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A SIMPLE MODEL OF A VACUUM-TUBING SYSTEM FOR COLLECTING MAPLE SAP

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Introduction

To produce maple syrup, one must first collect the sap from the tree. Traditionally, maple trees were tapped with spiles that had a bucket attached to capture the sap as it flowed out. However, research and advances in technology have given rise to a more efficient method of sap extraction: vacuum-tubing systems. This technique of sap extraction leverages the physics that allows for sap flow in the first place, creating larger yields from the tree throughout the season. Sap exudation is caused by the freeze thaw cycle that takes place in the early spring season. The xylem from a maple tree contains sap (a byproduct of photosynthesis) and gas bubbles, and acts as a pipeline for transporting water throughout the tree.[1] When temperatures drop below freezing, there is a negative pressure in the tree relative to the atmosphere, which draws water in from the roots.[2] When a thaw occurs, there is a positive pressure in the tree and the gas expands in the xylem. This expansion coupled with an osmotic sugar concentration gradient causes sap to flow out from the fibers.[3] When one taps into the xylem, there is a larger wound for the sap to flow out of, and more sap can escape from the tree. By attaching a tubing system to the tap, and removing air, one can increase this pressure gradient between the tree and the tap hole, which both increases the range of temperatures sap will flow, and the flow rate of sap during those runs.[4]

The following model was created to estimate a conservative yet realistic vacuum tubing model for sugarmakers of varying sizes, for the purpose of assessing the energy and emissions impacts of the entire production process. The methods are adapted from The New York State Maple Tubing and Vacuum Notebook (NYS Notebook) out of the Cornell University Extension Cooperative.[5] Their methods were altered to create a tubing model for different producer archetypes (characterizations of the industry based on production scale), meaning they are modeled without sugarbush data. Below is a breakdown of the assumptions made when modeling these archetypes, and the physical principles that underlie them. While producers may be able to use this as a tool to optimize a tubing network, this model is primarily used to estimate the amount and diameter of tubing used at different production scales.

Methods

To set up a general model of a vacuum-tubing system, some assumptions need to be made about the physical characteristics of the sugarbush (e.g., slope, spatial density of tappable trees, and acreage). Initial decisions were made with the guidance of the NYS Notebook, which were then modified to reflect a system with a single collection point and multiple independent lines. In the guide, all parameters are established in terms of acres (two-dimensional), however all branches of tubing are linear (one-dimensional) and independent of one another (individual lines stemming from the lone collection point). In practice, the NYS Notebook method estimates the number of trees a mainline serves per acre as an average, which holds independently for each acre. However, trying to visualize the scaling of a network of linear objects (tubing) in two-dimensional space lends itself to configurations that could use more tubing than one would reasonably expect. Figure 1 illustrates this issue, as each plot is scaled as one acre. As you try to access the area at the top end of the one-acre plot in Figure 1, a new line must run that length each time. This redundancy leads to more lines, each of which serve fewer trees when the sugarbush is longer than it is wide ($lm_2 < lm_1$). The most realistic model of a network of independent lines is to have each mainline serve as many trees as possible, rather than many repetitive (parallel) branches.

