Analysis of Control Barrier Function Framework for Safety-Critical Control of Connected Automated Vehicles

ENGINEERING HONORS PROGRAM
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Objective

Vehicle-to-vehicle (V2V) connectivity has gained traction in the automated vehicle space for its potential to improve congestion mitigation, fuel economy, and vehicle safety. V2V refers to when nearby automated vehicles exchange data to inform their driving and are thus 'connected'. However, the current lack of a control framework with provable safety guarantees for V2V connected vehicles prevents this form of automation from being applicable outside of the academic setting.

The objective of this project was to develop a V2V safety-critical controller via control barrier function (CBF) framework and apply this framework to models of increasing fidelity, from the 1 state to 4 state model case. Simulations of the CBF framework applied to various models were conducted in MatLab and characteristic vehicle behaviors were analyzed for varying parameter values and initial conditions. This allows us to take away insights on controller limitations and performance.

Methods

CBF Safety Function:

$$h(x) = d - r$$

d = distance from vehicle to obstacle center
r = radius of obstacle

Vehicle safe if $h(x) > 0$

https://innotechtoday.com/autonomous-cars/

The vehicle can be modeled in the state space form:

$$\dot{x} = f(x) + g(x)k_s(x)$$

$\dot{x}$ = Time derivative of "states" (position, heading angle, etc)
$k_s(x)$ = "Safe controller" inputs, dependent on $h(x)$

$$k_s(x) = k_n(x) + max\left\{0, \frac{-a(x)}{||b(x)||^2}\right\} b^T(x)$$

$$a(x) = \frac{\partial h(x)}{\partial x}(f(x) + g(x)k_n(x)) + \alpha(h(x))$$

$$b(x) = \frac{\partial h(x)}{\partial x} g(x)$$

$k_n(x)$ is nominal controller (when safety control is unnecessary)

Note: If $b(x) = 0$, $h(x)$ has no effect on vehicle dynamics. A solution is to extend $h(x)$ as follows:

$$h_e(x) = \frac{\partial h(x)}{\partial x} f(x) + \alpha(h(x))$$

The above state space system is solved in MatLab to create vehicle trajectory plots. Three models of increasing fidelity were explored: the integrator, unicycle, and bicycle. The impacts of system initial conditions (starting speed and position) and parameters on vehicle behavior were examined.

Dynamic Models Explored

Integrator (point)
(unconstrained movement)

Unicycle
(pivot in place)


https://www.walmart.com/ip/Fun-20-inch-Unicycle-with-Alloy-Rim-Blue/14699254

Bicycle (front wheel steer)


https://www.statebicycle.com/products/delfin-core-line

States: [Lat. Position; Lon. Position; Velocity; Heading angle]

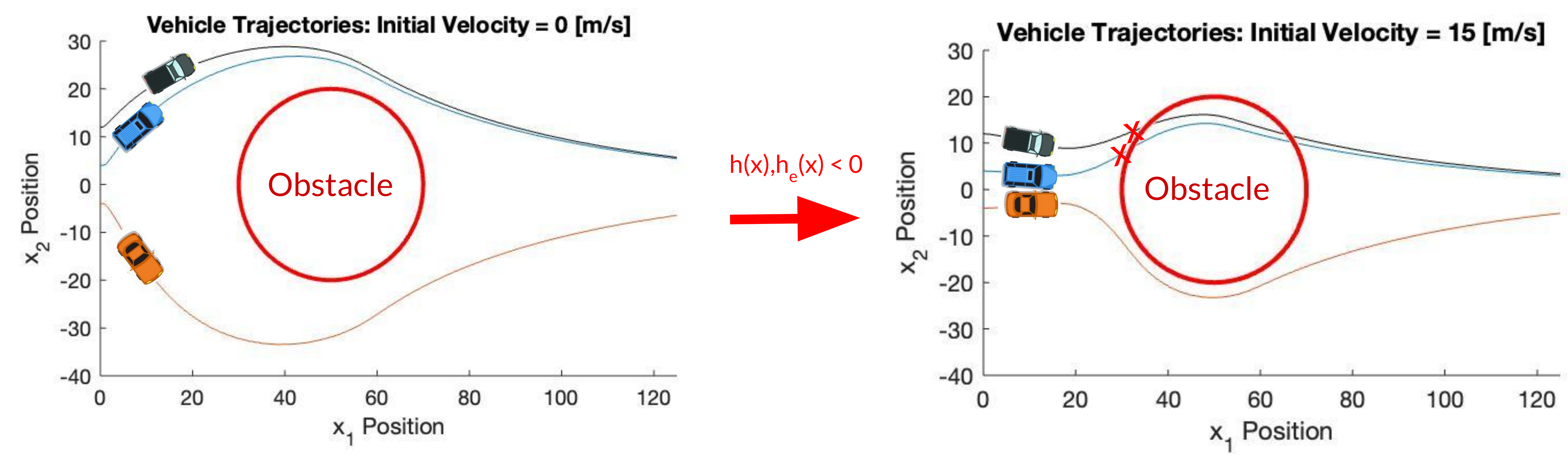
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}k_{s1}\\ k_{s2}\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}x_3\cos(x_4)\\ x_3\sin(x_4)\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}k_{s1}\\ k_{s2}\end{bmatrix}$$

