# Online Appendix

for

# Inflation Expectations and Nonlinearities in the Phillips Curve*

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**Appendix A. Structural Model**

To highlight how changes in structural parameters affect the Phillips curve in New Keynesian models and what types of nonlinearities can arise as a result, in this theoretical appendix we employ a basic structural model from Galí (2015, chap. 3). This model serves as a building block for more realistic medium-scale DSGE models (e.g., Smets and Wouters, 2007). Thus, many relationships derived in this small-scale model carry over to larger models. We assume rationality of individual consumers and firms, who make their allocation decisions optimally, with full information, and subject to resource constraints and exogenous frictions. These individual decisions form aggregate relationships between output, inflation, and interest rates. We further assume imperfect competition in the goods market, characterized by a constant elasticity of demand, prices sticky à la Calvo (1983), and flexible wages. Extensions featuring sticky nominal wages are discussed briefly. The model dynamics are characterized by three equations: a Phillips curve, an IS curve, and a monetary-policy reaction function.

We focus on the Phillips curve, which is described as follows:

$$\pi_t = \beta \, \mathbb{E}_t \, \pi_{t+1} + \lambda \, \widehat{mc}_t, \tag{A.1}$$

where $\pi_t$ is inflation in period $t$, $\widehat{mc}_t$ is the log-deviation from the steady-state of the aver-

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age real marginal cost, $0 < \beta < 1$ is the discount factor, and $\lambda > 0$ is the slope coefficient. Since the labor share defined in the main text is a direct measure of marginal cost, the linear slopes in columns (3) of Tables 1 and 2 in the paper correspond to $\lambda$ in Equation (A.1). Note that while $\mathbb{E}_t$ stands for the full-information rational-expectations operator, this equation also holds with nonrational expectations, as long as the operator corresponds to the expectations of price-setters (i.e., firms). While this equation does not include a backward-looking component explicitly, one can easily obtain such an extension with, for example, price indexation (e.g., Christiano, Eichenbaum, and Evans, 2005). Calibrating the model at a quarterly frequency, we obtain further that $\beta \approx 1$.

Note that the slope of the Phillips curve, $\lambda$, is a composite parameter. In terms of structural parameters, it can be written as

$$\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}, \tag{A.2}$$

where $0 < \theta < 1$ is the probability that a firm cannot change their price in a given period, $0 \leq \alpha < 1$ governs returns to scale, and $\epsilon > 1$ is the elasticity of substitution across varieties (and also demand elasticity). With constant returns to scale ($\alpha = 0$), $\theta$ is the main parameter governing the slope; with $\alpha > 0$, the demand elasticity, $\epsilon$, can also play a role.

If these structural parameters change during episodes of excess slack, the Phillips curve may exhibit nonlinearities. For example, if $\theta$ decreases in deep recessions (e.g., prices become more flexible because firms are more attentive during extreme events), the Phillips curve becomes steeper. Or if competition intensifies in recessions ($\epsilon$ goes up when demand goes down, consistent with the Kimball aggregator), the Phillips curve becomes flatter. This structural interpretation suggests that there are opposing forces affecting the slope of the Phillips curve during recessions. For that reason, several regions with different slopes could arise, and a flexible estimation approach is required. Moreover, allowing one structural parameter to change while keeping the others constant could lead to erroneous or partial results.

Note also that Equation (A.1) is a linear approximation of the model's solution around the steady-state. Because the exact relationship is nonlinear, the approximation's accuracy diminishes as the economy moves further from the steady-state, and the inherent nonlinearities can become more important quantitatively.

Two important assumptions require further clarification: zero trend inflation and flexible wages. First, the relationship above is still valid under constant, nonzero trend inflation (Cogley and Sbordone, 2008). Thus, a low, stable inflation targeted by many central banks (e.g., 2%) does not pose a problem. If trend inflation is time-varying but the degree of indexation is relatively high, our model is also likely to perform well. With no indexation, however, we need to explicitly control for the trend. Using lags of actual inflation and other predetermined variables that comove with trend inflation, we can mitigate this problem. Second, with sticky wages, as Galí (2015, p. 165) shows, the relationship between inflation and marginal cost is similar to the one above, whereas this is not necessarily the case for other forcing variables (ibid., p. 172). This is one reason to explore different forcing variables.

Next, with additional assumptions, one can also derive a Phillips curve that relates inflation to the output gap. Defining the output gap as a log-deviation of output from its natural level (i.e., the equilibrium under flexible prices), $\widetilde{y}_t = y_t - y_t^{\mathrm{n}}$, the Phillips curve can be written as follows:

$$\pi_t = \beta\, \mathbb{E}_t\, \pi_{t+1} + \kappa\, \widetilde{y}_t, \qquad \text{(A.3)}$$
$$\kappa = \lambda \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right),$$

where $\sigma > 0$ is the inverse elasticity of intertemporal substitution, and $\varphi > 0$ is the inverse Frisch elasticity of labor supply. If the elasticity of intertemporal substitution decreases during recessions (i.e., agents become more risk-averse), the Phillips curve is steeper in recessions than in expansions.

Note that the factors that affect the slope of the Phillips curve with marginal cost as a forcing variable ($\lambda$) also affect its slope with the output gap ($\kappa$). However, the two utility parameters ($\sigma$, $\varphi$) affect $\kappa$ but not $\lambda$. Hence, at least theoretically, it is possible to obtain nonlinearities for one representation of the curve but not for the other. This is another reason to explore different forcing variables.

To further derive the Phillips curve in terms of the unemployment gap, we need either an additional block allowing for idle labor (e.g., Galí, 2011) or a heuristic relationship between the output and unemployment gaps. For exposition, we discuss the latter approach. A natural heuristic relationship is Okun's law, postulating that a 1 percentage point deviation of unemployment from its natural rate (NAIRU) leads to a $\chi$% loss of (potential) output. Defining the unemployment gap as $\widetilde{u}_t = u_t - u_t^{\mathrm{n}}$, we obtain—admittedly, with slight abuse of notation—a version of Equation (1) in the paper:

$$\pi_t = \beta\, \mathbb{E}_t\, \pi_{t+1} - \xi\, \widetilde{u}_t, \qquad \text{(A.4)}$$

where $\xi = \chi\, \kappa > 0$. Note that while the exact slopes in Equations (A.1)–(A.4) differ from one another, the specification itself is independent of our choice of the forcing variable; that is, in all these cases, the Phillips curve contains the expectations term with the same coefficient $\beta \approx 1$. If, in recessions, the less productive workers lose their jobs before the more productive workers, the effect of changes in unemployment on output are larger when unemployment is already high, because firms are forced to lay off more productive workers. This conjecture, if true, would imply a larger $\chi$ and a steeper Phillips curve. While there could be additional interactions, this specification provides another example of the Phillips curve with a slope that depends on the state of the economy.

The examples in this appendix highlight that nonlinearities can stem from several structural parameters, and that recessions are likely to produce countervailing effects. While we focus on the potential state-dependence of model parameters for expositional reasons, our empirical approach can detect nonlinearities stemming from other sources as well as from the relationships examined above.

