To: **Dr. Carlos Cesnik, Professor of Aerospace Engineering, University of Michigan**
From: **Matt Foutter, Undergraduate Student Aerospace Engineering, University of Michigan**
Subject: **PID Controller for EASE Pitch Control**
Date: **4/24/21**
DIST: **Mateus De Freitas Virgilio Pereria, Graduate Student Research Assistant, Unv. of Michigan**

Foreword

Prof. Cesnik and his research laboratory require a controller to regulate the pitch of a model aircraft in wind tunnel testing. In design of a candidate controller, there is a great opportunity to learn advanced, graduate control theory techniques, and Prof. Cesnik requires a student to investigate controller design during the Fall 2021 semester. Under guidance from Mateus De Fretias Vrigilio Pereria, my task was to decide on a candidate controller architecture and develop an implementation. The purpose of this report is to present my investigations and analyze the results.

Summary

This report includes a discussion of two candidate PID architectures to meet the task of pitch control for the EASE aircraft in wind tunnel testing. First, to facilitate the design of each controller, a reduced order model was developed for the aircraft's dynamics using a residualization based on structural modal frequencies and actuator bandwidth limitations. This reduced order model was verified against the full order model, for each controller, in linear simulation through a range of expected flight speeds in the time and frequency domain. This project selected five design requirements to guide PID controller development. After linear simulation against the Full Order Model, this project chose to use a controller bandwidth of 0.16 Hz. Nonlinear simulation was then conducted to validate the controller's stability in the absence of the linear assumption used in its design. This project also found the controller to be stable in the presence of a gust magnitude up to __ m/s. Interestingly, the wing's stall angle turns out to be the limiting factor in stability with a gust perturbation. This project recommends Transitioning the controller's definition in MATLAB to a C++ architecture able to interact with the EASE aircraft. This implementation represents the final step before the controller can be used in a wind tunnel setting.

Introduction

Prof. Cesnik and the Active Aeroelasticity and Structures Research Laboratory require a digital controller to regulate the pitch of the EASE model aircraft in wind tunnel testing. The controller is required to maintain or restore the aircraft's trim pitch value by tracking a constant reference signal using the tail's elevator deflection as an input. This project investigates the design and simulates the performance of a PID controller architecture to achieve pitch regulation.

PID Controller Architecture

A PID controller is a feedback technique in control design to force a system's output to track a desired, reference signal. At a high level, a PID controller transforms the error in a system's output to a control input. The governing equation for a PID controller is presented below in Eq 1.0. Where K_p is the controller's proportional gain, K_d is the derivative gain, and K_i is the integral gain. Paired with each gain is a term derived from the error in the output.

e is the error between the output and reference signal while ė gives the time derivative of error and ē gives the time integral of error.

$$u = Kp * e + Kd * \dot{e} + Ki * \bar{e} \qquad \text{Eq 1.0}$$

In the context of this project, the measured output is the aircraft's pitch relative to zero body angle of attack and the tracked signal is the aircraft's trim pitch value.

Reduced Order Model Design

A common approach in control design with application to a nonlinear system is to use a surrogate, linear model. Mateus provided this project a linearized model of the EASE aircraft's dynamics. However, this full order model was not conducive to controller design since the included 163 states caused numerical issues in calculation of the system's transfer function. Methods to reduce the model's order, while preserving the full order model's performance, were investigated to enable control design. After consultation with Mateus, this project chose to reduce the model's size based on the frequency of its associated structural modes using a residualization.

First, the model - represented in continuous time - was transformed to the modal form using eigenvalue decomposition, providing the frequency of each structural state. Given the bandwidth of the EASE aircraft's actuators is 10 Hz, a structural mode with frequency greater than 10 Hz is unable to be tracked by the aircraft's actuators. A cutoff frequency of 10 Hz was set in model reduction such that all structural states with a frequency above 10 Hz were assumed to be in steady state. The system's dynamics were then compressed into the roughly nine structural states controllable by the EASE aircraft. The ability of this reduction technique to preserve the performance of the full order model is studied in a time response simulation to a step input of 0.5 rad in Figure 1. Please note that no controller is used in the open loop simulations below.

Figure 1: Reduced Order Model (ROM) Matches Full Order Model (FOM) Time Response

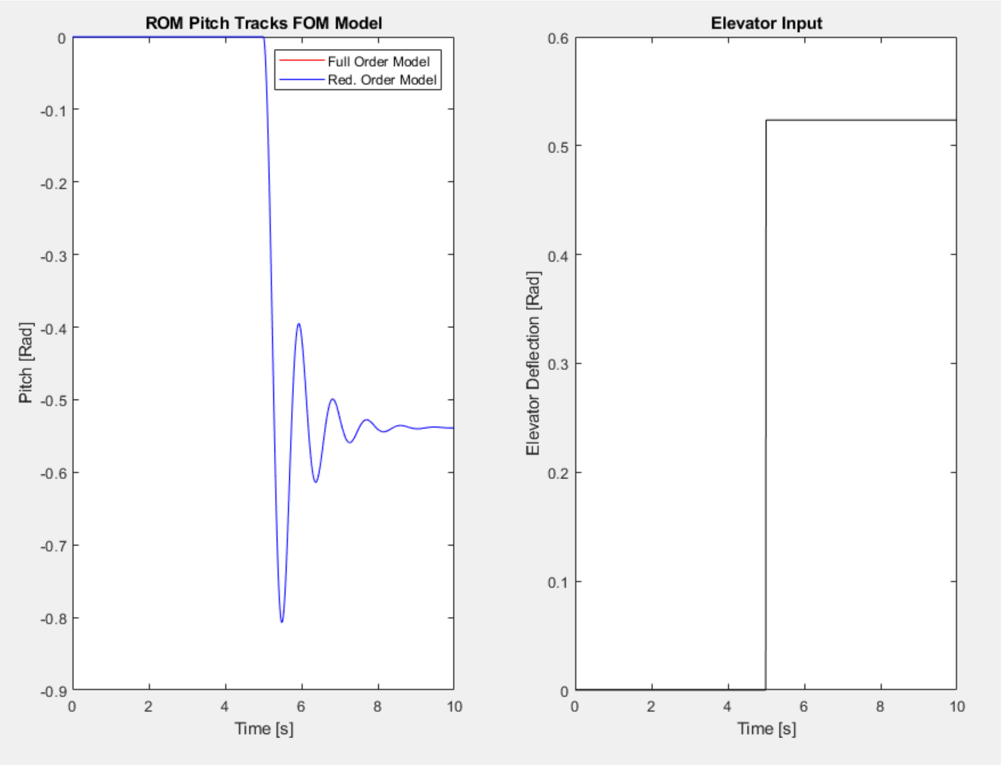
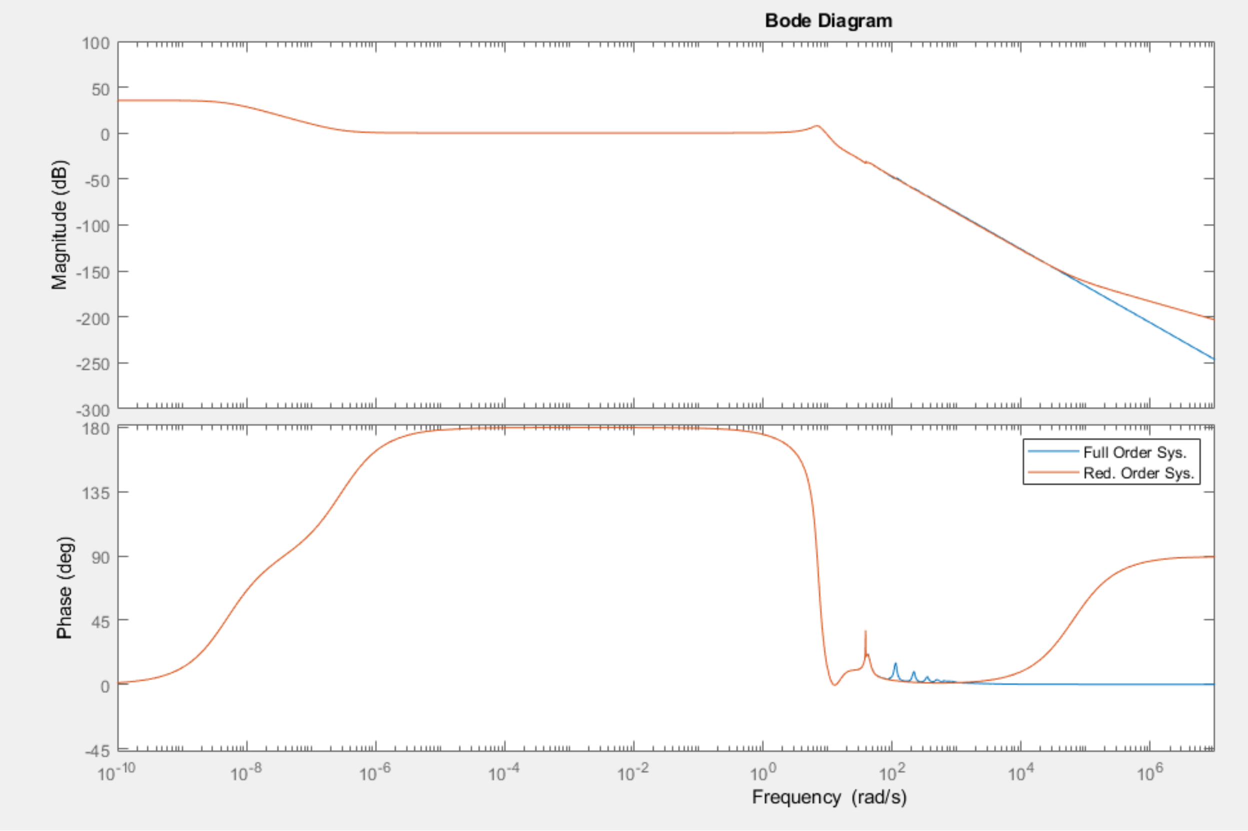


