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Toward an effective CFT_2 from $\mathcal{N} = 4$ super Yang-Mills and aspects of Hawking radiation

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Abstract: Using $\mathcal{N} = 4$ supersymmetric Yang-Mills theory we recover important aspects of the near-extremal thermodynamics of AdS_5 black holes including both the outer and the inner horizons with their corresponding entropy and energy. This $\mathcal{N} = 4$ supersymmetric Yang-Mills theory approach to black hole thermodynamics leads to an effective CFT_2 interpretation similar to the work by Callan and Maldacena. We corroborate this effective CFT_2 by implementing a particular near-horizon limit that geometrizes the Virasoro algebras as asymptotic symmetries. Using the effective CFT_2 picture, we discuss aspects of the Hawking radiation rate for a region of the near-extremal AdS_5 black hole quantum evolution.

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Contents

1 Introduction

Hawking established the legitimacy of associating a temperature to black holes by explicitly demonstrating that due to quantum effects they radiate thermally [1]. This development cemented the interpretation of black hole entropy proportional to horizon area [2] and, more generally, the thermodynamic nature of black holes [3–5]. The search for a statistical description of black hole thermodynamics in terms of microstates became paramount. String theory, in the works of Strominger and Vafa provided such an answer for a particular class of supersymmetric asymptotically flat black holes [6]. In this case an understanding of the rate of Hawking radiation was provided by Callan and Maldacena who identified an effective two-dimensional conformal field theory for the microscopic description [7]. Namely, they considered a near-extremal black hole configuration whose microscopic description involves a CFT with different temperatures in the right- and left-moving sectors [7]. Other effective CFT_2 descriptions were discussed at the time [8–10] and perhaps the apex of this approach was provided by Strominger who presented a microscopic derivation of the entropy of a large class of black holes using only properties of a near-horizon limit [11].

It is natural to attempt to match such understanding in the framework of the AdS/CFT correspondence [12]. For AdS_5 black holes the solutions have been constructed over a decade ago [13–17], the thermodynamical properties related to the inner and outer horizons have been discussed [18–26]. A number of recent works has provided microscopic foundations for the black hole entropy using the dual supersymmetric field theories [27–30]. Similar computations have been carried out for other asymptotically AdS black holes [31–35]. It is the right time to ask whether an effective explanation for these microscopic results can be derived.

In this paper we take some steps toward the construction of such a universal description of the microscopic foundations for asymptotically AdS black holes. Our picture provides a concrete path to the understanding of the rate of Hawking radiation for near-extremal asymptotically AdS_5 black holes from the dual $\mathcal{N} = 4$ SYM, which generalizes the previous work by Callan and Maldacena [7] for asymptotically flat black holes and the work by Gubser, Klebanov and Peet on near-extremal black D3-branes [36]. We elaborate on the approach of [37] to near-extremal AdS_5 black holes and obtain the entropy and energy at the outer and the inner horizons both from the gravity side and from the dual $\mathcal{N} = 4$ SYM. The results can be grouped into a left and a right sector. This left-right-structure signals an underlying effective CFT_2 and suggests the possibility of a locally AdS_3 near-horizon geometry. We, indeed, find such region via a near-horizon scaling for rotating black holes proposed in [38]. By applying the Cardy formula to the Virasoro algebra from the near-horizon AdS_3, we obtain the same result for the growth of states as from $\mathcal{N} = 4$ SYM. With this validation we discuss aspects of Hawking radiation only as detected in the near-horizon region.

This paper is organized as follows. In section 2 we review the non-extremal AdS_5 black hole solutions in the literature, discuss the near-extremal thermodynamical relations at the outer horizon, and then generalize these relations to the inner horizon. In section 3 we give a microscopic derivation of the thermodynamical relations for near-extremal AdS_5 black holes from the boundary $\mathcal{N} = 4$ SYM. In section 4 we demonstrate that the results obtained in the previous sections can be formulated as emerged from an effective CFT_2, which can also be justified by taking a special near-horizon limit similar to the Kerr/CFT correspondence. In section 5, we apply these notions to the early-time near-horizon Hawking radition, and find that indeed the radiation rate is thermal. Some possible directions for the future research are discussed in section 6.

2 Near-extremal AdS_5 black hole thermodynamics

Let us briefly review the non-extremal electrically charged rotating AdS_5 black holes, i.e. the non-extremal Kerr-Newman AdS_5 black holes. We follow closely [16], which discusses black holes with generic angular momenta J_1 and J_2 but degenerate electric charges $Q_1 = Q_2 = Q_3 \equiv Q$ within minimal gauged supergravity, which generalizes the non-extremal neutral rotating Kerr AdS_5 black holes discussed in [39] as well as some previously found special charged solutions [13–15]. More general solutions with arbitrary Q_I's were later constructed in [17].

The bosonic part of the 5d minimal gauged supergravity is given by the Lagrangian:

$$\mathcal{L} = (R + 12g^2) * \mathbb{I} - \frac{1}{2} * F \wedge F + \frac{1}{3\sqrt{3}} F \wedge F \wedge A \,, \tag{2.1}$$

where $F = dA$, and $g = L^{-1} > 0$ with L denoting the AdS_5 radius.

It was found in [16] that the equations of motion of the theory (2.1) have the following solution in the Boyer-Lindquist coordinates $x^\mu = (t,\, r,\, \theta,\, \phi,\, \psi)$:

$$
\begin{aligned}
ds^2 = &-\frac{\Delta_\theta\left[(1+g^2r^2)\rho^2 dt + 2q\nu\right]\, dt}{\Xi_a\Xi_b\rho^2} + \frac{2q\nu\omega}{\rho^2} + \frac{f}{\rho^4}\left(\frac{\Delta_\theta dt}{\Xi_a\Xi_b} - \omega\right)^2 + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta} \\
&+ \frac{r^2+a^2}{\Xi_a}\sin^2\!\theta\, d\phi^2 + \frac{r^2+b^2}{\Xi_b}\cos^2\!\theta\, d\psi^2\,, \qquad (2.2)
\end{aligned}
$$

$$
A = \frac{\sqrt{3}\,q}{\rho^2}\left(\frac{\Delta_\theta\, dt}{\Xi_a\Xi_b} - \omega\right)\,, \qquad (2.3)
$$

where

$$
\begin{aligned}
\nu &\equiv b\sin^2\!\theta\, d\phi + a\cos^2\!\theta\, d\psi\,,\\
\omega &\equiv a\sin^2\!\theta\,\frac{d\phi}{\Xi_a} + b\cos^2\!\theta\,\frac{d\psi}{\Xi_b}\,,\\
\Delta_\theta &\equiv 1 - a^2g^2\cos^2\!\theta - b^2g^2\sin^2\!\theta\,,\\
\Delta_r &\equiv \frac{(r^2+a^2)(r^2+b^2)(1+g^2r^2) + q^2 + 2abq}{r^2} - 2m\,, \qquad (2.4)\\
\rho^2 &\equiv r^2 + a^2\cos^2\!\theta + b^2\sin^2\!\theta\,,\\
\Xi_a &\equiv 1 - a^2g^2\,,\\
\Xi_b &\equiv 1 - b^2g^2\,,\\
f &\equiv 2m\rho^2 - q^2 + 2abqg^2\rho^2\,.
\end{aligned}
$$

