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# AN INVESTIGATION OF GALACTIC STRUCTURE IN A REGION OF CYGNUS

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1948

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

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## PREFACE

This dissertation deals with one of several Milky Way regions being investigated at the Warner and Swasey Observatory of the Case Institute of Technology. I am deeply grateful to Dr. J.J. Nassau, Director of that observatory, for making available to me its facilities and equipment, as well as for his continued encouragement and advice. Others at Case who have been very helpful are Dr. S.W. McCuskey and Dr. D.A. MacRae.

I am very grateful to Dr. Freeman D. Miller, who, since his appointment to the staff of the Observatory of the University of Michigan in 1946, has given freely of his time and advice. His encouraging assistance has been particularly helpful as Milky Way research is his specialty. I also wish to thank Dr. Dean B. McLaughlin, Chairman of my advisory committee, and Dr. Robley C. Williams, a member of that committee, for their aid and guidance, especially during my period of residence at the University of Michigan.

It is a pleasure to acknowledge the assistance of the administration and trustees of Baldwin-Wallace College, who granted me a sabbatical leave in connection with the preparation of this dissertation. Finally I wish to thank my wife, Betty Annear, for her help, particularly during the final stages of the work.

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## INTRODUCTION

This study of a region of Cygnus was undertaken with three principal goals in mind: (a) To determine the amount of obscuration due to interstellar material in that direction; (b) To determine the stellar density distribution in that direction; and (c) To determine the luminosity function in that direction.

The region is one of twelve galactic fields (varying in galactic longitude from twelve degrees to two-hundred and two degrees) being investigated at the Warner and Swasey Observatory of the Case Institute of Technology. Reports on two of these fields have been published by Dr. S.W. McCuskey and Dr. C. K. Seyfert.[1]

The instrument used for all studies is that observatory's twenty-four inch Schmidt-type telescope (the Burrell telescope), which has a thirty-six inch mirror and a focal ratio of three and five-tenths. The diameter of the field is five and three-tenths degrees, and the four-degree objective prism which is used to obtain the spectra has a dispersion which varies from 150 A/mm. at $\lambda$3700 to 280 A/mm. at H$\gamma$. The spectra are essentially complete to the twelfth magnitude and many are for stars fainter than this magnitude.

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[1] S.W. McCuskey and C.K. Seyfert, Astrophysical Journal, 106, 1, 1947.

The instrumentation has been described in detail by Dr. J.J. Nassau.[1]

The general approach is to obtain as much data as possible on color excesses and spectral types, and then proceed to analyze these data (with as few assumptions as possible) with the above goals in mind. It is hoped that the extensive and homogeneous data obtained will help to clear up some of the many ambiguities which exist in the field of galactic structure.

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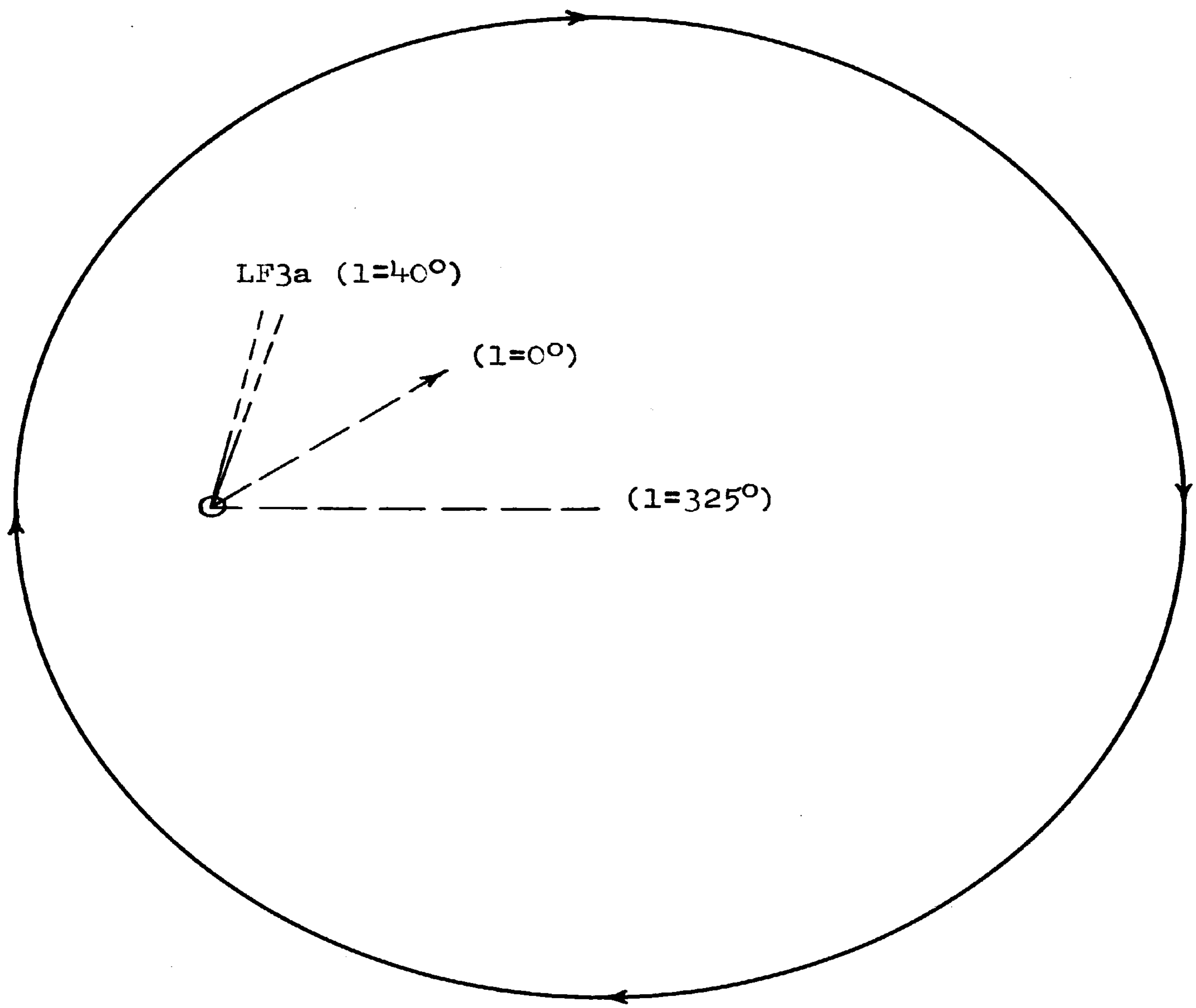
[1] J.J. Nassau, Astrophysical Journal, 101, 275, 1945.




Scale: One inch = Five kiloparsecs

Fig. 1--Plan view of LF3a region in plane of galaxy

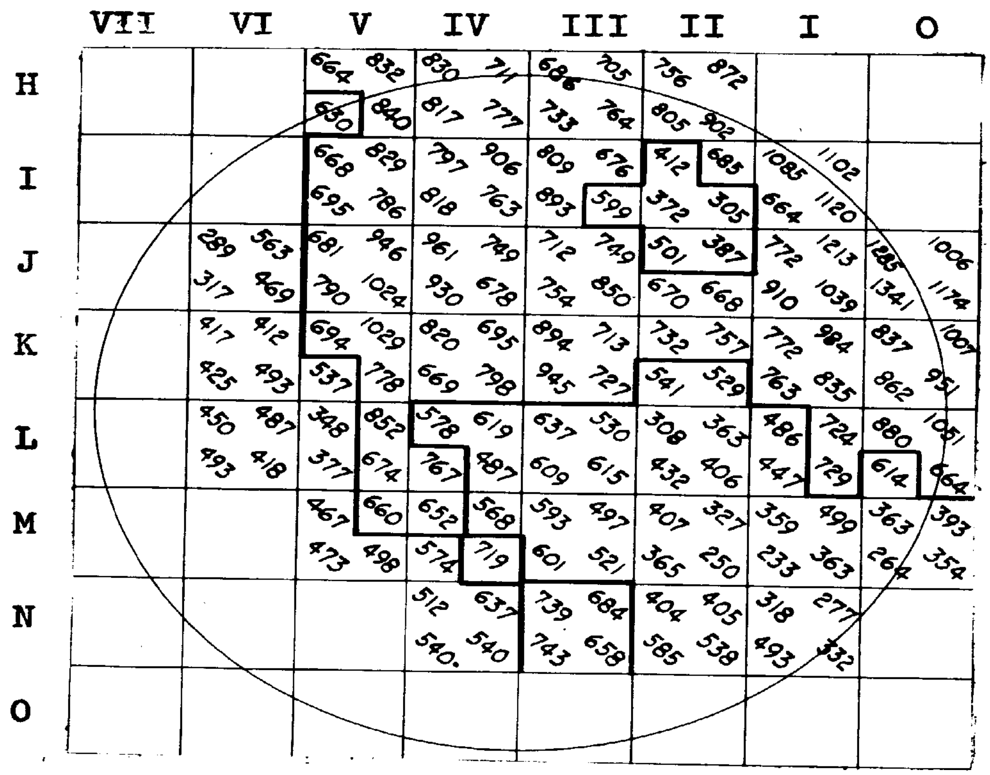


Fig. 2.--Star-counts to $m_b = 17.6$

PLATE I

LF3 LF3a LF3b

# CHAPTER II

## THE REGION INVESTIGATED

### Co-ordinates and Orientation

The field, to be designated as LF3a, is shown on Plate I, which is a reproduction of part of Ross Atlas Chart Number Seventeen. The abbreviations "LF" are for "luminosity function", and the "3a" signifies that the field is contiguous to the field known as LF3. Another region, LF3b, overlaps both of these; all are shown on Plate I, and their coordinates are summarized in Table 1.

TABLE 1

EQUATORIAL AND GALACTIC CO-ORDINATES

| LF Region | R.A.(1945) | Dec.(1945) | l | b |
|---|---|---|---|---|
| 3 . . . . . . . | $20^h\ 15^m$ | + 38°.0 | 44° | + 1° |
| 3a. . . . . . . | 20 04 | 35.4 | 40 | 1 |
| 3b. . . . . . . | 20 00 | 38.0 | 42 | 3 |

Although most of the galactic fields selected for the broad Case study are apparently uniform both in the distribution of stars and interstellar material, these three regions are decidedly non-uniform. Several investigators have found evidences of clustering of various interesting groups of stars. Dr. Nassau and Mr. Daniel Harris III have indicated in a preliminary report of their detailed study of LF3 that they have discovered an extensive cluster of B-type stars, with an indication that the cluster extends in the direction of LF3a.[1] A secondary goal of this study, therefore, was to determine whether this cluster extends further in the direction indicated. LF3b was added to the group because of the evidence of "transparent" areas, originally announced by Oort and Oosterhoff, in this vicinity.[2] Dr. Nassau is in the process of working up LF3b with Dr. D. A. MacRae, and with Dr. G. B. van Albada is completing what may be a far-reaching investigation of the distribution of the M-type stars in selected portions of this area, employing infra-red plates and filters.

Figure 1 shows a diagram in the plane of the galaxy with the LF3a longitude limits indicated, as is the accepted direction of the galactic center. The dashed lines in the direction of LF3a indicate the extrapolated range of the analysis; the uncertainty in the results increases with the distance.

Many investigations have been made in the constellation of Cygnus. It is a region of contrasts, including as it does the bright "Cloud", the dense absorption area known as the "Rift", and a variety of irregular dark markings. LF3a takes in roughly the east central portion of the Cloud (no implication of an actual star-cloud intended), part of the western portion of the Rift, and quite a number of the dark markings.

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[1] J.J. Nassau, Astronomical Journal, 53, 153, 1948.

[2] J.H. Oort and P.Th. Oosterhoff, Bulletin of the Astronomical Institutes of the Netherlands, 9, 325, 1942.

## Earlier Investigations

### Interstellar Absorption

Several investigators have found evidence that the portion of the Rift with which we are here concerned is due primarily to a dark nebula at a probable distance of about eight hundred parsecs and of absorbing power of about two magnitudes. Struve in 1927 found the first indication of this as a result of his study of Henry Draper and Henry Draper Extension stars of type O to B3 in this region.[1] He found a rather sharp edge in the distribution which agrees roughly with the boundary of the Rift as it appears in the reproduction of the Ross Atlas photograph.

Schalén provided further confirmation when he published two extensive papers dealing in part with this region.[2] The first of these, in 1928, consisted of a discussion of the B and A stars only, whereas the second, in 1932, covered the F, G, K, and M stars. (The methods involved in these investigations will be discussed in a later section).

Dr. F. D. Miller, who made an extensive series of counts of stars with a limiting magnitude of 15.0 to 16.0 over an area of seven hundred square degrees at the northern extremity of the Rift, also found results in general agreement with the above.[3] He found distances between five

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[1]O.Struve, Astronomische Nachrichten, 231, 17, 1927.

[2]Carl Schalén, Upsala Meddelanden, No. 37, (1928), and No. 55, (1932).

[3]F.D. Miller, Harvard Annals, 105, No. 15, 1937.

hundred and sixteen hundred parsecs and a total absorption of two to four magnitudes, with no evidence of strong absorption at distances less than two hundred or three hundred parsecs. He pointed out that the dark nebulae appear to be scattered at random through space within the range of the analysis, although systematically nearer the sun in some parts of the Rift than others. While admitting that the figures for the absorption should be considered as lower limits and that it is not possible to define the distribution in detail with this type of analysis, he concludes that the observed limits of the Cloud are determined by the distribution of dark nebulae at distances not less than five hundred parsecs.

Hubble, in his galaxy survey, found two each at the regions with galactic co-ordinates 40°, +5° and 45°, +3°, and none at two regions centered at 46°, +3°, and 43°, +2°, indicating rather heavy absorption.[1] The mean estimated distance of the four Cepheid variable stars in this region investigated by Oort and Oosterhoff, however, range from about seven thousand parsecs to twenty thousand parsecs, assuming that the photographic absorption is near one and one-half magnitudes (based on color excesses and proper motions).[2] They reported further that Baade had investigated the region for galaxies with no definite positive results. In an effort to harmonize these data, they suggested the possibility of a cloud of obscuring material just outside and around the galaxy.

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[1]E.P. Hubble, Astrophysical Journal, 79, 8, 1934.

[2]Oort and Oosterhoff, op. cit.

Stebbins, Huffer, and Whitford, in their well-known paper on the colors of B stars, measured colors for twenty-three stars near P Cygni.[1] They found considerable variation and an average absorption of about one magnitude per kiloparsec. Comparison of the present work with these results will be discussed in a later section.

In an extension of the work on the Cepheids, Heyden in 1947 found further evidence of patchy absorption in his investigation of seven Cepheids in Cygnus, two of which fall within the limits of LF3a (MW Cygni, CD Cygni).[2] For the former he obtained one and six-tenths magnitudes absorption at thirteen hundred parsecs, although some nearby B stars observed photo-electrically at greater distances show no appreciable reddening, and one star (HD 190603) shows two and one-half magnitudes at seven hundred parsecs. In the region of CD Cygni, he found an absorption of about one-half magnitude per kiloparsec.

With regard to the evidence furnished by the interstellar lines, Merrill in 1937 found indications of sodium clouds in this region (eleven stars at a mean distance of eighteen hundred parsecs).[3] Sanford, in the same year, found some correlation between the calcium absorption (intensity of K line) and the color excess.[4] In 1943, Adams found dense clouds of calcium in this direction, with varying radial velocities; he found as many as five components

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[1]J. Stebbins, R.C. Huffer, and A.E. Whitford, Astrophysical Journal, 91, 20, 1940.

[2]F.H. Heyden, Astrophysical Journal, 106, 325, 1947.

[3]P.W. Merrill, Astrophysical Journal, 86, 28, 1937.

[4]R.S. Sanford, Astrophysical Journal, 86, 136, 1937.

in the K line for one star (HD 199478).[1] Although the relation between the clouds of gas and the dark nebulae which absorb selectively has not yet been made clear, it seems noteworthy that the gas distribution also is irregular.

It seems quite probable that an "average absorption" has little meaning in LF3a, since the absorption varies a great deal in going from the Cloud to the Rift, and also since there is a good deal of irregular absorption. It is possible that most of the apparent disagreements which have arisen in this area can be explained by the highly irregular nature of the dark nebulae. It may be that the dark "globules" recently discussed by Bok are responsible for some of this.[2] In any event, it was thought when this present investigation was undertaken that the broad base line for the colors plus the penetrating power of the Burrell telescope should make possible a more detailed study of the absorption than had previously been made. The wide field of this telescope is an advantage in this work, of course, as well as its ability to reach to faint magnitude limits both in direct photography and with the objective prism.

## Stellar Distribution

General star-counts.-- Although the discussion of the various methods of determining stellar density distribution will be deferred to a later section, it might be well to discuss briefly the two general approaches to the problem at this point. In the method of general star-counts, all the stars in a given area are counted to successive limits

[1]W.S. Adams, Astrophysical Journal, 97, 105, 1943.

[2]B.J. Bok, Sky and Telescope, 6, No. 5, 1947.

of apparent magnitude. The density function is then computed, usually on a trial and error basis, utilizing an assumed luminosity function and an assumed or determined absorption for the area in question. One objection to this method is that the luminosity function does not seem to be constant, either in galactic latitude or longitude.[1] Another is that at great distances star-clouds or other structural features of the galaxy will be masked due to the "smoothing" of the process. The other general approach involves the determination of the spectral types of the stars in question. The direct method is more satisfactory in that it avoids the necessity of assuming a general luminosity function, but due to the difficulty of obtaining spectra of faint stars it is much more limited in its application with respect to distance.

Discussing first the surveys employing general star-counts, we find that Pannekoek, in a pioneering study, obtained a distance of forty thousand parsecs for a clustering of stars in this region in 1919.[2] He reduced this distance to eighteen thousand parsecs in 1922.[3] In 1929, however, he found a condensation of stars at fifteen hundred parsecs, which he associates with the Cloud, and another clustering at five hundred parsecs not identified with the Cloud.[4]

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[1]McCuskey and Seyfert, op. cit.

[2]A. Pannekoek, Monthly Notices of the Royal Astronomical Society, 79, 500, 1919.

[3]Pannekoek, Bulletin of the Astronomical Institutes of the Netherlands, No. 11, 1922.

[4]Pannekoek, Publications of the Observatory of Amsterdam, No. 2, 1929.

