Model Predictive Control for Ball Bouncing on a Robot Arm

Xingze Dai, HONORS – Capstone Project, Fall 2022

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Project Description

Robotic arms are ubiquitous nowadays in many applications and there is an increasing demand and few approaches in controlling a robotic arm dealing with high-speed tasks. We considered bouncing a ball by a paddle attached to a robot arm and sending it to a certain height as our example task. Our robot arm uses a Model Predictive Controller to wield a paddle while a computer vision module detects and tracks the ball. We first derive the dynamic model for the ball and the paddle and then design a Model Predictive Controller. Finally, we realize the control algorithm in Drake simulator and adapt it for hardware implementation.

Bouncing Model

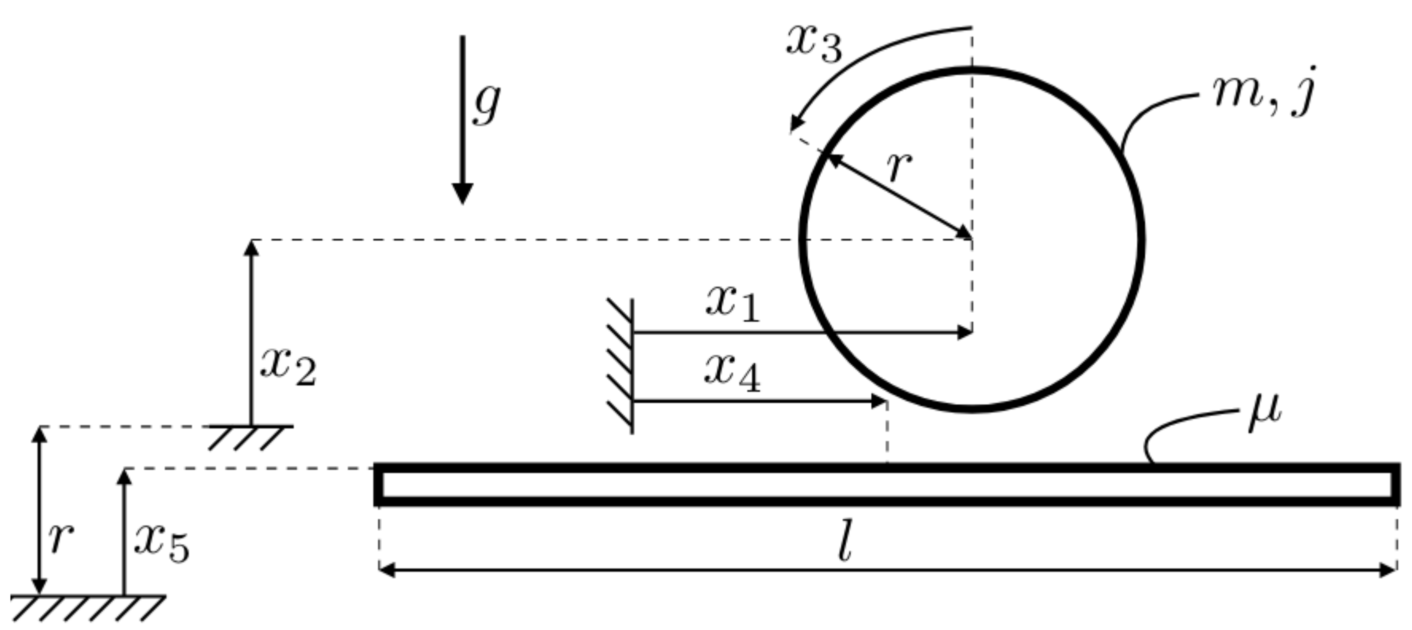

Figure 1. A modified 2D diagram of the model from Marcucci et al. [1] denoting geometries and dimensions. In the diagram, x_1, x_2 are ball's positions; x_3 is ball's rotation; x_4, x_5 are paddle's positions; *m, j, r* are ball's mass, moment of inertia, and the radius; *l*, μ are paddle's length and coefficient of restitution; *g* is the gravity acceleration.