Figure 1 tracks the EASE aircraft's pitch as simulated by the full and reduced order model to a 0.5 radian step input at the elevator. Importantly, each simulation captures the same trajectory in pitch. Next, the frequency response of each model is compared in Figure 2.

Figure 2: ROM Frequency Response Matches FOM below ~ 70 rad/s



Near roughly 100 rad/s, corresponding to 16 Hz, divergence is noticeable between the FOM and ROM in Figure 2's gain and phase plots. This divergence is to be expected, however, as the ROM is designed to track structural states below 10 Hz and is unable to capture the excitation of relatively high frequency states in the FOM. Given the ROM is able to follow the FOM's time and frequency response within the bounds of its design, this project concludes this residualization technique is able to create a surrogate, reduced order model for the EASE aircraft's dynamics. This ROM is used throughout the remainder of this project in controller design.

PID Design Requirements

With the ROM available and verified against the full order model, the PID's gain terms were sized to meet performance and stability design requirements. The controller's design requirements are listed below.

1. Demonstrate an overdamped step response with little oscillation, if any
2. Demonstrate a rise time to 90% the steady state value less than roughly 2 seconds
3. Demonstrate a 5% band settling time in the range of 4-6 seconds
4. Demonstrate a gain margin greater than 6 dB

5. Demonstrate a phase margin greater than 30 deg

The first design requirement is an attempt to reduce the magnitude of acceleration in pitch during a step response. Large accelerations can translate to a discrepancy in the controller's performance in numerical simulation and experiment due to model uncertainty. In the worst case, the controller could experience a loss of stability or cause the aircraft to unexpectedly stall. The second and third requirements encompass the desired time domain performance. A rise time of less than roughly 2 seconds and 5% band settling time around 4-6 seconds combine to ensure a relatively quick initial and steady-state controller response. A fast controller is desirable as a slow controller will experience difficulty responding to an unsteady free stream and incurs a time penalty in experiment. The fourth and fifth design requirements represent stability margins specified in the frequency domain. The specific values used for the gain and phase margin were determined through a recommendation from Mateus based on his previous experience. A gain and phase margin provide headroom on the controller's stability to account for uncertainty and imperfection in this project's aircraft dynamics model.

PID Controller Design

With the above design requirements, MATLAB's pidTuner function was utilized to size each gain term for a candidate PID controller. The input to pidTuner is the open loop transfer function from the system's input to the output and MATLAB returns the closed loop step response under control from a candidate PID architecture. The knobs available to tune the controller's gains are the bandwidth and phase margin.

This project found the controller's bandwidth is proportional to the speed of the time response but also the magnitude of oscillation. On the other hand, the prescribed phase margin is inversely proportional to the speed of the response and the magnitude of the oscillation. A balance, then, must be found between design requirements one through three. Initially, this project found a set of three candidate controllers, differentiated by bandwidth, with varying levels of oscillation and time response. The highest bandwidth controller was deemed to exhibit an acceptable level of oscillation and the fastest time response. The associated closed loop step response, open loop gain and phase margin plot are included below in Figure 3.

Figure 3: Closed Loop Step Response and Open Loop Bode Plot for Candidate Controller

