Connected Automated Vehicles

Analysis of Control Barrier Function Framework for Safety-Critical Control

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https://www.eetasia.com/creating-safer-roads-for-autonomous-vehicles/

Agenda

1. Background, V2V Connected Vehicles
2. Methods, Control Barrier Functions and Models
3. Results, Characteristic Limiting Behaviors
4. Significance and Next Steps
5. Acknowledgements

https://medium.com/@recogni/autonomous-vehicles-and-a-system-of-connected-cars-944f86275663

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Connected Automated Vehicles

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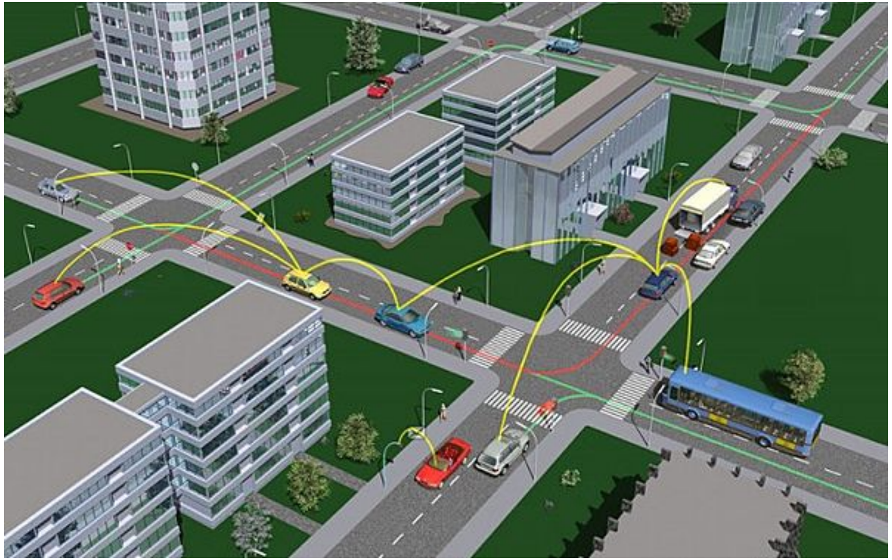
Vehicle-to-Vehicle (V2V) connectivity is a form of vehicle automation

- Refers to when nearby vehicles exchange data to inform driving
- Nearby vehicles are thus ‘connected’

(V2V) connectivity has the potential to:

- Mitigate traffic congestion
- Increase fuel economy
- Improve vehicle safety


https://blog.rgbsi.com/what-to-know-about-v2v-technology

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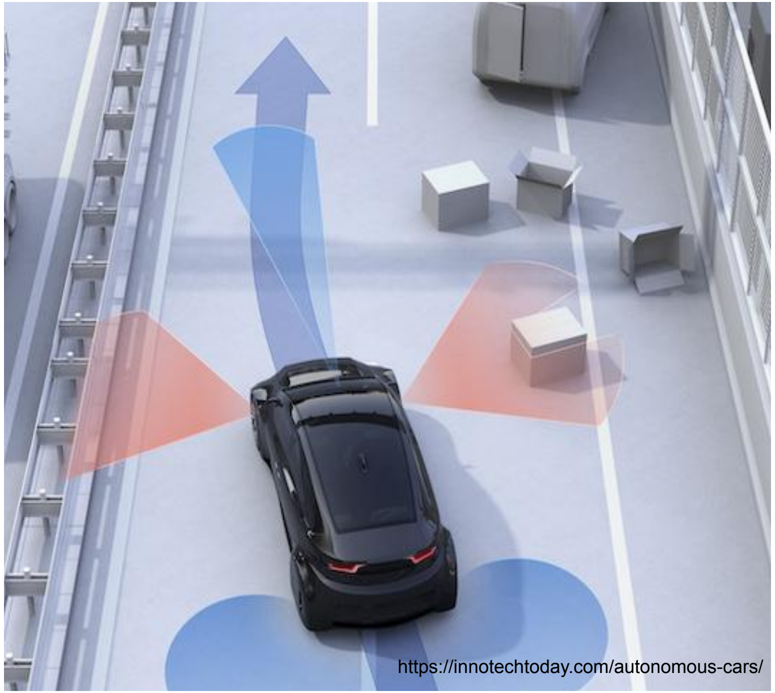
Problem:
Current lack of control framework with provable safety guarantees for V2V vehicles

Project Objectives:

1. Implement V2V safety-critical controller via control barrier function (CBF) framework
2. Apply CBF framework to vehicle models of higher fidelity
3. Simulate and evaluate controller in MatLab with varying models and parameters



Automated Vehicle Detecting Obstacle

Control Barrier Function Framework

Define CBF Safety Function:

$$h(x) = d - r$$

d = distance from vehicle to obstacle center
r = radius of obstacle

Vehicle safe if *h(x)* > 0

Characterization of Obstacle

Control Barrier Function Framework

h(x) implemented into the model through velocity/acceleration inputs.

