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AUTHOR Richard N. Lyon

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HEAT TRANSFER AT HIGH FLUXES IN CONFINED SPACES

by
Richard N. Lyon

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Committee in charge:

Professor Donald L. Katz, Chairman
Professor George G. Brown
Professor David M. Dennison
Associate Professor Alan S. Foust
Associate Professor Robert R. White

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CHAPTER I

INTRODUCTION

The purpose of this paper is to present the development and results of a theoretical investigation of the forced-convection heat transfer coefficient in tubes, to present two simple useable approximations for predicting the heat transfer with liquid metals in tubes and in annuli; and to describe experimental equipment and results with a sodium-potassium alloy which appear to support the approximations.

Except for the application of the Reynolds analogy to gases, the theoretical approach to heat transfer has found little use by practical engineers. This is because of the greater simplicity of the empirical relationships, and, until recently, because of their greater accuracy for materials other than gases.

Another reason for the general disinterest in the theoretical approach, particularly among chemical engineers, is that the nomenclature and arbitrary concepts used in this approach have come from the hydrodynamicists and are unfamiliar to many engineers.

Recently increasing interest has been shown in liquid metals as heat transfer media, due to the demand for higher temperatures in both the power and processing fields. Few heat transfer data have been obtained with liquid metals and those which have indicate that the usual empirical relationships cannot be applied. Application of the theoretical approach by Martinelli to liquid metals has led to extremely complicated relationships, and there has been no experimental evidence to support his results.

The theoretical approach used in this paper employs most of the basic assumptions of Martinelli, but it adheres as closely as possible to the nomenclature used by chemical engineers and follows lines of development with which they are more familiar. It is found that this offers no difficulty to the development, but rather, it appears to point the way toward simplifications which are not otherwise apparent.

The discovery of simple approximations of the theoretical predictions for the heat transfer coefficient with liquid metals in large-diameter, narrow annuli and in tubes makes the theoretical results applicable by practical engineers. The experimental evidence presented here lends support to these results and the approximations.

The author is indebted to many persons for suggestions and assistance in the work described here. Space permits only a few to be listed. The members of the author's doctoral committee, Professor D. L. Katz, Chairman; Professor G. G. Brown; Professor D. M. Dennison; Professor A. S. Faust; and Professor R. R. White have aided greatly in their suggestions and guidance.

The author is greatly indebted to Dr. M. C. Leverett, now associated with the Humble Oil Company, with whose help the problem was originally conceived, and who, as Technical Director of the Oak Ridge National Laboratory, made the necessary arrangements for the work to be carried out, as well as supplying many practical suggestions during its course.

Dr. Stuart McLain, formerly Associate Technical Director of the Laboratory and now affiliated with the Argonne National Laboratory, provided encouragement and suggestions which were invaluable.

Appreciation is felt for the opportunity to carry out the work and to publish this paper which has been provided by the administration of the Oak Ridge National Laboratory. The Laboratory is operated at present by the Carbide and Carbon Chemicals Corporation and was operated formerly as Clinton Laboratories by the Monsanto Chemical Company.

Numerous suggestions and helpful encouragement have been supplied as well by Dr. M. D. Peterson, Dr. C. E. Winters, Dr. R. M. Boarts and Mr. W. B. Harrison.

Assistance in calculations and in recording experimental data was provided by Mr. Charles C. Hurtt and Mr. Malcom Richardson, without whose aid the scale of this work would have been much more limited.

Finally the author would like to express his debt to the late Professor R. C. Martinelli of the University of California whose paper on liquid metal heat transfer which had been presented at the Sixth International Congress for Applied Mechanics at Paris in 1946, stimulated the writer to develop and use the theoretical approach which is presented here. Frequent references are made in the present paper to Professor Martinelli's work, and while it is hoped that the approach and conclusions shown here constitute a further advance and a partial rounding out of the subject, such results would have been impossible without the stimulus and foundation provided by the work of Professor Martinelli.

CHAPTER 2

PREVIOUS THEORETICAL INVESTIGATIONS

One of the earliest quantitative theoretical investigations of turbulent forced convection heat transfer was that by Reynolds[27]. He assumed that the turbulence extended to the walls of the tube, and that the friction forces (momentum transfer) and heat transfer were analogous. He related the heat transfer and fluid friction in a tube by an expression which may be written

(1) $$Nu = \frac{f}{2} Re$$

The term Nu represents the Nusselt modulus and is the heat transfer coefficient times the ratio of tube diameter over thermal conductivity.

(2) $$Nu = h \frac{2r_w}{k}$$

The term Re represents the Reynolds modulus:

(3) $$Re = \frac{2 r_w u_m \rho}{\mu}$$

Here u_m is the average velocity, ρ is the density, and μ is the viscosity of the fluid.

The term f represents the Fanning friction factor. It is a function of the Reynolds modulus and has been found experimentally[17] to fit the following equation:

(4) $$f = 0.046 \, Re^{-0.2}$$

Thus we may write equation (1) in the form:

(5) $Nu = 0.023\ Re^{.8}$

As stated earlier, Reynolds assumed that turbulence extended to the walls of the tube and turbulent conductance of momentum was the same as the turbulent conductance of heat.

It has since become apparent that the turbulence does not extend to the wall of the channel, but rather that a thin layer of fluid in laminar flow exists along the walls. This layer provides a barrier of low momentum conductivity and hence high velocity gradient compared with the bulk of the stream. In most fluids, it also provides a barrier of low thermal conductivity and hence one of high temperature gradient compared with the bulk of the stream.

Direct quantitative comparison of thermal conductivity with momentum conductivity (absolute viscosity) cannot be made because of the differences in the units. However, the comparison is easily made by converting to units of diffusivity which indicates the quantity transferred per unit driving force of concentration difference. This may be momentum concentration, heat concentration or material concentration. In all cases the units of diffusivity reduce to: $(\text{length})^2/\text{time}$.

The molecular (as opposed to turbulent) diffusivity of momentum is expressed by the kinematic viscosity:

(6) $\nu = \frac{\mu}{\rho}$

The molecular diffusivity of heat is obtained by dividing the thermal conductivity by the volume heat capacity, $c\rho$.

(7) $D_H = \frac{k}{c\rho}$

The ratio of molecular diffusivity of momentum to molecular diffusivity of heat is:

(8) $\frac{\mu}{\rho} \cdot \frac{c\rho}{k} = \frac{c\mu}{k} = Pr$, the Prandtl modulus.

The numerical value of this modulus for gases is close to unity, hence if we also assume that the eddy diffusivities of heat and momentum are similar, the Reynolds analogy should apply to gases regardless of whether the laminar layer is present or not.

When the Prandtl modulus is greater than unity, as is the case with most ordinary liquids, the molecular diffusivity of momentum is larger than the molecular diffusivity heat and the Reynolds analogy must be corrected.

Empirically it has been found [5] that an approximate correction may take the form of the Prandtl modulus of the laminar layer to the one-third power. Thus the corrected Reynolds equation becomes a form of the well known Colburn equation:

(9) $Nu = 0.023\ (Re)^{.8} \cdot (Pr)^{1/3}$

Attempts to obtain a satisfactory theoretical equation have met with complete success only recently. It will be seen that equation (9) is developed from equation (1) by use of one empirically determined relation for f and an empirically determined correction factor, $Pr^{1/3}$.

Prandtl[25] and Taylor[29] introduced the assumption of a laminar layer in the theoretical consideration and developed equations which agreed more closely with experimental heat transfer data than the Reynolds analogy, but both investigators were handicapped by lack of adequate knowledge of the velocity distribution in tubes. As a result the agreement is not satisfactory.

Recognizing the basic difficulty, Prandtl encouraged one of his students, J. Nikuradse, to conduct careful experimental investigations of the velocity distribution in a number of systems in turbulent flow. These studies resulted in a series of classic papers[20,21,22] which opened the door to a new realm of investigation in fluid flow, heat transfer and material transfer--a realm which has yet to be explored completely.

By using Nikuradse's data, Karman[13] introduced the concept of a transition or buffer layer between the laminar and buffer layer in which both turbulent and molecular transfer are prominent. This innovation permitted him to develop a relationship which fits experimental results with reasonable accuracy for fluids with a Prandtl modulus up to 25. The failure of his relationship to predict accurately the heat transfer in fluids of higher Prandtl modulus is attributed by Karman to the relatively high temperature drop across the laminar film and to poor knowledge of the actual thickness of the laminar layer in these materials where the overall thermal resistance of this layer is increasingly important.

To improve the situation, Reichardt[26] remeasured the velocity distribution in tubes using air. He essentially corroborated the distribution found by Nikuradse in the turbulent region, but he was able to obtain data into fringes of the laminar zone. He also attempted rather unsuccessfully to measure the velocity distribution while heat was being transferred, but he came to certain conclusions based on these measurements, on theoretical considerations, and on consideration of high Prandtl modulus heat transfer results which enabled him to predict accurately heat transfer rates over the entire measured range of heat transfer experience above the Prandtl modulus of gases.

Boelter, Martinelli, and Jonassen[4] have developed a correction for the equation of Karmen which accounts for the change in viscosity across the laminar layer and a subsequent change in its thickness with heat transfer. Their relationships also permit prediction of the heat transfer coefficient with an accuracy equal or greater than those of the usual empirical relationships such as equation (9).

In all of the preceding developments it was assumed that the molecular conductivity of the fully turbulent region is negligible. If by "fully turbulent region" is meant the region where molecular momemtum transfer is negligible compared with the turbulent or eddy transfer, such an assumption is justified for all cases where the Prandtl modulus is equal to or greater than unity.

In two papers,[18] Martinelli has pointed out that the molecular conductivity of heat in the turbulent core cannot be neglected when the Prandtl modulus becomes considerably less than unity. Such a case is found with liquid metals, because of their high molecular conductivity.

Accordingly, he has developed a correction to be applied to the previous type of theoretical heat transfer equation and this enables a prediction to be made for liquid metals. His investigation discloses that an extrapolation of the empirical relationships for ordinary fluids will probably not be accurate for liquid metals. He also presents a sufficient number of calculated values to enable graphs to be drawn for illustration and use of his conclusions in spite of the extremely lengthy and complex nature of his relationships.

Martinelli in his second paper also develops similar relationships for parallel plates with uniform heat flow through both sides of the channel. This work has recently been extended by Harrison and Menke[12] to the case of parallel plates where heat flows through only one of the parallel plates. Such a system is the limiting case of an annulus with large diameter compared to the distance between the walls. Reference will be made later in this paper to an approximation which it is possible to make for this case, and use will be made of it in the interpretation of experimental results.

While the theoretical study of heat transfer in fluids of high Prandtl modulus appears to be reasonably complete and to be supplemented by relatively simple empirical relationships, the results of Martinelli and of Harrison and Menke appear to be extremely complex and have yet to be given adequate experimental support.

The developments and experiments described in the remainder of this report are designed to assist in overcoming these difficulties, and hence to fill out the entire theoretical approach to heat transfer in turbulent fluids.

CHAPTER 3

THEORETICAL DEVELOPMENT FOR THE HEAT TRANSFER COEFFICIENT AND SIMPLIFIED APPROXIMATIONS FOR LIQUID METALS

In this chapter, a method for solution of the differential equations of idealized heat transfer by forced convection in circular tubes is presented. The resulting expression applies equally well to viscous or turbulent flow where end effects are not present. This method is then applied for turbulent flow using Nikuradse's smoothed data and numerical values of the Nusselt modulus are obtained for the case of liquid metals. A simple approximation is found for these results and those of Martinelli.

On the basis of the development by Harrison and Menke , an approximation is proposed for liquid metals in turbulent flow in annuli where the diameter of the annulus is reasonably large compared with its width.

The development of the general equation is entirely rigorous for the ideal system which has been chosen. The basic partial differential equation of heat flow contains four independent variables, three dimensions and time. By careful definition of our system, we have essentially reduced the independent variables to but one, the distance from the center of the tube, thus enabling us to treat our problem by means of ordinary differential equations. This reduction in the number of variables is implied in the developments by Karman and others, but the limitations which are imposed by such a simplification must be kept in mind to avoid misapplication of the lines of reasoning and the resulting relationships. Further discussion of this point will be found in the appendix.

Definition of the ideal system considered

The definition of the ideal system assumed in the development includes the following qualifications:

1.) Steady State-

This means that there are no changes in temperature, velocity, or fluid over a reasonable length of time at a given position in the fluid with respect to the tube wall. It does not eliminate the rapid fluctuations in these conditions due to eddying or molecular movement, since allowance for such fluctuations is made in the usual definitions of eddy diffusivity, density, viscosity, thermal conductivity, and heat capacity.

2.) System independent of angular displacement, θ, about the tube axis.

3.) Uniform radial heat flux at the wall-

This refers to the flux both as a function of distance along the wall (this is the z direction), and as a function of position around the circumference of the tube. Condition 2 also applies to the heat flux distribution around the tube wall.

4.) Total conductivity (molecular + eddy) not a function of distance parallel to tube axis.

5.) No end effects-

This means that the portion of the tube under consideration is far enough from the beginning and end of the tube's actual length and heated length that the velocity and temperature profile of the flowing stream does not change in shape with position along the tube length. In mathematical terms such a qualification together with the four which precede means that $\frac{\partial t}{\partial z}$ is a constant regardless of r, Θ , z, or time, where t is the temperature of the fluid passing through a given point with respect to the wall and r is the distance of that point from the center of the tube.

6.) Heat movement only by molecular conduction, eddy conduction, and forced convection-

This eliminates such unusual conditions as that where heat is transferred by radiation.

7.) Molecular conductivity at right angles to the flow of fluid uneffected by eddying of the fluid, by the velocity of the fluid, or by the gradient of this velocity with respect to the axes of reference.

8.) Constant physical properties of the fluid--

Such a qualification is met essentially by liquids under isothermal conditions, and by gases in an isothermal and isobaric condition.

9.) In the case of turbulent flow, the ratio between the eddy diffusivities of heat and momentum a constant, α.

In the developments by most of the earlier investigators, the value of this ratio was assumed to be unity. Martinelli substituted α, and attributed the practice to a suggestion by Boelter. It appears likely, however, on the basis of experimental evidence reported here and elsewhere that the ratio is constant and has a value close to one. Such a conclusion is also supported by the fact that the Reynolds analogy applies with accuracy to gases.

Development of the General Equation

$$(10) \qquad h = \frac{q_w}{A_w\,(t_w - t_m)}$$

In this equation q_w is the heat flowing per unit time through the surface of a unit length of tube toward the center line of the tube.

We have defined our system in such a way that $\frac{\partial t}{\partial z}$ is a constant, and therefore the net heat transferred longitudinally into one end of a small section of the tube per unit time is equal to the net heat transferred out longitudinally by the same means at the opposite end of the short section by molecular and eddy conductance. Hence by a heat balance, q_w must be equal to the heat carried away by the sensible heat increase of the fluid flowing per unit time through this unit length of tube.

(11) $q_w = \pi r_w^2 u_m c \rho \frac{dt}{dz}$,

where u_m is the average velocity, c the specified heat, and ρ the density of the fluid.

A_w in equation (10) is the area of the inner surface of a unit length of tube.

(12) $A_w = 2\pi r_w$

where r_w is the radius of the tube.

In equation (10) the term $(t_w - t_m)$ represents the difference between the wall temperature and the flow mean temperatures of the fluid. This mean temperature is that which would be obtained by catching all of the liquid flowing through a cross section of the tube at the point in question and mixing it thoroughly. For this reason it is frequently referred to as the "mixed mean" or "bulk average" temperature. It is not the same as the simple average of the temperatures of the fluid in the section of tube involved, but it is the average of fluid leaving the section of tube, hence it is the average temperature of the fluid in the tube weighted by the velocity at each point within the short length.

(13) $$(t_w - t_m) = t_w - \frac{\int_0^{r_w} 2\pi r_t \, u_t \, t \, dr_t}{\int_0^{r_w} 2\pi r_t \, u_t \, dr_t}$$

The term r_t indicates the value of r corresponding to temperature t; and u_t, represents the fluid velocity at distance r_t from the center, and hence at temperature t. While the use of t as a subscript appears to be tautological at this point in the development, its necessity will become obvious later.

We may rewrite equation (13):

$$(14) \qquad t_w - t_m = \frac{\int_0^{r_w} 2\pi r_t \, u_t \, (t_w - t) \, dr_t}{\int_0^{r_w} 2\pi r_t \, u_t \, dr_t}$$

The denominator of this equation equals $\pi r_w^2 u_m$, hence

$$(15) \qquad t_w - t_m = 2\int_0^1 \frac{r_t}{r_w} \frac{u_t}{u_m} (t_w - t) \, d\frac{r_t}{r_w}$$

Referring to Figure 1, it will be seen that the increase in temperature with radius at the radius, r_q, where the radial heat flow rate per unit length is designated by q, will be proportional to q and inversely proportional, r_q, and to the total conductivity (molecular plus eddy), K .

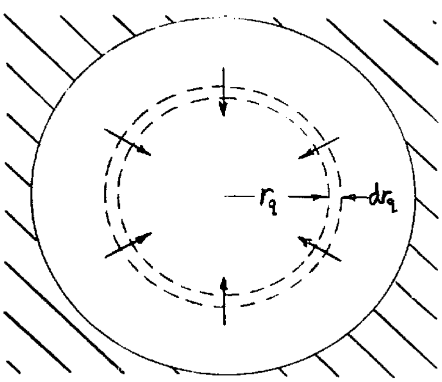

FIGURE 1

$$(16) \qquad \frac{d_t}{dr_q} = \frac{q}{2\pi r_q K}$$

Integration of equation (16) from $r_q = r_t$ to $r_q = r_w$ gives an expression for the term $(t_w - t)$ in terms of r_t

$$(17) \quad (t_w - t) = \int_{r_t}^{r_w} \frac{q}{2\pi r_q K} dr_q$$

The radial heat flow rate per unit length, q, at any radius r_q can be determined by a heat balance as was done for the specific case of q_w in equation (11), except that we are now discussing the cylinder of fluid of radius of r_q within the flowing stream.

$$(18) \quad q = \pi r_q^2 \, c\rho \frac{\partial t}{\partial z} \frac{\int_0^{r_q} 2\pi r \, u \, dr}{\int_0^{r_q} 2\pi r \, dr} = c\rho \frac{\partial t}{\partial z} \int_0^{r_q} 2\pi r \, u \, dr$$

Substitution of equation (18) into equation (17); (17) into (15); and (15), (12), and (11) into equation (10) gives with appropriate cancellations:

$$(19) \quad h = \frac{r_w u_m}{4 \int_0^1 \frac{r_t}{r_w} \frac{u_t}{u_m} \left[\int_{r_t}^{r_w} \frac{\int_0^{r_q} r \, u \, dr}{r_q K} dr_q \right] d \frac{r_t}{r_w}}$$

Multiplying both sides by $\frac{2 r_w}{k}$ with slight revision of equation (19) gives equation (20) where k represents molecular conductivity of heat and Nu represents the Nusselt modulus, $\frac{2 h r_w}{k}$.

$$(20) \quad Nu = \frac{1}{2\int_0^1 \frac{r_t}{r_w}\frac{u_t}{u_m}\int_{\frac{r_t}{r_w}}^{1} \frac{\int_0^{\frac{r_q}{r_w}} \frac{r}{r_w}\frac{u}{u_m}\, d\frac{r}{r_w}}{\frac{r_q}{r_w}\cdot\frac{K}{k}}\, d\frac{r_q}{r_w}\, d\frac{r_t}{r_w}}$$

For simplicity of notation we will define the relative velocity in the equation

$$(21) \quad V = \frac{u}{u_m}$$

and the relative distance to the wall in the equation

$$(22) \quad S = \frac{r}{r_w}$$

Equation (20) is a triple integral equation which may now be written in the form:

$$(23) \quad \frac{1}{Nu} = 2\int_0^1 \int_{S_t}^1 \int_0^{S_q} \frac{S_t\, V_t\, SV}{S_q\, \frac{K}{k}}\, d\,S_t\, d\,S_q\, dS$$

The order of the integrals in this equation may be changed by appropriate changes in the limits. Thus we may write:

$$(24) \quad \frac{1}{Nu} = 2\int_0^1 \int_0^{S_q} \int_0^{S_q} \frac{S_t\, V_t\, SV}{S_q\, \frac{K}{k}}\, d\,S_q\, d\,S_t\, dS$$

Equation (24) is equivalent to the equation:

$$(25) \quad \frac{1}{Nu} = 2\int_0^1 \frac{\left[\int_0^{S_q} VSdS\right]^2}{S_q \frac{K}{k}}\, d\,S_q$$

Since the term

$$\frac{\left[\int_0^{S_q} VSdS\right]^2}{S_q}$$

is a function only of Reynolds modulus and S_q, it is independent of any thermal conductivity considerations. Hence Equation (25) represents a considerable simplification in the theoretical equation for heat transfer from the standpoint of obtaining numerical answers.

