Progressive Failure Analysis of Unreinforced and Z-pin Reinforced Composite Pi Joints Subjected to Quasi-Static and Fatigue Loading

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Aerospace Engineering) in the University of Michigan 2023

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DEDICATION

To my family.
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ABSTRACT

Fiber reinforced composites have become the material of choice in aerospace applications where weight constraints require high performance materials and structures. Advantages of composite materials include their high strength-to-weight ratios, inherent damage tolerance, and tailorability. Bonded composite structures have the potential to promote broader application of these material in aerostructures, and therefore, further realization of their benefits. Challenges arise in the certification of bonded composite structures due to concerns of disbond and interlaminar failure. Damage arrestment features have been proposed to improve the interlaminar behavior of composite structures in order to meet certification requirements. Current certification approaches rely heavily on physical testing, which is generally expensive and time consuming. With advanced analysis methods and tools, faster development cycles and efficient certification programs are possible; however, confidence must be instilled in these progressive failure analysis tools. To this end, a progressive failure analysis modeling method for unreinforced and z-pin reinforced composite pi joints is developed and experimentally validated in this dissertation. The method utilizes meshing strategies and constitutive laws from the semi-discrete damage model (SD2M). Additionally, a smeared cohesive zone modeling approach was implemented to account for the effective bridging response of z-pin reinforcement. The model was first calibrated using experimental data obtained from pi joints subjected to pull-off and side-bend loading. Two rounds of blind predictions followed for increasingly complex pi joint configurations to assess the predictive capabilities of the model.

In addition to quasi-static investigations, a unified local cohesive fatigue model was developed and applied to analyze the fatigue response of z-pinned pi joints subjected to cyclic loading. The fatigue model operates within the same novel cohesive formulation used for quasi-static analyses and employs the cycle jump method to efficiently simulate high-cycle fatigue. The fatigue model was demonstrated with constant amplitude and block fatigue studies.
CHAPTER 1

Introduction

1.1 Motivation

Composite materials have emerged as an enabling technology offering key advantages over traditional materials. Fiber-reinforced composites exhibit high strength-to-weight ratios, inherent fatigue and damage resistance, and can be tailored to achieve desired performance. In future aircraft, these advantages have the potential to extend the range, to improve fuel efficiency, and to reduce maintenance costs. The push to incorporate more composite materials into aerospace structures has led to an increased interest in bonded composite structures. Bonded structures eliminate the need for mechanical fasteners, which in turn reduces part counts and leads to further reductions in structural weight. While bonded structures offer significant improvements over traditional fastening systems, such as rivets and bolts, the bond area of such structures is vulnerable to failure. Concerns regarding bond area failure (disbond) are evident in stringent certification requirements for bonded structures. One path to certification for bonded structures is through the use of damage arrestment features which act to arrest unstable damage/crack growth. The Fail-Safe Technologies for Bonded Unitized Composite Structures (FASTBUCS) research program aims to affordably certify bonded composite structural concepts containing damage arrestment features [1, 2]. Certification campaigns of aerospace structures follow the building block approach which consists of levels of experimental testing (Fig. 1.1) beginning with coupon level tests to establish fundamental material properties, leading into element and sub-component tests, and culminating with full-scale tests. In certification programs for new aircraft, extremely large costs and time investments are associated with the coupon and element testing levels of the building block pyramid (see Fig. 1.1). Smart testing is a concept intended to simplify and reduce costs associated with the certification of aerospace structures by leveraging analysis methods and computational tools within the building block approach. Structural
analysis methods have demonstrated capabilities to efficiently explore design spaces and guide experimental testing programs. Great strides have been made recently in the area of predictive structural modeling methods, and as confidence grows in these methods, virtual tests can begin to take the place of physical tests in conventionally empirically driven certification programs. This, in turn, leads to lower costs.

The bonded structural concept studied in this thesis is the pi joint [4, 5, 6, 7]. The pi joint is an adhesively bonded joint named after its resemblance to an inverted π symbol. The woven pi preform is used to join two perpendicularly orientated parts which are typically referred to as the skin and the web as shown in Fig. 1.2. The three-dimensional (3D) weave architecture of the preform can be woven flat and folded into shape to join planar components common in aerospace structures (Fig. 1.3). Adhesive is used to bond the web in the clevis of the preform as well as the skin to the base of the preform. This type of construction results in large bond areas between the adherends and provides structural redundancy through independent bondlines.
Figure 1.2: Pi joint [8].

Figure 1.3: Pi preform weave architecture and cross-sectional schematic with legs in upright position [9]
Consistent with laminated composite structures, delamination is a major concern and a common failure mode in bonded composite joints. The preform-skin bondline is responsible for load transfer and often the location of geometric discontinuities in the joint, making the adjacent interfaces susceptible to delamination. The prevalence of delamination failure in laminated composites has led to significant work and interest in the area of delamination resistance. A promising approach to improve delamination resistance is through-thickness reinforcement. Commonly used through-thickness reinforcement techniques include 3D weaving, braiding, stitching, tufting, and z-pinning. Uniquely, z-pinning can be used to reinforce prepreg laminates in the through-thickness direction and can be extended to large scale production. In the z-pinning process, thin fibrous composite or metal pins are inserted in the thickness direction of an uncured part (Fig. 1.4). The most common process for inserting z-pins is the Ultrasonically Assisted Z-Fiber (UAZ) technique which uses ultrasonic vibrations to drive z-pins into a part (Fig. 1.5). After curing, the z-pins secure layers together as a result of adhesion and friction between the z-pins and the surrounding material. Studies have shown that only a small density of z-pins (typically in the range of 0.5% to 4%) is needed to significantly improve through-thickness properties and delamination resistance [10, 11, 12, 13]. Z-pinning is the damage arrestment technique studied in this work.

Figure 1.4: Magnified image of z-pins made of carbon fiber composite material [14].
Figure 1.5: Schematic of the z-pin insertion process using the UAZ technique [15].
1.2 Review of modeling methods

1.2.1 Bonded joints models

With the goal of providing an overall understanding of modeling methods for bonded joints, a cursory summary of several computational studies on pi joints is provided. It is noted that the modeling strategies highlighted have been applied to other joint types [10, 16, 17]. Ji and Waas [4] developed a 2D pi joint finite element model using the discrete cohesive zone method (DCZM) to capture damage and failure of the preform-skin interface. The DCZM element, which was developed as a user element in Abaqus, is governed by a nodal traction-separation law and was shown to be effective in modeling interfacial delamination. Performing a design and reliability study, Flansburg and Engelstad [5] created a 2D pi joint model with a manufacturing flaw as well as a 3D model with impact damage. In the models, delamination was predicted using the extended finite element method (XFEM) for joints subjected to pull-off loading. In both [4] and [5], only inter-laminar failure (delamination) capabilities were included in the model. While inter-laminar damage alone may suffice for specific joint types and loading conditions, generally the failure progression of pi joints involves the interaction of both inter-laminar and intra-laminar damage mechanisms. In a computational study, Novak and Selvarathinam [18] introduced intra-laminar damage capabilities into a pi joint model and compared the behavior to inter-laminar only joint models subjected to pull-off loading. In the model, inter-laminar damage was modeled using Abaqus’s surface-based cohesive contact interaction, while intra-laminar damage was modeled using the XFEM approach within select skin plies. Results from the study showed significant differences in load-displacement responses and damage progression between models with inter-laminar damage only and those with inter- and intra-laminar damage. The model presented in this thesis incorporates both inter-laminar and intra-laminar damage using a single cohesive zone modeling (CZM) framework. The modeling approach has the unique advantage of capturing multiple and interacting failure mechanisms, while avoiding separate and sophisticated element formulations, which is a key to the computational efficiency of the model.
### 1.2.2 Z-pin models

Further adoption and implementation of z-pinned composites will rely heavily on numerical tools capable of efficiently predicting damage and failure of z-pin reinforced structures. Due to the difference in size scales between a structure and a z-pin, a variety of modeling approaches have been developed to model z-pinned composites depending on the level of fidelity sought. High-fidelity models where z-pins are explicitly modeled, such as the micromechanical model by Zhang et al [19], are useful for understanding the behavior and failure progression of single z-pins under various loading conditions. However, explicitly modeling individual z-pins is not suitable for structural-level models where hundreds or thousands of z-pins may be used. Methods aimed at accounting for the behavior of individual z-pins while reducing the complexity of the modeling approach typically use either nonlinear springs at pin locations [20] cohesive zones where two distinct cohesive laws are used to represent individual z-pins and the surrounding unpinned interface [20, 21]. In [20], Bianchi and Zhang used discrete 1D nonlinear spring elements to represent individual pins of a z-pinned end notched flexure (ENF) specimen. The use of discrete springs and concentrated forces, however, can lead to problematic stress singularities. CZM methods offer a robust alternative for modeling through-thickness reinforcements. In [21], a traction-separation law derived from a meso-mechanical model of the pin pull-out process governed the cohesive response at discrete pin locations for a z-pin reinforced double cantilever beam (DCB) model. Cohesive zone modeling approaches have produced encouraging results for z-pinned laminates; however, accounting for individual z-pins in this manner requires fine meshes and can become cumbersome for larger components or structures. An alternative method using cohesive zone modeling to account for z-pinning is to average the bridging effect over the entire reinforced area. Typically, an apparent fracture toughness is defined for the reinforced interface in order to mimic the bridging behavior of multiple z-pins. This averaging/smeared approach has been used by Huang and Waas in [22] and is suitable for models of complex and large structures.

### 1.3 Modeling framework used in thesis

The modeling framework used in this thesis relies on concepts developed in the semi-discrete damage model (SD2M) [23, 24, 25]. The semi-discrete method establishes an efficient compromise between continuum damage models and discrete damage models. Intra-ply damage
utilizes a separation of fiber and matrix failure modes through a deliberate meshing strategy. The meshing strategy creates thin strips of matrix-splitting elements between bulk elements (bulk elements are allowed to fail in fiber mode only, not matrix mode). The meshing approach is extended to cohesive interlayers to improve the representation of inter-laminar and intra-laminar damage interactions. Advanced constitutive models, including a novel mixed-mode formulation, govern the bulk material (continuum) and interfacial behavior.

1.3.1 Meshing strategy

A meshing strategy is used in the SD2M method to separate bulk material regions and intra-laminar cracking elements. As illustrated in Figure 4, bulk regions are partitioned to create thin strips along the fiber direction. Bulk elements are allowed to fail only in fiber failure modes; however, the thin strips of elements can fail in both fiber and matrix modes. In this way, the thin strips are able to capture the discreteness of intra-laminar cracking. Additionally, material non-uniformity is introduced into the splitting element strips by assigning randomized strength values to each strip. Requirements on the element shape in the bulk regions is also relaxed allowing for a free meshing approach, which can be accomplished using standard meshing algorithms. As shown in Fig. 1.6, the methodology takes advantage of the benefits of the free mesh, fiber-aligned mesh and discrete methods.

It is noted: for the work in this thesis, bulk regions were modeled as linear elastic (with an exception discussed in Chapter 3) because fiber failure was not a commonly observed failure mode in the pi joint. While the SD2M meshing strategy was used, matrix splitting failure was captured using cohesive elements, which is consistent with discrete damage methods. The modeling of the skin laminate is described in the chapters to follow.
1.3.2 Cohesive mixed mode formulation

The cohesive mixed-mode law is formulated in terms of an effective separation and allows the usage of arbitrary softening laws for each fracture mode. Additionally, the model enforces the physical requirement that all traction components must smoothly and simultaneously vanish for a fully developed crack. The principles of the mixed-mode law are illustrated in Fig. 1.7. Pure-mode and mixed-mode responses are shown in Fig. 1.7a. The initial response of each mode is linear elastic with stiffness $K_i$, where $i = I, II, III$ represent the fracture modes (opening mode I, shear modes II and III). The constitutive equation relating traction and separation components is given in Eq. 1.1. Transition from pre-peak response to post-peak softening occurs with the satisfaction of a damage initiation criterion. In the current work, the Hashin quadratic stress criterion is used (Eq. 1.2). The cohesive response following damage initiation is governed by the separation components and non-dimensionalized softening functions $\bar{f}_i$. Post-peak failure separations, $\hat{\delta}_i$, can be determined using Eq. 1.3 and Eq. 1.5 where $G_{ic}$ is the pure-mode fracture toughness value for $i = I, II, III$. In mixed mode loading, the softening response of each model is governed by the effective separation in Eq. 1.3, and all traction components vanish at $\delta_e = \hat{\delta}_e$. A complete description of the cohesive framework and novel mixed-mode law can be found in [25].
\[
\begin{bmatrix}
\sigma_I \\
\tau_{II} \\
\tau_{III}
\end{bmatrix} =
\begin{bmatrix}
(1 - d_I) & 0 & 0 \\
0 & (1 - d_{II}) & 0 \\
0 & 0 & (1 - d_{III})
\end{bmatrix}
\begin{bmatrix}
K_I & 0 & 0 \\
0 & K_{II} & 0 \\
0 & 0 & K_{III}
\end{bmatrix}
\begin{bmatrix}
\delta_I \\
\delta_{II} \\
\delta_{III}
\end{bmatrix}
\tag{1.1}
\]

\[
\left(\frac{\max(0, \sigma_I)}{\sigma_{Ic}}\right)^2 + \left(\frac{\tau_{II}}{\tau_{IIc}}\right)^2 + \left(\frac{\tau_{III}}{\tau_{IIIc}}\right)^2 \geq 1
\tag{1.2}
\]

\[
\delta_e = \sqrt{\max(0, \delta_I)^2 + \left(\frac{\delta_{II}}{\delta_{IIc}}\right)^2 + \left(\frac{\delta_{III}}{\delta_{IIIc}}\right)^2}
\tag{1.3}
\]

\[
\hat{\delta}_i = |\delta^*_i| + \delta_i^f
\tag{1.4}
\]

\[
F_i\delta_i^f \bar{g}_{ie}^* = G_{ie} - \frac{F_i^2}{2K_i}
\tag{1.5}
\]

\[
d_i = 1 - \frac{\delta_{ie}}{\dot{\delta}_e} \left(\frac{\delta_e - \delta^*_e}{\dot{\delta}_e - \delta^*_e}\right)
\tag{1.6}
\]
1.4 Organization of this thesis

This thesis focuses on computational studies of quasi-statically and fatigue loaded composite pi joints with and without z-pin reinforcement. Accompanying experimental studies will be briefly introduced and discussed; however, these studies were not performed by the author. The computational work aims to develop a predictive progressive failure analysis model that can be used in the certification process of bonded composite structures with and without damage arrestment features.

Because the majority of this work was done as part of the FASTBUCS research program, this thesis will follow the chronological order of that program. Quasi-static pi joint studies were divided into three stages. In each stage, specimen and loading condition complexities increased. Pi joint specimens for the three stages are shown in Fig. 1.8. In the first stage,
unpinned and z-pinned Narrow Elements were tested in pull-off and side-bend loading, and corresponding finite element models were developed and calibrated using experimental data. Material properties calibrated through the inverse method during stage 1 were subsequently used in stage 2. In the second stage, blind predictions of Wide Element Type 1 specimens (unpinned and z-pinned) were established allowing for an initial assessment of the predictive capabilities of the model. Following the blind predictions, a recalibration effort was also performed in stage 2 to improve correlation leading into stage three. In the third stage, final assessment of the predictive capabilities of the model was demonstrated for Wide Element Type 2 specimens (unpinned and z-pinned). The Wide Element Type 2 specimens were the largest specimens investigated and subjected to the most complex loading condition considered during the project (combined axial compression and push-off loading).

Following the quasi-static studies, fatigue investigations were performed using z-pin reinforced Narrow Elements subjected to cyclic pull-off loading.

Figure 1.8: Pi joint element types.

The organization of this thesis is as follows. Chapter 2 will present the non-reinforced Narrow Element model development as well as experimental and numerical comparisons for the unpinned configurations. Narrow Element specimens were subjected to pull-off and side-bend loading. The contents of Chapter 2 are published in [26]. The smeared approach for modeling the influence of z-pinning will be introduced in Chapter 3 and z-pin reinforced Narrow Element results will be presented. The contents of Chapter 3 are published in [27]. In Chapter 4, blind predictions of the unpinned and z-pinned Wide Element Type 1 configurations will be compared to experimental results and shortcomings of model input parameters will be addressed. The contents of Chapter 4 have been submitted for journal publication. Final demonstration of the predictive capabilities of the model will be shown in
Chapter 5 for the Wide Element Type 2 specimen (unpinned and z-pinned).

Chapter 6 will detail the development of a unified local cohesive fatigue model. The model will be validated through a double cantilever beam fatigue study and sensitivities of fatigue model input parameters will be explored. In Chapter 7, the fatigue model will be applied to study fatigue behavior of z-pinned Narrow Element specimens. The fatigue model was initially calibrated to constant amplitude fatigue crack growth data and subsequently used to establish blind predictions for two block fatigue spectra. The contents of Chapter 6 and Chapter 7 are being prepared for journal publication.

Finally, Chapter 8 will conclude the thesis with major findings, existing challenges, and topics for future research.
CHAPTER 2

Unreinforced Narrow Element Model
Development and Experimental Correlation

2.1 Introduction

The aim of this chapter is the development and calibration of a progressive failure analysis model of an unreinforced Narrow Element pi joint. A full 3D finite element model is developed using the commercially available finite element software Abaqus [28] and correlated to experimental results for pi joints subjected to pull-off loading as well as side-bend loading. The finite element model implements ply-level modeling techniques with both interlaminar and intralaminar damage capabilities in order to accurately capture the progression of damage and failure of the pi joint. Calibration of the model relies on inverse modeling at the element-level, thereby avoiding the need for coupon-level testing. The work presented in this chapter is part of a more extensive program where further validation of the modeling approach will be performed through blind predictions of pi joint specimens and load cases of greater complexity [29].

2.2 Experimental testing of pi joints

The skin and the web of the pi joint specimens were made from a toughened carbon-epoxy material system. The skin laminate was fabricated with unidirectional lamina and had a [29/57/14] composition, where 29 represents the percentage of 0° plies, 57 represents the percentage of ±45° plies, and 14 represents the percentage of 90° plies. A woven fabric was used to construct the web. The skin and the web were bonded to the pi preform using a film

1 Parts of this chapter are published in [26].
adhesive. Both co-bonded and co-cured specimens were manufactured. A portion of the specimens had an aluminum plate, of the same in-plane dimensions as the skin, bonded to the bottom of the laminate skin using a paste adhesive. The two joint skins are identified as L1 and L2 corresponding to the laminate skin only and the laminate skin bonded to an aluminum plate, respectively. Defected joints were manufactured by introducing a Teflon insert along the adhesive bond line between the adhesive and the skin (causing an initial disbond). The Teflon insert extended the full width of the joint and was in one of two locations. The D1 defect was centrally located with respect to the pi preform (in the throat region), and the D2 defect was located in one of the tip regions of the joint (near the termination of the adhesive layer). Throat and tip regions are indicated in Fig. 2.1.

![Figure 2.1: Schematic diagram of a pi joint.](image)

Pi joint specimens were tested at the National Institute for Aviation Research (NIAR). Four loading configurations were investigated: L1 four-point pull-off (L1-4pt-PO), L1 side-bend (L1-SB), L2 four-point pull-off (L2-4pt-PO), and L2 two-point pull-off (L2-2pt-PO). Figure 2.2 shows schematic representations of the four loading configurations. In pull-off loading, the skin was restrained while a tensile (pull-off) load was applied to the top of the web. For side-bend loading, the skin was constrained in a manner preventing rigid-body rotation while a transverse load was applied to the web above the pi preform. Co-cured joints were 50% wider than the co-bonded joints, however both types were subjected to identical loading conditions. Experimental results showed no significant differences in terms of width-normalized load-displacement responses nor failure modes between the co-bonded and co-cured specimens. For each configuration, tests were performed for
Figure 2.2: Loading configurations. a) Four-point pull-off loading (L1). b) Side-bend loading (L1). c) Four-point pull-off loading (L2). d) Two-point pull-off loading (L2).

pristine (no defect) and defective joints. For the L1-4pt-PO and L2-2pt-PO configurations, the defect was located in one of the tip regions of the joint. For the L1-SB and L2-4pt-PO configurations, the defect was in the throat region. Joints were loaded to complete failure. For all configurations, delamination was the dominate failure mode, however the location and interfaces experiencing delamination varied based on loading condition. While failure modes did not change significantly between pristine and defective specimens, joints with defects experienced strength knockdowns of approximately 30%-40% compared to pristine joints. Multiple specimens were tested, both pristine and defective, per loading condition with consistent results providing a strong data set for validation of the computational model.
2.3 Computational modeling and analyses

A full 3D finite element model was developed using Abaqus to investigate the structural response of the pi joint assembly and to ultimately arrive at a progressive damage and failure analysis model. To avoid convergence issues associated with implicit solvers for progressive failure analyses, dynamic explicit simulations were used. The model primarily used 3D continuum elements with reduced integration (C3D8R, C3D6R) for bulk material elements, however in regions of high load transfer (near the base of the pi preform), full integration elements (C3D8, C3D8I) were used. Away from the load transfer region of the joint, a coarse far-field mesh was used. Figure 2.3 shows the mesh of the joint model. To ensure calibrated properties were valid for all configurations of the study, the same mesh seeding was applied to all joint models.

In the model, the pi preform was modeled with homogenized orthotropic material properties. The adhesive layer between the base of the preform and the top of the skin was modeled using solid continuum elements, as opposed to finite-thickness cohesive elements.
Incremental plasticity was included in the adhesive material definition. Adhesive failure was accounted for with cohesive element interlayers connecting the pi preform base to the adhesive and the skin to the adhesive. The laminate web was modeled based on a set of two sub-laminate homogenization processes which homogenized repeated symmetric groups of laminas. The unidirectional laminate skin was modeled using a combined approach of both ply-level and sub-laminate homogenization modeling. The four skin plies nearest the pi preform were modeled with individual material orientations, while the remaining portion of the skin plate was subjected to a homogenization process and modeled as an equivalent orthotropic plate.

Teflon inserts representing defects were accounted for in the model by degrading the cohesive material properties (strength and critical energy release rate values) of the adhesive-skin interlayer in the area corresponding to the insert. By doing so, the corresponding cohesive elements failed very early in the analysis and the surface pair in the region subsequently defaults to the general contact formulation, which enforced frictional sliding and a hard contact penalty for penetration. Because the Teflon insert’s thickness was of the same order as the adhesive’s thickness, the stiffness of the solid material in the volume corresponding to the Teflon insert was reduced to be more representative of the Teflon material.

The two major damage modes observed experimentally were accounted for in the model: intra-laminar damage and inter-laminar damage.

### 2.3.1 Intra-laminar damage modeling

Intra-laminar damage, specifically matrix crack modeling, was included in the model through a novel meshing approach motivated by the work of Nguyen [23]. In the model, parts representing individual skin plies were first partitioned along the ply’s fiber-direction such that thin strips were created. Figure 2.4 illustrates the thin strip partitions for a 45° lamina. Adopting a discrete damage modeling approach, the thin strips were meshed with finite-thickness cohesive elements (COH3D8, COH3D6). Cohesive element behavior followed the novel mixed-mode law described in [25]. Triangle traction-separation laws were used, and the onset of damage was initiated following the Hashin quadratic stress criterion shown in Eq. (2.1). Damage evolution and failure was governed by an effective
separation comprised of individual separation components and shown in Eq. (2.2). Through
the definition of an effective separation, the smooth and simultaneous vanishing of all
cohesive traction components can be enforced. More details about the mixed-mode law and
definitions can be found in [25].

\[
\left( \frac{\max(0, \sigma_I)}{\sigma_{Ic}} \right)^2 + \left( \frac{\tau_{II}}{\tau_{IIc}} \right)^2 + \left( \frac{\tau_{III}}{\tau_{IIIc}} \right)^2 \geq 1
\]  

\(\delta_e = \sqrt{\max(0, \delta_I)^2 + \left( \frac{\delta_I}{\delta_{Ic}} - \delta_{II} \right)^2 + \left( \frac{\delta_I}{\delta_{III}} - \delta_{III} \right)^2}
\)

Cohesive materials with randomized strength and toughness values were assigned to
the matrix crack strips to introduce material non-uniformity into the model (depicted using
different colors in Fig. 2.4). Random cohesive properties were used for several reasons,
but the overarching motivation was to create a pi joint model that closely matched reality.
Not only do random properties break unrealistic symmetries in computational models, their
usage also allows for damage and failure localization. It is expected that when a crack
begins to develop, the material in the immediate vicinity will relax, preventing additional
cracking in the surrounding material. As damage localizes and the crack progresses towards
failure, the damaged region acts as a site for delamination initiation between plies. Correctly
capturing this interaction between matrix ply cracks and delamination is important for
progressive failure modeling. It has been shown in [23], that using a uniform probability
distribution to define the random properties results in good agreement with reality and
allows for accurate representation of the upper and lower bounds of the property.

