# Risk Analysis: Water Resources Education

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## I. Introduction

The concepts of risk, uncertainty, risk analysis, and risk-benefit analysis have become working tools of the water resource professional. In reality, hydrologists, flood control specialists, and water resource planners and engineers have estimated the magnitudes of a specified design storm, a specified design flood, or the magnitude of the maximum demand day for a water supply system. However, in addition to these more traditional professional specialties, interest in risk and reliability have expanded into other areas of the water resource profession. For example, following the enactment of the Clean Water Act in 1972, the regulatory agencies at both the federal and state levels have been interested in monitoring the performance of waste water treatment plants. Reliability in terms of plant performance has become a significant topic of interest. Facilities which lack reliability in terms of overall system performance may not only pose threats to human and ecosystem health but also may be subject to enforcement actions against the owners/operators of the facilities. Accordingly, it is important to familiarize water resource planners and engineers with working knowledge of engineering systems and associated risk and reliability studies.

A second major area to be examined in this paper examines risk in the context of limiting the concentration of toxic substances in surface waters. The process of establishing these limits is based upon the consideration of a number of important factors including the risk level, the weight of the human consuming fish which bio-concentrate the toxic chemical, the potency of the chemical to cause cancer (or other illness), the daily consumption of both fish and water, the Biocentration Factor for fish, and the food chain multiplier.

These two topic areas are presented as examples of the concept of risk and risk-related topics which are important elements to be included in the education of water resource professionals.

## II. Engineering Systems: Reliability and Risk

There are certain terms and definitions that are useful in working with engineering systems. for the purpose of this paper, consider as an initial example, a system which is in continuous operation and does not undergo repair. Later, we shall see how to extend the analysis by adding repair. The probability of failure of this system in $dt$ about $t$ :

$$f(t)dt = \lambda(t) \cdot dt[1 - F(t)] \qquad (1)$$

where $f(t)dt$ = the probability of failure in $dt$ about $t$

$\lambda(t)dt$ = the probability of failure in $dt$ about $t$, given that the system has survived to time $t$

$1 - F(t)$ = the probability that the device did not fail prior to time $t$

or one can express the relationship as follows:

$$f(t) = \lambda(t)[1 - F(t)] \qquad (2)$$

where $f(t)$ is the failure probability density; $\lambda(t)$ is the conditional failure rate-often called the hazard rate (failures/unit time)

If a system is known to be functioning at time $t = o$, the cumulative probability for failure between time=$O$ and time=$t$, $F(t)$ is related to the probability density for failure:

$$F(t) = \int_o^t f(t')dt' \tag{3}$$

where $t'$ represents a failure which may occur at anytime between $O$ and $t$.

$$f(t) = dF(t)/dt \tag{4}$$

$R(t)$ Reliability of the system is defined as the probability that a specified fault event has not occurred in a system for a given period of time $t$ and under specified operating conditions.

$$R(t) = 1 - F(t) \tag{5}$$

It can be shown that the following very useful equation relates $R(t)$ and the hazard rate =

$$R(t) = \exp[-\int_0^t \lambda(t')dt'] \tag{6}$$

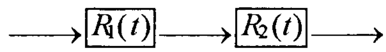
Table 1 shows a summary of the relationships between $\lambda(t), R(t), F(t), \text{and} f(t)$

| | | | | |
|---|---|---|---|---|
| Hazard rate | $\lambda(t)$ | $-(\frac{1}{R})dR/dt$ | $f(t)/1-F(t)$ | $\frac{f(t)}{R(t)}$ |
| Reliability | $R(t)$ | $\int_t^\infty f(\tau)d\tau$ | $1-F(t)$ | $\exp[-\int_o^t \lambda(\tau)d\tau]$ |
| Cumulative Failure Probability | $F(t)$ | $\int_0^t f(\tau)d\tau$ | $1-R(t)$ | $1-\exp[-\int_o^t \lambda(\tau)d\tau]$ |
| Failure Probability Density | $f(t)$ | $\frac{dF(t)}{dt}$ | $\frac{-dR(t)}{dt}$ | $\lambda(t)R(t)$ |