Dr. F. D. Miller found that the distribution of stars is similar for regions of low obscuration (including one in the Cloud) over a considerable range in galactic longitude.[1] The density is probably constant for distances up to six hundred or one thousand parsecs. He also cited confirmatory evidence by Balanovsky and Hase,[2] and Bok.[3] The clustering in the P Cygni (LF3 and LF3a) region was mentioned in this paper, as was the likelihood of error which would result from the counts if the presence of the additional O and B stars were not known.

In an extension of this work, which includes star-counts to the eighteenth magnitude, Dr. F. D. Miller and Dr. J. A. Hynek again found constant or slowly decreasing densities within two thousand parsecs of the sun, and hence no evidence of a star cloud within this distance.[4] Uncertainty as to the absorption at great distances kept the distance to the figure mentioned, and at lower latitudes reasonable solutions for the stellar density could be found only on the assumption of decreasing absorption beyond that distance.

Stellar distribution determined from stars of known spectral type.-- Another paper of historical interest only may be mentioned at this point; it is that of Kopff, who in 1922 found a distance for the Cloud of four thousand to six thousand parsecs under the assumption that the general

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[1] Miller, op. cit.

[2] P. Balanovsky and T. Hase, Bulletin de l'Observatoire Central a Poulkovo, 14, 2, 1935.

[3] Bok, Harvard Circular, No. 371, 1931.

[4] F.D. Miller and J.A. Hynek, Contributions from the Perkins Observatory, No. 13, 1939.

spectral character of the Cloud corresponds to type A to F, and that the absolute magnitudes of the Cloud stars are near zero.[1]

The sharp edge between the Rift and the Cloud found by Struve for stars of type O to B3 has been mentioned.[2] He obtained a distance of about eight hundred parsecs for the cluster but also made assumptions concerning the absolute magnitudes. This points up one of the weaknesses of the direct method--the uncertainties as to the values of the absolute magnitudes for stars of various spectral classes. Along this same line it might be mentioned that the adopted dispersion of these luminosities is also a factor of considerable importance in this approach.

Schalén concluded that there is no Cloud between fifteen hundred and four thousand parsecs, and that any such agglomeration of stars with relatively small extension in the line of sight would probably have to be at a distance of about eleven hundred parsecs.[3] He prefers to think of the region as one rich in stars rather than one which has a definite cluster.

Bok, in his 1937 summary of this region, agreed with the interpretation which holds that the Cloud is merely a region of contrast with surrounding dark material, and suggested that a spiral arm may run through the vicinity of the sun in the direction from Carina to Cygnus.[4] He further

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[1] A. Kopff, Astronomische Nachrichten, 216, 325, 1922.

[2] O. Struve, op. cit.

[3] Schalén, op. cit.

[4] Bok, Distribution of the Stars in Space, p. 119, 1937.

suggested that the clustering around P Cygni may represent a knot in the spiral arm.

Other investigations not strictly comparable with those under consideration in this section may well be included here because of their pertinence. Baade, from a study of eclipsing systems and long-period variables in 1934, found a distance for the Cloud of from fifteen hundred parsecs to twenty-five hundred parsecs.[1] Merrill and Burwell list the region of the Cloud as one rich in Be stars.[2] Reber has found the region of Cygnus to be one from which excessive amounts of radio "static" of various frequencies are received.[3] While the significance of this information is not fully understood, it perhaps tends to emphasize the peculiar character of this area of the sky.

It is seen that there are marked differences in the results of the analyses for this region. In view of the uncertainties mentioned, this perhaps should be expected. In the case of Baade's work, it might be that the distribution of the objects which he investigated is different from that of the stars in general. Although the areas involved in the above discussion are in general comparable, the fact that the absorption has been shown to be irregular may influence the results for regions of small angular separation.

As indicated before, it was hoped that the present investigation would obtain sufficiently faint colors and

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[1] W. Baade, Astrophysical Journal, 79, 475, 1934.

[2] P.W. Merrill and Cora Burwell, Astrophysical Journal, 78, 139, 1933.

[3] G. Reber, Astrophysical Journal, 100, 279, 1944.

spectra so that a significant contribution to the distribution in this area could be made. Some assumptions are still necessary, of course, notably those which involve the luminosities and their dispersion, but the general luminosity function need not be assumed.

# CHAPTER III

## OBSERVATIONAL MATERIAL

### Photometry

Most of the plates were obtained during the summer of 1946. Blue and red magnitudes were determined for those stars for which red indices were desired. For the blue magnitudes, which are on the usual international scale, Eastman IIa-O, III-O, and 103a-O plates were used. Later it was found that, contrary to sensitivity data received from the manufacturer, the effective wave-length of the III-O plates was somewhat shorter than the others, so it was necessary to discard practically all the blue magnitudes. As described in a later section, additional plates to replace those discarded were taken in the spring of 1947. Eastman 103a-E plates were used with a Wratten Number 22 filter to obtain the red magnitudes, the combination yielding an effective wave-length of about $\lambda$ 6200. Development was in a solution of D-19 for four minutes at sixty-five degrees Fahrenheit. The plates were developed in a tank, and were agitated throughout the process.

A magnitude sequence of twenty-five stars ranging in magnitude from 6.5 to 12.5 was established by triple exposures on each plate, in the sense (for all but one): field-comparison area—field. No appreciable pre-exposure effects were detected. The sequence stars are fairly widely distributed over the area although they do not cover the entire plate. For the blue magnitudes, three plates were compared with the polar sequence, and two with Harvard area C 10 (R.A. $19^h$ $05^m$, Dec. + $15^o$). The probable error of a single determination of the magnitude of a sequence star is ± 0.08 and that of the mean is ± 0.03, determined from the range in the determinations by Schlesinger's method.[1] For the red magnitudes, five plates were compared with the pole using the magnitudes for the polar sequence as determined by Dr. Nassau and Mrs. Virginia Burger Knight.[2] One plate was compared with Selected Area 39 (R.A. $19^h$ $48^m$, Dec. + $45^o$), using the standard magnitudes which have been set up there on the same scale by the same investigators (unpublished data--soon to be published). The same stars were used as for the blue sequence, and the corresponding probable errors are the same. The sequence is given in Table 2, which includes the spectral types determined in this investigation, as well as the blue and red magnitudes of the sequence stars. The sequence magnitudes and also the subsequent individual field star magnitudes were determined by the well-known method of visually comparing (with a low-power microscope) a graduated scale of images with the sequence stars and with the field stars which appear on the same plate. This method yields probable errors comparable to those obtained with the Schilt photometer.[3]

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[1] F. Schlesinger, Astronomical Journal, 46, 161, 1937.

[2] J.J. Nassau and Virginia Burger, Astrophysical Journal, 103, 25, 1946.

[3] Ibid, p. 30.

TABLE 2

MAGNITUDES AND SPECTRAL TYPES OF SEQUENCE STARS

| Star | $m_b$ | $m_r$ | Spectrum |
|---|---|---|---|
| 1 . . . . . . . | 6.54: | ---- | gG8 |
| 2 . . . . . . . | 7.34 | 7.30 | B0 |
| 3 . . . . . . . | 7.44 | 7.33 | A3 |
| 4 . . . . . . . | 7.64 | 7.92 | B2 |
| 5 . . . . . . . | 8.23 | 7.92 | B2 |
| 6 . . . . . . . | 8.22 | 8.51 | B2 |
| 7 . . . . . . . | 8.11 | 6.73 | gG8 |
| 8 . . . . . . . | 8.69 | 7.61 | gG8 |
| 9 . . . . . . . | 8.27 | 8.29 | B2 |
| 10 . . . . . . . | 8.70 | 7.87 | F0 |
| 11 . . . . . . . | 9.21 | 9.27 | A2 |
| 12 . . . . . . . | 9.18 | 9.40 | B9 |
| 13 . . . . . . . | 9.44 | 7.94 | gK2 |
| 14 . . . . . . . | 9.49 | 9.74 | B5 |
| 15 . . . . . . . | 9.58 | 9.93 | B2 |
| 16 . . . . . . . | 10.36 | 10.14 | A5 |
| 17 . . . . . . . | 10.68 | 10.96 | B8 |
| 18 . . . . . . . | 11.04 | 11.23 | A2 |
| 19 . . . . . . . | 11.61 | 9.21 | gK5 |
| 20 . . . . . . . | 11.27 | 11.58 | B9 |
| 21 . . . . . . . | 10.94 | 10.87 | A2 |
| 22 . . . . . . . | 12.30 | 12.11 | B8 |
| 23 . . . . . . . | 11.88 | 11.63 | G2: |
| 24 . . . . . . . | 12.38 | 12.22 | B8: |
| 25 . . . . . . . | 12.72 | 12.46 | -- |

For the blue magnitudes of the 1281 field stars, three III-O plates were used originally, and one additional plate (103a-O) for those stars which showed a large range in the determinations. The red excesses derived by comparing these magnitudes with the red magnitudes for the same stars were found to contain about sixty negative excesses; in other words, these sixty stars were found to be bluer than normal. This is a larger number than would be expected from the uncertainties in the colors. Also the late-type dwarf stars used to check the zero-point of the color system were much too red. When this problem was investigated, it was found, as mentioned previously, that the III-O plates had a shorter effective wave-length than the other plates--this was confirmed by the manufacturer, and fully explained the abnormal colors mentioned above. If the sequence magnitudes had depended only on III-O plates, the difficulty probably would not have arisen, but the six sequence plates had consisted of two plates of each type. Therefore the two III-O plates were discarded, and one additional sequence plate was taken. This was a 103a-O plate and was compared with the polar sequence, so that the final sequence in the blue magnitudes depends upon two IIa-O plates compared with Harvard region C 10, and three 103a-O plates compared with the pole. Two additional 103a-O plates were taken for the field star magnitudes, and the 103a-O plate which had been previously measured partially was completed, with some re-measures to determine whether there had been any systematic changes in the method. No changes were detected. Therefore the blue magnitudes for the 1281 stars depend upon the three 103a-O plates.

The longest exposures were about two minutes long, and these usually were made with a diaphragm (which reduces the incident light by three magnitudes) to avoid very short exposures.

Two 103a-E plates were used for the red magnitudes, with a third plate used for those stars for which the two measures differed in magnitude by more than four-tenths. Six hundred and eight stars of type O through AO were measured, and 111 of later types.

Mr. Edward Winkel has shown that no distance or color correction is necessary with the plates taken with the Burrell telescope; as a precaution the portion of the plate within two centimeters of the edge was not used.[1] One source of error is the possibility of varying transparency between the comparison field and the field under investigation. A visual inspection of the sky was maintained to avoid this as much as possible, and care was taken to see that the individual plates did not differ systematically from the others. Also no sequence plate was used which showed variation in the magnitudes derived from two images of the sequence star (one formed before the comparison exposure, and one formed after the comparison exposure). A correction was applied for differential extinction, based on the Rayleigh law of atmospheric scattering and the effective wave-length of the plates employed. Standard atmospheric conditions were assumed as well as the usual secant law of variation of the correction with the zenith distance. One-half the correction for the blue magnitudes was used for the red magnitudes. This is

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[1]E.F. Winkel, Unpublished M.S. Thesis, Department of Astronomy, Case Institute of Technology, 1942.

the usual correction,[1] and is an approximation based on Lundmark's[2] summary of the work on the extinction for photovisual magnitudes and the relation between the effective wave-lengths for the photovisual and the red magnitudes (red index = 1.33 color index).[3] The maximum differential extinction correction applied to the blue magnitudes was +0.22 (mean correction = +0.11), and to the red magnitudes +0.11 (mean correction = +0.08).

Table 3 exhibits various data concerning the precision and comparisons of the magnitudes. Columns 2 through 7 indicate various magnitude intervals, and column 8 shows the mean with the numbers in parentheses representing in each case the number of stars involved. The first part of the table refers to the blue magnitudes and shows in the first row the probable error of the mean of the three determinations, and in the second row that of a single observation. The next four rows indicate the residuals for those stars which LF3a has in common with the Henry Draper Catalog, its Extension, and the Upsala work of Schalen previously mentioned, with the early type stars compared first.[4] The residuals are all given in the sense LF3a minus comparison. There is some indication of a correlation of the residual with magnitude in the comparison data, in the sense that the LF3a magnitudes are too faint for the brighter stars. However this is not large and it is not considered to be serious in the light of other

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[1]B.J. Bok and Jean M. Rendall-Arons, Astrophysical Journal, 101, 282, 1945.

[2]K. Lundmark, Handbuch der Astrophysik, 5/1, 566, 1932.

[3]Nassau and Burger, op.cit., p.32. [4]Schalen, op.cit.

TABLE 3

PRECISION AND COMPARISONS OF MAGNITUDES

| | Blue Magnitude | | | | | | |
|---|---|---|---|---|---|---|---|
| | 7 | 8 | 9 | 10 | 11 | 12 | Mean |
| P.E. (Mean) . . | ±.07(11) | ±.07(40) | ±.06(83) | ±.07(214) | ±.05(548) | ±.07(385) | ±.06(1281) |
| P.E. (S.O.) . . | .13 | .12 | .11 | .12 | .09 | .12 | .11 |
| LF3a-HD . . . . | +.02(13) | -.13(35) | -.10(35) | +.10 ( 5) | | | -.08 (88) |
| LF3a-HDE . . . | | -.47 (3) | -.37(42) | -.20(172) | -.05(125) | -.04 (5) | -.16 (347) |
| LF3a-Ups. (B&A) | -.07 (9) | -.01(39) | +.09(68) | +.09 (14) | -.13(18) | | +.04 (148) |
| LF3a-Ups. (F-M) | +.04 (2) | -.10(19) | +.05(47) | +.12 (4) | -.12(5) | | +.01 (77) |
| LF3a-LF3 . . . | | -.05 (1) | +.28 (8) | +.46 (5) | +.22 (27) | +.01 (22) | +.17 (62) |
| LF3a-LF3b . . . | -.04 (2) | +.01 (6) | +.02(12) | +.08 (31) | +.07(108) | +.02 (55) | +.05 (214) |

| | Red Magnitude | | | | | | |
|---|---|---|---|---|---|---|---|
| | 7 | 8 | 9 | 10 | 11 | 12 | Mean |
| P.E. (Mean) . . | ±.05(16) | ±.07(40) | ±.08(66) | ±.06(170) | ±.05(254) | ±.06(173) | ±.06(719) |
| P.E. (S.O.) . . | .07 | .11 | .12 | .08 | .08 | .08 | .09 |
| LF3a-LF3 . . . | | | +.02 (8) | +.05 (8) | -.06 (25) | -.02 (21) | -.02 (63) |
| LF3a-LF3b . . . | +.03 (3) | +.20 (4) | +.13(11) | -.01 (26) | +.03 (49) | +.03 (22) | +.04(115) |

comparisons with these systems. Similarly the difference in zero-point between LF3a and the Henry Draper Extension magnitudes may be due to errors in the Extension, as similar differences have been found in LF3 and LF3b (unpublished data). The next two rows show the comparisons with LF3 and LF3b. The comparison with LF3 shows a rather large systematic difference of 0.17 in the sense that LF3a is too faint, whereas the comparison with LF3b shows but 0.05, in the same sense. The probable errors of these mean residuals are ±0.02 and ±0.005 respectively. The small size of the LF3b probable error and the large number of stars involved lend strength to the LF3a magnitudes; the reason for the rather poor agreement between LF3 and LF3a is not known. No corrections were applied to the blue magnitudes.

The second part of Table 3 shows the comparisons and probable errors for the red magnitudes. The probable errors are seen to be comparable with those obtained for the blue magnitudes, although only two plates were used for the majority of the stars. A third plate was used for those stars which differed by more than four-tenths in the two magnitude determinations. The only available comparisons are the magnitudes obtained in LF3 and LF3b; these are seen to be quite satisfactory. The probable errors for the mean residuals are also small; ±0.012 for LF3 and ±0.006 for LF3b. Therefore no corrections were applied to the red magnitudes.

## Red Indices

A red index is defined as the blue magnitude minus the red magnitude for each individual star. Red indices have been obtained for all (719) stars for which both blue and red magnitudes are available. Table 4 exhibits the observed red indices arranged according to magnitude and observed spectral type. These colors were obtained for all stars of class AO and earlier as they are the most luminous stars (generally speaking) and hence best fitted for a penetrating analysis of absorption characteristics, stellar space distribution, and the luminosity function. The colors for the later spectral types were determined largely for the purpose of checking the zero-point of the color scale.

The red indices are subject to the same errors which affect the magnitudes; these errors were discussed previously. From the given probable errors of these magnitudes, it is concluded that the internal probable error of a single red index is about ±0.09 (the square root of the sum of the squares of the magnitude errors). To obtain the external probable error, comparisons were made with the LF3 field (63 stars), LF3b (115 stars), and with the stars measured photoelectrically by Stebbins, Huffer, and Whitford (18 stars).[1] These comparisons each yielded a probable error for a single observation of ±0.12, neglecting the differences in zero-point. The mean red index residuals obtained by comparing with LF3 and LF3b are about what might be inferred from the magnitude comparisons: +0.20 ±0.015 for LF3 (in the sense LF3a minus LF3) and 0.00 ±0.011 for LF3b.

Returning to the work of Stebbins, Huffer, and Whitford, it was found that there were nineteen stars in

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[1]Stebbins, Huffer, and Whitford, <u>op. cit</u>.