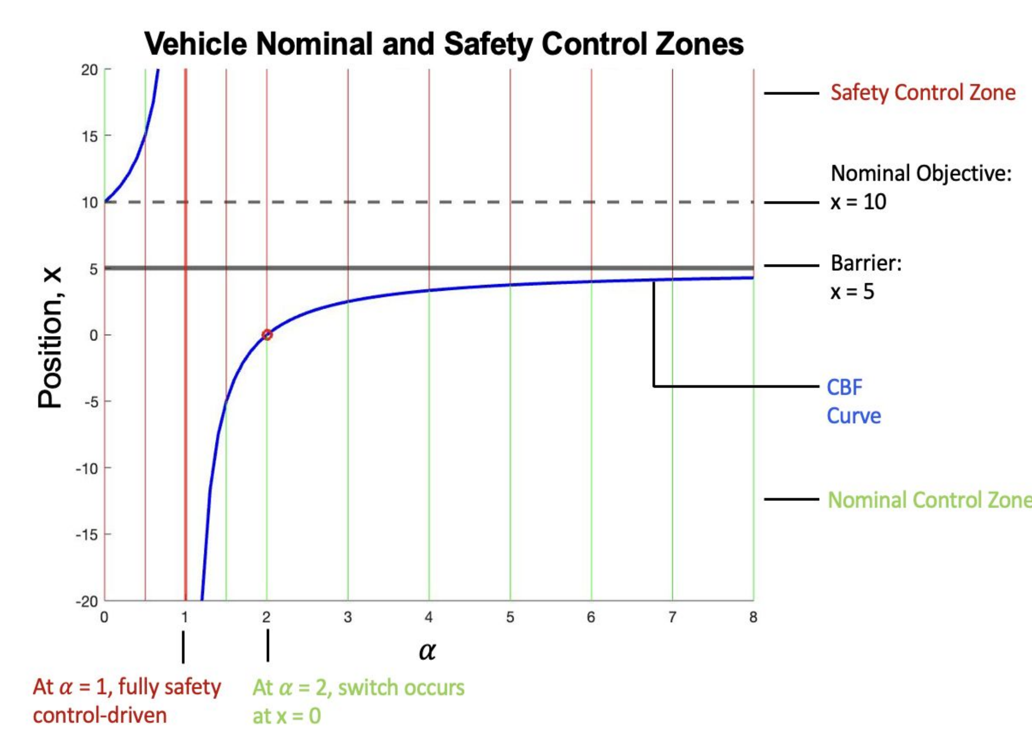
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}x_3\cos(x_4)\\ x_3\sin(x_4)\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & \frac{x_3}{l}\end{bmatrix}\begin{bmatrix}k_{s1}\\ k_{s2}\end{bmatrix}$$

