartness. This point connects back to his belief in how easy the math was and that he didn't learn anything from his peers. Jeremiah's conception of smartness is similar to his peers' conceptions in that many of them identified knowing a lot of math as a marker of smartness. However, he differed from the other focal students in that, for him, knowing a lot of math was the only conception that emerged clearly enough to name. While he may hold an additional conception related to internal motivation and perseverance, the majority of Jeremiah's data reveals a relatively traditional conception of smartness that hinges on external justifications.

## **4.7 Discussion**

Looking across the focal students' conceptions of smartness I identify one main theme: students' conceptions of smartness are complex. Here, I illustrate the complexities of students' conceptions of smartness by first discussing the ways that superficial similarities obscure substantive differences, next addressing the ways that their conceptions of smartness appear to function as coping strategies in mathematics classrooms, and lastly showing how students' conceptions may be malleable.

### **4.7.1 *Complex Conceptions***

Students have complex conceptions of smartness. By that, I mean that the conceptions about smartness that students have are not simple. An example of this was illustrated in this chapter by La'Rayne. La'Rayne believed being smart means getting correct answers, which is very common for students given the attention that is given to getting correct answers in class, quizzes, tests, and homework. However, La'Rayne described in her pre-survey that being smart meant answering big problems and that she wasn't smart because she didn't answer big questions. Later through interviews La'Rayne clarified her comments. She mentioned that she didn't think she was smart because the math was harder in the summer mathematics program in comparison to her previous classrooms in which the math was easier. So, in her previous classes there were instances when she thought she was smart because she could answer the problems and they were easy. However, in this class she felt she wasn't smart because she didn't get questions right and the math is harder. The complexity of La'Rayne's conception of being smart means getting right answers is embodied in what she described above. Instead of just saying smartness is about right and wrong answers, her response

provided more nuance. That is La'Rayne understood that the difficulty of the problems matters when thinking about smartness, which is a characteristic that is not obvious to be aware of nor is not connected to smartness.

Being able to find a mechanism for which to measure oneself is important. In math classrooms that can mean different things. It could be in the traditional sense of getting answers right, test scores, and grades. It could also be in relation to what the teachers and your peers say about you. It could also be in relation to your own personal criteria. Regardless, the ways in which students narrate their conceptions of smartness and represent themselves and their work are all important. Over the course of analyzing Chandler's artifacts multiple conceptions came up. In fact, she had six, which is more than the other students.

Having these different conceptions provided Chandler with multiple ways of evaluating herself on the spectrum of smartness. This allowed her to use different conceptions in the moment. She didn't always mention getting answers right. In fact, sometimes she got answers wrong. But she leveraged getting answers wrong as okay if you revise your thinking and learn from others. She also was very intentional about organizing notes, as she felt that was important. Chandler embodies the idea of having alternative measures of smartness which allows her to always see her smartness.

Overall, the examples highlight the complexity of students' conceptions of smartness. Below I unpack three themes that were found across students to provide more nuanced students' conceptions of smartness.

#### 4.7.1.1 Superficial Similarities, Substantive Differences

One feature of the complexity of students' conceptions of smartness is the way that superficial similarities obscure substantive differences in how they seem to think about and define smartness for themselves. By *superficial similarities, substantive differences* I mean that on the surface some students may appear to have the same conception(s) of smartness, but there are differences in how those conceptions are enacted by students. This finding is important because it highlights the need to not only identify conceptions students have, but to understand how these conceptions might be relevant to what students do in classrooms. Below I unpack this theme by comparing the conception *know a lot* shared among Arianna, Chandler, and Jeremiah.

Arianna, Chandler, and Jeremiah, share the conception that one is smart in math if they “know a lot.” Based on their survey and interview responses, “knowing a lot” seems to mean knowing math facts, getting answers right, getting good grades, and getting high test scores. These features of “knowing a lot” rely on external justifications to affirm one’s smartness. However, for Arianna and Chandler, knowing a lot is only one aspect of smartness. Their other conceptions seem to center on internal characteristics. For instance, both Arianna and Chandler express that trying and working hard is smart. Arianna also mentions the importance of taking your time on tasks, and Chandler highlights other actions like revising her thinking or working with others. These conceptions center the student’s ability and agency in *doing* smartness.

Whereas Arianna and Chandler had additional conceptions of smartness rooted in doing or enacting smartness, Jeremiah’s conception appeared to be very rigid,

depending almost entirely on external validation from right answers and test scores to justify his smartness to himself. For example, Jeremiah often mentioned the math he worked on in the summer mathematics program was easy and that he wanted it to be hard. His idea of “easy” is connected to the conception of knowing a lot, getting answers, and the math coming easy to him.

There is a lot to take away from these conceptions, however I will focus on the intersection of these conceptions with opportunities to learn that may or may not be available. For Chandler her conception could provide opportunities to see the value in her peers' thinking as she values working with others and listening to others' thinking. Additionally, because of her belief that one can learn from mistakes, she may be more inclined to share her thinking and see the strengths of her peers' work. For Arianna she may take advantage of opportunities with particularly challenging math tasks, giving her conceptions to work hard and try her best. Also, she may take advantage of working with other peers. With respect to Jeremiah, he could miss out on opportunities to work with and learn from his peers. Some of his artifacts highlight this possibility. For instance, Jeremiah repeatedly does not recognize any contributions that his peers make during mathematics instruction. In fact, when asked he often says he was not paying attention, or he could have done what his peers did. Given the nature of his conception it is not necessarily surprising that he does not recognize his peers' thinking because he relies upon getting correct answers and working independently as components of what it means to be smart. Whereas Arianna and Chandler have conceptions of smartness that include *doing* mathematics Jeremiah's conceptions are situated in traditional views

that may actually serve as an obstacle for him learning and therefore knowing a lot of math.

#### **4.7.1.2 Conceptions are Malleable**

Each of the students' demonstrated malleability in their conceptions of smartness. In other words, they seemed able to shift their conceptions or describe them in different ways which seems to be connected to different opportunities they had. La'Rayne and Kasim offer particularly vivid examples of malleability in their conceptions of smartness. Below I unpack their shifts and the context in which it seems they shifted. For La'Rayne, her shift or inclusion from a conception of smartness defined by *getting correct answers* to one that incorporated the need to *keep trying* embodies this malleability. One potential explanation for this shift could be the mathematical tasks that students worked on during the summer mathematics program. Students worked on several tasks that often-required multiple days to solve and were not easily solvable. The idea of *trying hard* or persevering came up repeatedly due to these tasks. In addition to the mathematical tasks another important element is it seems that all students to some extent found the tasks challenging. This point is particularly relevant because it highlights that the problems were challenging for everyone, not just a smaller subset of students. As a student recognizing the "collective struggle" could make it easier to move beyond getting correct answers only, to continuing to keep trying. This potential shift is important for La'Rayne's learning, as well as her peers, because it focuses on an aspect of smartness that often is not foregrounded in math. That is, "keep trying" is a part of *doing* mathematics and it is not something that only some students (e.g., lower students do).

Similarly, Kasim's initial conception centered on *consistently getting correct answers* and *getting correct answers quickly*. However, over the course of the program, his conceptions seemed to broaden somewhat--at least with respect to speed. That is, he began to shift his conception from *getting correct answers quickly* to *not giving up in pursuit of getting correct answers*, even if it takes a little longer. One potential reason for Kasim's suggested shift is his experiences in the summer mathematics program. For instance, the mathematical tasks were challenging. For example, Kasim was not able to solve the *Magic Triangle* right away; however, he did eventually solve it. The train problem which was a problem the class worked on for more than a week was another example of a problem that he didn't solve right away, although they eventually did solve them. Given Kasim's experiences in classrooms it seems he was accustomed to being the "smartest" in his classrooms and getting answers quickly. However, given the problems they worked on in this context and emphasis on the collective, getting answers correct and quickly was not the only priority. Kasim still valued getting answers correct, but he seemed to be less concerned with how quickly and more with getting it correct no matter how long it takes.

Both of these examples raise the questions: Under what circumstances are students' conceptions malleable and in what ways can/does the classroom context (including the teacher's practice support or hinder this malleability? These questions are different but are important as they do not put the malleability on students' conceptions only on students. For instance, these questions allow us to ask whether La'Rayne and Kasim value persevering? Additionally, is there something about the summer mathematics program that encouraged or supported different conceptions?



#### **4.7.1.3 Conceptions as Strategies That Push Back Against Anti-Blackness**

The third way that students' conceptions of smartness are complex is that they could serve as strategies that push back against antiBlackness. That is, the conceptions that students expressed, which are connected with their experiences in general and mathematics classrooms specifically, allow students to be themselves and find comfort in mathematics. AntiBlackness as defined by Dumas and Moore (2016) "refers to a broader antagonistic relationship between Blackness and (the possibility of humanity)" (p.429). The very nature of antiBlackness is systemic violence against Black people. As outlined by Martin, Price, and Moore (2019) in mathematics education there are three different forms of systemic violence: physical, symbolic, and epistemological. I focus specifically on symbolic violence which is characterized by Ferguson (2000) as "the painful, damaging, mortal wounds inflicted by the wielding of words, symbols, and standards." (p. 51). Below I unpack how the conceptions of smartness and ways of being for Black learners serve as examples of pushing back against systemic violence. Although all of the students to varying degrees embody this theme I focus on Jeremiah and La'Rayne.

Jeremiah and La'Rayne are students that I categorize as at risk to "slip through the cracks." By this I mean, that these students, because of their conceptions and ways of being, are likely to experience systemic violence in ways that result in consequences that include suspension, special education, and disenfranchisement from schools. Below I unpack the ways in which their conceptions and ways of being push back against antiBlackness.

Jeremiah's statements that "no one is as smart as me" and "everything I did was smart" might make him seem like a know-it-all. However, reframing Jeremiah's bluster as a coping strategy counter to antiBlackness, reveals that he has developed a conception of smartness that ensures that he not only sees himself as smart, but that he is the person that has the most say about dictating his own smartness. As discussed in chapters 1 and 2, Black learners are often not seen as smart or positioned as smart in traditional mathematics classrooms, reifying the notion that Black learners are unintellectual. Thus, dictating his own terms for what constitutes smartness is a way to resist the ways that traditional definitions of smartness, what is smart and who is smart, which is an example of epistemological violence, operate to undermine his own self-concept and identity.

La'Rayne had two conceptions of smartness: *getting answers correct* and *keep trying*. In some ways La'Rayne's conceptions seem to serve as a coping mechanism in the sense that it allowed her to be herself. Early in the program in a conversation with La'Rayne she made it clear that she did not like math and that she only attended the summer mathematics program because her dad made her. She also mentioned a previous teacher yelling at her and that she didn't like her. Given what La' Rayne said in the interviews and wrote in her notebooks, I wondered what her experiences might have been in prior classrooms. In the summer mathematics program, it is interesting that La'Rayne seemed to be very engaged. She often had and shared her answers, but she listened to her peers as well. She would often revoice what students said or even restate for students who spoke softer in class. She spoke her mind out of turn and was very blunt about her actions sometimes. For instance, one time she shared her thinking

and got it right, and just said she guessed. Given what La'Rayne was doing in class it was strange that she did not see herself as smart. It seems that given her actions, what she was saying, and possibly her prior experiences, being herself was a coping strategy. By being herself, I mean participating in the ways that felt natural to her. It seems likely given what she said and being a Black girl, her ways of being in other classrooms may not have been allowed or came with consequences. For instance, sharing without being called on or calling a teacher out publicly about her practice are things that are often not permitted in classrooms for Black learners. For La'Rayne she seems to be unapologetic about who she is, what she thinks, and what she does. La'Rayne pushes back on symbolic violence in that she does not assimilate to "normalized" behavior about when she should talk and how she should talk.

## **Conclusion**

The purpose of this chapter has been to answer the question: What were these Black learners' conceptions of smartness? Analyzing the artifacts provided us with the conceptions of smartness that each student had. On the surface some of the conceptions were expected. That is, one can imagine that students saying that getting answers right, getting high test scores, and getting good grades is what it means to be smart. However, some other conceptions were less expected. For instance, revising your thinking, trying your best, or listening to others, which are important characteristics, but often not emphasized. Looking across the students provided detail about the complexity of conceptions students have. The complexity of these conceptions suggests that although some conceptions among students seem similar, there are important differences, conceptions could shift, and conceptions might serve as strategies for

students to push back against antiBlackness. These points raise the question under what conditions might these conceptions that counter dominant patterns of anti-Blackness develop? The summer mathematics program, the environment in which students were situated, is a particular context in which there was intentionality about sending different signals about the qualities and capabilities of Black learners. In the next findings chapter, I focus on one part of the context, the teaching. Using my perspective as an observer and Black scholar, I focus on what the teacher does to create a context in which non-traditional conceptions of smartness may emerge and flourish.

## Chapter 5 Findings

To begin this chapter, it is important to reiterate given the historical and present experiences of Black learners (See Chapters 1 and 2) navigating anti-black spaces that this chapter will take a deep dive into the practice of one teacher. The goal is to identify practices that might interrupt common antiBlack messages about Black learners. I will not attend to the context specifically are attempted to evaluate or validate the summer mathematics program. Instead, I will focus on the teacher's practice and methods to better understand how messages about smartness are communicated.

In this chapter I address my second research question, "How does the teacher communicate smartness during a summer mathematics program?" I define *communicate* is defined as what the teacher says and does; this definition does not assume that students recognize or take up what the teacher is communicating. To answer my second question, I focus on and analyze the teacher's statements and actions to understand how she communicates smartness during the summer mathematics program. I argue that the teacher communicates smartness through messages that she signals through the moves she makes during mathematics instruction.

My analysis in Chapter 4 guided my approach to this chapter. That is, I examined students' responses to help identify key moments to analyze the teacher's practice. Additionally, I also used my conceptual framework to identify other moments

during instruction. I focused on moments in which the teacher seemed to be doing the following: (1) identifying students' competence, (2) identifying the competence of students who seemed to have lower status, and (3) publicly acknowledging the competence of those lower status students. To address this question, I identify messages the teacher signaled about smartness during five episodes which took place during whole-group instruction at different points throughout the program (see Table 5.1). In particular, I focus on what I call critical moments that occurred within each episode. Critical moments are moments that students alluded to in their artifacts as relevant or important with respect to defining "smartness" and/or moments that I determined were relevant or important to defining "smartness." Three of the moments took place during whole-group discussions that focused on mathematical tasks. Two of the moments took place during whole-group discussions that focused on students' behaviors. For each moment, I provide a framing of what has happened prior to the moment, describe the moment, and name the messages that the teacher communicated, and unpack how the moment illustrates the communication of a particular message. After describing the moments that occur within the five episodes, I identify the messages that came up most frequently and the teacher moves that supported those messages. At the end of the chapter, I introduce the idea of signaling methods to describe the ways the teacher communicates smartness during the summer mathematics program.

**Table 5.1***Overview of episodes*

Ep.	Problem	Kids	Key Messages
1	Cuisenaire Rods - Which rod is three times a light green?	Lauren La'Rayne	<ul style="list-style-type: none"> <li>• it's necessary to listen to peer's explanations for understanding</li> <li>• you must explain your thinking to convince peers</li> </ul>
2	Grey Rectangle Problem	Ryan Kasim Madison Michio	<ul style="list-style-type: none"> <li>• by explaining your thinking such that you convince your peers</li> <li>• listening to peers' explanations for understanding.</li> <li>• by having, and exercising agency to agree or disagree</li> <li>• by revising your thinking when you gain new knowledge or understanding</li> </ul>
3	Notebook Share Out	Miah Hamza Layla Ryan	<ul style="list-style-type: none"> <li>• writing clearly</li> <li>• writing a lot to explain your thinking</li> <li>• explaining your thinking for every answer</li> <li>• keeping track of your thinking as it changes over time</li> </ul>

			<ul style="list-style-type: none"> <li>• taking good notes even if the teacher doesn't ask you to</li> <li>• exercising your agency to do what helps you learn</li> <li>• using the conditions of the problem to solve problems and check your work</li> <li>• listening and learning from each other's thinking</li> <li>• taking your time and being thoughtful</li> </ul>
4	Reflection on Behavior	Arissa Layla Michio	<ul style="list-style-type: none"> <li>• being able to explain your thinking</li> <li>• sharing your thinking</li> <li>• explaining to convince your peers (e.g., using materials to explain and explaining for understanding)</li> <li>• sharing untried ideas</li> <li>• listening to a peer's explanation for understanding</li> <li>• identifying what helps you understand or learn</li> </ul>



5	Grey Rectangle Problem	Jeremiah La'Rayne Dior Chandler	That being smart in math means: <ul style="list-style-type: none"> <li>• listening, learning, and working with others</li> <li>• not being punished</li> <li>• exercising your agency</li> </ul>
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### 5.1 Episode #1

The first critical moment occurred on the very first day of class. During the first part of the morning, the teacher and students spent time familiarizing themselves with one another and with the space. After the break, they began their mathematics work, using Cuisenaire rods to reason about relationships and proportions. Although the materials were new to the children, they quickly became familiar with them. As the children worked with the rods, the teacher communicated two messages about what it means to be smart in math: (1) it's necessary to listen to peer's explanations for understanding, and (2) you must explain your thinking to convince peers.

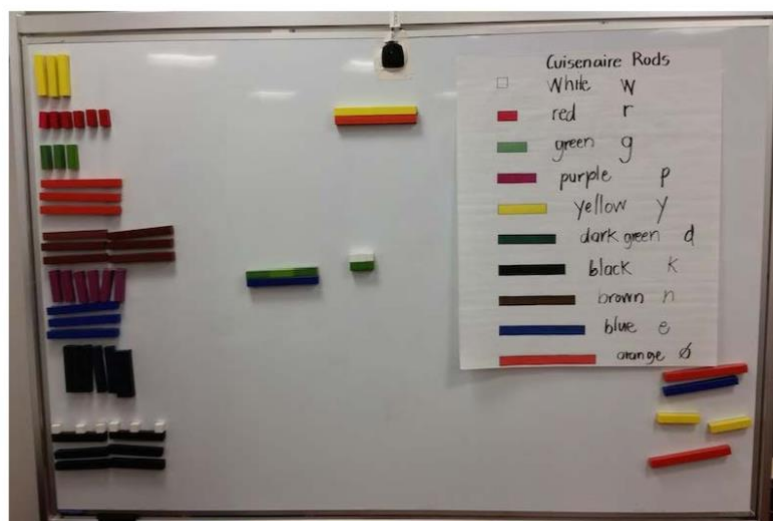
The teacher began the second part of class by offering several prompts: "Find a rod that's just as long as two of the white ones," the teacher said. "Hold up the one you think it is" (see Figure 5.1). The students held up the corresponding rods. Then, the teacher said, "Hold up the one that is the same length as two of the reds." The students again held up the corresponding rods.

Next, the teacher posed a question that was a bit more complex: "Which rod is three times a light green?" In other words, she wanted the students to identify which rod is the same length as three light greens. Initially, the teacher asked students to hold up

the rod and answer the question at their tables. However, perhaps after seeing some discrepancy in students' responses, she decided to have students come to the board.

### Figure 5.1

*Students' work on various Cuisenaire rod problems*



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After posing the question again, the teacher noticed Lauren, a Black girl, raised her hand and asked her to go to the board. As Lauren walked to the board, the teacher addressed the other students, “Where should your eyes be?” and “What are we doing right now?”