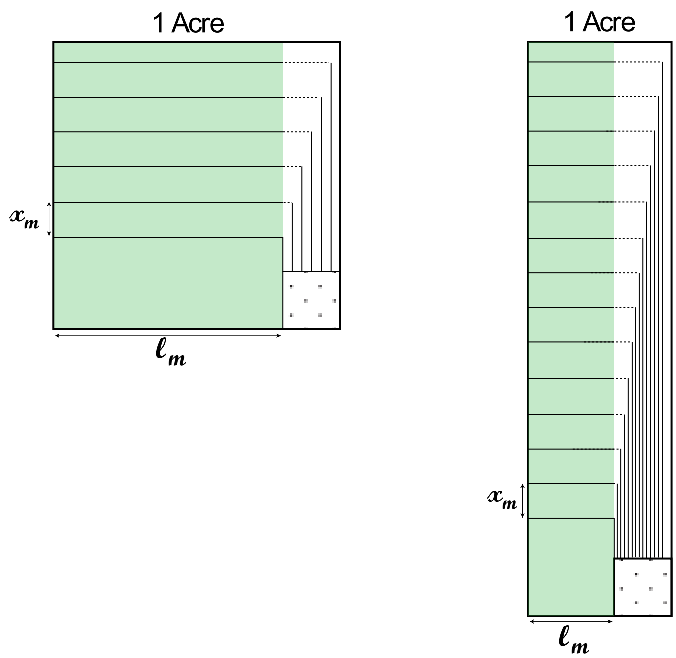

Figure 1 Scaling problem for mainline tubing in variable spatial plots. x_m represents the distance between mainlines, which is constant between the two plots (as it is based on the density of tappable trees). l_{m1} and l_{m2} represent the length of mainline serving the trees within the sugarbush. The dotted horizontal lines are assumed to be zero as all lines would run parallel and close together in this configuration.

The distance between mainlines (x_m) is constant between the two plots because it is based on the density of tappable trees in the sugarbush (see equation 5). Comparing the two plots in Figure 1, we can calculate the total amount of tubing by using eqn. 1:

$$N * (l_m) + \sum_{i=1}^{N} i * (x_m) \qquad (eqn.\ 1)$$

Notice that the discrete sum in eqn. 1 highlights the redundancy of tubing when the sugarbush is not a square, causing the right side of Figure 1 to use more than two times the amount of tubing per tap. In order to balance the length of mainline tubing with the number of lines, the sugarbush is approximated as a square.

Now that we have made this assumption, we can calculate the amount of mainline tubing per acre. A modified version[1] of how the NYS Notebook arrives at the mainline tubing per acre

[1] The NYS Notebook leverages that one knows the length of the mainline and is trying to figure out tappable trees per acre.

is by taking the square footage of an acre, dividing it by the assumed number of tappable trees per acre (100-120 being a good estimate), and taking the square root of this fraction to get the total linear distance of tappable trees in a one-acre area (eqn. 2):[6]

$$\frac{ft.\ mainline\ tubing}{acre} = \sqrt{\frac{\frac{sqft}{acre}}{\frac{tappable\ trees}{acre}}} \qquad (eqn. 2)$$

In theory, multiplying this number by the total acreage gives you the total length of tubing. However, as we have highlighted in Figure 1, density is an intensive quantity, meaning it does not scale with the size of the operation; doubling the acreage does not double the density of tappable trees but rather just the total number of tappable trees. So, as the operation gets bigger than one acre, the total amount of mainline tubing would increase, but the number of trees each mainline serves would remain the same (as would the diameter necessary to accommodate the volumetric flow of sap in each mainline), which is unrealistic. Going back to our approximation of the sugarbush as a square, this means that as we scale up the operation, it must increase equally in both dimensions to maintain shape. You would need to increase the diameter of these mainlines because the total volume of sap that they are moving is larger, and the length of each mainline would be longer (resulting in more frictional head loss).[7,8]

A more practical model assumes that you connect more trees to a mainline on the way to the fixed collection point, meaning you have fewer mainlines, with each serving more trees (see Figure 2).

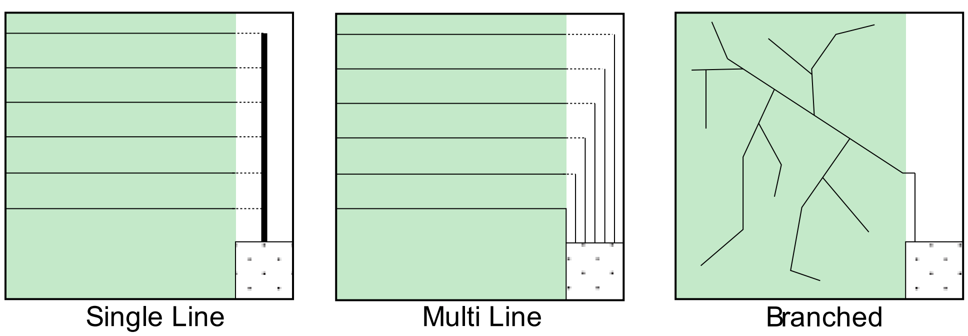

Figure 2 Different plausible configurations of a tubing system from most ordered (single-line) to most variable (branched). Figure derived from Figure 3.2 in NYS Notebook.[9]