Controlling velocity/accel. is not always enough for safety-critical behavior. In this case, must "extend" safety function:

$$h_e(x) = \frac{\partial h(x)}{\partial x} f(x) + \alpha(h(x))$$

Vehicle safe if $h_e(x)$ & $h(x) > 0$

Characterization of Obstacle

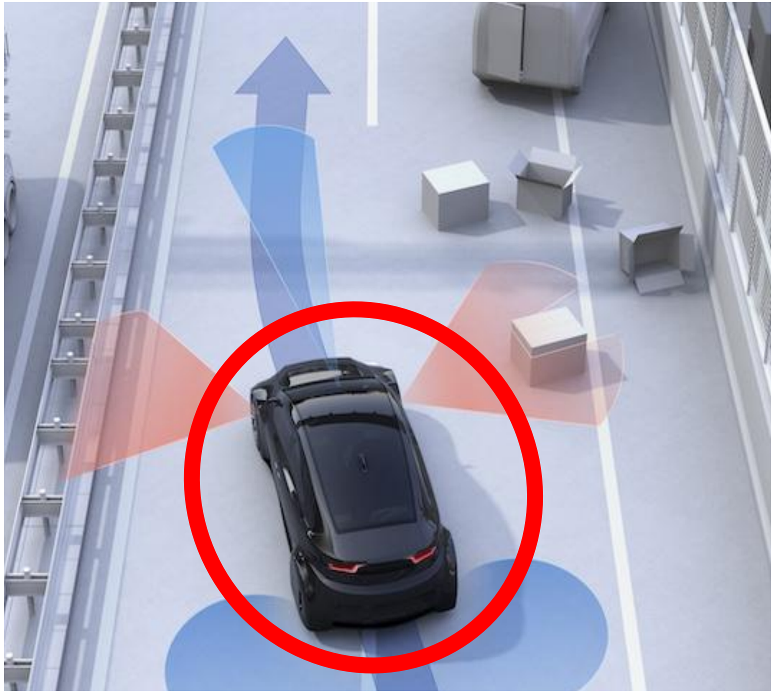
Control Barrier Function Framework

Need to apply the safety function to a vehicle model.

Cars are very complicated.

The vehicle can be more simply modeled as:

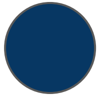
1. A point
2. A unicycle
3. A bicycle



Characterization of Vehicle

Vehicle Models

Integrator (point)
(unconstrained movement)



Unicycle
(pivot in place)

https://www.walmart.com/ip/Fun-20-inch-Unicycle-with-Alloy-Rim-Blue/14699254

Bicycle
(front wheel steer)

https://www.statebicycle.com/products/delfin-core-line

$$\begin{bmatrix}\text{Lat. Position}\\ \text{Lon. Position}\\ \text{Velocity}\\ \text{Heading angle}\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}k_{s1}\\ k_{s2}\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}x_3\cos(x_4)\\ x_3\sin(x_4)\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}k_{s1}\\ k_{s2}\end{bmatrix}$$

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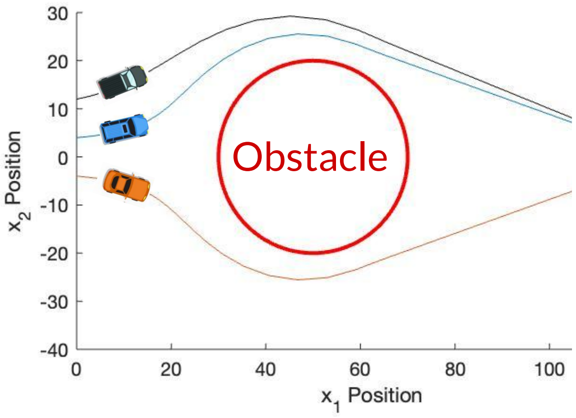
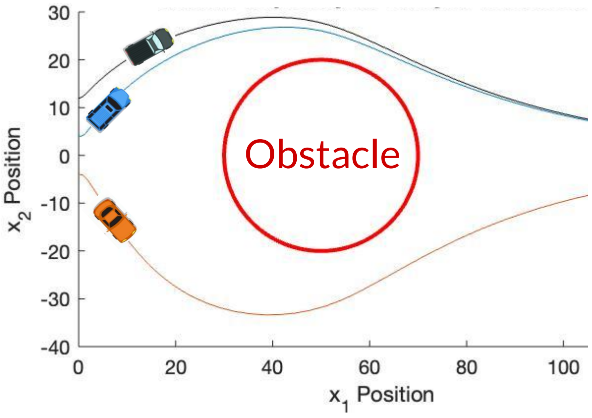
$k_s(x)$ = “Safe controller” inputs, dependent on *h(x)*

