Strategic New Product Media Planning under Emergent Channel Substitution and Synergy

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Abstract

New product and service introductions require careful joint planning of production and marketing campaigns. Consequently, they typically utilize multiple information channels to stimulate customer awareness and resultant word-of-mouth (WOM), availing of standard budget allocation tools. By contrast, when enacting *strategic* allocation decisions – which must align with other management imperatives – dividing expenditures across channels is far more complex. To this end, we formulate a multi-channel demand model for new products (or services), amenable to analysis of inter- and intra-channel interaction patterns and with the word-of-mouth process, without building such interactions directly into the modeling framework.

To address the notorious complexity of media planning over time, we propose a novel decomposition of the multi-channel dynamic programming problem into two distinct "tiers": the *strategic* tier addresses how to allocate total expenditure across channels, while the *tactical* tier studies how to allocate the channel-specific budgets (determined in the strategic tier) over time periods. This decomposition enables optimal media strategies to sidestep the curse of dimensionality and renders the model pragmatically estimable. Strategic tier analysis suggests a variety of novel insights, primarily that funds should not be allocated based on (relative) channel effectiveness alone, but also systematically aligned with WOM generation. Specifically, each channel can face a "chasm-crossing" threshold, abruptly transitioning the adoption process from lead-users to mass-market penetration. Moreover, the model provides actionable managerial insights into when, and which, channel interactions are synergistic vs. substitutive. Specifically, a channel's interactions are governed primarily by its own "leverage" (potential demand impact) and the WOM-based demand "momentum" (market penetration) it can generate, affording a novel basis for channel typography and firm action.

The modeling framework is illustrated by examining camera sales for two media channels (FSIs and radio) and their effects over 28 months. We use Bayesian machinery to estimate a highly-flexible diffusion-based model, along with forecasts, media plans, and both theoretical and empirically-based qualitative insights.

Keywords: Channel Strategy; Integrated Marketing Communications; Marketing Mix; Demand Diffusion of New Products; Channel Substitution and Synergy; Resource Allocation

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1 Introduction

Businesses continually assess the performance of marketing mix variables, media outlets, and channels, often apportioning funds as if their effects were essentially additive. Despite the prevalence and simplicity of this so-called “swim-lane analysis” (e.g., Nichols 2013), academic studies have long questioned whether its roughly proportionate allocation is justified. For example, empirical studies in marketing and operations have verified synergies among mix variables (Prasad & Ring 1976, Carpenter & Lehmann 1985, Naik et al. 2005); with salesforce spending (Narayanan et al. 2004); online and offline advertising (Naik & Raman 2003, Naik & Peters 2009); and that the strengths of such synergies are moderated by consumer-specific variables like brand familiarity (Pauwels et al. 2016).

Because ads in one medium can influence or assist those in another, failing to account for mix synergies can lead to ineffective allocation or over- / under-investment. Nichols (2013), calling for better analytics, recalls a company that presumed its ads – e.g., a TV spot and subsequent online search that leads to a clickthrough – seldom interact appreciably. Such assumptions are problematic enough for (relatively) stable established products, but are especially so for new products and services where media seek to stimulate social influence (Iyengar et al. 2011).

Successful media plans judiciously allocate the marketing budget across available communication channels and determine their intensities over time. Here, “channels” can include various media classes (TV, radio, online, etc.), venues within them (particular radio stations, social media placements), or sub-channels (e.g., a consistent ad time slot on a cable network). In practice, media planning decisions often span corporate tiers and are made in a loosely coordinated fashion (Joshi & Giménez 2014). Critically, at the “strategic” tier, marketing goals are aligned with other business imperatives, and both overall marketing spend and its allocation across channels are determined. These decisions lie at the seams between marketing and other C-suite functions it “competes” with for budget, such as IT, sales, finance, and new ventures.

Despite their practical importance, formal analyses of such strategic decisions remain relatively siloed, restricted to product subclasses with idiosyncratic characteristics. For example, several models (discussed later in detail) apply primarily to commodities, whose sales dynamics hardly typify new products in general. Analogously, the literature on new product sales models has rarely extended to “strategic” media planning, focusing mainly on pragmatic planning decisions like temporal expenditure patterns. For tractability and data availability reasons, such models typically analyze a single (aggregate) marketing channel, and rarely address inter-channel interactions. By contrast, managerial insight and pragmatic strategies for new product media planning require alignment between the strategic and tactical stages.

Strategic media planning would be challenging enough if it merely needed to determine the channel(s) with greatest (marginal) bang-for-the-buck. Its notorious complexity stems from needing to also manage how channels interact, both with one another and with the process of consumer-to-consumer information transfer – that is, word-of-mouth (WOM) – so as to stimulate and guide demand over the product's lifecycle. Empirical research supports the common real-world media planning belief that two elements are critical to a sound plan: substitution and synergy between different channels (e.g., Naik & Raman 2003; Goldfarb & Tucker 2011a,b). Channels "substitute" for one another when, roughly speaking, the more that's invested in one channel, the lower the incremental benefit of spending in another. For example, consider a firm advertising through both a TV commercial and a Facebook campaign; additional resources invested in TV ads may increase frequency and/or reach, in turn enhancing customer awareness, thereby rendering investments in Facebook ads less impactful. On the other hand, several channels acting in concert may enhance demand in a way not possible were these same channels employed separately, resulting in "synergy". Consider the perpetually multitasking modern consumer: because over 20% of TV viewers appear to be chatting on Facebook or Twitter while watching (Dredge 2012), a firm might benefit by reuniting customers' divided attention, advertising on TV and Facebook simultaneously.

Substitution and synergy are often presumed to work in opposing directions. Yet little is known about why some channels behave substitutively in one setting, but synergistically in another. This is among the primary issues we address: under what conditions does either effect – substitution or synergy – prevail? To answer this and related questions requires an analysis of the *interplay* between channels and customer WOM. To that end, we formulate a multi-channel demand model of new product adoption, one in which a potential customer's purchase decision results from either innovation-seeking behavior (purchasing 'independently' of other customers) or imitation-seeking behavior (being 'influenced' by others who have already purchased). These behaviors are, in turn, jointly influenced by the firm's marketing activities.

Our account of demand dynamics builds upon and expands the Generalized Bass Model (GBM) framework (Bass et al. 1994). The GBM not only provides excellent fit to sales data for a wide range of product and service categories (Krishnan & Jain 2006) and modeling marketing mix effects (Bass et al. 2000), but has been used to study the dynamics of new product development in operations (Carrillo 2005, Wu et al. 2017). We extend the domain of applicability of the GBM framework by incorporating the impact of multiple channels on demand, via a general formulation accounting for the influence of both contemporaneous and past marketing activities. The resulting model, despite its generality, allows the derivation of *strategic*-level insights into (optimal) media planning, while taking account of its linkage with critical tactical details, e.g., how ad spending should be allocated over time with respect to memorability or stickiness. Importantly, "synergy" is not baked into the model (e.g., via explicit interaction terms), but arises naturally from the GBM setting, in a way not anticipated by prior literature.

Our model applies to media planning over near-to-moderate timeframes – consistent with accelerating technological clock-speed (Carrillo 2005) – and where the product's or service's features enable a dedicated consumer base (or local monopoly); for example, in "situations in which the firm enjoys a patent protection, a proprietary technology, or a dominant market share" (Mesak & Clark 1998). Consistent with the GBM framework, each customer's ("purchase") decision is to adopt or not, in the sense of a conversion. Such scenarios are common: durables with long inter-purchase times; where repeat purchases are unlikely (e.g., experiential media like books or films); or businesses fostering customer retention for a consistent revenue stream (i.e., a "contractual" adoption setting; Fader & Hardie 2010).

Deriving optimal allocation plans is notoriously thorny, as it requires searching large spaces of (temporal) allocations across multiple channels, often falling into the class of non-separable, non-convex, NP-hard optimization problems (Horst et al. 1995); solving them 'exactly' cannot be done faster, loosely speaking, than searching through all possibilities across all channels and time periods. Therefore, as our first step, we provide a novel decomposition enabling optimal media strategies to be examined in two separate strategic and tactical tiers. This allows the media planning task to be apportioned into optimal *strategic* decisions (heuristically optimal when time discounting is present), and optimal *tactical* ones, while capturing both instantaneous and lagged marketing impacts. The optimal tactical plan is characterized for any given decision at the strategic level, greatly simplifying the problem at the strategic tier while avoiding multi-channel dynamic programming and the curse of dimensionality due to the time dimension.

Owing to the reduced dimensionality of the strategic problem, commercial solvers can find optimal media plans when the number of channels under consideration is modest. For practical purposes, however, the ever-growing number of social media and internet advertising channels requires analyst foresight to prune the set of channels under consideration before applying such algorithms. Yet a more fundamental issue – qualitative as opposed to quantitative – concerns gleaning managerial insight: algorithmically-determined media plans emerge from a Black Box, providing allocations without any sense of substantive context. In actual applications, strategic media planning requires a holistic managerial view, allowing fine-tuned coordination with other high-level organizational functions. Our explicit focus is precisely these sorts of insights: those that provide a structured, qualitative overview to strategic media planning, rather than a purely algorithmic approach (although many of our results can be useful for algorithmic development as well, despite not being generated for that purpose).

The ensuing analysis takes as its starting point the sort of swim-lane analysis common in the media planning industry (Nichols 2013), entailing an allocated level of spending in each of the channels under consideration. Such levels can take many forms; they can: be zero; represent the firm's current allocation practice; be obtained via an (aforementioned) algorithmic approach; denote a minimum channel spending level (e.g., one that ensures a desired level of ultimate market penetration); etc. We first evaluate the prudency of this allocated investment, and then analyze the nature of channel interactions, among themselves and with the WOM process.

The ensuing analysis sheds light on a number of issues in managerial practice only partially resolved by extant approaches. Five novel insights, in particular, stand out:

1. **Leverage**. Managers commonly allocate budget to a channel relative to its own ability to influence demand – referred to as channel's "leverage". By contrast, our analysis suggests that channel spending should also be aligned with how much "free advertising help" is generated from customer WOM.

2. **Channel Typography**. We show how both leverage and momentum can be used to profile channels: momentum quantifies the "mass market penetration" the channel can generate and its "chasm-crossing ability" (Moore 1991; Chandrasekaran & Tellis 2011); drops in leverage lower both chasm-crossing ability and mass market adoption, in turn weakening the channel's profitability.

3. **Channel Deletion**. A proportional allocation rule suggests dropping a channel only when it is completely ineffective. Yet our analysis suggests eliminating channels whose effectiveness is "dominated" by others or have exhausted their momentum-generation capability, alleviating the curse of dimensionality and simplifying the media planning task.

4. **Channel Interactions**. The interaction of one channel with others is either synergistic (enhances others) or substitutive (detracts or no influence). Optimizing channel portfolios therefore requires determining which effect emerges, and how both leverage and momentum affect such interactions. We find complex but explicit guidelines governing these factors: medium-leverage channels (tend to) act synergistically under low-momentum conditions and substitutively otherwise; while high-leverage channels cannot benefit from synergy at all, making them uniformly and dominantly substitutive. Importantly, a channel can behave synergistically in one setting while substitutively in another.

5. **Costs.** Media planning is essentially a trade-off between expenditures and effectiveness. We find that a higher-cost channel is associated with: reduced maximal profitable expenditure; tightened conditions for increasing spend beyond its allocated level; and increases the "boost" it needs from other channels before it can interact substitutively with them.

Our overarching goal is to understand how multiple channels interact, among themselves and with customer WOM, and how to manage these interactions via media planning for a new product or service introduction. As echoed by Nichols (2013), such "...insight represents the holy grail in marketing – knowing precisely how all the moving parts of a campaign collectively drive sales and what happens when you adjust them". To that end, the reminder of the paper is organized as follows. After reviewing relevant literature in Section 2, we discuss decomposing the media planning problem into strategic and tactical tiers in both the discounted and undiscounted cases in Section 3, along with a detailed empirical example of media planning for camera sales via Bayesian estimation. The strategic level problem – including the interaction of channels with one another and with WOM – which informs our managerial insights, appears in Section 4. Specifically, we start by analyzing the impact of channel leverage and demand momentum in Section 4.1, including results allowing a reduction in the number of channels under consideration. A two-way channel typology is developed in Section 4.2, characterizing inter-channel interactions based on both leverage and momentum; and illustrated numerically in Section 4.3 (sensitivity analyses appear in Appendix E). Lastly, our overall findings are summarized in Section 5, along with suggestions for future research.

2 Literature Review: Synergies, Interactions, and Media Planning

Synergies have long been recognized as critical in empirical marketing. Prasad & Ring's (1976) field experiment revealed interactions among mix variables – price, promotion, TV advertising – as key determinants of brand share. Scanner panel data allowed explicit modeling of mix interactions based on household choices, e.g., Carpenter & Lehmann (1985) incorporated effects of advertising, price, brand name, and form, reporting consistent evidence of price interactions. Narayanan, Desiraju, & Chintagunta (2004) verified the impact of mix variables, sales force expenditures, and their interactions for three antihistamine medications, finding synergistic demand effects and emphasizing "the importance of investigating firms' optimal budget allocation." Similarly, Naik et al. (2005) documented the need to account for interactions among advertising and promotion in planning mix strategies.

Detailed data on media types allowed similar econometric analyses to be applied to far more granular, channel-specific information. For example, Naik & Peters (2009) examine both online (television, print, and radio) and offline (banners and search) advertising, focusing on synergies both within and across media types, and studying (as we do here) optimal overall budget and proportional allocation. [Readers are directed to their paper for a detailed review and effects summaries for the literatures on both media synergies and multimedia allocation.] Pauwels et al. (2016) take synergies – both online and cross-channel – as a marketing fact, and further study how brand familiarity affects their strength, verifying that within-online synergies are stronger than online-offline ones for familiar brands, but not for unfamiliar ones. Synergies have been implicated for key metrics besides demand: Srinivasan et al. (2009) document relations between stock market valuation and interactions between marketing variables (e.g., advertising, promotions, quality) and measures of product innovativeness. In alignment with these empirical findings, our model allows for synergistic interactions between channels, but we do not build synergy directly into the model (e.g., via explicit interaction terms), rather exploring synergy that arises naturally from the structure of the demand model, in a manner distinct from prior literature.