The left-hand of Figure 2 illustrates this concept, where a large mainline acts as a highway, with each smaller mainline acting as an artery to that highway, and the laterals (not pictured) as capillaries. While this approach limits the amount of tubing, most sugarbushes are not as symmetrical as we have approximated, making a branched configuration more likely when sugarbush topography and shape are variable. To approximate the extra tubing needed for a branched layout, multi-line collection for a square shaped plot gives a conservative estimate while employing the symmetry we have used up to this point. To scale a multi-line system with the acreage growing in a square shape, we can apply the correction to eqn. 2 shown in eqn. 3:

$$\frac{mainline\ tubing}{width} = \sqrt{\frac{\frac{sqft}{acre}}{\frac{tappable\ trees}{acre}} \times \frac{total\ acreage}{total\ acreage}} \qquad (eqn. 3)$$

The result leverages the intensive property of the density of tappable trees, meaning the denominator will not change, but the total square footage will. Notice that the units of the results now change to being in units of tubing per width, as the square root of the square feet of total acreage equals the length and width of our square sugarbush. We now know how much tubing there is across the width (x-direction) of the sugarbush and can calculate the number of mainlines as a function of the distance between tappable trees in the lengthwise direction (y-direction) as well.

Using the information above we can also make decisions about the lateral lines. Research from UVM and Cornell Extension have found that 5/16" lateral tubing yields the best results overall, as it limits bacterial growth more than a 7/16" lateral and allows more flow than a 3/16".[6,10] Additionally, research has shown that six taps per lateral leads to the highest productivity per tap. The average length of a six-tap lateral can be calculated by first figuring out the number of laterals per mainline and the average distance between mainlines. To calculate the number of laterals per mainline, you can round down the product of the number of tappable trees per acre and the square root of the number of acres (eqn. 4). This will give you the number of taps along a given length, with that length being for the mainline.

$$\frac{tappable\ trees}{length} = rounddown\left(\frac{tappable\ trees}{acre} \times \sqrt{no.of\ acres}\right) \qquad (eqn. 4)$$

Now that we know how many taps there are per main line and taking the conservative estimate of one tap per tappable tree,[2] one can calculate how many laterals per mainline by dividing this number by six taps per lateral. Knowing how many taps per acre and the number of acres also gives us the total estimated number of taps, so we can calculate how many mainlines one would need to fulfill this. Because the number of laterals, taps, and mainlines are whole numbers, the rounding will lead to slightly fewer taps than initially calculated as a function of tap density. This provides a slightly more realistic result, as it is unlikely that there would be a uniform tappable tree density across the whole sugarbush. Knowing the length of one side of a square sugarbush is the square root of the number of total acres and dividing this number by the number of mainlines leads us to the average distance between mainlines (eqn. 5). This number can be used to calculate the average length of a lateral line.

[2] Exudation productivity may decrease per tap when more taps are added to a tree, making calculations regarding production harder to quantify.

$$Avg.\,Lateral\ Length = \sqrt{\frac{sugarbush\ width \times avg\ distance\ between\ main\ lines}{\frac{laterals}{main\ line} \times \frac{taps}{lateral}}} \times \left(\frac{taps}{lateral} - 0.5\right) \qquad (eqn.\,5)$$

The reason there is an extra 0.5 subtracted from the number of taps per lateral is that if a mainline is to run through any given area, it will split the distance between two tappable trees.[6] Thus, the distance from the mainline to the first tree averages half the distance between tappable trees. Now we have an intuition of the number of mainlines, the number of lateral lines, the density of tappable trees, the size of the sugarbush, the number of taps, the diameter of the lateral lines, and the average distance between mainline tubing. Next, we will focus on sizing mainline tubing to accommodate vacuum and estimating the resulting increase in yield.