Once at the board, Lauren asserted, “Three times a light green is blue because blue is the same size when you put three up against each other” (see Figure 5.2).

**Figure 5.2**

Lauren's explanation at the board



Below is the next part of the discussion:

**Teacher:** Can you turn and face the class now.

*[Lauren turns her body to face the board, then the teacher.]*

**Teacher:** Are you finished explaining?

**Lauren:** Yes.

**Teacher:** Okay. Does anyone have a question? Stay there. Does anyone have a question for Lauren? You can comment, La'Rayne.

**La'Rayne:** Uh, I think that it's a white.

**Teacher:** So you have it. Do you think it's a different answer?

**La'Rayne:** Yes.

**Teacher:** Okay. Can we, can we get right back to you in one second? Yeah, because that means you have- that you want to say it might be a different answer, right? Does anyone have a question? I'll change it yet. Okay. Leave what you have there a minute. Okay. Does anyone have a question for what Lauren said? Can someone say what Lauren said? You don't have to agree or disagree, but what did she say?

*[Lauren fingers the strap of her purse, which is draped across one shoulder.]*

The teacher asked Lauren to face the class and asked if she was finished explaining, to which Lauren responded that she was. As Lauren began to return to her seat, the teacher directed her to stay at the board, while the teacher asked the other students if they had any comments about what Lauren said. During this moment, the teacher was at the back of the classroom, while Lauren was at the front of the classroom at the board. This physical arrangement along with the teacher's moves (i.e., asking Lauren to face the class and keeping her at the board) signals to everyone that Lauren is the authority on her explanation. In addition, because Lauren is responsible for taking up comments and defending her thinking, this sequence also reinforces the idea that an important part of sharing your answer or explaining your thinking is convincing your peers. Finally, the expectation that students should comment on Lauren's explanation communicates that not only is Lauren supposed to take up feedback on her work, but that her peers are supposed to listen and provide that feedback. These moves are important departures from typical practice.

Next, the teacher called on La'Rayne, who raised her hand to respond to Lauren's explanation. However, La'Rayne wanted to offer a different answer rather than comment on Lauren's reasoning. The teacher tabled La'Rayne's comment by asking her, "Is it okay if I come back to you in a second?" Rather than immediately taking up La'Rayne's alternative answer, the teacher redirected the class' attention to Lauren's explanation.

With this move, the teacher communicated the importance of understanding a peer's explanation. The teacher's move refocused the class on understanding what Lauren just said instead of immediately moving to another answer. Also, as La'Rayne was the first student to comment on Lauren's explanation, this was an opportunity to model to other students that understanding a peer's thinking comes before proving or sharing your own answer.

The teacher then asked if anyone had questions about what Lauren said. She clarified that commenting on a peer's work does not mean you have to agree or disagree. When no one responded, she asked if anyone could restate what Lauren said. She called on Michio, a Latino boy, who said, "She said that if you put three of them together and the matching up with the blue it gets you the same length." The teacher checked with Lauren to confirm that's what she said. Lauren said it was, and the teacher directed Lauren to leave her work on the board and go back to her seat.

In this exchange, the teacher communicates that being smart at math means explaining work to your peers in ways that are convincing. At the same time, she shifts students' attention away from sheer correctness of an answer, as she specifically tells students that they don't have to agree or disagree with whether Lauren's answer is

right. Rather, she directs students' attention first toward understanding and then toward critiquing the quality of Lauren's mathematical explanation. Further, she underscores the value and importance of listening to others' thinking. Part of explaining one's thinking is being able to convince your peers. In this moment the teacher holds this expectation by having Lauren stay at the board, supporting Lauren's peers to provide commentary, and allowing Lauren to confirm her peers' interpretation of her explanation. Many teachers in this situation might have simply allowed Lauren to sit down after she explained her thinking; or they might just ask students if they agree with Lauren's answer, which is correct. These more typical responses would miss the opportunity to communicate important messages about smartness as described above.

After Lauren confirmed that her peers had accurately revoiced her thinking, the teacher went back to La'Rayne and asked what her answer was. La'Rayne stated, "I think it's three whites." La'Rayne then proceeded to the board. As she approached, the teacher repeated some of the same phrases she had when Lauren went to the board such as, "Who should we be listening to?" Once at the board, La'Rayne used the Cuisenaire rods to show her answer (see Figure 5.3). She stated, "I think that it's three whites that equal one green... Because if you put three white cubes on top of the green one, it's equal to one whole green cube."

### **Figure 5.3**

*La'Rayne's explanation at the board*



The teacher asked the class, "... Comments about what La'Rayne did? Is that right, that three whites are the same length as a green?" The class collectively responded yes. The teacher then said:

So when I said, which rod is the same as three greens, which one tells us which one's the same as three greens? And which one tells us something else? Because there's two different answers on the board, and they both tell us something important. Stay there, La'Rayne. You're showing us something really important. Let's figure out which question goes with which answer. Raise your hand if you think you know which question goes with which one of those. La'Rayne has one and Lauren has another. Jerone, what do you think?

In this moment we see the teacher reframe the original question, “Which rod is three times a light green”, to focus on what each student’s explanation tells the class. Further this reframing attend to what question does each student’s explanation answer.

Over the next couple of minutes, the teacher asked Jerone, a Black boy, to match the questions Lauren and La’Rayne were responding to with the explanations they had each provided. During this process, the teacher also checked in with La’Rayne to see if she agreed with Jerone’s comments. After some back and forth, La’Rayne agreed that her explanation answered the question, “Which rod is three times a white?” She also agreed that Lauren’s explanation answered the question, “Which rod is three times a light green?” Before the teacher closed the discussion, she had Deedrah name the two questions addressed one more time. The teacher concluded the discussion by stating, “Good job, La’Rayne. And good job, Lauren.”

In addition to explaining your work to peers to convince them, the teacher’s moves here also communicate that being smart in math means listening for understanding of a peer’s explanation. After Lauren and La’Rayne each present their thinking, the teacher asks students to comment on their thinking which includes voicing their thinking, highlighting what they did well, and/or agreeing or disagreeing with their explanation. Each type of comment requires students to provide evidence or justification for what they said rather than passively listen. Said another way, the teacher’s moves here demonstrate that listening does not mean merely being quiet or respectful when someone is talking. Instead, students are expected to ask questions, attend closely to their peers’ reasoning, and ultimately understand their peers’ thinking.



The teacher's reframing to ask which answer goes with which question de-emphasizes right versus wrong as the key quality of a contribution or the key purpose of a mathematical solution. This move is important because often in math class and during math discussions the main goal is to identify what is the right answer and who got the right answer. By focusing on the idea that each student is answering a different question--and identifying both answers as "tell[ing] us something important"--the teacher moves the discussion beyond right and wrong answers and allows students to make sense of explanations provided.

## **5.2 Episode #2**

On the third day of the program, the students worked on the Grey Rectangle Problem (see Figure 5.4).

### **Figure 5.4**

*Grey Rectangle Problem*

What fraction of the rectangle below is shaded gray? \_\_\_\_\_



What fraction of the rectangle below is shaded gray? \_\_\_\_\_



This problem has two parts: The first part uses an area model that is equally partitioned with one part shaded grey; the second part of the problem uses an unequally partitioned area model with one part shaded grey. In each part of the problem, the question remains the same: What fraction of the rectangle is shaded grey? This problem is important because it gets at foundational content knowledge around using area models to name fractions that is typically covered in elementary school. The first part of the problem is relatively easy because the rectangles have already been partitioned into equal parts. This allows students to focus on interpreting the total number of parts and parts shaded. The second part of the problem is different as the representation is not partitioned into equal parts. This prompts students to grapple with the need to partition the wholenecessity of the whole being partitioned into equal parts in order to name a fraction.

To highlight the messages that the teacher communicates about what smartness is during this episode, I break the episode down into the phases in which the work the teacher does takes place. By phase I mean different parts of the episode which I use to narrate overarching ideas in which I identify messages that are communicated.

### **5.2.1 Phase I: Ryan Initial Explanation**

The first phase focuses on the initial explanation of the first part of the Grey Rectangle Problem that Ryan, a Black boy, offers. The first phase is important because the teacher began to communicate an overarching message that collective explanation is a feature of smartness. Throughout this critical moment we will see that the teacher starts with Ryan's explanation and keeps that explanation in mind for the class as they complete the discussion of the grey rectangle problem.

**Teacher:** All right, time to look up. So, we have different answers for some of these. And what our goal now is we're going to talk about is everyone looking at me. Our goal is we're going to talk about different people's thinking and see if we can come to a way of agreeing about how to think about both these problems. So, when Ms. Bria walked around, she saw that people and I saw the same thing when I walked around. You have different ideas about the answers. You have to listen really carefully to other people's thinking and see if we can figure this out. This is very, very important, even though it looks kind of simple. This is a really basic idea that's very important for fifth grade and sixth grade and for doing all kinds of other mathematical thinking. So let's talk about the first problem first. This one who'd be willing to read the problem again and say

what you think the answer is and your reason. For the first one up here, Ryan. Read this, please.

**Ryan:** [Ryan read the problem from his seat]

**Teacher:** Can you come up and explain what you think? The answer is? You can write it on the board and explain it. Can everybody be looking at Ryan and trying hard to hear him. You want to write on the paper right next to the shape? Yeah, there you go. Oh are you writing your whole reason? Okay, Ryan, you know what, you can just write the number and talk your way through it. Okay. What number did you write in your notebook for how much was shaded? Okay, now can you turn around and explain to the class why you wrote, what number that is and why you wrote that?

**Ryan:** I think its  $\frac{1}{3}$  because...there are three

**Teacher:** One second Rayveion and Jerone are you listening to Ryan? Thank you.

**Ryan:** There are three squares and one is shaded.

**Teacher:** Can you turn and face the class and say it one more time because it was good but it was a little hard to hear you.

**Ryan:** There are three squares and one is shaded so I think its  $\frac{1}{3}$ .

**Teacher:** Okay. So stay right there, I'd like to hear if other people agree or disagree with what Ryan just said.

**Class:** Agree

During this phase of the discussion, the teacher communicates two ways to be smart in mathematics: (1) by explaining your thinking such that you convince your

peers, and (2) listening to peers' explanations for understanding. Together these two messages are central in reinforcing the development and importance of collective explanation. The teacher demonstrated the first message when Ryan gave an explanation at the board in which he stated, "There are three squares and one is shaded, so I think it's one-third." All of Ryan's classmates agreed with both his answer and explanation. The teacher did a quick check to confirm that the rest of the class came to the same answer, one-third. Next, she asked the class: "Did anybody have a different reason?" She then called on Kasim.

### **5.2.2 Phase II: What Makes a Mathematical Reason**

In the second phase, the teacher focuses explicitly on the idea of what makes a mathematical reason. This phase includes several explanations that were given by three students, Kasim, Madison, and Michio. As the discussion progresses, it becomes clearer what exactly counts as a mathematical explanation in this space. After Ryan's explanation about to the first part of the problem, the teacher raised the question "Did anybody have a different reason?" Kasim responded, "I said I think this one is right because I studied this in class for one month."

The teacher responded:

Because you studied it for a long time. So, when Kasim says he studied something for a long time, that's true and it's good. It's really important, but it's not the same thing as a reason. Because a reason would be explaining why it's true, and that's different than 'I studied it.' I said the same thing to you, La'Rayne, right? When I said, you can't just say because you learned it before. That's not really a reason ...

In this excerpt the teacher is clear in making the distinction between what is and what is not a mathematical reason. In particular the teacher leverages what students have already said in the class about what is a mathematical reason, to define what is a mathematical reason in the summer mathematics program.

The teacher's assertion that studying a problem or topic for a period of time is not a sufficient explanation or reason for an answer is important because it highlights the fact that a mathematical reason must explain the processes and logic one uses to get an answer. Although this message might not seem important in the context of a problem where one knows the correct answer, it becomes very important when one does not know the correct answer. For example, while the first part of the gray rectangle problem represents a relatively straightforward area model, the second part of the problem requires students to partition the whole into equal parts before naming the fraction. This is mathematically more complex and requires that students be able to reason and explain more thoroughly to arrive at the answer.

Starting with Kasim is important for this phase because he conveys a common view of what it means to explain or know. That is explaining or knowing something comes from the teacher and the amount of time spent studying something. The teacher moves to Kasim's explanation and subsequent explanations offer a different approach to explaining that runs counter to explanations that might be accepted in other spaces.

The teacher next takes up the second part of the problem with students. The teacher makes two moves to communicate what it means to be smart at math. The first move reinforces the message that a student's explanations must convince their peers.

The second move reinforces the message that students should listen to their peers' explanation for understanding.

The teacher opened this part of the discussion by stating:

The second one is where we have disagreements in the class. And I'll tell you what I saw. I saw people with two different answers in the class, and they are not both right. And so, we're going to have to figure out which one is right and how to think about it. Everybody ready to listen? Okay? So, I would like, let's see who would like to give an answer for the second one? Somebody who has not had a chance to be at the board yet in the summer class. Madison, you haven't had a chance yet?

The teacher calls on Madison, a Black girl, to come to the board to share her answer. Up until this point in the program, Madison hasn't had many opportunities to share her thinking at the board.

The teacher's acknowledgement that there are "two different answers" and her assertion that the class is going to "figure out which one is right and how to think about it" reinforces the message that the answer alone is not sufficient. The inclusion of how to think about the problem indicates her attention to students' thinking and the process behind their answers.

Once at the board Madison explained:

One-third. I still think it's- I think the bottom one is one-third. Because when you do fractions, you count, like, how many, like, squares they say there are. And then the

one that shaded you would put that there, so that's how you would know, if it was one-third.

After Madison's explanation the teacher followed up:

Okay, so a lot of people have the same answer as Madison. Put your thumb up if you had the same answer as Madison. *[The teacher pauses as students put their thumbs up in agreement.]* Does anyone have a question for Madison about that? *[The teacher waits to see if anyone has a question for Madison.]* So now someone who has a different answer can go to the board and explain your thinking, then we're going to try to figure it out. Thank you, Madison. Who has a different answer from Madison because some people had a different one? And I want to see if we can figure out how to think about this. Okay, Michio, you'd like to go to the board? I want you paying attention right now, Luiz. Where should your eyes be right now?

In this sequence we see Madison who had not previously gone to the board share her thinking at the board. Also, we see that it appears many of her peers agreed with her thinking as students raised their thumbs up and no one had questions for Madison.

The teacher communicates the importance of students listening to their peer, Madison, by (1) making the move to call Madison to go to the board in the first place (2) having the class raise their thumb if they agree, and (3) asking the class if they have questions about Madison's explanation. Madison had not been to the board prior to this.



Traditionally in math classrooms, students who go to the board frequently are considered smart. Therefore, by intentionally calling a student who hadn't been to the board before, the teacher communicates the message that all students' thinking is worth listening to.

The teacher continued to reinforce these messages when the next student, Michio, a Latino boy, provided an explanation to the same problem at the board. Below is the interaction between Michio and the teacher.

**Michio:** I think this is an improper fraction because, see how this one is bigger than both of these two ... There is no possible way that this would be correct because, see how all three of these are the same size that that's a proper fraction and that's a improper fraction because [inaudible]

**Teacher:** Okay, I see what you're saying is it can't be one-third, it can't be a fraction at all?

**Michio:** It can, you're just going to have to add a line right here to make it one-fourth.

**Teacher:** Okay, So I have something you can use for that. [The teacher hands Michio a black sticky line] Put that where you think it goes as a sticky line. [Michio adds black sticky line] Can everybody watch Michio now and let him finish his reason? So now explain what you're thinking.

**Michio:** Now, it's a proper fraction because they are all the same size. Now, it can be one-fourth.

**Teacher:** Can you write one-fourth up there then? [Michio writes one-fourth on the board] Deonte, Deonte, please look. Okay, go back and now turn around and tell the class what you said and what you just did.

**Michio:** I just added a line to make it a proper fraction. And I made it one-fourth by adding a line instead of one-third.

**Teacher:** And what does the line do? You said something, about what the line did.

**Michio:** The line made it a proper fraction.

**Teacher:** Why though? You said something else.

**Michio:** Because before this [the line added] these two used to be bigger than the other two, and now they're all the same size.

**Teacher:** Ok. So Michio is saying that all the parts have to be the same size in order to figure out the fraction. And therefore, once he makes the line and makes it the same size, it's one-fourth. I'd like to hear some other people comment about it. What Michio did? Or ask him a question. Deonte, can you see, okay, how about if you try to turn your body this way so you can see, or your head. Okay? Dior what do you think about what Michio said?

In this interaction, Michio explains that the parts have not been equally partitioned in the rectangle, and the rectangle must have equal sized parts in order to accurately identify the fraction. As Michio explains, the teacher makes the move to provide him with a “sticky” line to support his explanation visually. In this interaction, the teacher pushes Michio to explain why the sticky line is important and then asks the

class if they have any questions or comments for Michio. By providing Michio with the “sticky” line, the teacher supports the message that explaining your thinking means convincing your peers. The “sticky” line prompts Michio to make his reasoning explicit. The teacher’s questioning, including asking why he made particular moves with the sticky line, prompts him to provide clear logic as he explains.

In the next part of the discussion the teacher asked Dior “what do you think about what Michio said?” Below is the ensuing discussion.

**Dior:** Um I think he saying like, I think I know what he’s saying like...He's saying that there's four squares. So there’s four right now. And then once you shade in one, it takes away, it actually kind of takes away one square, so it's basically  $\frac{1}{4}$ .

**Teacher:** Okay. And what, Why do you remember what he said to us why there has to be another line?

**Dior:** Because if you if you don't put a line in there for four squares, they gone think it's just one square.

**Teacher:** Can you take the line back off for a minute, Michio? It just peels right back off. [Michio takes the black sticky line off the rectangle] Okay. So Dior can you now say why did Michio put the line there?

**Dior:** To make them equal.

**Teacher:** Can you say that louder? That's really important.

**Dior:** To make them...

**Teacher:** Equal. We use that word equal earlier today to make the parts equal. Is that what you were doing? Can you put it back? [Michio puts black sticky line back on the rectangle]

In this interaction the second message the teacher communicates is the importance of listening to peer's explanation for understanding. In the transcript above Dior along with the teacher probing and Michio removing the black sticky line restates Michio's idea emphasizing the idea of "equal parts". In the next segment of the discussion the teacher reiterates the message through Rayveion. Below the teacher first as a question to the class.

**Teacher:** Who else had one-fourth? You did, Rayveion? Do you want to go up and explain what you were thinking? You want me to take that line back off? Here, you can use it if you want it. Come on up. [Rayveion goes to the board] If you need it, you can use it. Please give your attention to Rayveion.

**Rayveion:** I thought it was  $\frac{1}{4}$  because the rectangle was bigger than both of these, so kind of the same thing as Michio...

**Teacher:** Could you say that again, a little bit louder Rayveion, please?

**Rayveion:** I think it was  $\frac{1}{4}$  because the big one is bigger than both of the squares, the shaded square and non-shaded square. So it's kind of the same as Michio's.

**Teacher:** So you are agreeing with Michio.

**Rayveion:** [Head nod yes]

**Teacher:** So do you want to put the line back on to show how you made the equal parts?

[Rayveion puts black sticky line back on the rectangle] Okay anyone have a question for Rayveion?