Characteristic Controller Behavior

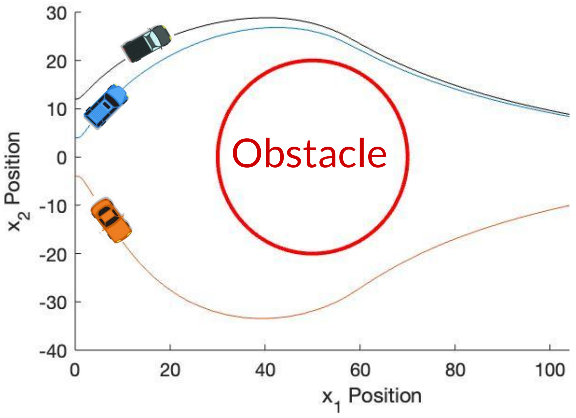
Integrator Model Trajectory



Unicycle Model Trajectory



Bicycle Model Trajectory



Characteristic Limiting Behaviors

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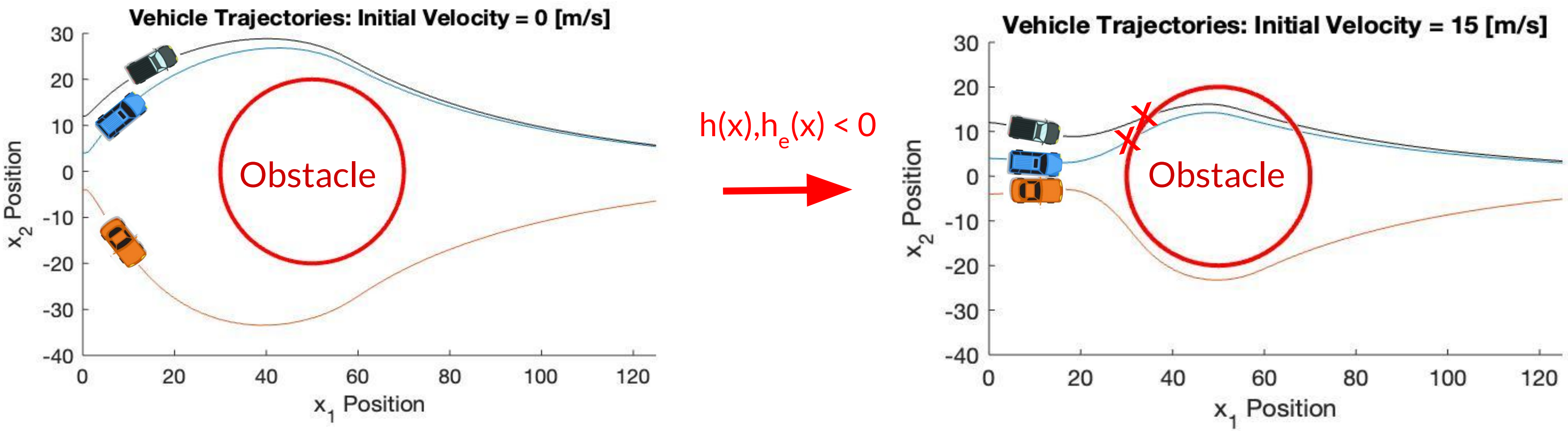
1. Initial Conditions That Do Not Satisfy the Safety Condition



- If $h_e(x),h(x) > 0$ initially, safety guaranteed for all time. If $h_e(x),h(x) < 0$, may have collision.
- $h(x) < 0$ only if vehicle occupies same space as obstacle.
- Initial velocity can cause $h_e(x) < 0$, violating safety condition.

