Reconstructing and Forecasting the Water Balance of Lake Victoria

COLLEGE OF ENGINEERING
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Abstract

Lake Victoria is one of the largest freshwater lakes in the world and serves as a critical resource to the region for energy production, the fishing industry, and agriculture. As one of the major sources of the Nile River, fluctuations in the water level of Lake Victoria have massive implications for the millions living in its basin. Here, we provide a comprehensive assessment of the Lake Victoria water balance using a combination of historical in situ data records, satellite observations, and a novel statistical model that has previously been applied to the Laurentian Great Lakes (in North America). More specifically, we employ this statistical model to develop historical estimates of Lake Victoria precipitation, evaporation, outflow, and tributary inflow that close the water balance over consecutive historical periods.

The Model

We assume that the change in the volume of water stored in a lake is equivalent to the sum of volume fluxes into the lake subtracted by the fluxes out of the lake. The proposed model uses a rolling time window (of length w, in months) over which observed changes in lake storage (ΔV) across a w month period are equated to the cumulative sum of water balance components over the same period.

$$\Delta V_{j,w} = V_{j+w} - V_j = \sum_{i=j}^{j+w-1} (P_i - E_i + R_i - Q_i + \epsilon_i)$$

Water Balance Components (all representing monthly totals, in mm over water surface)

P = Over-lake precipitation
E = Over-lake evaporation
R = Lateral tributary lake inflow (runoff)
Q = Outflow to Victoria Nile through dam
ε = Model error term

Data Assimilation (Bayesian likelihood functions and priors)

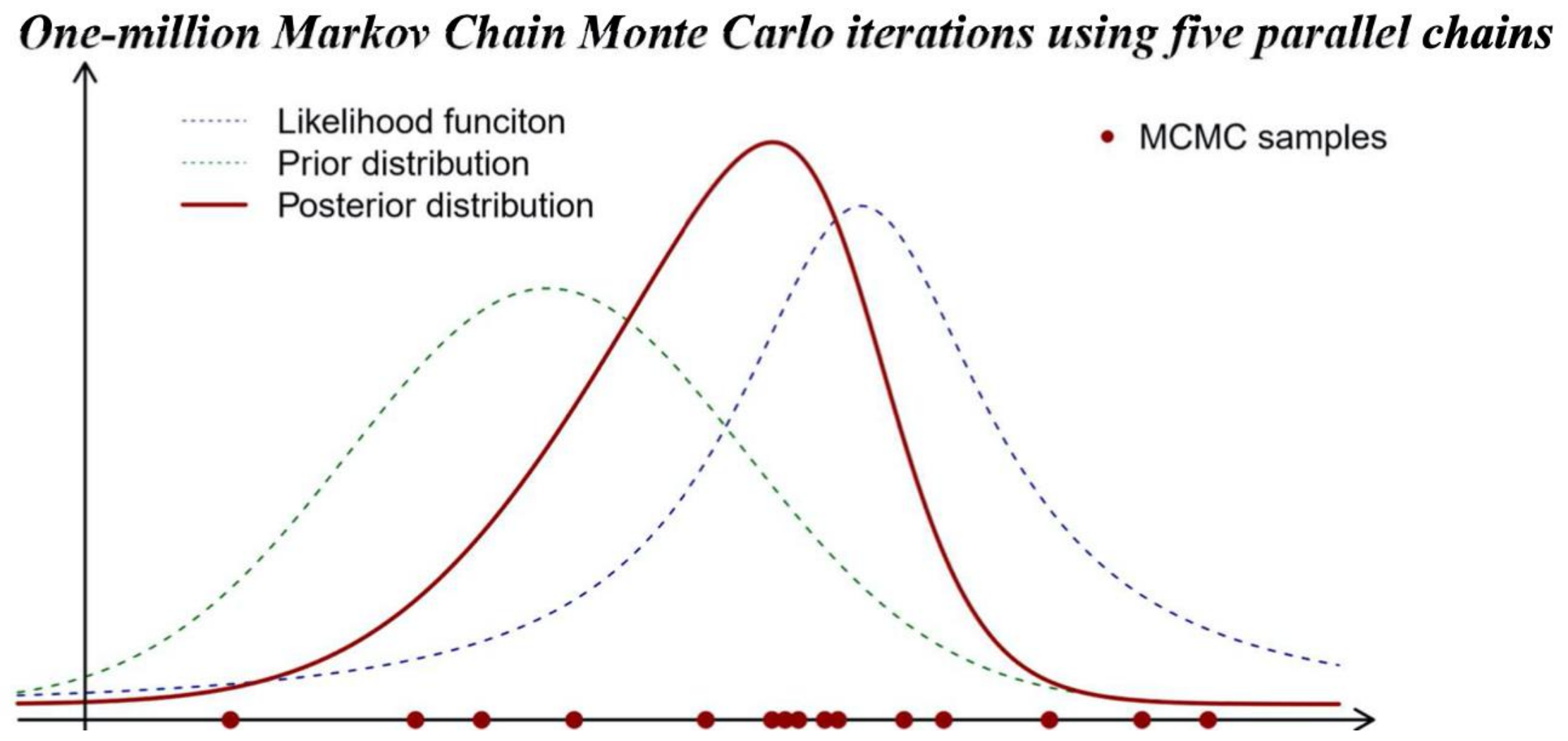
$$[y_{\Delta V}] \begin{bmatrix} y_P^1 \\ y_P^2 \\ \vdots \\ y_P^n \end{bmatrix} \begin{bmatrix} y_R^1 \\ y_R^2 \\ \vdots \\ y_R^n \end{bmatrix} \begin{bmatrix} y_E^1 \\ y_E^2 \\ \vdots \\ y_E^n \end{bmatrix} \cdots$$