2.3.2 Inter-laminar damage modeling

In the finite element model, inter-laminar damage was modeled using cohesive element
interlayers, using the same mixed-mode formulation introduced for the intra-laminar
damage. The cohesive element interlayers were used to connect adjacent parts using tie
constraints and were meshed following the strategy in [25]. Interlayer meshes were created
in a manner that aids in the coincident matching of interlayer element nodes with nodes of
Figure 2.4: Intra-laminar damage modeling for a 45° skin ply. The thin strips along the fiber direction represent matrix cracking elements and are assigned random cohesive properties.

the matrix ply-crack elements of the adjacent plies. Figure 2.5 illustrates an example of the interlayer meshing approach for a 45°/-45° interface. Compatible meshes were created by partitioning the interlayer with features of the surrounding plies which, when meshed using an Abaqus meshing algorithm, resulted in matching nodes and improved robustness of the cohesive elements. Interlayers were meshed with a slightly finer mesh than the surrounding plies to improve connectivity.

Cohesive interlayers were used to model the pi preform-adhesive interface and the adhesive-skin interface accounting for adhesive failure. Additionally, cohesive interlayers were used for individual skin ply interfaces to account for inter-laminar delamination. Triangle traction-separation laws were used for the ply-to-ply interlayers and a quasi-trapezoidal traction-separation law was used for the adhesive skin interlayer. For pull-off loading cases, a triangle traction-separation law was used for the pi preform-adhesive interlayer; however, for side-bend loading, a quasi-trapezoidal traction-separation law was used for the interface. The quasi-trapezoidal traction-separation law was shown to provide better results in side-bend loading; however, its effect on pull-off loading cases still needs to be considered. Obtaining agreement between experimental and computational results (load-displacement and damage patterns) for all configurations is challenging.

2.3.3 Inverse modeling and calibration

The pi joint model was created, in part, to calibrate the in-situ cohesive properties of the adhesive and the matrix material of the laminate. The model, therefore, relied heavily on experimental data. Beyond cohesive material properties, certain modeling details were
motivated by experimental results. For example, the number of interlayers within the laminate skin was chosen such that the delamination patterns observed experimentally could be fully captured without over-restricting the model, while at the same time avoiding unnecessary and computationally intensive features. Three inter-layers were included within the skin for pull-off loading models, while two were included for side-bend models. Another feature included based on experimental observations were adhesive tip cracks. Adhesive cracks develop in the tip region due to the material discontinuity at the termination of the pi preform and the adhesive ramp (Fig. 2.3). Similar to the matrix-crack elements included in the skin plies, cohesive element strips were used to model adhesive crack development in the tip regions.

Model calibration relied on interfacial stress analyses as well as sensitivity studies to arrive at suitable cohesive property sets. Dakota [30], which is an open source parametric analysis software available through Sandia National Laboratories, was used to perform Latin Hypercube Sampling (LHS) studies for the joint configurations of interest. The LHS studies provided sensitivities of structural response quantities (e.g. peak load) to model input parameters (cohesive strength and critical energy release rate values). Through correlation results from the sampling studies, it was possible to determine the influence each cohesive property had on the peak load predicted by the model for a given configuration. Calibration of the model was an iterative process in which the location of failure (interface and region) was first achieved, followed by the adjustment of material parameters (guided by LHS
sensitivities) to establish agreement with experimental critical loads.

2.4 Computational and experimental comparisons

Comparisons between the computational predictions and experimental data are presented in this section for pristine and defected joints subjected to pull-off loading and side-bend loading. Both co-bonded and co-cured experimental data (normalized by joint width) was used to calibrate the model. It is noted that model predictions are presented from simulations using the same material properties determined during the calibration process. The computational and experimental results are compared in terms of structural response, critical loads, and damage and failure modes. In the following numerical results, cohesive stiffness degradation (cohesive damage) is used to represent delamination in the cohesive interlayers (pi-adhesive, adhesive-ply 1, ply 1-ply 2, ply 2-ply 3, and ply 3-ply 4).

2.4.1 L1-4pt-PO

Figure 2.6 shows the force-displacement responses of the pristine L1-4pt-PO configuration. Both experimental and computational force-displacement curves remain linear up to peak load; however, the model response exhibits a slightly lower stiffness than the experimental responses. The peak load predicted by the model agrees well with experimental peak loads and was within 2% of the average experimental peak load. The model successfully captured the sudden and complete joint failure observed experimentally.
Figure 2.6: Comparison of experimental and computational force-displacement responses of L1-4pt-PO (pristine).

Figure 2.7: L1-4pt-PO (pristine) failure results. a) Delamination patterns predicted by the finite element model at joint failure. b) Typical failed specimen.

The delamination predictions at failure and a typical failed pristine specimen are shown.
in Fig. 2.7. Delamination is predicted in the adhesive-skin, ply 1-ply 2, and ply 2-ply 3 interfaces. Although occurring in rapid succession, the model first predicts delamination growth in the adhesive-skin interface, which progresses into skin ply interfaces through matrix ply cracks. As shown in Fig. 2.7b, the interfaces where delamination is predicted by the model are the same interfaces in which delamination was observed experimentally. Experimental delamination (Fig. 2.7b) initiated in one of the tip regions within the adhesive-ply 1 interface, and quickly progressed towards the opposite tip within the ply 1-ply 2 and the ply 2-ply 3 interfaces.

Defective joint responses are overlayed on the pristine (no defect) results in Fig. 2.8. For this configuration, the Teflon insert was in the right tip of the joint. The defect results in a significant strength knockdown as well as a slightly reduced stiffness. The model stiffness is about 5% lower than that recorded experimentally for both pristine and defective configurations. The peak load predicted by the model for the defective joint is 2.5% lower than the average peak load of the tested defective specimens. Compared to the pristine joints, a strength knockdown of 47% was observed experimentally as well as predicted computationally for defective joints. The post-peak response of the model does not predict complete failure of the joint which is inconsistent with test results. This prediction of a load rebound suggests that the calibrated critical energy release rate values used for the ply-to-ply cohesive interlayers of the skin were too high.

Computational and experimental failure results are shown in Fig. 2.9 for the tip defect case. As predicted by the model, delamination initiating from the right tip defect in the adhesive-ply 1 interface progresses into the ply 1-ply 2 interface of the skin. Experimental test specimens (Fig 2.9b) showed a similar failure progression from the defective tip; however, delamination was observed to be distributed in the ply 1-ply 2 and the ply 2-ply 3 interfaces (as opposed to primarily in the ply 1-ply 2 interface).
Figure 2.8: Comparison of experimental and computational force-displacement responses of L1-4pt-PO (D2 and pristine).

Figure 2.9: L1-4pt-PO (D2) failure results. a) Delamination patterns predicted by the finite element model immediately following peak load of defective joint. Defect was located in the right tip region and marked by the dark red rectangle in adhesive-ply 1 interface. b) Typical failed defective specimen.
2.4.2 L1-SB

Figure 2.10 shows the force-displacement curves of the pristine L1-SB configuration. In side-bend, nonlinearity was observed in the experimental responses as the joints approached peak load levels. Sharp load drops were seen immediately following peak load; however, the joints did not completely fail. Load rebounds were observed, and the joints were able to support further loading. This characteristic is also predicted by the model and agreement is seen between the structural response of the model and the experimental specimens. The peak load predicted by the model was 4% lower than the average peak load measured experimentally. The numbered points along the finite element model force-displacement curve in Fig. 2.10 correspond to snapshots of the delamination patterns shown in Fig. 2.11. The first load drop (point 1 – point 2) is caused by rapid delamination growth in the ply 1 – ply 2 interface near the center of the joint. After the initial load drop, delamination in the ply 1 – ply 2 interface grows slowly from the throat towards the right tip of the joint, during which further loading is supported (point 2 – point 3). Final failure occurs as delamination progresses to the right end of the bond area within the ply 1 – ply 2 interface. This progression of damage to failure is consistent with that observed experimentally.

Figure 2.10: Comparison of experimental and computational force-displacement responses of L1-SB (pristine).
Figure 2.11: Delamination patterns predicted by the finite element model during side-bend loading.

Figure 2.12: L1-SB (pristine) failure results. a) Delamination patterns predicted by the finite element model at joint failure. b) Typical failed experimental specimen.

Regarding the predicted force-displacement response, it is believed that the lack of non-linearity near peak load, as well as the higher load level at which the load rebound occurs, are discrepancies caused by diffuse damage in the throat of the pi preform. Experimentally, some configurations have shown diffuse damage develop in the pi preform throat; however,
the influence of this damage has yet to be fully investigated. In the model, the pi preform was modeled using linear material behavior, and therefore, preform damage is currently not accounted for in the model. It is also noted that the oscillatory behavior of the predicted response following the initial load drop is a result of dynamics in the explicit analysis, which is avoided in quasi-static tests.

Figure 2.12 compares the delamination patterns predicted at failure to a typical failed pristine specimen. Because the skin material was a toughened system, it was not uncommon to see intra-laminar delamination within the skin plies. However, discretization of the model was limited to the ply-level, and therefore, intra-laminar delamination predictions were not possible. While the experimental specimens show intra-laminar delamination of the first skin ply, the model correctly predicts the critical delamination occurring in the ply 1-ply 2 interface. Failed specimens show a triangular delaminated region in the pi preform-adhesive interface; however model predictions show this same pattern divided into the adhesive-ply 1 and ply 1-ply 2 interfaces.

Defective joint force-displacement curves are overlayed on the pristine results in Fig. 2.13. For side-bend loading, the defect was in the throat of the joint. Defective joints experienced an earlier load drop; however, it was much less severe than the initial load drop experienced by the pristine joints. The initial load drop predicted by the model occurs at a load 16% higher than the experimental average, but still within the spread of the experiments. Following the initial load drop, an extended period of reloading was observed before complete failure. The load rebound response of the model shows a higher level of residual stiffness compared to the experimental results, which is likely due to diffuse damage in the throat of the pi preform. Figure 2.14 shows delamination snapshots corresponding to several points along the predicted force-displacement curve in Fig. 2.13. The first load drop (point 1 – point 2), is a result of delamination initiating from the throat defect and moving into the ply 1-ply 2 interface. As the load rebounds, slow delamination growth is predicted moving towards the right tip. Complete failure occurs as delamination progresses through the right-half of the bond area in the adhesive-ply 1 and ply 1-ply 2 interface.

Figure 2.15 shows a typical failed defective specimen. Experimentally, defective joints show no significant difference from failed pristine joints. However, this observation was not captured by the model, as delamination predicted in the adhesive-ply 1 interface is
more prominent for the defective model than it is for the pristine model. Still, the predicted progression and delamination footprint at failure in the ply 1-ply 2 interface is consistent with experimental observations.

Figure 2.13: Comparison of experimental and computational force-displacement responses of L1-SB (D1 and pristine).
Figure 2.14: Delamination patterns predicted by the finite element model during side-bend loading. Defect was located in the throat region of the adhesive-ply 1 interface and marked by the dark red rectangle.

Figure 2.15: L1-SB (D1) failure results. a) Delamination patterns predicted by the finite element model at joint failure. Defect was located in the throat region and marked by the dark red rectangle in adhesive-ply 1 interface b) Typical failed defective specimen.
2.4.3 L2-2pt-PO

Figure 2.16 shows the force-displacement responses of the pristine L2-2pt-PO configuration. Both experimental and computational load-displacement curves remain linear up to peak load; however, the model response exhibits a slightly higher stiffness. The model prediction shows oscillatory behavior which is characteristic of dynamics in explicit simulations. These oscillations appear to be exacerbated by the wider support span of the two-point loading condition. The peak load predicted by the model is 13% higher than the experimental average; however, the prediction is still within the range of the experimental peak loads.

The delamination predictions at failure and a typical failed pristine specimen are shown in Fig. 2.17. Delamination is primarily predicted in the ply 2-ply 3 interface; however, the adhesive-skin and ply 1-ply 2 interfaces also experience delamination. Two-piece failure of test specimens typically shows intra-laminar delamination of the third skin ply as the primary failure mode. Because intra-laminar delamination is not included in the model, prediction of the large delaminated region in the ply 2-ply 3 interface is satisfactory. The model also predicts the tendency for delamination to initiate from one tip and progress through the center of the joint beyond the opposite tip, which was also commonly observed experimentally for this configuration.

Figure 2.16: Comparison of experimental and computational force-displacement responses of L2-2pt-PO (pristine).
Experimental and computational force-displacement responses of the tip defected joint are included in Fig. 2.18. The tip defect resulted in an average strength knockdown of 36% in the experiments and a 40% knockdown based on model predictions. The peak load predicted by the model for the defective joint was 6% higher than the experimental average. Like the pristine joints, the defective joints have a linear response up to peak load followed by sudden and complete failure.

Figure 2.19 shows the computational and experimental failure results. While the presence of the defect causes delamination to initiate from the defective tip, the final failure mode is not significantly different from that of the pristine joint. Model delamination predictions agree well with those observed experimentally, and in both cases, significant delamination is predicted in the first two interfaces of the laminate skin.
Figure 2.18: Comparison of experimental and computational force-displacement responses of L2-2pt-PO (D2 and pristine).

Figure 2.19: L2-2pt-PO (D2) failure results. a) Delamination patterns predicted by the finite element model at joint failure. Defect was located in the right tip region and marked by the dark red rectangle in adhesive-ply 1 interface. b) Typical failed defective specimen.
2.4.4 L2-4pt-PO

Figure 2.20 shows the computational force-displacement curves of the pristine and D1 L2-4pt-PO configurations. For this configuration, the Teflon insert was in the throat of the joint. Experimental force-displacement results included loading fixture compliance, and therefore only experimental peak loads (shown as horizontal lines) are displayed. The peak load predicted by the pristine joint model was within 2% of the average experimental peak load, and the predicted defective peak load was 2% higher than the average of the defective specimens tested. The strength knockdown due to the throat defect found experimentally and predicted computationally was 40%.

![Figure 2.20: Comparison of experimental peak loads and computational force-displacement responses of L2-4pt-PO (D1 and pristine).](image)

Delamination predictions from the pristine model at failure and a typical failed experimental specimen are shown in Fig. 2.21. The most critical delamination is predicted in the pi preform-adhesive interface which agrees well with experimental observations (adhesive is the light-colored material in Fig. 2.21b). While failed test specimens commonly showed less critical delamination in the adhesive-skin and ply 1-ply 2 interfaces, the model overpredicts the amount of delamination in these interfaces.

Figure 2.22 shows delamination predictions at failure and a typical failed defective spec-
imen. Experimentally observed and predicted failure modes show delamination primarily in the pi preform-adhesive interface. Compared to pristine failure, defective specimens show less delamination beyond the pi preform-adhesive interface and this observation is also captured by the model.

Figure 2.21: L2-4pt-PO (pristine) failure results. a) Delamination patterns predicted by the finite element model at joint failure. b) Typical failed specimen.
**Figure 2.22**: L2-4pt-PO (D2) failure results. a) Delamination patterns predicted by the finite element model at joint failure. Defect was located in the throat region and marked by the dark red rectangle in adhesive-ply 1 interface. b) Typical failed defective specimen.

## 2.5 Conclusion

A finite element model of a Narrow Element pi joint has been developed as a progressive failure analysis tool. The model serves two purposes: prediction of structural response and failure behavior of composite pi joints, as well as an inverse modeling tool for back-calculating cohesive properties. A novel mixed-mode cohesive formulation was implemented to govern the traction-separation response of both matrix ply crack elements and cohesive interlayers. Inclusion and interaction of intra-laminar and inter-laminar damage modes allowed for the accurate representation and progression of damage and failure observed in pi joint specimens subjected to pull-off loading and side-bend loading. Results using calibrated cohesive properties were shown to be in good agreement with experimental results for eight different test scenarios in terms of structural response, critical loads, damage progression, and failure modes.
CHAPTER 3

Progressive Failure Analysis of Z-Pin Reinforced Composite Pi Joints

3.1 Motivation

In the present chapter, a smeared cohesive zone approach similar to that developed in [22] is used to model the effect of z-pinning on composite pi joints. The smeared CZM approach is applied to the finite element model developed in Chapter 2 for the unpinned Narrow Element. The pi joint is a relatively complex structure; however, the smeared approach implemented in this work offers a practical balance between fidelity, modeling complexity, and computational cost. Cohesive properties of the z-pin reinforced areas were calibrated using experimental results of both pristine and defective z-pinned joints subjected to several loading conditions. Comparison of experimental and finite element analysis results are shown to be in good agreement, thereby validating the modeling approach.

3.2 Experimental testing of z-pinned pi joints

Experimental specimens were fabricated at Northrop Grumman Corporation (NGC). The skin and the web of the pi joint specimens were made from a toughened carbon-epoxy material system. The skin laminate was fabricated with unidirectional lamina and had a [29/57/14] percentage composition, where 29 represents the percentage of 0° plies, 57 represents the percentage of ±45° plies, and 14 represents the percentage of 90° plies. A woven fabric was used to construct the web. The skin and the web were bonded to the 3D

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1Parts of this chapter have been published in [27] and submitted to the International Journal of Solids and Structures.
woven pi preform using a film adhesive. Joints were reinforced with 2% z-pin areal density along the pi base/skin overlap (Fig. 3.1). After the z-pinning process was completed, the assembly was co-cured. A portion of the specimens had an aluminum plate of the same in-plane dimensions as the skin bonded to the bottom of the laminate skin using a paste adhesive. The two joint skins are identified as L1 and L2 corresponding to the laminate skin only and the laminate skin bonded to an aluminum plate, respectively. Defected joints were manufactured by introducing a Teflon insert along the adhesive bond line between the adhesive and the skin (causing an initial disbond). The Teflon insert extended the full width of the joint and was in one of two locations. The D1 defect was centrally located with respect to the pi preform (in the throat region), and the D2 defect was located in one of the tip regions of the joint (near the termination of the preform base).

Pi joint specimens were tested at the Air Force Research Laboratory (AFRL). Four loading configurations were investigated: L1 four-point pull-off (L1-4pt-PO), L1 side-bend (L1-SB), L2 four-point pull-off (L2-4pt-PO), and L2 two-point pull-off (L2-2pt-PO). Figure 3.2 shows schematic representations of the four loading configurations. In pull-off loading, the skin was restrained while a pull-off load was applied to the top of the web. For side-bend loading, the skin was constrained in a manner preventing rigid-body rotation while a transverse load was applied to the web above the pi preform legs. For the L1-4pt-PO and L1-SB configurations, tests were performed for pristine (no defect) and defective joints. For the L2-4pt-PO and L2-2pt-PO configurations, only defective specimens were tested. The defect for the L1-4pt-PO and L2-2pt-PO configurations was located in one of the tip
regions of the joint (D2). For the L1-SB and L2-4pt-PO configurations, the defect was in the throat region (D1). For all configurations, delamination was the dominate damage mode; however, diffuse damage in the pi preform was observed for the L1-SB and the L2-4pt-PO configurations, and in some cases failure of the pi preform occurred.

### 3.3 Numerical modeling

A full 3D finite element model was developed using Abaqus/Explicit [28] to investigate the structural response of the z-pinned pi joint and to ultimately arrive at a progressive damage analysis model. Using a smeared approach to model the influence of z-pinning, only modest modifications to the model developed in [26] were required. An abbreviated description of the finite element model is provided in this section; however, further details can be found in
In the model, the meso-scale details of the woven pi preform were not included, and instead, a macro-scale representation with homogenized orthotropic material properties was implemented. The adhesive film layer between the base of the pi preform and the top of the skin was modeled using an incremental plasticity material model and solid continuum elements. Cohesive element interlayers tying the pi preform base to the adhesive and the skin to the adhesive were used to capture adhesive failure. The unidirectional laminate skin was modeled using a combination of ply-level and sub-laminate homogenization modeling techniques. The four skin plies nearest the pi preform were modeled individually, while the remaining skin plies were subjected to a homogenization process and modeled as an equivalent orthotropic plate. Teflon inserts, representing defects, were accounted for in the model by reducing the cohesive material properties (strength and critical energy release rate values) of the adhesive-skin cohesive interlayer in the area corresponding to the insert. By doing so, the corresponding cohesive elements failed early in the analysis and subsequent interactions between the corresponding surface pair defaulted to the general contact formulation.

3.3.1 Intra-laminar damage modeling

Intra-laminar damage, specifically matrix cracking, was included in the model through a novel meshing approach motivated by the work of Nguyen and Waas [31]. In the model, parts representing individual skin plies were partitioned along the ply’s fiber-direction such that thin strips were created. Figure 3.3 illustrates the thin strip partitions for a 45° lamina. The thin strips were meshed with finite-thickness cohesive elements (COH3D8, COH3D6) while the bulk regions between strips were meshed with continuum elements. This method allows for sharp matrix cracks to be captured by the model. The cohesive element behavior followed the novel mixed-mode law described in [25]. Triangle traction-separation laws were used, and the onset of damage was initiated following the satisfaction of the Hashin quadratic stress criterion shown in Eq. 3.1. Damage evolution and failure (post-peak response) was governed by an effective separation comprised of individual separation components and shown in Eq. 3.2. Through the definition of an effective separation, the smooth and simultaneous vanishing of all cohesive traction components can be ensured. More details about the mixed-mode law and definitions can be found in [25].
\[
\left( \frac{\max(0, \sigma_I)}{\sigma_{Ic}} \right)^2 + \left( \frac{\tau_{II}}{\tau_{IIc}} \right)^2 + \left( \frac{\tau_{III}}{\tau_{IIIc}} \right)^2 \geq 1 \tag{3.1}
\]

\[
\delta_e = \sqrt{\max(0, \delta_I)^2 + \left( \frac{\dot{\delta}_I}{\Delta_{IIc}} \right)^2 + \left( \frac{\dot{\delta}_I}{\Delta_{IIIc}} \right)^2} \tag{3.2}
\]

Cohesive materials with randomized strength and toughness values were assigned to the matrix crack strips to introduce material non-uniformity into the model (illustrated using different colors in Fig. 3.3). In progressive failure models, non-uniform material properties allow for localization of damage and a more realistic development and progression of failure. It has been shown in [24], that using a uniform probability distribution to define the random properties results in good agreement with reality and allows for accurate representation of the upper and lower bounds of a property.

![Figure 3.3: Intra-laminar damage modeling for a 45° skin ply. The thin strips along the fiber direction represent matrix cracking elements and are assigned random cohesive properties.](image)

### 3.3.2 Inter-laminar damage modeling

Inter-laminar damage was modeled using cohesive element interlayers and governed by the same mixed-mode formulation used for intra-laminar damage. The cohesive element interlayers were used to connect adjacent parts using tie constraints and were meshed following the strategy in [25]. Interlayer meshes were created in a manner that promotes coincident matching of interlayer element nodes with nodes of the matrix ply-crack elements of the adjacent plies. Figure 3.4 illustrates an example of the interlayer meshing approach.
for a $45^\circ / -45^\circ$ interface. Compatible meshes were created by partitioning the interlayer with features of the surrounding plies which, when meshed, resulted in matching nodes and improved robustness of the tie constraints. Cohesive interlayers were used to model the preform-adhesive interface and the adhesive-skin interface accounting for adhesive failure. Additionally, cohesive interlayers were used for individual skin ply interfaces to account for inter-laminar delamination.

### 3.3.3 Z-pin modeling – smeared approach

Generally, with structural models it is not feasible to explicitly model individual z-pins. Such models are prohibitively complex and computationally expensive. Instead, a smeared approach has been implemented in this work to account for the z-pinning behavior in an averaged sense. To implement the smeared approach, all cohesive interlayers were partitioned creating z-pin reinforced and unreinforced areas. The z-pin reinforced areas had dimensions equal to the rectangular area containing the z-pin fields of the manufactured joint. Two distinct cohesive property sets were used to define the unreinforced and the z-pin reinforced cohesive responses. Figure 3.5 illustrates the smeared modeling strategy used for the z-pinned pi joint. For the unreinforced interfaces, cohesive properties previously calibrated from the unpinned joint experiments were used [26]. Z-pin reinforced cohesive properties were calibrated using experimental data from z-pinned joints through the inverse method. The effect of z-pinning on interlaminar properties, specifically fracture toughness, has
Characterization of apparent fracture toughness due to z-pinning at the coupon level, such as DCB [32, 33, 34, 35] and ENF [22, 36, 37], have been established and served as initial estimates in the calibration process. While it is accepted that z-pinning increases fracture toughness, the effect of z-pinning on interlaminar strength is less understood.