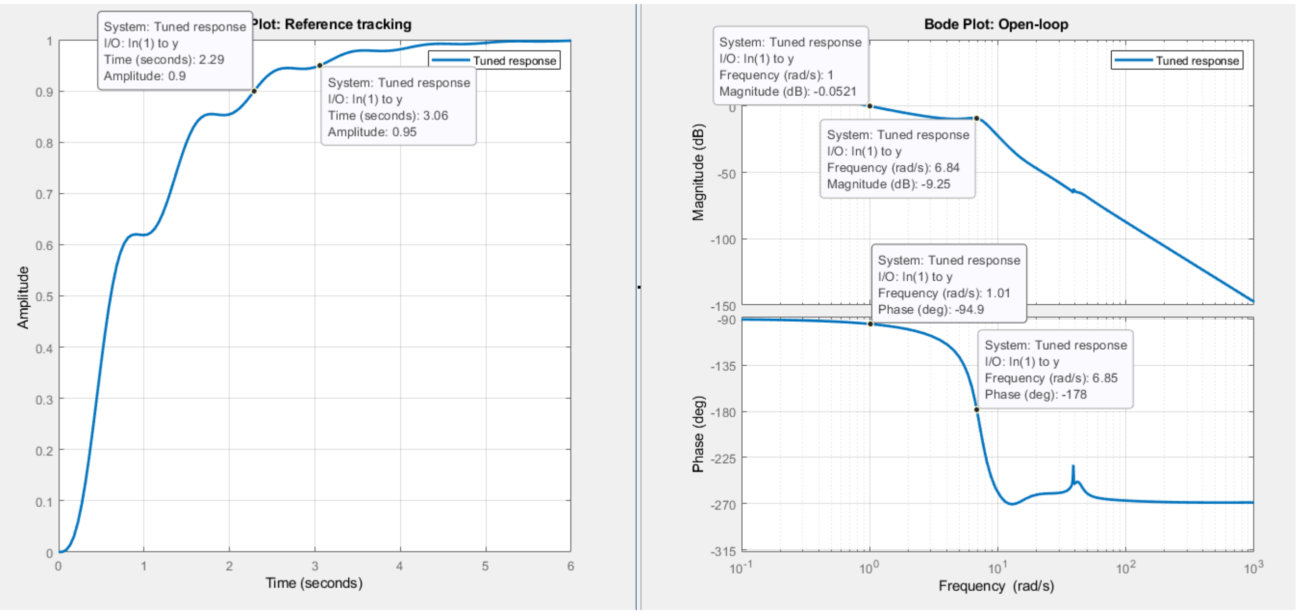


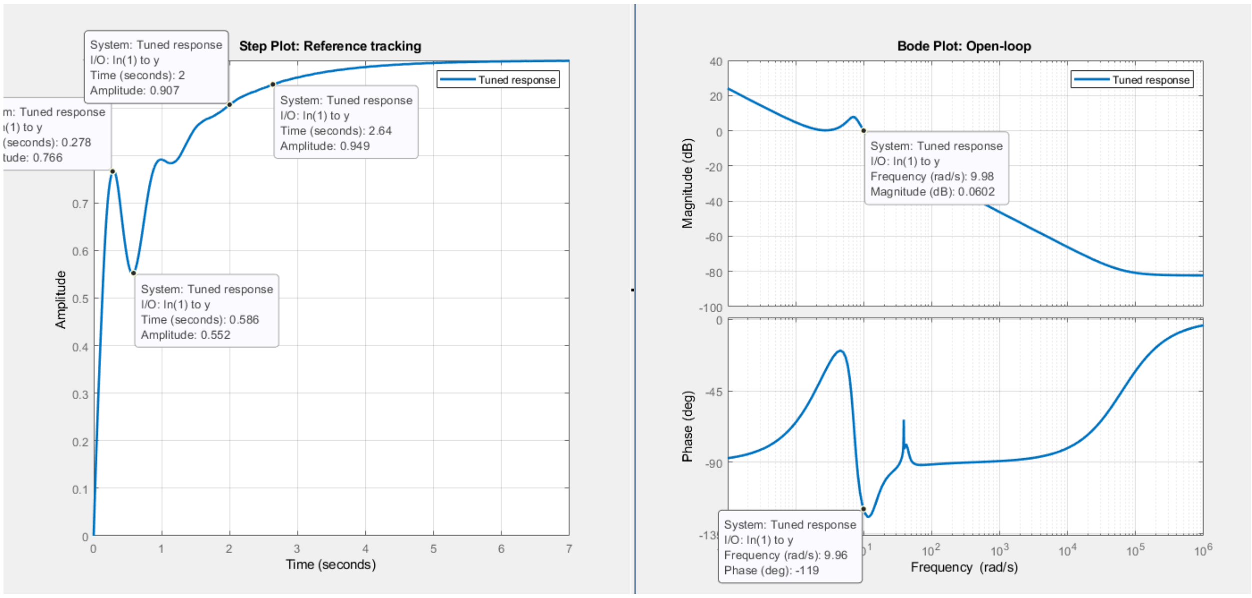
Figure 3 above demonstrates the time and frequency domain stability of the candidate controller. Though the response does exhibit some oscillation, through the full range of simulation, this oscillation was deemed acceptable given its magnitude relative to the step command input. The closed loop system above is tasked to track a reference value of one radian from an initial condition of zero. In the application of this project, this simulation framework would translate to pitch excursion of ~ 60 degrees - well beyond the limits of stall. This project expects to see a maximum pitch excursion of roughly 20 degrees, from -10 degrees to 10 degrees, before stall. Given the oscillation is well contained within a 0.1 radian bound on a one radian reference command, the oscillation in this project's application should be bounded above by 2 degrees for the maximum step command available. The open loop bode plot on the right of Figure 3 demonstrates a gain and phase margin of 9.25 dB and 85.1 deg, respectively, which meet the design requirements. The gain terms associated with this controller are presented below in Table 1. This controller has a bandwidth of 0.16 Hz.

Table 1: Gain Terms for 0.16 Hz Candidate PID Controller

Gain Term	K_P	K_D	K_I
Gain Value	0	0	-0.95

The controller architecture above is unconventional in that the derivative and proportional gain terms are zero. This project found that a bandwidth of ~ 0.19 Hz is the threshold for a controller with nonzero proportional and derivative gain terms. After consultation with Mateus, this project cannot offer an explanation as to why pidTuner suggests a controller without proportional and derivative feedback if the bandwidth is less than 0.19 Hz. The 0.16 Hz controller, however, ran into issues with exaggerated oscillation in nonlinear simulation to be discussed later in this report. For this reason, a second controller architecture is presented below in Figure 4 with nonzero proportional, derivative and integral feedback. The closed loop time response and bode plot for a second, traditional PID controller with 1.6 Hz bandwidth is presented below in Figure 4.

Figure 4: 1.6 Hz Candidate Controller Time Response and Bode Plot



Previously, a controller with the time response in Figure 4 was abandoned in favor of a response with a lower magnitude of oscillation. For reasons to be outlined in a discussion on nonlinear simulation, a controller with a traditional PID architecture and gradual steady state response was investigated. While the 1.6 Hz controller above does exhibit a ~12 deg oscillation magnitude, greater than the 0.16 Hz controller's oscillation, this oscillation is roughly one fifth the reference command size. Therefore, in the case of a 20 degree pitch excursion, the 1.6 Hz controller should exhibit oscillation contained within a 4 degree band, deemed to be acceptable for this project. This controller also reaches 90% the steady state command in 2 seconds and settles in a 5% band in about 2.5 seconds - both faster than the 0.16 Hz controller. Figure 4 demonstrates the stability of this candidate controller in the frequency domain as the phase plot does not intersect -180 degrees in which case the gain margin is positive infinity. At the gain's only crossover to negative gain, the phase is -119 degrees creating a phase margin of 61 degrees. A table of the gains for this candidate controller are presented below in Table 2.

Table 2: Gain Terms for 1.6 Hz Candidate PID Controller

Gain Term	K_P	K_D	K_I
Gain Value	-0.83	-0.11	-1.6

With two candidate controllers developed under a reduced order model assumption, each controller's performance was evaluated in simulation against the full order model through a range of expected flight speeds.

Linear Controller Validation

This section explores each candidate controller's time and frequency domain response with a varying input wind speed. This project selected a feasible range of flight speeds as 20 m/s - 30 m/s from Mateus' recommendation. For each flight speed, a unique, linearized dynamics model for the aircraft was generated and reduced to a lower order model. This report discusses the time response of each controller first.

Time Response

The pitch response in the reduced model is compared against the equivalent full order model response in Figure 5 below for the 0.16 Hz controller. Figure 5 shows that the controller is stable throughout the range of expected flight speeds; though the controller is designed under a reduced order model assumption, it is able to control the full order model. The response in each case follows the expected trajectory from controller design.

Similarly, the 1.6 Hz controller in Figure 6 is able to regulate the aircraft's pitch to trim from a zero initial condition using the reduced and full order model. In fact, the controller's closed loop response follows the expected trajectory from controller design. In the time domain, the controller's pitch response appears to be indifferent to flight speed as the time characteristics, like rise time and 5% band settling time, appear to be constant across the eleven simulations. Please continue to the next page for Figures 5 and 6.

Figure 5: 0.16 Hz Controller Stable in Closed Loop Linear Simulation through 20 - 30 m/s