The angular momenta J_i and the electric charge Q of the AdS_5 black hole can be obtained as follows. From the Komar integral one obtains the angular momenta:

$$
J_i = \frac{1}{16\pi}\int_{S^3} *dK_i\,, \qquad (2.5)
$$

where $i = 1,\, 2$, and

$$
K_1 = \frac{\partial}{\partial\phi}\,, \quad K_2 = \frac{\partial}{\partial\psi}\,. \qquad (2.6)
$$

More explicitly,

$$
\begin{aligned}
J_1 &= \frac{\pi\Big[2am + qb(1+a^2g^2)\Big]}{4\Xi_a^2\Xi_b}\,,\\
J_2 &= \frac{\pi\Big[2bm + qa(1+b^2g^2)\Big]}{4\Xi_b^2\Xi_a}\,.
\end{aligned} \qquad (2.7)
$$

From the Gaussian integral one obtains the electric charge:

$$
Q = \frac{1}{16\pi}\int_{S^3}\left(*F - \frac{1}{\sqrt{3}}F\wedge A\right)\,. \qquad (2.8)
$$

More explicitly,

$$
Q = \frac{\sqrt{3}\pi q}{4\,\Xi_a\Xi_b}\,. \qquad (2.9)
$$

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To obtain the energy (or conserved mass) E, we distinguish the outer and the inner horizons. At the outer horizon, the energy is obtained by integrating the first law of thermodynamics:

$$dE = T_+ dS_+ + \Omega_i^+ dJ_i + \Phi^+ dQ\,, \tag{2.10}$$

where $\Phi^+ \equiv \ell^\mu A_\mu$ is the electric potential at the outer horizon. The explicit expression of the energy is

$$E = \frac{m\pi(2\Xi_a + 2\Xi_b - \Xi_a\Xi_b) + 2\pi qabg^2(\Xi_a + \Xi_b)}{4\Xi_a^2\Xi_b^2}\,. \tag{2.11}$$

At the inner horizon, we can obtain the angular momenta J_i, the electric charge Q and the energy E in the same way, but the first law of thermodynamics at the inner horizon looks different:

$$dE = -T_- dS_- + \Omega_i^- dJ_i + \Phi^- dQ\,. \tag{2.12}$$

A similar expression of the first law has been seen in the previous study on the asymptotically flat black holes [25]. From a direct computation, we see that the expressions of the energy E, the angular momenta J_i and the electric charge Q remain the same at both the outer and the inner horizons.

In the BPS limit, the outer and the inner horizons coincide, i.e. $r_+ = r_-$, and the temperature becomes zero, i.e. $T_+ = T_- = 0$. Moreover, the BPS bound is saturated, i.e.,

$$E - gJ_1 - gJ_2 - 3Q = 0\,, \tag{2.13}$$

which is equivalent to the condition

$$q = \frac{m}{1 + ag + bg}\,. \tag{2.14}$$

To prevent unphysical naked closed timelike curves (CTC), it is shown in [16] that the BPS solutions should further satisfy the constraint:

$$m = \frac{1}{g}(a+b)(1+ag)(1+bg)(1+ag+bg)\,. \tag{2.15}$$

The positions of the horizons are simplified in the supersymmetric limit:

$$r^2\Delta_r = \frac{1}{g^2}\left(a + b + abg - gr^2\right)^2\left[(1+ag+bg)^2 + g^2r^2\right] = 0\,. \tag{2.16}$$

The outer horizon r_+ and the inner horizon r_- coincide in the BPS limit:

$$r_*^2 \equiv \frac{a + b + abg}{g}\,. \tag{2.17}$$

Considering a black hole slightly away from the BPS limit, the degeneracy of r_+ and r_- is lifted by a small change $0 < \delta r \ll r_*$, i.e.,

$$r_+ \longrightarrow r_* + \delta r\,, \qquad r_- \longrightarrow r_* - \delta r\,. \tag{2.18}$$

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One can prove that the vector

$$\ell(r) \equiv \frac{\partial}{\partial t} + \Omega_1(r)\frac{\partial}{\partial \phi} + \Omega_2(r)\frac{\partial}{\partial \psi} \tag{2.19}$$

$$\text{with} \quad \Omega_1(r) \equiv \frac{a(r^2+b^2)(1+g^2r^2)+bq}{(r^2+a^2)(r^2+b^2)+abq}\,, \qquad \Omega_2(r) \equiv \frac{b(r^2+a^2)(1+g^2r^2)+aq}{(r^2+a^2)(r^2+b^2)+abq}\,, \tag{2.20}$$

becomes null, i.e. $g_{\mu\nu}\ell^\mu\ell^\nu = 0$, when the equation $\Delta_r = 0$ holds. Therefore, the vectors $\ell(r_\pm)$ are null Killing vectors at the outer horizon r_+ and the inner horizon r_- respectively, defining Killing horizons. From these Killing vectors, $\ell(r_\pm)$, one can find the corresponding surface gravities $\kappa(r_\pm)$ obeying $\ell^\nu\nabla_\nu\ell^\mu = \kappa\ell^\mu$ in the Eddington-Finkelstein coordinates, which are proportional to $\Delta_r'(r_\pm)/2$, respectively. The temperatures associated with the horizons can be defined by the surface gravity κ at $r_\pm$:

$$T_\pm = \left|\frac{\kappa(r_\pm)}{2\pi}\right| = \frac{r_\pm^4\Big[1+g^2(2r_\pm^2+a^2+b^2)\Big]-(ab+q)^2}{2\pi r_\pm\Big[(r_\pm^2+a^2)(r_\pm^2+b^2)+abq\Big]}\,. \tag{2.21}$$

The entropies at the horizons are proportional to the surface area of the horizons:

$$S_\pm = \frac{\pi^2\Big[(r_\pm^2+a^2)(r_\pm^2+b^2)+abq\Big]}{2\Xi_a\Xi_b r_\pm}\,. \tag{2.22}$$

At the inner horizon we can either choose a positive temperature [25] or a negative temperature [26]. We choose a positive temperature at the inner horizon in this paper. With this choice, the first laws of thermodynamics look different at two horizons. At the outer horizon,

$$dE = T_+dS_+ + \Omega_i^+dJ_i + \Phi^+dQ\,, \tag{2.23}$$

while at the inner horizon,

$$dE = -T_-dS_- + \Omega_i^-dJ_i + \Phi^-dQ\,, \tag{2.24}$$

which differ by a sign.