2.1 Media Planning with Multiple Channels

Literature on optimal resource allocation among multiple marketing channels, especially at the strategic level, is relatively limited, focusing mainly on frequently-purchased products, e.g., for which panel data may be available. This stands in contrast to new products, whereby a diffusion process describes "adoption" rather than "consumption", with an upper bound on market saturation (Meade & Islam 2006). Because such products are, by their nature, relatively unfamiliar to customers, their sales over time rely on the build-up of social influences, such as customer WOM, in conjunction with marketing activities, which in turn aids operations (Cui et al. 2018). Thus, core concepts like adoption, market saturation, and WOM are less relevant for existing (henceforth, "commodity") products, while being crucial for new ones.

Media planning models for such commodity products date back many decades. Gensch (1968), for example, distinguished among non/linear programming, marginal analysis, and dynamic programming approaches, while Basu & Batra (1988) formulated ADSPLIT, which interactively allocates a pre-specified promotional budget. Yet tractability dictated fairly stringent assumptions, particularly so regarding media synergies: for example, (demand) response to advertising in each channel was ordinarily assumed linear or concave; no interaction was allowed among the various channels or with sales; and data limitations required precluding such important impacts as those of past advertising spend (on sales) and customer WOM.

Some of these early restrictions have been since alleviated. With respect to channel interactions, specifically, a number of studies empirically show or implicitly assume that channels (at least partially) substitute for one another in influencing demand (e.g., Goldfarb & Tucker 2011a,b; Bergemann & Bonatti 2011). By contrast, Naik & Raman (2003) show empirically that two advertising channels can interact synergistically to enhance sales of a commodity product, and test this via a model that includes an explicit multiplicative interaction term for marketing efforts in two channels. Raman & Naik (2004) further accommodate the impact of uncertainty, while Naik & Peters (2009) consider a hierarchical extension to study interaction between online and offline channels.

These models helped analysts understand channel interactions, including those with sales (Prasad & Sethi 2009). Although they validate substitution and synergy effects in the context of commodity (i.e., not new-to-market) products, their collective results remain difficult to reconcile, e.g., why a particular effect is observed in one study but not another. Although many (e.g., Prasad & Sethi 2009) focus on temporal allocation through specific dynamic or stochastic programming problems and ingenious, model-specific analyses, here we provide a decomposition of strategic and temporal allocations and then focus on the strategic tier: chronicling the nature of multi-channel policies for classes of response functions that obviate the need for the full arsenal of such techniques.

2.2 Media Planning for New Products

That previous research has focused nearly exclusively on commodities limits its use for new product media planning, for several reasons. First, new product markets are characterized by saturation, and "the basic diffusion process is terminated by a decay of the number of new adopters" (Peres et al., 2010). Extant models for commodities do not (need to) capture saturation effects typifying new product trajectories. Second, saturation, along with WOM, leads to S-shaped demand (Little 1979, Feinberg 2001), as opposed to the concave response for products past their "ramp up" phase. Lastly, combined demand response to advertising in these models (i.e., those that incorporate channel interactions) entails the curse of dimensionality, requiring $2^n - 1$ estimated quantities for n channels, a particular impediment for new products, given their scant data histories.

Allocation of marketing funds for new products has been studied mainly at the tactical level to describe the customer adoption process driven by social influences and the firm's current and past marketing efforts. This literature, which spans a range of activities and goals, is vast; excellent reviews are provided by Mahajan et al. (1990), Meade & Islam (2006), Chandrasekaran & Tellis (2007), and Peres et al. (2010). The impact of marketing efforts in diffusion models is ordinarily modeled for a single advertising channel, where price may or may not be controlled for; see, for example, Dockner & Jorgensen (1988), Horsky & Simon (1983), Mesak & Clark (1998), as well as the dedicated review of Peres et al. (2010). In essence, this approach aggregates the effects of all relevant advertising channels into a single one, providing guidance on total expenditure for this single (aggregated) channel over time; how to optimally allocate across multiple channels needs to be tackled by the analyst *post hoc*.

The only models that, to our knowledge, address multiple advertising channels for new product introductions are those of Swami & Khairnar (2006) and Abedi et al. (2014). The former considers the impact of two advertising channels on demand (one for awareness, one for availability), deriving optimal advertising policies under a specific logarithmic demand form. The latter analyzes a multi-market, multi-channel setting with a general form of demand diffusion, but the resulting optimal control problem is too analytically complex to afford managerial insight on optimal resource allocation or channel interactions.

None of these models considers the "customer journey" (see Tueanrat et al. 2021 and Lemon & Verhoef (2016) for recent reviews), wherein consumers progressively pass from awareness through purchase to potential advocacy. This lacuna in the new products literature may arise because early stages of new product adoption correspond to initial phases of the customer journey, with greater media emphasis on informational content pre-consumption (Demmers et al. 2020). Indeed, Lemon & Verhoef (2016) lament that aggregate sales models (like GBM) "can account for traditional media, but they do not model the individual customer journey," a topic to which we return later in Sections 3.1 Specification of Advertising Impact Over Time and 5 Discussion and Future Research.

The tactical-level granular view in the diffusion literature needs to enact strong assumptions (e.g., number of channels; form of sales response) so that the resulting resource allocation control problem is amenable to deriving a full media plan, as is required for strategic decision making. Here, we seek this sort of "high level" managerial insight on media planning synergies, *without severe limitations on the channels or the nature of their interactions among themselves and with other critical marketing elements*. To achieve this, as discussed earlier, we build a general account of multiple channels' demand impact into the GBM framework, specifically, one incorporating past advertising spending. This extension alleviates a number of shortcomings (as discussed later; see also Fruchter & Van den Bulte 2011) while allowing for interactions among channels and with customer WOM. As illustrated in Section 3.1, certain properties of this framework make it particularly useful in strategic media planning, while maintaining the all-important linkage with tactical objectives. We note that the Bass Model's flexibility has afforded various distinct extensions to specific operational scenarios, including for short lifecycle products (Chung et al. 2012) and the interplay between new and remanufactured products (Debo et al. 2006).

To reiterate, among our main goals is to unify the contrasting observations in previous research regarding under which conditions mainly substitutive, vs. synergistic, interactions between channels might arise. To that end, we next develop the GBM-based model for multi-channel media planning.

3 Model Development & Decomposition

Here, we extend the GBM framework to account for sales dynamics over time. The GBM relates the purchase decision of a potential customer at any time to two factors: purchasing independently of other customers at the "innovation" rate, $p \geq 0$, or being influenced by those who have already purchased (e.g., by WOM) at a rate of $qF(t)$, with $q \geq 0$ the "imitation" rate and $F(t)$ the fraction of cumulative adoptions by t; $F(\cdot)$ can also be interpreted as market penetration or share. These two effects combine to yield the purchase rate for a prospective customer, $p + qF(t)$, which is in turn influenced by the firm's marketing activities (Bass et al. 1994) and which we instantiate for channel planning purposes.

To enable media decision-making, the firm must plan over a given time horizon, T. This horizon typically ranges from a few days to a few months in most media planning applications; it can cover part of the product lifecycle, but could be extended to the full cycle, e.g., for fast-paced, technologically innovative products. Because the horizon can start after product launch, a fraction $x_0 \in [0,1)$ of customers may have already adopted at time 0. During this media planning period, this initial share grows to a fraction $F(T)$ of the potential market of size m, resulting in $m(F(T) - x_0)$ total additional sales over the horizon.

The firm can influence channel-specific levels of marketing effort over a set of potential channels, denoted by R. These levels, for each channel $r \in R$ and time t, are given by functions $u_r(t) \geq 0$, which can incorporate the impacts of current or past marketing expenditures, as elaborated in Section 3.1. The dependence of u_r on r allows for investments in distinct marketing channels to influence demand with different structures or with varying degrees of effectiveness.

Consistent with the GBM framework, the firm's overall marketing effort at time t is the sum of the efforts in each channel, i.e., $\sum_{r \in R} u_r(t)$. This 'separable' form agrees with that of Bass et al. (1994) (to combine the effects of a single advertising channel and price promotions) and of Swami & Khairnar (2006) (to combine the effects of the two advertising channels). Also consistent with GBM, marketing effort modifies the baseline purchase rate of a new customer multiplicatively, resulting in the 'instantaneous' purchase rate $[p + qF(t)](1 + \sum_{r \in R} u_r(t))$. The fraction of total customers adopting the product by time t is therefore described by the following differential equation[1] for "demand diffusion":

$$\frac{dF(t)}{dt} = \big(1 - F(t)\big)[p + qF(t)][1 + \textstyle\sum_{r \in R} u_r(t)]; \qquad F(0) = x_0. \tag{1}$$

Note that (1) reduces to the standard Bass model when there is no investment in marketing (so that $u_r(t) \equiv 0$), and results in the following closed-form solution (as per Bass et al. 1994):

$$F(T) = G(\textstyle\sum_{r \in R} \Phi_r),$$

where $G(U) = \dfrac{\left(1+x_0\frac{q}{p}\right) - (1-x_0)e^{-(p+q)(T+U)}}{\left(1+x_0\frac{q}{p}\right) + (1-x_0)\frac{q}{p}e^{-(p+q)(T+U)}}$ and $\Phi_r = \int_0^T u_r(t)dt.$ (2)

The expression $U = \sum_{r \in R} \Phi_r$ captures cumulative marketing effort over the horizon, itself composed of cumulative efforts across investments in the available channels. $G(U)$ is increasing and S-shaped in U, i.e., it is convex before its inflection point $\frac{1}{2}(1 - \frac{p}{q})$ and concave after. Thus, increases in cumulative marketing effort accelerate sales only when market penetration is relatively low.

It is important to note that (1) is linear in $\{u_r(t)\}$, that is, there are no explicit interaction terms of the sort often adopted in studies of media channels (e.g., Eqs. 3 and 6 in the seminal article by Naik & Raman 2003) and operations (e.g., Kovach et al. 2018) to account for synergies. By contrast, in our framework synergies arise from the "native" GBM setting; moreover, as demonstrated shortly, if spending is altered in one channel, optimal investments in other channels can increase or decrease, a feature that is not "hard-wired" into the model via interaction terms, although these can be incorporated by the analyst, as illustrated in Appendix D.

Section 3.1 presents a fairly general form of $u_r(\cdot)$ that encompasses many common in the literature, and also illustrates that the Φ_r resulting from a given pattern of temporal investing would be a concave increasing function of the total spending in channel r. These preliminaries in place, in Section 3.2 we formulate an (undiscounted) optimization problem for Detailed Media Planning (DMP), jointly addressing the two types of decisions: how much should be invested in each channel at each point in time. This represents the best a marketer can reasonably achieve through careful media

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[1] Seasonal fluctuations and/or other exogenous temporal effects can be incorporated by introducing the function $w(t)$, which captures *relative* deviation from baseline (i.e., $w(t) = 0.5$ means adoption rate at t is expected to be 50% of the average rate) and replacing the last term in (1) with $[w(t) + \sum_{r \in R} u_r(t)]$. Note that $w(t) \geq 0$, and averages to 1 over the horizon.

planning. We demonstrate that the optimal marketing strategy decomposes into two parts when the effect of time discounting is not large (as in most practical applications with relatively short media planning horizons): finding the optimal strategic plan, and finding the optimal tactical plan (specifying optimal spending in each channel over time) when total budget for each channel is determined at the strategic level.

We show that the optimal tactical plan follows typically observed patterns of advertising spending over time. It also results in the cumulative effectiveness Φ_r to be concave increasing in the total (per capita) expenditure in that channel (and hence referred to as $\Phi_r(E_r)$), not only under a pre-determined investment plan, but also when temporal spending is made optimally at the tactical level. In other words, *although marketing decisions are made at two separate tiers, they are fundamentally coordinated so long as the optimal strategy is sought in each*. We further show in Section 3.3 that, even under explicit time-discounting, implementing the above policy (as is typically approached in practice) is very close to optimal. A summary of all notation used in this section and the remainder also appears in Appendix A.

3.1 Specification of Advertising Impact Over Time

We first provide a specification for $u_r(t)$ under relatively mild conditions, then describe the structure of cumulative marketing effort Φ_r as a function of total channel spending.

As in real media planning, the advertising plan in each channel $r \in R$ is updated at certain time points; that is, the time horizon is partitioned into K_r time blocks, of lengths τ_r (i.e., $T = K_r\tau_r$). Depending on how frequently the firm can update its investment plan in each channel, the blocking can be relatively 'crude', e.g., a week or longer, or more granular (a day or even an hour). This results in a piecewise-constant spending pattern in each channel and accords with industry practice, where advertisers sell blocks guaranteeing a certain number of impressions be delivered, without specifying the way they are delivered *within* the block. Furthermore, empirical estimation of demand response to advertising is typically carried out in discrete time, reflecting how advertising and sales data are often made available. Therefore, we denote the advertising spend in channel $r \in R$ over the block of time $[(k-1)\tau_r, k\tau_r)$ to be a_{rk}, with $k = 1, \ldots K_r$, so total spending on channel r over the horizon is $\sum_{k=1}^{K_r} a_{rk}$.