Z-pins represent inclusions in the embedded material and often result in a mismatch of elastic constants. Therefore, inhomogeneous stress distributions and stress concentrations can arise near z-pins, dramatically affecting the initiation of damage and failure [38, 39, 40]. Interlaminar stress distributions in a z-pinned and unpinned laminate are sketched in Fig. 3.6. Figure 3.6a shows the interlaminar normal stress (or peel stress) for out-of-plane loading. Figure 3.6b shows the interlaminar shear stress for shear loading. The primary
stress components peak near the pin of the z-pinned laminate for both loading conditions. In contrast, stress distributions in the unpinned laminate are uniform and smooth. Z-pins are not explicitly modeled in the smeared approach; therefore, the z-pin material mismatch and the resulting interlaminar stress peaks that exist in reality are not captured by the model. Because stress concentrations commonly precipitate damage and failure, stress peaking near the z-pins should not be ignored. To account for this effect of z-pinning in the model, the cohesive strengths in the z-pin reinforced interfaces were reduced. In doing so, the uniform stress distributions observed in the z-pin reinforced cohesive interlayers of the model damage and fail more similarly to the z-pinned interfaces of the physical joint.

Lastly, it has been well documented that z-pinning reduces in-plane mechanical properties \cite{41, 42}. Elastic properties degrade with increasing pin density and diameter as a result of disturbances to the laminate microstructure (fiber waving and crimping). However, these reductions of elastic properties are usually under 10\% and were not considered in the model.

### 3.3.4 Crack band model

Because z-pinning acts to prevent a failure mode (delamination), there is a tendency for other damage and failure modes to emerge as the structure dissipates stored energy. In addition to delamination, experimental observations of z-pinned pi joints show diffuse damage within
Figure 3.7: a) Diffuse damage within pi preform throat region. b) Location of crack band bulk elements (blue region) in model.

the pi preform. This diffuse damage (crack development within the weave) was observed primarily for the L1-SB and the L2-4pt-PO test configurations where throat failure is common. Figure 3.7a shows diffuse damage in the pi throat region from a specimen subjected to side-bend loading. Modeling diffuse damage in the pi preform, however, is a challenge due to the complex architecture and inhomogeneity of the 3D weave. The crack band method [43] was implemented as the constitutive behavior of the pi preform material in the throat region to capture damage and failure without including details of the preform weave in the model. Crack band element behavior is divided into pre- and post-peak regimes. In the pre-peak regime, the crack band element behavior is linear elastic. Post-peak softening ensues upon satisfaction of transition criteria and is governed by the mixed-mode law proposed by Joseph and Waas [44] and extended by Lin and Waas [45]. Three fixed failure planes can be considered and are shown in Fig. 3.8. Considering the material orientation defined in Fig. 3.7b and the load scenarios, failure plane 1 was not active in this work. Strain-based damage initiation criteria were used for failure planes 2 and 3 and are given in Eq. 3.3 and Eq. 3.4, respectively. Strain values corresponding to damage initiation of failure plane 2 are $\varepsilon_{22}^{\text{ini,T}}$ (tension), $\varepsilon_{22}^{\text{ini,C}}$ (compression), $\gamma_{12}^{\text{ini}}$, and $\gamma_{23}^{\text{ini}}$. Similarly, strain values corresponding to damage initiation of failure plane 3 are $\varepsilon_{33}^{\text{ini,T}}$ (tension), $\varepsilon_{33}^{\text{ini,C}}$ (compression), $\gamma_{13}^{\text{ini}}$, and $\gamma_{23}^{\text{ini}}$.

\[
\left( \frac{\varepsilon_{22}}{\varepsilon_{22}^{\text{ini,T/C}}} \right)^2 + \left( \frac{\gamma_{12}}{\gamma_{12}^{\text{ini}}} \right)^2 + \left( \frac{\gamma_{23}}{\gamma_{23}^{\text{ini}}} \right)^2 \geq 1 \quad (3.3)
\]
Post-peak damage evolution employs a secant stiffness approach where moduli are degraded as energy is dissipated during the progression of damage. The failure criterion used in the mixed-mode law is given in Eq. 3.5. $G_{Ic}, G_{IIc}$, and $G_{IIIc}$ are the fracture toughness values of the three fracture modes, while $G^*_I, G^*_II$, and $G^*_III$ are the corresponding dissipated energy values. A more detailed writeup of the implemented crack band approach can be found in [45]. Because crack band was used to estimate the effective material behavior of complex damage within the weave of the pi preform, the input properties for the formulation were calibrated through the inverse method.

$$
\left( \frac{\varepsilon_{33}}{\varepsilon_{33}^{ini,T/C}} \right)^2 + \left( \frac{\gamma_{13}}{\gamma_{13}^{ini}} \right)^2 + \left( \frac{\gamma_{23}}{\gamma_{23}^{ini}} \right)^2 \geq 1
$$

(3.4)

$G^*_I/G_{Ic} + G^*_II/G_{IIc} + G^*_III/G_{IIIc} = 1$

(3.5)

3.4 Numerical and experimental comparisons

Comparisons between the numerical and experimental results for the six experimental configurations are presented in this section. It is noted that numerical results are obtained from simulations using the same material properties which were determined during the
calibration process. Because explicit dynamic analyses with mass scaling were performed, oscillatory behavior is common in the numerical load-displacement responses following load drops. To minimize oscillations, the applied loading rate was reduced prior to significant load drops using smooth step loading profiles. Due to suspected test fixture compliance in the L2-4pt-PO configuration measurements, fixed constraints applied to the top set of rollers were replaced with spring elements (in the vertical direction). The spring stiffness was calibrated using the L2-4pt-PO data; however, the same spring definition was also used in the L2-2pt-PO simulations (where little to no fixture compliance was observed) to verify the relaxed boundary condition would not introduce unwarranted compliance into the system. In the following load-displacement plots, numerical labels and circle markers correspond to numerical data points while alphabetical labels and square markers correspond to experimental data points. The numerical and experimental results are compared in terms of structural response, critical loads, and damage and failure modes.

3.4.1 L1-4pt-PO

Figure 3.9 shows the normalized force-displacement responses of the z-pinned pristine L1-4pt-PO configuration. Normalization was done with respect to the average experimental first peak load and the corresponding displacement. The numerical response captures the overall behavior observed experimentally. The slightly stiffer initial response of the experimental specimens was found to be a result of bulking in the z-pin field regions due to the additional z-pin material, which was not accounted for in the model. This stiffness discrepancy was also present between z-pinned and unpinned experimental specimens. The first peak load predicted by the model agrees well with the corresponding experimental peak loads (6% lower than the experimental average) and is also consistent with the ultimate load of the unpinned joints studied earlier. Following the first load drop, experimental responses exhibit several smaller load drops prior to reaching ultimate load which are characteristic of the slip-stick delamination growth due to z-pinning. The numerical response, however, displays smoother behavior following the second load drop which is consistent with the smeared approach used to account for the influence of z-pinning. Experimentally, the average ultimate load was over 50% higher than the average initial peak load and a more dramatic improvement was observed with respect to critical displacements. These improvements of ultimate load and displacement demonstrate the substantial toughness increase possible with z-pinning. The ultimate load predicted by the model agrees well with
experimental ultimate loads and was approximately 9% lower than the experimental average.

Figure 3.9: Comparison of experimental and numerical force-displacement responses of L1-4pt-PO (pristine) configuration.

Figure 3.10: Delamination patterns predicted by the finite element model during loading for the L1-4pt-PO (pristine) configuration (z-pin fields marked with gray rectangles).
Figure 3.11: Comparison of L1-4pt-PO (pristine) experimental edge-view crack growth (left) and delamination predicted by the finite element model in the ply 1 – ply 2 interface (right). Numerical delamination displayed for stiffness degradation of $\geq 99\%$.

Delamination snapshots corresponding to the numbered points along the finite element model force-displacement curve in Fig. 3.9 are shown in Fig. 3.10. The first load drop (point 1 – point 2) is caused by sudden delamination growth in the right unreinforced tip region (primarily in the adhesive – ply 1 and ply 1 – ply 2 interfaces). Shortly after the first load drop, a second load drop (point 3) is predicted in the unreinforced region of the left tip. Delamination progresses slowly through the z-pin reinforced CZMs (z-pin field areas are marked by gray rectangles) from point 3 to point 5 during which further loading is supported. Prior to point 5, a load drop occurs as delamination first extends into the throat region (primarily in the ply 1 – ply 2 interface). Joint failure occurs as delamination progresses
completely through the z-pin reinforced CZMs and into the unreinforced interfaces of the throat.

Comparison of experimental edge-view crack growth against delamination predicted in the ply 1 – ply 2 interface by the model is shown in Fig. 3.11. Experimental snapshots correspond to the black force-displacement response curve in Fig. 3.9. Following the first load drop (point B), delamination growth within the skin plies nearest the pi preform was observed up to the second row of z-pins in the right tip. Delamination of a similar extent was captured by the model (point 2) in the correct interfaces. Experimentally, delamination continues to grow through the right z-pin field up to point E; however, the model predicted delamination in the left tip region at point 3. This tendency for the model to predict balanced delamination growth while the experiment specimens show one-sided delamination growth is likely an artifact of the idealized model geometry used to represent test specimens that have geometry variations introduced during the manufacturing process. Eventually, delamination is observed experimentally in the second (left) tip at point E. While the specimen was able to support further loading beyond point E, a similar progression to final joint failure was observed between the experimental and numerical results. Considering the simplifications inherent to the smeared z-pin approach, experimental and numerical damage progression agreement is reasonable.

In addition to the pristine specimens, tip defective joints were also tested in the L1-4pt-PO configuration. Experimental force-displacement responses of the pristine and defective joints are compared in Fig. 3.12. Normalization with respect to pristine first peak load and corresponding displacement (from Fig. 3.9) is maintained in the force-displacement plot of Fig. 3.12. Except for the first load drop for each specimen type (for defective specimens, minor load drops were observed at a normalized force of approximately 0.64), the structural responses of pristine and defective joints were similar. The similar responses of the pristine and defective specimens extend to the ultimate load which was found to be insensitive to the presence of the defect in this study.
Figure 3.12: Pristine and defective experimental force-displacement responses for the L1-4pt-PO configuration.

Figure 3.13: Comparison of experimental and numerical force-displacement responses of L1-4pt-PO (D2) configuration.
Figure 3.14: Delamination patterns predicted by the finite element model during loading for the L1-4pt-PO (D2) configuration (z-pin fields marked with gray rectangles). Defect located in the right tip of the adhesive – ply 1 interface.

3.4.2 L1-SB

Figure 3.15 shows the normalized force-displacement responses of the z-pinned pristine L1-SB configuration. Normalization was done with respect to the average experimental peak load and the corresponding displacement. Both experimental and numerical responses display an initially linear response that begins to show non-linearity as peak load is approached. The peak load returned by the model was within 3% of the average experimental peak load. Following peak load, a sharp load drop occurs as a result of delamination in the throat region. Following the load drop, the load rebounds and remains approximately level. During this extended plateau of the load, little delamination growth is observed and instead diffuse damage within the pi weave accumulates.
Figure 3.15: Comparison of experimental and numerical force-displacement responses of L1-SB (pristine) configuration.

Figure 3.16: Delamination patterns predicted by the finite element model during loading for the L1-SB (pristine) configuration (z-pin fields marked with gray rectangles).
Figure 3.17: Comparison of L1-SB (pristine) experimental edge-view crack growth (left) and delamination footprint (cumulative delamination in the ply 1 – ply 2 and ply 2 – ply 3 interfaces) predicted by the finite element model (right). Numerical delamination displayed for stiffness degradation of $\geq 99\%$. 

Delamination snapshots corresponding to the numbered points along the finite element model force-displacement curve in Fig. 3.15 are shown in Fig. 3.16. The first load drop (point 1 – point 2) is caused by delamination growth in the unreinforced throat region (primarily in the ply 1 – ply 2 interface). Upon progressing into the reinforced CZMs, delamination is effectively arrested. The experimental edge-view snapshots in Fig. 3.17
also show delamination growth in the throat following the first peak (point A – point B). Experimentally, delamination was primarily observed in the adhesive – ply 1 and ply 1 – ply 2 interface; however, delamination also occurred between the pi base and adhesive layer. Following the load drop, the edge view shows crack growth extending through the second row of z-pins on the tensile side (loading applied from left to right) before arrestment. Experimental and numerical delamination results show slow delamination growth as the load rebounds and further lateral displacement is applied to the web (experimentally: point B – point E, numerically: point 2 – point 4). As the z-pinning impedes delamination growth, diffuse damage develops in the throat of the pi preform. The enlarged image of the pi throat in Fig. 3.17 (point E) shows significant cracking and debonding between the tows and resin. Experimentally, the joints continue to support loading beyond what is shown in Fig. 3.15; however, pi preform damage becomes the primary mode of energy dissipation. Because of the simplified approach for modeling pi preform damage, the model struggles to accurately represent the effective behavior as diffuse damage in the preform intensifies.

Figure 3.18: Comparison of experimental and numerical force-displacement responses of L1-SB (D1) configuration.

In addition to the pristine specimens, throat defective joints were also tested in the L1-SB configuration. Normalization with respect to pristine peak load and corresponding displacement (from Fig. 3.15) is maintained in the force-displacement plot of Fig. 3.18.
The first peak load of the defective joints was found to occur at approximately 80% of the pristine peak load. In contrast to the pristine joints, higher load levels were achieved following the initial load drop. While the model slightly underpredicts the initial peak load (10% lower than the experimental average), the overall response is in good agreement with the experimental responses. Following the initial load drop, defective specimens sustain higher load levels compared to pristine specimens and ultimately fail earlier. This same behavior is captured numerically.

Figure 3.19: Delamination patterns predicted by the finite element model during loading for the L1-SB (D1) configuration (z-pin fields marked with gray rectangles).
Figure 3.20: Comparison of L1-SB (D1) experimental edge-view crack growth (left) and delamination footprint (cumulative delamination in the adhesive – ply 1 and ply 1 – ply 2 interfaces) predicted by the finite element model (right). Numerical delamination displayed for stiffness degradation of $\geq 99\%$.

Snapshots of the model delamination patterns at the marked loading states in Fig. 3.18 are presented in Fig. 3.19. Delamination patterns and progression are similar to those predicted for the pristine joint. Consistent with the less dramatic first load drop of the defective joint compared to the pristine joint, the corresponding delamination predicted by
the model is less extensive. Experimental edge-view crack growth snapshots are shown alongside delamination footprints predicted by the model in Fig. 3.20. The experimental snapshots correspond to the black force-displacement response curve in Fig. 3.18. Both experimentally and numerically, delamination develops primarily in the adhesive – ply 1 and the ply 1 – ply 2 interfaces of the throat region. Delamination progresses slowly through the z-pin fields leading to the development of diffuse damage within the preform. Experimentally, final specimen failure occurred due to rupture of the pi preform near the pi preform leg-base intersection.

3.4.3 L2-4pt-PO

Figure 3.21 shows the normalized force-displacement responses of the z-pinned L2-4pt-PO throat defect configuration. A two-peak response with near-linear segments between was observed both experimentally and numerically. The peak loads predicted by the model were 7% higher and 5% lower than the experimental average of the first and second peak load, respectively. Oscillations are present in the model load-displacement response following the highly energetic first load drop. However, in an average sense, the subsequent model stiffness agrees well with experimental stiffnesses. Delamination between the pi base and the adhesive was the primary damage mechanism for all specimens tested. Ultimately, however, two of the three specimens failed as a result of pi preform rupture in the throat (the third specimen failed as a result of complete delamination along the pi – adhesive interface).
Figure 3.21: Comparison of experimental and numerical force-displacement responses of L2-4pt-PO (D1) configuration.

Figure 3.22: a) Delamination patterns predicted by the finite element model during loading for the L2-4pt-PO (D1) configuration (z-pin fields marked with gray rectangles). Defect located in the throat of the adhesive – ply 1 interface. b) Failed specimen.
Figure 3.23: Comparison of L2-4pt-PO (D1) experimental edge-view crack growth (left) and delamination predicted by the finite element model in the pi preform base – adhesive interface (right). Numerical delamination displayed for stiffness degradation of $\geq 99\%$.

Figure 3.22a presents delamination snapshots corresponding to points along the model force-displacement curve in Fig. 3.21. At the first peak (point 1), delamination is limited to the initial disbond of the Teflon insert in the adhesive – ply 1 interface. The first load drop (point 1 – point 2) is caused by sudden delamination growth initiating in the unreinforced throat region of the pi – adhesive interface. Delamination is arrested within the reinforced regions which allows for sustained loading up to point 3. Ultimate failure is predicted by the model due to complete delamination of the pi – adhesive interface. As can be seen in Fig. 3.22b, experimental delamination patterns are accurately captured by the model.

Comparison of experimental edge-view crack growth against delamination predicted in the pi – adhesive interface by the model is shown in Fig. 3.23. Experimental snapshots correspond to the black force-displacement response curve in Fig. 3.21. The first load drop (point B) was a result of abrupt delamination growth between the pi and the adhesive layer (white material). Although the Teflon insert was placed between the adhesive layer and
the skin, delamination primarily occurred in the pi – adhesive interface (which was also captured by the model). At point B, delamination was arrested by the second row of z-pins and limited delamination growth was observed up to point C. Immediately after point C, the pi base pulled free from the z-pins which mostly remained embedded in the laminate skin (Fig. 3.22b). The model accurately captures the location and progression of delamination; however, the extent of delamination leading up to failure may be overpredicted. It should be noted, drawing detailed conclusions from comparisons like those in Fig. 3.23 is not entirely appropriate considering the discrete bridging effect of z-pins and the smeared modeling approach. Furthermore, in many cases z-pins continue to provide bridging forces through frictional sliding after the surrounding interface has delaminated.

3.4.4 L2-2pt-PO

Normalized force-displacement responses of the z-pinned L2-2pt-PO tip defect configuration are shown in Fig. 3.24. Corresponding numerical delamination snapshots during the load history are presented in Fig. 3.25a. Similar to the L2-4pt-PO (D1) configuration, the first major load drop is associated with the highest peak load and further loading does not exceed first peak load levels. The peak load returned by the model agrees reasonably well with the corresponding experimental peak loads (7% higher than the experimental average). A minor load drop was observed both experimentally (point A – point B) and numerically (point 1 – point 2) resulting from small delamination growth initiating from the tip defect. Preceding the major load drop, delamination growth in the defective tip is largely arrested by the z-pins and the corresponding cohesive zone. Ultimately, delamination overcomes the z-pin field and progresses into the unreinforced throat region (primarily in the first two skin interfaces) causing the load drop following peak load. However, delamination is subsequently arrested by the second z-pin field preventing complete failure. Further loading is supported as delamination slowly progresses through the second z-pin field. The load rebound level predicted by the model is significantly higher than that observed experimentally which is possible due to an underestimation of the delaminated area following the major load drop. Because the model sustained a higher load level during the load rebound, complete failure was predicted earlier than the failures recorded experimentally. Figure 3.25b shows images of a failed specimen. The multi-interface delamination as well as the pattern observed experimentally was accurately captured by the model (failure column of Fig 3.25a).
Figure 3.24: Comparison of experimental and numerical force-displacement responses of L2-2pt-PO (D2) configuration.

Figure 3.25: a) Delamination patterns predicted by the finite element model during loading for the L2-2pt-PO (D2) configuration (z-pin fields marked with gray rectangles). Defect located in the right tip of the adhesive – ply 1 interface. b) Failed specimen.
Figure 3.26: Comparison of L2-2pt-PO (D2) experimental edge-view crack growth (left) and delamination footprint (cumulative delamination in the adhesive – ply 1, ply 1 – ply 2, and ply 2 – ply 3 interfaces) predicted by the finite element model (right). Numerical delamination displayed for stiffness degradation of $\geq 99\%$.

Experimental edge-view crack growth snapshots are shown alongside delamination footprints predicted by the model in Fig. 3.26. The edge-view crack growth follows the description above: delamination initiating from the Teflon insert, arrestment within the z-pin field up to peak load, and sudden delamination growth through the throat into the second z-pin field. For this configuration, the unbalanced delamination growth observed experimentally (initiating from defective tip and progressing through the throat to the opposite tip) was also captured by the model. Noting the discrepancy between the experimental and numerical load rebound levels, the experimental snapshot at point D shows approximately pure pull-off of the pi preform from the z-pins embedded in the skin. In such cases, z-pin snubbing, which increases the frictional force between the pin and the surrounding material, is largely absent and a softer structural response (like that seen experimentally) is expected.
A more refined approach to account for the mode-dependent behavior of the z-pin pull-out process, as well as the effects of localized rotation, could be an improvement suitable for the smeared modeling approach.

3.5 Conclusion

An experimentally correlated finite element model of a z-pinned composite pi joint was developed as a progressive failure analysis digital tool. A simple smeared approach was implemented to model the effects of z-pin reinforcement of the pi joint. The method utilized cohesive zone modeling to average the bridging behavior of z-pins over the entire reinforced area in each interface, resulting in an efficient and scalable approach for complex and large structural models. While fine details related to the pull-out/rupture process of individual z-pins were not attainable with the modeling technique, overall structural response and details of the failure modes were captured with sufficient accuracy in mixed-mode loading scenarios. Results using calibrated cohesive properties were shown to be in good agreement with experimental data for pi joints subjected to pull-off and side-bend loading.
CHAPTER 4

Extension of Analysis Methods to Unreinforced and Z-Pin Reinforced Wide Element Type 1 Specimens

4.1 Introduction

1 In the present chapter, a finite element modeling approach and previously established model parameters are extended to unreinforced (unpinned) and z-pin reinforced wide element pi joint specimens (Fig. 4.1) such that predictive capabilities of the model can be evaluated. A smeared cohesive zone modeling (CZM) approach to account for the z-pin reinforcement is implemented in which effective fracture toughness and cohesive strength values are defined in the areas corresponding to the z-pin fields. Cohesive material properties obtained at the Narrow Element level are maintained in the current model to establish blind predictions prior to experimental testing. Model predictions are compared to experimental responses of unpinned and z-pinned specimens subjected to pull-off loading. Experimental and numerical comparisons are made in terms of load-displacement response, critical loads, damage modes, and failure progression.

1Parts of this chapter have been submitted to the Journal of Applied Mechanics.
4.2 Methodology

In the FASTBUCS program, experimental testing and model development and verification followed an incremental approach divided into three stages [29]. In each of the three stages, specimen and loading condition complexities were increased. Pi joint specimens for the three stages are shown in Fig. 4.2. The stage 1 specimens are referred to as Narrow Elements, while the stage 2 and stage 3 specimens are referred to as Wide Element Type 1 and Wide Element Type 2, respectively. In the first stage, unpinned and z-pinned Narrow Elements were tested in pull-off and side-bend loading, and corresponding finite element models were developed utilizing experimental data [26, 27]. In the second stage, which is the focus of this chapter, material properties from stage 1 were used in models developed for the Wide Element Type 1 specimens (unpinned and z-pinned) and blind predictions were established. By comparing blind numerical predictions to experimental results, predictive capabilities of the modeling approach can be assessed. Following the initial experimental and numerical comparisons, model parameters were changed to improve correlation based on experimental data.
4.3 Experimental specimens and testing

The skin and the web of the pi joint specimens were made from a toughened carbon-epoxy material system. Unidirectional lamina were used to construct the skin which had a [29/57/14] composition, where 29 represents the percentage of 0° plies, 57 represents the percentage of ±45° plies, and 14 represents the percentage of 90° plies. The web was made of a woven fabric. The skin and the web were bonded to the 3D woven pi preform using a film adhesive, and the assembly was co-cured. A Teflon insert was placed between the adhesive and the skin resulting in an initial disbond extending the full length of the bond line (Fig. 4.2). Z-pinned specimens were reinforced with 2% z-pin areal density along the pi base/skin overlap in four locations centered around the Teflon insert. Six total specimens (three unpinned and three z-pinned) were manufactured.

Wide Element Type 1 specimens were tested at the Air Force Research Laboratory (AFRL). Specimens were loaded in four-point pull-off (4-pt PO) in which the skin was constrained while a pull-off load was applied to the top of the web (insert in Fig. 4.3 shows a schematic of the 4-pt PO loading condition). Figure 4.3 shows force-displacement responses for the unpinned and z-pinned experimental specimens. Normalization in the
force-displacement plot was done with respect to the average unpinned experimental peak load and the corresponding displacement. A dramatic increase in ultimate displacement (joint toughness) was observed for the z-pinned specimens compared to the unpinned specimens.