In the moment above the teacher reiterates the message that it is important to listen to peer's explanations for understanding but differently. Having Rayveion come up to the board and explain his thinking allowed him to also make connections to Michio's explanation. This point is important as demonstrates not only the value of listening to others, which is smart to do, but illustrates the idea of collective explanation. That is a part of developing collective explanation is the critical component of listening.

As the discussion continues, the teacher communicates two additional ways one can be smart in math: (1) by having, and exercising agency to agree or disagree, and (2) by revising your thinking when you gain new knowledge or understanding. The teacher asked:

You agree they have to be equal? Does anyone want to say it doesn't matter if they're equal or not? So, this is a very important idea about fractions, and that's what this problem was for. So, you think really carefully about, remember, the parts have to be equal to identify a fraction.

The teacher's message that students have agency to agree or disagree is subtle. To this point in the discussion, Madison and Michio have provided explanations that

seem to be convincing. The teacher's move to provide space for students to think about whether they agree or disagree gives them agency.

### **5.2.3 Phase III: *Bringing It All Together***

The third phase brings everything together. Bringing it altogether first requires examining the sequencing of tasks. Ryan's explanation first surfaced the idea that naming fractions depends on identifying equal parts. Next, Kasim's explanation leads the teacher to further elaborate what counts as mathematical reasoning. The teacher carries the explanation for students as they continue to explore and develop it.

In the final phase of this episode, the teacher communicates that being smart in math involves revising your thinking as you gain new knowledge or understanding and sharing your evolving thinking with others. She returns to Ryan, who launched their work with his initial response, "There are three squares, and one is shaded, so I think it's one-third." Other students had successively added to this, developing the key idea of "equal parts."

After hearing from Dior and Rayveion, the teacher circled back to Ryan, positioning him as a doer of mathematics and someone able to complete the class' work:

**Teacher:** So when Ryan gave his explanation, what word [do] we have to add to his explanation? He gave a good explanation. But what other word do you think we need in Ryan's explanation? Can you now say, Ryan, why this is one-third?

**Ryan:** Because there are three squares and only one is shaded, so that will make it one-third.

**Teacher:** Yeah, but can you add the word “equal” to your explanation? You have three what?

**Ryan:** There are three equal squares.

**Teacher:** Excellent. Can you tell that to the class? There are three equal squares, and ...

**Ryan:** And only one is shaded.

**Teacher:** And only one is shaded. So this one is one-third because there are three equal parts. One is shaded. How much is this one? One-fourth. Please work on your notebook now, and if you need to revise or change your answer, you can change it now. If you don't need to revise your answer, you can leave it.

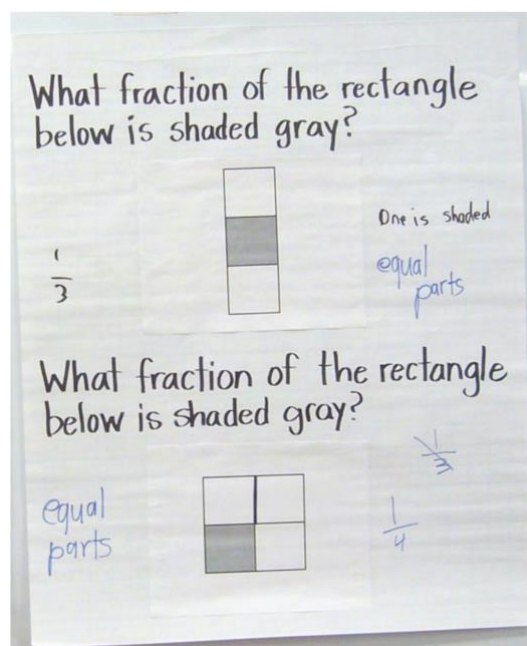
The teacher named Ryan's initial contribution to the discussion of the task and guided him to represent the ensuing work by adding to his initial contribution. He added the word “equal” to his explanation, and the teacher called this “excellent” and positioned him to provide this conclusion to the discussion. After he did this publicly, the teacher recorded the idea of equal parts on the board (see Figure 5.5) and encouraged others to “revise or change” their answers in their notebooks as needed. In doing so, the teacher signals that revision, as Ryan has just performed, is a useful move as a doer of mathematics. In addition, the teacher highlights that keeping records of one's thinking, including revisions along the way, is also important.

Across these phases, we see three messages about doing mathematics with smartness. One is the importance of working together with others, listening to and building on or challenging their ideas. A second is the practice of revising one's thinking,

and a third is keeping records of one's ideas and learning. These messages are distinct from normalized patterns wherein students see "getting the right answer" (in this case, "one-third") is primary, and having the "wrong answer" as something to make sure to correct on one's paper. These common patterns also position students as individual learners, whose own independent mathematical knowledge and skills are being developed. Here the doing of mathematics is explicitly collective, with conclusions and ideas being the product of multiple students' contributions, the teacher's guidance, and group and individual record-making.

**Figure 5.5**

*Day 3 Grey Rectangle Classroom Poster*



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The teacher's moves to communicate this message are important because they counter traditional views of being smart in mathematics that prioritize students getting answers right as the ultimate goal.

### **5.3 Episode #3**

On the fifth day of the summer mathematics program, after the warm-up, the teacher showed students' notebooks. The teacher asked students to come up to the document camera to share aspects of their notebooks that were particularly interesting or important for the way the class was working on mathematics together. Through this activity, the teacher signaled several messages about what it means to be smart in math. She signaled the importance of: (1) writing clearly, (2) writing a lot to explain your thinking, (3) explaining your thinking for every answer, (4) keeping track of your thinking as it changes over time, (5) taking good notes even if the teacher doesn't ask you to, (6) exercising your agency to do what helps you learn, (7) using the conditions of the problem to solve problems and check your work, (8) listening and learning from each other's thinking, and (9) taking your time and being thoughtful. Here, I highlight and unpack how the teacher communicated each message.

#### **5.3.1 *Writing Clearly and Writing a lot to Explain Your Thinking***

Before the teacher called upon students, she set up the activity:

Okay, excellent. I want to take a few minutes to look at some particularly good notebooks again today. Jeremiah, up here, eyes up here. So again, they were amazing notebooks. Again, yesterday I thought people wrote great end-of-class checks. Do you know, one of the things I thought was especially good on the end-of-class check

yesterday? I asked you some questions about train problem part two, and we just started it. But almost everybody got pretty difficult questions, correct on the end-of-class check because you had to know the conditions of the problem. And you wrote, you understood it, and you also wrote really good explanations. The other thing that was really good yesterday was a lot of good revising of your thinking. Like when we worked on fractions, I saw people change their mind and make better explanations. I also saw many of you organizing your work better yesterday. And I think you want to keep trying to find good ways to organize your work.

By setting up the activity in this way the teacher signals several messages that are worth highlighting. First, the teacher does indicate that students are getting some hard problems correct, even though they had just begun to work on them; however, she highlights that they were able to solve those problems because they knew the conditions of the problems. Identifying the conditions of problems is an important feature of tasks the class has been working on. The class has been using the conditions to not only understand what problems are asking, but to also track for themselves the progress they make on tasks. Second, the teacher highlights the importance of revising your thinking. Revising one's thinking is an important characteristic of doing mathematical work. The teacher clarifies that the value of revising one's thinking isn't just about changing an answer from wrong to right but is instead about developing a deeper understanding of the concept by indicating the improvement in students' explanations. Finally, the teacher highlights the ways students are getting better at organizing their work. She names this as relevant for students' own learning and

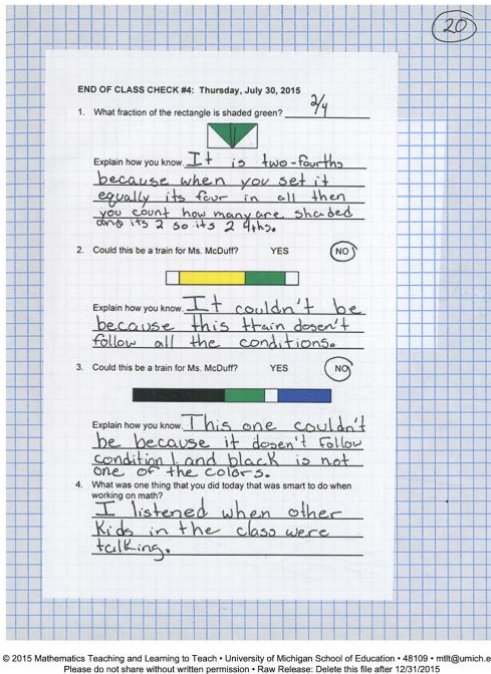
encourages them to continue to find better ways to organize their work. It is important to note that all of the messages communicated above focus on *doing* smartness.

The teacher then called on Miah, a Black girl, to share how she had recorded her explanation of one of the problems on the end-of-class check in her notebook:

So, the first person I'd like to come up is Miah. I'd like to show you something in Miah's notebook that you might like to try in your own notebook. Please look at, up at the screen because it will give your ideas for making your own notebook even better. It particularly would be good to show people your end-of-class check. So, I thought it would be useful to see Miah's end-of-class check partly because she writes very clearly and writes a lot to explain her thinking. And I wondered if you could explain your answers to 2 and 3 because that was the train problem. How did you write your explanation? Could people look up at Miah please? This one and this one [teacher points to questions 1 and 2]. (See Figure 5.6)

**Figure 5.6**

*Miah's End-of-class Check #4*



Here, the teacher spends a lot of time orienting students to Miah and her notebook before actually having Miah share. In addition, the teacher is clear that the purpose for doing this is to learn from Miah, positioning her as competent. The moves to orient students in this way reinforces the importance of listening and learning from others. Further, the teacher explicitly signals two specific messages about smartness which are writing clearly and writing a lot to explain your thinking. It is important to note here that “writing clearly” is not about legibility. Instead, “writing clearly” means writing in a way that communicates thinking and processes.

Miah began by explaining her response to question two on the end-of-class check. In this question, students were asked to examine a proposed solution to the train problem and say whether or not that solution is possible given the conditions of the problem. Miah stated, “I wrote it couldn't be because this train doesn't follow all of the

conditions.” The teacher then asked her peers to name good things that Miah did in her explanation. Ala pointed out that Miah used the word “conditions” in her explanation and Jerone said that she wrote clearly, which made it easier to read.

In this instance, the teacher could have named Miah’s strengths herself to ensure that students noticed particular things. However, having the other students highlight Miah’s strengths reinforces the importance of explanation as well as the importance of listening and learning from others. The move also reinforces the idea that students are accountable to their peers, by giving students agency and power to name their peer’s strengths.

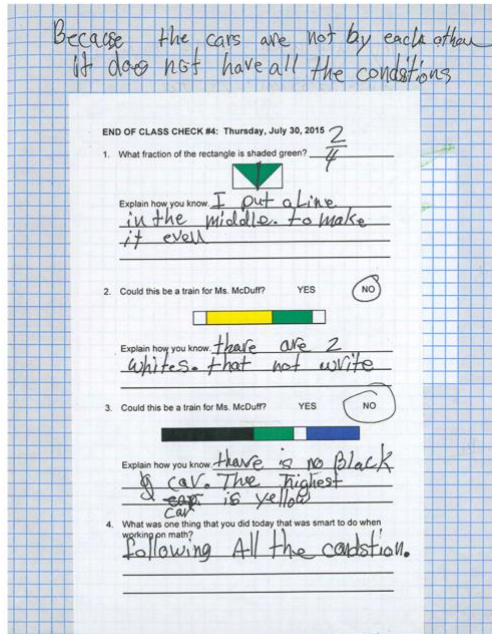
During this exchange, the teacher not only tells the class that writing clearly and writing a lot is important, but she also shows Miah’s notebook as an example of what those things can look like. Miah’s peers also acknowledge the strengths of her explanation that had been highlighted by the teacher. In this moment the teacher consistently reiterates that writing clearly and writing a lot are practices that one does that is smart.

### ***5.3.2 Explaining Your Thinking, Keeping Track as Your Thinking Changes Over Time, and Taking Good Notes***

Next, the teacher called up Hamza and asked him to explain his thinking for the third question on the end-of-class check. This question is also about the train problem and again requires that students use the conditions of the problem to analyze a proposed response. Hamza explained that the yellow rod is the longest rod allowed under the conditions of the problem, so the black rod cannot be used to create a train (see Figure 5.7).

**Figure 5.7**

*Hamza End-of-class Check #4*



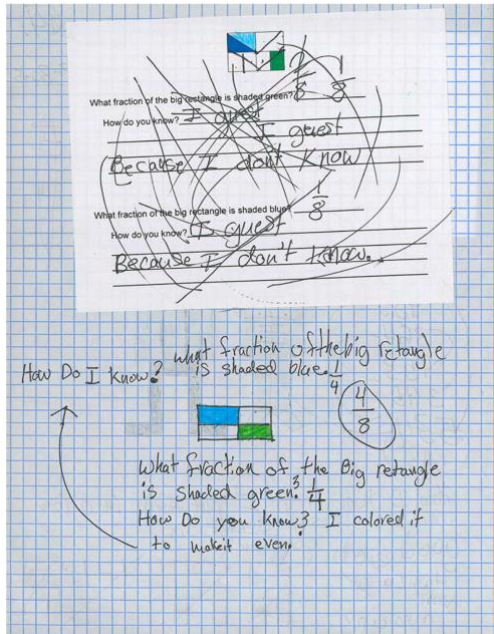
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After Hamza's explanation, the teacher asked one of his peers to name what he did well. The teacher then explicitly acknowledged strengths about Hamza's explanation and his notebook more generally:

... So, I think it was very clear and the whole end-of-class check. He really shows his thinking very clearly and lots of people did, but I think it's useful to see. Another thing he did, as you can see here, is that he kept a record of our work on the fraction work yesterday. He wasn't just sitting at his desk, but he was drawing some of the things and writing some of the different answers. And he tends to ... keep a lot of good notes in your notebook, which I think it's helping you to learn more math. Very nice job Hamza.

**Figure 5.8**

*Hamza notes from previous day*



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In this brief exchange, the teacher makes several moves to signal messages to the students about what constitutes smartness in mathematics. First, she offers Hamza's notebook as an example to demonstrate the importance of explaining your thinking using the conditions of the problem. One of the solutions he explains while at the document camera shows that the train in the problem cannot be made because it uses a black rod and the highest valued rod that could be used is yellow--that is, his explanation makes clear how the train violates the conditions of the problem. The teacher also points out his work from the previous day showing that he takes good notes and keeps track of his thinking. The other students can see that Hamza writes down his explanation, asks questions, and even draws a representation in his notes.

Although his answer is not right, his notes illustrate that he is thinking actively and critically about the problem and not “just sitting at his desk” during discussions. Hamza’s notebook also illustrates how he keeps track of his thinking by crossing out previous answers and explanations as his thinking changes and adding notes to reflect his new understanding. Doing this is important because it makes it possible for him to go back and see how his thinking evolved with respect to fractions. The work the teacher does to highlight Hamza and his notebook is important and communicates messages about taking good notes and keeping track of your thinking.

### ***5.3.3 Keeping Track of Your Ideas Over Time and Exercising Your Agency to Do What Helps You Learn***

Next the teacher called Layla up to the board. Instead of focusing on the end-of-class check, the teacher pointed out the notes Layla took the day before related to the Train Problem. The day prior the teacher had spent a lot of time ensuring students understood what the Train Problem asks. The teacher said:

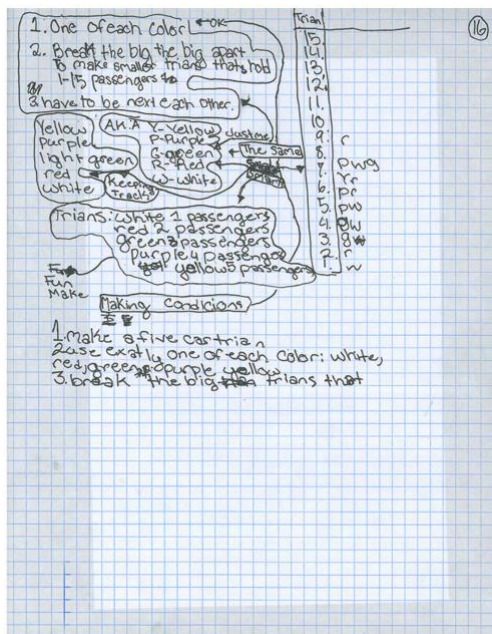
Um, Layla, would you come up and show your work on the train problem yesterday? Again, I'm doing this partly because I want you to see things people are trying, but I also want to encourage you to keep trying new things in your notebook to keep your own ideas organized and explain your thinking. I was thinking that this is interesting, beginning to keep track. So, this is the part we were learning about train problem part two, I wanted you to see the notes that Layla took. Just say what you were trying to keep track of to the class.



Instead of asking Layla to share an answer or an explanation, the teacher asked Layla to explain how she had been keeping track of her ideas in her notes. Layla explained that she wrote down the conditions of the train problem part two, and she wrote down some of the trains that they tried during the whole group discussion (see Figure 5.9).

**Figure 5.9**

*Layla's Notebook p. 16*



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The teacher then asked students what they noticed about Layla's notebook. She called on Chandler, who responded, "She wrote very clearly and wrote things that you [the teacher] didn't ask her to." The teacher took up Chandler's idea and elaborated:

Yeah. She wrote very clearly, and she kept notes. I think this is going to help you today when you work on train problem part two because you already took notes on it yesterday. So, you might try taking some notes when we're talking in whole group to keep track, even if I don't say write that word in your notebook. You could try doing some of what Layla did ... Thank you so much, Layla.

Layla's notes reflect her developing understanding of the Train Problem. First, in the middle of the page, she recorded the color of the rods that the class worked with during week one (white, red, light green, purple, and yellow). She also connected the numerical value of each Cuisenaire rod with the number of passengers associated with each color rod in the problem (i.e., white = 1 passenger, red = 2 passengers, 3 = passengers, 4 = passengers, or 5 passengers). She labeled and recorded the problem conditions, and she used note-taking conventions such as abbreviations, boxes, and arrows to capture her ideas. Although the teacher did not name these features of Layla's notes, students could see them on the document camera as the teacher elaborated on Chandler's idea.

In this exchange, one message the teacher signals about doing smartness is the importance of writing clearly and keeping track of your thinking, which is evidenced by Layla's work on page 16 in her notebook where she uses words, shapes, and arrows to track her thinking. The teacher also highlights the importance of students exercising their agency to do things that help you learn by elaborating on the second part of Chandler's idea about writing notes that the teacher did not say. Layla has done this in two places: she wrote the conditions of the task, and she kept track of the trains that

were discussed during the whole group discussion the day before. By having Layla share her notebook as an example, the teacher makes visible to the whole class how Layla exercised agency over her own learning and understanding. Positioning Layla as a positive example encourages other students to exercise their agency in similar ways.

#### ***5.3.4 Listening to and Learning from Each Other, Taking Your Time, and Being Detailed in Your Explanations***

The last student the teacher asked to come up to the document camera was Ryan. Instead of focusing on the questions related to fractions or the Train Problem Part 2, the teacher focused on Ryan's response to the reflection question, "What is one thing you did today that was smart to do when working on math?"