Let us focus on the near-extremal case in this paper. For such configurations, we should treat the thermodynamics at the outer and the inner horizons separately. As we have seen in (2.23) and (2.24), the first laws at the two horizons differ by a sign, similar to asymptotically flat black holes [25]. At the outer horizon, the following relations have been obtained from the near-extremal Kerr-Newman AdS_5 black holes in [37]:

$$S_+ = S_* + \left(\frac{C}{T}\right)_* T_+\,, \qquad E = E_* + \frac{1}{2}\left(\frac{C}{T}\right)_* T_+^2\,, \tag{2.25}$$

$$\left(\frac{C}{T}\right)_* = \frac{\pi^2 g^{-1}\Big[8(Q/g)^3+\frac{1}{4}N^4(J_1+J_2)\Big]}{3(Q/g)^2-\frac{1}{2}N^2\,(J_1+J_2)+\big(3Q/g+\frac{1}{2}N^2\big)^2}\,, \tag{2.26}$$

with Q and J_i taking the values for the BPS black holes. Above, $(C/T)_*$ is the heat capacity from the outer horizon on the gravity side which provides a counting for near extremal microstates.

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3 Microscopic $\mathcal{N} = 4$ SYM approach

3.1 Microstate counting of BPS AdS_5 black holes

It has recently been shown in [27–29], that entropy of electrically charged, rotating BPS AdS_5 black holes can be obtained from $\mathcal{N} = 4$ SYM by computing the superconformal index with complex chemical potentials, Δ_I:

$$\mathcal{I} = \text{Tr}\Big[(-1)^F e^{-\beta E} e^{-\Delta_I Q_I - \omega_i J_i}\Big] \,. \tag{3.1}$$

In the Cardy limit, for small angular velocites, $|\omega_i| \ll 1$, the leading results of the superconformal index is:

$$\mathcal{F} \equiv \log Z \simeq \log \mathcal{I} \simeq \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_1 \omega_2} \,, \tag{3.2}$$

subject to a constraint with $s_I, t_i \in \{\pm 1\}$:

$$\sum_I s_I \, \Delta_I - \sum_i t_i \, \omega_i = 2\pi i \,. \tag{3.3}$$

A more detailed discussion of the superconformal index in the context of $\mathcal{N} = 4$ SYM was presented in [40, 41]. The leading expression for the superconformal index has been obtained for generic $\mathcal{N} = 1$ supersymmetric field theories exploiting a Cardy-like limit in [42–44]. In two other works [45, 46], the leading contribution to the superconformal index of generic $\mathcal{N} = 1$ supersymmetric theories was computed without recourse to any Cardy limit. This was facilitated by using the Bethe-Ansatz formulation of the index [29, 47] which is based on the original work of [48]. A novel approach to the index, emphasizing modular properties, has recently been introduced in [30] and makes contact with aspects of the Bethe-Ansatz approach.

Let us briefly summarize the extremization process of the entropy function for the BPS AdS_5 black holes [27–29]. After the extremization, one obtains the entropy of $\frac{1}{16}$-BPS electrically charged rotating AdS_5 black holes. We follow closely [27, 37].

First, from the leading order of $\mathcal{N} = 4$ SYM's partition function, one can define the entropy function via a Legendre transformation:

$$S(\Delta_I,\, J_i,\, \Lambda) = -\frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_1 \omega_2} - Q_I \Delta_I - J_i \, \omega_i - \Lambda \left(\sum_I \Delta_I - \sum_i \omega_i - 2\pi i \right) , \tag{3.4}$$

where Λ is a Lagrange multiplier that imposes one choice of the BPS constraint $\sum_I s_I \Delta_I - \sum_i t_i \omega_i = 2\pi i$ with $s_I = t_i = +1$. The entropy function can be viewed as a functional of the electric potentials Δ_I and the angular momenta J_i.

Extremizing the entropy function (3.4) with respect to Δ_I and ω_i, we obtain the equations

$$\frac{\partial S}{\partial \Delta_I} = -\frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\Delta_I \omega_1 \omega_2} - (Q_I + \Lambda) = 0 \,, \tag{3.5}$$

$$\frac{\partial S}{\partial \omega_i} = \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_i \, \omega_1 \omega_2} - (J_i - \Lambda) = 0 \,. \tag{3.6}$$

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Combining the equations (3.5) and (3.6), we obtain

$$\prod_I (Q_I+\Lambda)+\frac{N^2}{2}\prod_i (J_i-\Lambda)=0\,, \tag{3.7}$$

which is a cubic equation for the Lagrange multiplier Λ. Moreover, simplifying the entropy function (3.4) using the extremization conditions (3.5) and (3.6), we find the BPS black hole entropy

$$S_*=2\pi i\Lambda\,. \tag{3.8}$$

In order to express the black hole entropy in terms of Q_I and J_i, we need to solve the cubic equation (3.7) to express Λ as a function of Q_I and J_i. We first write (3.7) as

$$\begin{aligned}&\Lambda^3+\Lambda^2\left(Q_1+Q_2+Q_3+\frac{1}{2}N^2\right)+\Lambda\left(Q_1Q_2+Q_2Q_3+Q_3Q_1-\frac{1}{2}N^2(J_1+J_2)\right)\\&+\left(Q_1Q_2Q_3+\frac{1}{2}N^2J_1J_2\right)=0\,,\end{aligned} \tag{3.9}$$

or equivalently,

$$\Lambda^3+A\Lambda^2+B\Lambda+C=0 \tag{3.10}$$

with

$$\begin{aligned}A&=Q_1+Q_2+Q_3+\frac{1}{2}N^2\,,\\B&=Q_1Q_2+Q_2Q_3+Q_3Q_1-\frac{1}{2}N^2(J_1+J_2)\,,\\C&=Q_1Q_2Q_3+\frac{1}{2}N^2J_1J_2\,.\end{aligned} \tag{3.11}$$

By requiring that the entropy is real, (3.8) implies that Λ has a purely imaginary root. Since Q_I and J_i are all real, the purely imaginary roots for Λ must appear in pairs for the cubic equation (3.10). Consequently, the cubic equation (3.10) should take the form

$$(\Lambda^2+B)(\Lambda+A)=\Lambda^3+A\Lambda^2+B\Lambda+AB=0\,. \tag{3.12}$$

Comparing (3.10) and (3.12), we therefore obtain the constraint $C=AB$, more explicitly,

$$\begin{aligned}&\left(Q_1+Q_2+Q_3+\frac{N^2}{2}\right)\left(Q_1Q_2+Q_2Q_3+Q_3Q_1-\frac{N^2}{2}(J_1+J_2)\right)\\&-\left(Q_1Q_2Q_3+\frac{N^2}{2}J_1J_2\right)=0\,.\end{aligned} \tag{3.13}$$

This is the constraint on Q_I and J_i for the BPS AdS_5 black holes. For generic values of Q_I and J_i we define a function as $h\equiv C-AB$. Hence, for the BPS case the constrait (3.13) becomes $h=0$.