- **Likelihood function**

$$\begin{bmatrix} y_{P,t}^1 \sim N(P_t + \eta_{P,c(t)}^1, \tau_{P,t}^1) \\ y_{P,t}^2 \sim N(P_t + \eta_{P,c(t)}^2, \tau_{P,t}^2) \\ \vdots \\ y_{P,t}^n \sim N(P_t + \eta_{P,c(t)}^n, \tau_{P,t}^n) \end{bmatrix}$$

- **Prior Distribution**

$$\pi(P) = Ga(\psi_1, \psi_2, \psi_3)$$



- **Fig 2:** Prior distribution and likelihood functions are combined to yield posterior distribution.

The prior distribution and each likelihood function are assimilated in a Bayesian framework to give a posterior distribution for each component value at every time step of interest (see Fig 2). Coupling of the likelihood function with the water balance equation to define biases results in a record of components that closes the water balance.

Study Area

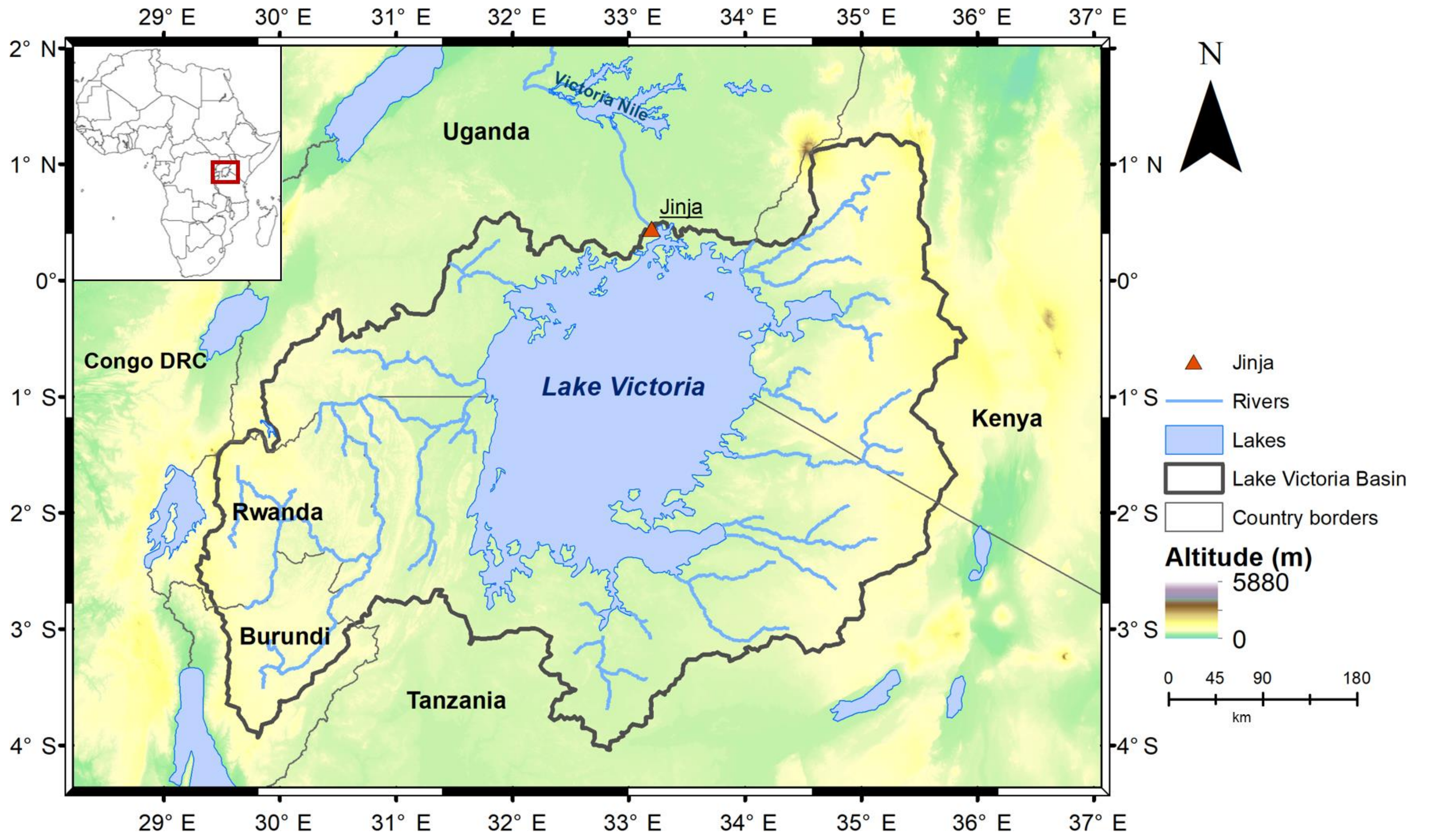


Figure 1. Map of Lake Victoria and its basin. Main tributaries are included along with the dam at Jinja. It's main tributary, the Kagera River, comes in from the west side of the lake (from Vanderkelen, 2018).

Data

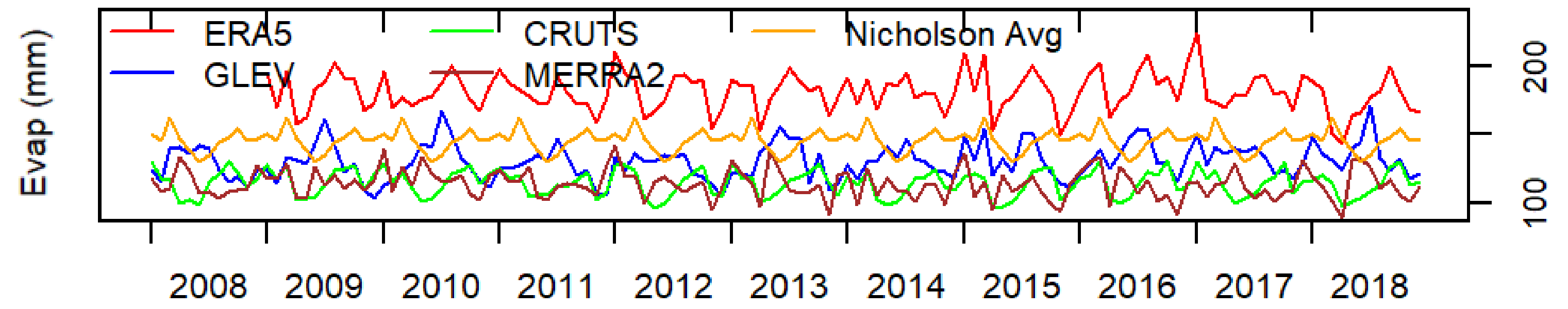


Figure 3. All evaporation datasets graphed for 2008-2018. These datasets are used to generate the likelihood functions and prior distributions. Similar datasets were compiled for the other water balance components.

