MICHIGAN ROSS

# Improving Portfolio Optimization Using Option Skewness Regimes

**by**

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**Abstract**

Portfolio optimization methods have had many different approaches and additional developments since the introduction of modern portfolio theory and the mean-variance optima. Research has shown that the financial derivative of options contains valuable future-looking implied information regarding the markets that can drastically improve the Sharpe ratios of portfolios. While the most common use case has been using option implied volatility to improve underlying variance estimations, higher moments such as skewness has been found to also provide portfolio improvements. However, these higher moments are very difficult to predict correctly, and thus their impact has been largely neglected. The purpose of this thesis will be to see if applying the machine learning method of Hidden Markov Models will be effective in predicting option skewness regimes rather than explicit values, evaluating effectiveness as the ability to improve portfolio performance. By predicting regimes of option skewness, the goal will be to gain greater accuracy in evaluating these higher moments rather than predicting explicit values, and thus derive more accurate information to feed into portfolio optimization. The method of optimization will be held constant to control for studying the effect of implied information, and effectiveness of combining higher moments with regime prediction will be how much the portfolio optimization process is improved.

**Contents**

## I. Introduction and Literature Review

### *Portfolio Optimization*

Markowitz first introduced the idea of mean-variance optimization for portfolios in 1952, focusing on knowing with certainty the expected returns and variance of each asset in the portfolio. In his paper, Markowitz breaks the optimization down into two stages, with the first stage using historical data and probabilistic models to predict the future returns and volatility, and the second stage then assuming the expected returns to be certain and deriving an optimal portfolio to balance these two features. While some investors are concerned with specific index performance or other investing goals, this approach seeks to address investors who are seeking the optimal balance between risk and reward. By determining all possible portfolio combinations, it's possible to find the entire efficient frontier, and the idea becomes that any investor can simply select a portfolio from their level of risk tolerance on this efficient frontier. Figure 1 below shows an example of an efficient frontier, and any portfolio located on the curve can be considered "optimal".

*****Please Insert Figure 1 Here*****

Specifically optimizing for the Sharpe Ratio, measured as the ratio between return and volatility, marks a point on the efficient frontier called the tangency portfolio as it is tangent to the line marking the risk-free rate. However, with Markowitz's method, there are two main limitations: being able to accurately predict future returns and risk, and efficiently determining this frontier. As computing power has expanded, the latter issue becomes less prominent, thus highlighting the importance of being able to accurately predict future returns and evaluate risk.

### *Higher Moments and Options in Portfolios*

In efforts to better understand future returns and risk of a portfolio, researchers have investigated measures beyond just the expected value and the standard deviation. In recent years, the focus has been on measuring tail events, such as VaR (value at risk) and CVaR (conditional value at risk). VaR is defined as the loss during the worst x% of cases, and CVaR is the expected value of the loss given that the portfolio is experiencing its tail 5% scenario. Along with VaR and CVaR, there has also been an ongoing debate as to the necessity of including higher moments such as skewness and kurtosis when the mean and variance are common practice. Skewness and kurtosis are important within the context of portfolio returns as it can help interpret likelihoods of returns and affects decision making under difference levels of risk tolerance. One of the biggest issues that research has shown is that higher moments only matter during rare events and are heavily influenced by these rare events, so historical data is not a strong indicator of future events (diBartolomeo, 2014). Additionally, skew especially has been hard to predict, where being wrong on the sign of the skew leads to extremely large estimation errors. Nonetheless, higher moments do contain value, especially in a Bayesian framework where a model is trying to constantly update its estimations, in selecting better performing portfolios in an adjusted Markowitz optimization schema (Harvey, 2004).

In efforts to augment historical data with forward looking indicators, researchers have found that options contain implied information about an asset that can greatly improve out-of-sample portfolio performance. Specifically, the option implied volatility, correlation, skewness, and risk premium for stochastic volatility have been found to be useful (McMillan, 2013). Option implied volatility refers to the level of volatility that the Black-Scholes equation would need to equate current option prices trading in the market. Correlation, skewness, and risk premium can then be derived on these measures of volatility. In previous approaches, this information has been used to rank stocks based on desirability through adjusting their variance levels, or as inputs for the parametric-portfolio methodology (DeMiguel, 2012). Regardless of how it's used, the information derived from options pricing is beneficial to portfolio optimization due to its forward-looking nature that encapsulates investor views and confidence. As such, this thesis will focus on investigating one feature of options, the skewness, to enhance predictions about the underlying asset and improve portfolio optimization.