Equation (25) is rigorous for our ideal system. It applies with equal validity to viscous and turbulent flow and for fluids of all Prandtl modulus.

The term $\frac{K}{k}$ becomes unity in viscous flow and where molecular conductivity is very high as in extremely low Prandtl modulus materials. Thus two limiting cases may be solved immediately*: 1) the case of viscous flow where $u_{\mathfrak{t}} - u$ is proportional to r^2 and where K equals the molecular conductivity, 2) the case where one assumes constant velocity, slug flow, and K is equal to molecular conductivity, an approximation for a very high conductivity material such as a liquid metal.

In the first case, the calculation readily gives:

$$(26) \quad Nu_{visc} = \frac{48}{11} = 4.36$$

* See Appendix C

THEORETICAL DEVELOPMENT FOR THE HEAT TRANSFER COEFFICIENT AND SIMPLIFIED APPROXIMATIONS FOR LIQUID METALS

In the case of slug flow and high conductivity the result is:

$$(27) \quad \left[Nu_{slug} \right]_{\frac{K}{k} \to 1} = 8$$

Slug flow approximates turbulent flow, but it will be seen later that the limiting case, for turbulent flow as the molecular conductivity becomes very large, is actually about:

$$(28) \quad \left[Nu_{turb} \right]_{\frac{K}{k} \to 1} = 7$$

It will also be found that liquid metals under most conditions have a molecular conductivity which only approaches the value required to mask the eddy conductivity.

<u>The Heat Transfer-Fluid Flow Analogy</u>-

Numerical solution of equation (25) requires a knowledge of the effect of eddying on the total thermal conductivity of the fluid at every point between the tube center and the wall. A means of estimating this eddy conductivity of heat is provided by a knowledge of the velocity distribution in the tube by means of the analogy between eddy conductivity of heat and eddy conductivity of momentum. Excellent presentations of this analogy are provided by Karman[13]; Boelter, et al[4]; and Martinelli[18]. For this reason it is only sketched briefly here. It has already been proposed that the ratio of the eddy diffusivities of momentum and heat be expressed as a dimensionless symbol, as suggested to Martinelli by Boelter:

$$(29) \quad \frac{\epsilon_M}{\epsilon_H} = \frac{1}{\alpha} \quad \text{or} \quad \epsilon_H = \alpha \epsilon_M$$

We have assumed in our system that α is a constant.

According to the Prandtl mixing length theory*, the eddy diffusivity of momentum, ϵ_M equals $\ell^2 \frac{du}{dy}$ where y is the distance from the wall and ℓ is the mixing length which may be thought of as proportional to the diameter of the eddies. By similar reasoning Prandtl[25], Karman[13], Boelter et al,[4] and Martinelli[18] show that the eddy diffusivity of heat ϵ_H, should be proportional to $\ell^2 \frac{du}{dy}$. The agreement of Reynolds , Prandtl , Karman, and Boelter et al with experimental data in the vicinity of Pr = 1 indicates that α is very close to unity. However α will be carried through the subsequent development, and will only be replaced by unity in comparing the approximate and more rigorous equations with experimental results.

In the use of equation (25) for the case of turbulent flow, the total conductivity:

$$(30) \quad K = k + E_H$$

where E_H is the eddy conductivity of heat.

* For an excellent description of this theory as related to fluid friction and velocity distribution see Bakhmeteff[2]

Converting to diffusivity,

$$(31)\quad \frac{K}{k} = \left(\frac{k}{c\rho} + \epsilon_H\right) \frac{c\rho}{k}$$

and dividing by molecular diffusivity of momentum or kinematic viscosity:

$$(32)\quad \frac{K}{k} = \left(\frac{1}{Pr} + \frac{\epsilon_H}{\nu}\right) Pr = \left(1 + \alpha Pr \frac{\epsilon_M}{\nu}\right)$$

Hence equation (25) becomes

$$(33)\quad \frac{1}{Nu} = 2\int_0^1 \frac{\left[\int_0^{S_q} VSdS\right]^2}{S_q\left(1 + \alpha Pr \frac{\epsilon_M}{\nu}\right)} d\,S_q$$

The results of Nikuradse's velocity investigation with smooth walled tubes can now be used to determine V and $\frac{\epsilon_M}{\nu}$. With these data numerical solutions to equation (33) can be obtained directly for various values of Re and αPr.

Numerical Results and Simplified Approximations

The numerical computations are listed in the appendix. They are performed by use of Nikuradse's data directly rather than by use of the approximate equations as in Martinelli's calculation. It is found, however, that the two sets of results agree closely.

An approximation is now proposed for liquid metal heat transfer in tubes:

$$(34)\quad Nu = 7 + 0.025\,(Pe)^{.8}.$$

The calculated values from equation (33) and from equation (34) are compared in Table I.

The values predicted by equation (34) and by Martinelli are compared in Table II.

TABLE I

COMPARISON OF NUSSELT MODULUS OBTAINED EQUATIONS (33) and (34)

Re	Pr	Pe	Nu, using Equation (33)	Nu, using Approximate Equation (34)
4×10^3	0	0	6.75	7.0
	10^{-3}	4	6.76	7.08
	10^{-2}	40	7.41	7.47
	10^{-1}	400	11.03	10.0
4.34×10^4	0	0	6.83	7.0
	10^{-3}	43.4	7.30	7.51
	10^{-2}	434	10.30	10.2
	10^{-1}	4,340	30.5	27.3
3.96×10^5	0	0	7.05	7.0
	10^{-3}	396	9.54	9.83
	10^{-2}	3,960	26.5	25.90
	10^{-1}	39,600	136.	127
3.24×10^6	0	0	7.17	7.0
	10^{-3}	3,240	20.8	21.6
	10^{-2}	32,400	100.	106.
	10^{-1}	324,000	613.	633.

TABLE II

COMPARISON OF EQUATION (34) WITH RESULTS OF MARTINELL FOR TUBES

Pe	Values of Nu According to Martinelli				Nu using Approximate Equation (34)
	$Pr = 10^{-4}$	$Pr = 10^{-3}$	$Pr = 10^{-2}$	$Pr = 10^{-1}$	
10	7.29	7.08			7.16
100	8.11	8.06	7.97		8.00
1000	12.5	13.3	14.5	14.00	13.28
10,000		43.3	48.7	53.00	46.6
100,000			244	271	257
10^6				1649	1580

NOTE: These values were obtained from a correction leaflet published by Dr. Martinelli and containing recalculated values of the results listed in his paper.

Equation (34) is seen to agree with all of the predicted values within reasonable engineering limits.

The term Pe represents the Peclet modulus which is defined in the equation:

$$(35)\quad Pe = \frac{2\, r_w\, u_m\, \rho c}{k} = Pr \cdot Re$$

It will be observed immediately that the viscosity does not enter in Equation (34). Since viscosity is the property which changes most rapidly with temperature, it may be that our assumption of uniform properties does not remove our development far from practical situations.

In Figure 2 will be found a plot of Equation (34). Plotted also are parallel broken lines which represent Equation (9), the empirical Colburn equation for fluids with a Prandtl modulus of one and greater. The remarkable change in relationships involving the heat transfer coefficient as the molecular diffusivity of heat becomes larger than the molecular diffusivity of momentum is at once apparent, and it is obvious that the empirical relationships for ordinary fluids should not be extrapolated for application to the case of liquid metals.

It should be borne in mind that the general equation, Equation (33), will give results when applied to ordinary fluids which are similar to those obtained by Karman[13], these correspond to experimental results for materials of Prandtl modulus up to 25. In addition, if the modifications of Reichardt[26] or Boelter[4] et al are adopted, Equation (33) should be capable of slight modification to fit all experimental results within the accuracy of equation (9), the emperical generalization.

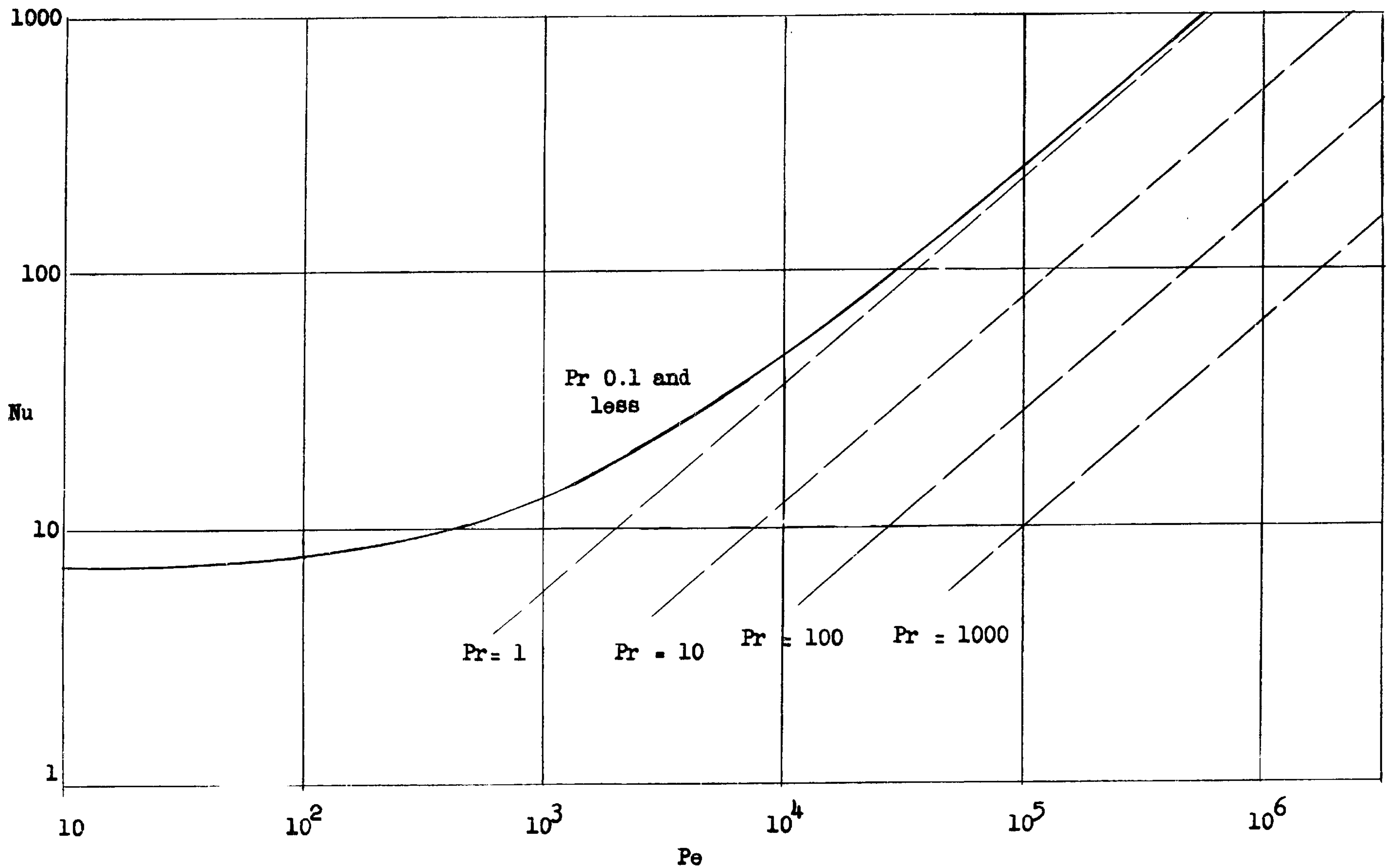

FIGURE 2

NUSSELT MODULUS VERSUS PECLET MODULUS

Harrison and Menke[12] have extended the development of Martinelli to the case where a fluid is flowing between two parallel plates, but where heat passes through only one of the plates. Such a situation is the case where an annulus with heat passing only through the inner wall has a large diameter compared with the space between the two walls.

Their results are found to be almost exactly seven tenths of the results for a tube of comparable hydraulic size. Thus we may write for liquid metals in large-diameter, narrow annuli:

$$(36) \quad Nu_{ann} = .7\, Nu_{tube} = 4.9 + 0.0175\, (Pe)^{.8}$$

In Table III a comparison of the approximation is made with the numerical results of Harrison and Menke.

TABLE III

COMPARISON OF EQUATION (36) WITH RESULTS OF HARRISON AND MENKE FOR ANNULI

Pe	Nu according to Harrison and Menke for Pr value of			Nu using Approximate Equation (36)
	0.005	0.01	0.05	
100	-	5.55	-	5.6
500	8.0	-	-	7.4
1,000	-	9.5	-	9.3
5,000	19.6	-	21.8	20.8
10,000	31.2	-	-	32.6

These approximations will be compared with experimental results in later chapters of this paper.

CHAPTER 4

COMPARISON OF THEORETICAL PREDICTIONS WITH PREVIOUS EXPERIMENTAL WORK

Four sources of experimental information on heat transfer with liquid metals have been found. The first of these is a reference by McAdams[17,6] to an equation which he attributes to Colburn. This equation is simply the empirical Colburn equation, (9), for ordinary fluids which has been modified to apply to a few experimental indications with mercury. It may be written in the form:

$$(37) \qquad Nu = 0.023\,(Re)^{.8}\,\frac{Pr}{0.05 + Pr^{2/3}}$$

No strong support for the modification has been supplied by McAdams or Colburn, and it may be concluded that it is advanced on a tentative basis only. In Figure 3, Equation (37) is plotted with Equation (34) for Prandtl moduli of 1, 10^{-1}, 10^{-2}, and 10^{-3}. In Equation (37) the value of Nu becomes zero when the molecular conductivity becomes very large. As already seen, this is at variance with the more analytical predictions.

Styrikovitch and Semenovker[28] have published their results using mercury in a vertical tube where the tube wall was not wet by the mercury. As shown in Figure 4, these data fall somewhat below the predicted values.

Musser and Page[19] have also published results with mercury and state that the Prandtl modulus used by Styrikovitch and Semenovker is incorrect. They find that their own data and those of Styrikovitch and Semenovker

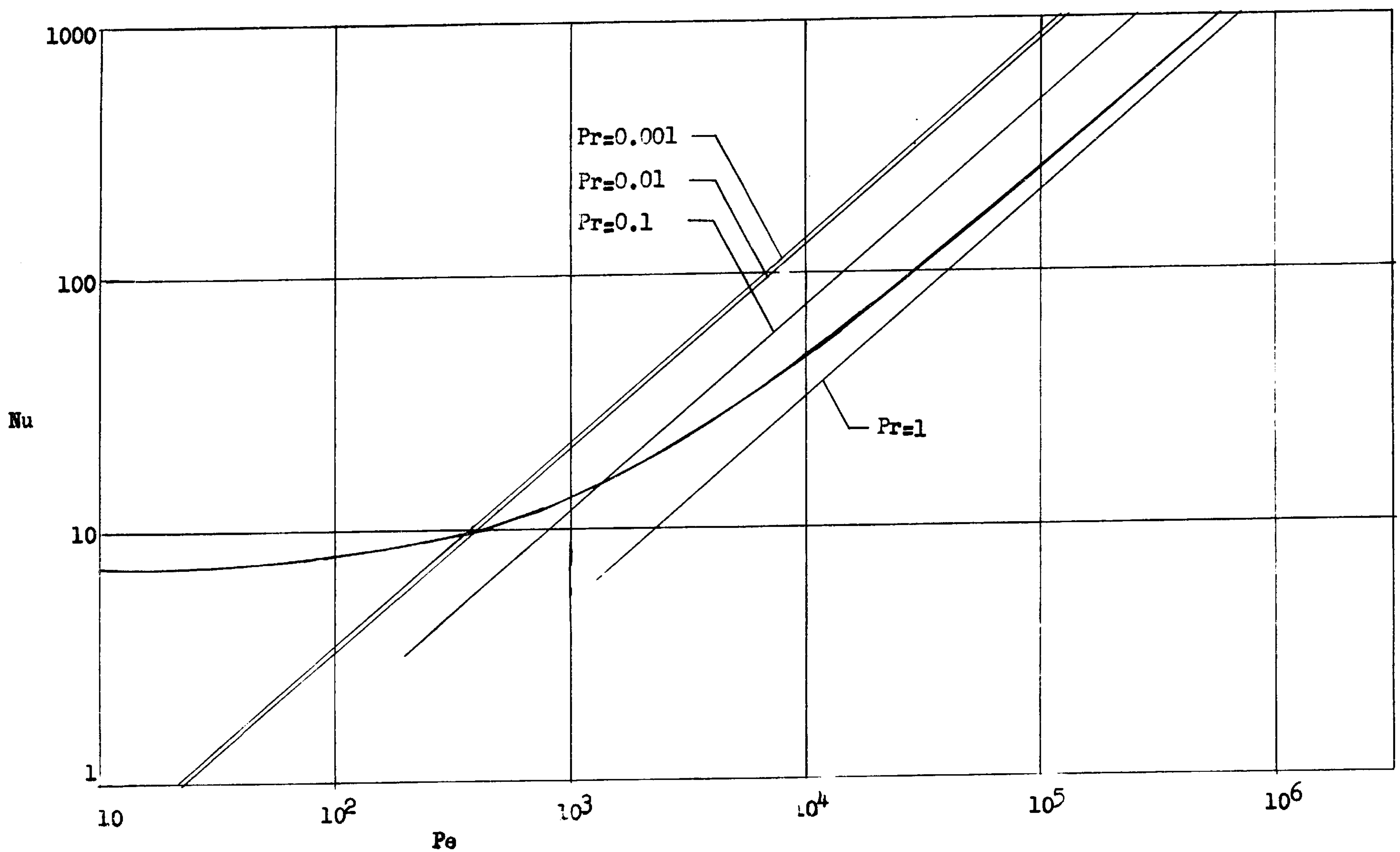

FIGURE 3

COMPARISON WITH MODIFIED COLBURN EQUATION

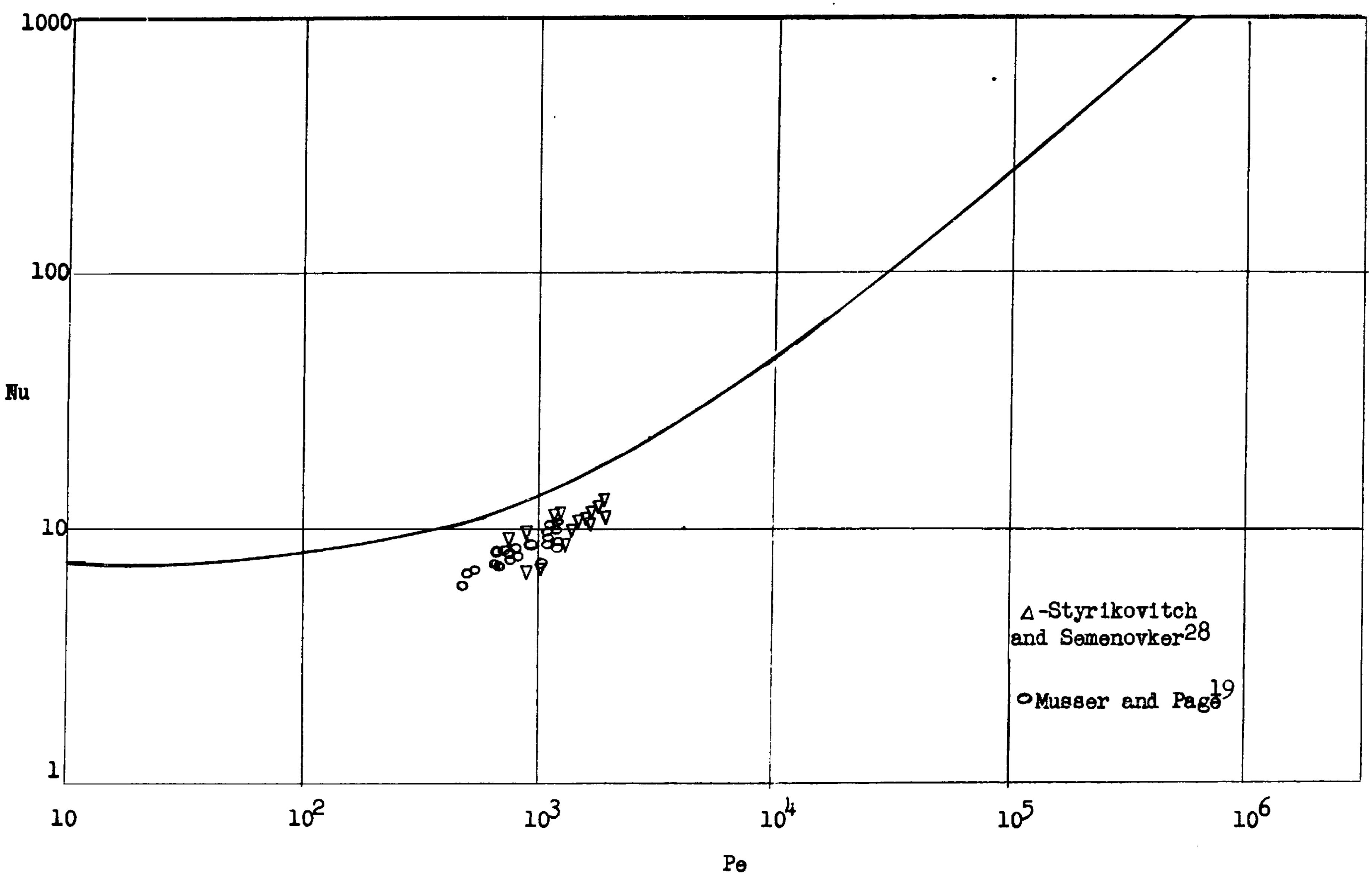

FIGURE 4

COMPARISON WITH EXPERIMENTAL DATA FOR MERCURY fit the predictions using Karman's equation which at their Prandtl and Reynolds moduli gives values for Nu about half those predicted by Martinelli and by Equation (34). They state however that in their case also, the walls were not wet, and call attention to the preliminary results of an investigation on wetting which indicate a definite impedance in heat transfer when the walls are not wet.