Based on a ball bouncing model due to Marcucci et al. [1] as shown in Figure 1, we derive a planar model to describe the ball and paddle's dynamics. The model is given below where *h* is the discretization step and *f* is a contact force with the subscripts p and c denoting the paddle and ceiling and t and n denoting the tangential and the normal components.

$$x_{k+} = x_k + h x_{(k+5)+}, \; k = 1, \dots, 5$$
$$x_{6+} = x_6 + h\, f_{pt}/m$$
$$x_{7+} = x_7 + h\, f_{pn}/m - hg$$
$$x_{8+} = x_8 + r\, h\, f_{pt}/j$$
$$x_{9+} = x_9 + h\, u_1$$
$$x_{10+} = x_{10} + h\, u_2$$

KINOVA KORTEX Gen3 6-DOF robot arm [3]

References

[1] T. Marcucci and R. Tedrake, "Mixed-integer formulations for optimal control of Piecewise-affine systems," 2019.
[2] C. Jones, F. Borrelli, and M. Morari, "Model Predictive Control," 2015.
[3] https://www.kinovarobotics.com/product/gen3-robots

Controller Implementation

The control algorithm is first implemented in Drake simulator as shown in Figure 2. The simulated ball and paddle trajectories are plotted in Figure 3.

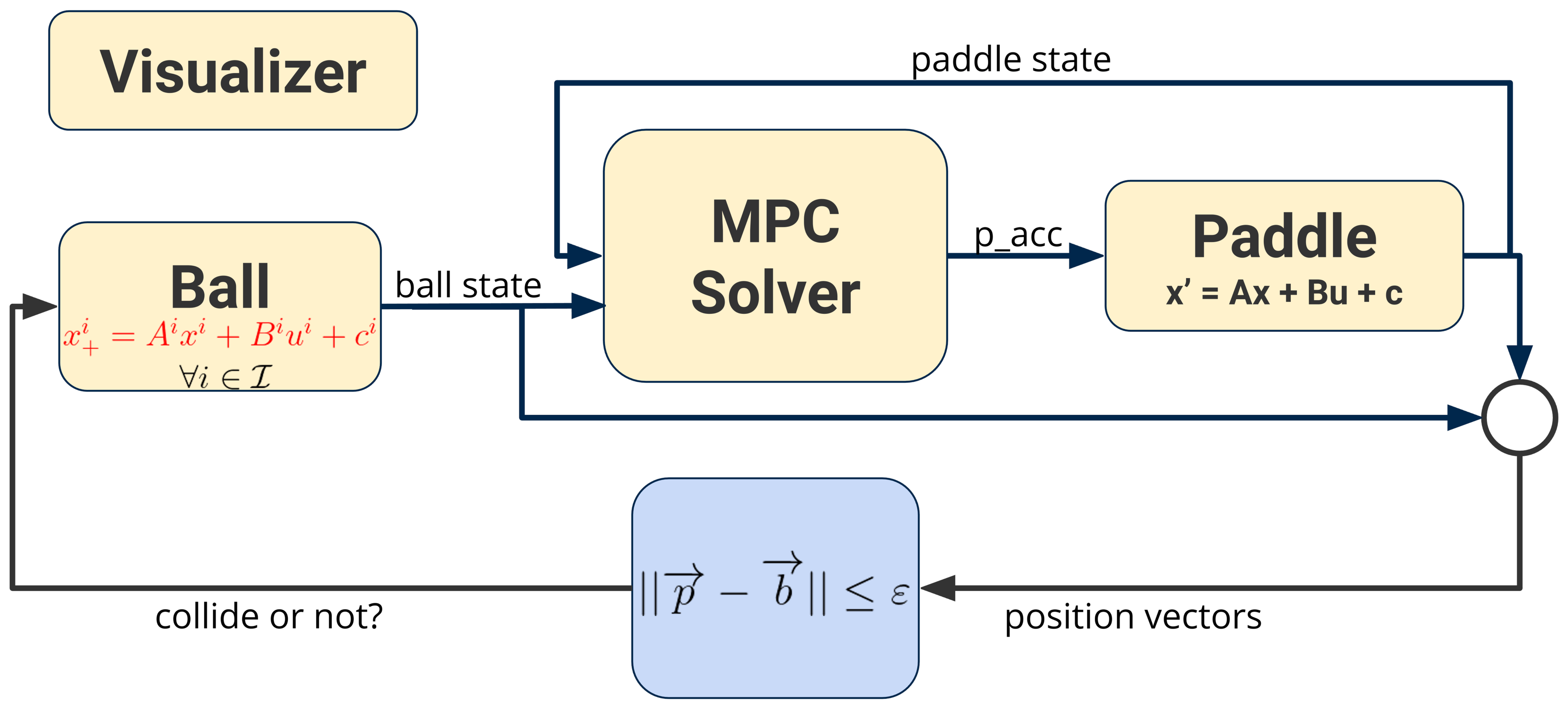

Figure 2. A diagram chart of the schematic implementation of the control algorithm in Drake simulator. *p_acc* is the paddle acceleration. The vectorized *p* represents the position of the paddle while the vectorized *b* represents the position of the ball. ε is the contact criterion which is a constant.

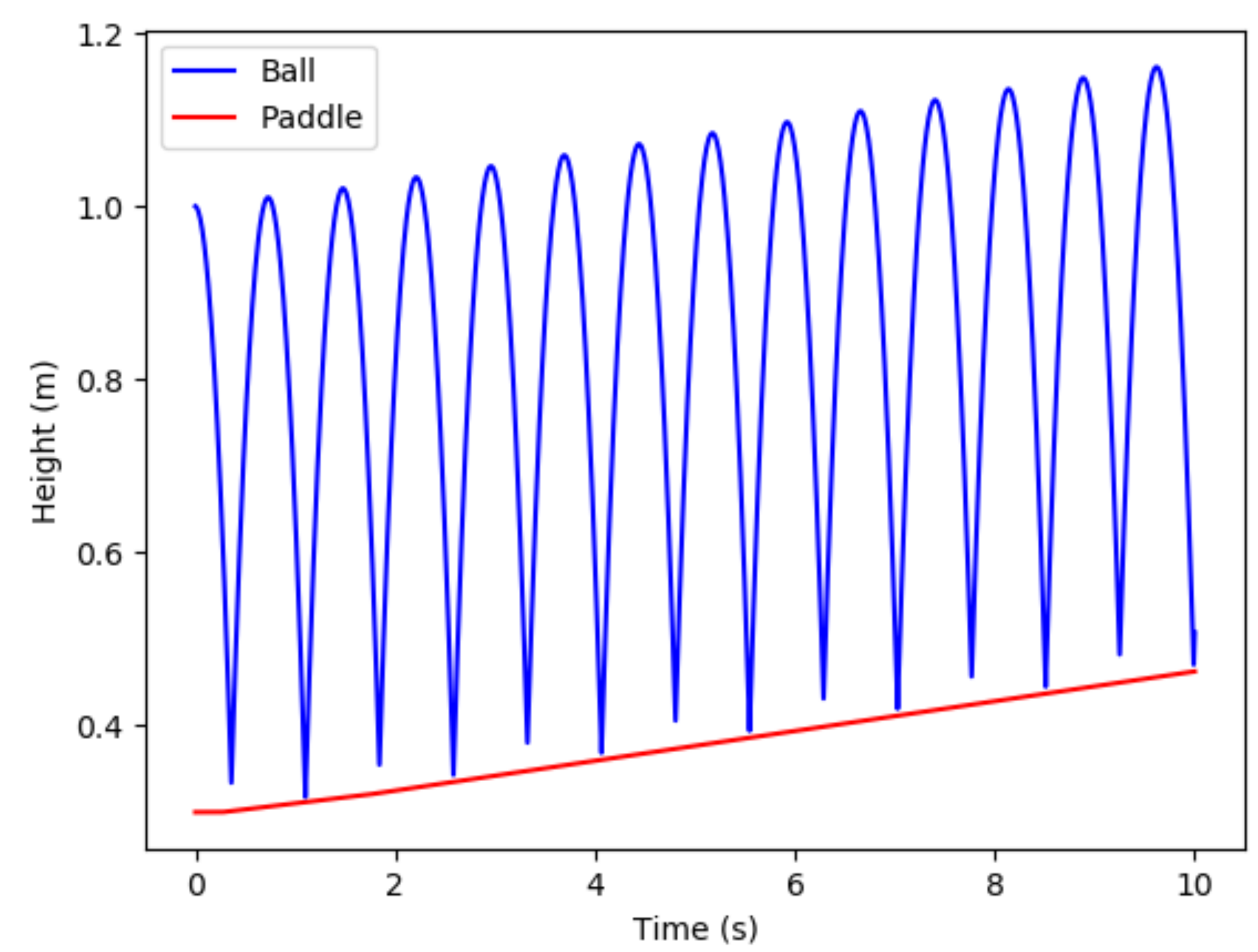

Figure 3. A simulation of the system using the configuration in Figure 2 with our model predictive controller. We successfully sustain ball bouncing to a target altitude.