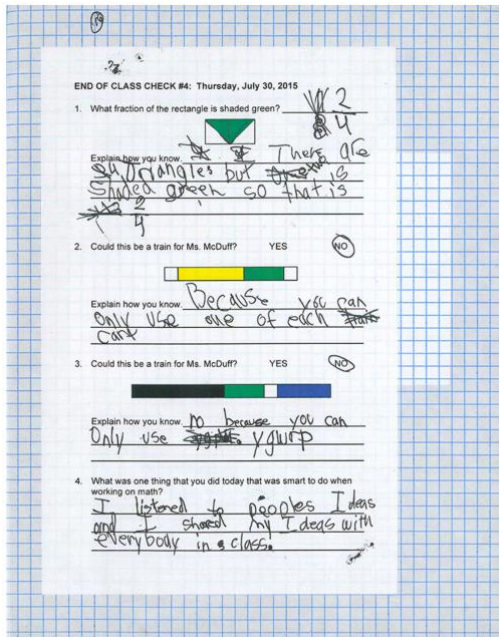
The teacher called Ryan up to the document camera and told him:

I was thinking your end-of-class check would be particularly good. Could you just pick one of them and say what you wrote? So I wanted you to [also see] Ryan's end-of-class check. Do you want to maybe talk about number four because I thought you did a good job of writing something you did that was smart.

Ryan opened his notebook and read, "I listened to people's ideas, and I shared my ideas with everybody in class" (see Figure 5.10). The teacher then asked the class why it is important to do what Ryan said. The teacher called on Michio, who said, "... You can learn from others... so others can learn from you, as well."

**Figure 5.10**

*Ryan's End-of-class Check #4*



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Here, we can see several things the teacher does to signal smartness. First the reflection question she asks Ryan to share has two built-in assumptions: (1) that students did at least one smart thing, and (2) that smartness is something you do. Additionally, pointing to this question publicly is a way to reinforce what it means to be smart in this space. Highlighting Ryan's notebook in particular reinforces the importance of listening and learning from others as well as sharing your own ideas. Moreover, having Michio revoice Ryan's thinking is also a strategic move. Until this point in the summer mathematics program, Michio has shared frequently and has presented at the board. In fact, he brought up the key idea of making the parts the same size when naming fractions on Day 3; several students mentioned that they learned from him in

their end-of-class check that day. By having Michio comment on why what Ryan wrote is particularly important, the teacher highlights that even students who may be considered smart by peers should exercise reciprocity by listening and learning from others just as others listen and learn from him.

The teacher then wrapped up the discussion by saying:

So, what I appreciated about what Ryan did is he was very thoughtful even though I know he probably wanted to go to lunch. He took the time to think about what he had done that he knows is smart to do in learning math. And other people wrote other good ideas. But I thought you really took your time to think of something you had done, and you did do those things yesterday and other things on his end-of-class check were extremely clear also. So, Ryan's another person who really thought about the conditions of the new problem. And it's very clear that you learned those already and are ready to work on them. Nice job ... Your notebook's been getting more and more detailed across the week. I really appreciate that.

During this wrap-up of the notebook activity, the teacher signals two additional messages. First, the teacher communicates the importance of taking your time. Ryan could have rushed through the end-of-class check just to get done. This would have aligned with traditional classroom norms that frame smartness in math as about speed. Instead, by highlighting the way that Ryan took his time, the teacher reframes mathematical smartness as tied to careful and thoughtful reflection. Second, the teacher acknowledges that Ryan's notebook has become more detailed. Although the teacher

did not specify where in Ryan's notebook there was more detail (e.g., warm-up problem, note-to-self, handouts, or end-of-class checks), her mentioning it reflects its importance.

### ***Summary***

The teacher uses the activity of sharing notebooks to signal messages about what it means to be smart in mathematics in this class. The teacher does this work in various ways which include carefully selecting which students she called on, the use of the notebook as a living artifact that other students can see and learn from, and the points she highlighted and how she highlighted them. For Miah, the teacher focuses on explanations, particularly highlighting how clear her writing and explanations are. With Hamza, the teacher highlights how clearly, he shows his thinking. She also highlights how Hamza's explanation explicitly referred to one of the conditions of the problem—a key feature of mathematical thinking and a practice that the class had been working on—as well as how he kept a record of the class' work on fractions from the previous day, which supports his ongoing learning. Next, instead of focusing on the end-of-class check, the teacher uses Layla's notebook to demonstrate how important her practice of writing down what she thought was useful is. Rather than only writing things that the teacher told her to write, Layla exercises her agency to keep track of things that will help her learn—a practice that the teacher wants the other students to see and consider taking up for themselves. Lastly, with Ryan, the teacher explicitly names his comment about listening to and learning from others and sharing his own thoughts as smart. Additionally, she further calls out the importance of learning by taking your time to write your thinking, knowing the conditions of the problem, and being detailed in your

explanations and notes. The exchange with Ryan is the only time the teacher uses the word “smart” during the notebook share.

Finally, the students the teacher highlights during this notebook share are those who often do not talk during whole group discussions or participate in easily observable ways. By publicly positioning them as worth learning from, the teacher builds up the value and voice of every person in the space. The moves the teacher makes are seem subtle, and they run counter to a traditional view of mathematics that not only focuses on speed and correctness but paints a picture and creates a narrative of the student who is smart in math. Instead of enacting these norms, the teacher sets up the notebook share as an opportunity to learn from one another, focuses on what students are doing in their notebooks to support their learning, and shares the notebooks of students who might not typically be seen as smart in math. Throughout, the teacher signals that keeping track of your thinking, listening and learning from others, and keeping your work organized—all things one *does*—are very much central to the learning of mathematics and cultivating smartness in mathematics. Additionally, the teacher not only communicates smartness as what one does, but also encourages students to exercise their agency that attend to the ways in which they learn.

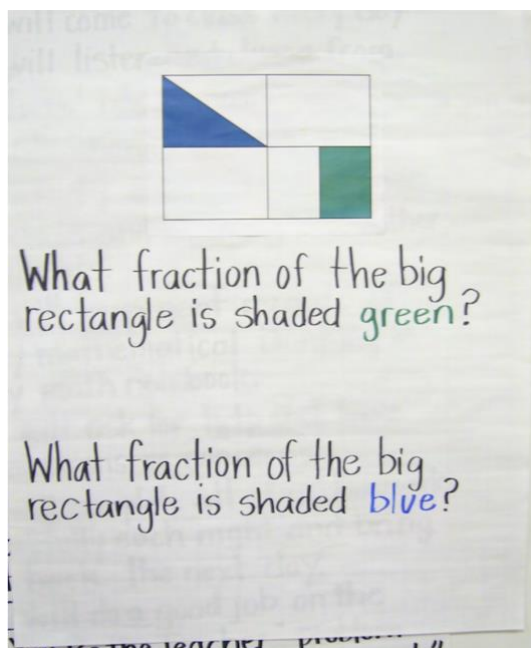
#### **5.4 Episode #4**

On the fifth day of the summer mathematics program, the teacher continued on the Blue Green Rectangle Problem that the class had begun the day before. This task is arguably one of the more difficult fraction tasks in the summer mathematics program. What makes the task challenging is that the whole rectangle has two different shaded parts (blue and green) which are two different shapes (triangle and rectangle) (see

Figure 5.11 and chapter 3). Hence, the blue triangle and green rectangle problem refers to the blue triangle and green rectangle that are parts of the larger rectangle that is referred to as the whole. Until this point in the program, students have only had to deal with area models that contain one shaded part which they could use to partition the whole. Including two shaded parts of two different shapes challenges students' conceptual understanding of fractions and area models.

### Figure 5.11

#### *Blue Green Rectangle Problem*



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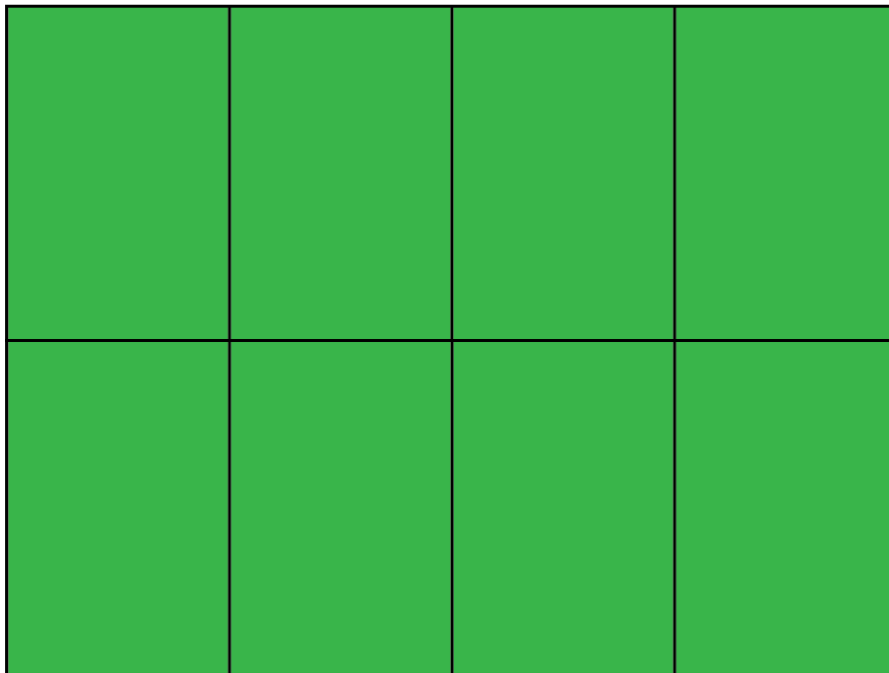
The day before, the class had taken up the first part of the problem, which was to determine what part of the whole rectangle was shaded green. Helen and Deonte were able to successfully identify that one-eighth of the fraction was shaded green. Using materials the teacher provided, Deonte covered the entire rectangle, the whole, using



eight green shaded parts. This showed the class that eight equally sized green parts made one whole (see Figure 5.12).

**Figure 5.12**

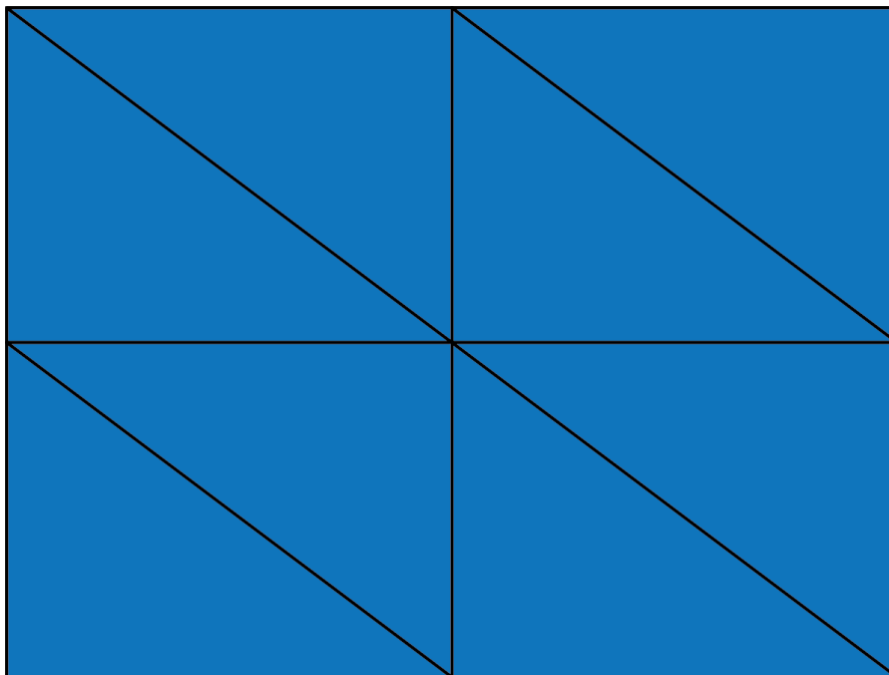
*Entire Rectangle Shaded Green*



On this day, the teacher began by highlighting what Helen and Deonte had done the day before, then discussed the second question in the problem, which asks which part of the whole rectangle is shaded blue. Kasim successfully went to the board and partitioned the whole rectangle into parts that correspond with the one blue shaded part and determined that one-eighth of the rectangle is shaded blue. Next, Deedrah covered the entire rectangle with eight of the blue shaded parts, similar to what Deonte had done the day prior with the green shaded parts.

**Figure 5.13**

*Entire Rectangle Shaded Blue*



Upon naming the fraction for both the blue shaded part and the green shaded part, the teacher posed this question:

Which is a bigger part of the ... rectangle, the blue or the green part? Don't say anything out loud. Which one is a bigger fraction of the rectangle, the blue part or the green part? Don't say anything. I'll write my question up here. Which is a larger fraction of the rectangle, the blue triangle or the green rectangle? Which one is, which one is more? A larger part of the rectangle? The blue or the green? Hey, turn to your neighbor and tell your neighbor what you think. Which one is bigger? Okay, so who would like to

repeat it? On the board, which has a larger fraction of the rectangle, the green rectangle here, or the blue triangle? Which one is a bigger fraction of the whole rectangle? Talk to your partner and see if you can agree.

During the 12-minute discussion that follows, the teacher communicates six different messages about what it means to be smart in math. Her moves communicate that being smart in math means: (1) being able to explain your thinking, (2) sharing your thinking, (3) explaining to convince your peers (e.g., using materials to explain and explaining for understanding), (4) sharing untried ideas, (5) listening to a peer's explanation for understanding, and (6) identifying what helps you understand or learn. As I unpack this episode, I focus on the messages of trying untried ideas and collective explanation as signals for what it means to be smart in math. I focus particularly on these ideas for two reasons. First, this is the first time that sharing untried idea has surfaced as an explicit message about smartness. Second, although collective explanation emerged during Episode 2, its appearance here is different in important ways. Unlike in Episode 2, the students will have already agreed on what the answer is, and the purpose of collective explanation is on how they prove the answer. Additionally, rather than using collective explanation to build conceptual understanding of mathematical content, the teacher uses collective explanation here to help students develop agency to decide which explanations help them to see, understand, and learn.

After the teacher gave students time to think about the question, she called on Dior, a Black boy, to begin the discussion. Dior started to describe the work that he and Luiz, a Latino boy, did but then asked Luiz to explain their thinking. Initially, Luiz stated

that he thought the blue triangle was bigger than the green rectangle. The teacher asked him to explain how he knew. At this point, Luiz changed his thinking and said they are both the same. After changing his mind, the teacher asked Luiz, "Why do you think they are the same?" to which Luiz responded, "Because they're both one whole."

The teacher continued:

Because they're both one whole? So, can someone say what Luiz just said? What did Luiz just ... Luiz and Dior first started by saying the green was more, but then, Luiz, you just changed your mind and said they're the same. Looks like you're thinking pretty hard about that right now. You're not sure why you would explain it that way?

Luiz said, "No."

In the interaction among the teacher, Dior, and Luiz, the students provided answers, albeit answers that shifted as they shared them. For both responses that Luiz offered, the teacher pressed him to try to explain how he knew. The teacher's focus on how the student knows regardless of the answer he shares suggests that her priority is the thought process that underlies the answer rather than the answer itself.

We see the same emphasis in the next interaction with Madison. The teacher asked, "Somebody else [want] to try to explain what you're thinking? Does anyone else think they're the same? Madison, you did?" Madison affirmed that she too thought the blue part was bigger; however, she has also changed her mind and now thinks both the

blue and green part are the same. Like Luiz, Madison did not yet have a reason to prove they are the same. In these exchanges with Dior, Luiz, and Madison, the teacher communicates that being smart in math means sharing your explanation along with your answer, with the emphasis being on the explanation rather than the answer.

#### **5.4.1 *Sharing Untried Ideas***

In this section I focus on the message of sharing untried ideas. Untried ideas are ideas that students either have not yet tried out (i.e., to prove or disprove) or have not yet fully formulated. Often in math classrooms it is expected and praised for students to only share fully formulated and/or correct ideas. As such, the teacher's encouragement for students to share untried ideas is a significant turn from typical practice. This message comes through the moves the teacher makes in her interactions with Arissa and Layla.

After hearing from Dior, Luiz, and Madison, the teacher pressed the class, "Does anyone else also think they are the same, or does somebody else think that one of them is more? Arissa, what do you think?" Arissa replied:

**Arissa:** They're the same because both equal one whole.

**Teacher:** I'm not sure I get what you mean by they both equal one whole ... This is the whole [*teacher points to the whole on board*]. So what do you mean by that? No, you might be saying, I just want to know more ... What are you thinking when you say they're both one whole?

**Arissa:** Like cause yesterday when we put all the green ones on there ... it equaled [inaudible]

**Teacher:** Who heard what Arissa said? I think Arissa is thinking about something that can really help us, but I'm not sure people could hear her. Stand up and say it one more time. That was helpful, what you said. Look at Arissa now and try to hear what she's saying and see if it can help us figure this out.

**Arissa:** I said that, um, 'cause yesterday we had put all the green ones on it, and it had equaled, um, one whole. And then we had the blue ones, and it equaled the same thing.

**Teacher:** So, you're talking about when we covered the whole thing with green. That's what Deonte did. And then today, Deedrah covered the whole thing with blue. And that's what makes you think they're the same. Okay. Does anyone have an idea of how we can make sure we can really prove that the green and the blue are each the same size and the same part of the whole. Anybody have any ideas for what we could do with our materials. Or our thinking? Anybody have an idea about that? Layla what do you think?

Arissa, a Black girl, had not often participated in observable ways in the class up to this point. Initially, as Arissa first tried to share her idea, she seemed to be taken aback by the teacher's response. The teacher did not understand what Arissa meant by the phrase "both equal one whole," but when she initially pressed Arissa for further explanation it seemed as though Arissa didn't want to explain anymore. In this moment Arissa may have believed that the teacher was evaluating what she was saying (e.g., interpreting what she was saying as wrong), which is a typical move that occurs in

classrooms. The teacher immediately clarified that she was not evaluating Arissa, rather she wanted to understand what Arissa was saying. Next, Arissa explained her thinking by referring to the work that was done yesterday, pointing out that, like yesterday when the whole rectangle was completely filled with green parts, the whole rectangle today was completely filled with blue parts. Hence that's why the blue and green parts are the same. The teacher then asked Layla to revoice Arissa's thinking.

The move to have Layla repeat Arissa's thinking does several things, including allowing her to think through her idea aloud, with each time her idea becoming more and more clear. After Arissa shares her thinking, Layla shares her thinking. In these interactions we see how the teacher continues to communicate the message of sharing untried ideas.

