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Sport Tickets Pricing Strategy with Home Team's Crowd Effect

Abstract: This paper investigates the impact of the crowd effect and financing constraints on pricing strategy by constructing an intertemporal model and introducing the crowd effect into a monopolistic home team's decision-making framework. The results demonstrate that a stronger crowd effect and a larger depreciation rate are always beneficial to the expected profits of the home team, and the home team may price along the inelastic portion of the static demand curve in period 1, 2 and 3, as long as the expected deferred marginal revenue and the additive price from the performance of the preceding match are sufficiently large.

***Keywords*:** Inelastic demand; Intertemporal sports pricing; Crowd effect

***JEL classification*:** C72; D01; D22; L11.

1. Introduction

It has been suggested that inelastic ticket pricing is a result of profit maximization, whereby tickets are underpriced to maximize ancillary revenue such as concessions and merchandise. DeSchriver and Jensen (2002) found that ticket price did not have a significant impact on attendance (Groza, 2010), which has often been referred in other sport demand contexts. It is commonly believed that a monopoly maximizes profit somewhere along the elastic portion of its demand curve. Anderson and Nielsen (2013) demonstrated that inelastic pricing may result from team risk aversion under uncertainty. Even though a large body of literature has attempted to demonstrate the validity of inelastic ticket pricing, Chang et al. (2016) argued that in a two-period setting, a monopolistic team sets a ticket price that will bring deferred strategic revenue from present game success. If the deferred benefit is sufficiently large, a forward-looking, profit-maximizing team prices along the inelastic portion of its static demand curve. An important consideration of Chang et al. (2016) is endogenizing the second-period ticket purchase, and identifying the conditions under which a monopolistic home team prices along the elastic (or inelastic) demand curve.

This paper elucidates the impact of the crowd effect and financing constraints on pricing strategy by constructing an intertemporal model and introducing the crowd effect into a monopolistic home team's decision-making framework. In a two-period model-setting under uncertainty, a monopolistic home team will determine the two-period price at the same time to be profit-maximizing, in which the present attendance level has a positive influence on future demand. The results show that a stronger crowd effect and weaker financing constraints (a larger depreciation rate) are always beneficial to the expected profits of the home team. It also demonstrates that the home team with the crowd effect may price along the inelastic portion of the static demand curve in period 1 and 2, as long as the expected deferred marginal revenue and the additive price from the performance of the preceding match are sufficiently large. In a three-period model-setting, the above results are robust, and the relative association between the attendance levels in period 2 and 3 is related to the crowd effect and the depreciation rate.

The rest of the paper is organized as follows. Section 2 introduces the basic two-period model setting. Section 3 provides the analysis of home team's intertemporal sports-ticket pricing strategy. Section 4 concludes the paper.

2. Basic Two-Period Model

Consider a home team that sells game tickets as a monopolist in an intertemporal model, in which it simultaneously determines the two-period price (present price and future price) to maximize dynamic profit (Alexander, 2001). Following Chang et al. (2016), we assume that a forward-looking home team will consider two components of marginal revenue for a given game: 1) direct revenue from match ticket sales; and 2) deferred revenue from match performance. In addition, attendance level influences present game performance through crowd pressure (Schwartz and Barsky, 1977; Agnew and Carron, 1994), and present game performance will influence future demand of the home game (Noll, 1974; Winfree et al., 2004), which we can regard as deferred home game revenue. We model this by letting the present attendance level enter the second-period inverse demand function through an additive shift in the second-period demand curve:

$$p_{H1} = p(q_{H1}) \tag{1}$$

$$p_{H2} = p(q_{H2}) + \alpha(w_{H1}(q_{H1}, \epsilon) - \bar{w}) \tag{2}$$

where $p(q_{Hi})$ denotes the static (immediate) inverse demand curve for the home team in period i $(i \in \{1,2\})$; $w_{H1}(q_{H1}, \epsilon)$ denotes the likelihood that the home team wins in period 1, given the attendance level; ϵ denotes the natural winning likelihood of the home team in that period, which is determined by numerous objective factors, such as ability differences between the home team and visiting team, weather on the match day, etc.; $\alpha(\ \alpha > 0)$ measures the intensity of the additive shift in the second-period demand curve under the influence of the first-period home team's game performance (it is assumed that a larger winning likelihood in period 1 will correspondingly produce a sharper additive shift of the inverse demand curve in period 2); $\bar{w}$ represents an unbiased expected winning likelihood of the home team (Fort, 2006); and $\bar{w} = E(\epsilon)$.

Thus, the home team's profit is specified as:

$$Max_{\{q_{H1}q_{H2}\}} E\pi(q_{H1}q_{H2})$$
$$= p(q_{H1})q_{H1} + \delta\big(p(q_{H2}) + \alpha(Ew_{H1}(q_{H1},\epsilon) - E(\epsilon))\big)q_{H2} - c(q_{H1})$$
$$- \delta c(q_{H2}) \tag{3}$$

where $E\pi(q_{H1}, q_{H2})$ denotes the expected present value of the aggregate home game profit for the representative home team in period 1 and 2; and δ $(0 < \delta \leq 1)$ is the depreciation rate, which measures the intensity of financing constraints experienced by the home team. Importantly, as δ decreases, the home team will value cash flow more and experience more intensive financing constraints (Fazzari et al., 1988). In addition, $c(q_{Hi})$ denotes the cost of providing attendance services in period i. It is also the case that $(\partial w_{H1}/\ \partial q_{H1}) > 0$, $c'(q_{Hi}) \geq 0$, and $p'(q_{Hi}) < 0$.