Characteristic Limiting Behaviors

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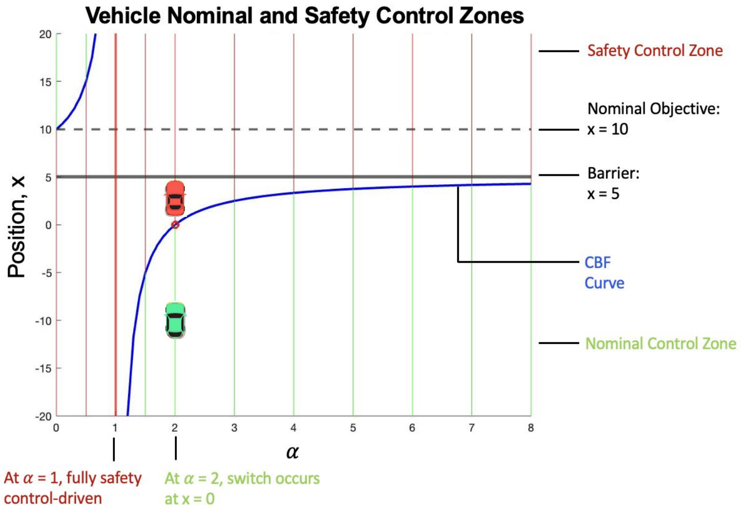
2. Controller Switching Position



- At some point, vehicle switches from nominal control $k_n(x)$ to safety control $k_s(x)$. $k_n(x)$ optimal in non safety-critical situation.
- Case shown: vehicle wants to drive to x = 10, but must stop at barrier at x = 5.

Characteristic Limiting Behaviors

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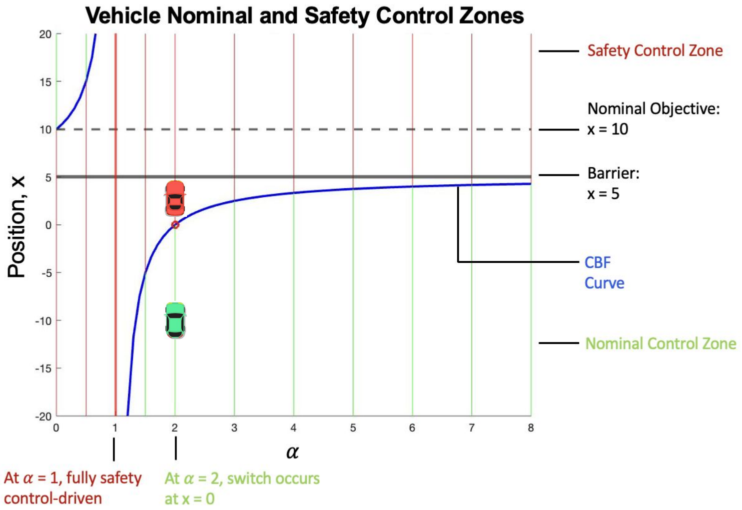

- The controller switches later with increasing α.
  - Large α could pose problem for real world vehicles, which cannot react immediately.
- For h(x) barrier, switching position only depends on position.
- For $h_e(x)$ barrier, switching position depends on position and velocity (time derivative).

Characteristic Limiting Behaviors

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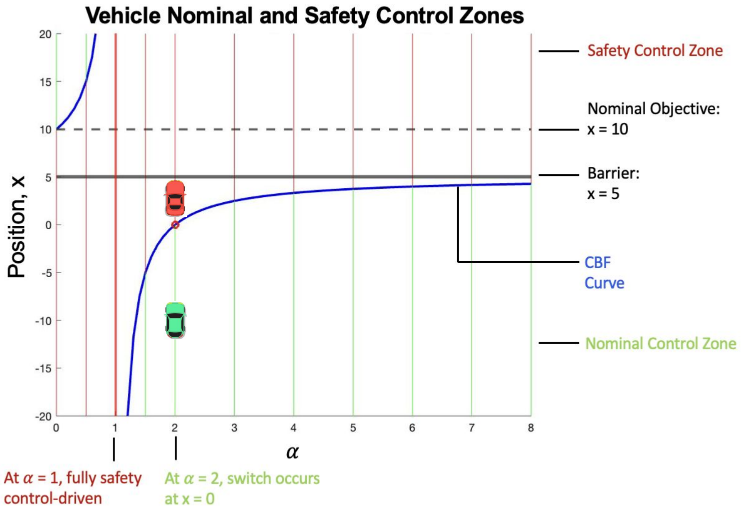