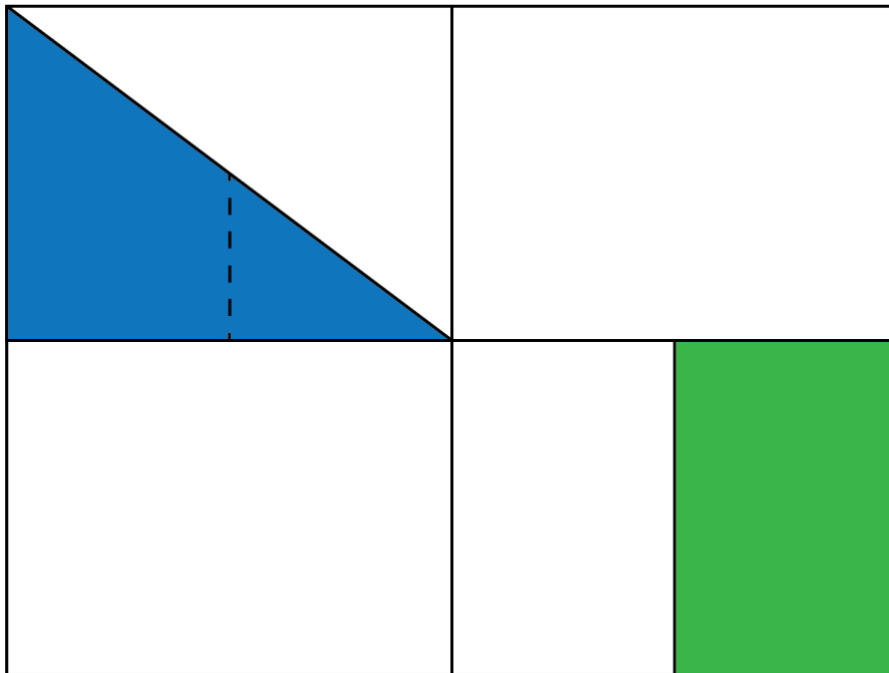
The teacher then called on Layla who said, "You could cut off the end of the triangle." The teacher responded:

You could cut off the end of the triangle? Can you come up? Can you come up here and show me what you mean? Layla thinks that we could cut the end of the triangle off to prove that they're the same size. Can you, can you figure out how you would do that but don't actually, you can draw on your paper. What would she have to cut to see if this is the same? Do you want to try it? *[The teacher gives Layla a blue triangle and scissors.]* How would you cut the blue? She says you can cut the end of the blue off and prove that it's the same size as the green. Can you make a sketch in your notebook and see if you can figure out what she would need to do? So, you just cut off the tip and now what are you going to do?

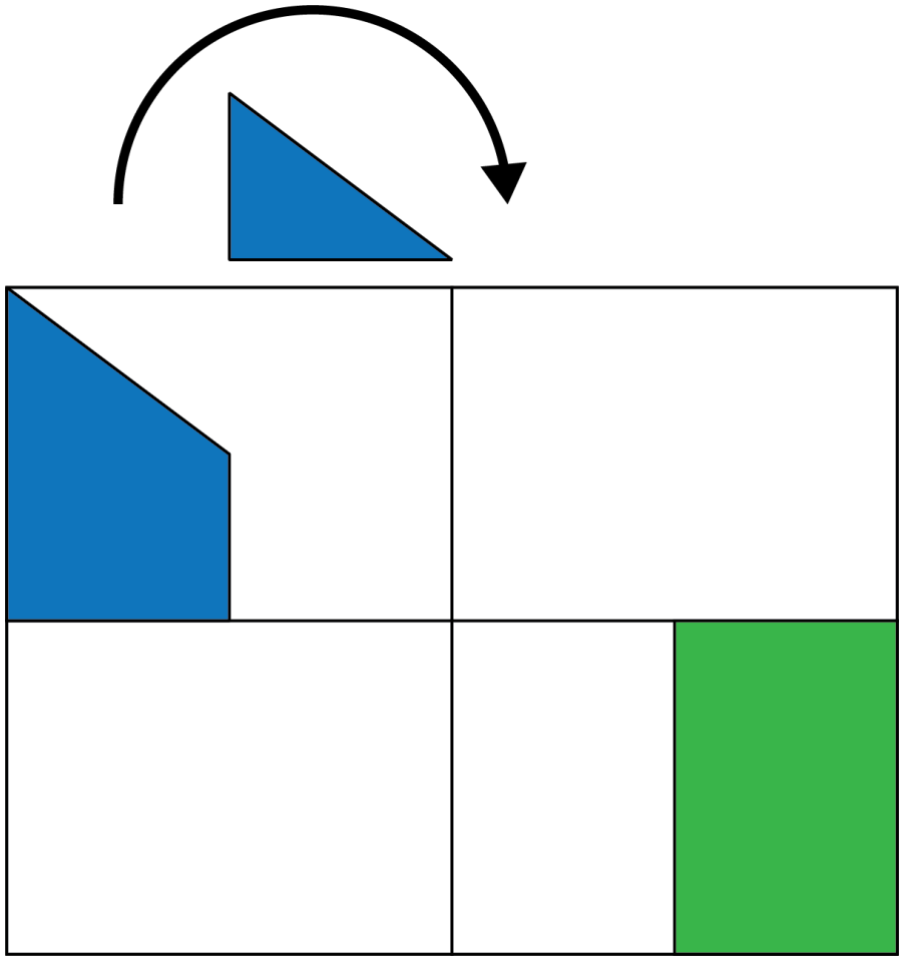
Layla cut one of the blue triangles and rearranged the cut pieces such that it made a small rectangle inside the whole (see Figure 5.14). The teacher also handed Layla a green rectangle to put beside the new blue rectangle. The teacher then asked Layla why she thinks they're the same. Layla said, "Because cutting off the end to make it look like a square. To make it look like the green one ..."

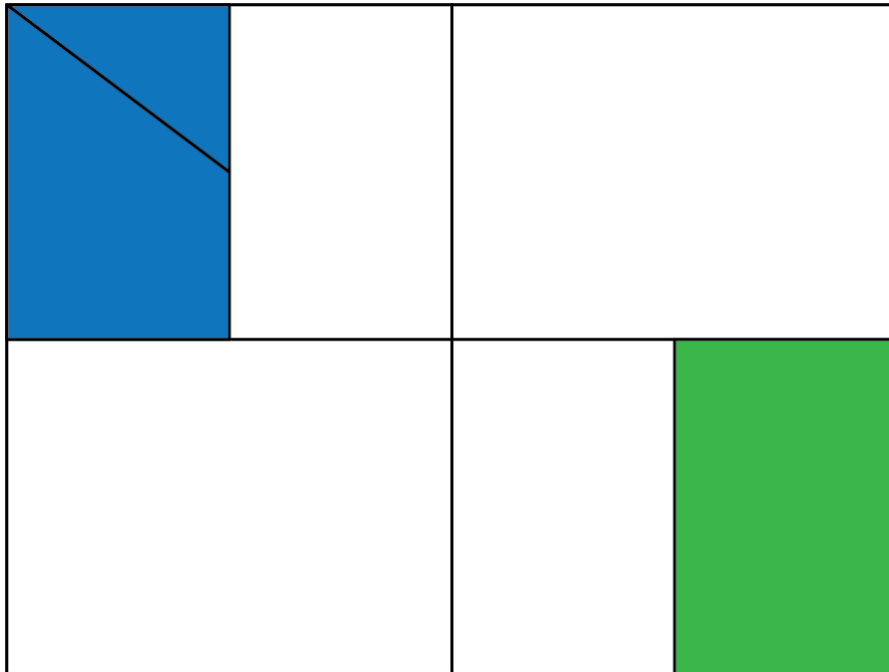
**Figure 5.14**

*Layla's explanation*









As Layla spoke, several students in the class said, “Oh.” The teacher then asked one of the students who said “Oh” to explain why. Chandler volunteered and revoiced the work that Layla did at the board.

In this moment, Layla has an idea to show that the blue triangle and green rectangle were the same but has not tried the idea out yet. That is, she has not cut the blue triangle and matched it up with the area model and green rectangle before sharing her idea with the class. By making the move to call Layla up anyway, the teacher communicates the importance of sharing untried ideas. Aside from Layla not often sharing her thinking publicly, the move to have her try an idea out publicly runs counter to moves a teacher may make in a traditional classroom. One could imagine that in a typical classroom the idea wouldn’t have been taken up because of concerns about time or that it might send the discussion down a trajectory the teacher had not planned for.

However, the teacher exercises her power to redefine what it means to be smart in this classroom by not only asking Layla to share her thinking but supporting her as she explains her untried idea. The students' "Oh/Aha" comments after Layla demonstrates her idea indicate that they have arrived at a fuller understanding of Layla's explanation than they had when she initially shared her untried idea.

#### **5.4.2 Collective Explanation**

Collective explanation is the idea that multiple students contribute to an explanation that helps the whole group understand a problem or concept. In Episode 2, the teacher used collective explanation to help students develop their conceptual understanding in order to arrive at an answer. Here, the focus is on how the group can work together to *prove* an answer they have already agreed on. In addition, the teacher prompts students to engage in the collective explanation in ways that reinforce their agency over their own learning.

After Layla's demonstration, the teacher asked, "What did anybody else see? What did you see, Michio?" Michio stated that he has a different way to explain the answer. Before he could offer his explanation, the teacher asked him to revoice what Layla did. Michio did so, and the teacher asked Layla if Michio revoiced her thinking correctly. Once Layla affirmed that he did, the teacher welcomed Michio to give his alternate explanation.

In this interaction among Michio Layla, and the teacher, the teacher underscores the importance of listening for understanding. After a student shares their thinking, it is quite common in typical classrooms to have a different student share their thinking. In this way, teachers get many ideas out on the table. However, one drawback of this

approach is that it assumes that everyone understands the first student's idea. It may also communicate that the idea is either incorrect or not worth unpacking. However, in this instance, before Michio could even share his idea, the teacher requires him to first show that he understands what Layla did, and Layla has to confirm that what he said was accurate. In this way, the teacher communicates to Michio, Layla, and the rest of the class that the expectation in the space is that students listen for understanding and that each student has the agency and authority to determine if classmates have interpreted their ideas accurately.

Next, Michio began his alternative explanation at his seat before the teacher invited him to share his thinking at the board. As Michio headed to the board, the teacher told Layla, "... Thank you very much. Did that work the way you thought it would work?" Layla responded, "Yeah." Then, the teacher stated:

Thank you very much for showing that. In a minute, I'm going to ask you if you think that the blue and the green are really the same or not. So, I want you to keep thinking, what did Layla show? And now pay attention to what Michio is going to show and see what you think about his, how he's going to explain it. What do you think of Michio?

As Michio walks to the board, the teacher makes a couple of important moves that are worth highlighting. First, the teacher checks in with Layla, offering another opportunity for Layla to use her agency to communicate how she thought the work at the board went. Then, the teacher orients the students to Michio's and Layla's

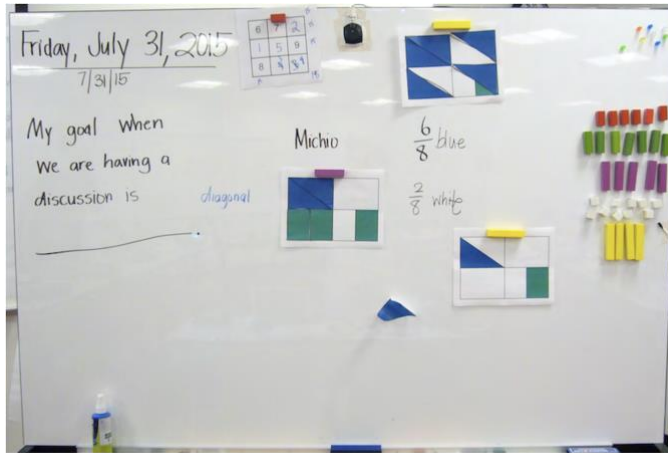
explanations, explicitly drawing their attention to what each explanation shows about the problem rather than whether or not one explanation or the other is correct. By directing students' attention in this way, the teacher communicates the idea that explaining effectively is about convincing your peers. Additionally, she also communicates that smartness is not necessarily about finding one right way to solve a problem. Instead, smartness is about communicating, thinking, and listening and learning from others to consider multiple possibilities.

Once at the board, Michio began using the materials available to arrange the different shaded parts on the whole rectangle. After a minute of watching him work, the teacher asked him if he could explain what he was doing. Michio responded:

Uh-huh. You see how when you put these two down and this one and that one together it equals one whole, one whole one of these, see. And then when you take a green one and you put it right here, it goes one whole as well. So, what I'm trying to say is that they're both, that they're both equal to one half of one of these. (See Figure 5.15)

**Figure 5.15**

*Michio's explanation*



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After Michio completed his explanation, the teacher called on Alex to ask what she thought about Michio's explanation. Alex, who was sitting near the front of the class to the left of the board responded, "Um I think they're saying ... but it's really hard for me to um, see um, see them both matched up because they're diagonal to each other, and I can't match them up." Michio had initially arranged the blue triangles and green rectangles so that they covered portions of the whole that were diagonal to one another. Upon hearing Alex's comment, he rearranged his work on the board. As he is rearranged the teacher stated, "That was a good comment, Alex, because sometimes in seeing what someone is explaining it helps if they can show it to you so you can see it more easily. That was a helpful comment." Michio then asked:

Is that better? See how they're both being equal size? That makes them both equal in size. See how those two are the same size, these two are the same size? Both of those combine to equal one whole square, and these two combine to equal one

whole square. That makes ... technically that makes them the exact same size but different shapes.

Micho's explanation in this moment drew upon and synthesized several contributions from multiple students across days on this problem. First, Helen and Deonte had identified that  $\frac{1}{8}$  of the whole was shaded green and the whole could be filled with green rectangles the day before. Then, on this day, Kasim and Deedrah did the same work but for the blue triangles. Dior, Luiz, and Madison realized that both the green rectangle and blue triangle were  $\frac{1}{8}$  of the whole but proving the two shapes were the same was more challenging. Arissa connected all of these ideas and explained that the whole could be filled with either blue triangles or green rectangles. Then, Layla's proof demonstrated that both the blue triangle and the green rectangle take up the same space when the blue triangle is physically altered by cutting it. Michio's explanation incorporated elements of each of these prior ideas to show that two blues and two greens cover the same portion of the whole, indicating that the blue triangle and the green rectangle are "the exact same size but different shapes."

Although the teacher never explicitly names that the class is engaged in collective explanation, her moves to involve multiple students in contributing elements of the completed proof highlights the collective nature of doing mathematical work in this space. This signals to students that smartness is not only about working on your own. By engaging students in collective explanation, the teacher defines smartness as something that the group can do together.

In the last part of the discussion, the teacher communicated to students that being smart in math meant identifying what helps you understand and/or learn. Before the teacher wrapped up the discussion, she set up the task that students would do to conclude this problem for the day. She said:

Okay, so what I would like for you to write in your notebook now is you saw two different explanations. Thank you very much for showing us ... Layla and Michio each showed you a way to see that the green and the blue are each the same size. This is Michio's. And this is Layla. You remember what Layla did? She cut. Pick one of these, you don't have to like one better than the other but pick one that you think is convincing to you. That means it makes you think that's right. And say, why did that one make you think that they are the same size? So, pick one of them and say, what is it about that that you thought was convincing or helped you to see it? Pick Layla or pick Michio. It's not a contest, it's just which one do you think helps you to see it? Is it when she cut it, did that help you to see it the best, or when Michio showed you that they cover the same amount of area that helped you to see it? I'd like to see everybody writing which one makes you understand that the green and the blue are the same size even though they're different shapes.

In the wrap-up the teacher directs students' attention to considering which explanation is more useful to them as learners. In doing so, she further de-emphasizes the importance of right and wrong answers in favor of strong reasoning. The day before and in the early part of this discussion the class concluded that the fraction covered by



either the blue triangle or the green rectangle was  $\frac{1}{8}$ . These interactions also appeared to establish that students had begun to believe that the two shapes were the same size. The focus of the discussion highlighted in this critical moment is therefore not about finding the right answer but is instead about proving that answer using mathematical reasoning. This emphasis on proving prioritizes understanding and learning, as well as suggests to students that reasoning for understanding is a key aspect of mathematical smartness. The teacher reiterates this point again at the end of the discussion when she prompts students to consider which explanation helped them more. She directs them to not decide which explanation is better or which explanation they prefer, but she leaves the agency and authority to consider their own learning and understanding up to the students.

## **5.5 Episode #5**

The start of the second week of the summer mathematics program was challenging. The students were not as focused or respectful toward one another as they had been in the first week of the program. On day eight, the teacher explicitly addressed her concerns with students. During this discussion the teacher communicated three messages. That is, being smart in math means: (1) listening, learning, and working with others; (2) not being punished; and (3) exercising your agency.

After unsuccessfully attempting to work on a mathematics problem together, the teacher called everyone to the rug. She opened the conversation as follows:

The reason I've asked us to come to the rug is that I'm really concerned ... We're not going to go on working this way today. I'm not- this is not going to be okay. We

cannot continue this way, and we're going to think about this for a minute. We have a student contract, and we have a teacher contract. I've tried really hard to do the things from the teacher contract. But many things on the student contract are actually not being done by people today. And it's making it super hard for us to learn anything. And we can't, we're not going to waste the rest of the morning this way because we can't get anything done.

Okay. So, we're going to talk about the student contract. I'm going to pick a few things. I'm going to read it to you. And I want you to be thinking while I read about which parts of the contract are a problem this morning: "I will come to class every day," and that's not the problem. But here's some others. "I will listen and learn from others' ideas. I will share my ideas with others by making contributions in class and listening to other students. I will keep neat records of my work in my notebook. I will try my very best on everything we do so that I can get my most out of being in this program."

What I'm concerned about is that when we started on Monday last week, I told you that you can't get things out of this program unless you agree to follow that contract. And I said at that time, do not sign the contract if you're not going to do that. And the last two days are disappointing because that contract is not being used. And that is a problem, that you asked me to do a whole bunch of things like listen to what you say, respect your ideas, help you learn to write. And I'm trying really hard to do those things. But I don't see everybody trying to do the things that are on the student contract right now. I'm going to get a couple of ideas from you about what we can do about this. And

then I'm going to give you a little time to figure out what you're going to do about the student contract and your own particular work this morning. Who has an idea of what we can do to get the class to be working together much better than we've been working the last two days. Who has an idea about that? Jeremiah, what do you think?

From the very beginning, the teacher framed the discussion around the students' failing to do what they agreed to in the student contract. The teacher named several items they agreed to, which include listening and learning from others and sharing your thinking. With these moves, the teacher uses the contract as an artifact to remind students what it means to be smart and successful in this space.

Several students then offered their suggestions. First, Jeremiah suggested that students engage in the breathing activity 7-11, where students breathe in for seven seconds and breathe out for eleven seconds as a way to improve their focus; he also suggested they play games. Next, Dior suggested taking more breaks. La'Rayne suggested that the teacher give students big packets for the next three days because that is what her teacher did in school when kids did not behave. The teacher then took a moment to address La'Rayne's suggestion:

Right. So, one reason that teachers give packets, look, La'Rayne is the first person who actually gave an idea of something to do. So, let's think about it for a minute. Teachers give packets because they think that kids can work really well by themselves and do their math on their own. But what we're trying to learn this summer is

how you work with other people's ideas and get a lot smarter because you're working together. So, if I give you each a packet, we won't get to learn that kind of thing.

Here, the teacher reiterates the goals for how the group should work together in the program and how this is tied to a different way of understanding smartness. She explicitly states that in this space being smart is about listening and learning from others and working together, which she explains is different from how teachers often want kids to work. So even though La'Rayne's suggestion is the first to address what the group can do about the problem they're having together, it runs counter to what it means to be smart in this space because it focuses on independence.

Next, Jerone suggested giving another warm-up problem in the morning and having them sit there instead of trying to write more as a way to help them calm down. Layla then offered a suggestion to have students come back more quickly from the restroom, which prompted the teacher to say that restroom breaks have not been an issue. Rayveion suggested that when it got close to breaks and lunch students could reflect on the work they had done in class. The teacher responded that students had been doing really well reflecting on their thinking. She repeated that students haven't been doing as good of a job listening to one another. Chandler then mentioned that the teacher could take breaks away from students who are not listening.