## Appendix B. Robustness and Additional Results

**Table B.1:** Data and Sources

| Variable | Notation (1) | Source (2) |
|---|---|---|
| *Panel A: Baseline* | | |
| *Inflation* | $\pi_t$ | |
| CPI, annualized quarterly growth rate | | Bureau of Labor Statistics (BLS) |
| *Slack* | $u_t$ | |
| Unemployment gap | | Authors' calculations: $ur_t - \bar{u}_t$ |
| Civilian unemployment rate | $ur_t$ | BLS |
| Natural rate of unemployment | $\bar{u}_t$ | Congressional Budget Office |
| Labor share | | |
| Nonfarm business | | BLS |
| Adjusted | | Armenter (2015); Authors' calculations |
| *Expectations* | $\mathbb{E}_t\, \pi_{t+1}$ | |
| UMSC: Mean expected inflation over next year | $\mathbb{E}_t^{\text{UMSC}}\, \pi_{t+1}$ | University of Michigan |
| SPF: Median expected GDP inflation over next quarter | $\mathbb{E}_t^{\text{SPF}}\, \pi_{t+1}$ | Federal Reserve Bank of Philadelphia |
| *Controls* | $\mathbf{z}_t$ | |
| Relative price of food and energy | | BLS; Authors' calculations |
| U.S. dollar nominal effective exchange rate | | Federal Reserve Board (FRB) |
| Price and wage control indicator | | Gordon (1982) |
| *Expectations process* | | |
| Real-time inflation | $\pi_{t-i\|t}$ | Federal Reserve Bank of Philadelphia |
| Nominal federal funds rate | $r_t$ | FRB |
| WTI Crude Oil Spot Price | $P_t^{\text{oil}}$ | Energy Information Administration |
| *Panel B: Robustness* | | |
| *Inflation* | | |
| Core CPI | | BLS |
| Headline and core PCE | | Bureau of Economic Analysis (BEA) |
| GDP deflator | | BEA |
| *Credit spreads* | | |
| Baa–Aaa spread | | Moody's/FRED |
| GZ and EBP spreads | | Gilchrist and Zakrajšek (2012); Favara et al. (2016), updated |

*Notes:* The sample period covers 1969:Q4 through 2019:Q4.

**Table B.2:** CPI Inflation; Dropped Consumer Expectations

| | Unemployment | | Labor Share | |
|---|---|---|---|---|
| | Gap | Rate | Raw | Adjusted |
| | (1) | (2) | (3) | (4) |
| *Panel A: Linear Model* | | | | |
| Slope, $\hat{\kappa}$ | $-0.31^{***}$ | $-0.19^{**}$ | $0.11^{***}$ | $0.25^{***}$ |
| | (0.10) | (0.08) | (0.03) | (0.05) |
| *Panel B: Threshold Model* | | | | |
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.63^{***}$ | −0.26 | 0.00 | 0.09 |
| | (0.19) | (0.20) | (0.05) | (0.09) |
| right, $\hat{\kappa}_R$ | 0.31 | −0.08 | $0.35^{***}$ | $0.49^{***}$ |
| | (0.27) | (0.22) | (0.12) | (0.15) |
| *Expected inflation* | | | | |
| SPF, $\hat{\alpha}_1$ | $0.73^{***}$ | $0.70^{***}$ | $1.00^{***}$ | $1.01^{***}$ |
| | (0.16) | (0.16) | (0.19) | (0.17) |
| Sum of lags, $\hat{\alpha}_0$ | 0.27 | 0.30 | 0.00 | −0.01 |
| | (0.16) | (0.16) | (0.19) | (0.17) |
| *Threshold*, $\hat{\gamma}$ | | | | |
| point estimate | 1.95 | 6.87 | −0.64 | −0.65 |
| 95% confidence interval | [−0.67, 2.93] | [4.23, 8.47] | [−6.18, 3.21] | [−4.27, 2.54] |
| *No. of thresholds, p-value* | | | | |
| 0 vs. 1, $H_0$: 0 | 0.03 | 0.94 | 0.09 | 0.12 |
| 1 vs. 2, $H_0$: 1 | 0.94 | 0.32 | 0.04 | 0.72 |
| $R^2$ | 0.74 | 0.71 | 0.73 | 0.74 |
| $N$ | 205 | 205 | 205 | 205 |

*Notes:* See notes to Table 1 in the paper. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table B.3:** PCE Inflation

| | Unemployment | | Labor Share | |
|---|---|---|---|---|
| | Gap | Rate | Raw | Adjusted |
| | (1) | (2) | (3) | (4) |
| *Panel A: Linear Model* | | | | |
| Slope, $\hat{\kappa}$ | $-0.15^{**}$ | −0.04 | $0.13^{***}$ | $0.24^{***}$ |
| | (0.06) | (0.05) | (0.03) | (0.04) |
| *Panel B: Threshold Model* | | | | |
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.36^{***}$ | −0.16 | 0.04 | $0.17^{***}$ |
| | (0.13) | (0.12) | (0.04) | (0.04) |
| right, $\hat{\kappa}_R$ | $0.24^{*}$ | 0.13 | $0.29^{***}$ | $0.47^{***}$ |
| | (0.14) | (0.13) | (0.08) | (0.16) |
| *Expected inflation* | | | | |
| UMSC, $\hat{\alpha}_2$ | $0.57^{***}$ | $0.62^{***}$ | $0.78^{***}$ | $0.68^{***}$ |
| | (0.08) | (0.09) | (0.09) | (0.08) |
| SPF, $\hat{\alpha}_1$ | 0.13 | 0.05 | $0.27^{*}$ | $0.39^{**}$ |
| | (0.16) | (0.16) | (0.14) | (0.15) |
| Sum of lags, $\hat{\alpha}_0$ | $0.30^{**}$ | $0.33^{***}$ | −0.04 | −0.06 |
| | (0.12) | (0.12) | (0.15) | (0.13) |
| *Threshold*, $\hat{\gamma}$ | | | | |
| point estimate | 1.95 | 6.87 | −1.13 | 0.64 |
| 95% confidence interval | [−0.82, 2.93] | [4.27, 8.47] | [−4.30, 3.21] | [−4.11, 2.54] |
| *No. of thresholds, p-value* | | | | |
| 0 vs. 1, $H_0$: 0 | 0.01 | 0.26 | 0.02 | 0.18 |
| 1 vs. 2, $H_0$: 1 | 0.72 | 0.17 | 0.23 | 0.18 |
| $R^2$ | 0.83 | 0.82 | 0.85 | 0.85 |
| $N$ | 205 | 205 | 205 | 205 |

*Notes:* See notes to Table 1 in the paper. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table B.4:** GDP Inflation

