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## PROBLEM

Autonomous vehicles offer a wide variety of benefits for the transportation sector, including increased efficiency, lower costs and wider access to transportation.

Current research at the Automotive Research Center at the University of Michigan aims to extend autonomous capabilities to off-road vehicles. Military vehicles operating in a priori unknown environments stand to realize gains in combat effectiveness and personnel safety through enhanced autonomous capabilities

A critical element of autonomous capabilities are path-planning algorithms that select a vehicle's trajectory towards the objective. These algorithms must consider Obstacle Avoidance, Path Optimization under various metrics as well as safety and computational limits for embedded hardware.

Reachability Based Trajectory Design (RTD) offers a promising approach for mathematically provable safe navigation algorithms. However, current implementations do not account for the unique dynamics of off-road vehicles which are affected by variations of terrain as well as tire-soil interactions.

The focus of this work will be to adapt Reachability Based Trajectory Design algorithms to off-road applications. We will aim to extend the mathematically provable safety guarantees offered by RTD to be compatible with more complex dynamics models that capture the behavior of off-road vehicles working in an *a priori* unknown area. We will work to factor in effects of terrain gradients as well as tire-soil interactions to the algorithms while ensuring feasible implementation with limited real-time processing needs

## REACHABILITY BASED TRAJECTORY DESIGN

### The Forward Reachable Set

Reachability Based Trajectory Design (RTD) is a technique developed at the University of Michigan's ROAHM Lab which aims to provide provably safe trajectory planning algorithms. RTD leverages a construct called the Forward Reachable Set (FRS) to implement this design

The Forward Reachable Set (FRS) is a key construct in the development of RTD Based Algorithms. Intuitively, the FRS describes a set of states that the system could starting within a given initial set within a specified time horizon. Formally. Given a state space and system described by dynamics

$$\frac{d}{dt}x = f(x, u)$$

Where  $u$  represents control inputs,  $x$  represents the state vector. Given an initial set of states  $\mathcal{L}$  and a planning horizon  $t_{plan}$  The FRS is defined as

$$F(t_{plan}) = \{x_i \mid \exists u, \underline{x}_0 \text{ s.t. } x(\bullet) = f(x, u), \underline{x}(0) \in \mathcal{L}, \underline{x}(t_{plan}) = x_i\}.$$

The definition of the FRS forms the basis of the safety guarantees offered by RTD. The computation of the FRS is a complex task and is the core of this work. He following factors are considered that are unique to this application

## Key Considerations for RTD and FRS Algorithms

Consideration	Details
Capability to work with Non-Linear Dynamics	Off-road application entails terrain effects, tire-soil interactions not captured by simplified linear models
Guaranteed Safety / Not-at-fault navigation	Mathematically provable safety important for use-cases involving key combat support operations
Computational Efficiency at Runtime	Limited computational capacity for on embedded hardware in vehicles

## REACHABILITY BASED TRAJECTORY DESIGN

The Reachability Based Trajectory Design workflow follows the process illustrated in this graphic