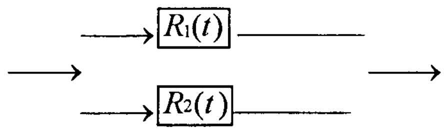
**Reliability Block Diagrams**

Units which comprise an engineering system of interest can be shown through a Reliability Block Diagram which demonstrates how the system functions. For the purpose of this paper, consider two independent units that operate either in series or in active-parallel. (Active-parallel means that no unit is held in standby waiting for other units in the parallel configuration to fail)

Series (1)




Active-Parallel (2)




It is important to see how system configuration impacts upon system reliability. Consider first the Reliability of the Series System. In this case the system fails when either unit 1 or unit 2 fails. Accordingly, the failure for the series system can be represented as follows:

$$F_{sys}(t) = F_1(t) \text{ or } F_2(t) \qquad (7)$$

$$F_{sys}(t) = F_1(t) + F_2(t) - F_1(t)F_2(t) \qquad (8)$$

and the reliability for the series systems follows:

$$1 - R_{sys}(t) = [(1 - R_1(t)) + (1 - R_2(t)) - (1 - R_1(t))(1 - R_2(t))] \qquad (9)$$

$$1 - R_{sys}(t) = 1 - R_1(t) + 1 - R_2(t) - 1 + R_1(t) + R_2(t) - R_1(t)R_2(t) \qquad (10)$$

$$1 - R_{sys}(t) = 1 - R_1(t)R_2(t) \qquad (11)$$

$$R_{sys}(t) = R_1(t)R_2(t) \qquad (12)$$

$$R_{sys}(t) = \exp[-(\lambda_1 + \lambda_2)t] \qquad (13)$$

The MTTF for the Series System is $\int_o^\infty R_{sys}(t)dt = \int_o^\infty \exp[-(\lambda_1 + \lambda_2)t]dt = \frac{1}{\lambda_1 + \lambda_2}$ (14)

Next consider the reliability of the system in Active-Parallel. The failure of this system requires that both units must fail for the system to fail. Accordingly

$$F_{sys}(t) = F_1(t)F_2(t) \qquad (15)$$

and the Reliability of the Active-Parallel system is as follows:

$$1 - R_{sys}(t) = [1 - R_1(t)] \cdot [1 - R_2(t)] \qquad (16)$$

$$1 - R_{sys}(t) = 1 - R_1(t) - R_2(t) + R_1(t)R_2(t) \qquad (17)$$

$$R_{sys}(t) = R_1(t) + R_2(t) - R_1(t)R_2(t) \qquad (18)$$

$$R_{sys}(t) = \exp[-\lambda_1 t] + \exp[-\lambda_2 t] - \exp[-(\lambda_1 + \lambda_2)t] \qquad (19)$$

The MTTF = $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2}$ (20)

Consider $\lambda_1 = 10^{-2}$ / hr

$\lambda_2 = 10^{-3}$ / hr

What is the reliability of each system at $t = 50$ hours

What is the mean time to failure of each system

For the Series System:

$$R(50) = \exp[-(.011)50] \quad (21)$$

$$R(50) = .58 \quad (22)$$

$$\text{MTTF(Series)} = \frac{1}{.011} = \underline{\underline{90.9 \text{ hours}}} \quad (23)$$

$$\underline{R(90) = .3716} \quad (24)$$

For the Active-Parallel System:

$$R(50) = \exp[-(.01 \cdot 50)] + \exp[-(.001 \cdot 50)] - \exp[-(.011 \cdot 50)] \quad (25)$$

$$= .6065 + .9512 - .58 \quad (26)$$

$$R(50) = .9777 \quad (27)$$

$$\text{MTTF Active Parallel} = \frac{1}{.01} + \frac{1}{.001} - \frac{1}{.011} \quad (28)$$

$$= 100 + 1000 - 90.9 \quad (29)$$

$$= \underline{1009 \text{ hours}} \quad (30)$$

$$\underline{R(1000) = .3679} \quad (31)$$

NOTE: In the Series System, the reliability of the system is less than the less reliable of the two units; in the Active-Parallel System, the reliability of the system is greater than the more reliable of the two units.

**Reliability and Availability of Engineering Systems with Repair**

The concept of reliability of a system, $R_{sys}(t)$, has been presented under a simple assumption of no repair. The system was either operating or it had failed. In the more general case, the system is either operating or it is under repair. What becomes of interest is the instantaneous availability, $A(t)$ of the system which is defined as the probability of a system performing a specified function or mission under given conditions at a prescribed time.

