Predictive Analysis, Uncertainty Modelling of Hall Thruster Propulsion

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Study Overview

- System Identification Task
- Learn delayed differential dynamics of electric propulsion data, interpret noise
- Parameter determination using Bayesian Inference (parameters govern dynamics)
- Characterize posterior of parameters using Markov Chain Monte Carlo (MCMC) variation → Adaptive Metropolis Hastings
- Estimated posterior used to predict various trajectories under different conditions, with **uncertainty**

Adaptive Metropolis Methodology

- Probabilistic model defined → represents posterior of parameters
- Posterior sampled via MCMC, begins with initial sample (preliminary guess of unknown parameters)
- Samples proposed, centered around adapting mean and covariance (update equations shown below)

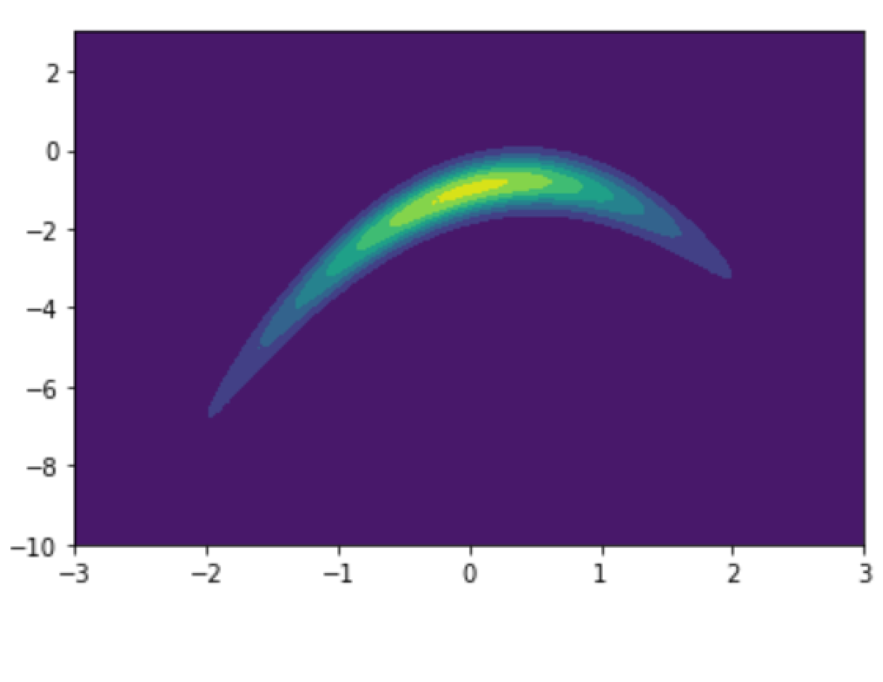
$$\bar{x}_k = \frac{1}{k+1}\left(x_{(k)} + k\bar{x}_{k-1}\right)$$

$$S_{k+1} = \frac{k-1}{k}S_k + \frac{s_d}{k}\left[\xi I + k\bar{x}_{k-1}\bar{x}_{k-1}^T - (k+1)\bar{x}_k\bar{x}_k^T + x_{(k)}x_{(k)}^T\right]$$

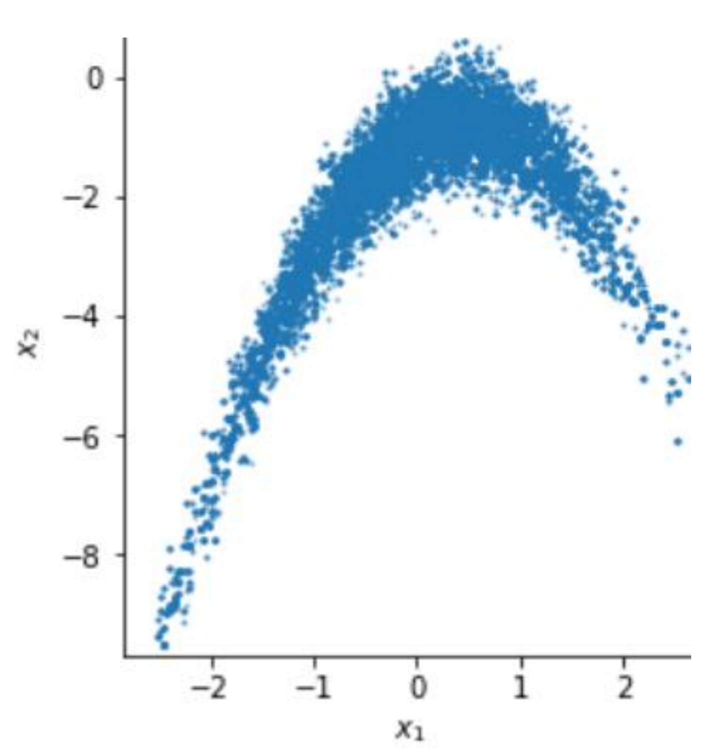
- Accepted with some probability based on accept-reject criteria relating to density of sample within distribution