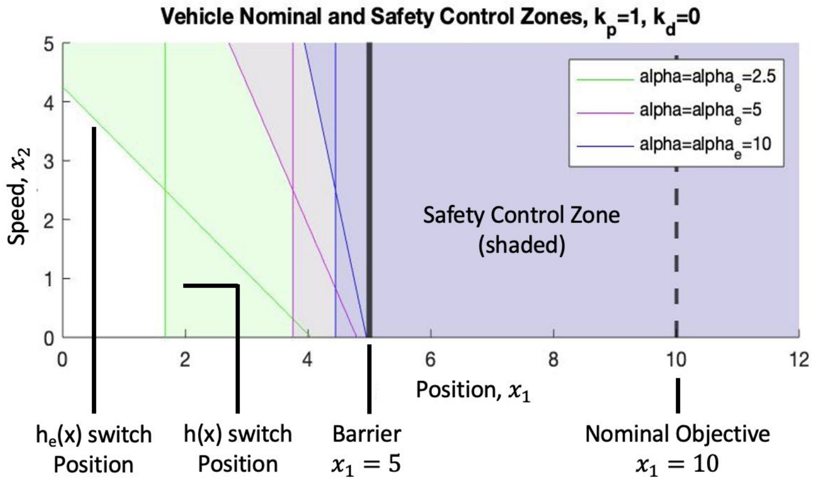

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Characteristic Limiting Behaviors

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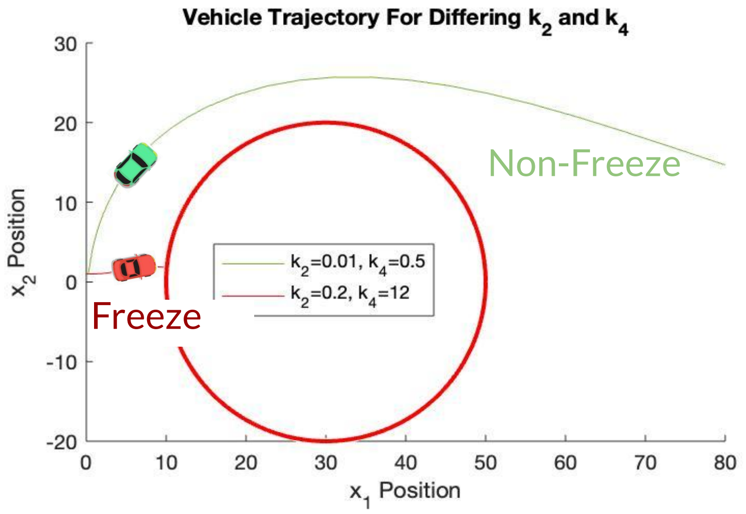
3. Controller Freezing Case



- Freezing case = vehicle stops at obstacle instead of driving around it. Safe but undesirable.
- Freezing is prominent when obstacle case is severe.
- Freezing is prominent when angular gain is high and lateral gain is ~0.1 to 0.2.

Characteristic Limiting Behaviors

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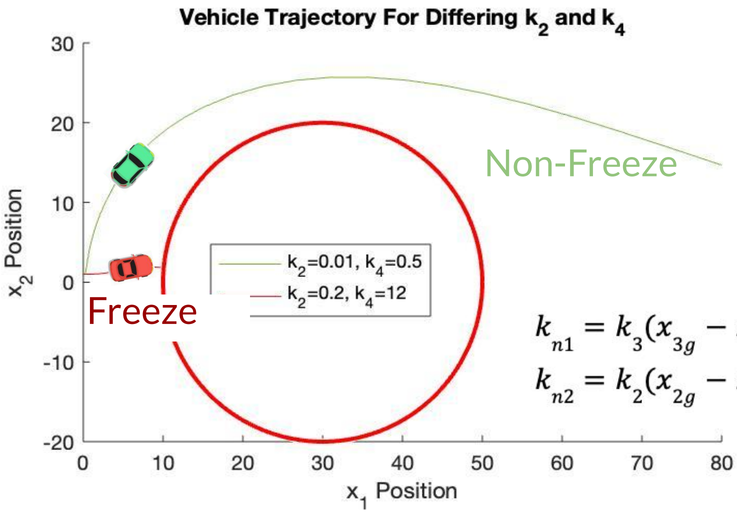
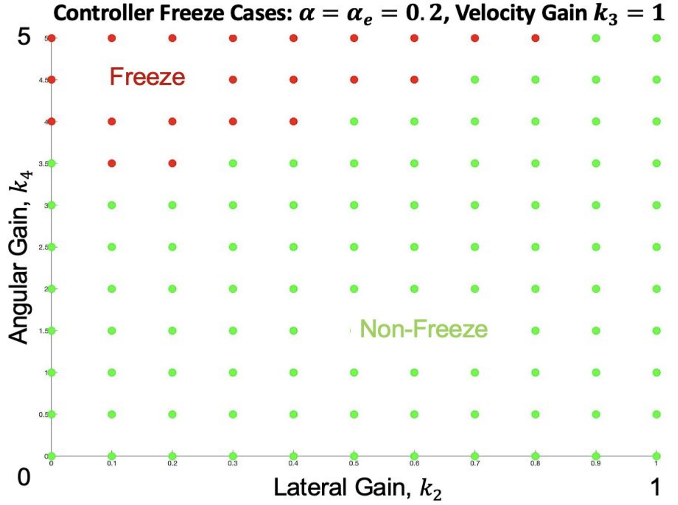