### ***Multi-Period Optimization***

While higher moments and options can aid in deriving a better prediction on future asset performances, another approach researchers have tested is using the same amount of information but factoring in regime shifts to segment expected behavior. A regime within the context of a stock can be defined by as a period with defined characteristics, such a bull versus bear regimes characterized by above and below average returns, respectively. By identifying regimes and changes in regimes using Hidden Markov Models (HMM), it becomes possible to not only get a probabilistic interpretation of what type of returns and volatility are expected, but also to classify the current regime and help guide portfolio selection (Kim, 2019). Hidden Markov Models are a machine learning method that allows for predicting unobservable variables based on a set of observable states. Thus, it becomes very applicable in the use case of identifying regimes, which is not directly observable, based on observable metrics of each regime, such as interest rate or volatility amongst many other possible measurable metrics. These efforts have led to multi-period optimization strategies, enabling more detailed and effective portfolio selection (Oprisor, 2020). Since each period can be understood to have its own optimal portfolio, multi-period strategies best mimic how markets can change over time and allow modeling of transitional probabilities and optimizing to get the best performance per regime rather than a blanket optimization schema regardless of regime.

The goal of this thesis will be to try and combine these two approaches, applying a multi-period approach to higher moments from option-implied metrics, specifically focusing on skewness of the options and its implications within asset performance. Since skewness of options is a very volatile number, simplifying the predictions to regimes of the options' skewness rather than explicit numerical interpretations can hopefully reduce estimation error while still providing tangible benefits to the portfolio optimization via additional information and segmentation.

## II. Theoretical Framework

There are going to be two main areas of research that this thesis aims to tackle and combine: ways to use options data and incorporate information about higher moments, and how to perform multi-period optimizations through regime detection.

### *Options Related Moments*

As described in the previous section, the goal will be to make use of the higher moments of options to get implied information about the underlying assets, focusing on option skewness measurements. One of the most common and basic ways to price options is using the Black-Scholes equation for pricing European options.

$$C = SN(d_1) - N(d_2)Ke^{-rt}$$

Where $C$ denotes the price of the European call option, $S$ denotes the current stock price, $N(d)$ denotes the standard normal distribution, $K$ denotes the strike price, $r$ denotes the risk-free rate, $t$ denotes the time to expiry, and $d_1 = \frac{\ln\left(\frac{S}{K}\right)+(r+s^2/2)t}{s\sqrt{t}}, d_2 = d_1 - s\sqrt{t}$.

Using the Black-Scholes equation, we can utilize the market prices of the at-the-money (ATM) contract to derive an implied volatility of the underlying stock by reversing and solving for what level of volatility is needed to get the market price that is currently trading. At the money refers to the contract trading closest to the current price of the stock, and it has a delta of 50. Delta measures the change in the price of the option with change in the underlying asset price, and a delta of 50 is atm while deltas of 100 are far in the money. One of the major assumptions of Black-Scholes is that the asset follows a log-normal distribution of returns, which would not account for any influence by higher moments. Under this assumption, the volatility curve would be perfectly symmetrical, since there is equal probability of the asset increasing and decreasing in price. Thus, instead of using the implied volatility alone, the goal will be to incorporate the skewness of the implied volatility, as that has been found to contain information about the higher moments of the option price and as a result, gives higher moment implied information about the underlying asset as well to improve portfolio optimization.

To calculate skewness, skewness must first be defined. Under the Black-Scholes equation, the implied volatility of a call and put equidistant from the strike price should be equal. However, in practice, options typically do not have the same implied volatility across all the strike prices and time to expiries. The financial explanation is one of supply and demand: across different strikes, there are differing levels of buying interest, thus leading to prices being higher or lower and as a result, differing levels of implied volatility. The result when plotting volatility is that of a volatility smile or smirk, and when time is factored in, a volatility surface.