$$a(x,y) = \min\left(1, \frac{\pi(y)}{\pi(x)}\right)$$

- After several sampling iterations, parameter distribution created



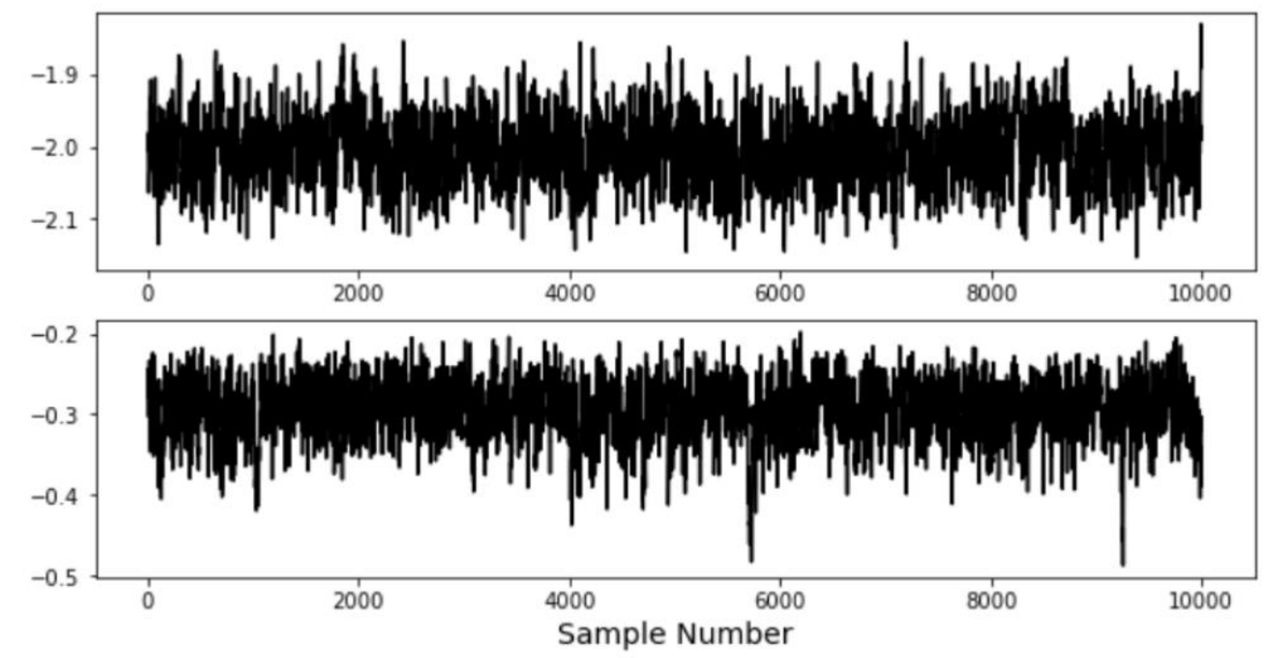
Truth



Sampled 1D Marginal

Mass-Spring-Damper System Setup

- Standard Differential Model: $\ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x$
- Wish to learn model term coefficients for synthesized data unknown parameters → scaling term, coefficient values
- Wish to learn coefficient relationship for synthesized data unknown parameters → coefficient placement
- Single trajectory case → single set of inputs
  Likelihood: $\log(\mathcal{L}(\theta)) = -\frac{1}{2}\frac{(y_i - M(x_i,\theta))^2}{\sigma^2}$
- Samples centered around distribution mean – generated randomly – accurately samples entire distribution
  - mimics white noise