Sizing Vacuum

Knowing the physical characteristics of the sugarbush allows for the proper sizing of tubing and vacuum to reduce losses along the lines. In a single wet line system, maple sap comes out of the tubing in a configuration that is approximated by open channel flow, meaning the pump is rarefying a layer of air above the sap, creating a vacuum.[11] Below the layer of air is liquid sap, which is flowing based on gravity and the pressure differential between the tree and the outflow. The most rigorous modeling of a complex tubing system such as this would include analytical solutions to the Navier-Stokes equations using computational fluid dynamics.[12] However, the goal of this model is to estimate the vacuum pump and tubing sizes necessary to facilitate laminar, stratified flow in the system. Starting with our anticipated outcome (we are trying to attain a specific pressure at the tap), some assumptions can be made about the flow to calculate a system that would facilitate such solutions. The main characteristics the system should have include:

1. The ability to handle average flow, which can be approximated as steady for the purposes of design.
2. The shape of the tubing limits flow in the radial and azimuthal directions
3. The flow is fully developed and can be approximated as a fluid moving between a stationary plate and a non-stationary plate moving in the direction of flow.
4. The no slip condition is obeyed at the bottom of each fluid layer.
5. Shear stress is equal at the boundary of the liquid and the gas.

From these outcomes, three of the four conditions needed to simplify the Navier-Stokes equations into the Hagen-Poiseuille equations are met.[13] The last condition is that the flow be axisymmetric. Because we simplified the cross section of the flow as being between two parallel plates, we can extend this assumption into three dimensions, where the shear along the boundary obeys the no slip condition for the liquid in contact with the gas and the tubing. This configuration would allow for two-dimensional shear (albeit at different strengths) in cylindrical coordinates, leading to a developed flow that can be approximated as axisymmetric. A simplified system can then be constructed using Hagen-Poiseuille to estimate flow rates based on pressure (eqn. 6):

$$Q_{sap} = \frac{\Delta P_{tree-tubing} \Pi R_{tap\ hole}^4}{8\mu_{sap} L_{tap\ hole}} \quad (eqn.\ 6)$$

(where Q_{sap} is the flow rate of sap, $R_{tap\ hole}$ is the radius of the tap hole, $L_{tap\ hole}$ is the depth of the tap hole, μ_{sap} is the dynamic viscosity of sap,[14] and $\Delta P_{tree-tubing}$ is the pressure gradient between the tree and the tubing system). The length of the tap hole was assumed to be 2 inches. The only unknown in this equation is the internal pressure of the tree, which varies by hour, day, season, and year depending on the ambient conditions. Based on experimental results of productivity increases from the addition of a vacuum, the highlighted band in Figure 3 represents the most likely average tree internal pressure throughout the day. Because flow rate is proportional to tree internal pressure, average pressure gives the average flow of sap through the system.