For both unpinned and z-pinned specimens, delamination near the adhesive – skin bond area (including adjacent skin plies) was the dominate failure mode. A failed unpinned specimen is shown in Fig. 4.4. Failure of the unpinned specimen was observed to be due to sudden delamination growth within the adhesive – skin and the nearest skin ply interfaces. Edge-view crack growth snapshots from a z-pinned specimen corresponding to the labeled points in Fig. 4.3 are shown in Fig. 4.5. The first load drop (point B) is caused by delamination growth in the left unreinforced tip region; however, the z-pin field quickly arrests unstable delamination growth. Delamination slowly progresses in the left tip, and a second significant load drop (point C) occurs due to delamination in the right tip region. This right tip delamination occurs approximately mid-width (not visible from the free edge) as evidenced by the change in the reflection of light from the surface of the pi base in that region. Nearing complete failure (point D), edge-view crack growth shows complete interfacial failure; however, z-pin bridging prevents two-piece failure. Final failure (point E) occurs as z-pins pull completely out of the surrounding material. Figure 4.6 shows a failed z-pinned specimen.
Figure 4.3: Experimental pull-off force-displacement responses (gray: unpinned, black: z-pinned).
Figure 4.4: Failed unpinned Wide Element Type 1 specimen.
Figure 4.5: Experimental edge-view crack growth of a z-pinned Wide Element Type 1 specimen.
4.4 Numerical modeling

A full 3D finite element model was developed using Abaqus/Explicit [28] to analyze the structural and failure behavior of the Wide Element Type 1 specimen. Following the approach described in [27], a smeared cohesive zone technique was used to incorporate the influence of z-pinning in the z-pin reinforced specimen model. An abbreviated description of the finite element model is provided in this section; however, further details can be found in [26, 27].

In the model, a macro-scale representation of the pi preform was employed using homogenized orthotropic material properties. The adhesive film layer between the base of the pi preform and the top of the skin was modeled using an incremental plasticity material model and solid continuum elements. Cohesive element interlayers connecting the pi preform base to the adhesive and the skin to the adhesive were used to capture adhesive
failure. The laminate skin was modeled using a combination of ply-level and sub-laminate homogenization modeling techniques. The four skin plies nearest the pi preform were modeled individually and cohesive interlayers were tied to adjacent plies. The remaining portion of the skin plate was homogenized and modeled as an equivalent orthotropic plate. The Teflon insert was accounted for in the model by reducing the cohesive material properties (strength and fracture toughness values) of the adhesive-skin cohesive interlayer in the area corresponding to the insert.

4.4.1 Intra-laminar and inter-laminar damage modeling

Intra-laminar damage, specifically matrix cracking, was included in the model through a novel meshing approach motivated by the work of Nguyen and Waas [31]. Individual skin plies were partitioned along the ply’s fiber-direction such that thin strips were created. Figure 4.7 illustrates the thin strip partitions for a 45° lamina. The thin strips were meshed using cohesive elements (COH3D8, COH3D6) while the bulk regions between strips were meshed with continuum elements. This method allows for sharp matrix ply crack representation. A novel mixed-mode law was implemented to govern cohesive element response [25]. Triangle traction-separation laws were used, and the onset of damage was initiated following the satisfaction of the Hashin quadratic stress criterion shown in Eq. 4.1. Damage evolution (softening behavior) was governed by an effective separation comprised of pure mode separation components and shown in Eq. 4.2. Through the definition of an effective separation, the smooth and simultaneous vanishing of all cohesive traction components can be ensured. More details about the mixed-mode law and definitions can be found in [25].

\[
\left( \frac{\max(0, \sigma_{II})}{\sigma_{Ic}} \right)^2 + \left( \frac{\tau_{II}}{\tau_{IIc}} \right)^2 + \left( \frac{\tau_{III}}{\tau_{IIIc}} \right)^2 \geq 1 \tag{4.1}
\]

\[
\delta_e = \sqrt{\max(0, \delta_I)^2 + \left( \frac{\dot{\delta}_I}{\delta_{II}} \delta_{II} \right)^2 + \left( \frac{\dot{\delta}_I}{\delta_{III}} \delta_{III} \right)^2} \tag{4.2}
\]

Material non-uniformity was included in the model by assigning randomized material properties (cohesive strength and toughness values) to matrix crack strips (illustrated
using different colors in Fig. 4.7). Non-uniform material properties are important in accurately capturing the stochastic and progressive nature of damage and failure in real structures.

Inter-laminar damage was modeled using cohesive element interlayers and governed by the same mixed-mode formulation governing intra-laminar damage. The cohesive element interlayers were used to connect adjacent parts using tie constraints and were meshed following the strategy in [25]. Interlayer meshes were generated in a manner that promoted coincident matching of interlayer element nodes with nodes of the matrix ply crack elements of the adjacent plies. Promoting mesh compatibility in this manner results in more robust tie constraints between layers and improves representation of matrix ply crack elements. Cohesive interlayers were used to model the pi preform-adhesive interface and the adhesive-skin interface accounting for adhesive failure. Additionally, cohesive interlayers were used for individual skin ply interfaces to account for inter-laminar delamination.

Figure 4.7: Intra-laminar damage modeling for a 45° skin ply. The thin strips along the fiber direction represent matrix cracking elements and are assigned random cohesive properties.
4.4.2 Z-pin modeling – smeared approach

With structural models it is not feasible to explicitly model individual z-pins. Such models are prohibitively complex and computationally expensive. Instead, a smeared approach was implemented in the model to account for the z-pinning behavior in an averaged sense. To implement the smeared approach, all cohesive interlayers were partitioned creating z-pin reinforced and unreinforced areas. The z-pin reinforced areas had dimensions equal to the rectangular area containing the z-pin fields of the manufactured joint. Two distinct cohesive property sets were used to define the unreinforced and the z-pin reinforce cohesive responses. Figure 4.8 illustrates the smeared modeling strategy used for the z-pinned pi joint. For the unreinforced interfaces, cohesive properties previously established from the unpinned Narrow Element specimen were used. Similarly, z-pin reinforced cohesive properties from the Narrow Element level for z-pinned specimens were used as input to the Wide Element Type 1 model. Good experimental and numerical correlation was achieved at the Narrow Element level by increasing only the fracture toughness of the reinforced region to represent an effective toughness due to z-pinning.
4.5 Narrow element model correlation prior to blind predictions

Prior to establishing blind predictions for the Wide Element Type 1 configurations, cohesive material properties were established at the Narrow Element level. Experimental data at the element level, as opposed to the coupon level, were used because previous extensions of material properties experimentally obtained at the coupon-level did not translate well to element-level models. Additionally, the element size in coupon-level meshes required to obtain material property values consistent with those reported experimentally are often prohibitively small when extended to element-level meshes. Models of eight unpinned (four pristine and four defective) and six z-pinned (two pristine and four defective) Narrow Element configurations were correlated with experimental data to arrive at material property
sets which were subsequently used to establish blind predictions for the Wide Element Type 1 configurations. Of the Narrow Element configurations, the tip defective specimen (unpinned and z-pinned) was the most similar to the Wide Element Type 1 specimen (unpinned and z-pinned), and both were subjected to 4-pt PO. Correlation between the experimental results and the model response for the unpinned and z-pinned tip defective Narrow Element specimen (prior to establishing blind predictions for the Wide Element Type 1) will be briefly summarized in this section. It is noted, beyond the difference in width, Teflon insert (defect) location and z-pin field locations were different between the Narrow Element and Wide Element Type 1 specimens (Fig. 4.2).

Figure 4.9 shows the normalized force-displacement responses of the unpinned tip defective Narrow Element subjected to 4pt-PO. Normalization was done with respect to the average experimental peak load and the corresponding displacement. Both experimental (gray) and numerical (orange) responses remain linear up to peak load, and the numerically predicted peak load was within 2\% of the average experimental peak load. Experimental failure was sudden and complete; however, the numerical failure behavior was slightly more gradual and less complete.

Z-pinned tip defective Narrow Element responses are included with the unpinned force-displacement responses in Fig. 4.10. Normalization with respect to unpinned peak load and corresponding displacement (from Fig. 4.9) is maintained in the force-displacement plot of Fig. 4.10. The numerical response (purple) captures the overall behavior observed experimentally (black) from the z-pinned specimens. The stiffer initial response of the z-pinned specimens was determined to be a result of bulking in the z-pin field regions due to the additional z-pin material and was not accounted for in the model. Experimentally, minor load drops were observed at a normalized force of approximately 1.25 (corresponding numerical load drop occurs at a normalized force of approximately 1). These first load drop events are consistent with failure of the unpinned joints (the approximately 25\% higher load of the experimental z-pinned joints was attributed to slight fabrication differences between the unpinned and z-pinned joints). Compared to the unpinned specimens, a dramatic increase in joint toughness and ultimate load was observed for the z-pinned specimens. Fracture toughness values defined in the reinforced cohesive interlayers were increased to numerical capture the toughening behavior observed experimentally.
Figure 4.9: Unpinned Narrow Element experimental and numerical force-displacement responses for the tip defective 4-pt PO configuration.

Figure 4.10: Unpinned and z-pinned Narrow Element experimental and numerical force-displacement responses for the tip defective 4-pt PO configuration. *Model responses represent the state of correlation prior to the blind predictions (blind prediction simulations used the same material properties).
4.6 Results and discussion - experimental and numerical and comparisons

Comparisons between the blind predictions and experimental results for the Wide Element Type 1 specimens are presented in this section. Because explicit dynamic analyses with mass scaling were performed, oscillatory behavior is common in the numerical load-displacement responses following load drops (especially for the z-pinned results). To minimize oscillations, the applied loading rate was reduced prior to significant load drops using smooth step loading profiles. The numerical and experimental results are compared in terms of structural response, critical loads, and damage and failure modes.

4.6.1 Blind predictions

4.6.1.1 Unpinned

Figure 4.11 shows the normalized force-displacement responses of the unpinned Wide Element Type 1 specimens and the corresponding blind prediction. Normalization was done with respect to the average experimental peak load and the corresponding displacement. Both experimental and blind prediction responses remain linear up to peak load with consistent stiffnesses. The peak load predicted by the model agrees well with the experimental peak loads and was 2% higher than the average experimental peak load. Experimental specimen failure was abrupt and complete; however, the blind prediction failure progression was more gradual as evidenced by the rounded peak and incomplete initial load drop. The slower progression to failure predicted by the model suggested fracture toughness values obtained at the Narrow Element level were too high.
4.6.1.2 Z-pinned

Figure 4.12 shows the normalized force-displacement responses of the z-pin reinforced configuration. Normalization was done with respect to the average experimental first peak load and the corresponding displacement. The slightly stiffer initial response of the experimental specimens compared to the blind prediction was consistent with previous results at the Narrow Element level (attributed to z-pin bulking and not accounted for in the model). This stiffness discrepancy is apparent experimentally between the z-pinned and unpinned specimens (Fig. 4.3). The first peak load predicted by the model agrees well with the experimental first peak loads and was 9% lower than the experimental average. Following the first load drop, further loading is predicted both experimentally and numerically as the z-pins (experimental) and reinforced cohesive interfaces (numerical) arrest unstable delamination growth. Experimentally, the load rebounds and remains approximately level (load does not exceed first peak value). The blind prediction response, however, exhibits a significantly tougher behavior and the ultimate load exceeds that of the initial peak.
4.6.2 New cohesive properties

The same finite element model used to establish the blind predictions was supplied with new cohesive properties in an effort to improve experimental and numerical agreement. No changes to the modeling method, mesh, nor element formulations were made, only cohesive material property inputs (cohesive strength and toughness) were changed to examine their effects on predicted results. In addition to force-displacement response comparisons, delamination pattern snapshots (including those established in the blind prediction phase) are provided in the following section.

Because these studies were guided by experimental and numerical agreement both at the Wide Element and Narrow Element level, a complete description of cohesive material properties changes during this phase of the study will be omitted to keep this paper succinct. While different cohesive property sets were used for the pi – adhesive, adhesive – skin, and interlaminar (ply – ply) cohesive interlayers, only the adhesive – skin and ply – ply interlayers affected the Wide Element Type 1 behavior. Based on the gradual failure predicted for the unpinned configuration, the mode I fracture toughness in the ply – ply interlayer was reduced to 50% the original value. This decrease also brought the mode
I fracture toughness value closer to reported values of common toughened carbon-epoxy material systems. The mode II/III fracture toughness was reduced to 85% the original value. Blind predictions for the z-pinned Wide Element configuration were established with the assumption that in the reinforced CZM regions, only the fracture toughness values needed to be changed to account for z-pinning (strengths were unchanged from the non-reinforced values). This assumption was based on good agreement for the z-pinned Narrow Element configuration. However, in a sensitivity analysis for the z-pinned Wide Element Type 1 model, it was discovered that reducing the cohesive strengths in the reinforced CZMs was effective in reducing the overly tough response of the blind prediction. This led to reductions of cohesive strength to 60% and 70%, mode I and mode II/III respectively, of the original value in all reinforced CZM regions.

Reductions of the cohesive strengths in the reinforced regions can be justified by noting that z-pins represent inclusions in the embedded material, and therefore lead to stress concentrations which can precipitate damage and failure. Interlaminar stress distributions in a z-pinned and unpinned laminate are sketched in Fig. 4.13. Figure 4.13a shows the interlaminar normal stress (or peel stress) for out-of-plane loading. Figure 4.13b shows the interlaminar shear stress for shear loading. The primary stress components peak near the pin of the z-pinned laminate for both loading conditions. In contrast, stress distributions in the unpinned laminate are uniform and smooth. In the smeared CZM approach, z-pins are not explicitly modeled and the interlaminar stress concentrations that exist in reality are not captured by the model. To account for this characteristic of z-pinning in the model, it was deemed appropriate to reduce cohesive strengths in the z-pin reinforced interlayers. A further explanation and more details can be found in [27].
Fig. 4.13: Interlaminar stress distributions of pinned and unpinned laminates (dashed vertical line marks the interface of the z-pin and surrounding material). Loading only applied to laminate blocks (not z-pins). a) Normal (peel) stress. b) Shear stress.

4.6.2.1 Unpinned

Force-displacement responses, including that established with the new CZM properties (blue curve), are shown in Fig. 4.14. The peak load with the new CZM properties maintains good agreement with experimental peak loads and was 4\% higher than the average experimental peak load. The new CZM properties, most noticeably, improve the failure behavior such that a more sudden and complete failure is predicted (consistent with experimental behavior).

Delamination patterns predicted and obtained using the new CZM properties at joint failure, as well as a post-test C-scan are shown in Fig. 4.15. In addition to delamination contours of individual interfaces, a cumulative view (including adhesive – ply 1, ply 1 – ply 2, and ply 2 – ply 3 interfaces) is provided for comparison to the experimental C-scan. Delamination was predicted primarily in the adhesive – ply 1 and the ply 1 – ply 2 interfaces in both the blind prediction and the new CZM results. Delamination extends from the Teflon insert in the adhesive – ply 1 interface into the ply 1 – ply 2 interface forming rectangular delamination regions below the intact bond area. Although the experimental C-scan does not clearly show the disbond associated with the Teflon insert (due to the Teflon insert being in the same plane as the web side skin surface), strong agreement between experimental and numerical (both blind prediction and new CZM results) delamination patterns was achieved.
Figure 4.14: Unpinned Wide Element Type 1 experimental and numerical (blind prediction and new CZM properties) force-displacement responses.

Figure 4.15: Delamination patterns at joint failure for the unpinned Wide Element Type 1 configuration.
4.6.2.2 Z-pinned

The force-displacement response obtained with the new CZM properties is overlayed on the z-pinned experimental and blind prediction responses in Fig. 4.16. The new response matches the overall behavior observed experimentally, and most notably, improves agreement of joint toughness. Experimental responses show three significant load drops during testing. This same three-load-drop response was captured with the new CZM properties. While the initial peak of the new response decreased to 90% of the experimental average, the ultimate load (8% higher than experimental average) and ultimate displacement (7% higher than experimental average) of the new response marked a significant improvement compared to the blind prediction results. Compared to the experimental behavior, the stiffness of the new response following the first two load drops is too high. This suggests the model is likely still too tough; however, the goal in using the new CZM properties was to improve correlation at the Wide Element Type 1 level while limiting detrimental effects on the correlation at the Narrow Element level.

Delamination snapshots corresponding to the numbered points along the blind prediction and new CZM force-displacement curves in Fig. 4.16 are shown in Fig. 4.17 (corresponding experimental C-scans are included as well). Numerical labels and circle markers correspond to numerical data points while alphabetical labels and square markers correspond to experimental data points. Snapshots are intended to compare similar events (as opposed to consistent force/displacement levels). The blind prediction shows the first load drop (point 1) being caused by sudden delamination growth primarily in the ply 1 – ply 2 interface forming in an anti-symmetric manner. Delamination with the new CZM properties, however, develops on the right side and is split between the adhesive – ply 1 and ply 1 – ply 2 interfaces. The experimental C-scan taken following the first load drop (point A) indicates one-sided delamination growth. In the blind prediction, the second load drop (point 2) is caused by anti-symmetric delamination developing in the adhesive – ply 1 interface, while with the new CZM properties this event is caused by delamination both developing on the left side and progressing in the right side. Experimental behavior at the second load drop (point B) varied between specimens. The C-scan in Fig. 4.17 at point B indicates further progression of delamination on the right side, however delamination in the intact side caused the second load drop for a different specimen (Fig. 4.5). Figure 4.18 shows numerical delamination patterns at joint failure and the corresponding post-test C-scan. Joint failure occurs as delamination progresses completely through the z-pin reinforced CZMs. While experimental
failure was found to be more dispersed within the adhesive-skin and the first several skin plies, numerical delamination patterns and interfaces (both blind prediction and new CZM results) were in good agreement.

Figure 4.16: Z-pinned Wide Element Type 1 experimental and numerical (blind prediction and new CZM properties) force-displacement responses.
Figure 4.17: Delamination pattern snapshots during loading for the z-pinned Wide Element Type 1 configuration (z-pin fields marked with gray rectangles).
4.6.3 Z-pin insertion depth and modeling assumptions

The ability to obtain material properties at the Narrow Element level and use those properties to establish blind predictions at the Wide Element level is predicated, in part, on the assumption that the z-pin insertion depth is maintained across all element levels. This work was based on that assumption holding true. As a consequence, new CZM properties used to improve the experimental and numerical agreement at the Wide Element level needed to be checked at the Narrow Element level to ensure agreement was reasonably maintained or improved. After completion of this work, further examination of the failed specimens revealed that the depth of z-pin penetration in the Wide Element Type 1 specimens was much less compared to the Narrow Element specimens (Fig. 4.19). Experimental work has shown that the bridging traction load generated by z-pins increases with increased embedded length (or insertion depth) [46, 47, 48, 49]. Therefore, the blind prediction of the z-pinned Wide Element Type 1 response should not be discredited and is consistent with expectation of a specimen with deeper z-pin insertion. This finding relaxes the constraint that the same cohesive properties need to be maintained between Narrow Element and Wide Element Type 1 models; and while not investigated here, better experimental
and numerical agreement at both levels would likely be possible. Establishing a relationship between z-pin insertion depth and the averaged smeared cohesive properties that are used in these modeling studies is the subject of an ongoing micro-mechanics-based study.

![Figure 4.19: Z-pin insertion depth. Left images taken from Narrow Element specimens. Right images taken from Wide Element Type 1 specimens. Note: pi preforms have been removed in the images of the Wide Element Type 1 specimens.](image)

### 4.7 Conclusion

The predictive capabilities of a progressive damage composite pi joint model were assessed through blind predictions of experimental responses for unpinned and z-pinned specimens. Blind predictions for the unpinned joint accurately predicted peak load and failure modes and locations. A slightly more gradual failure progression was predicted compared to the experimental results; however, only modest CZM property changes were needed to further improve agreement with experimental behavior. Blind predictions for the z-pinned configuration significantly overpredicted the toughness of the joint. In the smeared CZM approach used to model the effects of z-pinning, fracture toughness and cohesive strength values of the reinforced CZMs needed to be adjusted appreciably to capture experimental behavior better. The shortcomings of the model to accurately predict the z-pinned response point towards the relative immaturity of modeling through-thickness reinforcements (z-pins) at the subcomponent/structural level. The modeling approach and new CZM properties will
ultimately be extended to a more complex pi joint specimen subjected to combined loading (axial compression and pull-off) such that a final assessment of the predictive capabilities can be made.
5.1 Introduction

In this chapter, a previously calibrated finite element modeling approach is extended to a complex joint specimen allowing for the assessment of predictive capabilities through blind predictions. The static response of unreinforced (unpinned) and z-pin reinforced specimens is investigated. A smeared cohesive zone modeling approach is implemented in which effective fracture toughness and cohesive strength values are defined in the areas corresponding to the z-pin fields in the reinforced joint. Properties defining the cohesive responses were calibrated for simpler specimen configurations and maintained in the current model to establish blind predictions prior to experimental testing. Pi joint specimens were tested in combined axial compression and push-off loading. Experimental and numerical comparisons are made in terms of load-displacement response, critical loads, damage modes, and failure progression.

5.2 Methodology

In the research program, model development and testing followed an incremental approach divided into three stages [29]. Specimen and loading condition complexities increased at each stage. Pi joint specimens for the three stages are shown in Fig. 5.1. In the first stage, unpinned and z-pinned Narrow Elements were tested in pull-off and side-bend loading, and corresponding finite element models were developed and calibrated using experimental data [26, 27]. Material properties calibrated through the inverse method during stage 1 were
subsequently used in stage 2. In the second stage, blind predictions of Wide Element Type 1 specimens (unpinned and z-pinned) were established allowing for an initial assessment of the predictive capabilities of the model. Following the blind predictions, a recalibration effort was also performed in stage 2 to improve correlation leading into stage three. In the third stage, final assessment of the predictive capabilities of the model was demonstrated for Wide Element Type 2 specimens (unpinned and z-pinned). The Wide Element Type 2 specimens were the largest specimens investigated and subjected to the most complex loading condition considered during the project (combined axial compression and push-off loading). The focus of the work presented in this paper is on the model development and comparisons of the experimental and blind numerical predictions for the Wide Element Type 2 configuration (stage 3).

The main test and specimen design goals in stage 3 where to develop an efficient way to apply compressive and push-off loading to the specimen while also preventing buckling and non-delamination failure modes [50]. The design of the Wide Element Type 2 specimen deviates from that of the Wide Element Type 1 and Narrow Element designs in that non-rectangular sections were used for the skin and web components. Several sensitivity studies were performed in [50] to arrive at the final design for the Wide Element Type 2 specimen. The focus of these studies was placed on decoupling the skin from the end constraints in order to reduce the push-off actuator force and reducing strain concentrations to avoid laminate rupture during axial compression. The free body diagram of Fig. 5.2 shows the final geometry and the loading condition applied to the Wide Element Type 2 specimen.
5.3 Experimental specimens and testing

Experimental specimens were fabricated at Northrop Grumman Corporation (NGC). The skin and the web of the pi joint specimens were made from a toughened carbon-epoxy
material system. The skin and web laminate designs were kept the same for all stages as to not introduce unnecessary complexities. The skin laminate was fabricated with unidirectional lamina while a woven fabric was used to construct the web. The skin and the web were bonded to the 3D woven pi preform using a film adhesive, and the assembly was co-cured. A Teflon insert was placed between the adhesive and the skin resulting in an initial disbond extending the full length of the bond line (Fig. 5.1). Z-pinned specimens were reinforced with 2% z-pin areal density along the pi base/skin overlap in four locations centered around the Teflon insert. Twelve total specimens (six unpinned and six z-pinned) were manufactured.

Wide Element Type 2 specimens were tested at the AFRL. The test was designed to apply axial compression as well as push-off load. Fig. 5.3 shows the experimental test setup for the Wide Element Type 2 configuration. Specimens were first loaded in axial compression to a predetermined load level (percentage of a nominal maximum load). Compressive load levels varied such that representation of a push-off versus compression interaction envelope would be possible. While the compressive load was held in load control, push-off loading was applied symmetrically against the IML (inner model line) side of the skin (web side). The push-off load was controlled by a single actuator which displaced two arms connected to support rollers. Push-off loading was applied until failure of the specimen occurred. The six unpinned specimens were tested at 80% (2 specimens), 60%, 40%, 20%, and 10% of the nominal maximum compressive load. The six z-pinned specimens were tested at 60%, 40%, and 10% of the nominal maximum compressive load, with a repeat specimen at each compressive load level. Load-displacement data (frame and digital image correlation (DIC)) and strain data (strain gauges and DIC) was collected throughout the tests. Ultrasonic (UT) scans from the OML (outer model line) side of the skin (away from the web) were performed to assess damage following significant load drops and joint failure.
5.4 Numerical modeling

A progressive damage and failure finite element model of the Wide Element Type 2 specimen was built using Abaqus Explicit [28]. The modeling approach extended methods and features developed for the Narrow Element and Wide Element Type 1 models. For the z-pinned model, the smeared cohesive zone technique described in [27] was implemented to efficiently include the effects of z-pinning. An abbreviated description of the finite element model is provided in this section, however further details can be found in [26, 27].