Subject to the constraint (3.13), the purely imaginary root $\Lambda=-i\sqrt{B}$ gives the physical entropy for the $\frac{1}{16}$-BPS electrically charged rotating AdS_5 black holes

$$S=2\pi i\Lambda=2\pi\sqrt{Q_1Q_2+Q_2Q_3+Q_3Q_1-\frac{N^2}{2}(J_1+J_2)}\,. \tag{3.14}$$

This also agrees with the result on the gravity side [49]. Corresponding to the purely imaginary root $\Lambda = -i\sqrt{B}$, the solutions to the extremization equations (3.5) (3.6) are

$$\frac{\Delta_I}{2\pi i} = \frac{\prod_K (Q_K + \Lambda)}{Q_I + \Lambda} \frac{1}{2\Lambda(\Lambda + Q_1 + Q_2 + Q_3 + \frac{1}{2}N^2)} \,, \tag{3.15}$$

$$\frac{\omega_i}{2\pi i} = \frac{N^2}{2} \frac{\prod_k (J_k - \Lambda)}{J_i - \Lambda} \frac{1}{2\Lambda(\Lambda + Q_1 + Q_2 + Q_3 + \frac{1}{2}N^2)} \,. \tag{3.16}$$

3.2 Inner and outer horizons

In this subsection we discuss the extremization process for the near-BPS AdS_5 black holes, following closely [37]. To study the near-BPS AdS_5 black holes, we distinguish two classes: the near-extremal ones and the extremal near-BPS ones. The near-extremal configurations can be obtained by introducing a small temperature T, while the extremal near-BPS ones are characterized by the violation φ of the BPS constraint $\sum_I s_I \Delta_I - \sum_i t_i \omega_i = 2\pi i$.

Let us first consider the near-extremal configurations. Usually in field theory, the temperature is introduced as $T = \beta^{-1}$, where β denotes the period of the Euclidean time circle S^1. However, the $\frac{1}{16}$-BPS index of $\mathcal{N} = 4$ SYM does not depend on β, hence, β cannot be simply interpreted as T^{-1} in this case. Instead, we make use of the following relations to connect the temperature derivatives of the quantities on the gravity side, Φ_I and Ω_i, with the real parts of the quantities on the field theory side, Δ_I and ω_i:

$$\text{Re}(\Delta_I) = \frac{\partial \Phi_I}{\partial T} \,, \quad \text{Re}(\omega_i) = \frac{\partial \Omega_i}{\partial T} \,. \tag{3.17}$$

These relations were found in [27] and checked explicitly in [37].

If we assume that there is no phase transition between BPS black holes and near-extremal ones, that is, that the number of degrees of freedoms varies smoothly with a small temperature, then the spectrum of the microstates in the dual field theory should also change smoothly from BPS to near-extremal ones. The self-consistency of such crucial assumption will be verified *a posteriori* by the fact that there is no jump in the heat capacity from $T = 0$ to an infinitesimal $T > 0$. Based on this assumption, we can use the relations (3.17) to extend the BPS partition function (3.2) of the $\mathcal{N} = 4$ SYM to the one at an infinitesimal temperature $T > 0$ [37]:

$$\log Z = \frac{N^2}{2T} \frac{(\Phi_1 - \Phi_1^*)(\Phi_2 - \Phi_2^*)(\Phi_3 - \Phi_3^*)}{(\Omega_1 - \Omega_1^*)(\Omega_2 - \Omega_2^*)} \,, \tag{3.18}$$

and it is related to other thermodynamic quantities in the following way:

$$\log Z = S - \frac{1}{T}(M - M_*) - \frac{1}{T}(\Phi_I - \Phi_I^*) Q_I - \frac{1}{T}(\Omega_i - \Omega_i^*) J_i \,. \tag{3.19}$$

Moreover, $(\Phi_I - \Phi_I^*)$ and $(\Omega_i - \Omega_i^*)$ satisfy an additional constraint:

$$\sum_I (\Phi_I - \Phi_I^*) - \sum_i (\Omega_i - \Omega_i^*) = \varphi + 2\pi i T \,, \tag{3.20}$$

where φ denotes the violation of the BPS constraint $\sum_I s_I \Delta_I - \sum_i t_i \omega_i = 2\pi i$, that characterizes the extremal near-BPS case, while the near-extremal case is parameterized by T.

In the BPS limit, both φ and T vanish, hence, Φ_I and Ω_i will approach their BPS values Φ_I^* and Ω_i^* respectively.

We can define a free energy for the near-BPS AdS_5 black holes as the counterpart of the BPS entropy function (3.4):

$$\mathcal{F} \equiv (M - M_*) - TS - \Lambda\left[\sum_I (\Phi_I - \Phi_I^*) - \sum_i (\Omega_i - \Omega_i^*) - \varphi - 2\pi i T\right] \tag{3.21}$$

$$\begin{aligned} &= -\frac{N^2}{2}\frac{(\Phi_1 - \Phi_1^*)(\Phi_2 - \Phi_2^*)(\Phi_3 - \Phi_3^*)}{(\Omega_1 - \Omega_1^*)(\Omega_2 - \Omega_2^*)} - (\Phi_I - \Phi_I^*)Q_I - (\Omega_i - \Omega_i^*)J_i \\ &\quad - \Lambda\left[\sum_I (\Phi_I - \Phi_I^*) - \sum_i (\Omega_i - \Omega_i^*) - \varphi - 2\pi i T\right], \end{aligned} \tag{3.22}$$

where Λ is again a Lagrange multiplier that imposes the near-BPS constraint (3.20).