The investments in different marketing channels can influence not only the current demand, known as the "instant effect" of advertising, but also demand in the future, typically referred to as the "carry-over effect". Therefore, we consider a generalization of the Distributed Lagged Model of Koyck (1954), where investment in each channel r can impact current sales as well as sales up to

$s^r \geq 0$ blocks into the future. More formally, the effectiveness of expenditure a_{rk} in channel $r \in R$ during block k is given by $\phi_r^0(a_{rk})$, …, $\phi_r^{s^r}(a_{rk})$ for demand during blocks $k, \ldots, k+s^r$, where $\phi_r^i(\cdot)$ $(i = 0, \ldots, s^r)$ is a non-negative and smoothly differentiable function; $\phi_r^i(a_{rk})$ measures how much a non-adopter's baseline purchase rate increases during block $k+i$ for investment a_{rk} made i blocks ago. In accord with empirical research suggesting that demand response to advertising is positive with diminishing returns (e.g., Vakratsas 2005, Chae et al. 2019), we take $\{\phi_r^i(\cdot)\}$ to be strictly increasing and concave, but keep their structure general, to capture a variety advertising impact 'shapes' and our model applicable to disparate categories. If the media planning horizon starts sometime after launch (i.e., there are initial adoptions, $x_0 > 0$, and prior advertising spending), the overall effect of ad expenditures prior to $t = 0$ on demand in block k is set to γ_k, for $k = 1, \ldots, s^r$ (and, if $x_0 = 0$ and there is no pre-launch advertising, $\gamma_k = 0$). Consequently, the total impact of current and past marketing efforts in channel r during block k is given by:

$$u_r(t) = \gamma_k 1(k \leq s^r) + \sum_{i=0}^{\min\{k-1, s_r\}} \phi_r^i(a_{r,k-i}) \qquad \text{for } t \in [(k-1)\tau_r, k\tau_r) \text{ and } k = 1, \ldots, k^r \tag{3}$$

The structure of the marketing effort function $u_r(\cdot)$ can capture a variety of forms to incorporate instant and carry-over effects for each channel r, via specifying the functional form of $\phi_r^i(\cdot)$; for example:

- No carry-over effect: Set $\phi_r^i(\cdot) \equiv 0$ for $i \geq 1$. This form is similar to models in Dockner & Jorgensen (1988), Horsky & Simon (1983), and Mesak & Clark (1998).
- Exponentially decaying impact of past advertising expenses: Set $\phi_r^i(a_{rk}) = \delta^i a_{rk}^\rho$, with $\delta > 0$ and $s_r = \infty$; the exponent term $(0 < \rho \leq 1)$ can capture diminishing returns to advertising. In this case, the total impact of marketing effort in channel r up to block k is often referred to as the "stock of advertising goodwill," $u_r(t) = S_r(k)$ for $t \in [(k-1)\tau_r, k\tau_r)$ (Nerlove & Arrow 1962). The term $S_r(0)$ may be non-zero, capturing the impact of advertising spending prior to time 0, particularly if $t = 0$ is post-launch. For $k \geq 1$, we can write $S_r(k) = \gamma_k + \sum_{i=0}^{k-1} \phi_r^i(a_{r,k-i}) = \delta^k S_r(0) + \sum_{i=0}^{k-1} \delta^i a_{r,k-i}^\rho$: goodwill stock increases with new marketing effort, but declines at the "forgetting rate" of $1-\delta$, that is, $S_r(k) - S_r(k-1) = a_{rk}^\rho - (1-\delta)S(k-1)$, reducing to the model of Nerlove & Arrow (1962) when $\rho = 1$.
- Advertising memorability causes effectiveness to decline, not immediately, but after $z > 0$ periods: Set $s_r = \infty$ and consider $\delta > 0$. Then set $\phi_r^i(a_{rk}) = a_{rk}^\rho$ for $i = 1, \ldots z$, but $\phi_r^i(a_{rk}) = \delta^{i-z} a_{rk}^\rho$ for $i \geq z+1$ (again, $0 < \rho \leq 1$ captures diminishing returns). The total impact of marketing effort in channel r up to block k (i.e., over time interval $[(k-1)\tau_r, k\tau_r)$) resembles the stock of advertising goodwill $u_r(t) = S_r(k) = \delta^{max(k-z,0)} S_r(0) + \sum_{i=0}^{k-1} \phi_r^i(a_{r,k-i})$, and has the property that $S(k) - S(k-1) = a_{rk}^\rho - (1-\delta)S(k-z)$; i.e., goodwill stock increases immediately with new marketing effort, but declines at the "forgetting

rate” of $1-\delta$ after z periods. This model reduces to that of Aravindakshan & Naik (2015) when $\rho = 0.5$.

Despite the flexibility of the $u_r(\cdot)$ function, it deviates somewhat from the original form used in GBM in how carry-over is captured. In GBM, the percentage change in advertising at any point of time captures the impact of past advertising, for which “a behavioral rationale has never been articulated” and can lead to questionable optimal marketing strategies (Fruchter & Van den Bulte 2011). Therefore, we alter the form of carry-over in our tactical plan while retaining the general structure of GBM in how sales evolve over time, as expressed in (1).

To conclude, based on the definition of $u_r(t)$ above, cumulative marketing effort Φ_r over the planning horizon can be summarized as follows, and as a function of the total expenditure in channel r *per capita*, E_r (with $\Phi_0 = \tau_r \sum_{k=1}^{\min\{k,s_r\}} \gamma_k$ a constant capturing the effect of advertising prior to time 0):

$$\Phi_r(E_r) = \tau_r \sum_{k=1}^{K_r} \sum_{i=0}^{\min\{k-1,s_r\}} \phi_r^i\big(a_{r,k-i}\big) + \Phi_0, \quad \text{with } E_r = \frac{1}{m} \sum_{k=1}^{K_r} a_{rk}. \tag{4}$$

Note that, for ease of expression, a_{rk} represents dollar spending in each channel over each block, while E_r is dollar spending *per target customer*, adding up to total expenditure of mE_r in channel r.

When a pre-determined temporal investment plan is pursued, $\Phi_r(E_r)$ becomes concave increasing and smoothly differentiable in E_r. Specifically, consider a tactical plan $\{\hat{a}_{rk}\}$ $(k = 1, \dots K_r)$ to invest a total of \$1 in channel r (that is, $\sum_{k=1}^{K_r} \hat{a}_{rk} = 1$). If the total budget mE_r is spent in this channel proportional to the given tactical plan (i.e., investing $mE_r\hat{a}_{rk}$ over block k), it is easy to see that the cumulative effectiveness $\Phi_r(E_r) = \tau_r \sum_{k=1}^{K_r} \sum_{i=0}^{\min\{k-1,s_r\}} \phi_r^i\big(mE_r\, \hat{a}_{r,k-i}\big) + \Phi_0$ possesses the above characteristics as a function of E_r, based on the characteristics of $\phi_r^i(.)$ functions. We will illustrate a similar result when the optimal tactical plan is derives and analyzed in Section 3.2.

As will be seen later, the Φ_r functions link tactical and strategic decisions, whereas the ϕ_r^i can be viewed as potential linkages between tactical allocation and customer experience management. Advancements in digital advertising allow for a highly granular record of multiple customer touchpoints across the purchase funnel via multiple channels. However firm-level budget allocation often utilizes more aggregate measures to enable proper alignment with other business functions. Attribution models, such as introduced in Li & Kannan (2014) and Anderl et al. (2016), aggregate the customer experience at multiple touchpoints in different channels, and measure the effectiveness of each channel in generating sales (similar to ϕ_r^i), which can be used in tactical planning. Nevertheless, such models are mainly applied to established commodities, as opposed to new products, as discussed in Section 5.

3.2 Decomposing Optimal Strategic & Tactical Decisions: The Undiscounted Case

In the Detailed Media Planning (DMP) problem, the firm exerts full control over its marketing strategy and maximizes net profit over the horizon by deciding on expenditures in each marketing channel during each block (i.e., $a_{rk},\ r \in R,\ k = 1,\dots,K_r$). Since the media planning horizon in most applications is short relative to currency depreciation, the effect of discounting profits is usually minimal. Therefore, in our first formulation, we suppress an explicit discount factor, revisiting this case in Section 3.3. The "base" unit price, net of non-marketing variable costs, is set at P. Because the problem is a net profit maximization, no fixed budget need be pre-specified, although relevant constraints can be readily incorporated. The resulting optimization problem ("DMP") can be stated as follows:

$$\max_{a_{rk}, r \in R, k=1,\dots,K_r} m\, P(F(T) - x_0) - \sum_{r \in R} \sum_{k=1}^{K_r} a_{rk} \qquad \text{(DMP)}$$

Subject to: $\frac{dF(t)}{dt} = \big(1 - F(t)\big)[p + qF(t)][1 + u_r(t)]$; $\qquad F(0) = x_0$

$u_r(t) = \gamma_k\, 1(k \le s^r) + \sum_{i=0}^{\min\{k-1, s_r\}} \phi_r^i\big(a_{r,k-i}\big)$ for $t \in [(k-1)\tau_r, k\tau_r)$ and $k = 1,\dots,k^r$

$0 \le a_{rk} \le b_r,\quad \forall r \in R, k = 1,\dots,K_r.$

In practical marketing expenditure allocations, tactical planners have first-hand knowledge of the upper bound b_r for the spend on channel r; that is, they understand the available inventory of effective and appropriately priced advertising vehicles for a given channel. In the absence of upper bounds, b_r's can be set to ∞. Note that even though a fixed base price is assumed, price promotions can still be incorporated as part of the firm's marketing activities, i.e., as a separate marketing channel with its own cost and effectiveness function. Price P can also include the salvage value of the product at the end of the horizon[2].

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[2] In the GBM framework, Frutcher & Van den Bulte (2011) consider two specifications for product salvage value after horizon T. In the first, they consider the new product to still be available after time T and calculate the discounted value of remaining adoptions. In the second, they consider the product to evolve into a second generation, and account for the discounted profit if adopters form a potential market for that (2nd-gen) product when launched at time T. In both cases,

DMP is a highly non-linear NP-hard optimization: solving it for more than a few time periods and channels with typical non-linear solvers would be prohibitive, and the results could be strongly suboptimal. Solving DMP full-force for a large number of channels and time periods is highly impractical.

However, a key property of (2) is that *total market penetration over the decision horizon depends on the efforts in each channel only through the firm's cumulative marketing effort in that channel*, Φ_r, and not on the specific trajectory of effort, $u_r(t)$, $t \in [0, T]$. That is, any trajectory $u_r(t)$ (for each channel $r \in R$) leading to total cumulative effort Φ_r in that channel results in the same outcome, so long as the cumulative effort in *other* channels does not change. This affords a decomposition of the DMP problem into two tiers: (a) the *strategic* problem of how to achieve the best cumulative marketing effort in each channel (and in turn the best profit); and (b) the *tactical* problem of using available resources to achieve the required level of cumulative marketing effort for that channel. This decomposition property of the GBM-type models makes them particularly tractable and transparent for strategic media planning, a quality lacking for a wide spectrum of other frameworks in the new products literature.

We show in **Error! Reference source not found.**in Appendix B that the tactical expenditure plan for each channel over time can be represented in a much simpler form once the total expenditure in each of the channels are decided. Consequently, if E_r dollars are pre-allocated to channel $r \in R$ per capita, the optimal temporal investment plan for DMP maximizes cumulative marketing effort in channel r over the horizon, and this is independent of the plan for all the other channels and the WOM process. So, the Tactical Planning Problem (TPP) can be stated as follows:

$$\Phi_r(E_r) = \max_{\substack{0 \le a_{rk} \le b_r \\ k=1,\dots,K_r}} \tau_r \sum_{k=1}^{K_r} \sum_{i=0}^{\min\{k-1, s_r\}} \phi_r^i\left(a_{r,k-i}\right) + \Phi_r^0$$

(TPP)

$$\text{Subject to: } \sum_{k=0}^{K_r} a_{rk} \le mE_r.$$

If $\Phi_r(E_r)$ is determined by TPP, the DMP problem reduces to the following strategic Marketing Expenditure Allocation (MEA) problem, whose resolution is a primary focus in this paper:

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the authors show that total salvage value is linear in $F(T)$. Here, salvage value can be accounted for in the parameter P multiplying $F(T)$.

$$\max_{E_r} \Pi = m\,[P\;G(\sum_{r\in R}\;\Phi_r(E_r) - \mathrm{x}_0) - \sum_{r\in R}\;E_r] \qquad \text{(MEA)}$$

Subject to: $E_r \in [E_r^0, \bar{E}_r] \quad r \in R,$

where $\bar{E}_r = K_r b_r / m$, and $0 \le E_r^0 < \bar{E}_r$ represents the minimum "feasible" investment in channel $r \in R$ over the time horizon. Because there are often long-term minimal spend agreements in place for specific channels to ensure some advertising capacity is reserved, estimates of E_r^0 are available to the media planner (in their absence, the lower bound can be set to 0). Any budget constraint added to DMP would carry over directly to MEA. This two-tiered problem structure effectively 'disentangles' the decisions at the strategic and tactical levels.

Error! Reference source not found.in Appendix B further shows that *the resulting optimal cumulative effort, $\Phi_r(E_r)$, from solving TPP is concave and non-decreasing in E_r, and can be specified independently of the other channels and the WOM process. In addition, the optimal plan of investing in channel r is non-increasing over time, i.e., a_{rk}^* is non-increasing in k.* This pattern (for DMP and TPP) agrees with Fruchter & Van den Bulte's (2011) empirical analysis that "strongly suggests that the optimal strategy in real markets is likely to involve decreasing advertising over time, especially late in the diffusion process".

3.3 Decomposing Optimal Strategic & Tactical Decisions: The Discounted Case

Even though media planning horizons are typically brief, if a firm urgently desires sales earlier on, a version of the DMP problem with discount rate θ can be formulated as follows:

$$\max_{E_r\ r\in R}\ \max_{a_{rk}, r\in R, k=1,\dots,K_r} m\left[P\int_0^T \frac{dF(t)}{dt}e^{-\theta t}dt - \sum_{r\in R} E_r\right] \qquad (\mathrm{DMP-DISC})$$

Subject to: $\frac{dF(t)}{dt} = \big(1-F(t)\big)[p+qF(t)][1+u_r(t)]; \qquad F(0) = x_0$

$u_r(t) = \gamma_k\, 1(k \le s^r) + \sum_{i=0}^{\min\{k-1,s_r\}} \phi_r^i\big(a_{r,k-i}\big)$ for $t \in [(k-1)\tau_r, k\tau_r)$ & $k = 1,\dots,k^r$

$\sum_{k=0}^{K_r} a_{rk} = mE_r \quad \forall r \in R$

$0 \le a_{rk} \le b_r, \quad \forall r \in R, k = 1,\dots,K_r.$

In the discounted profit maximization, not only is the *volume* of sales important to the firm, but how they are obtained over time. Thus, the disentanglement of strategic and tactical decisions of Section 3.2 is no longer practically achievable, so that strategic budgeting and interaction between channels (and tactical implementation of the media plan) are interlinked at each point of time, and a division of tasks between organizational tiers is no longer possible. Because this greatly complicates media plan implementation, firms typically resort to a "rule-of-thumb" or "heuristic" approach. This is important because strategic decisions need to align with other business imperatives such as Finance, Operations, IT, etc., that cannot be readily captured in the firm's media plan optimization. Given that the optimal advertising spending in the undiscounted DMP problem in Section 3.2 possesses this modularity characteristic, we show that employing the optimal strategic policy from the undiscounted case would generate sales patterns that are very close to optimal even when sales are discounted over time.