Markov models provide an analytic technique to address the simple case of random failure and incorporate the repair of the failed component into the analysis.

In order to accomplish this task, it is necessary to introduce a conditional probability $\mu(t)\ dt$ that a unit which failed at time $t$ will be repaired in time $dt$ about $t$. Accordingly, $\mu(t)$ has units of (time)$^{-1}$ and serves to describe the instantaneous rate of repair, just as $\lambda(t)$ in units of (time)$^{-1}$ describes the instantaneous rate of failure. Also, if $\mu(t) = \mu$, ie the instantaneous rate of repair is constant, then this assumption on $\mu(t)$ is equivalent to assuming random repairs. This means that the repair is completed at a random time after it is started. Then the Mean Time to

Repair (MTTR) of a unit is $\frac{1}{\mu}$; this is similar to the Mean Time to Failure (MTTF) of a unit is $\frac{1}{\lambda}$. Finally, the Mean Time Between Failure (MTBF) is the sum of MTTF plus MTTR. In general, $\lambda << \mu$, ie the hazard rate is much smaller than the repair rate.

For the purpose of illustration, an example using a Markov model and Laplace Transforms will be presented. A complete presentation of the theory will be left to the reference. (McCormick, 1981)

Let us consider $\longrightarrow \boxed{1} \longrightarrow \boxed{2} \longrightarrow$ a very simple case, namely two identical units in series, each has a constant failure rate $\lambda$ and a constant repair rate $\mu$. It is further assumed that there is no simultaneous failure of the two units.

First: Consider the state transition diagram and the distinct system states.

0

1 1

m m

1 2

| System State | |
|---|---|
| 0 | Both units operating |
| 1 | Unit 1 failed (under repair)<br>system not operating |
| 2 | Unit 2 failed (under repair)<br>system not operating |

Next: The Transition Matrix M shows how the system goes from one state to another.

| M = | | | |
|---|---|---|---|
| | -21 | m | m |
| | 1 | -m | o |
| | 1 | o | -m |

Note: each of the diagonal terms shows how the system transitions out of the particular system state.

Next: the |sI-M| $= o$ to calculate the eigen values $s_n$

|sI-M| $= o$

$$= \begin{vmatrix} s+2\lambda & -\mu & -\mu \\ -\lambda & s+\mu & o \\ -\lambda & o & s+\mu \end{vmatrix} = o$$

$$(s+2\lambda)(s+\mu)(s+\mu)-(s+\mu)\lambda\mu-(s+\mu)\lambda\mu = o \tag{26}$$

$$s(s+\mu)(s+\mu+2\lambda) = o \tag{27}$$

$$s_o = o,\ s_1 = -\mu,\ s_2 = -(\mu+2\lambda) \tag{28}$$

$$\Delta = s(s+\mu)(s+\mu+2\lambda) \quad (29)$$

In this example, the system has three states. The zero (0) state where the system is operating; the one (1) state where unit 1 has failed and is under repair and the system is not operating; and the two (2) state where unit 2 has failed and is under repair and the system is not operating. Accordingly, we wish to calculate $P_o(t), P_1(t), P_2(t)$ i.e. the probability of being in the zero state at any time $t$, the probability of being in the one state at any time $t$, and the probability of being in the two state at any time $t$. It may be that we are only interested in the probability that the system is operating at a specified time $t$, then we would calculate the $P_o(t)$. If we were interested in the probability that the system is down or not operating at a specified time without wanting to know if unit 1 or unit 2 has failed, then the probability of being in a down state (either state one or state two) is simply $1-P_o(t)$.