The teacher responded to all of this with:

So, these are things I could do to make it--I could take away break time. My concern is that you aren't going to learn things just by listening to the teacher. There's

so much to learn from other kids in the class. So, Miah has good ideas, Lauren has good ideas. Arianna has good ideas. Dior has good ideas. But even right now while I'm talking, there are people talking. And one of our agreements is that we don't do that. So, we need some way to not have that be happening anymore. What do you think, Deedrah?

After Jerone, Rayveion and Chandler shared their suggestions the teacher again reiterates the importance of listening and learning from others. She repeatedly reminds the group of the things they've been doing really well at, like reflecting on their thinking, while refocusing them on the fact that they have not been doing a job of listening to one another. After Chandler's suggestion to take breaks away from students who are not listening, the teacher once again highlights for students that the goal isn't to learn just by listening to the teacher. She once again says that students can learn so much from their peer's ideas that they can't learn just by listening to her. These moves to keep the focus on listening and learning from others highlight the value the teacher places--and wants students to place--on this conception of smartness in math.

In response to the teacher's prompt, Deedrah proposed that they shouldn't work in partnerships because she noticed that partner work is when the class gets loud. The teacher acknowledged that what Deedrah said is true but asserted that fifth graders can do this kind of work successfully. Next, Madison suggested that students who are not listening should be sent out into the hallway. The teacher responded to Madison's comment by saying:

So, what's interesting to me is that the ideas you're giving me are all about how I could take things away from you, and, I don't know, that's not what I have in mind is to take lots of things away. Instead, I'm trying to have us have a really useful time together, which means giving you things, not taking away. But we're not using that very well.

In these exchanges with Deedrah and Madison, the teacher continues to push back on the deficit frames that the students are offering. While the teacher agrees with Deedrah's observation that the class gets louder during partner work, she argues that fifth graders can work together and listen and learn from one another thus rejecting Deedrah's implication that the class can't handle partner work. The teacher's response to Madison also responds to other students' suggestions that the class needs to be punished in some way. The teacher again rejects this framing, telling the class that learning together and being smart in math is not about punishment. La'Rayne's earlier comment that punishment (i.e., giving packets) is what her classroom teacher would do indicates that students are making suggestions that are built around punishment based on their experiences of how classrooms usually work. The teacher here expressly refuses to engage in those patterns of practice and instead is concerned with supporting students to exercise their agency to listen to each other and learn from each other in ways that might be new for them.

At this point, the teacher noticed students were still not listening to one another. She then sent students back to their seats and said:

I'm really concerned about this. I'm really serious about it. And I expect you to take this seriously too. There were lots of other kids who signed up for this program who didn't get into it. And you were picked to be in the program. And we just wasted a lot of time yesterday too. And that worries me a lot and doesn't make me happy at all. I put the student contract here because this is actually about you, not about me. You're here to help yourself have so many skills that will help you be a great learner in fifth grade and beyond. And when you're not concentrating, you're not getting that. And these were the things that we discussed that you need to do to get as much out of the program as possible. So, we're going to take about 10 minutes for you to seriously read the contract again. And I want you to write a serious note to yourself about what you, not what the rest of the class is going to do, or what I'm going to do. What you are going to do for the rest of the time we have the math program. And this isn't a time I want anybody talking to anybody else. I want you to write a note to yourself on a clean page in your notebook. And I don't want to hear, "Dr. Ball, I'm done." I don't want to see anything else. I just want to see writing and thinking. So, the whole list is right there. And I want you to write a serious note to yourself about what you're going to do.

In closing this discussion, the last message the teacher communicates is that being smart in math means exercising your agency. The teacher leverages the student contract, which was an agreement made between the teacher and each student in the classroom, to have students decide for themselves what they will work on. Rather than telling them what they need to do to improve, the teacher trusts them to identify an appropriate goal and to think about how they can work on it.

The teacher also makes two additional, subtle moves during this discussion to communicate messages about smartness. First, naming Miah, Arianna, and Dior as students who have important ideas to learn from seems purposeful. Miah, normally quiet in class, recently provided a proof for a warm-up problem, which the teacher called “Miah’s proof”; Arianna, who did not participate much during whole group in the first week, has begun to come to the board more often during the second week; and Dior has had his seat shifted numerous times to maximize his ability to focus. In highlighting these students, the teacher does not focus on those who always get answers right or are always at the board. Rather she calls the class’ attention to students who engage and participate in a variety of important ways that are not always celebrated in mathematics classrooms. By offering them and their ideas as examples of the importance of listening and learning from others, the teacher both sends the message that listening and learning from others is smart and expands the group’s ideas about which “others” are worth listening to and learning from.

The second message the teacher is communicating throughout this exchange is that students can do this work. She explicitly says this in response to Deedrah’s suggestion. In addition, implicit in all of the interactions described above is a belief that students are capable of listening, learning, and working together. She rejects their suggestions to shift to independent work or impose punishments because she believes they are capable of taking responsibility for their behavior and of engaging in the kind of work that they’ve outlined in the student contract. The moves she makes throughout this exchange are oriented towards supporting them to engage differently with one another and with mathematics learning.



## 5.6 Discussion

In this chapter, I address my second research question “How does the teacher communicate smartness during a summer mathematics program?” I took up the question by identifying five episodes and analyzing the critical moments within them to uncover what was being communicated, explicitly and implicitly, about what it means to be smart and a doer of mathematics. The teacher consistently communicated several key messages throughout the summer mathematics program. Some of these messages included that students should listen, learn and get ideas from their peers across episodes by having students revoice the thinking of others and critique the reasoning of others, as well as by explicitly stating the value of learning from peers. Additionally, the teacher frequently communicated the importance of explaining one’s thinking and doing so to convince your peers. The teacher supported students to explain their thinking through scaffolding as well as holding students accountable to explain their thinking and take comments from their peers. Another message that the teacher continuously communicated centered on the value of revising one’s thinking and reasoning. The teacher communicated this through highlighting when students shifted their thinking in their notebooks and through publicly having students name and revise their thinking.

Taken together, the messages the teacher communicated represent what it means to be smart as well as who is smart in this space. The teacher signaled these messages in several ways that I refer to as *signaling methods*. *Signaling methods* are the strategies and moves that the teacher used to signal what smartness is and what it means to be a doer of mathematics. Through my analysis I identified six signaling methods: (1) interrupting normalized patterns of classroom interaction, (2) scaffolding

students to explain their thinking and to orient their explanations to the rest of the class, (3) lifting up cases of revising one's thinking as positive and encouraging revising, (4) using the routines of "Notes to Self" and End-of-Class Checks as opportunities to reflect and focus on metacognitive development, (5) designing and using mathematics problems focused on key ideas and practices that challenge and surface multiple ways of thinking, and (6) strategically and intentionally acknowledging what and who is smart (acknowledging competence) (see Table 5.2).

In the following sections, I take up each of these signaling methods one by one, highlighting what seems to be involved, considering how each method compares with common practices or approaches in math classrooms, and connecting each method to examples from the episodes of specific messages about being a doer of math or what or who constitutes smartness.

### ***5.6.1 Interrupting Normalized Patterns of Classroom Interaction***

One of the methods for signaling smartness was interrupting normalized patterns of classroom interaction. By "normalized patterns of classroom interaction" I mean the type of classroom interactions that occur with regularity in typical math classrooms. An example of a normalized pattern is what I call the "spitball approach" used by teachers during whole group discussions. The spitball approach refers to teacher's prioritizing getting answers from students without necessarily orienting students to one another (e.g., ensuring everyone's explanation is properly heard; making contributions to support students in understanding one another). In this approach it is common for students to share their answers one by one until the teacher hears the right one, and, once that occurs, to move on.

In Episode 1, Lauren shared her answer to the question, “Which rod is three times a white?” Her answer was blue, which was correct. Instead of using the spitball approach, the teacher asked the class to comment on her thinking. La’Rayne proceeded to raise her hand and offer a different answer. Instead of taking up La’Rayne’s answer, the teacher tabled La’Rayne’s response momentarily to ensure that everyone understood Lauren’s explanation. Similarly, in Episode 4, Layla explained her thinking around the Blue-Green Rectangle Problem. Again, the teacher asked the class to comment on Layla’s explanation. Like La’Rayne in Episode 1, Michio wanted to offer a different answer. Instead of allowing Michio to offer his answer, the teacher first asked Michio to revoice Layla’s explanation. In both of these moments, the teacher’s practice of orienting students to one another’s ideas rather than moving on quickly to other ideas communicated two key messages about smartness: (1) that part of doing mathematics is to convince your peers, and (2) that part of doing mathematics is listening to and ultimately understanding one’s peers’ ideas.

On the surface it may appear that normalized patterns of interactions like the spitball approach are not problematic. By definition these are regular practices that teachers enact. However, following these approaches in the episodes highlighted here could lead to Lauren’s and Layla’s explanations not being fully understood by their peers. Further, focusing on the next student’s answers instead of attending to Lauren’s and Layla’s explanations first could signal to students that sharing your own thinking outweighs understanding your peers’ thinking. It is in these moments that ideas about who is smart and what it means to be smart are signaled to students. Normalized patterns of interaction contribute to a cycle of reinforcing narratives that are problematic

for learners in general and Black learners specifically because they have an individualistic and not collective focus of smartness. By interrupting normalized patterns, the teacher disrupts the status quo while redefining smartness in ways that provide opportunities for students to share their thinking and allow for students' strengths to be seen.

### ***5.6.2 Scaffolding Students to Explain Their Thinking and to Orient Their Explanations to the Rest of the Class***

Another signaling method the teacher used was scaffolding students' processes for explaining their thinking and orienting those explanations to their peers. By this I mean that the teacher frequently supported students in their explanations by asking questions or providing materials to guide them to explain their thinking in ways that helped to make their ideas clear to their peers. Explaining is a critical practice to develop in mathematics. Students enter classrooms with different expectations and beliefs about what it means to explain. For example, in Episode 2 Kasim's belief around explanation was that having studied something in school was sufficient justification for an answer. Such beliefs, commonly reinforced in math classes, deemphasize the value and importance of knowing and understanding content deeply in favor of deferring to the teacher's or the textbook's authority. The teacher's efforts to scaffold students' explanations and orient them to one another signaled that smartness in math is about more than simply deferring to others' authority.

In Episode 2, we see a clear example of this signaling method. In this episode, Michio was explaining his thinking on the second part of the Grey Rectangle Problem. As he was explaining, the teacher provided him with a sticky black line, which he used

to introduce the idea that all parts of the rectangle had to be the same or equal. Additionally, the teacher probed him to unpack his thinking. This is an example of the signaling method because providing Michio with the physical material necessary to illustrate his idea and probing his thinking allowed Michio to make his thinking clearer and more accessible to the class. Often in classrooms, teachers assume that students understand their peers' ideas, or the teacher simply revoices students' ideas for them. Instead, in this class, the teacher lifted up and supported individual student's reasoning and sought to make their explanations available for the class.

Similarly to Episode 2, in Episode 4 the teacher scaffolded and provided support for a variety of students as they explained their thinking, as well as orienting students to one another's ideas. First, in order for Kasim and Deedrah to show that eight blue triangles make one whole, the teacher provided them with sticky blue triangles and probed their thinking. Next, as Layla provided a proof, which required cutting, to show how the blue triangle and smaller green rectangle were the same, the teacher provided Layla with scissors to cut one of the blue triangle stickies. Then, she helped put the materials on the board to represent Layla's thinking as she explained. Similarly, the teacher provided green rectangles for Michio as he explained his thinking. These moves are examples of this signaling method because they both enabled students to make their explanations clearer and more accessible to their peers. The teacher was able to make these moves by anticipating students' thinking and having physical materials ready to use in the moment. In addition to having the materials available, the teacher supported students to use them to communicate their ideas, along with having students stay at the board and take comments. Together, these are examples of the moves the

teacher used in signaling the importance of listening to and hearing other students' ideas.

This signaling method—scaffolding students' explanations and orienting them to peers—was used to communicate several key messages. Some of the messages include explaining one's thinking to convince a peer and sharing untried ideas. Just as important as the messages that are communicated is how they are communicated. In this method there are three features that are important to how smartness is redefined. First, through this method there are multiple ways that students have to explain their thinking. Scaffolding provides students with different opportunities to convince their peers. Second, in providing different opportunities one must be able to anticipate students' thinking and provide resources that meet students where they are to support their thinking. Third, the method redefines smartness because it supports students to be oriented to one another, with respect to understanding the different ways that students draw upon (their own strengths) to explain their thinking.

### ***5.6.3 Encouraging Students to Revise Their Thinking***

The signaling method of encouraging students to revise their thinking was the third method used by the teacher. This method involved the teacher actively bringing attention to the importance of revising thinking as a vital mathematical practice that can be used for learning. Often the narrative in mathematics is an overemphasis on getting the right answer the first time, which is not productive or an accurate representation of what it means to do mathematics. Additionally, this over emphasis limits students' willingness to try more challenging problems especially if they do not think they will solve them right away. Leveraging moments where students do not get the right

answers or make mistakes as opportunities to learn provides a counter-narrative to this normative view.

In Episode 1, La'Rayne provided a wrong answer to the question "Which rod is three times a green?" on the first day of class. In this moment, the teacher responded by asking students to restate the explanations provided by Lauren and La'Rayne, come up with questions that fit the explanations that Lauren and La'Rayne offered, and asking La'Rayne to restate what her peers suggested about her answer and if she agreed. These moves embody the signaling method of lifting up cases of revising as positive and encouraging revising in a subtle way. The teacher emphasizes that providing an answer and knowing the question that is being answered is an important skill in mathematics. Often procedures are taught in math classrooms which leads to a focus on the answers, but not necessarily on understanding how they answer questions. However the practice of matching answers with questions is an example of revising one's thinking as there are opportunities to contextualize answers with the understanding of the question that students are answering. In the case of La'Rayne the discussion highlighted what was the original question the teacher asked and the answer to that question, as well as the question La'Rayne answered.

In Episode 2, Ryan gave his explanation to the first part of the Grey Rectangle Problem. In his explanation he mentioned that there were three parts, and one was shaded, which indicated the name of the fraction for the area model is  $\frac{1}{3}$ . Ryan's initial explanation did not include the words "equal" or "equal parts." After Michio's answer and explanation on the second part of the Grey Rectangle Problem, which brought up the idea of equal parts, the teacher went back to Ryan and asked if there is anything he

wanted to add to his explanation to the first part of the problem. Ryan then added the word “equal” to his explanation and wrote it on the board. Afterwards, the teacher acknowledged that on the second part of the Grey Rectangle Problem, 75% of the students got the problem wrong initially. The teacher encouraged students to change their answer if they now thought something different. The teacher’s moves in this sequence are an example of this signaling method as the teacher repeatedly and publicly supported students to revise their thinking if necessary as they gained new knowledge about the concepts they were working on.

In Episode 3, Hamza shared his notebook with the class. In that moment one of the things the teacher highlighted was the way that Hamza just crossed out his first answer and wrote a different answer when his understanding changed. She characterized this as a good move that could serve as a resource to go back to see his original thinking. In addition, although not mentioned in this chapter, on the previous day during a notebook share, the teacher mentioned that the reason the class wrote with pens is because it is important to honor and value mistakes. Both moves described revising as part of the learning process. The teacher also mentioned to just put one strikethrough so one could still see their thinking, again highlighting that mistakes or wrong answers are not something to be ashamed of. Instead, they are opportunities for learning. Collectively these moments and the moves by the teacher embody the signaling method as they focus on lifting up and encouraging mistakes and revision.

This signaling method as it attends to students as sensemakers first, which allows for understanding to be built on students own conceptions first. A second key feature of the method is finding opportunities in explanations to become more clear,



nuanced, and correct. The last key to the method is recording and keeping track of one's thinking, as well as accepting that ideas are subject to change. Features of this method is how messages of smartness are communicated and more specifically how smartness can be redefined.

#### ***5.6.4 Using the Routines of “Notes to Self” and End-of-Class Checks as Opportunities to Reflect and Focus on Metacognitive Development***

The teacher's fourth signaling method, using the routines of “Notes to Self” and End-of-Class Checks, offered students opportunities to reflect and focus on metacognitive development. The teacher used Note-to-Self and End-of-Class checks to not only understand what students were thinking from a content perspective but how they were making sense of instruction. Questions such as “Name something you did that was smart,” “Who did you learn from today,” or “How do you feel about where you are while working in the mathematics program,” inform the teacher about how learners are thinking about their own learning. Below I draw upon data to show how this signaling method was communicated through messages across different episodes during the summer mathematics program.

In Episode 3, the teacher called Ryan to the document camera to share his end-of-class check from the previous day with the class. In particular, the teacher focused on his response to the prompt, “Name one thing you did that was smart” to which Ryan responded that he listened to others' ideas and shared his own ideas with the group. After Ryan's response the teacher asked the class “Why is that important what he wrote and this question about what did you do that was smart to do in a math class today? Why are those things important?” This moment illustrates this signaling method in two

ways. First, the question that the teacher and Ryan focus on is an opportunity for Ryan to reflect on something he did as a learner that was smart. Second, the questions the teacher raised for the class offered an opportunity for students to reflect on why the actions Ryan named are important in mathematics class. The teacher's focus on metacognition and mathematical practices rather than a specific mathematical concept is valuable for two reasons. First, attending to *doing* mathematics is just as important as the mathematical content. *Doing* mathematics includes the key practices such as organizing your thinking and keeping track of your notes, as well as the value of listening and learning from others. The second part is relevant and important because it attends to the learner as the individual and critiquing their own processes and what they need. These things are just as important if not more than just attending to the mathematical content, which is often the center of interactions; meaning that everything is in service of the mathematics.

In Episode 4, after Layla and Michio shared their explanations for the Blue-Green Rectangle Problem, the teacher had the class write a Note-to-Self based on the question, "Which one makes you understand that the green and the blue are the same size even though they're different shapes?". The teacher emphasized that the Note-to-Self in this case was not about picking one explanation over the other or deciding if one explanation is right or wrong. Instead, it was about identifying for themselves which explanation helped them to better understand the problem they'd been working on. In other words, the teacher uses the Note-to-Self to have students reflect on their own learning and practice metacognition.

In Episode 5, the teacher called the students to the rug, the only time she did this during the summer mathematics program. During the discussion the teacher took suggestions from the students to find a solution to help the class work better together. After the discussion at the rug the teacher sent the students back to their seats and gave them a prompt for a Note-to-Self. In this Note-to-Self, students were tasked with identifying a goal from the student contract that they were going to work on and thinking about how they would meet that goal. As in Episode 4, the teacher used the Note-to-Self to engage students in reflecting on themselves as learners in order to support their ongoing development.

This signaling method is invaluable for redefining smartness. The method focuses on nurturing students' agency and awareness for identifying what best help them learn, as well as what important characteristics to be attuned to as doers of mathematics. The first point is relevant because each student has different needs to maximize their own learning. Understanding that and using their own agency to voice and advocate for what they need is relevant for defining what and who it means to be smart. The second part which highlights how students process who they are and where they are in doing mathematics focuses on students' metacognition. That is their processing of self and self in relation to their environment. Through this method very important messages about smartness are signaled and are critical to the redefining of smartness.