In doing so, a few formalities are in order. With a little abuse of notation, let $F[t, \{a_r|E_r\}]$ represent the market penetration by time t when the piecewise-constant advertising policy of $a_r(t)$ for $t \in [0, T]$ and $r \in R$ is used, which sums up to E_r per capita for channel r over the planning horizon. Also let $\Pi^D\{a_r|E_r\}$ and $\Pi^U\{a_r|E_r\}$ represent the discounted profit (from DMP-DISC) and the undiscounted profit (from DMP) respectively when the above advertising policy is employed. In these definitions, note that the tactical spending plan of $\{a_r\}$ is conditional on the budget allocated to each channel $\{E_r\}$, so that $\int_0^T a_r(t)dt = mE_r$. Therefore, if the budget allocated to a channel changes, the policy $a_r(.)$ would need to be adjusted to match the total budget. Further, let $\{a_r^D|E_r\}$ and $\{a_r^U|E_r\}$ represent the optimal spending plan in each of the channels over time based on the discounted (DMP-DISC) and undiscounted (DMP) problems, when the total budget spending in each channel r is constrained to be E_r. Lastly, $\{E_r^D\}$ and $\{E_r^U\}$ represent the optimal budget allocation for the discounted and undiscounted problems, respectively.

With the above definitions, the goal of the firm is to set both strategic and tactical plans to maximize profit, $\Pi^D\{a_r^D|E_r^D\}$. To facilitate this, the firm can follow two "rule-of-thumb" strategies. One is to fully utilize the strategic and tactical decisions derived from the undiscounted problem that yields a profit of $\Pi^D\{a_r^U|E_r^U\}$. Second, the firm could do somewhat better, and try to allocate total budget in each channel (which is a more complex part of DMP-DISC problem) based on the undiscounted problem, but to carry out the tactical plan based on the best strategy from DMP-DISC when the allocated budgets are taken as given. This second strategy would obtain a profit of $\Pi^D\{a_r^D|E_r^U\}$.

Even though optimizing the tactical plan of investment based on the discounted problem ($\{a_r^D|E_r^U\}$) is more profitable, it is less practical, since tactical plan of investments across channels are interlinked. We show in Appendix B (**Error! Reference source not found.**) that the profit obtained from using these two strategies are close to that of the optimal one, with the following worst-case error bound on profit:

$$0 \leq \quad \Pi^D\{a_r^D|E_r^D\} - \Pi^D\{a_r^D|E_r^U\} \quad \leq \quad \Pi^D\{a_r^D|E_r^D\} - \Pi^D\{a_r^U|E_r^U\}$$
$$\leq mP\left[(1 - e^{-\theta T})\left(F\left[T, \{a_r^U|E_r^U\}\right] - x_0\right) - \theta \int_0^T \left(F\left[t, \{a_r^U|E_r^U\}\right] - x_0\right)e^{-\theta t}dt\right] \qquad (5)$$

The total discounted share for the undiscounted policy has a similar worst-case error bound:

$$0 \leq \int_0^T \left(\frac{dF[t, \{a_r^D|E_r^U\}]}{dt} - \frac{dF[t, \{a_r^U|E_r^U\}]}{dt}\right)e^{-\theta t}dt$$
$$\leq \left(1 - e^{-\theta T}\right)(F[T, \{a_r^U|E_r^U\}] - x_0) - \theta \int_0^T (F[t, \{a_r^U|E_r^U\}] - x_0)e^{-\theta t}dt.$$

The above error bounds can be easily computed (numerically) from the trajectory of ad spending based on the undiscounted policy. Also note that these bounds govern the "theoretical" worst case scenario, but as illustrated in Section 3.4, the loss in profitability and discounted market share is much smaller "practically". The existence of such error bounds is particularly useful, as guarantees for many similar optimization problems are not available. In short, the firm can implement the undiscounted optimal strategic plan as described MEA confident about sacrificing neither profit nor total discounted sales.

3.4 Empirical Estimation & Illustration

We illustrate our model and method using real-world data on camera sales. The data stem from Gray's Photography[3], which at the time of collection had a new store with a "local monopoly" in a major North American city, but closed its brick-and-mortar operations in the mid-2000s. Both sales and ad expenditure data were available over 28 months, from which model parameters are estimated and subsequently used for media planning over a 12-month horizon. Using these results, we first illustrate how the functional form of the cumulative effectiveness function, $\Phi_r(.)$, can be obtained from past data, and investigate the effect of discounting on both tactical and strategic decisions. We revisit this case later in Section 4.3 for further illustration of media planning and managerial implications.

3.4.1 Data Description & Estimation

The data cover the period Sept 2003 to Dec 2005 and include total retail sales, as well as advertising spending in two channels. Because adoption data comprise dollar sales rather than units

[3] A pseudonym is used, as the actual firm, to whom we are indebted for these data, did not wish to be named.

purchased, we set normalized unit price to $P = \$1$. Advertising data for the first channel is dollars spent on free-standing inserts (FSI) in flyers, while the second pertains to aggregate expenditure on radio. Because advertising and sales data are provided monthly, we set $\tau_r = 1\ month$. Throughout, monetary figures are in units of \$1000.

We capture the form of influence of advertising spending a_{rk} on demand for each channel $r = 1,2$ (over time $k = 1,2,\ldots$) via the Nerlove-Arrow goodwill stock discussed in Section 3.1. In summary, we consider that advertising over each month can influence both current and (all) future sales, and set $\phi_r^i(a) = \alpha_r\, \delta_r^i\, a^{\rho_r}$ for $i = 0,1,\ldots$ and $r = 1,2$. This results in goodwill stock $u_r(t) = S_r(k) = \alpha_r \left(\delta_r^k S_r(0) + \sum_{i=0}^{k-1} \delta_r^i a_{r,k-i}^{\rho_r}\right)$ over time block (month) k, i.e., that coincides with $t \in [k-1, k)$.

The model's many parameters are estimated using Bayesian techniques, laid out fully in Online Appendix C: Bass-specific parameters p, q and m; channel-specific ad effectiveness (α_r), diminishing returns exponent (ρ_r), and ad remembering rate (δ_r) for both channels; and, because sales and ad data are made available starting a few months after the initial product offering, we also estimate x_0, $S_1(0)$ and $S_2(0)$. Online Appendix C also specifies: all estimated values, Highest Density Regions, and posterior statistics; all (diffuse) priors; all 1- and 2-dimensional "slices" of the joint 12-parameter posterior (showing all marginal densities were nearly unimodal and parameters were estimated relatively independently of one another); that all squared parameter correlations were well below 0.2; and the log-SE histogram was essentially bell-shaped. Moreover, multiple diagnostics indicated convergence, with all parameters having Effective Sample Size over 4000. For convenience, parameter means are given by:

Param.	Mean	Param.	Mean	Param.	Mean
p	0.0391	α_1	0.0124	δ_1	0.4038
q	0.5445	α_2	0.0081	δ_2	0.3602
m	6738.9	ρ_1	0.3777	$S_1(0)$	1.5781
x_0	0.0808	ρ_2	0.2722	$S_2(0)$	1.4068

3.4.2 Post-Estimation Media Planning Decisions

Subsequent to the 28-month observation period, we consider a 12-month window ("periods 29 to 40") for new media planning decisions. Starting at the estimated initial market penetration of $x_0 = 8.08\%$, the company has achieved a share of $F(28) = 44.22\%$ after the 28-month period. Also, given the ad spend in each channel over the 28 months, the goodwill stock has reached $S_1(28) = 2.742$ and $S_2(28) = 2.911$. The initial values of market penetration and goodwill for the media planning horizon are taken as the corresponding values after month 28; however, innovation and imitation rates, market potential, and channel-specific parameters remain at their estimated values. We still consider the possibility of updating the advertising plan once every month (indicating a piecewise-constant advertising plan), leading to $\tau_r = 1\, month$ and $K_r = 12$ for $r = 1,2$. We consider the firm to allocate a minimum spending of $E_1^0 = E_1^2 = 0.0002$ per target customer, which leads to total allocated spending of $mE_1^0 = mE_2^0 = \$1{,}348$. This allocated spend level would be equivalent to average spending of \$112 per month in each channel, considerably lower than average monthly spend of \$2,387 in each channel over the 28-month initial sales period. In our analysis, we use a moderately high annual discount rate of $\theta = 15\%$.

The best a media planner can do can be found by solving DMP-DISC (Section 3.3). This requires finding the optimal level of ad spending in each channel and each month – a dynamic optimization with 24 decision variables. We solve this problem in Matlab; the optimal tactical advertising strategy over time is depicted in Figure 1; total spend in each channel and total discounted profit are summarized in Table 1.

Even though the size of the undiscounted problem is not very large, high-grade commercial solvers, such as in Matlab, still struggle to locate a stable optimum. Because the problem is highly non-linear, the optimization needs to be repeated many times with different sets of initial solutions (the approach taken here), which is time-consuming even with only two channels. This makes sensitivity analysis or adjustment based on other unforeseeable factors very difficult, which in turn highlights the importance of structural results such as those derived earlier and subsequently in Section 4.

Advertising Plan	Total Discounted Profit (Π^D)	Total Spend in Channel 1 (mE_1)	Total Spend in Channel 2 (mE_2)
Optimal DISC-DMP plan $\{\boldsymbol{a}_r^D \mid \boldsymbol{E}_r^D\}$	\$ 1,505,493	\$ 195,480	\$ 42,472
Optimal tactical plan given Undiscounted allocation $\{\boldsymbol{a}_r^D \mid \boldsymbol{E}_r^U\}$	\$ 1,504,774	\$ 214,318	\$ 45,970
Using Optimal undiscounted DMP plan $\{\boldsymbol{a}_r^U \mid \boldsymbol{E}_r^U\}$	\$ 1,504,345	\$ 214,318	\$ 45,970

Table 1: Summary of optimal and heuristic media planning strategies.

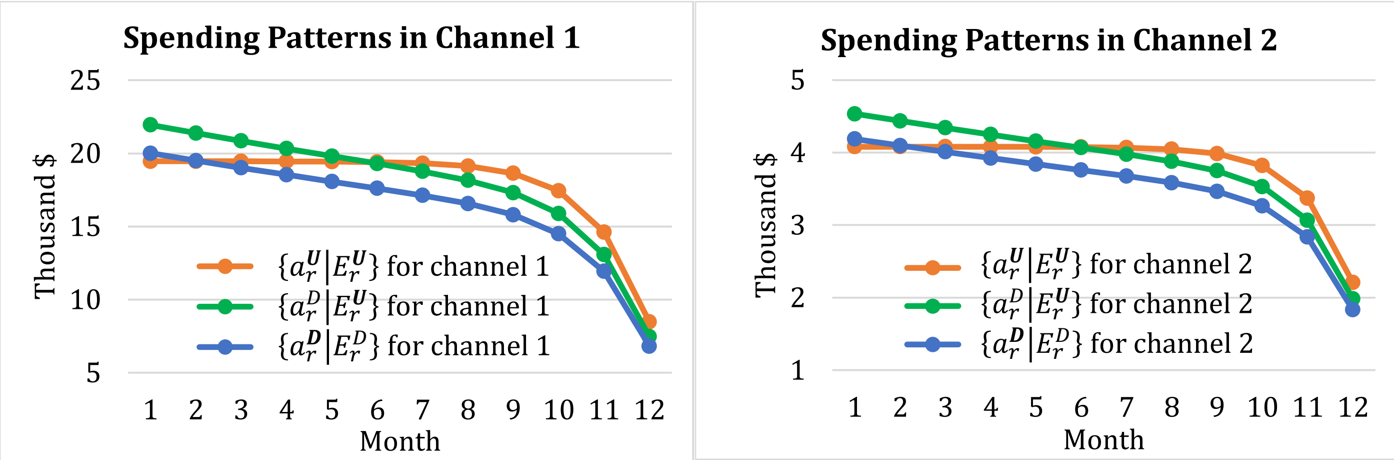

Figure 1: Comparison of spending patterns in each channel under different scenarios

To investigate the two heuristics of Section 3.3, we solve the undiscounted MEA problem, finding total undiscounted profit to be $\Pi^U[\{a_r^U|E_r^U\}] = \$1{,}633{,}607$, which translates to discounted profit $\Pi^D[\{a_r^U|E_r^U\}]$ (naturally lower than its undiscounted counterpart), and total channel spending listed in the third row of Table 1. In the second row of Table 1, the tactical discounted spending plan is optimized according to DMP-DISC, assuming the undiscounted budget allocation E_1^U and E_2^U as given.

Table 1 reveals that the discounted profit for optimal discounted solution ($\{\boldsymbol{a}_r^{\boldsymbol{D}}|\boldsymbol{E}_r^{\boldsymbol{D}}\}$) and the two undiscounted heuristics ($\{\boldsymbol{a}_r^{\boldsymbol{D}}|\boldsymbol{E}_r^{\boldsymbol{U}}\}$ and $\{\boldsymbol{a}_r^{\boldsymbol{U}}|\boldsymbol{E}_r^{\boldsymbol{U}}\}$) are very close, resulting in only \$719 (0.048%) and \$1,148 (0.076%) drops in discounted profit respectively. In contrast, the profit error bound (5) is computed numerically to be \$11,256 (0.748%): even though the guaranteed error bound is relatively small, the practical profit loss between discounted and undiscounted policies turn out dramatically smaller than the theoretical one. The optimal total spend in each of the channels is comparable as well, with the undiscounted case suggesting 9.39% higher expenditure; allocation of these total spends are also carried out quite similarly in the optimal discounted and the $\{\boldsymbol{a}_r^{\boldsymbol{D}}|\boldsymbol{E}_r^{\boldsymbol{U}}\}$ heuristic, as depicted in Figure 1. These spending patterns follow a declining pattern in advertising, as predicted in Section 3.2.