*****Please Insert Figure 2 Here*****

In general, the volatility is asymmetric, and that asymmetry is option skew. Due to investor interest, calls are generally sold more, and puts are bought more, so the volatility smile has a skew to it in favor of puts. The skew of options will be measured as the difference of equidistant implied volatilities, and this distance can be measured as the difference between equidistant implied volatility, with one very notable measurement detailed below as the difference between the implied volatilities of the 25-delta put and call, normalized by the volatility of the 50-delta option (Mixon, 2011).

$$Skew = \frac{IV_{-25\Delta} - IV_{25\Delta}}{IV_{50\Delta}}$$

Through these measurements, we will be able to calculate the skewness of the option over time in a standardized method. One major assumption that is kept is that the implied volatility can be accurately obtained, either through reverse solving the Black-Scholes equation, or provided via market data and interpolation.

### *Multi-Period Optimization*

The traditional Markowitz optimization problem simply seeks to minimize the difference between the portfolio variance and expected returns. The formulation is as follows: $\min_{w} w^T \Sigma w - w^T \mu$ such that $w^T e = 1$, where $w$ is the weights, $\mu$ is the expected returns, and $\Sigma$ is the covariance matrix. The only constraint that is leveraged is that the weights sum to 1, i.e., not allowing any short selling of assets. In this most basic version, there is no account of transaction costs or other desirable properties of portfolios.

After the parameters are set, the optimal weights can be solved for. The Markowitz optimization will serve as the basis of optimization that this thesis will focus on improving through additional inputs of option related moments. By holding the core optimization constant, the effect of the additional data can be tested and isolated.

To achieve multi-period optimization, regimes must be established, and one such method to do so is the use of Hidden Markov Models. In the figure below, the schema for an HMM is depicted, with hidden states being the unobservable underlying characteristics, and the observable states being ones that we can record. Hidden Markov Models follow the assumption that the current hidden state depends only on the previous hidden state, and the current observable variable depends only on the current unobservable state. In previous financial implications, the hidden states have been defined and periods of stagnancy, falling, or rising, and the observations have been market metrics such as Sharpe ratio or interest rate. To solve the HMM and obtain probabilities or to find the set of states that was most likely, The Baum-Welch or Viterbi algorithms, respectively, can be employed. These techniques have been widely used throughout machine learning and other HMM use cases.

*****Please Insert Figure 3 Here*****

Once the regimes are predicted, multi-period portfolio optimization can use the regime characteristics to select which assets to include in the portfolio. Since the Viterbi algorithm can give the most likely scenario, the transitional probabilities determined by the Baum-Welch algorithm can determine which assets are most likely to be in a favorable or unfavorable regime. By trimming down the asset selection process, the asset universe that is fed into the static weighting optimization will only allow investments in favorable assets and thus improve the performance of the portfolio.

*****Please Insert Figure 4 Here*****

## III. Methodology

### *Data Acquisition*

The data is acquired through OptionMetrics, a largely cited options data source, accessed via WRDS. The IvyDB US database within OptionMetrics contains volatility surfaces for options through many years. In this thesis, the data was used from 2014 to 2019, stopping shy of the hyper-volatile events of 2020. Information about the volatility surface and asset price were pulled daily for ten stocks: Apple (AAPL), Amazon (AMZN), Microsoft (MSFT), Alphabet (GOOG), Meta (FB), Tesla (TSLA), Nvidia (NVDA), Dow Jones Industrial Average (DIA), Aggregate Bond ETF (AGG), and SPDR Gold Trust (GLD). The technology stocks were chosen due to technology stocks historically having volatile option skew numbers and giving more information regarding option implied metrics, and the above stocks represent the biggest technology stocks in terms of market capitalization and trading volume. The other three stocks of DIA, AGG, and GLD were chosen as common hedges of technology stocks and have some of the largest trading volume as well, thus allowing portfolio optimization to select hedges and form the best and most representative portfolios.

After the data is pulled, the skewness can be calculated using the method Mixon described. OptionMetrics offers the implied volatilities of the interpolated 25 and 50 delta options for both calls and puts, so the skew can be calculated as the 25-delta put minus the 25-delta call divided by the implied volatility of the 50-delta option, taken as the average implied volatility of the 50-delta options. The term horizon was chosen as 30 days since the portfolio rebalancing is every 30 days, and monthly options are a common trading strategy that offers high liquidity. So, after accessing OptionMetrics, the daily information about 30 day option skewness and asset returns for each of the 10 stocks listed should be complete.