In the model, the pi preform was modeled as a homogenized part using orthotropic material properties. This simplified representation ignores material property and geometric variations (undulations) characteristic of the 3D woven architecture of the pi preform. The adhesive film layer between the base of the pi preform and the top of the skin was modeled using an incremental plasticity material model and solid continuum elements. Cohesive element interlayers connecting the pi preform base to the adhesive and the skin to the adhesive were used to capture adhesive failure. The laminate skin was modeled
using a combination of ply-level and sub-laminate homogenization modeling techniques. The four skin plies nearest the pi preform were modeled individually, while the remaining portion of the skin plate was modeled as an equivalent orthotropic plate. The Teflon insert was accounted for in the model by reducing the cohesive material properties (strength and critical energy release rate values) of the adhesive-skin cohesive interlayer in the area corresponding to the insert. As a result, the corresponding cohesive elements failed at the beginning of the simulation and subsequent interactions between the corresponding surface pair defaulted to the general contact formulation. The experimental end fixturing was also modeled. Aluminum blocks, including the potting material, were tied to the specimen’s ends. Rigid platens in conjunction with a contact interaction (hard contact and penalty friction) were used to apply the axial load to the potted ends, as well as mimic the boundary conditions of the test frame platens. Finally, a set of rigid rollers were used along the IML side of the skin (web side) to apply push-off loading.

5.4.1 Intra-laminar and inter-laminar damage modeling

Intra-laminar damage was included in the model using a novel meshing approach motivated by the work of Nguyen and Waas [31]. The approach integrates thin strips of elements between bulk elements to represent discrete matrix cracks. In the model, individual skin plies were partitioned along the ply’s fiber-direction such that thin strips were created (Fig. 5.4). The thin strips were meshed with finite-thickness cohesive elements (COH3D8, COH3D6) while the bulk regions between strips were meshed with continuum elements. Cohesive element constitutive behavior follows the novel cohesive formulation and mixed-mode law described in [25]. Triangle traction-separation laws were used, and the transition from the linear elastic regime to the softening response was based on the Hashin quadratic stress criterion shown in Eq. 5.1. Mixed-mode damage evolution and the softening of each mode is governed by the effective separation defined in Eq. 4.2. The mixed mode law ensures the smooth and simultaneous vanishing of all cohesive traction components. More details about the mixed-mode law and definitions can be found in [25].

\[
\left(\frac{\max(0, \sigma_I)}{\sigma_{Ic}}\right)^2 + \left(\frac{\tau_{II}}{\tau_{IIc}}\right)^2 + \left(\frac{\tau_{III}}{\tau_{IIIc}}\right)^2 \geq 1
\]  \hspace{1cm} (5.1)
\[ \delta_e = \sqrt{\max(0, \delta_I)^2 + \left(\frac{\dot{\delta}_I - \delta_{III}}{\delta_{II}}\right)^2 + \left(\frac{\dot{\delta}_I}{\delta_{II}} \delta_{III}\right)^2} \]  \hspace{1cm} (5.2)

Inter-laminar damage (delamination) was captured in the model using cohesive interlayers. The same cohesive formulation used to model intra-laminar damage was used to model inter-laminar damage. The cohesive element interlayers were used to connect adjacent parts using tie constraints and were meshed following the strategy in [25]. Interlayer meshes were created facilitating coincident matching of interlayer element nodes with nodes of ply-crack elements of the adjacent plies (Fig. 5.4). This meshing strategy improves the robustness of tie constraints and helps better represent the interaction between matrix ply-cracks and delamination. Cohesive interlayers were used to model the pi preform-adhesive interface and the adhesive-skin interface accounting for adhesive failure. Additionally, cohesive interlayers were used for individual skin ply interfaces to account for inter-laminar delamination.
5.4.2 Z-pin modeling – smeared approach

Due to the difference in size scales between a structure and a z-pin, as well as the discrete nature of the pins, it is a challenge to model a z-pinned structure. For structural models, it is impractical to explicitly model individual z-pins due to associated model complexities.
and computational requirements. Furthermore, details regarding the contribution of an individual z-pin to structural behavior are often not of primary concern and frequently lost in the higher scale response. For these reasons, a smeared cohesive zone approach was implemented in the model to account for the z-pinning behavior in an averaged sense [27]. In the proposed smeared approach, all cohesive interlayers were partitioned creating z-pin reinforced and unreinforced areas. The z-pin reinforced areas were rectangular and had dimensions equal to the area containing all z-pins in a z-pin field. Two distinct cohesive property sets were used to define the unreinforced and the z-pin reinforce cohesive responses. Fig. 5.5 illustrates the smeared modeling strategy used for the z-pinned pi joint.

The effect of z-pinning on interlaminar properties, specifically fracture toughness, has been well studied [33, 46, 22, 15]. It has been shown that z-pinning increases fracture toughness and that z-pinning characteristics (such as insertion depth) influence the degree to which the fracture toughness increases [46]. The effect of z-pinning on inter-laminar strength, however, is less understood. Z-pins represent inclusions in the embedded material and often result in a mismatch of elastic constants. Therefore, inhomogeneous stress distributions and stress concentrations can arise near z-pins, precipitating the initiation of damage and failure. Because z-pins are not explicitly modeled in the smeared approach, the material mismatch and resulting interlaminar stress peaks that exist in reality are not captured by the model. It was proposed in previous work [27] and maintained here, that an approach to account for this reality of z-pinning in the smeared model is to reduce the cohesive strengths in the z-pin reinforced interfaces. In this way, the smeared cohesive interlayers damage and fail more similarly to the z-pinned interfaces of the physical joint.
5.5 Numerical and experimental comparisons

Comparisons between the blind prediction and experimental results for the Wide Element Type 2 (unreinforced and z-pin reinforced) specimen are presented in this section. The models used to establish the blind predictions use the same material properties that were determined through recalibration following the Wide Element Type 1 evaluation (stage 2). Because explicit dynamic analyses with mass scaling were performed, oscillatory behavior is common in the numerical load-displacement responses following load drops. These oscillations are most prominent for the z-pinned configuration where delamination arrestment allows for load rebounding of kinetically energetic joints. In the following load-displacement plots, numerical labels and circle markers correspond to numerical data points while alphabetical labels and square markers correspond to experimental data points. The numerical and experimental results are compared in terms of structural response, critical
5.5.1 Unpinned

A summary of the push-off failure load versus axial compression interaction envelope for the unpinned Wide Element Type 2 configuration is shown in Fig. 5.6. Experimental test results are shown with solid markers while the blind predictions are shown with hollow markers. Linear fits to the experimental and blind prediction results are shown with solid and dashed lines, respectively. The interaction trend shows that as the axial compression load is increased, the push-off failure load decreases. Blind predictions agree well with both the absolute failure loads and the overall trend observed experimentally. For five of the six specimens, blind predictions of the push-off failure load were within 15% of the failure load found experimentally. A possible outlier was tested at 80% the maximum compressive load where the push-off failure load was observed to be 35% higher than the corresponding blind prediction. Because of this suspected outlier, a second specimen was tested at 80% the maximum compressive load, and the failure load for that specimen was within 10% of the prediction. Good agreement for five of the six specimens further supports the suspected outlier tested at 80% the maximum compressive load. Table 5.1 summarized the agreement between experimental and numerical push-off failure loads for each compressive load level tested.
Table 5.1: Push-off failure loads (unpinned).

<table>
<thead>
<tr>
<th>Compressive load (normalized)</th>
<th>Push-off failure ((P_{\text{pred.}}/P_{\text{exp.}}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.85</td>
</tr>
<tr>
<td>0.2</td>
<td>0.88</td>
</tr>
<tr>
<td>0.4</td>
<td>0.91</td>
</tr>
<tr>
<td>0.6</td>
<td>0.86</td>
</tr>
<tr>
<td>0.8</td>
<td>0.91</td>
</tr>
<tr>
<td>0.8</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Figure 5.7 shows normalized push-off force-displacement responses of the unpinned configuration tested at 20% maximum axial compression. The numerical response (blue) captures the overall behavior observed experimentally (gray). The peak load predicted by the model was 12% lower than the peak load recorded experimentally. Experimental and numerical results both show an incomplete failure as the clamping forces provided by
the block potting at the specimen’s ends and the web preserve residual structural integrity following peak load.

Figure 5.7: Comparison of experimental and blind prediction push-off force-displacement responses with 20% maximum axial compression load (unpinned).

Delamination snapshots corresponding to the numbered points along the predicted force-displacement curve in Fig. 5.7 are shown in Fig. 5.8. Interfaces labels are abbreviated as Pi/A (pi preform/adhesive), A/P1 (adhesive/ply-1), P1/P2 (ply-1/ply-2), P2/P3 (ply-2/ply-3), P3/P4 (ply-3/ply-4). At peak load (point 1), the only significant delamination predicted corresponds to the initial disbond of the Teflon insert in the adhesive/ply-1 interface. Immediately following peak load (point 2), sudden delamination growth is predicted in an anti-symmetric pattern within the ply-1/ply-2 interface. The most critical delamination observed experimentally was also in the ply-1/ply-2 interface. Fig. 5.9 compares experimental and predicted damage patterns at failure. Experimental UT scan data (Fig. 5.9a) shows delamination on one side of the joint, while the numerically predicted delamination pattern is anti-symmetric. This tendency for the model to predict balanced delamination is due to over-idealization of joint geometries and insufficient material non-uniformity in the model.
Figure 5.8: Delamination patterns predicted by the finite element model during loading for the unpinned configuration at 20% maximum axial compression load. Defect centrally located in the adhesive – ply 1 (A/P1) interface.
Figure 5.9: Delamination at failure for the unpinned configuration at 20% axial compression load. (a) Experimental UT scan (post-processed). (b) Model blind prediction (cumulative view).

Fig. 5.10 shows normalized push-off force-displacement responses of the unpinned configuration tested at 80% maximum axial compression. It is noted, the blind prediction response has been shifted horizontally in the force-displacement plot to account for how the push-off rollers were controlled experimentally during the initial axial compression step. In the tests, as axial compression was applied, the push-off rollers were set in load control to maintain a small load against the skin. In the model, the push-off rollers were fully constrained during the axial compression step. Therefore, as the skin deformed and reacted against the push-off rollers, load was registered. Experimental behavior following peak load at the 80% maximum axial compressive load level (as well at the 60% and 40% maximum compressive load levels) was characterized by complete and catastrophic failure as opposed to the incomplete fail observed for specimens tested at the 10% and 20% maximum compressive load levels. The blind prediction response for the 80% maximum axial compression case exhibits a brief load catch following peak load, but shortly after fails completely. This failure behavior discrepancy between experimental and predicted results is tentatively attributed to failure of the pi preform and web, which was observed experimentally (Fig. 5.12) but not considered in the model.
Figure 5.10: Comparison of experimental and blind prediction push-off force-displacement responses with 80% maximum axial compression load (unpinned). Suspected outlier specimen tested at 80% maximum axial compression load not included.

Interface-by-interface delamination predictions at joint failure are shown in Fig. 5.11. Large delamination areas were predicted the ply-1/ply-2 interface extending out free edges. Significant delamination was also predicted in the adhesive/ply-1 interface progressing from the Teflon insert region. A cumulative view of the predicted delamination at failure (included delamination in the adhesive/ply-1, ply-1/ply-2, and ply-2/ply-3 interfaces) is also provided in Fig. 5.11. A failed specimen tested at 80% of the maximum axial compression load is shown in Fig. 5.12. Delamination between the first and second skin plies as well as the adhesive and first skin ply was observed. While delamination was the primary failure mode, rupture of the pi preform and compressive failure of the web was also observed for the specimen shown in Fig. 5.12.
5.5.2 Z-pinned

A comparison of experimental and blindly predicted critical loads for the z-pinned Wide Element Type 2 configuration is shown in Fig. 5.13. In addition to the initial peak load, the
The ultimate load is also of significance (insert in the left plot of Fig. 5.13 distinguishes initial and ultimate load). The left plot of Fig. 5.13 summarized the push-off initial peak load versus axial compression interaction behavior. Blind prediction results for initial peak load agree very well with corresponding experimental results both in terms of absolute value and trend. Accurate predictions of the initial peak load for the z-pinned configuration was expected given the accuracy of the failure load predictions for the unpinned configuration. This expectation was based on the observation that initial peak loads for z-pinned joints approximately coincide with failure loads of corresponding unpinned joints (delamination initiates in non-reinforced interfaces). The right plot of Fig. 5.13 summarized the push-off ultimate load versus axial compression interaction behavior. Z-pinned specimens were tested at three compressive load levels with a repeat specimen for each load level. Experimental average ultimate loads were overpredicted by approximately 24-35%. Additionally, a slightly stronger interaction between axial compression and push-off ultimate load was predicted by the model compared to experimental observation. Table 5.2 summarized the agreement between experimental and numerical push-off initial and ultimate loads for the z-pinned configuration.

Figure 5.14 shows normalized push-off force-displacement responses of the z-pinned configuration tested at 60% maximum axial compression. Normalization was done with respect to the average experimental first peak load. The predicted response agrees well with the experimental response up to a normalized displacement value of approximately 2.5. Throughout the first half of the predicted response, the initial peak load, a second load drop, and the stiffness following the second load drop are all accurately captured. Experimentally, the push-off force-displacement softens as the joint nears complete failure, which occurs at a load level approximately two-times that of the initial peak load. This softening behavior was not captured by the predictions, and may suggest damage progression in the model is too slow at larger displacement values or a damage mechanism (e.g., compressive damage of the skin due to bending) is occurring experimentally that is not considered in the model. The ultimate load predicted by the model was 29% higher than the average experimental ultimate load for the 60% maximum axial compression case. Similar push-off force-displacement responses were observed for the 10% and 40% maximum axial compression cases.
Figure 5.13: Critical load predictions for the z-pinned Wide Element Type 2 configuration. Left: Push-off initial peak load versus compression load interaction envelope. Right: Push-off ultimate load versus compression load interaction envelope.

Table 5.2: Push-off critical loads (z-pinned).

<table>
<thead>
<tr>
<th>Compressive load (normalized)</th>
<th>Push-off initial peak load $P_{pred.}/P_{exp.}$</th>
<th>Push-off ultimate load $P_{pred.}/P_{exp.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.90</td>
<td>1.28</td>
</tr>
<tr>
<td>0.4</td>
<td>0.94</td>
<td>1.35</td>
</tr>
<tr>
<td>0.6</td>
<td>0.94</td>
<td>1.24</td>
</tr>
</tbody>
</table>
Delamination snapshots corresponding to the marked points along the push-off force-displacement responses in Fig. 5.14 are shown in Fig. 5.15. The first load drop predicted by the model (point 1) is caused by sudden delamination growth initiating from the initial disbond (Teflon insert) along the left unreinforced tip region (primarily in the adhesive/ply-1 and ply-1/ply-2 interfaces). Experimentally, following the first load drop (point A), a UT scan was taken of the specimen and the delamination area is included in Fig. 5.15. Comparison of the cumulative delamination predicted by the model and the UT scan following the first load drop highlight the agreement of predicted and experimental delamination area shape and size. Following the first load drop, a second load drop (point 2) was predicted due to delamination of the adhesive/ply-1 and ply-1/ply-2 interfaces along the right side of the bond area. Further loading is supported as delamination slowly progresses through the z-pin reinforced CZMs (z-pin field areas are marked by gray rectangles). Joint failure occurs as delamination overcomes the z-pin reinforced CZMs and advances unstably through the remaining unreinforced interfaces of the bond area (Fig. 5.16).
Figure 5.15: Comparison of experimental and blind prediction push-off force-displacement responses with 60% maximum axial compression load (z-pinned). Z-pin fields marked with gray rectangles.
5.5.2.1 Overprediction of ultimate load for z-pinned specimens

Overall, blind predictions for the Wide Element Type 2 specimens were successful. The largest error in the predictions was that of the ultimate load for the z-pinned configuration. Further investigations into the source of this error are ongoing, however, several possible explanations will be discussed here.

One possible explanation is related to z-pin insertion depth. Across element types, and even specimen-to-specimen within an element type, the depth of z-pin penetration in the skin was discovered to vary. Images showing z-pin insertion depth for all element types are shown in Fig. 5.17. Specimen WE2-15 (Wide Element Type 2) shows relatively shallow z-pin insertion for three of the five z-pin rows. This specimen was tested at 40% maximum axial compression and was found to have the lowest ultimate load of all z-pinned specimens (including those tested at 60% maximum axial compression). It was assumed in the model, however, that the z-pin insertion depth was consistent for all specimens and all element types. Without this modeling assumption holding true, establishing coherent predictions is a challenge.
Additionally, inaccuracies in the cohesive material properties calibrated at the Narrow Element and Wide Element Type 1 levels may have contributed to the overpredicted ultimate loads. Push-off loading of the Wide Element Type 2 specimens was applied with a roller support span longer than any prior tests for z-pinned specimens. A longer support span increases the shear proportion in mixed-mode delamination. Therefore, inaccurate calibrated cohesive properties of the shearing modes may have presented little to no detrimental effect on the Narrow Element and Wide Element Type 1 correlations; however, may have been more influential for the Wide Element Type 2 configuration where shear contributions were higher.

5.6 Conclusion

The predictive capabilities of a composite pi joint finite element model were assessed through blind predictions of experimental responses. Non-reinforced and z-pin reinforced pi joint specimens were subjected to a combined loading condition of axial compression and push-off loading. Blind predictions for the unpinned configuration were found to be in good agreement across the full range of experimentally tested compressive load. Comparisons between experimental and numerical structural responses, peak loads, and failure modes and locations demonstrated the high level of accuracy achieved by the model. While delamination near the pi preform – skin bond area was the primary failure mode for all joints, the joints subjected to high compressive loads also suffered pi preform and web material failure. Although these secondary failure modes were out-of-scope for
this work, inclusion of such damage capabilities in the model would further improve the predictions. Blind predictions for the z-pinned configuration demonstrated mixed success. Initial responses including initial peak loads were accurately predicted for the z-pinned joints; however, structural toughness and ultimate loads were consistently overpredicted for all compressive loads tested. Since these discrepancies occur late in the structural response, unmodeled experimental damage modes are a suspected source of disagreement. Furthermore, variations in z-pin insertion depth have been identified across the element levels tested, making blind predictions a greater challenge without such knowledge beforehand. The smeared CZM approach would benefit from a micro-mechanics-based z-pin model to help inform changes to smeared properties based on physical characteristics of the z-pinned structure.
6.1 Motivation

One of the most common and troubling failure mechanisms in laminated composite materials is delamination. Concerns and incomplete understanding regarding delamination behavior in composite materials results in overly conservative designs and over-engineered structures. Safe-life designs are subjected to strict certification requirements and disregard the natural damage tolerance of composite/hierarchical materials and structures. Instead, damage-tolerant design approaches are more appropriate for composite materials and leads to more efficient structures. However, damage tolerant approaches require a deeper understanding of damage initiation, damage propagation, and failure. Great efforts have gone into the experimental delamination characterization of composite materials in both quasi-static and fatigue loading conditions [51, 52, 53, 54, 55, 56, 57, 58]. Additionally, analysis methods for modeling and predicting quasi-static delamination have matured over recent years and have become common in advanced models [59, 60, 25, 61]. Cohesive zone modeling (CZM) has become a popular and efficient approach for modeling delamination and has proven successful in models with complex structures subjected to general loading conditions. Modeling of delamination in high-cycle fatigue loading, however, represents a unique challenge and has not yet reached a satisfactory level of maturity. Because the potential benefits of validated fatigue analysis methods are so great, most notably time and cost savings made possible through reduced fatigue characterization testing programs, the topic has become a vibrant area of research.

One of the most established approaches to analyze fatigue induced delamination is linear
elastic fracture mechanics (LEFM). LEFM approaches typically rely on a Paris law description [62], which relates crack growth rate, \( da/dN \) (increment of crack extension per increment of fatigue cycle), to the stress intensity factor, \( K \), or energy release rate, \( G \). The Paris law can be expressed in the form of Eq. 6.1,

\[
\frac{da}{dN} = D (G^n)
\]  

(6.1)

where \( D \) and \( m \) are empirical constants obtained from testing the material. The Paris law, however, requires the existence of an established crack, and therefore cannot describe the initiation phase of damage. Furthermore, extending the Paris law to mixed-mode loading scenarios is challenging, and such a description lacks the generality to describe crack progression in real structures.

Numerical methods have been leveraged to analyze fatigue; however, modeling each load cycle is computationally unfeasible for most fatigue studies. Accelerated approaches have been proposed to efficiently model cyclic fatigue by removing the need to simulate every cycle. Two common accelerated approaches are the envelope load method [63] and the cycle jump method [64].

In the envelope load approach, a quasi-static-like load, equal to the peak value of the cyclic load, is applied to the structure and held fixed. After stabilizing at the maximum load, fatigue degradation is activated. In this way, the cyclic history of the external load is represented in a pseudo-time domain where a mapping is defined between the pseudo-time-dependency of the constitutive model and fatigue cycle number. Examples of fatigue models using the envelope load approach can be found in [63, 65] and an illustration of the method is shown in Fig. 6.1. In the cycle jump approach, computational efficiency is achieved by only simulating a limited set of the full cyclic history. Following the simulation of a single fatigue cycle, damage is extrapolated in the structure by a discrete increment/jump and the next simulation is performed. The process is repeated until the full fatigue history is accounted for or structural failure occurs. By explicitly modeling loading cycles, the cycle jump approach can be used in fatigue analyses regardless of load and spectra complexities. Examples of the cycle jump approach can be found in [64, 66, 67, 68] and an illustration of the method is shown in Fig. 6.2. Because of the generality of the cycle jump approach and ease of implementation into commercial software, the method is applied in this work to develop a CZM-based fatigue model. The fatigue framework is founded on the fundamental
evolution of cohesive properties and can be applied to structures subjected to general loading conditions.

Figure 6.1: Pseudo-time representation of the cyclic load history [63].

![Figure 6.1](image1)

Figure 6.2: Illustration of the cycle jump principle [66].

![Figure 6.2](image2)

CZM methods combine fracture mechanics concepts with continuum damage mechanics in a numerical framework that can predict delamination initiation and progression in structures without an existing crack. Early extensions of the CZM approach to fatigue loading analyses include [63, 69, 70, 71, 72]. CZM-based fatigue models can be divided into one-of-two categories: models where the Paris law (or a variant) is relied upon to drive fatigue damage and models where the Paris law is not explicitly used as input. Paris
law-based fatigue models [63, 71, 72, 73, 74, 75] relate the strain energy release rate (or a similar quantity) at the crack tip to a crack propagation rate \((da/dN)\). However, such formulations must establish a link between non-local crack growth \((da/dN)\) and the local damage framework of CZM. Relating the crack growth rate to a local damage rate has been accomplished by calculating a characteristic length, for example the process zone length [63] or the cohesive length [72], which can be cumbersome in practice. In addition to the complexities of including spatial information in the damage model, Paris law-based formulations intimately depend on expensive experimental characterizations and lack generality in complex loading scenarios and with respect to structures not containing a pre-existing crack. Local damage models [76, 77, 78, 79] move away from reliance on the Paris law and instead the damage formulation operates within the pointwise constitutive relation between traction and separation. These models are inherently better suited to function within the CZM framework and are capable of predicting both fatigue damage onset and fatigue crack growth (often described as a “unified” model).