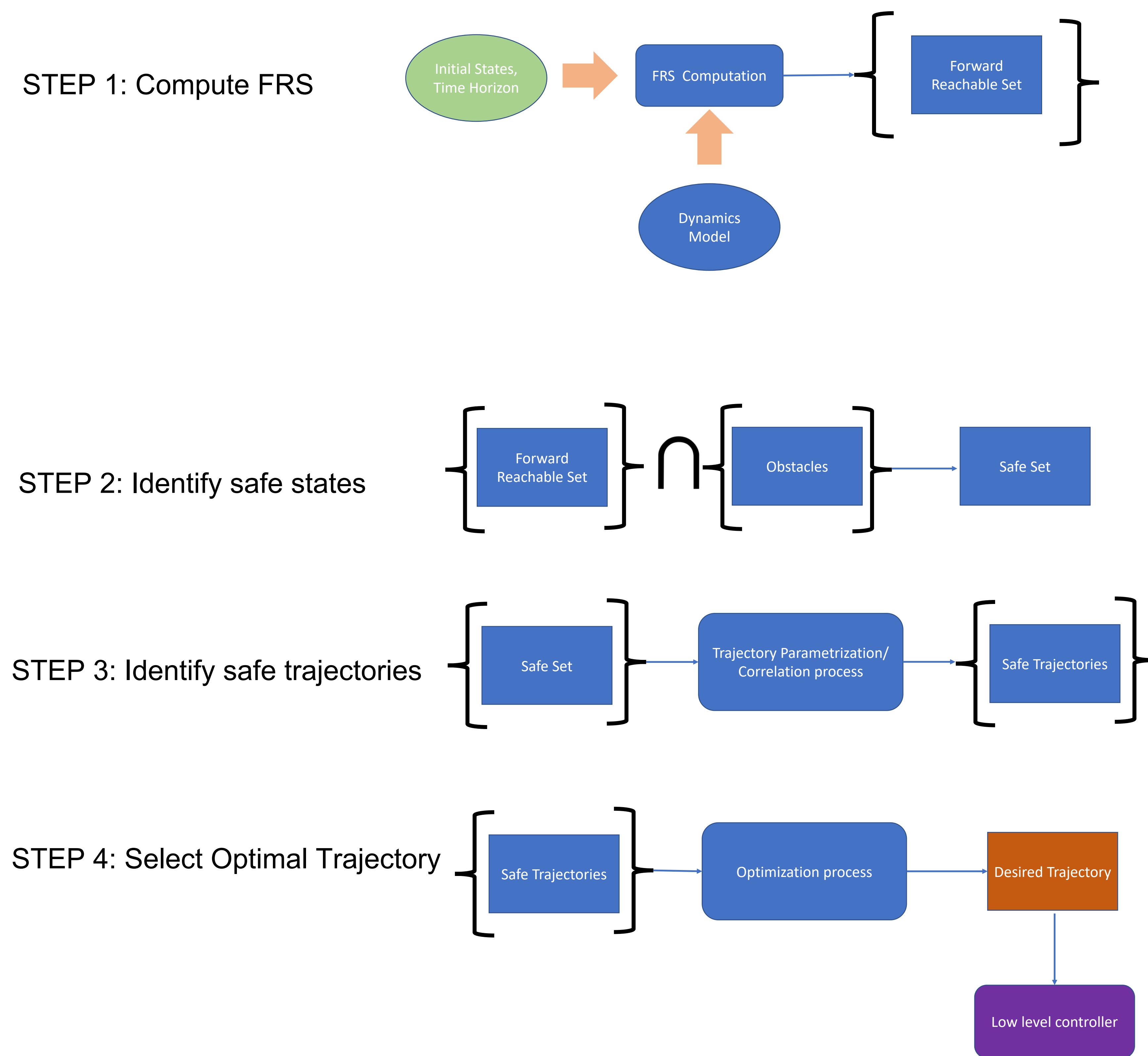


Figure 1: Diagrammatic stepwise representation of the RTD algorithm structure. This structure leverages a number of other important algorithms, including the FRS Computation and the Optimization Process

## PROPOSED SOLUTION

The proposed Solution aims to resolve the following issues with existing algorithms to compute the Forward Reachable Set

- Computational Efficiency:** The proposed application entails a very large number of states which makes standard methods unfeasible due to exponential growth of computational cost
- Assumptions of Simplified Dynamics:** The proposed application deals with very complex dynamics due to which some existing methods would not work

The proposed solution is an algorithm based on differential geometry that works around the problems of a large state space while trying to preserve fidelity of the trajectory planning model to the original high-fidelity/ground truth model.

## DESIGN CRITIQUE, DISCUSSION, AND FUTURE WORK

The design process involved considering a range of different implementations of RTD algorithms, as well as an analysis of past efforts to develop efficient solutions to the problem of Reachable Sets. The following works were analyzed in detail

- Neural Network Based Computation:** An attempt to compute the FRS based entirely on training data gathered from a ground truth model.
- Divide and Conquer implementation of FRS :** An algorithm that reformulates the problem of computing an FRS as a large number of optimization problems over small domains

Several other methods and approaches were also analyzed, but none of them were found to be similar to the approach proposed in this study. As we continue to develop and refine the method, we will continue to compare the advantages proposed with the newer methods being researched.

The next steps in this ongoing work are to expand the definition of the currently developed algorithm to higher dimensional models, and run verification with an implementation in MATLAB. Further development will follow toward eventual integration with the RTD structure.

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