We see that the near-BPS free energy (3.22) has an expression similar to the BPS entropy function (3.4). Hence, we can perform a similar extremization process:

$$\frac{\partial S}{\partial(\Phi_I - \Phi_I^*)} = -\frac{N^2}{2}\frac{(\Phi_1 - \Phi_1^*)(\Phi_2 - \Phi_2^*)(\Phi_3 - \Phi_3^*)}{(\Phi_I - \Phi_I^*)\omega_1\omega_2} - (Q_I + \Lambda) = 0\,, \tag{3.23}$$

$$\frac{\partial S}{\partial(\Omega_i - \Omega_i^*)} = \frac{N^2}{2}\frac{(\Phi_1 - \Phi_1^*)(\Phi_2 - \Phi_2^*)(\Phi_3 - \Phi_3^*)}{(\Omega_i - \Omega_i^*)(\Omega_1 - \Omega_1^*)(\Omega_2 - \Omega_2^*)} - (J_i - \Lambda) = 0\,. \tag{3.24}$$

These extremization equations have the solutions:

$$\frac{\Phi_I - \Phi_I^*}{\varphi + 2\pi i T} = \frac{\prod_K (Q_K + \Lambda)}{Q_I + \Lambda}\frac{1}{2\Lambda(\Lambda + Q_1 + Q_2 + Q_3 + \frac{1}{2}N^2)}\,, \tag{3.25}$$

$$\frac{\Omega_i - \Omega_i^*}{\varphi + 2\pi i T} = \frac{N^2}{2}\frac{\prod_k (J_k - \Lambda)}{J_i - \Lambda}\frac{1}{2\Lambda(\Lambda + Q_1 + Q_2 + Q_3 + \frac{1}{2}N^2)}\,, \tag{3.26}$$

which are similar to the solutions (3.15) (3.16) in the BPS case.

The extremal near-BPS case has been analyzed in more detail in [37], where it is shown that the following relations at the leading order can be obtained from the dual field theory:

$$S - S_* = \left(\frac{C_E}{T}\right)_* \left(\frac{\varphi}{2\pi}\right)\,, \tag{3.27}$$

$$E - E_* = \frac{1}{2}\left(\frac{C_T}{T}\right)_* \left(\frac{\varphi}{2\pi}\right)^2\,, \tag{3.28}$$

where

$$\left(\frac{C_T}{T}\right)_* = \frac{\pi^2\left[(Q_1 + Q_2)(Q_2 + Q_3)(Q_3 + Q_1) + \frac{1}{4}N^4(J_1 + J_2)\right]}{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{1}{2}N^2(J_1 + J_2) + \left(Q_1 + Q_2 + Q_3 + \frac{1}{2}N^2\right)^2}\,, \tag{3.29}$$

$$\left(\frac{C_E}{T}\right)_* = \frac{2\pi}{S_*}\left(\frac{C_T}{T}\right)_*\left(Q_1 + Q_2 + Q_3 + \frac{1}{2}N^2\right)\,. \tag{3.30}$$

These results are consistent with the ones obtained on the gravity side. However, the near-extremal case has not been considered from the dual field theory side in [37].

To derive the relations (2.25) for the near-extremal case from $\mathcal{N}=4$ SYM, the BPS cubic equation (3.12) gets modified:

$$\left[(\Lambda+\delta\Lambda)^2+B\right]\left[(\Lambda+\delta\Lambda)+A\right]+h=0\,, \tag{3.31}$$

with $h\neq 0$ for the near-extremal case. It means that a change in Λ, denoted by $\delta\Lambda$, will cause a small violation of the BPS constraint, $h_*=0$. Comparing (3.12) and (3.31), we obtain at the leading order in $\delta\Lambda$:

$$(3\Lambda^2+2A\Lambda+B)\delta\Lambda+h=0\,. \tag{3.32}$$

Since we consider the near-extremal case by perturbing the BPS case, whose black hole entropy corresponds to one of the purely imaginary roots satisfying $\Lambda^2+B=0$, the equation (3.32) can be simplified. Since only the imaginary part of $\delta\Lambda$ contributes to the real part of δS, assuming that h is purely imaginary, we obtain

$$\mathrm{Im}(\delta\Lambda)=-h\,\mathrm{Re}\left[\frac{1}{2\Lambda A+2\Lambda^2}\right]=\frac{h}{2(A^2-\Lambda^2)}=\frac{h}{2(A^2+B)}\,. \tag{3.33}$$

Consequently, the change of the black hole entropy is

$$\delta S\equiv S_+-S_*=2\pi i\,\delta\Lambda=\frac{i\pi h}{A^2+B}\,. \tag{3.34}$$

To find the entropy excess of the near-extremal black holes compared to the BPS ones, we should relate the function h in (3.34) to the temperatures $T_\pm$, similar to the extremal near-BPS case discussed in [37].

The change δh can be obtained from

$$\delta h=\frac{\partial h}{\partial Q_I}\delta Q_I+\frac{\partial h}{\partial J_i}\delta J_i\,, \tag{3.35}$$

by introducing the transformation parameter λ, as:

$$\delta Q_I=\lambda Q_I\,,\quad \delta J_i=\lambda J_i\,,\quad 2\pi i\,\delta T_+=2\lambda\,. \tag{3.36}$$

By varying $h=C-AB$ we obtain

$$\begin{aligned}\delta h&=-\left[(Q_1+Q_2)(Q_2+Q_3)(Q_3+Q_1)+\frac{1}{4}N^4(J_1+J_2)\right]\lambda\,,\\ &=-\pi i\left[(Q_1+Q_2)(Q_2+Q_3)(Q_3+Q_1)+\frac{1}{4}N^4(J_1+J_2)\right]\delta T_+\,,\end{aligned} \tag{3.37}$$

which is simplified using the BPS constraint $h_*=0$. Since for the near-extremal black holes away from the BPS ones ($T_*=0$, $h_*=0$) we have $\delta h=h$ and $\delta T_+=T_+$, (3.37) leads to

$$h=-\pi i\left[(Q_1+Q_2)(Q_2+Q_3)(Q_3+Q_1)+\frac{1}{4}N^4(J_1+J_2)\right]T_+\,, \tag{3.38}$$

where T_+ denotes the temperature corresponding to the outer horizon. In the degenerate limit $Q_1 = Q_2 = Q_3 \equiv Q$, combining (3.34) and (3.38), we obtain at the order $\mathcal{O}(T_+)$:

$$S_+ - S_* = \frac{\pi^2 \left[8Q^3 + \frac{1}{4}N^4(J_1+J_2)\right]}{3Q^2 - \frac{1}{2}N^2(J_1+J_2) + \left(3Q + \frac{1}{2}N^2\right)^2} T_+ \equiv \left(\frac{C}{T}\right)_* T_+ \,. \tag{3.39}$$

Integrating the first law at the outer horizon (2.23) with fixed values of Q and J_i, we obtain

$$E - E_* = \frac{1}{2}\left(\frac{C}{T}\right)_* T_+^2 \quad \text{with} \quad E_* \equiv J_1 + J_2 + 3Q \,. \tag{3.40}$$

These results exactly match the ones (2.25) (2.26) derived on the gravity side.