6.2 The Cohesive Zone Model for Fatigue

6.2.1 Cohesive formulation

The fatigue framework developed in this chapter is built into the existing cohesive formulation developed by Nguyen and Waas [25]. Arbitrary softening laws for each fracture mode are permitted in the model, however this work will be limited to triangular (bi-linear) traction-separation laws. Mixed-mode considerations are important in order to facilitate general loading conditions where delamination is likely to occur under a mixed-mode condition. The mixed-mode law developed in [31] guarantees the smooth and simultaneous vanishing of all traction components, which is a physical requirement of a fully formed crack. The basis for the cohesive formulation is shown in Fig. 6.3. Pure-mode and mixed-mode responses are illustrated in Fig. 6.3a. The initial response of each mode is linear elastic and defined using a penalty stiffness \(K_i\), where \(i = I, II, III\) represent the fracture modes (opening mode I, shear modes II and III). The constitutive equation relating traction and separation components is given in Eq. 6.2. Transition from pre-peak response to post-peak softening occurs with the satisfaction of a damage initiation criterion. In the current work, the Hashin quadratic stress criterion is used (Eq.
Mixed-mode damage evolution and the softening of each mode is governed by the effective separation defined in Eq. 6.4. Furthermore, damage variables are defined in Eq. 6.5, where $\delta^*_e$ is the effective separation at damage initiation, $\bar{f}_i$ is a non-dimensionalized softening function, and $\hat{\delta}_e$ is the effective separation at complete failure ($\hat{\delta}_e$ is defined as the pure-mode failure separation of the reference mode (mode I ($\hat{\delta}_I$) in this case). A complete description of the cohesive framework and novel mixed-mode law can be found in [25].

\[
\begin{bmatrix}
\sigma_I \\
\tau_{II} \\
\tau_{III}
\end{bmatrix} = 
\begin{bmatrix}
(1 - d_I) & 0 & 0 \\
0 & (1 - d_{II}) & 0 \\
0 & 0 & (1 - d_{III})
\end{bmatrix}
\begin{bmatrix}
K_I & 0 & 0 \\
0 & K_{II} & 0 \\
0 & 0 & K_{III}
\end{bmatrix}
\begin{bmatrix}
\delta_I \\
\delta_{II} \\
\delta_{III}
\end{bmatrix}
\]

(6.2)

\[
\left(\frac{\max(0, \sigma_I)}{\sigma_{Ic}}\right)^2 + \left(\frac{\tau_{II}}{\tau_{IIc}}\right)^2 + \left(\frac{\tau_{III}}{\tau_{IIIc}}\right)^2 \geq 1
\]

(6.3)

\[
\delta_e = \sqrt{\max(0, \delta_I)^2 + \left(\frac{\hat{\delta}_I}{\delta_{II}}\right)^2 + \left(\frac{\hat{\delta}_I}{\delta_{III}}\right)^2}
\]

(6.4)

\[
d_i = 1 - \frac{\delta^*_e}{\delta_e} \bar{f}_i \left(\frac{\delta_e - \delta^*_e}{\delta_e - \delta^*_e}\right)
\]

(6.5)
6.2.2 Fatigue damage model

As stated before, the fatigue model developed in this chapter is a local model capable of predicting delamination initiation and propagation within a single framework (unified). As implemented, fatigue is applied only to cohesive interfaces; and therefore, fatigue of material other than at the interface is not modeled. The fatigue damage formulation is based on a point-wise evolution of the fundamental cohesive material properties and in turn the evolving traction-separation law at a material point. The driving assumption is that cyclic loading causes pristine/undamaged cohesive material properties to degrade. The severity of this degradation at a material point is a function of the stress/separation state and the number of cycles of load at that material point. In this way, material points experiencing high load levels during fatigue will degrade faster than those experiencing low load levels. Further details of the fatigue damage model will be presented in the context of the cycle
jump method; however, the described formulation could be implemented in other numerical approaches.

In the cycle jump approach, fatigue cycle simulations are performed at deliberately chosen intervals and fatigue damage is extrapolated in between to efficiently account for the complete fatigue loading history (Fig. 6.2). The finite element implementation of the cycle jump method generally follows a version of the following steps:

1. Finite element simulation of one loading cycle.
2. From the stress/strain/separation results of the simulated cycle, identify the most critical element based on some measure of criticality (e.g. the element having reached the highest equivalent stress/strain/separation).
3. For the most critical element, calculate how many cycles are required to advance damage by a predetermined amount (e.g. 10%) using the fatigue damage law. This damage increment is a parameter that can be varied.
4. Extrapolate damage in all elements based on the cycle jump calculated in step 3 using the fatigue damage law. Therefore, a new set of properties for each element is determined and the simulation for that cycle is fully defined.
5. Repeat steps 1-4 until failure or total number of cycles has been reached.

In what follows, the proposed fatigue damage model is developed within a variation of the cycle jump steps listed above.

6.2.2.1 Step 1

It is assumed that no damage is incurred while the applied load is unloading. Therefore, only the loading up to maximum load is simulated. This half-cycle simulation technique is done to improve computational efficiency.
6.2.2.2 Step 2

Due to the stress distribution in a structure, fatigue damage will accumulate at different rates in different regions. Material points with low stress levels will remain nearly unchanged during fatigue loading, while material points with high stress levels will damage rapidly with successive cycles, thus redistributing stress. While it would be permissible to jump over a large number of cycles for the material points experiencing low stress levels, the same jump in cycles must be applied to the entire structure. Accuracy of the fatigue analysis and the ability to model stress redistributions due to damage requires cycle jumps that are not too large. Therefore, the most critical elements (those experiencing high stress/strain levels) limit and drive the cycle jump size. It follows that a definition of criticality must be established. In this work, an equivalent separation, $\tilde{\delta}_{eq}$, is used to quantify criticality and is given in Eq. 6.6. With each mode’s separation contributing in the definition of $\tilde{\delta}_{eq}$, the quantity is suitable for mixed-mode conditions ($\delta_{ic}$, where $i = I, II, III$, is the pure-mode separation at damage initiation).

\[
\tilde{\delta}_{eq} = \sqrt{\left(\frac{\max(0, \delta_I)}{\delta_{Ic}}\right)^2 + \left(\frac{\delta_{II}}{\delta_{IIc}}\right)^2 + \left(\frac{\delta_{III}}{\delta_{IIIc}}\right)^2}
\]  

(6.6)

6.2.2.3 Steps 3 and 4

Fatigue degradation laws define the evolution (fatigue damage) of cohesive material properties as a function of the number of cycles. These degradation laws apply to the fundamental material properties required to specify the state of the material at any given cyclic load state. These laws relate static values to fatigue damage values after N cycles. General fatigue laws are provided in Eqs. 6.7, 6.8, and 6.9, where $i = I, II, III$. These laws apply to the cohesive interfaces that are used to model delamination. Non-cohesive interfaces (bulk material) are assumed to remain unaffected by the cyclic loading. Based on these relations, fatigue damage of each cohesive material property (e.g., $G_{ic}$) can be characterized by damage variables of the form $d_{f,G_{ic}} = 1 - (G_{ic}^N / G_{ic}^0)$, where $d_{f,G_{I}} = 0$ represents the undamaged state and $d_{f,G_{I}} = 1$ represents the fully damaged state.

\[
K_i^N = K_i^0 f_i(N, \tilde{\delta}_{eq})
\]  

(6.7)

\[
\sigma_{ic}^N = \sigma_{ic}^0 g_i(N, \tilde{\delta}_{eq})
\]  

(6.8)
In Eqs. 6.7, 6.8, and 6.9, \( f_i, g_i, \) and \( h_i \) can be viewed as shape functions. The developed fatigue framework does not place restrictions on the types of functions used as shape functions. Notice that the fatigue damage laws form a surface in the \( X, N, \bar{\delta}_{eq} \) space, where \( X \) is a typical property. A point on this surface corresponds to a unique critical state. In this chapter, a variation of a function proposed by Sendeckyj [80] is used to describe the relationship between material property degradation and cycle number for a given value of equivalent separation. The Sendeckyj-like function adopted here is of the form

\[
X_N = X_0 \left( 1 - C \left( 1 - N \right) \right)^{-S}
\]

where \( X_0 \) and \( X_N \) are static and fatigue degraded (after \( N \) cycles) property values, respectively, and \( C \) and \( S \) are shape parameters. The parameter \( C \) controls the extent of the flat region of the curve, and the parameter \( S \) controls the asymptotic slope at large \( N \) (Fig. 6.5). For \( C = 1 \), Eq. 6.10 reduces to the classic power law. The Sendeckyj-like shape function was selected because it exhibits characteristics commonly observed in the behavior of composite materials using a limited number of parameters. As proposed in Eqs. 6.7, 6.8, and 6.9, fatigue degradation laws are also dependent on the equivalent separation. In this work, dependency on the local equivalent separation is included by multiplying the parameter \( S \) by a quantity \( \Gamma \), where

\[
\Gamma(\bar{\delta}_{eq}) = \frac{1 + \frac{a}{k}}{1 + e^{-k(\bar{\delta}_{eq} - a)}} - \frac{a}{k}
\]

The relation between \( \Gamma \) and \( \bar{\delta}_{eq} \) is shown in 6.4 for \( k = 10 \) and \( a = 0.35 \). The constants \( k \) and \( a \) control the shape of \( \Gamma \) as a function of \( \bar{\delta}_{eq} \) and hence influence the fatigue damage laws as shown in Eqs. 6.12, 6.13, and 6.14. Fatigue degradation laws used in this thesis are,
Figure 6.4: $\Gamma$ versus normalized equivalent separation.

\[
\frac{K_i^N}{K_i^0} = (1 - C_{K,i} (1 - N))^{-S_{K,i}} \Gamma \\
\frac{\sigma_{ic}^N}{\sigma_{ic}^0} = (1 - C_{\sigma,i} (1 - N))^{-S_{\sigma,i}} \Gamma \\
\frac{G_{ic}^N}{G_{ic}^0} = (1 - C_{G,i} (1 - N))^{-S_{G,i}} \Gamma
\]

(6.12) \hspace{2cm} (6.13) \hspace{2cm} (6.14)

Figure 6.6 shows an example fatigue degradation law. Each cohesive material property may have its own fatigue degradation law; however, each independent fatigue law introduces two parameters into the fatigue formulation.
Fatigue degradation laws are used to extrapolate damage across the cycles not simulated in the cycle jump method. Different approaches exist for determining the number of cycles to advance after each simulation; however, all approaches aim to balance computational efficiency (large cycle jumps) with analysis accuracy (requiring small jumps). In the approach presented here, a maximum allowed increase to the governing fatigue damage variable is imposed. It is noted, because multiple fatigue damage variables exist in this formulation (as many as one per property per mode), the analyst must choose one to govern damage extrapolation. For well understood problems, the damage variable associated with
the most influential material property should be used as the governing damage variable. For example, in the double cantilever beam (DCB) problem, the damage variable associated with the mode I fracture toughness ($G_{Ic}$) is most appropriate, while for end notched flexure (ENF) the damage variable associated with the mode II fracture toughness ($G_{IIc}$) would be most appropriate. For more complicated structures and/or loading conditions, it is recommended that the fatigue damage variable with the most severe degradation definition be used as the governing fatigue damage variable.

Using the fatigue degradation law of the governing damage variable, the local cycle jump ($\Delta N$) to advance damage by the maximum allowed increment is calculated for each integration point. For example, let $h(N, \bar{\delta}_{eq})$ be the fatigue degradation law of the governing damage variable. Then the local cycle jump can be determined at a particular integration point using its last known damage value ($d_N$) and equivalent separation ($\bar{\delta}_{eq}$), and imposing the maximum damage increase $\Delta d$ using Eqs. 6.15 and 6.16. It is noted that this is an approach that does not take into account the full damage-cycle history information, but instead uses only the last known damage state (details of how an integration point reached a damaged state characterized by $d_N$ are not considered in calculations for its subsequent damage state).

\[ 1 - d_N = h(N, \bar{\delta}_{eq}) \quad (6.15) \]

\[ 1 - (d_N + \Delta d) = h \left((N + \Delta N), \bar{\delta}_{eq}\right) \quad (6.16) \]

Due to the range of equivalent separation states at integration points, the local cycle jumps will not be the same for each point of the structure. However, the applied global cycle jump ($\Delta N_{\text{global}}$) must be the same for all points of the structure to maintain consistency. The simplest approach to determine the global cycle jump is to select the minimum of all local cycle jumps. While this approach helps to ensure accuracy, it may also cause the fatigue analysis to proceed too slowly. Therefore, the value of the global cycle jump is taken as that of the 10\textsuperscript{th} percentile of the local cycle jumps. This approach is believed to be a reasonable compromise between accuracy and computational efficiency. With the global cycle jump determined, it is supplied to all fatigue degradation laws such that fatigue degraded cohesive material properties following the cycle jump can be calculated at every integration point.
6.2.3 Post-peak damage

Cohesive elements applied in a fatigue analysis are subjected to two different damage types: fatigue degradation and post-peak (quasi-static) damage. Fatigue degradation is caused by cyclic loading. Post-peak damage is defined as the damage that is incurred by an element/integration point after it has satisfied quasi-static damage initiation (Eq. 6.17) and enters the softening regime of its cohesive response.

\[
\left( \frac{\max(0, \sigma_I)}{\sigma_{Ic}(N, \delta_{eq})} \right)^2 + \left( \frac{\tau_{II}}{\tau_{IIc}(N, \delta_{eq})} \right)^2 + \left( \frac{\tau_{III}}{\tau_{IIIC}(N, \delta_{eq})} \right)^2 \geq 1 \quad (6.17)
\]

Consider an integration point with the traction-separation law defined by the triangle ABC in Fig. 6.7b. During a loading-unloading cycle, the integration point response follows the line segments AB, BD, and DA. During this cycle, the quasi-static damage initiation criterion is satisfied at point B and loading continues while post-peak damage is incurred along the BD segment. Unloading from point D on the cohesive law occurs in a linear elastic manner at a reduced stiffness (segment DA). If immediately reloaded, the response is linear elastic along line segment AD until point D is reached, at which point the response again follows the cohesive law (line segment DC) and further post-peak damage is incurred through complete failure (zero traction). The reloading response following quasi-static damage initiation effectively follows a new traction-separation law given by triangle ADC, where the post-peak stiffness \( K_{pp} \) is the slope of line AD, the post-peak cohesive strength \( \sigma_{pp} \) corresponds to point D, and the post-peak fracture toughness \( G_{pp} \) is the area of triangle ADC.

In fatigue analyses, it is possible that the post-peak damage state is more severe than the damage state imposed by the fatigue degradation laws. While properties returned by the fatigue degradation laws take precedence over post-peak damage properties, material healing must always be prevented. In instances where one or more of the cohesive material properties returned by the fatigue degradation laws is less degraded/damaged than that returned from the post-peak damaged calculations, the violating fatigue degraded property (or properties) is replaced with the post-peak damaged property (or properties). Following this damage irreversibility check (and any necessary reconciliation), a consistent material
property set for each integration point is obtained and supplied as input for the next simulated fatigue cycle.

![Figure 6.7: (a) Traction-separation law evolution due to fatigue degradation. (b) Post-peak damaged cohesive properties.](image)

### 6.2.4 Implementation

The fatigue model was implemented in the novel cohesive framework described in [25]. The cohesive constitutive model exists as a user defined explicit material subroutine VUMAT in Abaqus. The material law is applied to 3D cohesive elements (COH3D8 and COH3D6).

### 6.3 Evaluation and validation of the fatigue model

As an initial validation of the proposed fatigue model, mode I fatigue driven delamination was investigated in a double cantilever beam (DCB) study. The DCB test is a standard method for measuring fracture toughness, as well as delamination onset and growth due to fatigue loading [81, 82]. The mode I fracture behavior of IM7/977-3 unidirectional laminae was characterized in an experimental effort led by the Air Force Research Laboratory [54]. DCB experiments were performed under constant displacement using guidelines in ASTM D6115 [82] and a NASA technical paper [52]. Maximum displacements used for fatigue loading were calculated to correspond to 50, 40, or 30% of the statically determined fracture toughness ($G_{Ic}$). The DCB specimen is shown in Fig. 6.8 and the corresponding nominal dimensions of the experimental specimens are provided in Table 6.1.
Table 6.1: Dimensions of DCB specimen.

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>a₀ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>25.4</td>
<td>1.5</td>
<td>54</td>
</tr>
</tbody>
</table>

Table 6.2: Elastic and failure model properties of IM7/977-3 unidirectional laminae

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$ (GPa)</td>
<td>164</td>
<td>Longitudinal modulus</td>
</tr>
<tr>
<td>$E_{22}$ (GPa)</td>
<td>8.98</td>
<td>Transverse modulus</td>
</tr>
<tr>
<td>$G_{12} = G_{13}$ (GPa)</td>
<td>5.01</td>
<td>In-plane shear modulus</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.32</td>
<td>Major Poisson's ratio</td>
</tr>
<tr>
<td>$\sigma_{Ic}$ (MPa)</td>
<td>80</td>
<td>Mode I cohesive strength</td>
</tr>
<tr>
<td>$\tau_{IIc} = \tau_{IIIc}$ (MPa)</td>
<td>100</td>
<td>Mode II/III cohesive strength</td>
</tr>
<tr>
<td>$G_{Ic}$ (kJ/m$^2$)</td>
<td>0.256</td>
<td>Mode I critical strain energy release rate</td>
</tr>
<tr>
<td>$G_{IIc} = G_{IIIc}$ (kJ/m$^2$)</td>
<td>0.65</td>
<td>Mode II/III critical strain energy release rate</td>
</tr>
</tbody>
</table>

### 6.3.1 Analytical solution of DCB fatigue

Based in LEFM, the Paris law has traditionally been used to characterize fatigue crack growth. Recalling Eq. 6.1, the Paris law is rewritten here for convenience

$$\frac{da}{dN} = D (G)^n$$

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The energy release for a DCB based on elementary beam theory is

\[ G_I = \frac{9E_{xx}I\delta^2}{4a^4} \quad (6.18) \]

where \( E_{xx} \) is the Young’s modulus in the fiber direction, \( a \) is the length of the crack, and \( I = bh^3/12 \) where \( h \) is the arm thickness and \( b \) is the specimen width. Substituting Eq. 6.18 into the Paris law (Eq. 6.1) yields

\[ \frac{da}{dN} = D \left( \frac{9E_{xx}I\delta^2}{4ba^4} \right)^n = D \left( \frac{9E_{xx}I\delta^2}{4b} \right)^n a^{-4n} \quad (6.19) \]

Rearranging Eq. 6.19 leads to

\[ a^{4n} da = D \left( \frac{9E_{xx}I\delta^2}{4b} \right)^n dN \quad (6.20) \]

It is customary to use the maximum strain energy release rate (\( G_{I_{\text{max}}} \)) obtained during the cyclic load in the Paris law. For displacement control fatigue, \( G_{I_{\text{max}}} \) corresponds to \( \delta_{I_{\text{max}}} \), which is a prescribed and therefore assumed constant. Equation 6.20 can be integrated as follows,

\[ \int_{a_0}^{a^*} a^{4n} da = \int_0^{N^*} D \left( \frac{9E_{xx}I\delta^2}{4b} \right)^n dN \quad (6.21) \]

\[ \left. \frac{a^{4n+1}}{4n+1} \right|_{a_0}^{a^*} = D \left( \frac{9E_{xx}I\delta^2_{I_{\text{max}}}}{4b} \right)^n N^* \quad (6.22) \]

\[ (a^*)^{4n+1} = (4n + 1) D \left( \frac{9E_{xx}I\delta^2_{I_{\text{max}}}}{4b} \right)^n N^* + a_0^{4n+1} \quad (6.23) \]

Removing the * superscript from \( a^* \) and \( N^* \), the relation between crack length and cycle number is given by,

\[ a(N) = \left[ (4n + 1) D \left( \frac{9E_{xx}I\delta^2_{I_{\text{max}}}}{4b} \right)^n N + a_0^{4n+1} \right]^{\frac{1}{4n+1}} \quad (6.24) \]

Using details from the experimental study referenced in this section (\( D = 528.36 \), \( n = 7.32 \)), for displacement control cyclic loading corresponding to 50%\( G_{Ic} \) (\( \delta_{I_{\text{max}}} = 3.68 \) mm), the fatigue crack growth solution of Eq. 6.24 is plotted in Fig. 6.9.
In this thesis, instead of using the Paris law that lead to the expression in Eq. 6.24, a fundamental approach to obtain the crack length versus cycle curve ($a$ versus $N$) is provided. Therefore, from that result it is possible to derive the effective Paris law as well become evident in the next sections.

### 6.3.2 Finite element model of DCB specimen

A DCB model was created in Abaqus/Explicit corresponding to the experimental specimen described in [54]. A strip model with a width 1/20th of that of the physical specimen was used to reduce computational requirements and enable efficient exploration of model parameters. The laminate beams were modeled using continuum shell elements (SC8R), and the interface between the two beams was modeled using cohesive elements (COH3D8). Material properties used in the numerical study are provided in Table 6.2. Consistent with the experimental approach, numerical analyses were conducted in displacement control. Simulations were carried out at maximum cyclic displacements, $\delta_{I_{\text{max}}}$, corresponded to initial maximum strain energy release rate ($G_{I_{\text{max}}}$) values equal to 50, 40, and 30\% of the fracture toughness ($G_{Ic}$). In displacement control studies, as the compliance increases due to crack advancement, $G_{I_{\text{max}}}$ will decrease. Maximum cyclic displacements were determined from the relationship

![Figure 6.9: LEFM solution of DCB fatigue crack growth versus cycle number ($a_0 = 54$ mm).](image)
\[
\frac{G_{I_{\text{max}}}}{G_{\text{ic}}} = \left( \frac{\delta_{I_{\text{max}}}}{\delta_{\text{ic}}} \right)^2
\]

(6.25)

where \(\delta_{\text{ic}}\) is the critical displacement at peak load of a quasi-static DCB load-displacement response.

### 6.3.2.1 Calculation of strain energy release rate and crack length using a double compliances method [83]

While the crack length, \(a\), can be obtained by inspection of the simulation output after each simulated cycle, such an approach is cumbersome and is not consistent with the experimental measurement method. Instead, a double compliances method proposed in [83] is used to determine the crack length and fracture toughness following each cycle. The necessary elements of the double compliances method will be outlined here; however, the reader is directed to [83] for a complete description.

According to Griffith [84], the energy release rate (\(G\)) can be expressed as

\[
G = \frac{1}{2b}P \frac{\partial C}{\partial a}
\]

(6.26)

where \(b\) is the width of the specimen, \(C\) is the compliance, and \(P\) and \(\delta\) are the applied load and resulting displacement, respectively. The energy release rate for a two-dimensional orthotropic DCB was given by Suo et al. [85] as

\[
G = \frac{P^2a^2}{bE_{xx}I} \left( 1 + 2\beta \frac{h}{a} + \beta^2 \frac{h^2}{a^2} \right)
\]

(6.27)

where

\[
\beta = \left[ 0.677 + 0.146(\rho - 1) - 0.0178(\rho - 1)^2 + 0.00242(\rho - 1)^3 \right] \left( \frac{E_{yy}}{E_{xx}} \right)^{(1/4)}
\]

and

\(^1C^1\) is used for compliance and the fatigue law shape parameter. Distinction can be made based on context.
\[ \rho = \sqrt{\frac{E_{xx}E_{yy}}{2G_{xy}}} - \sqrt{\nu_{xy}\nu_{yx}} \]

\( E_{xx}, E_{yy}, \) and \( G_{xy} \) are the Young's moduli and shear modulus, respectively; \( \nu_{xy} \) and \( \nu_{yx} \) are the Poisson's ratios. By integrating Eqs. 6.26 and 6.27, the relationship between the applied load and displacement of an orthotropic DCB is given as

\[
C = \frac{\delta}{P} = \frac{24}{bE_{xx}} \left( \frac{a_0^3}{3h^3} + \beta \frac{a_0^2}{h^2} + \beta^2 \frac{a_0}{h} \right) \quad (6.28)
\]

The unique advantage of the double compliance method is that \( E_{xx} \) (and therefore \( \beta \)) need not be known prior to the DCB test. Instead it was proposed that one compliance measurement \((C_0)\) from the initial slope of the load-displacement response and a second compliance measurement \((C_1)\) from the unloading response after the completed DCB test can be used to obtain \( E_{xx} \) and \( \beta \). With measured crack lengths \( a_0 \) and \( a_1 \) associated with compliances \( C_0 \) and \( C_1 \), respectively, substitution into Eq. 6.28 yields

\[
C_0 = \frac{\delta}{P} = \frac{24}{bE_{xx}} \left( \frac{a_0^3}{3h^3} + \beta \frac{a_0^2}{h^2} + \beta^2 \frac{a_0}{h} \right) \quad (6.29)
\]

\[
C_1 = \frac{\delta}{P} = \frac{24}{bE_{xx}} \left( \frac{a_1^3}{3h^3} + \beta \frac{a_1^2}{h^2} + \beta^2 \frac{a_1}{h} \right) \quad (6.30)
\]

Through the requirement that \( \beta \) be positive, \( E_{xx} \) and \( \beta \) can be solved for. It follows that a crack length can be calculated for a given load and displacement by using Eq. 6.28, which gives

\[
a = 3\sqrt{\beta^3 + \frac{\delta bE_{xx}}{8P} h} - \beta h \quad (6.31)
\]

And the calculated crack length from Eq. 6.31 can be substituted into Eq. 6.27 to calculate the strain energy release rate:

\[
G = \frac{P^2 h^2}{bE_{xx} I} \left( \beta^3 + \frac{\delta bE_{xx}}{8P} \right)^{2/3} \quad (6.32)
\]

Using Eq. 6.31 and knowledge of the total cycle number \((N)\), \( da/dN \) can be obtained from successive simulated cycles. With the addition of the maximum strain energy release rate calculated using Eq. 6.32, the familiar \( da/dN \) vs \( G_{\text{Imax}} \) plots for numerical data can be generated. It is noted, an intermediate fitting of compliance \((C)\) versus cycle number \((N)\)
data from the model is performed prior to the calculation of the crack length (Eq. 6.31) and the strain energy release rate (Eq. 6.32. This is done to avoid the generation of noisy numerical data that can occur due to differentiation (e.g., \(da/dN\)) of results calculated using data directly from an explicit solver. Figure 6.10 shows compliance versus cycle data obtained directly from the model and the corresponding fit used in the calculations of crack length and strain energy release rate.