In accordance with Andersen and Nielsen (2013), we assume the representative team's ticket prices are determined in advance of a game. Consider the marginal cost of supplying tickets as being positive. The first-order conditions of (3) are then specified and rearranged as follows:

$$\frac{\partial p(q_{H1})}{\partial q_{H1}} q_{H1} + p_{H1} = c'(q_{H1}) - \alpha\delta \frac{\partial Ew_{H1}}{\partial q_{H1}} q_{H2} \tag{4}$$
$$\frac{\partial p_{H2}}{\partial q_{H2}} q_{H2} + p_{H2} = c'(q_{H2}) \tag{5}$$

$$\frac{\partial p(q_{H2})}{\partial q_{H2}} q_{H2} + p(q_{H2}) = c'(q_{H2}) \ - \alpha(Ew_{H1} - E(\epsilon)) \tag{6}$$

The left-hand-side of equation (4) and (6) are the marginal revenues for the static demand curve in period 1 and 2, respectively; and equation (5) represents the marginal revenues for the dynamic demand curve in period 2. Since $c'(q_{Hi}) \geq 0$, in a general way, a myopic home team always chooses to price where the marginal revenue is positive (i.e., along the elastic portion of the demand curve) to maximize its own profit. However, a forward-looking team will locate along the inelastic portion of its static demand curve in period 1 and 2, if the expected deferred marginal revenue $[\alpha\delta \frac{\partial Ew_{H1}}{\partial q_{H1}} q_{H2}]$ and additive price $[\alpha(Ew_{H1} - E(\epsilon))]$ from attendance in period 1 are sufficiently large. Therefore, we have the following Lemma 1:

Lemma 1: *A monopolistic home team with intertemporal strategic considerations may price along the inelastic portion of its static demand curve in period 1 and 2 if:*

(i) *the expected deferred marginal revenue from the attendance level is greater than the marginal cost of providing attendance services in period 1; and*

(ii) *the additive price from the performance of the preceding match is greater than the marginal cost of providing attendance services in period 2.*

3. An Intertemporal Sports-Ticket Pricing Strategy

We now use the basic model to perform further investigations. Following Krautmann and Berri (2007), we assume a linear inverse demand function:

$$p(q_i) = a - bq_i \ , \ (a \geq b > 0)^1 \tag{7}$$

and consider that the winning likelihood of the home team increases in attendance

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[1] Obviously, the stadium maximum capacity q_i is $\frac{a}{b}$, i.e., $0 < q_i \leq \frac{a}{b}$. In addition, the maximum reserve price that sports fans are willing-to-pay is a. Even if the expected winning likelihood increased 100% in the first-stage game, the extent of the additive shift in the second-stage demand curve is no more than a, i.e., $0 < \alpha \leq a$.

level in period 1 and is influenced by the natural winning likelihood. Following Andersen and Vetter (2015), we specify a winning likelihood function as follows:

$$w_{Hi}(q_{Hi}, \epsilon) = f(q_{Hi}) + \epsilon, i = 2,3 \tag{8}$$

where $f(q_{Hi}) = k * b * \frac{q_{H1}}{a}$ denotes the part of winning likelihood determined by the crowd size of the home game in period 1. In which, $b * \frac{q_{Hi}}{a}$ denotes the crowd size in period i, and k $(k > 0)$ measures the strength of the effect of crowd size on the winning likelihood of the home team. Assume that the natural winning likelihood ϵ is subject to a uniform distribution function between 0 and 1 (i.e., the home team and visiting team are balanced in terms of competitiveness). We can then obtain $\overline{w} = E(\epsilon) = 0.5$ (Simmons, 2006). Furthermore, as $Ew_{Hi} \leq 1$ and crowd size ($b * \frac{q_{Hi}}{a}$) $\leq$ 1, we have k $\leq$1/2.

Certain costs, such as stadium cleanup and vendor labor (provided that the concession services are not outsourced), do depend on attendance (Boyd and Boyd, 1996). Therefore, we specify a linear cost function for simplicity: $C(q_{Hi}) = cq_{Hi}(a > c > 0)$[2].

3.1. Two-Period Setting

Considering two periods, the home team's objective can be simplified as follows:

$$Max_{\{q_{H1}\ q_{H2}\}} E\pi = (a - bq_{H1})q_{H1} + \delta\left(a - bq_{H2} + \alpha kb\frac{q_{H1}}{a}\right)q_{H2} - cq_{H1} - \delta cq_{H2} \tag{9}$$

For simplicity, we define the crowd effect, $\beta = \alpha k$, which measures the rising extent of the sports-ticket price in the second-stage when the crowd size increases by 100% (this is unrealistic, however, because the maximum value of crowd size is no greater than 100%). Obviously, this requires $0 < \beta \leq \frac{1}{2}a$. We can then rewrite

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[2] This conclusion will not change even if the fixed cost is considered.

equation (9) as:

$$Max_{\{q_{H1}\ q_{H2}\}} E\pi = (a - bq_{H1})q_{H1} + \delta\left(a - bq_{H2} + \beta b\frac{q_{H1}}{a}\right)q_{H2} - cq_{H1} - \delta cq_{H2} \quad (10)$$

As previously mentioned, the home team simultaneously determines q_{H1} and q_{H2} to maximize its own profit, specified in (10). By differentiation, we obtain:

$$\frac{\partial E\pi}{\partial q_{H1}} = a - c - 2bq_{H1} + \frac{b\beta\delta q_{H2}}{a} = 0 \quad (11)$$

$$\frac{\partial E\pi}{\partial q_{H2}} = -c\delta + \delta\left(a + \frac{b\beta q_{H1}}{a} - 2bq_{H2}\right) = 0 \quad (12)$$

Solving equations (11) and (12) simultaneously, we derive the following equilibrium outcomes:

$$q_{H1}{}^* = \frac{a(2a^2 - 2ac + a\beta\delta - c\beta\delta)}{b(4a^2 - \beta^2\delta)}, \qquad q_{H2}{}^* = \frac{a(2a^2 - 2ac + a\beta - c\beta)}{b(4a^2 - \beta^2\delta)}$$

Accordingly, we can obtain the optimal sports-ticket prices for period 1 and 2, respectively:

$$p_{H1}{}^* = p(q_{H1})^* = a - \frac{a(a - c)(2a + \beta\delta)}{4a^2 - \beta^2\delta}$$

$$p_{H2}{}^* = p(q_{H2})^* + \alpha(2bq_{H1}{}^* - 1) = c + \frac{a(a - c)(2a + \beta)}{4a^2 - \beta^2\delta}$$

$$E\pi^* = \frac{a(a - c)^2(a + a\delta + \beta\delta)}{b(4a^2 - \beta^2\delta)}$$

The equilibrium solutions mainly depend on crowd effect β and depreciation rate δ, indicating that they both play a significant role in affecting the home team's sports-ticket pricing strategies and expected profits. We have the following Lemma 2.