| | Unemployment | | Labor Share | |
|---|---|---|---|---|
| | Gap | Rate | Raw | Adjusted |
| | (1) | (2) | (3) | (4) |
| *Panel A: Linear Model* | | | | |
| Slope, $\hat{\kappa}$ | $-0.15^{***}$ | $-0.09^{**}$ | $0.07^{***}$ | $0.15^{***}$ |
| | (0.05) | (0.04) | (0.02) | (0.03) |
| *Panel B: Threshold Model* | | | | |
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.60^{***}$ | $-0.43$ | $0.03^{*}$ | $0.07^{**}$ |
| | (0.16) | (0.55) | (0.02) | (0.03) |
| right, $\hat{\kappa}_R$ | $-0.08$ | $-0.07^{*}$ | $0.47^{***}$ | $0.53^{***}$ |
| | (0.05) | (0.04) | (0.10) | (0.12) |
| *Expected inflation* | | | | |
| UMSC, $\hat{\alpha}_2$ | $0.29^{***}$ | $0.30^{***}$ | $0.38^{***}$ | $0.33^{***}$ |
| | (0.07) | (0.07) | (0.07) | (0.07) |
| SPF, $\hat{\alpha}_1$ | $0.24^{*}$ | 0.17 | $0.23^{**}$ | $0.36^{***}$ |
| | (0.12) | (0.12) | (0.10) | (0.11) |
| Sum of lags, $\hat{\alpha}_0$ | $0.47^{***}$ | $0.53^{***}$ | $0.40^{***}$ | $0.30^{***}$ |
| | (0.08) | (0.08) | (0.06) | (0.07) |
| *Threshold*, $\hat{\gamma}$ | | | | |
| point estimate | $-0.32$ | 4.23 | 1.94 | 0.80 |
| 95% confidence interval | $[-0.82, 1.27]$ | $[4.23, 8.47]$ | $[-3.45, 3.21]$ | $[-2.28, 2.22]$ |
| *No. of thresholds, p-value* | | | | |
| 0 vs. 1, $H_0$: 0 | 0.04 | 0.86 | 0.00 | 0.01 |
| 1 vs. 2, $H_0$: 1 | 0.77 | 0.69 | 0.11 | 0.10 |
| $R^2$ | 0.90 | 0.89 | 0.90 | 0.91 |
| $N$ | 205 | 205 | 205 | 205 |

*Notes:* See notes to Table 1 in the paper. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table B.5:** Core CPI Inflation

| | Unemployment | | Labor Share | |
|---|---|---|---|---|
| | Gap | Rate | Raw | Adjusted |
| | (1) | (2) | (3) | (4) |
| *Panel A: Linear Model* | | | | |
| Slope, $\hat{\kappa}$ | $-0.22^{***}$ | $-0.12^{**}$ | $0.10^{***}$ | $0.16^{***}$ |
| | (0.05) | (0.05) | (0.02) | (0.04) |
| *Panel B: Threshold Model* | | | | |
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.59^{**}$ | 0.02 | 0.01 | $0.14^{***}$ |
| | (0.24) | (0.09) | (0.04) | (0.04) |
| right, $\hat{\kappa}_R$ | $-0.19^{***}$ | $-0.46^{***}$ | $0.17^{***}$ | 0.61 |
| | (0.05) | (0.17) | (0.05) | (0.41) |
| *Expected inflation* | | | | |
| UMSC, $\hat{\alpha}_2$ | $0.35^{***}$ | $0.36^{**}$ | $0.48^{***}$ | $0.40^{***}$ |
| | (0.13) | (0.14) | (0.13) | (0.14) |
| SPF, $\hat{\alpha}_1$ | 0.16 | 0.11 | 0.18 | 0.25 |
| | (0.23) | (0.24) | (0.18) | (0.22) |
| Sum of lags, $\hat{\alpha}_0$ | $0.49^{***}$ | $0.53^{***}$ | $0.33^{***}$ | $0.36^{***}$ |
| | (0.13) | (0.14) | (0.12) | (0.13) |
| *Threshold*, $\hat{\gamma}$ | | | | |
| point estimate | $-0.82$ | 7.40 | $-3.29$ | 2.54 |
| 95% confidence interval | $[-0.82, 2.93]$ | $[4.73, 8.47]$ | $[-7.63, 3.21]$ | $[-3.44, 2.54]$ |
| *No. of thresholds, p-value* | | | | |
| 0 vs. 1, $H_0$: 0 | 0.69 | 0.20 | 0.40 | 0.66 |
| $R^2$ | 0.87 | 0.86 | 0.87 | 0.87 |
| $N$ | 205 | 205 | 205 | 205 |

*Notes:* See notes to Table 1 in the paper. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table B.6:** Core PCE Inflation

| | Unemployment | | Labor Share | |
|---|---|---|---|---|
| | Gap | Rate | Raw | Adjusted |
| | (1) | (2) | (3) | (4) |
| *Panel A: Linear Model* | | | | |
| Slope, $\hat{\kappa}$ | $-0.09^{***}$ | $-0.04$ | $0.06^{***}$ | $0.10^{***}$ |
| | (0.03) | (0.03) | (0.01) | (0.02) |
| *Panel B: Threshold Model* | | | | |
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.19^{***}$ | $-0.42$ | $0.03^{**}$ | $0.05^{**}$ |
| | (0.06) | (0.43) | (0.01) | (0.02) |
| right, $\hat{\kappa}_R$ | $0.19^{*}$ | $-0.02$ | $0.32^{***}$ | $0.45^{***}$ |
| | (0.12) | (0.03) | (0.07) | (0.11) |
| *Expected inflation* | | | | |
| UMSC, $\hat{\alpha}_2$ | $0.23^{***}$ | $0.25^{***}$ | $0.30^{***}$ | $0.25^{***}$ |
| | (0.05) | (0.05) | (0.05) | (0.05) |
| SPF, $\hat{\alpha}_1$ | $0.26^{**}$ | $0.25^{*}$ | $0.34^{***}$ | $0.41^{***}$ |
| | (0.11) | (0.14) | (0.10) | (0.10) |
| Sum of lags, $\hat{\alpha}_0$ | $0.51^{***}$ | $0.50^{***}$ | $0.36^{***}$ | $0.34^{***}$ |
| | (0.09) | (0.11) | (0.08) | (0.09) |
| *Threshold*, $\hat{\gamma}$ | | | | |
| point estimate | 2.47 | 4.23 | 1.78 | 1.27 |
| 95% confidence interval | [−0.80, 2.93] | [4.23, 8.47] | [−3.29, 3.21] | [−2.44, 2.54] |
| *No. of thresholds, p-value* | | | | |
| 0 vs. 1, $H_0$: 0 | 0.04 | 0.67 | 0.00 | 0.00 |
| 1 vs. 2, $H_0$: 1 | 0.36 | 0.17 | 0.72 | 0.95 |
| $R^2$ | 0.91 | 0.91 | 0.92 | 0.92 |
| $N$ | 205 | 205 | 205 | 205 |

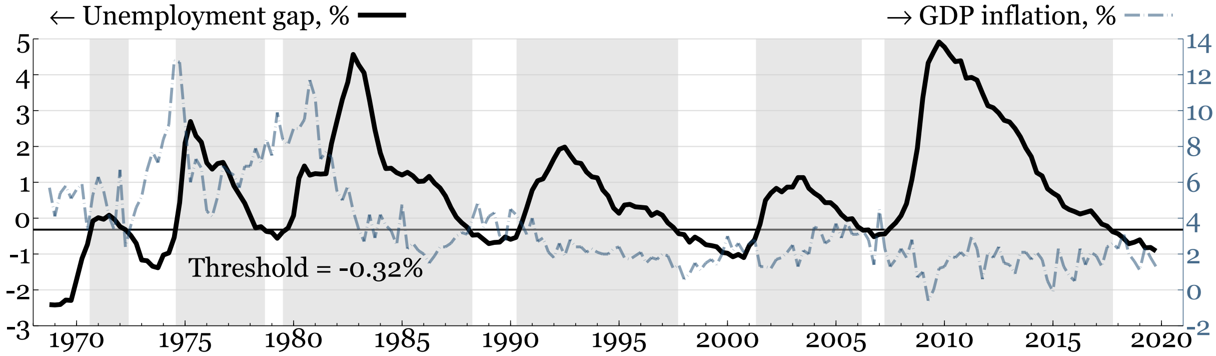
*Notes:* See notes to Table 1 in the paper. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table B.7:** Current Method CPI