The above analysis suggests that not much is lost if discounting is ignored at both the strategic and tactical levels (especially in terms of profitability), despite having a fairly high (15%) discount rate as in this example. However, ignoring discounting remarkably simplifies the complexity of the problem and allows more focused decision-making at each tier while maintaining the linkage between strategic and tactical decisions. This linkage is established through the cumulative effectiveness function $\Phi_r(E_r)$ for each channel. With the exponentially decaying form of $\phi_r^i(.)$

functions, the tactical plan from TPP problem can be solved in closed form, as follows, for given levels of the budget E_r for each channel:

$$a_{rk}^* = \frac{\left(1-\delta_r^{K_r+1-k}\right)^{\frac{1}{1-\rho_r}}}{\sum_{j=1}^{K_r}\left(1-\delta_r^{j}\right)^{\frac{1}{1-\rho_r}}} E_r \qquad and \qquad \Phi_r(E_r) = \beta_r E_r^{\rho_r} + \Phi_r^0 ,$$

$$with \quad \beta_r = \frac{\tau_r \alpha_r}{1-\delta_r}\left[\sum_{j=1}^{K_r}\left(1-\delta_r^{j}\right)^{\frac{1}{1-\rho_r}}\right]^{1-\rho_r} \qquad and \;\; \Phi_r^0 = \tau_r \alpha_r \frac{\delta_r - \delta_r^{K_r+1}}{1-\delta_r} S_r(0).$$

The above expressions confirm that the $\Phi_r(.)$ function has a power form when the $\phi_r^i(.)$ also do. For this empirical example, the constants corresponding to the 12-month media planning period would turn out to be $\beta_1 = 0.0926,\ \beta_2 = 0.0738,\ \Phi_1^0 = 0.0230,\ \Phi_2^0 = 0.0133$.

3.5 Summary of Model Decomposition & Next Steps in Strategic Decision-Making

We note that the derivations in this section allow us to transform and simplify the DMP problem optimally, or simplify the DMP-DISC problem near-optimally, into the strategic MEA problem. As detailed in their review of methodology specific to TV advertising, Singh et al. (2018) highlight the computational difficulties in solving media planning problems. In contrast, MEA is squarely focused on the strategic aspect of marketing allocation, which requires only determining how much should be allocated to each individual channel (E_r). Therefore, solving this problem is notably simpler (with $|R|$ decision variables), compared to either of the optimal control problems DMP or DMP-DISC (with $\sum_{r\in R} K_r$ decision variables), which focus on extracting specific time-paths to address tactical details. Therefore, most non-linear solvers would find the MEA problem far easier to solve quickly and accurately than either DMP or DMP-DISC.

The MEA problem can be regarded as strategic budget allocation across channels, incorporating how channels interact with one another and with the WOM process, while foreseeing that the budget allocated to each channel would be optimally spent. This structure further aligns with the practical implementation of media planning decisions (as referred to in Section 1). At the "strategic" tier, C-suite executive(s) align marketing with other business goals, and decide on both overall marketing spend and its allocation across channels. The decomposition of strategic and tactical decisions essentially means that the interaction of channels with one another and WOM needs to be captured mainly at the strategic level, without direct/constant involvement of lower-tier marketing managers who cannot assess how their efforts may impact overall firm profitability. Nevertheless, their feedback would be critical to assist C-level executives to properly capture and measure the effectiveness of each channel (i.e., $\Phi_r(.)$ functions).

The impact of lower-tier managerial effort is typically assessed through various observable metrics (other than profit), such as the number of impressions, click-through rates, etc., which can more readily be tactically accounted for by maximizing the overall effectiveness function, given an available budget. This practice highlights that multiple decision makers are involved in the overall implementation of a media plan who potentially pursue different objectives. However, *the decomposed structure shows that the incentives of the different decision-makers are still aligned to enhance profitability for the firm as a whole*, so long as channel interactions (with one another and with WOM) is properly managed at the strategic level, and the tactical planners optimally spend the allocated budget.

The main factor linking the strategic and tactical tiers are the functions $\{\Phi_r(E_r)\}$. In practice, each such function reflects the cumulative effort in channel r when the total budget E_r is spent either according to a "pre-determined marketing strategy", or "optimally" (based on TPP). The specific functional form of $\Phi_r(\cdot)$ would of course depend on how advertising impacts current and future sales at any point of time. For a given application and as illustrated in 3.4.1 Data Description & Estimation, the sales model from (1) and (3) can be fitted to data on past sales and advertising expenditures over time to estimate the functional form of the $\phi_r^i(.)$. Depending on using a pre-determined temporal marketing strategy or the optimal one, the $\Phi_r(.)$ functions can be derived/computed accordingly. The function $\Phi_r(E_r)$ from the TPP problem cannot generally be characterized in closed form unless more is known about the functional form of $\{\phi_r^i\}$ (as seen for example in 03.4.2 Post-Estimation Media Planning Decisions). Nevertheless, TPP is an instance of a concave Knapsack problem, and if no closed form solution is available, many algorithmic routines are available to quickly calculate $\Phi_r(.)$ and its numerical derivative, $\Phi_r'(.)$, with guaranteed optimality (e.g., Zipkin 1980, Moré & Vavasis 1990).

Despite not requiring the full arsenal of dynamic programming methods, the MEA Problem does present a number of technical hurdles. Specifically, it is a non-separable, non-convex non-linear program, an NP-hard global optimization (Horst et al. 1995). In practical terms, this means any algorithm optimizing media schedules in MEA would have computation time increasing exponentially in the number of channels (decision variables), and be feasible only if the dimensionality of MEA is kept small. In such cases, managerial guidelines narrowing down the set of relevant channels *a priori* are valuable in obtaining a good solution quickly enough to be useful in pragmatic media planning. While a world with such limited channel choice was once the norm, it no longer is: the number of available digital channels is both large and growing. Even if the "optimal" media plan can be realistically extracted from MEA, it must still make sense to marketing managers: strategic media decisions often involve large resource commitments unlikely to be delegated entirely to some algorithmic "black box". For these reasons, we focus on obtaining managerial insights beyond solely developing a new algorithmic approach to the MEA problem.

In the remainder of this paper, we derive guidelines regarding the role of each channel in the strategic media plan, and their interactions with both other channels and WOM. Some of the results help reduce the number of channels under consideration *a priori*, enhancing the usability of existing algorithms. We also outline conditions ensuring substitutive or synergistic channels interactions that help demystify these algorithmic black-box outputs and enable informed adjustments as needed, based on managerial considerations beyond the scope of the optimization problem.

4 Channel Interaction Typologies Based on Leverage and Momentum

In this section, we study how channels impact demand adoption and the WOM process, as well as how they interact with one another. As discussed earlier, we assume throughout that the firm has allocated (or committed to spend) $E_s^0 \geq 0$ on each channel $s \in R$ (of course, E_s^0 can be 0 as well).

In Section 4.1, we determine the extent to which each channel can individually influence sales, and classify channels based on their "leverage" (i.e., potential for impacting demand). We find that leverage is not the sole determining factor in how successfully a channel can impact sales; rather, leverage needs to be assessed in comparison to demand "momentum": the degree of market penetration ensured by the allocated expenditures, a primary driver of WOM's effects. The channel typology derived in this section also provides a number of high-level managerial insights. First, it allows us to identify low-leverage channels, in which any expenditures beyond their allocated levels are suboptimal, eliminating them from further consideration. Second, for the remaining channels, we identify those in which further expenditures may be reasonably considered, vs. those in which the (previously allocated) expenditures have already exhausted their momentum-generation capability, so that no further expenditures are necessary. This also allows us to reduce the set of channels considered, simplifying the media planning task.

In Section 4.2, we turn our focus to inter-channel interactions, which are revealed when the allocated investment in one or multiple channels are changed (potentially in response to unforeseen changes in business strategy or to fine-tune the media plan) and its impact on optimal investment in others is studied. Using the two-way (leverage and momentum) channel classification developed in Section 4.1, we show that channels in each class have different patterns of interaction with others, as well as identifying regions where a given channel's interactions are driven primarily by synergy vs. substitution. In Section 4.3, we continue analyzing the Camera Sales data (3.4 Empirical Estimation & Illustration) to illustrate the applicability of our results. To streamline exposition, detailed proofs of all results appear in Appendix B, and in Appendix E we further discuss how insights in this section are impacted when marketing resources become scarce as a result of total budget limitations or limited availability of advertising inventory.

4.1 The Structure of Influence of Individual Channels on Demand Adoption

Recall that the firm has already allocated a spend $E_s^0 \geq 0$ on each channel $s \in R$, resulting in a non-negative profit $\Pi[E^0] > 0$, where E^0 is the vector of allocated expenditures, and that $E^0 = 0$ corresponds to no prior allocation. Total cumulative marketing effort from initial investment in all channels is given by $C^0 = \sum_{r \in R} \Phi_r(E_r^0)$; it is convenient to notate cumulative initial-investment marketing effort in all channels except channel $s \in R$ using the usual "$-s$" subscript as $C_{-s}^0 = \sum_{r \in R-\{s\}} \Phi_r(E_r^0)$.

We begin by investigating whether any further investment beyond E_s^0 in a given channel $s \in R$ can profitably influence demand. We define $E_s^*(C_{-s}^0)$ as the optimal level of investment in channel s given cumulative effort in all other channels C_{-s}^0; this can be interpreted as the optimal "response" for channel s to C_{-s}^0. Since we assume that amount E_s^0 has already been allocated, we constrain $E_s^*(C_{-s}^0) \geq E_s^0$. When the value of C_{-s} is clear from context, we may drop the explicit dependence of E_s^* on C_{-s}.

4.1.1 Channel Classification Based on Leverage

It will be helpful for the ensuing analysis to derive an upper bound on the optimal investment in a given channel. Specifically, **Error! Reference source not found.** in Appendix B shows that the optimal investment in channel s is bounded from above by E_s^U, defined as follows:

$$E_s^U = \begin{cases} E_s^0 & \text{if } \Phi_s'(E_s^0) < \frac{4q}{P(q+p)^2}, \\ sup_{E_s \geq E_s^0} \left[\Phi_s'(E_s) \geq \frac{4q}{P(q+p)^2} \right] & \text{otherwise} \end{cases}.$$

As will be illustrated in Section 4.2, when E_s^U is finite, this upper bound is "tight" in the sense that, for certain allocated spend levels, it is optimal to invest E_s^U in channel s. Note that the value of E_s^U is independent of the spends or effectiveness of all other channels: it is driven only by the ability of channel s to influence demand. For this reason, E_s^U can serve as a measure of leverage or effectiveness for channel s, and we subsequently refer to it as such.

The expression for E_s^U, along with the concavity of Φ_s, affords a useful typology regarding its permissible values. Specifically, notating $E = [E_s^0, C_{-s}]$ as a spend vector with the same allocated spend on channel s as E_s^0 and total effort on all other channels C_{-s}, then one of the following cases occur – depending on whether E_s^U takes some internal value on $[E_s^0, \infty]$:

- **"Low Leverage"**, $E_s^U = E_s^0$. In this case $E_s^*(C_{-s}) = E_s^0$, i.e., it would never be optimal to allocate any additional expenditure in channel s, no matter how much spend is allocated to other channels.
- **"Medium Leverage"**, $E_s^0 < E_s^U < \infty$ and satisfies $\Phi_s'(E_s^U) = \frac{4q}{P(q+p)^2}$. In this case $E_s^*(C_{-s}) \in [E_s^0, E_s^U]$, i.e., it may be optimal to increase the investment in channel s to a level not exceeding E_s^U.
- **"High Leverage"**, $E_s^U = \infty$, i.e., $\lim_{E_s \to \infty} \Phi_s'(E_s) > \frac{4q}{P(q+p)^2}$, implying that the channel remains effective even when it is highly invested. In this case $E_s^*(C_{-s}) \in [E_s^0, \infty)$, that is, further positive investment in channel s may be warranted.

Intuitively, leverage classification suggests that low-leverage channels cannot influence demand effectively, so any marginal investment (beyond allocated amount E_s^0) is not justified. For medium and high leverage channels, further investment in channel s may be effective, but is not solely guaranteed by the given degree of channel leverage. As will be shown below, the suitability of additional investment in this channel depends on C_{-s}, i.e., how much has been allocated to all other channels under spend vector E and consequently how much boost in WOM they can collectively create to "help" channel s.

4.1.2 Channel Classification Based on Demand Momentum

Here we explore how much "help" WOM can provide to a channel's ability to generate sales. In addition to channel leverage, this second key factor has to do with the level of market penetration assured by allocated spend E^0 – namely, $G(C^0)$, where again G is as in the Bass formulation (2) and total cumulative marketing effort is $C^0 = \sum_{r \in R} \Phi_r(E_r^0)$ is. This level can be interpreted as the (pre-assured) market "momentum," the primary driver of the WOM effect, on which further expenditures in channel s can build.

Since we already know that no further expenditures in channel s can be justified for low-leverage channels, we assume that $E_s^0 < E_s^U$, i.e., that the leverage of channel s is medium or high. Then it is well-defined and useful to specify the following lower and upper market penetration thresholds:

$$G_s^1 := \frac{1}{2}\left(1 - \frac{p}{q}\right) - \frac{1}{2q}\sqrt{(q+p)^2 - \frac{4q}{P\Phi_s'(E_s^0)}} \tag{6}$$

$$G_s^2 := \frac{1}{2}\left(1 - \frac{p}{q}\right) + \frac{1}{2q}\sqrt{(q+p)^2 - \frac{4q}{P\Phi_s'(E_s^0)}} \; . \tag{7}$$

Observe that $\{G_s^1, G_s^2\}$ depend only on the parameters of the demand curve (innovation and imitation rates p, q), the base per-unit profit P and the initial expenditure E_s^0 on channel s, but not on expenditures in all other channels; and also that the two thresholds are symmetric around the inflection point $0.5(1 - p/q)$ of the demand penetration curve. **Error! Reference source not found.** in Appendix B shows that the threshold level G_s^2 is an upper bound on the market penetration level that can be achieved with investment vector $[E_s^*(C_{-s}^0), C_{-s}^0]$ – in which an optimal investment is made in channel s while keeping the expenditures in all other channels at their initial levels – and moreover that a three-way typology of channels (not of the low-leverage type) regarding the level of demand momentum a channel experiences is as follows:

- **"Low Momentum"**, $G(C^0) \leq G_s^1$: a small increase in expenditure in channel s results in a profit loss, i.e., there exists $E_s^{min} > E_s^0$ such that $\Pi[E_s, E_{-s}^0] < \Pi[E_s^0, E_{-s}^0]$ for all $E_s \in (E_s^0, E_s^{min}]$. However, a larger investment $E_s^* \in (E_s^{min}, E_s^U]$ may be profitable.
- **"Medium Momentum"**, $G_s^1 < G(C^0) < G_s^2$: it is always beneficial to increase investment in channel s, i.e., $E_s^*(C_{-s}^0) \in (E_s^0, E_s^U]$.
- **"High Momentum"**, $G(C^0) \geq G_s^2$: demand adoption with current expenditure E^0 is sufficiently high and further investment in channel s is not profitable, i.e., $E_s^*(C_{-s}^0) = E_s^0$.