***HMM Training***

Once the skewness of the options and underlying data metrics are calculated, the regimes prediction will be accomplished via HMM. The number of hidden states will be set as two, representing a bull and bear regime, where a bull regime can be expressed as one with a lower skewness measure. In general, a lower and more negative skew implies more investors seeking to own the asset, a bullish signal (Norland, 2019). The observation variable will be the skewness measure every 21 days, roughly one month in time as each month has 21 trading days. The HMM will assume a Gaussian distribution of skewness measures for each hidden state. The HMM is trained with the Baum-Welch algorithm with monthly observations of data from 2014-2018 inclusive, employing the use of the python package hmmlearn. Once the HMM is trained for a particular asset, it will then make a prediction on each month using the Viterbi algorithm. The confidence of the binary regime prediction will be calculated using the transition probabilities from the Baum-Welch algorithm. In this way, there is a deterministic ordering of which stocks are the most likely to belong to a regime that is "bullish", i.e. low skewness, for each month, according to the results of the HMM.

***Portfolio Weighting***

Within the portfolio optimization, the year 2019 will be the test year. The portfolio will be rebalanced monthly, with the objective function being maximization of Sharpe ratio. The expected returns will be an exponentially weighted average of returns with a 21-day half-life. The risk will be computed similarly as the exponentially weighted standard deviation, again with a half-life of one trading month. The optimization is done via the PyPortfolioOpt library in python, combined with data manipulation using Pandas.

The benchmark portfolio will be the above optimization applied to the universe of 10 stocks, without any trimming of asset class. The filtered performance considering the HMM results will be the performance of the same optimization but applied to only the top 5 (50% of total asset universe) most “bullish” stocks as deemed by the HMM regimes. Should all stocks in a particular month be in a bearish regime, it will be the 5 stocks that are least likely to be in those regimes. To test whether there has been any statistically significant improvement because of the filtering, a linear regression test for alpha can be used (Foster, 2011). Since the benchmark portfolio will be considered the market portfolio, the returns of the filtered assets can be modeled as $R = \mathrm{B}\mu + \alpha$, where $R$ denotes the filtered returns, $\mu$ denotes the benchmark returns, and $\alpha$ denotes a constant improvement. A statistically significant value of $\alpha$ would indicate that there has been an improvement in the portfolio performance.

## IV. Results

First conducting an exploratory data analysis, we confirmed that the skewness measures of most stocks do represent a relatively normal distribution. This result helps verify that the HMM model assumptions are somewhat valid, although more testing is needed to assess the extent to which the distribution is normal.

*****Please Insert Figure 5 Here*****

Next, the transitional probabilities for each asset were determined via the Baum-Welch algorithm, and the probability of staying within the current regime, i.e., not switching regimes the next month, was found to be 91.3% chance on average. While we can't say what degree accuracy this entails, it suggests that regimes last over several months and do not switch on short horizons, which would match previous literature and financial market findings that economic cycles have long horizons.

*****Please Insert Figure 6 Here*****

According the HMM results, the figure below shows which ones were the top 5 bullish stocks per month in 2019. 28 out of these 60 predictions truly had a top 5 average return in that month, and in 7/12 months the top returning stock was included in the list. These results raise some concerns, as the accuracy seems to be only about half, but since we were not trying to merely predict highest returns in the HMM but rather most optimal skew, the returns itself do not necessarily indicate anything. Having the highest returning assets are indeed indicative of good portfolio performance to come, but it does not validly measure the success of the HMM. Another interesting note is that the highest returning stocks in 2019, NVDA and AAPL, never made the top 5 most bullish stocks picked by the HMM, but further investigation is needed to determine if this marks any significance.

*****Please Insert Figure 7 Here*****

The results of the portfolio performance can be seen in the figure below. The benchmark portfolio has an annualized return of 13.62%, volatility of 10.26%, and a Sharpe ratio of 1.29. The portfolio with filtered assets has an annualized return of 17.70%, volatility of 14.11%, and a Sharpe ratio of 1.23. The Sharpe ratio decreased, despite having higher returns, indicating that there may not have been a significant improvement. The benchmark portfolio was found to have held 4.16 stocks on average with an average weight of 0.19, and the filtered portfolio held 2.58 stocks on average with an average weight of 0.37. This shows that there was nearly double the diversity in the benchmark portfolio, which is likely what led to the lower volatility and better Sharpe ratio. Note that in May, there was a steep drop in both portfolios, and this was due to the performance of FB, where both portfolios held large weights of that stock. Overall, the most held stock in the benchmark was AGG and GLD in the filtered, likely due to their hedging properties.