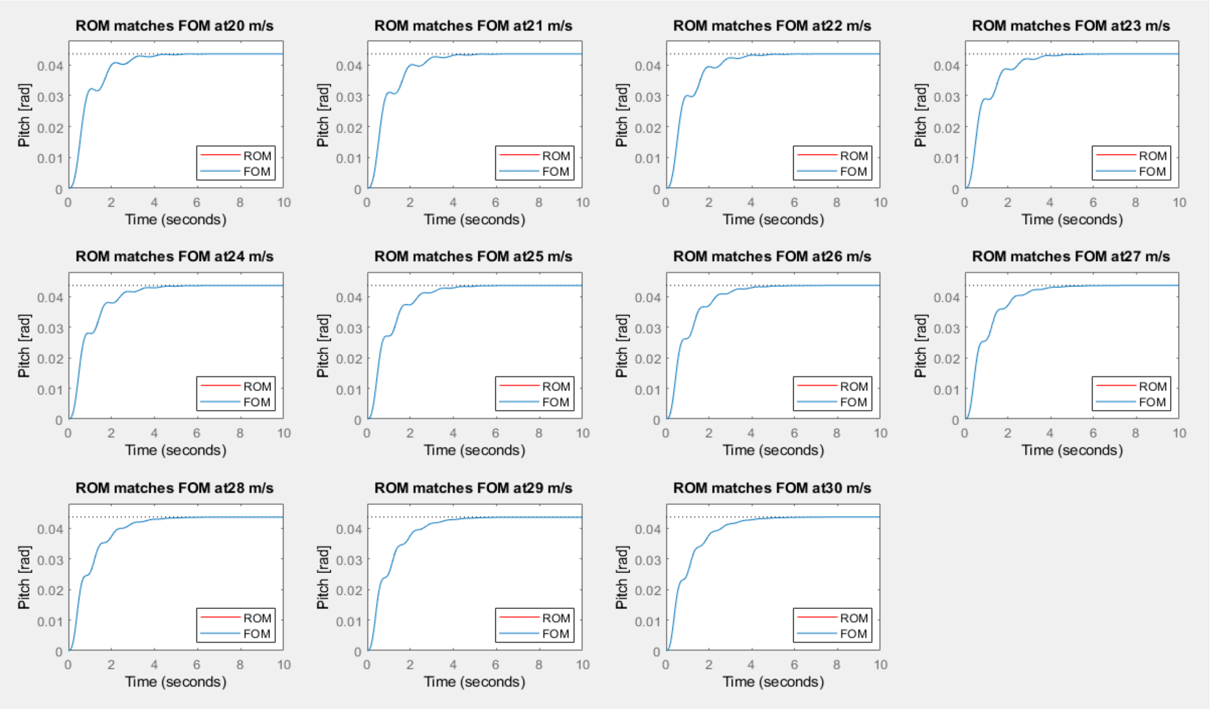
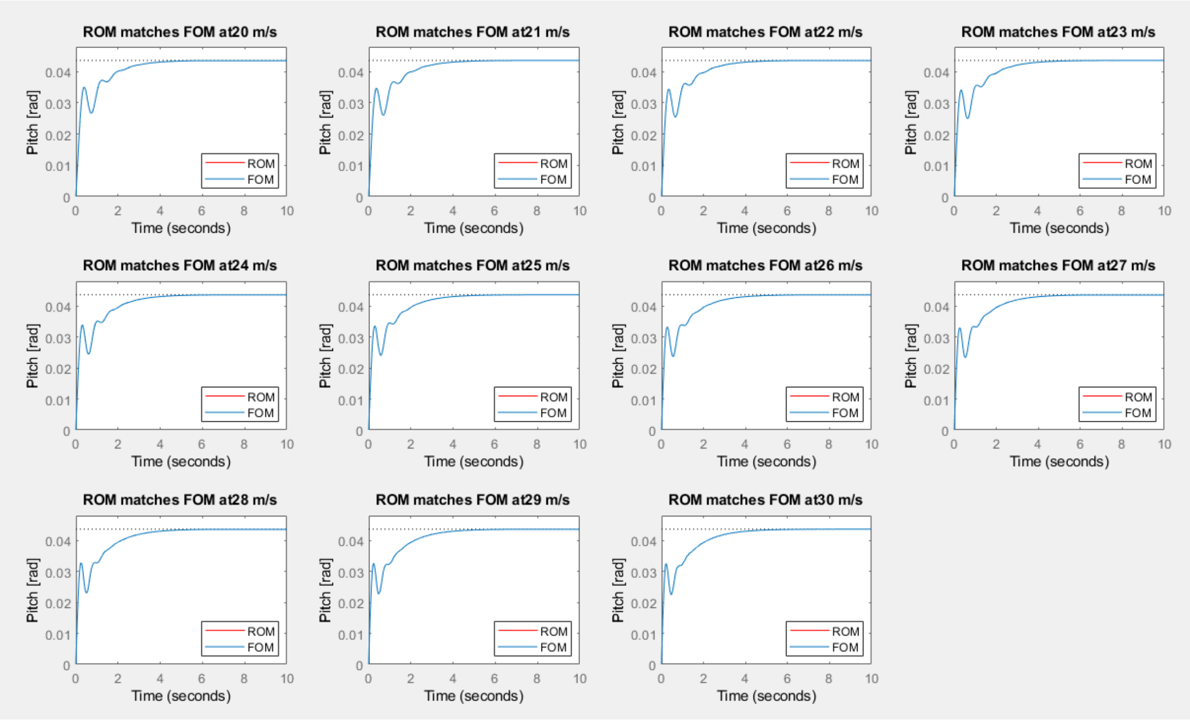


Figure 6: 1.6 Hz Controller Stable in Closed Loop Linear Simulation through 20 - 30 m/s



Frequency Response

Originally, this project only investigated the time response of a candidate controller before progressing to nonlinear simulation. After issues encountered in non-linear simulation, and recommendation from a member in the A^{2}SRL, the frequency response of each controller was investigated through the full range of flight speeds. This exercise provided new insight into controller robustness. Two plots are needed to investigate the frequency response of each model order: one for the model's gain and a second for the phase. Given this project uses two models and a range of eleven flight speeds, forty four plots are required to capture a controller's ROM response and FOM response. Given these bode plots follow a general trend, for the sake of brevity, a subset of the forty four plots for each controller are presented in the body of this report.

This project found an interesting result in studying the gain response of the 0.16 Hz controller with flight speed. In particular, the gain margin of the open loop system increased with flight speed. The gain margin begins at 4.47 dB, shown in Figure 7 below, and rises to 7.72 dB at 30 m/s. This result is important because the 0.16 Hz controller does not meet the gain margin design requirement at 20 m/s. Given this investigation was initiated after the 0.16 Hz architecture demonstrated issues in non-linear simulation, the gain margin's dependence on the flight speed was not caught in the original design. Please note in Figure 7 and 8 the full order model is pictured on the left while the reduced order model is pictured on the right.

The open loop system's gain margin returns to a value greater than 6 dB at 27 m/s, pictured in the Appendix. One possible explanation for this trend found is that a higher flight speed can add some stiffness to the aircraft's motion. Then, at a lower speed, and lower stiffness, uncertainties in the aircraft's dynamics model are accentuated.

Figure 7: Gain Margin Below Design Requirements at 20 m/s

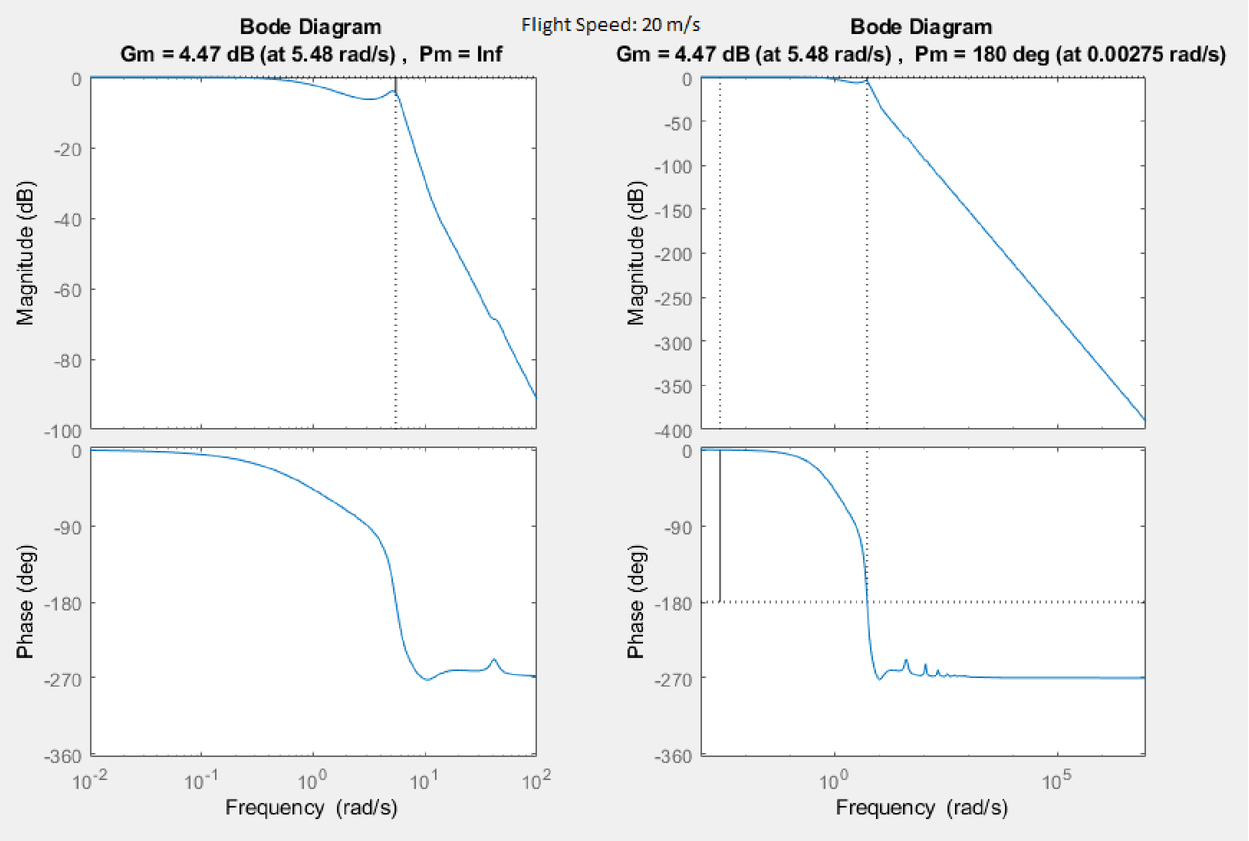
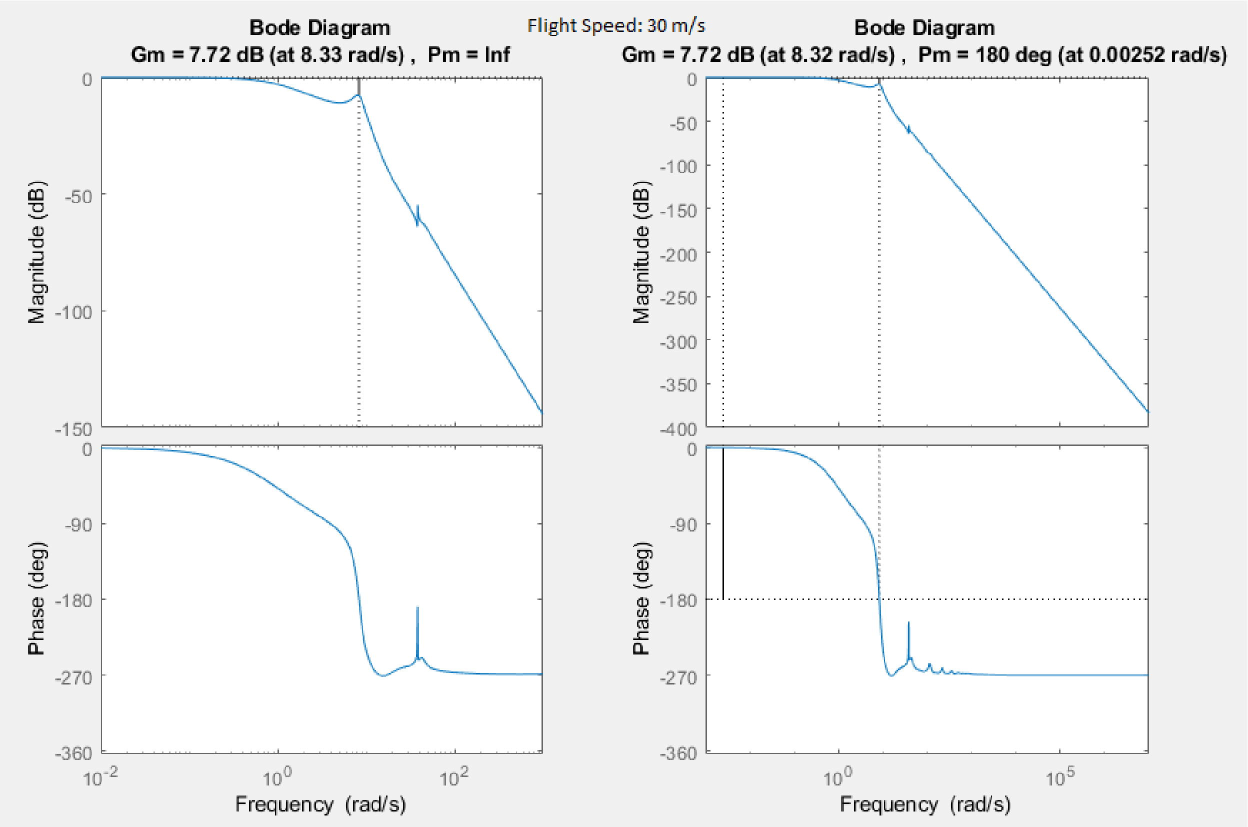