TABLE 4

OBSERVED RED INDICES

| Spectrum | $m_b$ <8.00 | 8.25 | 8.75 | 9.25 | 9.75 | 10.25 | 10.75 | 11.25 | 11.75 | 12.25 |
|---|---|---|---|---|---|---|---|---|---|---|
| O . . . . | -.09(3) | .21(1) | | | | | | | | |
| B0. . . . | -.03(3) | | .33(3) | .39(1) | | .28(2) | -.08(2) | | | |
| B2. . . . | -.06(4) | -.12(6) | -.10(3) | -.07(5) | -.08(12) | .12(19) | .13(15) | .13(13) | .26(6) | |
| B5. . . . | -.39(4) | -.41(1) | | -.23(3) | -.10(7) | .06(4) | .04(25) | .11(28) | .11(14) | -.02(7) |
| B8. . . . | -.14(2) | -.40(2) | -.37(6) | -.32(7) | -.22(5) | -.13(5) | -.01(18) | .06(33) | .14(28) | .20(21) |
| B9. . . . | -.27(3) | -.20(5) | -.37(7) | -.25(12) | -.21(10) | -.12(25) | -.02(25) | -.02(35) | .19(30) | .22(19) |
| A0. . . . | -.06(3) | -.16(2) | -.18(1) | -.18(2) | .02(6) | -.03(9) | .03(15) | .06(36) | .10(26) | .11(19) |
| A2,A3 . . | .11(1) | | .26(1) | .09(3) | | | .02(3) | .02(2) | .31(1) | |
| A5. . . . | | | | | | .22(1) | | .59(1) | | |
| F0,F2 . . | | .14(1) | .83(1) | .10(1) | .54(2) | .49(1) | .23(1) | .46(2) | | |
| F5. . . . | | | | | .28(3) | .80(1) | .46(1) | .21(1) | .26(2) | .20(1) |
| F8,G0 . . | | .53(1) | .69(2) | .54(1) | .52(4) | .37(1) | .44(1) | .49(7) | .29(1) | .78(1) |
| G2,dG5. . | | | | | | | .76(4) | .88(1) | 1.08(12) | |
| dG8,dK0 . | | | | | | | | | 1.42(6) | |
| dK2 . . . | 1.61(1) | | | | | | 2.03(2) | 1.89(2) | | |
| dM2,dM5 . | | | | | 2.07(1) | | | | | 2.70(2) |
| gG5 . . . | | .99(1) | | | | .85(1) | 1.07(1) | 1.45(1) | 1.38(2) | |
| gG8,gK0 . | | 1.56(2) | 1.08(1) | | | 2.19(1) | 1.60(3) | .81(1) | 2.11(2) | |
| gK2,gK3 . | | | | | 2.23(1) | 1.91(1) | 2.08(2) | | 2.62(1) | |
| gK5,gM0 . | | | | | 2.54(1) | | | 2.40(1) | | |
| gM2,gM5 . | | | 1.97(1) | 2.34(1) | | | 2.30(1) | | | 2.34(2) |

common.[1] The photoelectric $C_1$ was changed to a red index by means of the relation derived by Dr. Nassau (unpublished data--to be published soon):

$$R.I. = 0.12 + 3.00\,(C_1 - 0.03)$$

The relation was derived for early-type stars. The following alternative formula due to Seares yields values quite similar to those obtained by the above relations:[2]

$$R.I. = 2.53\,C_1 - 0.03$$

This relation was derived from comparisons with stars of type B8-A4, so is not strictly comparable. The derived red indices (using the first relation above) were then compared with those obtained in this investigation. The mean residual (LF3a minus Stebbins) is +0.11 ±0.03. One star, HDE 227460, yielded a residual of -1.08 and was omitted from the comparison; if it is included the mean residual is -0.04. As this star is also in LF3b, and the LF3a minus LF3b residual found in that comparison is -0.11, it appears that this star was misidentified in the photoelectric work. The small number of stars in common tends to diminish the significance of this comparison. A further comparison will be made, however, in the discussion of the photographic absorption.

## Spectral Types

Spectral types were determined for all 1281 stars. The spectral criteria used are those derived for the Burrell telescope and the four-degree objective prism by Dr. Nassau

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[1] *Ibid.*

[2] F.H. Seares and Mary C. Joyner, *Astrophysical Journal*, **98**, 244, 1943.

and Dr. Seyfert.[1] The principal criteria are the relative strengths of the He lines, H lines, the K line of Ca II, the G band, and the $\lambda$4227 line of Ca I. For stars of class G5 and later, giants and dwarfs were separated. The luminosity criteria consisted mainly of the CN bands at $\lambda$3883 and $\lambda$4215, and, for the classes later than K5, the intensity of the continuum between $\lambda$4227 and the G band. Much of this work represents an extension of the criteria used at Harvard University, the University of Chicago, and in Sweden. Vyssotsky at the McCormick Observatory uses essentially these same criteria.[2]

Five plates were taken with the four-degree objective prism, with exposures varying from thirty seconds to thirty minutes. One of these plates was taken with the prism turned at a five-degree angle from that usually employed, in an effort to separate overlapping spectra. In addition, one two-degree prism plate was used, mainly as a check plate. In nearly every case there were at least two four-degree prism plates used for each star, and usually two independent estimates were made for each plate. Except for overlapping spectra, which are numerous in the crowded regions, it was determined that the spectra are complete to about the twelfth magnitude, although many spectra were obtained for fainter stars. This limit was obtained by comparing the counted numbers of stars to various magnitude

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[1]Nassau and Seyfert, op. cit.

[2]A.N. Vyssotsky, Astrophysical Journal, 91, 425, 1943.

limits for small areas of the plate with the numbers of spectra obtained to various magnitude limits for the same number of small areas.

Frequent comparisons were made with Dr. Nassau and Mr. Harris, while the classification of both LF3 and LF3a was in process, on the overlapping stars, in order that the two areas would be on the same system. The other comparisons available are indicated in Table 5, and in Figures 3 to 7 inclusive.

There are seventy-eight stars common to the Henry Draper Catalog and LF3a. The table indicates that the LF3a system is somewhat earlier for the early types; this is further shown by the plot of the two systems in Figure 3. A similar situation exists in the comparisons with the Henry Draper Extension in this field, as shown in the table and also in Figure 4. In this case the LF3a system seems to be somewhat earlier for the later types as well. This agrees with conclusions reached in the case of LF3,[1] LF1 and LF2,[2] and comparisons between the Henry Draper and Henry Draper Extension systems as determined by Shapley.[3] The lack of a G8 class in these last systems tends to increase the apparent differences when compared with LF3a, as does the lack of the class A7 in the LF3a system. Vyssotsky found results which are of the same general type as those mentioned

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[1]Daniel L. Harris III, Unpublished M.S. thesis, Department of Astronomy, Case Institute of Technology, 1945.

[2]McCuskey and Seyfert, op. cit., p.2.

[3]H. Shapley, Harvard Circulars, 278, 1925.

**LF3a**

| LF3a \ HD | B0 | B2 | B5 | B8 | B9 | A0 | A2 | A3 | A5 | F0 | F2 | F5 | F8 | G0 | G2 | G5 | G8 | K0 | K2 | K3 | K5 | M0 | M2 | M5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| M5 | | | | | | | | | | | | | | | | | | | | | | | | 1 |
| M2 | | | | | | | | | | | | | | | | | | | | | 2 | 1 | | |
| M0 | | | | | | | | | | | | | | | | | | | | | | | | |
| K5 | | | | | | | | | | | | | | | | | | | 1 | | 1 | | | |
| K3 | | | | | | | | | | | | | | | | | | | | | 1 | | | |
| K2 | | | | | | | | | | | | | | | | | | | 4 | | 1 | | | |
| K0 | | | | | | | | | | | | | | | | | | 3 | 2 | | 1 | | | |
| G8 | | | | | | | | | | | | | | | | 2 | | 4 | 1 | | | | | |
| G5 | | | | | | | | | | | | | | | | | | 2 | | | | | | |
| G2 | | | | | | | | | | | | | | | | | | | | | | | | |
| G0 | | | | | | | | | | | | | | 1 | | | | | | | | | | |
| F8 | | | | | | | | | | | | 1 | 1 | | | | | | | | | | | |
| F5 | | | | | | | | | | | | | | | | | | | | | | | | |
| F2 | | | | | | | | | 1 | | | | | | | | | | | | | | | |
| F0 | | | | | | | | | | | | 1 | | | | | | | | | | | | |
| A5 | | | | | | | | | | | | | | | | | | | | | | | | |
| A3 | | | | | | 1 | 1 | | | | | | | | | | | | | | | | | |
| A2 | | | | | | 1 | | | | | | | | | | | | | | | | | | |
| A0 | | | | | | 2 | 3 | | | | | | | | | | | | | | | | | |
| B9 | | | | | | 28 | 3 | | | | | | | | | | | | | | | | | |
| B8 | | | | | | 25 | 1 | | 1 | | | | | | | (1) | | | | | | | | |
| B5 | | | | | | 1·33 | | | | | | | | | | | | | | | | | | |
| B2 | | 32 | 1 | | | | | | | | | | | | | | | | | | | | | |
| B0 | | 1 | | | | | | | | | | | | | | | | | | | | | | |

**HD**

Fig. 3 Comparison of LF3a and HD Spectra

```
 2                                                          1

MO

   LF3a

 5                                                       2  1

 3
 2                                              1 41 3 53 1

KO                                             13 11 4 3    1

 8                                        2     5 6  2 1

 5                                   2    5    11    1

 2                                   5    1     2

GO                                 2 9    2     1   (1)

 8                           1  4  9 5    3

 5                         2 2  5  4 1

 2                         4 1  2  1

FO                      1 1  2    2



 5                        1  1    1

 3             1 3  1 5 41 3    1
 2             3 7 10 6 3

AO          1 11 9   1
 9       5 13 29 5  2   1  1
 8       4  7 10 2

 5 11  2 3     1 1  2

 2 3111  4  1  1    1  1            (1)

BO1
  0 2  5 8  9  0  23   5    0 2  5   8 0 2  5  8  0 23 5   0 2
  B            A            F          G          K        M      HDE
```

Fig. 4.--Comparison of LF3a and HDE Spectra

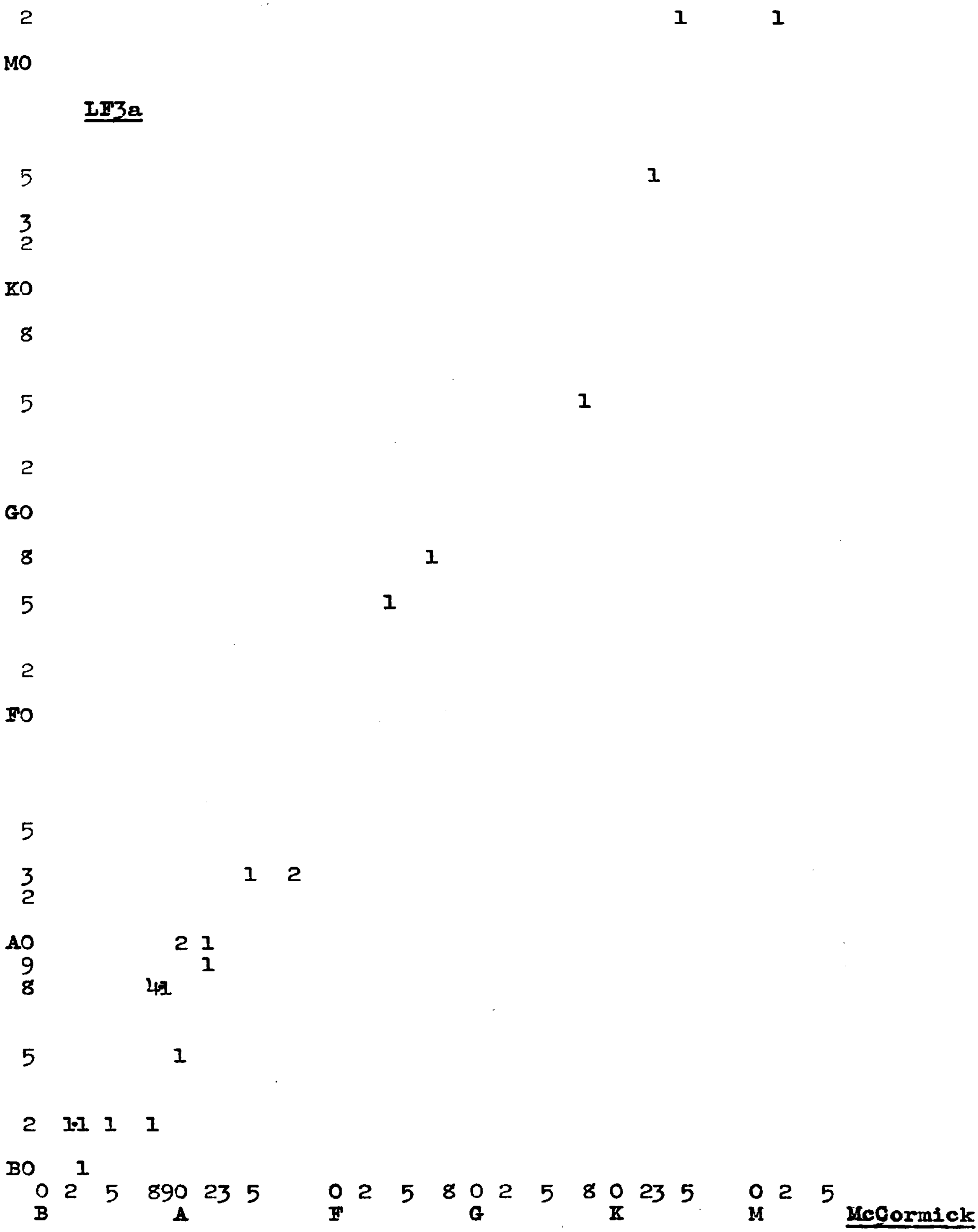


Fig. 5.--Comparison of McCormick and LF3a Spectra

| LF3a \ Upsala | B0 | B2 | B5 | B8 | A0 | A2 | A3 | A5 | F0 | F2 | F5 | F8 | G0 | G2 | G5 | G8 | K0 | K2 | K3 | K5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | | | | | | | | | | | | | | | | | | 1 | | 3 |
| 3 | | | | | | | | | | | | | | | | | | | | 1 |
| 2 | | | | | | | | | | | | | | | | | | 5 | | 3 |
| KO | | | | | | | | | | | | | | | 1 | 3 | 7 | 3 | | |
| 8 | | | | | | | | | | | | | | | 3 | 3 | 1 | 1 | | |
| 5 | | | | | | | | | | | | | 1 | | 1 | 1 | 2 | | | |
| 2 | | | | | | | | | | | | | 2 | | | 1 | | | | |
| GO | | | | | | | | | | | | 1 | 2 | | | | | | | |
| 8 | | | | | | | | | | | 1 | 9 | 1 | | | | | | | |
| 5 | | | | | | | | | | | 4 | 2 | | | | | | | | |
| 2 | | | | | | | | | 1 | 1 | 2 | | | | | | | | | |
| FO | | | | 1 | | | | | | 1 | 1 | 2 | | | | | | | | |
| 5 | | | | | | | 1 | | | | | 1 | | | | | | | | |
| 3 | | | | | 1 | | 2 | 1 | 1 | 1 | 1 | | | | | | | | | |
| 2 | | | | | 5 | 6 | 3 | 2 | | | 1 | | | | | | | | | |
| AO | | | | | 9 | 10 | | 1 | | | | | | | | | | | | |
| 9 | | | | 1 | 33 | 5 | | | | | | | | | | | | | | |
| 8 | | | | 1 | 17 | 1 | | | | | | | | | | | | | | |
| 5 | | | | | 9 | | | | | | | | | | | | | | | |
| 2 | | | | | 7 | 1 | | | | | | | | | | | | | | |
| BO | | | | | 2 | | | | | | | | | | | | | | | |

Fig. 6.--Comparison of LF3a and Upsala Spectra

**LF3a**

| LF3a \ LF3b | B0 | B2 | B5 | B8 | B9 | A0 | A2 | A3 | A5 | | F0 | F2 | F5 | F8 | G0 | G2 | G5 | G8 | K0 | K2 | K3 | K5 | M0 | M2 | M5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | | | | | | | | | | | | | | | | | | | | | | | | | 1 |
| 2 | | | | | | | | | | | | | | | | | | | | | | | | 1 | 1 |
| MO | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | | | | | | | | | 1 | | | |
| 3 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | | | | | | | 1 | 2 | 1 | | | |
| KO | | | | | | | | | | | | | | | | | | 1 | 6 | 1 | | | | | |
| 8 | | | | | | | | | | | | | | | | | 5 | 6 | 4 | | | | | | |
| 5 | | | | | | | | | | | | | | | | 1 | 6 | 5 | 1 | | | | | | |
| 2 | | | | | | | | | | | | | | 1 | 1 | | | | 2 | | | | | | |
| GO | | | | | | | | | | | | | | | 2 | | | | | | | | | | |
| 8 | | | | | | | | | | | | 1 | 3 | 4 | 2 | 1 | | | | | | | | | |
| 5 | | | | | | | | | 1 | | 1 | 2 | 9 | 1 | | | | | | | | | | | |
| 2 | | | | | | | | | 1 | | 2 | 2 | 2 | | | | | | | | | | | | |
| FO | | | | | | | | | | | 5 | | | | | | | | | | | | | | |
| 5 | | | | | | | | 1 | | | | | | | | | | | | | | | | | |
| 3 | | | | | | | 1 | 6 | 4 | 1 | | | | | | | | | | | | | | | |
| 2 | | | | | | 3 | 5 | 5 | 2 | 1 | | | | | | | | | | | | | | | |
| AO | | | 1 | 3 | | 6 | 6 | 3 | 1 | | | | | | | | | | | | | | | | |
| 9 | | | 3 | 2 | 7 | 7 | 1 | 1 | | | | | | | | | | | | | | | | | |
| 8 | | | 6 | 9 | 4 | 5 | 1 | | | | | | | | | | | | | | | | | | |
| 5 | | 1 | 2 | 2 | | 2 | | | | | | | | | | | | | | | | | | | |
| 22 | | 7 | 3 | | | | | | | | | | | | | | | | | | | | | | |
| BO | | 2 | | | | | | | | | | | | | | | | | | | | | | | |

**LF3b**

Fig. 7.--Comparison of LF3b and LF3a spectra

TABLE 5

SUMMARY OF SPECTRAL COMPARISONS

| System | LF3a | | |
|---|---|---|---|
| | Earlier | Equal | Later |
| H.D. (78). . . | 46(59%) | 21(27%) | 11(14%) |
| H.D.E. (321) . | 200(62%) | 86(27%) | 35(11%) |
| McC. (24). . . | 13(54%) | 10(42%) | 1 (4%) |
| Ups. B&A (108) | 81(75%) | 17(16%) | 10 (9%) |
| Ups. F-K (75). | 29(39%) | 34(45%) | 12(16%) |
| Ups. B-K (183) | 110(60%) | 51(28%) | 22(12%) |
| LF3b (199) . . | 75(38%) | 81(40%) | 43(22%) |

above.[1] The scanty material in common with that from the Leander McCormick Observatory is also indicated in Table 5 and in Figure 5 but is not considered significant.[2] Comparison with the Upsala spectra is also given in Table 5 and Figure 6.[3] In the group just later than A0, LF3a is again seen to be somewhat earlier. Comparison in the usual manner with the B stars is impossible as the Upsala spectra did not permit subdivision of these. Of the forty-five LF3a giants in common, forty-three are classified as such at Upsala. This same high order of agreement was found in LF3, as were the differences near A0.[4] There are no LF3a dwarfs among the stars in common. The spectral comparisons with LF3b are shown in Table 5 and Figure 7. The agreement is considered satisfactory, especially in view of the fact that various spectral subdivisions were grouped in the final discussion of the data. As four classes were used in the luminosity classification of LF3b, exact comparison is not possible. Of forty-one common stars, thirty-one may be said to agree in the luminosity classification. No systematic trend was detected in the ten stars for which the luminosity classifications differed.