*****Please Insert Figure 8 Here*****

Finally, the results of running a regression test for alpha show that there was not a statistically significant improvement in returns, with a p-value of 0.78. Thus, we find that the filtering done by HMM have a strongly correlated returns to the benchmark and do not actually offer any statistically significant improvement.

*****Please Insert Figure 9 Here*****

## V. Conclusions and Limitations

The HMM transition probability matrix predicts that volatility skewness regimes based on monthly options last over several months, which likely indicates that this methodology favors coupling longer term portfolio rebalancing with shorter-term options. So, any advantages of this methodology would likely lie in the term horizon difference. However, one major limitation of this method is that is very difficult to assess the accuracy of the HMM. The HMM algorithms can only tell us what the most likely sequence of regimes is, but it can't tell what extent it is likely. The assumption of two hidden states, first order dependence, and Gaussian emission of skew remain simplifying assumptions, and an ideal model performance would need further exploration and testing.

The increased volatility of the filtered portfolio performance is likely due to lower asset universe and thus lower number of assets in the portfolio and hedging properties no longer applying. The lack improvement in Sharpe Ratio suggests that this exact methodology of HMM using predicted option skewness doesn't aid portfolio optimization, and the alpha test showing insignificance likely indicates that there is not a pure outperformance by the filtered assets. A large limitation in this area is that the asset universe and option horizon was chosen based on previous literature, so more research is needed to determine the effect of smaller portfolio size versus efficacy of option skewness regimes in portfolio optimization. Expanding the asset universe and changing term horizons would be very natural next steps of exploration.

Thus, from this thesis, we can only conclude that using monthly option implied skewness regimes predicted via first-order two-state Gaussian HMMs do not enhance the Sharpe Ratio schema of portfolio optimization schema. As is the nature of portfolio construction, more testing is needed to make broader statements about the effects of HMMs and option skewness.