Lemma 2: *In an intertemporal sports-ticket pricing model with a home advantage, the equilibrium attendance level of a home game in period 1 and 2, and the price in period 2, are all increasing with an increase in the degree of the crowd effect or the*

depreciation rate, while the price in period 1 is decreasing. In addition, the expected present value of the home-game aggregate profits in period 1 and 2 is increasing.
(For the corresponding proof, see the Appendix).

The above lemma is intuitive and straightforward of understanding. Specifically, the stronger the crowd effect, the more aggressive the home team will be to increase the present attendance level by reducing the present price to obtain more deferred revenue. At the same time, the expected marginal revenue is more than the marginal cost in period 2 under a larger β. A rational home team will improve the attendance level in period 2. Although the price decreases as the attendance level rises in period 2, the price-increase effects induced by the crowd effect dominate the price-decrease effects through the improvement of attendance level, eventually leading to a higher price in period 2. In addition, a larger depreciation rate will also increase the expected deferred marginal revenue, motivating the home team to increase the present attendance level by reducing the present price. Hence, the mechanism underlying the depreciation rate is similar to the crowd effect.

Regarding the home team's aggregate expected profit, a larger crowd effect expands the demand in period 2, which will make the home team earn increased profits. Coupled with its impact, weaker financing constraints will further augment the present value of the second-stage profit of the home team. Thus, both of these effects together contribute to the improvement of the expected present value of the aggregate profits of the home team.

Since we assume that the home team will determine the two-period price at the same time to be profit-maximizing, we compare the prices in two periods. We have the following Proposition 1.

Proposition 1. *In an intertemporal sports-ticket pricing model with a home advantage, the present sports-ticket price is always lower than the future price, and the present attendance level is no more than the future attendance level, i.e.,* ${q_{H1}}^* < {q_{H2}}^*$ *when* $0 < \delta < 1$; ${q_{H1}}^* = {q_{H2}}^*$ *when* $\delta = 1$.
(For the corresponding proof, see the Appendix).

This result reflects the forward-looking pricing decision of the home team, and demonstrates that it is quite likely for a rational home team to choose the low-price strategy to attract audiences, i.e., "throwing a sprat to catch a herring". Specifically, as long as the deferred revenue exceeds the current profit loss, the home team is willing to sacrifice its temporary interest by reducing the present sports-ticket price, which may decrease the present sports-ticket price. Although the attendance level in period 1 and 2 is more than that without the home advantage, the attendance level in period 2 increases more than that in period 1 due to being motivated by the crowd effect under financing constraints ($0 < \delta < 1$). On the other hand, the attendance level in period 2 equals that in period 1 without financing constraints ($\delta = 1$).

The above proposition implies that the home team may determine a relatively lower price in the first-period, considering the long-term benefit. Similar to the basic model, the home team may also choose the inelastic portion of its static demand curve to price. Hence, whether and how will the home team determine the pricing strategy? We, respectively, compare ${q_{H1}}^*, {q_{H2}}^*$ with respect to $\frac{a}{2b}$ (the midpoint of the static demand function). We then obtain the following Proposition 2.

Proposition 2. *In an intertemporal sports-ticket pricing model with a home advantage, a forward-looking monopolistic home team may price along the inelastic portion of its home game static demand curve in period 1 and 2, as long as the net revenue increase* *in attendance-related purchases (i.e., deferred revenue) offsets the marginal costs of admittance. Specifically,* ${q_{H1}}^* > \frac{a}{2b}$*,* ${q_{H2}}^* > \frac{a}{2b}$ *when* $0 < b \leq a$ *and* $0 < c < \frac{a}{4}$ *and* $2c < \beta \leq \frac{a}{2}$ *and* $\frac{4ac}{2a\beta - 2c\beta + \beta^2} < \delta \leq 1$.

(For the corresponding proof, see the Appendix).

Given conditions $0 < b \leq a$ and $0 < c < \frac{a}{4}$, with a sufficiently larger crowd effect and depreciation rate, i.e., $2c < \beta \leq \frac{a}{2}$ and $\frac{4ac}{2a\beta - 2c\beta + \beta^2} < \delta \leq 1$, the expected deferred marginal revenue $(\frac{b\beta\delta {q_{H2}}^*}{a})$ is greater than the marginal cost (c) in period 1. In addition, the additive price from the performance of the preceding match $(\frac{b\beta {q_{H1}}^*}{a})$ is greater than the marginal cost (c) in period 2, which means that the marginal revenue of the static demand curve in period 1 and 2 $(a - 2b{q_{H1}}^*, a - 2b{q_{H2}}^*)$ is less than zero. In other words, the home team prices along the inelastic portion of its static demand curve in period 1 and 2.