We can similarly use $\mathcal{N}=4$ SYM to derive the entropy and the energy relations at the inner horizon. However, (2.18) implies that $\delta T_- \propto -\delta r < 0$. Accoding to our choice (2.21) of positive T_- at the inner horizon, there should be $\delta T_- = -T_-$. Therefore, the relation between h and T_- has a sign change compare to the one between h and T_+ (3.38) and consequently,

$$S_- - S_* = -\left(\frac{C}{T}\right)_* T_- \,, \quad E - E_* = \frac{1}{2}\left(\frac{C}{T}\right)_* T_-^2 \,, \tag{3.41}$$

where for the second equation we integrate the first law at the inner horizon (2.24) with fixed values of Q and J_i. To our knowledge, these results with an explicit heat capacity at the inner horizon have not appeared in the literature before, and we provide the first derivation from $\mathcal{N}=4$ SYM.

4 An effective $\mathrm{CFT_2}$

4.1 An effective $\mathrm{CFT_2}$ near extremality

Let us now reorganize the results obtained for $\mathcal{N}=4$ SYM in a way that suggets an underlying $\mathrm{CFT_2}$ structure.

For the near-extremal case, $T_\pm$ coincide at the leading order. To uncover the existence of a left sector and a right sector, we define

$$S_L \equiv S_* \,, \quad S_R \equiv \left(\frac{C}{T}\right)_* T_+ = \left(\frac{C}{T}\right)_* T_- + \mathcal{O}(\delta r^2) \,, \tag{4.1}$$

$$T_L \equiv \frac{(2E_*/S_* + T_+) + (2E_*/S_* - T_-)}{2} = \frac{2E_*}{S_*} + \mathcal{O}(\delta r^2) \,, \tag{4.2}$$

$$T_R \equiv \frac{(2E_*/S_* + T_+) - (2E_*/S_* - T_-)}{2} = T_+ + \mathcal{O}(\delta r^2) \,. \tag{4.3}$$

Consequently,

$$S_\pm = S_L \pm S_R = c_L T_L \pm c_R T_R \,, \tag{4.4}$$

$$E_+ = E_- = \frac{1}{2} c_L T_L^2 + \frac{1}{2} c_R T_R^2 \,, \tag{4.5}$$

where $c_L \equiv S_*^2/(2E_*)$ and $c_R \equiv (C/T)_*$ denote the central charges of the left and the right sectors respectively, both of which are of the order $\mathcal{O}(N^2)$. These relations are consistent

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with the ones for the asymptotically flat black holes [18–25]. If the temperatures and the mode numbers are related in the following way:

$$T_L = 2\pi\sqrt{\frac{N_L}{c_L\, L}}\,, \quad T_R = 2\pi\sqrt{\frac{N_R}{c_R\, L}}\,, \tag{4.6}$$

the expressions (4.4) can be rewritten as

$$S_\pm = 2\pi\sqrt{\frac{c_L N_L}{L}} \pm 2\pi\sqrt{\frac{c_R N_R}{L}}\,. \tag{4.7}$$

Equation (4.6) will be supported by a different analysis.

4.2 Near-horizon metric and Cardy formula

The effective appearance of the left- and the right-moving modes in $\mathcal{N}=4$ SYM indicates the possibility of a geometric interpretation which we have, indeed, found. Let us consider a near-horizon limit of the metric (2.2) of the non-extremal AdS_5 black hole following the scaling introduced in [38]:

$$r \to r_* + \lambda\,\widetilde{r}\,, \quad t \to \frac{\widetilde{t}}{\lambda}\,, \quad \phi \to \widetilde{\phi} + g\frac{\widetilde{t}}{\lambda}\,, \quad \psi \to \widetilde{\psi} + g\frac{\widetilde{t}}{\lambda}\,. \tag{4.8}$$

In addition, for the near-extremal asymptotically AdS_5 black holes we do not impose the BPS condition (2.14). Instead, we expand the parameter q around its BPS value q_0 for near-extremal black holes as follows

$$q = q_0\,(1+\lambda^2\widetilde{q})\,. \tag{4.9}$$

By taking the limit $\lambda \to 0$, we find the following near-horizon metric:

$$\begin{aligned} ds^2 &= \frac{a}{2g(1+5ag)}\left[-(1+\hat{r}^2)d\hat{t}^2 + \frac{d\hat{r}^2}{1+\hat{r}^2}\right] \\ &\quad + \frac{2a}{g(1-ag)}\,d\theta^2 + \Lambda_{\mathrm{AdS}_5}(\theta)\left[d\widetilde{\phi} + \frac{3a(1-ag)}{2(1+5ag)\sqrt{a\left(a+\frac{2}{g}\right)}}\,\hat{r}\,d\hat{t}\right]^2 \\ &\quad + \frac{a\Big(4-ag+3ag\cos(2\theta)\Big)\cos^2\theta}{2g(1-ag)^2}\left[d\widetilde{\psi} + \frac{6ag\sin^2\theta}{4-ag+3ag\cos(2\theta)}d\widetilde{\phi} + \hat{V}(\theta)\,\hat{r}\,d\hat{t}\right]^2\,, \end{aligned} \tag{4.10}$$

where

$$\hat{r} \equiv \widetilde{r}\sqrt{\frac{g(1+5ag)}{a\,\widetilde{q}(1+ag)(1+2ag)}}\,, \qquad \hat{t} \equiv 2\widetilde{t}\sqrt{\frac{g\,\widetilde{q}(1+2ag)(1+5ag)}{a(1+ag)}}\,, \tag{4.11}$$

$$\Lambda_{\mathrm{AdS}_5}(\theta) \equiv \frac{4a(2+ag)\sin^2\theta}{g(1-ag)(4-ag+3ag\cos(2\theta))}\,, \quad \hat{V}(\theta) \equiv \frac{3a(1-ag)\sqrt{g\left(\frac{2}{a}+g\right)}}{(1+5ag)(4-ag+3ag\cos(2\theta))}\,. \tag{4.12}$$