| | Unemployment | | Labor Share | |
|---|---|---|---|---|
| | Gap | Rate | Raw | Adjusted |
| | (1) | (2) | (3) | (4) |
| *Panel A: Linear Model* | | | | |
| Slope, $\hat{\kappa}$ | $-0.16^{*}$ | $-0.07$ | $0.14^{***}$ | $0.25^{***}$ |
| | (0.09) | (0.08) | (0.04) | (0.07) |
| *Panel B: Threshold Model* | | | | |
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.46^{**}$ | $-0.28$ | $0.09^{*}$ | $0.54^{***}$ |
| | (0.20) | (0.18) | (0.05) | (0.20) |
| right, $\hat{\kappa}_R$ | 0.28 | 0.18 | $0.48^{***}$ | 0.15 |
| | (0.18) | (0.15) | (0.16) | (0.09) |
| *Expected inflation* | | | | |
| UMSC, $\hat{\alpha}_2$ | $0.66^{***}$ | $0.69^{***}$ | $1.05^{***}$ | $0.95^{***}$ |
| | (0.19) | (0.20) | (0.21) | (0.20) |
| SPF, $\hat{\alpha}_1$ | $0.38^{**}$ | $0.39^{**}$ | $0.46^{***}$ | 0.29 |
| | (0.19) | (0.20) | (0.16) | (0.18) |
| Sum of lags, $\hat{\alpha}_0$ | $-0.05$ | $-0.08$ | $-0.51^{**}$ | $-0.24$ |
| | (0.18) | (0.17) | (0.23) | (0.22) |
| *Threshold*, $\hat{\gamma}$ | | | | |
| point estimate | 1.95 | 6.87 | 0.34 | −3.11 |
| 95% confidence interval | [−0.38, 3.31] | [4.27, 8.83] | [−7.75, 2.42] | [−4.61, 0.48] |
| *No. of thresholds, p-value* | | | | |
| 0 vs. 1, $H_0$: 0 | 0.05 | 0.20 | 0.08 | 0.44 |
| $R^2$ | 0.71 | 0.70 | 0.73 | 0.72 |
| $N$ | 163 | 163 | 163 | 163 |

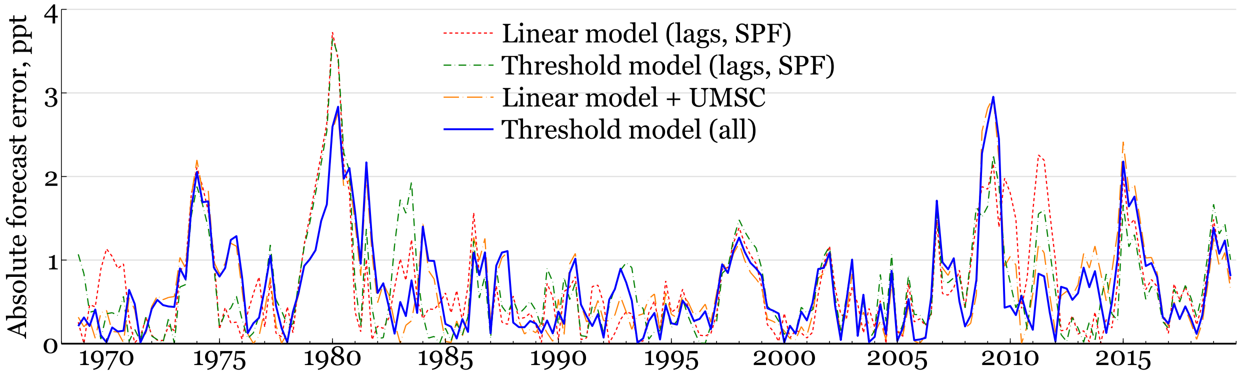
*Notes:* See notes to Table 1 in the paper. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Figure B.1:** Regimes in the Nonlinear Model with GDP Inflation




*Notes:* The shaded areas correspond to the periods when the unemployment gap is above the estimated threshold at −0.32% for the GDP inflation specification.

**Figure B.2:** In-sample Fit: Full Estimation Period




*Notes:* This figure shows in-sample fit of the models considered in the paper in the full estimation period. For better visibility, the errors are averaged over the previous four quarters.

*Measuring Innovations in Inflation Expectations*

To distinguish the role played by consumer expectations in historical episodes, we isolate the innovation component in consumers' inflation expectations (i.e., the inflation expectations that cannot be forecast by data available in the previous quarter). To this end, we allow consumer expectations to depend on the lags of real-time inflation, the federal funds rate, the Survey of Professional Forecasters (SPF) forecast, and the change in oil prices.[1] We also add four lags of consumer inflation expectations, since Fuhrer (2018) shows that, at a micro level, University of Michigan's Surveys of Consumers (UMSC) participants tend to revise their inflation forecasts in response to the lagged central tendency of survey inflation expectations. Such a mechanism should render persistence in survey expectations.[2] That is, we estimate the following specification:

$$\mathbb{E}_t^{\text{UMSC}}\,\pi_{t+1} = a + \sum_{i=1}^{4} \rho_i\,\mathbb{E}_{t-i}^{\text{UMSC}}\,\pi_{t-i+1} + \sum_{i=1}^{4} b_i\,\pi_{t-i|t} + c\,r_{t-1} + d\,\mathbb{E}_{t-1}^{\text{SPF}}\,\pi_t + f\,\frac{\Delta P_t^{\text{oil}}}{P_{t-1}^{\text{oil}}} + \varepsilon_t^{\text{UMSC}}, \qquad \text{(B.5)}$$

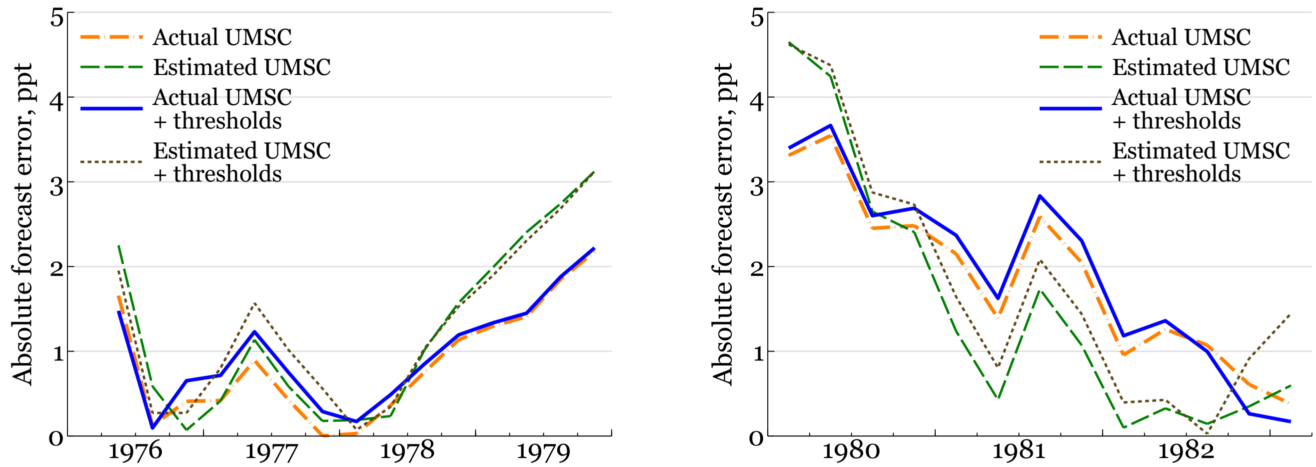
where $\pi_{t-1|t}$ is real-time inflation in period $t-1$ as observed in period $t$, $r_t$ is the nominal federal funds rate, and $P_t^{\text{oil}}$ is the oil price. With this inflation expectations process in mind, we estimate the model with either the actual UMSC variable, $\mathbb{E}_t^{\text{UMSC}}\,\pi_{t+1}$, or its fitted value, $\widehat{\mathbb{E}}_t^{\text{UMSC}}\,\pi_{t+1}$.