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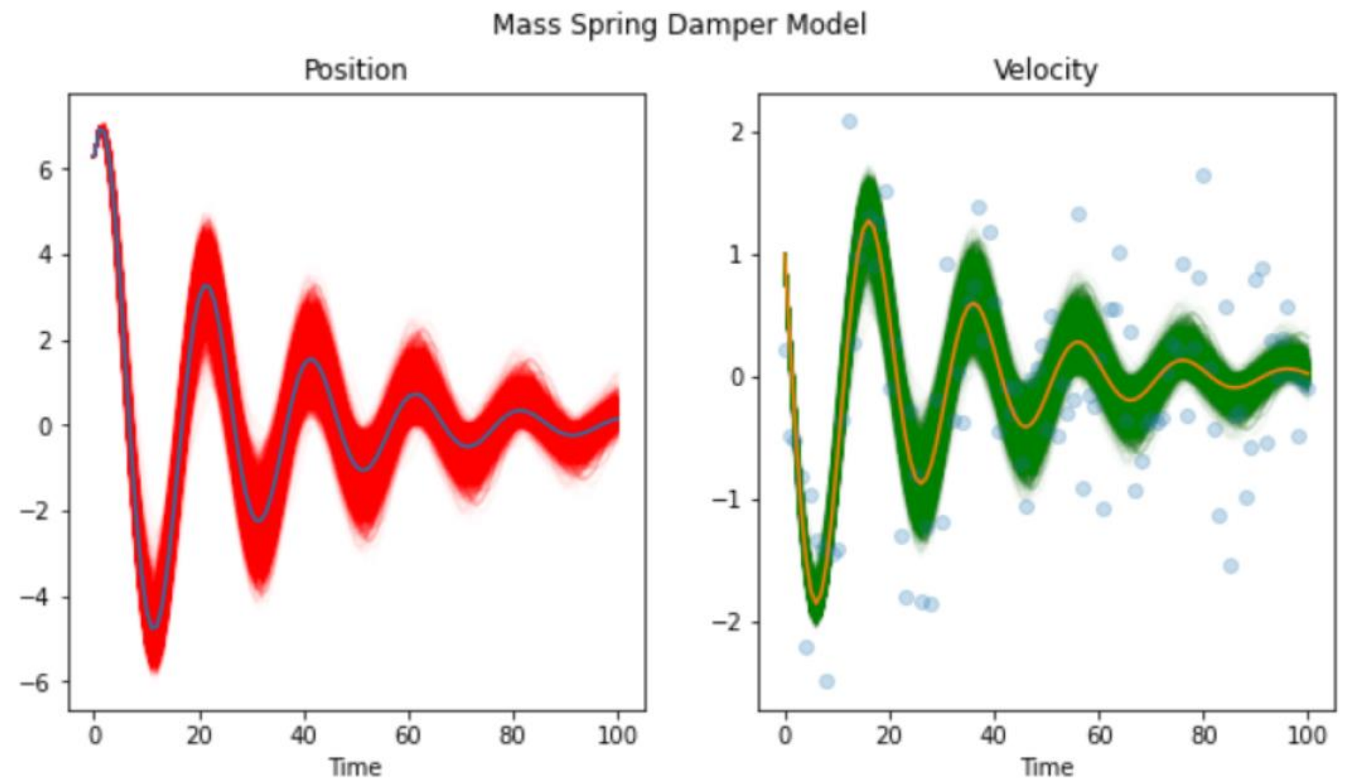
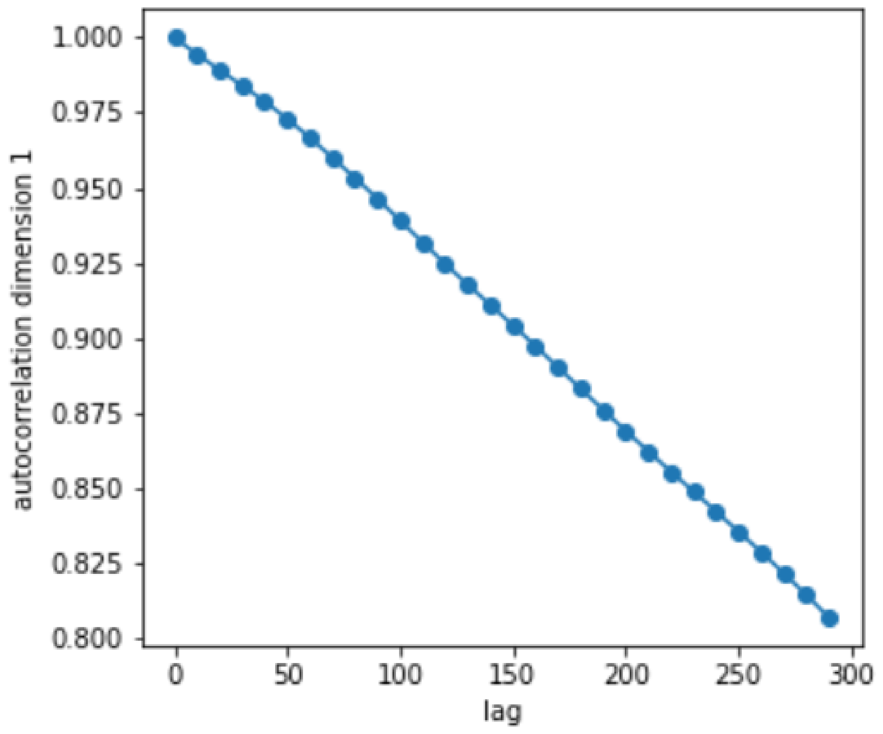
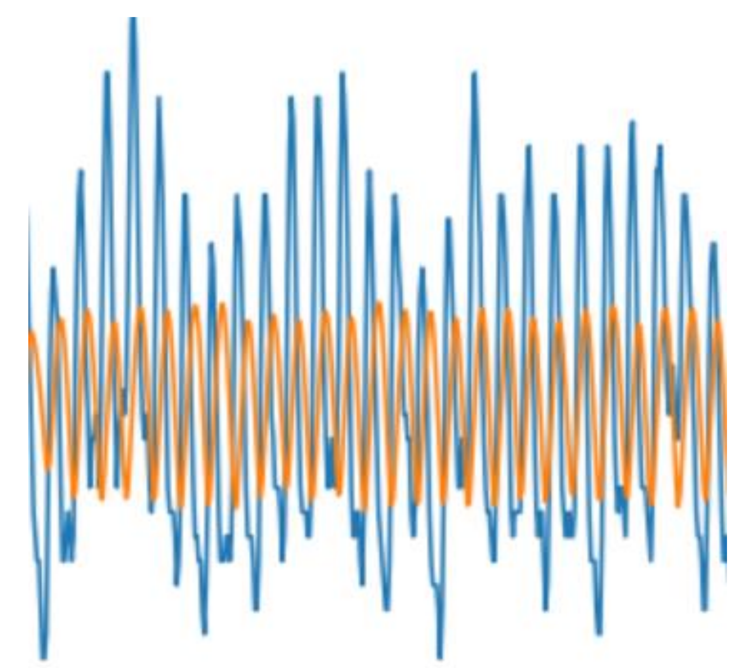