Table 1. Summary of datasets used to construct likelihood functions

Variable	Dataset Source	Dates Used
ERA5	Copernicus Climate Change Service	2009-2018
CRUTS	CEDA Archive	1950-2018
Nicholson Avg	Yin and Nicholson, 1998	1950-2018
GLEV	Zenodo	1985-2018
MERRA2	NASA GES DISC	1980-2018

Data Evaluation Example: Evaporation

An array of datasets was collected to represent each water balance component – evaporation is shown above. This time series can also be seen in the results panel, along with model estimates sampled from the posterior distribution. Precipitation is the only component regularly measured on the lake; therefore, we make use of satellite observations for the other components. Equations such as the Penman Equation are used to estimate evaporation from satellite observations.

$$E_{\mathrm{PEN}} = \frac{\Delta}{\Delta+\gamma} \cdot \frac{(R_{\mathrm{n}})}{\lambda} + \frac{\gamma}{\Delta+\gamma} \cdot \frac{6.43(f_{\mathrm{U}})D}{\lambda}$$

Equation 1. A simplified version of the Penman equation. E_{PEN} is the potential open-water evaporation, R_n is the net radiation at the surface, Δ is the slope of the saturation vapor pressure curve, γ is the psychrometric coefficient, λ is latent heat of vaporization, and f_U is a wind function.