[1] The real-time data go back to 1994:Q3. We use revised data for the period when real-time data are not available.

[2] Binder (2017) finds that many respondents round their forecasts to the nearest zero or five. If inflationary shocks are small, this mechanism can also generate persistence in the measured expectations.

**Figure B.3:** Model Performance During the Late 1970s and Early '80s

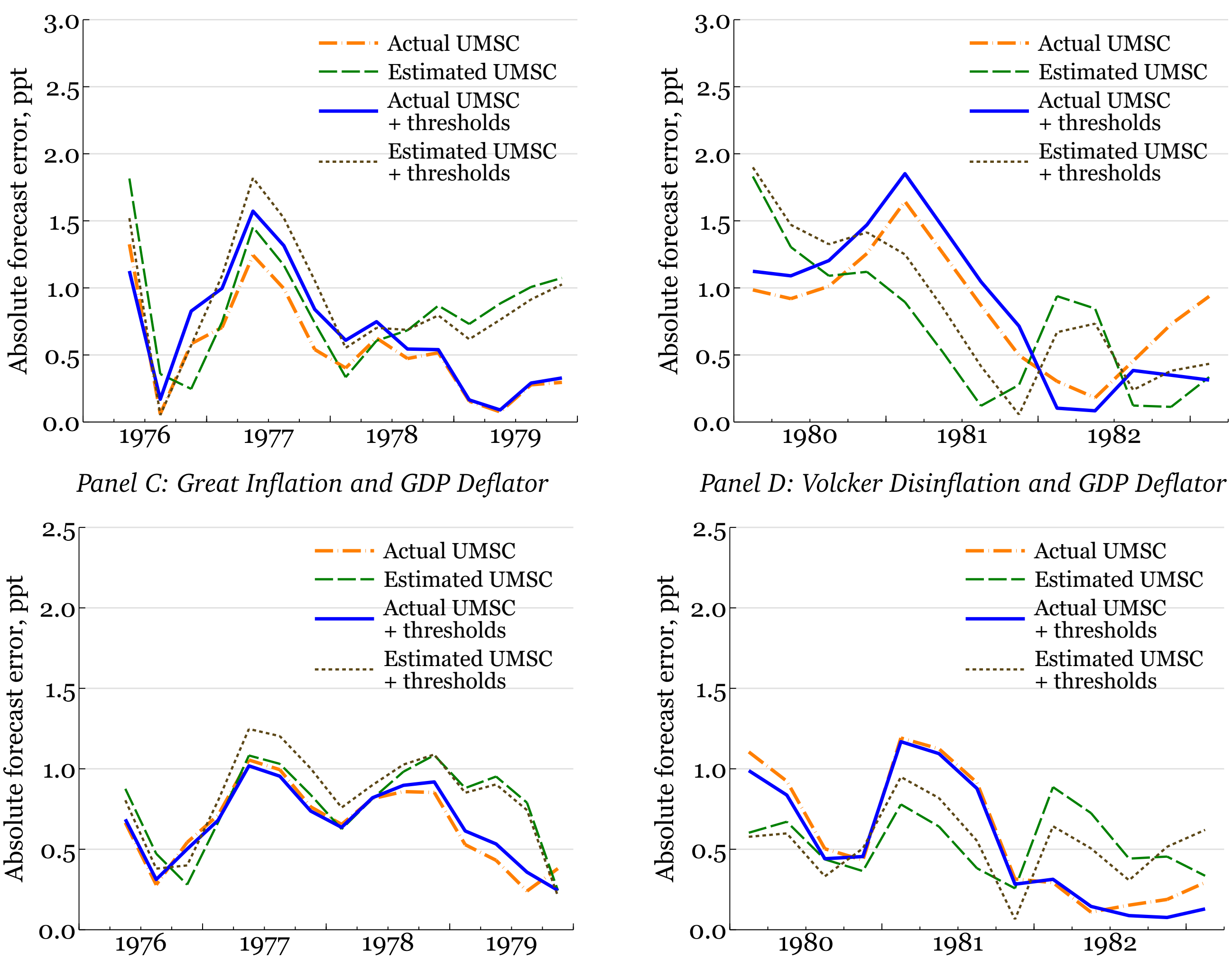
*Panel A: Great Inflation* *Panel B: Volcker Disinflation*




*Notes:* The points on the graph are the absolute values of the difference between the inflation forecasts over the previous four quarters and the corresponding measure of CPI inflation. In Panel A, dynamic forecasts start in 1976:Q2; in Panel B, they start in 1980:Q1.

**Figure B.4:** Dynamic Forecasts by Inflation Measure and Episode

*Panel A: Great Inflation and PCE* *Panel B: Volcker Disinflation and PCE*




*Notes:* See notes to Figure 2 in the paper.

**Figure B.5:** Do Nonlinearities Help Explain the Missing Disinflation?


*Notes:* This figure shows a scatterplot of the quarterly deviations of the CPI inflation rates from the previous four quarters' averages (vertical axis) and the unemployment gaps (horizontal axis). The sample period is 1968:Q4 through 2019:Q4. To enhance visibility, the large negative value corresponding to 2008:Q4 is excluded from the scatterplot but it is included in the computation of the fit lines. The black line represents the linear fit for the full sample. The gray dashed vertical line indicates the threshold. The green and red lines correspond to the two regimes of the piecewise-linear model, while the green hollow circles depict the quarters with an unemployment gap below the threshold and the red circles above the threshold. The red *solid* circles correspond to the 2009–2013 missing disinflation. The ratio of the red solid dots above and below the linear fit (black line) is 14 to 6 (i.e., the linear model predicts disinflation that did not occur) and above/below the piecewise-linear fit (red line) 11 to 9 (i.e., no missing disinflation according to the threshold model).

**Figure B.6:** Threshold Model without the Continuity Constraint


*Notes:* See notes to the previous figure.

**Table B.8:** Forecast RMSE During the Missing Disinflation

| | Estimated UMSC expectations (1) | Actual expectations (2) |
|---|---|---|
| Linear model | 1.90 | 1.69 |
| Threshold model | 1.66 | 1.58 |

*Notes:* The in-sample forecast RMSEs are computed for the period 2009:Q1 through 2013:Q4. The calculations are based on the estimates in column (1) of Table 1 in the paper.