The simulated control algorithm is modified to work on the hardware as shown in Figure 4. In this project, KINOVA KORTEX Gen3 robot arm is used as our experimental platform.

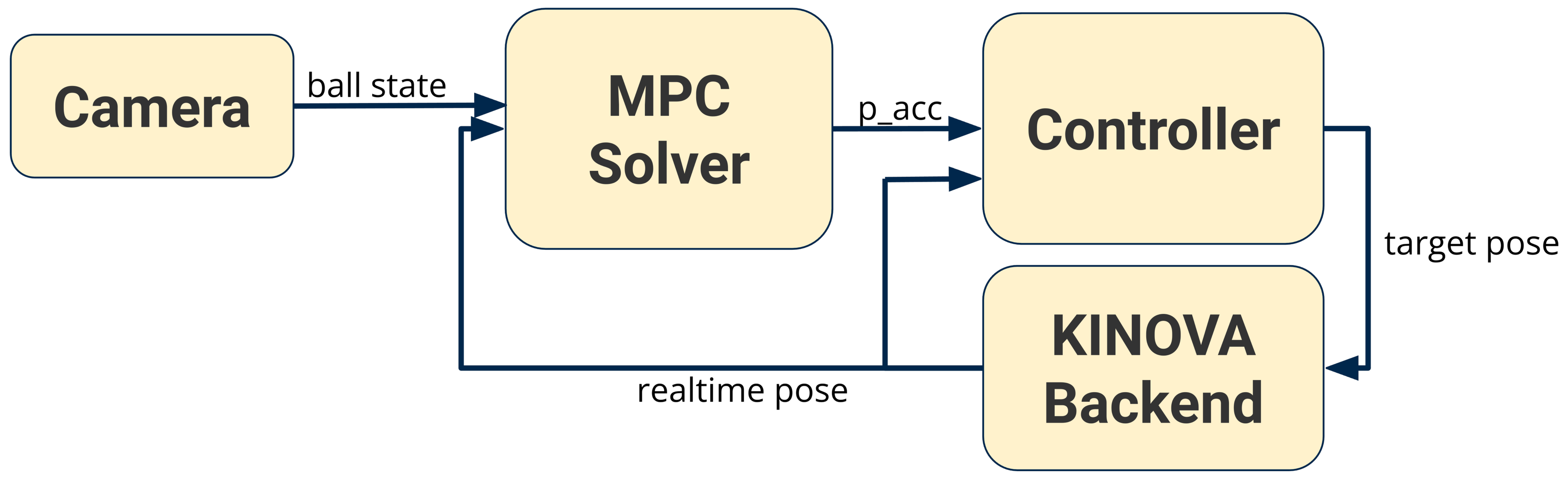

Figure 4. A diagram chart of the schematic implementation of the control algorithm on the hardware.

Model Predictive Controller

Given the state space model, a quadratic cost function is designed and the model predictive controller is computed by solving an optimization problem [2] of the following form at each time step, minimizing the quadratic cost.

Quadratic cost function

$$J_0(x(0), U_0) = x_N' P x_N + \sum_{k=0}^{N-1} x_k' Q x_k + u_k' R u_k$$

with $P \succeq 0,\ Q \succeq 0,\ R \succ 0$.

Constrained Finite Time Optimal Control problem (CFTOC).

$$\boxed{\begin{array}{lll} J_0^*(x(0)) = & \min\limits_{U_0} & J_0(x(0), U_0) \\ & \text{subj. to} & x_{k+1} = Ax_k + Bu_k,\ k = 0, \dots, N-1 \\ & & x_k \in \mathcal{X},\ u_k \in \mathcal{U},\ k = 0, \dots, N-1 \\ & & x_N \in \mathcal{X}_f \\ & & x_0 = x(0) \end{array}}$$

N is the time horizon and $\mathcal{X}$, $\mathcal{U}$, $\mathcal{X}_f$ are polyhedral regions.