Recently the results obtained with sodium by F. C. Bennett[3] of the Dow Chemical Company have become available. These have been delayed for a long time because of the difficulty involved in their interpretation. An attempt was made in the experimental work to vary the flow independently in the annulus and tube of a double-tube, figure-of-eight heat exchange system, by means of a by-pass. It was hoped at the time (1945) that a Wilson line type of plot could be used in separating the individual coefficients. Unfortunately the data do not bear out the hope and widely varying values are obtained for the inner tube coefficient. These range from 142,000 Btu/(hr)(sq.ft.)(^{o}F) to 5400 Btu/(hr)(sq.ft.)(^{o}F).

The report presents reasonable values for overall coefficient, however, and these may provide much useful information.

An excellent description of the handling methods for liquid and solid sodium, is given in the report, and reference in this regard will be made again in the following chapter.

It appears then that no satisfactory experimental information is available on heat transfer with liquid metals, with which to compare our theory. To help to fill the need for experimental liquid metal heat transfer data, equipment was set up and tests were made using an alloy of sodium and potassium. The equipment and procedure employed, and the results obtained are discussed in the following chapters.

CHAPTER 5

EQUIPMENT FOR OBTAINING HEAT TRANSFER INFORMATION WITH SODIUM POTASSIUM ALLOY

In order to obtain experimental data for tubes with wetted walls and to obtain data for annuli to compare with Equations (34) and (36), equipment was built for the circulation of an alloy of about 52 Wt% sodium and 48% potassium. An alloy of these metals was chosen for several reasons. Among these is the fact that it wets the walls of its containers, which, as was pointed out in the last chapter, appears to have an influence on the overall heat transfer rate. Since our theory provides for wetted walls it should be compared with data obtained under these conditions.

The alloy chosen is liquid at room temperature; it melts at about 15° C. Hence no special facilities are required in the equipment to melt the metal.

In addition, the vapor pressure of the alloy is negligible at the temperatures of operation, and no toxicity problem is involved.

Authorities[3,9,16] agree that as long as a few simple precautions are observed, the alloy is no less safe to handle than other chemicals. While small quantities of the alloy, both hot and cold, have been spilled during operation of the equipment described here, in no case did a hazardous situation arise.

Flow Circuit

The circulating equipment is outlined in Figure 5. It consisted of a relatively large sump tank, a sump pump, a flow-indicator, a heat exchange unit, a heating tank, an evaporator-cooler and a flow rate catch tank.

The liquid was pumped into the header by the sump pump, the header flow was controlled by means of an unpacked valve which was inside the sump tank. By means of a series of mechanical linkages and a stuffing box in the top of the sump tank, this throttling valve was controlled from a panel board at the front of the equipment.

From the valve the liquid passed out of the tank through the electromagnetic flowmeter to the inlet of the annulus side of the double tube heat exchanger. It was heated in the annulus and from there flowed through the heating tank where it could receive enough additional heat to raise its temperature as desired above the temperature at the outlet of the annulus. After being heated, the alloy passed back through the central tube of the heat exchanger where it lost heat to the same quantity of liquid flowing countercurrent in the annulus. After leaving the exchanger, the liquid was cooled further in the evaporator-cooler and spilled into the flow rate catch tank from which it drained back into the sump.

Materials of Construction

Metals in contact with the liquid alloy were mild steel, stainless steel, nickel and inconel. Asbestos packing was used for the drain valves handling cold alloy. Teflon will react with hot sodium or sodium-potassium;

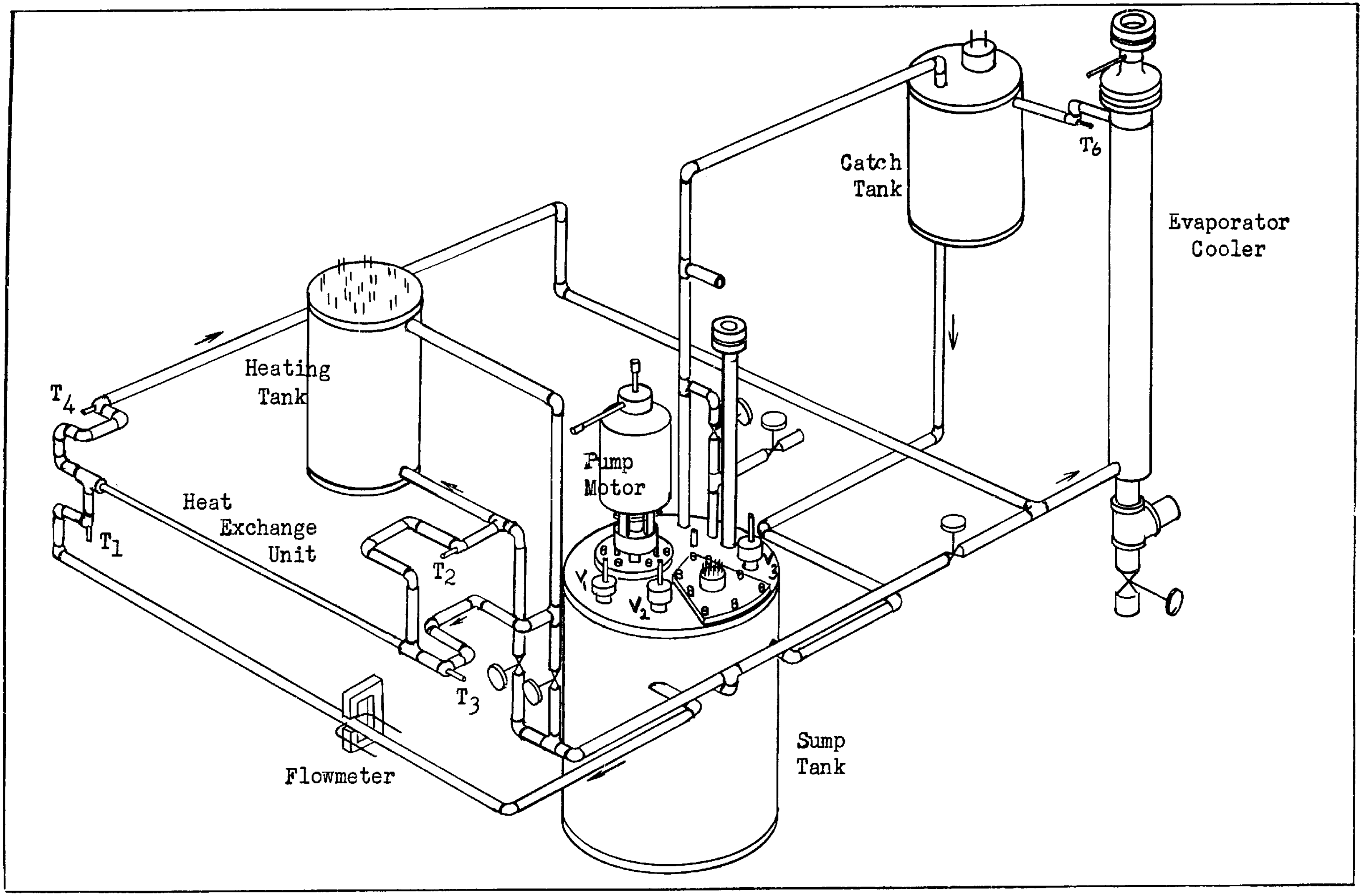

FIGURE 5
FLOW CIRCUIT

however, because of its excellent properties as a packing, it was used for those situations, such as the seal where the pump shaft enters the sump tank, in which alloy did not come in direct contact with the packing material.

The Sump Tank

This was a two foot length of mild steel, series 40, twenty four-inch pipe with a one inch thick plate welded on the top and bottom. Three one-inch pipes were welded through the wall of the sump about ten inches from the top and were fitted with valves inside the tank.

Valves

In the sump tank, ordinary stainless steel globe valves were used with all packing removed. Leakage around the stems was not large, and what leakage did occur dropped back into the sump. The valves were equipped with a linkage to connect with extension handles passing through oil cooled packing glands in the top of the sump tank. The purpose of the linkages was to allow for any misalignment of the valve and extension to absorb the rise and fall of the valve stem as the valve was operated.

One valve was connected with the outlet of the pump and controlled the flow rate through the system. Another valve controlled the flow back into the sump, and was used to back the alloy up into the flow rate catch tank when flow rate measurements were made.

The third valve was used in draining the system into the sump. It was supplemented by three valves outside the sump which were kept closed during operation of system. These were maintained at a low enough temperature to prevent excessive attack by the alloy on their asbestos packing.

Other Accessories on Sump Tank

In addition to the glands for the three valve stem extensions, the top of the sump tank supported the pump motor, a two inch pressure relief line equipped with a 250 psi rupture disc, the sump filling and emptying line with a pressure equalizing line, a thermocouple well, a ten probe level indicator unit, a pressure equalizing line from the catch tank which is used also for adding or removing gas from the atmosphere in the system, and a hand hole of approximately 100 square inches.

The Pump

This was a vertical shaft centrifugal unit which was built around the motor and liquid-cooled shaft from a "Gusher" liquid metal pump manufactured by the Ruthman Machinery Company of Cincinnati. The impellor was enlarged and changed from semi-closed with back-curved vanes to a completely closed impellor with radial vanes. Use of a closed impellor permitted the sealing clearances between high and low pressure liquid to be made between cylindrical surfaces parallel with the shaft axis, rather than between plane surfaces at right angles to the shaft. Thus the clearances could be considerably closer and greater allowance could be made for differential expansion of the uncooled pump casing support and the cooled shaft.

Radial vanes provided for a flatter head characteristic with flow rate, thus assuring maximum utilization of impellor diameter.

Flowmeter

A flowmeter for indicating the relative flow rate was installed on the inlet to the heat exchange annulus. It was an adaptation of the flowmeter reported by Kolin[15] and used the EMF produced by the liquid alloy flowing through a magnetic field. In this case a permanent Alnico V magnet was used across one-inch stainless steel (non magnetic) pipe, and 3/32" stainless steel welding rods, tack-welded on either side between the poles of the magnet, acted as electrodes. The D.C. voltage thus produced was indicated automatically by a slide wire potentiometer powered by a standard Brown-instrument 4 pole self balancing motor and a standard Brown phase-shifting amplifier. The resulting indicator had a sensitivity of about 2-3 microvolts, which was more than ample for our particular unit.

Heat Exchange Units

Results on four different heat exchange units will be reported here. The specifications for these are listed in Table IV. In Heat Exchanger A, a bellows expansion joint was provided between the two tubes and elaborate measures were taken to ensure good alignment of the inner and outer tubes. In the remaining three exchangers, no provision was made for differential expansion, and alignment was ensured by a few short bits of 1/16" welding rod welded upright on the surface of the inner tube. These appeared to center the tubes well without materially influencing the heat transfer.

The Heating Tank

This consisted of an 18 inch length of twelve inch series 40 iron pipe with 1 inch plate welded on top and bottom. The bottom was slightly inclined

TABLE IV

SPECIFICATIONS FOR EXPERIMENTAL HEAT EXCHANGERS

Heat Exchanger A

Material: Commercially pure nickel

Length: 48 inches;	Mounted vertically
Inner Tube	Outer Tube
O.D. - 0.500 inches	O.D. - 0.759 inches
I.D. - 0.432 inches	I.D. - 0.715 inches
Wall - 0.034 inches	Wall - 0.022 inches

Heat Exchanger B

Material: Commercially pure nickel

Length: 69 inches:	Mounted horizontally
Inner Tube	Outer Tube
O.D. - 0.757 inches	O.D. - 1.001 inches
I.D. - 0.703 inches	I.D. - 0.931 inches
Wall - 0.027 inches	Wall - 0.035 inches

Heat Exchanger C

Material: Commercially pure nickel

Length: 33 inches:	Mounted horizontally
Inner Tube	Outer Tube
O.D. - 0.500 inches	O.D. - 0.754 inches
I.D. - 0.434 inches	I.D. - 0.684 inches
Wall - 0.033 inches	Wall - 0.035 inches

Heat Exchanger D

Material: Commercially pure nickel

Length: 69 inches;	Mounted horizontally
Inner Tube	Outer Tube
O.D. - 0.500 inches	O.D. - 0.754 inches
I.D. - 0.434 inches	I.D. - 0.684 inches
Wall - 0.033 inches	Wall - 0.035 inches

to allow drainage. Alloy entered the bottom and was removed very close to the top. Heat was supplied by 45 U shaped Chromalox electric heaters with inconel sheaths rated at 1 KW each. For control of the heaters, they were divided into five individually controlled banks of nine heaters each. Each bank was wired as a delta with three heaters on a side to a three phase 440 volt power line. For fine control, the No. 5 bank was connected through three saturable core reactors. The controlling DC current for the three reactors was about 1 ampere total at about 30-40 volts. It was obtained by use of selenium rectifiers with a small variac. In this way about 40 watts was used to control sixteen to seventeen kilowatts. Total power available in the heating tank from the five banks was about 80 kilowatts.

Evaporator Cooler

The alloy was cooled in a vertical, double tube heat exchanger, in which water flowed as a falling film down the eight foot length of the inside 2 inch tube and the sodium-potassium alloy rose in the annulus formed by a three inch pipe. To minimize the explosion hazard from mixing of the alloy with water, the inner tube was of seamless stainless steel, and the steam which formed was removed at the bottom of the evaporator rather than the top. The top was closed by a 250 psi rupture disc. Drips from the evaporator were caught in a trough of sand which was drained at the far side. It is believed that even if a leak had occurred through the 1/8th inch seamless stainless steel wall, the possibility of serious explosion was remote, since the alloy would first have met the water as a thin film surrounded by an atmosphere of steam. Hence the hydrogen formed could not burn, and ample room was available for the sudden expansion of the water which would have vaporized. Water did not accumulate in the bottom of the evaporator, but dripped immediately onto the sand where it sank to the bottom of the trough. Any alloy dripping out of the evaporator would find no large accumulation of water, and presumably would be held on top of the sand by the crust of oxidation and hydration products which would form immediately. In addition, water to the evaporator could be stopped quickly by means of a solenoid valve. Differential expansion between the inner and outer tubes was absorbed by a large bellows expansion joint.

<u>The Catch Tank</u>

This consisted of a two foot length of twelve inch pipe, with a one inch flat plate welded on the top and bottom. The tank was divided into two sections by a baffle which was open at both top and bottom. The entering stream was fed into a side of the catch tank near the top and withdrawn at the lower end of the inclined bottom from where it was piped through wall of the sump tank and discharged from the outlet valve inside the sump. The top of the catch tank was equipped with a vent line to the top of the sump. Two probes were provided on the opposite side of the baffle from the inlet. These probes were of different lengths and were insulated from the tank. During operation they were charged with 110 volts through a relay system which operated a series of lights and a timer. Closing the outlet valve in the sump backed the alloy up into the catch tank. As the liquid metal rose it made contact with the longer probe and started the timer.

When the liquid made contact with the shorter probe it stopped the timer and sounded a buzzer. The outlet valve was then opened, and the catch tank was allowed to drain. The time required to fill the tank between probes was recorded and used to calculate the flow rate through the system.

Temperature Measurements

These were made with thermocouples of Leeds and Northrup glass covered #30 iron-constantan duplex wire which were read by means of a 12 point 0-200° C Brown electronic temperature indicator. A bucking potentiometer with voltage steps corresponding to about 100° C each was installed in series with the input to the indicator to increase its range to about 600° C. The thermocouples, instrument and potentiometers were all calibrated.

Auxiliaries

Auxiliaries for this test system included a cooling oil system, means for evacuating the circulating system and for maintaining at other times an atmosphere of argon at slight positive pressure. An attempt was made to install dial gages to measure the temperature of the inner tube of the exchanger by means of its thermal expansion. Very erratic results were obtained, however, and interpretation was found to be impossible so that this attempt was abandoned.

It was found that at low flow rates and at high temperatures, radiation from the main part of the circulating system was more than enough to remove the heat put in the heating tank. As a result a two-burner torch was made from 1 inch screwed pipe fittings and, when supplied with gas and compressed air, this was used to heat the sump tank.

The electrical controls for the pump, for the heaters, and for the cooling-water solenoid valve were interlocked, so that the heaters could not be turned on if the pump motor was not running, and the entire system could be stopped by a master switch at either end of the operating panel board.

The control panel included all of the operating controls and instruments. While this was provided primarily to protect the operator in case of a large leak in the circulating system, it also made the operation of the equipment remarkably easy.

Procedure

To start up this equipment, the following steps were taken:

1. Close auxiliary drain valves.
2. Turn on flowmeter, catch tank relay system, and temperature indicator.
3. Start cooling oil circulation.
4. Start alloy circulation.
5. Close outlet valve (Valve #3 in Figure 5).
6. Open outlet valve when buzzer sounds. (This indicated that the system had filled with alloy, and that heaters could be turned on. They burned out if not surrounded by alloy).
7. Turn on heaters and light burners under sump if required.

8. Adjust inlet valve (Valve #1 in Figure 5) heaters, burners, and water in evaporator cooler to provide operating conditions required.

Data were taken by going through the following steps after equilibrium was reached:

1. Close outlet valve.

2. Record temperatures 1, 4, 2, 3, and 6 in that order.

3. Open outlet valve when buzzer sounds.

4. Record time indicated by timer.

5. When light system indicated the level was below the lower probe in the catch tank, set timer to zero and reset relay system.