We now discuss all of the four possible cases, in which: (i) ${p_{H1}}^*$ and ${p_{H2}}^*$ are both inelastic;[3] (ii) ${p_{H1}}^*$ and ${p_{H2}}^*$ are both elastic; (iii) ${p_{H1}}^*$ is elastic, but ${p_{H2}}^*$ is inelastic; and (iv) ${p_{H1}}^*$ is inelastic, but ${p_{H2}}^*$ is elastic. For simplicity, we assume $b = a = 1$ and $c = \frac{1}{8}$. Then, the midpoint of the static demand function is $q_{Hi} = \frac{1}{2} (i = 1,2)$, and we have: (i) ${q_{H1}}^* > \frac{1}{2}$, ${q_{H2}}^* > \frac{1}{2}$ when $\frac{1}{4} < \beta \leq \frac{1}{2}$, $\frac{2}{7\beta + 4\beta^2} < \delta \leq 1$(see Figure 1); (ii)${q_{H1}}^* < \frac{1}{2}$, ${q_{H2}}^* < \frac{1}{2}$ when $0 < \beta < \frac{1}{4}, 0 < \delta \leq 1$ or $\frac{1}{4} < \beta < \frac{2}{7}, 0 < \delta < \frac{2 - 7\beta}{4\beta^2}$ (see Figures 2 and 3); (iii) ${q_{H1}}^* < \frac{1}{2}, {q_{H2}}^* > \frac{1}{2}$

[3] Note that we call ${p_{Hi}}^* (i = 1,2)$ inelastic only if ${q_{Hi}}^*$ is above the midpoint of its static demand function $(\frac{a}{2b})$; if not, we call ${p_{Hi}}^*$ elastic.

when $\frac{1}{4} < \beta \leq \frac{2}{7}$, $\frac{2-7\beta}{4\beta^2} < \delta < \frac{2}{7\beta+4\beta^2}$ or $\frac{2}{7} < \beta \leq \frac{1}{2}, 0 < \delta < \frac{2}{7\beta+4\beta^2}$ (see Figures 4 and 5) and (iv) it is impossible to obtain inelastic ${p_{H1}}^*$ and elastic ${p_{H2}}^*$ (For the corresponding proof, see the Appendix).

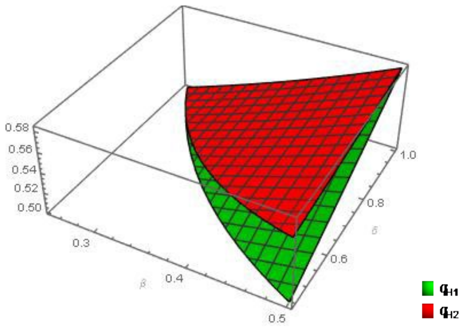

Figure 1. ${p_{H1}}^*, {p_{H2}}^*$ are both inelastic.

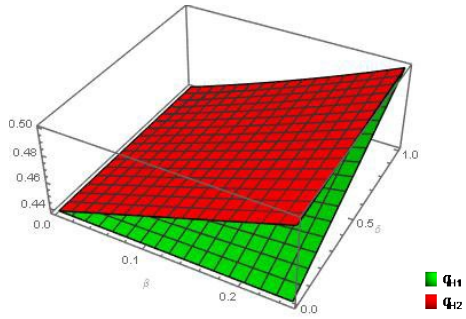

Figure 2. ${p_{H1}}^*, {p_{H2}}^*$ are both elastic.

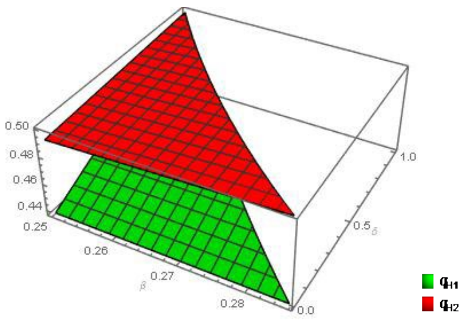

Figure 3. ${p_{H1}}^*, {p_{H2}}^*$ are both elastic.

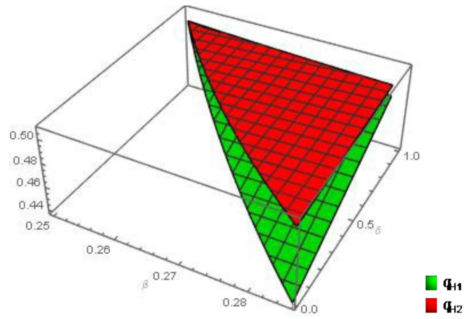

Figure 4. ${p_{H1}}^*$ is elastic, but ${p_{H2}}^*$ is inelastic.

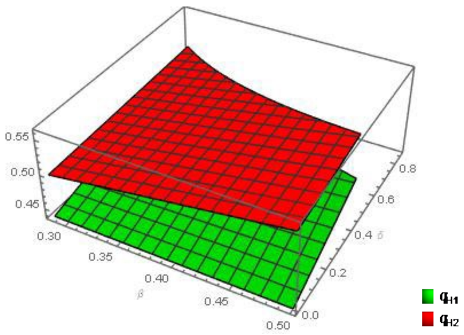

Figure 5. ${p_{H1}}^*$ is elastic, but ${p_{H2}}^*$ is inelastic.

Inelastic price in period 1 or 2 depends on both the crowd effect and depreciation rate. Specifically, when the crowd effect and depreciation rate are relatively large ($\frac{1}{4} <$

$\beta \leq \frac{1}{2}, \frac{2}{7\beta+4\beta^2} < \delta \leq 1$), the home team will price along the inelastic portion of its static demand curve in period 1 and 2, due to being motivated by a relatively larger deferred revenue in period 1 and additive price in period 2. On the contrary, the home team may price along the elastic portion of its home game static demand curve in period 1 and 2 when the crowd effect and depreciation rate are relatively smaller ($0 < \beta < \frac{1}{4}, 0 < \delta \leq 1$ or $\frac{1}{4} < \beta < \frac{2}{7}, 0 < \delta < \frac{2-7\beta}{4\beta^2}$), so that the expected deferred marginal revenue ($\frac{7b\beta\delta(2+\beta)}{8(4-\beta^2\delta)}$) is smaller than the marginal cost ($\frac{1}{8}$) in period 1 and the additive price from the performance of the preceding match ($\frac{7b\beta(2+\beta\delta)}{8(4-\beta^2\delta)}$) is smaller than marginal cost ($\frac{1}{8}$) in period 2. A similar analysis can be applied to the third case. It is worth noting that an inelastic ${p_{H1}}^*$ in period 1 usually means relatively larger deferred revenue in period 1 and additive price in period 2. In this condition, the home team will increase the attendance level over the midpoint of the static demand function in period 2 ($\frac{1}{2}$) to realize maximum profits. Therefore, it is impossible to determine an elastic price in period 2 for a forward-looking home team given an inelastic price in period 1. Of note, the above figures show that the present sports-ticket price is always lower than the future price, and the present attendance level is no more than the future attendance level, whether it is elastic pricing or inelastic pricing. It is the same with Proposition 1.