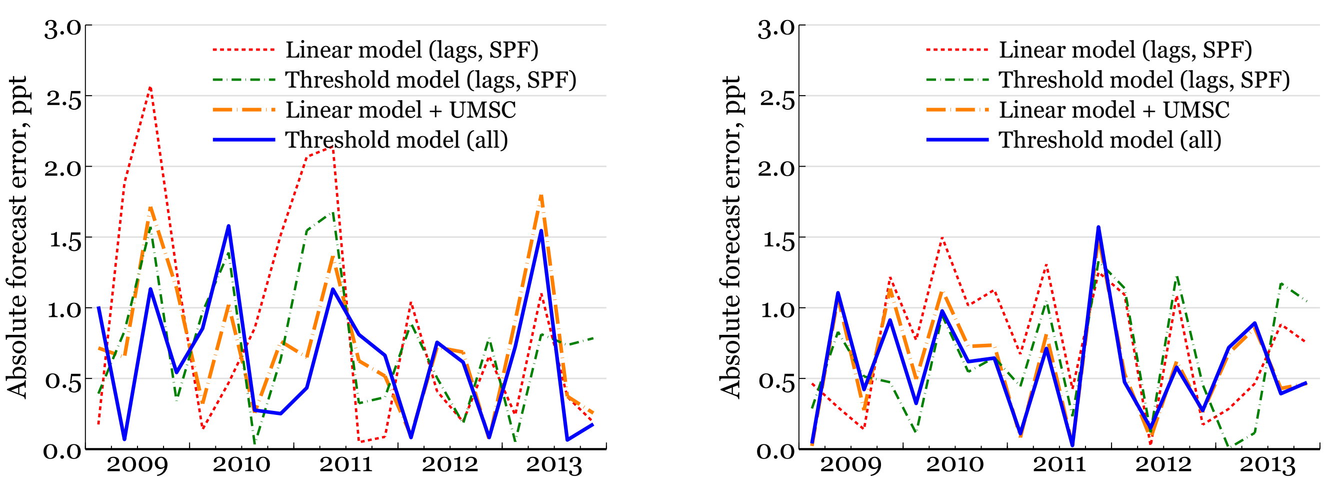
**Table B.9:** Model Performance with Alternative Measures of Inflation

| | Great Inflation | | Volcker Disinflation | | Missing Disinflation |
|---|---|---|---|---|---|
| Inflation | Peak-to-Trough Change, ppt (1) | RMSE (2) | Peak-to-Trough Change, ppt (3) | RMSE (4) | RMSE (5) |
| | | *Panel A: PCE* | | | |
| PCE inflation | 9.1 | | −9.0 | | |
| *Linear model* | | | | | |
| estimated UMSC expectations | 4.3 | 1.49 | −6.3 | 1.26 | 1.13 |
| actual expectations | 5.9 | 1.09 | −8.2 | 1.39 | 1.11 |
| *Threshold model* | | | | | |
| estimated | 4.4 | 1.57 | −5.2 | 1.32 | 0.91 |
| actual | 5.9 | 1.20 | −7.1 | 1.38 | 1.00 |
| | | *Panel B: GDP Deflator* | | | |
| GDP inflation | 4.9 | | −5.6 | | |
| *Linear model* | | | | | |
| estimated UMSC expectations | 4.2 | 1.08 | −5.2 | 0.99 | 0.94 |
| actual expectations | 5.3 | 1.04 | −6.6 | 1.04 | 0.84 |
| *Threshold model* | | | | | |
| estimated | 4.2 | 1.12 | −4.6 | 1.03 | 0.83 |
| actual | 5.1 | 1.02 | −6.3 | 1.00 | 0.77 |

*Notes:* See notes to Table B.8 and Table 4 in the paper.

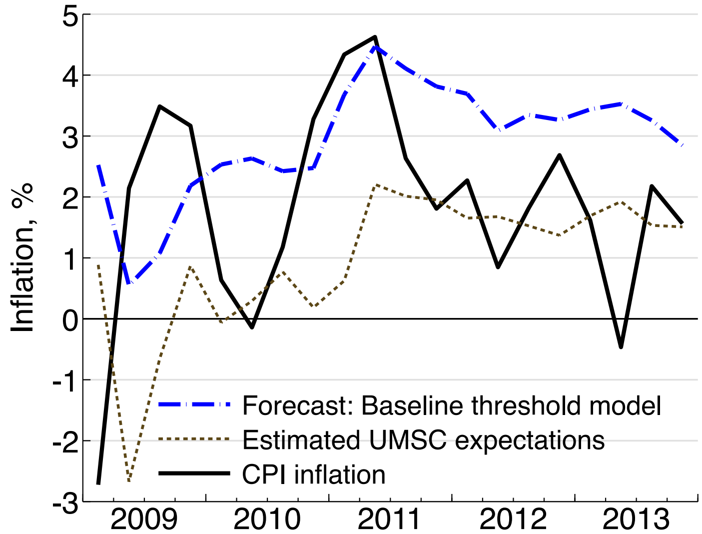
**Figure B.7:** Alternative Measures of Inflation: In-sample Fit

*Panel A: PCE* *Panel B: GDP Deflator*




*Notes:* See notes to Figure 2 in the paper.

**Figure B.8:** Out-of-sample Dynamic Forecasts During the Missing Disinflation Episode




*Notes:* This figure shows dynamic forecasts (with respect to the dependent variable) of inflation made as of 2007:Q4. That is, for the lags of inflation we use out-of-sample dynamic forecasts, while for other variables we use actual realizations. The blue dash-dot line and the brown dotted line represent the threshold models with the actual and estimated consumer expectations, respectively. The calculations are based on the model estimated in column (1) of Table 1. The estimated expectations rely on the out-of-sample forecasts from Equation (B.5). The black thick line represents the actual, seasonally adjusted, annualized CPI inflation rates.

**Table B.10:** Credit Spreads with Alternative Measures of Inflation

| | Baa–Aaa Spread | | GZ Credit Spread | | Excess Bond Premium | |
|---|---|---|---|---|---|---|
| | GDP | PCE | GDP | PCE | GDP | PCE |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| *Panel A: Linear Model* | | | | | | |
| Slope, $\hat{\kappa}$ | $-0.14^{***}$ | $-0.09^{**}$ | $-0.07^{*}$ | $-0.05$ | $-0.07^{*}$ | $-0.03$ |
| | (0.05) | (0.05) | (0.04) | (0.06) | (0.04) | (0.05) |
| *Panel B: Threshold Model* | | | | | | |
| *Slopes* | | | | | | |
| left, $\hat{\kappa}_L$ | $-0.57^{***}$ | $-0.28^{**}$ | $-1.71^{***}$ | $-2.52^{**}$ | $-1.69^{***}$ | $-2.42^{***}$ |
| | (0.16) | (0.11) | (0.62) | (0.99) | (0.61) | (0.92) |
| right, $\hat{\kappa}_R$ | $-0.08$ | $0.26^{**}$ | $-0.05$ | $-0.00$ | $-0.05$ | 0.01 |
| | (0.05) | (0.12) | (0.04) | (0.05) | (0.04) | (0.05) |
| *Expected inflation* | | | | | | |
| UMSC, $\hat{\alpha}_2$ | $0.33^{***}$ | $0.74^{***}$ | $0.33^{***}$ | $0.76^{***}$ | $0.34^{***}$ | $0.80^{***}$ |
| | (0.07) | (0.08) | (0.07) | (0.10) | (0.07) | (0.11) |
| SPF, $\hat{\alpha}_1$ | $0.20^{*}$ | 0.11 | $0.30^{**}$ | 0.15 | $0.29^{**}$ | 0.14 |
| | (0.12) | (0.14) | (0.12) | (0.21) | (0.12) | (0.21) |
| Sum of lags, $\hat{\alpha}_0$ | $0.47^{***}$ | 0.16 | $0.37^{***}$ | 0.09 | $0.37^{***}$ | 0.07 |
| | (0.08) | (0.15) | (0.07) | (0.20) | (0.07) | (0.21) |
| Spread coefficient | 0.02 | $0.11^{***}$ | 0.01 | $0.10^{***}$ | 0.01 | $0.10^{***}$ |
| | (0.02) | (0.03) | (0.02) | (0.03) | (0.02) | (0.03) |
| *Threshold*, $\hat{\gamma}$ | | | | | | |
| point estimate | $-0.32$ | 1.95 | $-0.71$ | $-0.67$ | $-0.71$ | $-0.67$ |
| 95% confidence interval | [−0.82, 2.12] | [−0.46, 2.93] | [−0.71, 1.76] | [−0.71, 1.95] | [−0.71, 1.98] | [−0.71, 1.95] |
| *No. of thresholds, p-value* | | | | | | |
| 0 vs. 1, $H_0$: 0 | 0.07 | 0.01 | 0.06 | 0.06 | 0.06 | 0.05 |
| 1 vs. 2, $H_0$: 1 | 0.79 | 0.82 | 0.89 | 0.62 | 0.88 | 0.54 |
| $R^2$ | 0.90 | 0.85 | 0.92 | 0.85 | 0.92 | 0.85 |
| $N$ | 205 | 205 | 188 | 188 | 188 | 188 |