6. Adjust to new flow or heat conditions if desired.

Auxiliary data were also recorded from time to time. These include:

Flowmeter reading
Line voltage
Current flowing in each phase of heater power line
Cooling oil temperature from the various cooled portions of equipment
Temperature of pump stuffing box
Miscellaneous NaK temperatures in various parts of the system

Effectiveness of the Equipment

In general the equipment operated with excellent reliability. The most serious criticism of it is the low pressure output of the pump (about 20 feet of head).

For future operation at sump tank temperatures above about 250° C the equipment should be more completely insulated, since at present, the sump tank can be raised to this point only by using strong heating directly on it. An alternative is a more effective method of heating the sump, perhaps by means of resistance heaters as in the heating tank. It is interesting to observe that the liquid leaving the heating tank might be above 500° C (900° F) while the cooler side of the system heated very slowly. Because of the effectiveness of the heat exchanger between the hot and cold ends of the system, the heat remained bottled at the hot end. This was in spite of the fact that the heat exchanger usually contained less than one square foot of surface.

CHAPTER 6

EXPERIMENTAL RESULTS

Tests were run with sodium-potassium alloy in the four heat exchangers described in the preceding chapter at velocities ranging from about two feet per second to about twelve feet per second in the tubes, and from about six tenths of a foot per second to about four feet per second in the annulus. While greater flow rates would have been desirable, it was impossible to obtain them with the particular pump used in these tests.

Reynolds moduli ranged from about 15,000 to 90,000 in the tube and from about 8,000 to 40,000 in the annulus.

A complete tabulation of the experimental results will be found in the Appendix. The results are also plotted in Figures 6A, B, C, and D. These plots show the ratio of observed overall coefficient to the overall coefficient predicted by the use of Equation (34) for the tube side and of Equation (36) for the annulus side to which is added the resistance of the nickel wall of the inner tube of the heat exchanger.

Since efforts to measure the inner tube wall temperature failed, it is necessary to rely on both theoretical equations to compare the data with either one. Fortunately the agreement between the experiment and the theory, appears to be within the errors of the experiment and the uncertainty of the physical properties of the alloy.

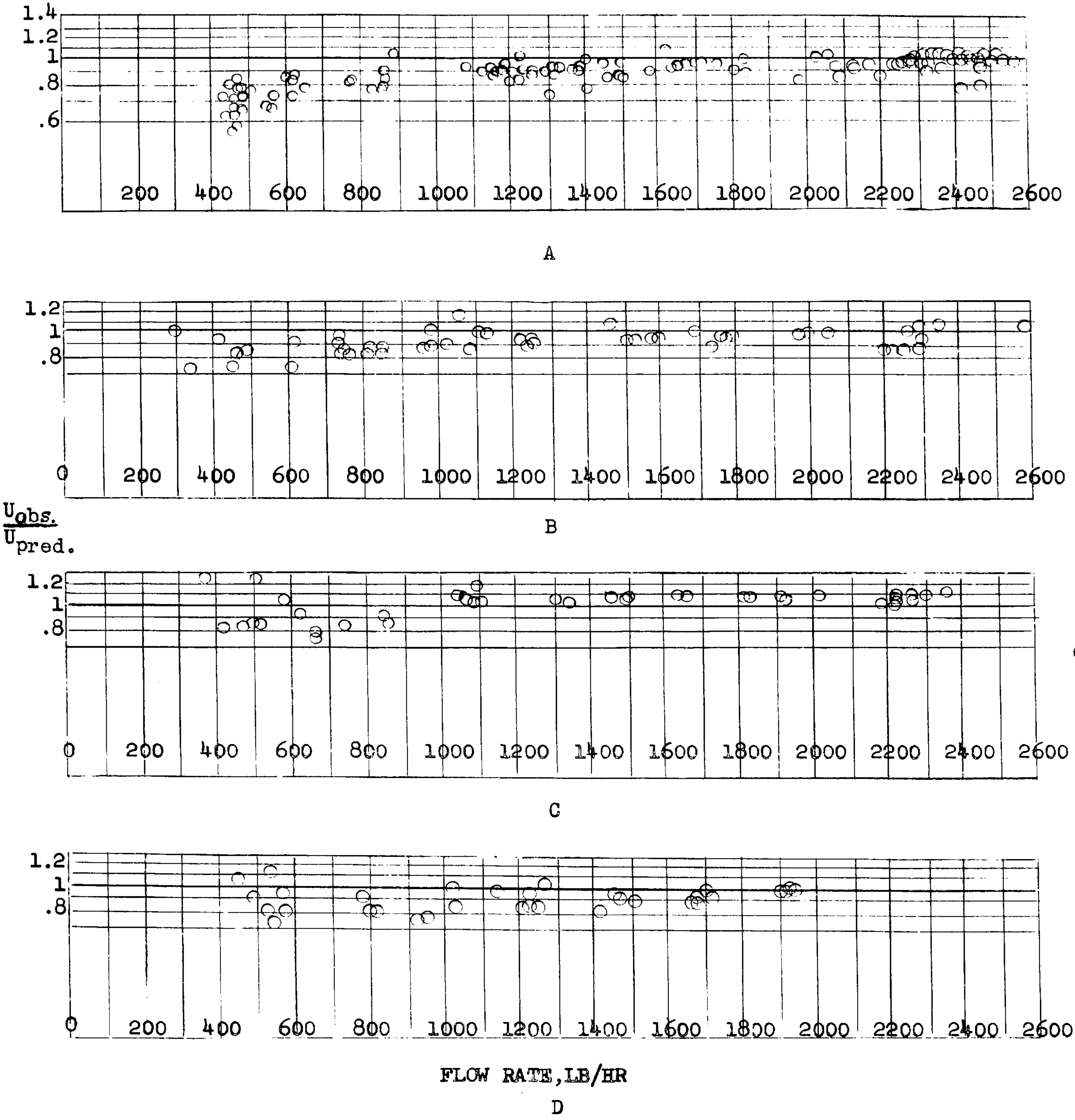

FIGURE 6

COMPARISON OF OBSERVED COEFFICIENT WITH

PREDICTED COEFFICIENT

Errors in the Experiments

Temperature measurements in heat transfer work are usually the subject of considerable suspicion. In view of the difficulties of accurate temperature measurement, particularly at high temperatures, the suspicion may be justified. Most important in these tests is the comparison of each thermocouple with the others. To accomplish this an extra thermocouple was connected to the indicator. This was then installed successively in each well with the indigenous thermocouple. Comparisons in the readings of the two thermocouples in no case amounted to more than half a degree centigrade. The indicating instrument was calibrated with a Leeds and Northrup type K potentiometer at the beginning of these tests and again near the end of the experimental period. The corrections required did not change appreciably during this period. The thermocouple wire was compared with Leeds and Northrup standardized wire, but no serious discrepancies appeared.

Ahead of each thermocouple, the pipe was passed around three or four right angle bends. The purpose of this was to provide for adequate mixing of the streams before the temperature was measured. To check the effectiveness of this measure, four thermocouples were brazed to the outside of each of the two outlet pipes at quarter points just opposite the thermocouples in the wells. The differences in temperature measured between the inside and outside thermocouples, and between the outside thermocouples themselves were recorded as the amount of heat transferred in the exchanger was increased and the temperature indicated by the inside thermocouple was held constant.

The results show essentially no change in these temperature differences over a range of heat transfer rates from zero to several times that recorded for experimental data. Hence we may conclude that proper mixing was taking place ahead of the thermocouples.

Heat balances were consistently off by about 2-3% in the same direction. At low flows this corresponds to a possible error in temperature difference between the two streams of as much as fifteen to twenty percent. This may account in part for the greater scattering of data and the general drift from the predicted values at low flows.

Reliable heat capacity and thermal conductivity values for sodium potassium alloys have not yet been published, hence it was necessary to use values which appeared to be the most probable average of existing information.

Flow rate measurements depended on an accurate knowledge of the catch tank cross section and of the distance between the probes. It also depended on the assumption that no droplets of liquid metal adhere to either probe. This last question could not be settled, however, because removal of the probes always caused sufficient jarring to dislodge adhering material. The flow rate timer was calibrated and found to be amply accurate.

Because of impurities in the system, it is quite possible that a scale may have formed on the tube surfaces. The impurities which were present were finely divided iron (possibly present as a ferrite) and the various oxides of sodium and potassium. In addition, sodium hydride, which Bennett[3] found particularly troublesome, was undoubtedly present.

It was observed that the results appeared to fluctuate, and that frequently the coefficients at low flows were considerably higher if taken immediately after runs at high flows. No real consistency in this regard was found, however.

As a result of all of these uncertainties, accuracy within 20-30% may not be claimed for these data. However, while the data cannot be interpreted as conclusively proving the theory, they certainly lend a large measure of support to it. Thus one may use Equation (34) for liquid metals in tubes and Equation (36) for liquid metals in large-diameter, narrow annuli, with reasonable confidence.

CHAPTER 7

DISCUSSION

The theory, with its approximate confirmation as presented in this paper, opens an interesting realm of speculation on the mechanics of heat transfer with fluids in turbulent, forced convection in tubes. While no attempt will be made here to explore this realm completely, a number of directions can be pointed out in which the prospect is particularly intriguing.

It appears that in all heat transfer involving fluids in tubes, some of the heat is transferred all the way into the main stream by molecular conduction, even though in most cases this effect is masked in the turbulent core by the greater heat conducting ability of the turbulence. The effect of the molecular conductance into the main stream may be expressed by the equation for the case where the molecular conductivity is very large:

$$Nu \doteq \text{approximately } 7$$

The actual value varies with changes in velocity profile from 4.36 for a parabolic profile, in the case of purely viscous flow, to 8 for the case of perfect slug flow. The value of 7 has been seen to be a reasonable average for the case of turbulent flow, however.

As the effect of eddy conductivity becomes more pronounced, the total heat transfer rate becomes more affected by this additional means of transporting heat which becomes pronounced, first in the central regions of the tube, and we must add some term to our equation to account for it. We have seen that an approximation of this new term is given by the expression: $0.025\,(Pe)^{.8}$.

Why the expression should be a function of the Peclet modulus becomes more clear if we think of it as written in the following equation:

$$(38) \qquad Pe = \frac{c\rho}{k} \cdot 2r_w\,u_m$$

The term $\frac{c\rho}{k}$ is recognized at once as the reciprocal of the molecular diffusivity of heat. The term $2\,r_w\,u_m$ also has the units of diffusivity, that is $(\text{length})^2$ per time, and we are brought to the conclusion that $2\,r_w\,u_m$, or the diameter times the mean velocity is proportional to the mean eddy diffusivity of the fluid in the tube. The Peclet modulus is then proportional to the ratio of the mean eddy diffusivity to the molecular diffusivity of heat, which we have assumed to be constant across the stream.

Another interesting point is disclosed by a brief glance at the Reynolds modulus, in the following equation:

$$(39) \qquad Re = \frac{\rho}{\mu} \cdot 2\,r_w\,u_m \simeq \frac{1}{\nu} \cdot 2\,r_w\,u_m$$

Here the Reynolds modulus is seen to be proportional to ratio of mean eddy diffusivity to molecular diffusivity of momentum.

It seems logical that to the term representing the heat transfer due to molecular conductivity of heat, we should add a term which is a function of the ratio between eddy diffusivity and molecular diffusivity of heat.

These two terms, one representing the effect due to molecular conductivity, and one representing the additional effect of eddying, appear to be sufficient for Prandtl moduli up to values close to unity. The similarity with the Reynolds analogy, Equation 1 has already been pointed out:

(1) $Nu = f/2\ Re =$ approximately $0.023\ Re^{.8}$

The agreement with Nusselt's empirical equation for gases, obtained in 1909[24], is startling:

(40) $Nu = 0.0255\ (Pe)^{.786}$

It will be observed that at Prandtl moduli less than unity, and at low velocities, hence low Peclet modulus, the molecular conductivity may be important well into the turbulent core and the Nusselt modulus may be close to 7; while for the same system and material at high velocity, the turbulence is great enough to carry most of the heat practically from the edge of the buffer region, just as in the case of Prandtl modulus unity. The only change which has been made is in the $2r_w\ u_m$ term and hence in the mean eddy diffusivity.

As the value of the Prandtl modulus is raised to unity, the region of dominance of turbulence as a means of transferring heat expands to the edge of the turbulent region; since at Prandtl modulus unity, the molecular diffusivity of heat is the same as that of momentum, and since we have seen that the eddy diffusivities of heat and momentum are essentially equal, under these conditions the region dominated by eddy conductivity of momentum would be dominated by eddy conductivity of heat, the region, near the wall, dominated by molecular conductivity of momentum would still be dominated by molecular conductivity of heat, and the temperature profile will coincide with the velocity profile.

At Prandtl modulus above unity, the dominance of eddy conductivity has been established in the turbulent core, and the molecular heat conductivity of the material is low enough that the critical resistance to heat flow is in the thin laminar and buffer regions near the tube walls.

The behavior of fluids near walls has been the object of very extensive studies, but the lack of agreement among investigators is typified by the fact that Karman[13] concludes that the true laminar region is two and one half times thicker than Reichardt[26] believes it to be.

Velocity distribution information except close to the center of the channel is well substantiated in the turbulent region, where it is important in predicting liquid metal heat transfer.

Fluid flow information close to the wall is difficult to obtain experimentally, and it becomes evident that heat transfer information may offer a means of studying fluid flow close to the wall, rather than attempting to use the present laminar and buffer region velocities to study heat transfer of ordinary fluids. Such an idea has been implied or expressed by previous investigators, and Reichardt has at least partially succeeded in learning more about the laminar region by this means.

The empirical Colburn equation:

$$(9) \quad Nu = 0.023\ (Re)^{.8}\ Pr^{1/3}$$

and similar equations appear to be the best means available at present for predicting heat transfer of ordinary fluids.

We will not attempt to predict whether this equation can be tied to Equation (34) to give a general approximation covering Pr values from zero to the limit of experiments. A theoretical utility may exist for such a combination, but for practical predictions of heat transfer it appears preferable to leave the equations separated and more simple--applying each to its own realm.

The more rigorous development provides impetus for further study as well. While Nikuradse's actual data have been used in computations in this paper, the use of Equation (33) with the generalized velocity distribution equations may produce a simpler analytical solution than that presented by Martinelli, since only two integrations will be involved instead of three.

The experimental work presented here is not conclusive. Additional work is needed, with liquid metals both in annuli and tubes. Velocity distribution data are also needed for further development of the fluid flow, heat transfer analogy.

It is hoped that the use of a nomenclature familiar to most chemical engineers will create more interest among them in this type of development; and that it will lead in this way to a better understanding not only of heat transfer, but to fluid flow and the diffusion of material as well.

CHAPTER 8

CONCLUSIONS

In the course of this study:

1) A general integral equation has been developed for heat transfer in ideal tubular system--Equation (33).

2) Numerical solutions of the equation for the case of liquid metals agree with solutions to Martinelli's equation and have led to an approximate equation for liquid metals in turbulent flow --Equation (34).

3) An approximate equation for liquid metals in annuli has been found, based on an extention of Martinelli's work by Harrison and Menke -- Equation (36)

4) Equipment for measuring heat transfer rates, using sodium-potassium alloy, has been designed and constructed.

5) Experimental data have been obtained which appear to confirm the approximations within the limits of the errors of the experiments and the uncertainty regarding the physical properties.

6) Brief consideration has been given to the physical implications of the theoretical and experimental results.

It may be concluded from the study that:

1) Liquid metals are excellent heat transfer media, requiring less area, velocity and temperature difference to transfer a given amount of heat to or from a solid surface.

2) Empirical equations based on experience with ordinary fluids cannot be used with assurance to predict heat transfer coefficients with liquid metals.

3) A relatively simple analytical approach can be used in the study of heat transfer for all materials over the entire range of Prandtl numbers.

4) Approximations for heat transfer for liquid metals in tubes and certain annuli, which are even easier to use than the empirical equations for ordinary fluids, may be employed with reasonable accuracy.

5) Heat transfer in liquid metals is relatively independent of the viscosity of the fluid, in turbulent flow, as long as the turbulent regime is well established.

6) The heat transfer coefficient with liquid metals is, in general, less sensitive to velocity changes of the fluid stream than with ordinary materials. The actual sensitivity is a function of the Peclet modulus, $\frac{2r_w u_m c\rho}{k}$. Hence the Wilson line approach cannot be used for correlating experimental data.

It may also be concluded that additional studies in this field are needed:

1) To establish values for the physical properties of liquid metals more accurately.

2) To obtain more accurate experimental heat transfer data, preferably on the individual coefficients, rather than the overall coefficient.

3) To determine the effect on heat transfer, if any, of non-wetting by liquid metals of heat exchanger walls.

4) To obtain velocity distribution data in annuli, with and without heat transfer*.

Such information is especially needed near the center of the stream to determine whether there is actually a reduction in the eddy diffusivity in this region as predicted by the mixing length theory.

* The author looks forward with interest to the results of work at the University of Michigan directed toward this end[14].

5) To apply the velocity distribution approximate equations of Reichardt, Nikuradse, and Karman to the general Equation (33) developed in this paper. Such an application may offer a method of simplifying the analytical results of Martinelli and Boelter et al.

6) To investigate still further the analytical approach to heat transfer, and to fluid flow and material transfer as well.

APPENDIX A

REDUCTION OF THE BASIC HEAT FLOW EQUATION TO ONE VARIABLE

The simplicity of the development in Chapter 3 is derived largely from the fact that only one independent variable need be considered, and hence that ordinary differential equations may be used instead of the partial differential equations usually associated with the solution of the equations based on the laws of heat flow.

The basic laws in general have four independent variables, three dimensions and time. These have been reduced to one by choosing a particular ideal system. It is important that this system be clearly understood to avoid misuse of the results obtained.

The basic equation for heat flow due only to conduction is the Fourier equation which is usually expressed:

$$\text{(A1)} \qquad \nabla^2 t = - \frac{c\rho}{k} \frac{\partial t}{\partial \lambda}$$

where λ represents time and ∇^2, the Laplacian operator.

The development of such an equation involves the assumption that the thermal conductivity, k, is a constant. If we use a variable conductivity K , we must write instead:

$$\text{(A2)} \qquad \text{div}\,(K\, \underline{\nabla t}) = -c\rho \frac{\partial t}{\partial \lambda}$$

where the underlined term is a vector quantity.

Such an equation applies if we are considering a small element of matter and if we assume that heat is transferred in or out only by the conductivity K .

When, as was done in Chapter 3, we are considering an element of space through which matter is passing, equations (A1) and (A2) must be modified to account for the change in sensible heat of the matter as it passes through:

$$\text{(A3)} \qquad -\operatorname{div}(K\ \underline{\nabla t}) = c\rho\frac{\partial t}{\partial \lambda} + c\rho\ \underline{u}\cdot\underline{\nabla t}$$

Equation (A3) is a fundamental equation for convective heat transfer.

The first term on the right hand side of Equation (A3) drops out in our development since we specify that there is no change with time.

By considering a tube whose axis coincides with the z axis of our coordinate system, the term $\underline{u} \cdot \underline{\nabla t}$ becomes $u \dfrac{\partial t}{\partial z}$.

Writing out the equation with these changes, we obtain:

$$\text{(A4)} \qquad \frac{\partial K \partial t}{\partial x^2} + \frac{\partial K \partial t}{\partial y^2} + \frac{\partial K \partial t}{\partial z^2} = c\rho u \frac{\partial t}{\partial z}$$

One of the conditions of our ideal system is that it is far enough inside the ends of the exchanger to ensure that the temperature profile of our system is established*.

* A study of the case where the temperature profile has not been established is presented for viscous flow by Norris and Streid[23].

We have also specified that the heat transferred through the wall is uniform through the wall regardless of the position along it, in the z direction. By specifying that the physical properties of the fluid remain constant we complete the conditions which force $\frac{\partial t}{\partial z}$ to remain constant regardless of x, y, or z.