We need to go through additional several steps to obtain $P_o(t)$. Using the Laplace Transform method, we first calculate the following:

$$P_n(s) = [cof(s\mathbf{I}-\mathbf{M})^{\mathrm{T}}]_{no} \quad n = o, N \quad (30)$$

Since we have three states 0, 1, 2, each of the expressions for $P_n(s)$ will have three terms. A partial fraction decomposition which will be illustrated enables us to obtain the following:

$$P_n(s) = \sum_{j=o}^{N} b_{nj}(s-s_j)^{-1} \quad (31)$$

where the $b_{nj}$ are constants. From the Laplace Transform, we obtain $P_n(t)$ as follows:

$$P_n(t) = \sum_{j=o}^{N} b_{nj}\exp(s_j t) \quad n = o, N \quad (32)$$

Note: the $b_{nj}$ constants are the same for both $P_n(s)$ and $P_n(t)$. Let us now carry out the remaining steps to obtain $P_o(t)$ which is the probability of being in the "up" or operating state at any time $t$.

$$[s\mathbf{I}-\mathbf{M}]^T = \begin{matrix} S+2\lambda & -\lambda & -\lambda \\ -\mu & s+\mu & o \\ -\mu & o & s+\mu \end{matrix}$$

$$[cof(s\mathbf{I}-\mathbf{M})^{\mathrm{T}}]_{oo} = (s+\mu)^2 \quad (33)$$

Since the zero (0) state is the only "up" state for the system, the $P_i(s)$ for the up state is as follows:

$$P_o(s) = \frac{[cof(s\mathbf{I}-\mathbf{M})^{\mathrm{T}}]_{oo}}{\Delta} = \frac{(s+\mu)^2}{(s)(s+\mu)(s+\mu+2\lambda)} \quad (34)$$

(Use Partial Fraction Decomposition)

$$P_o(s) = \frac{b_{oo}}{s} + \frac{b_{o1}}{s+\mu} + \frac{b_{o2}}{s+\mu+2\lambda} \quad (35)$$

$$P_o(s) = b_{oo}(s+\mu)(s+\mu+2\lambda) + b_{o1}(s)(s+\mu+2\lambda) + b_{o2}(s)(s+\mu) = (s+\mu)^2 \quad (36)$$

(1) let $s = s_o = o \qquad b_{oo} = \frac{\mu}{\mu + 20}$

(2) let $s = s_1 = -\mu \qquad b_{o1} = o$

(3) let $s = s_2 = -(\mu + 2\lambda) \qquad b_{o2} = \frac{2\lambda}{\mu + 2\lambda}$

Now having $b_{oo}, b_{o1}, b_{o2}$ and $s_o, s_1, s_2$ we can write down the equation for

recall: $$P_o(t) = \sum_{j=o}^{N} b_{oj}\, e^{(s_j t)} \quad (37)$$

$$P_o(t) = \frac{\mu}{\mu+2\lambda} \cdot e^{(ot)} + o \cdot e^{-\mu t} + \frac{2\lambda}{\mu+2\lambda} e^{-(\mu+2\lambda)t} \quad (38)$$

$$P_o(t) = \frac{\mu}{\mu+2\lambda} + \frac{2\lambda}{\mu+2\lambda} e^{-(\mu+2\lambda)t} \quad (39)$$

A. Check at $t = o \qquad P_o(t) = 1.0$

$t = \infty \qquad P_o(t) = \frac{\mu}{\mu + 2\lambda}$

B. if $\lambda = 10^{-3}$ / *hour*

if $\mu = 10^{-1}$ / *hour*

$t = 15{,}000$ hours

$$P_o(15{,}000) = \frac{10^{-1}}{10^{-1} + 2 \cdot 10^{-3}} + \frac{2 \cdot 10^{-3}}{10^{-1} + 2 \cdot 10^{-3}} e^{-(10^{-1} + 2 \cdot 10^{-3})15000}$$

$\cong$ ~ <u>.9804</u> steady state availability

<u>Observation</u>: Explicit consideration of repair long term availability - Simple two unit (identical) series system highly dependent upon interaction between failure rate and repair rate.

<u>Note</u>: If no repair were allowed, ie if $\mu = o$, then the resulting $Po(t)$ would be $e^{-(2\lambda)t}$ which is the result obtained in the no repair example.

## III. Risk Application: Regulation of Toxic Substances in Surface Waters

In contrast to risk and reliability as applied to engineering systems, another example which is also of increasing importance is the determination of limits of toxic substances in surface waters. The role of risk in these situations is associated with the degree of risk the public is willing to accept and the degree of risk that the regulators are willing to specify.

This presentation will focus upon toxic substances which are being discharged into bodies of water where there is a long residence time and in which there are fish present who have the characteristics needed to bioaccumulate the specific toxic substances. The potential threat to humans comes from eating the fish that have bioaccumulated the toxic substances and from drinking the water. At least in the Great Lakes region, the component of the human health risk that is of most concern is from the eating of the fish that bioaccumulate the toxic substances.