3.2. Three-Period Setting

Furthermore, we continue to investigate the ticket pricing strategy in three periods.[4] The home team's objective function then is expressed as:

$$Max_{\{q_{H1}\ q_{H2}, q_{H3}\}} E\pi = (a - bq_{H1})q_{H1} + \delta\left(a - bq_{H2} + \beta b\frac{q_{H1}}{a}\right)q_{H2}$$

[4] Following the reviewers' suggestions, we consider the case in three periods to further clarify pricing strategy of the home team with the crowd effect.

$$+\delta^2\left(a - bq_{H3} + \beta b\frac{q_{H2}}{a}\right)q_{H3} - cq_{H1} - \delta cq_{H2} - \delta^2 cq_{H3} \tag{13}$$

Following the subsection 3.1, the home team simultaneously chooses the prices in three periods to maximize its profit. Thus, taking the partial derivative of (13) with respect to prices, respectively, we have:

$$\frac{\partial E\pi}{\partial q_{H1}} = a - c - 2bq_{H1} + \frac{b\beta\delta q_{H2}}{a} = 0 \tag{14}$$

$$\frac{\partial E\pi}{\partial q_{H2}} = -c\delta + \delta\left(a + \frac{b\beta q_{H1}}{a} - 2bq_{H2}\right) = 0 \tag{15}$$

$$\frac{\partial E\pi}{\partial q_{H3}} = \frac{(a^2 - a(c + 2bq_{H3}) + b\beta q_{H2})\delta^2}{a} = 0 \tag{16}$$

Solving (14)-(16), we obtain the equilibrium outcomes:

$$q_{H1}^* = \frac{(a-c)(4a^2+2a\beta\delta-\beta^2\delta(1-\delta))}{8a^2b-4b\beta^2\delta},\quad q_{H2}^* = \frac{a(a-c)(2a+\beta+\beta\delta)}{4a^2b-2b\beta^2\delta},$$

$$q_{H3}^* = \frac{(a-c)(4a^2+2a\beta+\beta^2(1-\delta))}{8a^2b-4b\beta^2\delta}.$$

We then have the optimal sports-ticket prices for every period and expected profit:

$$p_{H1}^* = a - \frac{(a-c)(4a^2+2a\beta\delta-\beta^2(1-\delta)\delta)}{8a^2-4\beta^2\delta},$$

$$p_{H2}^* = \frac{2a^2-c\beta(1-\delta)+a(2c+\beta-\beta\delta)}{4a},$$

$$p_{H3}^* = \frac{4a^3+2a^2(2c+\beta)-c\beta^2(1+3\delta)-a\beta(2c-\beta+\beta\delta)}{8a^2-4\beta^2\delta},$$

$$E\pi^* = \frac{(a-c)^2(4a\beta\delta(1+\delta)+4a^2(1+\delta+\delta^2)-\beta^2(1-\delta)^2\delta)}{8b(2a^2-\beta^2\delta)}.$$

The above equilibrium outcomes mainly depend on crowd effect β and depreciation rate δ. The crowd effects and the effects of depreciation rate on all equilibrium outcome are similar to the case in two-period setting, which further verities the explanation that the stronger the crowd effect, the more aggressive the home team will be to increase the present attendance level by reducing the recent price to obtain more deferred revenue. Of note, when the depreciation rate is very low, the home team will increase the attendance in period 2. The main reasoning is that when the depreciation rate is very low, the depreciated revenue in period 3 is very small, hence, the home team will increase the attendance in period 2 for maximizing its profit.

Furthermore, in three-period setting, we compare ${q_{H1}}^*, {q_{H2}}^*, {q_{H3}}^*$ with respect to $\frac{a}{2b}$ (the midpoint of the static demand function) and then have the same results as in Proposition 2. In order to visualize the results, we assume $a = b = 1$ and $c = \frac{1}{8}$ and have the following five scenarios:

(1)$q_{H1}^* > \frac{1}{2}$, $q_{H2}^* > \frac{1}{2}$ and $q_{H3}^* > \frac{1}{2}$, when $\frac{(\sqrt{113}-7)}{16} < \beta < \frac{1}{2}$, $\frac{\sqrt{308+9\beta(28+9\beta)}-14-9\beta}{14\beta} < \delta < 1$(see Figure 6);

(2) $q_{H1}^* < \frac{1}{2}$, $q_{H2}^* > \frac{1}{2}$ and $q_{H3}^* > \frac{1}{2}$, when $\frac{(\sqrt{113}-7)}{16} < \beta < \frac{1}{4}$, $\frac{4-7\beta(2+\beta)}{9\beta^2} < \delta < \frac{\sqrt{308+9\beta(28+9\beta)}-14-9\beta}{14\beta}$; or $\frac{1}{4} < \beta < \frac{1}{2}$, $\frac{2-7\beta}{7\beta+8\beta^2} < \delta < \frac{\sqrt{308+9\beta(28+9\beta)}-14-9\beta}{14\beta}$(see Figure 7 and 8);