![Figure 6.10: Compliance versus cycle fit for 50\%G_{Ic}.](image)

### 6.3.3 Experimental and numerical comparisons

Delamination onset results are shown in Fig. 6.11. Each experimental data point represents an individual specimen and is defined as the initial \(G_{I_{\text{max}}}\) of the test versus the number of loading cycles to delamination onset. Static fracture toughness values are included for reference. Delamination onset within the finite element model (FEM) was defined as the number of cycles at which the compliance increased by 5\%. Model delamination onset results for the three fatigue load levels are consistent with corresponding test results and are contained within the experimental scatter.
Experimental and numerical crack growth rates are shown in Fig. 6.12. Results from three different applied displacement loads corresponding to initial maximum strain energy release rate ($G_{I\text{max}}$) values equal to 50, 40, and 30% of the fracture toughness ($G_{Ic}$) are included. The Paris law fit to the experimental data is included as a solid black line. In displacement control fatigue, $G_{I\text{max}}$ decreases as the crack grows, and therefore, the progression of data in Fig. 6.12 is from the upper right to the lower left. The approximately vertical initial crack growth rate of the model results corresponds to cycling prior to delamination onset during which damage accumulates but crack growth is insignificant. As fatigue degradation continues, delamination growth rates predicted by the model reduce and progression in a linear manner (noting the logarithmic scale of the horizontal axis) that is in very good agreement with the experimental Paris law fit. These results demonstrate the capability of the fatigue model to produce the Paris law without utilizing it as an input to the model as is customarily done. This aspect was illustrated in the example problem in Section 6.3.1. Additionally, delamination onset was predicted which is not considered in the Paris law. The fatigue degradation law for the mode I fracture toughness used to generate the delamination
onset and progression results shown in Fig. 6.11 and Fig. 6.12, respectively, is provided in Fig. 6.13. The law is defined with $C = 0.11$ and $S = 0.17$. It is noted, a similar law was used for the mode I cohesive strength; however, it has been shown that the mode I fracture toughness is the primary driving cohesive parameter in the DCB response [60].

Figure 6.12: Comparison of experimental and numerical crack growth rates for DCB fatigue.
Finally, the crack growth predicted by the fatigue model can be compared to the analytical DCB fatigue crack growth solution in Eq. 6.24 from section 6.3.1. Figure 6.14 compares crack growth versus cycle number results for the 50%$G_{lc}$ load level case (displacement control). In addition to the solution in Eq. 6.24, which was based on an elementary beam theory description of the strain energy release rate of a DCB (Eq. 6.18), the analytical solution using the strain energy release rate given in (Eq. 6.32) is also included. The LEFM solution using elementary beam theory exhibits faster crack growth than the other two solutions, which are in good agreement. This discrepancy results from the displacement - applied load ($\delta - P$) relation (6.33) taken from elementary beam theory to approximate the DCB response which was used to arrive at the energy release rate given in Eq. 6.18. Ultimately, this shows the capability of the fatigue model to capture behavior predicted by LEFM but in a more general framework.

\[
\delta = \frac{2Pa^3}{3E_{xx}I} \tag{6.33}
\]
6.3.4 Sensitivity studies

6.3.4.1 Fatigue degradation law parameters

To assess the ability of the fatigue model to capture a wide range of fatigue crack growth behaviors, sensitivity studies were performed. These sensitivity studies investigated changes to the fatigue degradation law shape which is controlled by parameters $C$ and $S$. Changes were limited to the mode I fracture toughness degradation law. Analyses were performed with an applied displacement corresponding to $50\%G_{Ic}$.

Figure 6.15 shows crack growth and crack growth rate sensitivities for fatigue degradation law shape changes in the high-cycle regime. Fatigue degradation laws used to generate the results are shown in Fig. 6.15c. Law 2 prescribed the slowest degradation of $G_{Ic}$, and therefore, the slowest crack growth of the three analyses was observed using Law 2 (Fig. 6.15a). Law 3 described the most severe degradation of $G_{Ic}$, and as a result, Law 3 produced the fastest crack growth of the three analyses. Crack growth rate versus $G_{I_{max}}$ responses (Fig. 6.15b) shift vertically downwards as the fatigue degradation severity is reduced; however, the slopes of the crack growth rate responses are similar.
Figure 6.15: Sensitivities of fatigue law changes in the high-cycle regime. (a) Crack length versus cycle. (b) Crack growth rate. (c) Normalized $G_{Ic}$ fatigue degradation laws ($\Gamma = 1$).

Figure 6.16 introduces two more fatigue degradation law shapes (in addition to Law 1) where low-cycle degradation behavior was varied. Law 4 describes an initially slower degradation of the mode I fracture toughness, while Law 5 describes an initially aggressive degradation of the property (Fig. 6.16c). The number of cycles to delamination onset predicted by Law 1 and Law 4 was similar (note the origin of the linear crack growth rate response in Fig. 6.16b); however, the propagation rate using Law 1 was slower. While Law 5 initially prescribes the fastest degradation of $G_{Ic}$, degradation slows before delamination onset causing the vertical offset (downward) of the crack growth rate versus $G_{Imax}$ response and the slower propagation rate (Fig. 6.16b).
Figure 6.16: Sensitivities of fatigue law changes in the low-cycle regime. (a) Crack length versus cycle. (b) Crack growth rate. (c) Normalized $G_{Ic}$ fatigue degradation laws ($\Gamma = 1$).

### 6.3.4.2 Cycle jump damage increment

Computational resources and time considerations are intensified in fatigue analyses. In the cycle jump scheme, the size of cycle jumps drives the analysis time. As implemented in this work, the cycle jump size is determined based on a maximum allowed increase of fatigue damage/degradation ($\Delta d_f$). Increasing the allowed damage increment leads to larger cycle jumps, and therefore quicker analyses; however, the accuracy of the solution and ability to capture stress redistributions is compromised. In Fig. 6.17, fatigue crack extensions predicted using different damage increments of the mode I fracture toughness are compared through the first 100,000 fatigue cycles. Crack extension behavior converges to the result obtained using the smallest damage increment ($\Delta d_f = 5\%$). Crack extension predictions using damage increments of 10\% and 20\% were within 1\% and 3\%, respectively, of the of the crack extension result using a damage increment of 5\%. Compared to the fatigue
analysis using a damage increment of 5%, which took 277 simulations to reach 100,000 cycles, the fatigue analyses using damage increments of 10% and 20% required 135 and 65 simulations, respectively. While the analysis using a damage increment of 40% only required 53 simulations to reach 100,000 cycles, the predicted crack extension was 56% of that when a damage increment of 5% was used. It can be concluded that a damage increment of 40% results in too large of cycle jumps to accurately capture delamination progression. Results from the study are summarized in Table 6.3.

Table 6.3: Summary of cycle jump damage increment study.

<table>
<thead>
<tr>
<th>Damage increment, $G_{Ic}$</th>
<th>Crack extension $(a - a_0)$</th>
<th>Number of simulated cycles (percentage of experimental cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>4.78 mm</td>
<td>277 (0.28%)</td>
</tr>
<tr>
<td>10%</td>
<td>4.73 mm</td>
<td>135 (0.14%)</td>
</tr>
<tr>
<td>20%</td>
<td>4.66 mm</td>
<td>64 (0.06%)</td>
</tr>
<tr>
<td>40%</td>
<td>2.68 mm</td>
<td>53 (0.05%)</td>
</tr>
</tbody>
</table>

Figure 6.17: Effect of cycle jump damage increment on crack extension versus cycle number (50%$G_{Ic}$ load level).
6.4 Conclusion

A unified local cohesive fatigue model was developed within the same novel cohesive formulation used for quasi-static analyses in this thesis. As implemented in the cycle jump method, the proposed fatigue model can efficiently simulate high cycle fatigue. The driving assumption of the model is that cyclic loading causes pristine/undamaged cohesive material properties to degrade. Characteristics of the fatigue model were investigated by numerically studying fatigue driven delamination in DCB specimens and comparing results with experimental data. The model was able to produce consistent crack propagation rates without relying on Paris law-related inputs. Sensitivity studies demonstrated the capability of the model to represent a broad range of fatigue crack growth behaviors.
CHAPTER 7

Fatigue Analysis of a Z-Pin Reinforced Composite Pi Joint

7.1 Motivation

Fatigue is a unique challenge for both experimental testing programs and numerical analyses. Fatigue testing campaigns are expensive and require a significant time investment. Similarly, fatigue modeling methods are relatively immature, uncommon in commercial software, and often computationally resource intensive. Because of these challenges associated with fatigue, current industry design practices preclude fatigue from being a critical design consideration [86]. Instead, it is accepted that other (non-fatigue) composite design allowables lead to a level of conservatism such that fatigue is not a major concern. As new design approaches emerge for more efficient structures, current knockdowns and safety factors may be reduced such that fatigue behavior becomes a critical design driver. In such cases, fatigue analysis methods will be critically important to explore design spaces and derisk physical testing.

Bonded unitized structures are proposed as weight saving and efficient designs for primary airframe structures. In addition to weight reductions, bonded structures have the potential to reduced part/fastener counts and promote more widespread adoption of composite materials. Certification of bonded structures, however, is a challenge due to concerns about delamination and bondline failure. To overcome these certification challenges, through-thickness reinforcement techniques, such as z-pins, are being studied as damage arrestment features for bonded structures, with the objective of transitioning from the current no-damage-growth certification approach to a slow-damage-growth approach for composite structures. To achieve this goal, predictive progressive damage and failure
analysis methods for bonded structures containing damage arrestment features are required.

In this chapter, a novel fatigue model is applied to a z-pinned composite pi joint subjected to cyclic pull-off loading. The fatigue model used here is a unified local cohesive zone model capable of capturing delamination onset and propagation for structures subjected to general loading conditions. The chapter is organized as follows: Section 7.2 provides a brief (non-exhaustive) overview of composite fatigue modeling, as well as a review of the cohesive fatigue model used in this work. In Section 7.3, the experimental approach for the z-pinned pi joint fatigue study is presented. In Section 7.4, the finite element model is introduced and assumptions within the modeling approach are addressed. Finally, in Section 7.5, experimental and numerical results are presented. Model calibration using experimental constant amplitude fatigue data is summarized in Section 7.5.1, and model blind predictions of constant amplitude block fatigue data follows in Section 7.5.2.

7.2 Fatigue modeling

7.2.1 Overview

In [87], a classification of existing fatigue models for fiber-reinforced composite materials is proposed and consists of three major categories: fatigue life models; phenomenological models for residual stiffness/strength; and progressive damage models. Fatigue life models use engineering S-N curves or Goodman-type diagrams [88] and introduce a fatigue failure criterion without considering physical degradation mechanisms [89, 90, 91, 92, 93]. These models are borrowed from metal fatigue and require extensive experimental work. The second class of models are phenomenological models for residual stiffness/strength. These models use evolution laws, which govern the degradation of the elastic properties (stiffness) or strength in terms of macroscopically observable properties [94, 64, 95, 96, 97]. Progressive damage models (or mechanistic models), on the other hand, use evolution laws which describe damage accumulation in terms of damage-specific variables [98, 99, 100, 101, 102, 67].

Within the progressive damage class, cohesive zone model-based (CZM) fatigue approaches have been proposed to model fatigue-driven delamination. These CZM-based
fatigue models can be divided into two categories: models that directly enforce the Paris law (or a Paris law variant) and models that do not. Models that directly enforce the Paris law are plagued by challenges associated with non-locality and lack generalization. Furthermore, the Paris law assumes that the crack growth rate is a function of only the applied energy release rate (or stress intensity factor). This assumption does not support the initiation of new cracks and leads to the conclusion that the crack growth rate is independent of the structure. Local damage models, however, attempt to predict crack progression with only pointwise information. These local damage models are rooted in a more fundamental description of damage mechanics and are generally easier to implement. The fatigue model used in this work is a unified local cohesive formulation. Unified refers to the ability to account for delamination progression as well as initiation. Stress-driven initiation preserves the generality of such models allowing for their usage for specimens without a preexisting crack.

7.2.2 Cohesive fatigue model

The unified local cohesive fatigue law proposed in the previous chapter was used to numerically study the fatigue behavior of z-pinned composite pi joints. A brief review of the fatigue model will be provided in this section. The fatigue damage formulation is built within the novel cohesive formulation developed by Nguyen and Waas [25] and is based on a point-wise evolution of fundamental cohesive material properties. In this way, the cohesive formulation remains unaltered (static behavior is preserved) and only the material properties provided as input to the cohesive law are evolved to account for fatigue loading. The basic assumption of the fatigue model is that fatigue loading causes pristine/undamaged cohesive material properties to degrade. The rate of fatigue degradation is determined by the local stress/separation state at element integration points such that points experiencing high-load levels will degrade faster than those experiencing low load levels during fatigue.

7.2.2.1 Fatigue degradation

Delamination often occurs in mixed-mode conditions, and therefore the usefulness of the fatigue model is tied to its ability to support general loading conditions. Inspired by the strategy in mixed-mode cohesive laws, an equivalent separation, \( \bar{\delta}_{eq} \), was defined to quantify the local load state at an integration point. The equivalent separation is given in Eq. 7.1. \( \delta_t \),
\(\delta_{II}\), and \(\delta_{III}\) are separation components for the opening and two shear modes, respectively. \(\delta_{Ic}\), \(\delta_{IIc}\), and \(\delta_{IIIc}\) are the pure-mode separation components at the transition from the pre-peak regime to the post-peak regime of the traction-separation law.

\[
\bar{\delta}_{eq} = \left(\frac{\max(0, \delta_I)}{\delta_{Ic}}\right)^2 + \left(\frac{\delta_{II}}{\delta_{IIc}}\right)^2 + \left(\frac{\delta_{III}}{\delta_{IIIc}}\right)^2
\]

(7.1)

Fatigue degradation laws are used to define the degradation of cohesive material properties as a function of the number of cycles and local load state. General fatigue laws are provided in Eqs. 7.2, 7.3, and 7.4, where \(i = I, II, III\). Based on these relations, fatigue damage of each material property (e.g., \(G_{Ic}\)) can be characterized by damage variables of the form \(d_{f,G_{Ic}} = 1 - \left(G_{Ic}^N/G_{Ic}^0\right)\), where \(d_{f,G_i} = 0\) represents the undamaged state and \(d_{f,G_i} = 1\) represents the fully damaged state.

\[
K_i^N = K_i^0 f_i(N, \bar{\delta}_{eq})
\]

(7.2)

\[
\sigma_{ic}^N = \sigma_{ic}^0 g_i(N, \bar{\delta}_{eq})
\]

(7.3)

\[
G_{ic}^N = G_{ic}^0 h_i(N, \bar{\delta}_{eq})
\]

(7.4)

For this work, a variation of a function proposed by Sendeckyj [80] is used to describe the relationship between material property degradation and cycle number. The Sendeckyj-like function is of the form:

\[
X_N = X_0 \left(1 - C \left(1 - N\right)\right)^{-S}
\]

(7.5)

where \(X_0\) and \(X_N\) are static and fatigue degraded (after \(N\) cycles) property values, respectively; and \(C\) and \(S\) are shape parameters. Figure 7.1 shows an example fatigue degradation law and illustrates dependencies on cycle number \((N)\) and equivalent separation \((\bar{\delta}_{eq})\) through \(\Gamma\) where

\[
\Gamma(\bar{\delta}_{eq}) = \frac{1}{1 + e^{k(\bar{\delta}_{eq} - a)}}
\]

(7.6)
While Eqs. 7.2, 7.3, and 7.4 describe the cycle-by-cycle evolution of cohesive material properties, it is computationally infeasible to model each individual cycle in a high-cycle fatigue analysis. Therefore, the cohesive fatigue model operates within the cycle jump scheme [66]. In the cycle jump scheme, simulations are only performed for a limited set of fatigue cycles in the full loading history and fatigue damage is extrapolated across discrete increments/jumps (Fig. 7.2). Fatigue degradation laws (Eqs. 7.2, 7.3, and 7.4) provide the link between the evolution of damage in the structure and the advancement of loading cycles. A complete description of how the cycle jump scheme is implemented in this work can be found in Chapter 6.

7.2.2.2 Post-peak damage

Because the fatigue model operates within a cohesive formulation originally developed for quasi-static loading, two mechanisms can contribute to the damaged/degraded state
at an integration point: fatigue degradation and post-peak (quasi-static) damage. Fatigue degradation is caused by cyclic loading. Post-peak damage is defined as the damage that is incurred by an element/integration point after it has satisfied quasi-static damage initiation (Eq. 7.7) and enters the softening regime of its cohesive response. While the damage variables associated with fatigue degradation and post-peak damage are kept independent in the cohesive framework, damage irreversibility must not be violated.

\[
\left( \frac{\max(0, \sigma_I)}{\sigma_{Ic}(N, \delta_{eq})} \right)^2 + \left( \frac{\tau_{II}}{\tau_{IIC}(N, \delta_{eq})} \right)^2 + \left( \frac{\tau_{III}}{\tau_{IIIc}(N, \delta_{eq})} \right)^2 \geq 1 \quad (7.7)
\]

Consider an integration point with the traction-separation law defined by the triangle \( ABC \) in Fig. 7.3. During a loading-unloading cycle, the integration point response follows the line segments \( AB, BD, \) and \( DA \). During this cycle, the quasi-static damage initiation criterion is satisfied at point \( B \) and loading continues while post-peak damage is incurred along the \( BD \) segment. Unloading from point \( D \) on the cohesive law occurs in a linear elastic manner at a reduced stiffness (segment \( DA \)). If immediately reloaded, the response is linear elastic along line segment \( AD \) until point \( D \) is reached, at which point the response again follows the cohesive law (line segment \( DC \)) and further post-peak damage is incurred through complete failure (zero traction). That is, the reloading response following quasi-static damage initiation effectively follows a new traction-separation law given by triangle \( ADC \). This post-peak damaged traction-separation law is defined by the post-peak stiffness (\( K_{pp} \): slope of line \( AD \)), the post-peak cohesive strength (\( \sigma_{pp} \): vertical height of point \( D \)), and the post-peak fracture toughness (\( G_{pp} \): area of triangle \( ADC \)).

Following a simulated fatigue cycle, it is possible that the post-peak damage state is more severe than the damage state imposed by the fatigue degradation laws for the subsequent simulation. In such instances, the post-peak damaged property (or properties) replaces the fatigue degraded property (or properties) such that a consistent set of input material properties is established prior to the next simulation.
7.3 **Experimental approach**

Fatigue tests were performed on z-pinned Narrow Element specimens. All specimens contained a Teflon insert (causing an initial disbond) in one of the tip regions between the film adhesive and the skin. An illustration of the specimen is shown in Fig. 7.4. Pi joint specimens were tested at the Air Force Research Laboratory (AFRL) and subjected to four-point pull-off loading. In four-point pull-off loading, the skin was restrained by two sets of rollers while a vertical (pull-off) load was applied to the top of the web (Fig. 7.5).
7.3.1 Quasi-static generation of pre-crack

Prior to fatigue testing, specimens were first quasi-statically loaded to generate a pre-crack\textsuperscript{1} from the Teflon insert. The load at which the pre-crack formed was used as the reference load to define maximum cyclic loads in the fatigue tests. Figure 7.6 shows normalized pull-off force-displacement responses for specimens loaded to failure (grey) as well as the pre-crack generation response of a fatigue specimen (black) prior to fatigue loading. Initial delamination growth from the Teflon insert consistently occurred at a normalized load of approximately 1. Figure 7.7a shows the reference state of a fatigue specimen and the location of the Teflon insert prior to pre-crack generation loading, while Fig. 7.7b shows the specimen immediately following the minor load drop associated with the pre-crack formation. Fatigue tests were conducted in load control at a range of maximum cyclic loads defined as a percentage of the specimen’s reference load (peak load associated with pre-crack delamination initiation).

\textsuperscript{1}The terms “delamination” and “crack” will be used interchangeably throughout this chapter.
Figure 7.6: Experimental pre-crack generation loading compared to quasi-static loading to failure.
Figure 7.7: Pre-crack generation and final crack extension. (a) Unloaded/reference state. (b) Initial crack growth from Teflon insert (pre-crack). (c) Final fatigue driven crack state.

7.3.2 Fatigue testing

Fatigue tests were conducted in either constant amplitude fatigue or constant amplitude block fatigue. Constant amplitude fatigue tests were conducted at three different maximum cyclic loads corresponding to 80%, 70%, and 60% of the reference load ($P_{ref}$). Because reference loads were unique to each specimen, fatigue load levels were specimen specific. The ratio of minimum load to maximum load (R-ratio) was 0.1. Specimens were cycled until delamination had progressed between the third and fourth row of z-pins (Fig. 7.7c)
or up to 1,000,000 cycles. It is noted, the crack length \((a)\) was measured from the internal edge of the Teflon insert. For the constant amplitude block fatigue tests, two fatigue spectra were investigated. The first spectrum, designated “lo-hi”, began with a low-load block at \(60\%P_{ref}\) for 150,000 cycles, followed by a high-load block at \(80\%P_{ref}\) for 10,000 cycles. The second spectrum, designated “hi-lo”, began with a high-load block at \(80\%P_{ref}\) for 10,000 cycles, followed by a low-load block at \(60\%P_{ref}\) for 150,000 cycles. An R-ratio of 0.1 was maintained throughout. The lo-hi and hi-low spectra are illustrated in Fig. 7.8. All fatigue tests were concluded with a quasi-static overload to failure in order to assess residual strength.

Fatigue crack growth was tracked using digital image correlation (DIC) as well as periodic full emersion Ultrasonic (UT) scans. DIC data were acquired from both edges of the specimen, and by its nature, was only representative of the crack position at the free edge. Full emersion UT scans were performed at less frequent increments (compared to DIC crack tracking), but the scans provided insight regarding internal crack position. While the edge crack position was found to consistently be slightly ahead of the internal crack front, the narrow width of the specimens limited this discrepancy. Additionally, load and displacement data were collected throughout the tests.
7.4 Modeling approach

7.4.1 Strip model and crack definitions

Given the computational requirements of fatigue analyses and the need to calibrate the model, the numerical study was performed using a strip model of the pi joint (Fig. 7.9). In addition to being a strip model (1/15th the nominal Narrow Element width), further
simplifications of the z-pinned pi joint model developed in [27] were made. These simplifications included replacing most cohesive interlayers, which were used in the higher-order model to capture delamination, with tie constraints. The cohesive interlayer used to model delamination in the ply 1 – ply 2 interface was preserved as this was the interface where fatigue delamination occurred experimentally. In an attempt to make the model representative of the average fatigue specimen, generation of the pre-crack was not explicitly simulated, but instead the average experimental pre-crack length was prescribed in the model. Both the Teflon insert (between the adhesive layer and the skin) and the pre-crack (ply 1 – ply 2 interface) were modeled using a contact interaction preventing penetration and allowing frictional sliding between surface pairs. Finally, plane strain-like boundary conditions were applied to the free edges of the strip model prevent translation in the width direction (x-direction as shown in Fig. 7.9).

Figure 7.9: Pi joint strip model.

Recalling the smeared CZM approach used to model the z-pin reinforcement [27], an inconsistency arises regarding the definition of a crack. Experimentally, a crack can extend beyond intact and activated z-pins (Fig. 7.7c). In the smeared CZM model, z-pins are not explicitly modeled, and the numerical crack length will be shorter as z-pin bridging
is accounted for in an averaged sense (Fig. 7.10). To reconcile this, a compliance-based approach was proposed to establish a relationship between a crack length in the smeared model to the experimental definition of a crack. To this end, a repeat unit strip pi joint model was created where a single lengthwise row of z-pins was explicitly modeled (Fig. 7.11). The model was prescribed crack lengths of various size and then loaded to load levels consistent with those studied experimentally (after scaling by width). For each prescribed crack length, the associated compliance was recorded. Compliance versus crack length results are shown in Fig. 7.11. By fitting the data, an equation relating compliance and crack length (as defined experimentally) was established. It follows that compliance results from the smeared model can be supplied to the fit equation to calculate crack lengths consistent with the experimental definition.

In the model, the interaction between elastic z-pins and the surrounding material was modeled using a contact interaction with basic behavioral properties (hard contact and frictional sliding). More advanced considerations when explicitly modeling z-pins, such as initial interference, bonding, and residual stresses, were not included.