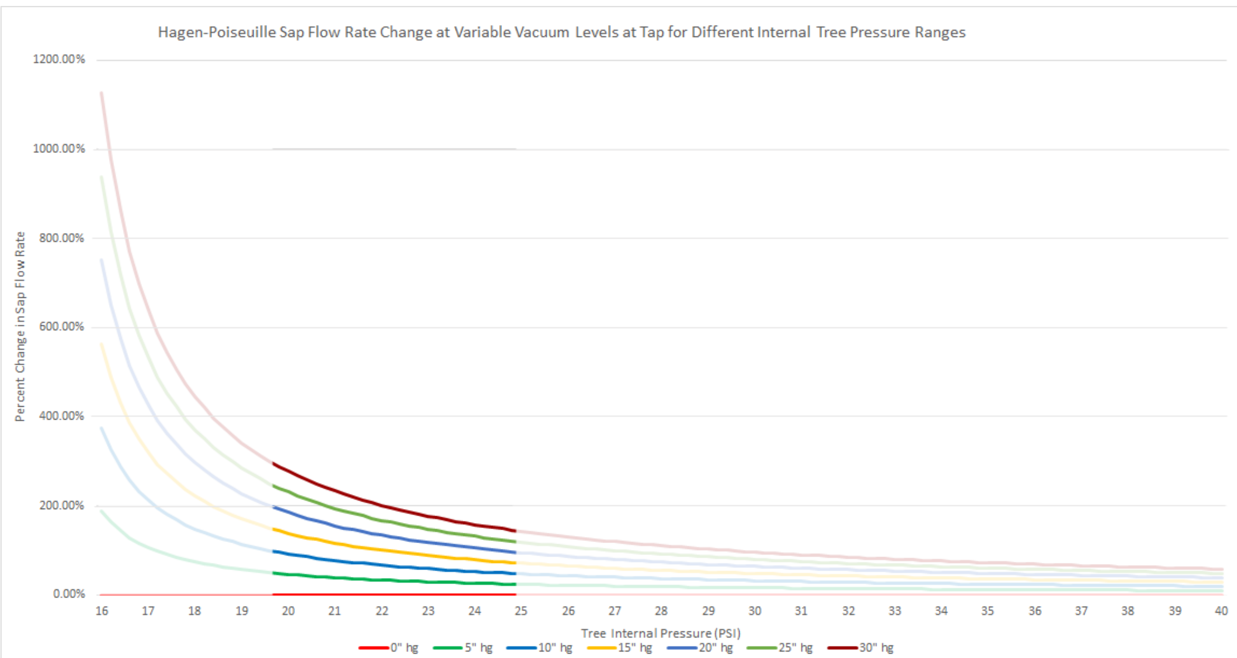

Figure 3 Changes in sap flow based on pressure differential between the tree and tap hole at different vacuum levels. Changes are measured as a percentage with respect to flow at atmospheric conditions. Vacuum is measured on inches hg removed from the tubing (making 0 "hg atmospheric conditions).

The question then becomes: why not max out on vacuum to yield the highest results? As per the guidance of the NYS Notebook, the economics of vacuum pump sizing plays a role in this decision. The marginal benefit you might get from producing more syrup may be offset by the energy costs of running the pump, as well as the upfront capital investment in a larger pump. A best practice rule of thumb is to size the pump to achieve 15" Hg vacuum at the tap. While many pumps are rated for 29-29.9" Hg, losses along the line from both friction and leakage (1.5 cfm per lead to a reduction in vacuum at the most distal tap).[11] Determining these losses mathematically will help to determine the length for each mainline diameter for each size of operation.

We can again assume ideal system conditions to size the system. Setting 15" Hg (50% air removal) as the target pressure at the tap, we can select a pump that matches the size rating of the system. The Becker catalog[15] was used to select pumps within the range of taps at a given size. The rate of air removal was also factored to ensure that the capacity of the pump would be enough to hold vacuum over the lines including expected connector losses. For every 100 taps, one can expect 1-1.5 cfm of air leakage into the system.[11] For smaller sugarbushes, this number is closer to 1 and for larger sugarbushes, this number is closer to 1.5; (for medium sized sugarbushes, we chose 1.25 cfm leakage).[11] Taking leakage losses into account, we allocated the rest of the pump's capacity and evenly distributed it among the lines, representing the flow rate of air at the tap. Knowing the flow rate of air and the specifications of the lateral lines from the previous section, we can leverage this information to calculate line loss along the lateral. This pressure drop was used in the Hazen-Williams head loss equation[3] (eqn. 7) to determine what the change in pressure is across the mainline only:

$$\mathrm{H} = \frac{10.583LQ^{1.85}}{C^{1.85}d^{4.87}} \qquad (eqn. 7)$$

Once the frictional losses [H] are known along the mainline (a function of length [L], flow rate [Q], material [C], and diameter [d]) we can iterate by allocating the total vacuum from the pump proportional to the cross-sectional area, as more air would be removed from larger tubing diameters. The derivation below shows the process of obtaining the pressure losses from each step in the tubing line.

Pressure Derivation:

By setting up the parameters of the sugarbush as square in dimension and uniform in slope, the critical path can be defined as the longest line from pump to tap (see Figure 4).