If we also specify that K be independent of z, we find that $\frac{\partial K \partial t}{\partial z^2}$ is equal to zero, and we have essentially reduced our basic equation to but two independent variables in the equation:

$$\text{(A5)} \quad \frac{\partial (K \partial t)}{\partial x^2} + \frac{\partial (K \partial t)}{\partial y^2} = c \rho u \frac{dt}{dz}$$

In this equation the system is described in terms of two rectangular coordinates. We have made the specification that when the system is expressed in terms of polar coordinates, r and θ, it is independent of θ. This enables us to write.

$$\text{(A6)} \quad \frac{d(Krdt)}{dr^2} = c \rho ur \frac{dt}{dz}$$

in which K , t and u are functions of r, the only independent variable.

Equation (A6) is the basic differential equation which is solved in Chapter 3.

APPENDIX B

CALCULATION OF THE PREDICTED VALUES OF NUSSELT MODULUS FOR LIQUID METALS

Equation (33), given in Chapter 3 of this report, is used in these calculations.

$$\text{(33)}\quad \text{(B1)}\qquad \frac{1}{Nu} = 2\int_0^1 \frac{\left[\int_0^{S_q} V_R\, S\, dS\right]^2}{S_q\left(1 + \frac{\epsilon_M}{\nu}(\alpha Pr)\right)}\, d\,S_q$$

The stages of the calculations at four different Reynolds moduli are shown in Tables V-a— V-d.

The values of S are the values of $\frac{r}{r_w}$ at which each velocity is measured. These are listed in Nikuradse's paper in terms of distance from the wall, y, hence S is found to be $\left(1 - \frac{y}{r_w}\right)$. Values of S are given in Column #1 of the calculation tables.

The relative velocity V, is found by dividing the actual velocity by the average velocity as given by Nikuradse. The values of V are listed in Column #2 of the calculation sheets.

The product of V times S is listed in Column #3 and plotted for the four Reynolds moduli in Figure 7.

Column #4 lists the values of $\int_0^{S_q}$ V S dS for various values of S_q as listed in Column #1. The integrations were carried out by a modified mean-ordinate method using plots similar to those in Figure 7.

In the next column, Column #5, are listed values of $\dfrac{\left[\int_o^{S_q} V\, S\, dS\right]^2}{S_q}$ obtained by squaring the values in Column #4 and dividing by the corresponding values in Column #1.

Column #6 contains values of $\dfrac{\epsilon_M}{\nu} + 1$. These are obtained from Nikuradse's report as values of "$\dfrac{\epsilon}{v^* r}$".

In our nomenclature:

$$\text{(B2)} \quad "\frac{\epsilon}{v^* r}" = \frac{\epsilon_M + \nu}{v^* r_w} = \left(\frac{\epsilon_M}{\nu} + 1\right)\frac{u_m}{Re\ v^*}$$

Hence $\dfrac{\epsilon_M}{\nu} + 1$ is obtained by multiplying "$\dfrac{\epsilon}{v^* r}$" by the term $\dfrac{Re\ v^*}{u_m}$, where v^* is "friction velocity" defined in the nomenclature at the end of this report and listed by Nikuradse.

The quotient:

$$\frac{\int_o^{S_q} V\, S\, dS}{S\left[1 + \frac{\epsilon_M}{\nu}(\alpha Pr)\right]}$$

is listed for values of αPr of 0.1, 0.01, and 0.001 in Columns #7, #8, and #9 respectively.

The term $\left[1 + \dfrac{\epsilon_M}{\nu}(\alpha Pr)\right]$ is obtained by substracting 1 from Column #6, adding $\dfrac{1}{(\alpha Pr)}$, and then multiplying the result by (αPr).

$$\left\{\left[\left(\frac{\epsilon_M}{\nu} + 1\right) - 1\right] + \frac{1}{\alpha Pr}\right\} \alpha Pr$$

Since values of (α Pr) have been chosen which are integral negative powers of 10, the manipulation can be performed mentally.

In Figures 8, 9, 10, and 11 the values listed in Columns #5, #7, #8, and #9 are plotted for the various Reynolds moduli considered.

Column #5 corresponds to the case of (α Pr) = 0 which would arise with very high conductivity fluid. This case has already been mentioned in the main body of this report.

$$\text{(B3)} \quad \frac{1}{Nu_{Pr}} = 2 \int_0^1 \frac{\left[\int_0^{S_q} V \; SdS\right]^2}{S_q} \, d\,S_q$$

Integration of curves similar to those in Figures 8, 9, 10 and 11, gives values of $\frac{0.5}{Nu}$ at the Prandtl and Reynolds numbers chosen. In a number of cases it is desirable to expand the right hand side of the curve to give better accuracy. Approximate interpolation can be obtained by using the equations of Karman[13] and Nikuradse[20] :

from $y^+ = 0$ to $y^+ = 5$

(B4) $u^+ = y^+$; $\quad \frac{\epsilon_M}{\nu} + 1 = 1$

from $y^+ = 5$ to $y^+ = 30$

(B5) $u^+ = 3.05 + 5 \ln_e y^+$

(B6) $\frac{\epsilon_M}{\nu} + 1 = .2 y^+ S$

above $y^+ = 30$

(B7) $u^+ = 5.5 + 2.5 \ln_e y^+$

(B8) $\frac{\epsilon_M}{\nu} + 1 = .4 y^+ S$

The term y^+ is Nikuradse's dimensionless wall distance and u^+ is his dimensionless velocity.

(B9) $y^+ = \frac{y}{\nu} v^* = \frac{y}{r_w} \frac{Re}{2} \frac{v^*}{u_m}$

(B10) $u^+ = \frac{u}{v^*}$

The results of the integrations are given in Table V-e, together with the resulting value of the Nusselt modulus.

TABLE V-a

CALCULATION SHEET

FOR

Re = 4000

u_m = 54.5 cm/sec ; v^* = 3.82 cm/sec ; r_w = 0.5 cm

S or S_q	V	V S	$\int_0^{S_q} V\,S\,dS$	$\frac{\left[\int_0^{S_q}\right]^2}{S}$	$\frac{\epsilon_M}{\nu} + 1$	$\left[\int_0^{q} VSdS\right]^2 \frac{k}{S\,K}$ for Pr: 0.1	0.01	0.001
#1	#2	#3	#4	#5	#6	#7	#8	#9
1.00	(.370)	.370	.4965	.2465	1	.2465	.2465	.2465
.99	.514	.509	.4920	.2445	-	.2445	.2445	.2445
.98	.642	.629	.4861	.2411	1.68	.2257	.2395	.2409
.96	.730	.701	.4727	.2328	3.05	.1932	.2281	.2323
.93	.815	.758	.4507	.2184	4.78	.1585	.2104	.2176
.90	.862	.776	.4277	.2033	6.18	.1349	.1933	.2022
.85	.921	.783	.3887	.1778	8.27	.1030	.1657	.1765
.80	.967	.774	.3505	.1536	9.62	.0825	.1414	.1523
.70	1.039	.727	.2780	.1104	11.81	.0531	.0996	.1092
.60	1.092	.655	.2088	.0727	12.95	.0331	.0649	.0718
.50	1.132	.566	.1476	.0436	13.19	.0196	.0389	.0431
.40	1.165	.466	.0960	.0230	12.73	.0106	.0206	.0227
.30	1.193	.358	.0547	.0100	11.60	.0049	.0090	.0099
.20	1.218	.244	.0245	.0030	9.90	.0016	.0028	.0030
.10	1.237	.124	.0062	.0004	7.10	.0002	.0004	.0004
.04	1.246	.050	-	-	4.55	-	-	-
.02	1.248	.026	-	-	-	-	-	-
.00	1.250	.000	.000	-	-	-	-	-

TABLE V-b

CALCULATION SHEET

FOR

Re = 43,400

u_m = 258.2 cm/sec ; v* = 3.82 cm/sec ; r_w = 1.0 cm

S or S_q	V	V S	$\int_o^{S_q} V\,Sd_S$	$\frac{\left[\int_o^{S_q}\right]^2}{S}$	$\frac{\epsilon_M}{\nu}+1$	$\left[\int_o^q VSdS\right]^2 \frac{k}{\bar{S}K}$ for Pr. 0.1	0.01	0.001
#1	#2	#3	#4	#5	#6	#7	#8	#9
1.00	.395	.395	.5022	.2522	0.0	.2522	.2522	.2522
.99	.600	.594	.4971	.2496	2.3	.2209	.2463	.2492
.98	.709	.695	.4906	.2455	4.5	.1822	.2372	.2447
.96	.786	.755	.4760	.2360	18.1	.0871	.2015	.2320
.93	.852	.792	.4526	.2203	31.6	.0543	.1686	.2137
.90	.891	.802	.4287	.2041	45.3	.0376	.1415	.1955
.85	.939	.798	.3886	.1776	58.7	.0262	.1126	.1679
.80	.976	.781	.3490	.1523	67.1	.0200	.0917	.1430
.70	1.034	.724	.2735	.1068	82.4	.0117	.0589	.0988
.60	1.078	.647	.2047	.0698	92.1	.0076	.0365	.0640
.50	1.111	.556	.1444	.0417	93.9	.0041	.0216	.0382
.40	1.140	.456	.0936	.0219	89.8	.0022	.0116	.0201
.30	1.162	.349	.0532	.0094	83.0	.0010	.0052	.0087
.20	1.183	.237	.0238	.0028	69.1	.0004	.0017	.0026
.10	1.195	.120	.0060	.0004	50.8	.0001	.0002	.0004
.04	1.199	.048	.0009	-	33.0	-	-	-
.02	1.200	.024	.0002	-	22.8	-	-	-
.00	1.202	.000	-	-	0.0	-	-	-

TABLE V-c

CALCULATION SHEET

FOR

Re = 396,000

u_m = 732 cm/sec ; v^* = 30.4 cm/sec ; r_w = 2.5 cm

S or S_q	V	V S	$\int_0^{S_q} V\, S d_S$	$\left[\int_0^{S_q}\right]^2 / S$	$\frac{\epsilon_M}{\nu}+1$	$\left[\int_0^q VSdS\right]^2 \frac{k}{S\,K}$ for Pr:		
						0.1	0.01	0.001
#1	#2	#3	#4	#5	#6	#7	#8	#9
1.00	.492	.492	.499	.2492	1	.2492	.2492	.2492
.99	.669	.662	.493	.2459	32.0	.0592	.1889	.2385
.98	.746	.731	.486	.2413	66.6	.03191	.1457	.2264
.96	.816	.783	.471	.2311	126.5	.01705	.1025	.2053
.93	.863	.803	.447	.2147	206.2	.00097	.07026	.1782
.90	.902	.812	.423	.1987	277.7	.00634	.05274	.1557
.85	.943	.802	.382	.1720	377.2	.00446	.03613	.1250
.80	.975	.780	.343	.1469	458.5	.00314	.02635	.1008
.70	1.036	.725	.268	.1026	568.6	.00178	.01538	.06588
.60	1.063	.638	.200	.0666	649.0	.00101	.00890	.04043
.50	1.092	.546	.141	.0396	658.0	.00006	.00523	.02390
.40	1.115	.446	.0909	.0207	630.2	.00032	.00284	.01269
.30	1.135	.341	.0515	.0088	583.4	.00015	.00129	.00555
.20	1.150	.230	.0230	.0027	489.7	.00005	.00045	.00177
.10	1.163	.116	.0058	.0003	357.4	.00001	.00007	.00025
.04	1.167	.047	.0010	-	228.4	-	-	.00002
.02	1.168	.023	.0002	-	160.4	-	-	-
.00	1.169	.000	.0000	.0000	-	.00000	.00000	-

TABLE V-d

CALCULATION SHEET

FOR

Re = 3,240,000

u_m = 2430 cm/sec ; v^* = 83.1 cm/sec ; r_w = 5.0 cm

S or S_q	V	V S	$\int_0^{S_q} V\, Sd_S$	$\left[\int_0^{S_q}\right]^2 / S$	$\frac{\epsilon_{M+1}}{\nu}$	$\left[\int_0^q VSdS\right]^2 \frac{k}{S\,K}$ for Pr. 0.1	0.01	0.001
#1	#2	#3	#4	#5	#6	#7	#8	#9
1.00	.621	.621	.5004	.2505	1	.2505	.2505	.2505
.99	.738	.731	.4937	.2463	222	.01062	.07650	.2016
.98	.778	.762	.4863	.2414	432	.00546	.04536	.1686
.96	.834	.801	.4706	.2308	788	.00293	.02600	.1291
.93	.881	.819	.4463	.2142	1349	.00159	.01479	.0911
.90	.918	.826	.4216	.1975	1782	.00111	.01037	.0710
.85	.957	.813	.3806	.1705	2450	.00069	.00668	.0494
.80	.986	.789	.3405	.1451	2997	.00048	.00468	.0363
.70	1.026	.718	.2651	.1004	3868	.00026	.00253	.0206
.60	1.056	.634	.1974	.0650	4301	.00015	.00147	.0123
.50	1.079	.540	.1386	.0385	4423	.00009	.00090	.0071
.40	1.099	.440	.0895	.0200	4246	.00005	.00046	.0039
.30	1.114	.334	.0508	.0086	3963	.00002	.00021	.0018
.20	1.126	.225	.0228	.0026	3302	.00001	.00008	.0006
.10	1.134	.113	.0057	.0003	2370	-	.00001	.0001
.04	1.137	.045	.0009	-	1554	-	-	-
.02	1.137	.023	.0001	-	1093	-	-	-
.00	1.138	.000	.0000	-	-	-	-	-

TABLE V-e

RESULTS OF CALCULATIONS

Re	Pr	$\frac{1}{2\,Nu}$ from Integration	Nu from Integration	Nu from Approx. Eq.
4.0×10^3	0.000	.0741	6.75	7.0
	0.001	.07395	6.76	7.0
	0.01	.06905	7.41	7.47
	0.1	.04533	11.03	10.0
4.34×10^4	0.000	.0732	6.83	7.0
	0.001	.0685	7.30	7.51
	0.01	.0486	10.3	10.2
	0.1	.0164	30.5	27.3
3.96×10^5	000	.0709	7.05	7.0
	0.001	.0524	9.54	9.83
	.01	.0189	26.5	25.90
	0.1	.0379	132.	127
3.24×10^6	000	.0697	7.17	7.0
	0.001	.0240	20.8	21.6
	0.01	.0050	100	106
	0.1		613.4	633