(3) $q_{H1}^* < \frac{1}{2}$, $q_{H2}^* < \frac{1}{2}$ and $q_{H3}^* > \frac{1}{2}$, when $\frac{(\sqrt{113}-7)}{16} < \beta < \frac{1}{4}$, $0 < \delta < \frac{2-7\beta}{7\beta+8\beta^2}$; or $\frac{1}{4} < \beta < \frac{(\sqrt{77}-7)}{7}$, $0 < \delta < \frac{4-7\beta(2+\beta)}{9\beta^2}$(see Figure 9 and 10);

(4) $q_{H1}^* < \frac{1}{2}$, $q_{H2}^* > \frac{1}{2}$ and $q_{H3}^* < \frac{1}{2}$, when $\frac{(\sqrt{113}-7)}{16} < \beta < \frac{1}{4}$, $\frac{2-7\beta}{7\beta+8\beta^2} < \delta < \frac{4-7\beta(2+\beta)}{9\beta^2}$; or $\frac{(\sqrt{65}-7)}{8} < \beta < \frac{(\sqrt{113}-7)}{16}$, $\frac{2-7\beta}{7\beta+8\beta^2} < \delta < 1$(see Figure 11 and 12);

(5) $q_{H1}^* < \frac{1}{2}$, $q_{H2}^* < \frac{1}{2}$ and $q_{H3}^* < \frac{1}{2}$, when $\frac{(\sqrt{113}-7)}{16} < \beta < \frac{1}{4}$, $0 < \delta < \frac{2-7\beta}{7\beta+8\beta^2}$; or $\frac{1}{4} < \beta < \frac{(\sqrt{77}-7)}{7}$, $0 < \delta < \frac{4-7\beta(2+\beta)}{9\beta^2}$; or $0 < \beta < \frac{(\sqrt{65}-7)}{8}$, $0 < \delta < 1$(see Figure 13, 14 and 15).

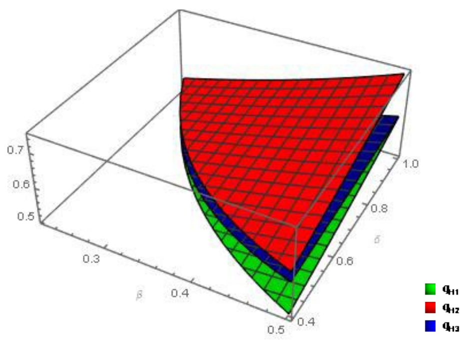

Figure 6. $p_{H1}{}^{*}, p_{H2}{}^{*}, p_{H3}{}^{*}$ are both inelastic.

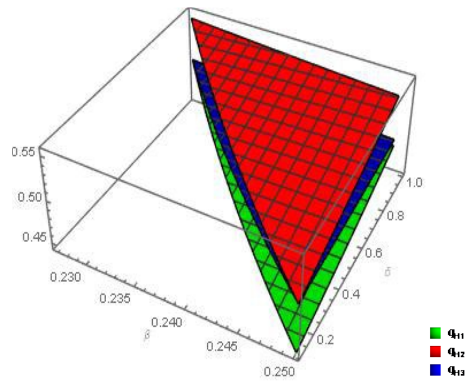

Figure 7. $p_{H1}{}^{*}$ is elastic, but $p_{H2}{}^{*}, p_{H3}{}^{*}$ are both inelastic.

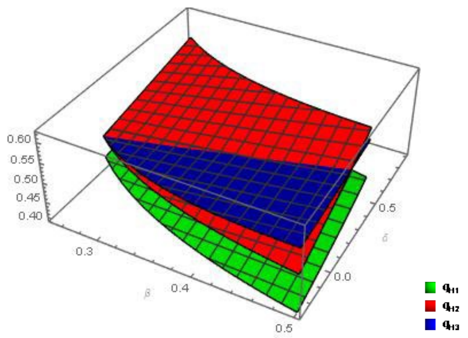

Figure 8. $p_{H1}{}^{*}$ is elastic, but $p_{H2}{}^{*}, p_{H3}{}^{*}$ are both inelastic.

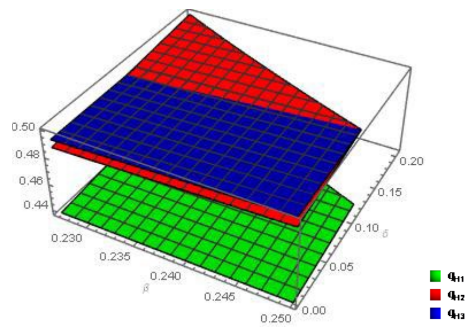

Figure 9. $p_{H1}{}^{*}, p_{H2}{}^{*}$ are both elastic, but $p_{H3}{}^{*}$ is inelastic.

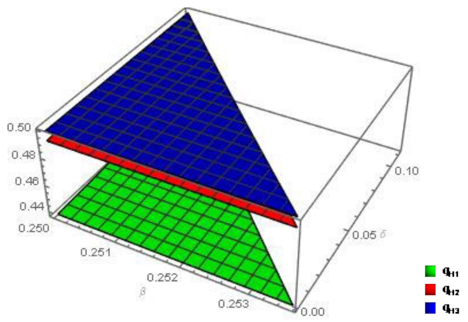

Figure 10. $p_{H1}{}^{*}, p_{H2}{}^{*}$ are both elastic, but $p_{H3}{}^{*}$ is inelastic.

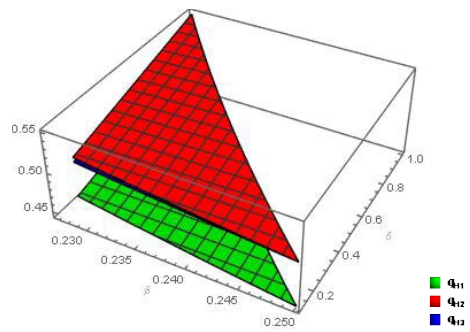

Figure 11. $p_{H1}{}^{*}, p_{H3}{}^{*}$ are both elastic, but $p_{H2}{}^{*}$ is inelastic.