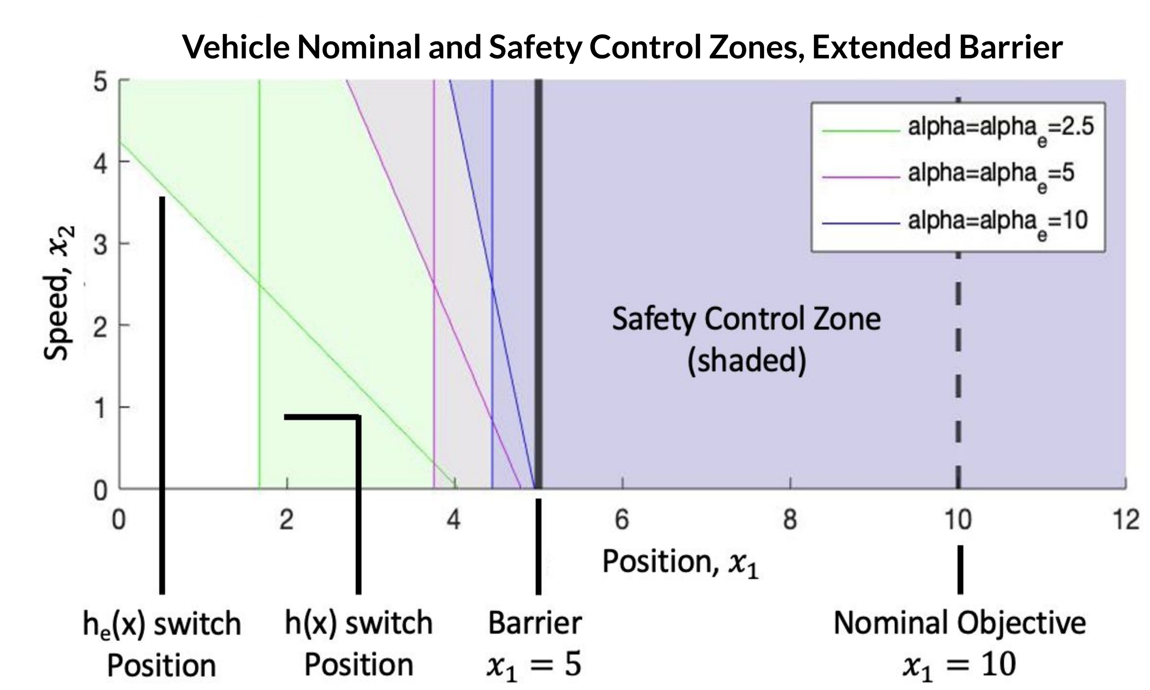
Characteristic Controller Limiting Behaviors