In the results pane, we can see that our datasets for evaporation disagree significantly compared to those for precipitation and runoff. One reason for this is the volatility of the Penman equation; small differences in the value of its variables can result in highly varying evaporation. Large differences in the likelihood functions and prior distributions for these datasets can lead to higher uncertainty in the model's prediction for change in water level.

Results

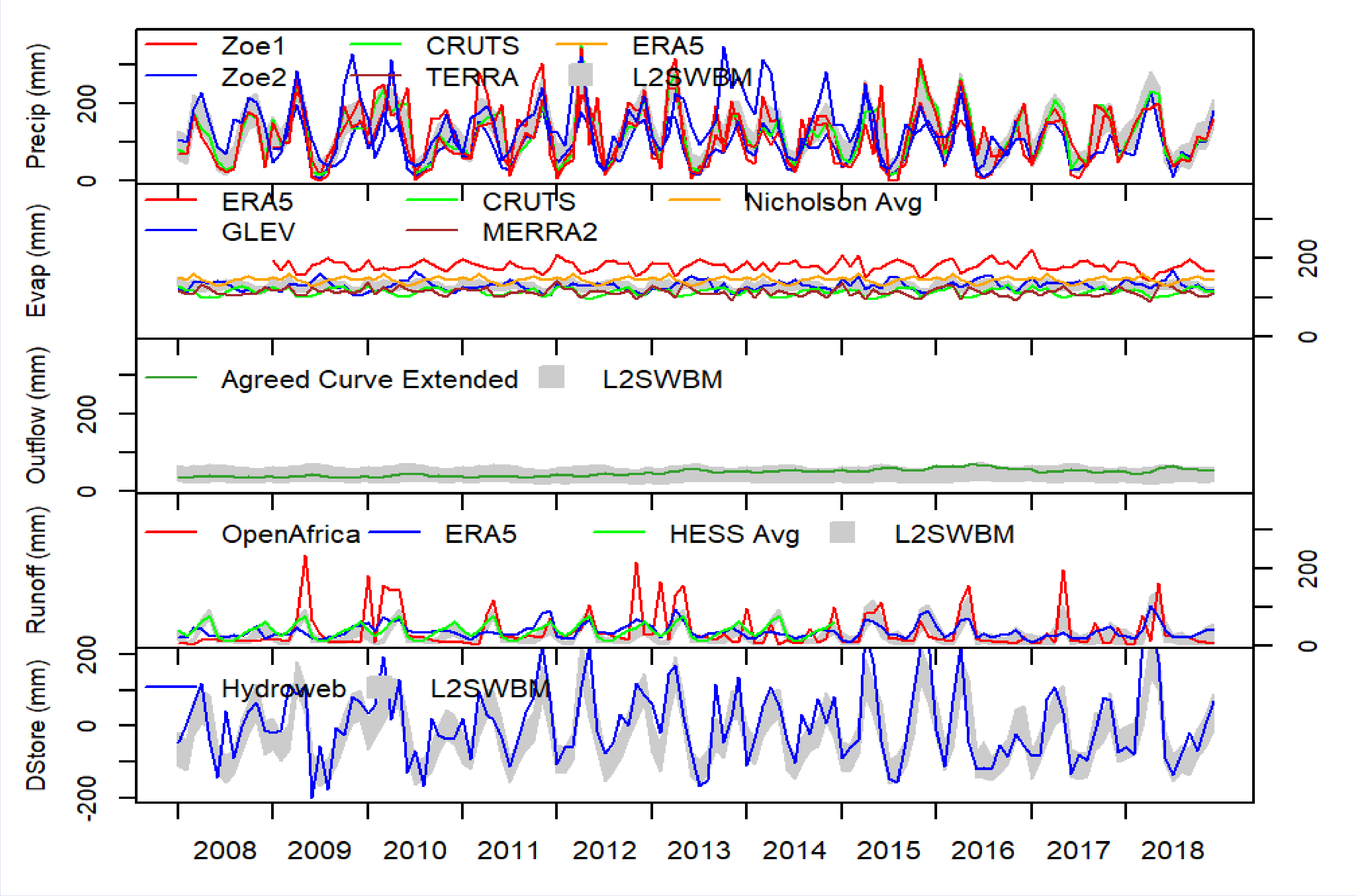


Figure 4. Model results with water balance component observations from Lake Victoria 2008-2018

Model results are expressed as a 95% confidence interval for each component at each month. Model ranges are plotted on top of the observations, represented by the grey bar (Fig 4). In this model run, we were able to keep our uncertainty in the change in storage, shown in the bottom row, to around ±20mm.