*Notes:* See notes to Tables 1 and 5 in the paper for estimation details. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table B.11:** Robustness to Sample Period: Pre-1990 Sample

| | Unemployment | | Labor Share | |
|---|---|---|---|---|
| | Gap | Rate | Raw | Adjusted |
| | (1) | (2) | (3) | (4) |
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | −0.16 | −0.13 | 0.39 | 0.22 |
| | (0.18) | (0.16) | (0.30) | (0.14) |
| right, $\hat{\kappa}_R$ | −0.59* | −0.59* | −0.03 | −0.17 |
| | (0.33) | (0.32) | (0.16) | (0.31) |
| *Expected inflation* | | | | |
| UMSC, $\hat{\alpha}_2$ | 0.77*** | 0.78*** | 0.86*** | 0.82*** |
| | (0.19) | (0.18) | (0.14) | (0.14) |
| SPF, $\hat{\alpha}_1$ | −0.05 | −0.06 | −0.11 | −0.02 |
| | (0.26) | (0.26) | (0.21) | (0.23) |
| Sum of lags, $\hat{\alpha}_0$ | 0.28** | 0.28** | 0.25** | 0.20 |
| | (0.12) | (0.11) | (0.12) | (0.13) |
| *Threshold*, $\hat{\gamma}$ | | | | |
| point estimate | 2.47 | 8.53 | 1.30 | 1.91 |
| 95% confidence interval | [−1.19, 2.47] | [4.80, 8.53] | [−0.31, 3.99] | [−2.94, 3.01] |
| *No. of thresholds, p-value* | | | | |
| 0 vs. 1, $H_0$: 0 | 0.79 | 0.74 | 0.67 | 0.77 |
| $R^2$ | 0.88 | 0.88 | 0.87 | 0.88 |
| $N$ | 85 | 85 | 85 | 85 |

*Notes:* The estimation sample is 1968:Q4 through 1989:Q4. For estimation details, see the notes to Table 1 in the paper. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

## Appendix C. Thresholds in Subsamples: A Wild-Bootstrap Test

To test the significance of nonlinearities against the null of a linear model, we use the Hansen (2017) method. Here, we describe how we use this algorithm to test the null hypothesis of a linear model against a one-threshold model in historical subsamples. To do this, we slightly alter the procedure described in the main text. As before, we fit both the linear and one-threshold models in the entire sample and, for each subsample $i$, generate $F$-statistics of the form:

$$F_i = n_i \frac{S_{0,i} - S_{1,i}}{S_{1,i}},$$

where $n_i$ is the number of observations in subsample $i$, and $S_{0,i}$, $S_{1,i}$ are the residual sums of squares in the subsample for the linear model and for the one-threshold model, respectively. We generate a distribution for each $F_i$ using the same bootstrap procedure as for the full sample (i.e., by drawing residuals with replacement from the entire sample and refitting both models on each iteration). We then use $F_i$ distributions to generate $p$-values for the given subsample.

The test results are presented in Table C.1. During the early period (the Great Inflation and Volcker disinflation), we cannot reject the null of a linear model at conventional significance levels. Hence, the linear model is preferred during these periods. For the missing disinflation, however, we reject the null at a 5% significance level, implying that the one-threshold model should be preferred.

**Table C.1:** Testing for Nonlinearities in Subsamples

| | $p$-value<br>(1) |
|---|---|
| Great Inflation | 0.94 |
| Volcker disinflation | 0.97 |
| Missing disinflation | 0.05 |
| Missing inflation | 0.95 |

*Notes:* The null hypothesis is no thresholds (linear model); the alternative is a one-threshold model. The test is based on the specification in column (1) of Table 1 in the paper.

## Appendix D. Identification with External Shocks: Additional Results

Because the process of combining two-stage least squares with a grid search is not yet fully understood, we supplement our main results with an additional exercise. Here, we fix the threshold exogenously at the level corresponding to our baseline threshold estimate. At 1.95, this threshold is somewhat higher than the ones obtained with the grid search and reported in the paper (about 1.1). In Table D.1, columns (1) and (2) show the 2SLS results for the two versions of the Romer and Romer shocks considered in the paper, and column (3) shows the OLS results for the corresponding sample period. Note that in the linear model, the slope estimates, by construction, are exactly the same as in Panel A of Table 3 in the paper. The results in the threshold model (Panel B) are qualitatively similar to those presented in the paper: there is no statistical difference between the slopes in the two regimes. However, when we fix the threshold exogenously, the left slope is estimated to be lower in magnitude than in the baseline and closer to the right slope. The coefficients on the expectations components are all close to their baseline values.

**Table D.1:** Instrumental Variables versus OLS: A Model with a Fixed Threshold

| | 2SLS | | OLS |
|---|---|---|---|
| Shocks | Federal funds rate | Shadow rate | |
| | (1) | (2) | (3) |
| *Panel A: Linear Model* | | | |
| Slope, $\hat{\kappa}$ | −0.27 | −0.30 | −0.29*** |
| | (0.18) | (0.19) | (0.09) |
| *Panel B: Threshold Model* | | | |
| *Slopes* | | | |
| left, $\hat{\kappa}_L$ | −0.15 | −0.28 | −0.53** |
| | (0.32) | (0.31) | (0.25) |
| right, $\hat{\kappa}_R$ | −0.35 | −0.12 | 0.08 |
| | (0.26) | (0.22) | (0.21) |
| *Expected inflation* | | | |
| UMSC, $\hat{\alpha}_2$ | 0.92*** | 0.89*** | 0.85*** |
| | (0.21) | (0.20) | (0.18) |
| SPF, $\hat{\alpha}_1$ | −0.30 | −0.30 | −0.30 |
| | (0.37) | (0.36) | (0.35) |
| Sum of lags, $\hat{\alpha}_0$ | 0.38** | 0.41** | 0.45** |
| | (0.15) | (0.15) | (0.15) |
| *Different slopes, p-value* | | | |
| $H_0 : \hat{\kappa}_L = \hat{\kappa}_R$ | 0.71 | 0.75 | |
| $N$ | 168 | 168 | 168 |