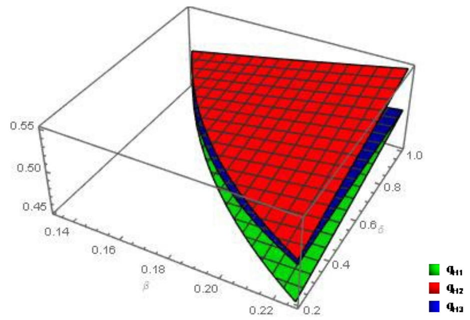

Figure 12. $p_{H1}{}^{*}, p_{H3}{}^{*}$ are both elastic, but $p_{H2}{}^{*}$ is inelastic.

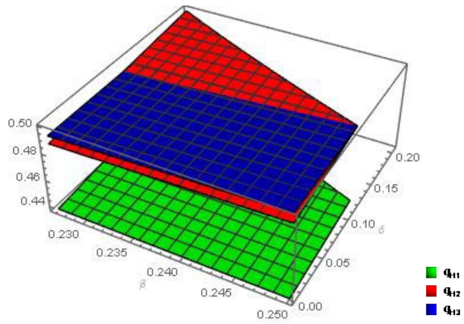

Figure 13. $p_{H1}{}^{*}, p_{H2}{}^{*}, p_{H3}{}^{*}$ are both elastic.

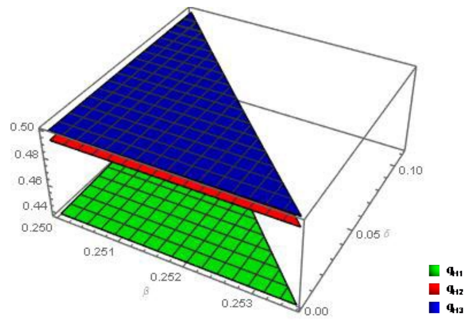

Figure 14 $p_{H1}{}^{*}, p_{H2}{}^{*}, p_{H3}{}^{*}$ are both elastic.

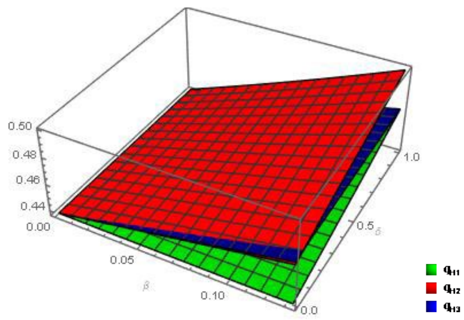

Figure 15. $p_{H1}{}^{*}, p_{H2}{}^{*}, p_{H3}{}^{*}$ are both elastic.

In three-period setting, inelastic price in every period is dependent on the crowd effect and depreciation rate. When the crowd effect and depreciation rate are relatively large ($\frac{(\sqrt{113}-7)}{16} < \beta < \frac{1}{2}$, $\frac{\sqrt{308+9\beta(28+9\beta)}-14-9\beta}{14\beta} < \delta < 1$), the home team will price along the inelastic portion of its static demand curve in every period. On the contrary, the home team may price along the elastic portion of its home game static demand curve in every period, when the crowd effect and depreciation rate are relatively smaller ($\frac{(\sqrt{113}-7)}{16} < \beta < \frac{1}{4}$, $0 < \delta < \frac{2-7\beta}{7\beta+8\beta^2}$; or $\frac{1}{4} < \beta < \frac{(\sqrt{77}-7)}{7}$, $0 < \delta < \frac{4-7\beta(2+\beta)}{9\beta^2}$; or $0 < \beta < \frac{(\sqrt{65}-7)}{8}$, $0 < \delta < 1$). Hence, a forward-looking monopolistic home team may price along the inelastic portion of its home game static demand curve in every period, as long as the net revenue increase in attendance-related purchases (i.e., deferred revenue) offsets the marginal costs of admittance. It is worth noting that the price strategy does not affect the result that the present sports-ticket price is always lower than the future price, and the present attendance level is no more than the future attendance levels. However, the differences are that the future attendance level in period 2 may be larger or smaller than the future attendance level in period 3, which is related to the relatively magnitude of the crowd effect and the depreciation rate.

The major events as we observed are that the team in the season pays a higher salary "poaching" a star from competing teams, to strengthen the team's future performance. The loyal fans will then have more confidence on the winning likelihood, and the crowd effect becomes larger, resulting in a high attendance level, vice versa.

4. Conclusions

This paper constructed an intertemporal model with the crowd effect into a monopolistic home team's decision-making framework, and illustrated some crucial impacts of the crowd effect and financing constraints on pricing strategy, and further compared the pricing characteristics of two stages. It demonstrated that a stronger crowd effect and weaker financing constraints (a larger depreciation rate) are always beneficial to the expected profits of the home team. Furthermore, the sports-ticket price in period 1 is always lower than that in period 2, and the attendance level in period 1 is no more than that in period 2. Moreover, it is shown that the home team may price along the inelastic portion of the static demand curve in period 1 and 2, as long as the expected deferred marginal revenue and the additive price from the performance of the preceding match are sufficiently large. In three periods, the above results are robust; however, the relationship between the attendance levels in period 2 and 3 is dependent on the relatively magnitude of the crowd effect and the depreciation rate. For instance, a news about poaching a star from competing team will increase the future attendance level.