1. Initial Conditions that Violate Safety Condition

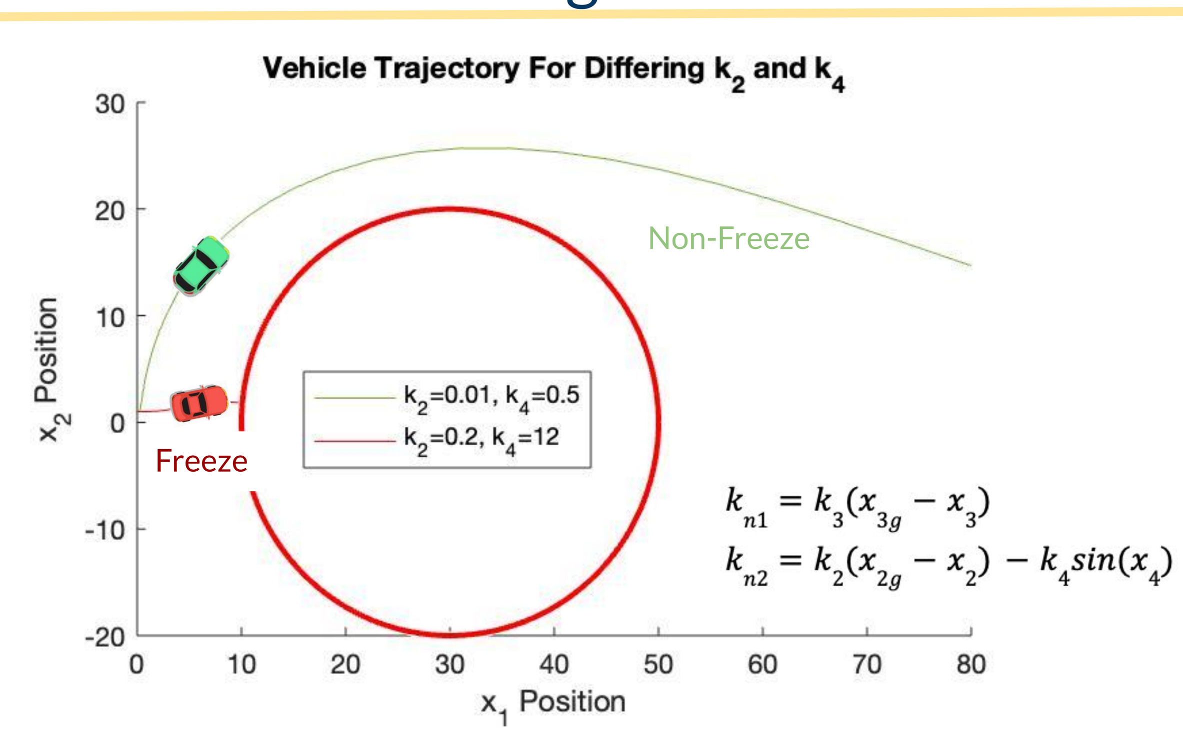


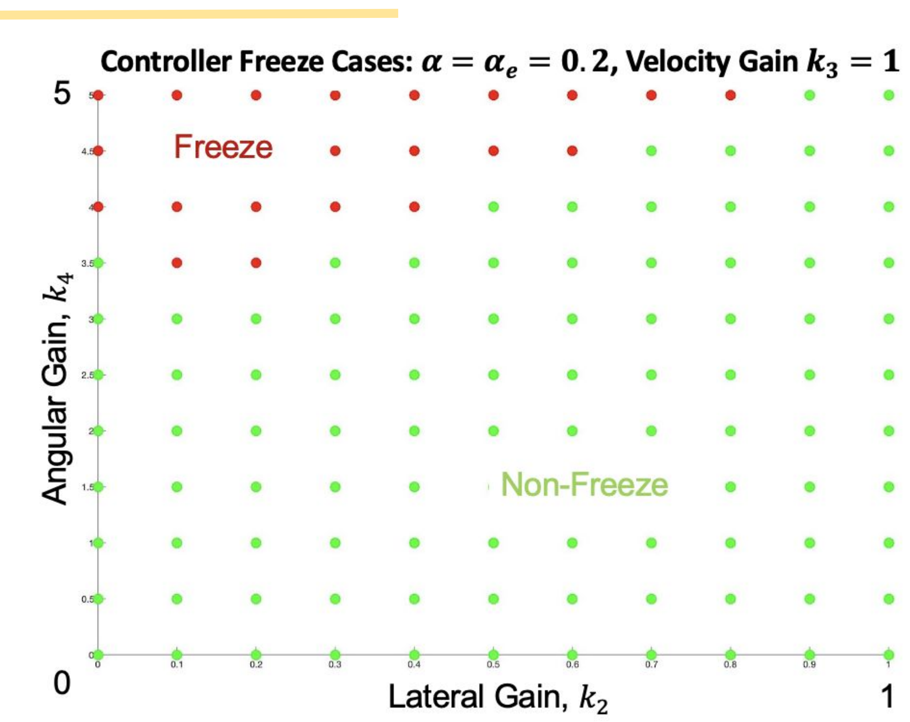
2. Controller Switching Position





3. Controller Freezing Case





Significance

By applying a CBF safety-critical controller to V2V vehicle models from the 1 state to 4 state case and simulating results, we characterize key controller behaviors for varying parameter conditions. Through this characterization, we identify conditions under which the CBF framework is effective. This gives a better understanding of how to engineer safety guarantees for V2V vehicles. Key findings were:

1. Controller guarantees theoretical safety if the system's initial conditions satisfy the condition: $h(x)$ and $h_e(x) > 0$.
   - If this condition is not satisfied, the vehicle trajectory may collide with the obstacle.
2. Within relevant space, higher α increases controller aggressiveness (i.e. controller switch to safety control later).
   - From how h(x) is defined, switch position is only position dependent.
   - Extending safety function to $h_e(x)$ makes switch position also dependent on velocity, the time derivative of position.
3. Holding other parameters constant, vehicle may freeze when:
   - Angular gain is high and lateral gain is within certain range.
   - The obstacle case is more severe. A larger obstacle or an obstacle closer to the vehicle starting position is much more likely to cause freezing.

References

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[2] A. Alan, A. J. Taylor, C. R. He, G. Orosz, and A. D. Ames. Safe controller synthesis with tunable input-to-state safe control barrier functions. *IEEE Control Systems Letters*, **6**:908-913, 2022.

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