$$k_{n1} = k_3(x_{3g} - x_3)$$
$$k_{n2} = k_2(x_{2g} - x_2) - k_4 sin(x_4)$$



- Freezing case = vehicle stops at obstacle instead of driving around it. Safe but undesirable.
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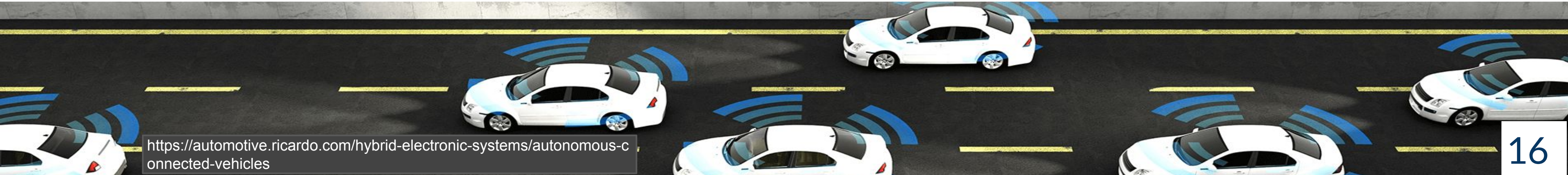
Significance

Identification of parameter conditions under which control barrier function (CBF) framework is/isn’t desirable is useful for engineering V2V safety guarantees.

Key Findings:

- Results are largely general, not model-based.
- For extended barrier, initial velocity may cause violation of safety condition.
- Higher α causes later switch to safety-critical control.
- High angular gain and ~0.1 to 0.2 lateral gain may cause freezing.



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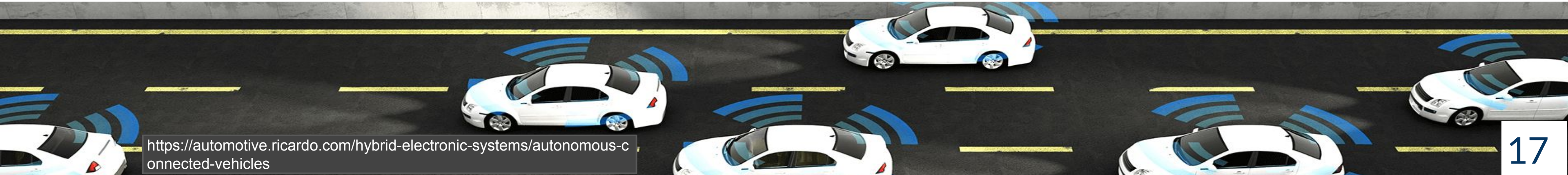
Next Steps

- Apply CBF framework to models beyond bicycle model
- Characterize behavior for multiple and dynamic obstacles
- Analytical characterization of switching and freezing surfaces for higher fidelity models
- Investigate why certain lateral gain is conducive to freezing



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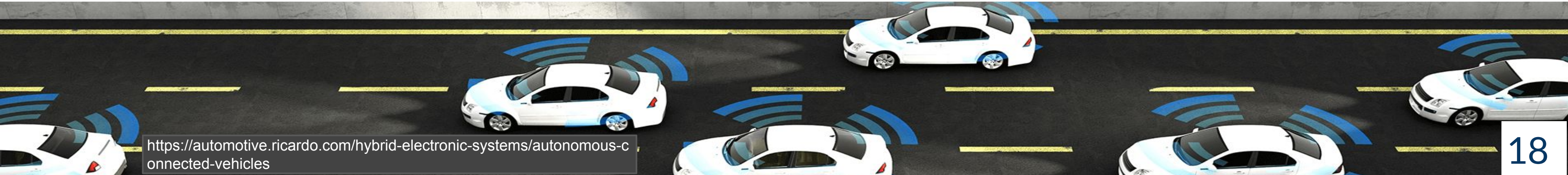