#### ***5.6.5 Strategically and Intentionally Acknowledge Competence***

A critical signaling method the teacher used throughout the summer mathematics program was strategically and intentionally acknowledging competence. The purpose of

acknowledging competence is to ensure that students who may have lower status in classrooms be positioned in relation to their peers as individuals who make valuable contributions (Featherstone et. al 2011). Many students are not labeled as smart based upon their race, gender, class, or intersections of these identities (Shah, 2017). Implicit in the signaling method of acknowledging competence is the strengths-based assumptions that the teacher makes about each student. That is, the teacher must assume that each student has strengths. Acknowledging their competence publicly allows others to see the competence the student already has. This last point attends specifically to strategically acknowledging competence as those who make contributions and the contributions themselves cannot be arbitrary. Across the episodes discussed in this chapter, I unpack two such examples to highlight this signaling method.

In Episode 3, the teacher leveraged the activity of student notebook share outs to strategically acknowledge the competence of Miah, Hamza, Layla, and Ryan. The teacher highlighted different aspects of each of their notebooks, including how they took notes, the quality of their explanations, and how they revised their thinking. These moments are representative of the signaling method as the students that the teacher selected for this activity were students who normally did not share out during whole group discussions. In addition, the teacher highlighted things in each student's notebook that extended beyond mathematical content specifically and included things they'd *done* as learners. For example, Layla took clear notes to help track her thinking.

The episode and moments within it are strategic in that the teacher attended to the students in the class who normally do not share their thinking during whole group and acknowledge contributions that often are not recognized during whole group. The

activity of sharing notebooks is different from whole group discussion because one has access to things that one normally wouldn't. For instance, during a whole group the focus is normally on answers and to some degree explaining your thinking. However, in a notebook one can see work from different time points, reflection, and through processes that may not otherwise be available. Therefore, the teacher is strategic in what is acknowledged by doing the acknowledgement during notebook share outs.

In Episode 4, during the Blue-Green Rectangle Problem discussion, the teacher encouraged several students, including Arissa, to share. Arissa was initially apprehensive about sharing, but after the teacher encouraged her, she connected ideas from the work that the class had done the day before and earlier in this same discussion to make an argument for why the blue and green parts were the same. The teacher acknowledged Arissa's competence by restating her idea for the class. In the next part of the discussion, the teacher asked students to elaborate why they said "ohhhh" once Layla explained her thinking about why the blue triangle and the green rectangle are the same. Then, the teacher had Michio restate Layla's explanation before offering his own idea.

These moments embody the signaling method of acknowledging competence in several ways. First, Arissa was a student who did not typically share her thinking during whole group discussions. By calling on her, supporting her to share her thinking, and highlighting the importance of what Arissa shared, the teacher positioned her as a competent mathematical thinker whom other students could learn from. In fact, Michio later leveraged part of Arissa's idea of covering the whole rectangle in his proof. The teacher's selection of Layla was equally strategic. Like Arissa, Layla had shared

infrequently prior to this; further, in this case, she had an idea to share that she had not tried out yet. In other words, the teacher saw that Layla had a great idea and made space for it as a way to advance the group's understanding and address Layla's status within the class. By having Michio restate Layla's explanation before sharing his own idea, the teacher leveraged a student whom others already saw as smart (Michio) to further spotlight the value of Layla's contributions.

The signaling method of acknowledging competence is important because it places attention on students who typically do not share their thinking during whole-group discussions, or students whose contributions may not be heard or valued. The moves the teacher used comprised the activity of "Notebook Share Out," highlighting particular explanations and backgrounding others, as well as explicitly naming and calling attention to the class about certain students and their mathematical contributions.

This signaling method is critical in redefining smartness as it attends to several important features. First, it requires that the teacher is attuned to the students in the classroom. Particularly students' strengths and areas of improvement as well as their characteristics in relation to other students. The other aspect that is critical is naming contributions that extend beyond traditional markers of smartness. Lastly the method requires purposefully selecting students. It is through these features how messages about smartness get communicated.

### ***5.6.6 Designing and Using Tasks Focused on Key Ideas and Practices that Challenge and Surface Multiple Ways of Thinking***

The last method of signaling focuses on the mathematics problems students worked on in the summer mathematics program. In mathematics classrooms, the problems students work on are an important signal about what it means to be smart or do smartness in math. The mathematics problems students work on may be rich enough to provide opportunities for students to understand and do smartness in a range of ways, or they may limit the types of messages about smartness that could be communicated. Below I draw on two episodes to highlight this signaling method and its connection to communicating messages around smartness.

In Episode 2, the Grey Rectangle Problem was designed to get students thinking about fractions and, in particular, to help students identify the need for equal parts and what the whole is when naming fractions. The first problem uses a straightforward area model that is already equally partitioned, enabling everyone in the class to name the fraction. The second part of the problem uses an area model that is not equally partitioned to challenge students' conceptions about fractions. For the second problem, the students' initial disagreement about the fraction represented and the ensuing discussion illustrates how the Grey Rectangle Problem was designed to surface, challenge, and leverage multiple ways of thinking. That is the building on the first part, the second part was designed to challenge one's definition of what is required to name a fraction using an area model.

In the first representation where the area model is partitioned into equal parts, one may name the fraction of  $\frac{1}{3}$  with the understanding that there are three total parts,

and one is not shaded. This is reasonable and a common pattern of thinking for students. It is important to note when the whole is partitioned into equal parts that reasoning will allow one to name the correct fraction. However, in the second problem applying that same reasoning will not work, as it is incomplete for the context in which the area model is not partitioned into equal parts. The unequal partition is an important design feature of the problem because it's connected to the previous problem but challenges common patterns of student thinking, pushing them to develop a more robust definition and understanding of naming fractions generally as well as specifically to area models.

In Episode 4, the Blue-Green Rectangle Problem has three parts: determining the fraction represented by the green rectangle, determining the fraction represented by the blue triangle, and determining whether the green part and the blue equal the same fraction. First, the problem surfaced students' understanding of fractions. Students figured out how to partition the green part and blue part differently within the same whole in order to properly name the fraction. Second, the problem challenged their understanding of how to determine if the fractions are the same. This aspect of the problem was particularly challenging because, despite knowing each shaded part represented  $\frac{1}{8}$  of the rectangle, it was not intuitive that they were the same because they were different shapes. The design of the problem allowed multiple different ways of thinking about the proof to come up throughout the discussion, including Layla's and Michio's explanations.

Collectively the problems used in the summer mathematics program highlight the value of designing tasks that challenge and surface multiple ways of thinking for



communicating messages about smartness. The design of the mathematical problems created contexts that enabled particular messages to be communicated. Further, a critical feature of this signaling method is that it depends on the teacher for both design and enactment. In other words, the teacher's moves during enactment with respect to particular problems communicated clear messages about what it meant to be smart and do smartness in this space. Key features about the method include having problems that are accessible to students while also challenging conceptions that students may have. In addition to the mathematical content the ways in which students engage with the content is just as important. This method is valuable for redefining smartness in that it allows for messages to be communicated in the process of actually doing mathematics work. In comparison to the initial three methods described it has potential to communicate a wider range of messages.

## **5.7 Summary**

In this chapter I have identified five episodes and critical moments within them that highlighted a variety of messages about smartness the teacher communicated. Additionally, I argue that these messages were communicated through six signaling methods. The signaling methods allow for messages to be communicated and are distinct in how they support redefining smartness during instruction. Four of the signaling methods are geared toward interactions that take place in classroom instruction; while two of the signaling methods do not place as much emphasis on the interactions, but rather some of the other aspects that are important for defining smartness during instruction. In the next chapter I discuss the findings from this chapter

in relation to chapter 4. I also describe limitations of this study and implications for future research.

## Chapter 6 Discussion

### 6.1 Introduction

In this chapter I provide an overview of what I set out to do in my dissertation. My goal was to understand Black learners' constructions of smartness, examining how upper elementary Black children talk about what it means to "be smart" when doing and learning math in school. I also sought to examine how a teacher might communicate what it means to be smart and who is smart.

My focus on Black learners is connected to my experiences as a Black man navigating the discipline. My experiences, both positive and negative, are common for Black learners in mathematics more generally. This can be seen in the data that show that Black learners are overrepresented in special education and under-identified and selected for gifted education programs. These contemporary patterns are reflective of master narratives about the intellectual inferiority of Black people and date back to slavery. These cultural narratives undergird institutionalized structures, including tracking and testing. These master narratives also are pushed through racial storylines and racial narratives about Black learners. This pervasiveness of antiBlackness, and my direct experiences with it as a learner of mathematics, form the backdrop of my dissertation work.

With this in mind I set out to hear the voices of Black learners, with the goal of reimagining what smartness in mathematics could be for Black learners. Similar to Howard (2001), I wanted to let Black learners "tell their side of the story." In addition, I

also considered it vital to attend the teacher's practice with respect to constructing smartness in mathematics. Although classrooms are embedded in and permeated by these broader cultural narratives and structures, still, a great deal can take place during instruction as teachers exercise their discretion in both equitable and inequitable ways (Ball, 2018). With regards to how being smart in math is defined, valued, named, and communicated, teachers have the power to reinforce or reimagine. In my work I attempted to simultaneously identify and interrogate the mathematical work, students' voices, and teachers' practice to uncover how these components fit together and what they can tell us about the teaching and learning of mathematics for Black learners.

## **6.2 Overview of the Chapter**

I divide this chapter into four sections. In the first section I return to the framing of my study, and explicitly connect the dots from the first chapter to the third chapter, highlighting what I did and what my analyses helped me to name and unpack. In this section I briefly discuss the problem space, the literature, my conceptual framework, and the methods that I used to answer my research questions.

In the second section I turn to focus specifically on the discussion sections of chapter 4 and chapter 5. I discuss explicitly how those chapters are connected, which is critical for understanding the overall goals of this work. It is in this section I examine the connections among the different students' narratives and ideas, as well as consider how these relate to the messages and signaling methods the teacher uses. I consider how students' perspectives interact with and compare with the signals represented in the teaching.

In the third section, I talk about the limitations of this work and how it connects to the field of mathematics education. No study is perfect, and in this section, I describe the study's imperfections for the purposes of future research. In the fourth and final section, I turn to implications of this work and steps for future research. It is my hope that at the conclusion of this chapter the reader will have a better understanding of the dissertation.

### **6.3 Revisiting Key Aspects of the Literature, Conceptual Framework, and Methods**

In this study I sought to understand the conceptions of smartness that the Black learners had and the ways in which the teacher was communicating what smartness in math involves. I began with what I could see in the literature about Black learners and about conceptions of “competence” or smartness in math. Studies in this domain reinforce that a deficit perspective in which Black learners are positioned as intellectually inferior to their counterparts. This is evident in the frames that are used to study Black children as learners that consistently portray them as “behind,” with “gaps” compared with white children. This is also reinforced in the empirical studies that, using data and evidence grounded in these theoretical frames, shows Black children as “underachieving” and “low.” Some scholars, though, show that Black learners are aware of the narratives that surround them and other racial groups, which provides insight as to how they construct notions of smartness. Still other studies suggest that classroom practice can contribute to shifting Black learners' constructions of smartness. Most studies do not consider how race intersects with definitions of smartness in mathematics. Despite this finding, the existing literature does highlight the ways that

smartness is both socially and locally constructed in classrooms. That is, smartness is defined within classrooms (contextually) and based on social dynamics. However, relatively few studies focus on Black learners specifically, highlighting the need to attend to these students' voices when conceiving of smartness in mathematics. Considering that teachers are an important part of constructing smartness locally and socially, another key finding is that teachers construct smartness through their practice in ways that can be either productive or unproductive. Based on my close study of the literature, I argue that there is a need to capture, understand, and incorporate young Black learners' constructions of smartness and identify the ways in which teachers construct smartness through their practice. Understanding the ways smartness has been discussed in the literature allows for critique and a reimagining/redefining of the construct that specifically attends to the needs of Black learners.

Grounding my study in the literature helped to sharpen my questions. I constructed the conceptual and theoretical framing of this study using critical race theory, philosophical perspectives on mathematical practice, conceptualization of mathematical competence and instructional “tasks,” and a theory of instruction as situated in environments. Each of these contributed crucial tools for my analyses. A key takeaway from CRT is the ways in which Black learners' conceptions of smartness are traditional, as well as the ways in which the messages and methods signaled by the teacher might disrupt those traditional patterns. Knowing that students are brilliant is relevant for seeing the conceptions that students described in the data that are non-traditional, as well as interpreting students' voices to understand the conceptions that students have. Because instruction incorporates many different components, it became

clear that the mathematical tasks on which students work are connected to students' conceptions and messages communicated by the teacher. The tasks that students worked on tended to be tasks that required students to work together, required different mathematical skills, and took longer to solve. Understanding that the mathematical tasks themselves provide opportunities for signaling what competence or smartness involve contributes to analyzing the types of messages teachers communicate. Teachers' messages contribute to shaping a normative identity for students in classrooms. Collectively, these messages set up what it means to be smart and who is seen as smart in the classroom. Not only are the messages communicated, but the method the teacher uses to communicate them are also relevant. One of those ways is through the practice of acknowledging competence. As can be seen in Episode 3 from chapter four, what competencies the teacher acknowledges and about whom is one way a teacher signals messages about smartness.

Connecting the literature and creating the conceptual framework helped to generate the questions 1) How do black learners describe what it means to be smart in mathematics? and 2) How does a teacher communicate that smartness is important? These are crucial questions that my study set out to begin to answer.

#### **6.4 Understanding Smartness with Black Learners, Mathematical Tasks, and Signaling Methods in Mind**

The research question on which I focus in chapter 4 is "How do Black learners describe what it means to be smart in mathematics?" In that chapter I use students' artifacts and what they told me to try to characterize and understand their conceptions of themselves as math learners and what they thought about smartness in math. I

examine different ideas that permeate the children's thinking and I identify three important themes— namely, (1) that there although there are superficial similarities, these mask substantive differences; (2) children's conceptions of smartness shift across time and instances and thus are malleable, and (3) that children's articulated conceptions of mathematical smartness seemed at times to function as coping strategies

In chapter 5 I turn to address the teacher's practice, identifying episodes of what I call "critical moments" that signal or communicate messages about mathematical competence. I analyze different messages in the teacher's practice. I also considered what it means to "communicate," "signal," or send "messages" about smartness in practice. I identify six signaling methods along with moves the teacher used to signal messages.

Undergirding both of these chapters is the mathematical task analysis in chapter 3. In chapter 3 mathematical tasks they came up during student artifacts and the episodes in chapter 5 provided insight to the nature of the tasks. In particular, I found that the tasks students worked on were both above and below grade level — i.e., "grade level" was not a reliable metric for the complexity of the problems. They varied in the tools and contexts provided to students (e.g., manipulatives), were inherently more collaborative in nature, and were sequenced in ways that allowed for key concepts and practices to be used and developed.

In conducting my study, I developed several key learnings. I set out to study what students' conceptions of smartness are. In conducting the study, I learned that students' conceptions of smartness are complex and that students can have multiple conceptions



of smartness. Some of these conceptions are grounded in traditional notions such as speed and getting correct answers. Others are nontraditional, such as trying hard or listening to others. The conceptions that students described are connected to their prior experiences as well as their current experiences and constructions. Their prior experiences seemed to have a profound impact on students' conceptions of smartness. Through the study I found that students' prior interactions impacted the conceptions students carried into this classroom and also the conceptions that students believed or enacted. Arianna presented one such example. It seemed that, in her prior experiences, because students had laughed at her incorrect answers, there was a period in the summer program that she tested out this new context to figure out how she could participate, but also, how her peers participated and interacted with one another about math. After the first week she began to participate in more observable ways which she attributed to feeling more comfortable. She was not the only one. For instance, Kasim described being the top star in his prior classrooms, the smartest kid in his classes, and not learning from anyone. Yet there were moments in the summer program where he could see that he could learn from others and found it beneficial to work with others. This came up a few times including when he mentioned learning improper fractions from Michio and enjoying working with his classmates Jerone and Deonte. Arianna and Kasim represent two cases of the prior experiences they brought into the classroom and navigating those experiences in this context.

Students' conceptions of smartness and their malleability is connected to the opportunities mathematically students have; specifically, the mathematical tasks that students work on. There are two examples that highlight that point. La'Rayne first

mentioned that she did not feel smart in math class. When probed, she explained that in prior classrooms the math was easy, and she was able to solve problems. However, in the summer mathematics program the math problems were harder, and she was unable to solve them. She articulated that when thinking about smartness one must consider the challengingness of the content. Later in the summer mathematics program when asked which problems she felt smart on, one of the problems she mentioned was the Train Problem. It's fascinating she mentioned the train problem as it was not easy, and she was not able to solve it quickly. However, her rationale for thinking she was smart on it was that she continued to work hard on it (persevere) and got it sometime (make trains that held some number of passengers). That is, the tasks she was working on appeared to have her thinking less about right and wrong and more about difficulty and persevering as measures for smartness. Kasim had a similar point. He often associated smartness with going to the board getting answers, as well as getting answers right quickly and consistently. Yet by the end of the program, he described being smart no matter how long it took as long as you eventually got it right. It seems that it could be connected to several mathematical tasks. One could have been the triangle problem, about which he wrote after the first day, "I will solve this problem," which he eventually did. Another influence might have been the train problem which took them two weeks to solve. Based on Kasim's engagement and experiences, it seemed that the mathematical tasks provided an opportunity to shift the conception students had.

The last key learning discussed from this study is the teacher's practice in signaling messages and signaling methods. It appears that it is not just important what the teacher does, but also when and how. There are a few examples that highlight this

point. I start with Arianna. In Arianna's student artifact she mentioned that she got the same answer as La'Rayne in episode one. That is, she thought the answer to the teacher's question was three whites equal to a light green. However, in that episode we saw that the teacher reframed that moment from right and wrong to which explanation answers which question. This moment is an example of the signaling method of disrupting normalized practice in that instead of focusing on correctness the teacher focused on understanding along with other practices. At least one of the messages signaled is that everyone could make important contributions. This could be important and resonate for a student such as Arianna as it seems her prior experiences being in math class students laughed at her for wrong answers. Therefore, this moment could be an example of Arianna seeing that this space was different. As the summer mathematics program went on, Arianna participated in more observable ways as evidenced when she was the only student to go to the board to revoice Miah's proof in a way that captured and synthesized Miah's and her peers' thinking in a concise way. The next day, she shared a solution to a problem at the board. Another example of teacher's signaling messages and signaling method can be seen in La'Rayne. La'Rayne mentioned not liking her previous math teacher and even describing an instance when a teacher screamed at her. However, in the summer mathematics program it appeared that La'Rayne liked the teacher. She often credited or mentioned that the teacher was the reason she got smarter and that she felt she could learn and be smarter [in the summer mathematics program]. I do not pinpoint one specific moment per se, however in chapter 5 there are several episodes that could be connected to why La'Rayne made this comment. In chapter 5 episode 2 both La'Rayne and Kasim had the notion that

studying something for a long period of time was an explanation for their answer. However, in that episode the teacher signaled the message that part of explaining is convincing your peers and the method in which one might do that is disrupting normalized patterns by attending to revising one's thinking. This episode and the one before are episodes that implicitly and explicitly involve La'Rayne and in each they were examples of teachable moments. More specifically they were examples where the teacher's message and method were aligned in a way that could be important for students (the teacher matching her walk with her talk). La'Rayne was not the only student who felt that they got smarter as a result of the summer mathematics program. Other students wrote things that were similar. I brought up these two instances that it seems that it's not just about the messages that are signaled although they are important, it seems that the signaling method, which occurs in critical moments is important for students as evidence to support what the teacher is communicating.