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[3] Hazen-Williams head loss is an empirical formula. The SI version of the formula was used in calculations. For the coefficient of friction (C) for LDPE, the high-end estimate of 140 was used.[16]

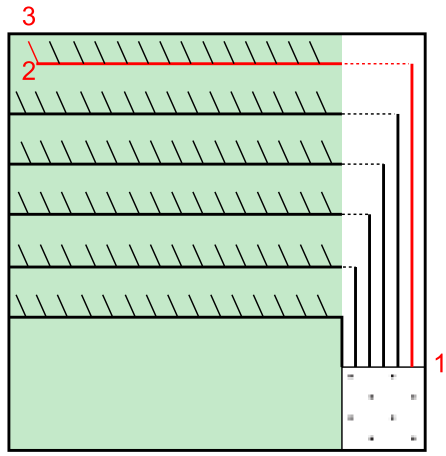


Figure 4 Map of critical path for a multi-line mainline configuration. X represents the collection point (1), (2) represents the mainline lateral junction, and (3) represents the tap hole at the most distal node from the collector. Note that the amount of mainline tubing vs. lateral line tubing is not to scale.

If the allowable drop in pressure is satisfied for the critical path, it is satisfied for all the other lines in an ideal system. The pressure loss along this path from the tap to the collector is calculated as:

$$\Delta P_{1\rightarrow 3} = \frac{8\mu_{air} L_{lateral} Q_{air,tap}}{\Pi R_{lateral}^4} + \frac{8\mu_{air} L_{mainline} Q_{air,junction}}{\Pi R_{mainline}^4} = P_{pump} - P_{tap} \qquad (eqn. 8)$$

(where P is the pressure, R is the radius, μ is the dynamic viscosity, Q is the flow rate, and L is the length). The only unknown in eqn. 8 is the radius of the mainline ($R_{mainline}$). The flow rate of air determined by the pump is split proportional to the cross-sectional area of the mainlines. However, the conditions of the system do not significantly alter density, as the associated expansion between the lateral line and the mainline is minor and no heat is flowing into or out of the system. Therefore, we can justify using the volumetric flow rate continuity equation over the mass flow rate (eqn. 9):

$$Q_{air,tap} = \frac{Q_{pump}}{\#\,main\ lines} - \left(\frac{Q_{leakage}}{junction} \times \frac{\#\,junctions}{main\ line}\right) \qquad (eqn. 9)$$

(where $Q_{air,\,tap}$ is the flow rate of air at the tap, Q_{pump} is the flow rate of air out of the tubing supplied by the pump, and $Q_{leakage}$ is the flow rate of air into the tubing). The number of junctions is the number of places a leakage could occur. We can further simplify the model of this system to approximate all laterals connecting at one junction, instead of at different points along the mainline. By modeling the system in this way, one junction incorporates the losses that would be incurred across all laterals of a given mainline. Even though there is a change in size (and a corresponding expansion minor loss) when the lateral connects to the mainline at an individual node, there needs to be enough space to accommodate all the air outflows for the system, making the continuity at an individual point a decent approximation for the physical behavior across the whole length of the mainline.

Eqn. 8 can also be expressed as the sum of the pressure drop from the pump to the junction of the lateral and mainline (1→2 in Figure 4), and the pressure drop from the junction to the tap (2→3 in Figure 4). Then, the lateral line from 2→3 can be rewritten as eqn. 10:

$$\Delta P_{2\to 3} = \Delta P_{1\to 3} - \Delta P_{1\to 2} \qquad (eqn. 10)$$

We already allocated the junctions of the lateral lines and their subsequent losses (leakage rate) to the mainline section of the system (1→2). This means that at point two in the diagram above, there should only be pressure losses attributable to friction. All the guiding assumptions for Hagen-Poiseuille also allow for the use of the Bernoulli equation[17] (eqn. 11):

$$\frac{P_3}{\gamma_{air}} + z_3 + \frac{\left(\frac{Q_{air,tap}}{A}\right)^2}{2g} = \frac{P_2}{\gamma_{air}} + z_2 + \frac{\left(\frac{Q_{air,junction}}{A}\right)^2}{2g} + \frac{10.67\left(Q_{pump}\right)^{1.852}}{C_{LDPE}^{1.852}\, d_{lateral}^{4.8704}} \qquad (eqn. 11)$$

(where *P* is the pressure at a specific point denoted by the subscript, *z* is the elevation in units of distance, *g* is gravitational acceleration, and *γ* is the specific weight of the substance—density times gravitational acceleration). Attributing continuity between the flow rates, we can relate the change in slope, change in pressure, and the Hazen-Williams head loss formulation to determine the change in pressure across the lateral *(eqn. 6)*.