The conclusion drawn from the comparisons is that there are no serious systematic errors indicated. No appreciable difference in the limiting magnitude for the various spectral classes was detected in the course of the study.

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[1] Vyssotsky, *op. cit.*

[2] Vyssotsky, *Publications of the Leander McCormick Observatory, VII*.

[3] Schalén, *op. cit.*

[4] Harris, *op. cit.*

It is considered possible that the B stars have a slightly fainter limit than the others because of the favorable distribution in the continuous background and easily recognized characteristics.

The spectral groups are summarized by magnitude intervals in Table 6. A comparison of the spectral data with corresponding material in LF1, LF2, LF3, and LF3b is shown in Table 7. Columns two through six show the percentages of the various spectral groups, in each area. It is noteworthy that both LF3 and LF3a contain a much higher percentage of BO-B2 stars than either LF1 or LF2. LF3 and LF3a seem to be in substantial agreement except for the B8-Ao group, where LF3a seems to be more in accord with the LF1-LF2 groups than with LF3. In columns seven through eleven, a similar comparison is made omitting those spectral classes earlier than A2. When this is examined the resultant distribution is seen to be similar for all five groups.

The many unusual spectra which characterize this region made the classification work of special interest. Six O stars (including five Wolf-Rayet), ten M stars, five known variable stars (including one of class N and one of class S), and several Be stars were noted in the original list before any discards were made. One planetary nebula, just outside the usable limit of the plate, has been reported by Dr. Seyfert.[1]

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[1]Seyfert, Publications of the Astronomical Society of the Pacific, 59, 34, 1947.

TABLE 6

OBSERVED DISTRIBUTION OF SPECTRAL TYPES

| $\bar{m}_b$ | Spectral Type | | | | | | | | | | | | All Types |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | O-B2 | B5 | B8-A0 | A2-A5 | F0-F5 | F8-G2 | G5 | | G8-K3 | | K5-M5 | | |
| | | | | | | | g | d | g | d | g | d | |
| 6.75 . . . . | | | 1 | | | | | | 1 | | | | 2 |
| 7.25 . . . . | 4 | 1 | 3 | 1 | | | | | | | | | 9 |
| 7.75 . . . . | 6 | 3 | 4 | | | 1 | | | 1 | | | | 15 |
| 8.25 . . . . | 7 | 1 | 9 | 1 | 1 | 1 | | | 3 | 1 | | | 24 |
| 8.75 . . . . | 6 | | 14 | 3 | 1 | 2 | 2 | | 6 | | 1 | | 35 |
| 9.25 . . . . | 6 | 3 | 21 | 7 | 2 | 1 | | | 7 | | 2 | | 49 |
| 9.75 . . . . | 12 | 7 | 21 | 17 | 8 | 9 | 4 | | 8 | | 3 | 1 | 90 |
| 10.25 . . . . | 21 | 4 | 39 | 14 | 5 | 17 | 4 | | 19 | | | | 123 |
| 10.75 . . . . | 17 | 25 | 58 | 43 | 29 | 23 | 9 | 3 | 22 | 2 | 1 | | 232 |
| 11.25 . . . . | 13 | 28 | 104 | 50 | 37 | 44 | 11 | 2 | 24 | 2 | 1 | | 316 |
| 11.75 . . . . | 6 | 14 | 84 | 47 | 28 | 34 | 11 | 13 | 22 | 6 | 1 | | 266 |
| 12.25 . . . . | | 7 | 59 | 19 | 11 | 10 | 2 | | 7 | | 3 | 2 | 120 |
| Total. . . | 98 | 93 | 417 | 202 | 122 | 142 | 43 | 18 | 120 | 11 | 12 | 3 | 1281 |

TABLE 7

COMPARISON OF SPECTRAL DISTRIBUTIONS

| Spectrum | Percentages | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | All Types | | | | | Type A2 and Later | | | | |
| | LF1 | LF2 | LF3 | LF3a | LF3b | LF1 | LF2 | LF3 | LF3a | LF3b |
| B0-B2. . . . | 0.1 | 0.2 | 7.2 | 7.6 | 4.0 | | | | | |
| B5 . . . . . . | 1.9 | 1.1 | 12.1 | 7.3 | 7.5 | | | | | |
| B8-A0. . . . | 38.9 | 25.8 | 19.1 | 32.5 | 17.7 | | | | | |
| A2-A5. . . . | 9.4 | 21.8 | 13.7 | 15.8 | 18.8 | 15.8 | 29.9 | 22.3 | 30.0 | 26.6 |
| F0-F5. . . . | 10.5 | 13.9 | 16.5 | 9.5 | 19.2 | 17.7 | 19.9 | 26.7 | 18.1 | 27.2 |
| F8-G2. . . . | 11.9 | 10.6 | 14.5 | 11.1 | 15.7 | 20.1 | 14.5 | 23.4 | 21.1 | 22.1 |
| dG5. . . . . | 4.0 | 3.4 | 1.7 | 1.4 | 1.2 | 6.7 | 4.6 | 2.8 | 2.7 | 1.7 |
| gG5. . . . . | 6.3 | 1.4 | 1.9 | 3.4 | 2.8 | 10.6 | 2.0 | 3.1 | 6.4 | 4.0 |
| dG8-dK3. . . | 1.3 | 4.3 | 1.5 | 0.9 | 0.4 | 2.2 | 5.9 | 2.5 | 1.6 | 0.6 |
| gG8-gK3. . . | 12.3 | 14.6 | 10.4 | 9.4 | 11.5 | 21.6 | 20.0 | 16.9 | 17.9 | 16.2 |
| dK5-dM8. . . | 0.3 | 0.2 | 0.0 | 0.2 | 0.1 | 0.6 | 0.3 | 0.1 | 0.4 | 0.1 |
| gK5-gM8. . . | 2.5 | 2.5 | 1.4 | 0.9 | 1.1 | 4.3 | 3.5 | 2.3 | 1.8 | 1.5 |

## Intrinsic Red Indices

The normal or intrinsic color indices were taken from the paper by Nassau and Seyfert.[1] As pointed out there, these are in substantial agreement with those of Seares and Joyner.[2] They may also be compared with those derived by Morgan and Bidelman; the difference is again found to be slight.[3] The color indices first mentioned were converted to intrinsic red indices by multiplying by 1.33, and are shown in Table 8.[4]

TABLE 8

INTRINSIC RED INDICES

| Spectral Type | Intrinsic Red Index |
|---|---|
| B0 | -0.60 |
| B2 | -0.55 |
| B5 | -0.50 |
| B8 | -0.42 |
| B9 | -0.32 |
| A0 | -0.20 |
| A2,A3 | -0.07 |
| A5 | 0.00 |
| F0,F2 | +0.18 |
| F5 | +0.28 |
| F8,G0 | +0.43 |
| dG2 | +0.60 |
| dG5 | +0.84 |
| dG8,dK0 | +1.07 |
| dK2,dK3 | +1.24 |

A check on these colors may be obtained by the red excesses of the late-type main-sequence and dwarf stars, which supposedly are close enough to preclude appreciable space-reddening effects. (The red excess is obtained by subtracting the intrinsic red index from the observed red

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[1]Nassau and Seyfert, op. cit., p. 133.

[2]Seares and Joyner, op. cit.

[3]W.W. Morgan and W.P. Bidelman, Astrophysical Journal, 104, 245, 1946.

[4]Nassau and Burger, op. cit., p. 32.

index). Sixty-four stars from classes F2-dK2 inclusive were measured for this purpose, and the mean red excess for these stars is +0.19. However as the spectral comparison data and the spread in these observed red indices had indicated the possibility that some giants had incorrectly been classified as dwarfs, a more significant selection might well be those stars from classes F2-G2 only, as there is much less likelihood of giants occurring in this spectral range. The stars in this group yielded a mean red excess of +0.08 (thirty-eight stars). Since this is comparable with the probable error of measurement, and also since a small amount of material absorbing two-tenths of a magnitude at a distance of one or two hundred parsecs could account for this result, it was concluded that the zero-point of the color system is not seriously in error.

## Red Excesses

In order to determine the probable error (due to all causes) of a single red excess, the internal probable error of a single red index, ±0.09, was combined with that due to the spread in the intrinsic red indices (estimated to be ±0.05), and that due to mis-classification (also estimated to be ±0.05). Squaring these and extracting the square root of the sum yields a probable error of a single red excess of ±0.13. It is similar to the previously derived external probable error of a single red index, ±0.12.

## Star-counts and Galaxy Searches

Because of the irregular nature of the absorption and also because of the known clustering of high-luminosity stars in this region, it was not considered feasible to make star-counts to successive magnitude limits for the general star-count analysis. However, in an effort to learn more concerning the obscuration, it was decided to count one well-exposed plate to the plate limit. This was a thirty minute exposure on a IIa-O plate and it was found to have a limiting magnitude of about 18.1, using some unpublished magnitude sequences based on the North Polar Sequence and obtained at the Warner and Swasey Observatory. The counts were not made to the actual limit of the plate; they extend to magnitude 17.6 approximately. The counts were made by squares which are about eight minutes of arc on a side. Four of these squares were summed for each square shown in Figure 2. These large (twenty millimeters on the original plate) squares extend a pattern which was used in LF3; the central line (between rows IV and V) was oriented along the edge of the Rift. These star-counts confirm the extrapolated fifteenth magnitude counts of Miller,[1] and also the eighteenth magnitude counts of Miller and Hynek,[2] both of which apply to areas within this field.

The plate used for the counts and another similar one were examined for galaxies--none was found on either plate. This is in agreement with Hubble's results, previously discussed.

Due to the fact that the form of the catalog (which is to include all the stars investigated in this series) has not been definitely settled as of the time of this writing, it was not thought desirable to include a catalog

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[1]Miller, op. cit.

[2]Miller and Hynek, op. cit.

of the individual stars in LF3a in this paper. Such a catalog will appear as part of the larger one soon. The various tables and figures in this paper summarize the essential data which will appear in the catalog.

## CHAPTER IV

## DISSECTION OF REGION INTO SUB-AREAS

Returning to Figure 2, it is seen by comparison with Plate I that the counts reflect rather well the apparent distribution of stars and obscuring material in the Ross Atlas reproduction. To emphasize this point, a "contour line" has been drawn to delineate those squares which have more (or less) than 640 stars. The Rift appears in row VI and parts of rows IV and V, while the dark region near the "vertex" of the parabolic dark segment is largely in row II, columns I and J. Surrounding this portion, and covering most of the central region, is a relatively bright area, generally considered to be part of the Cygnus Cloud; the remainder of the plate consists of the dark, broadening portion of the parabolic dark segment, largely along rows I and II.

The counts should be compared with Figures 8, 9, and 10, which show the distribution of the red excesses averaged over the same areas as the counts (one-quarter of a large square). Figure 8 illustrates this distribution for those stars with uncorrected distance moduli of 10 or 11 (between 9.5 and 11.5), Figure 9 for moduli of 12 or 13, and Figure 10 for moduli of 14 or 15. The figure in parentheses gives, as usual, the number of stars involved in each case (if more than one). In order to determine whether a correlation exists between the counts and the red

| | VII | VI | | V | | IV | | III | | II | | I | | O | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H | | | | 21 | | | | 36(2 | 33 | 20 | | | | | |
| I | | | 29 | | | 48(2 | 16(2 | 44 | 14(2 | 09(2 | | 22(4 | | | |
| | | 54 | | 40(2 | | 44 | 14 | 33 | | 12 | 32(2 | 07 | 47 | | |
| J | | | 26(2 | 26 | 37(4 | | 30 | 24 | 46 | | | | | | |
| | 37(2 | 22 | 28 | 06 | 21(3 | 39(2 | 28 | 37 | 28(2 | 66(2 | | 41(2 | | 26(4 | 30 |
| K | 25 | | 31(3 | | | 69 | 33(2 | 04(2 | | | | 16 | 17 | 41 | |
| | 03 | 22(2 | | | | 26(2 | 19 | 14(2 | | 21 | 61 | | | | |
| L | | | 01 | 08(2 | | | | 26 | | 09 | | | 13 | 03(2 | 34 |
| | | | -01 | | | 19 | | -07 | | 18(2 | 38(2 | 07(2 | | 46(2 | |
| M | | 18 | 23 | | | | 25 | 16 | 15(3 | 18 | 11 | 45 | 68 | -07 | |
| | | | | 15 | 46 | | | | 16(2 | 07 | 41 | 13(3 | | | |
| N | | | 08(4 | 08(2 | 04 | 53 | | 08(4 | 03 | | 31(2 | | 12 | | |
| | | | 02 | 16(2 | | 14 | 15 | -02 | | 03 | 07(3 | | | | |
| O | | | | | | 26 | -03(2 | -15 | 17 | 23 | 29(2 | -11 | | | |
| | | | | | | | 52 | 16(2 | | -03 | | | | | |

Fig. 8--.Distribution of red excesses for stars of uncorrected distance modulus ten and eleven.

| | VII | VI | | V | | IV | | III | | II | | I | | 0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H | | | | | | | 38(2 | 51(3 | 40(2 | | | | | | |
| | | | | 49(5 | 70(2 | 49 | 22(5 | 43(5 | 30(2 | 39(2 | 66 | | | | |
| I | | | | 50(2 | 75(7 | 44(2 | 32(5 | 48(2 | 26(2 | 28 | | | | | |
| | | | | | 59(3 | 50(7 | 43(5 | 40(4 | | 38(3 | 74(2 | 61 | 63(6 | | |
| J | | 60(2 | 96(3 | 71(6 | 71 | | 37(5 | 41(4 | 52(3 | | 50 | | 36(3 | 68(3 | |
| | | 56(2 | 76(2 | 56(2 | 48(4 | 40(3 | 49(2 | 36(5 | 49(3 | 51(6 | 42(5 | 41(2 | | | |
| K | 97 | 80 | 119 | 50(3 | 10(2 | 64 | 60 | 41(6 | 80(2 | | 84(3 | 54 | 36 | 48 | |
| | | | | 55(2 | 22 | 32(3 | 43(3 | 35(7 | 45(5 | 70 | 35(2 | 31 | 32(2 | 30(2 | 17(2 |
| L | 82 | | | 34(2 | 32(2 | 33 | 52(3 | 42(6 | 36(2 | 60(4 | 70(3 | | | 64 | |
| | | 92(3 | 67 | 28 | 26(5 | 37 | 47(3 | 30(3 | 29(2 | | 71(2 | 42 | 04 | 59 | |
| M | 89 | | | 34(2 | 01 | 37(3 | 30(3 | | 27 | | | 54(2 | 76(2 | | |
| | | | | 36(2 | 58(2 | | 28(2 | 32(2 | 26 | | | 60 | | 29 | |
| N | | | | 31 | 72(2 | 44 | 39(3 | 24(2 | 32 | 42 | 41 | | | | |
| | | | 16 | | | 05(2 | 22(2 | | 55(3 | | | | | | |
| O | | | | | | | 30(2 | 41(3 | 16(3 | 186: | 17(2 | | | | |
| | | | | 20 | | 09 | 40(2 | 22 | 32(2 | | | | | | |

Fig. 9--.Distribution of red excesses for stars of uncorrected distance modulus twelve and thirteen.

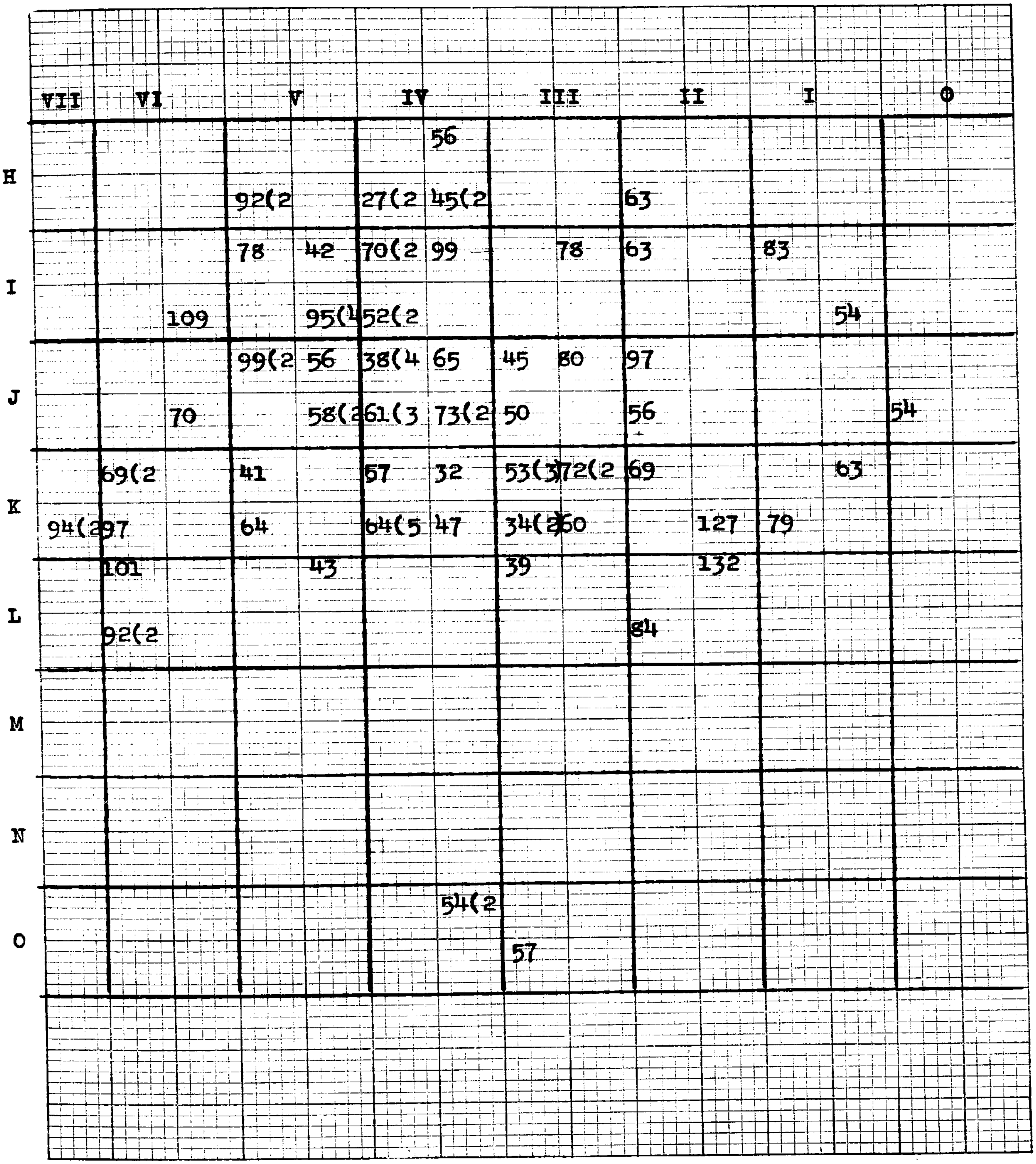


Fig. 10--.Distribution of red excesses for stars of uncorrected distance modulus fourteen and fifteen.

excesses, plots were made of these quantities for stars of a given uncorrected distance modulus range (10 or 11, 12 or 13, 14 or 15, as before), using the smallest areal unit--one sixteenth of a large twenty millimeter square. From these it appeared that there is no correlation between the two quantities plotted, except possibly in the Rift. In order to investigate this in another manner, plots were made of the distribution of the red excess for each uncorrected distance modulus range, again using the unit of one-sixteenth of a large square rather than the quarter-square units used in Figures 8, 9, and 10. The scatter in the first of these figures (that for the least distance modulus) was about what would be expected from the probable error of the measures, but was somewhat greater than expected in the other two diagrams.