## VI. Figures

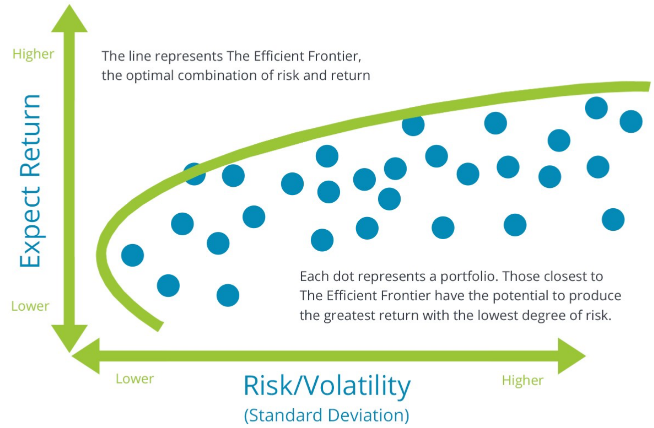


Figure 1: Efficient Frontier from Modern Portfolio Theory

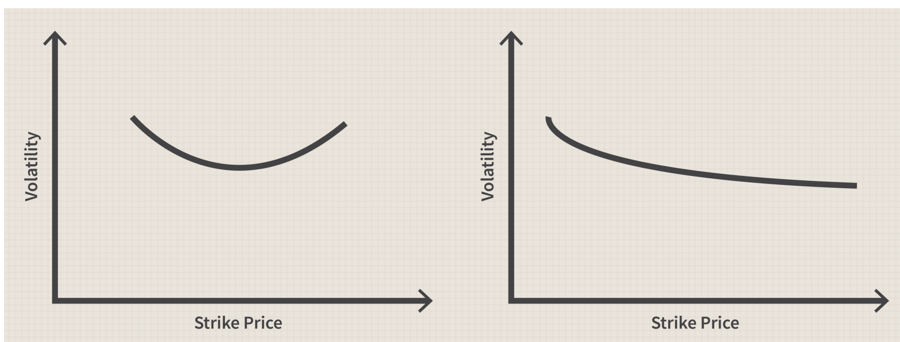


Figure 2: Symmetric Implied Volatility Curve

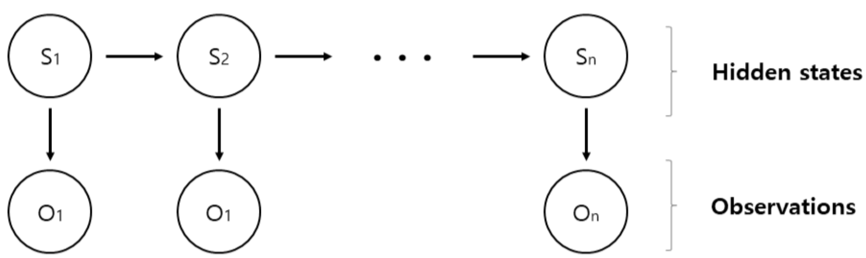


Figure 3: Depiction of a First Order HMM

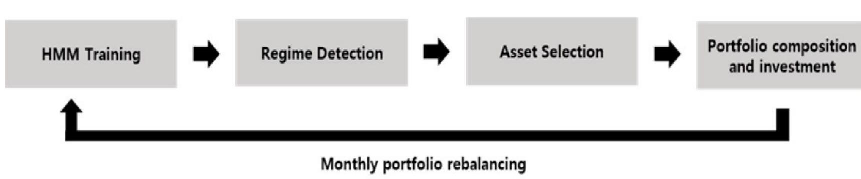


Figure 4: Process of Using HMM Regime Information in Portfolio Optimization

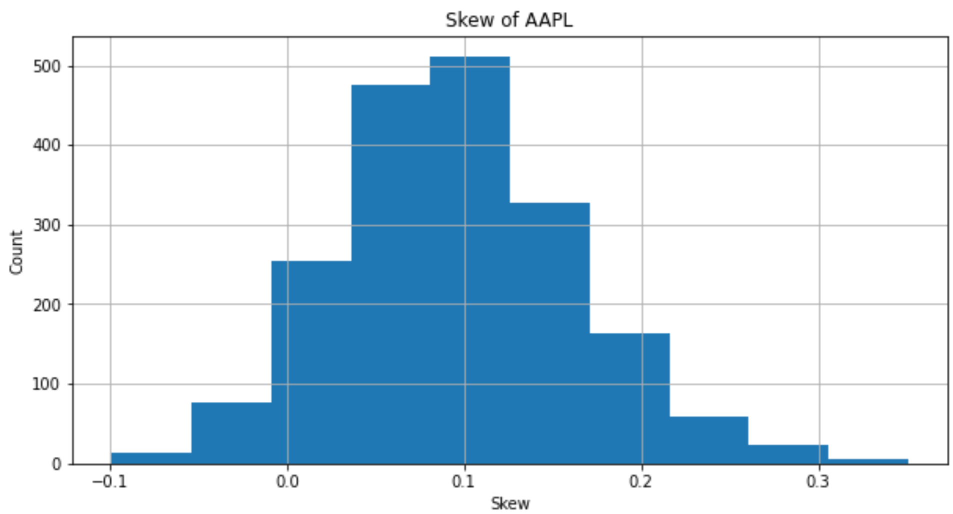


Figure 5: Skewness Distribution of AAPL

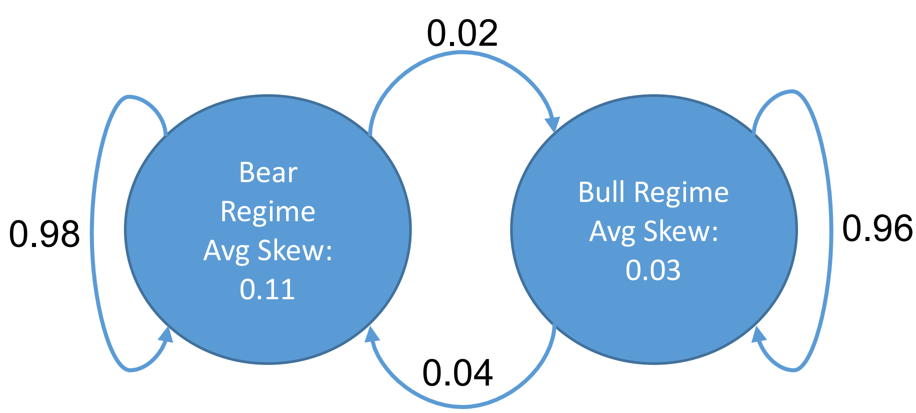


Figure 6: Transitional Probabilities of AAPL Skew Regimes

| | Top “Bull” Stocks in 2019 | | | | |
|---|---|---|---|---|---|
| **Month 1** | FB | GLD | DIA | MSFT | GOOG |
| **Month 2** | FB | GLD | DIA | MSFT | GOOG |
| **Month 3** | FB | GLD | AMZN | DIA | MSFT |