As mentioned at the outset, this typology characterizes how much "help" is available from the guaranteed WOM process for channel s to be further successful in generating sales, and how much capability channel s has to generate additional momentum with this help. Therefore, the threshold levels of demand momentum G_s^1 and G_s^2 depend on the current effectiveness of channel s as well.

4.1.3 Dual Typologies, Crossing the Chasm, and Mass Market Penetration

The two typologies derived earlier are summarized in Table 2, which illustrates that neither leverage nor momentum can *individually* classify a channel, thus requiring a dual typology.

<table>
<tr><th colspan="2"></th><th colspan="3">Guaranteed Demand Momentum by Allocated Spending, $G(C^0)$</th></tr>
<tr><th colspan="2"></th><th>Low</th><th>Medium</th><th>High</th></tr>
<tr><td rowspan="3">Leverage</td><td>Low</td><td colspan="3">No further channel spending</td></tr>
<tr><td>Medium</td><td rowspan="2">Further spending may be needed</td><td rowspan="2">Increase channel spending</td><td rowspan="2">No further channel spending</td></tr>
<tr><td>High</td></tr>
</table>

Table 2: Structure of influence of individual channels on demand

The "high momentum" case occurs when (the already-assured) demand penetration $G(C^0)$ is above the upper threshold G_s^2, so the allocated spend levels position the product's ultimate market share in the flatter branch of the S-shaped penetration curve. This could be either because of high allocated investment levels and/or because of partial build-up of WOM prior and during the media planning horizon. As demand momentum is already quite high (compared to what channel s needs, measured by G_s^2), increasing the penetration level further requires prohibitive effort, leading to $E_s^* = E_s^0$. At a first glance, it may seem surprising, as per Table 2, that channels facing high momentum behave similarly to the low-leverage channels: neither one warrants any expenditure beyond the already-allocated amount E_s^0. However, the reasons behind this outcome are very different. In the high-momentum case, channel s can in a sense now "free-ride" on customer WOM and allocated investments, so does not require further support. This would, of course, change if the allocated investments in other channels were lower, causing $G(C^0)$ to drop. This contrasts with a low-leverage channel unable to sufficiently influence demand cost-effectively independent of investments in other channels. Thus, no expenditure beyond E_s^0 is warranted even if investment in other channels in E^0 were to be reduced.

The medium momentum case occurs for demand momentum in the "sweet spot" range between G_s^2 and G_s^1: WOM can well accompany increased investment in channel s, and thus it is optimal to increase channel s spending, i.e., $E_s^* > E_s^0$. By contrast, in the low-momentum case the allocated spend levels put the product's ultimate market share in its initial slow-growth region when WOM is not well developed, requiring a relatively large exertion (in the form of additional investment in channel s) to properly influence demand; indeed such exertion may be cost-prohibitive, especially if it is not supported by investment in other channels to raise demand momentum. Thus $E_s^* = E_s^0$ cannot be ruled out. However, when a cost-efficient expenditure level exists, substantial further investments in channel s may be warranted to ensure that ultimate demand penetration is increased to a more desirable level. We also note that it follows from the definition of G_s^1 that $G_s^1 < 0$ when $\Phi_s'(E_s^0) > \frac{1}{Pp}$, so the low momentum case cannot occur if the leverage of channel s is not too low. This suggests that if the allocated investment in channel s has been sufficiently effective, it has already created a substantial level of demand momentum.

Based on the above discussion, the threshold G_s^1 can be interpreted as the "chasm-crossing ability" of channel s, that is, the ability of the channel in transitioning the ultimate fate of the adoption process from selling only to "early adopters" to reaching mass market penetration (Moore 1990; Van den Bulte & Joshi 2007). As described by Chandrasekaran & Tellis (2011), a chasm "separates the early adopters from the early majority" who may "have different characteristics and needs"; although we are agnostic here on whether the former do not form a good WOM reference point for the latter, or whether it leads to an explicit saddle (e.g., Goldenberg et al. 2002). Such chasms have been implicated, for example, in the sequential unfolding of consumer and developer segments in software platform deployment (Mehra et. al. 2014). Consequently, it would be possible to cross the chasm if enough marketing support is allocated initially so as to raise the market share above $\min_{s \in R} G_s^1$.

Analogously, the threshold G_s^2 can be interpreted as the "mass market penetration level" that can be achieved if channel s is best utilized. That is, the firm would be able to achieve a market share of at most $\max_{s \in R} G_s^2$ if proper marketing support is provided. Assuming s is not a low-leverage channel, we further observe that, as $\Phi_s'(E_s^0)$ decreases to $\frac{4q}{P(q+p)^2}$, the difference $G_s^2 - G_s^1$ decreases to 0; that is, as the leverage of the channel decreases to the low-level threshold, the "sweet spot" region of the demand curve defining medium momentum shrinks, thus reducing the region where channel s can individually influence demand penetration. Indeed, the interplay between the channel's leverage and demand momentum is a key determinant of how channels interact with each other; this will be further explored, and illustrated graphically, in Section 4.2. However, we first exploit some of the consequences of the previous result to further "prune" the set of channels under consideration.

It is critical to note that the initial market share of the product x_0 (before the media planning horizon) does not influence the thresholds in the typography of Table 2. This means that if media planning decisions are done at different stages of the product lifecycle, the same metrics of assessment still hold. The main impact of the initial market share would however be in the magnitude of $G(C^0)$, that is, the higher is the prior adoption of customers before media planning decisions, the higher is the chance that channel s would experience a higher momentum.

4.1.4 Eliminating Dominated Channels From the Media Planning Problem

Reducing the set of channels needing to be considered by the media planner is important not only from a computational perspective – non-linear optimization for MEA entails the curse of dimensionality – but a managerial one: there are nontrivial cognitive, accounting, and time costs to continually consider ineffective channels. Table 2 suggests that two types of channel can be eliminated from further consideration: low leverage or high momentum. While these general prescriptions are useful, it's possible to go further by considering channels *pairwise*, eliminating any channel that "dominates" another. Put more rigorously, if there is a channel $r \in R$ whose effectiveness always dominates that of channel s (that is, $\Phi_s'(E_s) < \Phi_r'(E_r)$ for all $E_s \in [E_s^0, E_s^U]$ and all $E_r \in [E_r^0, E_r^U]$) then it is never optimal to invest in channel s beyond the initial level, E_s^0 (**Error! Reference source not found.** in Appendix B). Practically speaking, this means that for each additional dollar invested in channel r and s, the former generates higher "bang for the buck". Moreover, this condition is easily verifiable: the concavity of both Φ_s and Φ_r makes it equivalent to $\Phi_s'(E_s^0) < \Phi_r'(E_r^U)$. Despite its simplicity, this observation has an important and non-obvious implication: it is never optimal to invest in a less effective channel despite its potential synergistic interaction with others.

In terms of solving the media planning problem, we can then narrow down R to a subset R^A of "active" channels that may warrant investments beyond the current vector of allocations E^0:

$$R^A = \left\{ s \in R \;\middle|\; \begin{array}{l} s \text{ has medium or high leverage} \\ G(C^0) < G_s^2 \text{ (low or medium momentum)} \\ s \text{ not dominated by any other channel } r \in R \end{array} \right\}.$$

4.2 Structure of Channel Interactions: Substitution & Synergy Implications

In this section, we explore how channels mutually interact. These interactions cannot be inferred by only looking statically at how influential each channel is for a given spend vector like E^0; rather, one must examine what happens when allocated investments in some of the channels change and the optimal response of others is characterized. The analysis is, somewhat surprisingly, entirely tractable if, for any feasible channel (i.e., $s \in R^A$) and any positive offset, $z \geq 0$, the marketing effectiveness function Φ_s satisfies at least one of the following conditions (where, as always, P is base unit price):

(a) $G(\Phi_s(E_s) + z)$ is concave or S-shaped in E_s

(b) $P \cdot G(x + z) - \Phi_s^{-1}(x)$ is concave or S-shaped in x

Intuitively, both conditions guarantee uniqueness: that if it is optimal to increase the investment in channel s (while keeping the investment in all other channels fixed), the optimal magnitude of the increase (i.e., the optimal response) can be obtained uniquely. In conditions (a) and (b), z stands in for the cumulative marketing effort of all channels other than s, so it suffices to check either condition for internal values, that is, for $0 \leq z \leq \Sigma_{r \in R^A - \{s\}} \Phi_r(E_r^U)$. While (a) and (b) depend on both demand adoption and effectiveness structure of channels $s \in R^A$, they have a simple associated sufficient condition, one based on channel effectiveness functions only; specifically, *they hold if* $(\Phi_s^{-1}(x))'$ *is convex in* x *for all* $x \geq 0$ *and* $s \in R^A$ (**Error! Reference source not found.** in Appendix B). Fortunately, this sufficient condition holds for the most common forms of effectiveness functions in the literature, e.g., $\Phi_s(E_s) = E_s$, $\ln(1 + E_s)$ and $\sqrt{E_s}$, among others.

With these preliminaries in place, channel interactions can be analyzed. To do so efficiently, we focus on a single channel $s \in R^A$ and study its interaction with potentially all remaining channels $R - \{s\}$. Specifically, given that a spend of E_s^0 has been allocated to channel s and combined cumulative effort $C_{-s}^0 = \sum_{r \in R-\{s\}} \Phi_r(E_r^0)$ to all other channels, we study how the optimal spend $E_s^* \geq E_s^0$ behaves as cumulative effort from other channels increases to C_{-s} from its initial value, C_{-s}^0. Note that if C_{-s}^0 is increased by investing in only one channel, the pairwise interaction of channel s with this channel can be inferred; but if the increase in C_{-s}^0 is achieved by investing in multiple or all channels other than s, the collective interaction of channel s with those channels can still be characterized.

As discussed in Section 2, several studies show that channels interact mainly substitutively, while others find synergistic behavior between them. However, for a given allocation of marketing resources (represented by spend vector E), the key question is: under what conditions is synergy vs. substitution observable (or dominant) as opposed to the other? Recall that substitution means investing more in one channel reduces the ability of another channel to (additionally) improve demand. Therefore, when the substitution effect dominates, one expects that, when the level of investment in channels $R - \{s\}$ is increased, it would be optimal to reduce the investment in channel s. This intuition generally holds for models of substitutable products or resources (e.g., Chapter 3 of Topkis 1988). By contrast, synergy means that channels work together better than the sum of each taken separately. Thus, when synergy dominates, increased spending on channels $R - \{s\}$ should trigger a spending increase in channel s (from Theorem 2.8.5 of Topkis 1988), a result that also holds for the synergy model of Naik and Raman (2003).

These considerations lead to some terminological shorthand that will prove useful. For a channel $s \in R^A$, if the optimal spend E_s^* is increasing in C_{-s} for some range $C_{-s} \in [C_{-s}^1, C_{-s}^2]$, where $C_{-s}^0 \leq C_{-s}^1 < C_{-s}^2$, then we say that the interaction between s and all other channels is ***"dominantly synergistic"*** in this range. If, on the other hand, E_s^* is decreasing for $C_{-s} \in [C_{-s}^1, C_{-s}^2]$, then the interaction between s and the other channels is ***"dominantly substitutive"*** in this range. Put simply, dominant synergy means that a channel's optimal investment goes up with the cumulative effort in other channels (in a particular range), while dominant substitution means the opposite.

The study of the dominant pattern of interaction of a given channel s with all others requires considering both its leverage-based and momentum-based categories. Focusing only on channels in R^A removes the low leverage and high momentum groups, allowing us to partition R^A into four distinct classes: $R^{H/M}, R^{H/L}, R^{M/M}, R^{M/L}$, where the first letter refers to the channel's leverage and the second one to its momentum. The next two sections detail the different interaction pattern of each class with other channels. To aid in the ensuing analysis, we define the following critical quantities for channel $s \in R^A$:

$$\begin{aligned} C_{-s}^{trans} &= \inf\{C_{-s} \geq C_{-s}^0 \mid E_s^*(C_{-s}) > E_s^0\} \\ C_{-s}^{LM} &= \inf\{C_{-s} \geq C_{-s}^0 \mid G(C_{-s} + \Phi_s(E_s^0)) \geq G_s^1\} \\ C_{-s}^{peak} &= \inf\{C_{-s} \geq C_{-s}^0 \mid G(C_{-s} + \Phi_s(E_s^U)) \geq \frac{1}{2}(1 - \frac{p}{q})\} \\ C_{-s}^{\max} &= \inf\{C_{-s} \geq C_{-s}^0 \mid G(C_{-s} + \Phi_s(E_s^0)) \geq G_s^2\} \end{aligned}$$

4.2.1 Medium Leverage Channels

Recall that for a medium leverage channel $s \in R^A$, the upper bound on its optimal expenditure, E_s^U, is well defined and finite. For such a channel, we start by analyzing the case wherein it creates a low market momentum, i.e., $s \in R^{M/L}$. Later in this section, we will show that the results for the case that channel s creates a medium momentum (i.e., $s \in R^{M/M}$) can be viewed as a special case of the $R^{M/L}$ results.