Unless the space distribution is decidedly non-uniform, it would seem that there are two possible interpretations to be made of the fact that the red excess in any small area does not represent the distribution as deduced from the counts. One is that the obscuration is so uneven that it varies considerably within the smallest areal unit, and the other is that the obscuration which causes the variation in the counts is so far away that it does not show in the color data. Van de Kamp has found, for stars of galactic latitude 0° and visual magnitude sixteen, a mean secular parallax of 0".0078, which yields a mean distance of 1260 parsecs.[1] As the mean visual magnitude of

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[1] P. Van de Kamp, <u>Annals of the New York Academy of Sciences</u>, <u>42</u>, 176, 1941.

the stars involved in the counts in this paper is probably fainter than sixteen, this may be thought of as a minimum distance. As will be seen presently, many of the stars for which excesses are available probably extend far beyond this distance. It seems likely that both of the above interpretations apply to some degree.

As previously indicated, some evidence of an inverse correlation between the counts and the red excesses was found in the Rift region. Consequently it was decided to investigate this region independently. Although the line of demarcation is not very definite, it seemed desirable on the basis of the distribution of the red excesses to include rows VI and VII in this region, as well as squares H, I, and J in row V. This will be designated as area B; the remaining portion of the field will be called area A. It was felt that further subdivision of the field was not justified by the color data, although it was recognized that the two areas are not strictly homogeneous within themselves. Area A contains 10.0 square degrees and 1015 stars; area B 2.4 square degrees and 266 stars.

CHAPTER V

# THE INTERSTELLAR ABSORPTION

## Possible Methods of Analysis

Most of the methods of determining the distance and absorbing power of a given dark nebula have involved the assumption that two regions are to be observed, one of which is relatively free from obscuration. Pannekoek[1] derived the fundamental equations for this type of analysis; modifications of it have been used by Schalén[2] and Miller[3] in the papers previously discussed. The method first used by Wolf[4], which consisted simply in plotting the logarithm of the stars counted (per square degree, usually) against the apparent magnitude for the two regions and then determining the distance and absorbing power by noting where the two curves diverge, has been shown by Miller[5] and others to be oversimplified. As there are no obviously clear regions in LF3a, no comparison method seemed feasible. An estimate of the minimum total photographic absorption to the edge of the galaxy in this direction may be obtained from the fact that no external galaxies could be detected. From the

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[1]Pannekoek, Proc. Kon. Akad. Amsterdam, 23, part 5, 1921.

[2]Schalén, op. cit. [3]Miller, op. cit.

[4]M. Wolf, Astronomische Nachrichten, 219, 109, 1923.

[5]Miller, Unpublished Ph. D. dissertation, Department of Astronomy, Harvard University, 1934.

known surface distribution of galaxies in the relatively clear regions in high galactic latitudes, and on the assumption that this same distribution would continue in low latitudes if it were not for the absorption in the galactic plane, a minimum total photographic absorption to the edge of the galaxy in this direction of about three magnitudes is derived.

## Interstellar Absorption Derived from Red Excesses

The procedure for determining the photographic absorption from the observed red excess consists simply in multiplying the excess by the proper factor. This factor is subject to considerable uncertainty, although the factor 3.0 has been used quite commonly.[1,2] As mentioned previously, however, Dr. Nassau and Mr. Harris have determined the value 2.6 to be somewhat better. In the following conversion both values will be used so that the nature of the difficulties as well as the approximate value of the differences obtained by the two factors may be perceived.

The first step was to determine the mean photographic absolute magnitudes to be adopted for the early-type stars involved in order to be able to compute distance moduli for various apparent magnitude groups. This, as Bok has pointed out, is "extremely difficult".[3] Previous determinations consulted include those of Malmquist[4], Seares

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[1] Bok and Rendall-Arons, op. cit., p.285.

[2] Bok and Wright, Astrophysical Journal, 101, 304, 1945.

[3] Bok and Rendall-Arons, op. cit., p. 281.

[4] K. Malmquist, Meddelanden Lund, Serie 2, No. 32, 1924.

and Joyner,[1] Schalén,[2] Bok,[3] Bok and Rendall-Arons,[4] Stebbins, Huffer, and Whitford,[5] Wilson,[6] and van Rhijn.[7] The adopted absolute magnitudes were ultimately modified slightly to agree with those used by McCuskey and Seyfert, for the sake of uniformity.[8] The adopted values are shown in Table 9.

TABLE 9

ADOPTED MEAN ABSOLUTE PHOTOGRAPHIC MAGNITUDES (FOR GIVEN APPARENT MAGNITUDE)

| Spectral Type | $\bar{M}_{pg}$ |
|---|---|
| B0 | -3.5 |
| B2 | -2.5 |
| B5 | -1.0 |
| B8 | -0.5 |
| B9 | 0.0 |
| A0 | +0.5 |

The red excesses were then averaged over an appropriate distance modulus interval for each spectral type, and for each area; the distance modulus is not corrected for absorption. As there are very few B0 stars, they were included with the B2 stars in the averages. The red excesses were then plotted against the uncorrected distance modulus for each area, as shown in Figure 11. The curves indicate higher excesses in area B as expected; it should again be emphasized that there are about four times as many stars in area A and hence that curve is stronger.

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[1] F.H. Seares and Mary C. Joyner, Astrophysical Journal, 98, 261, 1943.

[2] Schalén, op. cit.

[3] Bok, Distribution of the Stars in Space, p. 23, 1937.

[4] Bok and Rendall-Arons, op. cit.

[5] Stebbins, Huffer, and Whitford, op. cit.

[6] R.E.Wilson, Astrophysical Journal, 94, 12, 1941.

[7] P.van Rhijn, Groningen Publication 51, 1946.

[8] McCuskey and Seyfert, op. cit., p. 7.

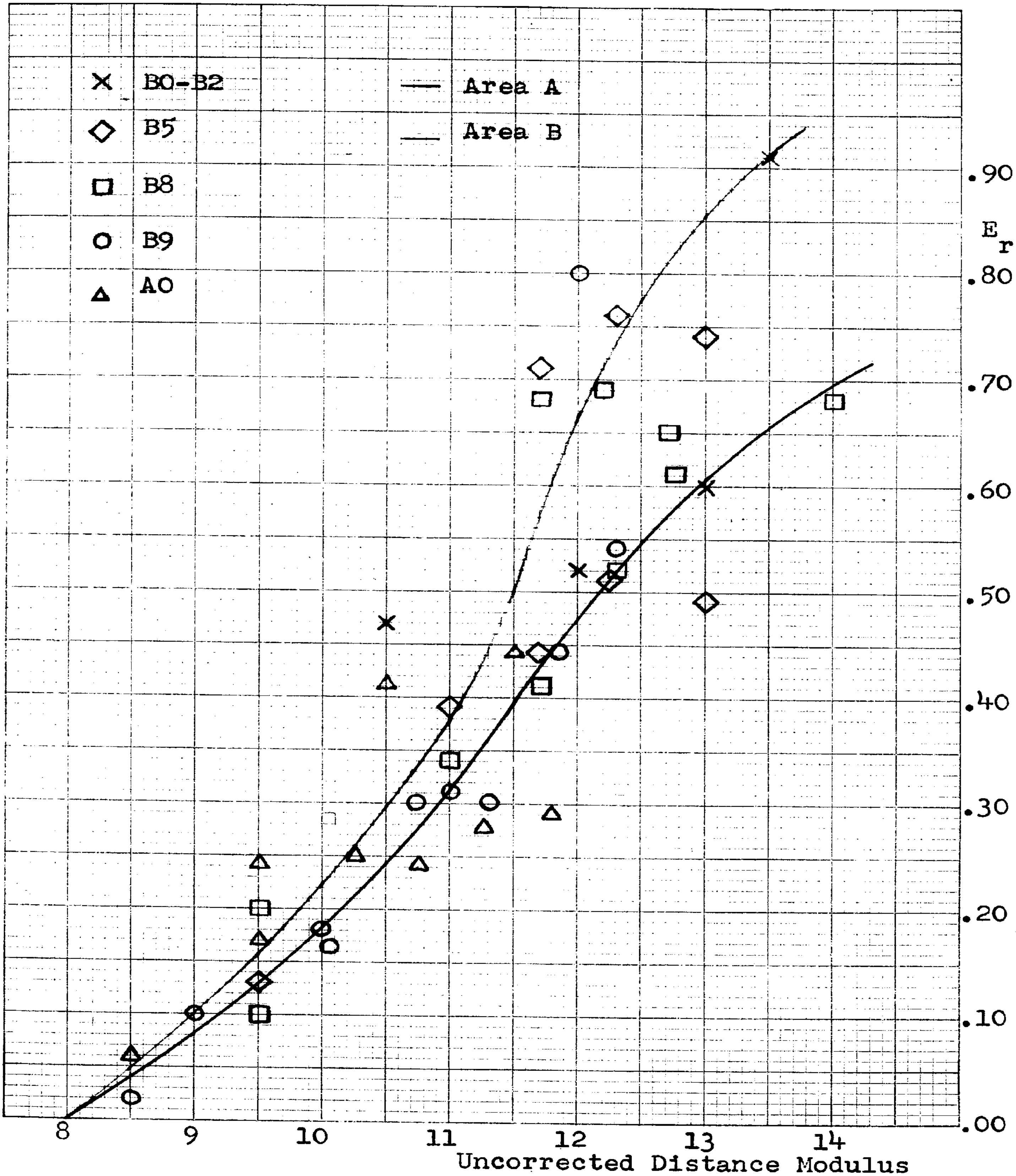


Fig. 11.--Plot of red excess against uncorrected distance modulus.

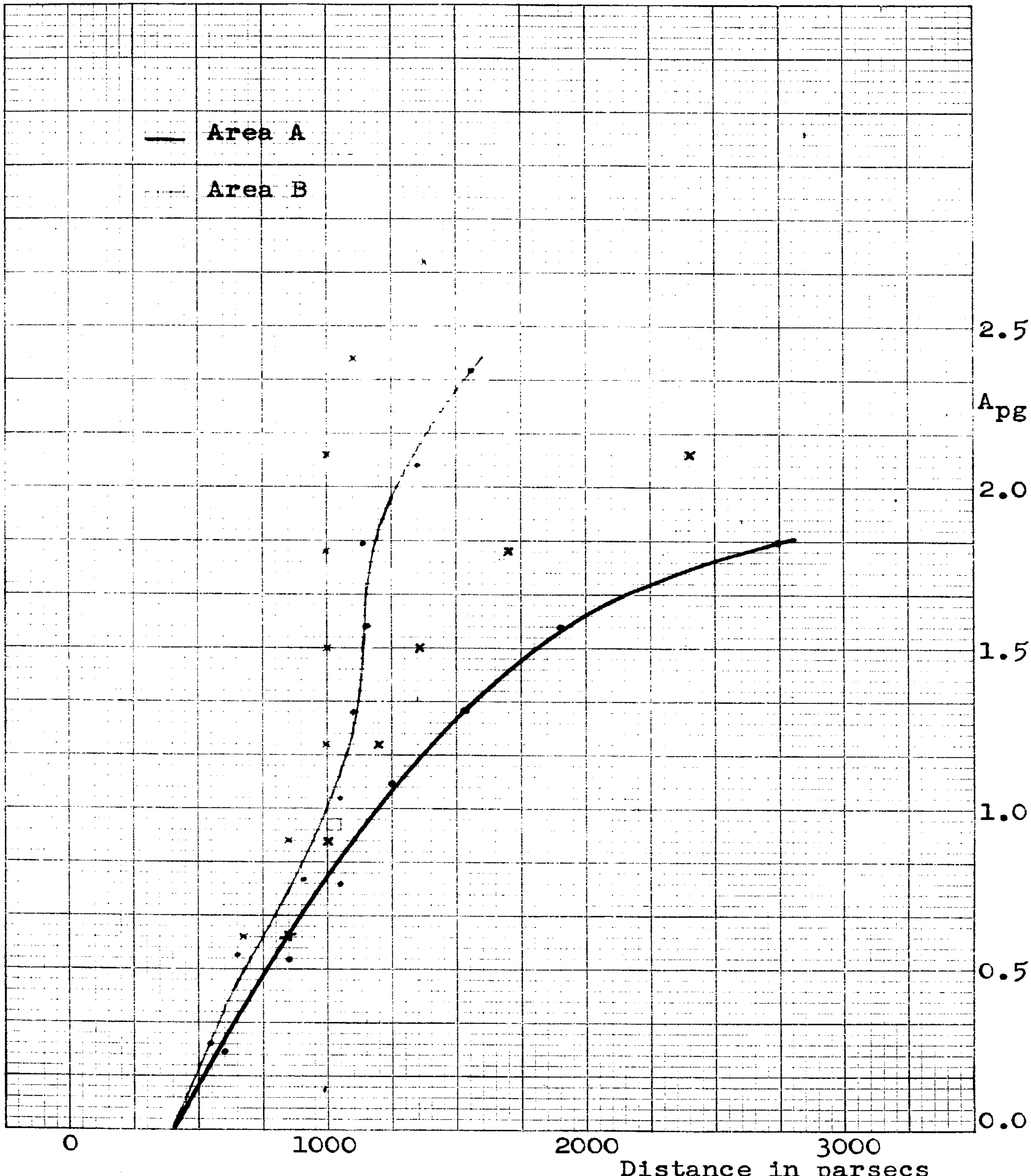


Fig. 12.--Plot of absorption in magnitudes against distance in parsecs.

The distance moduli were then corrected by multiplying the red excess for various distance moduli along the curve by the factor 2.6 to convert to total photographic absorption. The absorption thus obtained was then plotted against the corrected distance (or "true" distance) and is indicated for each area by the full lines in Figure 12. In order to illustrate the effect of multiplying the red excesses by 3.0 rather than by 2.6, the broken lines in the same figure show the absorption obtained by use of the higher factor. It is seen that in area A the absorption derived by this higher factor leads to a vertical portion of the curve at one thousand parsecs. A similar unreasonably steep curve in LF3 was a factor in the adoption of the lower factor of 2.6 for the conversion from red excesses to photographic absorption.

It is seen from the curves that the absorption begins to be appreciable at about five hundred parsecs. For area A it increases to about 0.75 magnitudes at one thousand parsecs, 1.5 at two thousand parsecs, and appears to level off at about 2.0 magnitudes at three thousand parsecs, although this is an extrapolation. In area B the absorption increases somewhat more sharply between five hundred and one thousand parsecs, then increases very sharply to about thirteen hundred parsecs, where there is some indication of leveling off.

## Comparison with Other Results

Comparing these results first with the Rift region of LF3, it is found that the absorption sets in there also at about four hundred parsecs, and increases to about four and five-tenths magnitudes at two thousand parsecs. Thus it may be said to agree with area B to the limit of the present data. In the clearest region of LF3, the absorption increases gradually to about two and five-tenths magnitudes at seven thousand parsecs; a result not in serious disagreement with area A of the present paper.

It is interesting to note that the results for area A compare quite well with those for LF1 and LF2, despite the differences in longitude and latitude.[1]

If comparison is made with the various papers mentioned in the historical survey carried out in an earlier section of this dissertation, it is seen that there is no serious discrepancy, if allowance is made for the spotty nature of the obscuration. There it was pointed out that the evidence indicated that the absorption in the Rift (area B) is due to a dark nebula at about eight hundred parsecs, and that the absorption amounts to about two magnitudes. Figure 12 yields a somewhat larger value for the distance to the Rift, perhaps eleven hundred or twelve hundred parsecs. This should be compared with Miller's results, which indicated a distance between five hundred and sixteen hundred parsecs, and an absorption between two and four magnitudes.[2]

Thus it would seem that the Rift sets in rather sharply along the line indicated in the photograph of the region, at a distance of about one thousand parsecs. The parabolic region also sets in quite sharply, as judged from the star-counts, but at a distance probably somewhat greater

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[1]McCuskey and Seyfert, op. cit., p.8.

[2]Miller, opcit.

than fifteen hundred parsecs. At this distance the data in this paper probably would be inadequate to detect it with certainty; its large angular dimensions would indicate an upper limit near this figure.

Returning to the photoelectric work of Stebbins, Huffer, and Whitford,[1] and adopting the value eight and two-tenths[2] as the ratio of the total photographic absorption to their $E_1$, a mean residual (LF3a minus Stebbins) of +0.13 ±0.05 was found for the eighteen stars in common (again neglecting the star HDE 227460).

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[1] Stebbins, Huffer, and Whitford, op. cit.

[2] Van Rhijn, Groningen Publication No. 51, 1946.

## CHAPTER VI

## THE STELLAR POPULATION

### Possible Methods of Analysis

The various methods of analysis which may be used for the determination of the distribution of the stellar population in space have been ably summarized by Bok.[1] The basic problem is to find the density function from an observed series of counted numbers of stars per unit of surface area, on the assumption (or derivation from observation) of a luminosity function. Allowance must be made for absorption of light in space. The luminosity function, $\phi(M)dM$, is defined as the number of stars per cubic parsec with absolute magnitudes between M and M + dM, in the vicinity of the sun, where M as usual represents the absolute magnitude of the stars. Formerly the density function was assumed to have a definite analytical form, and this was combined with an assumed form of the luminosity function until a fit with observations was obtained, making allowance always for the absorption present. However it soon became apparent that the analytical form for the density function was too restricting and that there are marked irregularities in the density distribution. Consequently numerical methods, which allow greater flexibility, have been used more and more frequently. For the details of the method to be used here, known generally as the (m, log pi) type of analysis,

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[1]Bok, Distribution of the Stars in Space, pp. 10-37, 1937.

reference may be made to Appendix A.