$$\Delta P_{2\to 3} = \frac{10.67\left(Q_{air.tap}\right)^{1.852}\gamma_{air}}{C_{LDPE}^{1.852}\, d_{lateral}^{4.8704}} - \Delta z_{lateral} \qquad (eqn. 12)$$

This will allow us to accurately gauge losses across the mainline only, as we already know the specifications of the lateral line. Relating *(eqn. 8), (eqn. 10),* and *(eqn. 12),* we get:

$$\Delta P_{1\to 2} = P_{pump} - P_{tap} - \left(\frac{10.67\left(Q_{tap}\right)^{1.852}\gamma_{air}}{C_{LDPE}^{1.852}\, d_{lateral}^{4.8704}} - \Delta z_{lateral}\right) = P_{pump} - P_{junction} \qquad (eqn. 13)$$

Reapplying Bernoulli across this section of mainline from point 1 to point 2, we can see the relation in equation 13 equals:

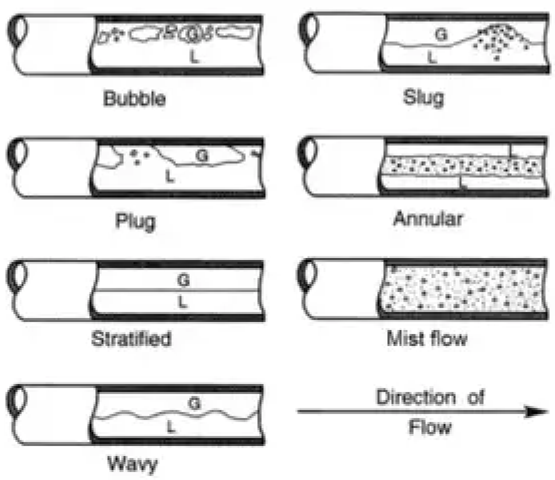
$$\Delta P_{1\to 2} = \frac{10.67\left(Q_{pump}\right)^{1.852}\gamma_{air}}{C_{LDPE}^{1.852}\, d_{main\ line}^{4.8704}} - \Delta z_{main\ line} \qquad (eqn. 14)$$

Note that the flow rate has changed from *(eqn. 12)* and is now Q_{pump}, so as not to double count the leakage from the laterals. Additionally, the change in height between the ends of the mainline is also reflected in *(eqn. 14)* as well as the new diameter, $d_{mainline}$. Choosing a starting value for the mainline equal to that of the lateral, one can now iterate like the Hardy-Cross method, to arrive at the correct diameter for each individual mainline. Because the vacuum flow rate is proportional to the cross-sectional area of air passing through the pipe for each mainline, the larger mainline diameters will command a larger share of the total volumetric flow.[6] Once the difference between iterative terms has converged at a critically small difference (<0.02%), the cross-sectional area allocated for air in each pipe has been optimized for the simple system and the best tubing diameter is now known.

Two Phase Flow

Now, the cross-sectional area for sap needs to be considered to correctly size the tubing for holding vacuum pressure.[11] The ratio of gas velocities to liquid velocities can determine the type of flow regime that sap will behave as within the tubing. We can best approximate this as a 2-phase flow (See Figure 5).[18]

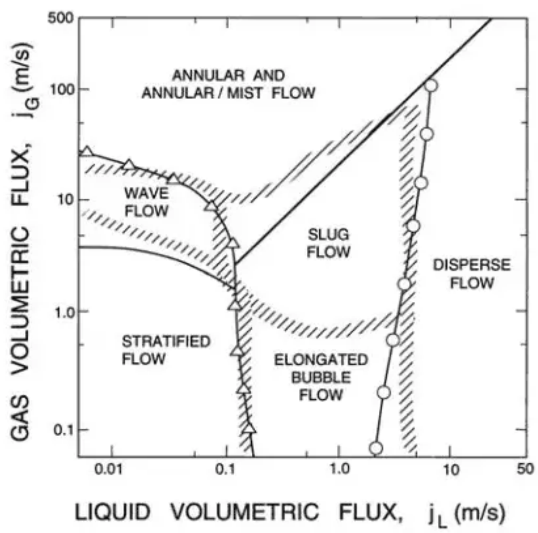
Figure 5 Sketches of flow regimes for two-phase flow in a horizontal pipe. Source: Weisman, J. Two-phase flow patterns. Chapter 15 in Handbook of Fluids in Motion, Cheremisinoff N.P., Gupta R. 1983, Ann Arbor Science Publishers. Source Credit[18]