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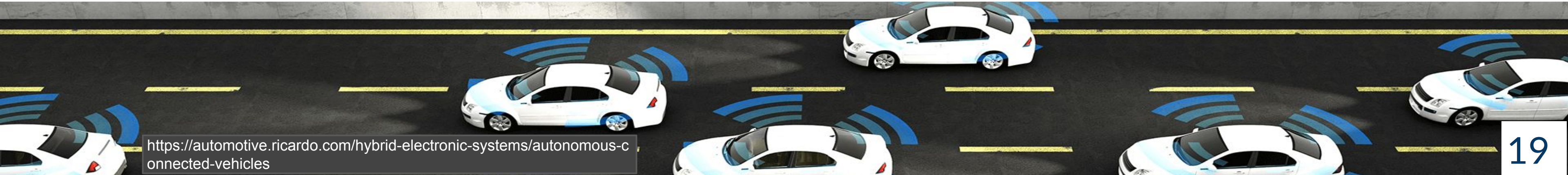
References

[1] T. G. Molnar, R. Cosner, A. Singletary, W. Ubellacker, and A. D. Ames, “Model-Free Safety-Critical Control for Robotic Systems”, *IEEE Robotics and Automation Letters*, Vol. 7, No. 2, 2022

[2] A. Alan, A. J. Taylor, C. R. He, G. Orosz, and A. D. Ames.
Safe controller synthesis with tunable input-to-state safe control barrier functions. *IEEE Control Systems Letters*, **6**:908-913, 2022.

[3] A. D. Ames, and P. Tabuada. Lectures on Nonlinear Dynamics and Control. 2022



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Control Barrier Function Framework

Vehicle can be modeled in the state space form:

$$\dot{x} = f(x) + g(x)k_s(x)$$

$\dot{x}$ = Time derivative of "states" (position, heading angle, etc)

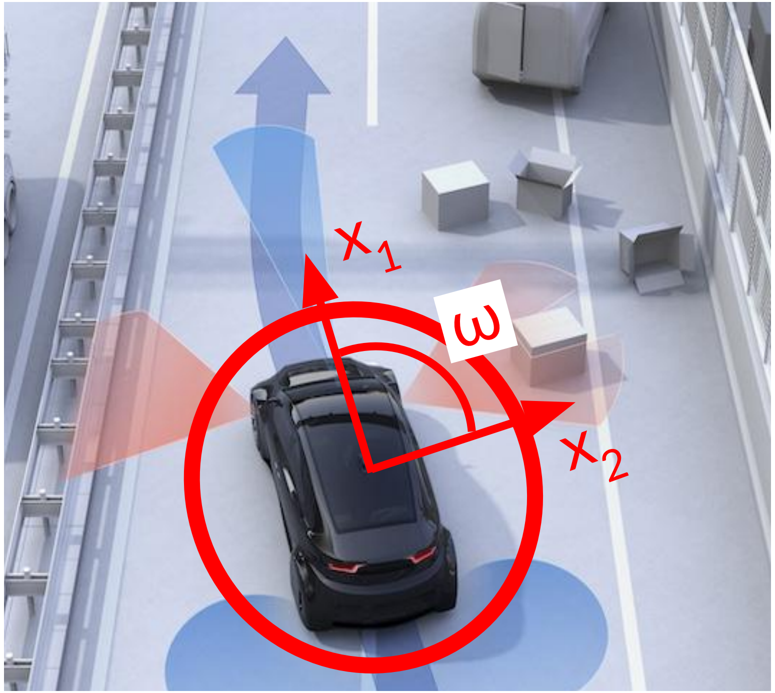
$k_s(x)$ = "Safe controller" inputs, dependent on $h(x)$

$$k_s(x) = k_n(x) + max\{0, \frac{-a(x)}{||b(x)||^2}\}b^T(x)$$

$$a(x) = \frac{\partial h(x)}{\partial x}(f(x) + g(x)k_n(x)) + \alpha(h(x))$$

$$b(x) = \frac{\partial h(x)}{\partial x}g(x)$$

*$k_n(x)$ is nominal controller (when safety control is unnecessary)