## Analysis of Data

### Strengthening of Bright Star Data

The spectral summations for area A and area B are shown separately in Tables 10 and 11. The summations are by magnitude intervals similar to Table 6.

In order to strengthen the data for the bright stars, two steps were taken. In the first place, counts of stars by spectral types for those with apparent magnitude less than seven have been taken from Seydl's compilation of the stars in the Henry Draper Catalog.[1] His tabulations for the zone of galactic latitude $0^{\circ}$ to $\pm 10^{\circ}$ at all latitudes are shown in Table 12, in the form of log A'(m), where A'(m) represents the number of stars with $m - \frac{1}{4}$ to $m + \frac{1}{4}$ per one hundred square degrees of the sky. The spectral groups shown in this table (and subdivided further as to luminosity in later tables) were adopted in an effort to increase the numbers per spectral group for statistical purposes and to minimize the errors of spectral classification. This involves a slight sacrifice as to homogeneity in luminosity, of course, but this is not considered to be serious.

Overlapping the faint end of the above counts and extending approximately to magnitude nine and five-tenths are counts made in similar fashion in the Henry Draper Catalog for an area of one hundred square degrees centered on LF3a. These counts are shown in Table 13. It is of inter-

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[1] Seydl, Publications of Prague, No. 6, 1929.

TABLE 10

OBSERVED DISTRIBUTION OF SPECTRAL TYPES IN AREA A

| $\bar{m}_b$ | Spectral Type | | | | | | | | | | | | All Types |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | O-B2 | B5 | B8-A0 | A2-A5 | F0-F5 | F8-G2 | G5 | | G8-K3 | | K5-M5 | | |
| | | | | | | | g | d | g | d | g | d | |
| 6.75 . . . . | | | 1 | | | | | | 1 | | | | 2 |
| 7.25 . . . . | 3 | | 2 | | | | | | | | | | 5 |
| 7.75 . . . . | 5 | 3 | 2 | | | | | | 1 | | | | 11 |
| 8.25 . . . . | 6 | 1 | 6 | | 1 | | | | 2 | 1 | | | 17 |
| 8.75 . . . . | 5 | | 10 | 2 | 1 | 1 | 2 | | 3 | | 1 | | 25 |
| 9.25 . . . . | 6 | 2 | 15 | 6 | 2 | | | | 6 | | 2 | | 39 |
| 9.75 . . . . | 10 | 7 | 13 | 15 | 6 | 7 | 3 | | 4 | | 1 | 1 | 67 |
| 10.25 . . . . | 17 | 3 | 30 | 10 | 3 | 16 | 4 | | 15 | | | | 98 |
| 10.75 . . . . | 13 | 16 | 50 | 32 | 25 | 20 | 6 | 2 | 20 | 2 | 1 | | 187 |
| 11.25 . . . . | 11 | 18 | 88 | 41 | 29 | 38 | 10 | 1 | 19 | 2 | 1 | | 258 |
| 11.75 . . . . | 3 | 9 | 68 | 38 | 25 | 28 | 7 | 13 | 13 | 5 | 1 | | 210 |
| 12.25 . . . . | | 5 | 52 | 14 | 9 | 8 | 1 | | 3 | | 3 | 1 | 96 |
| Total. . . | 79 | 64 | 337 | 158 | 101 | 118 | 33 | 16 | 87 | 10 | 10 | 2 | 1015 |

TABLE 11

OBSERVED DISTRIBUTION OF SPECTRAL TYPES IN AREA B

| $\bar{m}_b$ | Spectral Type | | | | | | | | | | | | All Types |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | O-B2 | B5 | B8-A0 | A2-A5 | F0-F5 | F8-G2 | G5 | | G8-K3 | | K5-M5 | | |
| | | | | | | | g | d | g | d | g | d | |
| 6.75 . . . . | | | | | | | | | | | | | |
| 7.25 . . . . | 1 | 1 | 1 | 1 | | | | | | | | | 4 |
| 7.75 . . . . | 1 | | 2 | | | 1 | | | | | | | 4 |
| 8.25 . . . . | 1 | | 3 | 1 | | 1 | | | 1 | | | | 7 |
| 8.75 . . . . | 1 | | 4 | 1 | | 1 | | | 3 | | | | 10 |
| 9.25 . . . . | | 1 | 6 | 1 | | 1 | | | 1 | | | | 10 |
| 9.75 . . . . | 2 | | 8 | 2 | 2 | 2 | 1 | | 4 | | 2 | | 23 |
| 10.25 . . . . | 4 | 1 | 9 | 4 | 2 | 1 | | | 4 | | | | 25 |
| 10.75 . . . . | 4 | 9 | 8 | 11 | 4 | 3 | 3 | 1 | 2 | | | | 45 |
| 11.25 . . . . | 2 | 10 | 16 | 9 | 8 | 6 | 1 | 1 | 5 | | | | 58 |
| 11.75 . . . . | 3 | 5 | 16 | 9 | 3 | 6 | 4 | | 9 | 1 | | | 56 |
| 12.25 . . . . | | 2 | 7 | 5 | 2 | 2 | 1 | | 4 | | | 1 | 24 |
| Total. . . | 19 | 29 | 80 | 44 | 21 | 24 | 10 | 2 | 33 | 1 | 2 | 1 | 266 |

TABLE 12

LOG A'(m) FOR BRIGHT STARS

| $\bar{m}_b$ | Spectral Type | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | B0-B2 | B5 | B8-A0 | A2-A5 | F0-F5 | F8-G0 | G5 | K0-K2 | K3-M |
| <4.00 . . . . | 9.635 | 8.843 | 9.320 | 8.047 | 9.320 | 8.922 | 8.445 | 9.506 | 9.320 |
| 4.25 . . . . | 9.258 | 9.144 | 9.223 | 9.399 | 9.098 | 9.258 | 8.621 | 9.422 | 8.843 |
| 4.75 . . . . | 9.797 | 9.258 | 9.563 | 9.542 | 9.223 | 9.258 | 8.844 | 9.649 | 9.374 |
| 5.25 . . . . | 9.943 | 9.486 | 9.922 | 9.777 | 9.635 | 9.320 | 9.399 | 9.892 | 9.591 |
| 5.75 . . . . | 9.907 | 9.575 | 0.299 | 9.964 | 9.843 | 9.422 | 9.599 | 0.234 | 9.606 |
| 6.25 . . . . | 0.185 | 9.860 | 0.605 | 0.216 | 0.117 | 9.675 | 9.816 | 0.490 | 9.922 |
| 6.75 . . . . | 0.264 | 9.976 | 0.808 | 0.451 | 0.379 | 0.036 | 0.146 | 0.673 | 0.103 |

TABLE 13

DISTRIBUTION OF SPECTRAL TYPES NEAR LF3a FROM HENRY DRAPER CATALOG

| $\bar{m}_b$ | Spectral Type | | | | | | | | | All Stars |
|---|---|---|---|---|---|---|---|---|---|---|
| | B0-B2 | B5 | B8-A0 | A2-A5 | F0-F5 | F8-G0 | G5 | K0-K2 | K3-M | |
| 6.75 . . . . | 7 | | 21 | 4 | 3 | 1 | 3 | 1 | 2 | 42 |
| 7.25 . . . . | 6 | 3 | 21 | 8 | 1 | | 2 | 5 | 1 | 47 |
| 7.75 . . . . | 15 | 2 | 39 | 9 | 3 | | 4 | 5 | 3 | 80 |
| 8.25 . . . . | 7 | 1 | 58 | 18 | 9 | 4 | 4 | 8 | 1 | 110 |
| 8.75 . . . . | 15 | 1 | 66 | 30 | 9 | 6 | 17 | 17 | 8 | 169 |
| 9.25 . . . . | 2 | | 23 | 18 | 7 | 6 | 11 | 21 | 15 | 103 |

est to note that the spectral classes B0 through A0 contain somewhat more than one-half of the total in Table 24, whereas in Table 6 these classes comprised somewhat less than one-half.

Separation of Giant and Dwarf Stars

Because of the wide difference in luminosity between giants and dwarfs, it is necessary to determine the percentage of stars in each spectral group beginning with the F8-G2 stars. The G5 stars are treated separately because of the rapid decrease in the percentage of dwarfs with advancing spectral class in the range G2-G8, as found by Dr. Nassau and Dr. Seyfert in their study of spectra in the region of the North Celestial Pole.[1] It was decided to break up the data by magnitudes also, into three groups as follows: those with magnitude less than 8.5, between 8.5 and 10.5, and greater than 10.5. Little difference was found between the two faintest groups. For the brightest group, the percentages adopted by Dr. McCuskey and Dr. Seyfert in LF1 and LF2 have been used, for uniformity.[2] For the F8-G2 group, however, the percentages found in the overlapping region LF3b have been followed (as being more representative of this region of the sky), using unpublished data kindly furnished by Dr. Nassau and Dr. MacRae. They used the luminosity criteria derived by Dr. Nassau and Dr. van Albada; LF3a had been classified before these criteria were derived and hence they were not used in the

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[1]Nassau and Seyfert, op. cit., p. 132.

[2]McCuskey and Seyfert, op. cit., p. 10.

spectral classification of LF3a.[1] For the two remaining magnitude groups of G5 and G8-K3 stars, the LF3b data were combined with the percentages of dwarfs as derived in LF3a. All stars of class K5-M5 were considered to be giants. Table 14 shows the final adopted percentages of dwarfs.

TABLE 14

ADOPTED PERCENTAGES OF DWARFS

| Spectral Type | $m_b$ | | |
|---|---|---|---|
| | <8.5 | 8.5-10.5 | >10.5 |
| F8-G2 . . . . . . . | 32 | 80 | 86 |
| G5 . . . . . . . . | 20 | 40 | 40 |
| G8-K3 . . . . . . . | 4 | 8 | 10 |

Adopted Surface Distribution

In combining the bright-star data and the observed numbers, the counts from the Henry Draper Catalog for one hundred square degrees centered on LF3a were converted to log A'(m), as were the observed numbers, for each of the spectral groups. These values for log A'(m) were then plotted against the magnitude. Slight corrections were made to the magnitudes taken from the Draper catalog to allow for the differences between the color indices used in converting these magnitudes from visual to photographic, and the values of the color indices used in the present paper. Smooth curves were then drawn through these points, making allowance for the fact that the Draper catalog is probably not complete beyond the eighth magnitude. For the later

[1]Nassau and van Albada, Astrophysical Journal, 106, 20, 1947.

spectral types, the percentages of dwarfs previously discussed were used. Table 15 exhibits the adopted values of log A'(m) for half-magnitude intervals and for the various spectral groups, for area A. Using the same bright star data, it was found that the curves fitted the observed numbers for area B quite satisfactorily also, except for two spectral groups. For the B5 stars, an excess over area A was found near the faint end of the curve, and for the B8-AO stars an excess amounting to a factor of two was found between magnitudes seven and eleven. These results may seem somewhat surprising in view of the difference in appearance of the two areas; however it should be emphasized that the samll number of stars in area B and its small areal extent (two and four-tenths square degrees) make the results rather uncertain.

Derived Density Functions

Preliminary considerations.--Table 16 shows the adopted mean absolute photographic magnitudes and dispersions, per unit volume of space, for the different spectral groups. Except for the BO-B2 group, these have been taken with slight modifications from LFl and LF2.[1] Appendix A exhibits the procedure followed in applying the (m, log pi) type of analysis to the log A'(m) values of Table 15, in computing the stellar space densities (number of stars per one thousand cubic parsecs).

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[1]McCuskey and Seyfert, <u>op. cit</u>., p. 11.

TABLE 15

ADOPTED VALUES OF LOG A'(m) FOR AREA A

| $\bar{m}_0$ | Spectrum | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | B0-B2 | B5 | B8-A0 | A2-A5 | F0-F5 | F8-G2 | | G5 | | G8-K3 | | K5-M |
| | | | | | | g | d | g | d | g | d | |
| 4.0 . . | 9.40 | 9.16 | 9.15 | 9.35 | 9.18 | 9.03 | 8.71 | 8.50 | 7.90 | 9.30 | 8.00 | 8.90 |
| 4.5 . . | 9.57 | 9.30 | 9.51 | 9.51 | 9.37 | 9.07 | 8.75 | 8.77 | 8.18 | 9.50 | 8.25 | 9.12 |
| 5.0 . . | 9.75 | 9.45 | 9.90 | 9.70 | 9.56 | 9.12 | 8.82 | 9.13 | 8.50 | 9.70 | 8.50 | 9.35 |
| 5.5 . . | 9.93 | 9.61 | 0.22 | 9.93 | 9.74 | 9.24 | 8.92 | 9.40 | 8.77 | 9.94 | 8.72 | 9.55 |
| 6.0 . . | 0.12 | 9.78 | 0.56 | 0.15 | 9.92 | 9.45 | 9.14 | 9.64 | 9.10 | 0.18 | 8.92 | 9.76 |
| 6.5 . . | 0.40 | 0.00 | 0.90 | 0.42 | 0.13 | 9.68 | 9.45 | 9.91 | 9.37 | 0.40 | 9.10 | 9.97 |
| 7.0 . . | 0.67 | 0.23 | 1.20 | 0.64 | 0.37 | 9.90 | 9.74 | 0.14 | 9.60 | 0.60 | 9.27 | 0.16 |
| 7.5 . . | 0.92 | 0.47 | 1.44 | 0.90 | 0.57 | 0.10 | 0.03 | 0.39 | 9.83 | 0.82 | 9.48 | 0.35 |
| 8.0 . . | 1.16 | 0.81 | 1.66 | 1.14 | 0.78 | 0.28 | 0.32 | 0.63 | 0.06 | 1.01 | 9.64 | 0.54 |
| 8.5 . . | 1.40 | 1.15 | 1.84 | 1.36 | 0.98 | 0.51 | 0.67 | 0.87 | 0.30 | 1.22 | 9.83 | 0.70 |
| 9.0 . . | 1.61 | 1.37 | 2.03 | 1.60 | 1.22 | 0.72 | 1.22 | 1.08 | 0.55 | 1.43 | 0.00 | 0.84 |
| 9.5 . . | 1.83 | 1.53 | 2.20 | 1.87 | 1.46 | 0.92 | 1.63 | 1.30 | 0.77 | 1.62 | 0.14 | 0.98 |
| 10.0 . . | 2.05 | 1.70 | 2.36 | 2.10 | 1.70 | 1.12 | 1.92 | 1.50 | 0.98 | 1.82 | 0.26 | 1.11 |
| 10.5 . . | 2.28 | 1.87 | 2.51 | 2.30 | 1.98 | 1.30 | 2.16 | 1.68 | 1.18 | 2.02 | 0.37 | 1.22 |
| 11.0 . . | 2.48 | 2.02 | 2.68 | 2.47 | 2.26 | 1.46 | 2.33 | 1.85 | 1.43 | 2.22 | 0.46 | 1.33 |
| 11.5 . . | 2.60 | 2.18 | 2.84 | 2.63 | 2.50 | 1.63 | 2.47 | 2.01 | 1.66 | 2.43 | 0.54 | 1.43 |
| 12.0 . . | 2.67 | 2.31 | 2.97 | 2.76 | 2.70 | 1.78 | 2.62 | 2.10 | 1.90 | 2.61 | 0.61 | 1.53 |
| 12.5 . . | 2.77 | 2.43 | 3.07 | 2.87 | 2.82 | 1.95 | 2.74 | 2.17 | 2.13 | 2.73 | 0.67 | 1.62 |

TABLE 16

ADOPTED MEAN ABSOLUTE PHOTOGRAPHIC MAGNITUDES AND DISPERSIONS (PER UNIT VOLUME OF SPACE)

| Sp. Type | $M_o$ | Stand. Dev. | Sp. Type | $M_o$ | Stand. Dev. |
|---|---|---|---|---|---|
| BO-B2 . . . | -2.0 | ±1.0 | dF8-dG2 . . | +5.0 | ±1.2 |
| B5 . . . . | -1.0 | ±1.0 | dG5 . . . . | +6.0 | ±1.0 |
| B8-AO . . . | +0.5 | ±1.0 | dG8-dK3 . . | +7.0 | ±1.5 |
| A2-A5 . . . | +2.0 | ±1.0 | gF8-gM5 . . | +2.0 | ±1.0 |
| FO-F5 . . . | +3.5 | ±1.0 | | | |

Density functions for different spectral types.--Figure 13 exhibits the density functions in area A for the BO-B2, B5, and B8-AO stars. The BO-B2 group shows a decrease to five hundred parsecs, followed by an increase to a maximum at two thousand parsecs. There is then a gradual decrease to four thousand parsecs, which is considered to be near the safe observable limit of the data. The B5 group shows a similar trend, although the maximum for this group comes at about thirteen hundred parsecs. The B8-AO maximum occurs still closer, at about four hundred parsecs. The four-fold increase in the scale of the ordinate should be noted.

Figure 14 exhibits in a similar fashion the density functions for area A for the A2-A5, FO-F5, and gF8-gM5 stars. The change in co-ordinates should again be noted. Each of the groups shows a decrease to a minimum, an increase to a maximum, then a decrease. The minima are all near six hundred parsecs.

Finally the dwarf stars in area A are shown in Figure 15. Due to their low luminosity and hence to the small numbers observed, these curves are perhaps weakest.