A further change of coordinates

$$g\,\rho = \hat{r} + \sqrt{1+\hat{r}^2}\cos(\hat{t})\,, \quad g^{-1}\,\tau = \frac{\sqrt{1+\hat{r}^2}\,\sin(\hat{t})}{\hat{r}+\sqrt{1+\hat{r}^2}\cos(\hat{t})} \tag{4.13}$$

can bring the global coordinates $(\hat{t},\,\hat{r})$ into the Poincaré coordinates $(\tau,\,\rho)$. Consequently, the near-horizon metric (4.10) becomes

$$\begin{aligned} ds^2 =& \frac{a}{2g(1{+}5ag)}\left[-\rho^2\,d\tau^2{+}\frac{d\rho^2}{\rho^2}\right]{+}\frac{2a}{g(1{-}ag)}\,d\theta^2 \\ &+\Lambda_{\text{AdS}_5}(\theta)\left[d\hat{\phi}{+}\frac{3a(1{-}ag)}{2(1{+}5ag)\sqrt{a\left(a{+}\frac{2}{g}\right)}}\,\rho\,d\tau\right]^2 \\ &+\frac{a\Big(4{-}ag{+}3ag\cos(2\theta)\Big)\cos^2\theta}{2g(1{-}ag)^2}\left[d\hat{\psi}{+}\frac{6ag\sin^2\theta}{4{-}ag{+}3ag\cos(2\theta)}d\hat{\phi}{+}\hat{V}(\theta)\,\rho\,d\tau\right]^2\,, \end{aligned} \tag{4.14}$$

where

$$\hat{\phi} \equiv \widetilde{\phi} - \frac{3a(1-ag)}{2(1+5ag)\sqrt{a\left(a+\frac{2}{g}\right)}}\,\gamma\,, \quad \hat{\psi} \equiv \widetilde{\psi} - \frac{3a(1-ag)}{2(1+5ag)\sqrt{a\left(a+\frac{2}{g}\right)}}\,\gamma\,, \tag{4.15}$$

with

$$\gamma \equiv \log\left(\frac{1+\sqrt{1+\hat{r}^2}\,\sin(\hat{t})}{\cos(\hat{t})+\hat{r}\,\sin(\hat{t})}\right)\,. \tag{4.16}$$

We see that in the near-horizon metric (4.14) the coordinates $(\tau,\rho,\hat{\phi},\hat{\psi})$ form two U(1)'s fibered over AdS_2, similar to the situation in the Kerr/CFT correspondence [50, 51]. Alternatively, the near-horizon metric (4.14) can also be written in the standard form:

$$ds^2 = f_0(\theta)\left(-\rho^2\,d\tau^2+\frac{d\rho^2}{\rho^2}\right) + f_\theta(\theta)\,d\theta^2 + \gamma_{ij}(\theta)\,\left(dx^i + k^i\rho\,d\tau\right)\,\left(dx^j + k^j\rho\,d\tau\right) \tag{4.17}$$

with $x^i \in \{\hat{\phi},\,\hat{\psi}\}$, and the coefficients $f_0(\theta)$, $f_\theta(\theta)$, $\gamma_{ij}(\theta)$ and k_i's can be easily obtained by comparing (4.14) with (4.17).

To illustrate the idea, we compute the central charges of the Virasoro algebras for the parameters $a = b$ of the non-extremal black holes for simplicity. The left central charge c_L can be computed by applying the standard Kerr/CFT correspondence [50, 51] to the near-horizon metric (4.10) in the global coordinates:

$$\frac{1}{8\pi G}\int_{\partial\Sigma} k_{\zeta_{(m)}}[\mathcal{L}_{\zeta_{(n)}}g,\,g] = -\frac{i}{12}c_L\,(m^3+\alpha m)\,\delta_{m+n,\,0}\,, \tag{4.18}$$

with the mode expansion of a diffeomorphism generating vector ζ as

$$\zeta_{(n)} = -e^{-in\widetilde{\phi}}\frac{\partial}{\partial\widetilde{\phi}} - inre^{-in\widetilde{\phi}}\frac{\partial}{\partial\hat{r}}\,, \tag{4.19}$$

while k_ζ stands for a 2-form defined for a general perturbation $h_{\mu\nu}$ around the background metric $g_{\mu\nu}$:

$$k_\zeta[h,\, g] \equiv -\frac{1}{4}\epsilon_{\alpha\beta\mu\nu}\Big[\zeta^\nu D^\mu h - \zeta^\nu D_\sigma h^{\mu\sigma} + \zeta_\sigma D^\nu h^{\mu\sigma} + \frac{1}{2}h D^\nu \zeta^\mu - h^{\nu\sigma} D_\sigma \zeta^\mu \\ + \frac{1}{2}h^{\sigma\nu}(D^\mu \zeta_\sigma + D_\sigma \zeta^\mu)\Big] dx^\alpha \wedge dx^\beta\,, \tag{4.20}$$

and $\mathcal{L}_\zeta$ denotes the Lie derivative with respect to ζ:

$$\mathcal{L}_\zeta g_{\mu\nu} \equiv \zeta^\rho \partial_\rho g_{\mu\nu} + g_{\rho\nu}\partial_\mu \zeta^\rho + g_{\mu\rho}\partial_\nu \zeta^\rho\,. \tag{4.21}$$

The right central charge c_R can be obtained by applying the quasi-local charge approach in [52–54] to the near-horizon metric in the standard form (4.17):

$$\frac{c_R}{12} = \frac{1}{8\pi G_N}\int d\hat{\phi}\, d\hat{\psi}\, d\theta\, \frac{k_i k_j \gamma_{ij}(\theta)\, \sqrt{\gamma(\theta)\, f_\theta(\theta)}}{2\Lambda_0 f_0(\theta)}\,, \tag{4.22}$$

where the parameter Λ_0 denotes a UV cutoff in r. The explicit expressions for the central charges are

$$c_L = \frac{9\pi a^2}{g(1-ag)(1+5ag)}\,, \tag{4.23}$$

$$c_R = \frac{27\pi a^{\frac{5}{2}}\sqrt{a+\frac{2}{g}}}{2g^2(1-ag)^2(1+5ag)\Lambda_0}\,, \tag{4.24}$$

where we will set the UV cutoff Λ_0 to be $9r_*/\,[2(3-ag)\lambda]$.

Besides the central charges, the Frolov-Thorne temperatures for the near-extremal case are given by

$$T_L = \frac{T_H}{g - \Omega_1(r_*)}\,, \quad T_R = \frac{T_H}{\lambda}\,, \tag{4.25}$$

where T_H is the Hawking temperature. The temperatures in (4.25) are consistent with the previous ones (4.2) (4.3).

With the explicit results of the central charges and the Frolov-Thorne temperatures, the near-extremal AdS_5 black hole entropy can be obtained by using the Cardy formula:

$$S_\pm = c_L T_L \pm c_R T_R\,, \tag{4.26}$$

which reproduces the result (4.4) obtained from $\mathcal{N} = 4$ SYM with $c_R = (C/T)_*$ given by (2.26).