Characterization of Vehicle

Control Barrier Function Framework

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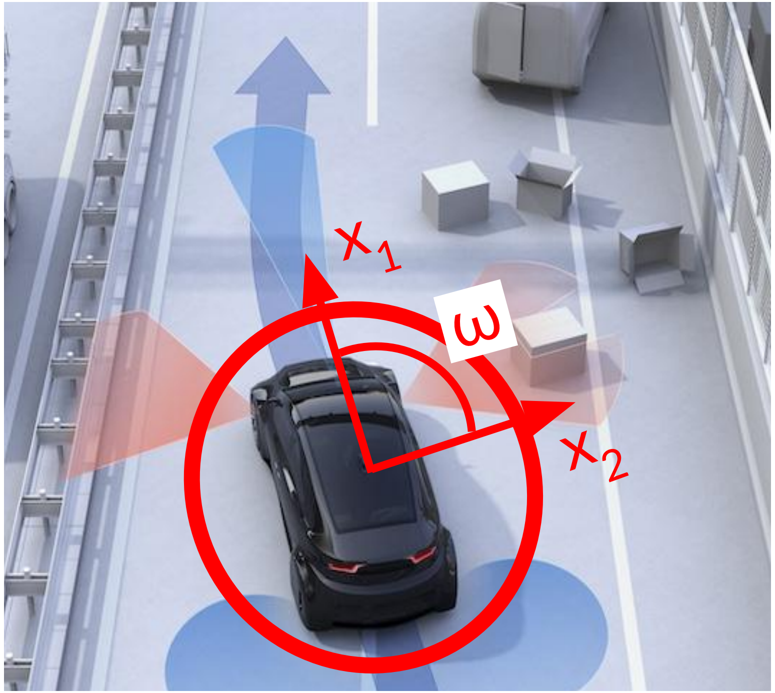
$$k_s(x) = k_n(x) + max\{0, \frac{-a(x)}{||b(x)||^2}\} b^T(x)$$

$$a(x) = \frac{\partial h(x)}{\partial x}(f(x) + g(x)k_n(x)) + \alpha(h(x))$$

$$b(x) = \frac{\partial h(x)}{\partial x} g(x)$$

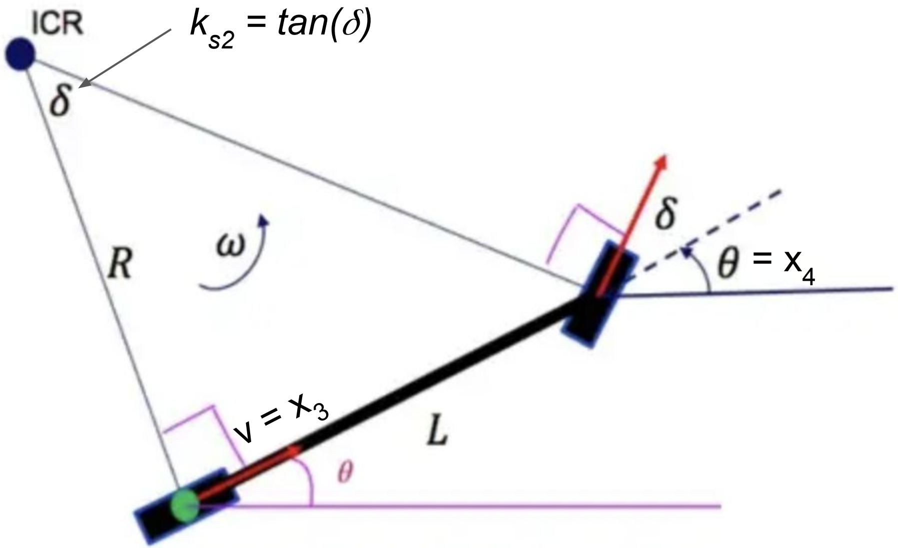
If $b(x) = 0$, then $k_s(x) = k_n(x)$!

- In this case, safe controller has no effect
- Need to “extend” the barrier in this case



Characterization of Vehicle

Dynamics of Bicycle Model



https://dingyan89.medium.com/simple-understanding-of-kinematic-bicycle-model-81cac6420357

Model:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}x_3\cos(x_4)\\ x_3\sin(x_4)\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & \frac{x_3}{l}\end{bmatrix}\begin{bmatrix}k_{s1}\\ k_{s2}\end{bmatrix}$$