Figure 8: Gain and Phase Margin Meet Design Requirements at 30 m/s



The result that the 0.16 Hz controller does not meet this project's design requirements at all flight speeds is further motivation to investigate an alternative controller architecture. Also, of note, the frequency response between the reduced order and full order model is noticeably different at relatively high frequencies, which follows from the cut off frequency used in model reduction. While the reduced model is not designed to capture the high frequency states in the full model, the reduced model is still able to demonstrate a comparable gain and phase margin.

Figure 9 and 10 below give the gain and phase margin for the 1.6 Hz controller open loop system at 20 m/s and 30 m/s, respectively. Please note the figures below are formatted in a similar fashion as Figures 7 and 8 above as results for the full order model are pictured to the left while results for the reduced order model are presented on the right. The 1.6 Hz controller demonstrates an infinite gain margin at 20 m/s and 30 m/s, which is a positive sign for the stability of this controller. While this project cannot explain the dramatic difference in gain and phase margin between controllers differentiated by 1 Hz of bandwidth, the traditional PID architecture in the second candidate controller could be a factor.

Figure 9: 1.6 Hz Controller Infinite Gain and 116 deg Phase Margin at 20 m/s

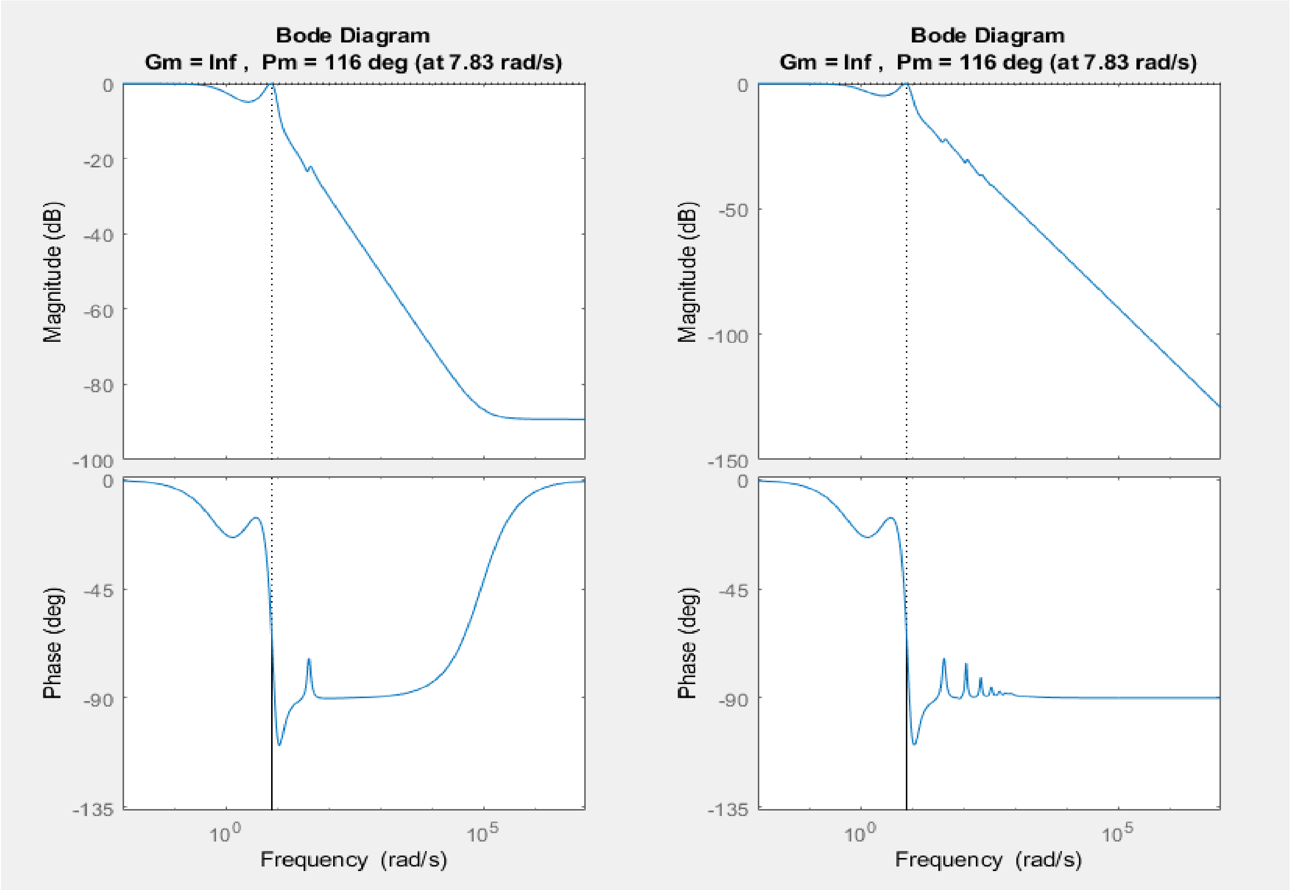
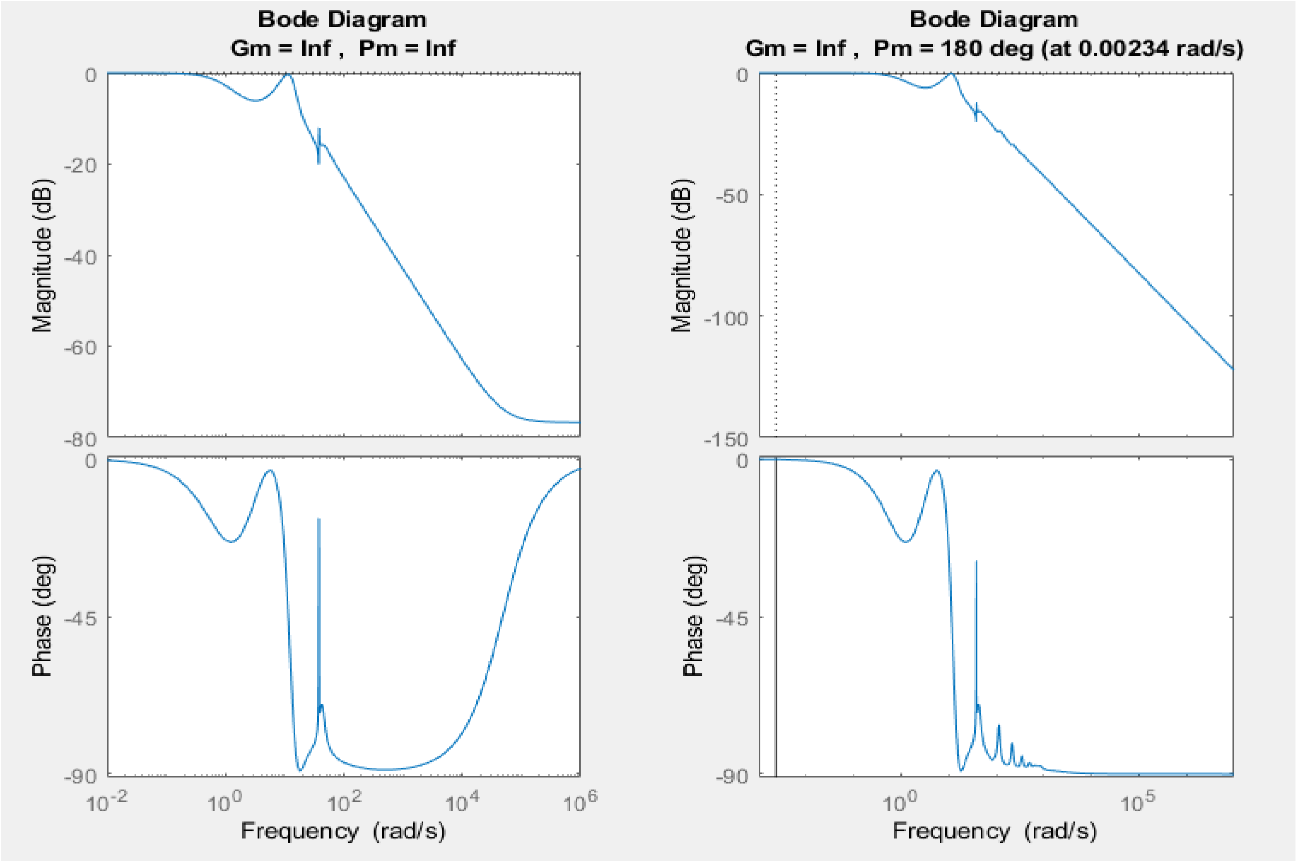