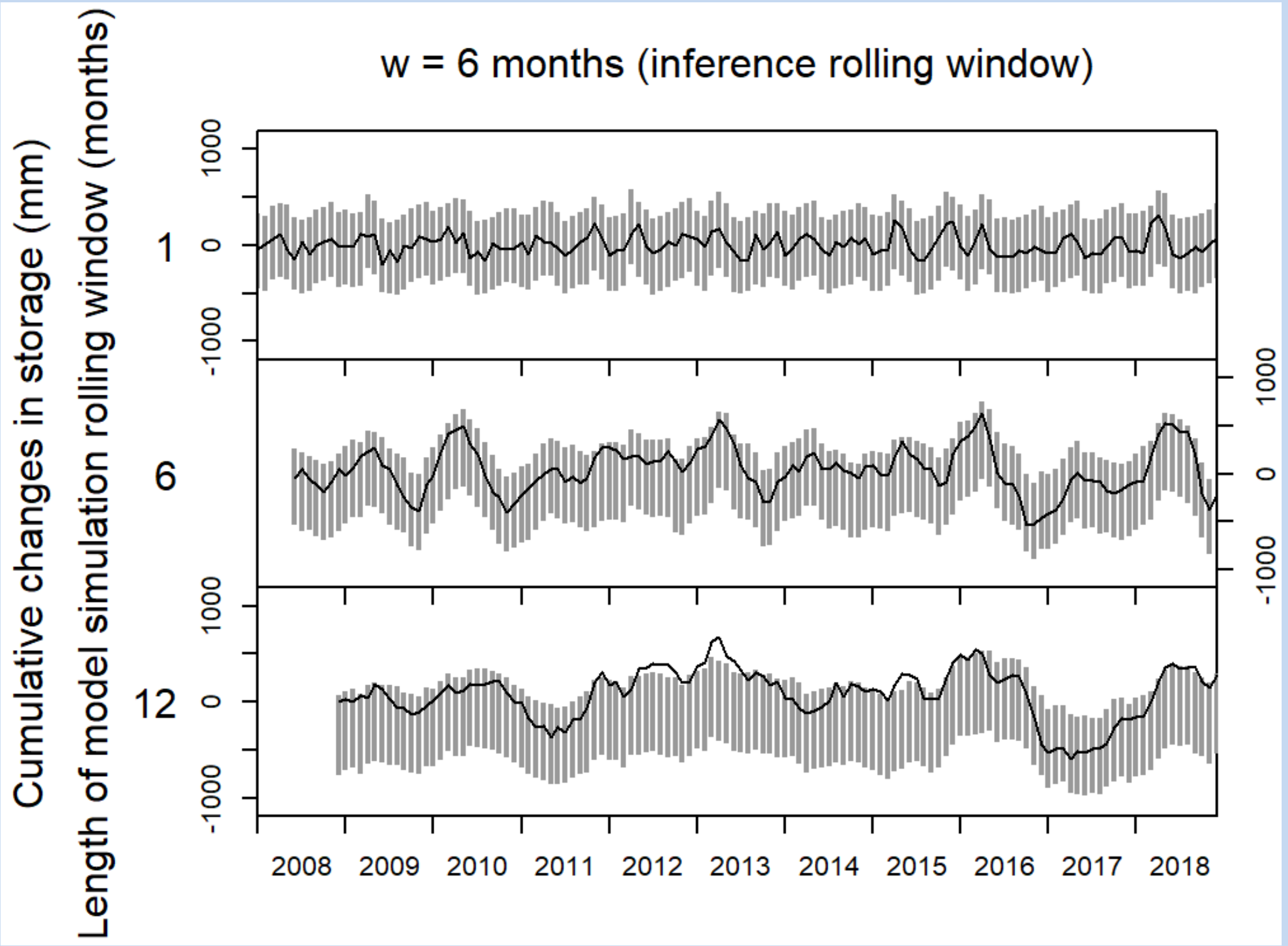


Figure 5. Observed (black line) and simulated (grey 95% credible intervals) changes in lake storage over periods of 1, 6, and 12 months. Model results are presented for a rolling inference window of 6 months.

After we run the model, we want to assess the fidelity of our estimates relative to long-term changes in lake storage by using our new estimates to simulate changes in lake storage over 1, 6, and 12 month periods. As we attempt to close the water balance for longer time periods, we can see the accuracy begin to decrease.

Acknowledgements & References

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