This analysis solidifies the decomposition of the $\mathcal{N} = 4$ SYM results into the left and the right sectors, which becomes manifest from the near-horizon CFT_2. As discussed in [55], the same approach can be obtained to other asymptotically AdS black holes in various dimensions. Besides the near-horizon limit (4.8) and the near-extremal Kerr/CFT correspondence discussed in this subsection, one can also directly study the AdS_2 in the near-horizon region and apply the AdS_2/CFT_1 correspondence to the near-extremal black holes [56, 57].

5 Hawking radiation rate

Having motivated the appearance of an effective CFT_2 from the point of view of $\mathcal{N} = 4$ SYM and from the near-horizon geometry, let us discuss, following closely [7–9], one final implication — the rate of Hawking radiation. The key observation is that any CFT_2 comes naturally equipped with the "momentum" operator which breaks the symmetry between the left- and the right-moving sectors. For the near-extremal case, we consider the following physical limit of the left-moving and the right-moving modes:

$$N_L = N_L^* + \delta N_L \,, \quad N_R = \delta N_R \,, \quad \text{with} \quad \delta N_L = \delta N_R \ll N_L^* \,. \tag{5.1}$$

Assuming that the right-moving modes form a canonical ensemble, then the partition function of the right sector is

$$Z_R = \sum_{N_R} q^{N_R} d(N_R) = \sum_{N_R} q^{N_R} e^{S_R} = \sum_{N_R} q^{N_R} e^{2\pi\sqrt{c_R T_R}} \,. \tag{5.2}$$

Performing a saddle-point approximation with respect to N_R, we obtain the number of right-moving modes:

$$\delta N_R = N_R = q \frac{\partial}{\partial q} \log Z \approx \frac{\pi^2 c_R}{(\log(q))^2} \,, \quad \text{with} \quad \log(q) < 0 \,. \tag{5.3}$$

The occupation number in the right sector is given by the Bose-Einstein statistics:

$$\rho_R(k_0) = \frac{q^n}{1 - q^n} = \frac{e^{-\frac{k_0}{T_R}}}{1 - e^{-\frac{k_0}{T_R}}} \,, \tag{5.4}$$

where n denotes the momentum quantum number of a mode, that moves in the presence of a time circle for the near-horizon region of AdS_5 black holes. From (5.3) and (5.4) we obtain

$$T_R = \frac{k_0}{\pi n} \sqrt{\frac{\delta N_R \, L}{c_R}} = 2\pi \sqrt{\frac{\delta N_R}{c_R \, L}} = 2\pi \sqrt{\frac{N_R}{c_R \, L}} \,, \tag{5.5}$$

where we used $2\pi/L = k_0/(\pi n)$. A similar expression holds for T_L. Since we consider the limit $k_0 \sim T_R \ll T_L$, the occupation number in the left sector is

$$\rho_L(k_0) = \frac{e^{-\frac{k_0}{T_L}}}{1 - e^{-\frac{k_0}{T_L}}} \approx \frac{T_L}{k_0} = \frac{2\pi}{k_0} \sqrt{\frac{N_L}{c_L \, L}} \,. \tag{5.6}$$

As discussed in [7], the Hawking radiation for 5d asymptotically flat near-extremal black holes can be viewed as a scattering process between the left-moving and the right-moving modes in the D1-D5 CFT. Based on our discussion, it is tempting to interpret the Hawking radiation in asymptotically AdS_5 near-extremal black holes as a scattering process between the left-moving and the right-moving modes of the open strings living on a stack of black D3-branes. Near-extremal black D3-branes have been studied in [36] and a derivation of Hawking radiation from black D3-brane dynamics was provided. he Hawking radiation rate for asymptotically flat black holes has also been studied using Kerr/CFT in [58]. Now we are able to find the Hawking radiation from the dual $\mathcal{N}=4$ SYM on the boundary, as expected from the AdS/CFT correspondence [12]. Following [7] one can evaluate the radiation rate as

$$d\Gamma \sim \frac{d^4k}{k_0}\frac{1}{p_0^L\,p_0^R}|\mathcal{A}|^2\,c_L\,\rho_L(k_0)\,\rho_R(k_0)\,, \tag{5.7}$$

where $\mathcal{A}$ denotes the disc amplitude of strings, and the central charge c_L accounts for the degrees of freedom for a given momentum n. Due to (5.4) and (5.6), we see that

$$c_L\,\rho_L(k_0) \sim S_L \propto \text{(horizon area)}\,, \tag{5.8}$$

and consequently,

$$d\Gamma \sim \text{(horizon area)}\cdot\frac{e^{-\frac{k_0}{T_R}}}{1-e^{-\frac{k_0}{T_R}}}d^4k\,. \tag{5.9}$$

Hence, the radiation rate is proportial to the area of the black hole, and has a spectrum obeying the Bose-Einstein statistics with the Hawking temperature $T_H \equiv T_R$. According to this picture, the scattering of modes is unitary, hence there is no information loss during the Hawking radiation process. Clearly, global aspects of asymptotically AdS_5 spacetimes play an important role in tracking Hawking radiation. In particular, it is clear that radiation eventually bounces at the asymptotic boundary of the spacetime and returns to the center. Therefore, the description we provide here should be understood exclusively as an approximate description valid in the region close to the horizon in the sense we have presented and should not be construed to address the late-time aspects of black hole evaporation.

6 Discussions

In this paper, we studied the thermodynamics of electrically charged rotating near-extremal AdS_5 black holes both from the gravity and from the dual $\mathcal{N}=4$ SYM. We started from the BPS AdS_5 black holes previously studied in the literature [27–29] and considered perturbations around this configuration by introducing a small temperature. We carefully distinguished the outer and the inner horizons and found first laws similar to asymptotically flat black holes [25, 26]. We reinterpreted these laws in terms of the left- and right-moving sectors of an effective CFT_2 arising directly from $\mathcal{N}=4$ SYM. We corroborated the existence of the effective CFT_2 by showing its geometric appearance as the set of asymptotic symmetries of a certain near-horizon limit. Armed with this effective CFT_2, we provided a path for understanding aspects of the Hawking radiation rate in near-extremal AdS_5 black holes similar to the asymptotically flat case [7, 8, 10].

Our results suggest that the universality of black hole quantum dynamics in AdS_5 can be related holographically to the existence of an AdS_3 region near the horizon [55, 59, 60]. It would be precisely because of this region that the effective CFT_2 description arises and can be, in principle, understood from the relation between some 4d superconformal algebra and its corresponding 2d CFT. It is also plausible that similar near-extremal relations and effective CFT_2 interpretations exist for asymptotically AdS black holes in other dimensions, but each case deserves more detailed analysis.

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