Figure 10: 1.6 Hz Controller Infinite Gain and Phase Margin at 30 m/s



The difference in the frequency response at relatively high frequencies is very noticeable in Figure 9 and Figure 10 above. However, given that the full model and reduced model are able to achieve comparable gain and phase margins, the high frequency is not considered a concern.

Given the 1.6 Hz controller is able to demonstrate a gain and phase margin within this project's design requirements, this controller is preferable to the 0.16 Hz controller in the frequency domain on the basis of robustness. The relative performance of each controller's time domain response in nonlinear simulation will be discussed in the next section.

Nonlinear Simulation Validation

I finished linear simulation and the controller's initial design in Fall 2021. Beginning Winter 2022, I planned to simulate the controller above in a nonlinear environment. However, upon returning to campus, I learned design changes had been made to the EASE model which were not reflected in my previous model used in PID controller design. I redesigned the controller with the new model following the same steps outlined above. For the sake of brevity and to avoid redundancy, I will omit the results from this controller's redesign from this report. The controller now demonstrates a 0.16 Hz bandwidth and follows a similar pitch trajectory in linear simulation.

In the scope of this project, evaluating each controller in nonlinear simulation represents the final step in controller design before an implementation is developed in software to test in a wind tunnel. Non-linear simulation is also the most rigorous evaluation step, as the controller is simulationed in the absence of the linear model assumption used in the controller's original design. A full nonlinear testing regime includes the full range of expected flight speeds, as done in linear simulation above. A flight envelope of 20-30 m/s is investigated in combination with an initial pitch array of -10 deg to 10 deg.

For the brevity of this report, nonlinear simulation results are presented at a single flight speed with multiple initial pitch conditions. Results at 20 m/s are highlighted in the body of this report since the damping in the system is lowest and the control effort is greatest. Results at 30 m/s are included in the appendix of the report for the audience's review.

Figures 11-13 are significant because the controller is able to demonstrate adequate pitch control to return the aircraft's pitch to the equilibrium position in a timely manner. Though the trajectory between linear and nonlinear simulation are slightly different, the nonlinear trajectory does meet specified design requirements. In short, through the full pitch envelope of -10 deg to 10 deg, the controller is able to restore the trim pitch. This trajectory is true for all speeds between 20 m/s and 30 m/s.