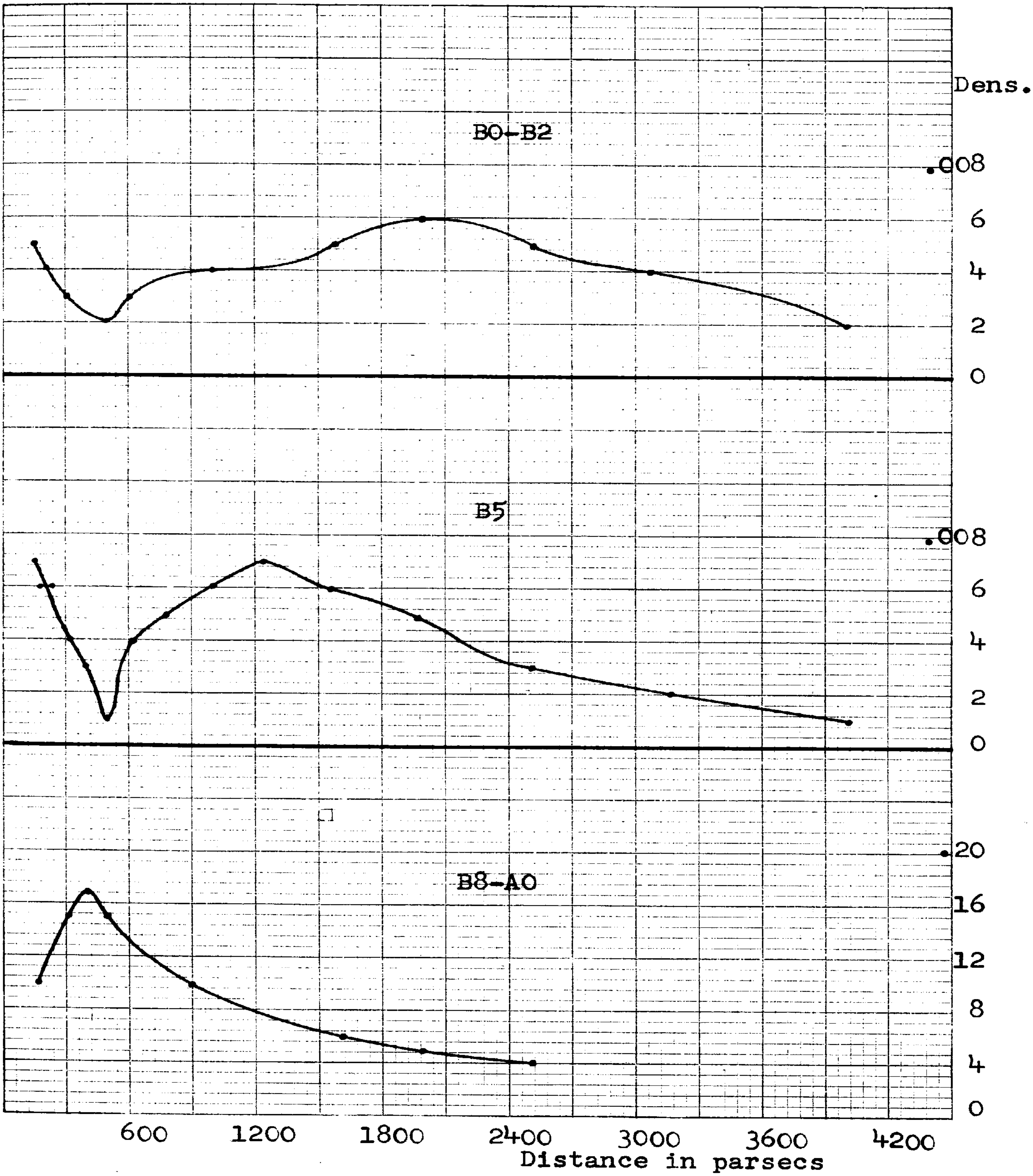


Fig. 13.--Space densities in area A for B0-A0 stars.

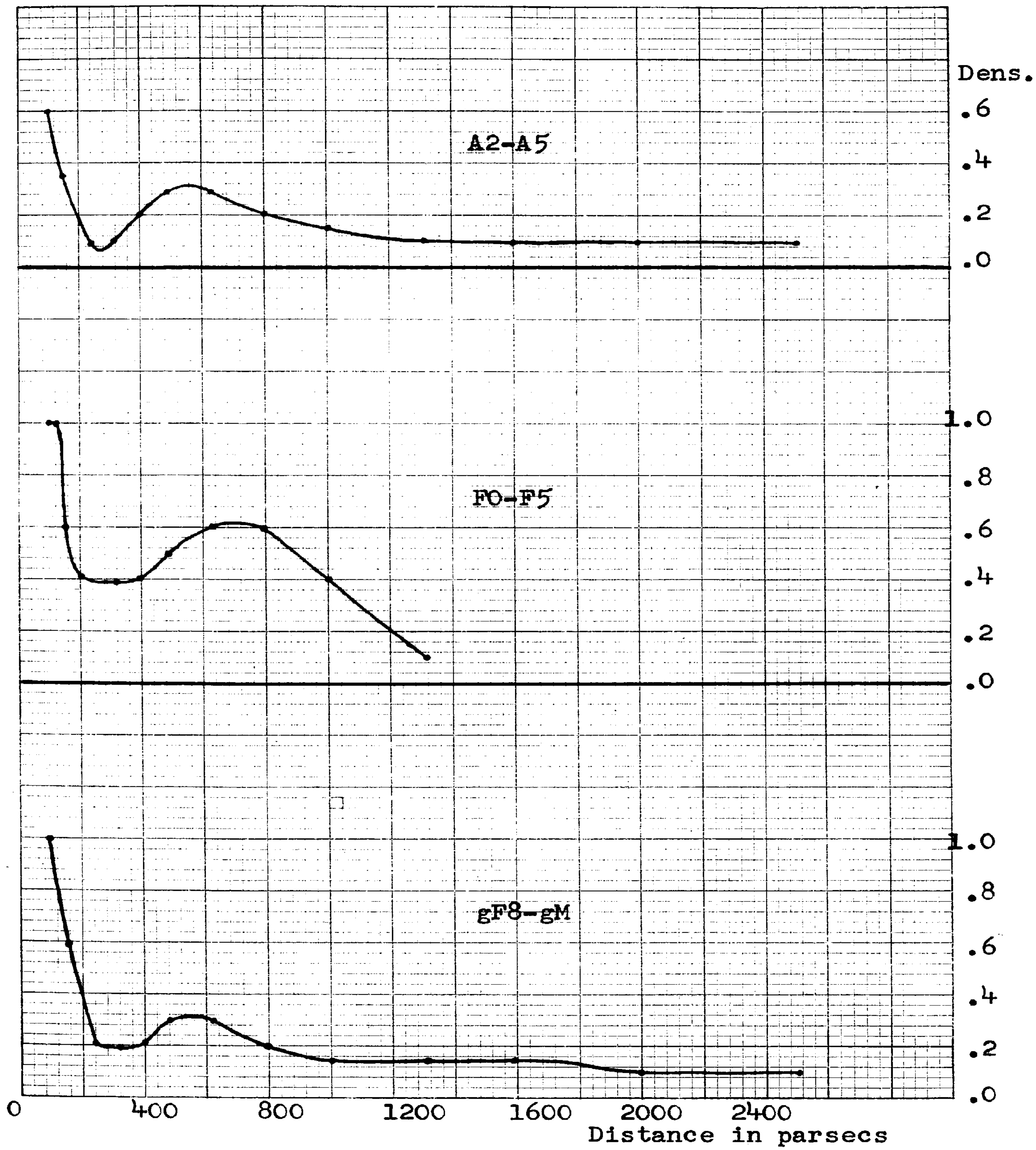


Fig. 14.--Space densities in area A for A2-F5 and gF8-gM stars.

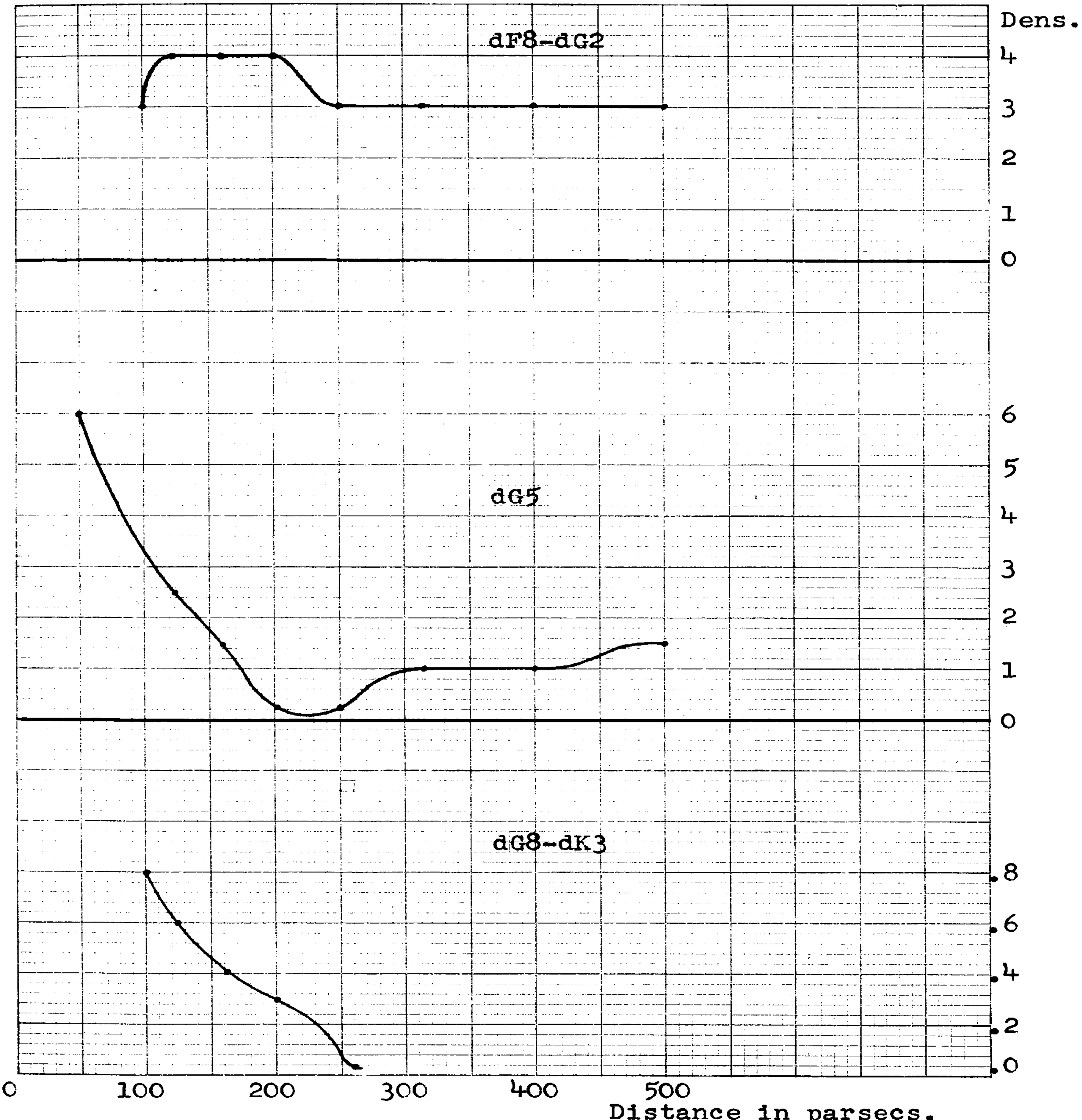


Fig. 15.--Space densities in area A for dF8-dK3 stars.

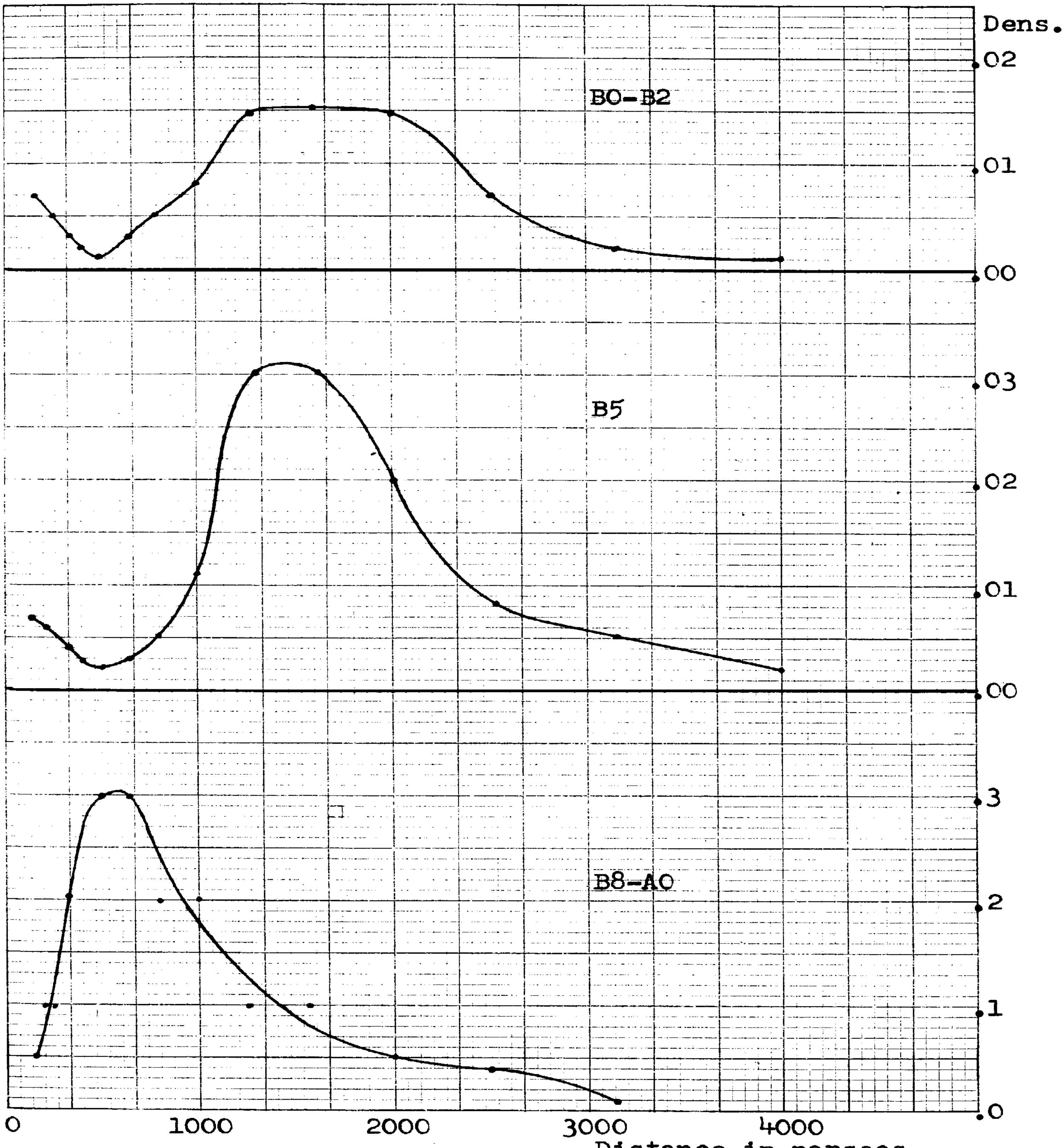


Fig. 16.--Space densities in area B for B0-A0 stars.

It was felt worth-while to obtain density functions in area B for the groups BO-AO only. These are shown in Figure 16. In the case of the BO-B2 stars, it is seen that there is a minimum near five hundred parsecs and a maximum near two thousand parsecs, just as in area A, except that the maximum in area B is higher by a factor of two. Similarly the curve for the B5 stars in area B shows the same trends as in area A, except that the maximum is about four times as high. Finally it should be observed that the B8-AO maximum is nearly at the same distance (five hundred parsecs) as in area A but again is about twice as high.

As an effect similar to the above could be caused by over-correction for absorption in area B, it seemed desirable to check this possibility. Accordingly the density function which represents area A was applied in an (m, log pi) analysis, using the absorption values determined for area B. If the density functions are really similar, as would seem likely in two adjacent regions, the values of log A'(m) computed as above should agree with the observed values. The computed values were found to fall below the observed, however, near the apparent magnitude at which the absorption becomes effective. The possibility remains, therefore, that the absorption determined for area B is somewhat high--possibly because of the relatively few stars which were available in that area for absorption determination.

It should be pointed out that the derived densities should be considered as representing minimum conditions, as several overlapping spectra were discarded, particularly in crowded regions.

Comparison with other results.--A preliminary report on the LF3 region indicates a condensation or clustering of B stars at about two thousand parsecs.[1] Unpublished results dealing with LF3 are quoted in the paper on LF1 and LF2 as yielding a high concentration of B8-AO stars at about thirteen hundred parsecs.[2] Except for the fact that the B8-AO stars in the present investigation were found to have a maximum much nearer (at about four hundred parsecs) in area A, these two regions may be said to be in rough agreement; it will be recalled that this was one of the reasons for investigating this region.

No comparison earlier than B8 is possible for the LF1 and LF2 regions as there were insufficient stars there for the analysis of the earlier spectral types. The B8-AO group is in general agreement, particularly for LF2, although many more stars in this group are indicated for LF3a. Rather good agreement is found in the steep decrease in density near the sun for the A2-A5, FO-F5, and gF8-gM5 groups. Some differences are noted for the lower luminosity groups remaining but may not be significant.

As to the results discussed earlier in the paper, it would seem that there is no definite evidence here for the existence of a large star-cloud, although there is evidence of clustering at different distances. These clusterings, plus the contrast effect of the dark nebulae, may well be responsible for the so-called Cygnus Cloud.

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[1] Nassau, Astronomical Journal, 53, 153, 1948.

[2] McCuskey and Seyfert, op. cit., p. 13.

As pointed out in the paper on LF1 and LF2[1] and also in the paper on Monoceros by Bok and Rendall-Arons,[2] the accuracy in these density analyses is only about thirty to fifty percent for a given region. However it was also pointed out in the first of these papers that the relative results of this series of papers done at the Warner and Swasey Observatory should have some validity, as every effort has been made to keep the work homogeneous.

## Luminosity Function at Different Distances

In continuing this effort to keep the results comparable, a method of analysis of the luminosity function used in the LF1 and LF2 paper will be followed here. In this method a comparative (m, log pi) table is formed, in which absorption is not involved (Table 17). There are really two tables here; one of them represents a summation of the tables used in the density analyses already discussed. The other one is formed by use of the standard van Rhijn Ø(M) with the densities in each shell arranged so that the surface distribution observed in LF3a is well represented. The ratio of the entry in the observed table is called R, and log R is used to represent the departure of the observed luminosity function at various distances from the standard luminosity function. These data are summarized in Figure 17, where the numbers to the right of each curve represent corrected distances.

Figure 17 indicates that there is an excess of luminous stars (M between minus four and plus one) in LF3a as

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[1] Ibid.

[2] Bok and Rendall-Arons, op. cit., p. 281.