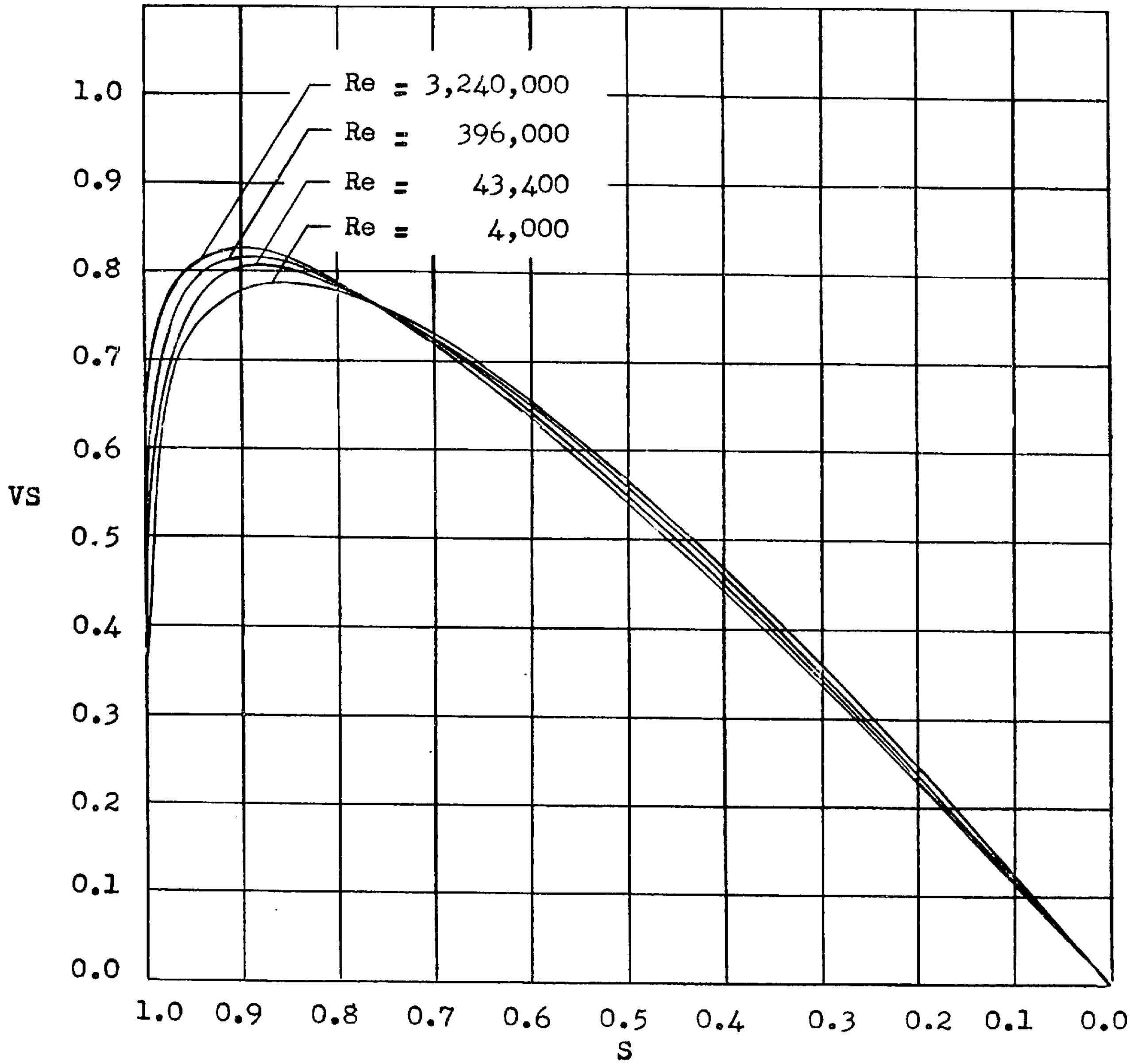

FIGURE 7

CURVES USED IN FIRST INTEGRATION
VS vs S

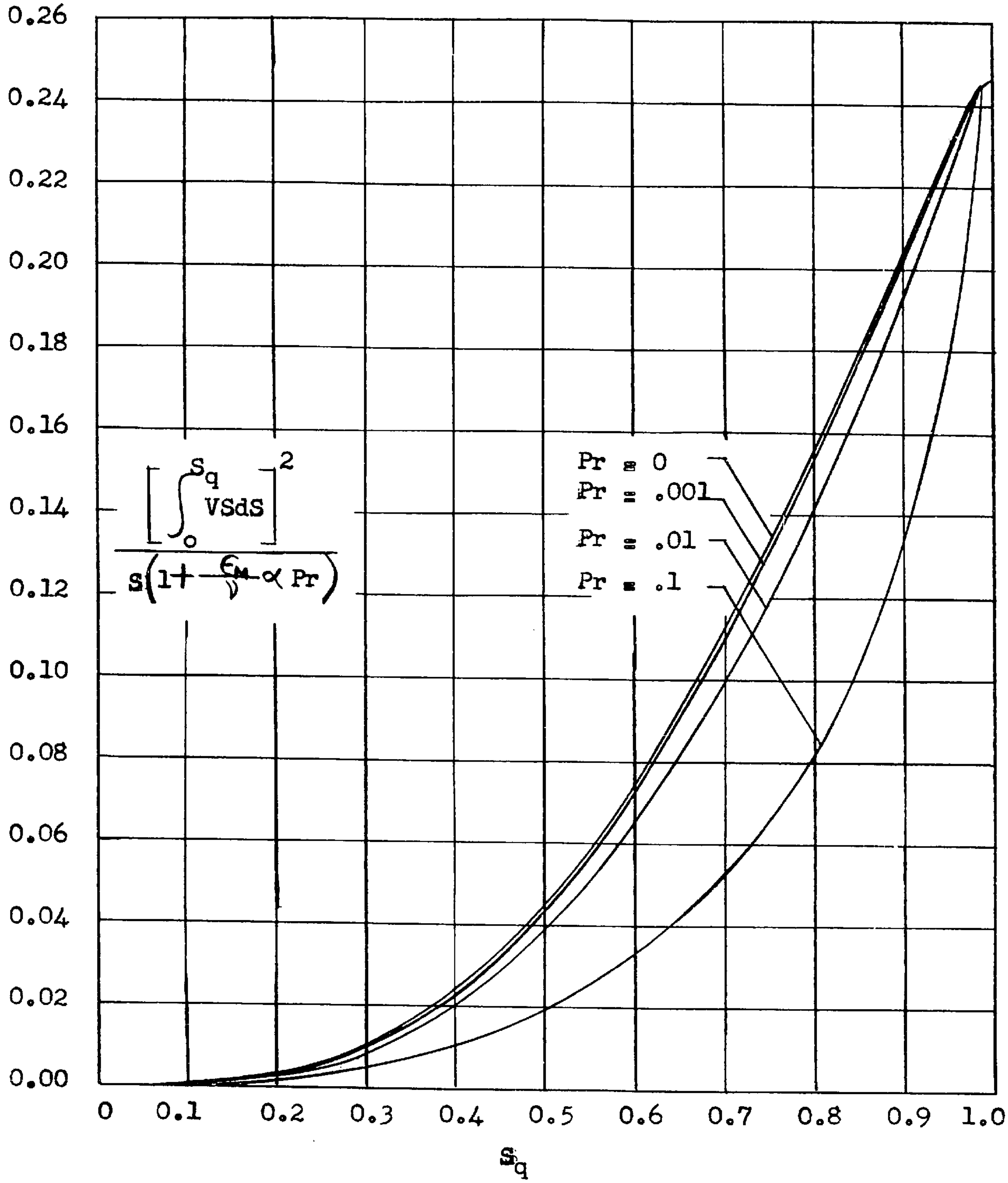

FIGURE 8

CURVE FOR SECOND INTEGRATION
WITH
RE = 4000

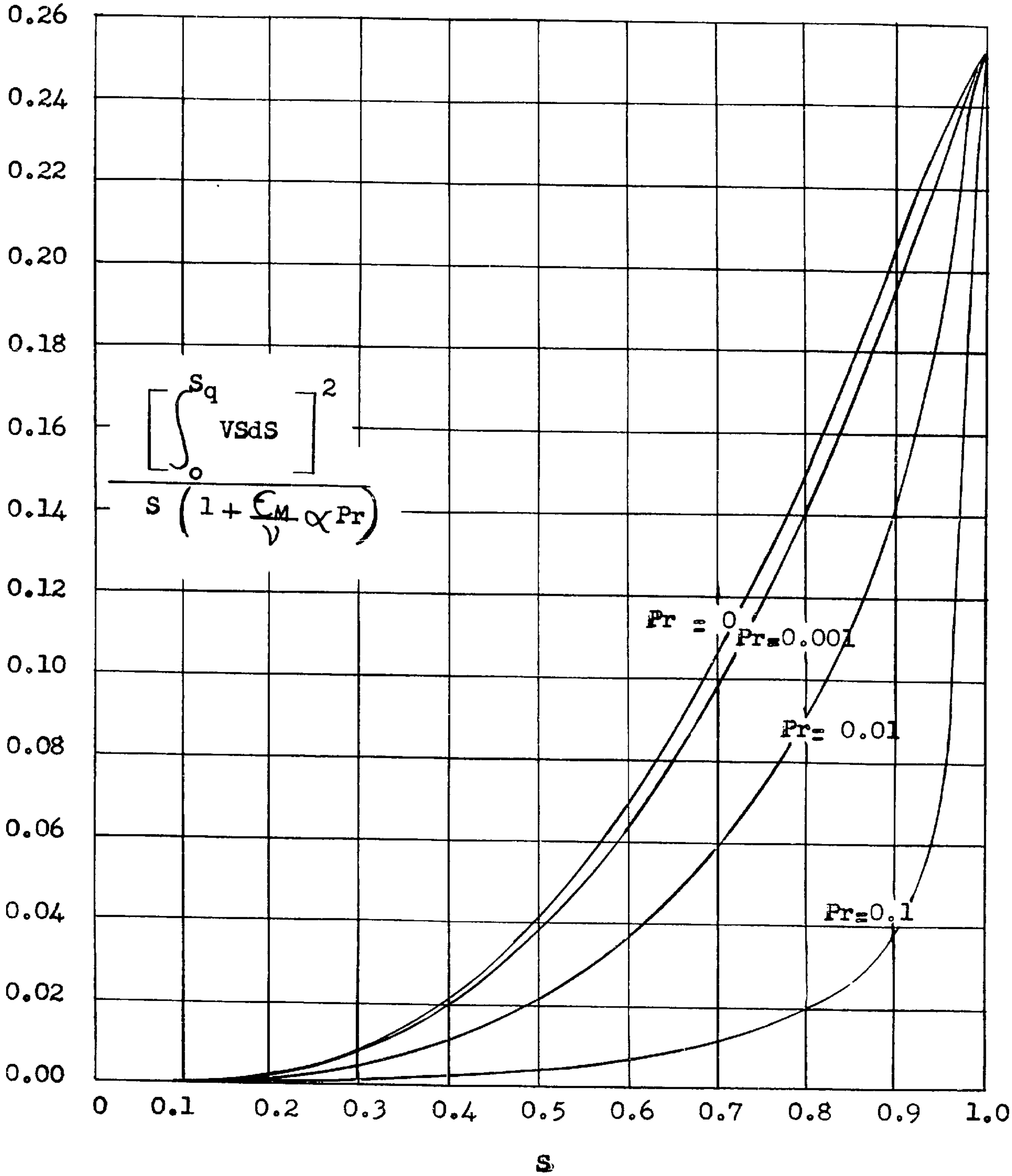

FIGURE 9

CURVES FOR SECOND INTEGRATION
AT
RE = 43,400

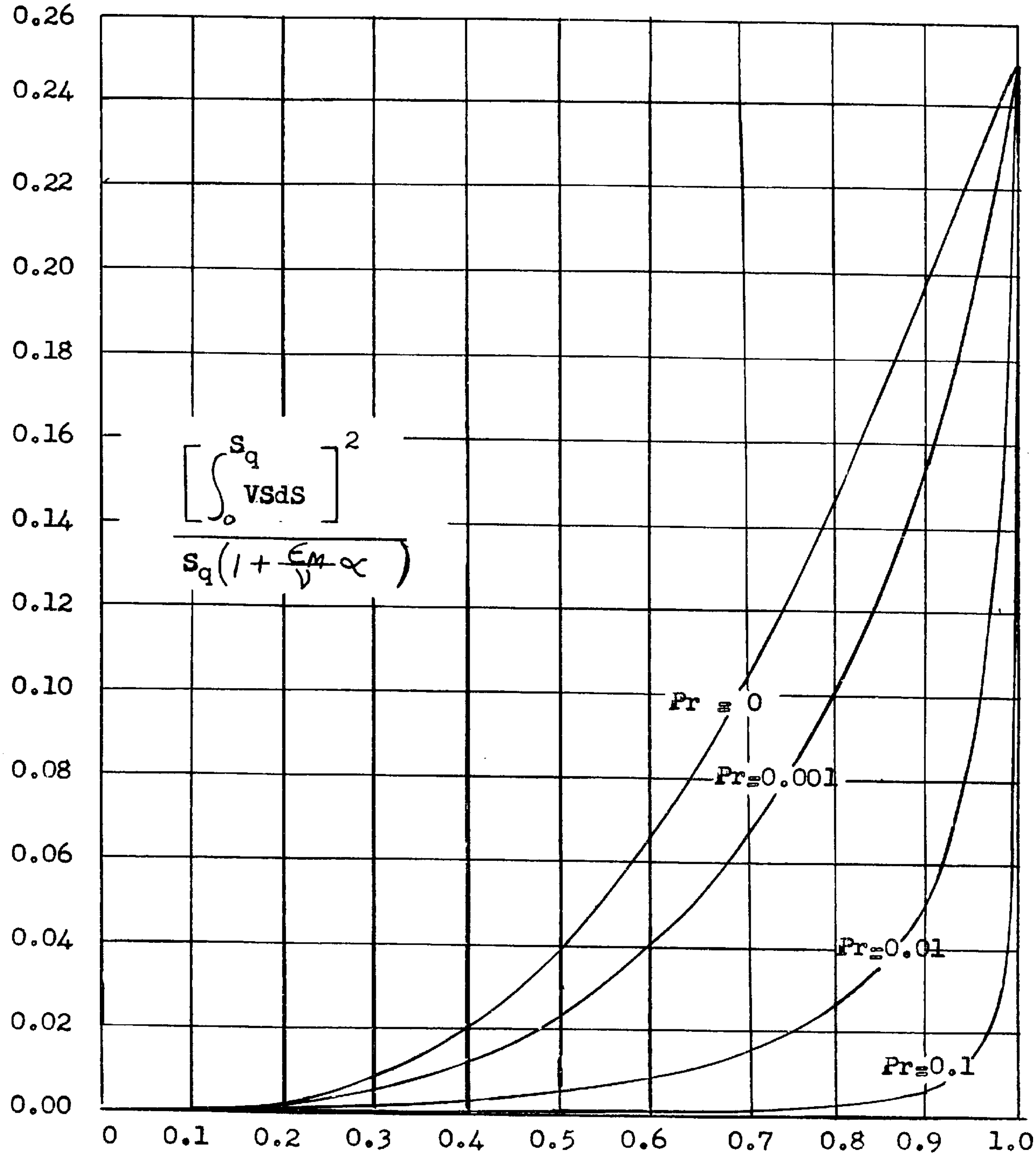

FIGURE 10

CURVES FOR SECOND INTEGRATION
FOR
RE = 396,000

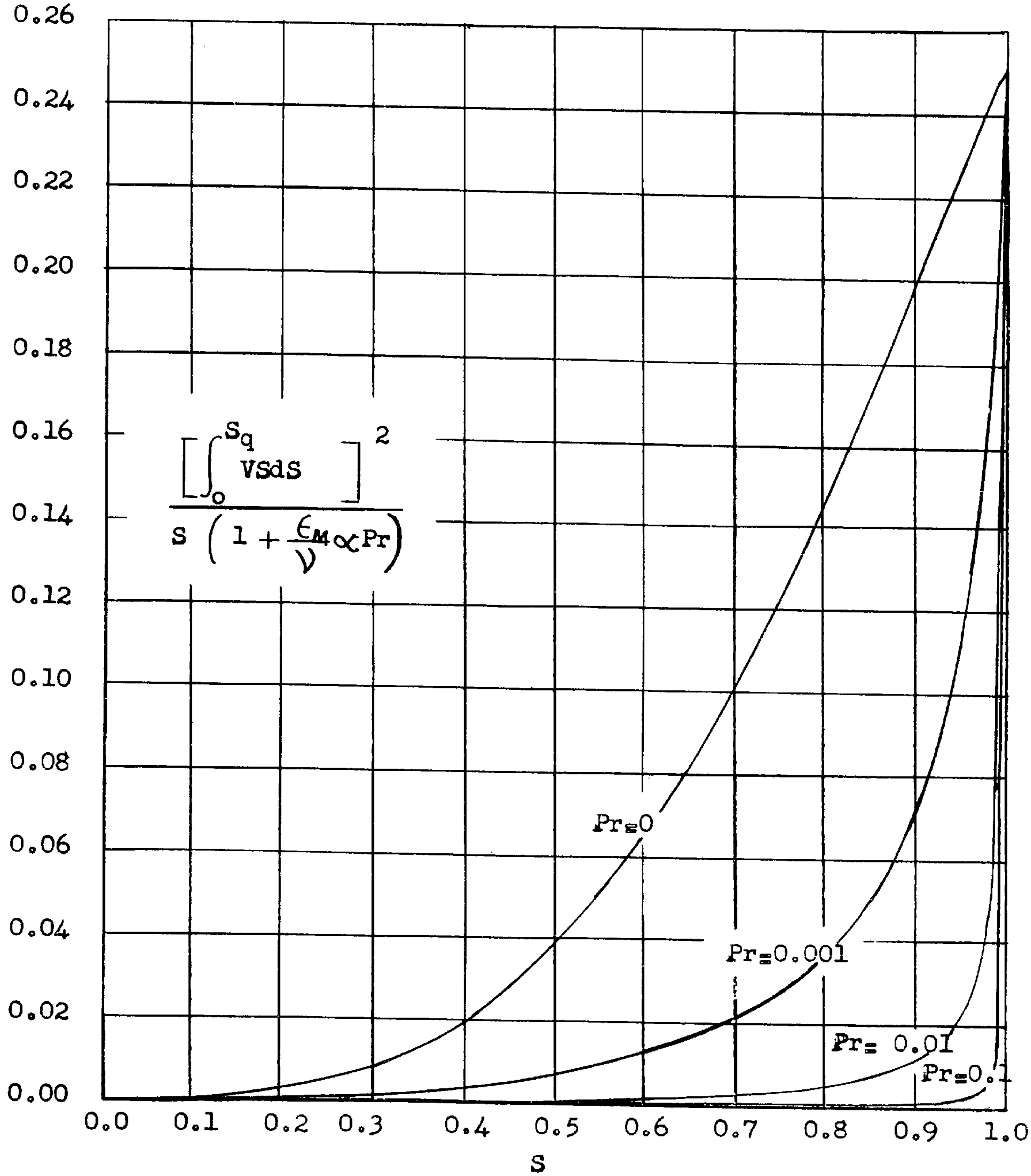

FIGURE 11

CURVES FOR SECOND INTEGRATION
AT
RE = 3,240,000

APPENDIX C

SLUG FLOW AND VISCOUS FLOW

As mentioned in Page 18 of Chapter 3 Equation (25) may be solved easily when the total conductivity of the entire stream is due to molecular conductivity, and where the relative velocity, V, is a simple function of the relative distance from the center, S. Since in this case $\frac{K}{k}$ becomes unity, Equation (25) may be written:

$$\text{(C1)} \qquad \frac{1}{Nu} = 2 \int_o^1 \frac{\left[\int_o^{S_q} V \, S dS\right]^2}{S_q} dS_q$$

Slug Flow

In the case of slug flow, the relative velocity, V, is unity at all points in the system, and Equation (C1) becomes:

$$\text{(C2)} \qquad \frac{1}{Nu} = 2 \int_o^1 \frac{\left[\int_o^{S_q} S dS\right]^2}{S_q} dS_q$$

Solution of this equation proceeds easily:

$$\text{(C3)} \qquad \frac{1}{Nu} = \int_o^1 \frac{S_q^{\,4}}{S_q} dS_q = \frac{1}{8}$$

or

$$\text{(C4)} \qquad Nu = 8$$

<u>Viscous Flow</u>

Viscous flow is characterized by a parabolic velocity profile:

(C5) $(u_{\text{℄}} - u) = br^2$

where $u_{\text{℄}}$ represents the velocity at the center of the tube and b is a constant for the particular system and defined in the equation

(C6) $u_{\text{℄}} = b\, r_w^2$ or $b = \dfrac{u_{\text{℄}}}{r_w^2}$

The mean velocity is found by solving the equation:

(C7) $$u_m = \frac{2\pi \int_0^{r_w} \left[u_{\text{℄}} - (u_{\text{℄}} - u)\right] r\, dr}{2\pi \int_0^1 r\, dr}$$

This is done by the following steps:

(C8) $$u_m = \frac{2\pi \int_0^{r_w} \left[u_{\text{℄}} - b\, r^2\right] r\, dr}{2\pi \int_0^1 r\, dr}$$

(C9) $$u_m = \frac{u_{\text{℄}}\; r^2_w - 1/2\, b\, r^4_w}{r^2_w} = u_{\text{℄}} - 1/2\, b\, r^2_w$$

or

(C10) $u_m = u_{\text{℄}} - 1/2\, u = 1/2\, u_{\text{℄}}$

The relative velocity, V, becomes

(C11) $$V = \frac{u}{u_m} \quad \frac{2u}{u_{\text{℄}}} = \frac{2(u_{\text{℄}} - b\, r^2)}{u_{\text{℄}}} = 2 - 2S^2$$

(C12) $$\frac{1}{Nu} = 2\int_0^1 \frac{\left[\int_0^{S_q}(2 - 2S^2)\, S\, dS\right]^2}{S_q}\, d\,S_q$$

Solution of Equation (C12) gives

(C13) $$\frac{1}{Nu} = 2\int_0^1 \frac{\left[S_q^{\ 2} - S_q^{\ 4}/2\right]^2}{S_q}\, d\,S_q =$$

(C15) $$\frac{1}{Nu} = 2\int_0^1 \left(S_q^{\ 3} - S_q^{\ 5} + S_q^{\ 7}/4\right) d\,S_q$$

(C16) $$\frac{1}{Nu} = \frac{1}{2} - \frac{1}{3} + \frac{1}{16} = \frac{24 - 16 + 3}{48} = \frac{11}{48}$$

(C17) $$Nu = \frac{48}{11} = 4.36$$

APPENDIX D

EXPERIMENTAL RESULTS AND CALCULATIONS

Physical Properties

In the comparison of the experimental results with the theoretical predictions from Equations (34) and (36), values for the following physical properties are required:

k, the thermal conductivity

c, the heat capacity

ρ, the density

The viscosity is not required, since it does not occur in either equation, and it is not used in calculations with the experimental data.

Of the three properties, the density appears to be the most accurately known. A compilation of known information on various compositions of sodium-potassium alloy is listed by Ewing, Atkinson, and Rice[10]. The agreement among investigators appears to be good. The values for the alloy in question are shown in Figure 12.

The heat capacity information for all sodium-potassium alloys appears to be conflicting and meager. Some data are presented by Ewing and Hartman[11] which emphasize the uncertainty. An average of their data for the composition involved and over the temperature range in which the heat transfer equipment was operated is 0.292 cal/gm °C or BTU/lb. °F. Hence this value was used in all calculations.

The thermal conductivity has been reported by Deem and Russell[7]. An approximate average of their results over the range of operation is 16.6 BTU/(hr)(ft^2)(°F/ft). This value was used in all calculations.

Results and Calculations

The experimental heat transfer results and calculations are tabulated in Table VI. Column #1 lists the run serial number and a letter which indicates the heat exchanger used for that particular run as described in Table IV of Chapter 5.

Columns #2, #3, #4, #5, and #6 of Table VI list temperatures T_1, T_2, T_3, T_4, and T_6 respectively in degrees Centigrade. The temperatures as listed have been corrected for errors in the temperature recorder as calibrated, and for the effect of the bucking potentiometer which was also calibrated.

In Column #7 is listed the time required to fill the catch tank between the two probes, in seconds.

Column #8 contains the corresponding values of the flow rate in pounds per hour. This is obtained by multiplying the density of the alloy in pounds per cubic foot at temperature T_6 by .364, the volume in cubic feet of between the probes, times 3600, the number of seconds in an hour and dividing the result by the time to fill between the probes in seconds.

The heat flux, listed in Column #9 is obtained by multiplying the heat capacity, 0.292 by the average temperature change of the two streams, times the flow rate, and divided by the inside area of the inner tube*.

The observed overall coefficient listed in Column #10 was obtained by dividing the flux by the average temperature difference between the streams. Since the same amount of liquid was flowing on each side of the heat exchanger, and since it is assumed that the heat capacity does not vary, it must be assumed that variations in temperature at one end from that at the other must be due to errors in temperature measurements or to heat losses in the two streams between the points of temperature measurement. On this basis there is no justification for using a log mean temperature difference.