Figure 11: Controller Stable at 20 m/s with Positive IC

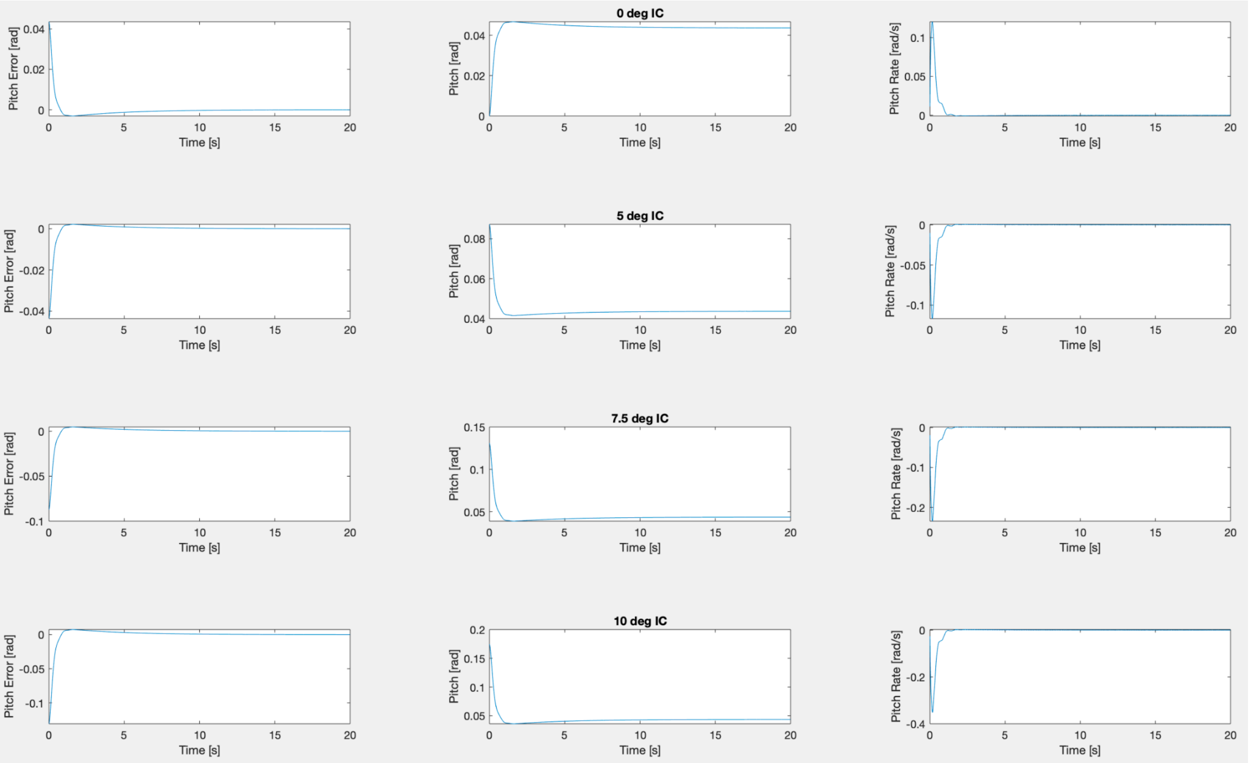


Figure 12: Controller Stable at 20 m/s Trim IC

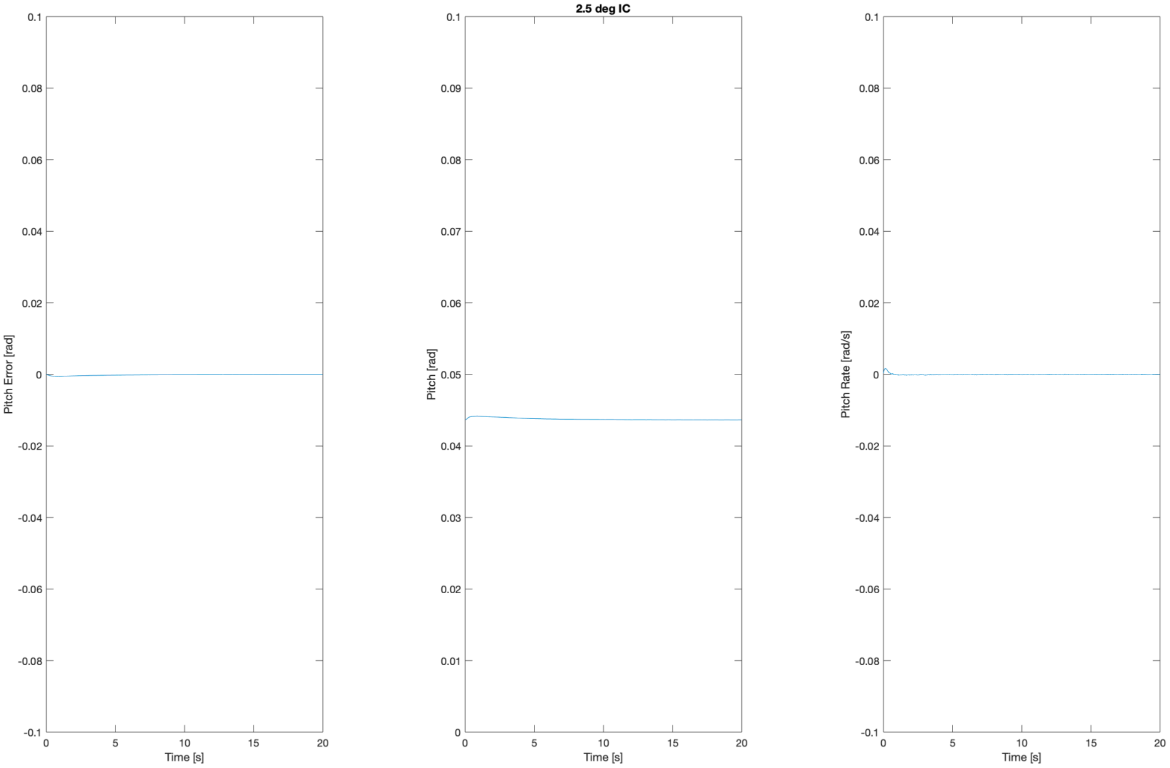
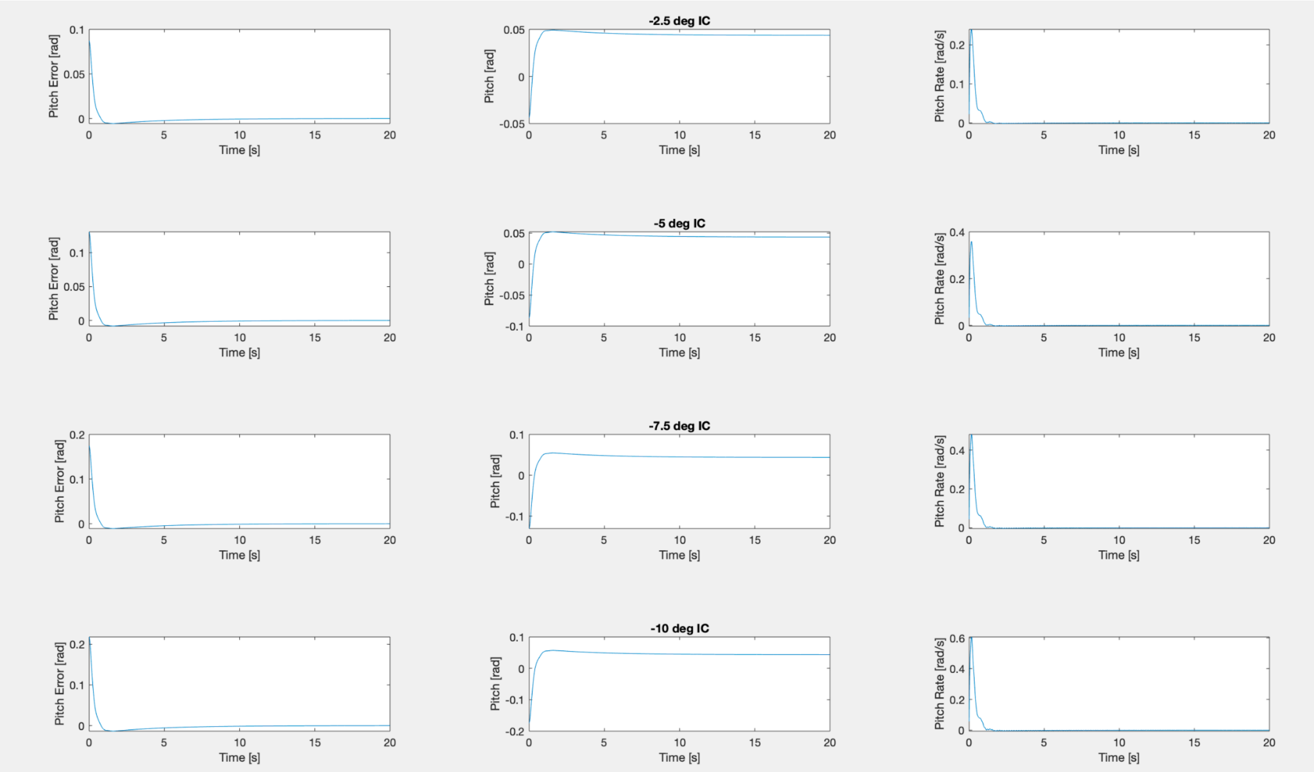