TABLE 17

COMPARATIVE (m, LOG $\pi$) TABLES FOR AREA A

| Log | $m_b$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | From Observed Spectra | | | | | From Standard $\phi$ (M) | | | | |
| | 8 | 9 | 10 | 11 | 12 | 8 | 9 | 10 | 11 | 12 |
| -2.0 . . . . . . . | 0.15 | 0.23 | 0.40 | 0.51 | 0.23 | 0.33 | 0.69 | 1.02 | 1.44 | 1.89 |
| 2.2 . . . . . . . | 0.28 | 0.48 | 0.76 | 1.01 | 0.81 | 0.71 | 0.87 | 1.86 | 2.70 | 3.80 |
| 2.4 . . . . . . . | 0.29 | 0.58 | 1.16 | 2.25 | 2.62 | 0.52 | 1.41 | 1.73 | 3.72 | 5.37 |
| 2.6 . . . . . . . | 0.65 | 1.29 | 2.09 | 4.13 | 6.80 | 0.23 | 1.04 | 2.81 | 3.46 | 7.39 |
| 2.8 . . . . . . . | 0.42 | 1.34 | 2.84 | 4.45 | 6.29 | 0.22 | 0.91 | 4.16 | 11.20 | 13.77 |
| 3.0 . . . . . . . | 0.19 | 0.76 | 2.86 | 8.43 | 15.05 | 0.26 | 0.53 | 2.17 | 9.93 | 26.73 |
| 3.2 . . . . . . . | 0.19 | 0.56 | 1.65 | 5.09 | 10.39 | 0.11 | 0.34 | 0.71 | 2.88 | 13.2 |
| 3.4 . . . . . . . | 0.10 | 0.47 | 1.78 | 6.03 | 14.54 | 0.07 | 0.26 | 0.80 | 1.69 | 6.90 |
| 3.6 . . . . . . . | 0.04 | 0.32 | 1.33 | 3.50 | 8.54 | 0.02 | 0.08 | 0.34 | 1.08 | 2.24 |
| 3.8 . . . . . . . | 0.01 | 0.13 | 0.81 | 2.70 | 6.10 | 0.01 | 0.04 | 0.17 | 0.68 | 2.14 |
| -4.0 . . . . . . . | 0.00 | 0.06 | 0.72 | 3.90 | 10.26 | - | - | 0.18 | 0.68 | 2.69 |
| A(m) comp. . . . | 2.3 | 6.2 | 16.4 | 42. | 82. | 2.5 | 6.2 | 16.0 | 40. | 86. |
| A(m) obs.. . . . | 2.1 | 6.0 | 16.5 | 39. | 76. | 2.1 | 6.0 | 16.5 | 39. | 76. |

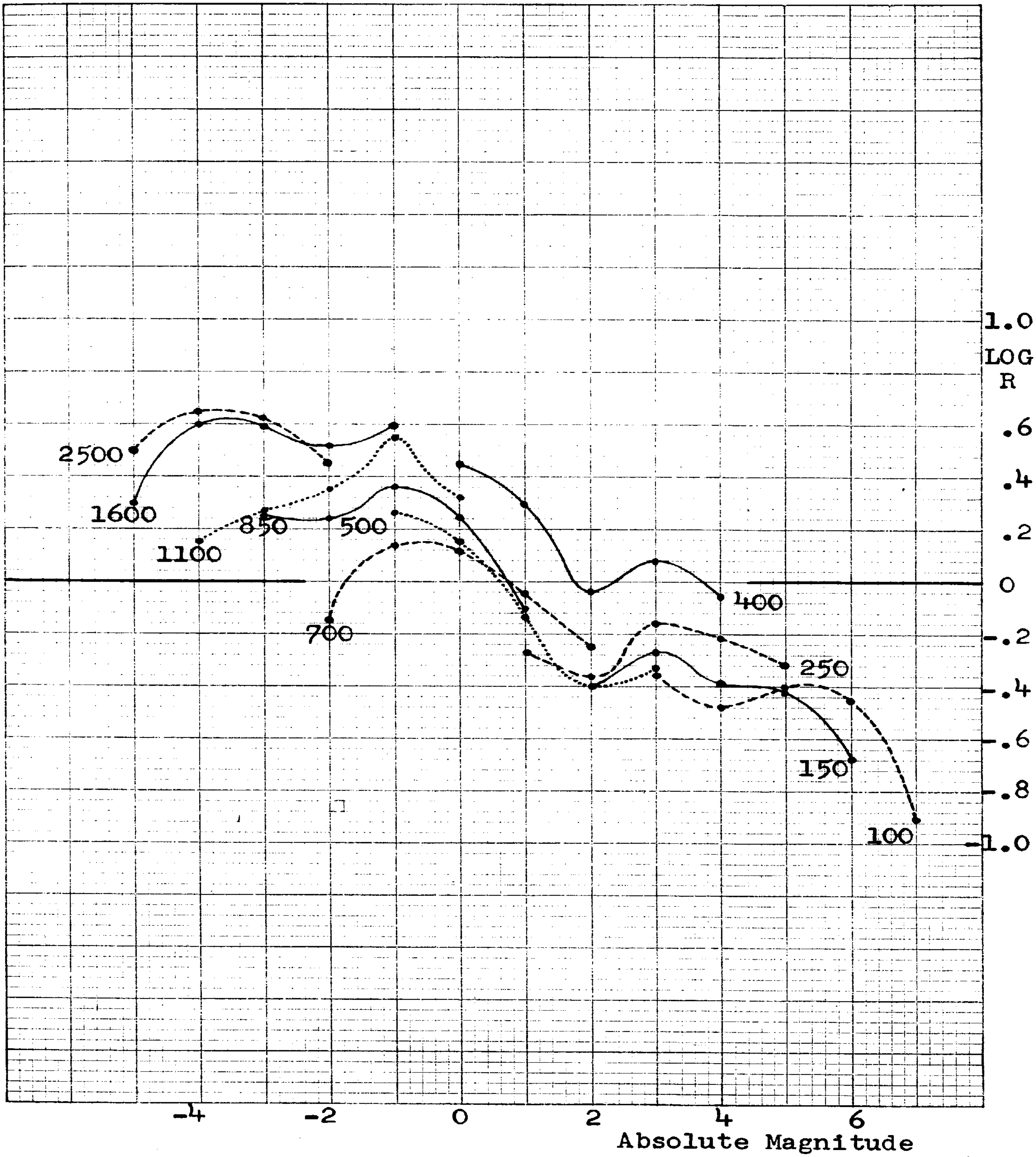


Fig. 17.--Log R for various distances, indicating variation of luminosity function.

compared with the standard Ø (M), and that there is a deficiency of less luminous stars (M between plus one and plus four). In each case an average factor of two is indicated, with a rather sharp change at M = +1.

These results are in accord with those found in LF1 and LF2, and also in general agreement with other results from other fields which have been summarized in that paper.

# CHAPTER VII

## SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

### Summary

The rather large uncertainties present in this type of analysis should be kept in mind while drawing conclusions, as well as the amount of "smoothing" necessary. As to the absorption in the Rift area, it would seem that it sets in at about five hundred parsecs and increases rapidly to at least two and one-half magnitudes at fifteen hundred parsecs. In the other area, which is considerably more heterogeneous in appearance, the absorption seems to amount to about two magnitudes at three thousand parsecs. The dark "parabolic" section of this area probably does not cause large absorption at distances less than fifteen hundred parsecs.

The density analysis in this same area reveals clustering of the early-type stars, as might be expected from previous studies. The principal difference between LF3a and the overlapping region (LF3) in this respect is that in LF3a the B8-A0 stars show clustering at about five hundred parsecs, a distance much less than in LF3. The spectral groups A2-F5 and the F8-M giants all show rapid decreases in density going outward from the sun in this direction. This same result has been found for other areas, particularly for the F-type stars. Due to the small number of stars, it was thought advisable to carry out a density analysis in the Rift area for the early-type stars only. Clustering was detected at approximately the same distances as in the less obscured area, but the indicated density in the clusters is greater. It may be that the absorption used for this analysis in the Rift area is too high.

In comparing the luminosity function in this area with the standard van Rhijn luminosity function, it was found (as in LF1 and LF2) that there is an excess of luminous stars (absolute magnitude between -4 and +1) in this region and a deficiency of stars with absolute magnitude between +1 and +4. The first of these results is not surprising in view of the known concentration of luminous stars in this vicinity.

## Suggestions for Future Research

The immediate task at hand is the assembling of the data collected in the various LF regions thus far completed into a catalog. Then one or more of the following projects might be carried out in this region:

1. A search for M-type stars, along the lines of the work by Dr. Nassau and Dr. van Albada in LF3b.
2. A search for faint diffuse nebulae.
3. A search for faint peculiar spectra.
4. A search for faint variable stars.
5. A search for small dark nebulae, of the "globule" type.
6. A study of the intensity of interstellar lines on the objective-prism plates.

Further projects will doubtless suggest themselves as a result of the comparison of the present data with the detailed account of the investigations in LF3 and LF3b, which will soon be published.

# APPENDIX A

## FORMATION AND USE OF (m, log pi) TABLES

### Fundamental Equations

The first part of this appendix will illustrate the derivation of the fundamental equations used in the analysis of space densities; the second part will indicate the modifications employed in adapting the method to the derivation of space densities in LF3a.

The three assumptions involved in the first approach to the solution of the problem are as follows:

1. All stars have the same absolute magnitude.
2. The stellar space density is constant.
3. There is no absorption of light in space.

Then if N(m) is used to indicate the number of stars per square degree with apparent magnitude equal to or brighter than m, and x indicates an increment in m, the ratio N(m+x)/N(m) will equal the ratio between the volumes of two spheres whose radii are equal to the distances of the stars with apparent magnitudes m+x and m. From the definition of the magnitude system and the inverse square law of the diminution of light, the ratio of the radii is equal to $(2.512)^{x/2}$, and the ratio of the volumes to $(2.512)^{3x/2}$. Therefore:

$$N(m+x)/N(m) = (2.512)^{3x/2} = (3.98)^{x}$$

In other words, the number of stars to successive integral magnitude limits should be four times the number of stars included in the previous limits. In logarithmic form:

$$\log N(m+x)/N(m) = 0.60x \quad \text{or} \quad N(m) = k\,(10)^{0.60m}$$

It is often more convenient to use the quantity A(m), which is defined as follows:

$$A(m) = dN(m)/dm$$

Or in words, A(m) represents the number of stars per square degree between successive magnitude limits. Differentiating the expression for N(m) yields:

$$A(m) = (10)^{0.60m}\,(k)\,(0.60)/\log_{10} e$$

Again:

$$\text{Log } A(m+x)/A(m) = 0.60x$$

The assumption that the absolute magnitudes are constant is now changed to allow a spread in the luminosities, but the frequency distribution is assumed to be identical for any two large elements of volume. This luminosity function, $\emptyset(M)dM$, is defined as the number of stars per cubic parsec with absolute magnitudes between M and M+dM, near the sun. The number of stars per cubic parsec with absolute magnitude between M and M+dM at a distance r from the sun, is then:

$$D(r)\ \emptyset(M)dM$$

in which D(r) is the density function and is equal to unity near the sun. In order to convert this to an expression involving the apparent magnitude m, the relation which follows is used:

$$M = m + 5 - 5 \log r$$

The number of stars per square degree falling within a spherical shell of inner radius r and thickness dr, and between apparent magnitudes m and m+dm, is then:

$$a(m,r)\,dm\,dr = \frac{4\pi}{41,253} r^2 D(r)\, \phi(m + 5 - 5 \log r)\, dm\, dr$$

It follows that:

$$A(m) = \frac{4\pi}{41,253} \int_0^\infty r^2 D(r)\, \phi(m + 5 - 5 \log r)\, dr$$

This is sometimes called the fundamental equation of stellar statistics. If D(r) is set equal to unity, the above equation will not include that term explicitly. It can be shown that, as before:

$$A(m+x)/A(m) = 10^{0.60x}$$

## Procedure Followed in Setting up the Tables for LF3a

In the fundamental equation of stellar statistics derived above, A(m) is an observed quantity, Ø(M) is known (approximately), and D(r) is the desired quantity. The guiding principle, due to Kapteyn, is to divide the volume of space in question into a definite number of spherical shells, concentric with the sun, and then replace the fundamental integral by the sum of a limited number of terms (shells). Bok has developed this method extensively and his work will be followed closely here.[1]

As general star-counts were not considered feasible because of the great irregularities in the surface distribution, the density function for each spectral class was computed separately. As pointed out earlier, this limits the distance reached but decreases the uncertainty of the results somewhat. To illustrate the procedure in a concrete manner, the computations necessary for setting up the table for the B5 stars in area A will be indicated. For these

[1]Bok, B.J., Distribution of the Stars in Space, pp. 26-31, 1937.

stars the adopted mean absolute photographic magnitude per unit volume of space is -1.0; the adopted dispersion (standard deviation) is ±1.0. Absorption is momentarily neglected.

As there are relatively few stars closer than one hundred parsecs, these will be neglected. The stars beyond ten thousand parsecs will also be disregarded, as there are few stars luminous enough to show through the absorbing clouds at that distance. The logarithm of the parallax of the central portion of the first shell considered will be -2.0, the next -2.1, etc. If these distances are represented by distance moduli, the first will be 5.0, the next 5.5, the next 6.0, etc. The apparent magnitudes will start at 4.0 and increase by half-magnitude intervals to 12.5--this represents a half-magnitude extrapolation of the observed data. Let us assume that the distribution in space of the stars of apparent magnitude 8.0 is desired. Obviously if there were no dispersion in the absolute magnitude (and no absorption), these stars would all have distance modulus 9.0. Due to the increasing volume of the successive shells, however, the distribution will not even be symmetrical about distance modulus 9.0. To facilitate the computation, the areal unit adopted is one hundred square degrees, rather than one square degree. The volume increment for each distance modulus may then be computed, over the solid angle of one hundred square degrees. Assuming that the distribution of the luminosities for this spectral subdivision follows the Gaussian curve, and using the adopted standard deviation of ±1.0, the relative number of stars with absolute magnitude of -1.0 ±0.25 may be computed. This is then multiplied by the volume increment to give the entry in that cell in the table. This figure represents the number of stars of mean apparent magnitude 8.0 and with mean distance modulus of 9.0, under the assumptions mentioned. The unit of density chosen is $10^{-6}$ stars per cubic parsec. This procedure is repeated for each row under the column headed apparent magnitude 8.0, using successive one-half magnitude intervals. The entries in this column then give the distribution in space of the stars of this apparent magnitude, provided that the given assumptions are valid.

In order to obtain adjacent columns, use is made of a relationship previously mentioned--namely, that an increase in apparent magnitude of one-half doubles the number of stars for that cell. Thus each entry may be moved to the right and down for increasing apparent magnitude and to the left and up for decreasing apparent magnitude; in the first case the original cell entries are multiplied by two and in the second case they are divided by two. By trial and error, each row is then multiplied by a number representing the density at that distance until good agreement is obtained with the observed A'(m)--the number of stars per one hundred square degrees between $m-\frac{1}{4}$ and $m+\frac{1}{4}$.

In order to account for the absorption another property of the table is employed. If in a given shell, there is an absorption of one-half magnitude, all the stars in that shell will appear one-half magnitude fainter than they would be if the region were clear; in other words, each entry in that shell will be shifted one column to the right. A graphical method is used to interpolate on this same principle, as usually the absorption will differ from multiples of one-half magnitude. Those shells not affected by absorption will be unchanged, and the density function will be computed by trial and error as before. For distance moduli greater than 12.0, the absorption has been extrapolated.

Table 18 shows the table (absorption included) for the B5 stars in area A.

TABLE 18

(m, LOG π) TABLE FOR B5 STARS IN AREA A ($M_0$ = -1.0, $\sigma_0$ = ±1.0)

| True m–M | $m_b$ | | | | | | | | | | | | | | | | | | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | | 11 | | 12 | | |
| 5 | .002 | .001 | .001 | | | | | | | | | | | | | | | | 7 |
| | .002 | .003 | .002 | | | | | | | | | | | | | | | | 7 |
| 6 | .004 | .005 | .006 | .01 | | | | | | | | | | | | | | | 7 |
| | .004 | .007 | .010 | .01 | .01 | .01 | | | | | | | | | | | | | 6 |
| 7 | .003 | .008 | .014 | .02 | .02 | .02 | .01 | .01 | | | | | | | | | | | 6 |
| | .002 | .006 | .015 | .03 | .04 | .04 | .04 | .03 | .02 | | | | | | | | | | 4 |
| 8 | .002 | .004 | .012 | .03 | .05 | .08 | .09 | .08 | .06 | .03 | | | | | | | | | 3 |
| | | | | | | .1 | .1 | .2 | .2 | .1 | .1 | | | | | | | | 1 |
| 9 | | | | | | .1 | .2 | .2 | .3 | .4 | .3 | .2 | .1 | | | | | | 4 |
| | | | | | | | .1 | .2 | .4 | .6 | .7 | .6 | .4 | .2 | .1 | | | | 5 |
| 10 | | | | | | | | .1 | .3 | .7 | 1.1 | 1.4 | 1.4 | 1.1 | .7 | .3 | .1 | | 6 |
| | | | | | | | | | .1 | .4 | .9 | 1.6 | 2.4 | 2.8 | 2.5 | 1.8 | 1. | .5 | 7 |
| 11 | | | | | | | | | | .1 | .4 | 1.1 | 2.3 | 3.9 | 5.2 | 5.5 | 4.6 | 3. | 6 |
| | | | | | | | | | | | .1 | .5 | 1.4 | 3.2 | 6.2 | 9.3 | 11. | 10.3 | 5 |
| 12 | | | | | | | | | | | | .1 | .6 | 1.9 | 5.1 | 10.4 | 16.8 | 21.5 | 3 |
| | | | | | | | | | | | | | .2 | .7 | 2.5 | 7.2 | 16. | 28.7 | 2 |
| 13 | | | | | | | | | | | | | | .5 | 1.4 | 4.5 | 13.1 | 29.8 | 1 |
| | | | | | | | | | | | | | | | .9 | 2.7 | 9.0 | 26.1 | 1 |
| 14 | | | | | | | | | | | | | | | | 2. | 5. | 18. | 1 |
| | | | | | | | | | | | | | | | | | 4. | 11. | 1 |
| 15 | | | | | | | | | | | | | | | | | | 7. | 1 |
| Com. | .11 | .19 | .30 | .46 | .49 | 1.08 | 1.89 | 3.02 | 6.16 | 12.4 | 20.6 | 33 | 51 | 74 | 107 | 149 | 200 | 287 | |
| Obs. | .14 | .20 | .28 | .41 | .60 | 1.00 | 1.70 | 2.95 | 6.46 | 14.1 | 23.4 | 34 | 50 | 74 | 105 | 151 | 204 | 269 | |

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