We next describe how the M/L (medium-leverage, low-momentum) channel s interacts with all other channels by characterizing the behavior of $E_s^*(C_{-s})$ as C_{-s} is increased from its initial level C_{-s}^0. Recall than an increase in C_{-s} can be achieved by increasing the investment in one or multiple channels in $R - \{s\}$. **Error! Reference source not found.** in Appendix B states that for $s \in R^{M/L}$, the critical quantities $C_{-s}^{trans}, C_{-s}^{LM}, C_{-s}^{peak}$and $C_{-s}^{\max}$ exist and obey the order restrictions $C_{-s}^0 \leq C_{-s}^{trans} \leq C_{-s}^{peak} < C_{-s}^{\max}$ and $C_{-s}^{trans} \leq C_{-s}^{LM} < C_{-s}^{\max}$. Moreover, we can characterize the behavior of $E_s^*(C_{-s})$ in each of the associated regions as follows:

Value of C_{-s}	**Behavior of $E_s^*(C_{-s})$ for $s \in R^{M/L}$**
$\in [C_{-s}^0, C_{-s}^{trans})$	$E_s^*(C_{-s}) = E_s^0$
$\in [C_{-s}^{trans}, C_{-s}^{peak}]$	$E_s^*(C_{-s})$ **increases** smoothly in C_{-s}, with maximum E_s^U at C_{-s}^{peak}. The interaction of s with others is **dominantly synergistic**.
$\in [C_{-s}^{peak}, C_{-s}^{\max}]$	$E_s^*(C_{-s})$ **decreases** smoothly in C_{-s}, with minimum E_s^0 at $C_{-s}^{\max}$. The interaction of s with others is **dominantly substitutive**.
$\geq C_{-s}^{\max}$	$E_s^*(C_{-s}) = E_s^0$

Proceeding downward through the ranges outlined above illustrates how the interactions of channel s with other channels change as investment in other channels is increased. This pattern is also illustrated in Figure 2, where a typical curve, together with various regions of interactions, are depicted. To begin with, cumulative effort from all channels except s must reach a threshold C_{-s}^{trans} for additional investment in channel s to be profitable. This makes intuitive sense: since channel s does not have much leverage, it needs a certain minimum level of momentum (market penetration) to be built up by the other channels before investments in s become cost-effective.

When C_{-s} rises above C_{-s}^{trans}, two different interaction patterns emerge. First when $C_{-s} \in [C_{-s}^{trans}, C_{-s}^{peak}]$, the interaction of channel s with other channels is dominated by the **synergy effect**. In this range, the support of channel s is required to complement the limited investments made in other channels in order to improve market momentum and WOM, but channel s must work in concert with the other channels in order to be effective. As a result, the firm is only motivated to increase its channel s investment when it is supported by increased spending in at least one other channel. This increasing pattern continues until the peak spend of E_s^U is reached at C_{-s}^{peak}, where the combined marketing effort from all channels (including s) ensures that the inflection point $\frac{1}{2}(1 - \frac{p}{q})$ of the demand curve can ultimately be reached.

The second pattern of interaction occurs once peak expenditure level is reached at C_{-s}^{peak}, at which point the interaction between channel s and other channels is dominated by the **substitution effect**. Here the cumulative efforts by all other channels is high enough to build a substantial level of market momentum, and hence expenditures on channel s are less and less economically justified as cumulative marketing effort increases. Eventually once C_{-s} exceeds C_{-s}^{max}, no further expenditures on s can be justified.

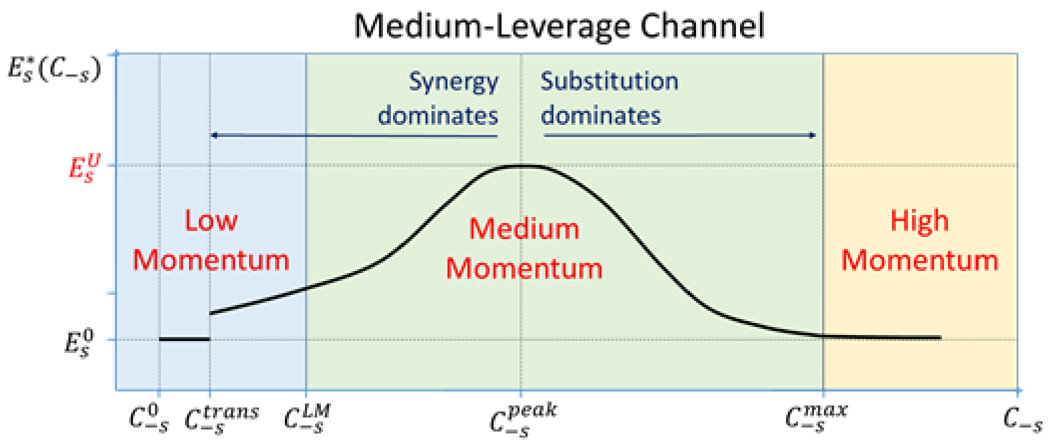

Figure 2: Typical pattern of interaction for a channel with initial classification of "M/L"

As suggested in the preceding discussion (and indicated in Figure 2), while channel s starts out as low-momentum, market momentum "transitions" as the cumulative effort C_{-s} is increased; this means that channel s can generate a moderate level of momentum when additionally supported by other channels. Indeed, as C_{-s} increases to above C_{-s}^{LM}, channel s transitions to the medium momentum category, which cannot occur before the optimal spend level increases beyond E_s^0, given that $C_{-s}^{LM} \geq C_{-s}^{trans}$. The transition to medium momentum can happen before or after C_{-s}^{peak}; however, in our numerical experiments we observed $C_{-s}^{LM} \leq C_{-s}^{peak}$, even though the opposite cannot be ruled out. Also, as C_{-s} further increases beyond $C_{-s}^{\max}$, it transitions to the high momentum region, where no increase in channel s spending is justified. These observations suggest that channels belonging to the initial category $R^{M/M}$ can be regarded as a special case of that for $R^{M/L}$: their pattern of interactions with all other channels follows the one described in the tabular display for $R^{M/L}$ when initial cumulative effort $C_{-s}^0 \geq C_{-s}^{LM}$. This can be analogously summarized for $R^{M/M}$ as follows with the corresponding critical quantities of $C_{-s}^0 = C_{-s}^{LM} \leq C_{-s}^{peak} < C_{-s}^{max}$:

Value of C_{-s}	**Behavior of $E_s^*(C_{-s})$ for $s \in R^{M/M}$**
	If $C_{-s}^0 < C_{-s}^{peak}$
$\in [C_{-s}^0, C_{-s}^{peak}]$	$E_s^*(C_{-s})$ **increases** smoothly in C_{-s}, with maximum E_s^U at C_{-s}^{peak}. The interaction of s with other channels is **dominantly synergistic**.
$\in [C_{-s}^{peak}, C_{-s}^{\max}]$	$E_s^*(C_{-s})$ **decreases** smoothly in C_{-s}, from E_s^U to its minimum E_s^0 at $C_{-s}^{\max}$. The interaction of s with others is **dominantly substitutive**.

$\geq C_{-s}^{\max}$	$E_s^*(C_{-s}) = E_s^0$
If $C_{-s}^0 = C_{-s}^{peak}$	
$\in [C_{-s}^0, C_{-s}^{max}]$	$E_s^*(C_{-s})$ **decreases** smoothly in C_{-s}, with minimum E_s^0 at $C_{-s}^{\max}$. The interaction of s with other channels is **dominantly substitutive**.
$\geq C_{-s}^{\max}$	$E_s^*(C_{-s}) = E_s^0$

To summarize, the results above show that medium-leverage channels $R^{M/L} \cup R^{M/M}$ have two regimes of interactions with other channels – one dominated by the synergy effect, one by the substitution effect – and which regime is dominant depends on the cumulative marketing effort of all other channels. In the following section we provide analogous analyses for high-leverage channels, showing that their patterns of channel interaction are notably distinct.

4.2.2 High Leverage Channels

Recall that "high leverage" refers to channels for which $E_s^U = \infty$, that is, even large investments in the channel remain effective on demand. We start the analysis by considering channel s with the initial classification of high-leverage, low-momentum, i.e., $s \in R^{H/L}$. As before, we will see that the results for high-leverage, medium-momentum (i.e., $R^{H/M}$) channels can be viewed as a special case of the $R^{H/L}$ results. For the $R^{H/L}$ case, **Error! Reference source not found.** in Appendix B states that we have $C_{-s}^0 \leq C_{-s}^{trans} = C_{-s}^{peak} \leq C_{-s}^{LM} < C_{-s}^{max}$. Moreover, spending in channel $s \in R^{H/L}$ can be categorized as follows:

Value of C_{-s}	**Behavior of $E_s^*(C_{-s})$ for $s \in R^{H/L}$**
$\in [C_{-s}^0, C_{-s}^{trans})$	$E_s^*(C_{-s}) = E_s^0$
$\in [C_{-s}^{trans}, C_{-s}^{max}]$	$E_s^*(C_{-s})$ **decreases** smoothly in C_{-s}, from its maximum at C_{-s}^{trans} to the minimum E_s^0 at C_{-s}^{max}. The interaction of s with others is **dominantly substitutive**.
$\geq C_{-s}^{\max}$	$E_s^*(C_{-s}) = E_s^0$

Note that a H/L channel behaves somewhat differently from its medium-leverage counterpart (M/L): while there may exist an initial interval $[C_{-s}^0, C_{-s}^{trans})$ where no additional spend is cost effective, once C_{-s} reaches the threshold $C_{-s}^{trans} = C_{-s}^{peak}$, it faces a substantial enough level of market momentum to jump-start investment in channel s up to its maximum value. Due to high leverage in this channel, it can impact the penetration curve on its own, that is, not requiring "help" from the other channels. Thus, there is no "synergy" interval. Marketing effort in other channels beyond C_{-s}^{trans} results in investment in channel s to be gradually reduced to its minimum level, E_s^0, so that interaction with other channels is dominated by substitution throughout the whole positive spend region.

As cumulative effort C_{-s} increases to C_{-s}^{LM}, channel s transitions to medium momentum; and, as it further increases to C_{-s}^{max}, it transitions to high momentum. If C_{-s}^{0} is altered to be larger than C_{-s}^{LM} (i.e., transition threshold to medium momentum), the interaction pattern of channel s with other channels can be regarded as a special case of that observed in the $R^{H/L}$ case. Therefore, the initial "no-spend" region does not occur as the channel faces a substantially higher level of market momentum. Results for $R^{H/M}$ channels can thus be summarized as follows and depicted in Figure 3 with corresponding critical quantities of $C_{-s}^{0} = C_{-s}^{LM} = C_{-s}^{peak} < C_{-s}^{max}$:

Value of C_{-s}	**Behavior of $E_s^*(C_{-s})$ for $s \in R^{H/M}$**
$\in [C_{-s}^{0}, C_{-s}^{max}]$	$E_s^*(C_{-s})$ **decreases** smoothly in C_{-s} from its maximum at C_{-s}^{0} to the minimum E_s^0 at C_{-s}^{max}. The interaction of s with others is **dominantly substitutive**.
$\geq C_{-s}^{max}$	$E_s^*(C_{-s}) = E_s^0$

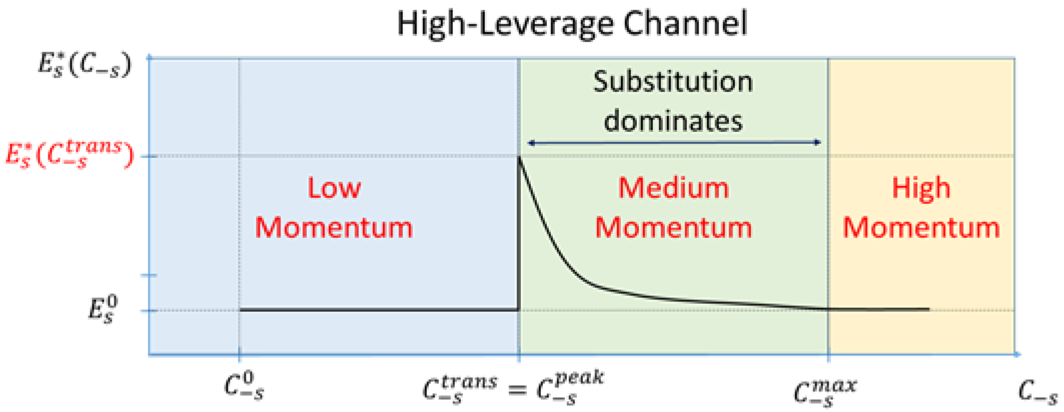

Figure 3: Pattern of interactions for a channel with initial classification of "H/L"

In summary, for high-leverage channels, the interaction with other channels is dominantly substitutive until C_{-s} reaches C_{-s}^{max}, at which point no further spend beyond the allocated amount E_s^0 can be justified. Combining the results of this section and the previous one, we conclude that if a channel has limited leverage and spending maximally cannot assure strong enough market momentum to ultimately develop, it works *dominantly synergistically* with other channels to alleviate these limitations; otherwise, the dominant interaction of the channel with others is *substitutive*.

4.3 Camera Sales Case: Empirical Analysis and Implications

Equipped with the results of Sections 4.1 and 4.2, we complete our analysis of the camera media planning case of Section 3.4, along with all critical calculations and procedures. Recall that, at the start of the 12-month media planning horizon, the firm had already obtained $F(28) = 44.22\%$ market share, having allocated at least \$0.0002 per customer in each channel ($E_1^0 = E_2^0 = 0.0002$).

To investigate channel interactions at the strategic level, one needs to specify the structure of the $\Phi_s(.)$ functions that links the tactical and strategic decisions. At the tactical level in Section 3.4, we take the sales impact of advertising to have the commonly-used "power form," with customers recalling a limited amount of past marketing effort. Recall from that section that the resulting cumulative effectiveness function $\Phi_s(E_s)$, which allocates the budget E_s optimally over time in channel s, has a similar power form:

$$\Phi_s(E_s) = \beta_s {E_s}^{\rho_s} + \Phi_s^0 \quad \text{for } s = 1,2 \text{ with } \beta_1, \beta_2 > 0. \tag{8}$$

Note that $\Phi_s(E_s)$ is a concave increasing function of E_s. Based on the estimates and media planning setting of Section 3.4, the constants were obtained as $\beta_1 = 0.0926$, $\beta_2 = 0.0738$, $\Phi_1^0 = 0.0230$, $\Phi_2^0 = 0.0133$. We see that $\beta_1 > \beta_2$ and $\Phi_1^0 > \Phi_2^0$, indicating that Channel 1 is relatively more 'effective' and that relatively more advertising goodwill is built in this channel prior to the start of the media planning horizon.