Figure 13: Controller Stable at 20 m/s Negative IC

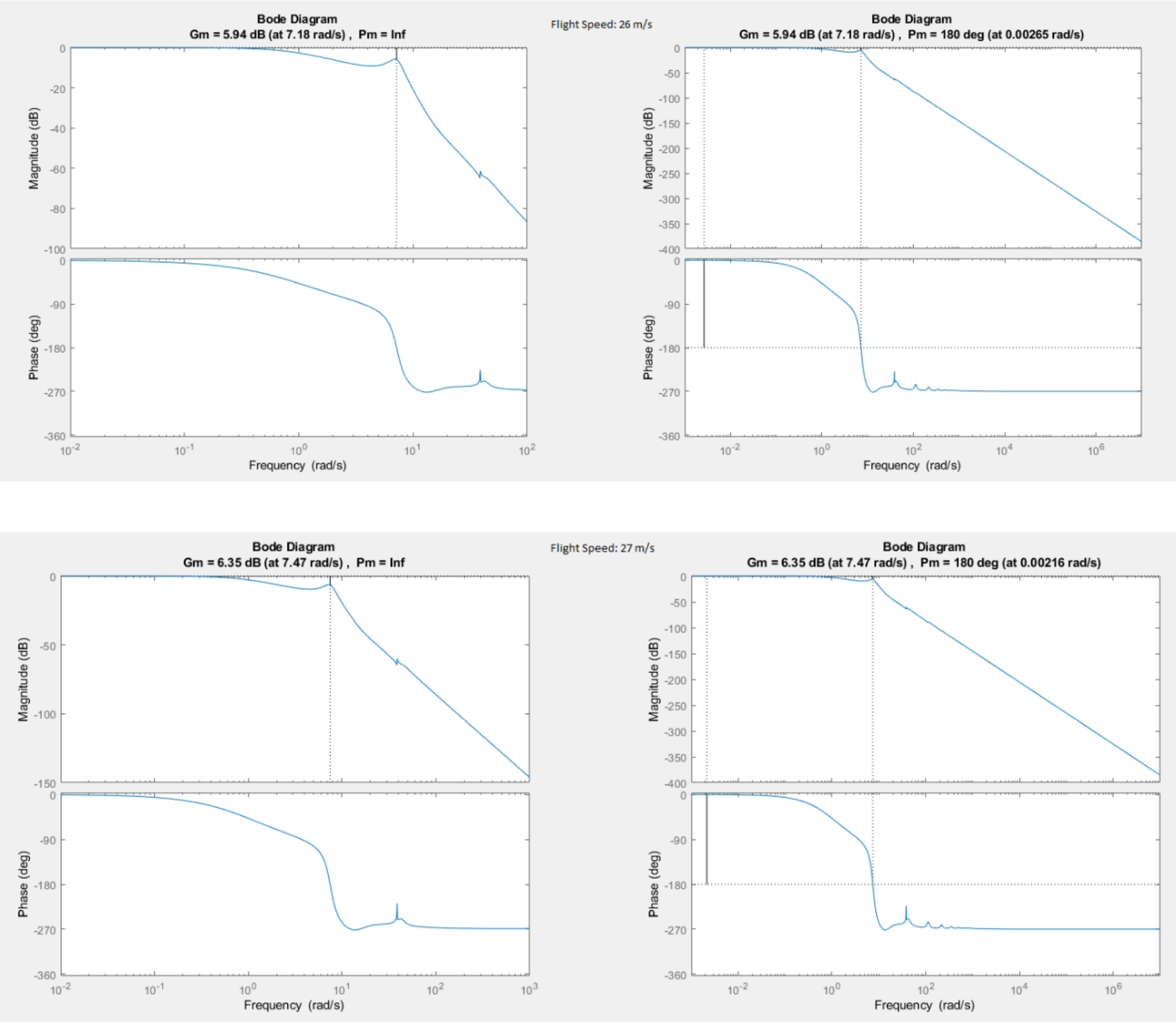


Conclusion

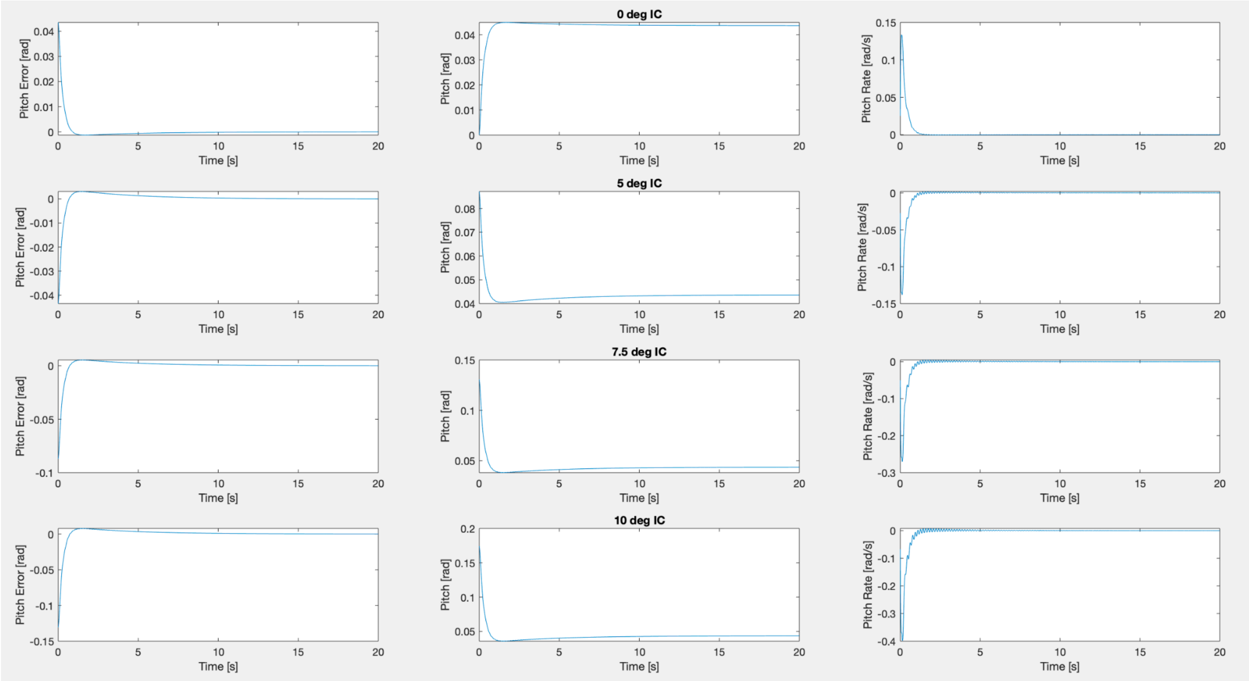
This project investigated multiple PID controller architectures to satisfy pitch control for the EASE aircraft model in wind tunnel testing. Two controllers were isolated from MATLAB’s pidTuner function based on the ability to meet time and frequency domain requirements and the magnitude oscillation in the controller’s response. A detailed analysis in linear and nonlinear simulation determined the 0.16 Hz controller is suitable for pitch control in the specified flight envelope.

Appendix

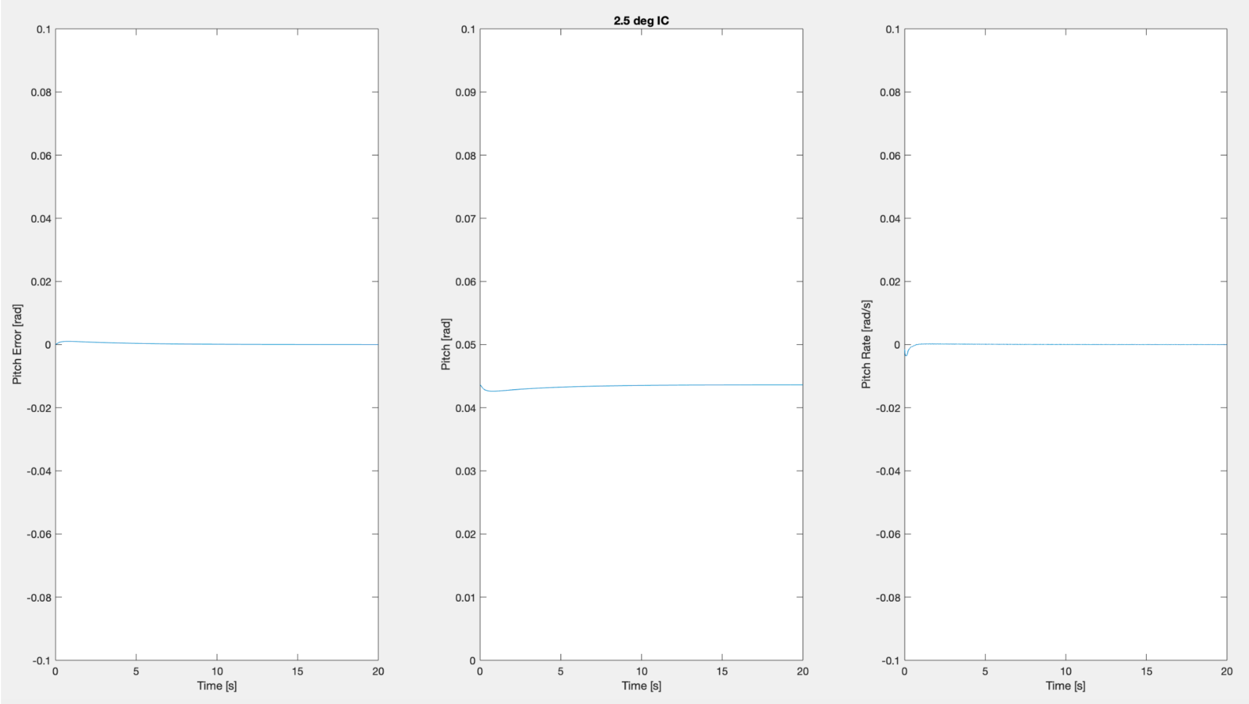
Appendix Figure 1: Gain Margin Crosses 6 dB after 26 m/s



Appendix Figure 2: Controller Stable at 30 m/s Positive Initial Condition



Appendix Figure 3: Controller Stable at 30 m/s Trim Initial Condition



Appendix Figure 4: Controller Stable at 30 m/s Negative Initial Condition