*Notes:* The estimation sample is 1969:Q1 through 2015:Q4. The slack and regime variable is the unemployment gap. The instruments in the 2SLS specifications include 20 lags of the Romer and Romer (2004) shocks, extended by Wieland and Yang (2020). In column (1), we use the shocks based on the original regressions with the federal funds rate as the left-hand-side variable. In column (2), we use the Wu and Xia (2016) shadow rate instead. Column (3) shows OLS estimates for the same sample as in columns (1) and (2). The thresholds in Panel B are exogenously fixed at the values estimated in Table 1 of the paper. Newey–West standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

## Appendix E. Regional Analysis: Additional Results

To complement our baseline results using metropolitan level data, we estimate Equation (4) in the paper for a threshold fixed at the level $\gamma = 7\%$, which roughly corresponds to the baseline value in column (1) of Table 1 in the paper and which is close to the one estimated in column (3) of Table 3. This allows us to use a slightly larger panel than in the baseline, because with a fixed threshold we no longer require a balanced panel. We estimate this model for 29 MSAs during the period 1990:H1–2019:H2 at a semiannual frequency. Table E.1 reports our estimates for different combinations of fixed effects. Our preferred specification in column (4) includes both location and time effects. It shows that the metro area Phillips curve is steeper to the left of the threshold than it is to the right, similar to our baseline results. The difference between the slopes is marginally significant at the 10% level. However, the coefficient on the UMSC expectations is close to zero and highly insignificant, suggesting that expectations in a broad region are not a particularly strong proxy for metro area expectations. However, when we do not absorb aggregate effects (columns 1 and 2), consumer inflation expectations appear important and the slopes on either side of the threshold are not statistically different from one another.

When we do not include consumer expectations (Table E.2), the estimates remain similar for the specifications with time effects (columns 3 and 4). Without time effects, the difference between the slopes becomes larger (columns 1 and 2). While this difference remains statistically insignificant, it appears that not accounting for consumer inflation expectations puts nonlinearities in a more favorable light. This finding is again similar to the one obtained with aggregate data.

**Table E.1:** A Metro Area Phillips Curve with a Fixed Threshold

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.13^{***}$ | $-0.16^{***}$ | $-0.26^{***}$ | $-0.48^{***}$ |
| | (0.03) | (0.04) | (0.05) | (0.07) |
| right, $\hat{\kappa}_R$ | $-0.15^{***}$ | $-0.18^{***}$ | $-0.18^{***}$ | $-0.32^{***}$ |
| | (0.04) | (0.04) | (0.04) | (0.07) |
| *Expected inflation* | | | | |
| UMSC, $\hat{\alpha}$ | $0.31^{***}$ | $0.36^{***}$ | $-0.02$ | 0.01 |
| | (0.06) | (0.06) | (0.10) | (0.10) |
| Lag, $\hat{\delta}$ | $0.32^{***}$ | $0.29^{***}$ | $0.14^{***}$ | $0.06^{*}$ |
| | (0.04) | (0.04) | (0.04) | (0.03) |
| *Fixed effects* | | | | |
| Time | No | No | Yes | Yes |
| MSA | No | Yes | No | Yes |
| *Different slopes, p-value* | | | | |
| $H_0 : \hat{\kappa}_L = \hat{\kappa}_R$ | 0.71 | 0.80 | 0.27 | 0.09 |
| Observations | 1,491 | 1,491 | 1,491 | 1,491 |

*Notes:* The table shows estimates of Equation (4) in the paper. Each column corresponds to a different combination of fixed effects. The dependent variable is the core CPI inflation rate. The slack and regime variable is the unemployment rate. The unit of observation is MSA $i$ and semester $t$. The sample period is 1990:H1 through 2019:H2. The threshold is fixed at the 7% unemployment rate, corresponding to the baseline estimate in column (1) of Table 1 in the paper. Standard errors are clustered at the MSA level. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

**Table E.2:** A Metro Area Phillips Curve *without* UMSC Expectations

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| *Slopes* | | | | |
| left, $\hat{\kappa}_L$ | $-0.10^{***}$ | $-0.12^{***}$ | $-0.26^{***}$ | $-0.48^{***}$ |
| | (0.03) | (0.04) | (0.05) | (0.07) |
| right, $\hat{\kappa}_R$ | $-0.16^{***}$ | $-0.19^{***}$ | $-0.18^{***}$ | $-0.32^{***}$ |
| | (0.04) | (0.04) | (0.04) | (0.07) |
| *Expected inflation* | | | | |
| Lag, $\hat{\delta}$ | $0.34^{***}$ | $0.32^{***}$ | $0.14^{***}$ | $0.06^{*}$ |
| | (0.04) | (0.04) | (0.04) | (0.03) |
| *Fixed effects* | | | | |
| Time | No | No | Yes | Yes |
| MSA | No | Yes | No | Yes |
| *Different slopes, p-value* | | | | |
| $H_0 : \hat{\kappa}_L = \hat{\kappa}_R$ | 0.27 | 0.27 | 0.29 | 0.09 |
| Observations | 1,491 | 1,491 | 1,491 | 1,491 |

*Notes:* See notes to Table E.1. Standard errors are clustered at the MSA level. $^{*}$ $p < 0.1$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

## References


Armenter, Roc. 2015. "A Bit of a Miracle No More: The Decline of the Labor Share." *FRB Philadelphia Business Review* 2015 (Q3): 1–9.

Binder, Carola C. 2017. "Measuring Uncertainty Based on Rounding: New Method and Application to Inflation Expectations." *Journal of Monetary Economics* 90: 1–12.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383–398.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45.

Cogley, Timothy, and Argia M. Sbordone. 2008. "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve." *American Economic Review* 98 (5): 2101–2126.

Favara, Giovanni, Simon Gilchrist, Kurt F. Lewis, and Egon Zakrajšek. 2016. "Updating the Recession Risk and the Excess Bond Premium." FEDS Notes, Board of Governors of the Federal Reserve System.

Fuhrer, Jeffrey C. 2018. "Intrinsic Expectations Persistence: Evidence from Professional and Household Survey Expectations." Working Paper No. 18-9, Federal Reserve Bank of Boston.

Galí, Jordi. 2011. "The Return of the Wage Phillips Curve." *Journal of the European Economic Association* 9 (3): 436–461.

———. 2015. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications*, second ed. Princeton, N.J.: Princeton University Press.

Gilchrist, Simon, and Egon Zakrajšek. 2012. "Credit Spreads and Business Cycle Fluctuations." *American Economic Review* 102 (4): 1692–1720.

Gordon, Robert J. 1982. "Price Inertia and Policy Ineffectiveness in the United States, 1890–1980." *Journal of Political Economy* 90 (6): 1087–1117.

Hansen, Bruce E. 2017. "Regression Kink with an Unknown Threshold." *Journal of Business & Economic Statistics* 35 (2): 228–240.

Romer, Christina D., and David H. Romer. 2004. "A New Measure of Monetary Shocks: Derivation and Implications." *American Economic Review* 94 (4): 1055–1084.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97 (3): 586–606.

Wieland, Johannes F., and Mu-Jeung Yang. 2020. "Financial Dampening." *Journal of Money, Credit and Banking* 52 (1): 79–113.

Wu, Jing Cynthia, and Fan Dora Xia. 2016. "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound." *Journal of Money, Credit and Banking* 48 (2-3): 253–291.