We further impose constraints on the states of the system to guarantee safety of the arm during the manipulation task. Both these constraints and the target set are modeled with polytopes in the state space as shown in Figure 5.

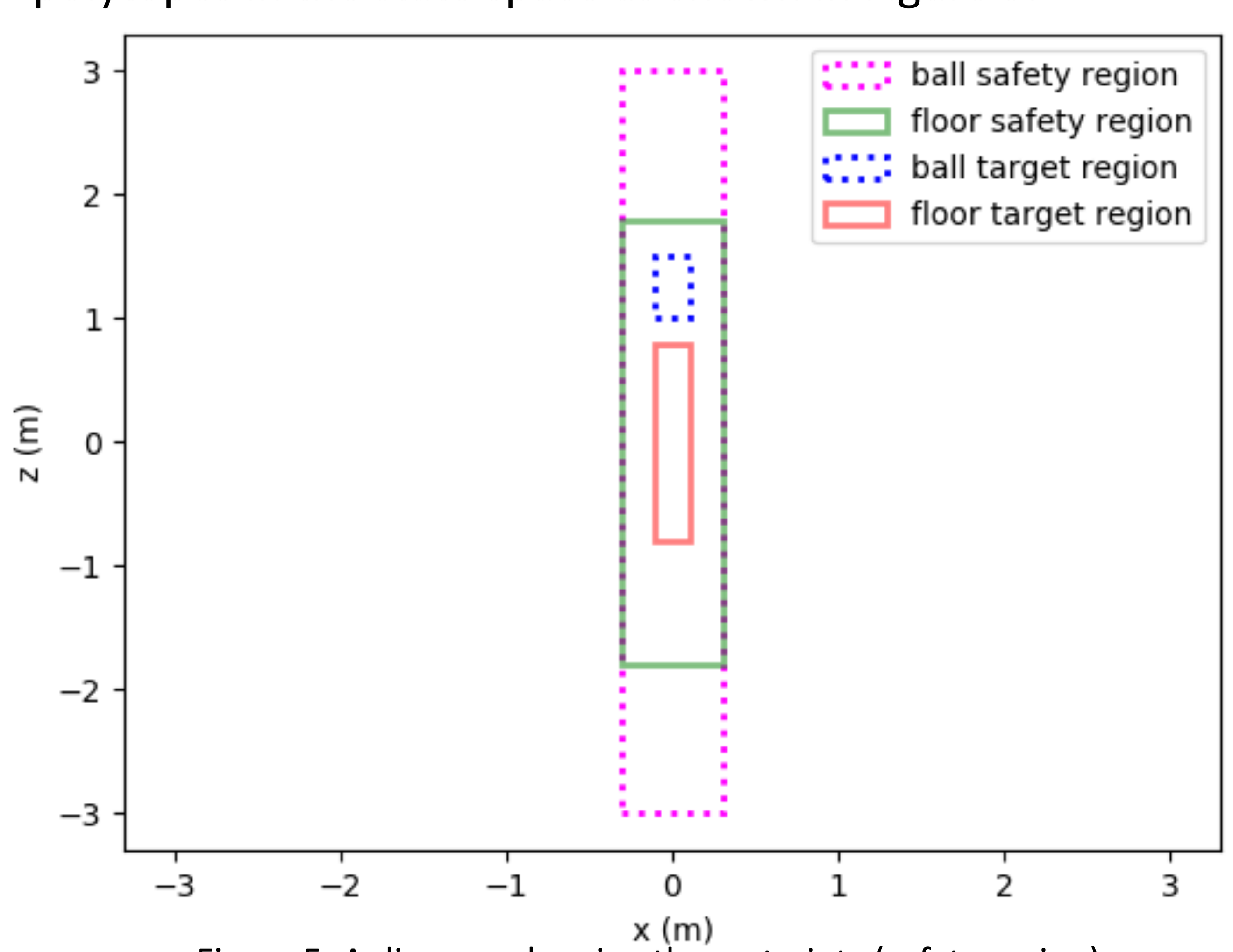

Figure 5. A diagram showing the restraints (safety region) and target regions on the ball and the paddle

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