### **6.5 What Are Some Key Limitations of this Study?**

The focus of my study was to understand how Black learners in my sample seemed to think what it meant to be "smart" in math smartness and how the teacher in this classroom communicated smartness during instruction. My analyses offer windows into how these children saw their own smartness as well as what they thought was involved in being smart or doing smart things in math class. I was also able to uncover and identify a complex of signals about doing and being smart in math within this class. I was struck by the multiple channels along which mathematical smartness was constructed by the learners and the teacher, in the context of broader cultural and systemic messages about who and what is being smart at math. I do not make claims

about whether and how the learners in my study interpreted these signals, and my analyses of the signals are from my perspective. Still, lifting the cover off this important and complex set of messages and ideas about “smartness” offers a crucial step into understanding the interplay of classroom-level experiences and the systemic patterns and racialized and reductive tropes around who and what it means to be smart. In this section, I consider key limitations of the study, based on insights I gained from my analyses, and knew were in many cases fruitful next steps for research on Black learners’ experiences of and identification with mathematical smartness.

I turn first to the concept of identity, an idea mined by several scholars who study Black learners. Although I did not begin with this orientation, through my analyses, I came to consider conceptions of “smartness” in relation to oneself and one’s context to constitute aspects of identity. However, because I focused on the students only during a two-week period, the data were not well-suited to analyze or make claims about students’ identities with respect to smartness. However, future research could build on this work to study students’ identities. I see how this work is and could be connected to identity work. For instance, if smartness is an identity, then how does it connect with a racial, gender, academic, or disciplinary identity (Valeras & Martin, 2013)? To what extent do they overlap or not and how might that impact students’ learning? What factors intersect to shape and influence students’ mathematical identities? Given the malleability in students’ ideas about smartness in math, what are promising ways to shape positive identities about oneself, and about what and who is “smart ” in mathematics. Connected to this notion of malleability are questions about how robust or fragile these identities might be (McGee, 2015).

Another interesting issue is what it means to “do” or enact conceptions of smartness. One thing I was not able to attend to specifically in this study is the relationship between what students say and what they do. The students described smartness in particular ways but understanding how that is connected to what they do in the classroom or what opportunities they have to do in the classroom is a direction for future research. One of my goals was to reimagine smartness as a verb. In reimagining smartness as a verb, it includes identifying actions and practices that one does that is important or relevant. This point extends, specifies, and critiques work by scholars around growth mindset and grit are useful in providing theoretical constructs to explain human action. However, these theories can be taken up in problematic ways especially with respect to Black learners. Often this work is rooted in racialized conceptions of what it means to have “grit” and who lacks it. Specifically, the work often reinforces historical narratives about Black learners, arguing that they are “struggling learners” and that what they need to be successful is to develop more grit, more perseverance in the face of challenges. Conceiving instead of smartness as something one does instead is— a set of habits and practices—is an opportunity to move beyond labeling students as having a growth mindset or not or having grit or not. This theoretical reimagining affords a strengths-based approach for Black learners, one that is based in their capacity to persevere and their persistence. This direction could build on work in the field about Black resilience and brilliance. For example, O’Connor and colleagues (2007) describes the considerations for thinking about the experiences of Black learners theoretically and methodologically. They argue that often race has been described through a cultural lens or as a variable. However, both perspectives have their limitations. This study provides

important examples of the heterogeneity within Black learners. All the students in this study were Black learners and the analyses show significant similarities and differences among them. Further understanding the relationship among smartness, race, and gender in mathematics classrooms would be fruitful moving forward. The students describe very nuanced views of their constructions of smartness, which seem to be connected to their prior experiences, but also highlight how dynamic their conceptions are. Understanding the circumstances under which these constructions are developing becomes paramount for reimagining smartness in mathematics in school.

A third limitation rests with the incomplete methods we have for probing the thinking of young learners. Although I worked carefully to connect with the children, even emphasizing my connections with them as a Black man and engaging in friendly and relaxed conversations with each of them, I am aware that methods of learning about children's perspectives are a fruitful area for continued work. Focusing on the language we use as researchers in asking questions (e.g., what does it mean to be smart instead of what it means to be good) or interpreting students' language (e.g., being smart means to do good) are two such examples that came up in this work. From the learners in this study, it is clear that students have a lot to say about their conceptions of smartness specifically and their experiences in mathematics classrooms more broadly. Additionally, learners' ideas about these important topics can change and hinge upon the questions asked and the opportunities provided, as well as who asks the questions and how learners might relate to those adults or to the methods used. Capturing their ideas is challenging but is necessary and important work. Continuing to build and use more complete methods is critical for being able to both

capture and make use of students' perspectives in mathematics education research and mathematics classrooms.

A fourth limitation connected to the previous one is member checking. Originally, my goal was to also follow the students into their fifth-grade classroom, however I was unable to do that. Overall, I was unable to follow-up with the students to see if they agreed with the findings in the study or if they could provide their perspective. I don't see this limitation as one that is specific to my study. Instead, I see it as something that is relevant to the field when interviewing students. To what extent is member checking done and how? What is the relationship between student development and member checking? Is member checking more effective with older students because they might be more mature? If younger students do not agree with outcomes what does that mean for findings? The limitations described here provide the field of mathematics education with important questions to being able to conduct more informed research which has implications for both research and practice.

## **6.6 Next Steps**

Given the findings of my study, there are several avenues that seem both important and promising for future research. One of them is thinking about the relationships between the conceptions students have and what they actually do in the classroom. There is some evidence of this in my study as the students would discuss either in the interviews or their artifacts what they did. However, comparing that with how they participated in class could provide a more holistic picture of students' conceptions of smartness. One way to capture this could be using a tool like Equity QUantified in Participation (EQUIP) as described by Reinholz and Shah (2018) to code



whole group discussions. This would then allow me to compare what they said and what they did. As research has shown there is not necessarily a one-to-one correspondence between what we say and what we do or what we believe and what we do. Studying this from the students' perspective is a parallel example and could have implications for practice.

As the study unfolded it became clearer that students were drawing from the previous mathematical experiences when thinking about smartness and thinking about smartness during the summer mathematics program. A next study could focus on studying students in different mathematical environments more critically. In Martin (2012) he introduced a framework for connecting the interpersonal and intrapersonal in mathematics education research with respect to Black learners. Chapter 4 and chapter 5 represent the interpersonal and intrapersonal levels, however a deeper analysis of the classroom specifically and contexts more generally would provide an opportunity to connect the interpersonal and intrapersonal. This deeper analysis could include, how antiBlackness is manifesting inside of classrooms, naming practices that are connected to Black literature that support the teaching and learning of Black learners and connecting students and their phenomenal realities to the context being studied. For example, I could study a student over time, beginning in fourth grade, then following them into the summer, and then into fifth grade. This study will allow me to see the differences in students in different classrooms along with the different opportunities they had in different classrooms. As described in the literature smartness is locally and socially constructed. With that in mind it seems relevant to see students across

mathematical environments to understand how they negotiate smartness, which could offer suggestions for teacher practice.

In this study I created an open ended pre- and post-survey to try to capture the students thinking about smartness. Building on this it might be useful to create an instruction that measures students' conceptions of smartness quantitatively. This type of measure could be very insightful in at least two ways. First, for teachers attempting to plan. For instance, if a teacher knew that students had particular types of conceptions how would that impact their practice? What picks certain problems? Do they arrange the groups in particular ways? Do they interact with students differently? These are important questions to consider from an affective perspective instead of an evaluative perspective (e.g., test scores). Second, it could provide insight to how students participate. For instance, do particular conceptions of smartness mean that students are more or less likely to participate in class? In what ways does the environment impact students' conceptions? These questions allow us to be able to examine the role of environments in students' conceptions of smartness.

Another study I am thinking about is showing students video clips of classroom instruction to get their perspectives or interpretations of teacher practice. This is connected to Howard (2001) where he focused on letting students tell their side of the stories with respect to culturally responsive teachers. This builds on the work in chapter 5. As described, I considered communication one-way. However, this study would allow one to think about communication as two-way. Or said another way it would be the opportunity to think about students recognizing the messages that are communicated. This would be a critical step to investigate students taking up, resisting, or both. This is

connected to Hand (2010) who studied a teacher's practice in a high school classroom and found that students resisted the reform practices that the teacher used. While there is a shift in research that teachers practice should be more equitable, not taking into consideration that students have been conditioned and socialized in ways that contradict the very practices that need attention when considering students' conceptions of smartness.

### **Conclusion**

As a Black boy now a Black man there are several things, I have grappled with in my mathematics education experiences. Some of those experiences have been positive such as skipping the fourth grade, passing math courses at a community college while I was in high school, and graduating with a B.S. in mathematics from Morehouse College. However, some of my experiences have also been negative such as having a racist teacher in fourth and sixth grade, the latter of which led to me doing my work in a third-grade class, not doing well in my Real Analysis II course, and not completing my master's degree in mathematics because of repeated harmful experiences in my courses and in interactions with peers and instructors. These experiences, along with what I have learned from the literature, are vital to my journey. That journey has resulted in a passion, motivation, and curiosity to know what it means to be smart in mathematics from the perspective of Black learners, what can be done to reimagine smartness in the ways that are more inclusive of Black voices and productive for Black learners' learning of mathematics content and practices. This dissertation begins to address these questions and offers some initial findings. While I know more now than I did when I started this journey, there is still much to learn. However, the same passion,

commitment, and curiosity I have will be used to continue the commitment I have to the teaching and learning of Black learners.

In this dissertation I use the real names and pictures of Black learners. I had IRB approval and the families' consent to do this research and to represent the children with their real names and images. However, students' last names were not included and pictures that were used did not include contextual information that identified students (e.g., names on clothing). My decision to include Black children's names, instead of pseudonyms, as well as pictures is not common. I decided to include students' names and pictures for three reasons. First, often Black children are adultified (Epstein, Blake, & González, 2017), meaning they are not thought of as children, but as adults, thus robbing them of their innocence and childhood. Including Black children's names and images makes them real. That is, you can see children for who they are. Second, the inclusion of images was strategic. Finding images that focused on the selected student and captured them in the class, was an opportunity for the reader to see the children in relation to what they were describing in the data. I thought this helped to see the nuance and complexity that students have. Third, and last, the names and images were included to humanize Black learners (Turner, 2021). The complexity of the conceptions of smartness that students describe were sophisticated and connecting what students were saying with their images were ways of connecting and affirming their ordinary brilliance.

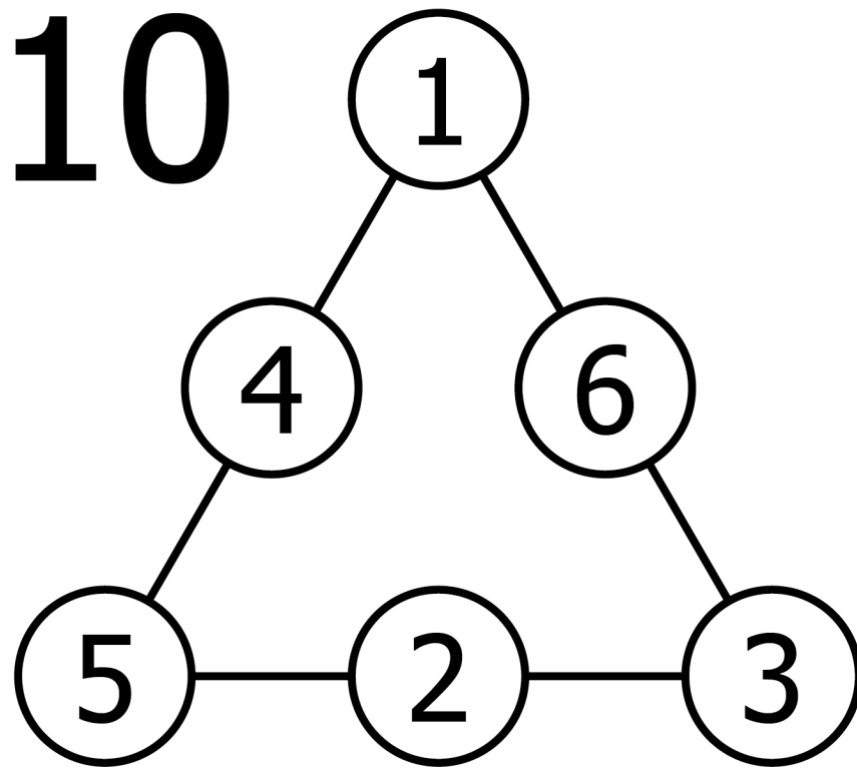
I acknowledge that readers may have ethical concerns about using Black learner's name and images. This perspective could be the result of wanting to protect Black children. Historically Black learners have not always been positioned in a positive

light. In fact, it is more common to see Black learners both historically and presently depicted from deficit perspectives. Examples of this include the work of Jackson (2018), which builds on the work of others (Mullings, 1997; West, 2008; Ladson-Billings, 2009; Collins, 2001; Townsend, 2010; Gordon, 2008) that highlight how Black girls and women are often depicted in research in problematic ways and in ways that do not align with how Black girls and women feel or think about themselves. Additionally, the work, of Howard, Flenbaugh, and Terry Sr. (2012) synthesizes the depiction and social imagery of Black males highlighting the evolution of problematic characterizations that include Black males as "...pimps, thugs, hustlers, and law-breaking slicksters who were not to be trusted, were not worthy of equal treatment, and needed to be marginalized because they were a "menace to society," prone to violence, and constantly involved in gangs and drugs (Bogle, 2001; Diawara, 1993)" p. 89 and Black boys as "...defiant, disruptive, disrespectful, and profane" p. 46 (Ferguson, 2001). These are a few examples that affirm the urgency and importance of protecting Black children's bodies and minds. From this perspective one might ask does mathematics education research or the society writ large deserve to see the brilliance of Black children? Especially when images and names can be taken to reinforce and perpetuate narratives that are not intended by the author. Although this perspective is fair, I believe that the benefits of my decision to include the names and pictures of children outweigh the drawbacks, particularly the need to provide positive examples.

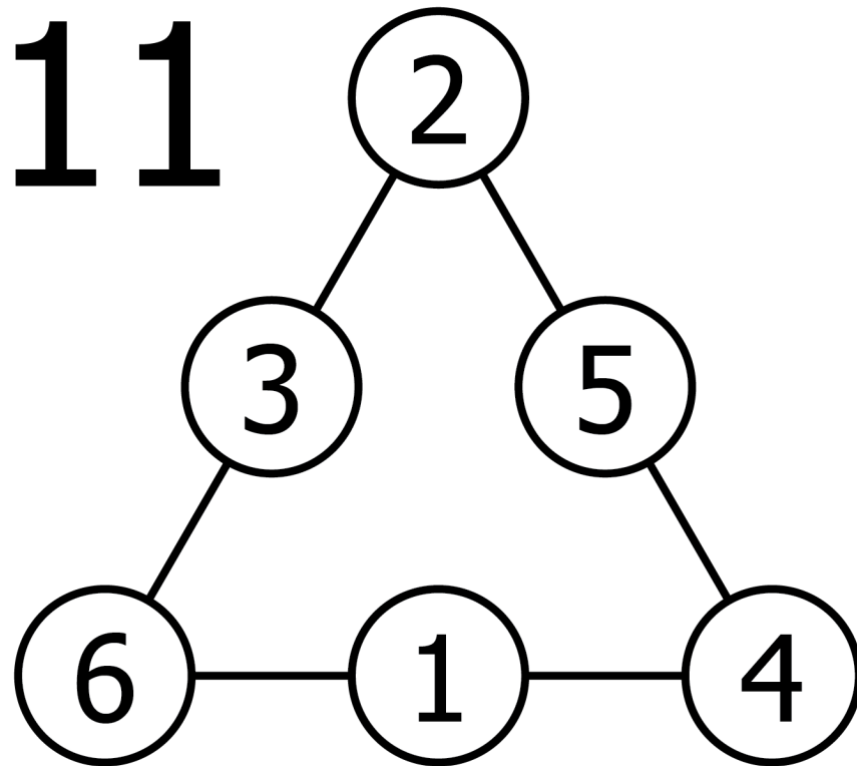
However, with that in mind, I name the steps I took as a researcher to protect the Black learners described in my dissertation. I took care to ensure that participants' last names were not included. I chose images that painted students in a positive light. While

collecting the data I ensured that students had agency and could exercise that agency, which included not wanting to participate (e.g., Chandler did not want to do one of the interviews), I included notes or text to support the student artifacts included and I wrote about students in ways that took a strengths-based perspective. As a researcher one can never fully control how research is taken up, but there are things that can be done to attempt to avert possible harm to the children who made this study possible. These steps represent my process to do that. Protecting Black children is crucial and raises questions about conducting research with Black children. At the time of this study, I did not follow up with the participants in this study. However, conducting member checks and following up with students could be another step in the process that future researchers should consider. Although I had IRB approval to share students' first names and images in class, adding an additional layer could be critical to ensuring research is done in a way that protects and provides agency to Black children and their families.

Appendix



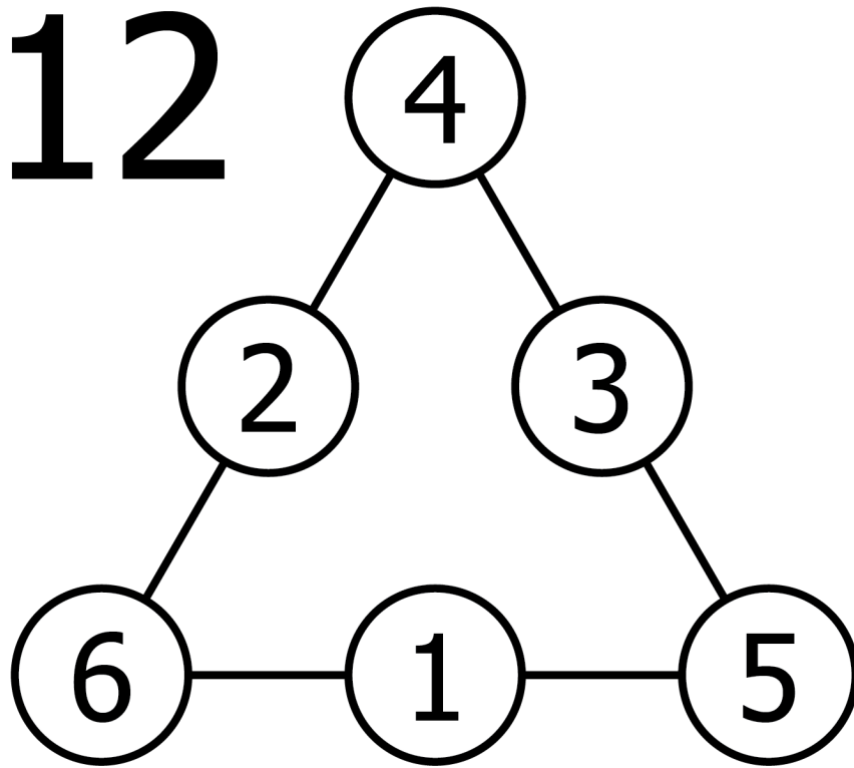
Appendix Figure 1 Magic Triangle Solution for 10



Appendix Figure 2 Magic Trigangle Solution for 11



12



Appendix Figure 3 Magic Triangle Solution for 12

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