To determine the predicted coefficient the individual coefficients were calculated for the particular tube and annulus using Equations (34) and (36) respectively. The coefficient in the annulus was then corrected to the area of the inside wall by multiplying by the ratio of outer to inner diameter of the inner tube. The reciprocals of the coefficients were then added together with 8×10^{-5} which was found to be the approximate value for the resistance of the wall using an average of the information

* All fluxes and coefficients are based on inside area of the inner tube.

given by McAdams[17], The Driver Harris Company[8] and the Metals Handbook[1]. In most cases the resistance of the metal wall represented about ten to thirty percent of the total resistance.

The last column, Column #12, in Table VI lists the ratio of observed to predicted coefficients.

Sample Calculation: Run 203-B

$T_1 = 125^\circ$ C; $T_2 = 257^\circ$ C; $T_3 = 300^\circ$ C; $T_4 = 166^\circ$ C; $T_6 = 139^\circ$ C

Density at 139° C = .873 x 62.4 = 54.4 lb/cu. ft.

Time = 31.5 seconds

$$W \text{ or Flow Rate} = \frac{0.364}{31.5} \times 54.4 \times 3600 = 2260 \text{ lb/hr.}$$

$$\text{Area of tube} = \frac{0.703\pi \times 69}{144} = 1.06 \text{ sq. ft.}$$

$$\text{Average Temp. change} = \frac{257 - 125 + 300 - 166}{2} \times 1.8 = 248^\circ \text{ F}$$

$$\text{Heat Flux} = \frac{248 \times 2260 \times .292}{1.06} = 1.54 \times 10^5$$

$$\text{Average Temp. drop} = \frac{166 - 125 + 300 - 257}{2} \times 1.8 = 75^\circ \text{ F}$$

$$\text{Observed coefficient} = \frac{1.54}{75} \times 10^5 = 2040 \text{ BTU/(hr)(ft}^2\text{)(}^\circ\text{F)}.$$

$$Pe_{tube} = \frac{Du\rho c}{k} = \frac{4}{\pi} \frac{c}{k} \frac{W}{D} = 0.0224 \frac{W}{D}$$

$$Pe_{ann} = \frac{D_{eq} u \rho c}{k} = \frac{4}{\pi} \frac{c}{k} \frac{W}{(D_o + D_i)} = 0.0224 \frac{W}{(D_o + D_i)}$$

(D_o is inner diameter of outer tube; D_i is outer diameter of of inner tube)

For tube:

$$Pe = 0.0224 \; \frac{2260}{.703} \times 12 = 865$$

$$Nu = 7 + 0.025 \times 223$$

$$= 7 + 5.57 = 12.57$$

$$h = \frac{12/57 \times 16.6 \times 12}{.703} = 3560 \; BTU/(hr)(ft^2)(^oF)$$

$$\frac{1}{h} = 2.81 \times 10^{-4} \; \frac{(hr)(ft^2)(^oF)}{BTU}$$

For annulus:

$$Pe = 0.0224 \; \frac{2260}{1.688} \times 12 = 361$$

$$Nu = 4.9 + 0.0175 \times 111 = 6.84$$

$$h = \frac{6.84 \times 16.6 \times 12}{.1615} = 8,440 \; BTU/(hr)(ft^2)(^oF)$$

$$\frac{1}{h} \text{ corrected} = \frac{1}{8,440} \times \frac{.703}{.757} = 1.18 \times 10^{-4} \; \frac{(hr)(ft^2)(^oF)}{BTU}$$

Total Resistance $= 2.8 + 1.18 + .80 = 4.78 \times 10^{-4}$

Calculated coefficient $= \frac{10^4}{4.78} = 2090 \; BTU/(hr)(ft^2)(^oF)$

Ratio $\frac{\text{observed}}{\text{calculated}} = \frac{2040}{2090} = 0.98$

TABLE VI

EXPERIMENTAL RESULTS

Run Serial Number Heat Exchanger	Temp. #1 °C	Temp. #2 °C	Temp. #3 °C	Temp. #4 °C	Temp. #6 °C	Time to Fill Catch Tank - Seconds	Flow Rate Lbs./Hr.	Heat Flux-BTU/Hr (Sq.Ft) x 10^{-5}	Observed Overall-BTU/Hr. (Sq.Ft)(°F) Coefficient	Predicted Overall-BTU/Hr. (Sq.Ft)(°F) Coefficient	Ratio - $\frac{U_{obs}}{U_{calc.}}$
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
1A	152	234	275	190	172	34.7	2037	1.98	2780	2770	1.00
2A	156	239	281	194	175	34.2	2064	2.03	2800	2780	1.01
3A	153	243	281	186	157	48.9	1452	1.55	2440	2550	0.96
4A	144	242	276	172	150	68.4	1039	1.23	2170	2370	0.92
5A	217	327	386	270	249	32.2	2127	2.79	2770	2810	0.99
6A	217	324	381	270	250	29.5	2343	2.97	3040	2880	1.05
7A	212	321	368	249	230	42.7	1628	2.16	2830	2630	1.08
8A	209	347	400	251	210	60.5	1156	1.93	2250	2440	0.92
9A	110	233	298	171	111	31.5	2282	3.32	2920	2860	1.02
10A	113	232	296	172	122	31.0	2311	3.26	2970	2870	1.03
11A	112	239	291	159	128	49.3	1450	2.18	2460	2550	0.96
12A	109	256	293	136	119	124.5	576	1.02	1750	2140	0.82
13A	109	257	294	136	118	124.5	576	1.03	1770	2140	0.83
14A	111	239	283	150	138	65.1	1097	1.67	2220	2400	0.93
15A	115	251	293	150	136	82.0	871	1.41	2030	2290	0.89
16A	114	252	293	149	136	82.0	871	1.43	2090	2290	0.91
17A	217	327	387	271	237	28.8	2408	3.16	3090	2900	1.07
18A	214	327	389	270	236	28.7	2417	3.26	3090	2900	1.07
19A	210	334	384	252	229	49.5	1405	2.09	2500	2530	0.99
20A	214	342	391	253	228	58.3	1193	1.84	2340	2440	0.96
21A	209	347	386	233	225	115.2	604	1.02	1810	2160	0.84
22A	222	358	398	241	254	137.9	501	0.85	1600	2100	0.76
23A	214	334	373	243	212	79.2	882	1.26	2100	2300	1.03
24A	133	226	277	181	138	32.8	2175	2.38	2700	2840	0.95
25A	129	234	292	182	139	30.8	2322	2.90	2870	2870	1.00

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	$\frac{U_{obs}}{U_{pred}}$
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
26A	128	234	292	183	140	30.7	2330	2.91	2870	2870	1.00
27A	126	234	292	182	132	30.3	2360	2.99	2930	2880	1.02
28A	125	230	291	177	128	30.3	2360	3.01	2960	2880	1.03
29A	122	234	295	180	144	31.6	2270	2.99	2830	2850	0.99
30A	126	237	296	190	150	30.7	2320	2.92	2640	2870	0.92
31A	118	231	293	177	140	30.2	2380	3.15	2900	2890	1.00
32A	119	233	295	178	141	29.7	2420	3.26	3010	2900	1.04
33A	107	224	288	168	124	30.1	2380	3.27	2940	2890	1.02
34A											
35A											
36A											
37A											
38A											
39A											
40A											
41A			These runs were discarded because of								
42A			erroneous temperature measurements.								
43A											
44A											
45A											
46A											
47A											
48A											
49A											
50A	192	229	251	213	208	29.4	2380	1.03	2700	2890	0.93
51A	191	228	251	212	202	28.4	2470	1.08	2790	2920	0.96
52A	191	229	251	212	202	28.3	2470	1.10	2840	2920	0.97
53A	191	230	253	213	203	28.3	2470	1.14	2790	2920	0.96
54A	191	230	253	213	103	28.3	2580	1.19	2920	2930	0.99
55A	192	231	254	214	103	29.0	2490	1.14	2930	2930	1.00
56A	192	231	255	214	104	28.2	2560	1.20	2900	2950	0.98
57A	192	232	255	215	105	28.5	2530	1.18	2890	2940	0.98
58A	265	301	322	284	270	28.4	2420	1.03	2870	2910	0.98
59A	242	296	325	273	259	27.9	2470	1.43	2920	2920	1.00
60A	243	293	322	270	256	27.8	2480	1.48	2950	2920	1.01

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	U_{obs}/U_{pred}
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
61A	Not completed, error in temperature measurement.										
62A	242	265	277	255	245	27.6	2510	0.67	2960	2930	1.01
63A	238	263	276	252	240	27.7	2500	0.72	2950	2930	1.01
64A	231	256	270	244	234	27.7	2510	0.75	3060	2930	1.04
65A	229	259	277	246	234	30.0	2320	0.82	2600	2870	0.90
66A	223	255	275	240	229	28.0	2480	0.96	2890	2930	0.99
67A	225	253	278	242	233	28.1	2470	0.92	2350	2920	0.80
68A	219	254	274	238	227	28.0	2490	1.02	2930	2930	1.00
69A	223	258	279	241	231	29.8	2330	0.99	2820	2880	0.98
70A	Errors in Temperature Measurements.										
71A											
72A											
73A											
74A											
75A	116	268	300	138	113	155.6	462	0.88	1520	2080	0.73
76A	114	263	298	142	115	154.7	464	0.82	1480	2080	0.71
77A	115	261	296	143	115	154.2	466	0.81	1420	2080	0.68
78A	118	261	296	144	116	148.8	[illegible]	0.83	1520	2090	0.72
79A	118	260	296	145	118	150.6	476	0.81	1450	2090	0.69
80A	118	261	298	145	113	151.4	474	0.82	1420	2090	0.68
81A	121	262	297	148	117	151.6	473	0.80	1430	2080	0.69
82A	122	259	295	148	122	151.2	474	0.78	1640	2090	0.78
83A	120	259	295	150	132	153.3	468	0.77	1190	2080	0.57
84A	127	265	296	154	122	151.7	472	0.77	1510	2080	0.73
85A	128	263	296	154	125	149.1	480	0.77	1470	2090	0.70
86A	122	260	296	149	125	161.8	443	0.73	1300	2070	0.63
87A	115	260	298	142	114	155.9	461	0.80	1380	2080	0.66
88A	122	261	296	149	118	152.5	470	0.78	1410	2080	0.68
89A	121	255	289	148	123	161.7	444	0.71	1290	2070	0.62
90A	117	256	289	135	116	151.3	475	0.81	1760	2090	0.84
91A	115	256	293	143	113	156.2	460	0.76	1360	2080	0.65
92A	118	265	300	143	122	152.8	469	0.83	1520	2080	0.73
93A	121	262	295	147	117	151.2	475	0.80	1510	2090	0.72
94A	119	264	298	143	113	156.2	460	0.80	1540	2080	0.74
95A	109	230	295	170	117	33.8	2130	3.03	2670	2810	0.95

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	$\frac{U_{obs}}{U_{pred}}$
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
96A	111	230	293	171	161	33.8	2080	2.90	2620	2790	0.94
97A	115	231	293	175	121	29.9	2400	3.27	2990	2900	1.03
98A	114	229	291	173	119	30.3	2370	3.21	2960	2890	1.02
99A	112	228	290	171	158	30.4	2330	3.18	2890	2880	1.00
100A	117	229	290	175	130	29.8	2670	3.52	3290	2990	1.10
101A	112	266	295	137	114	158.3	454	0.82	1670	2080	0.80
102A	109	260	292	135	112	164.3	437	0.78	1500	2070	0.72
103A	110	261	294	135	111	152.5	471	0.85	1630	2090	0.78
104A	113	264	298	137	110	151.9	473	0.86	1640	2090	0.78
105A	114	230	295	175	127	30.5	2350	3.22	2850	2910	0.98
106A	113	228	292	175	128	30.2	2370	3.29	2820	2890	0.98
107A	114	228	291	174	129	29.8	2400	3.23	2920	2900	1.01
108A	115	229	293	175	129	30.4	2350	3.28	2870	2880	1.00
109A	115	231	297	176	129	30.0	2390	3.39	2890	2890	1.00
110A	116	231	294	176	130	30.4	2350	3.29	2870	2880	1.00
111A	112	259	294	138	115	155.0	463	0.82	1470	2080	0.71
112A	117	259	296	144	117	149.2	479	0.80	1390	2090	0.67
113A	118	261	296	144	121	150.1	477	0.82	1540	2090	0.74
114A	115	261	297	143	128	150.3	476	0.83	1450	2090	0.69
115A	113	259	293	139	121	147.6	486	0.85	1570	2100	0.75
116A	110	258	296	139	118	150.1	478	0.85	1420	2090	0.68
117A	116	262	300	144	141	147.7	483	0.85	1450	2090	0.69
118A	119	260	298	149	122	145.8	491	0.83	1360	2100	0.65
119A	114	255	289	141	124	147.0	487	0.82	1500	2100	0.71
120A	111	256	297	142	120	150.7	488	0.85	1320	2100	0.63
121A	109	256	293	136	121	152.8	472	0.84	1450	2090	0.69
122A	111	259	297	134	114	139.1	516	0.94	1720	2110	0.82
123A	108	247	283	134	112	153.7	517	0.87	1590	2110	0.75
124A	111	252	291	140	114	154.7	464	0.81	1130	2080	0.54
125A	110	252	287	138	117	151.6	474	0.80	1420	2090	0.68
126A	111	253	291	140	117	155.6	460	0.78	1290	2080	0.62
127A	120	244	293	165	148	55.0	1300	1.89	2220	2490	0.89
128A	114	240	290	160	147	56.4	1260	1.87	2160	2470	0.87
129A	115	243	293	161	150	56.1	1260	1.91	2220	2480	0.90
130A	119	242	291	163	141	57.6	1230	1.79	2500	2460	1.02

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	$\frac{U_{obs}}{U_{pred}}$
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
131A	115	249	289	147	135	113.4	630	1.00	1530	2170	0.71
132A	113	254	293	146	131	109.2	655	1.09	1690	2180	0.78
133A	111	247	283	143	129	116.3	615	0.99	1610	2160	0.75
134A	117	243	290	162	148	57.6	1240	1.82	2230	2460	0.91
135A	117	237	282	161	148	51.2	1390	1.92	2310	2530	0.91
136A	121	233	279	163	148	51.2	1390	1.84	2320	2530	0.92
137A	118	250	300	163	147	59.1	1210	1.82	2200	2440	0.90
138A	118	249	298	162	150	60.4	1130	1.83	2200	2460	0.89
139A	119	250	299	163	150	60.3	1180	1.83	2200	2460	0.89
140A	121	250	299	165	150	60.2	1180	1.81	2160	2460	0.88
141A	123	250	297	165	152	60.0	1180	1.78	2190	2460	0.89
142A	114	255	293	146	127	123.6	578	0.97	1560	2140	0.73
143A	114	255	295	146	126	130.3	549	0.92	1440	2130	0.68
144A	114	255	292	146	126	126.4	566	0.93	1510	2130	0.66
145A	114	229	295	176	147	30.3	2350	3.19	2800	2880	0.97
146A	120	233	296	180	135	30.6	2340	3.11	2820	2880	0.98
147A	121	235	299	181	134	31.7	2260	3.04	2750	2850	0.97
148A	120	231	293	178	144	30.9	2310	3.04	2810	2870	0.98
149A	128	237	297	185	174	30.9	2290	3.03	2790	2860	0.97
150A	133	241	300	189	179	30.4	2320	2.93	2570	2870	1.00
151A	119	230	292	178	168	31.4	2250	2.96	2720	2850	0.95
152A	124	232	291	180	170	31.1	2270	2.89	2780	2860	0.97
153A	213	331	397	274	258	30.7	2250	3.13	2770	2850	0.97
154A	218	333	396	278	260	30.1	2290	3.12	2820	2860	0.99
155A	219	333	296	278	261	30.0	2300	2.97	2820	2870	0.98
156A	220	333	296	279	262	28.4	2430	3.24	2980	2910	1.02
157A	222	339	297	280	255	28.3	2440	3.24	2990	2910	1.03
158A	112	239	299	168	154	50.3	1410	2.07	1990	2540	0.78
159A	114	239	298	172	156	54.2	1310	1.91	1830	2490	0.74
160A	121	231	296	182	129	34.2	2090	2.72	2420	2800	0.86
161A	119	232	298	181	127	36.2	1980	2.63	2300	2750	0.84
162A	116	228	293	178	139	34.0	2100	2.77	2440	2800	0.87
163A	124	235	294	178	130	39.5	1810	2.49	2460	2690	0.91
164A	121	234	293	176	128	38.9	1840	2.79	2740	2700	1.01
165A	119	234	294	175	126	38.9	1840	2.51	2410	2710	0.89

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	U_{obs}/U_{pred}
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
166A	117	243	300	171	143	47.3	1510	2.22	2240	2570	0.87
167A	118	243	300	171	143	47.7	1500	2.20	2220	2570	0.86
168A	119	243	300	171	143	48.4	1470	2.16	2200	2560	0.86
169A	116	246	299	165	132	58.9	1210	1.86	2030	2450	0.83
170A	115	247	300	164	131	59.0	1210	1.86	2040	2450	0.83
171A	114	245	298	163	131	59.5	1200	1.86	2040	244C	0.83
172A	111	250	298	154	138	85.7	8320	1.37	1740	2270	0.77
173A	112	250	296	153	138	81.4	876	1.43	1820	2290	0.80
174A	111	248	295	152	138	82.8	861	1.40	1780	2290	0.78
175A	210	327	392	272	256	28.7	2410	3.30	2270	2900	0.78
176A	212	328	392	274	258	28.1	2450	3.34	2960	2920	1.01
177A	218	331	394	278	246	29.1	2380	3.16	2870	2890	0.99
178A	217	331	394	277	246	27.9	2480	3.30	2990	2920	1.02
179A	216	340	397	270	250	39.3	1760	2.57	2570	2670	0.96
180A	214	339	395	267	249	40.5	1700	2.50	2560	2650	0.97
181A	210	337	396	264	245	43.8	1580	2.38	2340	2600	0.90
182A	209	337	397	263	245	41.1	1680	2.56	2510	2640	0.95
183A	216	341	399	269	249	42.2	1640	2.43	2450	2630	0.93
184A	216	340	398	268	249	41.5	1670	2.45	2490	2640	0.94
185A	213	342	396	263	233	52.7	1320	2.01	2170	2490	0.87
186A	212	339	392	259	236	50.8	1370	2.06	2300	2540	0.91
187A	211	339	393	258	235	56.3	1230	1.88	2070	2470	0.84
188A	214	349	399	259	231	59.6	1170	1.86	2180	2430	0.90
189A	214	348	400	261	234	60.0	1160	1.83	2070	2420	0.86
190A	222	334	397	282	239	27.9	2480	3.28	2970	2930	1.01
191A	220	337	397	277	237	27.9	2480	3.36	3090	2930	1.05
192A	115	235	295	175	122	31.4	2280	3.14	2820	2860	0.99
193A	114	235	296	175	130	31.7	2260	3.13	2790	2850	0.98
194A	120	237	297	180	138	33.8	2130	2.87	2600	2800	0.93
195A	119	230	291	178	139	30.7	2330	3.03	2790	2870	0.97
196A	126	241	294	175	160	42.6	1670	2.27	2480	2640	0.94
197A	127	240	292	175	162	41.6	1710	2.28	2550	2650	0.96
198A	122	247	297	167	156	54.0	1320	1.94	2280	2490	0.92
199A	123	246	296	168	156	53.6	1330	1.93	2280	2500	0.91
200A	116	255	297	156	142	85.4	835	1.35	1850	2270	0.81