Solving the MEA problem, the optimal channel spends are $E_1^* = 0.0318$ and $E_2^* = 0.0068$, equivalent to the levels on the last row of Table 1 (when multiplied by m). These levels suggest spending considerably more on the relatively more effective channel. However, in competitive response to other external factors, these investments may need to be revised. For example, if a competitor is heavily investing in Channel 2 and large investment in Channel 1 is not considered compatible with the brand's image, a manager might wish to learn how lowering investment in Channel 1 to a suboptimal level would impact the marketing plan. S/he would be interested to know if savings on Channel 1's spending can create more budget for Channel 2's spending and create a better barrier to entry; or would lowering Channel 1's spending make Channel 2's less effective? In such a scenario, understanding the pattern of interaction between the two channels can help meaningfully adjust their investment levels, which we subsequently explore in the context of this case, specifically, how optimal spend in Channel 2 varies with changes in Channel 1 expenditure (the reverse argument being analogous).

The first step is to check the uniqueness of optimal increase condition in Section 4.2. It is readily verified that the sufficient condition – i.e., $(\Phi_s^{-1}(x))'$ is convex in x for all $x \geq 0$ and $s \in R^A$ – only holds for $\rho_s \in (0,0.5)$, which aligns with estimated values of ρ_1 and ρ_2. However, numerical verification shows the condition does hold for all $\rho_s \in (0,1]$. Thus, the results of Section 4.2 apply to this example.

Next, we classify Channel 2 with respect to leverage and the market momentum faced based on the results of Section 4.1. Comparing the value of $\Phi_2'(E_2^0) = 108.9$ with the threshold level of $\frac{4q}{P(p+q)^2} = 6.39$, we see that Channel 2 does not have low leverage. We further find that the maximum optimal spend is bounded by $E_2^U = 0.0098 < \infty$, indicating that Channel 2 has "medium leverage". To determine the demand momentum of Channel 2 at the initial spend vector $E^0 = [0.0002, 0.0002]$, we compute the two penetration thresholds, $G_2^1 = -5.6\%$ and $G_2^2 = 98.4\%$, as well as the initial penetration level $G(E^0) = 61.0\%$. Since $G(E^0)$ is between G_2^1 and G_2^2, the channel's momentum is "medium". The negative value of G_2^1 essentially indicates that Channel 2 would never be a low-momentum channel for any spend level. Consequently, the interaction of Channel 2 with Channel 1 is governed by the typology specified earlier for $R^{M/M}$ class. Figure 4 depicts the optimal Channel 2 spend $E_2^*(E_1)$ for different levels of spending in Channel 1. Since there are only two channels in this example, there is a one-to-one correspondence between $C_{-s} = \Phi(E_1)$ and E_1, and using E_1 for the x-axis instead of C_{-s} results in a bijective re-scaling of the axis.

From Figure 4, we see that the optimal spend in Channel 2 decreases with increase in Channel 1 spend, indicating a dominantly substitutive interaction for reasonable ranges of Channel 1 spending. Given that $G([E_1^0, E_2^U]) = G([0.0002, 0.0098]) = 64.9\%$ is larger than the threshold of $\frac{1}{2}\left(1 - \frac{p}{q}\right) = 46.4\%$ (from the definition of C_{-s}^{peak}; i.e., the highest spending in Channel 2 can guarantee the ultimate market share passing the demand inflection point), we have $E_1^{trans} = E_1^{LM} = E_1^{peak} = E_1^0$. Therefore, the decreasing pattern of optimal Channel 2 spend starts at a lower level than E_2^U, meaning that Figure 4 depicts the right portion of Figure 2 after the peak. The optimal Channel 2 spend stays well above the allocated value of $E_2^0 = 0.0002$, but when Channel 1 spending reaches the inefficiently high value of $E_2^{max} = 10.058$, Channel 2 transitions to the high momentum category and optimal Channel 2 spend drops to the allocated value. From the figure on optimized profit, we can see that the highest profit coincides with the previously found optimal spend plan of $E^* = [0.0318, 0.0068]$. Interestingly, the optimized profit is somewhat flat around the highest level so that when Channel 1 spending is in the range $[0.0116, 0.0636]$, the optimal profit does not drop more than 3%. Returning to the question initially raised, we can conclude that lowering the investment in Channel 1 from its optimal level of $E_1^* = 0.0318$ slightly reduces the total optimal profit, but Channel 2 is interacting mainly substitutive with Channel 1, so that reduction in Channel 1 spending opens up some budget and would necessitate increased Channel 2 spending.

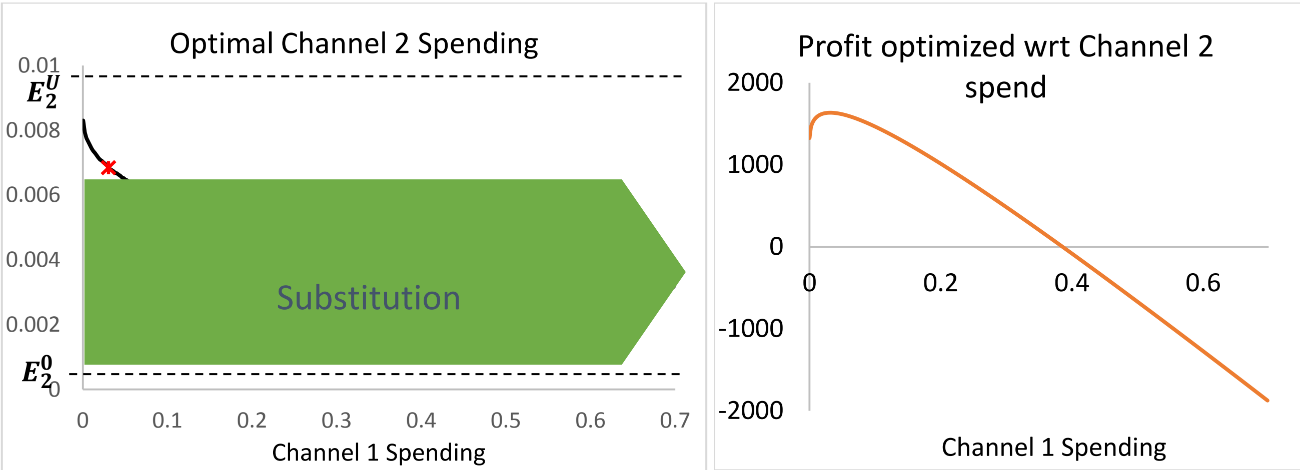

Figure 4: Optimal expenditure on Channel 2 and optimal corresponding profit with respect to changes in Channel 1 spend; *: optimal spend plan

The interaction of Channel 2 with Channel 1 is mainly substitutive in this case, which is partly dependent on the media planning settings, e.g., the demand boost (from WOM and advertising) over the initial 28 months. To illustrate, imagine that the 12-month media planning horizon were to hypothetically start with initial settings of 28 months ago, i.e., with the initial market share of 8.08% and initial advertising goodwill of 1.578 and 1.407 for channels 1 and 2 respectively. Figure 5 illustrates the optimal Channel 2 spending with respect to Channel 1 expenditures.

Both synergistic and substitutive patterns can now be observed in Figure 5. When $E_1 < E_1^{peak} = 0.3982$, the two channels interact synergistically until Channel 2 spending reaches its maximum value of E_2^U at E_1^{peak}. For higher Channel 1 spend levels, the two channels interact substitutively. The optimal spend plan (marked with *) falls in the synergistic portion of interaction of the two channels, in contrast to that observed in Figure 4. Therefore, if the firm were to reduce the optimal spend in Channel 1 under these settings, it would need to cut back on the spending in Channel 2 as well, as Channel 2 is relatively less effective in ultimate demand adoption.

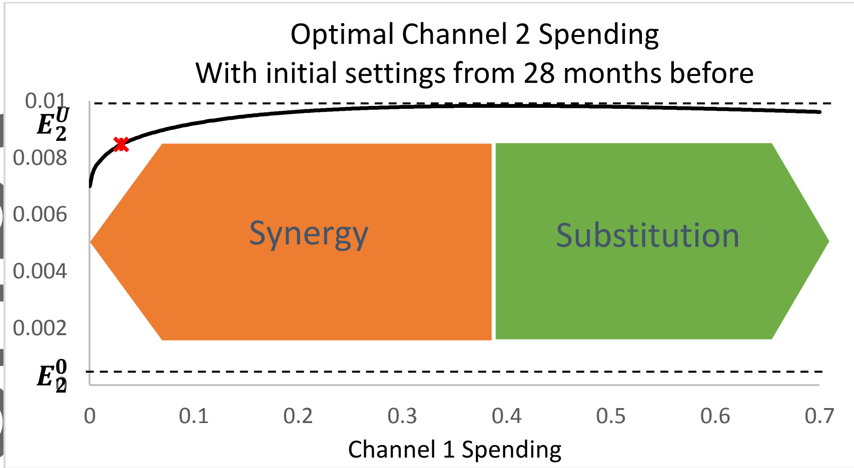

Figure 5: Optimal expenditure on Channel 2 when the initial media plan values are set to those from 28 months ago; *: the corresponding optimal spend plan

The type of interaction between the two channels also depends on the nature of demand response to advertising. For example, we illustrate in Appendix D how a change in ρ_1, or potential inclusion of explicit interaction term between the two channels (to force in substitution or synergy) can influence the resulting interaction between the two channels.

5 Discussion and Future Research

A great deal of managerial and academic attention has focused on improving media budget allocation, a problem exacerbated by dramatic recent proliferation of online media venues. As underscored by Weinberg & Pehlivan (2011), there remains “a fair degree of uncertainty with respect to allocating marketing effort and budget”. Here, we analyzed media planning decisions for multiple marketing channels over time to support a new product or service introduction. In contrast to a “swim-lane analysis,” where each channel operates more-or-less independently, our main focus has been on interactions between channels and with the WOM process typically driving new product sales. In such situations, channels can enhance or detract from one another’s effectiveness, leading, to synergy or substitution. Building upon the marketing literature on new product diffusion, we extended the Generalized Bass Model (GBM) framework in a manner allowing high-level insight without the complexities of explicit dynamic programming or the specific assumptions and functional forms required for tractability. We also showed how to apply the model empirically, specifically, to camera sales for a major metropolitan retailer, via Bayesian estimation of a highly flexible, two-channel set-up for radio ads and flyers.

The ensuing analysis suggests several broad insights. Perhaps most directly relevant for media planners is that patterns of channel interactions are governed by two factors: each channel’s leverage and the momentum built up by all remaining channels; the latter, being the primary driver of WOM, is especially critical for new product launches. The modeling framework allows the derivation of specific conditions under which channel interaction is dominantly synergistic vs. dominantly substitutive, irrespective of functional forms chosen for various elements of market response. Specifically, if a channel has limited leverage and faces insufficient support (from other channels) to build market momentum, it works dominantly synergistically with other channels to alleviate its predicament; otherwise, the dominant interaction pattern for the channel is substitutive. The framework thereby provides a usable typology based on low/medium/high levels of leverage and momentum, including the identification of specific points past which a channel's adoption process can "cross the chasm" from lead users to mass market penetration.

In terms of practical media planning, extant literature suggests how certain classes of channels might be categorized. For example, Lemon & Verhoef (2016) describe how mobile channels, which offer location-based, time-sensitive opportunities to create touchpoints, can directly interfere and interact with other channels, especially with the prevalence of "showrooming". Their high per-dollar effectiveness suggests medium-to-high leverage; consequently, customers' exodus to mobile channels can incentivize increased substitution of the expenditures from other channels. On the other hand, content-separated online ads (i.e., having little relation to the medium's content) are found to be less effective than content-integrated channels, resulting in lower leverage, possibly because they feel more intrusive (De Haan et al. 2016). If expenditures in such channels are differentially utilized at the early stages of product adoption (leading to low momentum), they may require support from other channels, thereby interacting more synergistically. Generally speaking, however, our results illustrate that channel leverage and momentum are dependent on product and channel characteristics as well as the product adoption stage. That is, a channel can behave synergistically in one setting and substitutive in another, making "global" categorization of channels along leverage and momentum (or emergent substitution / synergy patterns) potentially misleading.

A number of results facilitate computing optimal spend allocations for a target firm. First, deriving optimal temporal spend patterns allows the original optimal control problem to be formulated as a non-linear program, to which numerical solvers can be readily applied. We also derive results allowing some "dominated" channels to be eliminated from the planning problem *a priori*, helping to alleviate the curse of dimensionality in crowded media channel spaces. Notably, the model allows the direct incorporation of relative channel marketing costs, leading to optimality conditions that apply even when various channels are priced differently, as is common in nearly all media planning platforms. Such conditions enable enhanced allocation using identical numerical methods, without adding computational overhead.

Several directions for future research present themselves. Although our modeling framework greatly reduces the complexities of dynamic programming, a critical subsequent area of inquiry concerns (numerical) optimization, as the MEA problem structure may hinder the use of non-linear programming solvers when dimensionality – the number of media channels – is large, as it could be in practice. Special-purpose algorithms exploiting the structural properties of the model can be constructed; conversely, specific functional forms amenable to direct analysis by optimal control, as per Prasad & Sethi (2009), is a fertile area for exploration. Similarly, the model can be extended to consider not only competition among channels, but among products. While this is critical for mature (commodity) markets, firms cannot assume that, just because their product is "new", it will retain its local monopoly power indefinitely.

Important avenues for expansion involve particular distinctions within the model or data sources. For example, the GBM framework presumes "complete mixing" in that all customers are equally likely to "innovate" and then "imitate" from one another. One could therefore posit that p and q be channel-specific; while a seemingly straightforward generalization, GBM is an *aggregate* model, so accommodating this would require additional information on individual-level exposure. Similarly, "media", while characterized by associated channel-specific parameters, are not treated as fundamentally different; the Customer Journey literature suggests that, even for new products, different media may best be deployed at different junctures. And the Bass and GBM frameworks address first, not repeat, purchases, which depend on intermediate satisfaction; a model addressing channel interactions for follow-up (like Fader et al. 2005 in the CLV literature) or continent purchases (as in Abedi et al.'s 2014 example of Nespresso machines and capsules), would enrich the model's purview. Also, the effects of both firm-initiated touchpoints (for advertising; Li & Kannan 2016) and customer-initiated ones (for social effects) could inform attribution modeling for new products specifically. Lastly, although our flexibly-parameterized model can capture a wider variety of effects / shapes than many in the literature, it cannot "learn" these forms in the sense of Machine Learning; we see great potential for the application of nonparameterics (e.g., Gaussian Processes; Dew & Ansari 2018), which have not appeared thus far in the empirical diffusion literature, to large-scale media planning data, particularly as regards channel interaction.

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