The U.S. Environmental Protection Agency has developed the Human Carcinogen Criterion (HCC) as a means to protect humans from an unreasonable incremental risk of developing cancer from contact with, or ingestion of surface waters, and from ingestion of aquatic organisms taken from surface waters. (Foran, 1993)

The Human Cancer Criteria specifies that allowable concentration of the specific toxic substance in the surface water in mg/l.

$$HCC = \frac{(RL * WT)}{q_1 * [WI + FC * (FM * BCF)]}$$

| | | | |
|---|---|---|---|
| where | HCC | = | Human Cancer Criterion (mg/l) |
| | RL | = | Risk Level |
| | WT | = | Weight of Average Adult (kg) |
| | $q_1$* | = | Carcinogenic Potency Factor (kg-day/mg) |
| | WI | = | Average Adult Water Intake (2L/day) |
| | FC | = | Daily Fish Consumption ( kg/day) |
| | FM | = | Food Chain Multiplier |
| | BCF | = | Bioconcentration Factor for Fish with 3% lipid (L/kg) |

It is clear from the factors that comprise the Human Cancer Criterion (HCC) that there are many areas which require judgment and careful thought in its application. For example, what is the appropriate Risk Level (RL) to be utilized. The risk level is a measure of the additional expected cases of cancer to occur over a 70-year lifetime as a consequence of ingesting this fish and drinking this water in the quantities specified. The RL's may vary from 1 in 10,000; 1 in 100,000; 1 in 1,000,000. Each of these RL's shown are reduced by one order of magnitude. Accordingly, all other elements of the HCC being the same the risk level specified many vary the allowable concentration of the toxic substance by one or more orders of magnitude. In the State of Michigan, the RL used is 1 in 100,000. Beyond the RL, another critical factor is the <u>weight</u> of the individual. If one protects for an adult at 70 kg, one has a different concentration than if one protects for a child at say 15 kg. The allowable concentration will be reduced by a factor of 4.6667. Another important factor to be determined in the daily fish consumption in kg/day, the FC. Should one use an overall average fish consumption figure for this factor or does one use a FC factor which may be more representative of a population at risk ie urban or rural or native Americans whose diet may include the consumption of large quantities of fish relative to other segments of the populations.

Table 2 shows the impact of different food consumption quantities, different individuals (adults and children), and different risk levels on the calculation of an allowable concentration of chlordane in mg/l for surface waters. Note that the allowable concentration varies by four (4) orders of magnitude in this simple calculation.

## IV. Observation

These examples are provided to show several potential applications of risk and reliability to water resource issues. It is important that students in all areas of water resources develop working familarity with the ideas of concepts of risk and reliability. This paper is presented to enable an exchange of information to take place.

Table 2

Human Cancer Criterion (HCC) for Chlordane*

| | | Risk Level | | |
|---|---|---|---|---|
| Grams Fish/Day | Human Weight (kg) | $10^{-4}$ | 10-5 | 10-6 |
| 6.5 | 70 | 0.20 | 0.02 | 0.002 |
| | 15 | 0.043 | 0.0043 | 0.00043 |
| 20.0 | 70 | 0.07 | 0.007 | 0.0007 |
| | 15 | 0.015 | 0.0015 | 0.00015 |
| 90.0 | 70 | 0.016 | 0.0016 | 0.00016 |
| | 15 | 0.003 | 0.0003 | 0.00003 |
| 180.0 | 70 | 0.008 | 0.0008 | 0.00008 |
| | 15 | 0.002 | 0.0002 | 0.00002 |

*In $\mu g / L$; $q^{*}_1$ = 1.3 / mg / kg / day; BCF = 3804

NOTE: Water Quality Criterion for Chlordane (µg/L) based on cancer risk levels of $10^{-4}$ to $10^{-6}$, various fish consumption rates (grams fish/day), and two human weight levels (70 kg adult, 15 kg child).

**References**

Reliability and Risk Analysis, Norman J. McCormick, Academic Press, 1981.

Regulating Toxic Substances in Surface Waters, Jeffrey A. Foran, Lewis Publishers, 1993.

Relative Risk Reduction Project, The Science Advisory Board, U.S. EPA, 1990.