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	$\frac{U_{abs}}{U_{pred}}$
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
201A	117	251	293	154	140	81.4	876	1.39	1960	2300	0.85
202B	117	246	295	162	150	31.1	2290	1.54	1820	2100	0.87
203B	125	257	300	166	139	31.5	2260	1.54	2040	2090	0.98
204B	112	246	294	158	120	32.6	2200	1.52	1790	2080	0.86
205B	112	246	295	158	119	31.9	2250	1.56	1820	2090	0.87
206B	110	249	299	158	131	32.3	2210	1.58	1800	2080	0.87
207B	112	249	298	159	131	31.1	2300	1.75	2030	2100	0.97
208B	121	279	296	134	112	146.5	491	0.40	1360	1570	0.87
209B	116	277	296	132	111	161.5	445	0.37	1180	1550	0.76
210B	110	274	291	125	110	158.9	452	0.38	1330	1570	0.85
211B	125	284	299	136	108	217.7	330	0.28	1170	1500	0.77
212B	110	270	297	134	116	99.3	723	0.62	1390	1660	0.84
213B	114	271	296	137	118	98.8	727	0.60	1400	1660	0.85
214B	115	274	299	139	119	117.9	609	0.51	1170	1610	0.73
215B	121	267	288	140	121	97.9	733	0.57	1580	1660	0.95
216B	122	272	296	192	122	97.4	737	0.59	1480	1660	0.89
217B	114	267	296	140	127	84.6	846	0.69	1410	1700	0.83
218B	110	270	298	136	123	86.9	825	0.70	1460	1690	0.86
219B	109	263	292	137	125	74.6	961	0.79	1540	1740	0.88
220B	110	256	285	136	124	71.8	979	0.76	1530	1740	0.88
221B	113	245	271	138	126	69.0	1037	0.73	1570	1760	0.89
222B	114	245	275	142	131	57.9	1235	0.86	1650	1820	0.91
223B	117	250	281	146	135	57.2	1248	0.88	1650	1830	0.90
224B	120	253	284	149	138	56.8	1259	0.89	1590	1830	0.87
225B	127	247	279	158	147	45.1	1580	1.00	1780	1920	0.93
226B	133	259	292	164	153	46.6	1530	1.02	1757	1900	0.92
227B	136	266	299	168	156	47.1	1510	1.04	1754	1900	0.92
228B	144	260	298	176	165	40.8	1740	1.09	1718	1960	0.88
229B	148	262	294	179	168	39.4	1800	1.09	1893	1970	0.96
230B	151	264	296	182	170	39.6	1790	1.07	1908	1970	0.97
231B	153	266	298	183	170	39.6	1790	1.07	1919	1970	0.97
232B	157	258	289	187	177	34.4	2050	1.11	2012	2040	0.99
233B	157	258	289	186	176	35.4	1990	1.08	2000	2030	0.99
234B	157	259	290	186	176	35.8	1970	1.07	1984	2020	0.98
235B	154	262	287	176	163	56.8	1250	0.77	1724	1830	0.94

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	U_{obs}/U_{pred}
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
236B	211	351	396	259	238	29.5	2350	1.75	2217	2110	1.05
237B	211	353	398	254	239	30.2	2300	1.73	2188	2100	1.04
238B	218	360	397	254	234	43.6	1600	1.20	1798	1920	0.94
239B	221	361	397	256	236	41.2	1690	1.25	1968	1950	1.01
240B	213	368	400	240	217	62.7	1110	0.93	1739	1780	0.98
241B	214	362	391	242	217	61.8	1130	0.89	1748	1790	0.98
242B	213	356	387	243	222	47.6	1460	1.11	1996	1890	1.05
243B	213	368	396	237	211	66.1	1060	0.93	2004	1770	1.13
244B	211	371	400	237	206	71.0	986	0.85	1733	1750	0.99
245B	62	159	194	92	77	27.6	2630	1.38	2331	2180	1.07
246B	63	160	194	95	78	27.6	2630	1.36	2277	2180	1.04
247B	66	161	195	97	74	28.0	2590	1.32	2272	2170	1.05
248B	59	171	191	77	54	89.0	822	0.49	1440	1690	0.85
249B	57	174	195	75	47	96.2	760	0.47	1351	1670	0.81
250B	43	168	189	63	40	90.7	808	0.53	1408	1690	0.83
251B	63	168	192	83	62	67.5	1080	0.61	1540	1780	0.86
252B	54	147	158	66	36	119.4	6150	0.30	1480	1620	0.91
253B	48	174	181	56	26	253.0	292	0.19	1490	1490	1.00
254B	42	184	198	52	25	182.4	403	0.30	1410	1530	0.92
255C	217	328	397	285	265	30.9	2230	4.16	3360	3020	1.11
256C	217	328	399	285	265	30.3	2270	4.28	3400	3030	1.12
257C	220	328	396	287	267	29.9	2300	4.18	3430	3050	1.12
258C	221	330	400	290	227	29.4	2370	4.32	3490	3070	1.14
259C	124	224	290	188	110	31.6	2280	3.84	3310	3030	1.09
260C	118	224	293	186	174	31.8	2220	3.96	3210	3010	1.07
261C	114	219	290	183	142	32.5	2200	3.89	3070	3000	1.02
262C	121	225	297	190	141	32.1	2220	3.92	3100	3010	1.03
263C	129	229	297	194	149	32.1	2220	3.76	3130	3010	1.04
264C	113	246	287	143	116	143.6	500	1.14	1940	2220	0.87
265C	111	257	291	142	120	168.8	424	1.05	1820	2170	0.84
266C	111	257	295	145	121	138.9	516	1.28	1940	2230	0.87
267C	110	254	289	143	121	151.3	474	1.15	1870	2200	0.85
268C	117	241	289	162	144	77.5	861	1.80	2150	2420	0.89
269C	117	248	296	163	146	83.3	854	1.87	2240	2410	0.93
270C	117	249	296	162	145	106.3	669	1.48	1790	2310	0.77

TABLE VI (CON'T)

Run	T_1	T_2	T_3	T_4	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	$\frac{U_{obs}}{U_{pred}}$
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
271C	117	243	290	160	143	94.7	752	1.61	2010	2360	0.85
272C	115	249	298	159	141	105.4	677	1.56	1870	2320	0.81
273C	212	369	395	239	185	137.5	512	1.34	2840	2220	1.28
274C	215	374	392	238	186	187.3	376	0.98	2700	2130	1.27
275C	212	366	400	248	191	119.5	588	1.51	2422	2270	1.07
276C	222	348	386	255	206	112.4	622	1.34	2080	2290	0.94
277C	211	339	386	260	228	64.4	1090	2.30	2640	2530	1.04
278C	211	353	399	259	225	63.9	1100	2.53	3040	2540	1.20
279C	210	339	387	261	229	63.3	1100	2.36	2640	2540	1.04
280C	215	335	386	266	245	63.7	1310	2.64	2860	2640	1.08
281C	232	396	395	287	261	45.7	1520	2.82	2980	2730	1.09
282C	226	343	396	283	260	45.8	1510	2.91	2920	2720	1.07
283C	228	334	391	289	269	35.8	1930	3.34	3150	2900	1.09
284C	210	336	376	257	231	66.1	1050	2.16	2770	2520	1.10
285C	220	346	390	267	230	64.9	1070	2.22	2700	2530	1.07
286C	217	334	386	271	246	51.0	1360	2.64	2770	2660	1.04
287C	217	335	386	273	251	47.3	1460	2.82	2910	2700	1.08
288C	220	334	390	280	258	41.4	1670	3.15	3060	2790	1.10
289C	224	330	390	283	257	42.0	1640	3.01	3060	2780	1.10
290C	227	336	391	286	258	37.7	1820	3.27	3140	2860	1.10
291C	225	338	394	286	258	37.7	1820	3.37	3140	2860	1.10
292C	224	333	391	288	259	34.2	2020	3.60	3270	2930	1.11
293C	227	332	388	288	252	34.7	1990	3.43	3230	2920	1.11
294D	210	357	398	250	231	48.9	1420	1.69	2220	2690	0.83
295D	226	355	391	263	244	41.0	1690	1.76	2560	2800	0.91
296D	220	358	398	259	243	35.8	1940	2.16	2920	2900	1.01
297D	216	355	395	256	240	36.0	1920	2.16	2840	2900	0.98
298D	214	353	393	253	238	35.4	1960	2.20	2920	2910	1.00
299D	212	354	396	252	237	36.0	1920	2.21	2850	2900	0.98
300D	214	370	396	230	185	133.2	529	0.68	1840	2230	0.83
301D	210	374	395	227	181	124.3	567	0.76	2140	2260	0.95
302D	213	376	396	227	185	144.1	488	0.65	2030	2200	0.92
303D	212	372	390	226	178	159.4	442	0.58	2360	2180	1.08
304D	220	370	398	238	192	89.7	782	0.97	2220	2380	0.93
305D	214	374	386	230	191	130.9	537	0.69	2560	2240	1.14

TABLE VI (CON'T)

Run	T_1	T_2	T_3	[illegible]	T_6	Sec.	Lb/Hr	Flux	U_{obs}	U_{pred}	$\frac{U_{obs}}{U_{pred}}$
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
306D	231	363	387	253	214	68.0	1030	1.10	2520	2500	1.01
307D	211	363	396	241	218	56.5	1230	1.53	2490	2600	0.96
308D	211	360	391	240	213	55.0	1270	1.54	2690	2620	1.03
309D	213	357	385	240	220	61.3	1140	1.33	2490	2560	0.97
310D	209	352	387	243	223	47.3	1470	1.66	2530	2710	0.93
311D	213	354	389	246	227	47.3	1470	1.69	2610	2710	0.96
312D	218	355	391	254	223	40.8	1700	1.89	2760	2810	0.98
313D	110	251	294	149	142	41.4	1720	1.99	2590	2810	0.92
314D	118	258	300	156	146	42.2	1690	1.94	2560	2800	0.91
315D	124	257	299	160	151	42.4	1670	1.84	2490	2790	0.89
316D	120	257	297	155	112	42.4	1580	1.71	2440	2730	0.89
317D	115	260	293	143	137	57.1	1250	1.48	2220	2610	0.85
318D	116	254	290	147	137	59.0	1210	1.37	2170	2590	0.84
319D	125	265	300	157	146	58.2	1220	1.39	2230	2600	0.86
320D	124	263	295	151	143	69.0	1030	1.18	2130	2510	0.85
321D	120	250	281	146	135	77.1	926	0.99	1890	2450	0.77
322D	116	254	286	143	131	74.0	955	1.08	1930	2460	0.78
323D	112	264	294	137	125	87.3	821	1.02	1970	2400	0.82
324D	109	261	291	133	121	88.9	807	1.01	1970	2390	0.82
325D	110	265	290	128	110	123.6	583	0.74	1860	2270	0.82
326D	123	272	296	140	114	135.2	532	0.65	1680	2240	0.75

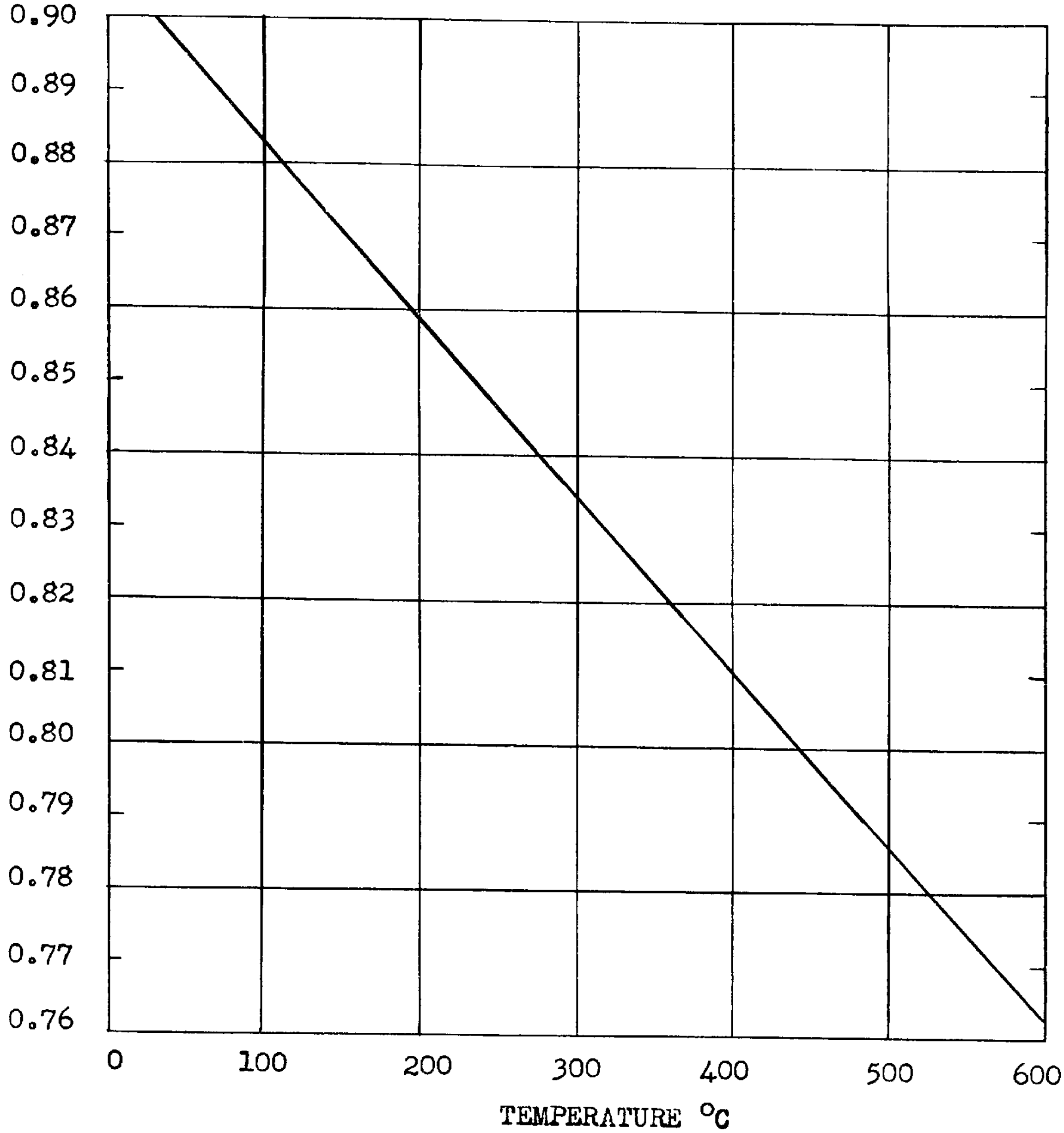

FIGURE 12

DENSITY OF 48 WT% KNa

APPENDIX E

REMARKS ON HYDRODYNAMIC RELATIONSHIPS

The hydrodynamic concepts developed by Nikuradse and used by him to present his "universal velocity distribution" are used in most of the discussions of fluid flow which apply to heat transfer. It is the purpose here to describe these and related concepts briefly, and to indicate their relationship to the concepts used in the development in Chapter 3.

Friction Velocity

The friction velocity is defined in the following equation:

$$(E1) \qquad v* = \sqrt{\frac{\tau_w}{\rho} g_c}$$

where τ_w is the shear at the wall; ρ is the density; and g_c is the gravitational constant, the conversion factor of mass to weight, and of practical to absolute units of force. When appropriate units of shear or density are used, g_c is not used in the definition.

It can easily be shown that the friction velocity is also represented in the equation:

$$(E2) \qquad v* = u_m \sqrt{f/2}$$

where f is the Fanning friction factor as defined by McAdams[17], and is a function of the Reynolds modulus.

REMARKS ON HYDRODYNAMIC RELATIONSHIPS

Dimensionless Velocity and Wall Distance

Nikuradse demonstrated that in the turbulent region, a general velocity distribution for all Reynolds moduli when a dimensionless velocity modulus is compared with a dimensionless function of the relative distance to the center of the tube and the Reynolds modulus.

The dimensionless velocity modulus was expressed by the symbol ϕ in Nikuradse's paper. It is now usually given the symbol u^+ and it is defined in the equation:

$$\text{(E3)} \quad u^+ = \frac{u}{v^*} = \frac{u}{u_m \sqrt{f/2}} = \frac{V}{\sqrt{f/2}}$$

The wall distance function was expressed by Nikuradse as η but recent investigators assign it the symbol y^+. It is defined in the equation:

$$\text{(E4)} \quad y^+ = \frac{y}{\nu} v^* = \frac{y}{r_w} \cdot \frac{r_w u_m}{\nu} \sqrt{f/2} = \frac{y}{2r_w} Re \sqrt{f/2}$$

At the centerline of the tube:

$$\text{(E5)} \quad y^+ = y^+_{℄} = \frac{Re}{2} \sqrt{f/2}$$

Differentiating u^+ with respect to y^+ and rearranging gives the following equation:

Eddy Diffusivity of Momentum

$$\text{(E6)} \quad g_c \frac{\tau_w}{\rho} = \frac{\nu}{du^+/dy^+} \frac{du}{dy}$$

Since

$$\text{(E7)} \qquad g_c \frac{\tau}{\rho} = (\epsilon_M + \nu) \frac{du}{dy}$$

and

$$\text{(E8)} \qquad \tau = S \tau_w$$

we find:

$$\text{(E9)} \qquad \frac{\epsilon_M + \nu}{\nu} \quad \text{or} \quad \frac{\epsilon_M}{\nu} + 1 = \frac{1}{S(du^+/dy^+)}$$

$$\text{(E10)} \qquad \frac{\epsilon_M}{\nu} = \frac{1}{S\, du^+/dy^+} - 1$$

<u>Application to Equation 33</u>

Appropriate substitutions may be may be made in Equation (33) to transform it into terms of u^+ and y^+. The resulting equation is more complicated, but it has the advantage of being solvable directly with Equations (B4) - (B8) listed in Appendix B. The numerical results will be essentially the same as Martinelli's, though perhaps slightly easier to obtain.

NOMENCLATURE

Term		Meaning	Units
A_w	-	Area on the surface of a unit length of tube	area
C	-	Specific heat	heat/(mass)(temperature)
D	-	Molecular Diffusivity	$(\text{length})^2$/time
D	-	Diameter of tube	length
E_H	-	Eddy conductivity of heat	heat/(time)(area) (temperature/length)
f	-	Fanning friction factor	none
g_c	-	Gravitational constant	Length/$(\text{time})^2$
h	-	Heat transfer coefficient	heat/(time)(area)(temperature)
k	-	Molecular conductivity of heat	heat/(time)(area) (temperature/length)
$\overline{K}$	-	Total conductivity of heat	heat/(time)(area) (temperature/length)
ℓ	-	Mixing length	length
q	-	heat flow toward the tube center	heat/time
r	-	distance from tube center	length
S	-	relative distance from tube center, $\frac{r}{r_w}$	none
t	-	temperature	temperature
T	-	temperature	temperature
u	-	velocity	ength/time
u^+	-	dimensionless velocity, $= \frac{u}{v^*}$	none
U	-	overall heat transfer coefficient	heat/(time)(area)(temperature)

Term		Meaning	Units
v^*	-	friction velocity = $\sqrt{\frac{\tau_w}{\rho} g} = u_m \sqrt{f/2}$	length/time
V		relative velocity = $\frac{u}{u_m}$	none
W		flow rate	weight/time
x, y, z	-	rectangular coordinate system	length
y		distance from wall	length
y^+	-	dimensionless wall distance = $\frac{y}{\nu} v^*$	none
z	-	distance along tube axis	length
Nu	-	Nusselt Modulus = $\frac{2r_w h}{k}$	none
Pe	-	Peclet Modulus = $\frac{2r_w C \rho u_m}{k}$	none
Pr	-	Prandtl Modulus = $\frac{c\mu}{k}$	none
Re	-	Reynolds Modulus = $\frac{2r_w \rho u_m}{\mu}$	none
α	-	ϵ_H/ϵ_M	none
ϵ	-	eddy diffusivity	$(\text{length})^2$/time
η	-	Nikuradse's term for y^+	none
θ	-	angular displacement	radius
λ	-	time	time
μ	-	absolute viscosity	mass/(length)(time)
ν	-	kinematic viscosity	$(\text{length})^2$/time
ρ	-	density	mass/$(\text{length})^3$
τ	-	shear	force/area

Term	Meaning	Units
ϕ	Nikuradse's term for u^+	none

Subscript		Meaning
H	-	of heat
i	-	inner
m	-	mean, (in the case of fluid temperature, the flow mean)
M	-	of momentum
o	-	outer
q	-	where heat flow toward center is q
t	-	where fluid temperature is t
w	-	at the wall
eq	-	equivalent
℄	-	at the axis of the tube

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