![Figure 7.10: Experimental ($a_{exp}$) and smeared CZM ($a_{smeared}$) crack definition inconsistencies.](image)
7.4.2 Fatigue model constraints

Fatigue simulations were conducted using the fatigue model proposed in the previous chapter and reviewed in Section 7.2. Fatigue material degradation was applied to the z-pin reinforced cohesive interlayer ahead of the pre-crack front in the ply 1 – ply 2 interface. This limited application of fatigue degradation was reasonable as experimental cracks did not progress beyond the z-pin field during fatigue loading. Additionally, the pre-crack extends into the z-pin reinforced cohesive interlayer (no non-reinforced cohesive interlayer in front of the crack). In the proposed fatigue degradation shape functions, cycle number \( (N) \) dependency on fatigue degradation is of the form given in Eq. 7.5 and rewritten as

\[
\frac{X_N}{X_0} = (1 - C (1 - N))^S
\]  

(7.8)

for static/pristine property \( X_0 \) and fatigue degraded property \( X_N \). Dependency on the local equivalent separation \( (\bar{\delta}_{eq}) \) is introduced by multiplying the parameter \( S \) by \( \Gamma \). Fatigue degradation laws then take the form

\[
\frac{K_i^N}{K_i^0} = (1 - C_{K_i} (1 - N))^{-S_{K_i}\Gamma}
\]  

(7.9)
\[
\frac{\sigma_{ic}^N}{\sigma_{ic}^0} = (1 - C_{\sigma_i} (1 - N))^{-S_{\sigma_i}\Gamma}
\]

(7.10)

\[
\frac{G_{ic}^N}{G_{ic}^0} = (1 - C_{G_i} (1 - N))^{-S_{G_i}\Gamma}
\]

(7.11)

where each property’s fatigue degradation law has two parameters \((C\) and \(S\)). While a unique parameter set can be used for each cohesive property, such an approach would lead to a large number of unknown parameters without sufficient data for calibration. To reduce the number of free parameters as well as facilitate the exploratory nature of this numerical study, two simplifying assumptions were initially made within the fatigue framework. The first assumption was that all fracture modes \((I, II,\) and \(III)\) degrade identically, that is, \(C_I = C_{II} = C_{III}\) and \(S_I = S_{II} = S_{III}\). The second assumption was that fracture toughness and cohesive strength degrade identically (e.g., \(C_{G_I} = C_{\sigma_I}\) and \(S_{G_I} = S_{\sigma_I}\)). As a result, only one \(C\) and \(S\) parameter set was required as input to the fatigue model. Because penalty stiffness \((K)\) lacks physical basis, minor fatigue degradation was applied to stiffness of each mode; however, post-peak damage of the stiffness remained fully accounted for.

The fatigue model (parameters \(C\) and \(S\)) was calibrated using experimental crack length versus cycle number data from the constant amplitude fatigue tests. Fatigue degradation parameters used successfully in an earlier double cantilever beam (DCB) fatigue study were used as initial estimates for \(C\) and \(S\).

7.5 Experimental and numerical comparisons

7.5.1 Constant amplitude fatigue

Crack length versus cycle number results for the constant amplitude fatigue study are shown in Fig. 7.12. Crack growth behavior is included for all three load levels. The variation in pre-crack lengths across test specimens is conveyed by the crack length scatter at \(N = 1\). The experimental crack growth for the 80% and 70%\(P_{ref}\) load levels was very similar; however a markedly slower crack growth was observed for the 60%\(P_{ref}\) load level. This distinct difference in crack growth behavior between the 60%\(P_{ref}\) load level and the 80% and 70%\(P_{ref}\) load levels was a challenge for the fatigue model to capture;
and therefore two separate (applied load specific) fatigue material degradation laws were used. Normalized material degradation laws used to obtain the numerical results included in Fig. 7.12 are shown in Fig. 7.13. Using a single fatigue law, crack growth response was sufficiently captured numerically for the 80% and 70%$P_{ref}$ load levels. At higher fatigue cycles (approximately $N > 50,000$), crack length results from the model begin to constantly overpredict experimental crack lengths for the 70%$P_{ref}$ load level. Using a distinct material degradation law for the 60%$P_{ref}$ load levels, reasonable experimental and numerical crack growth agreement was achieved. As can be seen in Fig. 7.13, the fatigue material degradation law for the 60%$P_{ref}$ load level defines a less severe degradation of cohesive properties (fracture toughness and strength) compared to the law used for the 80% and 70%$P_{ref}$ load levels.

Figure 7.12: Constant amplitude crack length versus fatigue cycle.
It is noted, during the constant amplitude fatigue calibration process, attempts were made to accurately capture the crack growth behavior at all three load levels using a single set of material degradation laws (as opposed to applied-load-specific material degradation laws). Early strategies included removing the constraint of identical degradation across fracture modes as well as removing the constraint of identical degradation of fracture toughness and cohesive strength properties. These studies did not indicate that similarities in material degradation law definitions were preventing the modeling from capturing experimental behavior. The variation in pre-crack length was also identified as a possible source of experimental and numerical inconsistencies. By allowing the modeled pre-crack length to vary between the range of pre-crack lengths observed across the test specimens, predicted crack growth behavior was found to change significantly. Crack length versus cycle number results for the $60\% P_{ref}$ load level are shown in Fig. 7.14 where the modeled pre-crack length was varied (the same material degradation laws were used for the three pre-crack lengths studied). It is apparent that the initial pre-crack length is an influential parameter with respect to predicted fatigue crack growth. Because of this sensitivity to pre-crack length, consistency between test specimens should be strived for.

Since pre-crack lengths were not known prior to establishing blind predictions of the constant amplitude block fatigue behavior, the predictions were ultimately made using the fatigue degradation laws in Fig. 7.13. Experimental and blind predictions for the two constant amplitude block fatigue spectra are presented in the following section.
7.5.2 Constant amplitude block fatigue

7.5.2.1 Lo-hi

Crack length versus cycle number experimental results and blind predictions for the lo-hi constant amplitude block fatigue spectrum are shown in Fig. 7.15. Experimental results (grey markers) are recorded edge-view crack lengths from three specimens. The predicted response captures the initially slow crack growth during the first 150,000 cycles at 60%\( P_{\text{ref}} \), as well as the rapid crack growth that follows during the high-load (80%\( P_{\text{ref}} \)) block. At the end of the low-load block, the crack length predicted by the model was 6% longer than the experimental average crack length. At the end of the high-load block, the predicted crack length was 3% shorter than the experimental average crack length. Therefore, the experimental crack growth slightly outpaced the predicted crack growth during the high-load block.
Following the cyclic loading, specimens were quasi-statically loaded to failure to assess residual strength. Figure 7.16 shows the normalized force-displacement responses of the fatigued specimens. Normalization was done with respect to the average experimental first peak load and the corresponding displacement. The blind predicted residual strength was 22% lower than the experimental average residual strength. Additionally, the predicted response was more compliant than the experimental responses. In previous work for this project, it was observed that the initial stiffness of z-pinned experimental specimens was approximately 10% higher than the initial stiffness of the model. This stiffness discrepancy was determined to be a result of bulking in the z-pin field regions due to the additional z-pin material, which was not accounted for in the model. Also included in Fig. 7.16 is the predicted response where the displacement has been scaled to increase the stiffness by 10% (dashed blue curve). The stiffened model response is more consistent with the experimental responses, however, a greater degree of softening non-linearity near peak load was predicted compared to what was observed experimentally. This more compliant response predicted by the model, in addition to the underpredicted residual strength, may allude to shortcomings in the mapping between model crack length and experimental crack length. The trend in
behavior suggests that experimentally consistent crack lengths calculated from the model (using the fit in Fig. 7.11), are representative of a longer crack lengths in reality (defined experimentally).

Figure 7.16: Quasi-static residual strength force-displacement responses following lo-hi fatigue loading (four-point pull-off loading).

7.5.2.2 Hi-lo

Crack length versus cycle number experimental results and blind predictions for the hi-lo constant amplitude block fatigue spectrum are shown in Fig. 7.17. The predicted crack growth during the high-load block (first 10,000 cycles) is slightly slower than that observed experimentally. At the end of the high-load block, the predicted crack length was 23% shorter than the experimental average crack length. Upon switching to the 60%$P_{ref}$ load level, crack growth slows considerably, and very limited crack extension was observed experimentally nor numerically. At the end of the low-load block, the predicted crack length was 24% shorter than the experimental average crack length. Because the error in the predicted crack length remains approximately unchanged after the high-load block, it is reasonable to concluded that the majority of the experimental and numerical discrepancy develops in the high-load block. The source of this error can be understood by recalling the constant amplitude data. Figure 7.18 includes the 80%$P_{ref}$ constant amplitude data (through
the first 10,000 cycles) with the hi-lo constant amplitude block fatigue spectrum crack growth results. Crack growth observed for the block fatigue specimens was consistently faster than that of the constant amplitude specimens. Because the fatigue model was calibrated using the constant amplitude data, the predicted crack growth, in turn, underestimates the high-load block crack growth. This finding, in addition to experimental pre-crack variability, highlights the challenges of scatter in fatigue test data.

Figure 7.17: Experimental and blind prediction crack length versus fatigue cycle results for the hi-lo constant amplitude block fatigue spectrum.
Figure 7.18: Constant amplitude (CA) crack length versus fatigue cycle at $80\%P_{ref}$ included with the hi-lo constant amplitude block fatigue results.

Normalized force-displacement responses from the residual strength tests are shown in Fig. 7.19. The blind predicted residual strength was 5\% lower than the experimental average residual strength. Again, the predicted response scaled to increase the stiffness 10\% is included to remove the stiffening effect of z-pin bulking. The underpredicted residual strength, to a lesser degree here, and the more compliant numerical response are consistent with the predictions established for the lo-hi spectrum and further support the conclusions drawn in that section.
Figure 7.19: Quasi-static residual strength force-displacement responses following hi-lo fatigue loading (four-point pull-off loading).

7.6 Conclusion

A novel cohesive fatigue damage model was evaluated as part of an experimental and numerical investigation into the fatigue behavior of z-pinned pi joints. The unified local fatigue model proposes fundamental descriptions of the evolution of cohesive material properties due to fatigue loading. As implemented, material degradation laws are used to drive fatigue damage accumulation within the cycle jump scheme. A strip finite element model of a z-pinned composite pi joint was created to efficiently investigate and assess the fatigue model’s ability to capture complex fatigue behaviors. First, fatigue model parameters were obtained through correlation with experimental data from constant amplitude fatigue tests. Crack length versus cycle number data from three maximum fatigue load levels was the primary behavior used to correlate the model. Following the correlation effort, blind predictions were established for constant amplitude block fatigue tests. Model predictions of crack growth for the two block fatigue spectra agree well with the experimental results. Model predictions of residual strength following fatigue loading were conservative by 5-22%. The model demonstrated the ability to capture critical fatigue behaviors of a non-traditional structure in a mixed-mode loading condition.
CHAPTER 8

Concluding Remarks and Topics for Future Work

8.1 Summary

The studies presented in this thesis establish a methodology to efficiently and accurately model the responses of non-reinforced and z-pin reinforced pi joints subjected to quasi-static and fatigue loading. Experimentally validated models have become increasingly important throughout the design and certification phases of advanced aerostructures. These tools can expedite and reduce the costs of development cycles and ultimately lead to better and more efficient structures. To fully realize the benefits of predictive analysis methods, maturation and confidence must be developed in the models.

The modeling approach presented in this thesis extends concepts from the semi-discrete damage model (SD2M), which has efficiently demonstrated high-fidelity predictions for coupon-level laminates, to structural element-level cases. Smart meshing strategies and a novel mixed-mode cohesive formulation help to capture experimentally observed failure mode interactions between intra-laminar cracking and delamination. The effects of z-pinning were included through a smeared cohesive modeling approach where effective fracture toughness and bridging strength are prescribed in the z-pin reinforced regions. This smeared method sufficiently represents the influence of z-pinning in a scalable manner. The modeling approach was first applied at the Narrow Element level and, through the inverse method, cohesive material properties were established. Subsequently, two rounds of blind predictions were performed on pi joint specimens of increasingly complex geometries and loading conditions.

A unified local cohesive fatigue model was developed within the same mixed-mode cohesive formulation used for the quasi-static models. The fatigue model operates within
the cycle jump scheme and is based on the point-wise degradation of fundamental cohesive material properties. This fundamental philosophy and framework allow the model to be applied to structures subjected to general loading conditions. The model is first investigated and validated in a double cantilever beam (DCB) fatigue study and is then used to study the fatigue behavior of z-pinned Narrow Element pi joint specimens.

8.2 Concluding remarks

In Chapter 2, a progressive failure model of the unpinned Narrow Element was developed. The novel mixed-mode cohesive formulation in [hn] was used as the constitutive model of both matrix ply crack elements and cohesive interlayers. Cohesive material properties were calibrated through the inverse method, which relied on Latin Hypercube Sampling studies to provide sensitivities of cohesive material properties on structural peak load responses. Good experimental and numerical correlation was achieved in terms of structural response and failure progression and location both pull-off and side-bend loading conditions, which demonstrated the capabilities of the modeling approach.

In Chapter 3, the Narrow Element model was extended to include the effects of z-pinning. To this end, a cohesive zone approach was implemented where effective fracture toughness and strength properties were smeared over the reinforce areas in each cohesive interlayer. Additionally, the continuum crack band method was used to account for diffuse damage and failure in the woven pi preform, which developed due to delamination suppression caused by the z-pinning. The model was calibrated using experimental data from pristine and defective z-pinned pi joints subjected to pull-off and side-bend loading. Comparisons of experimental and numerical results show good agreement in terms of structural response, critical loads, and failure modes.

In Chapter 4, non-reinforced and z-pin reinforced cohesive material properties obtained at the Narrow Element level were supplied to the Wide Element Type 1 model to establish blind predictions of specimens subjected to pull-off loading. Predictions for the unpinned configuration were found to be in good agreement overall; however, predictions for the z-pinned configuration overpredicted joint toughness. Based on parametric studies, the most influential cohesive parameters to joint toughness were updated to improve experimental
and numerical agreement before the final demonstration of predictive capabilities. Further examination of the failed specimens revealed that the depth of z-pin penetration in the Wide Element Type 1 specimens was much less compared to the Narrow Element specimens – this finding explained discrepancies in the blind prediction of the z-pinned configuration and revealed additional challenges in establishing coherent blind prediction of z-pinned structures.

In Chapter 5, blind predictions of the Wide Element Type 2 specimens were made as a final assessment of the predictive capabilities of the model. Wide Element Type 2 specimens were subjected to combined loading of axial compression and pull-off. Predicted structural responses, peak loads, and damage patterns were found to be in good agreement across a range of experimentally tested compressive loads for the unpinned configuration. For the z-pin reinforced joints, initial responses were accurately predicted; however, ultimate loads and the toughening effect of the z-pin reinforcement was overpredicted.

In Chapter 6, a unified local cohesive fatigue model was developed. As implemented in the cycle jump method, the proposed fatigue model can efficiently simulate high cycle fatigue. The driving assumption of the model is that cyclic loading causes pristine/undamaged cohesive material properties to degrade. Characteristics of the fatigue model were investigated by numerically studying fatigue driven delamination in DCB specimens and comparing results with experimental data. The model was able to produce consistent crack propagation rates without relying on Paris law-related inputs.

In Chapter 7, the fatigue model was challenged to capture the behavior of z-pinned Narrow Elements subjected to constant amplitude and constant amplitude block fatigue loading. The model was calibrated using experimental constant amplitude fatigue data, and then used to make blind predictions of constant amplitude block fatigue responses. Predictions of two block fatigue spectra captured overall crack growth behaviors and predicted residual joint strength with reasonable accuracy.
8.3 Future work

The work presented in this thesis can be extended upon to further mature analysis methods for bonded structures, as well as improve the representation of damage arrestment features in structural models. Furthermore, the proposed fatigue model has demonstrated potential – yet would benefit most from continued development and investigations.

1. **Model representation of pi preform.** In this work the preform was modeled as a homogenized orthotropic part, and its 3D woven architecture was ignored. This is a simplified representation of a complex and critically important component. Multi-scale methods have been successfully used for woven composites [103, 104] and would provide a more accurate representation of the preform. Accounting for spatial variations in material properties due to the coarseness of the weave would benefit accuracy.

2. **Comprehensive damage modeling.** While the modeling approach developed in this thesis aimed to achieve a balance between fidelity and computational efficiency, more comprehensive damage modeling in the skin and preform would likely improve experimental and numerical agreement. As z-pinning suppresses delamination, diffuse damage in the pi preform was observed for some configurations. This was partially considered using crack band element in the throat region of the preform for the z-pinned Narrow Elements; however, a more systematic approach utilizing material data from the pi preform (stress-strain curve) would be beneficial. Furthermore, due to the large deflections sustained by the z-pinned joints, compressive material failure as a result of skin bending is possible (observed for some Narrow Element configurations). This type of damage could be included in the model using the enhanced Schapery Theory [105, 106].

3. **Z-pin modeling.** The smeared CZM approach for modeling the influence of z-pinning is attractive as it is simple to implement and scalable. The approach, however, would benefit from an accompanying micro-mechanical model where several z-pins are explicitly modeled in a laminate. In this way, relationships between physical features, such as insertion depth, insertion angle, and z-pin density and their affect on the z-pin pull-out or rupture response could be accurately studied and guide adjustments of smeared properties. Additionally, such a model would be helpful to assess the effects of laminate loading on z-pin behavior – such as compression in the Wide Element.
Type 2 test. This work was started but has been left for future studies.

As implemented in this work, a uniform representation of the smeared cohesive properties was used in the reinforced cohesive regions. However, the discreteness of z-pins can be included within the smeared approach using non-uniform distributions of the cohesive properties based on z-pin locations.

4. **Model calibration using data-driven methods and machine learning.** As the industry moves towards reduced building block testing schemes, model material properties will be calibrated at the element level as opposed to obtained through experimental testing at the coupon level. Data-driven methods and reduced-order models can be developed to efficiently arrive at model parameters that accurately predict structural response, critical loads, and failure locations.

5. **Fatigue model.** While the fatigue model was limited to inter-laminar degradation, the framework can and should be extended to account for intra-laminar fatigue degradation (with continuum or cohesive elements).

6. **Fatigue degradation laws.** In its current state, fatigue degradation laws are implicitly dependent on quantities such as R-ratio and frequency. Ideally, the fatigue laws should be provided these quantities as direct inputs and be able to predict the difference in fatigue behavior. Future work should investigate the link between details of fatigue loading definitions and the local pointwise accumulation of fatigue degradation. Additionally, endurance limits should be included in the degradation laws. Efficient methods for calibrating fatigue degradation laws should also be explored.
APPENDIX A

Appendix: Micro-Mechanical Model of Z-pin Bridging

A.1 Introduction

Due to differences in scale, it is not feasible to model individual z-pins in a structural finite element model. Multi-scale modeling offers a link between complex single/multiple pin reinforcement mechanics and simplified modeling methods used to represent z-pin reinforcement in structural models. To this end, a micro-mechanical model of nine embedded z-pins was developed to provide insight in the definition of effective/average cohesive properties in a smeared cohesive zone approach. In the smeared approach, areas of cohesive element interfaces enclosing z-pin fields are assigned cohesive material properties intended to capture the effect of z-pinning. In this way, the macroscopic behavior of z-pin fields is modeled instead of considering the discrete behavior of individual z-pins. Because the structural model of interest in this work is a z-pin reinforced pi joint, the micro-mechanical model was developed to represent a reinforced section of the pi preform - laminate overlap (Fig. A.1). The modeling approach proposed here can be used to obtain bridging force-displacement responses of the pin pull-out process, which can be recast to arrive at a effective traction-separation responses.
A.2 Modeling method

The micro-mechanical model consisted of two dissimilar blocks and nine individual z-pins (Fig. A.1). The top block represented the pi preform base, adhesive layer, and the top skin ply. The bottom block represented the remaining sublaminate. The two blocks were bridged by the z-pins only (no interfacial bonding). Additionally, a small gap was placed between the first and second laminate plies separating the two blocks. This first and second ply interface was commonly the location of critical delamination in pi joint tests. The z-pins were modeled as linear elastic using the material properties given in [19].

A.2.1 Laminate modeling

Figure A.2 shows the bottom laminate block. The top three plies of the bottom block were modeled individually, while the remaining skin plies were modeled as an equivalent orthotropic plate. It has been shown that z-pins distort fibers in laminae resulting in eye-like resin rich pockets surrounding the z-pin (Fig. A.3). The major axis of this resin pocket is orientated along the fiber direction in each lamina. For each of the individually modeled plies, resin pockets were included and assigned matrix properties. Disturbances in the local fiber direction (in-plane fiber waviness) were not accounted for in the model.
A.2.2 Z-pin - block interface

Surface-based interactions were used to model the bonding and friction between z-pins and the surrounding laminate/preform. Cohesive contact was used to define the bonding behavior. Damage of the cohesive bond was initiated using a quadratic stress criterion, and damage evolution behavior was governed by a strain energy release rate power law. A
Coulomb friction interaction described the slipping between adjacent surfaces. Following failure of the cohesive bond, the pull-out response was solely governed by the friction interaction.

A.2.3 Analysis details

Pull-out analyses were performed in Abaqus with the implicit solver using coupled temperature-displacement steps. In the first step, a temperature differential was applied to the z-pins which initially fit perfectly in cavities within the preform and laminate blocks. A positive coefficient of thermal expansion was prescribed for the z-pin material causing radial expansion against the surfaces of the blocks. This step was performed to mimic the initial interference that exists when driving z-pins into a part. Additionally, this step produces forces normal to the circumferential surfaces of the z-pins, which is necessary to generate frictional sliding. In the second step, the top surface of the top block (excluding the z-pins) was displaced normally (pure mode I) away from the bottom block, which was held fixed along its bottom surface (excluding the z-pins). Figure A.4 shows a schematic of the mechanical loading step.
A.3 Results and discussions

Pull-out force displacement responses for three preliminary simulations are shown in Fig. A.5. The responses illustrate the contribution of cohesion alone, friction alone, and the combined effect of cohesion with friction to the pull-out behavior. All responses show an initially increasing bridging force with a high stiffness through peak load. Friction only and cohesion only both exhibit bilinear force-displacement responses. While the friction only response is activated throughout the complete pull-out process, the cohesion only response shows a sharp and brief interaction between pins and the surrounding material. The combined friction and cohesion response exhibits an approximately tri-linear response.
and is a near superposition of the friction only and the cohesion only responses.

Figure A.5: Bridging force-displacement responses with friction only, cohesion only, and combined friction and cohesion.

Figure A.6 shows snapshots of the cohesive interface damage at several points along the force-displacement response of the friction and cohesion simulation (black curve in Fig. A.5). Nearing peak load (point B), interfacial damage develops near the top and bottom surfaces of the pi preform section. At peak load (point C), z-pin – top block interfaces have nearly completely failed. At point D, cohesive interfaces have completely failed and the remaining response is governed by frictional sliding.
Because z-pin bridging is governed by cohesion and friction, the influence of corresponding model parameters was investigated using the micro-mechanical model. Figure A.7 shows bridging force-displacement responses for three shear cohesive strength values ($\tau_{IIc}$) defined for the z-pin – block interfaces. It is noted that the friction interaction definition was unchanged. Higher strength values lead to higher peaks of the force-displacement responses but have no effect on the frictional pull-out responses that follow the sharp load drops. The width of the cohesive response decreases for higher peak loads in order to maintain the strain energy requirement for creating a new surface.
Figure A.7: Influence of cohesive strength (shear modes) on bridging force-displacement response.

Figure A.8 shows bridging force-displacement responses for three friction coefficient values ($\mu$) defined for the z-pin – block interfaces. It is noted that the cohesive interaction definition was unchanged. The friction coefficient enhances the cohesive behavior associated with the peak load, as evidenced by the increasing peak load for larger friction coefficients. The friction coefficient, however, primarily influences the slope of the third linear portion of the tri-linear force-displacement response. By noting the area under the force-displacement responses, it can be concluded that the frictional behavior has significant influence on the effective toughness associated with z-pinning.
A.4 Future work

After validating the results predicted by the model, bridging force-displacement responses can be transformed into traction-separation laws, which can then be used in macroscopic cohesive representations of z-pinning behavior. It follows that physical features of z-pinning, such as insertion depth and angle, can be analyzed using the micro-mechanical model to understand the effect on larger-scale behavior.

The model described in this work is rather simple but serves as a foundation for a more detailed and physically-sound model. While the cohesive properties and friction definition used to describe the pin – block interfaces were uniform in this work, a more realistic approach would vary these properties to introduce non-uniformity into the model. More comprehensive damage modeling of both the blocks and z-pins would help to capture experimentally observed behaviors such as z-pin rupture. Additionally, pull-out responses very significantly depending on mode mixity; therefore, mixed mode behaviors should be investigated.
BIBLIOGRAPHY