As a gas moves over a liquid, its speed dictates the surface effects of the sap flow.[19] If there are large enough ripples along the surface, creating slug or dispersed bubble flow, there will be a blockage in the gas flow stream and reduce the effectiveness of the vacuum. Thus, the proper sizing of the sap area will help allow for the stratified or wavy flow necessary to optimize the vacuum set up. Again, the assumption that the cross-sectional area of sap, and therefore air, will remain constant will serve as a decent approximation in estimating the flow, as the hope is that perfectly stratified flow will occur in the ideal case. Remember that this is a simplified version of a tubing system at ambient conditions, so line maintenance issues, sharp bends in the tubing, and changes in the homeostasis of the system would alter these cross-sectional areas and cause more turbulent flow of both air and sap than the laminar assumption.

The length and radius of the tap hole are determined by the tapping guidelines set out by research at Cornell University, stating that 5/16" tap diameters and 2" bore length should be used for best practice. Here we can use the same approximation that all the laterals join at a single point and that the flow does not deform to determine the area. Using an engineering flow regime chart, we can determine the sap cross-sectional area that satisfies the range of proportions which fall within the stratified and wave flow regimes (see Figure 6).

Figure 6 A flow regime map for the flow of an air/water mixture in a horizontal, 2.5cm diameter pipe at 25◦C and 1bar. Solid lines and points are experimental observations of the transition conditions while the hatched zones represent theoretical predictions. Source: Mandhane, J.M., Gregory, G.A. and Aziz, K.A. (1974). A flow pattern map for gas-liquid flow in horizontal pipes. Int. J. Multiphase Flow Source Credit[18]



Taking the pump flow rate and dividing it by the air flow cross-sectional area found above, we can get the gas volumetric flux. Similarly, taking the flow rate out of the tree from *(eqn. 9),* and multiplying it by the number of laterals, we can find the max flow rate (all the sap from all the taps along the single mainline).

$$Q_{sap} = \frac{\Delta P_{tree-tubing} \Pi R_{tap\ hole}^{4}}{8\mu_{sap} L_{lateral}} \quad (eqn. 15)$$

Note that we are using $L_{lateral}$ instead L_{tap} like in equation 6. This choice was a result of the simplification of all the leakage coming from a single point at the end of the mainline junction. Because we assumed that only Bernoulli applied along this line, the effective length of the tap hole is the length of the lateral serving it.

Using the range of liquid fluxes allowed, we can solve for the liquid cross-sectional area. By adding the air and sap cross-sectional areas, we can now find the area of each section of mainline tubing in the system. The last step is to account for the fact that the system is not ideal. The nominal pipe size found in the ideal system calculations can be sized up to best approximate variable system conditions that would have to be met (including peak flow), as the calculations above are for ideal average flow.

Conclusion

The modeling effort above is a simplified version of reality and can be used as an auxiliary model to help benchmark system productivity, and total material used in the sugarbush. The main assumptions can be broken into three parts: sugarbush characteristics, system configuration, and fluid mechanics. The assumptions made about the sugarbush were used to simplify symmetry and uniformity for setting up a tubing system. Once the characteristics of the sugarbush were determined, the system was assumed to consist of mainlines connected to a central receiver (the pump). The configuration of the system led to simplifications regarding head loss, with uniform leakages, no significant bends, and minor losses from fittings or taps. Once this simplification was made, the flow could be best approximated by Hagen-Poiseuille and Bernoulli as the system was modeled as two disjoint pieces (sap and air) with different flow rates. For a more rigorous solution, one could solve the Navier-Stokes equations for all the pipes and use the Hardy-Cross method to calculate flow rate. However, for the purposes of assessing the sustainability of a tubing system or its performance, this simple model allows for realistic yet conservative assumptions of vacuum size, tubing diameter, and tubing length.

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