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Appendix

Proof of Lemma 2

Differentiating $p_{H1}{}^*$, $q_{H1}{}^*$, $p_{H2}{}^*$, $q_{H2}{}^*$ and $E\pi^*$ with respect to β and δ, respectively, we obtain:

$$\frac{\partial p_{H1}{}^*}{\partial \beta} = -\frac{a(a-c)\delta(4a(a+\beta)+\beta^2\delta)}{(-4a^2+\beta^2\delta)^2} < 0 \qquad (A1)$$

$$\frac{\partial q_{H1}{}^*}{\partial \beta} = \frac{a(a-c)\delta(4a(a+\beta)+\beta^2\delta)}{b(-4a^2+\beta^2\delta)^2} > 0 \qquad (A2)$$

$$\frac{\partial p_{H2}{}^*}{\partial \beta} = \frac{a(a-c)(4a^2+\beta(4a+\beta)\delta)}{(-4a^2+\beta^2\delta)^2} > 0 \qquad (A3)$$

$$\frac{\partial q_{H2}{}^*}{\partial \beta} = \frac{a(a-c)(4a^2+\beta(4a+\beta)\delta)}{b(-4a^2+\beta^2\delta)^2} > 0 \qquad (A4)$$

$$\frac{\partial E\pi^*}{\partial \beta} = \frac{a(a-c)^2(2a+\beta)\delta(2a+\beta\delta)}{b(-4a^2+\beta^2\delta)^2} > 0 \qquad (A5)$$

$$\frac{\partial p_{H1}{}^*}{\partial \delta} = -\frac{2a^2(a-c)\beta(2a+\beta)}{(-4a^2+\beta^2\delta)^2} < 0 \qquad (A6)$$

$$\frac{\partial q_{H1}{}^*}{\partial \delta} = \frac{2a^2(a-c)\beta(2a+\beta)}{b(-4a^2+\beta^2\delta)^2} > 0 \qquad (A7)$$

$$\frac{\partial p_{H2}{}^*}{\partial \delta} = -\frac{a(-a+c)\beta^2(2a+\beta)}{(-4a^2+\beta^2\delta)^2} > 0 \qquad (A8)$$

$$\frac{\partial q_{H2}{}^*}{\partial \delta} = -\frac{\beta^2(-2a^3+2a^2c-a^2\beta+ac\beta)}{b(4a^2-\beta^2\delta)^2} > 0 \qquad (A9)$$

$$\frac{\partial E\pi^*}{\partial \delta} = \frac{a^2(a-c)^2(2a+\beta)^2}{b(-4a^2+\beta^2\delta)^2} > 0 \qquad (A10)$$

Proof of Proposition 1

$$p_{H1}{}^* - p_{H2}{}^* = -\frac{(a-c)\beta(a+a\delta+\beta\delta)}{4a^2-\beta^2\delta} < 0 \longrightarrow p_{H1}{}^* < p_{H2}{}^* \qquad (A11)$$

$q_{H1}{}^{*} - q_{H2}{}^{*} = \frac{a(a-c)\beta(-1+\delta)}{b(4a^2-\beta^2\delta)} \le 0 \longrightarrow q_{H1}{}^{*} \le q_{H2}{}^{*}$ $(A12)$

Letting δ=1, we can obtain $q_{H1}{}^{*} = q_{H2}{}^{*}$

Proof of Proposition 2

Letting $q_{H1}{}^{*} = \frac{a(2a^2-2ac+a\beta\delta-c\beta\delta)}{b(4a^2-\beta^2\delta)} > \frac{a}{2b}$, $q_{H2}{}^{*} = \frac{a(2a^2-2ac+a\beta-c\beta)}{b(4a^2-\beta^2\delta)} > \frac{a}{2b}$, we can obtain

$$0 < b \le a, 0 < c < \frac{a}{4}, 2c < \beta \le \frac{a}{2} \text{ and } \frac{4ac}{2a\beta - 2c\beta + \beta^2} < \delta \le 1 \quad (A13)$$

Furthermore, we let $a = b = 1$ and $c = \frac{1}{8}$, we can obtain $q_{H1}{}^{*} = \frac{14+7\beta\delta}{32-8\beta^2\delta}$ and $q_{H2}{}^{*} = -\frac{7(2+\beta)}{8(-4+\beta^2\delta)}$.

$q_{H1}{}^{*} > \frac{1}{2}$ and $q_{H2}{}^{*} > \frac{1}{2}$, when $\frac{1}{4} < \beta \le \frac{1}{2}$ and $\frac{2}{7\beta+4\beta^2} < \delta \le 1$ $(A14)$

$q_{H1}{}^{*} < \frac{1}{2}$ and $q_{H2}{}^{*} < \frac{1}{2}$, when $0 < \beta < \frac{1}{4}, 0 < \delta \le 1$ or $\frac{1}{4} < \beta < \frac{2}{7}$, $0 < \delta < \frac{2-7\beta}{4\beta^2}$ $(A15)$

$q_{H1}{}^{*} < \frac{1}{2}$ and $q_{H2}{}^{*} > \frac{1}{2}$, when $\frac{1}{4} < \beta \le \frac{2}{7}, \frac{2-7\beta}{4\beta^2} < \delta < \frac{2}{7\beta+4\beta^2}$ or $\frac{2}{7} < \beta \le \frac{1}{2}, 0 < \delta < 2/(7\beta + 4\beta\text{^}2\ )$ $(A16)$

Letting $q_{H1}{}^{*} > \frac{1}{2}$, we can obtain a range{ $\frac{1}{4} < \beta \le \frac{1}{2}, \frac{2}{7\beta+4\beta^2} < \delta \le 1$}(Range 1); Letting $q_{H2}{}^{*} < \frac{1}{2}$, we can obtain another range $\{\left(0 < \beta < \frac{1}{4}, 0 < \delta \le 1\right), \left(\frac{1}{4} \le \beta < \frac{2}{7}, 0 < \delta < \frac{2-7\beta}{4\beta^2}\right)\}$ (Range 2). Since there are no intersections between Range 1 and Range 2, it is impossible to obtain $q_{H1}{}^{*} > \frac{1}{2}$ and $q_{H2}{}^{*} < \frac{1}{2}$ at the same time. Therefore, it is impossible to obtain inelastic $p_{H1}{}^{*}$ and elastic $p_{H2}{}^{*}$.