- Parametric case → multiple trajectories (multiple sets of inputs)
  Likelihood: $\log(\mathcal{L}(\theta)) = -\frac{1}{2}\Sigma\frac{(y_i - M(x_i,\theta))^2}{\sigma^2}$
- Allows for Bayesian Inference techniques to be applied to datasets containing multiple trials (various initial conditions, various stages in times, etc…)

Electric Propulsion Application

- Data generated by Plasmadynamics and Electric Propulsion Laboratory (PEPL)
- Had to develop model that governs data through handtuning → actual relationship unknown
  - double hidden layer, state-delay neural network
- Partial characterization, parameter approximation





*unable to share full model generation, data labels (PEPL's work)

- Autocorrelation needs work → lots of lag, samples closely related → trace doesn't not appear random
- Oscillations matched, correct order of magnitude, amplitude peak/amplitude variation requires finetuning

Future Work/Continuations

- Implement noise learning with Inverse Gamma distribution prior
- Prior: $log(f_\theta(\sigma^2, \alpha, \beta)) = -(\alpha+1)log(\sigma^2) - \frac{\beta}{\sigma^2}$
- Likelihood: $log(\mathcal{L}(\theta)) = -\frac{1}{2}[log|\Gamma| + (y - M(x,\theta))\Gamma^{-1}(y - M(x,\theta))^T]$
- Delay-differential state model, data/neural network optimization
- Delayed Rejection MHMCMC variation (DRAM) optimization

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