| **Month 4** | FB | GLD | AMZN | DIA | MSFT |
| **Month 5** | FB | GLD | AMZN | DIA | MSFT |
| **Month 6** | FB | GLD | DIA | AMZN | MSFT |
| **Month 7** | GLD | DIA | MSFT | GOOG | AMZN |
| **Month 8** | GLD | DIA | MSFT | GOOG | AMZN |
| **Month 9** | GLD | DIA | AGG | MSFT | GOOG |
| **Month 10** | GLD | DIA | MSFT | GOOG | AMZN |
| **Month 11** | GLD | DIA | AGG | MSFT | GOOG |
| **Month 12** | GLD | TSLA | DIA | AGG | MSFT |

Figure 7: Top Bullish Stocks of 2019

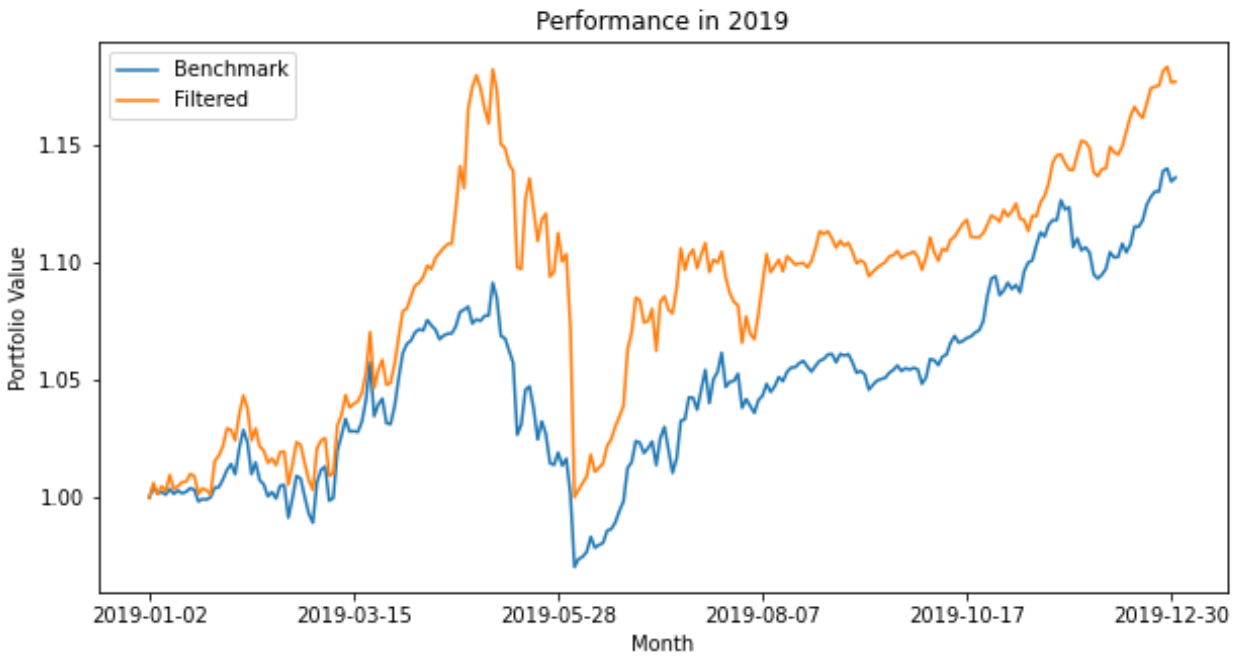


Figure 8: Portfolio Performances in 2019

Test for Constraints

| | coef | std err | t | P>\|t\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| c0 | 7.891e-05 | 0.000 | 0.256 | 0.798 | -0.001 | 0.001 |

Figure 9: Regression Results for Alpha Significance Test

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